

# Solutions of Triangle Location Construction Problems

## Part I - Wernick's corpus

Generated automatically by ArgoTriCS  
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Problem 0018	86
Problem 0019	90
Problem 0020	95
Problem 0021	100
Problem 0022	105
Problem 0023	109
Problem 0024	116
Problem 0025	122
Problem 0026	128
Problem 0027	134
Problem 0028	139
Problem 0029	145
Problem 0030	150
Problem 0031	155
Problem 0032	160
Problem 0033	165
Problem 0034	170
Problem 0035	175
Problem 0036	181
Problem 0037	186
Problem 0038	191
Problem 0039	196
Problem 0040	201
Problem 0041	206
Problem 0042	211

Problem 0043	216
Problem 0044	221
Problem 0045	226
Problem 0046	231
Problem 0047	236
Problem 0048	242
Problem 0049	243
Problem 0050	244
Problem 0051	251
Problem 0052	256
Problem 0053	261
Problem 0054	266
Problem 0055	271
Problem 0056	276
Problem 0057	283
Problem 0058	289
Problem 0059	295
Problem 0060	301
Problem 0061	307
Problem 0062	312
Problem 0063	317
Problem 0064	322
Problem 0065	327
Problem 0066	334
Problem 0067	340

Problem 0068	346
Problem 0069	352
Problem 0070	358
Problem 0071	364
Problem 0072	370
Problem 0073	376
Problem 0074	381
Problem 0075	387
Problem 0076	388
Problem 0077	389
Problem 0078	397
Problem 0079	403
Problem 0080	409
Problem 0081	416
Problem 0082	423
Problem 0083	430
Problem 0084	436
Problem 0085	442
Problem 0086	448
Problem 0087	454
Problem 0088	460
Problem 0089	467
Problem 0090	473
Problem 0091	480
Problem 0092	486

Problem 0093	492
Problem 0094	499
Problem 0095	504
Problem 0096	511
Problem 0097	512
Problem 0098	513
Problem 0099	514
Problem 0100	515
Problem 0101	522
Problem 0102	529
Problem 0103	535
Problem 0104	542
Problem 0105	548
Problem 0106	554
Problem 0107	559
Problem 0108	564
Problem 0109	568
Problem 0110	572
Problem 0111	577
Problem 0112	582
Problem 0113	587
Problem 0114	591
Problem 0115	598
Problem 0116	604
Problem 0117	610

Problem 0118	616
Problem 0119	622
Problem 0120	627
Problem 0121	633
Problem 0122	638
Problem 0123	643
Problem 0124	648
Problem 0125	653
Problem 0126	658
Problem 0127	664
Problem 0128	670
Problem 0129	675
Problem 0130	681
Problem 0131	686
Problem 0132	692
Problem 0133	697
Problem 0134	702
Problem 0135	707
Problem 0136	712
Problem 0137	717
Problem 0138	724
Problem 0139	730
Problem 0140	736
Problem 0141	742
Problem 0142	748

Problem 0143	753
Problem 0144	758
Problem 0145	763
Problem 0146	768
Problem 0147	773
Problem 0148	779
Problem 0149	780
Problem 0150	786
Problem 0151	787
Problem 0152	794
Problem 0153	799
Problem 0154	804
Problem 0155	809
Problem 0156	814
Problem 0157	821
Problem 0158	827
Problem 0159	833
Problem 0160	839
Problem 0161	845
Problem 0162	851
Problem 0163	857
Problem 0164	863
Problem 0165	868
Problem 0166	869
Problem 0167	875

Problem 0168	876
Problem 0169	884
Problem 0170	890
Problem 0171	896
Problem 0172	902
Problem 0173	909
Problem 0174	915
Problem 0175	922
Problem 0176	929
Problem 0177	935
Problem 0178	942
Problem 0179	949
Problem 0180	956
Problem 0181	963
Problem 0182	969
Problem 0183	975
Problem 0184	982
Problem 0185	988
Problem 0186	993
Problem 0187	1000
Problem 0188	1001
Problem 0189	1002
Problem 0190	1003
Problem 0191	1004
Problem 0192	1011



Problem 0193	1012
Problem 0194	1018
Problem 0195	1025
Problem 0196	1031
Problem 0197	1037
Problem 0198	1042
Problem 0199	1047
Problem 0200	1053
Problem 0201	1058
Problem 0202	1063
Problem 0203	1068
Problem 0204	1073
Problem 0205	1079
Problem 0206	1085
Problem 0207	1091
Problem 0208	1096
Problem 0209	1101
Problem 0210	1106
Problem 0211	1112
Problem 0212	1117
Problem 0213	1122
Problem 0214	1127
Problem 0215	1132
Problem 0216	1139
Problem 0217	1145

Problem 0218	1151
Problem 0219	1157
Problem 0220	1163
Problem 0221	1169
Problem 0222	1174
Problem 0223	1179
Problem 0224	1184
Problem 0225	1189
Problem 0226	1196
Problem 0227	1202
Problem 0228	1208
Problem 0229	1214
Problem 0230	1220
Problem 0231	1225
Problem 0232	1230
Problem 0233	1235
Problem 0234	1240
Problem 0235	1246
Problem 0236	1247
Problem 0237	1248
Problem 0238	1254
Problem 0239	1261
Problem 0240	1267
Problem 0241	1273
Problem 0242	1279

Problem 0243	1284
Problem 0244	1285
Problem 0245	1286
Problem 0246	1292
Problem 0247	1300
Problem 0248	1306
Problem 0249	1312
Problem 0250	1318
Problem 0251	1325
Problem 0252	1332
Problem 0253	1338
Problem 0254	1345
Problem 0255	1351
Problem 0256	1357
Problem 0257	1364
Problem 0258	1369
Problem 0259	1375
Problem 0260	1382
Problem 0261	1389
Problem 0262	1396
Problem 0263	1403
Problem 0264	1410
Problem 0265	1416
Problem 0266	1417
Problem 0267	1418

Problem 0268	1419
Problem 0269	1420
Problem 0270	1421
Problem 0271	1428
Problem 0272	1434
Problem 0273	1441
Problem 0274	1447
Problem 0275	1453
Problem 0276	1459
Problem 0277	1465
Problem 0278	1470
Problem 0279	1476
Problem 0280	1482
Problem 0281	1488
Problem 0282	1495
Problem 0283	1501
Problem 0284	1502
Problem 0285	1503
Problem 0286	1504
Problem 0287	1510
Problem 0288	1515
Problem 0289	1521
Problem 0290	1527
Problem 0291	1533
Problem 0292	1540

Problem 0293	1541
Problem 0294	1547
Problem 0295	1548
Problem 0296	1549
Problem 0297	1554
Problem 0298	1559
Problem 0299	1564
Problem 0300	1569
Problem 0301	1574
Problem 0302	1575
Problem 0303	1576
Problem 0304	1582
Problem 0305	1583
Problem 0306	1588
Problem 0307	1593
Problem 0308	1598
Problem 0309	1603
Problem 0310	1604
Problem 0311	1605
Problem 0312	1606
Problem 0313	1607
Problem 0314	1608
Problem 0315	1609
Problem 0316	1615
Problem 0317	1616

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Problem 0319	1618
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Problem 0322	1626
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Problem 0325	1629
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Problem 0329	1638
Problem 0330	1639
Problem 0331	1640
Problem 0332	1641
Problem 0333	1642
Problem 0334	1643
Problem 0335	1644
Problem 0336	1645
Problem 0337	1646
Problem 0338	1647
Problem 0339	1648
Problem 0340	1649
Problem 0341	1650
Problem 0342	1656

Problem 0343	1661
Problem 0344	1666
Problem 0345	1671
Problem 0346	1676
Problem 0347	1677
Problem 0348	1678
Problem 0349	1679
Problem 0350	1680
Problem 0351	1681
Problem 0352	1686
Problem 0353	1691
Problem 0354	1696
Problem 0355	1701
Problem 0356	1702
Problem 0357	1703
Problem 0358	1704
Problem 0359	1705
Problem 0360	1706
Problem 0361	1712
Problem 0362	1717
Problem 0363	1722
Problem 0364	1727
Problem 0365	1733
Problem 0366	1734
Problem 0367	1735

Problem 0368	1743
Problem 0369	1749
Problem 0370	1755
Problem 0371	1761
Problem 0372	1766
Problem 0373	1767
Problem 0374	1768
Problem 0375	1776
Problem 0376	1781
Problem 0377	1787
Problem 0378	1793
Problem 0379	1799
Problem 0380	1800
Problem 0381	1801
Problem 0382	1807
Problem 0383	1813
Problem 0384	1814
Problem 0385	1820
Problem 0386	1821
Problem 0387	1822
Problem 0388	1823
Problem 0389	1824
Problem 0390	1825
Problem 0391	1826
Problem 0392	1827



Problem 0393	1834
Problem 0394	1835
Problem 0395	1836
Problem 0396	1837
Problem 0397	1842
Problem 0398	1847
Problem 0399	1852
Problem 0400	1857
Problem 0401	1858
Problem 0402	1859
Problem 0403	1860
Problem 0404	1861
Problem 0405	1862
Problem 0406	1867
Problem 0407	1873
Problem 0408	1878
Problem 0409	1883
Problem 0410	1884
Problem 0411	1890
Problem 0412	1891
Problem 0413	1898
Problem 0414	1903
Problem 0415	1909
Problem 0416	1915
Problem 0417	1921

Problem 0418	1927
Problem 0419	1928
Problem 0420	1929
Problem 0421	1935
Problem 0422	1941
Problem 0423	1942
Problem 0424	1947
Problem 0425	1948
Problem 0426	1956
Problem 0427	1962
Problem 0428	1963
Problem 0429	1969
Problem 0430	1975
Problem 0431	1976
Problem 0432	1977
Problem 0433	1978
Problem 0434	1979
Problem 0435	1980
Problem 0436	1981
Problem 0437	1982
Problem 0438	1983
Problem 0439	1984
Problem 0440	1991
Problem 0441	1992
Problem 0442	1997

Problem 0443	2002
Problem 0444	2008
Problem 0445	2013
Problem 0446	2014
Problem 0447	2015
Problem 0448	2021
Problem 0449	2028
Problem 0450	2034
Problem 0451	2039
Problem 0452	2045
Problem 0453	2051
Problem 0454	2052
Problem 0455	2058
Problem 0456	2059
Problem 0457	2064
Problem 0458	2070
Problem 0459	2071
Problem 0460	2077
Problem 0461	2083
Problem 0462	2084
Problem 0463	2090
Problem 0464	2091
Problem 0465	2092
Problem 0466	2097
Problem 0467	2106

Problem 0468	2107
Problem 0469	2108
Problem 0470	2109
Problem 0471	2110
Problem 0472	2111
Problem 0473	2112
Problem 0474	2113
Problem 0475	2114
Problem 0476	2115
Problem 0477	2123
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Problem 0479	2125
Problem 0480	2130
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Problem 0482	2137
Problem 0483	2138
Problem 0484	2139
Problem 0485	2140
Problem 0486	2145
Problem 0487	2146
Problem 0488	2152
Problem 0489	2153
Problem 0490	2154
Problem 0491	2159
Problem 0492	2160

Problem 0493	2161
Problem 0494	2167
Problem 0495	2168
Problem 0496	2169
Problem 0497	2170
Problem 0498	2171
Problem 0499	2172
Problem 0500	2173
Problem 0501	2174
Problem 0502	2175
Problem 0503	2176
Problem 0504	2177
Problem 0505	2178
Problem 0506	2184
Problem 0507	2190
Problem 0508	2198
Problem 0509	2205
Problem 0510	2206
Problem 0511	2207
Problem 0512	2213
Problem 0513	2220
Problem 0514	2221
Problem 0515	2229
Problem 0516	2230
Problem 0517	2236

Problem 0518	2237
Problem 0519	2238
Problem 0520	2239
Problem 0521	2240
Problem 0522	2241
Problem 0523	2247
Problem 0524	2248
Problem 0525	2249
Problem 0526	2250
Problem 0527	2255
Problem 0528	2256
Problem 0529	2264
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Problem 0533	2280
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Problem 0536	2283
Problem 0537	2284
Problem 0538	2285
Problem 0539	2286
Problem 0540	2292
Problem 0541	2293
Problem 0542	2294

Problem 0543	2295
Problem 0544	2302
Problem 0545	2303
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Problem 0548	2306
Problem 0549	2307
Problem 0550	2308
Problem 0551	2314
Problem 0552	2315
Problem 0553	2316
Problem 0554	2317
Problem 0555	2318
Problem 0556	2319
Problem 0557	2320
Problem 0558	2321
Problem 0559	2328
Problem 0560	2335

# Problem 1

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## 1 Problem

Problem 1: Given a point  $A$ , a point  $B$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

Non-degenerate conditions: .

Determination conditions: .

Rules used: []

Lemmas used: []

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95  
point B 20 40  
point C 110 40
```

```
color 220 0 0  
fontsize 9
```

```
cmark_t A  
cmark_b B  
cmark_l C  
color 0 0 0  
fontsize 8
```

```
drawsegment A B  
drawsegment A C  
drawsegment B C
```

```
% Non-degenerate conditions:  
% Determination conditions:
```



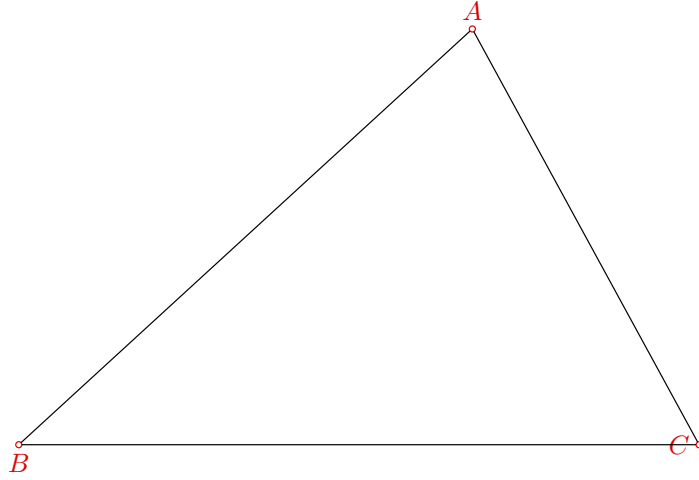


Figure 1: Illustration of the problem 0001

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.033 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.1.3 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.024 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

There are no ndg conditions.

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $B=B$

There are no ndg conditions.

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $C=C$

There are no ndg conditions.

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.3 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 2

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## 1 Problem

Problem 2: Given a point  $B$ , a point  $O$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
2. Choose freely a point  $A$  on the circle  $k(O, C)$  (rule WOncircle);
3. Choose freely a point  $C$  on the circle  $k(O, C)$  (rule WOncircle).

Non-degenerate conditions: points  $B$  and  $O$  are not the same.

Determination conditions: .

Rules used: [W06,WOncircle1]

Lemmas used: [D26,L12]

Solving time: 585.5 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point O 65 51.14
point A 80 95

color 220 0 0
fontsize 9

cmark_b B
cmark_t O
cmark_t A
color 0 0 0
```

```

fontsize 8

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Choosing randomly a point A on the circle with center O through point B
oncircle A O B
cmark_t A
color 200 200 200
drawcircle O B
color 0 0 0

% Choosing randomly a point C on the circle with center O through point A
oncircle C O A
cmark_l C
color 200 200 200
drawcircle O A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: points B and O are not the same
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

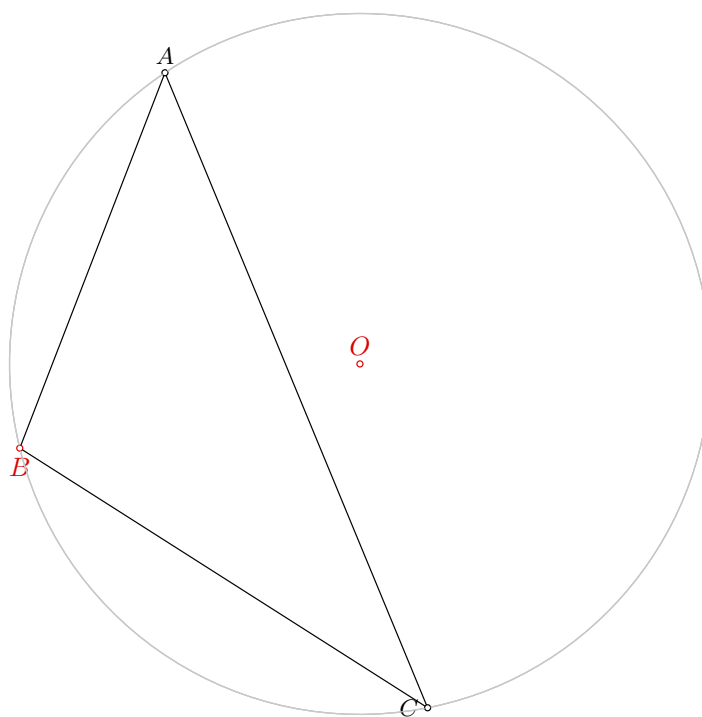


Figure 1: Illustration of the problem 0002

### 4.1.2 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 52 terms.

**Time Complexity:** Time spent by the prover is 0.506 seconds.

**NDG conditions** Points  $A$ ,  $B$  and  $C$  are not collinear

Line through points  $C$  and  $B$  is not perpendicular to line through points  $B$  and  $O$

### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{_M_aBC} \neq 0$  i.e., points  $_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$  i.e., points  $_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a-M_bF_{_m_b}^1} \neq S_{F_{_m_a}^0-M_bF_{_m_b}^1}$  i.e., lines  $_M_aF_{_m_a}^0$  and  $_M_bF_{_m_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $O=_O$

Proving failed

### 4.2.3 Proving $A=A$

NDG conditions are:

$S_{_M_aBC} \neq 0$  i.e., points  $_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$  i.e., points  $_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a-M_bF_{_m_b}^1} \neq S_{F_{_m_a}^0-M_bF_{_m_b}^1}$  i.e., lines  $_M_aF_{_m_a}^0$  and  $_M_bF_{_m_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $O=_O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 327 terms.

**Time Complexity:** Time spent by the prover is 0.450 seconds. There are no ndg conditions.

### 4.3.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $O=_O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 930 terms.

**Time Complexity:** Time spent by the prover is 49.020 seconds. There are no ndg conditions.

### 4.4.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.



# Problem 3

*Generated automatically by ArgoTriCS  
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## 1 Problem

Problem 3: Given a point  $A$ , a point  $B$  and a point  $M_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D21, GL03]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
```

```
point B 20 40
```

```
point M_{a} 65 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_t A
```

```
cmark_b B
```

```
cmark_r M_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point C such that BC/BM_{a}=2
```

```
towards C B M_{a} 2
```

```
cmark_l C
```

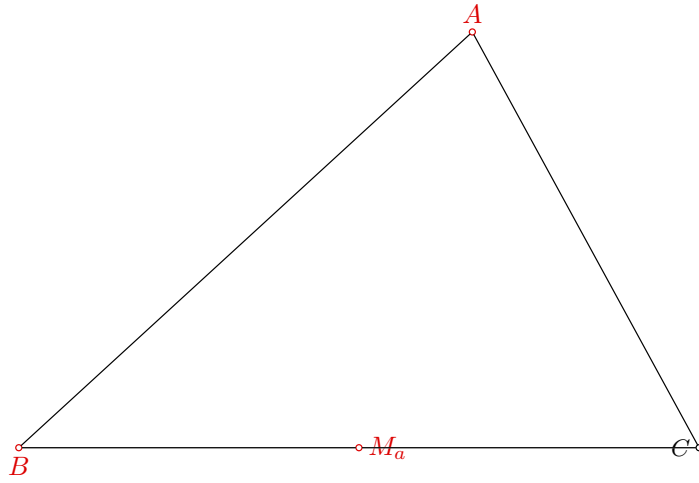


Figure 1: Illustration of the problem 0003

```
color 200 200 200
drawsegment B C
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions:
% Determination conditions:
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.035 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.034 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $M_a=_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.057 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

There are no ndg conditions.

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $B=B$

There are no ndg conditions.

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $M_a=_M_a$

There are no ndg conditions.

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $M_a=_M a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 19 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_a=_M a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 42 terms.

**Time Complexity:** Time spent by the prover is 0.090 seconds. There are no ndg conditions.

# Problem 4

*Generated automatically by ArgoTriCS  
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## 1 Problem

Problem 4: Given a point  $A$ , a point  $B$  and a point  $M_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D22, GL03, GL04]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point B 20 40
point M_{b} 95 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_b B
cmark_lt M_{b}
color 0 0 0
fontsize 8
```

```
% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
```

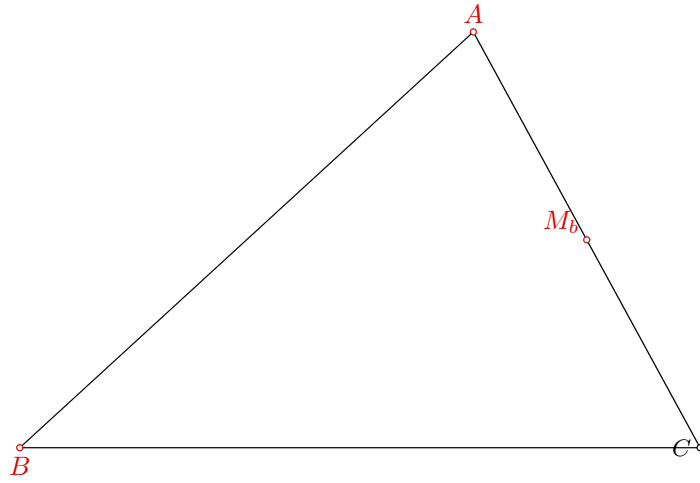


Figure 1: Illustration of the problem 0004

```
color 200 200 200
drawsegment A C
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions:
% Determination conditions:
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.034 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.033 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $M_b=_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.052 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

There are no ndg conditions.

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $B=B$

There are no ndg conditions.

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $M_b=_M_b$

There are no ndg conditions.

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $M_b=\neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_b=\neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.



# Problem 5

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 5: Given a point  $B$ , a point  $M_c$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is redundant.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01);
2. Choose freely a point  $C$  (rule free).

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01,free]

Lemmas used: [D20,GL03,GL04]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point M_{c} 50 67.5
point A 80 95
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_lt M_{c}
cmark_t A
color 0 0 0
fontsize 8
```

```
% Constructing a point A such that BA/BM_{c}=2
```

```

towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

% Constructing a free point C
point C 110 40

cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.042 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c = M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.053 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.024 seconds.

**NDG conditions** There are no NDG conditions for this theorem

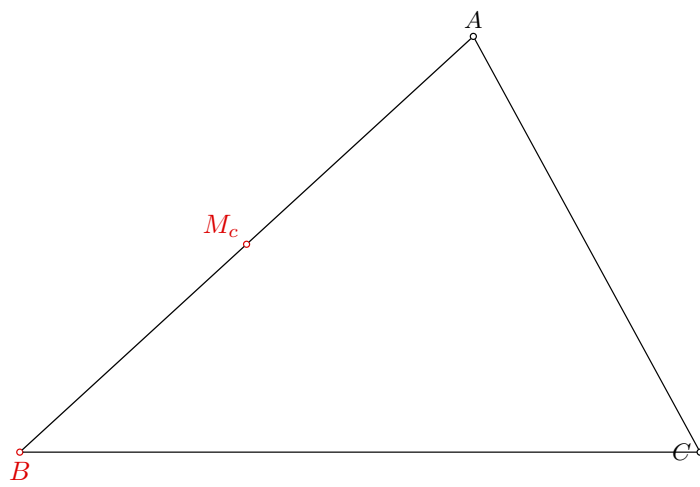


Figure 1: Illustration of the problem 0005

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

There are no ndg conditions.  
Total number of proof steps: 1  
Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_c=_M M_c$

There are no ndg conditions.  
Total number of proof steps: 42  
Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $A=A$

There are no ndg conditions.  
Total number of proof steps: 1  
Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_c=_M M_c$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $A=A$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $M_c=_M M_c$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 6

*Generated automatically by ArgoTriCS  
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## 1 Problem

Problem 6: Given a point  $A$ , a point  $B$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $G$ , construct a point  $M_b$  (rule W01); ;
2. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D22,GL03,GL04,L56]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point B 20 40
point G 70 58.33
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_b B
cmark_t G
color 0 0 0
fontsize 8
```

```
% Constructing a point  $M_b$  such that  $BM_b/BG=1.5$ 
```

```

towards M_{b} B G 1.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

```

```

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $G=_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 15 terms.

**Time Complexity:** Time spent by the prover is 0.429 seconds.

**NDG conditions** Line through points  $A$  and  $_M a$  is not parallel with line through points  $B$  and  $_M b$   
Points  $A$  and  $_M a$  are not identical

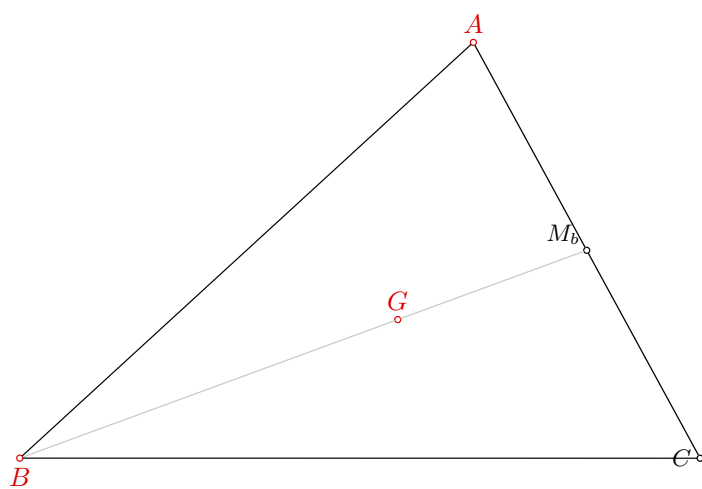


Figure 1: Illustration of the problem 0006



## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{AB\_M_b} \neq S_{M_aB\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $B=B$

NDG conditions are:

$S_{AB\_M_b} \neq S_{M_aB\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $G=_G$

NDG conditions are:

$S_{AB\_M_b} \neq S_{M_aB\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (cancellation assumption)

Total number of proof steps: 613

Time spent by the prover: 0.200 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $G=_G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 67 terms.

**Time Complexity:** Time spent by the prover is 0.150 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $G=_G$

Proving failed

# Problem 7

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 7: Given a point  $B$ , a point  $H_a$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $H_a$  are not the same;
2. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
3. Choose freely a point  $A$  on the line  $h_a$  (rule WOnline2);
4. Choose freely a point  $C$  on the line  $a$  (rule WOnline1) .

Non-degenerate conditions: .

Determination conditions: points  $B$  and  $H_a$  are not the same.

Rules used: [W02,W10b,WOnline1,WOnline2]

Lemmas used: [D5,D8,GD01]

Solving time: 1085.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point H_{a} 80 40
point A 80 95

color 220 0 0
fontsize 9

cmark_b B
cmark_r H_{a}
```

```

cmark_t A
color 0 0 0
fontsize 8

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% Generating random value V[_G6398]
random V[_G6398]

% Calculating value V[_G6419] using formula V[_G6398]*20
expression V[_G6419] { V[_G6398]*20 }

% Constructing a point A which is a point for which holds  $H_{a}A = V[_G6419]$  and angle  $BH_{a}A = 90$ 
turtle A B H_{a} 90 V[_G6419]
cmark_t A

% Choosing randomly a point C on the line BH_{a}
online C B H_{a}
cmark_l C
color 200 200 200
drawline B H_{a}
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions: points B and H_{a} are not the same

```

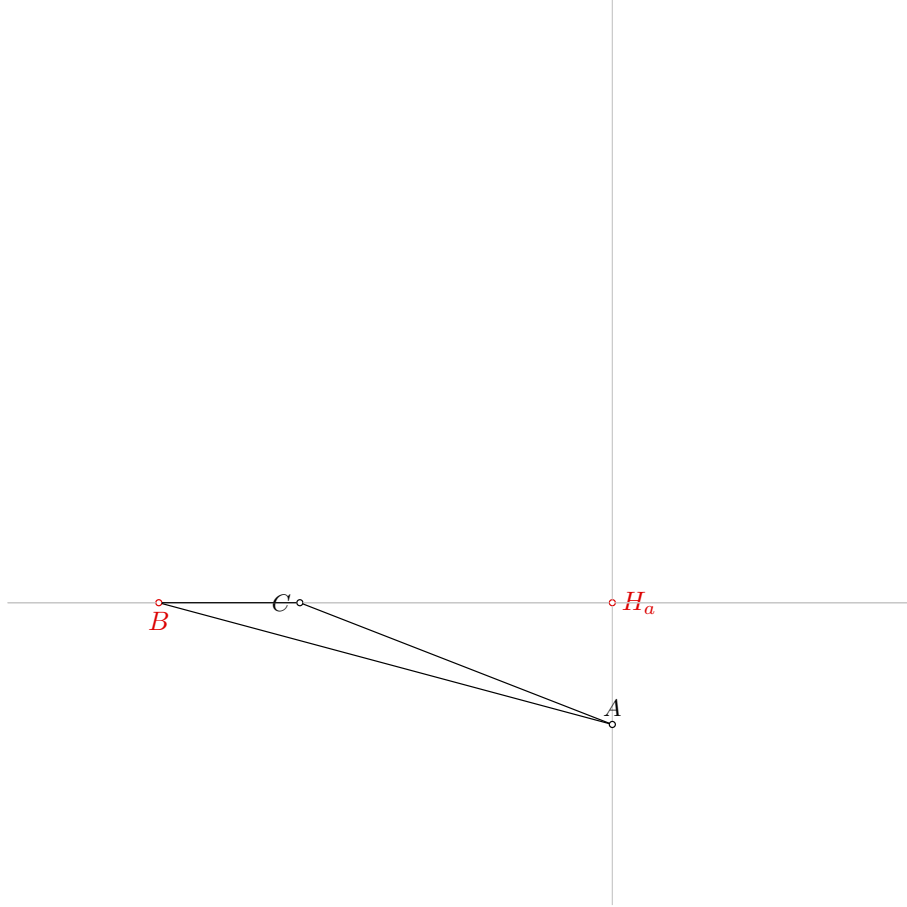


Figure 1: Illustration of the problem 0007

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.023 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a=H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.008 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^2} \neq S_{CAF_{h_a}^2}$  i.e., lines  $BC$  and  $AF_{h_a}^2$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_a=\neg H_a$

Proving failed

### 4.2.3 Proving $A=A$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_a=\neg H_a$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $A=A$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 31.390 seconds. There are no ndg conditions.

#### 4.4.3 Proving $A = A$

Proving failed

# Problem 8

*Generated automatically by ArgoTriCS  
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## 1 Problem

Problem 8: Given a point  $B$ , a point  $H_b$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_b$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H_b$  are not the same;
2. Using the point  $H_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
3. Choose freely a point  $A$  on the line  $b$  (rule WOnline2);
4. Choose freely a point  $C$  on the line  $b$  (rule WOnline1) .

Non-degenerate conditions: .

Determination conditions: points  $B$  and  $H_b$  are not the same.

Rules used: [W02,W10a,WOnline1,WOnline2]

Lemmas used: [D6,D9]

Solving time: 1205.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point H_{b} 89.36 77.83
point A 80 95

color 220 0 0
fontsize 9

cmark_b B
cmark_l H_{b}
```



```

cmark_t A
color 0 0 0
fontsize 8

% DET: points B and H_{b} are not the same
% Constructing a line h_{b} which passes through point B and point H_{b}
line h_{b} B H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
perp b H_{b} h_{b}

color 200 200 200
drawline b
color 0 0 0

% Generating random value V[_G5970]
random V[_G5970]

% Calculating value V[_G5991] using formula V[_G5970]*20
expression V[_G5991] { V[_G5970]*20 }

% Constructing a point A which is a point for which holds H_{b}A = V[_G5991] and angle BH_{b}A = 90
turtle A B H_{b} 90 V[_G5991]
cmark_t A

% Choosing randomly a point C on the line AH_{b}
online C A H_{b}
cmark_l C
color 200 200 200
drawline A H_{b}
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions: points B and H_{b} are not the same

```

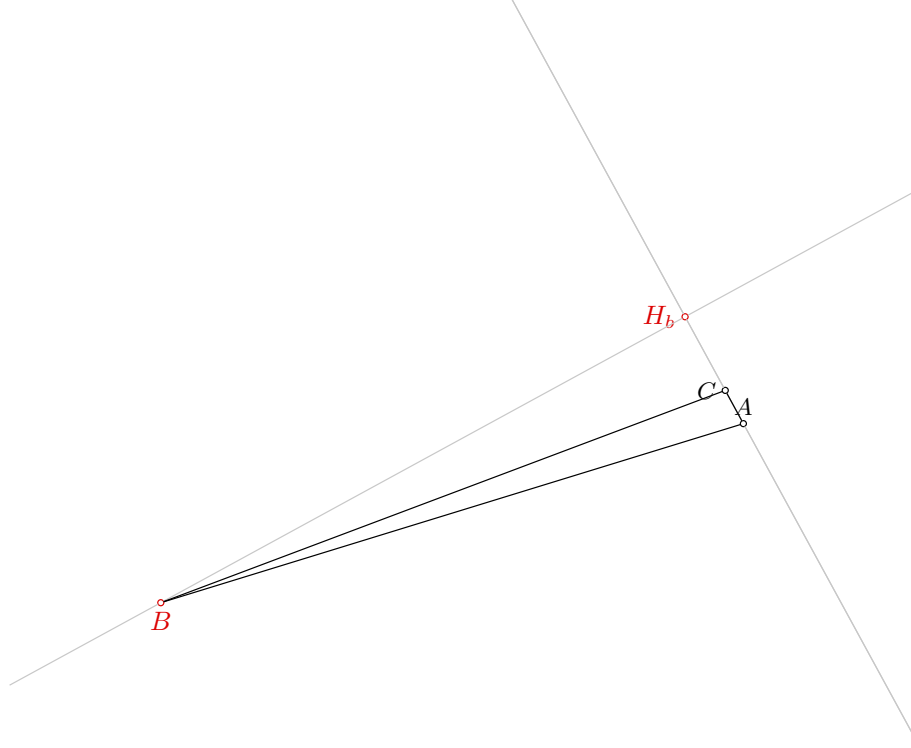


Figure 1: Illustration of the problem 0008

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b=H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.008 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\perp H_b}^2} \neq S_{CBF_{\perp H_b}^2}$  i.e., lines  $AC$  and  $BF_{\perp H_b}^2$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_b=\_H_b$

Proving failed

### 4.2.3 Proving $A=A$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

Proving failed

### 4.3.2 Proving $H_b=\_H_b$

Proving failed

### 4.3.3 Proving $A=A$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

Proving failed

### 4.4.2 Proving $H_b=\_H_b$

Proving failed

### 4.4.3 Proving $A=A$

Proving failed

# Problem 9

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 9: Given a point  $B$ , a point  $H_c$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $H_c$  are not the same;
2. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) ;
3. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
4. Choose freely a point  $C$  on the line  $h_c$  (rule WOnline2).

Non-degenerate conditions: .

Determination conditions: points  $B$  and  $H_c$  are not the same.

Rules used: [W02,W10b,WOnline1,WOnline2]

Lemmas used: [D10,D7,GD01]

Solving time: 198.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point H_{c} 68.91 84.83
point A 80 95

color 220 0 0
fontsize 9

cmark_b B
cmark_rt H_{c}
```

```

cmark_t A
color 0 0 0
fontsize 8

% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

color 200 200 200
drawline c
color 0 0 0

% Choosing randomly a point A on the line BH_{c}
online A B H_{c}
cmark_t A
color 200 200 200
drawline B H_{c}
color 0 0 0

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0

% Generating random value V[_G6764]
random V[_G6764]

% Calculating value V[_G6785] using formula V[_G6764]*20
expression V[_G6785] { V[_G6764]*20 }

% Constructing a point C which is a point for which holds  $H_{c}C = V[_G6785]$  and angle  $AH_{c}C = 90$ 
turtle C A H_{c} 90 V[_G6785]
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions: points B and H_{c} are not the same

```

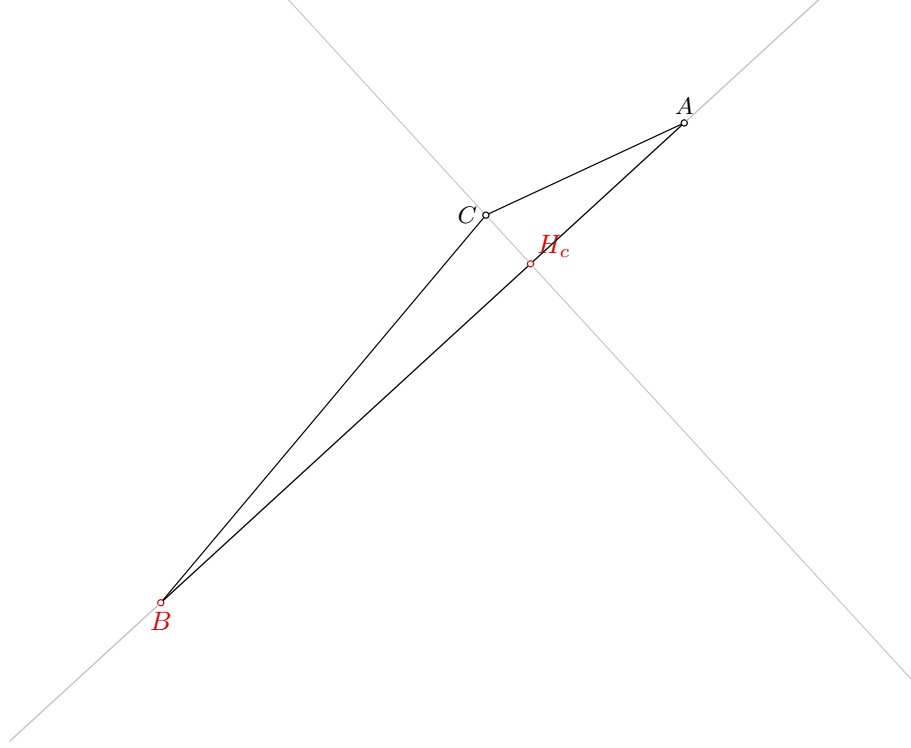


Figure 1: Illustration of the problem 0009

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.02 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c=H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.007 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF^2_{\neg H_c}} \neq S_{BCF^2_{\neg H_c}}$  i.e., lines  $AB$  and  $CF^2_{\neg H_c}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_c=\neg H_c$

Proving failed

### 4.2.3 Proving $A=A$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF^2_{\neg H_c}} \neq S_{BCF^2_{\neg H_c}}$  i.e., lines  $AB$  and  $CF^2_{\neg H_c}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_c=\neg H_c$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $H_c=-H_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 31.380 seconds. There are no ndg conditions.

### 4.4.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.



# Problem 10

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 10: Given a point  $A$ , a point  $B$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H$  are not the same;
2. Using the point  $B$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H$  are not the same;
3. Using the point  $A$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
4. Using the point  $B$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
5. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel.

Determination conditions: lines  $b$  and  $a$  are not the same; points  $B$  and  $H$  are not the same; points  $A$  and  $H$  are not the same.

Rules used: [W02,W03,W10a]

Lemmas used: [D3,D8,D9,GD01]

Solving time: 1.2 seconds.

### 3.2 Construction in GCLC language

dim 120 120

point A 80 95  
point B 20 40  
point H 80 72.73

```

color 220 0 0
fontsize 9

cmark_t A
cmark_b B
cmark_rt H
color 0 0 0
fontsize 8

% DET: points A and H are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point A and point H
line  $h_{\{a\}}$  A H

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% DET: points B and H are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point B and point H
line  $h_{\{b\}}$  B H

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% Constructing a line  $b$  which is perpendicular to line  $h_{\{b\}}$  and which passes through point A
perp b A  $h_{\{b\}}$ 

color 200 200 200
drawline b
color 0 0 0

% Constructing a line  $a$  which is perpendicular to line  $h_{\{a\}}$  and which passes through point B
perp a B  $h_{\{a\}}$ 

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $b$  and  $a$  are not parallel% DET: lines  $b$  and  $a$  are not the same
% Constructing a point  $C$  which belongs to line  $b$  and line  $a$ 
intersec C b a
cmark_l C

```

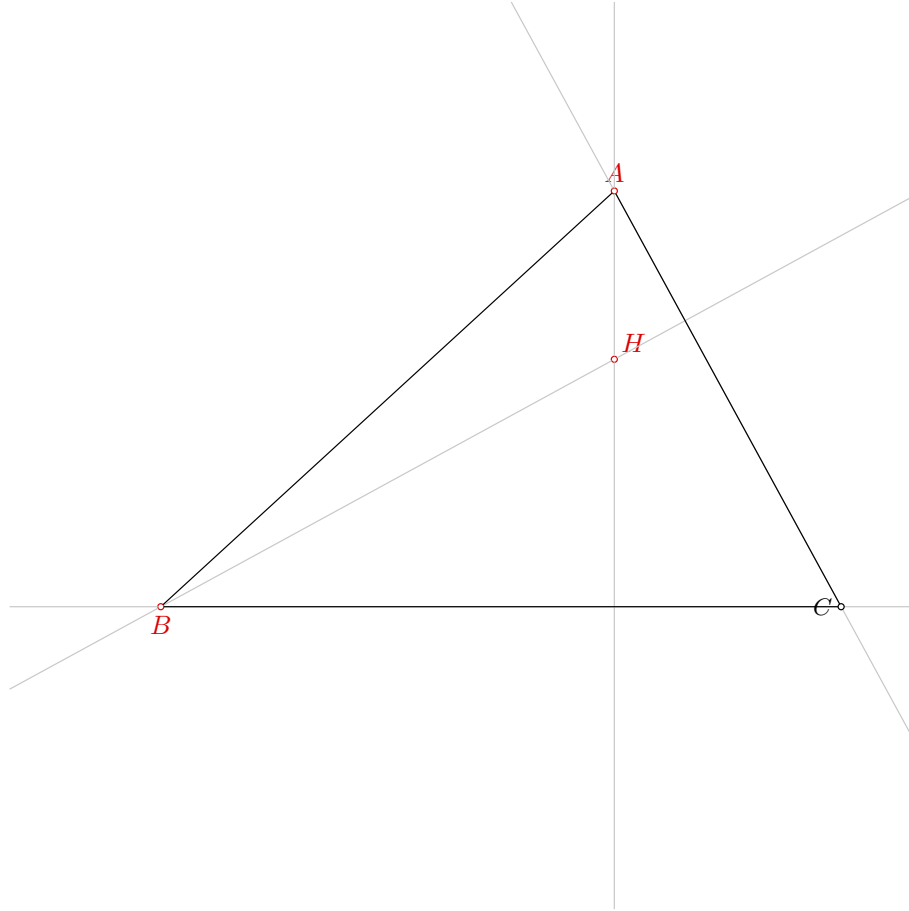


Figure 1: Illustration of the problem 0010

```
drawsegment A B
drawsegment A C
drawsegment B C
```

*% Non-degenerate conditions: lines b and a are not parallel*

*% Determination conditions: lines b and a are not the same; points B and H are not the same; points A and H are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $H=_H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 14 terms.

**Time Complexity:** Time spent by the prover is 0.331 seconds.

**NDG conditions** Points  $A$  and  $H$  are not identical

Points  $A$  and  $B$  are not identical

Points  $B$  and  $H$  are not identical

Points  $A$  and  $C$  are not identical

Line through points  $C$  and  $B$  is not perpendicular to line through points  $B$  and  $A$

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{ABH} \neq 0$  i.e., points  $A$ ,  $B$  and  $H$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAH} \neq 0$  i.e., points  $B$ ,  $A$  and  $H$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_a^1} \neq S_{F_b^0 BF_a^1}$  i.e., lines  $AF_b^0$  and  $BF_a^1$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{F_{-h_a}^2 BF_{-h_b}^3}$  i.e., lines  $AF_{-h_a}^2$  and  $BF_{-h_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $B=B$

NDG conditions are:

$S_{ABH} \neq 0$  i.e., points  $A$ ,  $B$  and  $H$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAH} \neq 0$  i.e., points  $B$ ,  $A$  and  $H$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_a^1} \neq S_{F_b^0 BF_a^1}$  i.e., lines  $AF_b^0$  and  $BF_a^1$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{F_{-h_a}^2 BF_{-h_b}^3}$  i.e., lines  $AF_{-h_a}^2$  and  $BF_{-h_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $H=_H$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $H=_H$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 24 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $H=_H$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 39 terms.

**Time Complexity:** Time spent by the prover is 1.020 seconds. There are no ndg conditions.

# Problem 11

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 11: Given a point  $A$ , a point  $B$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $B$  are not the same;
2. Using the point  $B$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $T_a$  are not the same;
3. Using the point  $T_a$  and the line  $c$ , construct a circle  $k(T_a, foot[T_a, c])$  (rule W11); % NDG: point  $T_a$  is not incident to the line  $c$ ;
4. Using the circle  $k(T_a, foot[T_a, c])$ , the point  $A$ , the point  $T_a$  and the line  $c$ , construct a line  $b$  (rule W13); % NDG: point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ;
5. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ; point  $T_a$  is not incident to the line  $c$ .

Determination conditions: lines  $b$  and  $a$  are not the same; points  $B$  and  $T_a$  are not the same; points  $A$  and  $B$  are not the same.

Rules used: [W02,W03,W11,W13]

Lemmas used: [D23,GD01,GD02,GL10,GL11,L8]

Solving time: 4.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point B 20 40
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_t A
cmark_b B
cmark_rb T_{a}
color 0 0 0
fontsize 8

% DET: points A and B are not the same
% Constructing a line c which passes through point A and point B
line c A B

color 200 200 200
drawline c
color 0 0 0

% DET: points B and T_{a} are not the same
% Constructing a line a which passes through point B and point T_{a}
line a B T_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: point T_{a} is not incident to the line c
% Constructing a point P_{\_G7001} which is a foot of the point T_{a} on the line c
foot P_{\_G7001} T_{a} c
cmark_r P_{\_G7001}
color 200 200 200
drawline T_{a} P_{\_G7001}
color 0 0 0

% Constructing a circle k(T_{a},foot[T_{a},c]) whose center is at point T_{a} and which passes
    through point P_{\_G7001}
circle k(T_{a},foot[T_{a},c]) T_{a} P_{\_G7001}

color 200 200 200
drawcircle k(T_{a},foot[T_{a},c])
color 0 0 0

% NDG: point A is outside the circle k(T_{a},foot[T_{a},c])
```

```

% Constructing a point  $P_{\{a\}}$  which is a foot of the point  $T_{\{a\}}$  on the line  $c$ 
foot  $P_{\{a\}}$   $T_{\{a\}}$   $c$ 
cmark_r  $P_{\{a\}}$ 
color 200 200 200
drawline  $T_{\{a\}}$   $P_{\{a\}}$ 
color 0 0 0

% Constructing a line  $L_{\{a\}}$  which passes through point  $T_{\{a\}}$  and point  $A$ 
line  $L_{\{a\}}$   $T_{\{a\}}$   $A$ 

color 200 200 200
drawline  $L_{\{a\}}$ 
color 0 0 0

% Constructing a point  $P_{\{a\}}$  which is an image of the point  $P_{\{a\}}$  in the symmetry to
point/line  $L_{\{a\}}$ 
sim  $P_{\{a\}}$   $L_{\{a\}}$   $P_{\{a\}}$ 
cmark_r  $P_{\{a\}}$ 

% Constructing a line  $b$  which passes through point  $A$  and point  $P_{\{a\}}$ 
line  $b$   $A$   $P_{\{a\}}$ 

color 200 200 200
drawline  $b$ 
color 0 0 0

% NDG: lines  $b$  and  $a$  are not parallel% DET: lines  $b$  and  $a$  are not the same
% Constructing a point  $C$  which belongs to line  $b$  and line  $a$ 
intersec  $C$   $b$   $a$ 
cmark_l  $C$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(T_{\{a\}}, foot[T_{\{a\}}, c])$ ; point  $T_{\{a\}}$  is not incident to the line  $c$ 
% Determination conditions: lines  $b$  and  $a$  are not the same; points  $B$  and  $T_{\{a\}}$  are not the same; points  $A$  and  $B$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format



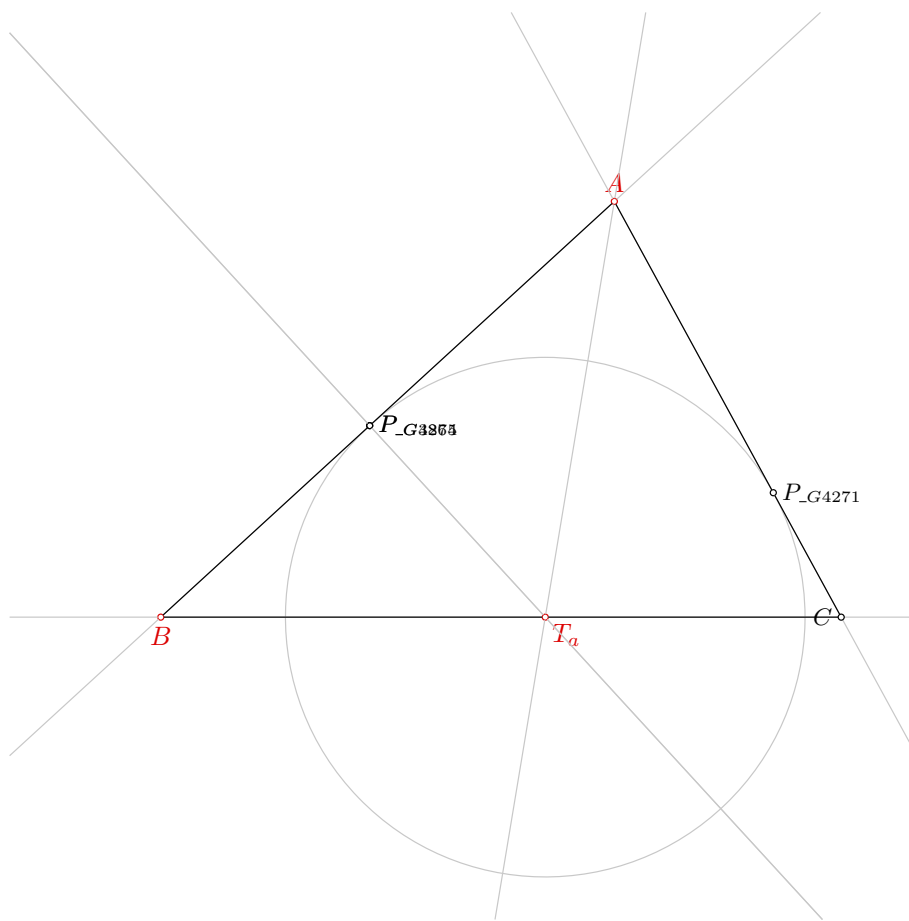


Figure 1: Illustration of the problem 0011

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.021 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $T_a=T_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{ABT_a} \neq S_{P_{G5220}BT_a}$  i.e., lines  $AP_{G5220}$  and  $BT_a$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $B=B$

NDG conditions are:

$S_{ABT_a} \neq S_{P_{G6728}BT_a}$  i.e., lines  $AP_{G6728}$  and  $BT_a$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $T_a=T_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_a = \neg T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 30 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_a = \neg T_a$

Proving failed

# Problem 12

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 12: Given a point  $A$ , a point  $B$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $B$  are not the same;
2. Using the point  $A$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $T_b$  are not the same;
3. Using the point  $T_b$  and the line  $c$ , construct a circle  $k(T_b, foot[T_b, c])$  (rule W11); % NDG: point  $T_b$  is not incident to the line  $c$ ;
4. Using the circle  $k(T_b, foot[T_b, c])$ , the point  $B$ , the point  $T_b$  and the line  $c$ , construct a line  $a$  (rule W13); % NDG: point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ;
5. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ; point  $T_b$  is not incident to the line  $c$ .

