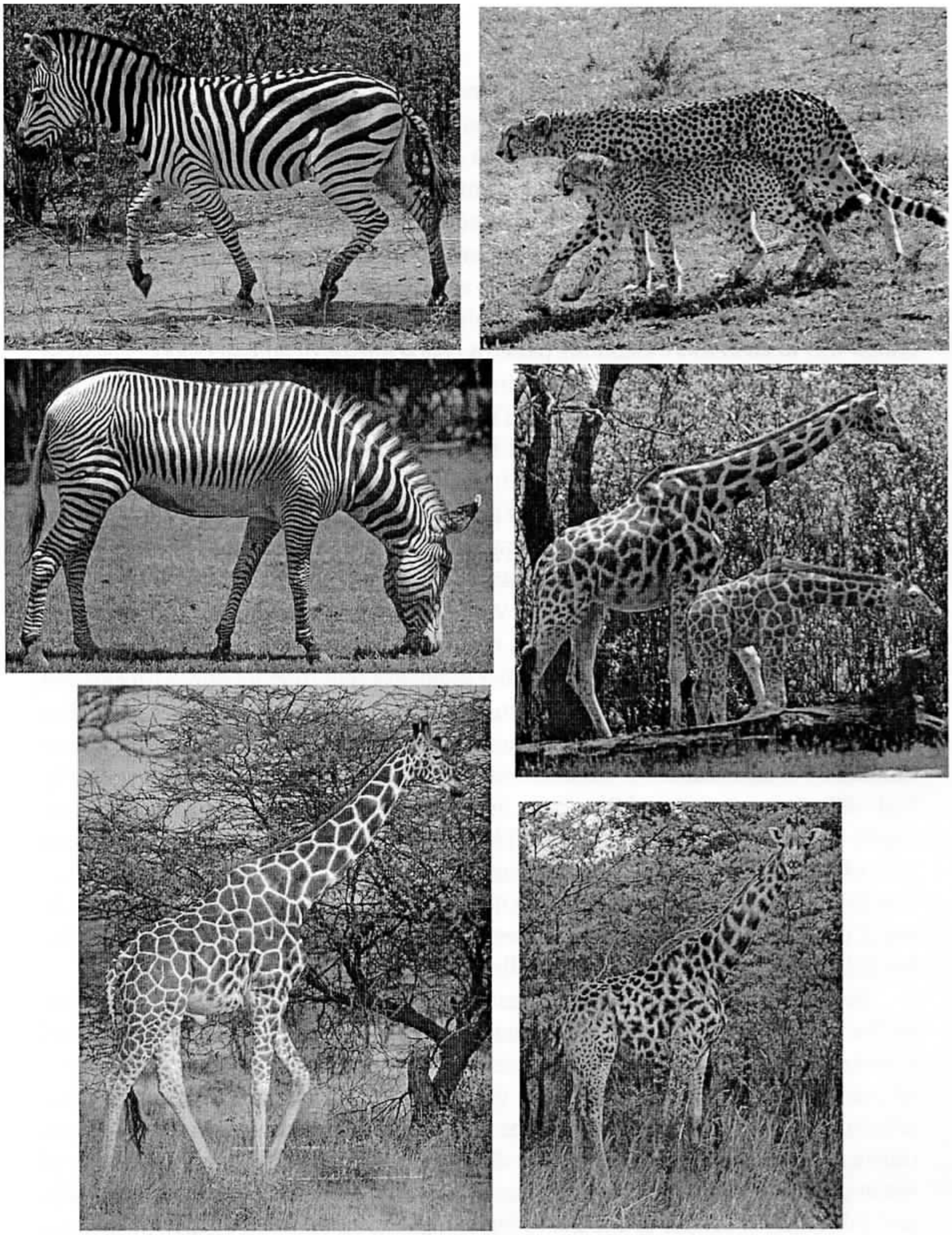


**Figure 7.2.1** Photographs showing examples of pigment patterns on animal skins. From top left by row: Grant's zebra, South African cheetah, Grevy's zebra, Uganda giraffe, reticulated giraffe and Masai giraffe. These patterns arise from a process that requires differentiation between regions that contain pigment-producing cells and those that do not. The study of such patterns captures one of the essential processes involved in various stages of development that require differentiation in order to form structures and organs that form a functioning physiology. ∎

