Friday, October 15, 2021
9:10 AM

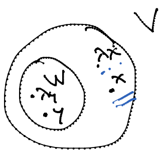
$V_1=[1,1,1]$, $V_2=[1,2,3]$, $V_3=[0,0,0]$

$$\alpha_1 V_1+\alpha_2 V_2+\alpha_3 V_3=0$$

$$\alpha_1[1,1,1]+\alpha_2[1,2,3]+\alpha_3[0,0,0]=[0,0,0]$$

$$\left.\begin{aligned}\alpha_1+\alpha_2&=0\\ \alpha_1+2\alpha_2&=0\\ \alpha_1+3\alpha_2&=0\end{aligned}\right\}\quad \alpha_1=0=\alpha_2 \qquad (\alpha_1,\alpha_2,\alpha_3) = (0,0,\alpha_3),\ \alpha_3\in\mathbb{R}$$

$(0,0,0)$, $(0,0,79)$, $(0,0,-123)$, ...



(*) $A=\begin{bmatrix}1 & 2\\ 1 & 1\end{bmatrix}$, $B=\begin{bmatrix}-2 & 4\\ 2 & -2\end{bmatrix}$

$A\cdot B=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$, $B\cdot A=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$ $\Rightarrow AB=BA$

(*) $A=\begin{bmatrix}1 & 3\\ 2 & -1\end{bmatrix}$, $B=\begin{bmatrix}2 & -1\\ 0 & 2\end{bmatrix}$

$A\cdot B=\begin{bmatrix}2 & 5\\ 4 & -4\end{bmatrix}$, $B\cdot A=\begin{bmatrix}0 & 7\\ 4 & -2\end{bmatrix}$ $\Rightarrow AB\neq BA$

(*) $A=\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}$, $B=\begin{bmatrix}2 & 4\\ 2 & 3\end{bmatrix}$, $C=\begin{bmatrix}1 & -2\\ -1 & 2\end{bmatrix}$

$A\cdot C=\begin{bmatrix}-2 & 4\\ -1 & 2\end{bmatrix}=BC$ $\Rightarrow \underline{AC}=\underline{BC}$ ali $A\neq B$

(*) $A=\begin{bmatrix}a_{11} & \dots & a_{1n}\\ \vdots & & \\ a_{m1} & \dots & a_{mn}\end{bmatrix}$, $A+\begin{bmatrix}0 & \dots & 0\\ \vdots & & \\ 0 & \dots & 0\end{bmatrix}_{m\times n}=A$

(*) $A_{2\times 4}$

$A_{2\times4}\cdot I_{4\times4}=A_{2\times4}$ $\quad I_{4\times4}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$

$I_{2\times2}\cdot A_{2\times4}=A_{2\times4}$ $\quad I_{2\times2}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$

Friday, October 15, 2021
9:10 AM

(*) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

(*) $\det(A) = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix}$

$= a_{11} \cdot a_{22} \cdot a_{33} + a_{12} \cdot a_{23} \cdot a_{31} + a_{13} \cdot a_{21} \cdot a_{32}$

$- a_{13} \cdot a_{22} \cdot a_{31} - a_{11} \cdot a_{23} \cdot a_{32} - a_{12} \cdot a_{21} \cdot a_{33}$

(*) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 8 & 7 & 6 \\ 5 & 4 & 3 & 2 \end{bmatrix}$ $\begin{vmatrix} 2 & 3 \\ 8 & 7 \end{vmatrix}$ - minor reda 2

(*) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 8 & 7 & 6 \\ 5 & 4 & 3 & 2 \end{bmatrix}$ $M_{23} = \begin{vmatrix} 1 & 2 & 4 \\ 9 & 8 & 6 \\ 5 & 4 & 2 \end{vmatrix}$

a_{23}

$C_{23} = (-1)^{2+3} \cdot M_{23} = -M_{23}$

$\det(A) = a_{11} \cdot C_{11} + a_{12} \cdot C_{12} + a_{13} \cdot C_{13} + a_{14} \cdot C_{14}$

$= 1 \cdot C_{11} + 2 \cdot C_{12} + 3 \cdot C_{13} + 4\, C_{14}$

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1) $\begin{vmatrix} a_{11} & \dots & a'_{1j} & \dots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ a_{n1} & & a'_{nj} & \dots & a_{nn} \end{vmatrix} + \begin{vmatrix} a_{11} & \dots & a''_{1j} & \dots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ a_{n1} & \dots & a''_{nj} & \dots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & \dots & a'_{1j}+a''_{1j} & \dots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ a_{n1} & \dots & a'_{nj}+a''_{nj} & \dots & a_{nn} \end{vmatrix}$

