

$x_j = \frac{2\pi j}{n}, j = 0, \dots, n-1$ ekvidistantne tačke iz $[0, 2\pi)$

Uslov: $f(x_j) = \sum_{k=0}^{n-1} c_k e^{-ik \frac{2\pi j}{n}}, j = 0, \dots, n-1$.

Knjiga:

Oznaka: $w = e^{i \frac{2\pi}{n}}$.

Furijeova matrica:

$$F = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & w & w^2 & \dots & w^{(n-1)} \\ 1 & w^2 & w^4 & \dots & w^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & w^{(n-1)} & w^{2(n-1)} & \dots & w^{(n-1)^2} \end{bmatrix}$$

$F = \text{conj}(\text{dftmtx}(n))/n$

$f = F^* c \Rightarrow c = (F^*)^{-1} f = \frac{1}{n} F f$

tj. $c_k = \frac{1}{n} \sum_{j=0}^{n-1} w^{jk} f_j$

$c = \text{ifft}(f)$

Inverzna DFT:

$f_j = \sum_{k=0}^{n-1} \bar{w}^{jk} c_k, \bar{w} = e^{-i \frac{2\pi}{n}}$

$f = \text{fft}(c)$

MATLAB:

Oznaka: $w = e^{-i \frac{2\pi}{n}}$.

Furijeova matrica:

$$F = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & w & w^2 & \dots & w^{(n-1)} \\ 1 & w^2 & w^4 & \dots & w^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & w^{(n-1)} & w^{2(n-1)} & \dots & w^{(n-1)^2} \end{bmatrix}$$

$F = \text{dftmtx}(n)$

$c = F f$

tj. $c_k = \sum_{j=0}^{n-1} w^{jk} f_j$

$c = \text{fft}(f)$

Inverzna DFT:

$f_j = \frac{1}{n} \sum_{k=0}^{n-1} \bar{w}^{jk} c_k, \bar{w} = e^{i \frac{2\pi}{n}}$

$f = \text{ifft}(c)$

Množenje polinoma

$$p_1(x) = \sum_{j=0}^{n_1} f_j x^j, p_2(x) = \sum_{j=0}^{n_2} g_j x^j, p(x) = p_1(x)p_2(x) = \sum_{j=0}^n c_j x^j$$

$f = [f_0, f_1, \dots, f_{n_1}, 0, \dots, 0]$ i $g = [g_0, g_1, \dots, g_{n_2}, 0, \dots, 0]$ oba vektora dopunjena nulama do dimenzije koja odgovara stepenu proizvoda polinoma, tj. duzine $n+1 = n_1 + n_2 + 1$. Vektor $c = [c_0, \dots, c_n]$ sadrži koeficijente proizvoda polinoma.

Oznake: \hat{f} je DFT od vektora f , \hat{g} je DFT od vektora g , \hat{c} je DFT od vektora c .

$$\hat{f} = \frac{1}{n} F f \Rightarrow \hat{f}_k = \frac{1}{n} \sum_{j=0}^n w^{jk} f_j = \frac{1}{n} p_1(w^k)$$

$$\hat{g} = \frac{1}{n} F g \Rightarrow \hat{g}_k = \frac{1}{n} \sum_{j=0}^n w^{jk} g_j = \frac{1}{n} p_2(w^k)$$

$$\hat{c} = \frac{1}{n} F c \Rightarrow \hat{c}_k = \frac{1}{n} \sum_{j=0}^n w^{jk} c_j = \frac{1}{n} p(w^k)$$

$$\hat{f}_k \cdot \hat{g}_k = \frac{1}{n^2} p_1(w^k) p_2(w^k) = \frac{1}{n^2} p(w^k)$$

$$(\hat{f} \cdot \hat{g})_k = \frac{1}{n^2} p(w^k) = \frac{1}{n} \hat{c}_k$$

Napomena: gornje množenje (oznaka \cdot) sa leve strane jednakosti je tačka po tačka (operator \cdot u MATLAB-u).

$\hat{f} \cdot \hat{g} = \frac{1}{n} \hat{c} = \frac{1}{n^2} F c$ (pomnožimo levu i desnu stranu sa F^*)

$$F^*(\hat{f} \cdot \hat{g}) = \frac{1}{n^2} F^* F c = \frac{1}{n^2} n I c = \frac{1}{n} c$$

$$c = n F^*(\hat{f} \cdot \hat{g}) = f * g$$

$$\hat{f} = F f \Rightarrow \hat{f}_k = \sum_{j=0}^n w^{jk} f_j = p_1(w^k)$$

$$\hat{g} = F g \Rightarrow \hat{g}_k = \sum_{j=0}^n w^{jk} g_j = p_2(w^k)$$

$$\hat{c} = F c \Rightarrow \hat{c}_k = \sum_{j=0}^n w^{jk} c_j = p(w^k)$$

$$\hat{f}_k \cdot \hat{g}_k = p_1(w^k) p_2(w^k) = p(w^k)$$

$$(\hat{f} \cdot \hat{g})_k = p(w^k) = \hat{c}_k$$

Napomena: gornje množenje (oznaka \cdot) sa leve strane jednakosti je tačka po tačka (operator \cdot u MATLAB-u).

$\hat{f} \cdot \hat{g} = \hat{c} = F c$ (pomnožimo levu i desnu stranu sa F^*)

$$F^*(\hat{f} \cdot \hat{g}) = F^* F c = n I c$$

$$c = \frac{1}{n} F^*(\hat{f} \cdot \hat{g}) = f * g$$