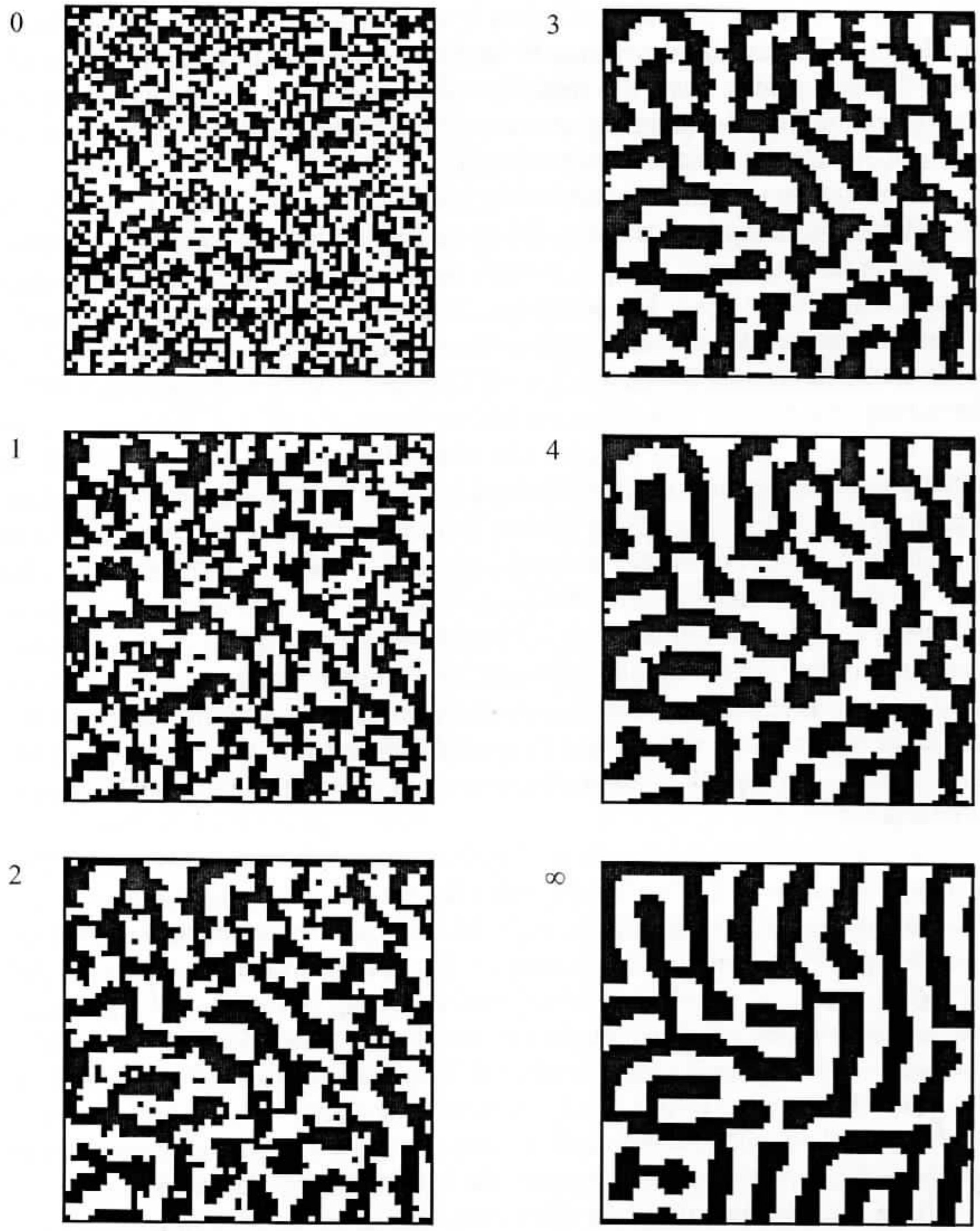



**Figure 7.2.3** A simulation of a CA model of pattern formation. ON cells (black) produce pigment and OFF cells (white) do not. The initial conditions assign cells to be ON or OFF at random with probability 1/2. Five updates are shown and then the unchanging stable limit that is reached after about 20 updates. The parameters are $R_1 = 1$, $R_2 = 6$, $J_1 = 1$, $J_2 = -0.1$, and $h = 0$. ∎

