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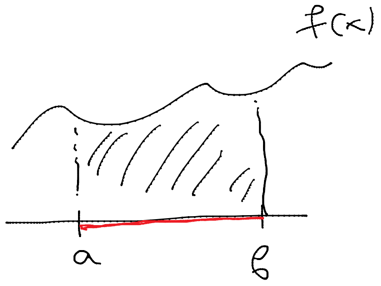
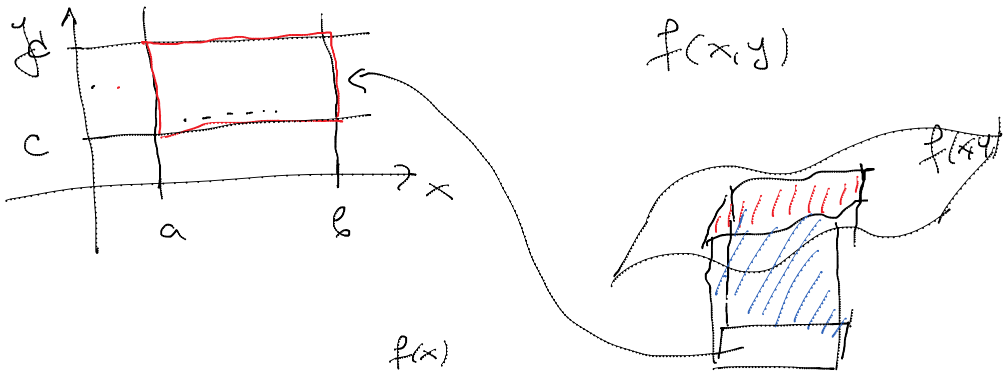
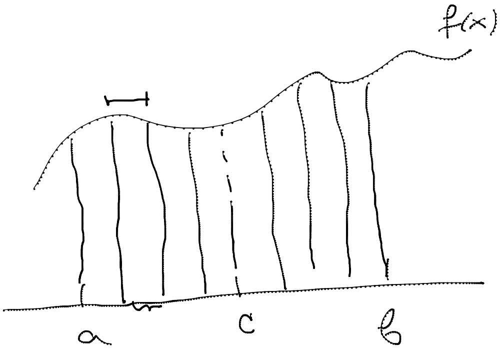
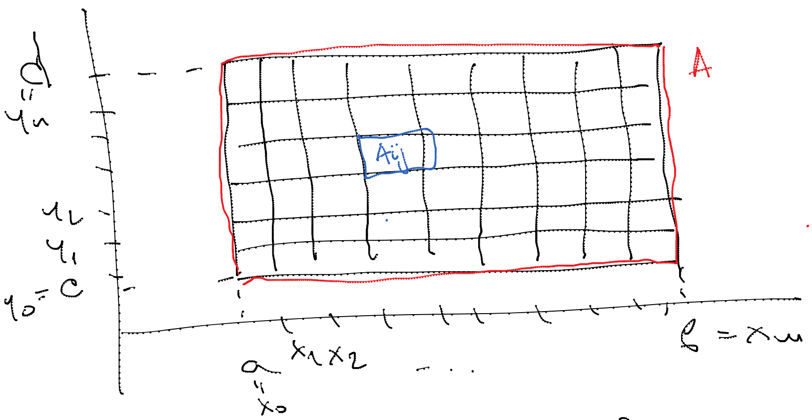
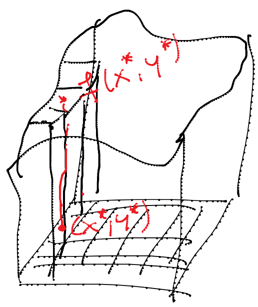
$$\int_a^b f(x)dx$$











$$A = [a,b] \times [c,d] = \{(x,y) \mid a \leq x \leq b,\ c \leq y \leq d\}$$

$$a = x_0 < x_1 < \dots < x_m = b$$
$$c = y_0 < y_1 < \dots < y_n = d$$

$$A_{ij} = [x_{i-1}, x_i] \times [y_{j-1}, y_j] = \{(x,y) \mid x_{i-1} \leq x \leq x_i,\ y_{j-1} \leq y \leq y_j\}$$
$m \cdot n \quad , i = 1,\dots,m,\ j = 1,\dots,n$

$$P_{[a,b]\times[c,d]} = \{A_{ij} \mid 1 \leq i \leq m,\ 1 \leq j \leq n\}$$

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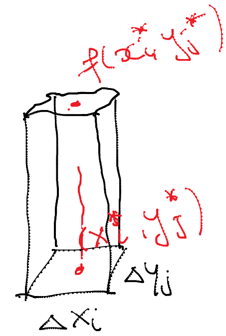


