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Facility location and scale decision problem with customer preference

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A B S T R A C T

This paper addresses the facility location problem that aims to optimize the location and scale of a new facility in consideration of customer restrictions, including customer preference and the minimum number of customers required to open the facility. In a classic covering problem, the customer is assumed to be covered if he/she is located within the critical distance zone around the facility and is otherwise not covered. This problem is caused by customer facility selection, which differs from the classic covering problem in which services are determined only by proximity. This paper proposes a mixed integer programming formulation based on customer restrictions and also develops a heuristic solution procedure using Lagrangian relaxation. The suggested solution procedure is shown to yield acceptable results in a reasonable computation time.

1. Introduction

The location problem regarding the choice of the best location for a facility given a set of potential sites with a minimum total cost or maximum total service has been extensively studied since the 1960s. This problem has been applied in the location decisions of hospitals, fire stations, schools, distribution centers, and computer networks. Brandeau and Chiu's (1989) survey paper demonstrates that the model can be extended for more than 50 related problem types.

The maximal covering location problem (MCLP) introduced by Church and ReVelle (1974) determines which facilities can maximize the overall demand of covered customers. In the MCLP model, customers are considered to be served, or covered, by the facility if it is located within the specified distance. In other words, decisions on the installation of the facility are based only on the distance between the customer and the facility. However, both customer preference and the number of customers must be practically considered when determining the location and scale of the service facility. For example, hospitals, restaurants, shopping malls, and theaters are often visited based on customer satisfaction; the brand value, accessibility, and convenience are also taken into consideration. From an investment point of view, the number of customers to be served by the facility is one of the vital factors affecting the decisions on opening a new facility.

The MCLP is extended by introducing two other factors into the location decision of the service facility: the minimal number of customer required (MCR hereafter) to open a new facility and customer preference, where MCR is included in the model as a service provider consideration for the minimal requirement of profit, while customer preference plays a role as a service recipient consideration. The model suggested in this paper, denoted by the facility location and scale decision problem (FLSDP), incorporates two factors simultaneously, with the locations of the facilities and their scales to be determined.

In the model suggested, the location and scale for a new facility is determined in consideration of the profitability. For example, assuming that a service facility, such as a Starbucks or McDonalds, would like to open a new branch in some area based on the market research including floating population, propensity to consume, and accessibility, the location and the scale of the facilities can be determined. The service type, the capacity, and the available operating budget for the facility should also be considered. One example of the FLSDP can be described as in Fig. 1. Suppose that there are two potential facilities, x_1 and x_2, either of which could be opened depending on the budget constraint. The expected preferences to sites $x_1(x_2)$ are 0.5(0.9), respectively, assuming that the requirement level value to open a new facility is 4, where the level can be defined by multiplying the number of covered customers by the facility preference index. The level value guarantees the profit of the facility, and the scale of the newly opened facility is determined based on the value. The circle in Fig. 1 indicates the coverage of each facility candidate x_1 and x_2, which implies that $x_1(x_2)$ can serve 6 (5) customers, respectively. While x_1 is favored in terms of proximity, the coverage based on customer preference favors x_2 since the coverage of $x_2(4.5)$ is greater than that of $x_1(3)$. Based on the required coverage value of 4, only facility x_2 can be opened. The profitability of the model has to be considered with respect to customer preference and facility proximity.

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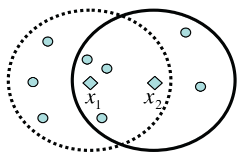


Fig. 1. An example of FLSDP.

This paper is organized as follows: in Section 2, the literature review is introduced. In Section 3, the mathematical formulation of the problem is suggested. In Section 4, a heuristic using Lagrangian relaxation is developed. The results of the computational experiments are analyzed and discussed in Section 5, and Section 6 contains some concluding remarks.

2. Literature review

Facility location problems have received a great deal of attention from researchers, many of whom have chosen the MCLP as the most useful facility location model both from theoretical and practical points of view. The MCLP introduced by Church and ReVelle (1974) and its extensions have been applied both in public and private sectors. Darskin (1983) studied the MCLP in the public sector, where servers are not always available to answer calls immediately. A tree network MCLP is studied in Megiddo, Zemel, and Hakimi (1983), in which a solution algorithm was presented. Extensions such as multiple locations, excess facilities, and backup sites on the MCLP are investigated in Daskin, Hogan, and ReVelle (1988). An MCLP to determine ambulance deployment for emergency services in Santo Domingo is studied in Eaton, Hector, Sanchez, Latingua, and Morgan (1986). An MCLP of the private sector, such as that for locating bank branches, is investigated in Pastor (1994). More applications of the MCLP can be found in Dwyer and Evans (1986) and in Hougland and Stephens (1976). The hierarchical covering location problem (HCLP) in the health care service area by Moore and ReVelle (1982) extends MCLP with introducing various kinds of services, in which customers are assumed to be open and served by the facility as long as it is located within the specified distance of the facility. MCLP is generalized in Berman and Krass (2002), and in Karasakal and Karasakal (2004), by allowing the partial coverage, in which customers are partially covered or satisfied if the facility is located farther than a pre-specified distance. HCLP is also generalized by including the partial coverage of customers in Lee and Lee (2010). As seen from the modeling of MCLP and HCLP, the location decisions are made by the coverage or the accessibility from the service facility. Although it is more practical if the decisions on the location and location's scale is determined by the customer preference, it is rare to find those models in the literature. One of them can be found in Drezner (1994) where the locations are decided in consideration of the facility utility level. Zhang (2001) has suggested the location model to maximize the profit, in which the warehouse facility can be installed in the competition environment of product reservation and proper product price.