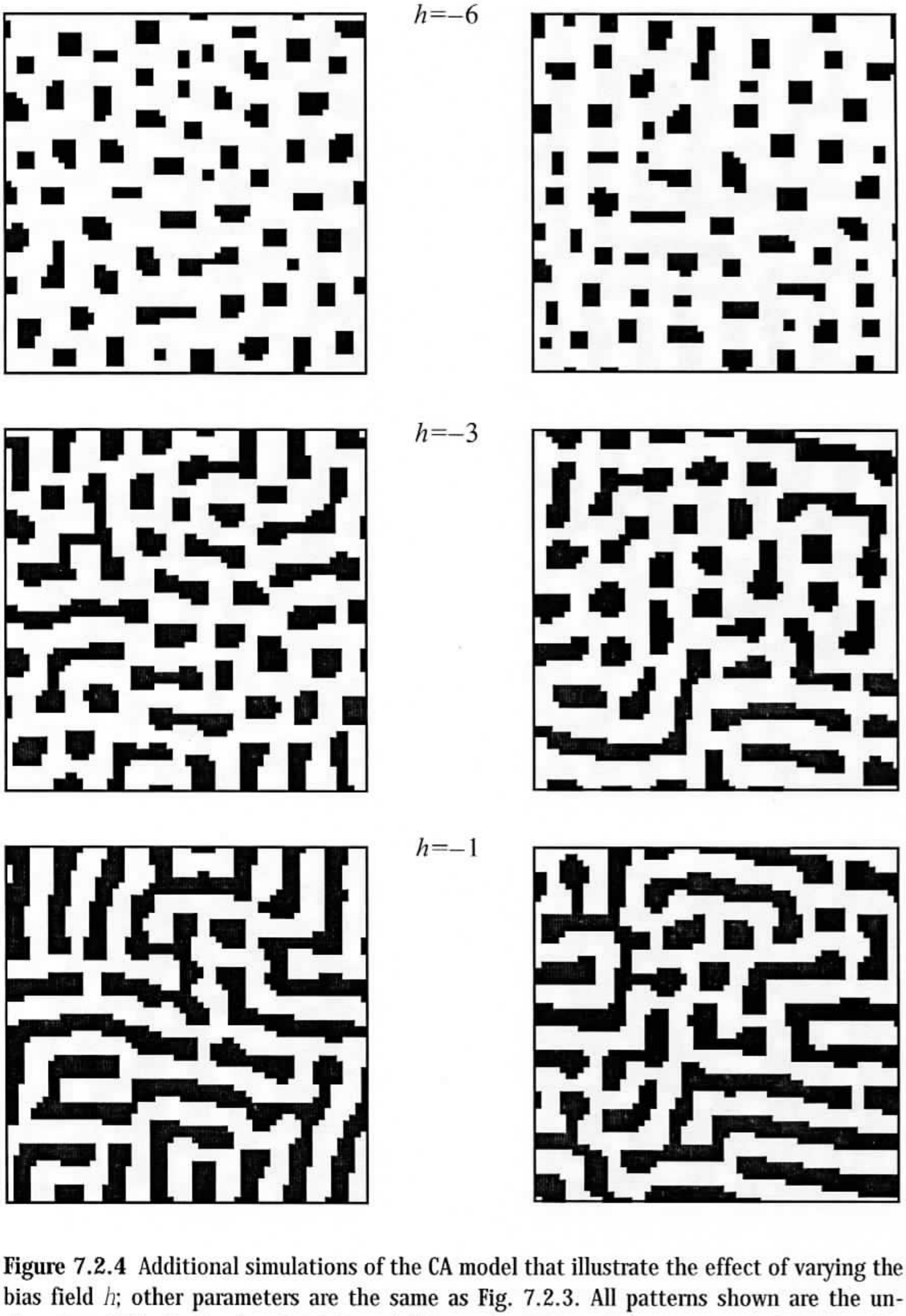



**Figure 7.2.4** Additional simulations of the CA model that illustrate the effect of varying the bias field $h$; other parameters are the same as Fig. 7.2.3. All patterns shown are the unchanging stable limit of a simulation. $h$ biases the system to have more or less ON cells. Varying $h$ results in patterns with black spots on a white background, white and black stripes, or white spots on a black background. The left and right panels differ only in the initial conditions of the simulation. All of the left panels start with the initial condition shown