Δx_i dužinu $[x_{i-1}, x_i]$

Δy_j $[y_{j-1}, y_j]$

$\Delta A_{ij} = \Delta x_i \cdot \Delta y_j$ (površina)

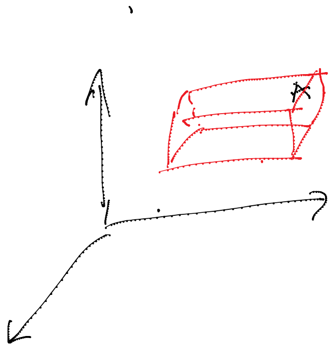
zapremina: $f(x_i^*, y_j^*) \cdot \Delta x_i \Delta y_j$

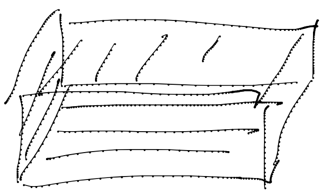
$$\sum_{i=1}^{m} \sum_{j=1}^{n} f(x_i^*, y_j^*) \cdot \Delta x_i \cdot \Delta y_j = R(f, P)$$

Rimanova int. suma

$\|P\|$ dijagonala A_{ij}

$\|P\| \to 0$ $(\Delta x_i \to 0, \Delta y_j \to 0)$





$f(x, y, z)$

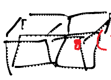
$f: \mathbb{R}^3 \to \mathbb{R}$, $A \subseteq \mathbb{R}^3$

$A = [a,b] \times [c,d] \times [e,f]$

$a = x_0 < x_1 < \dots < x_m = b$

$c = y_0 < y_1 < \dots < y_n = d$

$e = z_0 < z_1 < \dots < z_s = f$

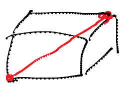


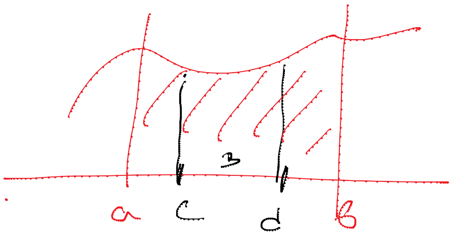
(x_i^*, y_j^*, z_k^*) $\Delta x_i, \Delta y_j, \Delta z_k \to 0$

$A_{ijk} = \Delta x_i \cdot \Delta y_j \cdot \Delta z_k$

$$R(f, R) = \sum_{i=0}^{m} \sum_{j=0}^{n} \sum_{k=0}^{s} f(x_i^*, y_j^*, z_k^*) \cdot \Delta x_i \cdot \Delta y_j \Delta z_k$$

