Wednesday, December 15, 2021
11:26 AM

$y'' + y = 0$, $F(x, y, y'') = 0$

$y = f(x)$?

$y = \sin(x)$ rešenje

$y' = \cos(x)$

$y'' = -\sin(x)$

$\sin x - \sin x = 0$ ✓

$y = \cos x$

$y' = -\sin x$

$y'' = -\cos x$

$-\cos x + \cos x = 0$ ✓

$y = 3\sin x + 4\cos x$ ✓

$y'^2 + xy' + 6x^2 = 0$ $F(x, y') = 0$, $y = ?$

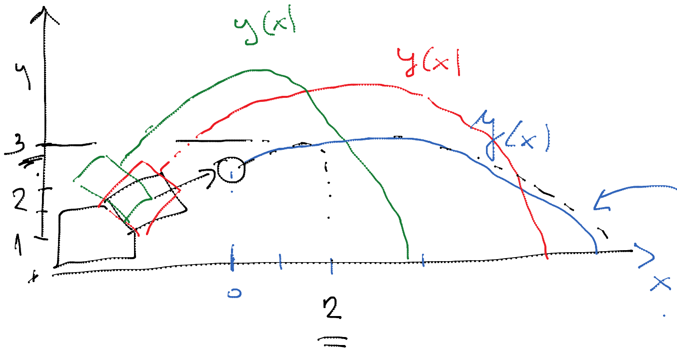
$x \cdot \frac{\partial z}{\partial x} + y \cdot \frac{\partial z}{\partial y} = 0$ $z = f(x, y)$ 2 prom.

$\underset{\sim}{z}$ nepoznata

parcijalna DJ

$xy' + y = 0$ rešenje: $\underbrace{y(x) = \frac{c}{x} \quad , c \in \mathbb{R}}$

∞ mnogo rešenja



$y(x) = \ldots$

$x = 2$, $y(2) = 3$

$x = x_0$ $y|_{x=2} = 3 = y_0$

početni uslov: $y|_{x=1} = 2$, $y(1) = 2 = y_0$

opšte rešenje: $y(x) = \frac{c}{x}$, $c \in \mathbb{R}$ (∞ mnogo)

$y = \varphi(x, c)$

$\frac{c}{x}$

partikularno rešenje: $2 = \frac{c}{1} \Rightarrow c = 2 \cdot 1 = 2 \equiv c_0$

$y(x) = \frac{c}{x} = \frac{2}{x}$ (jedno!)

$\varphi(x, c_0)$

Wednesday, December 15, 2021
11:26 AM

$$y' = \frac{x+1}{y^2} \qquad y' = f(x,y)$$

$$\frac{dy}{dx} = \frac{x+1}{y^2} \qquad / \cdot y^2 dx$$

$$\underbrace{y^2}_{g(y)} dy = \underbrace{(x+1)}_{f(x)} dx \qquad / \int$$

(sa razdvojenim prom.)

$$\int y^2 dy = \int (x+1)\, dx$$

$$\frac{y^3}{3} + C_1 = \frac{x^2}{2} + x + C_2$$

$$y^3 = \left(\frac{x^2}{2} + x + \underbrace{C_2 - C_1}_{C}\right) \cdot 3$$

$$y = \sqrt[3]{3\left(\frac{x^2}{2} + x + C\right)} \qquad \text{OR}, \quad C \in \mathbb{R}$$

$$f(x,y) = \frac{x^2+y^2}{x^2+xy} \qquad \text{homogena?}$$

$$f(kx, ky) = \frac{(kx)^2 + (ky)^2}{(kx)^2 + (kx)\cdot(ky)} = \frac{k^2x^2 + k^2y^2}{k^2x^2 + k^2\cdot xy} = \frac{\cancel{k^2}(x^2+y^2)}{\cancel{k^2}(x^2+xy)}$$

$$= \frac{x^2+y^2}{x^2+xy} = f(x,y) \quad \checkmark$$

$$\underbrace{y' = \underbrace{\frac{x^2+y^2}{x^2+x\cdot y}}_{\text{hom. f-ja}}}$$

hom. DJ

Wednesday, December 15, 2021
11:26 AM

$y' = f(x,y) = f(kx, ky) \quad , \forall k \in R$

uzmimo $k = \frac{1}{x}$

$\Rightarrow f(x,y) = f\left(\frac{1}{x}\cdot x , \frac{1}{x}\cdot y\right) = f\left(1, \frac{y}{x}\right) \equiv g\left(\frac{y}{x}\right) = y'$

smena: $u = \frac{y}{x}$

$y = u \cdot x \Rightarrow y'_x = u'_x \cdot x + u \cdot 1$

$\frac{dy}{dx} = \underbrace{\frac{du}{dx} \cdot x + u}_{g(u)}$

$g(u) = \frac{du}{dx} \cdot x + u$

$g(u) - u = \frac{du}{dx} \cdot x \qquad / \cdot \frac{dx}{x} \qquad / \cdot \frac{1}{g(u)-u}$

$\frac{dx}{x} = \frac{du}{g(u)-u}$ (DJ sa razdv. prom. po u)

$\int \frac{du}{g(u)-u} = \int \frac{dx}{x} + C$ rešimo po u

$u = \dots = ♡$

Vratimo smenu $u = \frac{y}{x} \Rightarrow y = u \cdot x$

$y = ♡ \cdot x$

OR

$y' = x$

$\frac{dy}{dx} = x$

$dy = x \cdot dx \quad / \int$

$\int dy = \int x\,dx + C$

$y = \frac{x^2}{2} + C$

$\int x\,dx = \frac{1}{2}x^2$

$\int$