Because of the complexity of the MCLP various heuristics are developed such as the liner programming relaxation, greedy heuristics, and Lagrangian relaxation. The linear programming relaxation of the 0–1 integer programming formulation, greedy adding with substitution, and relaxed linear programming followed by branch and bound, are proposed in Church and ReVelle (1974). A constructive generic algorithm to solve real world data with up to 500 vertices is considered in Arakaki and Lorena (2001). The meta-heuristic heuristic concentration is explored to solve MCLP in ReVelle, Scholssberg, and Williams (2008), in which the new version of the heuristic concentration called HC:1–2–1 is demonstrated, and the usefulness of that meta-heuristic method for a large MCLP is illustrated. A decomposition heuristic for the MCLP is proposed by Senne, Pereira, and Lorena (2010), in which a cluster partitioning technique to solve large scale problem is used. Lagrangian relaxation is applied to location problems in Mazzola and Neebe (1999) and Nozick (2001). Mazzola and Neebe's solution procedure presents a multiproduct-capacity facility location problem in which the demand for several different product groups must be supplied from a set of facility sites. Two heuristics based on the Lagrangian relaxation, in which a greedy algorithm is used to calculate upper bounds and sub-gradient optimization is used to calculate lower bounds, are suggested in Nozick (2001). Lagrangian heuristic for MCLP is developed to find better bounds for the large scale problems in Galvao and ReVelle (1996), in which solutions are searched in the range of upper and lower bounds attempting to improve both bounds in the iterative procedure. In Galvao, Espejo, and Boffey (2000), the heuristic based on Lagrangian obtained by dualizing the cover constraints is compared by the surrogate relaxation approach constructed by combining the cover constraints into a single knapsack constraint. A Lagrangian/surrogate heuristic using Hillsman's edition is developed in Lorena and Pereira (2002). The unified linear model developed by Hillsman to adapt the distance coefficients of a p-median problem is transformed to solve the MCLP.

3. Model formulation

FLSDP model takes the profitability of the location decisions into account in two ways: the objective function to be maximized with the total customer preferences including the demand size, and the location installation validity which is measured by total customer preferences of customers covered. Both the scale and location of the facility are determined, where the service types of the facility depend on the scale or level of the facility. A first level facility may provide type 1 service, whereas a second level facility can provide both type 1 and type 2 services. It is noticeable that the scale of the facility may determine the types of the services, and the customer may determine the facility from which the type of services required can be provided. Hence, the facility scale and the types of services are determined in FLSDP, while they are predetermined for candidate facilities in HCLP. The FLSDP is formulated as follows:

Indices

i: index for potential facility site ($i = 1, \ldots, I$)
j: index for customer node ($j = 1, \ldots, J$)
k: index for service type ($k = 1, \ldots, K$)
s: index for facility scale ($s = 1, \ldots, S$)

Decision variables

$$x_{ijk}\begin{cases} 1, & \text{if a customer in node } j \text{ is served with service } k \\ & \text{in a facility located at site } i \\ 0, & \text{otherwise} \end{cases}$$

$$y_{is}\begin{cases} 1, & \text{if a facility of scale } s \text{ is opened at } i \\ 0, & \text{otherwise} \end{cases}$$

Data

p_{ij}: the preference of a person for the facility (=the proportion of persons in node j who could potentially choose facility site i)
d_{jk}: the population requesting service k in node j
t_{isk}: the capacity of service k in the facility of scale s opened at site i
b_{is}: the cost for opening a new facility of scale s at site i
V: the total budget for opening new facilities
MCR_s: the minimum number of potential customers required to open the new facility of the scale s

$$a_{ij}\begin{cases}1, & \text{if a customer in node } j \text{ is covered by a facility opened at site } i\\ 0, & \text{otherwise}\end{cases}$$

Maximize

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}p_{ij}d_{jk}x_{ijk} \tag{1}$$

Subject to

$$\sum_{j=1}^{J}d_{jk}x_{ijk}\leqslant\sum_{s=k}^{S}t_{isk}y_{is}\quad\forall i,k \tag{2}$$

$$x_{ijk}\leqslant a_{ij}\quad\forall i,j,k \tag{3}$$

$$\sum_{i=1}^{I}x_{ijk}\leqslant 1\quad\forall j,k \tag{4}$$

$$MCR_s\times y_{is}\leqslant\sum_{j=1}^{J}\sum_{k=1}^{s}p_{ij}d_{jk}a_{ij}\quad\forall i,s \tag{5}$$

$$\sum_{i=1}^{I}\sum_{s=1}^{S}b_{is}y_{is}\leqslant V \tag{6}$$

$$\sum_{s=1}^{S}y_{is}\leqslant 1\quad\forall i \tag{7}$$

$$y_{is}\in\{0,1\}\quad\forall i,s \tag{8}$$

$$x_{ijk}\in\{0,1\}\quad\forall i,j,k \tag{9}$$

The objective function (1) maximizes the number of potential customers to be served, in other words, the number of customers served by the facility multiplied by the preference index. Constraint (2) allows service allocation within the capacity of the facility, and constraint (3) implies that the customers are served if they are located within the critical distance of the facility. Constraint (4) imposes that the customer can be served by, at most, one of the facilities, constraint (5) states that the minimum number of potential customers is required in order to establish a new facility, and constraint (6) ensures that the budget can account for the total number of facilities opened. Constraint (7) states that only one facility can be opened on each site, and constraints (8) and (9) define the binary variables.

