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MATEMATIKA 2

U DONJEM LEVOM UGLU IMATE DUGME "CHAT". KLIKOM NA NJEGA OTVARA SE PROZOR U KOM MOŽETE I DA PIŠETE

POČINJEMO U 12:15h

Kolokvijum : 25 zadaci + 10 test

25 + 10

25 5

30 - 43 → 6
44 - 57 → 7
58 - 70 → 8

usmeni 30

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$(x_1, \dots, x_n) + \boxed{(0, \dots, 0)}$
$\shortparallel$
$(x_1, \dots, x_n)$

(*) $\mathbb{R}^n = \{(x_1, \dots, x_n) \mid \underline{x_i \in \mathbb{R}}\}$ ✓

0) $\exists\, \mathbf{0} = (0, \dots, 0) \in \mathbb{R}^n$ neprazan

1) $x = (x_1, \dots, x_n)$, $y = (y_1, \dots, y_n)$, $x, y \in \mathbb{R}^n \overset{?}{\Rightarrow} x + y \in \mathbb{R}^n$

$x + y = (x_1, \dots, x_n) + (y_1, \dots, y_n) = (\underbrace{\overset{\in\mathbb{R}}{x_1} + \overset{\in\mathbb{R}}{y_1}}_{\in\mathbb{R}}, \dots, \underbrace{x_n + y_n}_{\in\mathbb{R}})$ ✓

2) $x \in \mathbb{R}^n \overset{?}{\Rightarrow} \alpha \cdot x \in \mathbb{R}^n$, $\alpha \in \mathbb{R}$

$\alpha \cdot x = \alpha \cdot (x_1, \dots, x_n) \doteq (\underbrace{\overset{\in\mathbb{R}}{\alpha} \cdot \overset{\in\mathbb{R}}{x_1}}_{\in\mathbb{R}}, \dots, \underbrace{\alpha \cdot x_n}_{\in\mathbb{R}})$ ✓

(a) – (h) ......

(*) $\mathbb{Z}$, $\alpha \in \mathbb{R}, \mathbb{C}$ $\qquad a \in \mathbb{Z} \quad a + \underline{0} = a$

0) $0 \in \mathbb{Z}$ ✓ neprazan

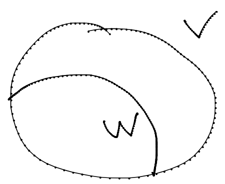
1) $x, y \in \mathbb{Z} \overset{?}{\Rightarrow} x + y \in \mathbb{Z}$ ✓

2) $\alpha \cdot x \overset{?}{\in} \mathbb{Z}$

$\alpha \in \mathbb{R}$: $\alpha = \frac{1}{2}$: $\frac{1}{2} \cdot x \notin \mathbb{Z}$ $\quad$ ↯ $\quad$ nije VP

$\frac{1}{2} \cdot 1 = \frac{1}{2}$

$\alpha = i$: ...



$V = \mathbb{R}^3 = \{(x_1, \underline{x_2}, x_3) \mid x_i \in \mathbb{R}\}$

$W = \{(x_1, \underset{\equiv}{0}, x_3) \mid x_{1,3} \in \mathbb{R}\}$, $W \subseteq V$

0) $(0, 0, 0) \overset{?}{\in} W$ ✓

1) $x = (x_1, 0, x_3)$, $y = (y_1, 0, y_3)$, $x, y \in W \overset{?}{\Rightarrow} x + y \in W$

$x + y = (x_1, 0, x_3) + (y_1, 0, y_3) = (\underbrace{x_1 + y_1}_{\in\mathbb{R}}, \underbrace{0 + 0}_{=0}, \underbrace{x_3 + y_3}_{\in\mathbb{R}}) \in W$