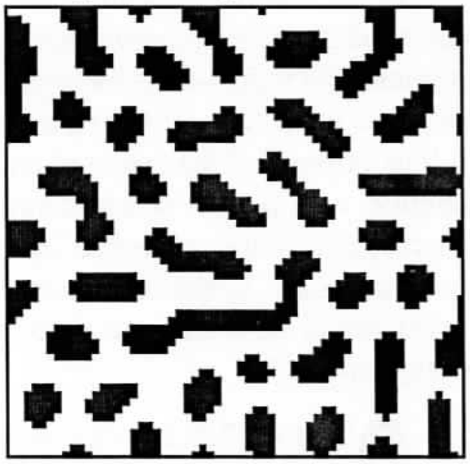
$R_2$=6.0 $R_1$=1.0 $h$=0

$R_2$=7.0 $R_1$=1.0 $h$=0

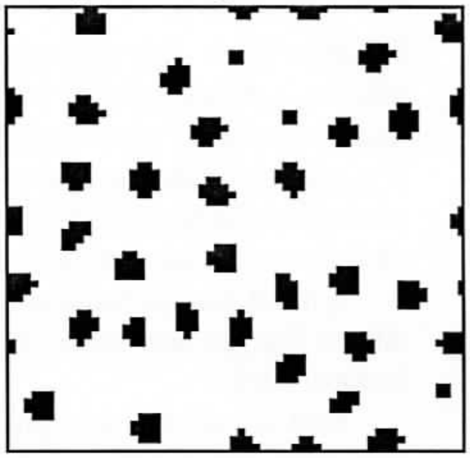
$R_2$=8.0 $R_1$=1.0 $h$=0

$R_2$=6.0 $R_1$=1.5 $h$=0

$R_2$=6.0 $R_1$=1.5 $h$=–3



$R_2$=6.0 $R_1$=1.5 $h$=–6



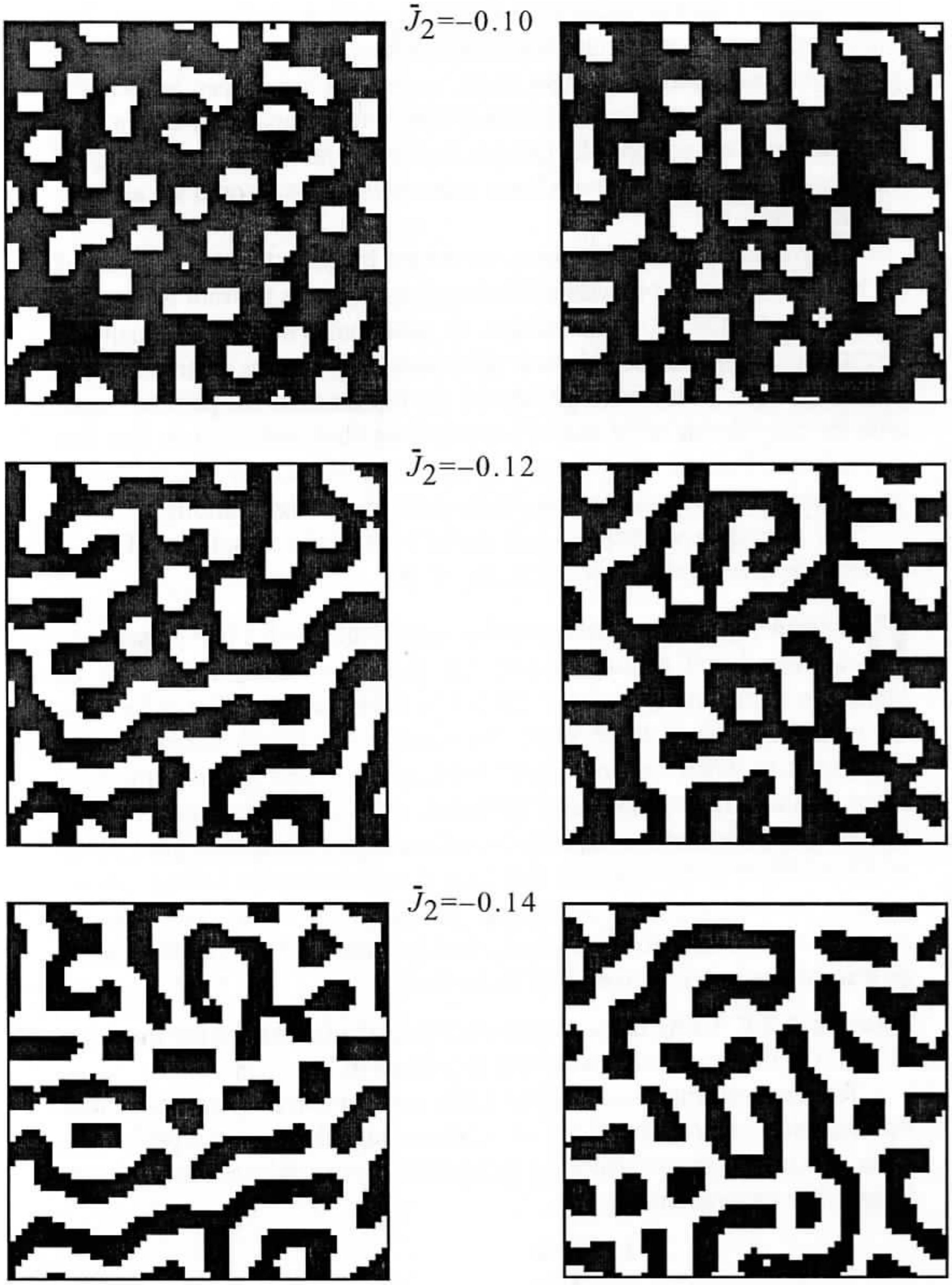



**Figure 7.2.6** Using a different parametrization of the CA model for pattern formation with $\bar{s}_i = 0, 1$ and $\bar{h} = 0$ in Eq. 7.2.3, we generate patterns that are similar to Fig. 7.2.3 by varying the strength of the inhibition $\bar{J}_2$. The other parameters were taken to be $\bar{R}_1 = 1$, $\bar{R}_2 = 6$, $\bar{J}_1 = 1$. Left and right panels use different initial random conditions similar to Fig. 7.2.4 (see Question 7.2.2). ▮

