Friday, November 05, 2021
9:43 AM

nizovi

$f_1(x), f_2(x), \ldots$

$(f_n(x))_{n\in\mathbb{N}} \to f(x)$ $\qquad N(\varepsilon, x)$

$(f_n(x))_{n\in\mathbb{N}} \rightrightarrows f(x)$ $\qquad N(\varepsilon)$

redovi

$$\sum_{n=1}^{\infty} f_n(x) = f_1(x) + f_2(x) + \ldots$$

$$(S_n(x))_{n\in\mathbb{N}} \longrightarrow S(x) = \sum_{k=1}^{\infty} f_k(x) \quad K$$

$$(S_n(x))_{n\in\mathbb{N}} \rightrightarrows S(x) = \sum_{k=1}^{\infty} f_k(x) \quad UK \quad \left(\begin{array}{c}\text{dodatne}\\ \text{osobine}\end{array}\right)$$

$$\frac{\partial^2 f}{\partial t^2} = k^2 \frac{\partial^2 f}{\partial x^2}, \quad f(x,t) = A \sin at \sin bt$$

$f(x,t) \to$ zbir.

$$f(x,t) = \sum_{1}^{\infty} \sin \ldots$$

$$f(x) \rightsquigarrow \frac{a_0}{2} + \sum_{1}^{\infty} a_k \cos kx + b_k \sin x$$

$$f = f_1 + f_2 + \ldots$$

nep. $\qquad$ nepr.

$$f' = f_1' + f_2' + \ldots$$

$$\sum f_n \rightsquigarrow f(x)$$

$$f(x) \;☹\; \rightsquigarrow \sum f_n$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

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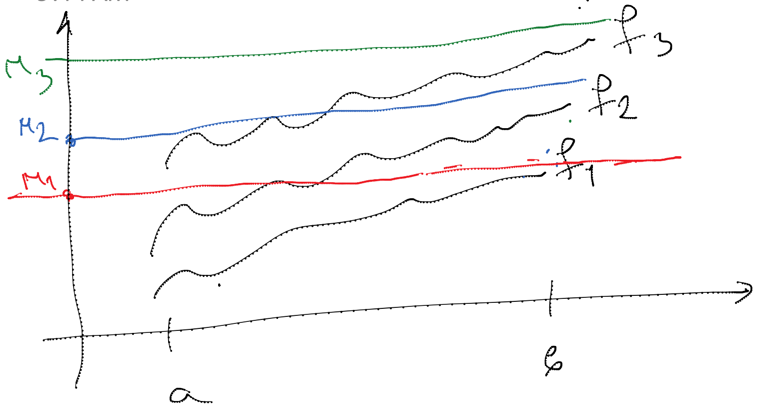
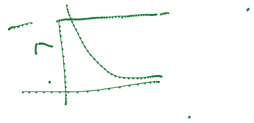




$|f_1(x)| < M_1$

$|f_2(x)| < M_2$

$|f_3(x)| < M_3$

$\downarrow$

$(M_n)_{n\in\mathbb{N}}$ brojni niz

$M_1 + M_2 + M_3 + \dots$

$\sum_{k=1}^{\infty} M_k$ (brojni red)

$$\sum M_k \ (K) \Rightarrow \sum f_k \ (UK)$$

$f_k(x) = \frac{x^k}{2^k}$, $\sum_{k=0}^{\infty} f_k(x)$ UK? $x \in [-1,1]$

$$S_n = \sum_{k=1}^{n} \frac{x^k}{2^k} \overset{?}{\rightrightarrows} S(x)$$

$$|f_k(x)| = \left|\frac{x^k}{2^k}\right| \overset{?}{<} M_k$$

$$\left|\frac{x^k}{2^k}\right| < \frac{1}{2^k}$$

$$M_k = \frac{1}{2^k}$$

$\sum_{k=0}^{\infty} M_k = \sum_{k=0}^{\infty} \frac{1}{2^k}$ geom. red (K)

$\Downarrow$ Vajerstras

$\sum_{k=0}^{\infty} f_k(x)$ (UK)

Stepeni red

Def: Funkcionalni red oblika $\sum_{\substack{k=1 \\ k=0}}^{\infty} a_k x^k$, $a_k \in \mathbb{R}$, $\forall k \in \mathbb{N}$ zove se stepeni red. (centriran u 0)

$x = 0$: $\underbrace{a_0 \cdot 0^0}_{\text{nedef.}}$

$0^0 \overset{\text{def}}{=} 1$.