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2) $x=(x_1,0,x_3)\in W$, $\alpha\in\mathbb{R}$, $\alpha\cdot x\overset{?}{\in}W$

$$\alpha\cdot x=\alpha\overset{\text{iz } V}{\cdot}(x_1,0,x_3)=(\underbrace{\alpha\cdot x_1}_{\in\mathbb{R}},\underbrace{\alpha\cdot 0}_{=0},\underbrace{\alpha\cdot x_3}_{\in\mathbb{R}})\quad ✓$$

$W\subseteq V$, i važi 0, 1, 2 $\Rightarrow$ W V. p-prostor od V

(*) $V=\mathbb{R}^3=\{(x_1,x_2,x_3)\mid x_i\in\mathbb{R}\}$

$W=\{(x_1,1,x_3)\mid x_1,x_3\in\mathbb{R}\}$

$W\subseteq V$ ✓   0) da li je neprazan? $\boxed{(0,0,0)\notin W}$

$(17,1,23)\in W$ ✓ neprazan

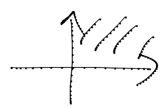
1) $x=(x_1,1,x_3)$, $y\in(y_1,1,y_3)$

$$x+y=(\underbrace{x_1+y_1}_{\in\mathbb{R}},\underbrace{1+1}_{=2\neq 1},\underbrace{x_3+y_3}_{\in\mathbb{R}})$$

↯ $\Rightarrow$ $x+y\notin W$

$\Rightarrow$ W nije V. P prostor od V

(*) $V=\mathbb{R}^2=\{(x_1,x_2)\mid x_i\in\mathbb{R}\}$ nad $\mathbb{R}$ $(\alpha_i\in\mathbb{R})$



$W=\{(x_1,x_2)\mid \cancel{x_1\geq 0},\ \underline{x_2\geq 0}\}$

$W\subseteq V$ ✓   0) neprazan W? $\exists(0,0)\in W$

1) $x+y=(\underbrace{x_1+y_1}_{\geq 0},\underbrace{x_2+y_2}_{\geq 0})$ $\Rightarrow$ $x+y\in W$

2) $\alpha\cdot x\overset{?}{\in}W$, $\alpha\in\mathbb{R}$

$\alpha\geq 0$ ✓

$\alpha<0$, $(\alpha=-1)$ $\alpha\cdot x=-1(x_1,x_2)=(\underbrace{-\underset{\geq 0}{x_1}}_{\leq 0},\underbrace{-x_2}_{\leq 0})$ ↯

$\Rightarrow$ W nije VPp od V

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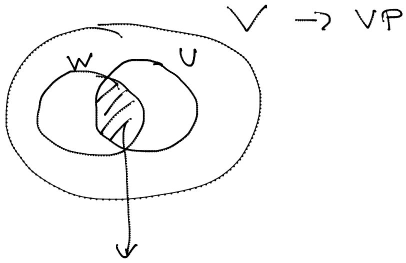

$W \cap U$

0) U, W su pp. od $V \Rightarrow$ neprazni $(\exists 0) \Rightarrow 0 \in W \cap U$

1) $v_1, v_2 \in W \cap U \overset{?}{\Rightarrow} v_1 + v_2 \in W \cap U$

$\Downarrow$

$v_1, v_2 \in W$ i $v_1, v_2 \in U$, $v_1 + v_2 \in W$ i $v_1 + v_2 \in U$

$\Downarrow$

$v_1 + v_2 \in W \cap U$

2) $v_1 \in W \cap U$

$$\underbrace{\alpha \cdot v_1 \in W \quad \wedge \quad \alpha \cdot v_1 \in U}$$

$\Downarrow$

$\alpha \cdot v_1 \in W \cap U$

(*) $e_1 = (1,0,0)$, $e_2 = (0,1,0)$, $e_3 = (0,0,1)$

$e_1 = u_1$, $e_2 = u_2$, $e_3 = u_3$

$$V = (1,2,3) = \underbrace{1}_{\alpha_1} \cdot \underbrace{(1,0,0)}_{u_1} + \underbrace{2}_{\alpha_2} \cdot \underbrace{(0,1,0)}_{u_2} + \underbrace{3}_{\alpha_3} \cdot \underbrace{(0,0,1)}_{u_3}$$

(*) $v = (1,3,1)$, $u_1 = (0,1,2)$, $u_2 = (1,0,-5)$

$\alpha_1 = 3$, $\alpha_2 = 1$: $\alpha_1 \cdot u_1 + \alpha_2 u_2 = 3 \cdot (0,1,2) + 1 \cdot (1,0,-5)$

$= (1,3,1) = v$

(*) v, u_1, u_2 lin. zav. ili nez?