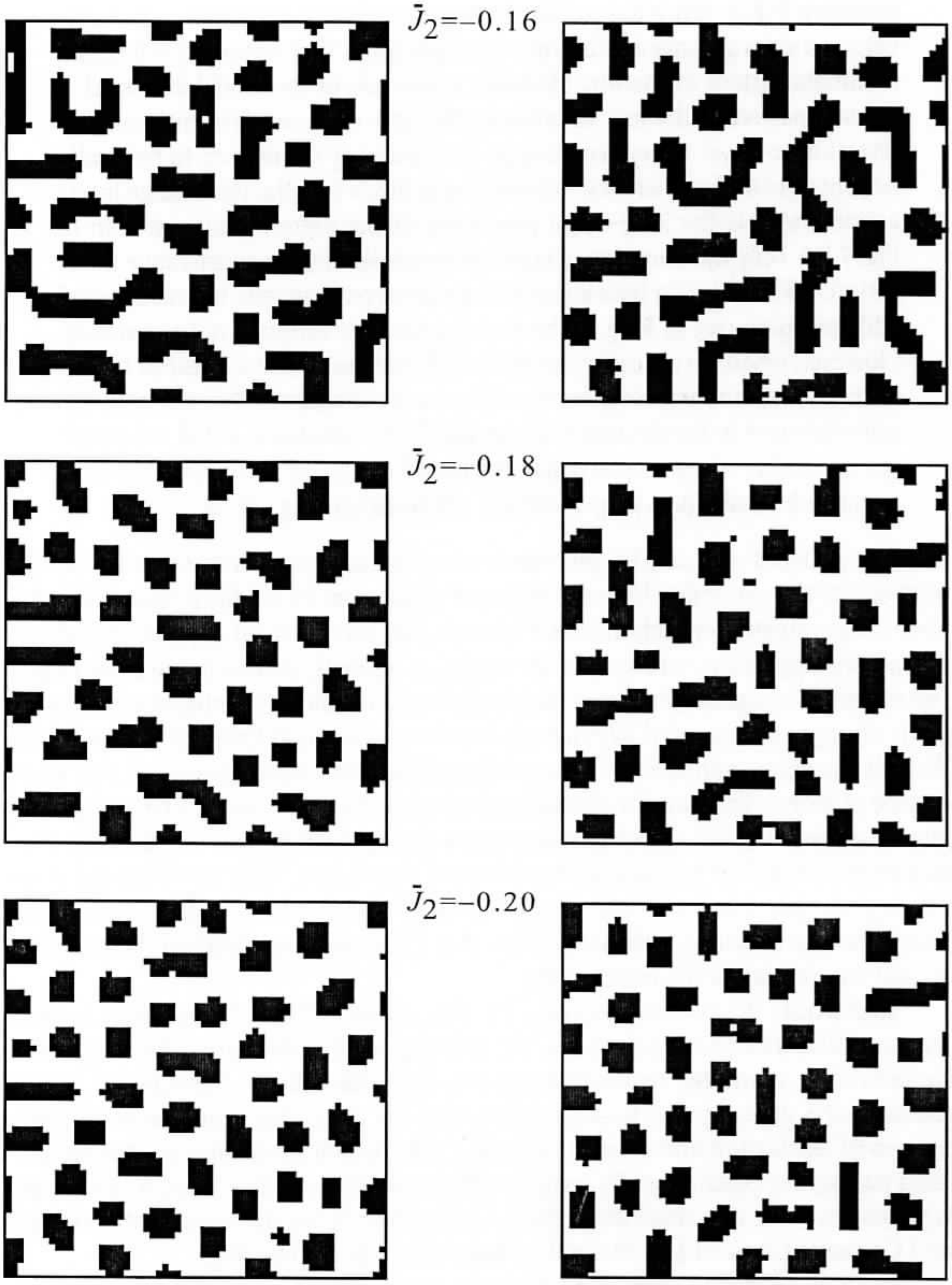



**Figure 7.2.6** (*continued*)

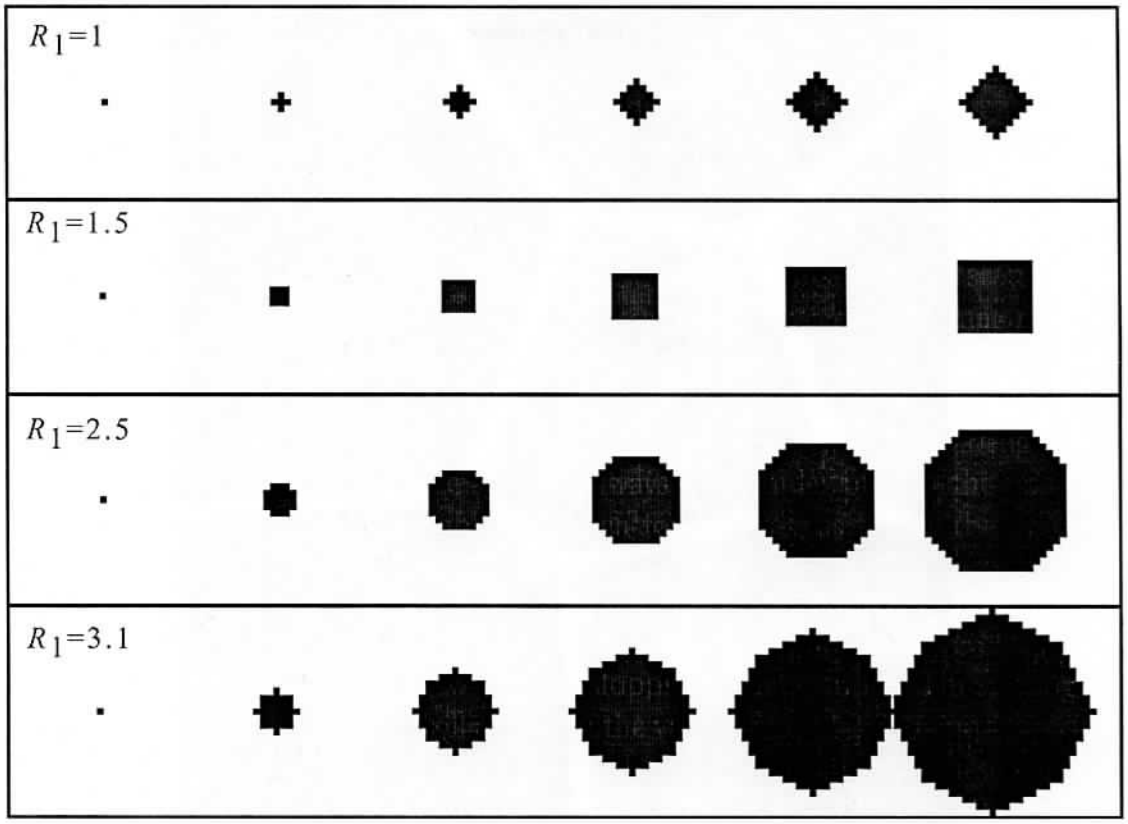



**Figure 7.2.8** Starting from a seed pigment cell we can grow outward using a rule that sets a cell ON if there are any ON cells in its neighborhood. However, the shape of the growing region on a square lattice depends on the way we grow it. Here, growth of a region is shown for various sizes of the neighborhood given by its radius $R_1$. A larger $R_1$ leads to more circular pigmented areas. ∎

OFF cells. To achieve the desired result, we can take a due from the previous model of pattern formation and set up two distances,a distance $R_1$ over which the growth is determined, and a distance $R_2$ over which the stopping is determined. Thus, using binary variables $s_i = 0,1$ we turn a cell ON when the value of Eq.(7.2.3) is positive, with parameter values of $R_1 = 2.5$, $R_2 = 4.3$, $J_1 = 1$ and $J_2 = -.5$. The values of these parameters can be adjusted by trial and error.