Friday, October 15, 2021
9:10 AM

2) $D_1 = \begin{vmatrix} a_{11} & \dots & \lambda a_{1j} & \dots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ a_{n1} & \dots & \lambda a_{nj} & \dots & a_{nn} \end{vmatrix} = \lambda \cdot \begin{vmatrix} a_{11} & \dots & a_{1j} & \dots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ a_{n1} & \dots & a_{nj} & \dots & a_{nn} \end{vmatrix}$

$$D_1 = \sum_{k=1}^{n} \lambda \cdot a_{jk} C_{jk} = \lambda \cdot \underbrace{\sum a_{jk} \cdot C_{jk}} = \lambda \cdot \det(A)$$

$$\begin{vmatrix} \lambda a_{11} & \dots & \lambda a_{1n} \\ \vdots & & \vdots \\ \lambda a_{n1} & \dots & \lambda a_{nn} \end{vmatrix} = \lambda \cdot \begin{vmatrix} a_{11} & \lambda a_{12} & \dots & \lambda a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & \lambda a_{n2} & \dots & \lambda a_{nn} \end{vmatrix}$$

$$= \lambda \cdot \lambda \cdot \begin{vmatrix} a_{11} & a_{12} & \lambda \cdot a_{13} & \dots & \lambda a_{1n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \lambda a_{n3} & \dots & \lambda a_{nn} \end{vmatrix}$$

$$= \lambda^n \cdot \begin{vmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \\ a_{n1} & \dots & a_{nn} \end{vmatrix}$$

$$A = \begin{bmatrix} 2 & 8 & 14 \\ 4 & 10 & 16 \\ 6 & 12 & 18 \end{bmatrix} \qquad \det(A) = 2 \cdot \begin{vmatrix} 1 & 8 & 14 \\ 2 & 10 & 16 \\ 3 & 12 & 18 \end{vmatrix}$$

$$= 2 \cdot 2 \cdot \begin{vmatrix} 1 & 4 & 14 \\ 2 & 5 & 16 \\ 3 & 6 & 18 \end{vmatrix}$$

$$= 2 \cdot \begin{vmatrix} 1 & 4 & 2 \cdot 14 \\ 2 & 5 & 2 \cdot 16 \\ 3 & 6 & 2 \cdot 18 \end{vmatrix}$$

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i-ta vrsta: $| a_{i1} \dots a_{ik} \dots a_{il} \dots a_{in} |$

$= | a_{i1} \dots a_{ik} + a_{il} \dots a_{il} \dots a_{in} |$ $\qquad \cancel{a_{il}} - a_{ik} - \cancel{a_{il}}$

$= | a_{i1} \dots a_{ik} + a_{il} \dots -a_{ik} \dots a_{in} |$ $\qquad \cancel{a_{ik}} + a_{il} - \cancel{a_{ik}}$

$= | a_{i1} \dots a_{il} \dots \underbrace{-a_{ik}}_{\lambda = -1} \dots a_{in} |$

$= \underbrace{-1} \cdot | a_{i1} \dots a_{il} \dots a_{ik} \dots a_{in} |$

Friday, October 15, 2021
9:10 AM

$$(*)\quad \begin{vmatrix} 1 & -1 & 0 & 2 \\ 2 & 0 & 1 & 1 \\ -1 & 2 & 1 & 0 \\ 3 & 1 & 2 & 2 \end{vmatrix} \overset{\text{det}}{=} 1\cdot C_{11} + (-1)\cdot C_{12} + 0\cdot C_{13} + 2\cdot C_{14}$$

$$= \begin{vmatrix} 1 & 0 & 0 & 2 \\ 2 & 2 & 1 & 1 \\ -1 & 1 & 1 & 0 \\ 3 & 4 & 2 & 2 \end{vmatrix} \xrightarrow{\cdot(-2)} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 1 & -3 \\ -1 & 1 & 1 & 2 \\ 3 & 4 & 2 & -4 \end{vmatrix}$$

$$\overset{\text{det}}{=} 1\cdot C_{11} + 0\cdot C_{12} + 0\cdot C_{13} - 0\cdot C_{14}$$

$$= \underbrace{1\cdot(-1)^{1+1}}_{1}\cdot M_{11} = \begin{vmatrix} 2 & 1 & -3 \\ 1 & 1 & 2 \\ 4 & 2 & -4 \end{vmatrix}$$

→ Sarusovo pravilo

→ po det.1 ili Laplasov razvoj $(2\cdot C_{11} + 1\cdot C_{12} + (-3)\cdot C_{13})$

$$= \begin{vmatrix} -1 & 1 & -3 \\ 3 & 1 & 2 \\ 0 & 2 & -4 \end{vmatrix} \xrightarrow{\cdot 2} = \begin{vmatrix} -1 & 1 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 0 \end{vmatrix}$$

Laplasov razvoj

$$= 0\cdot C_{31} + 2\cdot C_{32} + 0\cdot C_{33}$$

$$= 2\cdot(-1)^{3+2}\cdot M_{32}$$

$$= -2\cdot M_{32}$$

$$= -2\cdot \begin{vmatrix} -1 & -1 \\ 3 & 4 \end{vmatrix}$$

$$= -2\cdot(-1\cdot 4 - (-1)\cdot 3)$$

$$= 2$$