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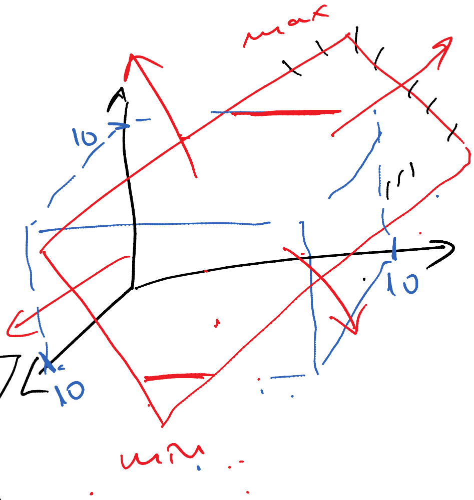
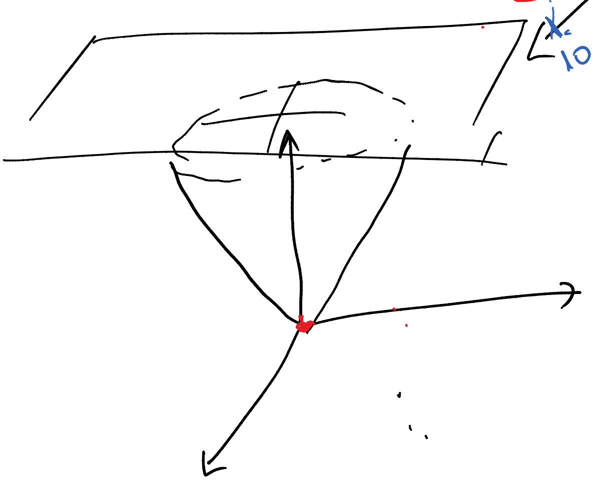
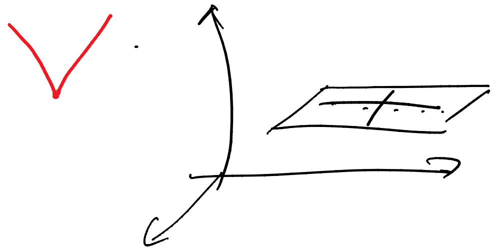
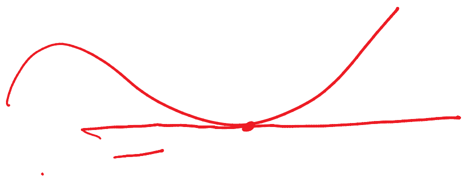








Neophodni uslov: x_0 lok. extrem $\Rightarrow$ x_0 stac. tačka

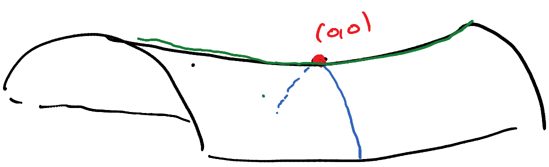
$$\not\Leftarrow$$

A, B, C stac. $\Rightarrow$ A, B lok. extr.
C nije

$f(x,y) = 2x^2 - 3y^2$

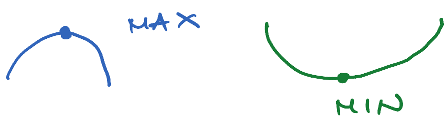
stac. tačka: $\left.\begin{array}{l} \frac{\partial f}{\partial x} = 4x = 0 \\ \frac{\partial f}{\partial y} = 6y = 0 \end{array}\right\}$ $(0,0)$ stac. tačka

SEDLASTA TAČKA



$y=0: \ f(x,0) = 2x^2 > 0$

$x=0: \ f(0,y) = -3y^2 < 0$



diferencijal drugog reda

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$$d^2f = \underbrace{\frac{\partial^2 f}{\partial x^2}}_{A} dx^2 + 2 \underbrace{\frac{\partial^2 f}{\partial x \partial y}}_{B} dx\,dy + \underbrace{\frac{\partial^2 f}{\partial y^2}}_{C} dy^2$$

$$d^2f = A\,dx^2 + 2B\,dx\,dy + C\,dy^2$$

diskriminanta $\Delta = AC - B^2 = \frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$

$$f(x,y) = x^3 + y^3 - 3xy$$

stac. točke: $\left.\begin{aligned} \frac{\partial f}{\partial x} &= 3x^2 - 3y = 0 \\ \frac{\partial f}{\partial y} &= 3y^2 - 3x = 0 \end{aligned}\right\}$ $\left.\begin{aligned} &A\ (1,1) \\ &B\ (0,0) \end{aligned}\right\}$ stacionarne \& extremi ???