Determination conditions: lines  $a$  and  $b$  are not the same; points  $A$  and  $T_b$  are not the same; points  $A$  and  $B$  are not the same.

Rules used: [W02,W03,W11,W13]

Lemmas used: [D24,GD01,GD02,GL10,GL11,L9]

Solving time: 4.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point B 20 40
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_t A
cmark_b B
cmark_t T_{b}
color 0 0 0
fontsize 8

% DET: points A and B are not the same
% Constructing a line c which passes through point A and point B
line c A B

color 200 200 200
drawline c
color 0 0 0

% DET: points A and T_{b} are not the same
% Constructing a line b which passes through point A and point T_{b}
line b A T_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: point T_{b} is not incident to the line c
% Constructing a point P_{\_G4420} which is a foot of the point T_{b} on the line c
foot P_{\_G4420} T_{b} c
cmark_r P_{\_G4420}
color 200 200 200
drawline T_{b} P_{\_G4420}
color 0 0 0

% Constructing a circle k(T_{b},foot[T_{b},c]) whose center is at point T_{b} and which passes
    through point P_{\_G4420}
circle k(T_{b},foot[T_{b},c]) T_{b} P_{\_G4420}

color 200 200 200
drawcircle k(T_{b},foot[T_{b},c])
color 0 0 0

% NDG: point B is outside the circle k(T_{b},foot[T_{b},c])
```

```

% Constructing a point  $P_{\{G4811\}}$  which is a foot of the point  $T_{\{b\}}$  on the line  $c$ 
foot  $P_{\{G4811\}}$   $T_{\{b\}}$   $c$ 
cmark_r  $P_{\{G4811\}}$ 
color 200 200 200
drawline  $T_{\{b\}}$   $P_{\{G4811\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G4814\}}$  which passes through point  $T_{\{b\}}$  and point  $B$ 
line  $L_{\{G4814\}}$   $T_{\{b\}}$   $B$ 

color 200 200 200
drawline  $L_{\{G4814\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G4817\}}$  which is an image of the point  $P_{\{G4811\}}$  in the symmetry to
point/line  $L_{\{G4814\}}$ 
sim  $P_{\{G4817\}}$   $L_{\{G4814\}}$   $P_{\{G4811\}}$ 
cmark_r  $P_{\{G4817\}}$ 

% Constructing a line  $a$  which passes through point  $B$  and point  $P_{\{G4817\}}$ 
line  $a$   $B$   $P_{\{G4817\}}$ 

color 200 200 200
drawline  $a$ 
color 0 0 0

% NDG: lines  $a$  and  $b$  are not parallel% DET: lines  $a$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $a$  and line  $b$ 
intersec  $C$   $a$   $b$ 
cmark_l  $C$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; point  $B$  is outside the circle  $k(T_{\{b\}},$ 
foot $[T_{\{b\}},c])$ ; point  $T_{\{b\}}$  is not incident to the line  $c$ 
% Determination conditions: lines  $a$  and  $b$  are not the same; points  $A$  and  $T_{\{b\}}$  are not the same;
points  $A$  and  $B$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

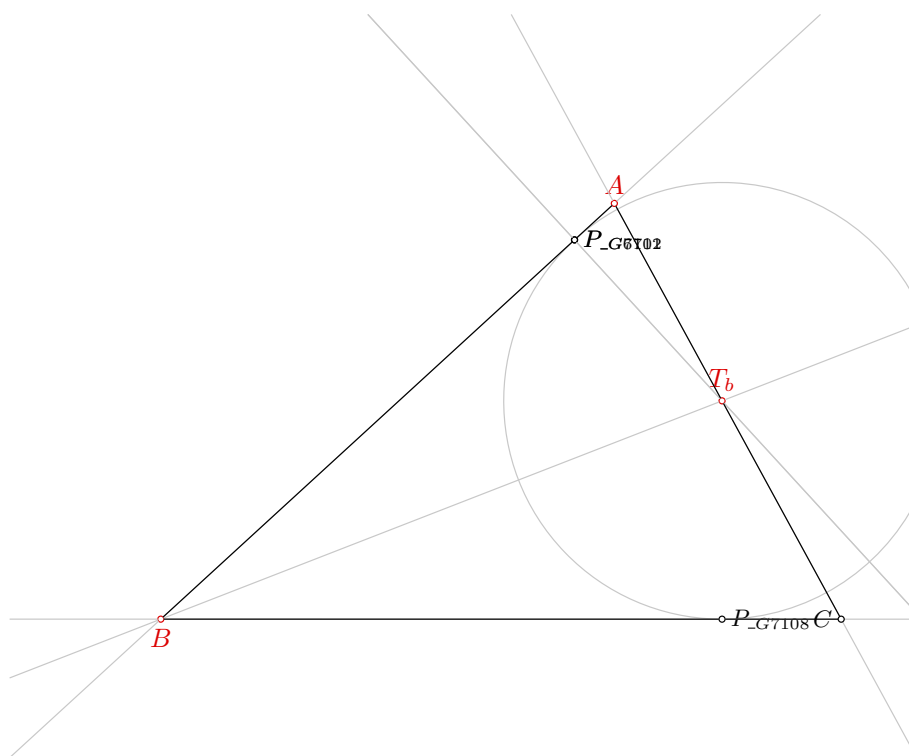


Figure 1: Illustration of the problem 0012

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.019 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $T_b=\neg T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{BAT_b} \neq S_{P_{G2530}AT_b}$  i.e., lines  $BP_{G2530}$  and  $AT_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $B=B$

NDG conditions are:

$S_{BAT_b} \neq S_{P_{G4041}AT_b}$  i.e., lines  $BP_{G4041}$  and  $AT_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $T_b=\neg T_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.



#### 4.3.3 Proving $T_b = \neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 16 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_b = \neg T_b$

Proving failed

# Problem 13

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 13: Given a point  $B$ , a point  $T_c$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $T_c$  are not the same;
2. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) ;
3. Using the point  $A$ , the point  $B$ , the point  $T_c$  and the line  $c$ , construct a point  $T'_c$  (rule W19);  
% NDG: points  $A$  and  $B$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $\text{midpoint}([A, T_c])$  are not the same;
4. Choose freely a point  $C$  on the circle  $k_{over}(T_c, T'_c)$  (rule WOncircle).

Non-degenerate conditions: points  $A$  and  $B$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $\text{midpoint}([A, T_c])$  are not the same.

Determination conditions: points  $B$  and  $T_c$  are not the same.

Rules used: [W02,W19,WOncircle2,WOnline1]

Lemmas used: [D25,GD01,L79]

Solving time: 105.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point T_{c} 55.38 72.43
point A 80 95

color 220 0 0
fontsize 9
```

```

cmark_b B
cmark_rt T_{c}
cmark_t A
color 0 0 0
fontsize 8

% DET: points B and T_{c} are not the same
% Constructing a line c which passes through point B and point T_{c}
line c B T_{c}

color 200 200 200
drawline c
color 0 0 0

% Choosing randomly a point A on the line BT_{c}
online A B T_{c}
cmark_t A
color 200 200 200
drawline B T_{c}
color 0 0 0

% NDG: points A and B are not the same; points B and T_{c} are not the same; points B and midpoint
      ([A,T_{c}]) are not the same
% Constructing a point P_{\_G2081} which is an image of the point B in a rotation around the point
      T_{c} for the angle 90
rotate P_{\_G2081} T_{c} 90 B
cmark_r P_{\_G2081}
color 200 200 200
drawarc_p T_{c} B 90
color 0 0 0

% Constructing a line L_{\_G2084} which passes through point T_{c} and point P_{\_G2081}
line L_{\_G2084} T_{c} P_{\_G2081}

color 200 200 200
drawline L_{\_G2084}
color 0 0 0

% Constructing midpoint P_{\_G2087} of the segment BP_{\_G2081}
midpoint P_{\_G2087} B P_{\_G2081}
cmark_r P_{\_G2087}

% Constructing a line L_{\_G2090} which passes through point A and point P_{\_G2087}
line L_{\_G2090} A P_{\_G2087}

color 200 200 200
drawline L_{\_G2090}
color 0 0 0

```

```

% Constructing a line  $L_{G2093}$  which passes through point B and point  $P_{G2087}$ 
line  $L_{G2093}$  B  $P_{G2087}$ 

color 200 200 200
drawline  $L_{G2093}$ 
color 0 0 0

% Constructing a point  $P_{G2096}$  which belongs to line  $L_{G2084}$  and line  $L_{G2090}$ 
intersec  $P_{G2096}$   $L_{G2084}$   $L_{G2090}$ 
cmark_r  $P_{G2096}$ 

% Constructing a point  $P_{G2099}$  which belongs to line  $L_{G2084}$  and line  $L_{G2093}$ 
intersec  $P_{G2099}$   $L_{G2084}$   $L_{G2093}$ 
cmark_r  $P_{G2099}$ 

% Constructing a line  $L_{G2102}$  which passes through point A and point  $P_{G2099}$ 
line  $L_{G2102}$  A  $P_{G2099}$ 

color 200 200 200
drawline  $L_{G2102}$ 
color 0 0 0

% Constructing a line  $L_{G2105}$  which passes through point B and point  $P_{G2096}$ 
line  $L_{G2105}$  B  $P_{G2096}$ 

color 200 200 200
drawline  $L_{G2105}$ 
color 0 0 0

% Constructing a point  $P_{G2108}$  which belongs to line  $L_{G2102}$  and line  $L_{G2105}$ 
intersec  $P_{G2108}$   $L_{G2102}$   $L_{G2105}$ 
cmark_r  $P_{G2108}$ 

% Constructing a line  $L_{G2111}$  which passes through point  $P_{G2087}$  and point  $P_{G2108}$ 
line  $L_{G2111}$   $P_{G2087}$   $P_{G2108}$ 

color 200 200 200
drawline  $L_{G2111}$ 
color 0 0 0

% Constructing a point  $T'_c$  which belongs to line  $L_{G2111}$  and line c
intersec  $T'_c$   $L_{G2111}$  c
cmark_r  $T'_c$ 

% Constructing midpoint  $P_{G2936}$  of the segment  $T_c T'_c$ 
midpoint  $P_{G2936}$   $T_c$   $T'_c$ 
cmark_r  $P_{G2936}$ 

% Choosing randomly a point C on the circle with center  $P_{G2936}$  through point  $T_c$ 
oncircle C  $P_{G2936}$   $T_c$ 

```

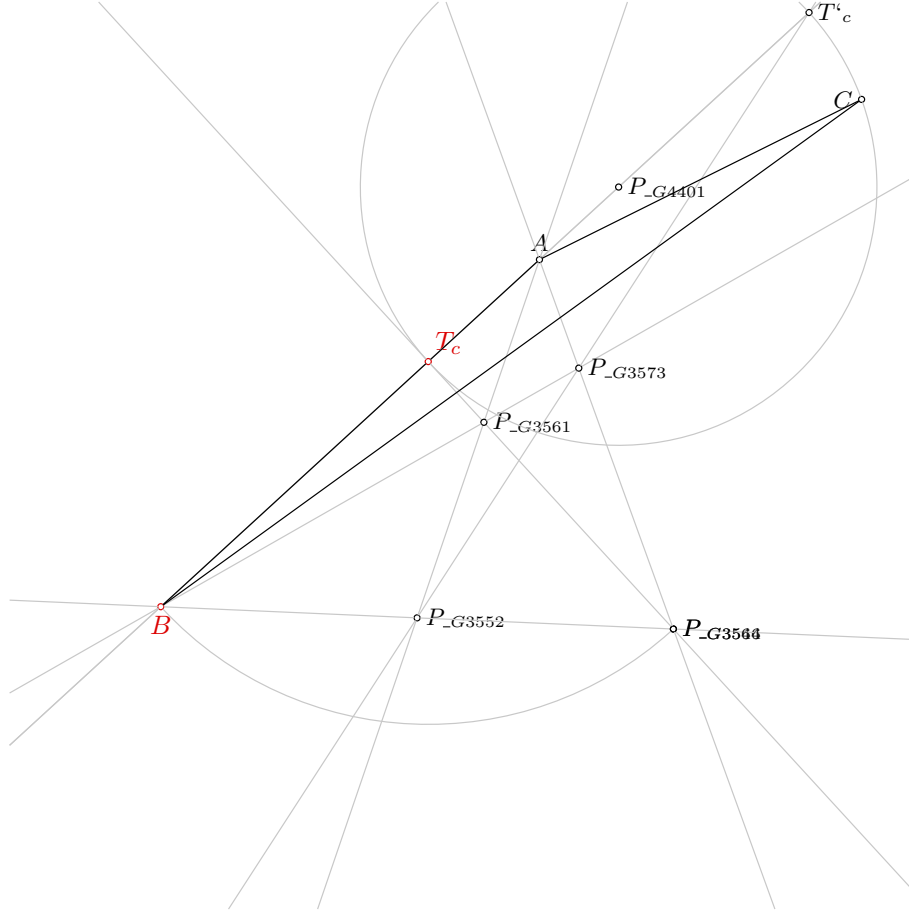


Figure 1: Illustration of the problem 0013

```

cmark_1 C
color 200 200 200
drawcircle P_{\_G2936} T_{c}
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions: points A and B are not the same; points B and T_{c} are not the same;
%                             points B and midpoint([A,T_{c}]) are not the same
% Determination conditions: points B and T_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.039 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_c = \neg T_c$

Proving failed

#### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.015 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{T_cAP_{G5031}} \neq S_{P_{G5025}AP_{G5031}}$  i.e., lines  $T_cP_{G5025}$  and  $AP_{G5031}$  are not parallel (construction based assumption)

$S_{T_cBP_{G5031}} \neq S_{P_{G5025}BP_{G5031}}$  i.e., lines  $T_cP_{G5025}$  and  $BP_{G5031}$  are not parallel (construction based assumption)

$S_{ABP_{G5040}} \neq S_{P_{G5043}BP_{G5040}}$  i.e., lines  $AP_{G5043}$  and  $BP_{G5040}$  are not parallel (construction based assumption)

$S_{P_{G5031}BT_c} \neq S_{P_{G5052}BT_c}$  i.e., lines  $P_{G5031}P_{G5052}$  and  $BT_c$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $T_c = \neg T_c$

Proving failed

#### 4.2.3 Proving $A=A$

NDG conditions are:

$S_{T_cAP_{G3474}} \neq S_{P_{G3468}AP_{G3474}}$  i.e., lines  $T_cP_{G3468}$  and  $AP_{G3474}$  are not parallel (construction based assumption)

$S_{T_cBP_{G3474}} \neq S_{P_{G3468}BP_{G3474}}$  i.e., lines  $T_cP_{G3468}$  and  $BP_{G3474}$  are not parallel (construction based assumption)

$S_{ABP_{G3483}} \neq S_{P_{G3486}BP_{G3483}}$  i.e., lines  $AP_{G3486}$  and  $BP_{G3483}$  are not parallel (construction based assumption)

$S_{P_{G3474}BT_c} \neq S_{P_{G3495}BT_c}$  i.e., lines  $P_{G3474}P_{G3495}$  and  $BT_c$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $B=B$**

Proving failed

#### **4.3.2 Proving $T_c=\neg T_c$**

Proving failed

#### **4.3.3 Proving $A=A$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $B=B$**

Proving failed

#### **4.4.2 Proving $T_c=\neg T_c$**

Proving failed

#### **4.4.3 Proving $A=A$**

Proving failed

# Problem 14

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 14: Given a point  $A$ , a point  $B$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $B$  are not the same;
2. Using the point  $I$  and the line  $c$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $c$ ;
3. Using the circle  $k(I, P_a)$ , the point  $A$ , the point  $I$  and the line  $c$ , construct a line  $b$  (rule W13); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
4. Using the circle  $k(I, P_a)$ , the point  $B$ , the point  $I$  and the line  $c$ , construct a line  $a$  (rule W13); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
5. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; point  $B$  is outside the circle  $k(I, P_a)$ ; point  $A$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $c$ .

Determination conditions: lines  $b$  and  $a$  are not the same; points  $A$  and  $B$  are not the same.

Rules used: [W02,W03,W11,W13]

Lemmas used: [D27,GD01,L59,L60,L61]

Solving time: 17.0 seconds.

### 3.2 Construction in GCLC language



```

dim 120 120

point A 80 95
point B 20 40
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_t A
cmark_b B
cmark_b I
color 0 0 0
fontsize 8

% DET: points A and B are not the same
% Constructing a line c which passes through point A and point B
line c A B

color 200 200 200
drawline c
color 0 0 0

% NDG: point I is not incident to the line c
% Constructing a point  $P_{\setminus G7090}$  which is a foot of the point I on the line c
foot  $P_{\setminus G7090}$  I c
cmark_r  $P_{\setminus G7090}$ 
color 200 200 200
drawline I  $P_{\setminus G7090}$ 
color 0 0 0

% Constructing a circle  $k(I, P_{\setminus a})$  whose center is at point I and which passes through point  $P_{\setminus G7090}$ 
circle k(I,  $P_{\setminus a}$ ) I  $P_{\setminus G7090}$ 

color 200 200 200
drawcircle k(I,  $P_{\setminus a}$ )
color 0 0 0

% NDG: point A is outside the circle  $k(I, P_{\setminus a})$ 
% Constructing a point  $P_{\setminus G7297}$  which is a foot of the point I on the line c
foot  $P_{\setminus G7297}$  I c
cmark_r  $P_{\setminus G7297}$ 
color 200 200 200
drawline I  $P_{\setminus G7297}$ 
color 0 0 0

% Constructing a line  $L_{\setminus G7300}$  which passes through point I and point A
line  $L_{\setminus G7300}$  I A

color 200 200 200
drawline  $L_{\setminus G7300}$ 

```

```

color 0 0 0

% Constructing a point  $P_{\backslash\_G7303}$  which is an image of the point  $P_{\backslash\_G7297}$  in the symmetry to
point/line  $L_{\backslash\_G7300}$ 
sim  $P_{\backslash\_G7303}$   $L_{\backslash\_G7300}$   $P_{\backslash\_G7297}$ 
cmark_r  $P_{\backslash\_G7303}$ 

% Constructing a line  $b$  which passes through point  $A$  and point  $P_{\backslash\_G7303}$ 
line b A  $P_{\backslash\_G7303}$ 

color 200 200 200
drawline b
color 0 0 0

% NDG: point  $B$  is outside the circle  $k(I, P_{\backslash\_a})$ 
% Constructing a point  $P_{\backslash\_G7632}$  which is a foot of the point  $I$  on the line  $c$ 
foot  $P_{\backslash\_G3043}$  I c
cmark_r  $P_{\backslash\_G3043}$ 
color 200 200 200
drawline I  $P_{\backslash\_G3043}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G7635}$  which passes through point  $I$  and point  $B$ 
line  $L_{\backslash\_G3046}$  I B

color 200 200 200
drawline  $L_{\backslash\_G3046}$ 
color 0 0 0

% Constructing a point  $P_{\backslash\_G3049}$  which is an image of the point  $P_{\backslash\_G3043}$  in the symmetry to
point/line  $L_{\backslash\_G3046}$ 
sim  $P_{\backslash\_G3049}$   $L_{\backslash\_G3046}$   $P_{\backslash\_G3043}$ 
cmark_r  $P_{\backslash\_G3049}$ 

% Constructing a line  $a$  which passes through point  $B$  and point  $P_{\backslash\_G3049}$ 
line a B  $P_{\backslash\_G3049}$ 

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $b$  and  $a$  are not parallel% DET: lines  $b$  and  $a$  are not the same
% Constructing a point  $C$  which belongs to line  $b$  and line  $a$ 
intersec C b a
cmark_l C

drawsegment A B
drawsegment A C

```

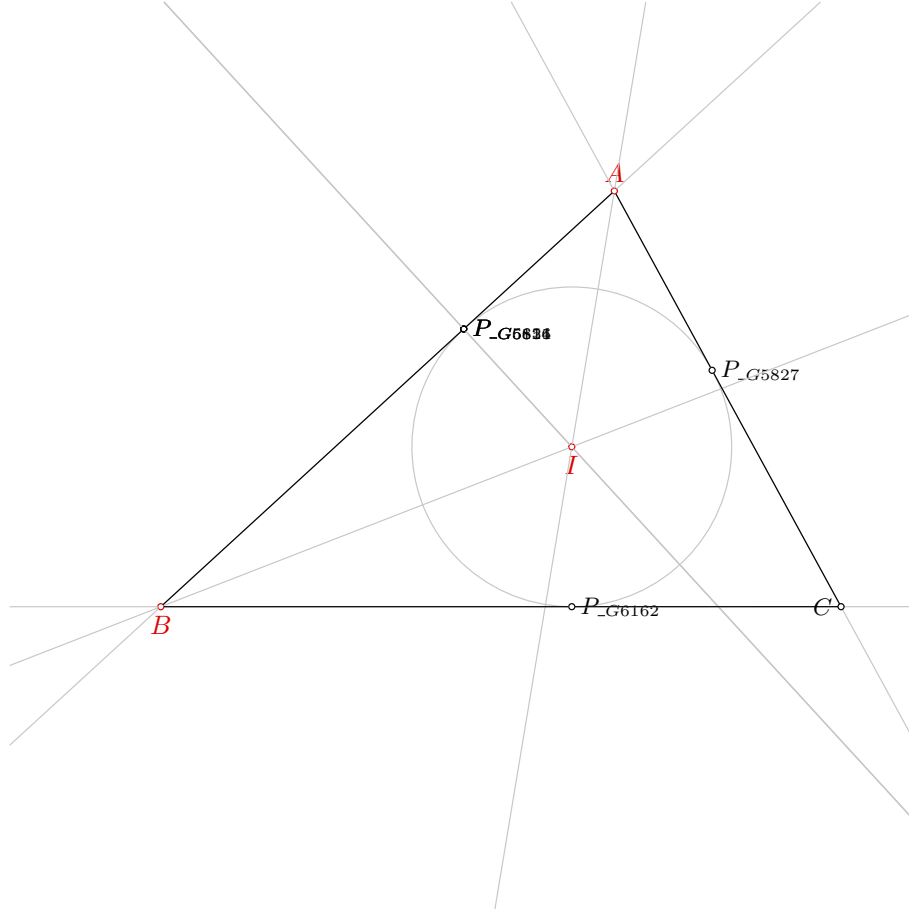


Figure 1: Illustration of the problem 0014

drawsegment B C

*% Non-degenerate conditions: lines b and a are not parallel; point B is outside the circle  $k(I, P_{\{a\}})$ ; point A is outside the circle  $k(I, P_{\{a\}})$ ; point I is not incident to the line c*  
*% Determination conditions: lines b and a are not the same; points A and B are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

Proving failed

#### 4.1.2 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{ABP_{G7168}} \neq S_{P_{G6833}BP_{G7168}}$  i.e., lines  $AP_{G6833}$  and  $BP_{G7168}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $B=B$

NDG conditions are:

$S_{ABP_{G4483}} \neq S_{P_{G4148}BP_{G4483}}$  i.e., lines  $AP_{G4148}$  and  $BP_{G4483}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I=_I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 23 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.3 Proving $I=_I$

Proving failed

# Problem 15

*Generated automatically by ArgoTriCS  
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## 1 Problem

Problem 15: Given a point  $C$ , a point  $O$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
2. Choose freely a point  $A$  on the circle  $k(O, C)$  (rule WOncircle);
3. Choose freely a point  $B$  on the circle  $k(O, C)$  (rule WOncircle).

Non-degenerate conditions: points  $C$  and  $O$  are not the same.

Determination conditions: .

Rules used: [W06,WOncircle1]

Lemmas used: [D26]

Solving time: 457.7 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point O 65 51.14
point A 80 95

color 220 0 0
fontsize 9

cmark_l C
cmark_t O
cmark_t A
color 0 0 0
```

```

fontsize 8

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Choosing randomly a point A on the circle with center O through point C
oncircle A O C
cmark_t A
color 200 200 200
drawcircle O C
color 0 0 0

% Choosing randomly a point B on the circle with center O through point A
oncircle B O A
cmark_b B
color 200 200 200
drawcircle O A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: points C and O are not the same
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.02 seconds.

**NDG conditions** There are no NDG conditions for this theorem

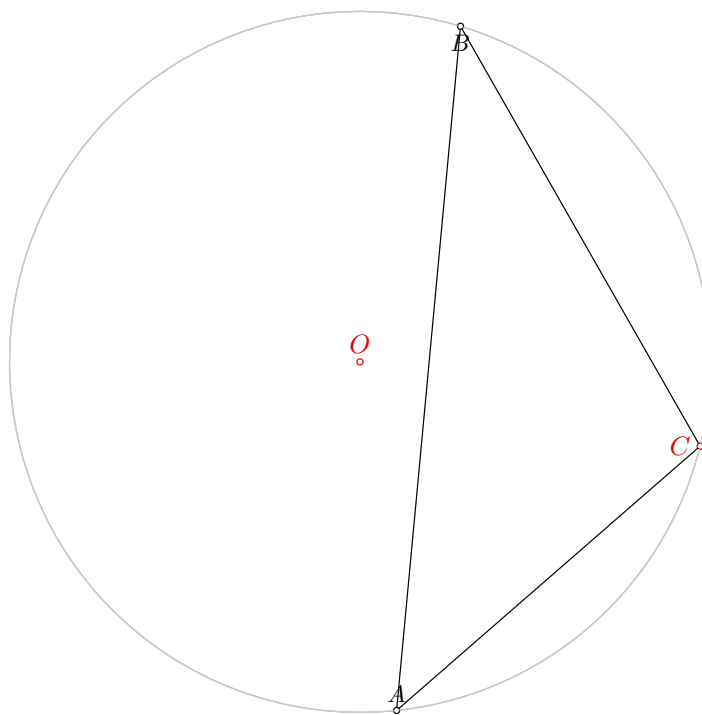


Figure 1: Illustration of the problem 0015



### 4.1.2 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 31 terms.

**Time Complexity:** Time spent by the prover is 0.417 seconds.

**NDG conditions** Points  $A$ ,  $B$  and  $C$  are not collinear

Line through points  $O$  and  $C$  is not perpendicular to line through points  $C$  and  $B$

### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{_M_aBC} \neq 0$  i.e., points  $_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$  i.e., points  $_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a-M_bF_{_m_b}^1} \neq S_{F_{_m_a}^0-M_bF_{_m_b}^1}$  i.e., lines  $_M_aF_{_m_a}^0$  and  $_M_bF_{_m_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $O=_O$

Proving failed

### 4.2.3 Proving $A=A$

NDG conditions are:

$S_{_M_aBC} \neq 0$  i.e., points  $_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$  i.e., points  $_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a-M_bF_{_m_b}^1} \neq S_{F_{_m_a}^0-M_bF_{_m_b}^1}$  i.e., lines  $_M_aF_{_m_a}^0$  and  $_M_bF_{_m_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $O=_O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 80 terms.

**Time Complexity:** Time spent by the prover is 0.030 seconds. There are no ndg conditions.

#### 4.3.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $O=_O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 42 terms.

**Time Complexity:** Time spent by the prover is 0.150 seconds. There are no ndg conditions.

#### 4.4.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 16

*Generated automatically by ArgoTriCS  
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## 1 Problem

Problem 16: Given a point  $A$ , a point  $C$  and a point  $M_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D21, GL03, GL04]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
```

```
point C 110 40
```

```
point M_{a} 65 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_t A
```

```
cmark_l C
```

```
cmark_r M_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point B such that CB/CM_{a}=2
```

```
towards B C M_{a} 2
```

```
cmark_b B
```

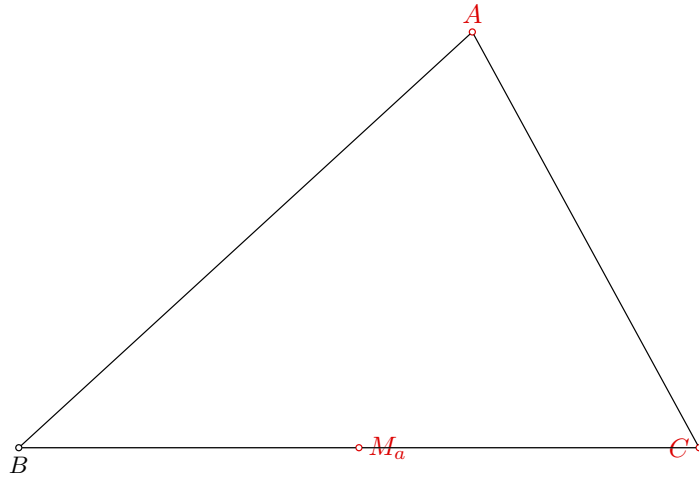


Figure 1: Illustration of the problem 0016

```
color 200 200 200
drawsegment C B
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions:
% Determination conditions:
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.033 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.033 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $M_a=_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.055 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

There are no ndg conditions.

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $C=C$

There are no ndg conditions.

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $M_a=_M_a$

There are no ndg conditions.

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $M_a=-M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 19 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_a=-M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 42 terms.

**Time Complexity:** Time spent by the prover is 0.090 seconds. There are no ndg conditions.

# Problem 17

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 17: Given a point  $C$ , a point  $M_b$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is redundant.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); ;
2. Choose freely a point  $B$  (rule free).

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01,free]

Lemmas used: [D22,GL03]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point M_{b} 95 67.5
point A 80 95
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_lt M_{b}
cmark_t A
color 0 0 0
fontsize 8
```

```
% Constructing a point A such that CA/CM_{b}=2
```

```

towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

% Constructing a free point B
point B 20 40

cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.034 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b = M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.062 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.022 seconds.

**NDG conditions** There are no NDG conditions for this theorem



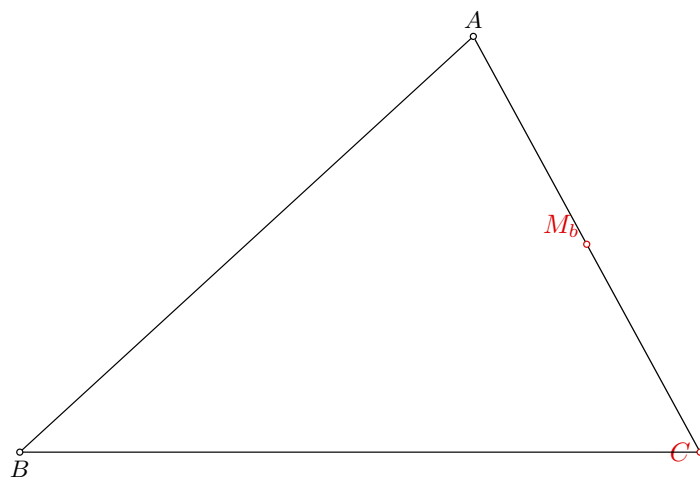


Figure 1: Illustration of the problem 0017

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

There are no ndg conditions.  
Total number of proof steps: 1  
Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_b=_M M_b$

There are no ndg conditions.  
Total number of proof steps: 41  
Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $A=A$

There are no ndg conditions.  
Total number of proof steps: 1  
Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_b=_M M_b$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $A=A$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $M_b=_M M_b$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 18

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 18: Given a point  $A$ , a point  $C$  and a point  $M_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D20, GL03]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point C 110 40
point M_{c} 50 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_l C
cmark_lt M_{c}
color 0 0 0
fontsize 8
```

```
% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
```

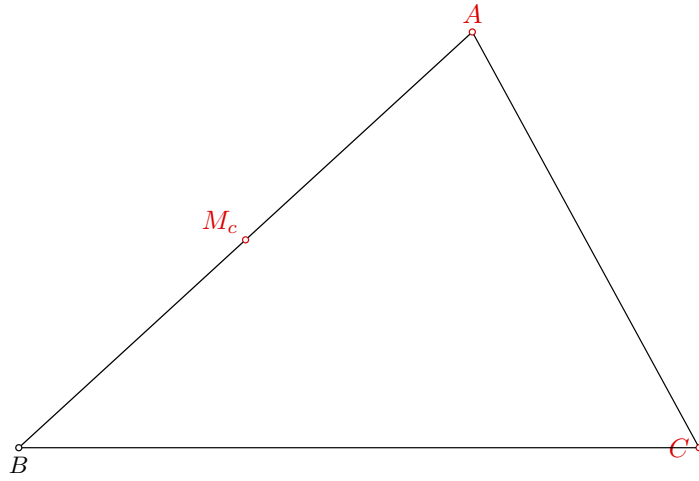


Figure 1: Illustration of the problem 0018

```
color 200 200 200
drawsegment A B
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions:
% Determination conditions:
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.035 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.033 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $M_c=_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.053 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

There are no ndg conditions.

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $C=C$

There are no ndg conditions.

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $M_c=_M_c$

There are no ndg conditions.

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $M_c=_M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_c=_M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

# Problem 19

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 19: Given a point  $A$ , a point  $C$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $G$ , construct a point  $M_c$  (rule W01); ;
2. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D20,GL03,L57]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point C 110 40
point G 70 58.33
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_l C
cmark_t G
color 0 0 0
fontsize 8
```

```
% Constructing a point  $M_{\{c\}}$  such that  $CM_{\{c\}}/CG=1.5$ 
```



```

towards M_{c} C G 1.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $G=_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 24 terms.

**Time Complexity:** Time spent by the prover is 0.371 seconds.

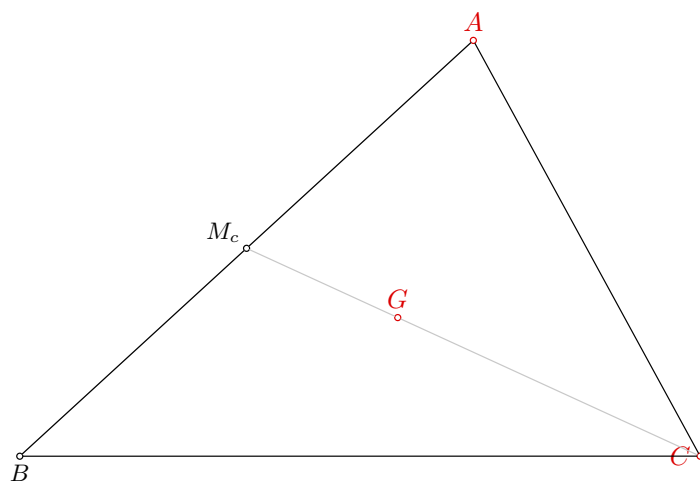


Figure 1: Illustration of the problem 0019

**NDG conditions** Line through points  $A$  and  $M_a$  is not parallel with line through points  $B$  and  $M_b$   
Points  $C$ ,  $M_a$  and  $M_b$  are not collinear

## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{AB-M_b} \neq S_{M_a B-M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $C=C$

NDG conditions are:

$S_{AB-M_b} \neq S_{M_a B-M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $G=_G$

NDG conditions are:

$S_{AB-M_b} \neq S_{M_a B-M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (cancellation assumption)

Total number of proof steps: 675

Time spent by the prover: 0.200 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $G=_G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 38 terms.

**Time Complexity:** Time spent by the prover is 0.090 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $G=_G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 223 terms.

**Time Complexity:** Time spent by the prover is 3.390 seconds. There are no ndg conditions.

# Problem 20

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 20: Given a point  $C$ , a point  $H_a$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $H_a$  are not the same;
2. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
3. Choose freely a point  $A$  on the line  $h_a$  (rule WOnline2);
4. Choose freely a point  $B$  on the line  $a$  (rule WOnline1) .

Non-degenerate conditions: .

Determination conditions: points  $C$  and  $H_a$  are not the same.

Rules used: [W02,W10b,WOnline1,WOnline2]

Lemmas used: [D5,D8,GD01]

Solving time: 1290.8 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{a} 80 40
point A 80 95

color 220 0 0
fontsize 9

cmark_l C
cmark_r H_{a}
```

```

cmark_t A
color 0 0 0
fontsize 8

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% Generating random value V[_G7442]
random V[_G7442]

% Calculating value V[_G7463] using formula V[_G7442]*20
expression V[_G7463] { V[_G7442]*20 }

% Constructing a point A which is a point for which holds  $H_{a}A = V[_G7463]$  and angle  $CH_{a}A = 90$ 
turtle A C H_{a} 90 V[_G7463]
cmark_t A

% Choosing randomly a point B on the line CH_{a}
online B C H_{a}
cmark_b B
color 200 200 200
drawline C H_{a}
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions: points C and H_{a} are not the same

```

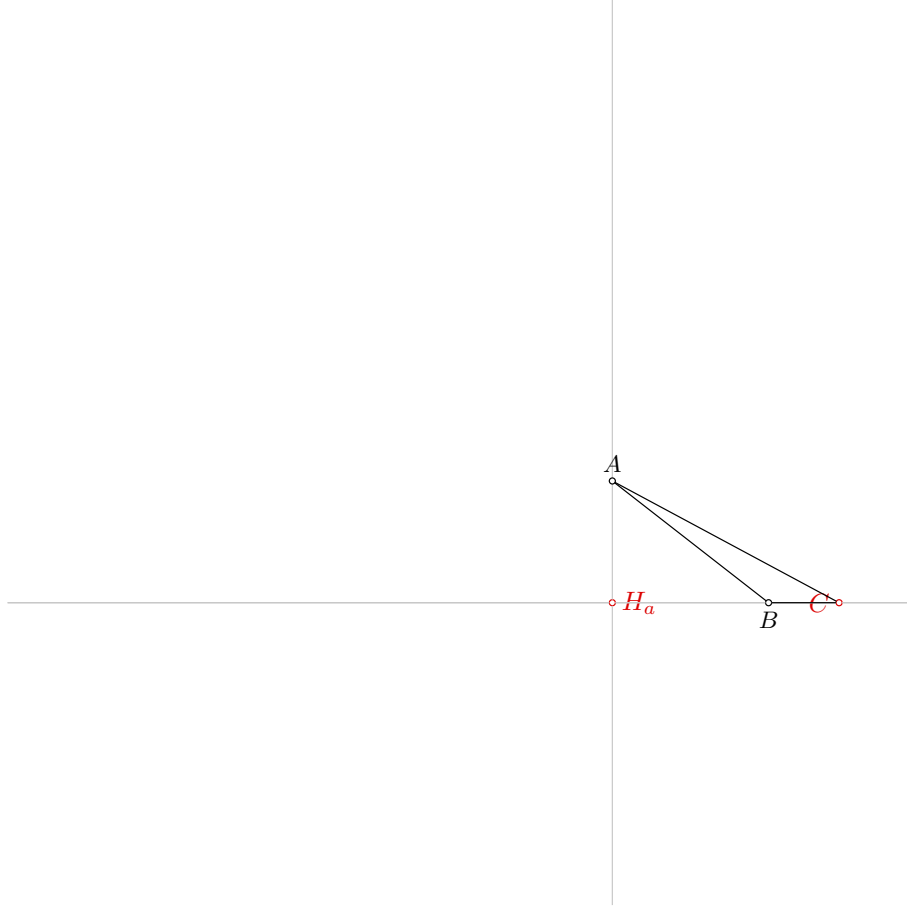


Figure 1: Illustration of the problem 0020

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a=H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.008 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.018 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^2} \neq S_{CAF_{h_a}^2}$  i.e., lines  $BC$  and  $AF_{h_a}^2$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_a=\neg H_a$

Proving failed

### 4.2.3 Proving $A=A$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_a=\neg H_a$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $A=A$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.



#### 4.4.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 31.670 seconds. There are no ndg conditions.

#### 4.4.3 Proving $A = A$

Proving failed

# Problem 21

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 21: Given a point  $C$ , a point  $H_b$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $H_b$  are not the same;
2. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) ;
3. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
4. Choose freely a point  $B$  on the line  $h_b$  (rule WOnline2).

Non-degenerate conditions: .

Determination conditions: points  $C$  and  $H_b$  are not the same.

Rules used: [W02,W10b,WOnline1,WOnline2]

Lemmas used: [D6,D9,GD01]

Solving time: 239.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{b} 89.36 77.83
point A 80 95

color 220 0 0
fontsize 9

cmark_1 C
cmark_1 H_{b}
```

```

cmark_t A
color 0 0 0
fontsize 8

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% Choosing randomly a point A on the line CH_{b}
online A C H_{b}
cmark_t A
color 200 200 200
drawline C H_{b}
color 0 0 0

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0

% Generating random value V[_G6131]
random V[_G6131]

% Calculating value V[_G6152] using formula V[_G6131]*20
expression V[_G6152] { V[_G6131]*20 }

% Constructing a point B which is a point for which holds  $H_{b}B = V[_G6152]$  and angle  $AH_{b}B = 90$ 
turtle B A H_{b} 90 V[_G6152]
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions: points C and H_{b} are not the same

```

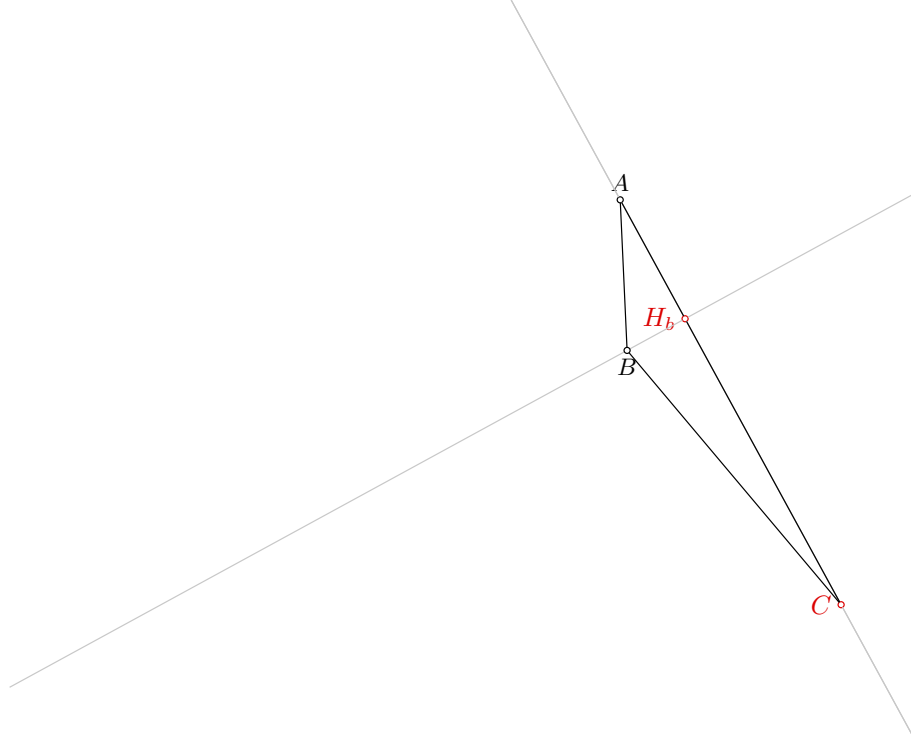


Figure 1: Illustration of the problem 0021

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b=H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.008 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.019 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^2_{\neg h_b}} \neq S_{CBF^2_{\neg h_b}}$  i.e., lines  $AC$  and  $BF^2_{\neg h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_b=\neg H_b$

Proving failed

### 4.2.3 Proving $A=A$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^2_{\neg h_b}} \neq S_{CBF^2_{\neg h_b}}$  i.e., lines  $AC$  and  $BF^2_{\neg h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_b=\neg H_b$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $H_b=-H_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 31.360 seconds. There are no ndg conditions.

### 4.4.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 22

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 22: Given a point  $C$ , a point  $H_c$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_c$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H_c$  are not the same;
2. Using the point  $H_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
3. Choose freely a point  $A$  on the line  $c$  (rule WOnline2);
4. Choose freely a point  $B$  on the line  $c$  (rule WOnline1) .

Non-degenerate conditions: .

Determination conditions: points  $C$  and  $H_c$  are not the same.