$(x-5)^2 + (x-5)^1 + \dots$

$$\sum_{k=0}^{\infty} a_k (x - x_0)^k, \quad x_0 \in \mathbb{R} \qquad \text{stepeni}$$

(centriran u x_0)

$$t = x - x_0 \implies \sum_{k=0}^{\infty} a_k \cdot t^k$$

$$\sum_{k=0}^{\infty} a_k x^k = \cancel{a_0 x^0} + a_1 x^1 + a_2 x^2 + a_3 \cdot x^3 + \dots$$

$a_0 \cdot \underset{1}{0^0}$

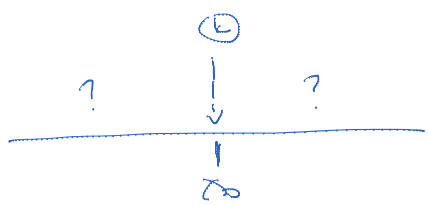
$$x = 0: \quad \cancel{a_0} \quad \underbrace{a_1 \cdot 0^1}_{0} + \underbrace{a_2 \cdot 0^2}_{0} + \underbrace{a_3 \cdot 0^3}_{0} + \dots = 0 \quad = a_0$$

$$\boxed{\sum_{k=1}^{\infty} a_k x^k \underset{x=0}{=} 0 \qquad \text{(stepeni red uvek konvergira za } x = 0)}$$

$$\sum_{k=1}^{\infty} a_k (x - x_0)^k = a_1 (x - x_0)^1 + a_2 (x - x_0)^2 + a_3 (x - x_0)^3 + \dots$$

$$x = x_0: \quad a_1 \underbrace{(x_0 - x_0)^1}_{0} + a_2 \underbrace{(x_0 - x_0)^2}_{0} + a_3 \underbrace{(x_0 - x_0)^3}_{0} + \dots = 0$$

$$\boxed{\sum_{k=1}^{\infty} a_k (x - x_0)^k = 0 \qquad \text{(stepeni red konv. za } x = x_0)}$$



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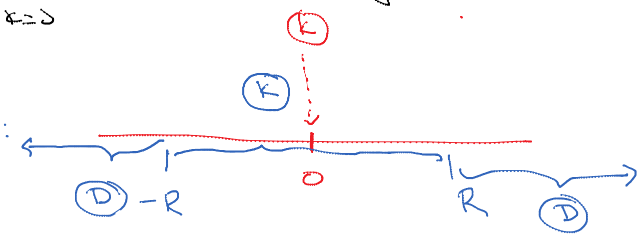
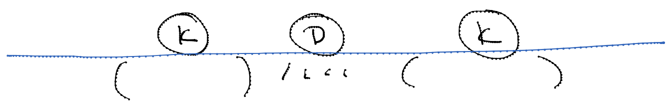
$a_k(x-x_0)^k$

Teorema: Za svaki red $\sum_{k=0}^{\infty} a_k x^k$ postoji broj R $(0<R<\infty)$ takav da važi:

1) Red $\sum_{k=0}^{\infty} a_k x^k$ apsolutno konvergira za $|x|<R$ $\Rightarrow$ $-R<x<R$

2) Red $\sum_{k=0}^{\infty} a_k x^k$ divergira za $|x|>R$





R - poluprečnik konvergencije
$(-R, R)$ - interval konvergencije
} stepenog reda

$R = ?$

$\boxed{x=R\ ?}$ ↳ $\sum \underbrace{a_k x^k}_{\text{broj}}$ $\rightarrow$ K ? D ?

$\sum a_k R^k$

$x=-R$

$\sum a_k(-R)^k$ $\rightarrow$ K ? D ?

$(-R, R)$
$[-R, R]$
$(-R, R]$
$[-R, R)$

$\boxed{R = ?}$

Dalamber :

$$\left| \frac{f_{n+1}(x)}{f_n(x)} \right| = \left| \frac{a_{n+1} \cdot x^{n+1}}{a_n \cdot x^n} \right| = \left| \frac{a_{n+1}}{a_n} \cdot x \right|$$

$$= \left| \frac{a_{n+1}}{a_n} \right| \cdot |x|$$

$\sum a_k x^k$ (K) ako $\lim\limits_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \cdot |x| < 1$

$$|x| \cdot \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$$

$$|x| < \frac{1}{\lim\limits_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|} = \frac{1}{\frac{\lim |a_{n+1}|}{\lim |a_n|}}$$

$$= \frac{\lim |a_n|}{\lim |a_{n+1}|} = \underbrace{\lim_{n\to\infty} \left| \frac{a_n}{a_{n+1}} \right|}_{R}$$

$$|x| < \lim_{n\to\infty} \left| \frac{a_n}{a_{n+1}} \right| = R$$

Kosi :

$$\sqrt[n]{|f_n(x)|} = \sqrt[n]{|a_n \cdot x^n|} = \sqrt[n]{|a_n| \cdot |x|^n}$$

$$= |x| \cdot \sqrt[n]{|a_n|}$$

$\sum a_k x^k$ (K) ako $\lim\limits_{n\to\infty} |x| \cdot \sqrt[n]{|a_n|} < 1$

$$|x| \cdot \lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$$

$$|x| < \frac{1}{\lim\limits_{n\to\infty} \sqrt[n]{|a_n|}} = R$$