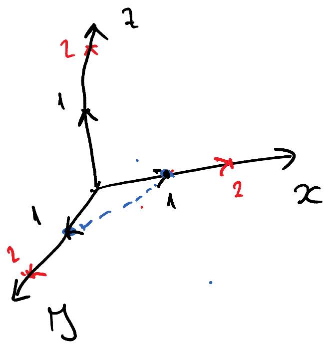
Wednesday, November 17, 2021
11:39 AM



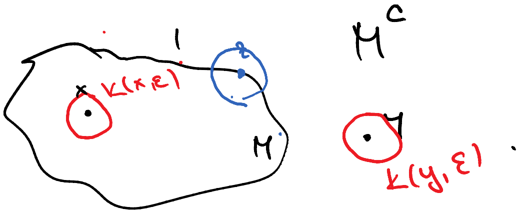
$e_1 = (1,0,0)$

$e_2 = (0,1,0)$

$e_3 = (0,0,1)$

$\|e_1\| = \sqrt{1^2+0^2+0^2} = 1$

$d(e_1, e_2)$



$K(x,\varepsilon) \subseteq M$

$K(y,\varepsilon) \subseteq M^c$

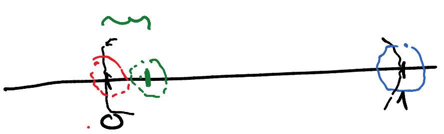
$\not\subseteq M$

ext M

$K(z,\varepsilon) \not\subseteq M$
$K(z,\varepsilon) \not\subseteq M^c$ } ръбна

∞ много бр.

$M = [0,1)$



$0 \in \operatorname{int} M$?

$\notin \operatorname{int} M$
$\notin \operatorname{ext} M$ } $\Rightarrow 0$ ръбна

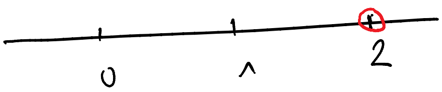
$1 \notin \operatorname{int} M$
$1 \notin \operatorname{ext} M$ } $\Rightarrow 1$ ръбна

$\operatorname{int} M = (0,1)$

$\{0,1\}$ ръбни

$\operatorname{ext} M = (-\infty,0) \cup (1,\infty)$

$M = [0,1) \cup \{2\}$



$2 \in M$, $K(2,\varepsilon) \setminus \{2\} \not\subseteq M$

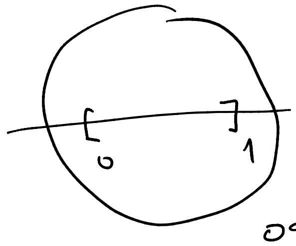
$\Rightarrow 2$ ръбна

ръбни $\{0,1,2\}$

Wednesday, November 17, 2021
11:39 AM

$(0,1)$ otvoren

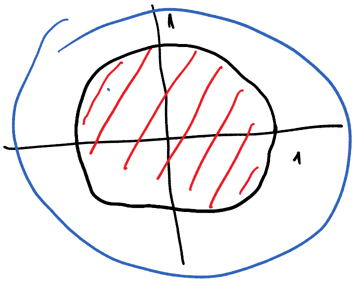
$[0,1]$ zatvoren



ograničen

kompaktan

$M = \{(x,y) \mid x^2+y^2 \le 1\}$

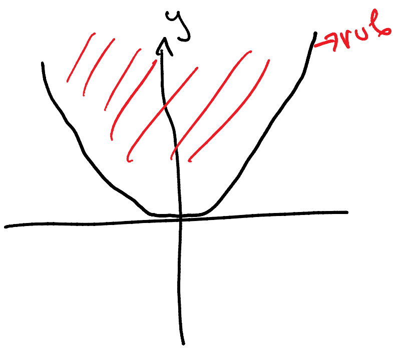


rubne : kružnica

zatvoren

ograničen $\Big\} \Rightarrow$ kompaktan

$M = \{(x,y) \mid y > x^2\}$



otvoren

nije ograničen!