Rules used: [W02,W10a,WOnline1,WOnline2]

Lemmas used: [D10,D7]

Solving time: 1324.8 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{c} 68.91 84.83
point A 80 95

color 220 0 0
fontsize 9

cmark_l C
cmark_rt H_{c}
```

```

cmark_t A
color 0 0 0
fontsize 8

% DET: points C and H_{c} are not the same
% Constructing a line h_{c} which passes through point C and point H_{c}
line h_{c} C H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0

% Generating random value V[_G5252]
random V[_G5252]

% Calculating value V[_G5273] using formula V[_G5252]*20
expression V[_G5273] { V[_G5252]*20 }

% Constructing a point A which is a point for which holds  $H_{c}A = V[_G5273]$  and angle  $CH_{c}A = 90$ 
turtle A C H_{c} 90 V[_G5273]
cmark_t A

% Choosing randomly a point B on the line AH_{c}
online B A H_{c}
cmark_b B
color 200 200 200
drawline A H_{c}
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions: points C and H_{c} are not the same

```



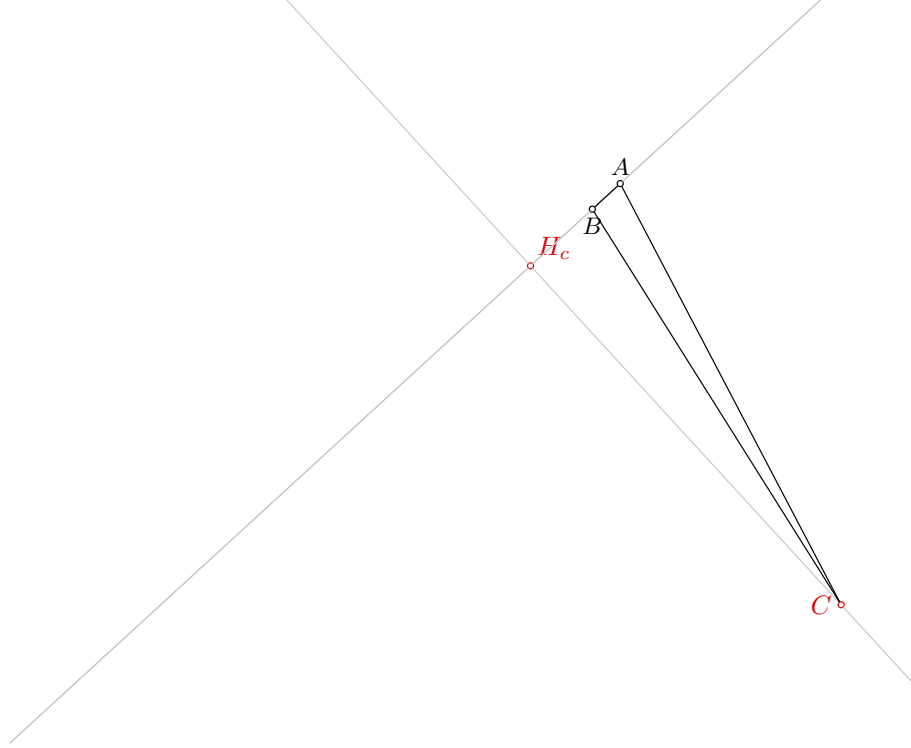


Figure 1: Illustration of the problem 0022

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.041 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c=H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{-h_c}^2} \neq S_{BCF_{-h_c}^2}$  i.e., lines  $AB$  and  $CF_{-h_c}^2$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_c=_Hc$

Proving failed

### 4.2.3 Proving $A=A$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

Proving failed

### 4.3.2 Proving $H_c=_Hc$

Proving failed

### 4.3.3 Proving $A=A$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

Proving failed

### 4.4.2 Proving $H_c=_Hc$

Proving failed

### 4.4.3 Proving $A=A$

Proving failed

# Problem 23

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 23: Given a point  $A$ , a point  $C$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $C$ , construct a point  $M_b$  (rule W01); ;
2. Using the point  $A$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H$  are not the same;
3. Using the point  $C$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H$  are not the same;
4. Using the point  $A$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $A$  and  $M_b$  are not the same;
5. Using the circle  $k(M_b, C)$ , the line  $h_a$ , the point  $M_b$  and the point  $A$ , construct a point  $H_a$  (rule W05); % NDG: line  $h_a$  and circle  $k(M_b, C)$  intersect % DET: points  $A$  and  $H_a$  must be different;
6. Using the point  $H_a$  and the point  $C$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $C$  are not the same;
7. Using the circle  $k(M_b, C)$ , the line  $h_c$ , the point  $M_b$  and the point  $C$ , construct a point  $H_c$  (rule W05); % NDG: line  $h_c$  and circle  $k(M_b, C)$  intersect % DET: points  $C$  and  $H_c$  must be different;
8. Using the point  $H_c$  and the point  $A$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $A$  are not the same;
9. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; line  $h_c$  and circle  $k(M_b, C)$  intersect; line  $h_a$  and circle  $k(M_b, C)$  intersect; points  $A$  and  $M_b$  are not the same.

Determination conditions: lines  $a$  and  $c$  are not the same; points  $H_c$  and  $A$  are not the same; points  $C$  and  $H_c$  must be different; points  $H_a$  and  $C$  are not the same; points  $A$  and  $H_a$  must be different; points  $C$  and  $H$  are not the same; points  $A$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D22,D3,D5,D7,D8,GD01,GD02,GL04,L3,L43,L44,L45]

Solving time: 4.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
```

```
point C 110 40
```

```
point H 80 72.73
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_t A
```

```
cmark_l C
```

```
cmark_rt H
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point M_{b} such that AM_{b}/AC=0.5
```

```
towards M_{b} A C 0.5
```

```
cmark_lt M_{b}
```

```
color 200 200 200
```

```
drawsegment A C
```

```
color 0 0 0
```

```
% DET: points A and H are not the same
```

```
% Constructing a line h_{a} which passes through point A and point H
```

```
line h_{a} A H
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```
% DET: points C and H are not the same
```

```
% Constructing a line h_{c} which passes through point C and point H
```

```
line h_{c} C H
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```

% NDG: points A and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point A
circle k(M_{b},C) M_{b} A

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line h_{a} and circle k(M_{b},C) intersect% DET: points A and H_{a} must be different
% Constructing a point P_{\G7361} which is a foot of the point M_{b} on the line h_{a}
foot P_{\G7361} M_{b} h_{a}
cmark_r P_{\G7361}
color 200 200 200
drawline M_{b} P_{\G7361}
color 0 0 0

% Constructing a point H_{a} which is an image of the point A in the symmetry to point/line P_{\G
7361}
sim H_{a} P_{\G7361} A
cmark_r H_{a}

% DET: points H_{a} and C are not the same
% Constructing a line a which passes through point H_{a} and point C
line a H_{a} C

color 200 200 200
drawline a
color 0 0 0

% NDG: line h_{c} and circle k(M_{b},C) intersect% DET: points C and H_{c} must be different
% Constructing a point P_{\G7599} which is a foot of the point M_{b} on the line h_{c}
foot P_{\G7599} M_{b} h_{c}
cmark_r P_{\G7599}
color 200 200 200
drawline M_{b} P_{\G3204}
color 0 0 0

% Constructing a point H_{c} which is an image of the point C in the symmetry to point/line P_{\G
7599}
sim H_{c} P_{\G3204} C
cmark_rt H_{c}

% DET: points H_{c} and A are not the same
% Constructing a line c which passes through point H_{c} and point A
line c H_{c} A

color 200 200 200
drawline c
color 0 0 0

```

```

% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and c are not parallel; line h_{c} and circle k(M_{b},C)
% intersect; line h_{a} and circle k(M_{b},C) intersect; points A and M_{b} are not the same
% Determination conditions: lines a and c are not the same; points H_{c} and A are not the same;
% points C and H_{c} must be different; points H_{a} and C are not the same; points A and H_{a}
% must be different; points C and H are not the same; points A and H are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $H=_H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 36 terms.

**Time Complexity:** Time spent by the prover is 1.921 seconds.

**NDG conditions** Points  $M_b$  and  $H$  are not identical

Points  $M_b$  and  $H$  are not identical

Points  $A$ ,  $C$  and  $H$  are not collinear

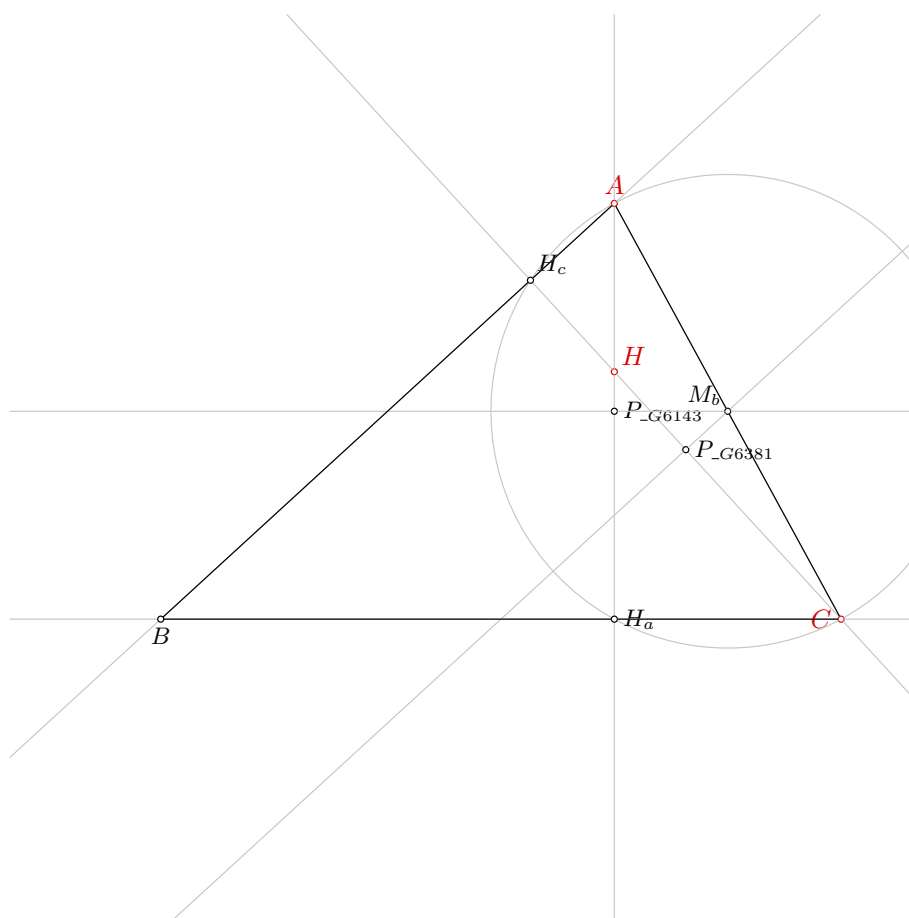


Figure 1: Illustration of the problem 0023

Points  $A$ ,  $C$  and  $H$  are not collinear

Line through points  $A$  and  $H_c$  is not parallel with line through points  $C$  and  $H_a$

Points  $A$  and  $H_a$  are not identical

Points  $M_b$ ,  $A$  and  $B$  are not collinear

## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_a H_c A} \neq S_{C H_c A}$  i.e., lines  $H_a C$  and  $H_c A$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a} BF^1_{-h_b}}$  i.e., lines  $AF^0_{-h_a}$  and  $BF^1_{-h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $C=C$

NDG conditions are:

$S_{H_a H_c A} \neq S_{C H_c A}$  i.e., lines  $H_a C$  and  $H_c A$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a} BF^1_{-h_b}}$  i.e., lines  $AF^0_{-h_a}$  and  $BF^1_{-h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $H=_H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $H=_H$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 9 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.



## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.3 Proving $H=_H$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 24

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 24: Given a point  $A$ , a point  $C$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $C$  are not the same;
2. Using the point  $C$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $T_a$  are not the same;
3. Using the point  $T_a$  and the line  $b$ , construct a circle  $k(T_a, foot[T_a, c])$  (rule W11); % NDG: point  $T_a$  is not incident to the line  $b$ ;
4. Using the circle  $k(T_a, foot[T_a, c])$ , the point  $A$ , the point  $T_a$  and the line  $b$ , construct a line  $c$  (rule W13); % NDG: point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ;
5. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ; point  $T_a$  is not incident to the line  $b$ .

Determination conditions: lines  $c$  and  $a$  are not the same; points  $C$  and  $T_a$  are not the same; points  $A$  and  $C$  are not the same.

Rules used: [W02,W03,W11,W13]

Lemmas used: [D23,GD01,GD02,GL10,GL11,L8]

Solving time: 4.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point C 110 40
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_t A
cmark_l C
cmark_rb T_{a}
color 0 0 0
fontsize 8

% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C

color 200 200 200
drawline b
color 0 0 0

% DET: points C and T_{a} are not the same
% Constructing a line a which passes through point C and point T_{a}
line a C T_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: point T_{a} is not incident to the line b
% Constructing a point P_{\_G4339} which is a foot of the point T_{a} on the line b
foot P_{\_G4339} T_{a} b
cmark_r P_{\_G4339}
color 200 200 200
drawline T_{a} P_{\_G4339}
color 0 0 0

% Constructing a circle k(T_{a},foot[T_{a},c]) whose center is at point T_{a} and which passes
    through point P_{\_G4339}
circle k(T_{a},foot[T_{a},c]) T_{a} P_{\_G4339}

color 200 200 200
drawcircle k(T_{a},foot[T_{a},c])
color 0 0 0

% NDG: point A is outside the circle k(T_{a},foot[T_{a},c])
```

```

% Constructing a point  $P_{\{a\}}$  which is a foot of the point  $T_{\{a\}}$  on the line  $b$ 
foot  $P_{\{a\}}$   $T_{\{a\}}$   $b$ 
cmark_r  $P_{\{a\}}$ 
color 200 200 200
drawline  $T_{\{a\}}$   $P_{\{a\}}$ 
color 0 0 0

% Constructing a line  $L_{\{a\}}$  which passes through point  $T_{\{a\}}$  and point  $A$ 
line  $L_{\{a\}}$   $T_{\{a\}}$   $A$ 

color 200 200 200
drawline  $L_{\{a\}}$ 
color 0 0 0

% Constructing a point  $P_{\{a\}}$  which is an image of the point  $P_{\{a\}}$  in the symmetry to
    point/line  $L_{\{a\}}$ 
sim  $P_{\{a\}}$   $L_{\{a\}}$   $P_{\{a\}}$ 
cmark_r  $P_{\{a\}}$ 

% Constructing a line  $c$  which passes through point  $A$  and point  $P_{\{a\}}$ 
line  $c$   $A$   $P_{\{a\}}$ 

color 200 200 200
drawline  $c$ 
color 0 0 0

% NDG: lines  $c$  and  $a$  are not parallel% DET: lines  $c$  and  $a$  are not the same
% Constructing a point  $B$  which belongs to line  $c$  and line  $a$ 
intersec  $B$   $c$   $a$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(T_{\{a\}},$ 
     $foot[T_{\{a\}},c])$ ; point  $T_{\{a\}}$  is not incident to the line  $b$ 
% Determination conditions: lines  $c$  and  $a$  are not the same; points  $C$  and  $T_{\{a\}}$  are not the same;
    points  $A$  and  $C$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

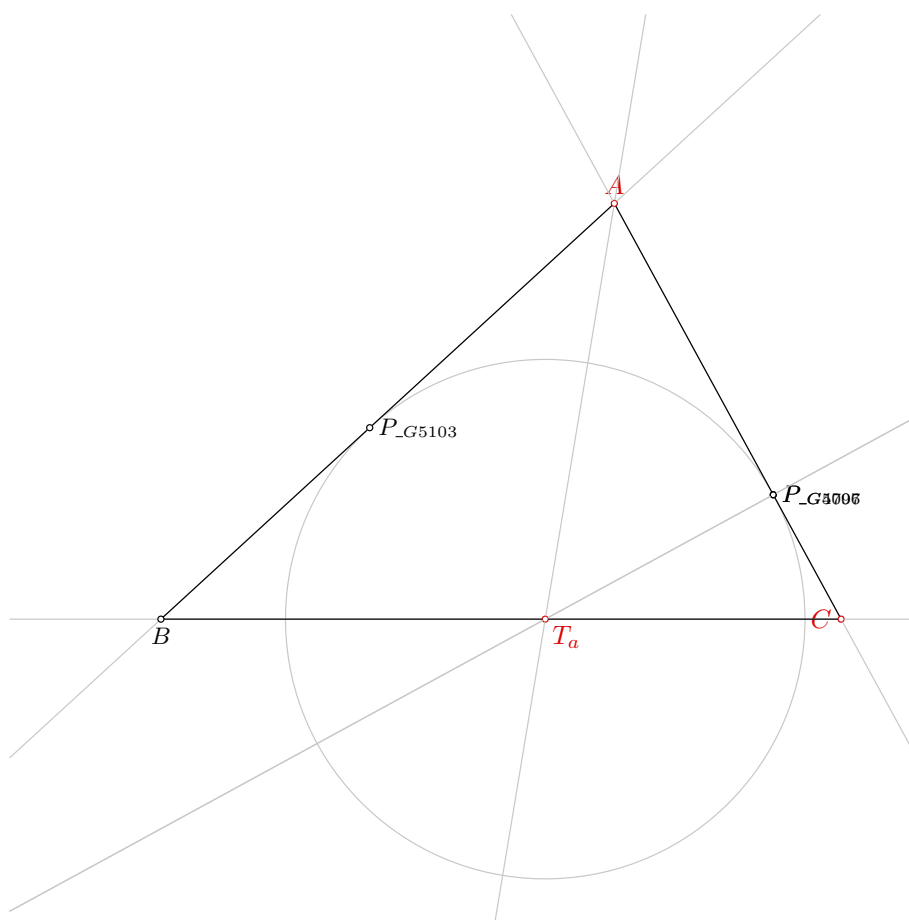


Figure 1: Illustration of the problem 0024

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $T_a=_T T_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{ACT_a} \neq S_{P_{G6052}CT_a}$  i.e., lines  $AP_{G6052}$  and  $CT_a$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $C=C$

Proving failed

#### 4.2.3 Proving $T_a=_T T_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $C=C$

Proving failed

#### 4.3.3 Proving $T_a=_T T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 30 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $C=C$

Proving failed

### 4.4.3 Proving $T_a=T_a$

Proving failed

# Problem 25

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 25: Given a point  $C$ , a point  $T_b$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $T_b$  are not the same;
2. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) ;
3. Using the point  $A$ , the point  $C$ , the point  $T_b$  and the line  $b$ , construct a point  $T'_b$  (rule W19);  
% NDG: points  $A$  and  $C$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $\text{midpoint}([A, T_b])$  are not the same;
4. Choose freely a point  $B$  on the circle  $k_{over}(T_b, T'_b)$  (rule WOncircle).

Non-degenerate conditions: points  $A$  and  $C$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $\text{midpoint}([A, T_b])$  are not the same.

Determination conditions: points  $C$  and  $T_b$  are not the same.

Rules used: [W02,W19,WOncircle2,WOnline1]

Lemmas used: [D24,GD01,L78]

Solving time: 114.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point T_{b} 94.25 68.88
point A 80 95

color 220 0 0
fontsize 9
```



```

cmark_l C
cmark_t T_{b}
cmark_t A
color 0 0 0
fontsize 8

% DET: points C and T_{b} are not the same
% Constructing a line b which passes through point C and point T_{b}
line b C T_{b}

color 200 200 200
drawline b
color 0 0 0

% Choosing randomly a point A on the line CT_{b}
online A C T_{b}
cmark_t A
color 200 200 200
drawline C T_{b}
color 0 0 0

% NDG: points A and C are not the same; points C and T_{b} are not the same; points C and midpoint
      ([A,T_{b}]) are not the same
% Constructing a point P_{\_G4407} which is an image of the point C in a rotation around the point
      T_{b} for the angle 90
rotate P_{\_G4407} T_{b} 90 C
cmark_r P_{\_G4407}
color 200 200 200
drawarc_p T_{b} C 90
color 0 0 0

% Constructing a line L_{\_G4410} which passes through point T_{b} and point P_{\_G4407}
line L_{\_G4410} T_{b} P_{\_G4407}

color 200 200 200
drawline L_{\_G4410}
color 0 0 0

% Constructing midpoint P_{\_G4413} of the segment CP_{\_G4407}
midpoint P_{\_G4413} C P_{\_G4407}
cmark_r P_{\_G4413}

% Constructing a line L_{\_G4416} which passes through point A and point P_{\_G4413}
line L_{\_G4416} A P_{\_G4413}

color 200 200 200
drawline L_{\_G4416}
color 0 0 0

```

```

% Constructing a line  $L_{\backslash\_G4419}$  which passes through point  $C$  and point  $P_{\backslash\_G4413}$ 
line  $L_{\backslash\_G4419}$   $C$   $P_{\backslash\_G4413}$ 

color 200 200 200
drawline  $L_{\backslash\_G4419}$ 
color 0 0 0

% Constructing a point  $P_{\backslash\_G4422}$  which belongs to line  $L_{\backslash\_G4410}$  and line  $L_{\backslash\_G4416}$ 
intersec  $P_{\backslash\_G4422}$   $L_{\backslash\_G4410}$   $L_{\backslash\_G4416}$ 
cmark_r  $P_{\backslash\_G4422}$ 

% Constructing a point  $P_{\backslash\_G4425}$  which belongs to line  $L_{\backslash\_G4410}$  and line  $L_{\backslash\_G4419}$ 
intersec  $P_{\backslash\_G4425}$   $L_{\backslash\_G4410}$   $L_{\backslash\_G4419}$ 
cmark_r  $P_{\backslash\_G4425}$ 

% Constructing a line  $L_{\backslash\_G4428}$  which passes through point  $A$  and point  $P_{\backslash\_G4425}$ 
line  $L_{\backslash\_G4428}$   $A$   $P_{\backslash\_G4425}$ 

color 200 200 200
drawline  $L_{\backslash\_G4428}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G4431}$  which passes through point  $C$  and point  $P_{\backslash\_G4422}$ 
line  $L_{\backslash\_G4431}$   $C$   $P_{\backslash\_G4422}$ 

color 200 200 200
drawline  $L_{\backslash\_G4431}$ 
color 0 0 0

% Constructing a point  $P_{\backslash\_G4434}$  which belongs to line  $L_{\backslash\_G4428}$  and line  $L_{\backslash\_G4431}$ 
intersec  $P_{\backslash\_G4434}$   $L_{\backslash\_G4428}$   $L_{\backslash\_G4431}$ 
cmark_r  $P_{\backslash\_G4434}$ 

% Constructing a line  $L_{\backslash\_G4437}$  which passes through point  $P_{\backslash\_G4413}$  and point  $P_{\backslash\_G4434}$ 
line  $L_{\backslash\_G4437}$   $P_{\backslash\_G4413}$   $P_{\backslash\_G4434}$ 

color 200 200 200
drawline  $L_{\backslash\_G4437}$ 
color 0 0 0

% Constructing a point  $T'_{\{b\}}$  which belongs to line  $L_{\backslash\_G4437}$  and line  $b$ 
intersec  $T'_{\{b\}}$   $L_{\backslash\_G4437}$   $b$ 
cmark_r  $T'_{\{b\}}$ 

% Constructing midpoint  $P_{\backslash\_G5262}$  of the segment  $T_{\{b\}}T'_{\{b\}}$ 
midpoint  $P_{\backslash\_G5262}$   $T_{\{b\}}$   $T'_{\{b\}}$ 
cmark_r  $P_{\backslash\_G5262}$ 

% Choosing randomly a point  $B$  on the circle with center  $P_{\backslash\_G5262}$  through point  $T_{\{b\}}$ 
oncircle  $B$   $P_{\backslash\_G5262}$   $T_{\{b\}}$ 

```

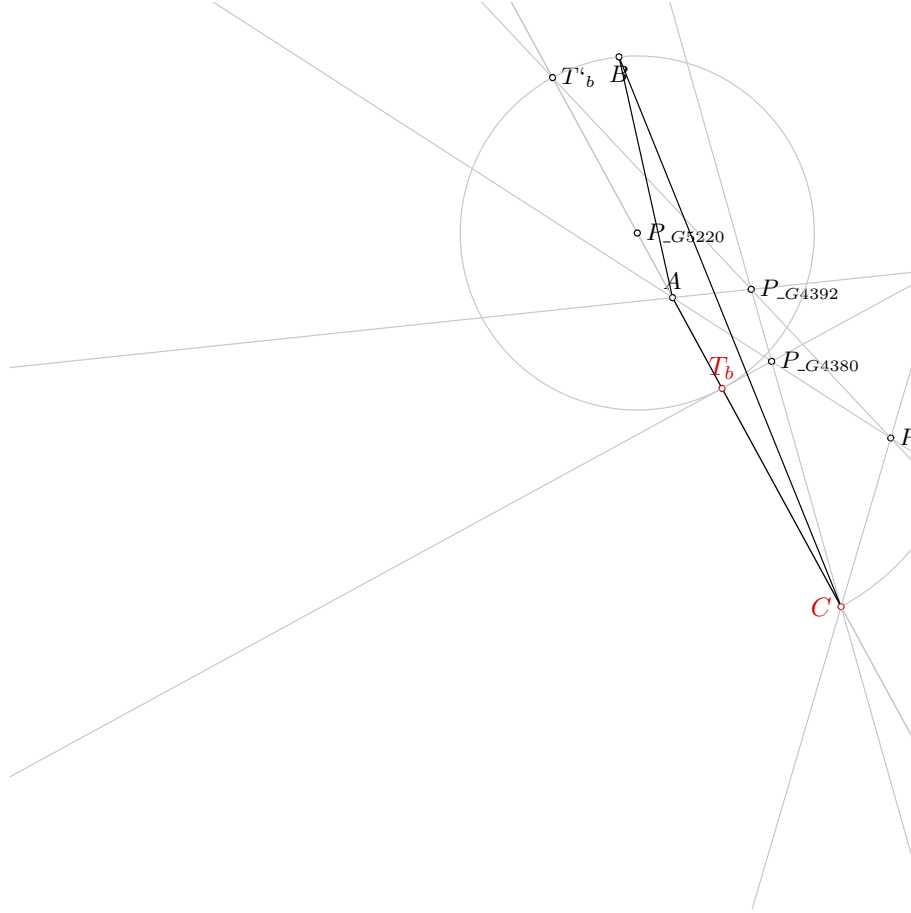


Figure 1: Illustration of the problem 0025

```

cmark_b B
color 200 200 200
drawcircle P_{\_G5262} T_{b}
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions: points A and C are not the same; points C and T_{b} are not the same;
%                             points C and midpoint([A,T_{b}]) are not the same
% Determination conditions: points C and T_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.015 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{T_bAP_{G5850}} \neq S_{P_{G5844}AP_{G5850}}$  i.e., lines  $T_bP_{G5844}$  and  $AP_{G5850}$  are not parallel (construction based assumption)

$S_{T_bCP_{G5850}} \neq S_{P_{G5844}CP_{G5850}}$  i.e., lines  $T_bP_{G5844}$  and  $CP_{G5850}$  are not parallel (construction based assumption)

$S_{ACP_{G5859}} \neq S_{P_{G5862}CP_{G5859}}$  i.e., lines  $AP_{G5862}$  and  $CP_{G5859}$  are not parallel (construction based assumption)

$S_{P_{G5850}CT_b} \neq S_{P_{G5871}CT_b}$  i.e., lines  $P_{G5850}P_{G5871}$  and  $CT_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.2.3 Proving $A=A$

NDG conditions are:

$S_{T_bAP_{G4941}} \neq S_{P_{G4935}AP_{G4941}}$  i.e., lines  $T_bP_{G4935}$  and  $AP_{G4941}$  are not parallel (construction based assumption)

$S_{T_bCP_{G4941}} \neq S_{P_{G4935}CP_{G4941}}$  i.e., lines  $T_bP_{G4935}$  and  $CP_{G4941}$  are not parallel (construction based assumption)

$S_{ACP_{G4950}} \neq S_{P_{G4953}CP_{G4950}}$  i.e., lines  $AP_{G4953}$  and  $CP_{G4950}$  are not parallel (construction based assumption)

$S_{P_{G4941}CT_b} \neq S_{P_{G4962}CT_b}$  i.e., lines  $P_{G4941}P_{G4962}$  and  $CT_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $C=C$**

Proving failed

#### **4.3.2 Proving $T_b=\neg T_b$**

Proving failed

#### **4.3.3 Proving $A=A$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $C=C$**

Proving failed

#### **4.4.2 Proving $T_b=\neg T_b$**

Proving failed

#### **4.4.3 Proving $A=A$**

Proving failed

# Problem 26

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 26: Given a point  $A$ , a point  $C$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $C$  are not the same;
2. Using the point  $A$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $T_c$  are not the same;
3. Using the point  $T_c$  and the line  $b$ , construct a circle  $k(T_c, foot[T_c, b])$  (rule W11); % NDG: point  $T_c$  is not incident to the line  $b$ ;
4. Using the circle  $k(T_c, foot[T_c, b])$ , the point  $C$ , the point  $T_c$  and the line  $b$ , construct a line  $a$  (rule W13); % NDG: point  $C$  is outside the circle  $k(T_c, foot[T_c, b])$ ;
5. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; point  $C$  is outside the circle  $k(T_c, foot[T_c, b])$ ; point  $T_c$  is not incident to the line  $b$ .

Determination conditions: lines  $a$  and  $c$  are not the same; points  $A$  and  $T_c$  are not the same; points  $A$  and  $C$  are not the same.

Rules used: [W02,W03,W11,W13]

Lemmas used: [D25,GD01,GD02,GL10,GL11,L10]

Solving time: 1.7 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point C 110 40
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_t A
cmark_l C
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C

color 200 200 200
drawline b
color 0 0 0

% DET: points A and T_{c} are not the same
% Constructing a line c which passes through point A and point T_{c}
line c A T_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: point T_{c} is not incident to the line b
% Constructing a point P_{\_G5151} which is a foot of the point T_{c} on the line b
foot P_{\_G5151} T_{c} b
cmark_r P_{\_G5151}
color 200 200 200
drawline T_{c} P_{\_G5151}
color 0 0 0

% Constructing a circle k(T_{c},foot[T_{c},b]) whose center is at point T_{c} and which passes
    through point P_{\_G5151}
circle k(T_{c},foot[T_{c},b]) T_{c} P_{\_G5151}

color 200 200 200
drawcircle k(T_{c},foot[T_{c},b])
color 0 0 0

% NDG: point C is outside the circle k(T_{c},foot[T_{c},b])
```

```

% Constructing a point  $P_{\{c\}}$  which is a foot of the point  $T_{\{c\}}$  on the line  $b$ 
foot  $P_{\{c\}}$   $T_{\{c\}}$   $b$ 
cmark_r  $P_{\{c\}}$ 
color 200 200 200
drawline  $T_{\{c\}}$   $P_{\{c\}}$ 
color 0 0 0

% Constructing a line  $L_{\{c\}}$  which passes through point  $T_{\{c\}}$  and point  $C$ 
line  $L_{\{c\}}$   $T_{\{c\}}$   $C$ 

color 200 200 200
drawline  $L_{\{c\}}$ 
color 0 0 0

% Constructing a point  $P_{\{c\}}$  which is an image of the point  $P_{\{c\}}$  in the symmetry to
point/line  $L_{\{c\}}$ 
sim  $P_{\{c\}}$   $L_{\{c\}}$   $P_{\{c\}}$ 
cmark_r  $P_{\{c\}}$ 

% Constructing a line  $a$  which passes through point  $C$  and point  $P_{\{c\}}$ 
line  $a$   $C$   $P_{\{c\}}$ 

color 200 200 200
drawline  $a$ 
color 0 0 0

% NDG: lines  $a$  and  $c$  are not parallel% DET: lines  $a$  and  $c$  are not the same
% Constructing a point  $B$  which belongs to line  $a$  and line  $c$ 
intersec  $B$   $a$   $c$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; point  $C$  is outside the circle  $k(T_{\{c\}},$ 
foot[ $T_{\{c\}}$ , $b$ ]); point  $T_{\{c\}}$  is not incident to the line  $b$ 
% Determination conditions: lines  $a$  and  $c$  are not the same; points  $A$  and  $T_{\{c\}}$  are not the same;
points  $A$  and  $C$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format



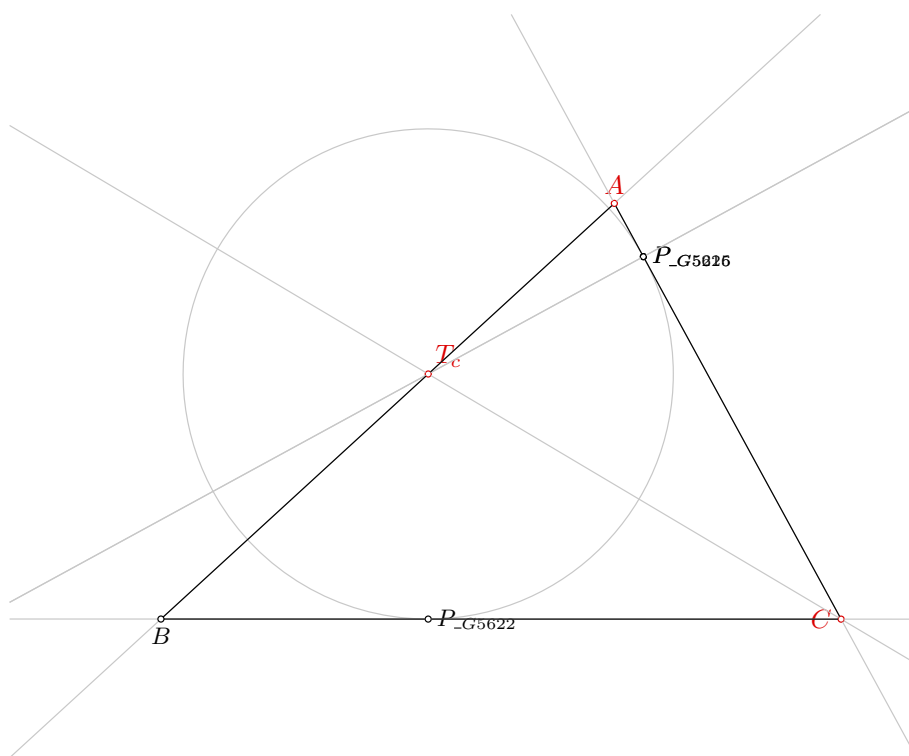


Figure 1: Illustration of the problem 0026

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $T_c=\neg T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{CAT_c} \neq S_{P_{G6574}AT_c}$  i.e., lines  $CP_{G6574}$  and  $AT_c$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $C=C$

NDG conditions are:

$S_{CAT_c} \neq S_{P_{G3508}AT_c}$  i.e., lines  $CP_{G3508}$  and  $AT_c$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $T_c=\neg T_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_c = \neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 16 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_c = \neg T_c$

Proving failed

# Problem 27

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 27: Given a point  $A$ , a point  $C$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $C$  are not the same;
2. Using the point  $I$  and the line  $b$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $b$ ;
3. Using the circle  $k(I, P_a)$ , the point  $A$ , the point  $I$  and the line  $b$ , construct a line  $c$  (rule W13); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
4. Using the circle  $k(I, P_a)$ , the point  $C$ , the point  $I$  and the line  $b$ , construct a line  $a$  (rule W13); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
5. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; point  $C$  is outside the circle  $k(I, P_a)$ ; point  $A$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $b$ .

Determination conditions: lines  $c$  and  $a$  are not the same; points  $A$  and  $C$  are not the same.

Rules used: [W02,W03,W11,W13]

Lemmas used: [D27,GD01,L59,L60,L61]

Solving time: 32.0 seconds.

### 3.2 Construction in GCLC language

```

dim 120 120

point A 80 95
point C 110 40
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_t A
cmark_l C
cmark_b I
color 0 0 0
fontsize 8

% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C

color 200 200 200
drawline b
color 0 0 0

% NDG: point I is not incident to the line b
% Constructing a point  $P_{\setminus G6735}$  which is a foot of the point I on the line b
foot  $P_{\setminus G6735}$  I b
cmark_r  $P_{\setminus G6735}$ 
color 200 200 200
drawline I  $P_{\setminus G6735}$ 
color 0 0 0

% Constructing a circle  $k(I, P_{\setminus a})$  whose center is at point I and which passes through point  $P_{\setminus G6735}$ 
circle k(I,  $P_{\setminus a}$ ) I  $P_{\setminus G6735}$ 

color 200 200 200
drawcircle k(I,  $P_{\setminus a}$ )
color 0 0 0

% NDG: point A is outside the circle  $k(I, P_{\setminus a})$ 
% Constructing a point  $P_{\setminus G6942}$  which is a foot of the point I on the line b
foot  $P_{\setminus G6942}$  I b
cmark_r  $P_{\setminus G6942}$ 
color 200 200 200
drawline I  $P_{\setminus G6942}$ 
color 0 0 0

% Constructing a line  $L_{\setminus G6945}$  which passes through point I and point A
line  $L_{\setminus G6945}$  I A

color 200 200 200
drawline  $L_{\setminus G6945}$ 

```

```

color 0 0 0

% Constructing a point P_{\_G6948} which is an image of the point P_{\_G6942} in the symmetry to
    point/line L_{\_G6945}
sim P_{\_G6948} L_{\_G6945} P_{\_G6942}
cmark_r P_{\_G6948}

% Constructing a line c which passes through point A and point P_{\_G6948}
line c A P_{\_G6948}

color 200 200 200
drawline c
color 0 0 0

% NDG: point C is outside the circle k(I,P_{a})
% Constructing a point P_{\_G7277} which is a foot of the point I on the line b
foot P_{\_G7277} I b
cmark_r P_{\_G7277}
color 200 200 200
drawline I P_{\_G7277}
color 0 0 0

% Constructing a line L_{\_G7280} which passes through point I and point C
line L_{\_G7280} I C

color 200 200 200
drawline L_{\_G7280}
color 0 0 0

% Constructing a point P_{\_G7283} which is an image of the point P_{\_G7277} in the symmetry to
    point/line L_{\_G7280}
sim P_{\_G7283} L_{\_G7280} P_{\_G7277}
cmark_r P_{\_G7283}

% Constructing a line a which passes through point C and point P_{\_G7283}
line a C P_{\_G7283}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

drawsegment A B
drawsegment A C

```

Figure 1: Illustration of the problem 0027

drawsegment B C

*% Non-degenerate conditions: lines c and a are not parallel; point C is outside the circle k(I,P\_{a  
}); point A is outside the circle k(I,P\_{a}); point I is not incident to the line b  
% Determination conditions: lines c and a are not the same; points A and C are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $I=I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{ACP\_G4850} \neq S_{P\_G4515CP\_G4850}$  i.e., lines  $AP\_G4515$  and  $CP\_G4850$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $C=C$

NDG conditions are:

$S_{ACP\_G6679} \neq S_{P\_G6344CP\_G6679}$  i.e., lines  $AP\_G6344$  and  $CP\_G6679$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $I=_I$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $I=_I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 185 terms.

**Time Complexity:** Time spent by the prover is 0.080 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.3 Proving $I=_I$

Proving failed



# Problem 28

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 28: Given a point  $A$ , a point  $O$  and a point  $M_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $M_a$ , construct a point  $G$  (rule W01); ;
2. Using the point  $O$  and the point  $G$ , construct a point  $H$  (rule W01); ;
3. Using the point  $A$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H$  are not the same;
4. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
5. Using the point  $M_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $A$  and  $H$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D21,D26,D3,D8,GD01,GL03,GL04,GL09,L11,L12,L55,L58]

Solving time: 3.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point O 65 51.14
point M_{a} 65 40
```

```

color 220 0 0
fontsize 9

cmark_t A
cmark_t 0
cmark_r M_{a}
color 0 0 0
fontsize 8

% Constructing a line L_{\_G5685} which passes through point A and point M_{a}
line L_{\_G5685} A M_{a}

color 200 200 200
drawline L_{\_G5685}
color 0 0 0

% Constructing a point P_{\_G5786} with coordinates (0,0)
point P_{\_G5786} 0 0
cmark_r P_{\_G5786}

% Constructing a point P_{\_G5710} such that AP_{\_G5710}/AP_{\_G5786}=2
towards P_{\_G5710} A P_{\_G5786} 2
cmark_r P_{\_G5710}
color 200 200 200
drawsegment A P_{\_G5710}
color 0 0 0

% Constructing a point P_{\_G5755} such that AP_{\_G5755}/AP_{\_G5786}=3
towards P_{\_G5755} A P_{\_G5786} 3
cmark_r P_{\_G5755}
color 200 200 200
drawsegment A P_{\_G5755}
color 0 0 0

% Constructing a line L_{\_G5716} which passes through point M_{a} and point P_{\_G5755}
line L_{\_G5716} M_{a} P_{\_G5755}

color 200 200 200
drawline L_{\_G5716}
color 0 0 0

% Constructing a line L_{\_G5679} which contains the point P_{\_G5710} and is parallel to the line
L_{\_G5716}
parallel L_{\_G5679} P_{\_G5710} L_{\_G5716}

color 200 200 200
drawline L_{\_G5679}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G5679} and line L_{\_G5685}
intersec G L_{\_G5679} L_{\_G5685}

```

```

cmark_t G

% Constructing a point H such that  $OH/OG=3$ 
towards H O G 3
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0

% DET: points A and H are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point A and point H
line  $h_{\{a\}}$  A H

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point A
circle  $k(O,C)$  O A

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% Constructing a line a which is perpendicular to line  $h_{\{a\}}$  and which passes through point  $M_{\{a\}}$ 
perp a  $M_{\{a\}}$   $h_{\{a\}}$ 

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle  $k(O,C)$  intersect
% Constructing points C and B which are in intersection of  $k(O,C)$  and a
intersec2 C B  $k(O,C)$  a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle  $k(O,C)$  intersect; points A and O are not the same
% Determination conditions: points A and H are not the same

```

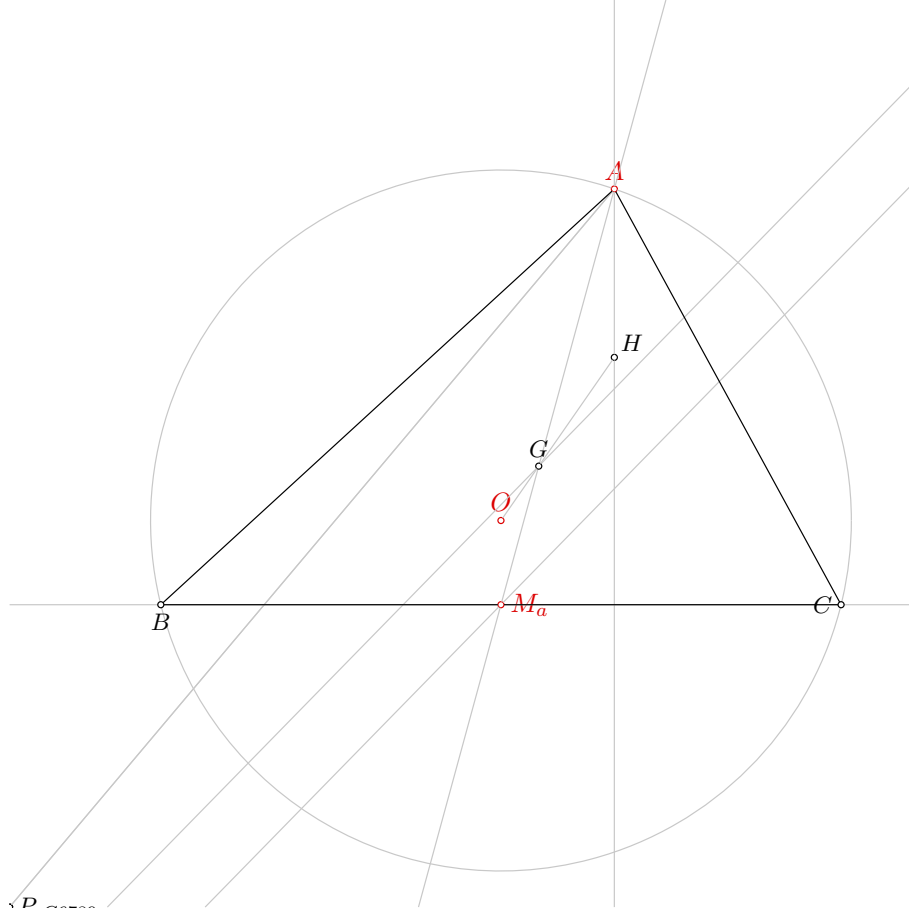


Figure 1: Illustration of the problem 0028

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 684 terms.

**Time Complexity:** Time spent by the prover is 2.856 seconds.

**NDG conditions** Points  $P_{G6655}$ ,  $A$  and  $M_a$  are not collinear

Line through points  $P_{G6655}$  and  $M_a$  is not parallel with line through points  $A$  and  $O$

Points  $A$  and  $H$  are not identical

Points  $A$ ,  $H$  and  $O$  are not collinear

Points  $A$ ,  $B$  and  $C$  are not collinear

Line through points  $A$  and  $B$  is not perpendicular to line through points  $B$  and  $O$

#### 4.1.3 Proving $M_a = \neg M_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A = A$

NDG conditions are:

$S_{P_{G3129}AM_a} \neq S_{P_{L_{G3098}}^{0}AM_a}$  i.e., lines  $P_{G3129}P_{L_{G3098}}^0$  and  $AM_a$  are not parallel (construction based assumption)

$S_{M_aAH} \neq 0$  i.e., points  $M_a$ ,  $A$  and  $H$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aBC} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aM_bF_{M_b}^3} \neq S_{F_{M_a}^2M_bF_{M_b}^3}$  i.e., lines  $M_aF_{M_a}^2$  and  $M_bF_{M_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O = \neg O$

Proving failed

#### 4.2.3 Proving $M_a = \neg M_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $O = \neg O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 11731 terms.

**Time Complexity:** Time spent by the prover is 53.780 seconds. There are no ndg conditions.

#### 4.3.3 Proving $M_a = \neg M_a$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4.2 Proving $O=_O$

Proving failed

### 4.4.3 Proving $M_a=_M a$

Proving failed

# Problem 29

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 29: Given a point  $O$ , a point  $M_b$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $M_b$ , construct a line  $m_b$  (rule W02); % DET: points  $O$  and  $M_b$  are not the same;
2. Using the point  $M_b$  and the line  $m_b$ , construct a line  $b$  (rule W10a); ;
3. Choose freely a point  $A$  on the line  $b$  (rule WOnline2);
4. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
5. Choose freely a point  $B$  on the circle  $k(O, C)$  (rule WOncircle).

Non-degenerate conditions: .

Determination conditions: points  $O$  and  $M_b$  are not the same.

Rules used: [W01,W02,W10a,WOncircle1,WOnline2]

Lemmas used: [D1,D12,D22,GD01,GL03,GL04,GL09]

Solving time: 470.5 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point O 65 51.14
point M_{b} 95 67.5
point A 80 95
```

```
color 220 0 0
fontsize 9
```

```

cmark_t 0
cmark_lt M_{b}
cmark_t A
color 0 0 0
fontsize 8

% DET: points O and M_{b} are not the same
% Constructing a line m_{b} which passes through point O and point M_{b}
line m_{b} 0 M_{b}

color 200 200 200
drawline m_{b}
color 0 0 0

% Constructing a line b which is perpendicular to line m_{b} and which passes through point M_{b}
perp b M_{b} m_{b}

color 200 200 200
drawline b
color 0 0 0

% Generating random value V[_G5454]
random V[_G5454]

% Calculating value V[_G5475] using formula  $V[_G5454]*20$ 
expression V[_G5475] { V[_G5454]*20 }

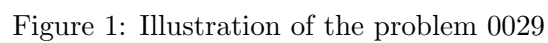
% Constructing a point A which is a point for which holds  $M_{b}A = V[_G5475]$  and angle  $OM_{b}A = 90$ 
turtle A 0 M_{b} 90 V[_G5475]
cmark_t A

% Constructing a point C such that  $AC/AM_{b}=2$ 
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% Choosing randomly a point B on the circle with center O through point A
oncircle B 0 A
cmark_b B
color 200 200 200
drawcircle 0 A
color 0 0 0

```





```
% Non-degenerate conditions:
% Determination conditions: points  $O$  and  $M_{\{b\}}$  are not the same
```

Illustration of the constructed figure is given in Figure 1

## 169

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 49 terms.

**Time Complexity:** Time spent by the prover is 0.352 seconds.

**NDG conditions** Line through points  $C$  and  $A$  is not perpendicular to line through points  $A$  and  $M_b$   
Line through points  $M_b$  and  $A$  is not perpendicular to line through points  $A$  and  $O$

#### 4.1.2 Proving $M_b=_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.036 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.008 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $O=_O$

Proving failed

#### 4.2.2 Proving $M_b=_M_b$

Proving failed

#### 4.2.3 Proving $A=A$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $O=_O$

Proving failed

#### 4.3.2 Proving $M_b=_M_b$

Proving failed

#### 4.3.3 Proving $A=A$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $O=_O$

Proving failed

### 4.4.2 Proving $M_b=_M M_b$

Proving failed

### 4.4.3 Proving $A=A$

Proving failed

# Problem 30

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 30: Given a point  $O$ , a point  $M_c$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $M_c$ , construct a line  $m_c$  (rule W02); % DET: points  $O$  and  $M_c$  are not the same;
2. Using the point  $M_c$  and the line  $m_c$ , construct a line  $c$  (rule W10a); ;
3. Choose freely a point  $A$  on the line  $c$  (rule WOnline2);
4. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); ;
5. Choose freely a point  $C$  on the circle  $k(O, C)$  (rule WOncircle).

Non-degenerate conditions: .

Determination conditions: points  $O$  and  $M_c$  are not the same.

Rules used: [W01,W02,W10a,WOncircle1,WOnline2]

Lemmas used: [D13,D20,GD01,GL03,GL09,L1]

Solving time: 456.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point O 65 51.14
point M_{c} 50 67.5
point A 80 95
```

```
color 220 0 0
fontsize 9
```

```

cmark_t 0
cmark_lt M_{c}
cmark_t A
color 0 0 0
fontsize 8

% DET: points O and M_{c} are not the same
% Constructing a line m_{c} which passes through point O and point M_{c}
line m_{c} 0 M_{c}

color 200 200 200
drawline m_{c}
color 0 0 0

% Constructing a line c which is perpendicular to line m_{c} and which passes through point M_{c}
perp c M_{c} m_{c}

color 200 200 200
drawline c
color 0 0 0

% Generating random value V[_G3383]
random V[_G3383]

% Calculating value V[_G3404] using formula  $V[_G3383]*20$ 
expression V[_G3404] { V[_G3383]*20 }

% Constructing a point A which is a point for which holds  $M_{c}A = V[_G3404]$  and angle  $OM_{c}A = 90$ 
turtle A 0 M_{c} 90 V[_G3404]
cmark_t A

% Constructing a point B such that  $AB/AM_{c}=2$ 
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% Choosing randomly a point C on the circle with center O through point A
oncircle C 0 A
cmark_l C
color 200 200 200
drawcircle 0 A
color 0 0 0

```

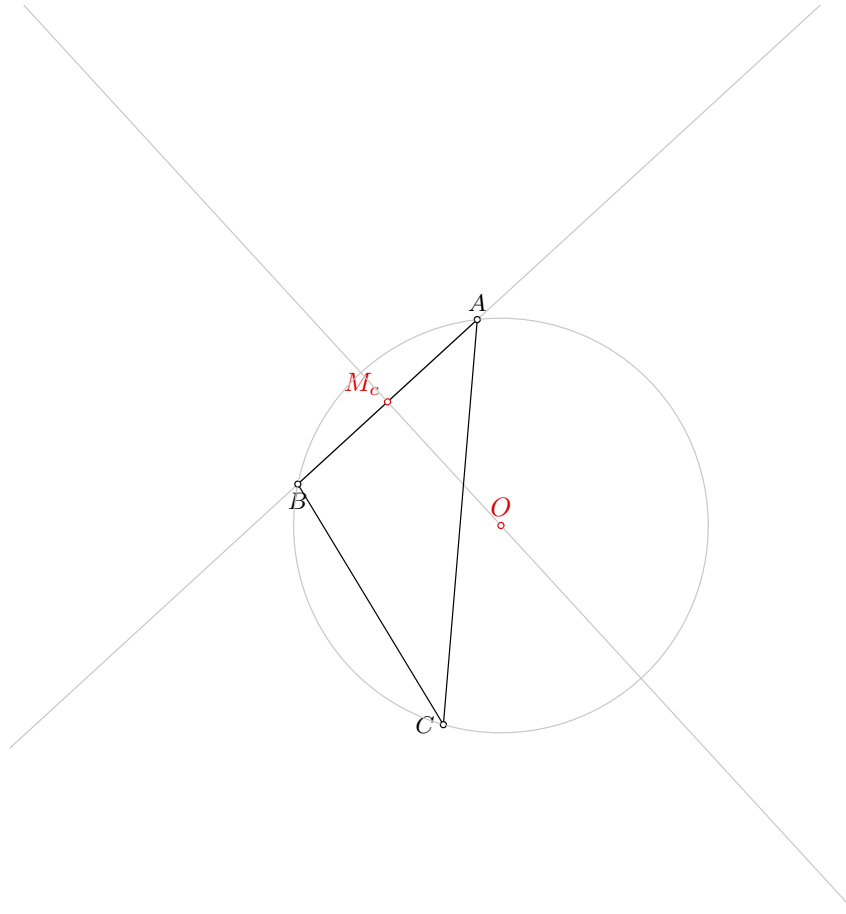


Figure 1: Illustration of the problem 0030

```
drawsegment A B
drawsegment A C
drawsegment B C
```

*% Non-degenerate conditions:*

*% Determination conditions: points  $O$  and  $M_{\{c\}}$  are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 134 terms.

**Time Complexity:** Time spent by the prover is 0.849 seconds.

**NDG conditions** Line through points  $A$  and  $B$  is not parallel with line through points  $C$  and  $M_c$   
Points  $A$ ,  $C$  and  $M_c$  are not collinear

#### 4.1.2 Proving $M_c=_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.036 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.015 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $O=_O$

Proving failed

#### 4.2.2 Proving $M_c=_M_c$

Proving failed

#### 4.2.3 Proving $A=A$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $O=_O$

Proving failed

#### 4.3.2 Proving $M_c=_M_c$

Proving failed

#### 4.3.3 Proving $A=A$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $O=_O$

Proving failed

### 4.4.2 Proving $M_c=_M M_c$

Proving failed

### 4.4.3 Proving $A=A$

Proving failed



# Problem 31

*Generated automatically by ArgoTriCS  
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## 1 Problem

Problem 31: Given a point  $A$ , a point  $O$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $G$ , construct a point  $M_a$  (rule W01); ;
2. Using the point  $O$  and the point  $G$ , construct a point  $H$  (rule W01); ;
3. Using the point  $A$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H$  are not the same;
4. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
5. Using the point  $M_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $A$  and  $H$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D21,D26,D3,D8,GD01,GL03,GL04,GL09,L11,L12,L55,L58]

Solving time: 1.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point O 65 51.14
point G 70 58.33
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_t O
cmark_t G
color 0 0 0
fontsize 8
```

```
% Constructing a point M_{a} such that AM_{a}/AG=1.5
towards M_{a} A G 1.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0
```

```
% Constructing a point H such that OH/OG=3
towards H O G 3
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0
```

```
% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0
```

```
% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0
```

```
% Constructing a line a which is perpendicular to line h_{a} and which passes through point M_{a}
perp a M_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0
```

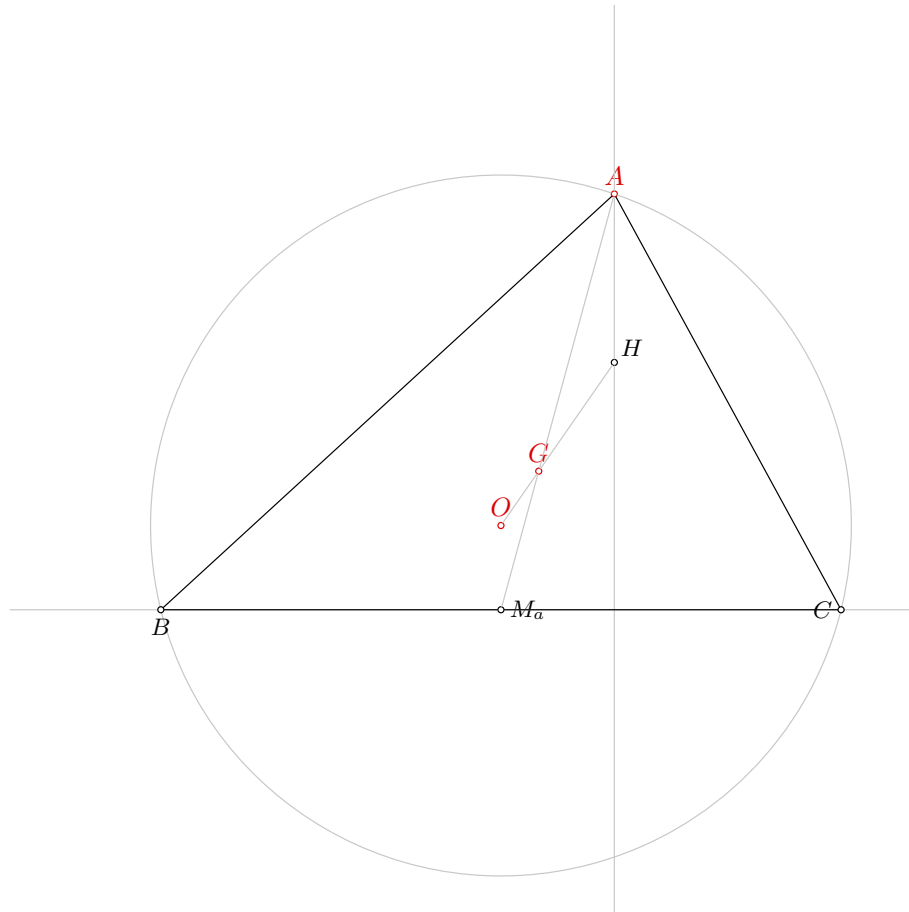


Figure 1: Illustration of the problem 0031

```
% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same
% Determination conditions: points A and H are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.03 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 1638 terms.