The patterns generated in Fig. 7.2.10 using this approach are reminiscent of the patterns of giraffes;however, they are not entirely satisfactory. While some of the regions follow the convex shape that we expect,other regions are more convoluted. By looking carefully at the patterns, we see that this occurs because the separations between the initial ON cells vary in distance. This would not occur if the starting points were more regularly spaced. There are many ways to consider placing the points at more regular intervals. A reasonable approach for this case is to use the previous method of creating patterns using activation-inhibition to generate a pattern of spots such as those shown in Fig. 7.2.4 and then to apply the growth starting from these spots. This is illustrated in Fig. 7.2.11, where the initial pattern is generated

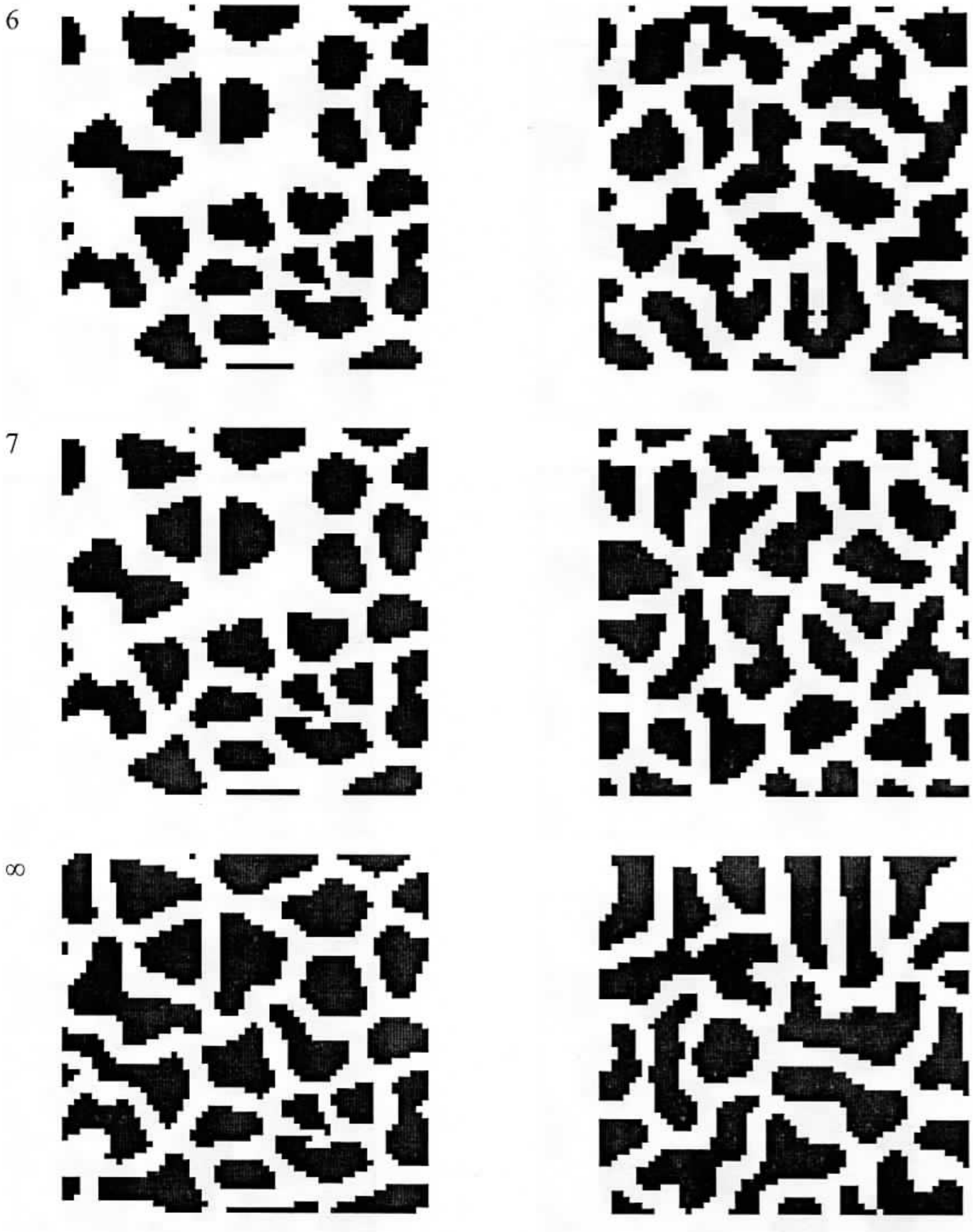



**Figure 7.2.10** (*continued*)

from a CA activation-inhibition model, resulting in more regular but still randomly placed spots. By growing out into the OFF regions we form a pattern that is closer to the patterns on the giraffe coats. More specifically, this coloration is similar to that of the Uganda giraffe (Fig. 7.2.1). Two other kinds of giraffe—the reticulated giraffe and the Masai giraffe—would require additional tuning of parameters. The reticu-