The customer preference index, p_{ij}, in the formulation above can be defined as follows:

$$p_{ij}=q_{ij}(c_i\times\exp(A\times r_{ij}))^{-1}$$

where q_{ij} is the customer satisfaction index in node j for potential facility site i, c_i is the service expense index of potential facility site i, A is the coefficient for distance adjustment, and r_{ij} is the distance between the potential facility site i and the customer in node j.

4. Lagrangian heuristic algorithm

The Lagrangian relaxation method is commonly used as a heuristic, especially when the problem becomes relatively easy to solve by relaxation of some difficult constraints. Lagrangian multipliers are introduced in the objective functions, and adequate solutions can be found in the range of the upper (lower) bound for the maximization (minimization) problem. In the FLSDP model, constraints (2) and (6) can be relaxed. If constraint (2), containing x_{ijk} and y_{is}, is relaxed, then the problem can be partitioned into two independent sub-problems: the service allocation problem defined by x_{ijk} and the facility location problem defined by y_{is}. The following Lagrangian relaxation of the problem is obtained by relaxing constraints (2) and (6) with multipliers λ_{ik} and γ, respectively.

$$\begin{aligned}(LR)\theta(\lambda,\gamma)=Max&\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}p_{ij}d_{jk}x_{ijk}\\&+\sum_{i=1}^{I}\sum_{k=1}^{K}\lambda_{ik}\left(\sum_{s=k}^{S}t_{sk}y_{is}-\sum_{j=1}^{J}d_{jk}x_{ijk}\right)\\&+\gamma\left(1-\frac{\sum_{i=1}^{I}\sum_{s=1}^{S}b_{is}y_{is}}{V}\right)\end{aligned} \tag{10}$$

Subject to

$$x_{ijk}\leqslant a_{ij}\quad\forall i,j,k \tag{3}$$

$$\sum_{i=1}^{I}x_{ijk}\leqslant 1\quad\forall j,k \tag{4}$$

$$MCR_s\times y_{is}\leqslant\sum_{j=1}^{J}\sum_{k=1}^{s}p_{ij}d_{jk}a_{ij}\quad\forall i,s \tag{5}$$

$$\sum_{s=1}^{S}y_{is}\leqslant 1\quad\forall i \tag{7}$$

$$y_{is}\in\{0,1\}\quad\forall i,s \tag{8}$$

$$x_{ijk}\in\{0,1\}\quad\forall i,j,k \tag{9}$$

$$\lambda_{ik}\leqslant 0\quad\forall i,k \tag{11}$$

$$\gamma\leqslant 0 \tag{12}$$

Problem *LR* can be further decomposed into two sub-problems *LR1* and *LR2*:

$$(LR1)\ \text{Max}\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}d_{jk}(p_{ij}-\lambda_{ik})x_{ijk} \tag{13}$$

subject to constraints (3), (4), (9), (11), and

$$(LR2)\ \text{Max}\ \sum_{i=1}^{I}\left(\sum_{k=1}^{K}\sum_{s=k}^{S}t_{sk}\lambda_{ik}y_{is}-\sum_{s=1}^{S}\frac{b_{is}}{V}\gamma y_{is}\right)+\gamma \tag{14}$$

subject to constraints (5), (7), (8), (11), (12).

Given the value of the multipliers, the optimal solutions for *LR1* and *LR2* can be calculated, the sum of which is the upper bound for the objective value of the original problem. Based on the solution found in the relaxed problem, a feasible solution can be constructed to determine a lower bound. Using the upper and lower bounds, the Lagrangian multipliers are updated according to the calculated step size and direction, and new solutions to the two sub-problems can be calculated. These procedures are repeated until the optimal solutions are identified or until a stop criterion is reached, such as the pre-specified number of iterations or the step-size.

In the upper bound procedure, the best upper bound is obtained according to $\theta=\max_{\lambda\leqslant 0,\gamma\leqslant 0}\theta(\lambda,\gamma)$. The problem *LR* is decomposed into two sub-problems: a set of service allocation problems and a set of facility location problems. The sub-problems are solved independently using CPLEX 7.0. For fixed values of λ and γ, the optimal values of the x variables in the relaxed problem can be determined independently using the optimal values of the y variables.

To calculate the service allocation variables, the problem *LR1* is decomposed into a set of $n_j\times n_k$ service allocation problems, where

n_j is the number of customer nodes and n_k is the number of service types.

$$\theta_1(\lambda,k,j) = \text{Max}\sum_{i=1}^{I} d_{jk}(p_{ij}-\lambda_{ik})x_{ijk} \tag{15}$$

$$x_{ijk} \leqslant a_{ij} \quad \forall i \tag{16}$$

$$\sum_{i=1}^{I} x_{ijk} \leqslant 1 \tag{17}$$

$$x_{ijk} = \{0,1\} \quad \forall i \tag{18}$$

$$\lambda_{ik} \leqslant 0 \quad \forall i \tag{19}$$

Ignoring constraint (16), the 0–1 knapsack problems are defined by (15), (17), and (18) and can be solved easily by setting $\theta_1(\lambda,k,j) = \max\{\max_{i\in I}\{d_{jk}(p_{ij}-\lambda_{ik})\},0\}$.