**Time Complexity:** Time spent by the prover is 4.669 seconds.

**NDG conditions** Points  $A$  and  $H$  are not identical

Points  $A$ ,  $H$  and  $O$  are not collinear

Points  $A$ ,  $B$  and  $C$  are not collinear

Line through points  $A$  and  $B$  is not perpendicular to line through points  $B$  and  $O$

#### 4.1.3 Proving $G=_G$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{M_aAH} \neq 0$  i.e., points  $M_a$ ,  $A$  and  $H$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_aBC} \neq 0$  i.e., points  $_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$  i.e., points  $_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_aM_bF_{_m_b}^2} \neq S_{F_{_m_a}^1M_bF_{_m_b}^2}$  i.e., lines  $_M_aF_{_m_a}^1$  and  $_M_bF_{_m_b}^2$  are not parallel (construction based assumption)

$S_{ABM_b} \neq S_{_M_aB_M_b}$  i.e., lines  $A_M_a$  and  $B_M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $G=_G$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $O=_O$

Proving failed

#### 4.3.3 Proving $G=_G$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $O=_O$

Proving failed

#### 4.4.3 Proving $G=_G$

Proving failed

# Problem 32

*Generated automatically by ArgoTriCS  
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## 1 Problem

Problem 32: Given a point  $A$ , a point  $O$  and a point  $H_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H_a$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H_a$  are not the same;
2. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
3. Using the point  $H_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
4. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $A$  and  $H_a$  are not the same.

Rules used: [W02,W04,W06,W10a]

Lemmas used: [D26,D5,D8,GD01,L11,L12]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
```

```
point O 65 51.14
```

```
point H_{a} 80 40
```

```
color 220 0 0
```

```
fontsize 9
```

```

cmark_t A
cmark_t O
cmark_r H_{a}
color 0 0 0
fontsize 8

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
perp a H_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same
% Determination conditions: points A and H_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

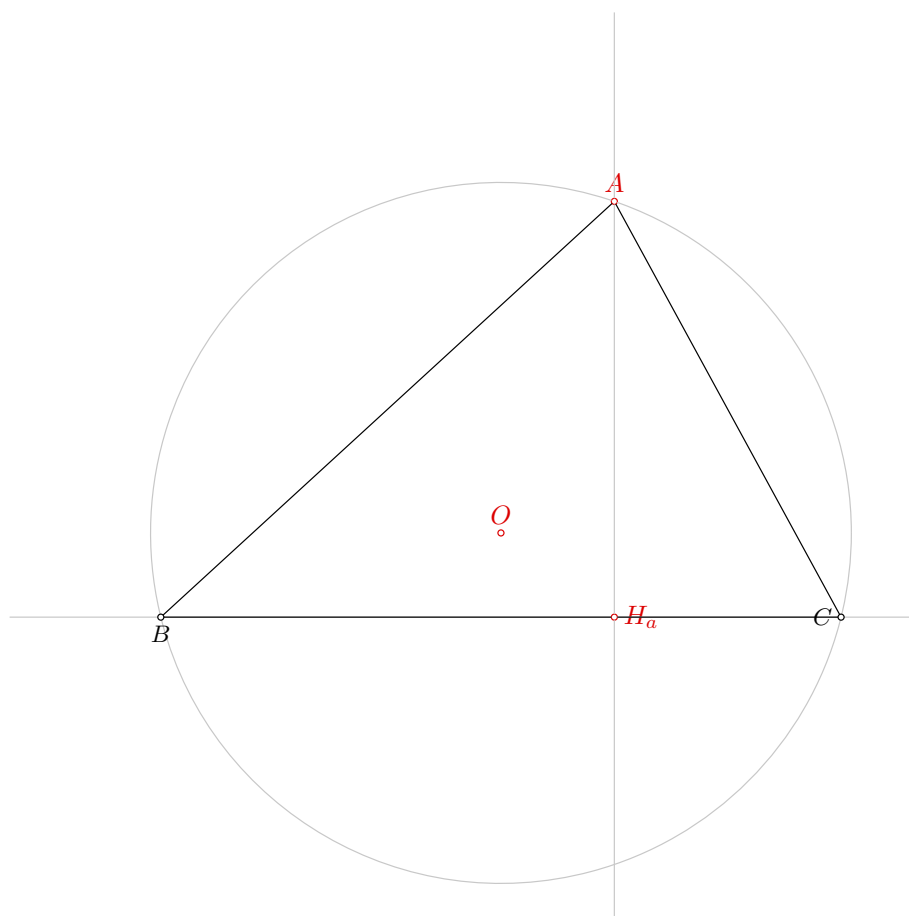


Figure 1: Illustration of the problem 0032



## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 201 terms.

**Time Complexity:** Time spent by the prover is 1.424 seconds.

**NDG conditions** Points  $A$  and  $H_a$  are not identical

Points  $A$  and  $H_a$  are not identical

Points  $A$ ,  $B$  and  $C$  are not collinear

Line through points  $A$  and  $B$  is not perpendicular to line through points  $B$  and  $O$

#### 4.1.3 Proving $H_a=_H_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{M_aBC} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aM_bF_{m_b}^2} \neq S_{F_{m_a}^1M_bF_{m_b}^2}$  i.e., lines  $M_aF_{m_a}^1$  and  $M_bF_{m_b}^2$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^3} \neq S_{CAF_{h_a}^3}$  i.e., lines  $BC$  and  $AF_{h_a}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $H_a=_H_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $O=_O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 5579 terms.

**Time Complexity:** Time spent by the prover is 13.070 seconds. There are no ndg conditions.

#### 4.3.3 Proving $H_a=_H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 5989 terms.

**Time Complexity:** Time spent by the prover is 4.710 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $O=_O$

Proving failed

#### 4.4.3 Proving $H_a=_H_a$

Proving failed

# Problem 33

*Generated automatically by ArgoTriCS  
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## 1 Problem

Problem 33: Given a point  $A$ , a point  $O$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $H_b$  are not the same;
2. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
3. Using the circle  $k(O, C)$ , the line  $b$ , the point  $O$  and the point  $A$ , construct a point  $C$  (rule W05); % NDG: line  $b$  and circle  $k(O, C)$  intersect % DET: points  $A$  and  $C$  must be different;
4. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
5. Using the circle  $k(O, C)$  and the line  $h_b$ , construct a point  $B_k$  and a point  $B$  (rule W04); % NDG: line  $h_b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $h_b$  and circle  $k(O, C)$  intersect; line  $b$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $A$  and  $C$  must be different; points  $A$  and  $H_b$  are not the same.

Rules used: [W02,W04,W05,W06,W10b]

Lemmas used: [D26,D39,D6,D9,GD01,L11,L12]

Solving time: 4.5 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
```

```
point O 65 51.14
```

```

point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_t A
cmark_t 0
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line b and circle k(O,C) intersect% DET: points A and C must be different
% Constructing a point P_{\_G7417} which is a foot of the point O on the line b
foot P_{\_G7417} O b
cmark_r P_{\_G7417}
color 200 200 200
drawline O P_{\_G7417}
color 0 0 0

% Constructing a point C which is an image of the point A in the symmetry to point/line P_{\_G7417}
sim C P_{\_G7417} A
cmark_l C

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{b} and circle k(O,C) intersect
% Constructing points B_{k} and B which are in intersection of k(O,C) and h_{b}

```

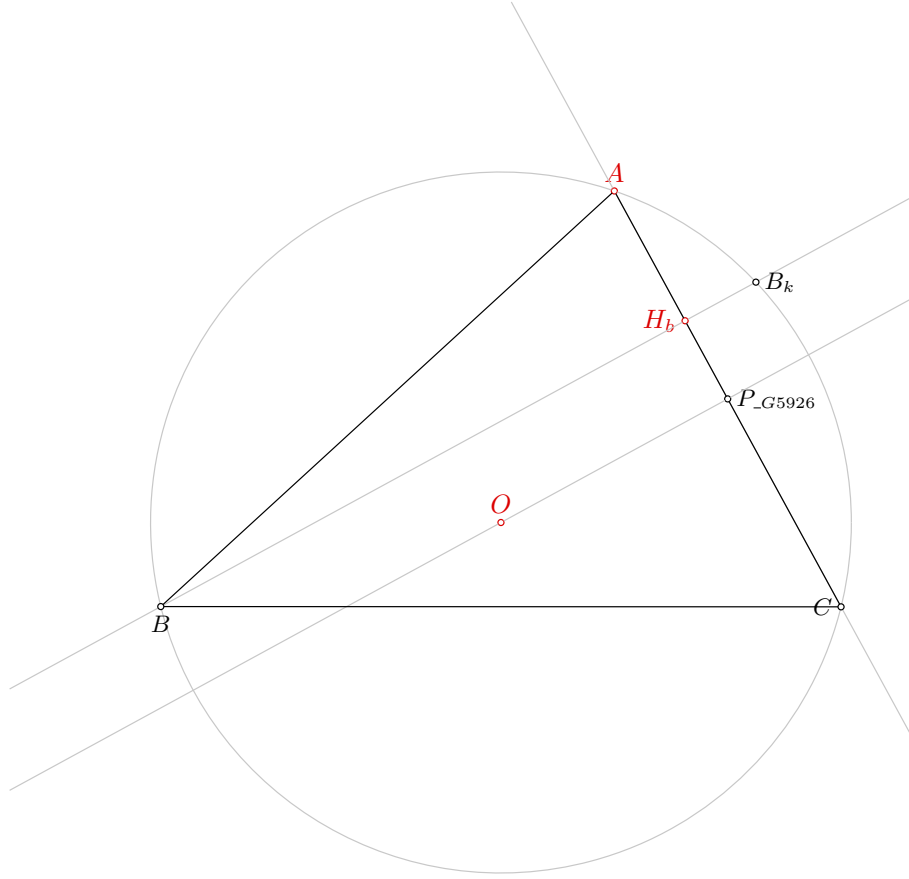


Figure 1: Illustration of the problem 0033

```

intersec2 B_{k} B k(O,C) h_{b}
cmark_r B_{k}
cmark_b B

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

*% Non-degenerate conditions: line  $h_{\{b\}}$  and circle  $k(O,C)$  intersect; line  $b$  and circle  $k(O,C)$  intersect; points  $A$  and  $O$  are not the same*  
*% Determination conditions: points  $A$  and  $C$  must be different; points  $A$  and  $H_{\{b\}}$  are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 201 terms.

**Time Complexity:** Time spent by the prover is 1.521 seconds.

**NDG conditions** Points  $H_b$  and  $O$  are not identical

Points  $H_b$  and  $O$  are not identical

Points  $A$ ,  $B$  and  $C$  are not collinear

Line through points  $A$  and  $B$  is not perpendicular to line through points  $B$  and  $O$

#### 4.1.3 Proving $H_b=_H H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 17 terms.

**Time Complexity:** Time spent by the prover is 0.355 seconds.

**NDG conditions** Points  $H_b$  and  $O$  are not identical

Points  $H_b$  and  $O$  are not identical

Points  $A$  and  $C$  are not identical

Points  $A$  and  $C$  are not identical

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{M_a BC} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_b AC} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_a M_b F_{M_a}^2} \neq S_{F_{M_a}^1 M_b F_{M_b}^2}$  i.e., lines  $M_a F_{M_a}^1$  and  $M_b F_{M_b}^2$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{H_b}^3} \neq S_{CBF_{H_b}^3}$  i.e., lines  $AC$  and  $BF_{H_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $H_b=_H H_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

Proving failed

#### 4.3.2 Proving $O=_O$

Proving failed

#### 4.3.3 Proving $H_b=_Hb$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A=A$

Proving failed

#### 4.4.2 Proving $O=_O$

Proving failed

#### 4.4.3 Proving $H_b=_Hb$

Proving failed

# Problem 34

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 34: Given a point  $A$ , a point  $O$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $H_c$  are not the same;
2. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
3. Using the circle  $k(O, C)$ , the line  $c$ , the point  $O$  and the point  $A$ , construct a point  $B$  (rule W05); % NDG: line  $c$  and circle  $k(O, C)$  intersect % DET: points  $A$  and  $B$  must be different;
4. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
5. Using the circle  $k(O, C)$  and the line  $h_c$ , construct a point  $C$  and a point  $C_k$  (rule W04); % NDG: line  $h_c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $h_c$  and circle  $k(O, C)$  intersect; line  $c$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $A$  and  $B$  must be different; points  $A$  and  $H_c$  are not the same.

Rules used: [W02,W04,W05,W06,W10b]

Lemmas used: [D10,D26,D40,D7,GD01,L11,L12]

Solving time: 4.5 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
```

```
point O 65 51.14
```



```

point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_t A
cmark_t 0
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line c and circle k(O,C) intersect% DET: points A and B must be different
% Constructing a point P_{\_G3838} which is a foot of the point O on the line c
foot P_{\_G3838} O c
cmark_r P_{\_G3838}
color 200 200 200
drawline O P_{\_G3838}
color 0 0 0

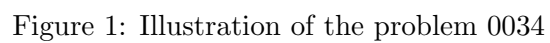
% Constructing a point B which is an image of the point A in the symmetry to point/line P_{\_G3838}
sim B P_{\_G3838} A
cmark_b B

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: line h_{c} and circle k(O,C) intersect
% Constructing points C and C_{k} which are in intersection of k(O,C) and h_{c}

```



```
% Non-degenerate conditions: line h_{c} and circle k(O,C) intersect; line c and circle k(O,C)
    intersect; points A and O are not the same
% Determination conditions: points A and B must be different; points A and H_{c} are not the same
```

Illustration of the constructed figure is given in Figure 1

## 194

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 201 terms.

**Time Complexity:** Time spent by the prover is 1.506 seconds.

**NDG conditions** Points  $A$ ,  $H_c$  and  $O$  are not collinear

Points  $A$ ,  $H_c$  and  $O$  are not collinear

Points  $A$ ,  $B$  and  $C$  are not collinear

Line through points  $A$  and  $B$  is not perpendicular to line through points  $B$  and  $O$

#### 4.1.3 Proving $H_c=_Hc$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 17 terms.

**Time Complexity:** Time spent by the prover is 0.377 seconds.

**NDG conditions** Points  $A$ ,  $H_c$  and  $O$  are not collinear

Points  $A$ ,  $H_c$  and  $O$  are not collinear

Points  $A$  and  $B$  are not identical

Points  $A$  and  $B$  are not identical

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{_M_aBC} \neq 0$  i.e., points  $_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$  i.e., points  $_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a_M_bF_{_m_b}^2} \neq S_{F_{_m_a}^1_M_bF_{_m_b}^2}$  i.e., lines  $_M_aF_{_m_a}^1$  and  $_M_bF_{_m_b}^2$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{_Hc}^3} \neq S_{BCF_{_Hc}^3}$  i.e., lines  $AB$  and  $CF_{_Hc}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $H_c=_Hc$

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $A=A$**

Proving failed

#### **4.3.2 Proving $O=_O$**

Proving failed

#### **4.3.3 Proving $H_c=_Hc$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $A=A$**

Proving failed

#### **4.4.2 Proving $O=_O$**

Proving failed

#### **4.4.3 Proving $H_c=_Hc$**

Proving failed

# Problem 35

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 35: Given a point  $A$ , a point  $O$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $H$ , construct a point  $G$  (rule W01); ;
2. Using the point  $A$  and the point  $G$ , construct a point  $M_a$  (rule W01); ;
3. Using the point  $A$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H$  are not the same;
4. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
5. Using the point  $M_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $A$  and  $H$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D21,D26,D3,D8,GD01,GL03,GL04,GL09,L11,L12,L55,L58]

Solving time: 1.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point O 65 51.14
point H 80 72.73
```

```

color 220 0 0
fontsize 9

cmark_t A
cmark_t 0
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a line  $L_{\backslash\_G5271}$  which passes through point  $O$  and point  $H$ 
line  $L_{\backslash\_G5271}$  0 H

color 200 200 200
drawline  $L_{\backslash\_G5271}$ 
color 0 0 0

% Constructing a point  $P_{\backslash\_G5372}$  with coordinates  $(0,0)$ 
point  $P_{\backslash\_G5372}$  0 0
cmark_r  $P_{\backslash\_G5372}$ 

% Constructing a point  $P_{\backslash\_G5296}$  such that  $OP_{\backslash\_G5296}/OP_{\backslash\_G5372}=1$ 
towards  $P_{\backslash\_G5296}$  0  $P_{\backslash\_G5372}$  1
cmark_r  $P_{\backslash\_G5296}$ 
color 200 200 200
drawsegment 0  $P_{\backslash\_G5296}$ 
color 0 0 0

% Constructing a point  $P_{\backslash\_G5341}$  such that  $OP_{\backslash\_G5341}/OP_{\backslash\_G5372}=3$ 
towards  $P_{\backslash\_G5341}$  0  $P_{\backslash\_G5372}$  3
cmark_r  $P_{\backslash\_G5341}$ 
color 200 200 200
drawsegment 0  $P_{\backslash\_G5341}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G5302}$  which passes through point  $H$  and point  $P_{\backslash\_G5341}$ 
line  $L_{\backslash\_G5302}$  H  $P_{\backslash\_G5341}$ 

color 200 200 200
drawline  $L_{\backslash\_G5302}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G5265}$  which contains the point  $P_{\backslash\_G5296}$  and is parallel to the line
 $L_{\backslash\_G5302}$ 
parallel  $L_{\backslash\_G5265}$   $P_{\backslash\_G5296}$   $L_{\backslash\_G5302}$ 

color 200 200 200
drawline  $L_{\backslash\_G5265}$ 
color 0 0 0

% Constructing a point  $G$  which belongs to line  $L_{\backslash\_G5265}$  and line  $L_{\backslash\_G5271}$ 
intersec G  $L_{\backslash\_G5265}$   $L_{\backslash\_G5271}$ 

```

```

cmark_t G

% Constructing a point M_{a} such that AM_{a}/AG=1.5
towards M_{a} A G 1.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Constructing a line a which is perpendicular to line h_{a} and which passes through point M_{a}
perp a M_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same
% Determination conditions: points A and H are not the same

```

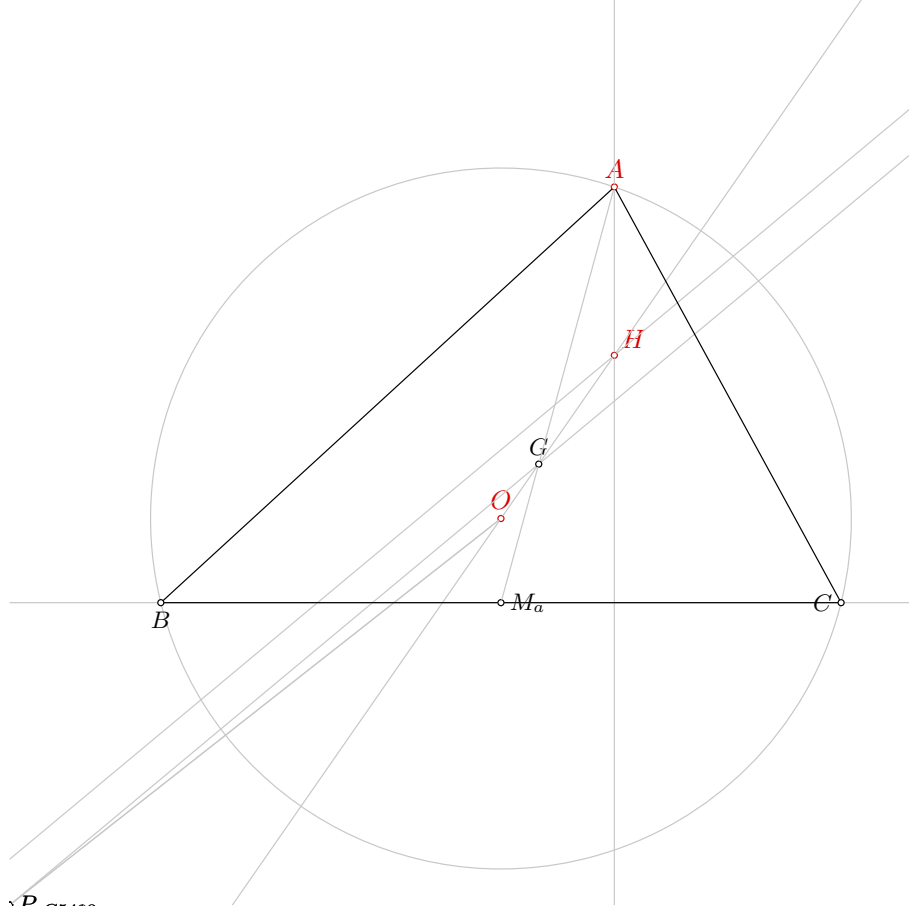


Figure 1: Illustration of the problem 0035

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.032 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

**Status:** Theorem has been proved.



**Space Complexity:** The biggest polynomial obtained during prover execution contains 424 terms.

**Time Complexity:** Time spent by the prover is 2.372 seconds.

**NDG conditions** Points  $P_{G4719}$ ,  $H$  and  $O$  are not collinear

Line through points  $P_{G4719}$  and  $H$  is not parallel with line through points  $A$  and  $O$

Points  $A$ ,  $H$  and  $O$  are not collinear

Points  $A$ ,  $H$  and  $O$  are not collinear

Points  $A$ ,  $B$  and  $C$  are not collinear

Line through points  $A$  and  $B$  is not perpendicular to line through points  $B$  and  $O$

#### 4.1.3 Proving $H=_H$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{P_{G6455}OH} \neq S_{P_{L_{G6424}}^{0}OH}$  i.e., lines  $P_{G6455}P_{L_{G6424}}^{0}$  and  $OH$  are not parallel (construction based assumption)  
 $S_{M_aAH} \neq 0$  i.e., points  $M_a$ ,  $A$  and  $H$  are not collinear (foot is not the point itself; construction based assumption)  
 $S_{M_aBC} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)  
 $S_{M_bAC} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)  
 $S_{M_aM_bF_{M_b}^3} \neq S_{F_{M_a}^2M_bF_{M_b}^3}$  i.e., lines  $M_aF_{M_a}^2$  and  $M_bF_{M_b}^3$  are not parallel (construction based assumption)  
 $S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)  
 $S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)  
 $S_{ABF_{M_b}^5} \neq S_{F_{M_a}^4BF_{M_b}^5}$  i.e., lines  $AF_{M_a}^4$  and  $BF_{M_b}^5$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $H=_H$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $O=_O$

Proving failed

#### 4.3.3 Proving $H=_H$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4.2 Proving $O=_O$

Proving failed

### 4.4.3 Proving $H=_H$

Proving failed

# Problem 36

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 36: Given a point  $A$ , a point  $O$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $T_a$ , construct a line  $s_a$  (rule W02); % DET: points  $A$  and  $T_a$  are not the same;
2. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
3. Using the circle  $k(O, C)$ , the line  $s_a$ , the point  $O$  and the point  $A$ , construct a point  $N_a$  (rule W05); % NDG: line  $s_a$  and circle  $k(O, C)$  intersect % DET: points  $A$  and  $N_a$  must be different;
4. Using the point  $N_a$  and the point  $O$ , construct a line  $m_a$  (rule W02); % DET: points  $N_a$  and  $O$  are not the same;
5. Using the point  $T_a$  and the line  $m_a$ , construct a line  $a$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; line  $s_a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $N_a$  and  $O$  are not the same; points  $A$  and  $N_a$  must be different; points  $A$  and  $T_a$  are not the same.

Rules used: [W02,W04,W05,W06,W10a]

Lemmas used: [D1,D11,D23,D26,D47,GD01,L11,L12,L25,L5]

Solving time: 1.1 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point O 65 51.14
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_t A
cmark_t O
cmark_rb T_{a}
color 0 0 0
fontsize 8

% DET: points A and T_{a} are not the same
% Constructing a line s_{a} which passes through point A and point T_{a}
line s_{a} A T_{a}

color 200 200 200
drawline s_{a}
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line s_{a} and circle k(O,C) intersect% DET: points A and N_{a} must be different
% Constructing a point P_{\_G6876} which is a foot of the point O on the line s_{a}
foot P_{\_G6876} O s_{a}
cmark_r P_{\_G6876}
color 200 200 200
drawline O P_{\_G6876}
color 0 0 0

% Constructing a point N_{a} which is an image of the point A in the symmetry to point/line P_{\_G
6876}
sim N_{a} P_{\_G6876} A
cmark_b N_{a}

% DET: points N_{a} and O are not the same
% Constructing a line m_{a} which passes through point N_{a} and point O
line m_{a} N_{a} O

color 200 200 200
```

```

drawline m_{a}
color 0 0 0

% Constructing a line a which is perpendicular to line m_{a} and which passes through point T_{a}
perp a T_{a} m_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; line s_{a} and circle k(O,C)
% intersect; points A and O are not the same
% Determination conditions: points N_{a} and O are not the same; points A and N_{a} must be
% different; points A and T_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

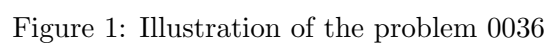
**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=O$

Proving failed

#### 4.1.3 Proving $T_a=T_a$

Proving failed



## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{T_a N_a O} \neq 0$  i.e., points  $T_a$ ,  $N_a$  and  $O$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_a BC} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_b AC} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_a M_b F_{M_a}^2} \neq S_{F_{M_a}^1 M_b F_{M_b}^2}$  i.e., lines  $M_a F_{M_a}^1$  and  $M_b F_{M_b}^2$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $O=_O$

Proving failed

### 4.2.3 Proving $T_a=_T T_a$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $O=_O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 5579 terms.

**Time Complexity:** Time spent by the prover is 6.300 seconds. There are no ndg conditions.

### 4.3.3 Proving $T_a=_T T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6468 terms.

**Time Complexity:** Time spent by the prover is 5.160 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $O=_O$

Proving failed

### 4.4.3 Proving $T_a=_T T_a$

Proving failed

# Problem 37

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 37: Given a point  $A$ , a point  $O$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $T_b$  are not the same;
2. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
3. Using the circle  $k(O, C)$ , the line  $b$ , the point  $O$  and the point  $A$ , construct a point  $C$  (rule W05); % NDG: line  $b$  and circle  $k(O, C)$  intersect % DET: points  $A$  and  $C$  must be different;
4. Using the point  $C$  and the point  $A$ , construct a point  $M_b$  (rule W01); ;
5. Using the point  $O$  and the point  $M_b$ , construct a line  $m_b$  (rule W02); % DET: points  $O$  and  $M_b$  are not the same;
6. Using the circle  $k(O, C)$  and the line  $m_b$ , construct a point  $N_b$  and a point  $N_{bk}$  (rule W04); % NDG: line  $m_b$  and circle  $k(O, C)$  intersect;
7. Using the point  $N_b$  and the point  $T_b$ , construct a line  $s_b$  (rule W02); % DET: points  $N_b$  and  $T_b$  are not the same;
8. Using the circle  $k(O, C)$ , the line  $s_b$ , the point  $O$  and the point  $N_b$ , construct a point  $B$  (rule W05); % NDG: line  $s_b$  and circle  $k(O, C)$  intersect % DET: points  $N_b$  and  $B$  must be different.

Non-degenerate conditions: line  $s_b$  and circle  $k(O, C)$  intersect; line  $m_b$  and circle  $k(O, C)$  intersect; line  $b$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.



Determination conditions: points  $N_b$  and  $B$  must be different; points  $N_b$  and  $T_b$  are not the same; points  $O$  and  $M_b$  are not the same; points  $A$  and  $C$  must be different; points  $A$  and  $T_b$  are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D1,D12,D22,D24,D26,D48,D51,GD01,L11,L12,L26,L6]

Solving time: 2.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point O 65 51.14
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_t A
cmark_t O
cmark_t T_{b}
color 0 0 0
fontsize 8

% DET: points A and T_{b} are not the same
% Constructing a line b which passes through point A and point T_{b}
line b A T_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line b and circle k(O,C) intersect% DET: points A and C must be different
% Constructing a point P_{\_G4981} which is a foot of the point O on the line b
foot P_{\_G4981} O b
cmark_r P_{\_G4981}
color 200 200 200
drawline O P_{\_G4981}
color 0 0 0

% Constructing a point C which is an image of the point A in the symmetry to point/line P_{\_G4981}
sim C P_{\_G4981} A
```

```

cmark_l C

% Constructing a point  $M_{\{b\}}$  such that  $CM_{\{b\}}/CA=0.5$ 
towards M_{b} C A 0.5
cmark_lt M_{b}
color 200 200 200
drawsegment C A
color 0 0 0

% DET: points O and  $M_{\{b\}}$  are not the same
% Constructing a line  $m_{\{b\}}$  which passes through point O and point  $M_{\{b\}}$ 
line m_{b} O M_{b}

color 200 200 200
drawline m_{b}
color 0 0 0

% NDG: line  $m_{\{b\}}$  and circle  $k(O,C)$  intersect
% Constructing points  $N_{\{b\}}$  and  $N_{\{bk\}}$  which are in intersection of  $k(O,C)$  and  $m_{\{b\}}$ 
intersec2 N_{b} N_{bk} k(O,C) m_{b}
cmark_rb N_{b}
cmark_r N_{bk}

% DET: points  $N_{\{b\}}$  and  $T_{\{b\}}$  are not the same
% Constructing a line  $s_{\{b\}}$  which passes through point  $N_{\{b\}}$  and point  $T_{\{b\}}$ 
line s_{b} N_{b} T_{b}

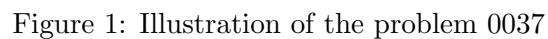
color 200 200 200
drawline s_{b}
color 0 0 0

% NDG: line  $s_{\{b\}}$  and circle  $k(O,C)$  intersect% DET: points  $N_{\{b\}}$  and B must be different
% Constructing a point  $P_{\{\_G5391\}}$  which is a foot of the point O on the line  $s_{\{b\}}$ 
foot P_{\_G5391} O s_{b}
cmark_r P_{\_G5391}
color 200 200 200
drawline O P_{\_G5391}
color 0 0 0

% Constructing a point B which is an image of the point  $N_{\{b\}}$  in the symmetry to point/line  $P_{\{\_G5391\}}$ 
sim B P_{\_G5391} N_{b}
cmark_b B

drawsegment A B
drawsegment A C

```



```
% Non-degenerate conditions: line s_{b} and circle k(O,C) intersect; line m_{b} and circle k(O,C)
% intersect; line b and circle k(O,C) intersect; points A and O are not the same
% Determination conditions: points N_{b} and B must be different; points N_{b} and T_{b} are not
% the same; points O and M_{b} are not the same; points A and C must be different; points A and T
% _{b} are not the same
```

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

### 4.1.1 Proving $A=A$

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**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.032 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

Proving failed

#### 4.1.3 Proving $T_b=_T T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{_M_a B C} \neq 0$  i.e., points  $_M_a, B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_b A C} \neq 0$  i.e., points  $_M_b, A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a _M_b F_{_m_b}^1} \neq S_{F_{_m_a}^0 _M_b F_{_m_b}^1}$  i.e., lines  $_M_a F_{_m_a}^0$  and  $_M_b F_{_m_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $T_b=_T T_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

Proving failed

#### 4.3.2 Proving $O=_O$

Proving failed

#### 4.3.3 Proving $T_b=_T T_b$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A=A$

Proving failed

#### 4.4.2 Proving $O=_O$

Proving failed

#### 4.4.3 Proving $T_b=_T T_b$

Proving failed

# Problem 38

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 38: Given a point  $A$ , a point  $O$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $T_c$  are not the same;
2. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
3. Using the circle  $k(O, C)$ , the line  $c$ , the point  $O$  and the point  $A$ , construct a point  $B$  (rule W05); % NDG: line  $c$  and circle  $k(O, C)$  intersect % DET: points  $A$  and  $B$  must be different;
4. Using the point  $B$  and the point  $A$ , construct a point  $M_c$  (rule W01); ;
5. Using the point  $O$  and the point  $M_c$ , construct a line  $m_c$  (rule W02); % DET: points  $O$  and  $M_c$  are not the same;
6. Using the circle  $k(O, C)$  and the line  $m_c$ , construct a point  $N_c$  and a point  $N_{ck}$  (rule W04); % NDG: line  $m_c$  and circle  $k(O, C)$  intersect;
7. Using the point  $N_c$  and the point  $T_c$ , construct a line  $s_c$  (rule W02); % DET: points  $N_c$  and  $T_c$  are not the same;
8. Using the circle  $k(O, C)$ , the line  $s_c$ , the point  $O$  and the point  $N_c$ , construct a point  $C$  (rule W05); % NDG: line  $s_c$  and circle  $k(O, C)$  intersect % DET: points  $N_c$  and  $C$  must be different.

Non-degenerate conditions: line  $s_c$  and circle  $k(O, C)$  intersect; line  $m_c$  and circle  $k(O, C)$  intersect; line  $c$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $N_c$  and  $C$  must be different; points  $N_c$  and  $T_c$  are not the same; points  $O$  and  $M_c$  are not the same; points  $A$  and  $B$  must be different; points  $A$  and  $T_c$  are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D13,D20,D25,D26,D49,D52,GD01,GL04,L1,L11,L12,L27,L7]

Solving time: 4.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point O 65 51.14
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_t A
cmark_t O
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points A and T_{c} are not the same
% Constructing a line c which passes through point A and point T_{c}
line c A T_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line c and circle k(O,C) intersect% DET: points A and B must be different
% Constructing a point P_{\_G4776} which is a foot of the point O on the line c
foot P_{\_G4776} O c
cmark_r P_{\_G4776}
color 200 200 200
drawline O P_{\_G4776}
color 0 0 0

% Constructing a point B which is an image of the point A in the symmetry to point/line P_{\_G4776}
sim B P_{\_G4776} A
```

```

cmark_b B

% Constructing a point  $M_{\{c\}}$  such that  $BM_{\{c\}}/BA=0.5$ 
towards M_{c} B A 0.5
cmark_lt M_{c}
color 200 200 200
drawsegment B A
color 0 0 0

% DET: points O and  $M_{\{c\}}$  are not the same
% Constructing a line  $m_{\{c\}}$  which passes through point O and point  $M_{\{c\}}$ 
line m_{c} O M_{c}

color 200 200 200
drawline m_{c}
color 0 0 0

% NDG: line  $m_{\{c\}}$  and circle  $k(O,C)$  intersect
% Constructing points  $N_{\{c\}}$  and  $N_{\{ck\}}$  which are in intersection of  $k(O,C)$  and  $m_{\{c\}}$ 
intersec2 N_{c} N_{ck} k(O,C) m_{c}
cmark_b N_{c}
cmark_r N_{ck}

% DET: points  $N_{\{c\}}$  and  $T_{\{c\}}$  are not the same
% Constructing a line  $s_{\{c\}}$  which passes through point  $N_{\{c\}}$  and point  $T_{\{c\}}$ 
line s_{c} N_{c} T_{c}

color 200 200 200
drawline s_{c}
color 0 0 0

% NDG: line  $s_{\{c\}}$  and circle  $k(O,C)$  intersect% DET: points  $N_{\{c\}}$  and C must be different
% Constructing a point  $P_{\{\backslash\_G5186\}}$  which is a foot of the point O on the line  $s_{\{c\}}$ 
foot P_{\backslash\_G5186} O s_{c}
cmark_r P_{\backslash\_G5186}
color 200 200 200
drawline O P_{\backslash\_G5186}
color 0 0 0

% Constructing a point C which is an image of the point  $N_{\{c\}}$  in the symmetry to point/line  $P_{\{\backslash\_G5186\}}$ 
sim C P_{\backslash\_G5186} N_{c}
cmark_l C

drawsegment A B
drawsegment A C

```

Figure 1: Illustration of the problem 0038

drawsegment B C

*% Non-degenerate conditions: line s\_{c} and circle k(O,C) intersect; line m\_{c} and circle k(O,C) intersect; line c and circle k(O,C) intersect; points A and O are not the same*  
*% Determination conditions: points N\_{c} and C must be different; points N\_{c} and T\_{c} are not the same; points O and M\_{c} are not the same; points A and B must be different; points A and T\_{c} are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.032 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

Proving failed

#### 4.1.3 Proving $T_c=_Tc$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{_M_aBC} \neq 0$  i.e., points  $_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$  i.e., points  $_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a _M_b F_{_m_b}^1} \neq S_{F_{_m_a}^0 _M_b F_{_m_b}^1}$  i.e., lines  $_M_a F_{_m_a}^0$  and  $_M_b F_{_m_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $T_c=_Tc$

Proving failed



### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $A=A$**

Proving failed

#### **4.3.2 Proving $O=_O$**

Proving failed

#### **4.3.3 Proving $T_c=_T T_c$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $A=A$**

Proving failed

#### **4.4.2 Proving $O=_O$**

Proving failed

#### **4.4.3 Proving $T_c=_T T_c$**

Proving failed

# Problem 39

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 39: Given a point  $A$ , a point  $O$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $I$ , construct a line  $s_a$  (rule W02); % DET: points  $A$  and  $I$  are not the same;
2. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
3. Using the circle  $k(O, C)$ , the line  $s_a$ , the point  $O$  and the point  $A$ , construct a point  $N_a$  (rule W05); % NDG: line  $s_a$  and circle  $k(O, C)$  intersect % DET: points  $A$  and  $N_a$  must be different;
4. Using the point  $I$  and the point  $N_a$ , construct a circle  $k(N_a, C)$  (rule W06); % NDG: points  $I$  and  $N_a$  are not the same;
5. Using the circle  $k(O, C)$  and the circle  $k(N_a, C)$ , construct a point  $C$  and a point  $B$  (rule W07); % NDG: circles  $k(O, C)$  and  $k(N_a, C)$  intersect % DET: circles  $k(O, C)$  and  $k(N_a, C)$  are not the same.

Non-degenerate conditions: circles  $k(O, C)$  and  $k(N_a, C)$  intersect; points  $I$  and  $N_a$  are not the same; line  $s_a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: circles  $k(O, C)$  and  $k(N_a, C)$  are not the same; points  $A$  and  $N_a$  must be different; points  $A$  and  $I$  are not the same.

Rules used: [W02,W05,W06,W07]

Lemmas used: [D2,D26,D47,GD02,L11,L12,L31,L32,L5]

Solving time: 0.7 seconds.

## 3.2 Construction in GCLC language

```

dim 120 120

point A 80 95
point O 65 51.14
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_t A
cmark_t O
cmark_b I
color 0 0 0
fontsize 8

% DET: points A and I are not the same
% Constructing a line  $s_{\{a\}}$  which passes through point A and point I
line  $s_{\{a\}}$  A I

color 200 200 200
drawline  $s_{\{a\}}$ 
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point A
circle  $k(O,C)$  O A

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $s_{\{a\}}$  and circle  $k(O,C)$  intersect% DET: points A and  $N_{\{a\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G4296\}}$  which is a foot of the point O on the line  $s_{\{a\}}$ 
foot  $P_{\{\backslash\_G4296\}}$  O  $s_{\{a\}}$ 
cmark_r  $P_{\{\backslash\_G4296\}}$ 
color 200 200 200
drawline O  $P_{\{\backslash\_G4296\}}$ 
color 0 0 0

% Constructing a point  $N_{\{a\}}$  which is an image of the point A in the symmetry to point/line  $P_{\{\backslash\_G4296\}}$ 
sim  $N_{\{a\}}$   $P_{\{\backslash\_G4296\}}$  A
cmark_b  $N_{\{a\}}$ 

% NDG: points I and  $N_{\{a\}}$  are not the same
% Constructing a circle  $k(N_{\{a\}},C)$  whose center is at point  $N_{\{a\}}$  and which passes through point I
circle  $k(N_{\{a\}},C)$   $N_{\{a\}}$  I

color 200 200 200

```

```
drawcircle k(N_{a},C)
color 0 0 0
```

```
% NDG: circles k(O,C) and k(N_{a},C) intersect% DET: circles k(O,C) and k(N_{a},C) are not the same
% Constructing points C and B which are in intersection of k(O,C) and k(N_{a},C)
intersec2 C B k(O,C) k(N_{a},C)
cmark_l C
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: circles k(O,C) and k(N_{a},C) intersect; points I and N_{a} are not
the same; line s_{a} and circle k(O,C) intersect; points A and O are not the same
% Determination conditions: circles k(O,C) and k(N_{a},C) are not the same; points A and N_{a} must
be different; points A and I are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

Proving failed

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{M_aBC} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)  
 $S_{M_bAC} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)  
 $S_{M_aM_bF_{-m_b}^1} \neq S_{F_{-m_a}^0M_bF_{-m_b}^1}$  i.e., lines  $M_aF_{-m_a}^0$  and  $M_bF_{-m_b}^1$  are not parallel (construction based assumption)

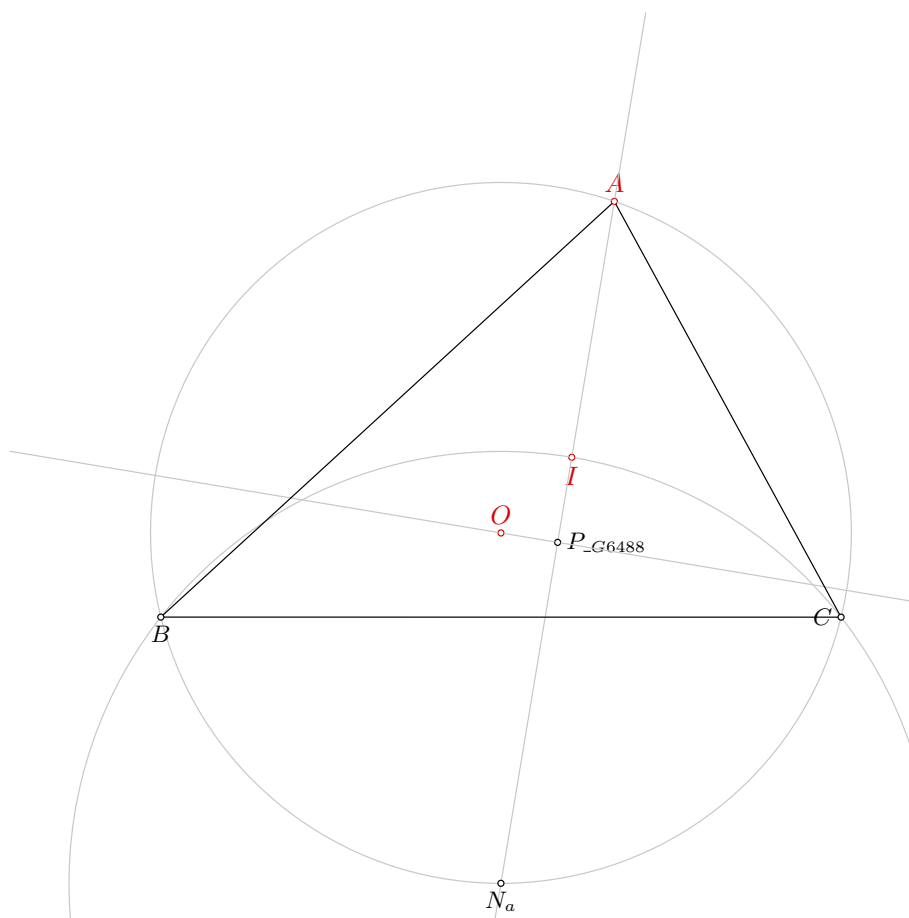


Figure 1: Illustration of the problem 0039

Total number of proof steps: 1  
Time spent by the prover: 0.000 seconds

#### **4.2.2 Proving $O=_O$**

Proving failed

#### **4.2.3 Proving $I=_I$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $A=A$**

Proving failed

#### **4.3.2 Proving $O=_O$**

Proving failed

#### **4.3.3 Proving $I=_I$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $A=A$**

Proving failed

#### **4.4.2 Proving $O=_O$**

Proving failed

#### **4.4.3 Proving $I=_I$**

Proving failed

# Problem 40

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 40: Given a point  $A$ , a point  $M_a$  and a point  $M_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
2. Using the point  $M_a$  and the point  $C$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D21,D22,GL03,GL04]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point M_{a} 65 40
point M_{b} 95 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r M_{a}
cmark_lt M_{b}
color 0 0 0
fontsize 8
```

```
% Constructing a point C such that AC/AM_{b}=2
```

```

towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% Constructing a point B such that M_{a}B/M_{a}C=-1
towards B M_{a} C -1
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.033 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a=_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.173 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $M_b=_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.06 seconds.

**NDG conditions** There are no NDG conditions for this theorem



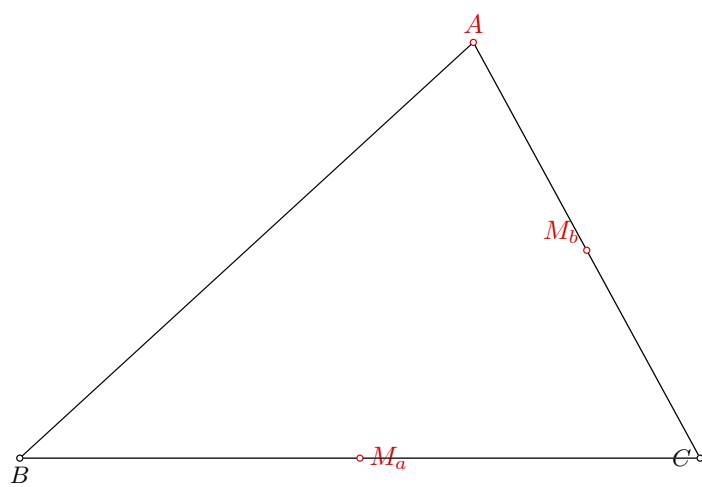


Figure 1: Illustration of the problem 0040

## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

There are no ndg conditions.  
Total number of proof steps: 1  
Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_a=_M M_a$

There are no ndg conditions.  
Total number of proof steps: 44  
Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $M_b=_M M_b$

There are no ndg conditions.  
Total number of proof steps: 42  
Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_a=_M M_a$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 19 terms.  
**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.3 Proving $M_b=_M M_b$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.  
**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $M_a=_M M_a$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 67 terms.  
**Time Complexity:** Time spent by the prover is 0.540 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_b = -M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 67 terms.

**Time Complexity:** Time spent by the prover is 0.620 seconds. There are no ndg conditions.

# Problem 41

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 41: Given a point  $A$ , a point  $M_a$  and a point  $M_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); ;
2. Using the point  $M_a$  and the point  $B$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D20,D21,GL03,GL04]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point M_{a} 65 40
point M_{c} 50 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r M_{a}
cmark_lt M_{c}
color 0 0 0
fontsize 8
```

```
% Constructing a point B such that AB/AM_{c}=2
```

```

towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% Constructing a point C such that M_{a}C/M_{a}B=-1
towards C M_{a} B -1
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.034 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a=_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.177 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $M_c=_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.055 seconds.

**NDG conditions** There are no NDG conditions for this theorem

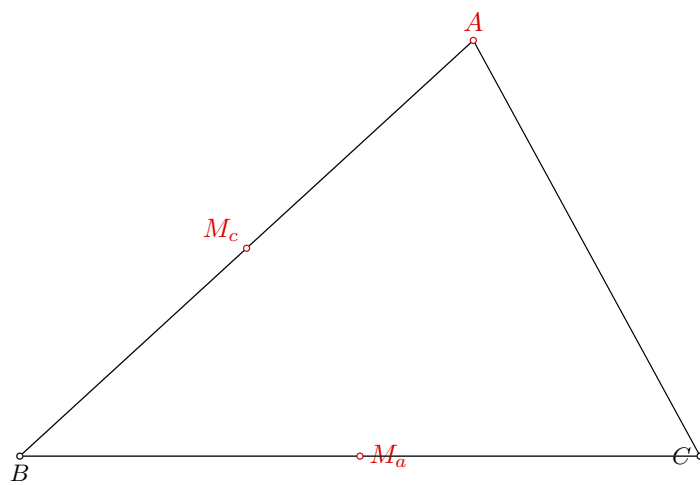


Figure 1: Illustration of the problem 0041

## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

There are no ndg conditions.  
Total number of proof steps: 1  
Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_a=_M M_a$

There are no ndg conditions.  
Total number of proof steps: 41  
Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $M_c=_M M_c$

There are no ndg conditions.  
Total number of proof steps: 41  
Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_a=_M M_a$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 19 terms.  
**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

### 4.3.3 Proving $M_c=_M M_c$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $M_a=_M M_a$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 67 terms.  
**Time Complexity:** Time spent by the prover is 0.620 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_c = M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 67 terms.

**Time Complexity:** Time spent by the prover is 0.500 seconds. There are no ndg conditions.



# Problem 42

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 42: Given a point  $M_a$ , a point  $G$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is redundant.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $G$ , construct a point  $A$  (rule W01); ;
2. Choose freely a point  $B$  (rule free);
3. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01,free]

Lemmas used: [D21,GL03,GL04,L55]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40
```

```
point G 70 58.33
```

```
point A 80 95
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r M_{a}
```

```
cmark_t G
```

```
cmark_t A
```

```
color 0 0 0
```

```
fontsize 8
```

```

% Constructing a point A such that  $M_{\{a\}A}/M_{\{a\}G}=3$ 
towards A M_{a} G 3
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

```

```

% Constructing a free point B
point B 20 40

```

```

cmark_b B

```

```

% Constructing a point C such that  $BC/BM_{\{a\}}=2$ 
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = \_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.054 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $G = \_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 77 terms.

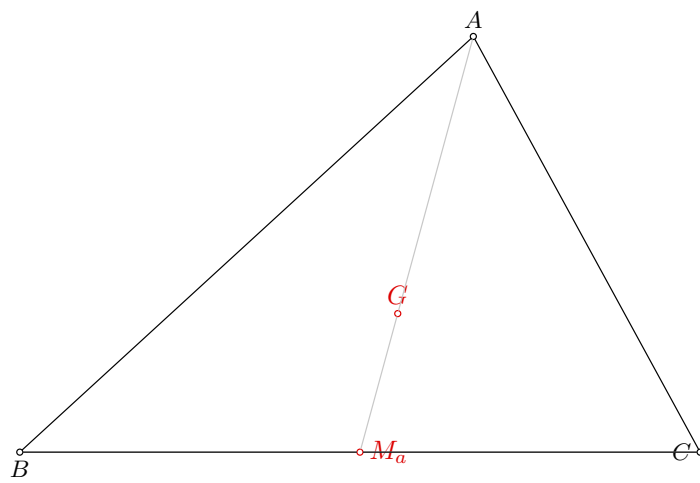


Figure 1: Illustration of the problem 0042

**Time Complexity:** Time spent by the prover is 0.784 seconds.

**NDG conditions** Line through points  $A$  and  $\neg M_a$  is not parallel with line through points  $B$  and  $\neg M_b$   
Points  $\neg M_a$ ,  $G$  and  $M_a$  are not collinear

### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_a=\neg M_a$

NDG conditions are:

$S_{AB\neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A\neg M_a$  and  $B\neg M_b$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $G=\neg G$

NDG conditions are:

$S_{AB\neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A\neg M_a$  and  $B\neg M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (cancellation assumption)

Total number of proof steps: 588

Time spent by the prover: 0.120 seconds

### 4.2.3 Proving $A=A$

NDG conditions are:

$S_{AB\neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A\neg M_a$  and  $B\neg M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_a=\neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 12 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.2 Proving $G=\neg G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 939 terms.

**Time Complexity:** Time spent by the prover is 1.420 seconds. There are no ndg conditions.

### 4.3.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 319 terms.

**Time Complexity:** Time spent by the prover is 15.460 seconds. There are no ndg conditions.

### 4.4.2 Proving $G = \neg G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 319 terms.

**Time Complexity:** Time spent by the prover is 15.470 seconds. There are no ndg conditions.

### 4.4.3 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 43

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 43: Given a point  $M_a$ , a point  $H_a$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $H_a$  are not the same;
2. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
3. Choose freely a point  $A$  on the line  $h_a$  (rule WOnline2);
4. Choose freely a point  $B$  on the line  $a$  (rule WOnline1) ;
5. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: points  $M_a$  and  $H_a$  are not the same.

Rules used: [W01,W02,W10b,WOnline1,WOnline2]

Lemmas used: [D21,D5,D8,GD01,GL03,GL09]

Solving time: 194.7 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40
point H_{a} 80 40
point A 80 95
```

```
color 220 0 0
fontsize 9
```

```

cmark_r M_{a}
cmark_r H_{a}
cmark_t A
color 0 0 0
fontsize 8

% DET: points M_{a} and H_{a} are not the same
% Constructing a line a which passes through point M_{a} and point H_{a}
line a M_{a} H_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% Generating random value V[_G7077]
random V[_G7077]

% Calculating value V[_G7098] using formula V[_G7077]*20
expression V[_G7098] { V[_G7077]*20 }

% Constructing a point A which is a point for which holds  $H_{a}A = V[_G7098]$  and angle  $M_{a}H_{a}A = 90$ 
turtle A M_{a} H_{a} 90 V[_G7098]
cmark_t A

% Choosing randomly a point B on the line M_{a}H_{a}
online B M_{a} H_{a}
cmark_b B
color 200 200 200
drawline M_{a} H_{a}
color 0 0 0

% Constructing a point C such that  $BC/BM_{a}=2$ 
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C

