• $\boxed{0001} = 1 \rightarrow$ inverz je $\underline{1}$ $\qquad F_{16} = F_2[x]/(x^4+x+1)$

jer je $1 \cdot 1 = 1$

• $\boxed{0100} = x^2$

$NZD(x^2, x^4+x+1)$

$x^4 + x + 1 = \underline{x^2} \cdot x^2 + \underline{(x+1)}$ ... $(x^4+x+1) : x^2 = \underline{x^2}$

x^4

$= \underline{x+1}$

$x^2 = \underline{(x+1)} \cdot (x+1) + \underline{1}$ .... $x^2 : (x+1) = x + 1$

$x^2 + x$

$= x$

$x + 1$

$= 1$

$1 = x^2 + (x+1) \cdot (x+1)$

$= x^2 + (x+1) \cdot \left[(x^4 + x + 1) + x^2 \cdot x^2 \right]$

$= (x+1)(x^4+x+1) + x^2 + (x+1) \cdot x^2 \cdot x^2$

$= (x+1)(x^4+x+1) + (x^3 + x^2) \cdot x^2 + x^2$

$= (x+1)(x^4+x+1) + \underbrace{(x^3+x^2+1)} \cdot x^2$

$(x^2)^{-1} = x^3 + x^2 + 1 = \underline{1101}$

• $\boxed{0101} = x^2 + 1$

$NZD(x^4+x+1, x^2+1)$

$x^4 + x + 1 = \underline{(x^2+1)} \cdot (x^2+1) + \underline{x}$ ... $(x^4+x+1) : (x^2+1) = \underline{x^2+1}$

$x^4 + x^2$

$= x^2 + x$

$x^2 + 1$

$= \underline{x} + x + x$

$x^2 + 1 = \underline{x} \cdot (x) + \underline{1}$ ...

$1 = (x^2+1) + x \cdot x$

$= (x^2+1) + x \cdot \left[(x^4+x+1) + (x^2+1)(x^2+1) \right]$ .... $(x^2+1) : x = \underline{x}$

$= (x^2+1) + x \cdot (x^4+x+1) + x \cdot (x^2+1)(x^2+1)$ $\quad x^2$

$= 1$

$= x \cdot (x^4+x+1) + (x^3+x)(x^2+1) + (x^2+1)$

$= x \cdot (x^4+x+1) + \underbrace{(x^3+x+1)}(x^2+1)$

$(x^2+1)^{-1} = x^3 + x + 1 = \underline{1011}$

1000 $= x^3$

$NZD(x^4+x+1, x^3)$

$x^4+x+1 = \underline{x} \cdot x^3 + \underline{(x+1)}$ ... $(x^4+x+1) : x^3 = x$, x^4 // $x+1$

$x^3 = \underline{x^2+x+1} \cdot (x+1) + \underline{1}$ ... $x^3 : (x+1) = x^2+x+1$, x^3+x^2 // x^2, x^2+x // x, $x+1$ // 1

$1 = x^3 + (x^2+x+1)\cdot(x+1)$

$= x^3 + (x^2+x+1)\left[(x^4+x+1) + x\cdot x^3\right]$

$= (x^2+x+1)(x^4+x+1) + (x^2+x+1)\cdot x\cdot x^3 + x^3$

$= (x^2+x+1)(x^4+x+1) + (x^3+x^2+x)\cdot x^3 + x^3$

$= (x^2+x+1)(x^4+x+1) + \underline{(x^3+x^2+x+1)}\cdot x^3$

$(x^3)^{-1} = x^3+x^2+x+1 = 1111$

1010 $= x^3+x$

$NZD(x^4+x+1, x^3+x)$

$x^4+x+1 = \underline{x} \cdot (x^3+x) + \underline{(x^2+x+1)}$ ... $(x^4+x+1) : (x^3+x) = x$, x^4+x^2 // x^2+x+1

$x^3+x = \underline{(x+1)} \cdot (x^2+x+1) + \underline{(x+1)}$ ... $(x^3+x) : (x^2+x+1) = x+1$, x^3+x^2+x // x^2 // , x^2+x+1 // $x+1$

$x^2+x+1 = \underline{x} \cdot (x^2+x) + \underline{1}$

$1 = (x^2+x+1) + x\cdot(x+1)$

$= (x^2+x+1) + x\cdot\left[(x^3+x) + (x+1)\cdot(x^2+x+1)\right]$

$= (x^2+x+1) + x\cdot(x^3+x) + (x^2+x)\cdot(x^2+x+1)$

$= (x^2+x+1)(x^2+x+1) + x\cdot(x^3+x)$

$= (x^2+x+1)\left[(x^4+x+1) + x\cdot(x^3+x)\right] + x\cdot(x^3+x)$

$= (x^2+x+1)(x^4+x+1) + (x^3+x^2+x+x)(x^3+x)$

$(x^3+x)^{-1} = x^3+x^2 = 1100$

• $\boxed{1100} = x^3 + x^2$

$NZD(x^4+x+1,\ x^3+x^2)$

$x^4 + x + 1 = \underline{x} \cdot (x^3+x^2) + \underline{(x^3+x+1)}$

$x^3 + x^2 = \underline{1} \cdot (x^3+x+1) + \underline{(x^2+x+1)}$

$x^3 + x + 1 = \underline{x} \cdot (x^2+x+1) + \underline{x^2+1}$

$x^2 + x + 1 = \underline{1} \cdot (x^2+1) + \underline{x}$

$x^2 + 1 = \underline{x} \cdot x + \underline{1}$

$(x^4+x+1) : (x^3+x^2) = x$
$x^4 + x^3$
x^3+x+1

$(x^3+x^2) : (x^3+x+1) = 1$
x^3+x+1
x^2+x+1

$(x^3+x+1) : (x^2+x+1) = x$
x^3+x^2+x
x^2+1

$(x^2+x+1) : (x^2+1) = 1$
x^2+1
x

$(x^2+1) : x = x$
x^2
1

$1 = (x^2+1) + x \cdot x$

$= (x^2+1) + x \cdot [(x^2+x+1) + (x^2+1)]$

$= x \cdot (x^2+x+1) + (x+1)(x^2+1)$

$= x \cdot (x^2+x+1) + (x+1) \cdot [(x^3+x+1) + x \cdot (x^2+x+1)]$

$= (x+1)(x^3+x+1) + (x^2+x^2+x)(x^2+x+1)$

$= (x+1)(x^3+x+1) + x^2 \cdot (x^2+x+1)$

$= (x+1)(x^3+x+1) + x^2 \cdot [(x^3+x^2) + (x^3+x+1)]$

$= x^2 \cdot (x^3+x^2) + (x^2+x+1)(x^3+x+1)$

$= x^2 \cdot (x^3+x^2) + (x^2+x+1) \cdot [(x^4+x+1) + x \cdot (x^3+x^2)]$

$= (x^2+x+1)(x^4+x+1) + (x^3+x^2+x+x^3)(x^3+x^2)$

$= (x^2+x+1)(x^4+x+1) + (x^3+x)(x^3+x^2)$

$(x^3+x^2)^{-1} = x^3 + x =$ ~~1000~~ 1010