Constraint (16) can be included in order to choose customers covered by a facility opened at i .

To calculate the facility location variables, *LR2* is decomposed into a set of n_i facility location problems, where n_i is the number of potential facilities, and γ:

$$\theta_2(\lambda,\gamma,i) = \text{Max}\sum_{k=1}^{K}\sum_{s=k}^{S} t_{sk}\lambda_{ik}y_{is} - \sum_{s=1}^{S}\frac{b_{is}}{V}\gamma y_{is} \tag{20}$$

$$MCR_s \times y_{is} \leqslant \sum_{j=1}^{J}\sum_{k=1}^{s} p_{ij}d_{jk}a_{ij} \quad \forall s \tag{21}$$

$$\sum_{s=1}^{S} y_{is} \leqslant 1 \tag{22}$$

$$y_{is} = \{0,1\} \tag{23}$$

$$\lambda_{ik} \leqslant 0 \quad \forall k \tag{24}$$

$$\gamma \leqslant 0 \tag{25}$$

In this procedure, if the total population requesting service is greater than the minimum number of potential customers required to open the new facility of the scale s according to constraint (21), then $\theta_2(\lambda,\gamma,i) = \max\{\sum_{k=1}^{K}\sum_{s=k}^{S} t_{sk}\lambda_{ik}y_{is} - \sum_{s=1}^{S}\frac{b_{is}}{V}\gamma y_{is},0\}$. Clearly, the solutions to the Lagrangian sub- problems, *LR1* and *LR2*, provide upper bounds to the optimal solutions for the FLSDP model: $\theta(\lambda,\gamma) = \sum_{j=1}^{J}\sum_{k=1}^{K}\theta_1(\lambda,j,k) + \sum_{i=1}^{I}\theta_2(\lambda,\gamma,i) + \gamma$.

The solution obtained in the relaxed problem may not be feasible in the original problem, since the facility capacity and budget limitation are relaxed. Based on the solution of the relaxed problem, the lower bound procedure requires the identification of a feasible solution to the original problem. Given the facility locations in the upper bound procedure, the customer nodes are allocated to the open facilities. The customer within the service distance of an open facility is selected arbitrarily and is allocated to the facility with the highest preference index (*PI*). This step is repeated until all of the customer nodes are allocated.

When determining the capacity of the open facility, the facility is chosen arbitrarily. If the total population of the customers located within the service distance of the facility requires service that exceeds the capacity of the facility, the customer node with the smallest population is deleted. The deletion process is continued until the total population is less than or equal to the capacity. This step is terminated after analyzing all of the open facilities.

When checking the budget, the total opening cost (TOC) for all of the possible facilities is calculated. If this cost is greater than the budget, the efficiency values of each facility are calculated. The facility with the lowest efficiency is deleted until the TOC is less than or equal to the budget. The efficiency of the scale s facility at site i can be defined as the value representing the customer coverage of the facility per unit cost as follows:

$$Eff_{is} = \frac{\sum_{j=1}^{J}\sum_{k=1}^{K} p_{ij}d_{jk}x_{ijk}}{b_{is}} \quad \forall i,s$$

The numerator indicates the total demand served, hence Eff_{is} represents the utility value of the facility.

Since the facility capacity and budget limitations are relaxed in the upper bound procedure, some facilities and customer nodes may be deleted in steps 2 and 3 of determining a feasible solution. Some deleted customer nodes may be served by the open facility, and the reallocation of these customer nodes to the open facility is necessary in order to maximize the objective value. This process begins with determining whether the non-served customer nodes exist within the service distance of the open facility. If the non-served customer nodes exist, the reallocation to the facility with the proper capacity is executed repeatedly until all of the customer nodes are addressed. This process is known as the deletion and reallocation algorithm and is summarized as follows.

⟨Deletion and reallocation algorithm⟩

Step 1. Given the facility locations, allocate customer nodes.

1.1 Classify i into the selected (open) facility set I^* or the non-selected set I $\forall i$.

1.2 Make the set of customers allocated to the open facility $i(J_i^* = \phi)$ $\forall i \in I^*$.

1.3 Select an arbitrarily customer j from the set $J_i = \{j|a_{ij} = 1\}\forall i \in I^*$.

1.4 Allocate selected customer j to $i \in I^*$ such that $PI_{ij} = \underset{i\in I^*}{Max}[p_{ij}]$,
remove allocated customer j from the set J_i,
and add customer j to the set J_i^*.

1.5 If $J_i \neq \varphi \forall i \in I^*$, then go to 1.3,
otherwise go to step 2.

Step 2. Check the capacity of the open facility.

2.1 If $I^* \neq 0$, then choose $i \in I^*$ arbitrarily and remove i from set I^*.
otherwise go to step 3.

2.2 If $\sum_{j=1}^{J} d_{jk}x_{ijk} \leqslant t_{isk}\forall k$, then go to 2.1,
otherwise calculate $j = \underset{j\in J_i^*}{Min}[\sum_k d_{jk}]$, remove j from set J_i^*, and go to 2.2.

Step 3. Check the budget.