```

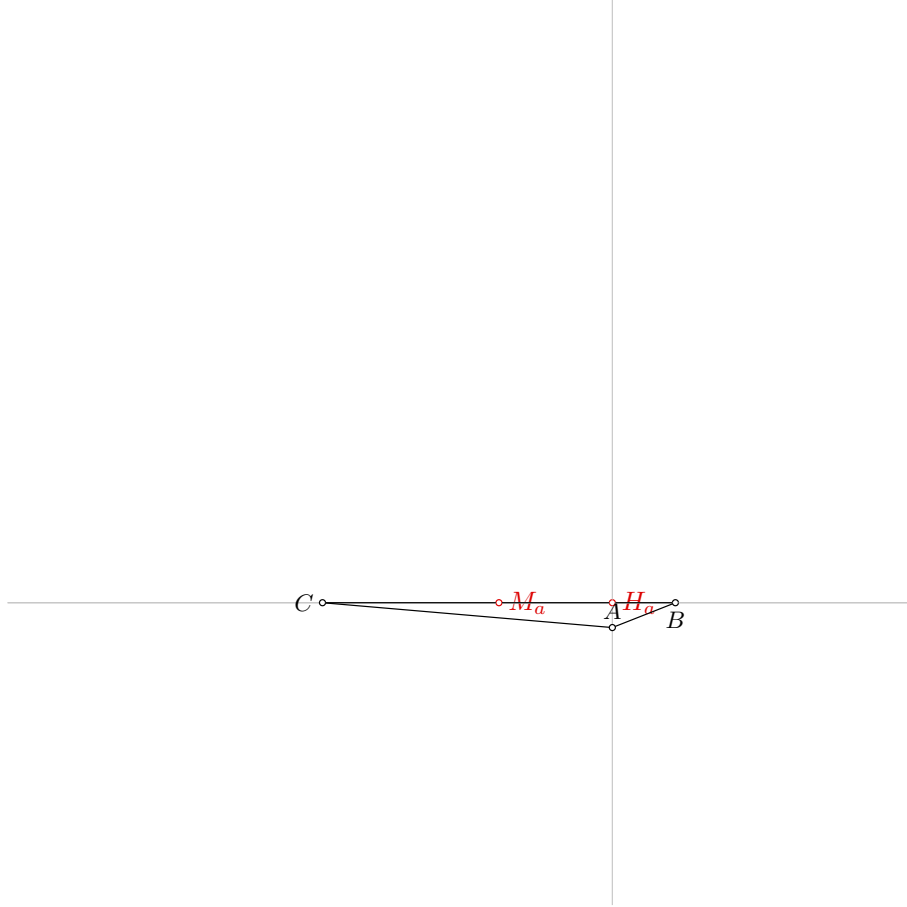


Figure 1: Illustration of the problem 0043

```
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions:
```

```
% Determination conditions: points  $M_{\{a\}}$  and  $H_{\{a\}}$  are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format



## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = \neg M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.032 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a = \neg H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.088 seconds.

**NDG conditions** Line through points  $C$  and  $B$  is not perpendicular to line through points  $B$  and  $M_a$

#### 4.1.3 Proving $A = A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{\neg h_a}^2} \neq S_{CAF_{\neg h_a}^2}$  i.e., lines  $BC$  and  $AF_{\neg h_a}^2$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.2.3 Proving $A = A$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $A = A$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 2 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $A = A$

Proving failed

# Problem 44

*Generated automatically by ArgoTriCS  
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## 1 Problem

Problem 44: Given a point  $A$ , a point  $M_a$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $H_b$  are not the same;
2. Using the point  $H_b$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_b$  and  $M_a$  are not the same;
3. Using the circle  $k(M_a, B)$ , the line  $b$ , the point  $M_a$  and the point  $H_b$ , construct a point  $C$  (rule W05); % NDG: line  $b$  and circle  $k(M_a, B)$  intersect % DET: points  $H_b$  and  $C$  must be different;
4. Using the point  $M_a$  and the point  $C$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: line  $b$  and circle  $k(M_a, B)$  intersect; points  $H_b$  and  $M_a$  are not the same.

Determination conditions: points  $H_b$  and  $C$  must be different; points  $A$  and  $H_b$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D21,D6,GD01,GD02,GL03,GL04,L37,L38]

Solving time: 0.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point M_{a} 65 40
point H_{b} 89.36 77.83
```

```

color 220 0 0
fontsize 9

cmark_t A
cmark_r M_{a}
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points H_{b} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{b}
circle k(M_{a},B) M_{a} H_{b}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line b and circle k(M_{a},B) intersect% DET: points H_{b} and C must be different
% Constructing a point P_{\_G5909} which is a foot of the point M_{a} on the line b
foot P_{\_G5909} M_{a} b
cmark_r P_{\_G5909}
color 200 200 200
drawline M_{a} P_{\_G5909}
color 0 0 0

% Constructing a point C which is an image of the point H_{b} in the symmetry to point/line P_{\_G5909}
sim C P_{\_G5909} H_{b}
cmark_l C

% Constructing a point B such that M_{a}B/M_{a}C=-1
towards B M_{a} C -1
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

drawsegment A B

```

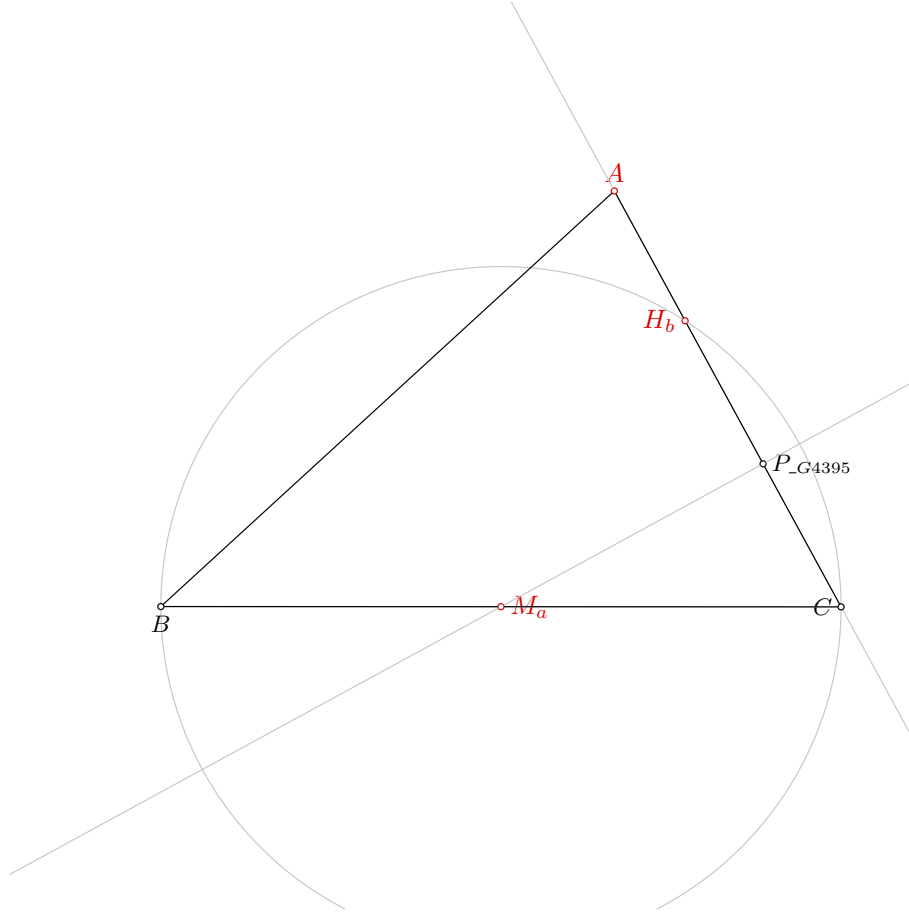


Figure 1: Illustration of the problem 0044

```
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line b and circle k(M_{a},B) intersect; points H_{b} and M_{a} are not
the same
% Determination conditions: points H_{b} and C must be different; points A and H_{b} are not the
same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a = \neg M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.166 seconds.

**NDG conditions** Points  $H_b$ ,  $A$  and  $M_a$  are not collinear

Points  $H_b$ ,  $A$  and  $M_a$  are not collinear

#### 4.1.3 Proving $H_b = \neg H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 39 terms.

**Time Complexity:** Time spent by the prover is 0.518 seconds.

**NDG conditions** Points  $H_b$ ,  $A$  and  $M_a$  are not collinear

Points  $H_b$ ,  $A$  and  $M_a$  are not collinear

Points  $A$  and  $C$  are not identical

Points  $A$  and  $C$  are not identical

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A = A$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^0_{\neg H_b}} \neq S_{CBF^0_{\neg H_b}}$  i.e., lines  $AC$  and  $BF^0_{\neg H_b}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^0_{\neg H_b}} \neq S_{CBF^0_{\neg H_b}}$  i.e., lines  $AC$  and  $BF^0_{\neg H_b}$  are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $H_b = \neg H_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A = A$

Proving failed

#### 4.3.2 Proving $M_a = \neg M_a$

Proving failed

### **4.3.3 Proving $H_b = \neg H_b$**

Proving failed

## **4.4 GCLC - Grobner basis method**

### **4.4.1 Proving $A = A$**

Proving failed

### **4.4.2 Proving $M_a = \neg M_a$**

Proving failed

### **4.4.3 Proving $H_b = \neg H_b$**

Proving failed

# Problem 45

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 45: Given a point  $A$ , a point  $M_a$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $H_c$  are not the same;
2. Using the point  $H_c$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_c$  and  $M_a$  are not the same;
3. Using the circle  $k(M_a, B)$ , the line  $c$ , the point  $M_a$  and the point  $H_c$ , construct a point  $B$  (rule W05); % NDG: line  $c$  and circle  $k(M_a, B)$  intersect % DET: points  $H_c$  and  $B$  must be different;
4. Using the point  $M_a$  and the point  $B$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: line  $c$  and circle  $k(M_a, B)$  intersect; points  $H_c$  and  $M_a$  are not the same.

Determination conditions: points  $H_c$  and  $B$  must be different; points  $A$  and  $H_c$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D21,D7,GD01,GD02,GL03,GL04,L39]

Solving time: 0.4 seconds.

### 3.2 Construction in GCLC language

dim 120 120

```
point A 80 95
point M_{a} 65 40
point H_{c} 68.91 84.83
```



```

color 220 0 0
fontsize 9

cmark_t A
cmark_r M_{a}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points H_{c} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{c}
circle k(M_{a},B) M_{a} H_{c}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line c and circle k(M_{a},B) intersect% DET: points H_{c} and B must be different
% Constructing a point P_{\_G5522} which is a foot of the point M_{a} on the line c
foot P_{\_G5522} M_{a} c
cmark_r P_{\_G5522}
color 200 200 200
drawline M_{a} P_{\_G5522}
color 0 0 0

% Constructing a point B which is an image of the point H_{c} in the symmetry to point/line P_{\_G5522}
sim B P_{\_G5522} H_{c}
cmark_b B

% Constructing a point C such that M_{a}C/M_{a}B=-1
towards C M_{a} B -1
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

drawsegment A B

```

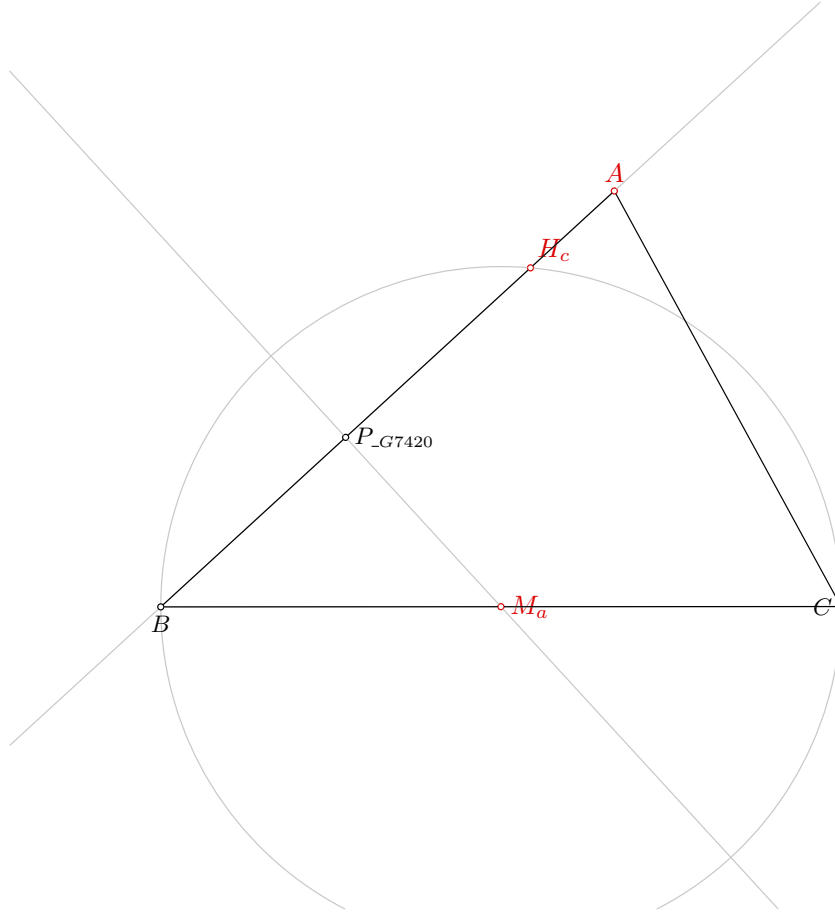


Figure 1: Illustration of the problem 0045

```
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line c and circle k(M_{a},B) intersect; points H_{c} and M_{a} are not
the same
% Determination conditions: points H_{c} and B must be different; points A and H_{c} are not the
same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a = \neg M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.163 seconds.

**NDG conditions** Points  $H_c$  and  $M_a$  are not identical

Points  $H_c$  and  $M_a$  are not identical

#### 4.1.3 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 39 terms.

**Time Complexity:** Time spent by the prover is 0.548 seconds.

**NDG conditions** Points  $H_c$  and  $M_a$  are not identical

Points  $H_c$  and  $M_a$  are not identical

Points  $A$  and  $B$  are not identical

Points  $A$  and  $B$  are not identical

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A = A$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{H_c}^0} \neq S_{BCF_{H_c}^0}$  i.e., lines  $AB$  and  $CF_{H_c}^0$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{H_c}^0} \neq S_{BCF_{H_c}^0}$  i.e., lines  $AB$  and  $CF_{H_c}^0$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $H_c = \neg H_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A = A$

Proving failed

#### 4.3.2 Proving $M_a = \neg M_a$

Proving failed

### **4.3.3 Proving $H_c = \neg H_c$**

Proving failed

## **4.4 GCLC - Grobner basis method**

### **4.4.1 Proving $A = A$**

Proving failed

### **4.4.2 Proving $M_a = \neg M_a$**

Proving failed

### **4.4.3 Proving $H_c = \neg H_c$**

Proving failed

# Problem 46

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 46: Given a point  $A$ , a point  $M_a$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $M_a$ , construct a point  $G$  (rule W01); ;
2. Using the point  $H$  and the point  $G$ , construct a point  $O$  (rule W01); ;
3. Using the point  $A$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H$  are not the same;
4. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
5. Using the point  $M_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $A$  and  $H$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D21,D26,D3,D8,GD01,GL03,GL09,L11,L12,L55,L58]

Solving time: 3.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point M_{a} 65 40
point H 80 72.73
```

```

color 220 0 0
fontsize 9

cmark_t A
cmark_r M_{a}
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a line L_{\_G5108} which passes through point A and point M_{a}
line L_{\_G5108} A M_{a}

color 200 200 200
drawline L_{\_G5108}
color 0 0 0

% Constructing a point P_{\_G5209} with coordinates (0,0)
point P_{\_G5209} 0 0
cmark_r P_{\_G5209}

% Constructing a point P_{\_G5133} such that AP_{\_G5133}/AP_{\_G5209}=2
towards P_{\_G5133} A P_{\_G5209} 2
cmark_r P_{\_G5133}
color 200 200 200
drawsegment A P_{\_G5133}
color 0 0 0

% Constructing a point P_{\_G5178} such that AP_{\_G5178}/AP_{\_G5209}=3
towards P_{\_G5178} A P_{\_G5209} 3
cmark_r P_{\_G5178}
color 200 200 200
drawsegment A P_{\_G5178}
color 0 0 0

% Constructing a line L_{\_G5139} which passes through point M_{a} and point P_{\_G5178}
line L_{\_G5139} M_{a} P_{\_G5178}

color 200 200 200
drawline L_{\_G5139}
color 0 0 0

% Constructing a line L_{\_G5102} which contains the point P_{\_G5133} and is parallel to the line
L_{\_G5139}
parallel L_{\_G5102} P_{\_G5133} L_{\_G5139}

color 200 200 200
drawline L_{\_G5102}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G5102} and line L_{\_G5108}
intersec G L_{\_G5102} L_{\_G5108}

```

```

cmark_t G

% Constructing a point O such that HO/HG=1.5
towards O H G 1.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Constructing a line a which is perpendicular to line h_{a} and which passes through point M_{a}
perp a M_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same
% Determination conditions: points A and H are not the same

```

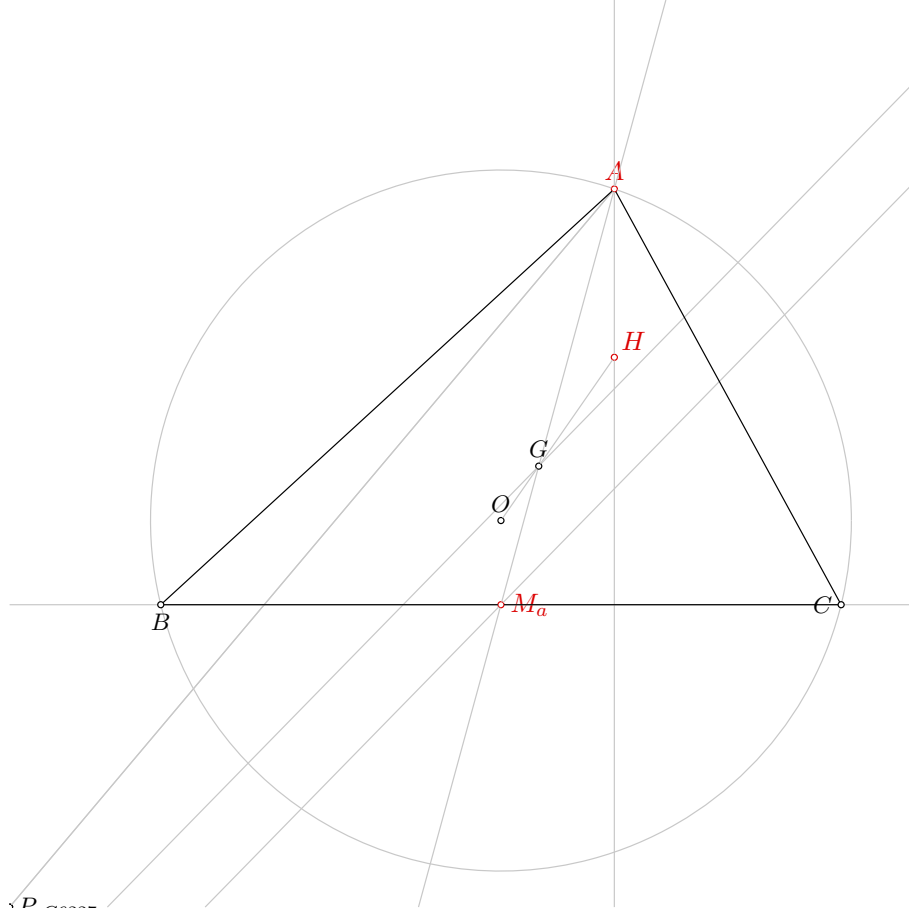


Figure 1: Illustration of the problem 0046

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.035 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a = M_a$

Proving failed



### 4.1.3 Proving $H=_H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 412 terms.

**Time Complexity:** Time spent by the prover is 7.034 seconds.

**NDG conditions** Points  $P_{G6662}$  and  $G$  are not identical

Points  $A$ ,  $H$  and  $M_a$  are not collinear

Points  $A$ ,  $H$  and  $M_a$  are not collinear

Points  $A$ ,  $B$  and  $C$  are not collinear

Line through points  $A$  and  $B$  is not perpendicular to line through points  $B$  and  $M_a$

## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

Proving failed

### 4.2.2 Proving $M_a=_M M_a$

Proving failed

### 4.2.3 Proving $H=_H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

Proving failed

### 4.3.2 Proving $M_a=_M M_a$

Proving failed

### 4.3.3 Proving $H=_H$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

Proving failed

### 4.4.2 Proving $M_a=_M M_a$

Proving failed

### 4.4.3 Proving $H=_H$

Proving failed

# Problem 47

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 47: Given a point  $A$ , a point  $M_a$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $T_a$ , construct a line  $s_a$  (rule W02); % DET: points  $A$  and  $T_a$  are not the same;
2. Using the point  $M_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $T_a$  are not the same;
3. Using the point  $M_a$  and the line  $a$ , construct a line  $m_a$  (rule W10b); ;
4. Using the line  $m_a$  and the line  $s_a$ , construct a point  $N_a$  (rule W03); % NDG: lines  $m_a$  and  $s_a$  are not parallel % DET: lines  $m_a$  and  $s_a$  are not the same;
5. Using the point  $A$  and the point  $N_a$ , construct a line  $m(AN_a)$  (rule W14); % DET: points  $A$  and  $N_a$  are not the same;
6. Using the line  $m(AN_a)$  and the line  $m_a$ , construct a point  $O$  (rule W03); % NDG: lines  $m(AN_a)$  and  $m_a$  are not parallel % DET: lines  $m(AN_a)$  and  $m_a$  are not the same;
7. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
8. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same; lines  $m(AN_a)$  and  $m_a$  are not parallel; lines  $m_a$  and  $s_a$  are not parallel.

Determination conditions: lines  $m(AN_a)$  and  $m_a$  are not the same; points  $A$  and  $N_a$  are not the same; lines  $m_a$  and  $s_a$  are not the same; points  $M_a$  and  $T_a$  are not the same; points  $A$  and  $T_a$  are not the same.

Rules used: [W02,W03,W04,W06,W10b,W14]

Lemmas used: [D1,D11,D21,D23,D26,D47,GD01,GL01,GL09,L11,L12,L25,L5]

Solving time: 2.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point M_{a} 65 40
point T_{a} 70.86 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r M_{a}
cmark_rb T_{a}
color 0 0 0
fontsize 8
```

```
% DET: points A and T_{a} are not the same
% Constructing a line s_{a} which passes through point A and point T_{a}
line s_{a} A T_{a}
```

```
color 200 200 200
drawline s_{a}
color 0 0 0
```

```
% DET: points M_{a} and T_{a} are not the same
% Constructing a line a which passes through point M_{a} and point T_{a}
line a M_{a} T_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% Constructing a line m_{a} which is perpendicular to line a and which passes through point M_{a}
perp m_{a} M_{a} a
```

```
color 200 200 200
drawline m_{a}
color 0 0 0
```

```
% NDG: lines m_{a} and s_{a} are not parallel% DET: lines m_{a} and s_{a} are not the same
% Constructing a point N_{a} which belongs to line m_{a} and line s_{a}
intersec N_{a} m_{a} s_{a}
cmark_b N_{a}
```

```

% DET: points A and N_{a} are not the same
% Constructing bisector m(AN_{a}) of the segment AN_{a}
med m(AN_{a}) A N_{a}

color 200 200 200
drawline m(AN_{a})
color 0 0 0

color 200 200 200
drawsegment A N_{a}
color 0 0 0

% NDG: lines m(AN_{a}) and m_{a} are not parallel% DET: lines m(AN_{a}) and m_{a} are not the same
% Constructing a point O which belongs to line m(AN_{a}) and line m_{a}
intersec O m(AN_{a}) m_{a}
cmark_t O

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

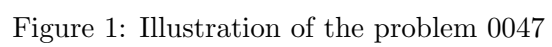
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% lines m(AN_{a}) and m_{a} are not parallel; lines m_{a} and s_{a} are not parallel
% Determination conditions: lines m(AN_{a}) and m_{a} are not the same; points A and N_{a} are not
% the same; lines m_{a} and s_{a} are not the same; points M_{a} and T_{a} are not the same;
% points A and T_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format



## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.036 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a=_M M_a$

Proving failed

#### 4.1.3 Proving $T_a=_T T_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{M_a A T_a} \neq S_{T_{m_a}^0 A T_a}$  i.e., lines  $M_a T_{m_a}^0$  and  $A T_a$  are not parallel (construction based assumption)

$S_{M_{m(AN_a)}^1 M_a T_{m_a}^0} \neq S_{T_{m(AN_a)}^2 M_a T_{m_a}^0}$  i.e., lines  $M_{m(AN_a)}^1 T_{m(AN_a)}^2$  and  $M_a T_{m_a}^0$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_a=_M M_a$

Proving failed

#### 4.2.3 Proving $T_a=_T T_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_a=_M M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 7807 terms.

**Time Complexity:** Time spent by the prover is 7.800 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_a=_T T_a$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4.2 Proving $M_a=_M a$

Proving failed

### 4.4.3 Proving $T_a=_T a$

Proving failed

## Problem 48

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 48: Given a point  $A$ , a point  $M_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.



## Problem 49

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 49: Given a point  $A$ , a point  $M_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 50

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 50: Given a point  $A$ , a point  $M_a$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $I$ , construct a line  $IM_a$  (rule W02); % DET: points  $M_a$  and  $I$  are not the same;
2. Using the point  $I$  and the point  $M_a$ , construct a circle  $k_{over}(I, M_a)$  (rule W09); % NDG: points  $I$  and  $M_a$  are not the same;
3. Using the point  $A$  and the line  $IM_a$ , construct a line  $AP'_a$  (rule W16); ;
4. Using the point  $M_a$ , the line  $AP'_a$  and the point  $A$ , construct a line  $h_{M_a, -1/1}(AP'_a)$  (rule W15); ;
5. Using the circle  $k_{over}(I, M_a)$  and the line  $h_{M_a, -1/1}(AP'_a)$ , construct a point  $A_{fo}$  and a point  $P_a$  (rule W04); % NDG: line  $h_{M_a, -1/1}(AP'_a)$  and circle  $k_{over}(I, M_a)$  intersect;
6. Using the point  $P_a$  and the point  $I$ , construct a circle  $k(I, P_a)$  (rule W06); % NDG: points  $P_a$  and  $I$  are not the same;
7. Using the circle  $k(I, P_a)$ , the point  $A$  and the point  $I$ , construct a line  $c$  and a line  $b$  (rule W12); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
8. Using the circle  $k(I, P_a)$ , the point  $M_a$  and the point  $I$ , construct a line  $x1$  and a line  $a$  (rule W12); % NDG: point  $M_a$  is outside the circle  $k(I, P_a)$ ;
9. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same;
10. Using the point  $M_a$  and the point  $B$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; point  $M_a$  is outside the circle  $k(I, P_a)$ ; point  $A$  is outside the circle  $k(I, P_a)$ ; points  $P_a$  and  $I$  are not the same; line  $h_{M_a, -1/1}(AP'_a)$  and circle  $k_{over}(I, M_a)$  intersect; points  $I$  and  $M_a$  are not the same.

Determination conditions: lines  $c$  and  $a$  are not the same; points  $M_a$  and  $I$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W09,W12,W15,W16]

Lemmas used: [D21,D27,D59,D85,GD01,GL02,GL03,GL04,GL09,L59,L60,L61,L62,L74]

Solving time: 5.8 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point M_{a} 65 40
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_t A
cmark_r M_{a}
cmark_b I
color 0 0 0
fontsize 8

% DET: points M_{a} and I are not the same
% Constructing a line IM_{a} which passes through point M_{a} and point I
line IM_{a} M_{a} I

color 200 200 200
drawline IM_{a}
color 0 0 0

% NDG: points I and M_{a} are not the same
% Constructing midpoint P_{\_G12887} of the segment IM_{a}
midpoint P_{\_G12887} I M_{a}
cmark_r P_{\_G12887}

% Constructing a circle k_{over}(I, M_{a}) whose center is at point P_{\_G12887} and which passes
    through point I
circle k_{over}(I, M_{a}) P_{\_G12887} I

color 200 200 200
drawcircle k_{over}(I, M_{a})
color 0 0 0

% Constructing a line AP'_{a} which contains the point A and is parallel to the line IM_{a}
parallel AP'_{a} A IM_{a}
```

```

color 200 200 200
drawline AP'_{a}
color 0 0 0

% Constructing a point  $P_{\backslash\_G13188}$  such that  $M_{\{a\}}P_{\backslash\_G13188}/M_{\{a\}}A=-1$ 
towards P_{\backslash\_G13188} M_{\{a\}} A -1
cmark_r P_{\backslash\_G13188}
color 200 200 200
drawsegment A P_{\backslash\_G13188}
color 0 0 0

% Constructing a line  $h_{\{M_{\{a\}},-1/1\}}(AP'_{\{a\}})$  which contains the point  $P_{\backslash\_G13188}$  and is parallel
to the line  $AP'_{\{a\}}$ 
parallel h_{\{M_{\{a\}},-1/1\}}(AP'_{\{a\}}) P_{\backslash\_G13188} AP'_{\{a\}}

color 200 200 200
drawline h_{\{M_{\{a\}},-1/1\}}(AP'_{\{a\}})
color 0 0 0

% NDG: line  $h_{\{M_{\{a\}},-1/1\}}(AP'_{\{a\}})$  and circle  $k_{\text{over}}(I,M_{\{a\}})$  intersect
% Constructing points  $A_{\{fo\}}$  and  $P_{\{a\}}$  which are in intersection of  $k_{\text{over}}(I,M_{\{a\}})$  and  $h_{\{M_{\{a\}},-1/1\}}(AP'_{\{a\}})$ 
intersec2 A_{fo} P_{\{a\}} k_{\text{over}}(I,M_{\{a\}}) h_{\{M_{\{a\}},-1/1\}}(AP'_{\{a\}})
cmark_r A_{fo}
cmark_r P_{\{a\}}

% NDG: points  $P_{\{a\}}$  and  $I$  are not the same
% Constructing a circle  $k(I,P_{\{a\}})$  whose center is at point  $I$  and which passes through point  $P_{\{a\}}$ 
circle k(I,P_{\{a\}}) I P_{\{a\}}

color 200 200 200
drawcircle k(I,P_{\{a\}})
color 0 0 0

% NDG: point  $A$  is outside the circle  $k(I,P_{\{a\}})$ 
% Constructing midpoint  $P_{\backslash\_G13937}$  of the segment  $AI$ 
midpoint P_{\backslash\_G13937} A I
cmark_r P_{\backslash\_G13937}

% Constructing a circle  $C_{\backslash\_G13940}$  whose center is at point  $P_{\backslash\_G13937}$  and which passes through
point  $A$ 
circle C_{\backslash\_G13940} P_{\backslash\_G13937} A

color 200 200 200
drawcircle C_{\backslash\_G13940}
color 0 0 0

```

```

% Constructing points  $P_{\{ \_G13943 \}}$  and  $P_{\{ \_G13946 \}}$  which are in intersection of  $C_{\{ \_G13940 \}}$  and  $k$ 
  ( $I, P_{\{ a \}}$ )
intersec2  $P_{\{ \_G13943 \}}$   $P_{\{ \_G13946 \}}$   $C_{\{ \_G13940 \}}$   $k(I, P_{\{ a \}})$ 
cmark_r  $P_{\{ \_G13943 \}}$ 
cmark_r  $P_{\{ \_G13946 \}}$ 

% Constructing a line  $c$  which passes through point  $A$  and point  $P_{\{ \_G13943 \}}$ 
line c A  $P_{\{ \_G13943 \}}$ 

color 200 200 200
drawline c
color 0 0 0

% Constructing a line  $b$  which passes through point  $A$  and point  $P_{\{ \_G13946 \}}$ 
line b A  $P_{\{ \_G13946 \}}$ 

color 200 200 200
drawline b
color 0 0 0

% NDG: point  $M_{\{ a \}}$  is outside the circle  $k(I, P_{\{ a \}})$ 
% Constructing midpoint  $P_{\{ \_G14349 \}}$  of the segment  $M_{\{ a \}}I$ 
midpoint  $P_{\{ \_G14349 \}}$   $M_{\{ a \}}$  I
cmark_r  $P_{\{ \_G14349 \}}$ 

% Constructing a circle  $C_{\{ \_G14352 \}}$  whose center is at point  $P_{\{ \_G14349 \}}$  and which passes through
  point  $M_{\{ a \}}$ 
circle  $C_{\{ \_G14352 \}}$   $P_{\{ \_G14349 \}}$   $M_{\{ a \}}$ 

color 200 200 200
drawcircle  $C_{\{ \_G14352 \}}$ 
color 0 0 0

% Constructing points  $P_{\{ \_G14355 \}}$  and  $P_{\{ \_G14358 \}}$  which are in intersection of  $C_{\{ \_G14352 \}}$  and  $k$ 
  ( $I, P_{\{ a \}}$ )
intersec2  $P_{\{ \_G14355 \}}$   $P_{\{ \_G14358 \}}$   $C_{\{ \_G14352 \}}$   $k(I, P_{\{ a \}})$ 
cmark_r  $P_{\{ \_G14355 \}}$ 
cmark_r  $P_{\{ \_G14358 \}}$ 

% Constructing a line  $x1$  which passes through point  $M_{\{ a \}}$  and point  $P_{\{ \_G14355 \}}$ 
line x1  $M_{\{ a \}}$   $P_{\{ \_G14355 \}}$ 

color 200 200 200
drawline x1
color 0 0 0

% Constructing a line  $a$  which passes through point  $M_{\{ a \}}$  and point  $P_{\{ \_G14358 \}}$ 
line a  $M_{\{ a \}}$   $P_{\{ \_G14358 \}}$ 

color 200 200 200
drawline a

```

```
color 0 0 0
```

```
% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B
```

```
% Constructing a point C such that  $M_{\{a\}}C/M_{\{a\}}B=-1$ 
towards C  $M_{\{a\}}$  B -1
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines c and a are not parallel; point  $M_{\{a\}}$  is outside the circle  $k(I, P_{\{a\}})$ ; point A is outside the circle  $k(I, P_{\{a\}})$ ; points  $P_{\{a\}}$  and I are not the same; line  $h_{\{M_{\{a\}}, -1/1\}}(AP'_{\{a\}})$  and circle  $k_{\text{over}}(I, M_{\{a\}})$  intersect; points I and  $M_{\{a\}}$  are not the same
% Determination conditions: lines c and a are not the same; points  $M_{\{a\}}$  and I are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a = -M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 10 terms.

**Time Complexity:** Time spent by the prover is 0.292 seconds.

**NDG conditions** Points A and I are not identical

Points A and tempPoint-649 $AP'_a$  are not identical

Points  $P_{G8432}$ ,  $M_a$  and tempPoint-649 $AP'_a$  are not collinear

Points A and  $M_a$  are not identical

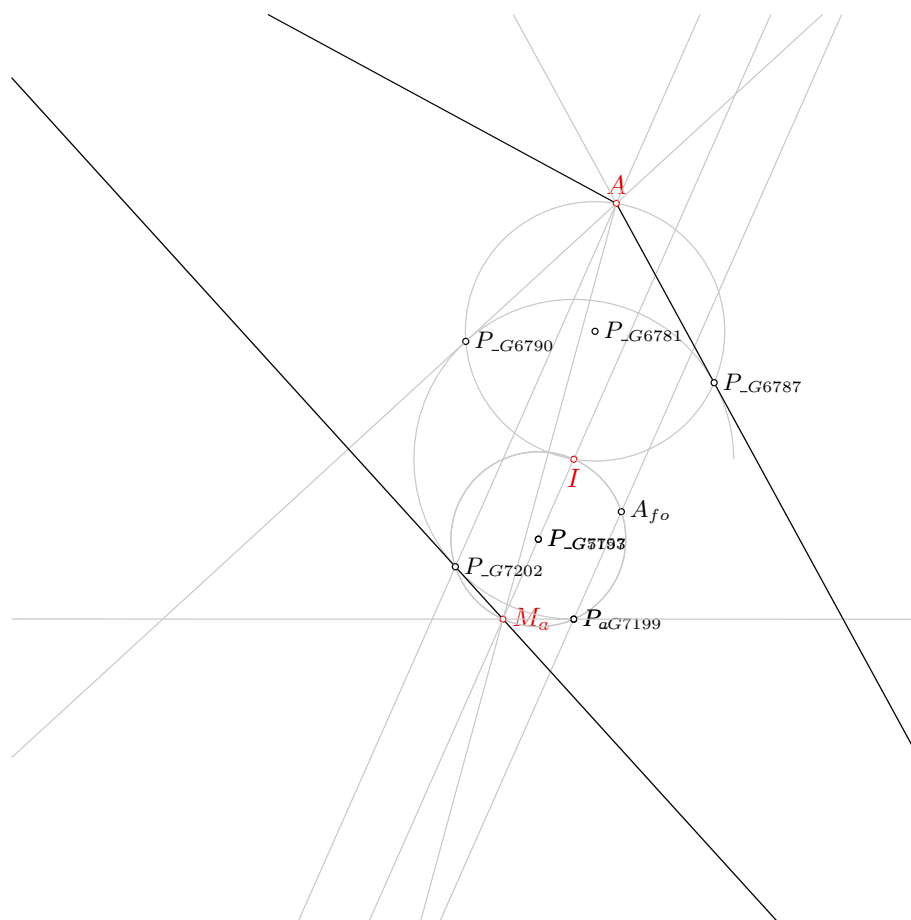


Figure 1: Illustration of the problem 0050

### 4.1.3 Proving $I=_J$

Proving failed

## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{AM_aP_{G9366}} \neq S_{P_{G8951}M_aP_{G9366}}$  i.e., lines  $AP_{G8951}$  and  $M_aP_{G9366}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_a=_M_a$

Proving failed

### 4.2.3 Proving $I=_J$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.030 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_a=_M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 189 terms.

**Time Complexity:** Time spent by the prover is 0.380 seconds. There are no ndg conditions.

### 4.3.3 Proving $I=_J$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 2377 terms.

**Time Complexity:** Time spent by the prover is 9.060 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.030 seconds. There are no ndg conditions.

### 4.4.2 Proving $M_a=_M_a$

Proving failed

### 4.4.3 Proving $I=_J$

Proving failed



# Problem 51

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 51: Given a point  $A$ , a point  $M_b$  and a point  $M_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
2. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D20,D22,GL03,GL04]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point M_{b} 95 67.5
point M_{c} 50 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_lt M_{b}
cmark_lt M_{c}
color 0 0 0
fontsize 8
```

```
% Constructing a point C such that AC/AM_{b}=2
```

```

towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.033 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b=_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.049 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $M_c=_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.056 seconds.

**NDG conditions** There are no NDG conditions for this theorem

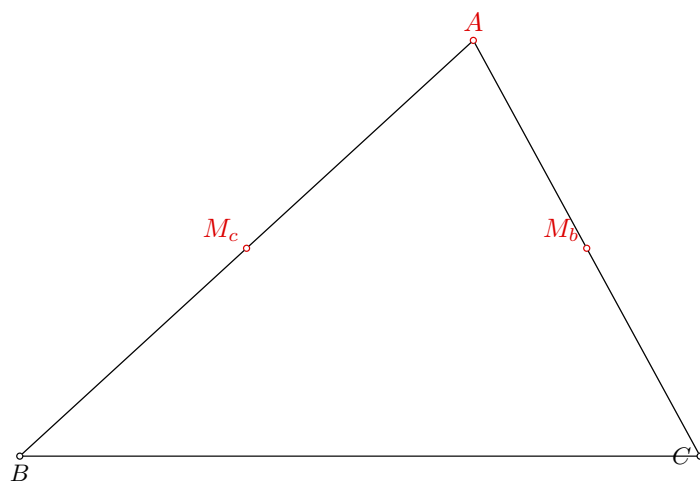


Figure 1: Illustration of the problem 0051

## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

There are no ndg conditions.  
Total number of proof steps: 1  
Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_b=_M M_b$

There are no ndg conditions.  
Total number of proof steps: 42  
Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $M_c=_M M_c$

There are no ndg conditions.  
Total number of proof steps: 41  
Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_b=_M M_b$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $M_c=_M M_c$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $M_b=_M M_b$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.  
**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_c = -M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

# Problem 52

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 52: Given a point  $A$ , a point  $M_b$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
2. Using the point  $M_b$  and the point  $G$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D22, GL03, GL04, L56]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point M_{b} 95 67.5
point G 70 58.33
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_lt M_{b}
cmark_t G
color 0 0 0
fontsize 8
```

```
% Constructing a point C such that AC/AM_{b}=2
```

```

towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% Constructing a point B such that M_{b}B/M_{b}G=3
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b=_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.056 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $G=_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 25 terms.

**Time Complexity:** Time spent by the prover is 0.363 seconds.

**NDG conditions** Line through points  $A$  and  $_M_a$  is not parallel with line through points  $B$  and  $_M_b$   
Points  $A$  and  $_M_a$  are not identical

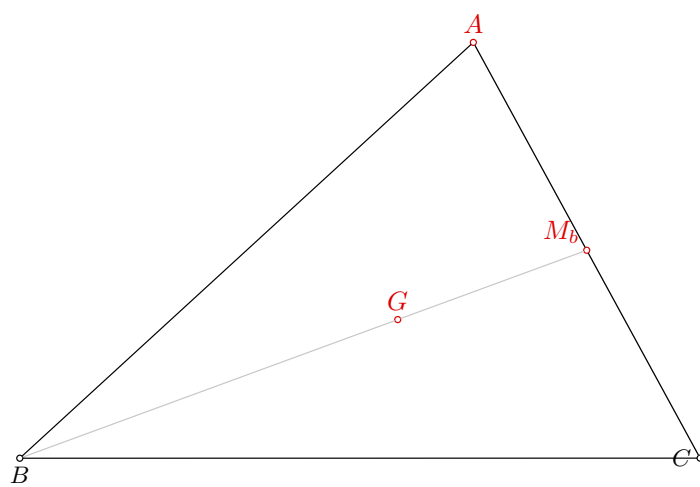


Figure 1: Illustration of the problem 0052



## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{AB\_M_b} \neq S_{M_aB\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_b=M_b$

NDG conditions are:

$S_{AB\_M_b} \neq S_{M_aB\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $G=G$

NDG conditions are:

$S_{AB\_M_b} \neq S_{M_aB\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (cancellation assumption)

Total number of proof steps: 698

Time spent by the prover: 0.100 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_b=M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $G=G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 40 terms.

**Time Complexity:** Time spent by the prover is 0.090 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_b = M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 294 terms.

**Time Complexity:** Time spent by the prover is 1.830 seconds. There are no ndg conditions.

#### 4.4.3 Proving $G = G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 294 terms.

**Time Complexity:** Time spent by the prover is 2.180 seconds. There are no ndg conditions.

# Problem 53

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 53: Given a point  $M_b$ , a point  $H_a$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_a$  and  $M_b$  are not the same;
2. Choose freely a point  $A$  on the circle  $k(M_b, C)$  (rule WOncircle);
3. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
4. Using the point  $H_a$  and the point  $C$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $C$  are not the same;
5. Choose freely a point  $B$  on the line  $a$  (rule WOnline1) .

Non-degenerate conditions: points  $H_a$  and  $M_b$  are not the same.

Determination conditions: points  $H_a$  and  $C$  are not the same.

Rules used: [W01,W02,W06,WOncircle1,WOnline1]

Lemmas used: [D22,D5,GD01,GD02,GL03,GL04,L44]

Solving time: 929.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point H_{a} 80 40
point A 80 95

color 220 0 0
fontsize 9
```

```

cmark_lt M_{b}
cmark_r H_{a}
cmark_t A
color 0 0 0
fontsize 8

% NDG: points H_{a} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{a}
circle k(M_{b},C) M_{b} H_{a}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% Choosing randomly a point A on the circle with center M_{b} through point H_{a}
oncircle A M_{b} H_{a}
cmark_t A
color 200 200 200
drawcircle M_{b} H_{a}
color 0 0 0

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points H_{a} and C are not the same
% Constructing a line a which passes through point H_{a} and point C
line a H_{a} C

color 200 200 200
drawline a
color 0 0 0

% Choosing randomly a point B on the line H_{a}C
online B H_{a} C
cmark_b B
color 200 200 200
drawline H_{a} C
color 0 0 0

```

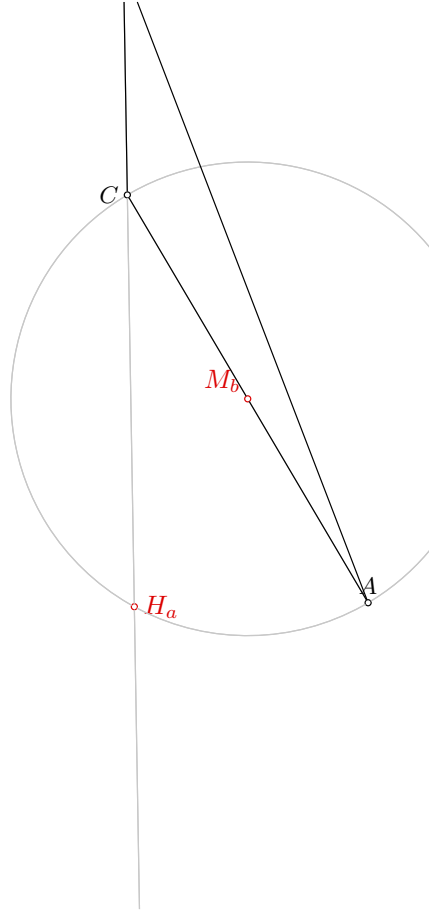


Figure 1: Illustration of the problem 0053

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: points  $H_{\{a\}}$  and  $M_{\{b\}}$  are not the same
% Determination conditions: points  $H_{\{a\}}$  and  $C$  are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.111 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.1.3 Proving $A = A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.03 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF^1_{h_a}} \neq S_{CAF^1_{h_a}}$  i.e., lines  $BC$  and  $AF^1_{h_a}$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.2.3 Proving $A = A$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF^1_{h_a}} \neq S_{CAF^1_{h_a}}$  i.e., lines  $BC$  and  $AF^1_{h_a}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 17 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 280 terms.

**Time Complexity:** Time spent by the prover is 0.270 seconds. There are no ndg conditions.

### 4.3.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_b=_M M_b$

Proving failed

### 4.4.2 Proving $H_a=_H H_a$

Proving failed

### 4.4.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 54

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 54: Given a point  $M_b$ , a point  $H_b$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $H_b$  are not the same;
2. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) ;
3. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
4. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
5. Choose freely a point  $B$  on the line  $h_b$  (rule WOnline2).

Non-degenerate conditions: .

Determination conditions: points  $M_b$  and  $H_b$  are not the same.

Rules used: [W01,W02,W10b,WOnline1,WOnline2]

Lemmas used: [D22,D6,D9,GD01,GL03,GL04,GL09]

Solving time: 156.8 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point H_{b} 89.36 77.83
point A 80 95

color 220 0 0
fontsize 9
```



```

cmark_lt M_{b}
cmark_l H_{b}
cmark_t A
color 0 0 0
fontsize 8

% DET: points M_{b} and H_{b} are not the same
% Constructing a line b which passes through point M_{b} and point H_{b}
line b M_{b} H_{b}

color 200 200 200
drawline b
color 0 0 0

% Choosing randomly a point A on the line M_{b}H_{b}
online A M_{b} H_{b}
cmark_t A
color 200 200 200
drawline M_{b} H_{b}
color 0 0 0

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0

% Generating random value V[_G11325]
random V[_G11325]

% Calculating value V[_G11346] using formula V[_G11325]*20
expression V[_G11346] { V[_G11325]*20 }

% Constructing a point B which is a point for which holds H_{b}B = V[_G11346] and angle AH_{b}B =
90
turtle B A H_{b} 90 V[_G11346]

```

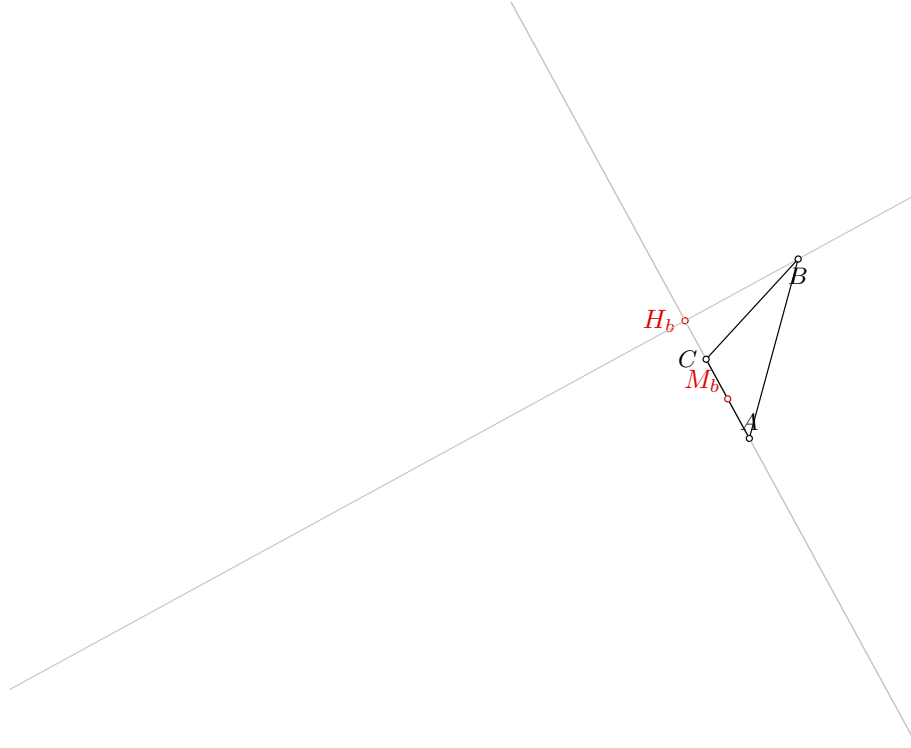


Figure 1: Illustration of the problem 0054

```
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

*% Non-degenerate conditions:*

*% Determination conditions: points  $M_{\{b\}}$  and  $H_{\{b\}}$  are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = \neg M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b = \neg H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.096 seconds.