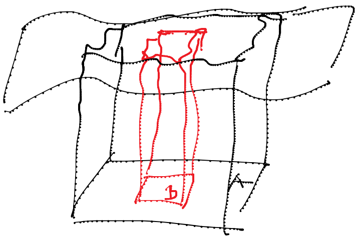
$R = \{ A_{ijk} \mid \dots \}$

$\|P\| \to 0$ 



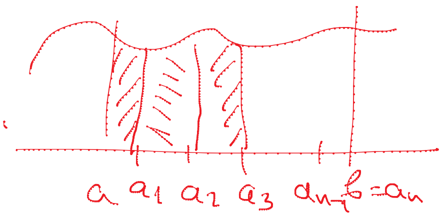
$$\int_a^b f(x)\,dx$$

$$\int_c^d f(x)\,dx$$



$$\iint_A f(x,y)\,dx\,dy$$

$$\iint_B f(x,y)\,dx\,dy$$



$$\int_a^{a_1} f(x)\,dx = C_1$$

$$\int_{a_1}^{a_2} f(x)\,dx = C_2$$

$$\vdots$$

$$\int_{a_{n-1}}^{a_n} f(x)\,dx = C_n$$

$$\int_a^b f(x)\,dx = C_1 + \dots + C_n$$

$$\iint_A f(x,y)\,dx\,dy = \int_c^d \left(\int_a^b f(x,y)\,dx \right) dy$$

$$A = \underset{x}{[a,b]} \times \underset{y}{[c,d]}$$

Integracija nad proizvoljnom oblašću

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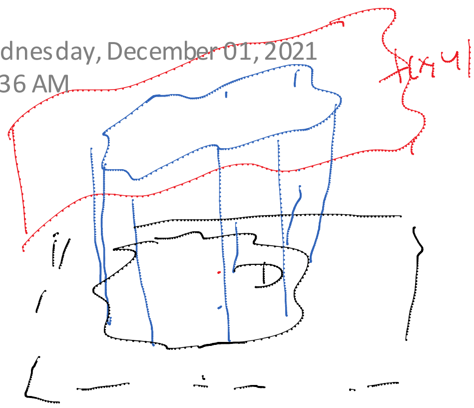
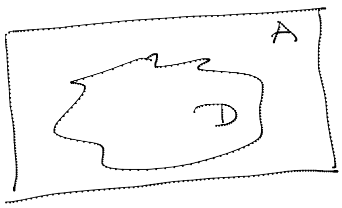


$f(x,y)$

$(x,y) \in D$

$$\iint\limits_D f(x,y)\,dxdy$$



$f: D \to \mathbb{R},\quad D \subseteq \mathbb{R}^2$

$\bar{f}: A \to \mathbb{R},\quad A \subseteq \mathbb{R}^2$

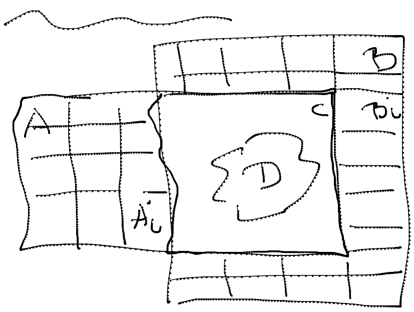
$D \subseteq A$

$$\bar{f} = \begin{cases} f(x), & x \in D \\ 0, & x \notin D \end{cases}$$

Def: $\iint\limits_D f(x,y)\,dxdy = \iint\limits_A \bar{f}(x,y)\,dxdy$

A – pravougaonik koji sadrži D

$\bar{f}$ – proširenje od f



$A \cap B = C$

$A = C \cup A_1 \cup \dots \cup A_p$

$B = C \cup B_1 \cup \dots \cup B_q$

$\bar{f}(x,y) = 0,\quad \forall x \notin D$

$\bar{f}(x,y) = 0 \quad \forall x \in A_1, \dots, A_p, B_1, \dots, B_q$

$$\iint\limits_A \bar{f}(x)\,dx = \iint\limits_C \bar{f}(x,y)\,dxdy + \underbrace{\iint\limits_{A_1} \bar{f}(x,y)\,dxdy}_{=0} + \dots + \underbrace{\iint\limits_{A_p} \bar{f}(x,y)\,dxdy}_{=0}$$

$$= \iint\limits_C \bar{f}(x,y)\,dxdy$$

$$= \iint\limits_B \bar{f}(x,y)\,dxdy = \iint\limits_C \bar{f}(x,y)\,dxdy + \underbrace{\iint\limits_{B_1} \bar{f}(x,y)\,dxdy}_{0} + \dots + \underbrace{\iint\limits_{B_q} \bar{f}(x,y)\,dxdy}_{0}$$

$$= \iint_B \bar{f}(x,y)\,dx\,dy = \iint_C \bar{f}(x,y)\,dx\,dy + \underbrace{\iint_{B_1} \underbrace{\bar{f}(x,y)}_{0}\,dx\,dy}_{=0} + \dots + \underbrace{\iint_{B_q} \underbrace{\bar{f}(x,y)}_{0}\,dx\,dy}_{=0}$$

$$= \iint_C \bar{f}(x,y)\,dx\,dy$$

$$\iint_A \bar{f}(x,y)\,dx\,dy = \iint_B \bar{f}(x,y)\,dx\,dy - \iint_C \bar{f}(x,y)\,dx\,dy$$