3.1 Calculate $TOC = \sum_{i=1}^{I}\sum_{s=1}^{S} b_{is}y_{is}$.

3.2 If $TOC \leqslant V$, then go to Step 4,
otherwise calculate $Eff_{is}\,\forall i, s$.

3.3 Find $i = \underset{i\in I^*}{Min}[Eff_{is}]$, remove i from set I^*, and go to 3.2.

Step 4. Reallocate the customer node to an open facility.

4.1 Establish the set JJ_i, where $JJ_i = \{j|a_{ij} = 1$ and non-allocated to $i\}$ $\forall i \in I^*$.

4.2 If $JJ_i \neq 0$, then calculate $j = \underset{j}{Max}[\sum_{k=1}^{K} d_{jk}]$,
otherwise go to the procedure for updating the Lagrangian multipliers.

4.3 Select $i \in I^*$ such that $PI_{ij} = \underset{i\in I^*}{Max}[p_{ij}a_{ij}]$.

4.3 If $\sum_{j=1}^{J} d_{jk}x_{ijk} \leqslant t_{isk}\,\forall k$, then remove j from set JJ_i,
add customer j to set J_i^*, and go to 4.2;
if there is another $i \in I^*$ with $a_{ij} = 1$, then go to 4.3;
otherwise go to 4.2.

The calculations of the movement direction and the step size are required in the updating of the Lagrangian multipliers. The process outlined in Crowder (1976) is used to calculate the direction and the step size at each iteration. The step size at the nth iteration is as follows:

$$t_n = \frac{\beta^n(UB^n - LB^n)}{\sum_{i=1}^{I}\sum_{k=1}^{K}\left(\sum_{s=1}^{S} t_{sk} y_{is} - \sum_{j=1}^{J} d_{jk} x_{ijk}\right)^2 + \left(1 - \frac{\sum_{i=1}^{I}\sum_{s=1}^{S} b_{is} y_{is}}{V}\right)^2}$$

where UB^n is the best upper bound found by solving the Lagrangian relaxation problem, LB^n is the objective value of the best known feasible solution, and β^n is a scalar with an initial value between 0 and 2. Whenever the solution of the relaxation problem fails to increase in a specified number of iterations, the value is reduced by a factor. The rule for the sequence β^n is determined by term $\Lambda(a, b, c, d)$. The sequence starts by setting β equal to a for b iterations. Then both a and b are divided by c to calculate new values. β is then equal to a for the next b iterations. This process continues until b is less than d. Once b is less than d, β is divided by c for every d iteration. In the next section, Λ (2, 2∗number of facility, 2, 20) is used for computational experiments.

The updating equations of the direction at the nth iteration are as follows:

For constraint (2):

$$v_{ik}^n = \varphi v_{ik}^{n-1} + \left(\sum_{s=1}^{S} t_{sk} y_{is} - \sum_{j=1}^{J} d_{jk} x_{ijk}\right) \quad \forall i, k$$

$$\lambda_{ik}^{n+1} = \min(0, \lambda_{ik}^n + t_n v_{ik}^n) \quad \forall i, k$$

For constraint (6):

$$w^n = \varphi w^{n-1} + \left(1 - \frac{\sum_{i=1}^{I}\sum_{s=1}^{S} b_{is} y_{is}}{V}\right)$$

$$\gamma^{n+1} = \min(0, \gamma^n + t_n w^n)$$

where ϕ is a weighting factor. Based on the experiments of Crowder (1976), $\phi = 0.3$ is used in all experiments in the next section, although experimentation indicates that the solution procedure is insensitive to the value of ϕ. However, this value is very useful in setting the parameter to be significantly less than 1 (2001). Also, all of the experiments begin with each Lagrangian multiplier initialized to zero.

The procedure will stop when a pre-specified number of iterations has been completed, when the gap between the lower and upper bounds is equal to or less than a pre-specified value, or when the step size in the Lagrangian multiplier update procedure becomes sufficiently small.

5. Computational results

The performances of the proposed solution procedure for FLSDP were tested on randomly generated instances by varying the key parameters, such as the MCR, the coverage distance of the facility, the number of customers, and potential facilities. The suggested solution procedure for FLSDP was programmed in MS C++ 6.0 and the CPLEX 7.0 library and was run on a Pentium IV PC with 3.0 GHz CPU and 1 GB RAM.

The experimental data for the performance evaluation of the heuristic algorithm are summarized in Table 1. The number of customers ranges from 50 to 1000, and the number of potential facilities ranges from 5 to 30. It is assumed that the facility can serve customers within the specific range, and the MCR are used for calculating each facility scale. There are two levels in the facility scale, small and large, and the minimum number of customers required to open each facility is given. Two dimensional x and y coordinates of customers and potential facilities are generated from a uniform distribution within the range [0, 100]. The budget limit (V) 1000 is chosen so that two large facilities and five small facilities are allowed to be open collectively. The opening cost is [100, 200] for the small facility and [300, 400] for the large facility. The demand of customers (or the population of customers in a node) is distributed on the range of [1, 5] in each service type, and the facility capacity is distributed on the range of [350, 550] for small facilities and [650, 850] for large facilities.

Table 1
The experimental parameters used in each case.

Case	J	I	s (b)	d
1	50	5, 7, 10	2 (4), 3 (6), 5 (10)	20, 25, 30
2	100	5, 7, 10	5 (10), 7 (15), 10 (20)	20, 25, 30
3	500	10, 20, 30	10 (20), 20 (40), 40 (80)	20, 25, 30
4	1000	10, 20, 30	10 (20), 20 (40), 50 (100)	20, 25, 30

J: the number of customer nodes.
I: the number of potential facility locations.
s: the minimum number of customers required to open a small facility.
b: the minimum number of customers required to open a large facility.
d: coverage distance.