I način (Prva teorema)

$$\frac{\partial^2 f}{\partial x^2} = 6x, \quad \frac{\partial^2 f}{\partial y^2} = 6y, \quad \frac{\partial^2 f}{\partial x \partial y} = -3$$

$$d^2 f(x) = \frac{\partial^2 f}{\partial x^2} dx^2 + 2 \frac{\partial^2 f}{\partial x \partial y} dx\,dy + \frac{\partial^2 f}{\partial y^2} dy^2$$

$$= 6x \cdot dx^2 + 2 \cdot (-3) \cdot dx\,dy + 6y\,dy^2$$

$\boxed{A}$: $d^2f(1,1) = 6dx^2 - 6\,dx\,dy + 6\,dy^2$

$$= 6\left[\underbrace{dx^2 - dx\,dy + \frac{dy^2}{4}} + \frac{3dy^2}{4}\right]$$

$$= 6\left[\underbrace{\left(dx - \frac{dy}{2}\right)^2}_{\geq 0} + \underbrace{\frac{3dy^2}{4}}_{\geq 0}\right] \geq 0$$

$$d^2f(1,1) \underset{=?}{\geq} 0$$

$$d^2f(1,1) \overset{?}{=} 0 \Leftrightarrow dx - \frac{dy}{2} = 0 \ ; \ \frac{3dy^2}{4} = 0$$

$$\Leftrightarrow dx = dy = 0$$

$$\Rightarrow d^2f(1,1) > 0 \ , \forall (dx, dy) \neq (0,0) \overset{T}{\Rightarrow} A \text{ lok. min.}$$

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$\boxed{B:}$ $d^2f(0,0) = 0\cdot dx^2 - 6dxdy + 0\cdot dy^2$

$= -6dxdy$

$dx<0,\ dy>0 \Rightarrow d^2f(0,0) > 0$
$dx>0,\ dy<0 \Rightarrow d^2f(0,0) > 0$
$dx>0,\ dy>0 \Rightarrow d^2f(0,0) < 0$
$dx<0,\ dy<0 \Rightarrow d^2f(0,0) < 0$

} $d^2f(0,0)$ мења знак

$\Downarrow$

B није extrem

II начин (II теорема)

$$\Delta(x) = \frac{\partial^2 f}{\partial x^2}(x)\cdot\frac{\partial^2 f}{\partial y^2}(x) - \left(\frac{\partial^2 f}{\partial x\partial y}(x)\right)^2$$

$$= 6x\cdot 6y - (-3)^2 = 36xy - 9$$

A: $\Delta(A) = \Delta(1,1) = 36 - 9 = 27 > 0 \Rightarrow$ A je extrem

$\frac{\partial^2 f}{\partial x^2}(1,1) = 6\cdot 1 = 6 > 0 \Rightarrow$ A je lok. min

B: $\Delta(B) = \Delta(0,0) = 36\cdot 0\cdot 0 - 9 = -9 < 0 \Rightarrow$ B није extrem

---

$f(x,y,z) = x^3 + y^2 + z^2$ (3 пром. NE може II теорема)

$\frac{\partial f}{\partial x} = 3x^2,\quad \frac{\partial f}{\partial y} = 2y,\quad \frac{\partial f}{\partial z} = 2z \Rightarrow A(0,0,0)$ stac.t.