$\alpha_1 v + \alpha_2 u_1 + \alpha_3 u_2 = 0$

$\alpha_1 \cdot (1,3,1) + \alpha_2 (0,1,2) + \alpha_3 \cdot (1,0,-5) = (0,0,0)$

$(\alpha_1 + 0 \cdot \alpha_2 + 1 \cdot \alpha_3 , 3\alpha_1 + 1 \cdot \alpha_2 + 0 \cdot \alpha_3 , 1 \cdot \alpha_1 + 2\alpha_2 - 5\alpha_3) = (0,0,0)$

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$$\left.\begin{aligned} \alpha_1 + \alpha_3 &= 0 \\ 3\alpha_1 + \alpha_2 &= 0 \\ \alpha_1 + 2\alpha_2 - 5\alpha_3 &= 0 \end{aligned}\right\} \quad \begin{aligned} \alpha_1 &= 1 \\ \alpha_2 &= -3 \\ \alpha_3 &= -1 \end{aligned}$$

$\exists$ netrivijalno rješ.
$\Rightarrow v, u_1, u_2$ lin. zavisni

(*) e_1, e_2, e_3

$\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 = 0$

$(\alpha_1, \alpha_2, \alpha_3) = (0, 0, 0) \Rightarrow \alpha_1 = \alpha_2 = \alpha_3 = 0$

TRIVIJALNO, jedno

$\Rightarrow e_1, e_2, e_3$ lin. NEZAVISNI

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(1) $\boxed{\Rightarrow}$ S lin. zavistan $\overset{?}{\Longrightarrow} v_i = \sum_{i \neq j} \alpha_j v_j$

S lin. zav. $\Rightarrow \exists\, \alpha_1, \dots, \alpha_k$ od kojih je bar jedan $\neq 0$

t.d. $\alpha_1 v_1 + \dots + \alpha_k v_k = \theta$

pp. $\alpha_1 \neq 0$

$\alpha_1 v_1 = -\alpha_2 v_2 - \alpha_3 v_3 - \dots - \alpha_k v_k \quad \backslash : \alpha_1 \neq 0$

$\underline{\underline{v_1}} = -\underbrace{\frac{\alpha_2}{\alpha_1}}_{\text{broj}} v_2 - \underbrace{\frac{\alpha_3}{\alpha_1}}_{\text{broj}} v_3 - \dots - \underbrace{\frac{\alpha_k}{\alpha_1}}_{\text{broj}} v_k \Rightarrow v_1$ je lin. komb. $v_2, \dots, v_k$

$\boxed{\Leftarrow}$ pp. $v_1 = \alpha_2 v_2 + \dots + \alpha_k v_k \overset{?}{\Longrightarrow} \{v_1, \dots, v_k\}$ lin. zav.

$\underline{-v_1} + \alpha_2 v_2 + \dots + \alpha_k v_k = 0$

$\underbrace{-1} \cdot v_1$

$\alpha_1 = -1 \neq 0 \Rightarrow \exists$ bar jedan $\alpha_i\ (\alpha_1) \neq 0$

$\Rightarrow \{v_1, \dots, v_k\}$ lin. zav. $\boxtimes$

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lin. zav (nije baza)

pokrivač

(*) $\{(1,0,0), (0,1,0), (0,0,1), (0,0,2)\}$

jeste pokrivač
jeste baza

$\alpha_1 \cdot (1,0,0) + \alpha_2 (0,1,0) + \alpha_3 (0,0,1) = (\alpha_1, \alpha_2, \alpha_3)$

