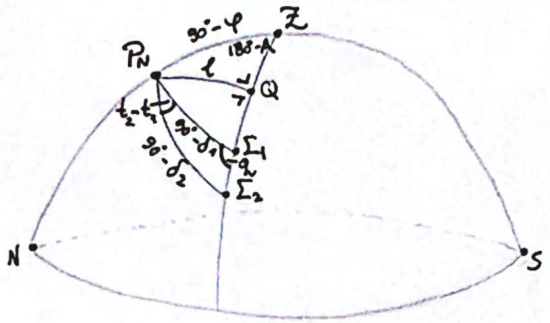
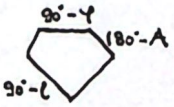
Grupa B – zadatak 1.



$\Sigma_1\Sigma_2 = \sigma$

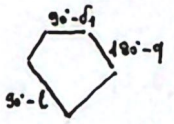
$\triangle P_N Z Q$:



$\cos(90°-l) = \sin(90°-\varphi)\sin(180°-A)$

$\sin l = \cos\varphi \sin A$ (1)

$\triangle P_N Q \Sigma_1$:



$\cos(90°-l) = \sin(90°-\delta_1)\sin(180°-q)$

$\sin l = \cos\delta_1 \sin q$ (2)

$\triangle P_N \Sigma_1 \Sigma_2$:

$\sin q \sin\sigma = \sin(90°-\delta_2)\sin(t_2-t_1)$

$\sin q = \cos\delta_2 \sin(\alpha_1-\alpha_2)\operatorname{cosec}\sigma$ (4)

(1) i (2): $\cos\varphi \sin A = \cos\delta_1 \sin q$ (3)

$s_1 = s_2 \Rightarrow t_1 + \alpha_1 = t_2 + \alpha_2$

$t_2 - t_1 = \alpha_1 - \alpha_2$

(3) i (4): $\cos\varphi \sin A = \cos\delta_1 \cos\delta_2 \sin(\alpha_1-\alpha_2)\operatorname{cosec}\sigma$

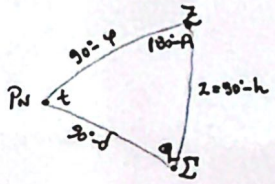
$\sin A \leq 1 \Rightarrow \sin A \cos\varphi \leq \cos\varphi$

$\Rightarrow \cos\varphi \sin\sigma > \cos\delta_1 \cos\delta_2 \sin(\alpha_1-\alpha_2)$ ∎

Grupa A – zadatak 1.

Treba pokazati: $t = \arccos\left(\sin 2\varphi \cos^2\frac{A}{2}\right)$

$(\varphi = h)$ *



$\cos(90°-h) = \cos(90°-\varphi)\cos(90°-\delta) + \sin(90°-\varphi)\sin(90°-\delta)\cos t$

$\sin h = \sin\varphi\sin\delta + \cos\varphi\cos\delta\cos t$

$\cos(90°-\delta) = \cos(90°-\varphi)\cos(90°-h) + \sin(90°-\varphi)\sin(90°-h)\cos(180°-A)$

$\sin\delta = \sin\varphi\sin h + \cos\varphi\cos h\cos A$

* → $\sin\varphi = \sin\varphi\sin\delta + \cos\varphi\cos\delta\cos t$ (1)

$\sin\delta = \sin^2\varphi - \cos^2\varphi\cos A$ (2)

(1) $\Rightarrow \cos t = \dfrac{\sin\varphi - \sin\varphi\sin\delta}{\cos\varphi\cos\delta} = \dfrac{\sin\varphi(1-\sin\delta)}{\cos\varphi\cos\delta} = \operatorname{tg}\varphi\,\dfrac{1-\sin\delta}{\cos\delta}$ (3)

(2) $\Rightarrow \sin\delta = \sin^2\varphi - (1-\sin^2\varphi)\cos A$

ili još lakše: $\sin\delta = 1 - \cos^2\varphi - \cos^2\varphi\cos A = 1 - \cos^2\varphi(1+\cos A)$

$\Rightarrow 1 - \sin\delta = \cos^2\varphi(1+\cos A)$ (4)

(3) i (4): $\cos t = \operatorname{tg}\varphi\,\dfrac{\cos^2\varphi(1+\cos A)}{\cos\delta}$

$\cos t = \dfrac{\sin\varphi}{\cos\varphi}\,\dfrac{\cos^2\varphi(1+\cos A)}{\cos\delta}$

$\cos t = \sin\varphi\cos\varphi(1+\cos A)\sec\delta \cdot \frac{2}{2}$

$\cos t = 2\sin\varphi\cos\varphi \cdot \dfrac{1+\cos A}{2}\sec\delta$

$\cos t = \sin 2\varphi \cdot \cos^2\frac{A}{2}\sec\delta$

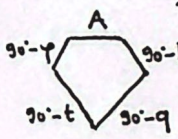
$\underbrace{\cos\delta}\cos t = \sin 2\varphi \cdot \cos^2\frac{A}{2}$

↓

da bi vrijedio treći izraz mora biti $\cos\delta = 1 \rightarrow \delta = 0°$

$\Rightarrow t = \arccos\left(\sin 2\varphi \cos^2\frac{A}{2}\right)$

skriveni uslov: $\delta = 0° \rightarrow$ $\triangle P_N Z \Sigma$ kvadratni



$\cos(90°-h) = \sin(90°-\varphi)\sin(90°-t)$

$\sin h = \cos\varphi\cos t \quad \{h = \varphi\}$

$\operatorname{tg}h = \cos t = \operatorname{tg}\varphi \qquad (\cos\varphi \neq 0)$

$\operatorname{tg}\varphi = \cos t \;;\; |\operatorname{tg}\varphi| \leq 1$

$\Rightarrow |\varphi| \leq 45° \;;\; \varphi = 45° \rightarrow \cos t = 1 \rightarrow t = 0°$ – granični slučaj

$\cos A = \operatorname{ctg}(90°-\varphi)\operatorname{ctg}(90°-h) = \operatorname{tg}^2\varphi$

$\cos^2\frac{A}{2} = \dfrac{1+\cos A}{2} = \dfrac{1+\operatorname{tg}^2\varphi}{2} = \dfrac{\frac{\sin^2\varphi+\cos^2\varphi}{\cos^2\varphi}}{2} = \dfrac{1}{2\cos^2\varphi}$

$\sin 2\varphi \cdot \cos^2\frac{A}{2} = 2\sin\varphi\cos\varphi \cdot \dfrac{1}{2\cos^2\varphi} = \operatorname{tg}\varphi$

$\Rightarrow \cos t = \operatorname{tg}\varphi$ – uslov ispunjen ...

$|\varphi| < 45°$