**NDG conditions** Line through points  $C$  and  $A$  is not perpendicular to line through points  $A$  and  $M_b$

#### 4.1.3 Proving $A = A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.015 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^2_{h_b}} \neq S_{CBF^2_{h_b}}$  i.e., lines  $AC$  and  $BF^2_{h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.2.3 Proving $A = A$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^2_{h_b}} \neq S_{CBF^2_{h_b}}$  i.e., lines  $AC$  and  $BF^2_{h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 2 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 55

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 55: Given a point  $M_b$ , a point  $H_c$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_c$  and  $M_b$  are not the same;
2. Choose freely a point  $A$  on the circle  $k(M_b, C)$  (rule WOncircle);
3. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
4. Using the point  $A$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $H_c$  are not the same;
5. Choose freely a point  $B$  on the line  $c$  (rule WOnline1) .

Non-degenerate conditions: points  $H_c$  and  $M_b$  are not the same.

Determination conditions: points  $A$  and  $H_c$  are not the same.

Rules used: [W01,W02,W06,WOncircle1,WOnline1]

Lemmas used: [D22,D7,GD01,GD02,GL03,GL04,L45]

Solving time: 821.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point H_{c} 68.91 84.83
point A 80 95

color 220 0 0
fontsize 9
```

```

cmark_lt M_{b}
cmark_rt H_{c}
cmark_t A
color 0 0 0
fontsize 8

% NDG: points H_{c} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{c}
circle k(M_{b},C) M_{b} H_{c}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% Choosing randomly a point A on the circle with center M_{b} through point H_{c}
oncircle A M_{b} H_{c}
cmark_t A
color 200 200 200
drawcircle M_{b} H_{c}
color 0 0 0

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0

% Choosing randomly a point B on the line AH_{c}
online B A H_{c}
cmark_b B
color 200 200 200
drawline A H_{c}
color 0 0 0

```

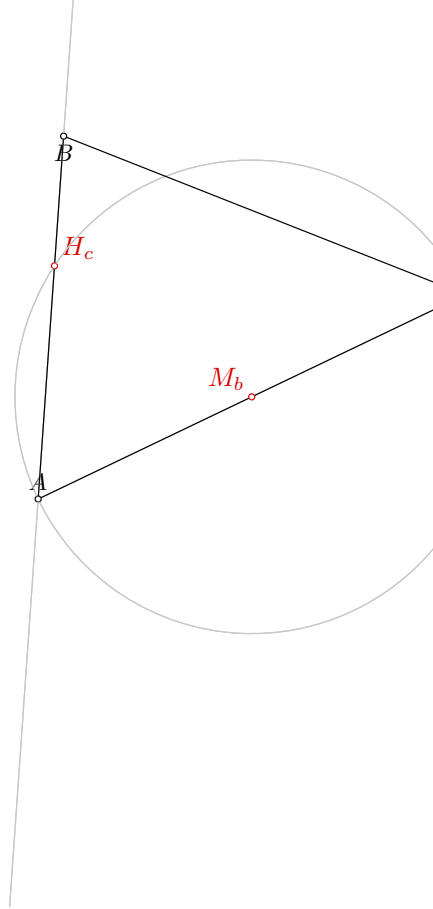


Figure 1: Illustration of the problem 0055

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: points  $H_{\{c\}}$  and  $M_{\{b\}}$  are not the same
% Determination conditions: points A and  $H_{\{c\}}$  are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.146 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 47 terms.

**Time Complexity:** Time spent by the prover is 0.63 seconds.

**NDG conditions** Points  $M_b$  and  $A$  are not identical

Points  $A$  and  $B$  are not identical

Points  $M_b$ ,  $A$  and  $B$  are not collinear

#### 4.1.3 Proving $A = A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^1} \neq S_{BCF_{h_c}^1}$  i.e., lines  $AB$  and  $CF_{h_c}^1$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.2.3 Proving $A = A$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^1} \neq S_{BCF_{h_c}^1}$  i.e., lines  $AB$  and  $CF_{h_c}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 17 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.



#### 4.3.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 280 terms.

**Time Complexity:** Time spent by the prover is 0.270 seconds. There are no ndg conditions.

#### 4.3.3 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 312 terms.

**Time Complexity:** Time spent by the prover is 1.970 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 312 terms.

**Time Complexity:** Time spent by the prover is 1.850 seconds. There are no ndg conditions.

#### 4.4.3 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 56

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 56: Given a point  $A$ , a point  $M_b$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
2. Using the point  $A$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H$  are not the same;
3. Using the point  $H$  and the point  $C$ , construct a line  $h_c$  (rule W02); % DET: points  $H$  and  $C$  are not the same;
4. Using the point  $A$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $A$  and  $M_b$  are not the same;
5. Using the circle  $k(M_b, C)$ , the line  $h_a$ , the point  $M_b$  and the point  $A$ , construct a point  $H_a$  (rule W05); % NDG: line  $h_a$  and circle  $k(M_b, C)$  intersect % DET: points  $A$  and  $H_a$  must be different;
6. Using the point  $H_a$  and the point  $C$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $C$  are not the same;
7. Using the circle  $k(M_b, C)$ , the line  $h_c$ , the point  $M_b$  and the point  $C$ , construct a point  $H_c$  (rule W05); % NDG: line  $h_c$  and circle  $k(M_b, C)$  intersect % DET: points  $C$  and  $H_c$  must be different;
8. Using the point  $H_c$  and the point  $A$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $A$  are not the same;
9. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; line  $h_c$  and circle  $k(M_b, C)$  intersect; line  $h_a$  and circle  $k(M_b, C)$  intersect; points  $A$  and  $M_b$  are not the same.

Determination conditions: lines  $a$  and  $c$  are not the same; points  $H_c$  and  $A$  are not the same; points  $C$  and  $H_c$  must be different; points  $H_a$  and  $C$  are not the same; points  $A$  and  $H_a$  must be different; points  $H$  and  $C$  are not the same; points  $A$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D22,D3,D5,D7,D8,GD01,GD02,GL03,GL04,L3,L43,L44,L45]

Solving time: 1.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point M_{b} 95 67.5
point H 80 72.73

color 220 0 0
fontsize 9

cmark_t A
cmark_lt M_{b}
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% DET: points H and C are not the same
% Constructing a line h_{c} which passes through point H and point C
line h_{c} H C

color 200 200 200
drawline h_{c}
color 0 0 0
```

```

% NDG: points A and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point A
circle k(M_{b},C) M_{b} A

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line h_{a} and circle k(M_{b},C) intersect% DET: points A and H_{a} must be different
% Constructing a point P_{\_G11277} which is a foot of the point M_{b} on the line h_{a}
foot P_{\_G11277} M_{b} h_{a}
cmark_r P_{\_G11277}
color 200 200 200
drawline M_{b} P_{\_G11277}
color 0 0 0

% Constructing a point H_{a} which is an image of the point A in the symmetry to point/line P_{\_G
11277}
sim H_{a} P_{\_G11277} A
cmark_r H_{a}

% DET: points H_{a} and C are not the same
% Constructing a line a which passes through point H_{a} and point C
line a H_{a} C

color 200 200 200
drawline a
color 0 0 0

% NDG: line h_{c} and circle k(M_{b},C) intersect% DET: points C and H_{c} must be different
% Constructing a point P_{\_G11515} which is a foot of the point M_{b} on the line h_{c}
foot P_{\_G11515} M_{b} h_{c}
cmark_r P_{\_G11515}
color 200 200 200
drawline M_{b} P_{\_G11515}
color 0 0 0

% Constructing a point H_{c} which is an image of the point C in the symmetry to point/line P_{\_G
11515}
sim H_{c} P_{\_G11515} C
cmark_rt H_{c}

% DET: points H_{c} and A are not the same
% Constructing a line c which passes through point H_{c} and point A
line c H_{c} A

color 200 200 200
drawline c
color 0 0 0

```

```

% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and c are not parallel; line h_{c} and circle k(M_{b},C)
% intersect; line h_{a} and circle k(M_{b},C) intersect; points A and M_{b} are not the same
% Determination conditions: lines a and c are not the same; points H_{c} and A are not the same;
% points C and H_{c} must be different; points H_{a} and C are not the same; points A and H_{a}
% must be different; points H and C are not the same; points A and H are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b=_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $H=_H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 224 terms.

**Time Complexity:** Time spent by the prover is 8.902 seconds.

**NDG conditions** Points  $A$  and  $H$  are not identical

Points  $A$  and  $H$  are not identical

Points  $C$  and  $H$  are not identical

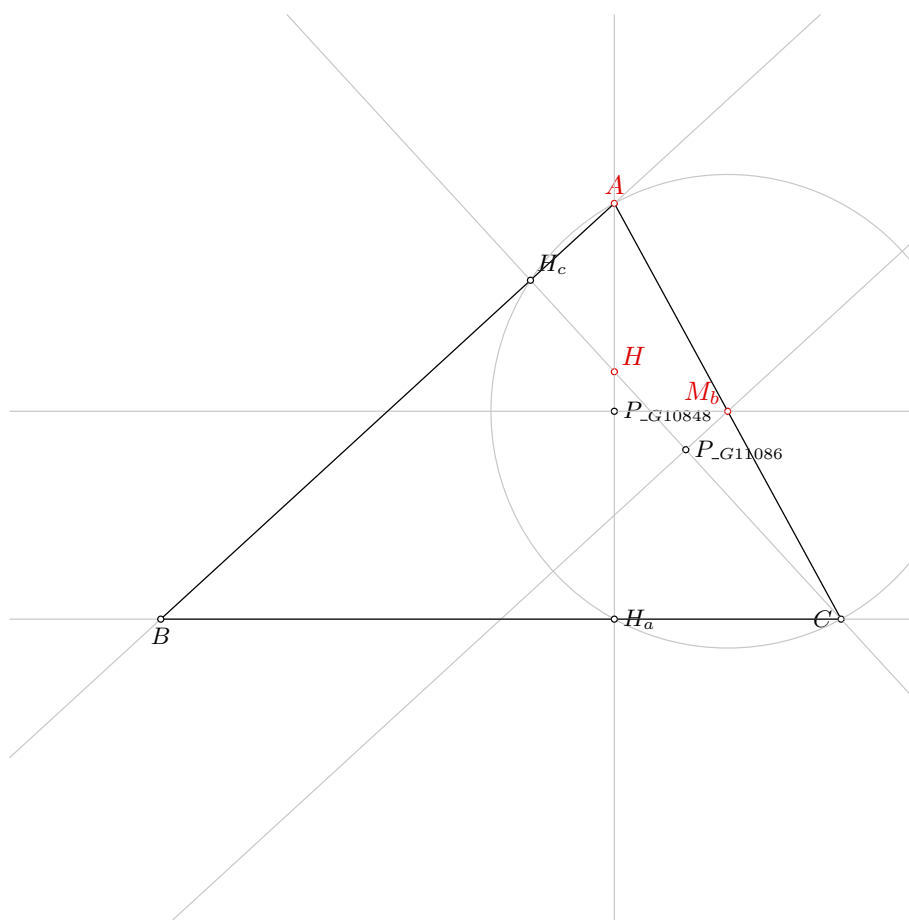


Figure 1: Illustration of the problem 0056

Line through points  $M_b$  and  $C$  is not perpendicular to line through points  $C$  and  $A$   
Line through points  $A$  and  $H_c$  is not parallel with line through points  $C$  and  $H_a$   
Points  $A$  and  $H_a$  are not identical  
Line through points  $M_b$  and  $B$  is not parallel with line through points  $A$  and  $C$   
Line through points  $M_b$  and  $B$  is not perpendicular to line through points  $B$  and  $A$

## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_a H_c A} \neq S_{CH_c A}$  i.e., lines  $H_a C$  and  $H_c A$  are not parallel (construction based assumption)  
 $S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)  
 $S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)  
 $S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$  i.e., lines  $AF_{-h_a}^0$  and  $BF_{-h_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_b=_M M_b$

Proving failed

### 4.2.3 Proving $H=_H H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_b=_M M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $H=_H H$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 9 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_b = M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.3 Proving $H = H$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.



# Problem 57

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 57: Given a point  $A$ , a point  $M_b$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
2. Using the point  $A$  and the point  $M_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $M_b$  are not the same;
3. Using the point  $T_a$  and the point  $C$ , construct a line  $a$  (rule W02); % DET: points  $T_a$  and  $C$  are not the same;
4. Using the point  $T_a$  and the line  $b$ , construct a circle  $k(T_a, foot[T_a, c])$  (rule W11); % NDG: point  $T_a$  is not incident to the line  $b$ ;
5. Using the circle  $k(T_a, foot[T_a, c])$ , the point  $A$ , the point  $T_a$  and the line  $b$ , construct a line  $c$  (rule W13); % NDG: point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ;
6. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ; point  $T_a$  is not incident to the line  $b$ .

Determination conditions: lines  $c$  and  $a$  are not the same; points  $T_a$  and  $C$  are not the same; points  $A$  and  $M_b$  are not the same.

Rules used: [W01,W02,W03,W11,W13]

Lemmas used: [D22,D23,GD01,GD02,GL03,GL04,GL09,GL10,GL11,L8]

Solving time: 1.8 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point M_{b} 95 67.5
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_t A
cmark_lt M_{b}
cmark_rb T_{a}
color 0 0 0
fontsize 8

% Constructing a point C such that  $AC/AM_{b}=2$ 
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points A and M_{b} are not the same
% Constructing a line b which passes through point A and point M_{b}
line b A M_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points T_{a} and C are not the same
% Constructing a line a which passes through point T_{a} and point C
line a T_{a} C

color 200 200 200
drawline a
color 0 0 0

% NDG: point T_{a} is not incident to the line b
% Constructing a point P_{\G7716} which is a foot of the point T_{a} on the line b
foot P_{\G7716} T_{a} b
cmark_r P_{\G7716}
color 200 200 200
drawline T_{a} P_{\G7716}
color 0 0 0

% Constructing a circle  $k(T_{a}, \text{foot}[T_{a}, c])$  whose center is at point T_{a} and which passes
    through point P_{\G7716}
```

```

circle k(T_{a},foot[T_{a},c]) T_{a} P_{\_G7716}

color 200 200 200
drawcircle k(T_{a},foot[T_{a},c])
color 0 0 0

% NDG: point A is outside the circle k(T_{a},foot[T_{a},c])
% Constructing a point P_{\_G8107} which is a foot of the point T_{a} on the line b
foot P_{\_G8107} T_{a} b
cmark_r P_{\_G8107}
color 200 200 200
drawline T_{a} P_{\_G8107}
color 0 0 0

% Constructing a line L_{\_G8110} which passes through point T_{a} and point A
line L_{\_G8110} T_{a} A

color 200 200 200
drawline L_{\_G8110}
color 0 0 0

% Constructing a point P_{\_G8113} which is an image of the point P_{\_G8107} in the symmetry to
point/line L_{\_G8110}
sim P_{\_G8113} L_{\_G8110} P_{\_G8107}
cmark_r P_{\_G8113}

% Constructing a line c which passes through point A and point P_{\_G8113}
line c A P_{\_G8113}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and a are not parallel; point A is outside the circle k(T_{a},
foot[T_{a},c]); point T_{a} is not incident to the line b
% Determination conditions: lines c and a are not the same; points T_{a} and C are not the same;
points A and M_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

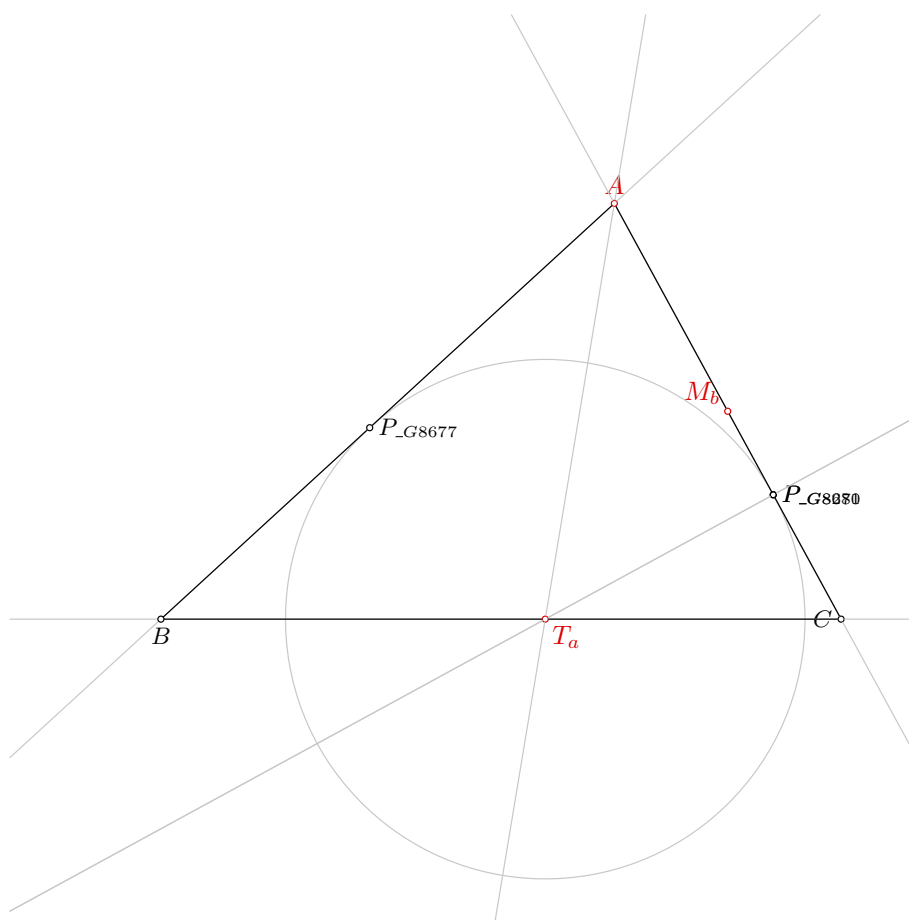


Figure 1: Illustration of the problem 0057

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b=M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.034 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $T_a=T_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{AT_aC} \neq S_{P_{G9703}T_aC}$  i.e., lines  $AP_{G9703}$  and  $T_aC$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_b=M_b$

Proving failed

#### 4.2.3 Proving $T_a=T_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_b=M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 5 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_a = \neg T_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 14 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_a = \neg T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 58

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 58: Given a point  $M_b$ , a point  $T_b$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $T_b$  are not the same;
2. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) ;
3. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
4. Using the point  $A$ , the point  $C$ , the point  $T_b$  and the line  $b$ , construct a point  $T'_b$  (rule W19);  
% NDG: points  $A$  and  $C$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $midpoint([A, T_b])$  are not the same;
5. Choose freely a point  $B$  on the circle  $k_{over}(T_b, T'_b)$  (rule WOncircle).

Non-degenerate conditions: points  $A$  and  $C$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $midpoint([A, T_b])$  are not the same.

Determination conditions: points  $M_b$  and  $T_b$  are not the same.

Rules used: [W01,W02,W19,WOncircle2,WOnline1]

Lemmas used: [D22,D24,GD01,GL03,GL04,GL09,L78]

Solving time: 88.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{b} 95 67.5  
point T_{b} 94.25 68.88  
point A 80 95
```

```

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_t T_{b}
cmark_t A
color 0 0 0
fontsize 8

% DET: points M_{b} and T_{b} are not the same
% Constructing a line b which passes through point M_{b} and point T_{b}
line b M_{b} T_{b}

color 200 200 200
drawline b
color 0 0 0

% Choosing randomly a point A on the line M_{b}T_{b}
online A M_{b} T_{b}
cmark_t A
color 200 200 200
drawline M_{b} T_{b}
color 0 0 0

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% NDG: points A and C are not the same; points C and T_{b} are not the same; points C and midpoint
% ([A,T_{b}]) are not the same
% Constructing a point P_{\_G5954} which is an image of the point C in a rotation around the point
% T_{b} for the angle 90
rotate P_{\_G5954} T_{b} 90 C
cmark_r P_{\_G5954}
color 200 200 200
drawarc_p T_{b} C 90
color 0 0 0

% Constructing a line L_{\_G5957} which passes through point T_{b} and point P_{\_G5954}
line L_{\_G5957} T_{b} P_{\_G5954}

color 200 200 200
drawline L_{\_G5957}
color 0 0 0

% Constructing midpoint P_{\_G5960} of the segment CP_{\_G5954}

```



```

midpoint P_{\_G5960} C P_{\_G5954}
cmark_r P_{\_G5960}

% Constructing a line L_{\_G5963} which passes through point A and point P_{\_G5960}
line L_{\_G5963} A P_{\_G5960}

color 200 200 200
drawline L_{\_G5963}
color 0 0 0

% Constructing a line L_{\_G5966} which passes through point C and point P_{\_G5960}
line L_{\_G5966} C P_{\_G5960}

color 200 200 200
drawline L_{\_G5966}
color 0 0 0

% Constructing a point P_{\_G5969} which belongs to line L_{\_G5957} and line L_{\_G5963}
intersec P_{\_G5969} L_{\_G5957} L_{\_G5963}
cmark_r P_{\_G5969}

% Constructing a point P_{\_G5972} which belongs to line L_{\_G5957} and line L_{\_G5966}
intersec P_{\_G5972} L_{\_G5957} L_{\_G5966}
cmark_r P_{\_G5972}

% Constructing a line L_{\_G5975} which passes through point A and point P_{\_G5972}
line L_{\_G5975} A P_{\_G5972}

color 200 200 200
drawline L_{\_G5975}
color 0 0 0

% Constructing a line L_{\_G5978} which passes through point C and point P_{\_G5969}
line L_{\_G5978} C P_{\_G5969}

color 200 200 200
drawline L_{\_G5978}
color 0 0 0

% Constructing a point P_{\_G5981} which belongs to line L_{\_G5975} and line L_{\_G5978}
intersec P_{\_G5981} L_{\_G5975} L_{\_G5978}
cmark_r P_{\_G5981}

% Constructing a line L_{\_G5984} which passes through point P_{\_G5960} and point P_{\_G5981}
line L_{\_G5984} P_{\_G5960} P_{\_G5981}

color 200 200 200
drawline L_{\_G5984}
color 0 0 0

% Constructing a point T'_{b} which belongs to line L_{\_G5984} and line b

```

```

intersec T'_{b} L_{\_G5984} b
cmark_r T'_{b}

% Constructing midpoint P_{\_G6809} of the segment T_{b}T'_{b}
midpoint P_{\_G6809} T_{b} T'_{b}
cmark_r P_{\_G6809}

% Choosing randomly a point B on the circle with center P_{\_G6809} through point T_{b}
oncircle B P_{\_G6809} T_{b}
cmark_b B
color 200 200 200
drawcircle P_{\_G6809} T_{b}
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: points A and C are not the same; points C and T_{b} are not the same;
% points C and midpoint([A,T_{b}]) are not the same
% Determination conditions: points M_{b} and T_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = \_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_b = \_T_b$

Proving failed

#### 4.1.3 Proving $A = A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.015 seconds.

**NDG conditions** There are no NDG conditions for this theorem

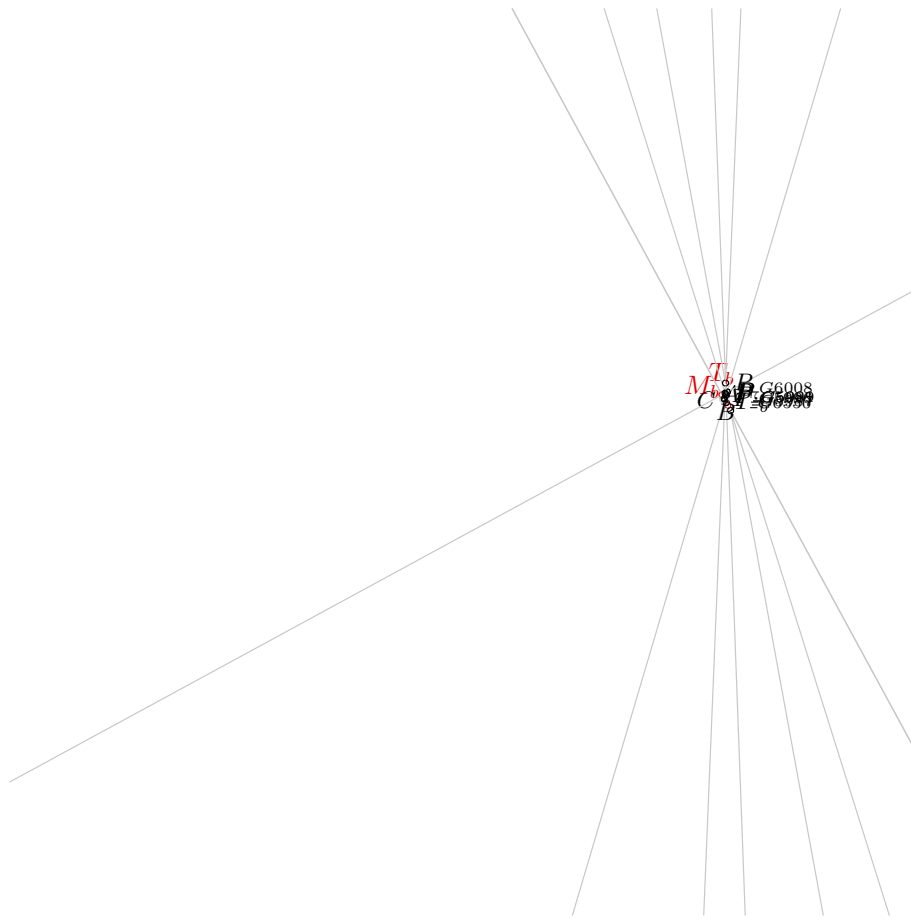


Figure 1: Illustration of the problem 0058

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_b = \neg M_b$

Proving failed

### 4.2.2 Proving $T_b = \neg T_b$

Proving failed

### 4.2.3 Proving $A = A$

NDG conditions are:

$S_{T_bAP_{G11439}} \neq S_{P_{G11433}AP_{G11439}}$  i.e., lines  $T_bP_{G11433}$  and  $AP_{G11439}$  are not parallel (construction based assumption)

$S_{T_bCP_{G11439}} \neq S_{P_{G11433}CP_{G11439}}$  i.e., lines  $T_bP_{G11433}$  and  $CP_{G11439}$  are not parallel (construction based assumption)

$S_{ACP_{G11448}} \neq S_{P_{G11451}CP_{G11448}}$  i.e., lines  $AP_{G11451}$  and  $CP_{G11448}$  are not parallel (construction based assumption)

$S_{P_{G11439}M_bT_b} \neq S_{P_{G11460}M_bT_b}$  i.e., lines  $P_{G11439}P_{G11460}$  and  $M_bT_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_b = \neg M_b$

Proving failed

### 4.3.2 Proving $T_b = \neg T_b$

Proving failed

### 4.3.3 Proving $A = A$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_b = \neg M_b$

Proving failed

### 4.4.2 Proving $T_b = \neg T_b$

Proving failed

### 4.4.3 Proving $A = A$

Proving failed

# Problem 59

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 59: Given a point  $A$ , a point  $M_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
2. Using the point  $A$  and the point  $M_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $M_b$  are not the same;
3. Using the point  $A$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $T_c$  are not the same;
4. Using the point  $T_c$  and the line  $b$ , construct a circle  $k(T_c, foot[T_c, b])$  (rule W11); % NDG: point  $T_c$  is not incident to the line  $b$ ;
5. Using the circle  $k(T_c, foot[T_c, b])$ , the point  $C$ , the point  $T_c$  and the line  $b$ , construct a line  $a$  (rule W13); % NDG: point  $C$  is outside the circle  $k(T_c, foot[T_c, b])$ ;
6. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; point  $C$  is outside the circle  $k(T_c, foot[T_c, b])$ ; point  $T_c$  is not incident to the line  $b$ .

Determination conditions: lines  $a$  and  $c$  are not the same; points  $A$  and  $T_c$  are not the same; points  $A$  and  $M_b$  are not the same.

Rules used: [W01,W02,W03,W11,W13]

Lemmas used: [D22,D25,GD01,GD02,GL03,GL04,GL09,GL10,GL11,L10]

Solving time: 1.9 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point M_{b} 95 67.5
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_t A
cmark_lt M_{b}
cmark_rt T_{c}
color 0 0 0
fontsize 8

% Constructing a point C such that  $AC/AM_{b}=2$ 
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points A and M_{b} are not the same
% Constructing a line b which passes through point A and point M_{b}
line b A M_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points A and T_{c} are not the same
% Constructing a line c which passes through point A and point T_{c}
line c A T_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: point T_{c} is not incident to the line b
% Constructing a point P_{\G14495} which is a foot of the point T_{c} on the line b
foot P_{\G14495} T_{c} b
cmark_r P_{\G14495}
color 200 200 200
drawline T_{c} P_{\G14495}
color 0 0 0

% Constructing a circle  $k(T_{c},foot[T_{c},b])$  whose center is at point T_{c} and which passes
    through point P_{\G14495}
```

```

circle k(T_{c},foot[T_{c},b]) T_{c} P_{\_G14495}

color 200 200 200
drawcircle k(T_{c},foot[T_{c},b])
color 0 0 0

% NDG: point C is outside the circle k(T_{c},foot[T_{c},b])
% Constructing a point P_{\_G14886} which is a foot of the point T_{c} on the line b
foot P_{\_G14886} T_{c} b
cmark_r P_{\_G14886}
color 200 200 200
drawline T_{c} P_{\_G14886}
color 0 0 0

% Constructing a line L_{\_G14889} which passes through point T_{c} and point C
line L_{\_G14889} T_{c} C

color 200 200 200
drawline L_{\_G14889}
color 0 0 0

% Constructing a point P_{\_G14892} which is an image of the point P_{\_G14886} in the symmetry to
point/line L_{\_G14889}
sim P_{\_G14892} L_{\_G14889} P_{\_G14886}
cmark_r P_{\_G14892}

% Constructing a line a which passes through point C and point P_{\_G14892}
line a C P_{\_G14892}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and c are not parallel; point C is outside the circle k(T_{c},
foot[T_{c},b]); point T_{c} is not incident to the line b
% Determination conditions: lines a and c are not the same; points A and T_{c} are not the same;
points A and M_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

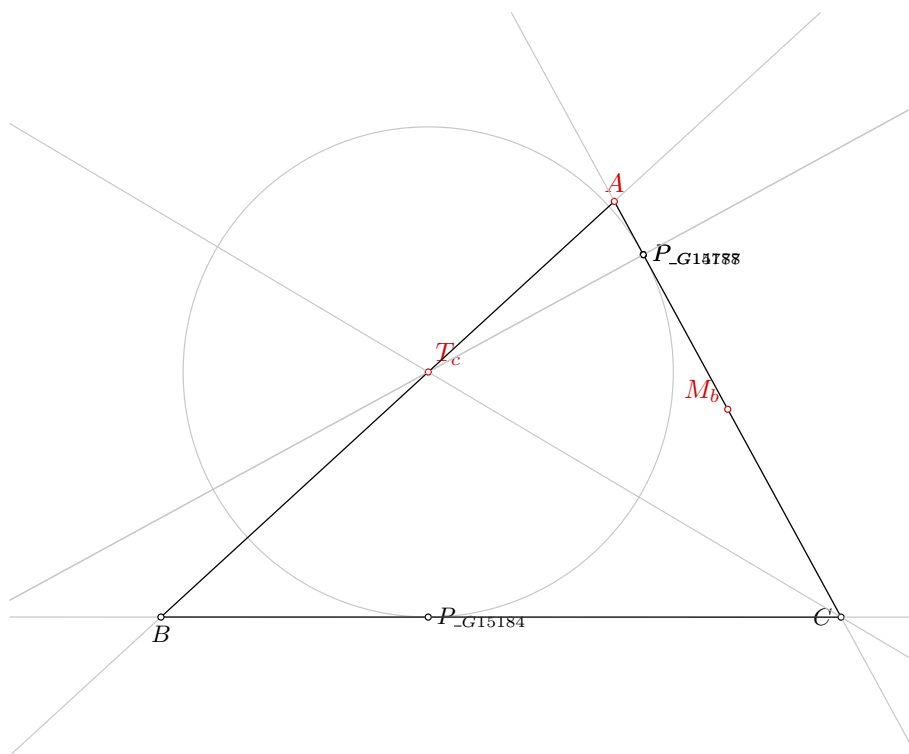


Figure 1: Illustration of the problem 0059



### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b=_M M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.032 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $T_c=_M T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$SC_{AT_c} \neq SP_{G6562AT_c}$  i.e., lines  $CP_{G6562}$  and  $AT_c$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_b=_M M_b$

Proving failed

#### 4.2.3 Proving $T_c=_M T_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_b=_M M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_c = \neg T_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 10 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 7 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_c = \neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 7 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

# Problem 60

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 60: Given a point  $A$ , a point  $M_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
2. Using the point  $A$  and the point  $M_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $M_b$  are not the same;
3. Using the point  $I$  and the line  $b$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $b$ ;
4. Using the circle  $k(I, P_a)$ , the point  $A$ , the point  $I$  and the line  $b$ , construct a line  $c$  (rule W13); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
5. Using the circle  $k(I, P_a)$ , the point  $C$ , the point  $I$  and the line  $b$ , construct a line  $a$  (rule W13); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
6. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; point  $C$  is outside the circle  $k(I, P_a)$ ; point  $A$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $b$ .

Determination conditions: lines  $c$  and  $a$  are not the same; points  $A$  and  $M_b$  are not the same.

Rules used: [W01,W02,W03,W11,W13]

Lemmas used: [D22,D27,GD01,GL03,GL04,GL09,L59,L60,L61]

Solving time: 17.3 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point M_{b} 95 67.5
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_t A
cmark_lt M_{b}
cmark_b I
color 0 0 0
fontsize 8

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points A and M_{b} are not the same
% Constructing a line b which passes through point A and point M_{b}
line b A M_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: point I is not incident to the line b
% Constructing a point P_{\_G7508} which is a foot of the point I on the line b
foot P_{\_G7508} I b
cmark_r P_{\_G7508}
color 200 200 200
drawline I P_{\_G7508}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G7508}
circle k(I,P_{a}) I P_{\_G7508}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point A is outside the circle k(I,P_{a})
% Constructing a point P_{\_G7715} which is a foot of the point I on the line b
```

```

foot P_{\_G7715} I b
cmark_r P_{\_G7715}
color 200 200 200
drawline I P_{\_G7715}
color 0 0 0

% Constructing a line L_{\_G7718} which passes through point I and point A
line L_{\_G7718} I A

color 200 200 200
drawline L_{\_G7718}
color 0 0 0

% Constructing a point P_{\_G7721} which is an image of the point P_{\_G7715} in the symmetry to
point/line L_{\_G7718}
sim P_{\_G7721} L_{\_G7718} P_{\_G7715}
cmark_r P_{\_G7721}

% Constructing a line c which passes through point A and point P_{\_G7721}
line c A P_{\_G7721}

color 200 200 200
drawline c
color 0 0 0

% NDG: point C is outside the circle k(I,P_{a})
% Constructing a point P_{\_G8050} which is a foot of the point I on the line b
foot P_{\_G8050} I b
cmark_r P_{\_G8050}
color 200 200 200
drawline I P_{\_G8050}
color 0 0 0

% Constructing a line L_{\_G8053} which passes through point I and point C
line L_{\_G8053} I C

color 200 200 200
drawline L_{\_G8053}
color 0 0 0

% Constructing a point P_{\_G8056} which is an image of the point P_{\_G8050} in the symmetry to
point/line L_{\_G8053}
sim P_{\_G8056} L_{\_G8053} P_{\_G8050}
cmark_r P_{\_G8056}

% Constructing a line a which passes through point C and point P_{\_G8056}
line a C P_{\_G8056}

color 200 200 200
drawline a
color 0 0 0

```

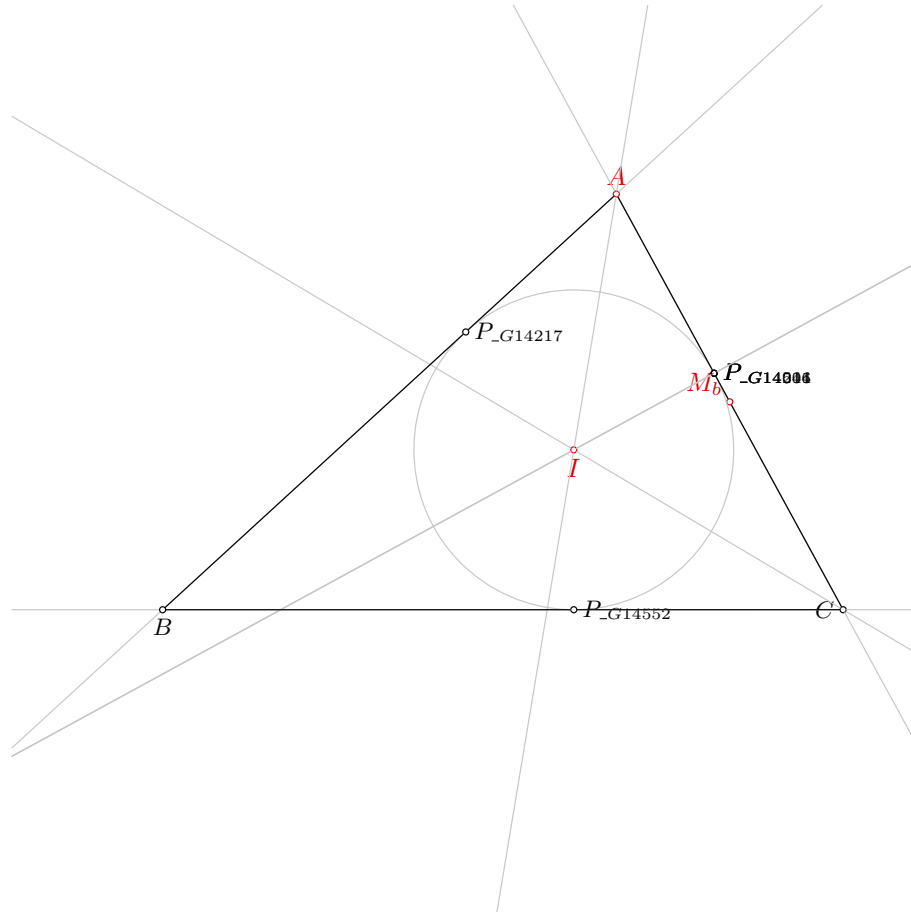


Figure 1: Illustration of the problem 0060

```
% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines c and a are not parallel; point C is outside the circle k(I,P_{a
}); point A is outside the circle k(I,P_{a}); point I is not incident to the line b
% Determination conditions: lines c and a are not the same; points A and M_{b} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b=_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.035 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{ACP\_G15635} \neq S_{P\_G15300CP\_G15635}$  i.e., lines  $AP\_G15300$  and  $CP\_G15635$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_b=_M_b$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 43 terms.

**Time Complexity:** Time spent by the prover is 0.030 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I = \neg I$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 115 terms.

**Time Complexity:** Time spent by the prover is 0.130 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 15 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

#### 4.4.3 Proving $I = \neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 15 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.



# Problem 61

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 61: Given a point  $A$ , a point  $M_c$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); ;
2. Using the point  $M_c$  and the point  $G$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D20,GL03,GL04,L57]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point M_{c} 50 67.5
point G 70 58.33
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_lt M_{c}
cmark_t G
color 0 0 0
fontsize 8
```

```
% Constructing a point B such that AB/AM_{c}=2
```

```

towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% Constructing a point C such that M_{c}C/M_{c}G=3
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.019 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c=_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.058 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $G=_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 25 terms.

**Time Complexity:** Time spent by the prover is 0.417 seconds.

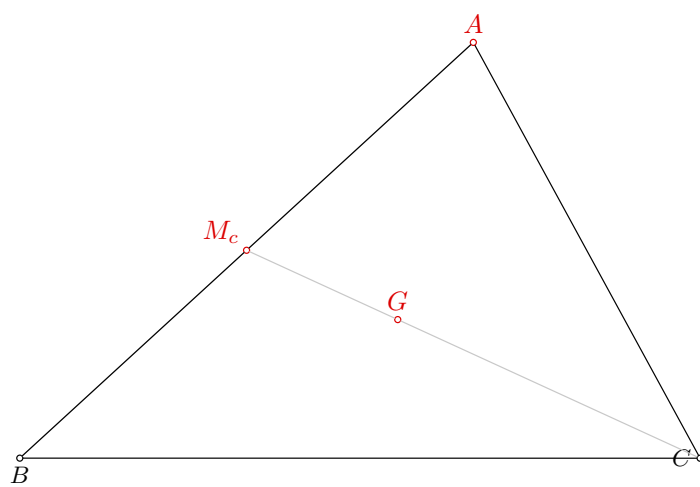


Figure 1: Illustration of the problem 0061

**NDG conditions** Line through points  $A$  and  $\_M_a$  is not parallel with line through points  $B$  and  $\_M_b$   
Points  $A$ ,  $B$  and  $\_M_a$  are not collinear

## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{AB\_M_b} \neq S_{\_M_a B\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_c=\_M_c$

NDG conditions are:

$S_{AB\_M_b} \neq S_{\_M_a B\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $G=\_G$

NDG conditions are:

$S_{AB\_M_b} \neq S_{\_M_a B\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (cancellation assumption)

Total number of proof steps: 630

Time spent by the prover: 0.200 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_c=\_M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.3 Proving $G=\_G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 84 terms.

**Time Complexity:** Time spent by the prover is 0.190 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_c = \neg M_c$

Proving failed

#### 4.4.3 Proving $G = \neg G$

Proving failed

# Problem 62

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 62: Given a point  $M_c$ , a point  $H_a$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_a$  and  $M_c$  are not the same;
2. Choose freely a point  $A$  on the circle  $k(M_c, A)$  (rule WOncircle);
3. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); ;
4. Using the point  $H_a$  and the point  $B$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $B$  are not the same;
5. Choose freely a point  $C$  on the line  $a$  (rule WOnline1) .

Non-degenerate conditions: points  $H_a$  and  $M_c$  are not the same.

Determination conditions: points  $H_a$  and  $B$  are not the same.

Rules used: [W01,W02,W06,WOncircle1,WOnline1]

Lemmas used: [D20,D5,GD01,GD02,GL03,L42]

Solving time: 951.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point H_{a} 80 40
point A 80 95

color 220 0 0
fontsize 9
```

```

cmark_lt M_{c}
cmark_r H_{a}
cmark_t A
color 0 0 0
fontsize 8

% NDG: points H_{a} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H
_{a}
circle k(M_{c},A) M_{c} H_{a}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% Choosing randomly a point A on the circle with center M_{c} through point H_{a}
oncircle A M_{c} H_{a}
cmark_t A
color 200 200 200
drawcircle M_{c} H_{a}
color 0 0 0

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% Choosing randomly a point C on the line H_{a}B
online C H_{a} B
cmark_l C
color 200 200 200
drawline H_{a} B
color 0 0 0

```

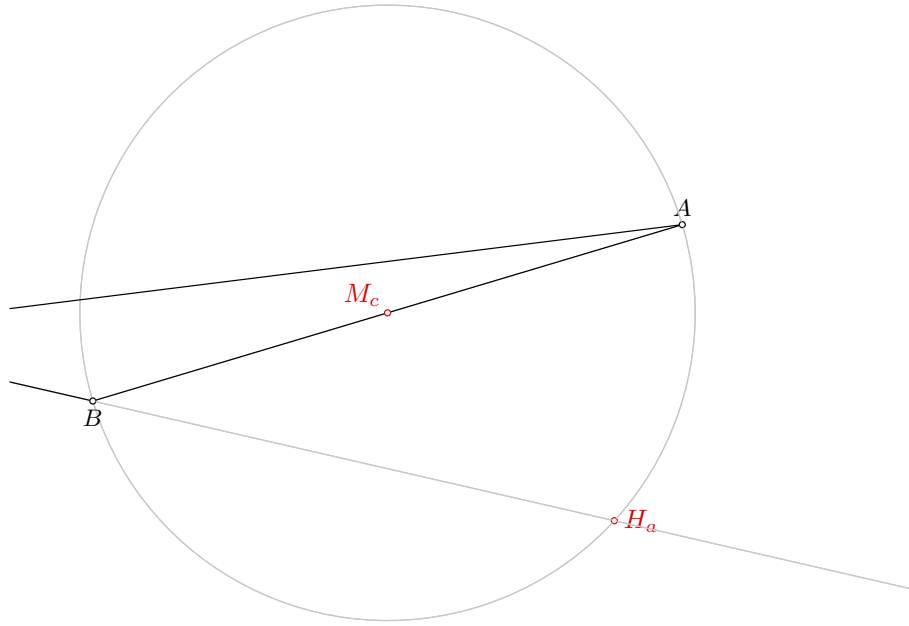


Figure 1: Illustration of the problem 0062

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: points  $H_{\{a\}}$  and  $M_{\{c\}}$  are not the same
% Determination conditions: points  $H_{\{a\}}$  and B are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = M_c$

**Status:** Theorem has been proved.



**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.118 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.1.3 Proving $A = A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.024 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF^1_{h_a}} \neq S_{CAF^1_{h_a}}$  i.e., lines  $BC$  and  $AF^1_{h_a}$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.2.3 Proving $A = A$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF^1_{h_a}} \neq S_{CAF^1_{h_a}}$  i.e., lines  $BC$  and  $AF^1_{h_a}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 17 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 280 terms.

**Time Complexity:** Time spent by the prover is 0.280 seconds. There are no ndg conditions.

#### 4.3.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_c = M_c$

Proving failed

#### 4.4.2 Proving $H_a = H_a$

Proving failed

#### 4.4.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 63

*Generated automatically by ArgoTriCS  
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## 1 Problem

Problem 63: Given a point  $M_c$ , a point  $H_b$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_b$  and  $M_c$  are not the same;
2. Choose freely a point  $A$  on the circle  $k(M_c, A)$  (rule WOncircle);
3. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); ;
4. Using the point  $A$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $H_b$  are not the same;
5. Choose freely a point  $C$  on the line  $b$  (rule WOnline1) .

Non-degenerate conditions: points  $H_b$  and  $M_c$  are not the same.

Determination conditions: points  $A$  and  $H_b$  are not the same.

Rules used: [W01,W02,W06,WOncircle1,WOnline1]

Lemmas used: [D20,D6,GD01,GD02,GL03,L41]

Solving time: 813.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point H_{b} 89.36 77.83
point A 80 95

color 220 0 0
fontsize 9
```

```

cmark_lt M_{c}
cmark_l H_{b}
cmark_t A
color 0 0 0
fontsize 8

% NDG: points H_{b} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{b}
circle k(M_{c},A) M_{c} H_{b}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% Choosing randomly a point A on the circle with center M_{c} through point H_{b}
oncircle A M_{c} H_{b}
cmark_t A
color 200 200 200
drawcircle M_{c} H_{b}
color 0 0 0

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% Choosing randomly a point C on the line AH_{b}
online C A H_{b}
cmark_l C
color 200 200 200
drawline A H_{b}
color 0 0 0

```

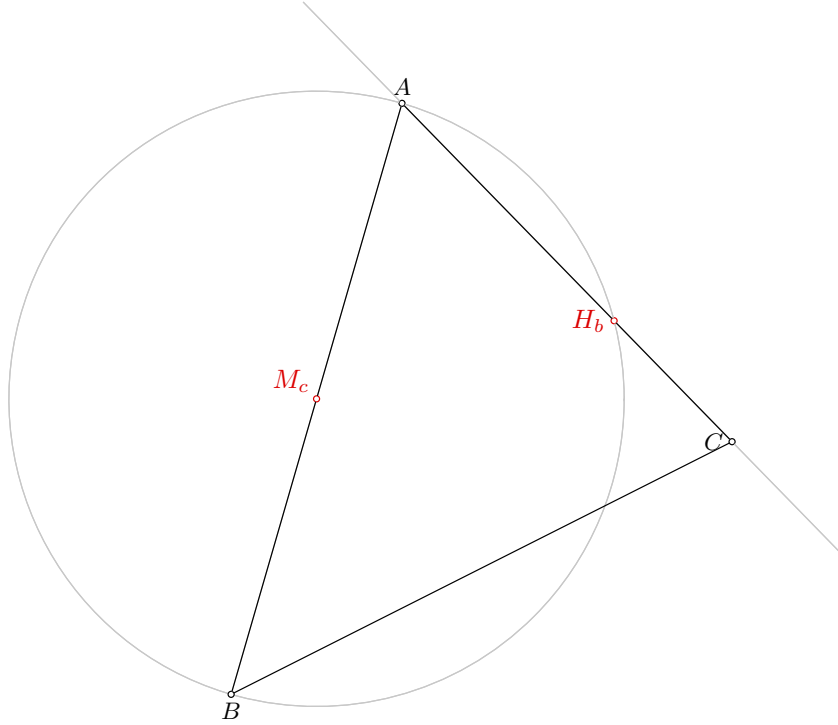


Figure 1: Illustration of the problem 0063

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: points  $H_{\{b\}}$  and  $M_{\{c\}}$  are not the same
% Determination conditions: points A and  $H_{\{b\}}$  are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.119 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b = \neg H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 47 terms.

**Time Complexity:** Time spent by the prover is 0.629 seconds.

**NDG conditions** Points  $A$  and  $M_c$  are not identical

Points  $A$  and  $C$  are not identical

Points  $A$ ,  $C$  and  $M_c$  are not collinear

#### 4.1.3 Proving $A = A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.024 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{\neg h_b}} \neq S_{CBF^1_{\neg h_b}}$  i.e., lines  $AC$  and  $BF^1_{\neg h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.2.3 Proving $A = A$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{\neg h_b}} \neq S_{CBF^1_{\neg h_b}}$  i.e., lines  $AC$  and  $BF^1_{\neg h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 17 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 280 terms.

**Time Complexity:** Time spent by the prover is 0.190 seconds. There are no ndg conditions.

#### 4.3.3 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 312 terms.

**Time Complexity:** Time spent by the prover is 1.960 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 312 terms.

**Time Complexity:** Time spent by the prover is 1.850 seconds. There are no ndg conditions.

#### 4.4.3 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 64

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 64: Given a point  $M_c$ , a point  $H_c$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $H_c$  are not the same;
2. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) ;
3. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); ;
4. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
5. Choose freely a point  $C$  on the line  $h_c$  (rule WOnline2).

Non-degenerate conditions: .

Determination conditions: points  $M_c$  and  $H_c$  are not the same.

Rules used: [W01,W02,W10b,WOnline1,WOnline2]

Lemmas used: [D10,D20,D7,GD01,GL03,GL09]

Solving time: 155.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point H_{c} 68.91 84.83
point A 80 95

color 220 0 0
fontsize 9
```



```

cmark_lt M_{c}
cmark_rt H_{c}
cmark_t A
color 0 0 0
fontsize 8

% DET: points M_{c} and H_{c} are not the same
% Constructing a line c which passes through point M_{c} and point H_{c}
line c M_{c} H_{c}

color 200 200 200
drawline c
color 0 0 0

% Choosing randomly a point A on the line M_{c}H_{c}
online A M_{c} H_{c}
cmark_t A
color 200 200 200
drawline M_{c} H_{c}
color 0 0 0

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0

% Generating random value V[_G6844]
random V[_G6844]

% Calculating value V[_G6865] using formula V[_G6844]*20
expression V[_G6865] { V[_G6844]*20 }

% Constructing a point C which is a point for which holds H_{c}C = V[_G6865] and angle AH_{c}C = 90
turtle C A H_{c} 90 V[_G6865]
cmark_l C

```

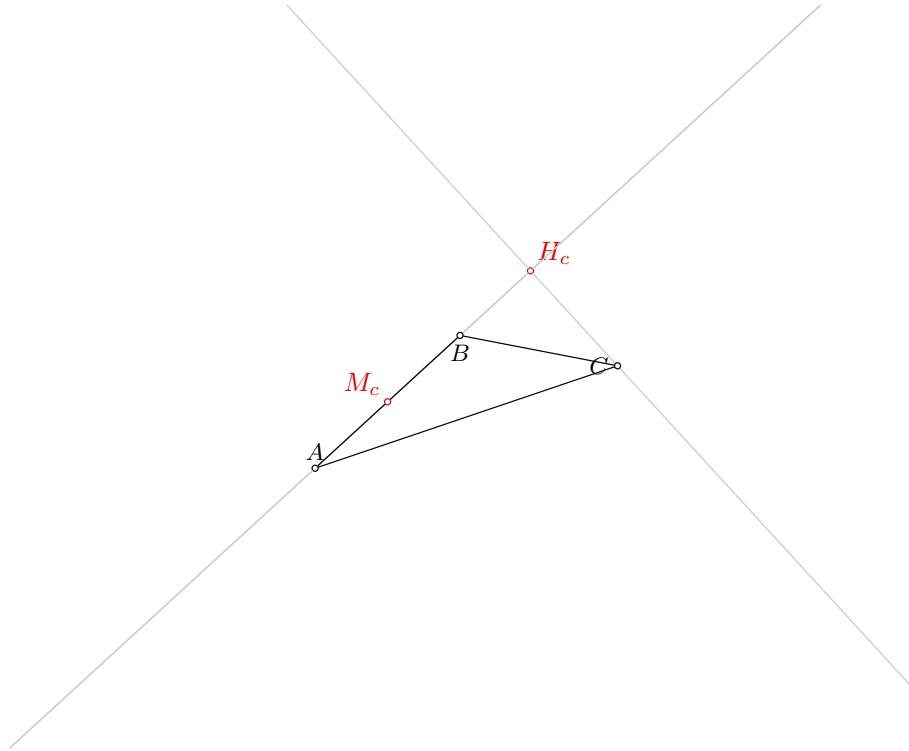


Figure 1: Illustration of the problem 0064

```
drawsegment A B
drawsegment A C
drawsegment B C
```

*% Non-degenerate conditions:*

*% Determination conditions: points  $M_{\{c\}}$  and  $H_{\{c\}}$  are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = \neg M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.032 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.098 seconds.

