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Uvod u interaktivno dokazivanje teorema

Vežbe 5

Zadatak 1 *Algebra skupova*

Diskutovati o sledećim termovima:

term $\{1, 2, 3\}$
term $\{1{::}nat, 2, 3\}$
term $(\in)$
term $(\cap)$
term $(\cup)$
term $(\cup)\ A$
term $A \cup B$
term $(A{::}'a\ set) - B$

Zadatak 2 *Isar dokazi*

Pokazati sledeća tvrđenja o skupovima pomoću jezika Isar.

Napomena: Dozvoljeno je korišćenje samo *simp* metode za dokazivanje pojedinačnih koraka.

lemma $-(A \cup B) = -A \cap -B$
proof
  show $-(A \cup B) \subseteq -A \cap -B$
  proof
    fix x
    assume $x \in -(A \cup B)$
    then have $x \notin A \cup B$ **by** *simp*
    then have $x \notin A \wedge x \notin B$ **by** *simp*
    then have $x \in -A \wedge x \in -B$ **by** *simp*
    then show $x \in -A \cap -B$ **by** *simp*
  qed
next
  show $-A \cap -B \subseteq -(A \cup B)$
  proof
    fix x
    assume $x \in -A \cap -B$
    then have $x \in -A \wedge x \in -B$ **by** *simp*
    then have $x \notin A \wedge x \notin B$ **by** *simp*
    then have $x \notin A \cup B$ **by** *simp*
    then show $x \in -(A \cup B)$ **by** *simp*
  qed
qed

Savet: Iskoristiti *find-theorems* - $\vee$ - $\longleftrightarrow$ - $\vee$ - za pronalaženje odgovarajuće teoreme.

lemma $A \cup B = B \cup A$
proof

show $A \cup B \subseteq B \cup A$
proof
fix x
assume $x \in A \cup B$
then have $x \in A \vee x \in B$ **by** *simp*
then have $x \in B \vee x \in A$ **using** *disj-commute*[*of* $x \in A$ $x \in B$] **by** *simp*
then show $x \in B \cup A$ **by** *simp*
qed
next
show $B \cup A \subseteq A \cup B$
proof
fix x
assume $x \in B \cup A$
then have $x \in B \vee x \in A$ **by** *simp*
then have $x \in A \vee x \in B$ **using** *disj-commute*[*of* $x \in A$ $x \in B$] **by** *simp*
then show $x \in A \cup B$ **by** *simp*
qed
qed

Savet: Iskoristiti aksiomu isključenja trećeg $A \vee \neg A$ u kontekstu operacije pripadanja $(\in) :: {'a} \Rightarrow {'a}\ set \Rightarrow bool$.

lemma $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (**is** *?left* $=$ *?right*)
proof
show *?left* $\subseteq$ *?right*
proof
fix x
assume $x \in$ *?left*
then have $x \in A \vee x \in B \cap C$ **by** *simp*
then show $x \in$ *?right*
proof
assume $x \in A$
then have $x \in A \cup B \wedge x \in A \cup C$ **by** *simp*
then show $x \in$ *?right* **by** *simp*
next
assume $x \in B \cap C$
then have $x \in B \wedge x \in C$ **by** *simp*
then have $x \in A \cup B \wedge x \in A \cup C$ **by** *simp*
then show $x \in$ *?right* **by** *simp*
qed
qed
next
show *?right* $\subseteq$ *?left*
proof
fix x
assume $x \in$ *?right*
then have $*$: $x \in A \cup B \wedge x \in A \cup C$ **by** *simp*
have $x \in A \vee x \notin A$ **by** *simp*
then show $x \in$ *?left*
proof
assume $x \in A$
then show $x \in$ *?left* **by** *simp*

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    next
      assume x ∉ A
      from this and * have x ∈ B ∧ x ∈ C by simp
      then have x ∈ B ∩ C by simp
      then show x ∈ ?left by simp
    qed
  qed
qed

lemma A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) (is ?left = ?right)
proof
  show ?left ⊆ ?right
  proof
    fix x
    assume x ∈ ?left
    then have *: x ∈ A ∧ x ∈ B ∪ C by simp
    then have x ∈ A by simp
    from * have x ∈ B ∪ C by simp
    then have x ∈ B ∨ x ∈ C by simp
    from this and ‹x ∈ A› have x ∈ A ∩ B ∨ x ∈ A ∩ C by simp
    then show x ∈ ?right by simp
  qed
next
  show ?right ⊆ ?left
  proof
    fix x
    assume x ∈ ?right
    then have x ∈ A ∩ B ∨ x ∈ A ∩ C by simp
    then show x ∈ ?left
    proof
      assume x ∈ A ∩ B
      then have x ∈ A ∧ x ∈ B by simp
      then have x ∈ A ∧ x ∈ B ∪ C by simp
      then show x ∈ ?left by simp
    next
      assume x ∈ A ∩ C
      then have x ∈ A ∧ x ∈ C by simp
      then have x ∈ A ∧ x ∈ B ∪ C by simp
      then show x ∈ ?left by simp
    qed
  qed
qed

lemma A − (B ∩ C ) = (A − B) ∪ (A − C ) (is ?left = ?right)
proof
  show ?left ⊆ ?right
  proof
    fix x
    assume x ∈ ?left
    then have *: x ∈ A ∧ x ∉ B ∩ C by simp
    then have x ∈ A by simp
```

from $*$ **have** $x \notin B \cap C$ **by** *simp*
then have $x \notin B \vee x \notin C$ **by** *simp*
then show $x \in$ *?right*
proof
assume $x \notin B$
with ‹$x \in A$› **have** $x \in A - B$ **by** *simp*
then show $x \in$ *?right* **by** *simp*
next
assume $x \notin C$
with ‹$x \in A$› **have** $x \in A - C$ **by** *simp*
then show $x \in$ *?right* **by** *simp*
qed
qed
next
show *?right* $\subseteq$ *?left*
proof
fix x
assume $x \in$ *?right*
then have $x \in A - B \vee x \in A - C$ **by** *simp*
then show $x \in$ *?left*
proof
assume $x \in A - B$
then have $x \in A \wedge x \notin B$ **by** *simp*
then have $x \in A \wedge x \notin B \cap C$ **by** *simp*
then show $x \in$ *?left* **by** *simp*
next
assume $x \in A - C$
then have $x \in A \wedge x \notin C$ **by** *simp*
then have $x \in A \wedge x \notin B \cap C$ **by** *simp*
then show $x \in$ *?left* **by** *simp*
qed
qed
qed