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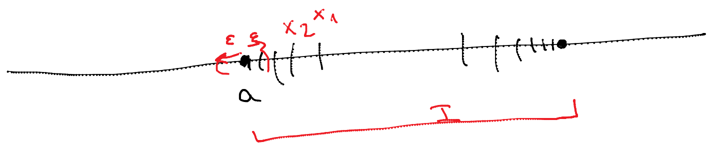
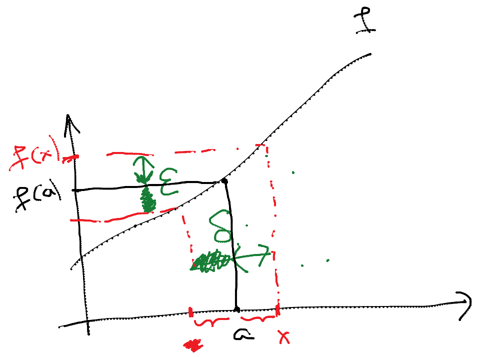
$f_{10}(x_1), f_{10}(x_2), \dots$

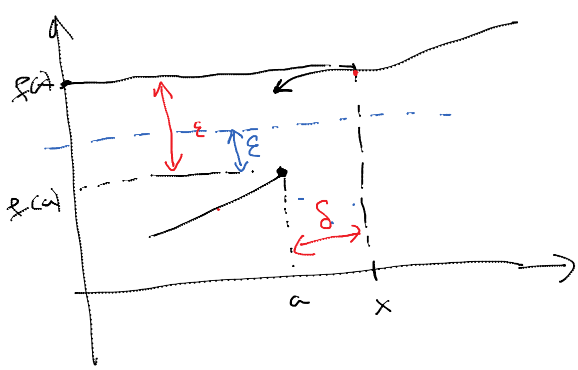
$x_i \to a$



$(a-\varepsilon, a+\varepsilon) \cap I \setminus \{a\}$

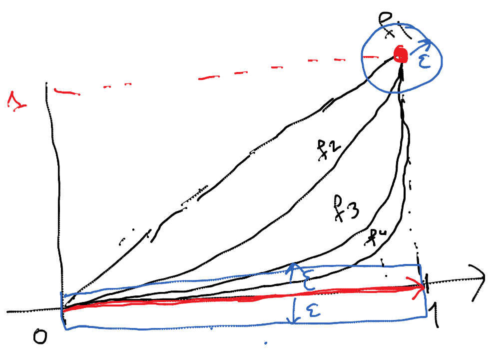


$(\forall \varepsilon > 0)\ (\exists \delta > 0)$



$\forall \varepsilon > 0$

$f_n(x) = x^n$, $f_n : [0,1] \to \mathbb{R}$



$f_1 = x^1,\ f_2 = x^2,\ f_3 = x^3,\ f_4 = x^4, \dots$

neprekidne

$x = 1 : \ f_n(x) = 1^n = 1$

$x < 1 :$

$0.9^1 = 0.9$

$0.9^2 = 0.81$

$0.9^3 = 0.73$

$0.9^4 = 0.6 \dots$

$\to 0$

$$\lim_{n \to \infty} f_n(x) = f(x) = \begin{cases} 0, & x < 1 \\ 1, & x = 1 \end{cases}$$

$f_n(x)$ nije unif. k.

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~~$f_k(x) = 1 + \sum_{k=2}^{\infty} x^{k-2}(x-1)$~~ $f_1(x) = 1$

$\sum f_k(x) = ?$

$f_1(x) = 1, \quad f_k(x) = x^{k-2}(x-1) \quad , k \geq 2$

$\sum_{k=1}^{\infty} f_k(x) = ?$

$f_2(x) = x^0 \cdot (x-1)$
$f_3(x) = x^1 (x-1)$
$f_4(x) = x^2 (x-1)$

$S_n(x) = f_1(x) + \dots + f_n(x)$

$= \cancel{1} + (\cancel{x} - \cancel{1}) + (\cancel{x^2} - \cancel{x}) + (x^3 - \cancel{x^2}) + \dots + (x^{n-1} - \cancel{x^{n-2}})$

$= x^{n-1}$

$(S_n(x))_{n \in \mathbb{N}}, \quad S_n^{(x)} = x^{n-1}$ konv. ? (funkc. niz)

$\Downarrow$ ako

$x_0 \in \mathbb{R}$ (tačka po tačka)

$(S_n(x_0))_{n \in \mathbb{N}}$ konv. (brojni niz)

brojni: $S_n(x_0) = x_0^{n-1}$

1) $x_0 = 1$: $S_n(x_0) = 1^{n-1} = 1$

2) $0 \leq x_0 < 1$: $S_n(x_0) = (\underset{<1}{\cdot})^{n-1} \xrightarrow[n \to \infty]{} 0$

3) $x_0 > 1$: $S_n(x_0) = (\underset{>1}{\cdot})^{n-1} \xrightarrow[n \to \infty]{} +\infty$

4) $x_0 = -1$: $S_n(x_0) = (-1)^{n-1} \nearrow 1$, n neparno
$\searrow -1$, n parno

2 tačke nag.

$\Downarrow$

divergira

5) $-1 < x_0 \leq 0$: $S_n(x_0) = (\underset{|\cdot| < 1}{\cdot})^{n-1} \xrightarrow[n \to \infty]{} 0$

6) $x_0 < -1$: $S_n(x_0) = (\underset{|\cdot| > 1}{\quad})^{n-1} \longrightarrow \pm\infty$

$\Rightarrow (S_n(x_0))$ konv. $\forall x_0 \in (-1, 1]$
brojni niz

$\Rightarrow (S_n(x))$ (K) na $I = (-1, 1]$

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$S_n(x)$ konv. $\Rightarrow \exists \lim_{n\to\infty} S_n = \underbrace{S(x)}_{?}$, $x \in (-1,1]$

$$S(x) = \begin{cases} 0, & x \in (-1,1) \\ 1, & x = 1 \end{cases}$$

$$|S_n(x) - S(x)| = \begin{cases} |x^{n-1} - 0| = |x^{n-1}| = |x|^{n-1}, & x \in (-1,1) \\ |x^{n-1} - 1| = |1^{n-1} - 1| = 0, & x = 1 \end{cases}$$

$x = 1$: $|S_n(x) - S(1)| = 0 < \varepsilon$ uvek

$x \in (-1,1)$: $|S_n(x) - S(x)| = |x|^{n-1} < \varepsilon \quad / \ln$

$$\ln|x|^{n-1} > \ln\varepsilon$$

$$(n-1)\cdot \ln|x| > \ln\varepsilon$$

$$n - 1 > \frac{\ln\varepsilon}{\ln|x|}$$

$$n > \underbrace{\boxed{\frac{\ln\varepsilon}{\ln|x|} + 1}}_{\text{zavisi i od } \varepsilon \text{ i od } x} = N(\varepsilon, x)$$

$\exists N(\varepsilon, x)$, $\forall n > N$ .... (def.)

zavisi i od
ε i od x

$\Rightarrow$ NE konv. unif.

$(S_n(x))$ konv. (obično) na $(-1,1]$
$\exists \lim_{n\to\infty} S_n(x) = S(x) = \sum_{k=1}^{\infty} f_k(x)$
(red konv.)

$(S_n(x))$ NE konv. unif. $\Rightarrow \sum_{k=1}^{\infty} f_k(x)$ ne konv.
unif.