Friday, November 26, 2021
9:38 AM

Jakobijeva matrica

$f: D \to \mathbb{R}^n$, $D \subseteq \mathbb{R}^m$

$f_j: D \to \mathbb{R}$, $D \subseteq \mathbb{R}^m$, $j = 1, \dots, n$

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1}(a) & \dots & \frac{\partial f_1}{\partial x_m}(a) \\ \frac{\partial f_2}{\partial x_1}(a) & \dots & \frac{\partial f_2}{\partial x_m}(a) \\ \vdots & & \\ \frac{\partial f_n}{\partial x_1}(a) & \dots & \frac{\partial f_n}{\partial x_m}(a) \end{bmatrix}_{n \times m} = f'(a) = J_f(a)$$

Jakobijeva matrica funkcije f u tački a

$n = m$, $\det(f'(a))$ Jakobijan

Friday, November 26, 2021
9:38 AM

Izvod složene funkcije

$f: D \to E, \quad D \subseteq \mathbb{R}^m$

$g: E \to \mathbb{R}, \quad E \subseteq \mathbb{R}^n$

$\cancel{f \circ g = g(f(x))}$

$g \circ f = g(f(x))$

$g \circ f : D \to \mathbb{R}$

$g =$

(T) Neka je f realna f-ja m realnih promenljivih na D, $f = f(g_1, \ldots, g_m)$, a g_i realne funkcije od n promenljivih na E $g_i = g_i(x_1, \ldots, x_n)$.
Ako $\forall x = (x_1, \ldots, x_n) \in E$, tačke $(g_1(x), \ldots, g_m(x)) \in D$ i ako f ima neprekidne parcijalne izvode po g_i i ako g_i imaju parcijalne izvode po x_j onda važi:

$$\frac{\partial f}{\partial x_j} = \frac{\partial f}{\partial g_1} \cdot \frac{\partial g_1}{\partial x_j} + \frac{\partial f}{\partial g_2} \cdot \frac{\partial g_2}{\partial x_j} + \ldots + \frac{\partial f}{\partial g_m} \cdot \frac{\partial g_m}{\partial x_j}$$

$g_i = g_i(t)$

$f(\underset{g_1}{x}, \underset{g_2}{y}, \underset{g_2}{z}) = x \cdot y^2 \cdot z^3$

$x(t) = e^t, \quad y(t) = t^2, \quad z(t) = \sin t$

$\frac{\partial f}{\partial t} = ?$

$f(t) = e^t \cdot (t^2)^2 \cdot (\sin t)^3$

$\frac{\partial f}{\partial x} = y^2 z^3$

$\frac{\partial f}{\partial y} = 2xyz^3$

$\frac{\partial f}{\partial z} = 3xy^2z^2$

$x'(t) = e^t \quad \left(= \frac{\partial x}{\partial t}\right)$

$y'(t) = 2t$

$z'(t) = \cos t$

$\frac{\partial f}{\partial t} = y^2 z^3 \cdot e^t + 2xyz^3 \cdot 2t + 3xy^2z \cdot \cos t$

$= t^2 \cdot (\sin t)^3 \cdot e^t + 2 \cdot e^t \cdot t^2 \cdot (\sin t)^3 \cdot 2t + 3 \cdot e^t \cdot (t^2)^2 \cdot \sin t \cdot \cos t$

Friday, November 26, 2021
9:38 AM

$f(x_1, x_2, x_3)$

$\frac{\partial f}{\partial x_1}$ $\quad \frac{\partial f}{\partial x_2}$ $\quad \frac{\partial f}{\partial x_3}$

$\frac{\partial^2 f}{\partial x_1^2}$ $\quad \frac{\partial^2 f}{\partial x_1 \partial x_2}$ $\quad \frac{\partial^2 f}{\partial x_1 \partial x_3}$ $\quad \frac{\partial^2 f}{\partial x_2 \partial x_1}$ $\quad \frac{\partial^2 f}{\partial x_2^2}$ $\quad \frac{\partial^2 f}{\partial x_2 \partial x_3}$ $\quad \frac{\partial^2 f}{\partial x_3 \partial x_1}$ $\quad \frac{\partial^2 f}{\partial x_3 \partial x_2}$ $\quad \frac{\partial^2 f}{\partial x_3^2}$

$f(x_1, x_2)$

$f(x,y,z) = x^y + y^z + z^x$

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = \left(y \cdot x^{y-1} + z^x \ln z,\ x^y \cdot \ln x + z \cdot y^{z-1},\ z^y \cdot \ln y + x \cdot z^{x-1}\right)$$

$$\frac{\partial f}{\partial x} = y \cdot x^{y-1} + z^x \ln z$$

$$\frac{\partial f}{\partial y} = x^y \cdot \ln x + z\, y^{z-1}$$

$$\frac{\partial f}{\partial z} = y^z \cdot \ln y + x \cdot z^{x-1}$$

$$= \left(y \cdot x^{y-1} + z^x \cdot \ln z\right) \cdot \vec{i} + (\dots) \cdot \vec{j} + (\dots) \cdot \vec{k}$$

$\vec{i} = (1,0,0)$

$\vec{j} = (0,1,0)$

$\vec{k} = (0,0,1)$

$A = (1,2,3)$

$\nabla f(1,2,3) = (*, *, *)$

$\dot{x}, \dot{y}$

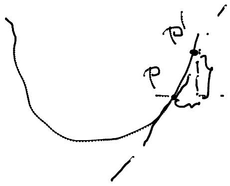
$(x, y) \qquad (x + \sigma \cdot \dot{x},\ y + \sigma \cdot \dot{y}) \quad , \sigma$

$$y = x^2 \longleftarrow \quad \cancel{y} + \cancel{\sigma} \cdot \dot{y} = (x + \sigma \cdot \dot{x})^2$$

$$= \cancel{x^2} + 2x\cancel{\sigma}\dot{x} + (\sigma\dot{x})^2 \to 0 \qquad / : \sigma \neq 0$$

$$\dot{y} = 2x$$

Friday, November 26, 2021
9:39 AM



$y = x^2$

$P(x, y)$

$P'(x+dx,\ y+dy)$ $\quad P' \in$ paraboli

$y + dy = (x+dx)^2$

$= x^2 + 2x\,dx + \cancel{dx^2} \to 0 \quad / : dx \neq 0$

$$\frac{dy}{dx} = 2x$$