The results of the computational experiment of the performances of the heuristic are summarized in Tables 2 and 3. For each iteration type, the number of customer nodes is denoted by J, the number of potential facility locations by I, the MCR by C, and coverage distance by d. Tables 2 and 3 also denote the number of iterations, the upper and lower bound, the gap between the upper and lower bound, and the computational time for the facilities opened.

The experimental results of the Lagrangian algorithm show that the gaps account for 1.66% of the total on average. The maximum gap for the experimental instances is 5.96%, and only five of the 108 cases are greater than 5%. The proposed algorithm can be performed in a reasonable computation time.

The results of the experiment to study the differences between FLSDP and MCLP are shown in Fig. 2. The experiment is designed with $J = 100$, $I = 10$, $d = 30$, and $C(s = 15, b = 30)$, and the scale of the facility in the MCLP model is equal to that of the large facility in the FLSDP. It is worth noting that the classical MCLP allows the opening of four facilities, while FLSDP predicts two small facilities 4 and 7 and one large facility 10, since the latter considers the MCR, the minimal number of customers necessary to validate opening a new facility.

The difference in facilities sited according to the change in the coverage distance is analyzed. If coverage distances are less than 20, the new facility may not be opened because of the lack of customers. According to the increase in the coverage distance, the number of small facilities opened decreased, while the opening ratio of large facilities increased. Additionally, the number of open facilities increased as the number of potential facilities increased. In particular, according to the increase in the MCR, the number of small facilities increased.

In the heuristic procedure, the two sub-problems relaxed by the Lagrangian relaxation guarantee rapid computation, and CPLEX in the calculation procedure of the upper bound improves the solution quality. In the long run, the proposed heuristic for FLSDP can simultaneously choose the location and scale of the new facility considering customer restrictions, such as customer preference, MCR, facility size, and budget restriction, within a reasonable computational time.

Table 2
The experimental results for cases 1 and 2.

J	I	C (s, b)	d	Iteration	Upper bound	Lower bound	Gap (%)	Time (sec)	Facility numbers
50	5	2, 4	20	300	87.812	85.653	2.458	36.703	B1, B3, S4
50	5	2, 4	25	300	151.043	148.850	1.452	35.968	B3, B5, S1, S4
50	5	2, 4	30	300	113.886	112.392	1.312	30.984	B3, B4
50	5	3, 6	20	300	98.081	96.595	1.515	33.078	S1, S3, S5
50	5	3, 6	25	300	101.076	99.580	1.479	30.453	B3, B5
50	5	3, 6	30	300	113.902	112.392	1.326	30.687	B3, B4
50	5	5, 10	20	300	98.076	96.595	1.510	37.969	S1, S3, S5
50	5	5, 10	25	300	151.987	148.850	2.064	36.390	S1, S3, S4, S5
50	5	5, 10	30	300	189.908	179.956	5.241	39.703	B3, S1, S4, S5
50	7	2, 4	20	300	111.149	109.029	1.908	45.250	B1, B2, S3, S4
50	7	2, 4	25	300	164.087	161.733	1.434	45.125	B1, B3, S4, S6
50	7	2, 4	30	300	125.978	123.884	1.663	47.218	B1, B3
50	7	3, 6	20	300	109.155	102.652	5.957	54.000	B1, B3, S4
50	7	3, 6	25	300	107.415	105.317	1.953	40.718	B1, B3
50	7	3, 6	30	300	139.352	135.499	2.765	42.406	B3, S1, S6
50	7	5, 10	20	300	69.600	67.534	2.969	39.593	S1, S2
50	7	5, 10	25	300	151.344	148.784	1.691	45.150	B1, B2, S3, S4
50	7	5, 10	30	300	183.984	181.726	1.227	45.453	B1, B3, S4, S6
50	10	2, 4	20	300	94.657	91.599	3.231	54.312	B1, S3, S5
50	10	2, 4	25	300	147.786	144.744	2.059	54.921	B1, B10, S6
50	10	2, 4	30	300	156.187	152.063	2.641	54.265	B5, B10, S6
50	10	3, 6	20	300	119.760	116.757	2.507	57.859	B1, S3, S4, S5
50	10	3, 6	25	300	141.987	138.965	2.128	56.718	B1, B10, S4
50	10	3, 6	30	300	156.187	152.063	2.641	59.093	B5, B10, S6
50	10	5, 10	20	300	113.534	110.564	2.617	57.343	S1, S2, S10
50	10	5, 10	25	300	135.006	131.979	2.242	59.328	B10, S3, S4, S5
50	10	5, 10	30	300	138.371	130.750	5.507	54.703	B10, S4, S5
100	5	5, 10	20	138	195.767	193.827	0.991	17.953	B2,B3, B5
100	5	5, 10	25	123	292.675	290.051	0.896	17.203	B3, B5, S1, S4
100	5	5, 10	30	202	329.483	326.225	0.989	26.000	B3, B5, S4
100	5	7, 15	20	227	224.196	222.417	0.793	30.312	S1, S2, S3, S5
100	5	7, 15	25	300	263.218	248.480	5.599	36.578	B3, B5, S4
100	5	7, 15	30	90	270.648	267.948	0.998	11.844	B3, B5
100	5	10, 20	20	227	224.196	222.417	0.793	30.203	S1, S2, S3, S5
100	5	10, 20	25	300	265.115	255.409	3.661	38.859	S1, S2, S3, S5
100	5	10, 20	30	202	329.483	326.225	0.989	26.468	B3, B5, S4
100	7	5, 10	20	176	233.148	230.841	0.990	32.312	B1, S2, S3, S4, S5
100	7	5, 10	25	300	222.183	219.069	1.402	48.093	B1, B3, S6
100	7	5, 10	30	300	313.143	306.848	2.010	46.859	B1, B4, S6
100	7	7, 15	20	300	233.389	221.329	5.167	48.656	B1, S3, S4, S5
100	7	7, 15	25	143	323.546	320.328	0.995	26.016	B1, S2, S3, S4, S5
100	7	7, 15	30	76	248.625	246.187	0.981	12.156	B1, B4
100	7	10, 20	20	300	111.214	109.127	1.876	43.484	S1, S2
100	7	10, 20	25	300	318.956	303.817	4.747	54.671	S1, S2, S3, S4, S5, S7
100	7	10, 20	30	131	339.154	335.784	0.994	23.109	B1, S4, S5, S6
100	10	5, 10	20	300	247.286	243.135	1.678	67.218	B10, S3, S4, S5, S9
100	10	5, 10	25	300	240.869	236.792	1.693	63.500	B5, B10, S6
100	10	5, 10	30	300	307.676	303.770	1.269	65.140	B6, B10, S5
100	10	7, 15	20	300	256.485	245.620	4.236	66.343	B10, S2, S4, S5, S9
100	10	7, 15	25	300	266.296	262.589	1.392	68.781	B10, S3, S4, S8, S9
100	10	7, 15	30	300	307.237	303.770	1.128	64.078	B5, B10, S6
100	10	10, 20	20	300	194.346	191.373	1.529	62.015	S1, S2, S10
100	10	10, 20	25	158	324.034	320.797	0.999	39.687	S1, S3, S4, S5, S9
100	10	10, 20	30	300	303.912	298.423	1.806	86.284	S5, S6, S10