$\frac{\partial^2 f}{\partial x^2} = 6x,\quad \frac{\partial^2 f}{\partial y^2} = 2,\quad \frac{\partial^2 f}{\partial z^2} = 2,\quad \frac{\partial^2 f}{\partial x\partial y} = 0 = \frac{\partial^2 f}{\partial x\partial z} = \frac{\partial^2 f}{\partial y\partial z}$

$$d^2 f = \frac{\partial^2 f}{\partial x^2}dx^2 + \frac{\partial^2 f}{\partial y^2}\cdot dy^2 + \frac{\partial^2 f}{\partial z^2}\cdot dz^2 + 2\frac{\partial^2 f}{\partial x\partial y}dxdy + 2\frac{\partial^2 f}{\partial x\partial z}dxdz + 2\frac{\partial^2 f}{\partial y\partial z}dydz$$

$d^2f(A) = d^2f(0,0,0) = 2dy^2 + 2dz^2 \geq 0$

$d^2f(A) \overset{?}{=} 0 \Leftrightarrow dy = dz = 0$ немамо услов $dx = 0$

dx ?

$d^2f(A) = 0 \Leftrightarrow (dx, dy, dz) = (\underset{\neq 0}{dx}, 0, 0)$ није extrem

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$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \\ a_{21} & a_{22} & a_{23} & \\ a_{31} & a_{32} & a_{33} & \\ & & & \ddots \end{bmatrix}$$

$$f(x,y,z) = x^3 + y^2 + z^2$$

$$H = \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial x \partial z} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial y \partial z} \\ \frac{\partial^2 f}{\partial z \partial x} & \frac{\partial^2 f}{\partial z \partial y} & \frac{\partial^2 f}{\partial z^2} \end{pmatrix} = \begin{pmatrix} 6x & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$H(A) = H(0,0,0) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

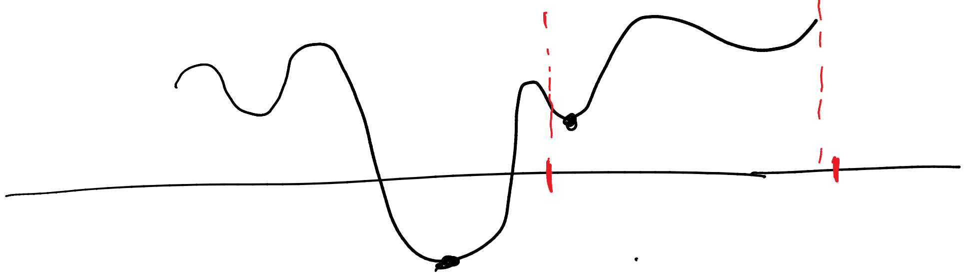
$M_1 = 0$, $M_2 = 0$, $M_3 = 0$

$M_i \not> 0$

M_i nisu razl. znaka

$\Rightarrow$ nije ni poz. ni neg. def.

$\Rightarrow A$ nije extrem.



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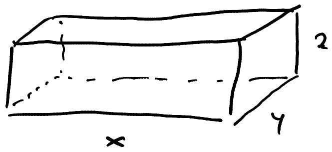

kutija bez poklopca

što manje materijala

$V = 500\, m^3$

$x, y, z > 0$

$$V = x \cdot y \cdot z = 500$$

$$P = x \cdot y + 2yz + 2xz \longrightarrow \min \quad (3 \text{ prom.})$$

$$z = \frac{500}{x \cdot y}$$

$$P = x \cdot y + 2 \cdot y \cdot \frac{500}{x \cdot y} + 2 \cdot x \cdot \frac{500}{x \cdot y} = x \cdot y + \frac{1000}{x} + \frac{1000}{y}$$

(2 prom.)

$$\left.\begin{aligned} \frac{\partial P}{\partial x} &= y - \frac{1000}{x^2} = 0, \\ \frac{\partial P}{\partial y} &= x - \frac{1000}{y^2} = 0 \end{aligned}\right\} \Rightarrow A(10, 10) \text{ stac. točka}$$

$\cancel{B(-10,-10)}$ $x, y, z > 0$

$$H = \begin{bmatrix} \frac{2000}{x^3} & 1 \\ 1 & \frac{2000}{y^3} \end{bmatrix}, \quad H(10,10) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\left.\begin{aligned} M_1 &= 2 > 0, \\ M_2 &= \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4 - 1 = 3 > 0 \end{aligned}\right\} \Rightarrow H \text{ poz. def.}$$

$\Downarrow$

u A je lok. min.

$$\left.\begin{aligned} x &= 10, y = 10 \\ z &= \frac{500}{10 \cdot 10} = 5 \end{aligned}\right\} \quad x = 10, y = 10, z = 5$$