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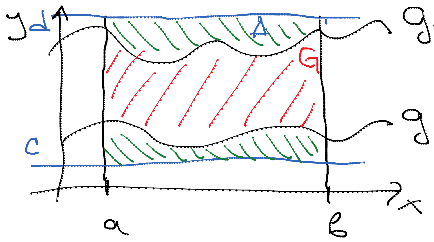
$\bar{f}(x,y) = 0 \quad \forall x \notin G$

$f: G \to \mathbb{R}, \quad G \subseteq \mathbb{R}^2$

$G = \{(x,y) \in \mathbb{R}^2 \mid a \le x \le b,\ g_1(x) \le y \le g_2(x)\}$



$\iint_G$ $\quad A = [a,b] \times [c,d]$

$G \subseteq A$

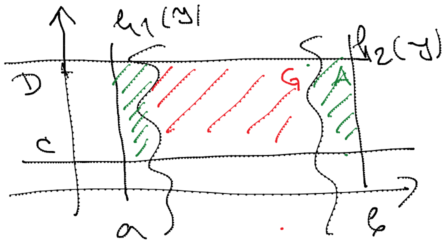
$$\boxed{\iint_G f(x,y)\,dx\,dy} = \iint_A \bar{f}(x,y)\,dx\,dy$$

$$= \int_a^b \Big(\underbrace{\int_c^d \bar{f}(x,y)\,dy}\Big)dx$$

$$= \int_a^b \Big(\underbrace{\underbrace{\int_c^{g_1(x)} \bar{f}(x,y)\,dy}_{0}}_{=0} + \int_{g_1(x)}^{g_2(x)} \underbrace{\bar{f}(x,y)}_{f(x,y)}\,dy + \underbrace{\int_{g_2(x)}^{d} \underbrace{\bar{f}(x,y)}_{0}\,dy}_{=0} \Big) dx$$

$$\boxed{= \int_a^b \Big(\int_{g_1(x)}^{g_2(x)} f(x,y)\,dy \Big) dx}$$

$G = \{(x,y) \in \mathbb{R}^2 \mid h_1(y) \le x \le h_2(y),\ c \le y \le d\}$



$$\iint_G f(x,y)\,dx\,dy \overset{\text{def}}{=} \iint_A \bar{f}(x,y)\,dx\,dy$$

$$(\text{slično}) =$$

$$= \ldots \int_c^d \Big(\int_{h_1(y)}^{h_2(y)} f(x,y)\,dx \Big) dy$$

$f(x,y) = e^{y^2}, \quad D = \{(x,y) \mid 0 \le x \le 1,\ \underset{g_1(x)}{x} \le y \le \underset{g_2(x)}{1}\}$

f neprekidna $\Rightarrow$ važi Fubinijeva t.

$$\iint_D f(x,y)\,dx\,dy = \int_0^1 \Big(\int_x^1 e^{y^2}\,dy \Big) dx \overset{?}{=} \int_x^1 \Big(\int_0^1 e^{y^2}\,dx \Big) dy$$

NE!!!!

spoljašnji MORA imati konstantne granice

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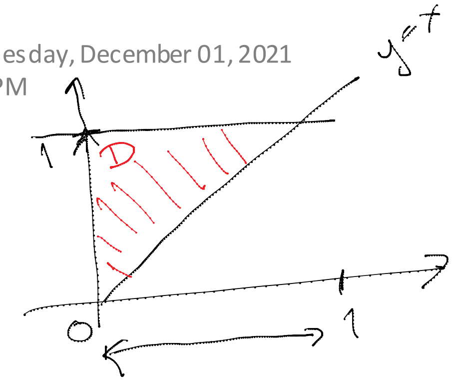

$$\iint_D f(x,y)\,dx\,dy$$

$$\shortparallel$$

$$\int_0^1 \left(\int_x^1 f(x,y)\,dy \right) dx$$

$$\shortparallel$$

$$\int_0^1 \left(\int_0^y f(x,y)\,dx \right) dy$$

$$\int_0^1 \left(\int_0^y \underbrace{e^{y^2}}_{f(x,y)} dx \right) dx = \int_0^1 e^{y^2} \left(\underbrace{\int_0^y dx}_{= x \big|_0^y = y - 0} \right) dy$$

$$= \int_0^1 e^{y^2} \cdot y\,dy = \begin{cases} t = y^2 \\ dt = 2y\,dy \end{cases}$$

$$= \ldots$$