J: the number of customer nodes.
I: the number of potential facility locations.
C: s = the minimum number of customers required to open a small facility.
b = the minimum number of customers required to open a large facility.
d: coverage distance.
B: large facility.
S: small facility.
Gap: 100(Upper bound − Lower bound)/Upper bound.

6. Conclusions

In this paper, the location decision problem with customer preference and the minimal number of customers required to open a new facility, is suggested. The FLSDP model can be used in public service areas, such as medical, education, and security services. This model can seek simultaneously to the best locations with proper level of the scale of the new facility on the various factors of customer preference, multiple facilities, elastic demand, facility size as well as the available budget.

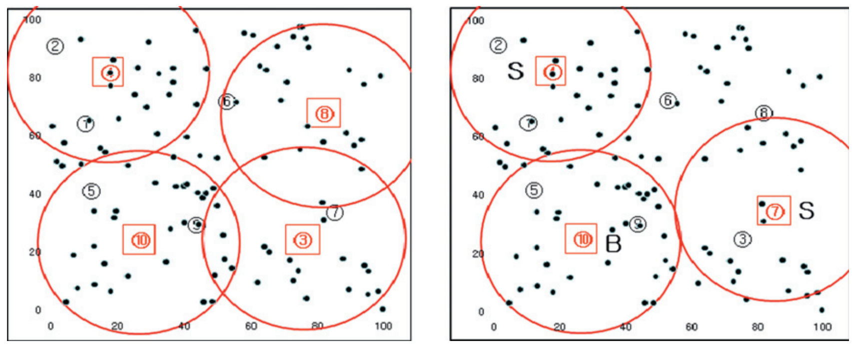
Table 3
The experimental results for cases 3 and 4.