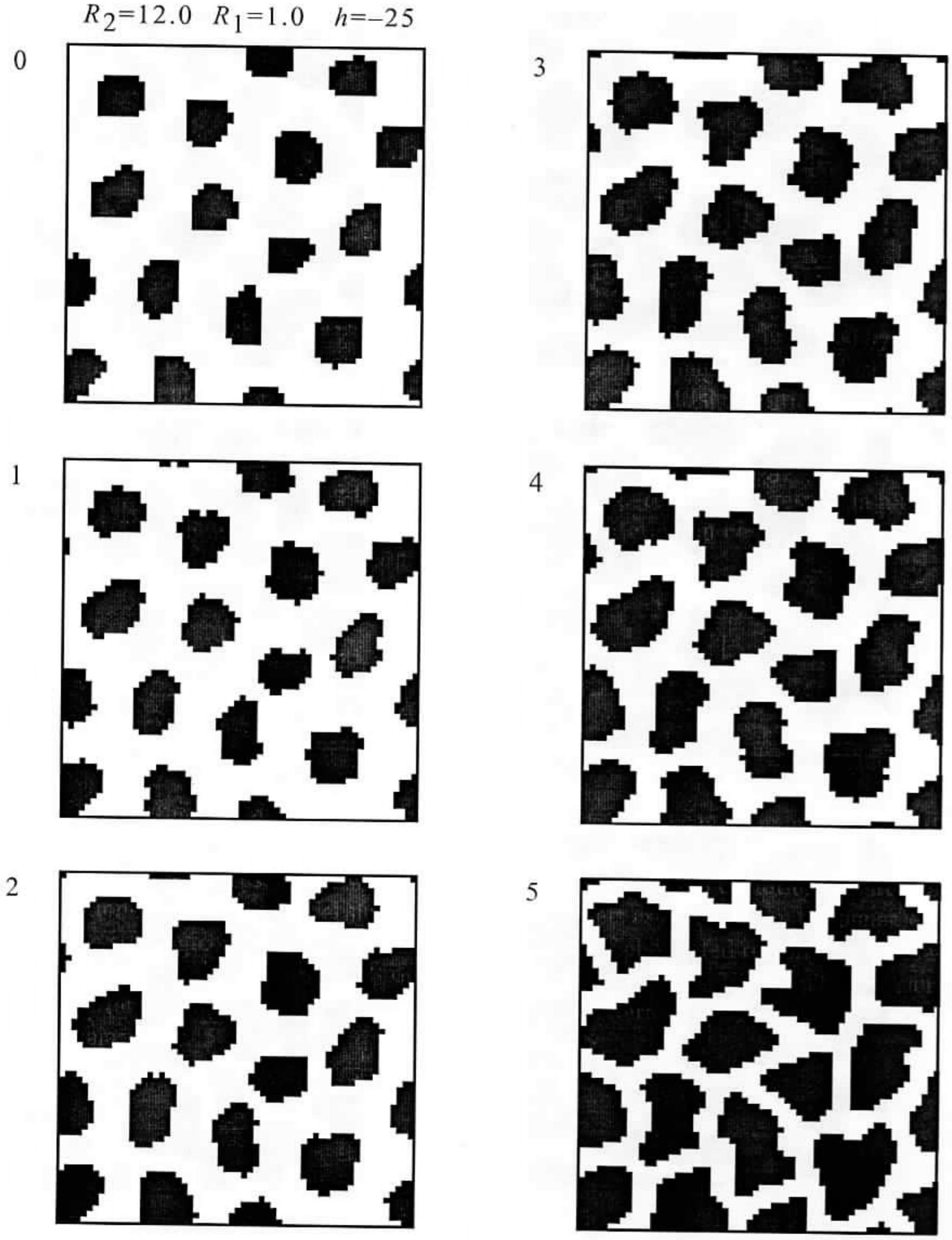



**Figure 7.2.11** The giraffe color patterns generated in Fig. 7.2.10 can be improved by starting from points that are more regularly spaced in the plane. They might be placed more regularly by several processes, one of which is illustrated here. The initial conditions result from an activation-inhibition CA model simulation with parameters as indicated on the upper left. This is the starting conformation for the growth outward of pigmented regions. The subsequent frames show updates using the same algorithm as Fig. 7.2.10. This results in a more regular pattern reminiscent of the Uganda giraffe. Other patterns can be generated by varying the parameters. ∎