$(2,2,2) = 2e_1 + 2 \cdot e_2 + 2 \cdot e_3$

$= 2 \cdot e_1 + 2 \cdot e_2 + 1 \cdot (0,0,2)$

(*) e_1, e_2, e_3 baza $\mathbb{R}^3$

$B = \{(1,2,3), (0,1,2), (-2,0,1)\}$

B lin. nez. $(\alpha_1 = \alpha_2 = \alpha_3 = 0)$

$\mathbb{R}^2$: $e_1 = (1,0)$, $e_2 = (0,1)$ (baza) (pok.)

$(3,0)$, $(0,7)$ (baza) (pok)

$(1,0), (3,7), (0,7)$ (nije baza) (pok.)

$(3,7) = \underbrace{3}_{\alpha_1} \cdot (1,0) + \underbrace{1}_{\alpha_2} \cdot (0,7) \Rightarrow$ lin. zav.

$(1,1), (0,7), (15,-23), (0,0), (2,0)$

DT S baza $\Rightarrow$ S pokrivač $\overset{\text{def.}}{\Rightarrow}$ $u \in V$
$\not\Leftarrow$
$u = \alpha_1 v_1 + \dots + \alpha_n v_n$

jedinstvenost:

pps. $u = \alpha_1 v_1 + \dots + \alpha_n v_n$
$u = \beta_1 v_1 + \dots + \beta_n v_n$ $\Big\}$ $\alpha_i \neq \beta_i$
$\ominus$

$u - u = 0 = \underbrace{\underbrace{(\alpha_1 - \beta_1)}_{\gamma_1} v_1 + \dots + \underbrace{(\alpha_n - \beta_n)}_{\gamma_n} v_n}$

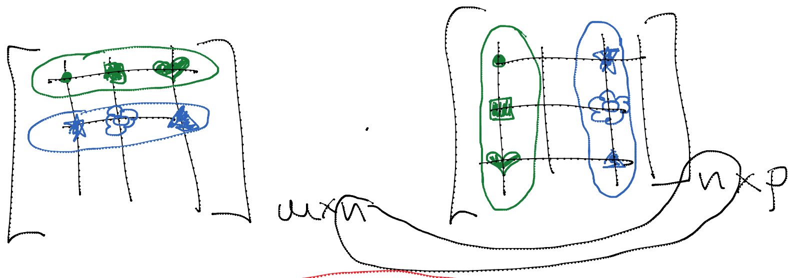
S baza $\overset{(\text{def.})}{\Rightarrow}$ $v_1, \dots, v_n$ lin. nez.

$\overset{(\text{def } L.N.)}{\Rightarrow}$ $\gamma_1 = \dots \gamma_n = 0$

S baza $\overset{(\text{def.})}{\Rightarrow} v_1, \ldots, v_n$ lin. nez.

$\overset{(\text{def } L.N.)}{\Rightarrow} \gamma_1 = \ldots = \gamma_n = 0$

$\alpha_i - \beta_i = 0 \Rightarrow \alpha_i = \beta_i$ $\boxtimes$



$\to m \times p$

$$(2 \times 3) \cdot (3 \times 4) = 2 \times 4$$

br. kolona br. vrsta.

$(2 \times 3) \cdot (2 \times 3)$ ↯

$c_{11} = ● \cdot ● + ■ \cdot ■ + ♥ \cdot ♥$

$c_{23} = ★ \cdot ★ + ✿ \cdot ✿ + △ \cdot △$

$$A_{2\times 3} \cdot B_{3\times 4} = C_{2\times 4}$$

$B_{3\times 4} \cdot A_{2\times 3}$ ↯ $(4 \neq 2)$

$$A_{3\times 3} \cdot B_{3\times 3} = C_{3\times 3}$$

$\neq$ može $=$, a i ne mora

$$B_{3\times 3} \cdot A_{3\times 3} = D_{3\times 3}$$