J	I	C (s, b)	d	Iteration	Upper bound	Lower bound	Gap (%)	Time (sec)	Facility numbers
500	10	10, 20	20	130	868.339	859.688	0.996	70.750	B5, B10, S9
500	10	10, 20	25	32	963.189	953.748	0.980	15.671	B5, B10
500	10	10, 20	30	75	1664.250	1647.820	0.987	50.109	B5, B10, S6, S9
500	10	20, 40	20	43	718.098	711.037	0.983	21.671	B5, B10
500	10	20, 40	25	35	963.127	953.748	0.974	16.953	B5, B10
500	10	20, 40	30	20	1261.160	1248.660	0.991	10.156	B5, B10
500	10	40, 80	20	55	1382.890	1369.140	0.994	33.890	S1, S2, S3, S5, S10
500	10	40, 80	25	75	1605.280	1589.470	0.985	47.531	B10, S3, S5, S8, S9
500	10	40, 80	30	20	1261.160	1248.660	0.991	10.359	B5, B10
500	20	10, 20	20	78	1089.490	1078.620	0.998	102.250	B5, B13, S6, S16
500	20	10, 20	25	56	953.833	944.376	0.991	56.390	B5, B13
500	20	10, 20	30	54	1424.280	1410.380	0.976	70.375	B13, B15, S6
500	20	20, 40	20	57	716.993	710.098	0.962	55.718	B5, B13
500	20	20, 40	25	56	953.833	944.376	0.991	53.406	B5, B13
500	20	20, 40	30	49	1107.560	1096.490	0.999	47.875	B13, B15
500	20	40, 80	20	69	1470.940	1456.360	0.992	99.078	S1, S5, S8, S10, S13, S15
500	20	40, 80	25	90	1318.630	1305.550	0.992	123.062	B5, S6, S10, S11, S15
500	20	40, 80	30	49	1107.560	1096.490	0.999	47.875	B13, B15
500	30	10, 20	20	300	686.338	677.372	1.306	775.453	B13, B30
500	30	10, 20	25	69	1246.880	1234.600	0.984	205.125	B15, B30, S19, S22
500	30	10, 20	30	64	1005.870	995.971	0.984	128.297	B15, B30
500	30	20, 40	20	71	1012.750	1002.640	0.998	247.968	B30, S6, S15, S19, S26
500	30	20, 40	25	300	838.638	829.630	1.074	778.625	B15, B30
500	30	20, 40	30	64	1005.870	995.971	0.984	127.188	B15, B30
500	30	40, 80	20	50	1672.840	1656.390	0.983	221.3120	S8, S10, S13, S23, S24, S30
500	30	40, 80	25	300	2193.390	2127.050	3.025	1189.750	S5, S10, S13, S15, S23, S30
500	30	40, 80	30	62	1626.390	1610.150	0.998	223.312	B30, S6, S15, S19, S26
1000	10	10, 20	20	54	1381.650	1367.840	0.999	57.562	B5, B10
1000	10	10, 20	25	55	1869.120	1850.860	0.977	58.625	B5, B10
1000	10	10, 20	30	300	2382.130	2303.100	3.317	471.921	B5, B10
1000	10	20, 40	20	54	1381.650	1367.840	0.999	57.328	B5, B10
1000	10	20, 40	25	55	1869.120	1850.860	0.977	57.317	B5, B10
1000	10	20, 40	30	300	2382.130	2303.100	3.317	572.245	B5, B10
1000	10	50, 100	20	56	2110.520	2089.510	0.996	97.015	B5, S4, S6, S8, S9
1000	10	50, 100	25	41	2720.290	2693.120	0.999	65.031	S5, S10, S6, S9
1000	10	50, 100	30	300	2382.130	2303.100	3.317	563.921	B5, B10
1000	20	10, 20	20	55	1406.420	1392.430	0.995	189.140	B5, B13
1000	20	10, 20	25	56	1839.440	1821.410	0.980	196.437	B5, B13
1000	20	10, 20	30	79	2202.850	2180.830	0.999	313.703	B5, B13
1000	20	20, 40	20	55	1406.420	1392.430	0.995	191.640	B5, B13
1000	20	20, 40	25	56	1839.440	1821.410	0.980	196.437	B5, B13
1000	20	20, 40	30	79	2202.850	2180.830	0.999	327.797	B5, B13
1000	20	50, 100	20	59	2034.580	2014.420	0.991	409.890	B5, S6, S11, S15, S16
1000	20	50, 100	25	56	2393.080	2369.640	0.980	319.140	B5, S6, S13
1000	20	50, 100	30	79	2202.850	2180.830	0.999	325.687	B5, B13
1000	30	10, 20	20	59	1302.560	1289.720	0.986	440.796	B10, B30
1000	30	10, 20	25	57	1508.060	1493.120	0.991	452.000	B15, B30
1000	30	10, 20	30	57	1801.980	1784.250	0.984	346.781	B15, B30
1000	30	20, 40	20	59	1302.560	1289.720	0.986	366.656	B10, B30
1000	30	20, 40	25	57	1508.060	1493.120	0.991	348.203	B15, B30
1000	30	20, 40	30	57	1801.980	1784.250	0.984	389.437	B15, B30
1000	30	50, 100	20	80	2272.640	2249.990	0.997	894.562	B30, S13, S15, S19, S26
1000	30	50, 100	25	57	2373.050	2349.490	0.993	542.718	B15, S30, S19, S22
1000	30	50, 100	30	56	2403.690	2379.800	0.994	460.078	B15, S30, S19

J: the number of customer nodes.
I: the number of potential facility locations.
C: s = the minimum number of customers required to open a small facility.
b = the minimum number of customers required to open a large facility.
d: coverage distance.
B: large facility.
S: small facility.
Gap: 100(Upper bound − Lower bound)/Upper bound.

A mathematical programming formulation is constructed for the problem, and a solution algorithm for large problems using the Lagrangian relaxation is addressed. The experimental results of this paper show that the solutions obtained using the suggested heuristic deviates by 1.65% in average. In most cases, the algorithm provides good solutions within a reasonable computational time. More specific modeling for the covering function of private companies, like chain restaurants, shopping centers, and distribution centers, based on individual characteristics is interesting, and it may provide decision makers with different perspectives on facility location management.

The modeling includes two different kinds of problem attributes: the covering location problem is a strategic and long-term range issue, while the preference index is dynamic. However, in



(a) Classical MCLP (b) FLSDP with MCR (s=15, b=30)

Fig. 2. Comparison results between MCLP and FLSDP.

the location decision, more factors that may impact business have to be included. Because more modern facilities are operated by installment and outsourcing, their managerial decisions are able to be updated as the market changes, especially with regard to customer preference or brand value. Practical and efficient implementation procedures can be developed at the operational level to maximize profit in a changing market.

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