Wednesday, November 17, 2021
11:39 AM

$$f(x,y) = \frac{x^4-y^4}{3x^2+3y^2}$$

$$\lim_{x\to x_0} f(x,y) \overset{?}{=} a \quad , x_0 = (0,0)$$

$D_f = \mathbb{R}^2 \setminus \{(0,0)\}$

$$a = \lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{3x^2+3y^2} = \lim_{(x,y)\to(0,0)} \frac{(x^2-y^2)\cancel{(x^2+y^2)}}{3\cancel{(x^2+y^2)}} = \lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{3} = \frac{0}{3} = 0$$

Def 1:

$|a \pm b| \le |a| + |b|$

$$|f(x,y) - a| = \left| \frac{x^4-y^4}{3x^2+3y^2} - 0 \right| = \frac{1}{3}|x^2-y^2| \le \frac{1}{3}(x^2+y^2)$$

$?(\forall \varepsilon > 0)(\exists \delta_\varepsilon > 0)$ t.d. $d(x,x_0) < \delta_\varepsilon \Rightarrow |f(x,y) - a| < \varepsilon$

$$d(x,x_0) = \sqrt{(x-x_0)^2+(y-y_0)^2} = \sqrt{(x-0)^2+(y-0)^2} = \sqrt{x^2+y^2} < \delta$$

$$\Rightarrow x^2+y^2 < \delta^2$$

$$|f(x,y)-a| \le \frac{x^2+y^2}{3} < \underbrace{\frac{\delta^2}{3} = \varepsilon}$$

$\forall \varepsilon$, $?\exists \delta_\varepsilon$: $\delta_\varepsilon = \sqrt{3\varepsilon}$

$\Rightarrow a$ jeste granična vr.

$$f(x,y) = \frac{xy}{x^2+y^2} \quad , \quad \lim_{x\to x_0} f(x,y) \overset{?}{=} a \quad , x_0 = (0,0)$$

Def 2 : $\forall \{x_n\} \subset D_f \setminus \{x_0\}$, $(x_n) \xrightarrow{n\to\infty} x_0 \Rightarrow (f(x_n)) \xrightarrow{n\to\infty} a$

$(x_n') = \left(\frac{1}{n}, \frac{1}{n}\right)$, $(x_n'') = \left(\frac{1}{n}, \frac{-1}{n}\right)$ — oba $\to x_0$

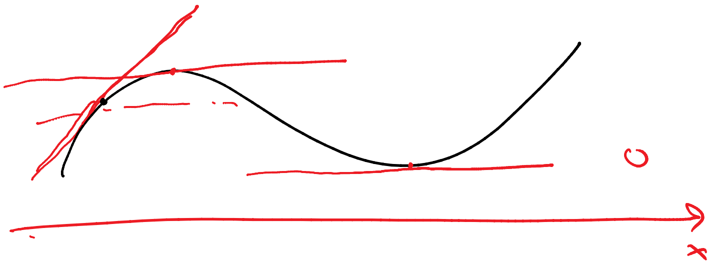
$\quad\quad (0 \,,\, 0) = x_0 \quad\quad\quad\quad (0 \,,\, 0) = x_0$

$$\lim_{n\to\infty} (f(x_n')) = \lim_{n\to\infty} \frac{\frac{1}{n}\cdot\frac{1}{n}}{\left(\frac{1}{n}\right)^2+\left(\frac{1}{n}\right)^2} = \lim_{n\to\infty} \frac{\frac{1}{n^2}}{\frac{2}{n^2}} = \frac{1}{2}$$

$$\lim_{n\to\infty} (f(x_n'')) = \lim_{n\to\infty} \frac{\frac{1}{n}\cdot\left(-\frac{1}{n}\right)}{\left(\frac{1}{n}\right)^2+\left(-\frac{1}{n}\right)^2} = \lim_{n\to\infty} \frac{-\frac{1}{n^2}}{\frac{2}{n^2}} = -\frac{1}{2}$$

$\neq$ $\quad \nexists \lim_{(x,y)\to(0,0)} f(x,y)$

Wednesday, November 17, 2021
11:39 AM



$f'(x) = 0$