**NDG conditions** Line through points  $A$  and  $H_c$  is not perpendicular to line through points  $H_c$  and  $B$

#### 4.1.3 Proving $A = A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.014 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF^2_{H_c}} \neq S_{BCF^2_{H_c}}$  i.e., lines  $AB$  and  $CF^2_{H_c}$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.2.3 Proving $A = A$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF^2_{H_c}} \neq S_{BCF^2_{H_c}}$  i.e., lines  $AB$  and  $CF^2_{H_c}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 2 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 65

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 65: Given a point  $A$ , a point  $M_c$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); ;
2. Using the point  $A$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H$  are not the same;
3. Using the point  $H$  and the point  $B$ , construct a line  $h_b$  (rule W02); % DET: points  $H$  and  $B$  are not the same;
4. Using the point  $A$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $A$  and  $M_c$  are not the same;
5. Using the circle  $k(M_c, A)$ , the line  $h_a$ , the point  $M_c$  and the point  $A$ , construct a point  $H_a$  (rule W05); % NDG: line  $h_a$  and circle  $k(M_c, A)$  intersect % DET: points  $A$  and  $H_a$  must be different;
6. Using the point  $H_a$  and the point  $B$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $B$  are not the same;
7. Using the circle  $k(M_c, A)$ , the line  $h_b$ , the point  $M_c$  and the point  $B$ , construct a point  $H_b$  (rule W05); % NDG: line  $h_b$  and circle  $k(M_c, A)$  intersect % DET: points  $B$  and  $H_b$  must be different;
8. Using the point  $H_b$  and the point  $A$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $A$  are not the same;
9. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; line  $h_b$  and circle  $k(M_c, A)$  intersect; line  $h_a$  and circle  $k(M_c, A)$  intersect; points  $A$  and  $M_c$  are not the same.

Determination conditions: lines  $a$  and  $b$  are not the same; points  $H_b$  and  $A$  are not the same; points  $B$  and  $H_b$  must be different; points  $H_a$  and  $B$  are not the same; points  $A$  and  $H_a$  must be different; points  $H$  and  $B$  are not the same; points  $A$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D20,D3,D5,D6,D8,D9,GD01,GD02,GL03,L40,L41,L42]

Solving time: 1.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point M_{c} 50 67.5
point H 80 72.73

color 220 0 0
fontsize 9

cmark_t A
cmark_lt M_{c}
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% DET: points H and B are not the same
% Constructing a line h_{b} which passes through point H and point B
line h_{b} H B

color 200 200 200
drawline h_{b}
color 0 0 0
```

```

% NDG: points A and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point A
circle k(M_{c},A) M_{c} A

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line h_{a} and circle k(M_{c},A) intersect% DET: points A and H_{a} must be different
% Constructing a point P_{\G8280} which is a foot of the point M_{c} on the line h_{a}
foot P_{\G8280} M_{c} h_{a}
cmark_r P_{\G8280}
color 200 200 200
drawline M_{c} P_{\G8280}
color 0 0 0

% Constructing a point H_{a} which is an image of the point A in the symmetry to point/line P_{\G
8280}
sim H_{a} P_{\G8280} A
cmark_r H_{a}

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% NDG: line h_{b} and circle k(M_{c},A) intersect% DET: points B and H_{b} must be different
% Constructing a point P_{\G8518} which is a foot of the point M_{c} on the line h_{b}
foot P_{\G8518} M_{c} h_{b}
cmark_r P_{\G8518}
color 200 200 200
drawline M_{c} P_{\G8518}
color 0 0 0

% Constructing a point H_{b} which is an image of the point B in the symmetry to point/line P_{\G
8518}
sim H_{b} P_{\G8518} B
cmark_l H_{b}

% DET: points H_{b} and A are not the same
% Constructing a line b which passes through point H_{b} and point A
line b H_{b} A

color 200 200 200
drawline b
color 0 0 0

```

```

% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_1 C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and b are not parallel; line h_{b} and circle k(M_{c},A)
% intersect; line h_{a} and circle k(M_{c},A) intersect; points A and M_{c} are not the same
% Determination conditions: lines a and b are not the same; points H_{b} and A are not the same;
% points B and H_{b} must be different; points H_{a} and B are not the same; points A and H_{a}
% must be different; points H and B are not the same; points A and H are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c=M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $H=H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 224 terms.

**Time Complexity:** Time spent by the prover is 9.216 seconds.

**NDG conditions** Points  $A$  and  $H$  are not identical

Points  $A$  and  $H$  are not identical

Points  $B$  and  $H$  are not identical



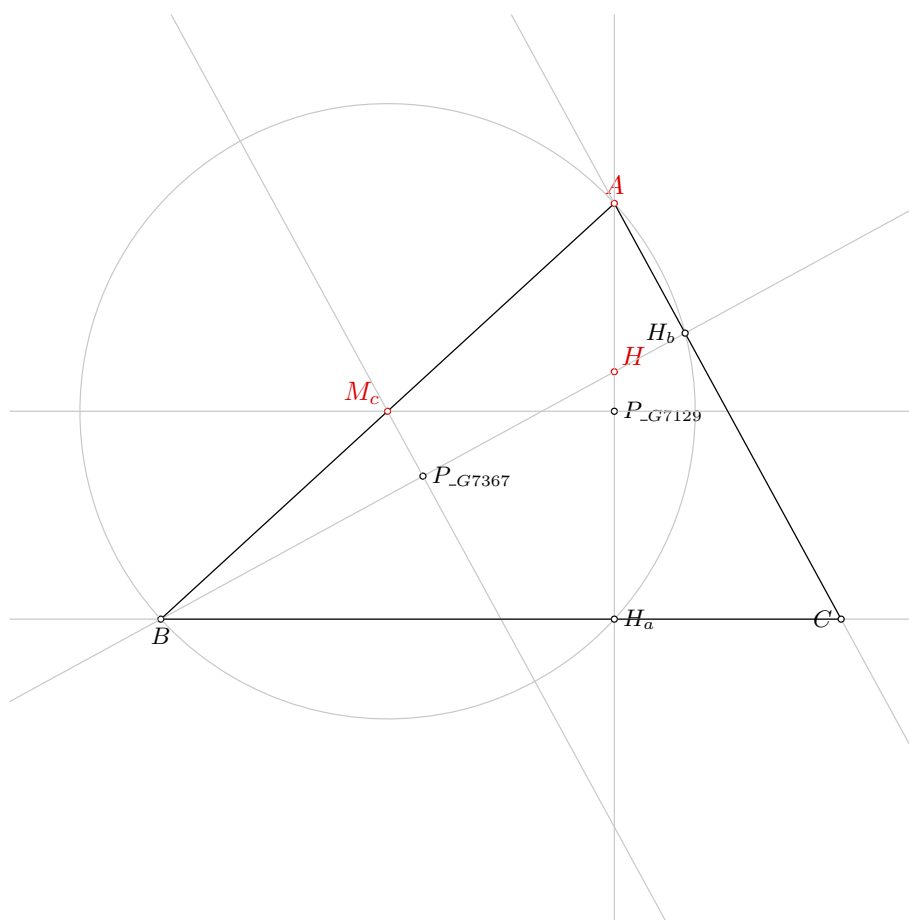


Figure 1: Illustration of the problem 0065

Line through points  $A$  and  $B$  is not perpendicular to line through points  $B$  and  $M_c$

Line through points  $H_b$  and  $A$  is not parallel with line through points  $B$  and  $H_a$

Points  $A$  and  $H_a$  are not identical

Line through points  $A$  and  $B$  is not parallel with line through points  $C$  and  $M_c$

Line through points  $A$  and  $B$  is not perpendicular to line through points  $B$  and  $M_c$

## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_a H_b A} \neq S_{B H_b A}$  i.e., lines  $H_a B$  and  $H_b A$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-H_b}^1} \neq S_{F_{-H_a}^0 BF_{-H_b}^1}$  i.e., lines  $AF_{-H_a}^0$  and  $BF_{-H_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_c=_M_c$

Proving failed

### 4.2.3 Proving $H=_H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_c=_M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $H=_H$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 9 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_c = M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.030 seconds. There are no ndg conditions.

#### 4.4.3 Proving $H = H$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 11 terms.

**Time Complexity:** Time spent by the prover is 0.040 seconds. There are no ndg conditions.

# Problem 66

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 66: Given a point  $A$ , a point  $M_c$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); ;
2. Using the point  $A$  and the point  $M_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $M_c$  are not the same;
3. Using the point  $T_a$  and the point  $B$ , construct a line  $a$  (rule W02); % DET: points  $T_a$  and  $B$  are not the same;
4. Using the point  $T_a$  and the line  $c$ , construct a circle  $k(T_a, foot[T_a, c])$  (rule W11); % NDG: point  $T_a$  is not incident to the line  $c$ ;
5. Using the circle  $k(T_a, foot[T_a, c])$ , the point  $A$ , the point  $T_a$  and the line  $c$ , construct a line  $b$  (rule W13); % NDG: point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ;
6. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ; point  $T_a$  is not incident to the line  $c$ .

Determination conditions: lines  $b$  and  $a$  are not the same; points  $T_a$  and  $B$  are not the same; points  $A$  and  $M_c$  are not the same.

Rules used: [W01,W02,W03,W11,W13]

Lemmas used: [D20,D23,GD01,GD02,GL03,GL09,GL10,GL11,L8]

Solving time: 1.8 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point M_{c} 50 67.5
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_t A
cmark_lt M_{c}
cmark_rb T_{a}
color 0 0 0
fontsize 8

% Constructing a point B such that  $AB/AM_{\{c\}}=2$ 
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points A and  $M_{\{c\}}$  are not the same
% Constructing a line c which passes through point A and point  $M_{\{c\}}$ 
line c A M_{c}

color 200 200 200
drawline c
color 0 0 0

% DET: points  $T_{\{a\}}$  and B are not the same
% Constructing a line a which passes through point  $T_{\{a\}}$  and point B
line a T_{a} B

color 200 200 200
drawline a
color 0 0 0

% NDG: point  $T_{\{a\}}$  is not incident to the line c
% Constructing a point  $P_{\{\backslash\_G9349\}}$  which is a foot of the point  $T_{\{a\}}$  on the line c
foot P_{\backslash\_G9349} T_{a} c
cmark_r P_{\backslash\_G9349}
color 200 200 200
drawline T_{a} P_{\backslash\_G9349}
color 0 0 0

% Constructing a circle  $k(T_{\{a\}}, foot[T_{\{a\}}, c])$  whose center is at point  $T_{\{a\}}$  and which passes
    through point  $P_{\{\backslash\_G9349\}}$ 
```

```

circle k(T_{a},foot[T_{a},c]) T_{a} P_{\_G9349}

color 200 200 200
drawcircle k(T_{a},foot[T_{a},c])
color 0 0 0

% NDG: point A is outside the circle k(T_{a},foot[T_{a},c])
% Constructing a point P_{\_G9740} which is a foot of the point T_{a} on the line c
foot P_{\_G9740} T_{a} c
cmark_r P_{\_G9740}
color 200 200 200
drawline T_{a} P_{\_G9740}
color 0 0 0

% Constructing a line L_{\_G9743} which passes through point T_{a} and point A
line L_{\_G9743} T_{a} A

color 200 200 200
drawline L_{\_G9743}
color 0 0 0

% Constructing a point P_{\_G9746} which is an image of the point P_{\_G9740} in the symmetry to
point/line L_{\_G9743}
sim P_{\_G9746} L_{\_G9743} P_{\_G9740}
cmark_r P_{\_G9746}

% Constructing a line b which passes through point A and point P_{\_G9746}
line b A P_{\_G9746}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and a are not parallel; point A is outside the circle k(T_{a},
foot[T_{a},c]); point T_{a} is not incident to the line c
% Determination conditions: lines b and a are not the same; points T_{a} and B are not the same;
points A and M_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

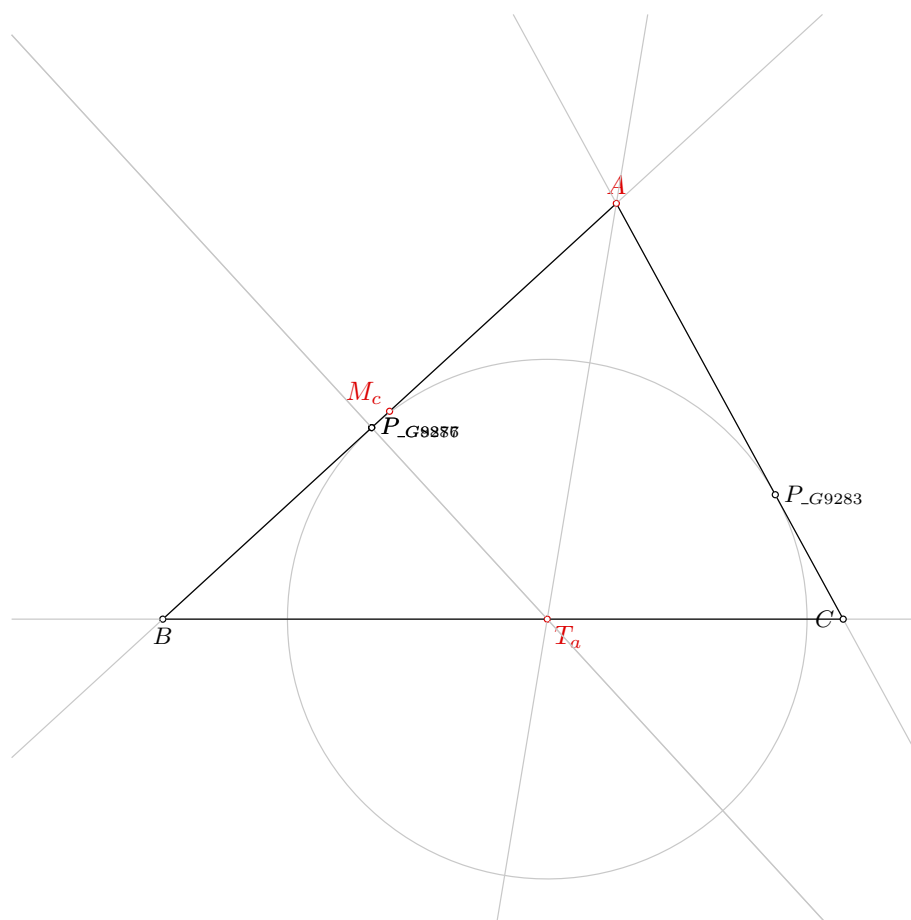


Figure 1: Illustration of the problem 0066

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c=_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $T_a=_T_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{AT_aB} \neq S_{P_{G10309}T_aB}$  i.e., lines  $AP_{G10309}$  and  $T_aB$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_c=_M_c$

Proving failed

#### 4.2.3 Proving $T_a=_T_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_c=_M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 5 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.



#### 4.3.3 Proving $T_a = \neg T_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 14 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_a = \neg T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 67

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 67: Given a point  $A$ , a point  $M_c$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); ;
2. Using the point  $A$  and the point  $M_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $M_c$  are not the same;
3. Using the point  $A$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $T_b$  are not the same;
4. Using the point  $T_b$  and the line  $c$ , construct a circle  $k(T_b, foot[T_b, c])$  (rule W11); % NDG: point  $T_b$  is not incident to the line  $c$ ;
5. Using the circle  $k(T_b, foot[T_b, c])$ , the point  $B$ , the point  $T_b$  and the line  $c$ , construct a line  $a$  (rule W13); % NDG: point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ;
6. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ; point  $T_b$  is not incident to the line  $c$ .

Determination conditions: lines  $a$  and  $b$  are not the same; points  $A$  and  $T_b$  are not the same; points  $A$  and  $M_c$  are not the same.

Rules used: [W01,W02,W03,W11,W13]

Lemmas used: [D20,D24,GD01,GD02,GL03,GL09,GL10,GL11,L9]

Solving time: 1.9 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point M_{c} 50 67.5
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_t A
cmark_lt M_{c}
cmark_t T_{b}
color 0 0 0
fontsize 8

% Constructing a point B such that  $AB/AM_{\{c\}}=2$ 
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points A and  $M_{\{c\}}$  are not the same
% Constructing a line c which passes through point A and point  $M_{\{c\}}$ 
line c A M_{c}

color 200 200 200
drawline c
color 0 0 0

% DET: points A and  $T_{\{b\}}$  are not the same
% Constructing a line b which passes through point A and point  $T_{\{b\}}$ 
line b A T_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: point  $T_{\{b\}}$  is not incident to the line c
% Constructing a point  $P_{\{\backslash\_G9457\}}$  which is a foot of the point  $T_{\{b\}}$  on the line c
foot P_{\_G9457} T_{b} c
cmark_r P_{\_G9457}
color 200 200 200
drawline T_{b} P_{\_G9457}
color 0 0 0

% Constructing a circle  $k(T_{\{b\}}, foot[T_{\{b\}}, c])$  whose center is at point  $T_{\{b\}}$  and which passes
    through point  $P_{\{\backslash\_G9457\}}$ 
```

```

circle k(T_{b},foot[T_{b},c]) T_{b} P_{\_G9457}

color 200 200 200
drawcircle k(T_{b},foot[T_{b},c])
color 0 0 0

% NDG: point B is outside the circle k(T_{b},foot[T_{b},c])
% Constructing a point P_{\_G9848} which is a foot of the point T_{b} on the line c
foot P_{\_G9848} T_{b} c
cmark_r P_{\_G9848}
color 200 200 200
drawline T_{b} P_{\_G9848}
color 0 0 0

% Constructing a line L_{\_G9851} which passes through point T_{b} and point B
line L_{\_G9851} T_{b} B

color 200 200 200
drawline L_{\_G9851}
color 0 0 0

% Constructing a point P_{\_G9854} which is an image of the point P_{\_G9848} in the symmetry to
point/line L_{\_G9851}
sim P_{\_G9854} L_{\_G9851} P_{\_G9848}
cmark_r P_{\_G9854}

% Constructing a line a which passes through point B and point P_{\_G9854}
line a B P_{\_G9854}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and b are not parallel; point B is outside the circle k(T_{b},
foot[T_{b},c]); point T_{b} is not incident to the line c
% Determination conditions: lines a and b are not the same; points A and T_{b} are not the same;
points A and M_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

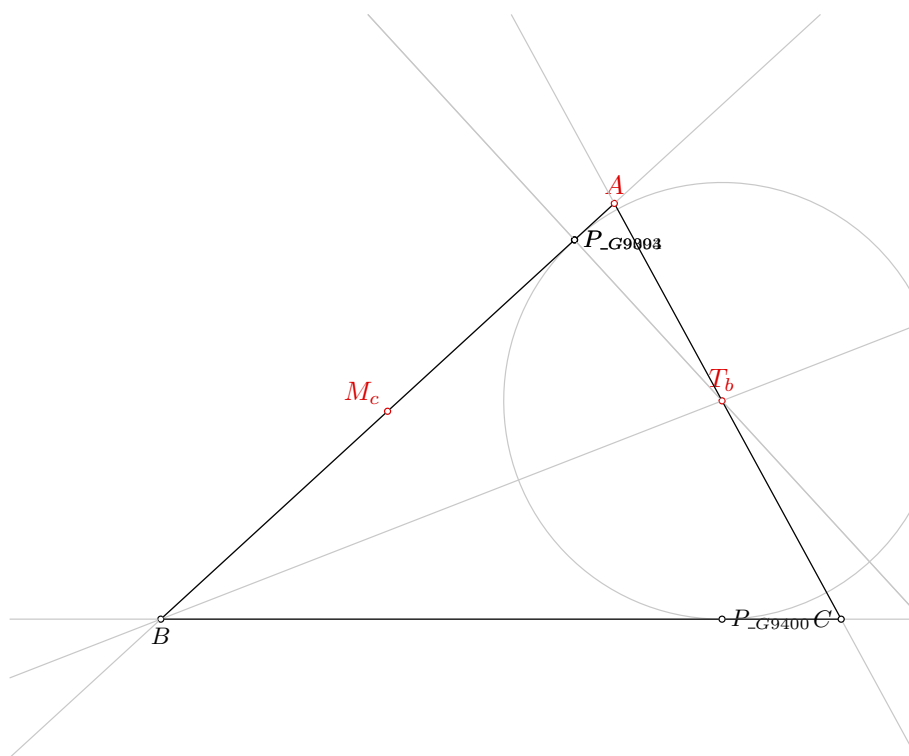


Figure 1: Illustration of the problem 0067

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c=M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $T_b=T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{BAT_b} \neq S_{P_{G10429}AT_b}$  i.e., lines  $BP_{G10429}$  and  $AT_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_c=M_c$

Proving failed

#### 4.2.3 Proving $T_b=T_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_c=M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_b = \neg T_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 10 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 7 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_b = \neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 7 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 68

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 68: Given a point  $M_c$ , a point  $T_c$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $T_c$  are not the same;
2. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) ;
3. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); ;
4. Using the point  $A$ , the point  $B$ , the point  $T_c$  and the line  $c$ , construct a point  $T'_c$  (rule W19);  
% NDG: points  $A$  and  $B$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $midpoint([A, T_c])$  are not the same;
5. Choose freely a point  $C$  on the circle  $k_{over}(T_c, T'_c)$  (rule WOncircle).

Non-degenerate conditions: points  $A$  and  $B$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $midpoint([A, T_c])$  are not the same.

Determination conditions: points  $M_c$  and  $T_c$  are not the same.

Rules used: [W01,W02,W19,WOncircle2,WOnline1]

Lemmas used: [D20,D25,GD01,GL03,GL09,L79]

Solving time: 89.8 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{c} 50 67.5  
point T_{c} 55.38 72.43  
point A 80 95
```



```

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_rt T_{c}
cmark_t A
color 0 0 0
fontsize 8

% DET: points M_{c} and T_{c} are not the same
% Constructing a line c which passes through point M_{c} and point T_{c}
line c M_{c} T_{c}

color 200 200 200
drawline c
color 0 0 0

% Choosing randomly a point A on the line M_{c}T_{c}
online A M_{c} T_{c}
cmark_t A
color 200 200 200
drawline M_{c} T_{c}
color 0 0 0

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% NDG: points A and B are not the same; points B and T_{c} are not the same; points B and midpoint
% ([A,T_{c}]) are not the same
% Constructing a point P_{\_G9562} which is an image of the point B in a rotation around the point
% T_{c} for the angle 90
rotate P_{\_G9562} T_{c} 90 B
cmark_r P_{\_G9562}
color 200 200 200
drawarc_p T_{c} B 90
color 0 0 0

% Constructing a line L_{\_G9565} which passes through point T_{c} and point P_{\_G9562}
line L_{\_G9565} T_{c} P_{\_G9562}

color 200 200 200
drawline L_{\_G9565}
color 0 0 0

% Constructing midpoint P_{\_G9568} of the segment BP_{\_G9562}

```

```

midpoint P_{\_G9568} B P_{\_G9562}
cmark_r P_{\_G9568}

% Constructing a line L_{\_G9571} which passes through point A and point P_{\_G9568}
line L_{\_G9571} A P_{\_G9568}

color 200 200 200
drawline L_{\_G9571}
color 0 0 0

% Constructing a line L_{\_G9574} which passes through point B and point P_{\_G9568}
line L_{\_G9574} B P_{\_G9568}

color 200 200 200
drawline L_{\_G9574}
color 0 0 0

% Constructing a point P_{\_G9577} which belongs to line L_{\_G9565} and line L_{\_G9571}
intersec P_{\_G9577} L_{\_G9565} L_{\_G9571}
cmark_r P_{\_G9577}

% Constructing a point P_{\_G9580} which belongs to line L_{\_G9565} and line L_{\_G9574}
intersec P_{\_G9580} L_{\_G9565} L_{\_G9574}
cmark_r P_{\_G9580}

% Constructing a line L_{\_G9583} which passes through point A and point P_{\_G9580}
line L_{\_G9583} A P_{\_G9580}

color 200 200 200
drawline L_{\_G9583}
color 0 0 0

% Constructing a line L_{\_G9586} which passes through point B and point P_{\_G9577}
line L_{\_G9586} B P_{\_G9577}

color 200 200 200
drawline L_{\_G9586}
color 0 0 0

% Constructing a point P_{\_G9589} which belongs to line L_{\_G9583} and line L_{\_G9586}
intersec P_{\_G9589} L_{\_G9583} L_{\_G9586}
cmark_r P_{\_G9589}

% Constructing a line L_{\_G9592} which passes through point P_{\_G9568} and point P_{\_G9589}
line L_{\_G9592} P_{\_G9568} P_{\_G9589}

color 200 200 200
drawline L_{\_G9592}
color 0 0 0

% Constructing a point T'_{c} which belongs to line L_{\_G9592} and line c

```

```

intersec T'_{c} L_{\G9592} c
cmark_r T'_{c}

% Constructing midpoint P_{\G10417} of the segment T_{c}T'_{c}
midpoint P_{\G10417} T_{c} T'_{c}
cmark_r P_{\G10417}

% Choosing randomly a point C on the circle with center P_{\G10417} through point T_{c}
oncircle C P_{\G10417} T_{c}
cmark_l C
color 200 200 200
drawcircle P_{\G10417} T_{c}
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: points A and B are not the same; points B and T_{c} are not the same;
% points B and midpoint([A,T_{c}]) are not the same
% Determination conditions: points M_{c} and T_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.023 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_c = T_c$

Proving failed

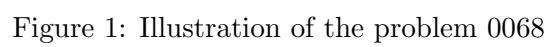
#### 4.1.3 Proving $A = A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.019 seconds.

**NDG conditions** There are no NDG conditions for this theorem



## 4.2 GCLC - Area method

### 4.2.1 Proving $M_c = M_c$

Proving failed

### 4.2.2 Proving $T_c = T_c$

Proving failed

### 4.2.3 Proving $A = A$

NDG conditions are:

$S_{T_cAP_{G13799}} \neq S_{P_{G13793}AP_{G13799}}$  i.e., lines  $T_cP_{G13793}$  and  $AP_{G13799}$  are not parallel (construction based assumption)

$S_{T_cBP_{G13799}} \neq S_{P_{G13793}BP_{G13799}}$  i.e., lines  $T_cP_{G13793}$  and  $BP_{G13799}$  are not parallel (construction based assumption)

$S_{ABP_{G13808}} \neq S_{P_{G13811}BP_{G13808}}$  i.e., lines  $AP_{G13811}$  and  $BP_{G13808}$  are not parallel (construction based assumption)

$S_{P_{G13799}M_cT_c} \neq S_{P_{G13820}M_cT_c}$  i.e., lines  $P_{G13799}P_{G13820}$  and  $M_cT_c$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_c = M_c$

Proving failed

### 4.3.2 Proving $T_c = T_c$

Proving failed

### 4.3.3 Proving $A = A$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_c = M_c$

Proving failed

### 4.4.2 Proving $T_c = T_c$

Proving failed

### 4.4.3 Proving $A = A$

Proving failed

# Problem 69

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 69: Given a point  $A$ , a point  $M_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); ;
2. Using the point  $A$  and the point  $M_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $M_c$  are not the same;
3. Using the point  $I$  and the line  $c$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $c$ ;
4. Using the circle  $k(I, P_a)$ , the point  $A$ , the point  $I$  and the line  $c$ , construct a line  $b$  (rule W13); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
5. Using the circle  $k(I, P_a)$ , the point  $B$ , the point  $I$  and the line  $c$ , construct a line  $a$  (rule W13); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
6. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; point  $B$  is outside the circle  $k(I, P_a)$ ; point  $A$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $c$ .

Determination conditions: lines  $b$  and  $a$  are not the same; points  $A$  and  $M_c$  are not the same.

Rules used: [W01,W02,W03,W11,W13]

Lemmas used: [D20,D27,GD01,GL03,GL09,L59,L60,L61]

Solving time: 17.3 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point M_{c} 50 67.5
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_t A
cmark_lt M_{c}
cmark_b I
color 0 0 0
fontsize 8

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points A and M_{c} are not the same
% Constructing a line c which passes through point A and point M_{c}
line c A M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: point I is not incident to the line c
% Constructing a point P_{\_G14929} which is a foot of the point I on the line c
foot P_{\_G14929} I c
cmark_r P_{\_G14929}
color 200 200 200
drawline I P_{\_G14929}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G14929}
circle k(I,P_{a}) I P_{\_G14929}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point A is outside the circle k(I,P_{a})
% Constructing a point P_{\_G15136} which is a foot of the point I on the line c
```

```

foot P_{\_G15136} I c
cmark_r P_{\_G15136}
color 200 200 200
drawline I P_{\_G15136}
color 0 0 0

% Constructing a line L_{\_G15139} which passes through point I and point A
line L_{\_G15139} I A

color 200 200 200
drawline L_{\_G15139}
color 0 0 0

% Constructing a point P_{\_G15142} which is an image of the point P_{\_G15136} in the symmetry to
point/line L_{\_G15139}
sim P_{\_G15142} L_{\_G15139} P_{\_G15136}
cmark_r P_{\_G15142}

% Constructing a line b which passes through point A and point P_{\_G15142}
line b A P_{\_G15142}

color 200 200 200
drawline b
color 0 0 0

% NDG: point B is outside the circle k(I,P_{a})
% Constructing a point P_{\_G15471} which is a foot of the point I on the line c
foot P_{\_G15471} I c
cmark_r P_{\_G15471}
color 200 200 200
drawline I P_{\_G15471}
color 0 0 0

% Constructing a line L_{\_G15474} which passes through point I and point B
line L_{\_G15474} I B

color 200 200 200
drawline L_{\_G15474}
color 0 0 0

% Constructing a point P_{\_G15477} which is an image of the point P_{\_G15471} in the symmetry to
point/line L_{\_G15474}
sim P_{\_G15477} L_{\_G15474} P_{\_G15471}
cmark_r P_{\_G15477}

% Constructing a line a which passes through point B and point P_{\_G15477}
line a B P_{\_G15477}

color 200 200 200
drawline a
color 0 0 0

```



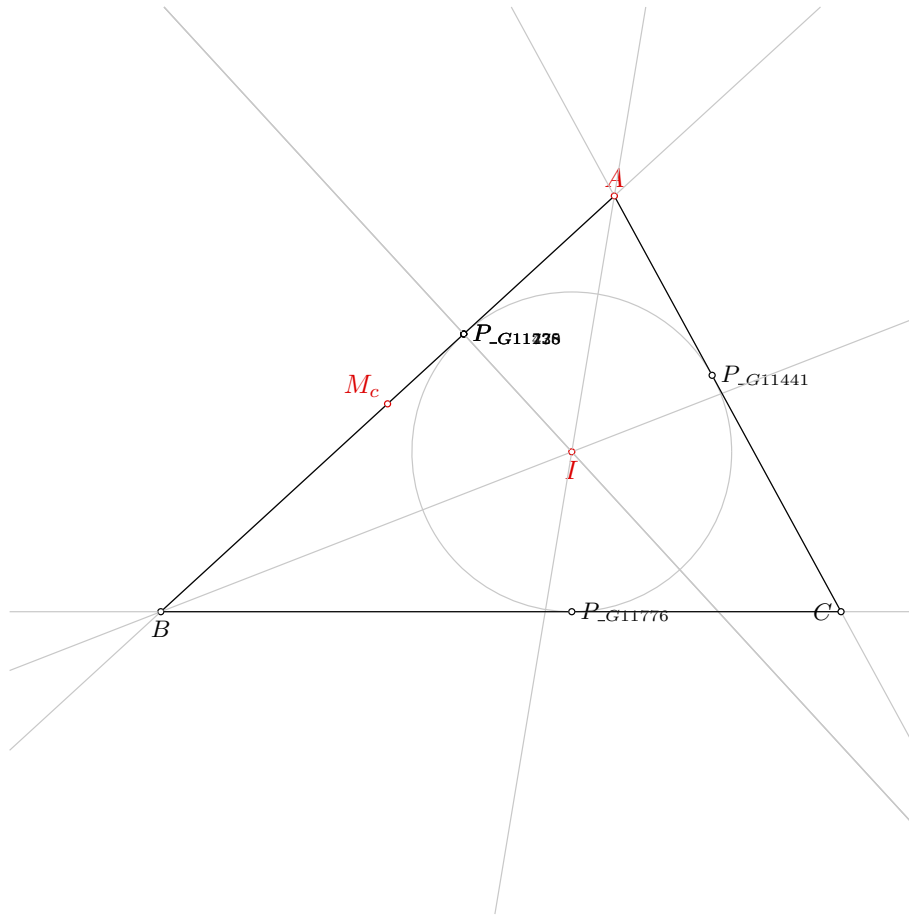


Figure 1: Illustration of the problem 0069

```
% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines b and a are not parallel; point B is outside the circle k(I,P_{a
}); point A is outside the circle k(I,P_{a}); point I is not incident to the line c
% Determination conditions: lines b and a are not the same; points A and M_{c} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.015 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c=_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.036 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{ABP_{G12859}} \neq S_{P_{G12524}BP_{G12859}}$  i.e., lines  $AP_{G12524}$  and  $BP_{G12859}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_c=_M_c$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 10 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I = \neg I$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 15 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $I = \neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

# Problem 70

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 70: Given a point  $A$ , a point  $H_a$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H_a$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H_a$  are not the same;
2. Using the point  $H_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
3. Using the point  $A$ , the line  $a$  and the point  $H_a$ , construct a line  $h_{A,2/3}(a)$  (rule W15); ;
4. Choose freely a point  $G$  on the line  $h_{A,2/3}(a)$  (rule WOnline4);
5. Using the point  $G$  and the point  $A$ , construct a point  $M_a$  (rule W01); ;
6. Choose freely a point  $B$  on the line  $a$  (rule WOnline1) ;
7. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: points  $A$  and  $H_a$  are not the same.

Rules used: [W01,W02,W10a,W15,WOnline1,WOnline4]

Lemmas used: [D21,D5,D8,GL03,GL04,L55]

Solving time: 191.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point H_{a} 80 40
point G 70 58.33
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r H_{a}
cmark_t G
color 0 0 0
fontsize 8
```

```
% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}
```

```
color 200 200 200
drawline h_{a}
color 0 0 0
```

```
% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
perp a H_{a} h_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% Constructing a point P_{\_G7646} such that  $AP_{\_G7646}/AH_{a}=0.6666666666666666$ 
towards P_{\_G7646} A H_{a} 0.6666666666666666
cmark_r P_{\_G7646}
color 200 200 200
drawsegment A H_{a}
color 0 0 0
```

```
% Constructing a line h_{A,2/3}(a) which contains the point P_{\_G7646} and is parallel to the line
a
parallel h_{A,2/3}(a) P_{\_G7646} a
```

```
color 200 200 200
drawline h_{A,2/3}(a)
color 0 0 0
```

```
% Constructing a point P_{\_G7953} such that  $AP_{\_G7953}/AH_{a}=0.6666666666666666$ 
towards P_{\_G7953} A H_{a} 0.6666666666666666
cmark_r P_{\_G7953}
color 200 200 200
drawsegment A H_{a}
color 0 0 0
```

```

% Generating random value V[_G7892]
random V[_G7892]

% Calculating value V[_G7913] using formula V[_G7892]*20
expression V[_G7913] { V[_G7892]*20 }

% Constructing a point G which is a point for which holds  $P_{\setminus\_G7953}G = V[_G7913]$  and angle  $AP_{\setminus\_G7953}G = 90$ 
turtle G A P_{\_G7953} 90 V[_G7913]
cmark_t G

% Constructing a point  $M_{\{a\}}$  such that  $GM_{\{a\}}/GA=-0.5$ 
towards M_{a} G A -0.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% Choosing randomly a point B on the line  $H_{\{a\}}M_{\{a\}}$ 
online B H_{a} M_{a}
cmark_b B
color 200 200 200
drawline H_{a} M_{a}
color 0 0 0

% Constructing a point C such that  $BC/BM_{\{a\}}=2$ 
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions: points A and  $H_{\{a\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

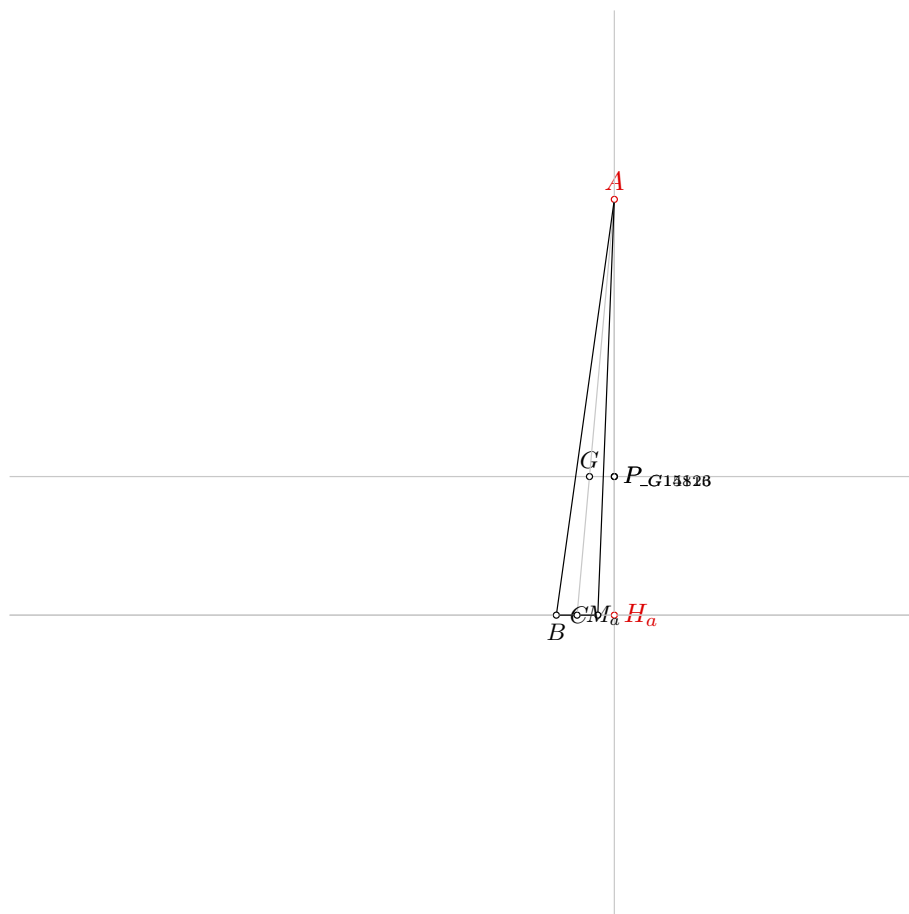


Figure 1: Illustration of the problem 0070

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a = \neg H_a$

##### Construction steps:

- Free point  $A$
- Free point  $H_a$
- Line  $h_a$  through two points  $A$  and  $H_a$
- Line  $a$  through point  $H_a$  perpendicular to line  $h_a$
- Segment division point  $P_{G13723}$  of segment  $AH_a$  with division coefficient 1.999999999999998
- Line  $h_{A,2/3}(a)$  through point  $P_{G13723}$  parallel with line  $a$
- Segment division point  $P_{G13855}$  of segment  $AH_a$  with division coefficient -0.6666666666666666
- Line  $L_{G13858}$  through point  $P_{G13855}$  parallel with line  $h_{A,2/3}(a)$
- Random point  $G$  from line  $L_{G13858}$
- Line  $L_{G14084}$  through two points  $G$  and  $A$
- Free point  $P_{G14087}$
- Segment division point  $P_{G14090}$  of segment  $GP_{G14087}$  with division coefficient -0.5
- Segment division point  $P_{G14093}$  of segment  $GP_{G14087}$  with division coefficient -2.0
- Line  $L_{G14096}$  through two points  $A$  and  $P_{G14093}$
- Line  $L_{G14099}$  through point  $P_{G14090}$  parallel with line  $L_{G14096}$
- Intersection point  $M_a$  of point sets  $L_{G14099}$  and  $L_{G14084}$
- Random point  $B$  from line  $a$
- Segment division point  $C$  of segment  $BM_a$  with division coefficient -2.0
- Line  $\neg a$  through two points  $B$  and  $C$
- Line  $\neg h_a$  through point  $A$  perpendicular to line  $\neg a$
- Intersection point  $\neg H_a$  of point sets  $\neg a$  and  $\neg h_a$

##### Theorem statement:

- Points  $H_a$  and  $\neg H_a$  are identical

**Info:** Attempting to add the construction of new random point tempPoint-425 $h_{A,2/3}(a)$  necessary for completion of construction of line  $L_{G13858}$

**Warning:** Generated new random point tempPoint-425 $h_{A,2/3}(a)$  on line  $h_{A,2/3}(a)$  in order to complete the construction of parallel line  $L_{G13858}$

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $A$  has been assigned following coordinates: (0, 0)



#### **4.1.3 Proving $G=_G$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $A=A$**

Proving failed

#### **4.2.2 Proving $H_a=_H_a$**

Proving failed

#### **4.2.3 Proving $G=_G$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $A=A$**

Proving failed

#### **4.3.2 Proving $H_a=_H_a$**

Proving failed

#### **4.3.3 Proving $G=_G$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $A=A$**

Proving failed

#### **4.4.2 Proving $H_a=_H_a$**

Proving failed

#### **4.4.3 Proving $G=_G$**

Proving failed

# Problem 71

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 71: Given a point  $A$ , a point  $G$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $G$ , construct a point  $M_a$  (rule W01); ;
2. Using the point  $A$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $H_b$  are not the same;
3. Using the point  $H_b$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_b$  and  $M_a$  are not the same;
4. Using the circle  $k(M_a, B)$ , the line  $b$ , the point  $M_a$  and the point  $H_b$ , construct a point  $C$  (rule W05); % NDG: line  $b$  and circle  $k(M_a, B)$  intersect % DET: points  $H_b$  and  $C$  must be different;
5. Using the point  $G$  and the point  $C$ , construct a point  $M_c$  (rule W01); ;
6. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: line  $b$  and circle  $k(M_a, B)$  intersect; points  $H_b$  and  $M_a$  are not the same.

Determination conditions: points  $H_b$  and  $C$  must be different; points  $A$  and  $H_b$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D20,D6,GD01,GD02,GL03,GL04,L37,L38,L55,L57]

Solving time: 0.2 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point G 70 58.33
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_t A
cmark_t G
cmark_l H_{b}
color 0 0 0
fontsize 8

% Constructing a point M_{a} such that AM_{a}/AG=1.5
towards M_{a} A G 1.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points H_{b} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{b}
circle k(M_{a},B) M_{a} H_{b}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line b and circle k(M_{a},B) intersect% DET: points H_{b} and C must be different
% Constructing a point P_{\G6812} which is a foot of the point M_{a} on the line b
foot P_{\G6812} M_{a} b
cmark_r P_{\G6812}
color 200 200 200
drawline M_{a} P_{\G6812}
color 0 0 0
```

```

% Constructing a point C which is an image of the point H_{b} in the symmetry to point/line P_{\_G
6812}
sim C P_{\_G6812} H_{b}
cmark_l C

% Constructing a point M_{c} such that GM_{c}/GC=-0.5
towards M_{c} G C -0.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k(M_{a},B) intersect; points H_{b} and M_{a} are not
the same
% Determination conditions: points H_{b} and C must be different; points A and H_{b} are not the
same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $G=_G$

Proving failed

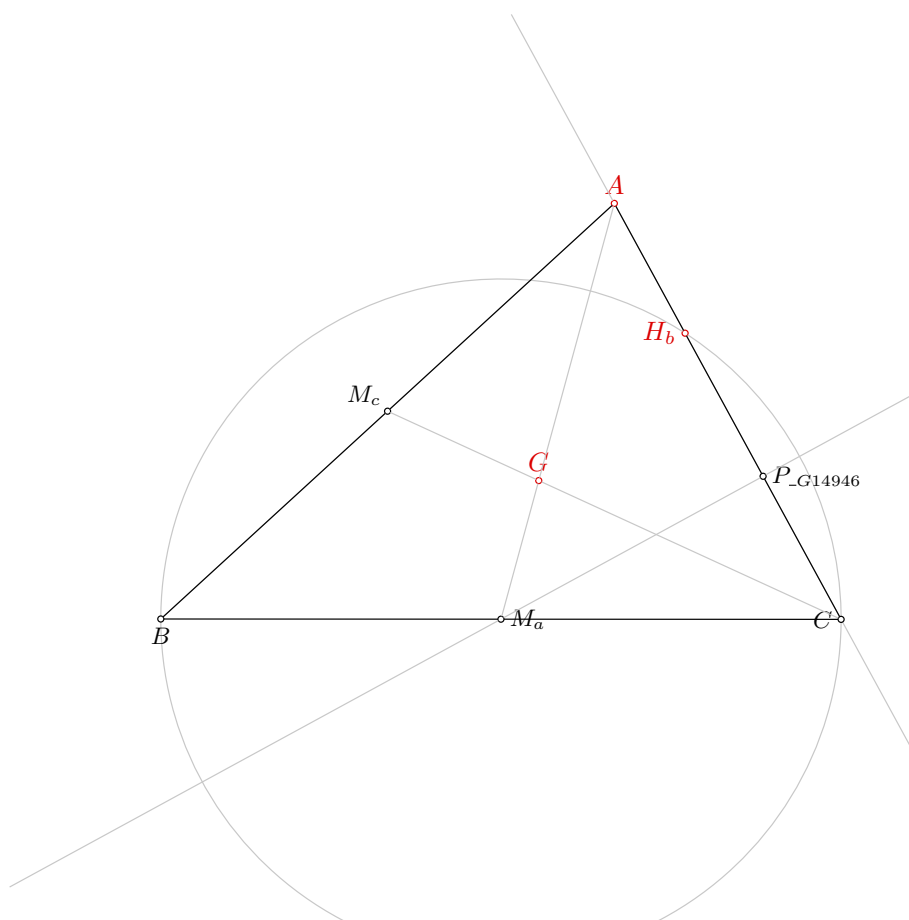


Figure 1: Illustration of the problem 0071

### 4.1.3 Proving $H_b = H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 615 terms.

**Time Complexity:** Time spent by the prover is 20.282 seconds.

**NDG conditions** Points  $H_b$  and  $G$  are not identical

Points  $H_b$  and  $G$  are not identical

Points  $P_{G13973}$ ,  $C$  and  $G$  are not collinear

Line through points  $P_{G13973}$  and  $C$  is not parallel with line through points  $A$  and  $G$

Points  $A$  and  $C$  are not identical

Points  $A$ ,  $C$  and  $G$  are not collinear

## 4.2 GCLC - Area method

### 4.2.1 Proving $A = A$

NDG conditions are:

$S_{AB_Mb} \neq S_{M_aB_Mb}$  i.e., lines  $A_Ma$  and  $B_Mb$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^0_{h_b}} \neq S_{CBF^0_{h_b}}$  i.e., lines  $AC$  and  $BF^0_{h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $G = G$

Proving failed

### 4.2.3 Proving $H_b = H_b$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A = A$

Proving failed

### 4.3.2 Proving $G = G$

Proving failed

### 4.3.3 Proving $H_b = H_b$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A = A$

Proving failed

### 4.4.2 Proving $G = G$

Proving failed

### 4.4.3 Proving $H_b = H_b$

Proving failed

# Problem 72

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 72: Given a point  $A$ , a point  $G$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $G$ , construct a point  $M_a$  (rule W01); ;
2. Using the point  $A$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $H_c$  are not the same;
3. Using the point  $H_c$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_c$  and  $M_a$  are not the same;
4. Using the circle  $k(M_a, B)$ , the line  $c$ , the point  $M_a$  and the point  $H_c$ , construct a point  $B$  (rule W05); % NDG: line  $c$  and circle  $k(M_a, B)$  intersect % DET: points  $H_c$  and  $B$  must be different;
5. Using the point  $G$  and the point  $B$ , construct a point  $M_b$  (rule W01); ;
6. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: line  $c$  and circle  $k(M_a, B)$  intersect; points  $H_c$  and  $M_a$  are not the same.

Determination conditions: points  $H_c$  and  $B$  must be different; points  $A$  and  $H_c$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D22,D7,GD01,GD02,GL03,GL04,L39,L55,L56]

Solving time: 0.2 seconds.



## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point G 70 58.33
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_t A
cmark_t G
cmark_rt H_{c}
color 0 0 0
fontsize 8

% Constructing a point M_{a} such that AM_{a}/AG=1.5
towards M_{a} A G 1.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points H_{c} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{c}
circle k(M_{a},B) M_{a} H_{c}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line c and circle k(M_{a},B) intersect% DET: points H_{c} and B must be different
% Constructing a point P_{\_G14797} which is a foot of the point M_{a} on the line c
foot P_{\_G14797} M_{a} c
cmark_r P_{\_G14797}
color 200 200 200
drawline M_{a} P_{\_G14797}
color 0 0 0
```

```

% Constructing a point B which is an image of the point H_{c} in the symmetry to point/line P_{\_G
14797}
sim B P_{\_G14797} H_{c}
cmark_b B

% Constructing a point M_{b} such that GM_{b}/GB=-0.5
towards M_{b} G B -0.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k(M_{a},B) intersect; points H_{c} and M_{a} are not
the same
% Determination conditions: points H_{c} and B must be different; points A and H_{c} are not the
same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $G=_G$

Proving failed

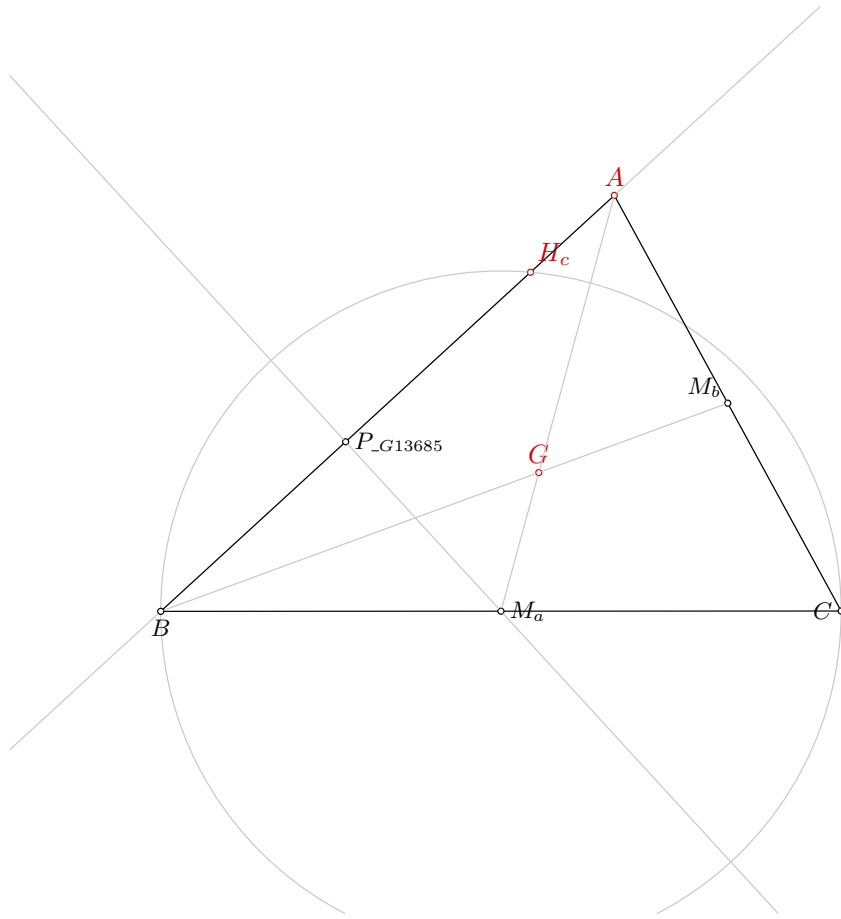


Figure 1: Illustration of the problem 0072

### 4.1.3 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 615 terms.

**Time Complexity:** Time spent by the prover is 19.793 seconds.

**NDG conditions** Points  $H_c$  and  $G$  are not identical

Points  $H_c$  and  $G$  are not identical

Points  $B$ ,  $P_{G12808}$  and  $G$  are not collinear

Line through points  $A$  and  $G$  is not parallel with line through points  $B$  and  $P_{G12808}$

Points  $A$  and  $B$  are not identical

Points  $A$  and  $B$  are not identical

## 4.2 GCLC - Area method

### 4.2.1 Proving $A = A$

NDG conditions are:

$S_{AB \neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A \neg M_a$  and  $B \neg M_b$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{\neg H_c}^0} \neq S_{BCF_{\neg H_c}^0}$  i.e., lines  $AB$  and  $CF_{\neg H_c}^0$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $G = \neg G$

NDG conditions are:

$S_{AB \neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A \neg M_a$  and  $B \neg M_b$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{\neg H_c}^0} \neq S_{BCF_{\neg H_c}^0}$  i.e., lines  $AB$  and  $CF_{\neg H_c}^0$  are not parallel (construction based assumption)

Total number of proof steps: 596

Time spent by the prover: 0.200 seconds

### 4.2.3 Proving $H_c = \neg H_c$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A = A$

Proving failed

### 4.3.2 Proving $G = \neg G$

Proving failed

### 4.3.3 Proving $H_c = \neg H_c$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A = A$

Proving failed

#### 4.4.2 Proving $G=_G$

Proving failed

#### 4.4.3 Proving $H_c=_H H_c$

Proving failed

# Problem 73

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 73: Given a point  $A$ , a point  $G$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $G$ , construct a point  $M_a$  (rule W01); ;
2. Using the point  $G$  and the point  $H$ , construct a point  $O$  (rule W01); ;
3. Using the point  $A$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H$  are not the same;
4. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
5. Using the point  $M_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $A$  and  $H$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D21,D26,D3,D8,GD01,GL03,GL04,GL09,L11,L12,L55,L58]

Solving time: 1.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95  
point G 70 58.33  
point H 80 72.73
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_t G
cmark_rt H
color 0 0 0
fontsize 8
```

```
% Constructing a point M_{a} such that AM_{a}/AG=1.5
towards M_{a} A G 1.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0
```

```
% Constructing a point O such that GO/GH=-0.5
towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0
```

```
% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0
```

```
% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0
```

```
% Constructing a line a which is perpendicular to line h_{a} and which passes through point M_{a}
perp a M_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0
```

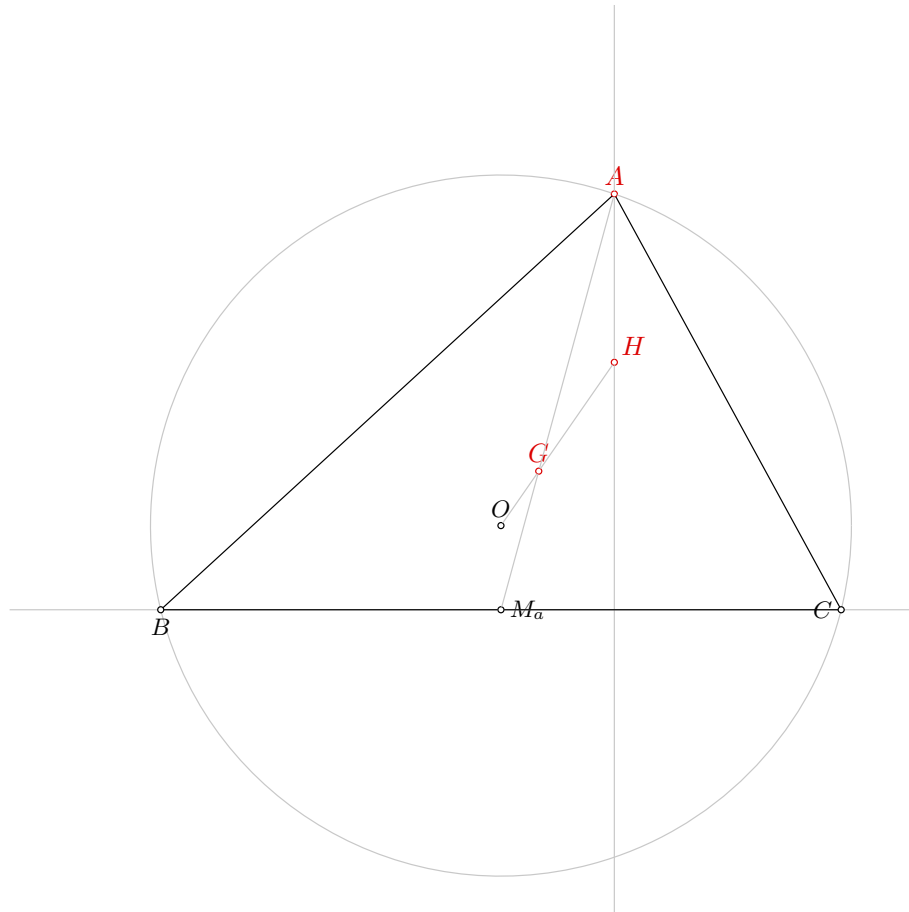


Figure 1: Illustration of the problem 0073

```
% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same
% Determination conditions: points A and H are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1



### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.017 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $G=_G$

Proving failed

#### 4.1.3 Proving $H=_H$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{M_aAH} \neq 0$  i.e., points  $M_a$ ,  $A$  and  $H$  are not collinear (foot is not the point itself; construction based assumption)

$S_{AB-M_b} \neq S_{M_aB-M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^2} \neq S_{F_{-h_a}^1BF_{-h_b}^2}$  i.e., lines  $AF_{-h_a}^1$  and  $BF_{-h_b}^2$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $G=_G$

Proving failed

#### 4.2.3 Proving $H=_H$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $G=_G$

Proving failed

### 4.3.3 Proving $H=_H$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $G=_G$

Proving failed

### 4.4.3 Proving $H=_H$

Proving failed

# Problem 74

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 74: Given a point  $A$ , a point  $G$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $G$ , construct a point  $M_a$  (rule W01); ;
2. Using the point  $A$  and the point  $T_a$ , construct a line  $s_a$  (rule W02); % DET: points  $A$  and  $T_a$  are not the same;
3. Using the point  $T_a$  and the point  $M_a$ , construct a line  $a$  (rule W02); % DET: points  $T_a$  and  $M_a$  are not the same;
4. Using the point  $M_a$  and the line  $a$ , construct a line  $m_a$  (rule W10b); ;
5. Using the line  $m_a$  and the line  $s_a$ , construct a point  $N_a$  (rule W03); % NDG: lines  $m_a$  and  $s_a$  are not parallel % DET: lines  $m_a$  and  $s_a$  are not the same;
6. Using the point  $A$  and the point  $N_a$ , construct a line  $m(AN_a)$  (rule W14); % DET: points  $A$  and  $N_a$  are not the same;
7. Using the line  $m(AN_a)$  and the line  $m_a$ , construct a point  $O$  (rule W03); % NDG: lines  $m(AN_a)$  and  $m_a$  are not parallel % DET: lines  $m(AN_a)$  and  $m_a$  are not the same;
8. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
9. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same; lines  $m(AN_a)$  and  $m_a$  are not parallel; lines  $m_a$  and  $s_a$  are not parallel.

Determination conditions: lines  $m(AN_a)$  and  $m_a$  are not the same; points  $A$  and  $N_a$  are not the same; lines  $m_a$  and  $s_a$  are not the same; points  $T_a$  and  $M_a$  are not the same; points  $A$  and  $T_a$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14]

Lemmas used: [D1,D11,D21,D23,D26,D47,GD01,GL01,GL03,GL09,L11,L12,L25,L5,L55]

Solving time: 1.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point G 70 58.33
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_t A
cmark_t G
cmark_rb T_{a}
color 0 0 0
fontsize 8

% Constructing a point M_{a} such that AM_{a}/AG=1.5
towards M_{a} A G 1.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% DET: points A and T_{a} are not the same
% Constructing a line s_{a} which passes through point A and point T_{a}
line s_{a} A T_{a}

color 200 200 200
drawline s_{a}
color 0 0 0

% DET: points T_{a} and M_{a} are not the same
% Constructing a line a which passes through point T_{a} and point M_{a}
line a T_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0
```

```

% Constructing a line  $m_{\{a\}}$  which is perpendicular to line  $a$  and which passes through point  $M_{\{a\}}$ 
perp  $m_{\{a\}}$   $M_{\{a\}}$   $a$ 

color 200 200 200
drawline  $m_{\{a\}}$ 
color 0 0 0

% NDG: lines  $m_{\{a\}}$  and  $s_{\{a\}}$  are not parallel% DET: lines  $m_{\{a\}}$  and  $s_{\{a\}}$  are not the same
% Constructing a point  $N_{\{a\}}$  which belongs to line  $m_{\{a\}}$  and line  $s_{\{a\}}$ 
intersec  $N_{\{a\}}$   $m_{\{a\}}$   $s_{\{a\}}$ 
cmark_b  $N_{\{a\}}$ 

% DET: points  $A$  and  $N_{\{a\}}$  are not the same
% Constructing bisector  $m(AN_{\{a\}})$  of the segment  $AN_{\{a\}}$ 
med  $m(AN_{\{a\}})$   $A$   $N_{\{a\}}$ 

color 200 200 200
drawline  $m(AN_{\{a\}})$ 
color 0 0 0

color 200 200 200
drawsegment  $A$   $N_{\{a\}}$ 
color 0 0 0

% NDG: lines  $m(AN_{\{a\}})$  and  $m_{\{a\}}$  are not parallel% DET: lines  $m(AN_{\{a\}})$  and  $m_{\{a\}}$  are not the same
% Constructing a point  $O$  which belongs to line  $m(AN_{\{a\}})$  and line  $m_{\{a\}}$ 
intersec  $O$   $m(AN_{\{a\}})$   $m_{\{a\}}$ 
cmark_t  $O$ 

% NDG: points  $A$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $A$ 
circle  $k(O,C)$   $O$   $A$ 

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $a$  and circle  $k(O,C)$  intersect
% Constructing points  $C$  and  $B$  which are in intersection of  $k(O,C)$  and  $a$ 
intersec2  $C$   $B$   $k(O,C)$   $a$ 
cmark_l  $C$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

```

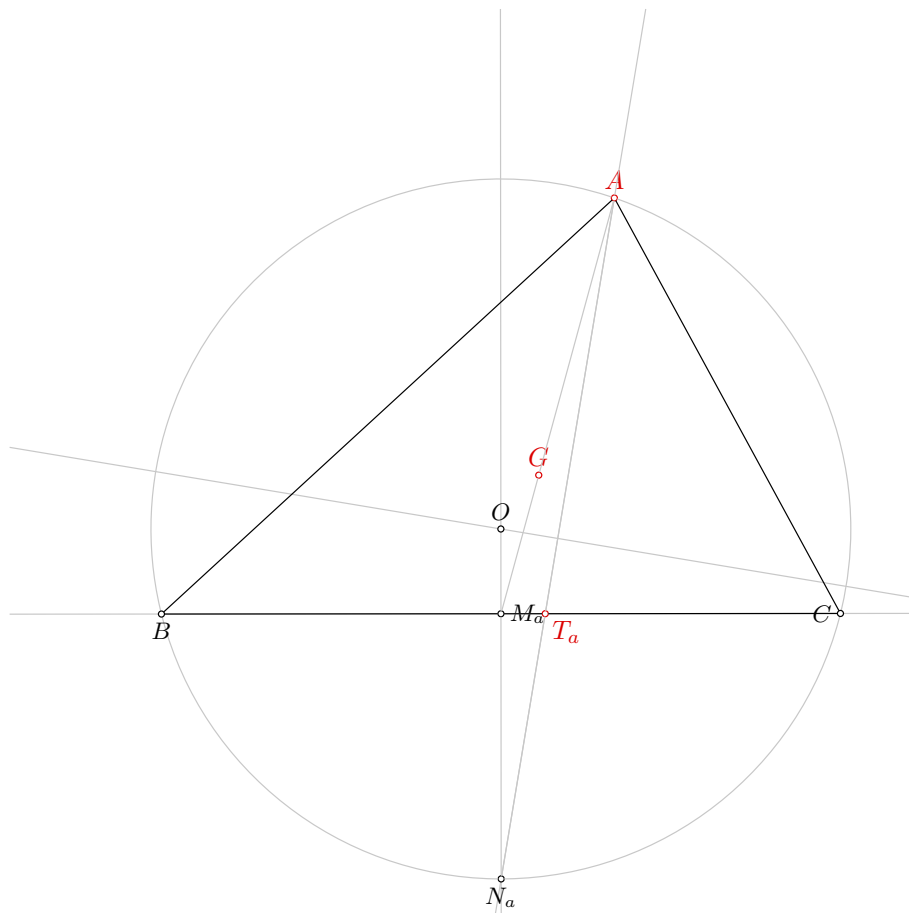


Figure 1: Illustration of the problem 0074

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% lines m(AN_{a}) and m_{a} are not parallel; lines m_{a} and s_{a} are not parallel
% Determination conditions: lines m(AN_{a}) and m_{a} are not the same; points A and N_{a} are not
% the same; lines m_{a} and s_{a} are not the same; points T_{a} and M_{a} are not the same;
% points A and T_{a} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

#### 4.1 OGP - Wu method

### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.021 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $G=_G$

Proving failed

#### 4.1.3 Proving $T_a=_T T_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{M_a AT_a} \neq S_{T_{m_a}^0 AT_a}$  i.e., lines  $M_a T_{m_a}^0$  and  $AT_a$  are not parallel (construction based assumption)

$S_{M_{m(AN_a)}^1 M_a T_{m_a}^0} \neq S_{T_{m(AN_a)}^2 M_a T_{m_a}^0}$  i.e., lines  $M_{m(AN_a)}^1 T_{m(AN_a)}^2$  and  $M_a T_{m_a}^0$  are not parallel (construction based assumption)

$S_{AB-M_b} \neq S_{-M_a B-M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $G=_G$

Proving failed

#### 4.2.3 Proving $T_a=_T T_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $G=_G$

Proving failed

#### 4.3.3 Proving $T_a=_T T_a$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.2 Proving $G=_G$

Proving failed

#### 4.4.3 Proving $T_a=_T T_a$

Proving failed



## Problem 75

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 75: Given a point  $A$ , a point  $G$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 76

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 76: Given a point  $A$ , a point  $G$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 77

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 77: Given a point  $A$ , a point  $G$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $G$ , construct a point  $M_a$  (rule W01); ;
2. Using the point  $I$  and the point  $M_a$ , construct a line  $IM_a$  (rule W02); % DET: points  $I$  and  $M_a$  are not the same;
3. Using the point  $I$  and the point  $M_a$ , construct a circle  $k_{over}(I, M_a)$  (rule W09); % NDG: points  $I$  and  $M_a$  are not the same;
4. Using the point  $A$  and the line  $IM_a$ , construct a line  $AP'_a$  (rule W16); ;
5. Using the point  $M_a$ , the line  $AP'_a$  and the point  $A$ , construct a line  $h_{M_a, -1/1}(AP'_a)$  (rule W15); ;
6. Using the circle  $k_{over}(I, M_a)$  and the line  $h_{M_a, -1/1}(AP'_a)$ , construct a point  $A_{fo}$  and a point  $P_a$  (rule W04); % NDG: line  $h_{M_a, -1/1}(AP'_a)$  and circle  $k_{over}(I, M_a)$  intersect;
7. Using the point  $P_a$  and the point  $I$ , construct a circle  $k(I, P_a)$  (rule W06); % NDG: points  $P_a$  and  $I$  are not the same;
8. Using the circle  $k(I, P_a)$ , the point  $A$  and the point  $I$ , construct a line  $c$  and a line  $b$  (rule W12); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
9. Using the circle  $k(I, P_a)$ , the point  $M_a$  and the point  $I$ , construct a line  $x1$  and a line  $a$  (rule W12); % NDG: point  $M_a$  is outside the circle  $k(I, P_a)$ ;
10. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same;
11. Using the point  $G$  and the point  $B$ , construct a point  $M_b$  (rule W01); ;

12. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; point  $M_a$  is outside the circle  $k(I, P_a)$ ; point  $A$  is outside the circle  $k(I, P_a)$ ; points  $P_a$  and  $I$  are not the same; line  $h_{M_a, -1/1}(AP_a)$  and circle  $k_{over}(I, M_a)$  intersect; points  $I$  and  $M_a$  are not the same.

Determination conditions: lines  $c$  and  $a$  are not the same; points  $I$  and  $M_a$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W09,W12,W15,W16]

Lemmas used: [D21,D22,D27,D59,D85,GD01,GL02,GL03,GL04,GL09,L55,L56,L59,L60,L61,L62,L74]

Solving time: 3.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point G 70 58.33
point I 74.37 61.15
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_t G
cmark_b I
color 0 0 0
fontsize 8
```

```
% Constructing a point M_{a} such that AM_{a}/AG=1.5
towards M_{a} A G 1.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0
```

```
% DET: points I and M_{a} are not the same
% Constructing a line IM_{a} which passes through point I and point M_{a}
line IM_{a} I M_{a}
```

```
color 200 200 200
drawline IM_{a}
color 0 0 0
```

```
% NDG: points I and M_{a} are not the same
% Constructing midpoint P_{\_G9289} of the segment IM_{a}
midpoint P_{\_G9289} I M_{a}
cmark_r P_{\_G9289}
```

```
% Constructing a circle k_{over}(I,M_{a}) whose center is at point P_{\_G9289} and which passes
    through point I
circle k_{over}(I,M_{a}) P_{\_G9289} I
```

```

color 200 200 200
drawcircle k_over(I,M_{a})
color 0 0 0

% Constructing a line AP'_{a} which contains the point A and is parallel to the line IM_{a}
parallel AP'_{a} A IM_{a}

color 200 200 200
drawline AP'_{a}
color 0 0 0

% Constructing a point P_{\G9590} such that M_{a}P_{\G9590}/M_{a}A=-1
towards P_{\G9590} M_{a} A -1
cmark_r P_{\G9590}
color 200 200 200
drawsegment A P_{\G9590}
color 0 0 0

% Constructing a line h_{M_{a},-1/1}(AP'_{a}) which contains the point P_{\G9590} and is parallel
to the line AP'_{a}
parallel h_{M_{a},-1/1}(AP'_{a}) P_{\G9590} AP'_{a}

color 200 200 200
drawline h_{M_{a},-1/1}(AP'_{a})
color 0 0 0

% NDG: line h_{M_{a},-1/1}(AP'_{a}) and circle k_over(I,M_{a}) intersect
% Constructing points A_{fo} and P_{a} which are in intersection of k_over(I,M_{a}) and h_{M_{a},-1/1}(AP'_{a})
intersec2 A_{fo} P_{a} k_over(I,M_{a}) h_{M_{a},-1/1}(AP'_{a})
cmark_r A_{fo}
cmark_r P_{a}

% NDG: points P_{a} and I are not the same
% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{a}
circle k(I,P_{a}) I P_{a}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point A is outside the circle k(I,P_{a})
% Constructing midpoint P_{\G10339} of the segment AI
midpoint P_{\G10339} A I

```

```

cmark_r P_{\_G10339}

% Constructing a circle C_{\_G10342} whose center is at point P_{\_G10339} and which passes through
point A
circle C_{\_G10342} P_{\_G10339} A

color 200 200 200
drawcircle C_{\_G10342}
color 0 0 0

% Constructing points P_{\_G10345} and P_{\_G10348} which are in intersection of C_{\_G10342} and k
(I,P_{a})
intersec2 P_{\_G10345} P_{\_G10348} C_{\_G10342} k(I,P_{a})
cmark_r P_{\_G10345}
cmark_r P_{\_G10348}

% Constructing a line c which passes through point A and point P_{\_G10345}
line c A P_{\_G10345}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line b which passes through point A and point P_{\_G10348}
line b A P_{\_G10348}

color 200 200 200
drawline b
color 0 0 0

% NDG: point M_{a} is outside the circle k(I,P_{a})
% Constructing midpoint P_{\_G10751} of the segment M_{a}I
midpoint P_{\_G10751} M_{a} I
cmark_r P_{\_G10751}

% Constructing a circle C_{\_G10754} whose center is at point P_{\_G10751} and which passes through
point M_{a}
circle C_{\_G10754} P_{\_G10751} M_{a}

color 200 200 200
drawcircle C_{\_G10754}
color 0 0 0

% Constructing points P_{\_G10757} and P_{\_G10760} which are in intersection of C_{\_G10754} and k
(I,P_{a})
intersec2 P_{\_G10757} P_{\_G10760} C_{\_G10754} k(I,P_{a})
cmark_r P_{\_G10757}
cmark_r P_{\_G10760}

% Constructing a line x1 which passes through point M_{a} and point P_{\_G10757}
line x1 M_{a} P_{\_G10757}

```

```

color 200 200 200
drawline x1
color 0 0 0

% Constructing a line a which passes through point M_{a} and point P_{\_G10760}
line a M_{a} P_{\_G10760}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

% Constructing a point M_{b} such that GM_{b}/GB=-0.5
towards M_{b} G B -0.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and a are not parallel; point M_{a} is outside the circle k(I,
P_{a}); point A is outside the circle k(I,P_{a}); points P_{a} and I are not the same; line h_{
M_{a},-1/1}(AP'_{a}) and circle k_{over}(I,M_{a}) intersect; points I and M_{a} are not the same
% Determination conditions: lines c and a are not the same; points I and M_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

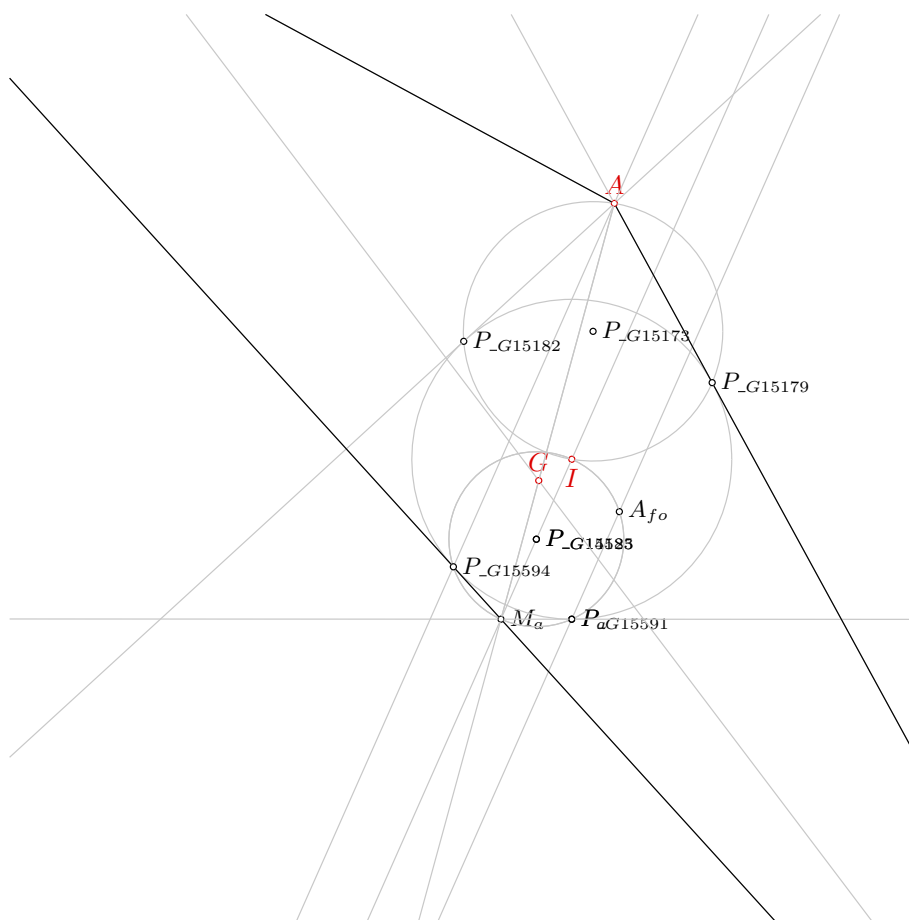


Figure 1: Illustration of the problem 0077



## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.02 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $G=_G$

Proving failed

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{AM_aP_{G10772}} \neq S_{P_{G10357}M_aP_{G10772}}$  i.e., lines  $AP_{G10357}$  and  $M_aP_{G10772}$  are not parallel (construction based assumption)

$S_{AB_{-}M_b} \neq S_{_{-}M_aB_{-}M_b}$  i.e., lines  $A_{-}M_a$  and  $B_{-}M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $G=_G$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.050 seconds. There are no ndg conditions.

#### 4.3.2 Proving $G=_G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 306 terms.

**Time Complexity:** Time spent by the prover is 0.970 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I=_I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6437 terms.

**Time Complexity:** Time spent by the prover is 58.980 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.050 seconds. There are no ndg conditions.

#### 4.4.2 Proving $G=_G$

Proving failed

#### 4.4.3 Proving $I=_I$

Proving failed

# Problem 78

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 78: Given a point  $A$ , a point  $H_a$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H_a$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H_a$  are not the same;
2. Using the point  $A$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $H_b$  are not the same;
3. Using the point  $H_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
4. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same;
5. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
6. Using the line  $a$  and the line  $h_b$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $h_b$  are not parallel % DET: lines  $a$  and  $h_b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $h_b$  are not parallel; lines  $a$  and  $b$  are not parallel.

Determination conditions: lines  $a$  and  $h_b$  are not the same; lines  $a$  and  $b$  are not the same; points  $A$  and  $H_b$  are not the same; points  $A$  and  $H_a$  are not the same.

Rules used: [W02,W03,W10a,W10b]

Lemmas used: [D5,D6,D8,D9,GD01]

Solving time: 1.2 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point H_{a} 80 40
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_t A
cmark_r H_{a}
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
perp a H_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_l C

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b
```

```

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines a and h_{b} are not parallel% DET: lines a and h_{b} are not the same
% Constructing a point B which belongs to line a and line h_{b}
intersec B a h_{b}
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and h_{b} are not parallel; lines a and b are not parallel
% Determination conditions: lines a and h_{b} are not the same; lines a and b are not the same;
  points A and H_{b} are not the same; points A and H_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.02 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a=H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.104 seconds.

**NDG conditions** Points  $H_b$  and  $H_a$  are not identical

Points  $H_b$  and  $H_a$  are not identical

Line through points  $A$  and  $H_a$  is not parallel with line through points  $B$  and  $C$

#### 4.1.3 Proving $H_b=H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 11 terms.

**Time Complexity:** Time spent by the prover is 0.188 seconds.

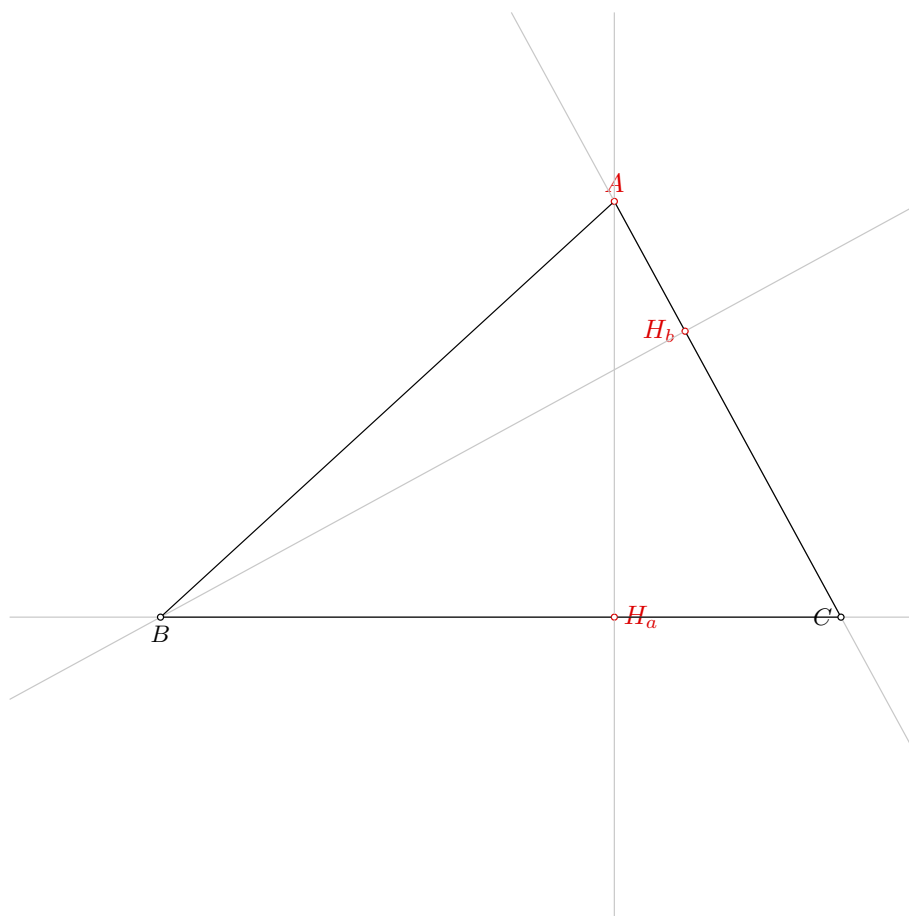


Figure 1: Illustration of the problem 0078

**NDG conditions** Points  $H_b$  and  $H_a$  are not identical

Points  $H_b$  and  $H_a$  are not identical

Point  $H_a$  is not the midpoint of segment with endpoints  $A$  and  $C$

Points  $A$ ,  $C$  and  $H_a$  are not collinear

## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_a A H_b} \neq S_{T_a^0 A H_b}$  i.e., lines  $H_a T_a^0$  and  $A H_b$  are not parallel (construction based assumption)

$S_{H_a H_b T_{h_b}^1} \neq S_{T_a^0 H_b T_{h_b}^1}$  i.e., lines  $H_a T_a^0$  and  $H_b T_{h_b}^1$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^2} \neq S_{CAF_{h_a}^2}$  i.e., lines  $BC$  and  $AF_{h_a}^2$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^3} \neq S_{CBF_{h_b}^3}$  i.e., lines  $AC$  and  $BF_{h_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_a = \neg H_a$

Proving failed

### 4.2.3 Proving $H_b = \neg H_b$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $H_b = \neg H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 23 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_a = H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $H_b = H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 15 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.



# Problem 79

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 79: Given a point  $A$ , a point  $H_a$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H_a$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H_a$  are not the same;
2. Using the point  $A$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $H_c$  are not the same;
3. Using the point  $H_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
4. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same;
5. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
6. Using the line  $a$  and the line  $h_c$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $h_c$  are not parallel % DET: lines  $a$  and  $h_c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $h_c$  are not parallel; lines  $a$  and  $c$  are not parallel.

Determination conditions: lines  $a$  and  $h_c$  are not the same; lines  $a$  and  $c$  are not the same; points  $A$  and  $H_c$  are not the same; points  $A$  and  $H_a$  are not the same.

Rules used: [W02,W03,W10a,W10b]

Lemmas used: [D10,D5,D7,D8,GD01]

Solving time: 1.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point H_{a} 80 40
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_t A
cmark_r H_{a}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
perp a H_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c
```

```

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines a and h_{c} are not parallel% DET: lines a and h_{c} are not the same
% Constructing a point C which belongs to line a and line h_{c}
intersec C a h_{c}
cmark_1 C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and h_{c} are not parallel; lines a and c are not parallel
% Determination conditions: lines a and h_{c} are not the same; lines a and c are not the same;
  points A and H_{c} are not the same; points A and H_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a=H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.114 seconds.

**NDG conditions** Points  $A$  and  $H_c$  are not identical

Points  $A$  and  $H_c$  are not identical

Line through points  $A$  and  $H_a$  is not parallel with line through points  $B$  and  $C$

#### 4.1.3 Proving $H_c=H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 11 terms.

**Time Complexity:** Time spent by the prover is 0.18 seconds.

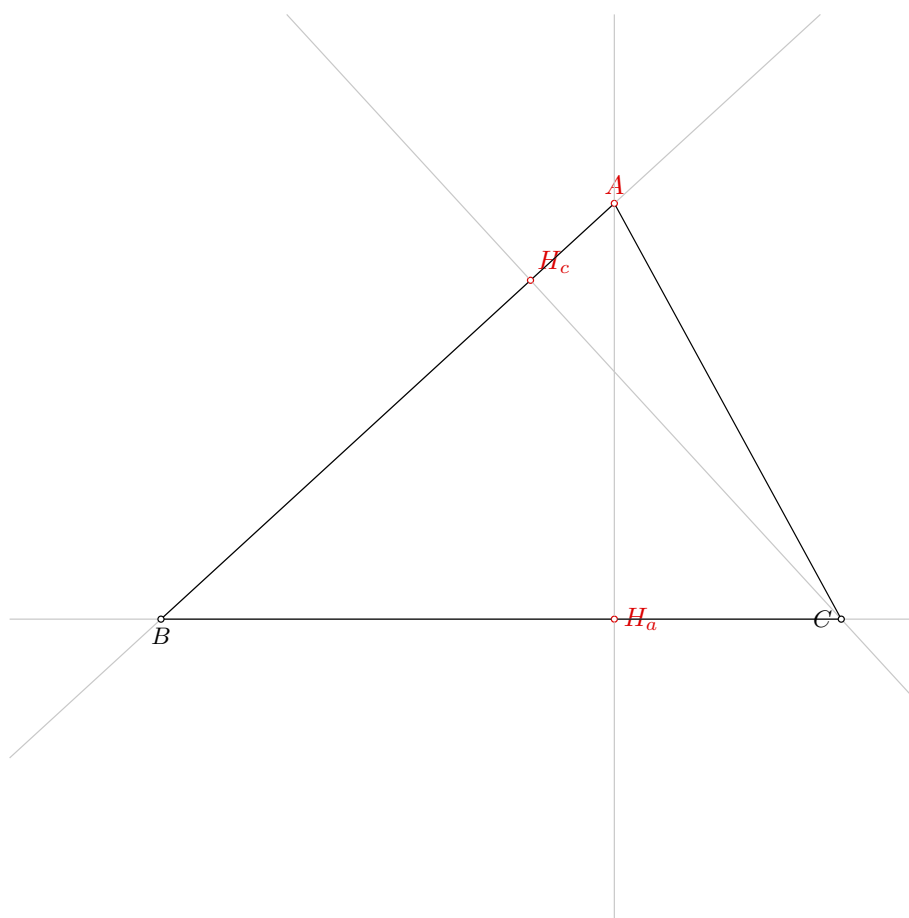


Figure 1: Illustration of the problem 0079

**NDG conditions** Points  $A$  and  $H_c$  are not identical

Points  $A$  and  $H_c$  are not identical

Points  $A$  and  $B$  are not identical

Points  $B$  and  $H_a$  are not identical

## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_a A H_c} \neq S_{T_a^0 A H_c}$  i.e., lines  $H_a T_a^0$  and  $A H_c$  are not parallel (construction based assumption)

$S_{H_a H_c T_{h_c}^1} \neq S_{T_a^0 H_c T_{h_c}^1}$  i.e., lines  $H_a T_a^0$  and  $H_c T_{h_c}^1$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^2} \neq S_{CAF_{h_a}^2}$  i.e., lines  $BC$  and  $AF_{h_a}^2$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^3} \neq S_{BCF_{h_c}^3}$  i.e., lines  $AB$  and  $CF_{h_c}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_a = \neg H_a$

Proving failed

### 4.2.3 Proving $H_c = \neg H_c$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 23 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_a = H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $H_c = H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 15 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

# Problem 80

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 80: Given a point  $H_a$ , a point  $H$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $H_a$  and  $H$  are not the same;
2. Choose freely a point  $A$  on the line  $h_a$  (rule WOnline1) ;
3. Using the point  $H_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
4. Choose freely a point  $B$  on the line  $a$  (rule WOnline2);
5. Using the point  $B$  and the point  $A$ , construct a point  $M_c$  (rule W01); ;
6. Using the point  $B$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H$  are not the same;
7. Using the point  $B$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $B$  and  $M_c$  are not the same;
8. Using the circle  $k(M_c, A)$ , the line  $h_b$ , the point  $M_c$  and the point  $B$ , construct a point  $H_b$  (rule W05); % NDG: line  $h_b$  and circle  $k(M_c, A)$  intersect % DET: points  $B$  and  $H_b$  must be different;
9. Using the point  $H_b$  and the point  $A$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $A$  are not the same;
10. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; line  $h_b$  and circle  $k(M_c, A)$  intersect; points  $B$  and  $M_c$  are not the same.

Determination conditions: lines  $a$  and  $b$  are not the same; points  $H_b$  and  $A$  are not the same; points  $B$  and  $H_b$  must be different; points  $B$  and  $H$  are not the same; points  $H_a$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D20,D3,D5,D6,D8,D9,GD01,GD02,GL04,L40,L41]

Solving time: 9.5 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{a} 80 40
```

```
point H 80 72.73
```

```
point A 80 95
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r H_{a}
```

```
cmark_rt H
```

```
cmark_t A
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{a} and H are not the same
```

```
% Constructing a line h_{a} which passes through point H_{a} and point H
```

```
line h_{a} H_{a} H
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```
% Choosing randomly a point A on the line H_{a}H
```

```
online A H_{a} H
```

```
cmark_t A
```

```
color 200 200 200
```

```
drawline H_{a} H
```

```
color 0 0 0
```

```
% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
```

```
perp a H_{a} h_{a}
```

```
color 200 200 200
```

```
drawline a
```

```
color 0 0 0
```



```

% Generating random value V[_G15239]
random V[_G15239]

% Calculating value V[_G15260] using formula V[_G15239]*20
expression V[_G15260] { V[_G15239]*20 }

% Constructing a point B which is a point for which holds  $H_{\{a\}B} = V[_G15260]$  and angle  $AH_{\{a\}B} = 90$ 
turtle B A H_{a} 90 V[_G15260]
cmark_b B

% Constructing a point  $M_{\{c\}}$  such that  $BM_{\{c\}}/BA=0.5$ 
towards M_{c} B A 0.5
cmark_lt M_{c}
color 200 200 200
drawsegment B A
color 0 0 0

% DET: points B and H are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point B and point H
line h_{b} B H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points B and  $M_{\{c\}}$  are not the same
% Constructing a circle  $k(M_{\{c\}},A)$  whose center is at point  $M_{\{c\}}$  and which passes through point B
circle k(M_{c},A) M_{c} B

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(M_{\{c\}},A)$  intersect% DET: points B and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G15732\}}$  which is a foot of the point  $M_{\{c\}}$  on the line  $h_{\{b\}}$ 
foot P_{\_G15732} M_{c} h_{b}
cmark_r P_{\_G15732}
color 200 200 200
drawline M_{c} P_{\_G15732}
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point B in the symmetry to point/line  $P_{\{\backslash\_G15732\}}$ 
sim H_{b} P_{\_G15732} B

```

```

cmark_l H_{b}

% DET: points H_{b} and A are not the same
% Constructing a line b which passes through point H_{b} and point A
line b H_{b} A

color 200 200 200
drawline b
color 0 0 0

% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and b are not parallel; line h_{b} and circle k(M_{c},A)
% intersect; points B and M_{c} are not the same
% Determination conditions: lines a and b are not the same; points H_{b} and A are not the same;
% points B and H_{b} must be different; points B and H are not the same; points H_{a} and H are
% not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_a = \_H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.159 seconds.

**NDG conditions** Points  $B$  and  $H$  are not identical

Points  $B$  and  $H$  are not identical

Line through points  $H_b$  and  $H$  is not perpendicular to line through points  $H$  and  $A$

Points  $B$ ,  $C$  and  $H$  are not collinear

#### 4.1.2 Proving $H = \_H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 31 terms.

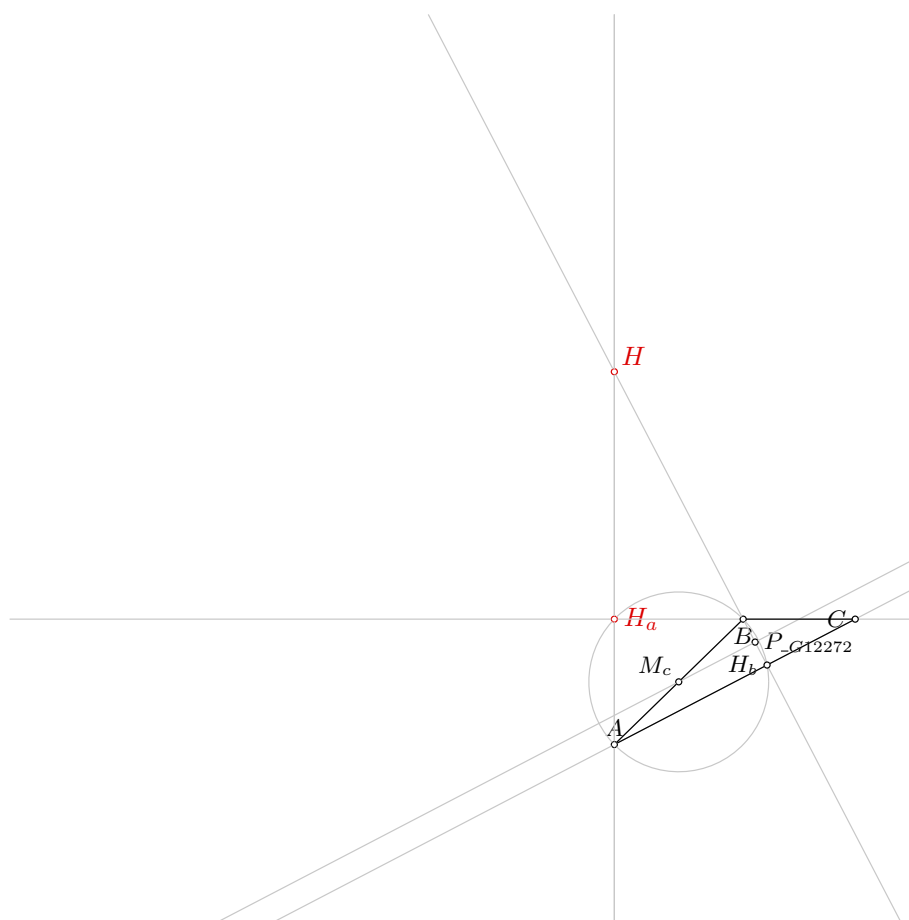


Figure 1: Illustration of the problem 0080

**Time Complexity:** Time spent by the prover is 0.558 seconds.

**NDG conditions** Points  $B$  and  $H$  are not identical

Points  $B$  and  $H$  are not identical

Line through points  $H_b$  and  $H$  is not perpendicular to line through points  $H$  and  $A$

Points  $B$ ,  $C$  and  $H$  are not collinear

Points  $A$  and  $B$  are not identical

### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.008 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $H_a = \neg H_a$

Proving failed

### 4.2.2 Proving $H = \neg H$

Proving failed

### 4.2.3 Proving $A=A$

NDG conditions are:

$S_{H_a H_b A} \neq S_{T_a^1 H_b A}$  i.e., lines  $H_a T_a^1$  and  $H_b A$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{\neg H_a}^2} \neq S_{CAF_{\neg H_a}^2}$  i.e., lines  $BC$  and  $AF_{\neg H_a}^2$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg H_b}^3} \neq S_{F_{\neg H_a}^2 BF_{\neg H_b}^3}$  i.e., lines  $AF_{\neg H_a}^2$  and  $BF_{\neg H_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $H_a = \neg H_a$

Proving failed

### 4.3.2 Proving $H = \neg H$

Proving failed

### 4.3.3 Proving $A=A$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $H_a = \neg H_a$

Proving failed

#### 4.4.2 Proving $H=_H$

Proving failed

#### 4.4.3 Proving $A=A$

Proving failed

# Problem 81

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 81: Given a point  $H_a$ , a point  $T_a$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $T_a$  are not the same;
2. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
3. Choose freely a point  $A$  on the line  $h_a$  (rule WOnline2);
4. Using the point  $A$  and the point  $T_a$ , construct a line  $s_a$  (rule W02); % DET: points  $A$  and  $T_a$  are not the same;
5. Using the point  $H_a$ , the point  $A$ , the point  $T_a$ , the line  $s_a$  and the line  $h_a$ , construct a line  $AO$  (rule W17); % NDG: points  $A$  and  $T_a$  are not the same; points  $H_a$  and  $A$  are not the same % DET: points  $A$  and  $T_a$  are not the same;
6. Choose freely a point  $B$  on the line  $a$  (rule WOnline1) ;
7. Using the point  $B$  and the point  $A$ , construct a point  $M_c$  (rule W01); ;
8. Using the point  $B$  and the point  $A$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $A$  are not the same;
9. Using the point  $M_c$  and the line  $c$ , construct a line  $m_c$  (rule W10b); ;
10. Using the line  $m_c$  and the line  $AO$ , construct a point  $O$  (rule W03); % NDG: lines  $m_c$  and  $AO$  are not parallel % DET: lines  $m_c$  and  $AO$  are not the same;
11. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;

12. Using the circle  $k(O, C)$ , the line  $a$ , the point  $O$  and the point  $B$ , construct a point  $C$  (rule W05); % NDG: line  $a$  and circle  $k(O, C)$  intersect % DET: points  $B$  and  $C$  must be different.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same; lines  $m_c$  and  $AO$  are not parallel; points  $A$  and  $T_a$  are not the same; points  $H_a$  and  $A$  are not the same.

Determination conditions: points  $B$  and  $C$  must be different; lines  $m_c$  and  $AO$  are not the same; points  $B$  and  $A$  are not the same; points  $A$  and  $T_a$  are not the same; points  $A$  and  $T_a$  are not the same; points  $H_a$  and  $T_a$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W10b,W17,WOnline1,WOnline2]

Lemmas used: [D13,D20,D23,D26,D5,D8,GD01,GL04,GL17,L1,L104,L12,L5]

Solving time: 20.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point H_{a} 80 40
point T_{a} 70.86 40
point A 80 95

color 220 0 0
fontsize 9

cmark_r H_{a}
cmark_rb T_{a}
cmark_t A
color 0 0 0
fontsize 8

% DET: points H_{a} and T_{a} are not the same
% Constructing a line a which passes through point H_{a} and point T_{a}
line a H_{a} T_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% Generating random value V[_G12417]
random V[_G12417]
```

```

% Calculating value  $V_{G12438}$  using formula  $V_{G12417} \cdot 20$ 
expression  $V_{G12438}$  {  $V_{G12417} \cdot 20$  }

% Constructing a point A which is a point for which holds  $H_{\{a\}}A = V_{G12438}$  and angle  $T_{\{a\}}H_{\{a\}}A = 90$ 
turtle A  $T_{\{a\}}$   $H_{\{a\}}$  90  $V_{G12438}$ 
cmark_t A

% DET: points A and  $T_{\{a\}}$  are not the same
% Constructing a line  $s_{\{a\}}$  which passes through point A and point  $T_{\{a\}}$ 
line  $s_{\{a\}}$  A  $T_{\{a\}}$ 

color 200 200 200
drawline  $s_{\{a\}}$ 
color 0 0 0

% NDG: points A and  $T_{\{a\}}$  are not the same; points  $H_{\{a\}}$  and A are not the same% DET: points A and  $T_{\{a\}}$  are not the same
% Constructing an angle  $V_{G12669}$  which is equal to the angle  $H_{\{a\}}AT_{\{a\}}$ 
angle_o  $V_{G12669}$   $H_{\{a\}}$  A  $T_{\{a\}}$ 

% Calculating value  $angle_{G12748}$  using formula  $1/\text{pow}(2,0) \cdot V_{G12669} + 0/\text{pow}(2,0) \cdot 180$ 
expression  $angle_{G12748}$  {  $1/\text{pow}(2,0) \cdot V_{G12669} + 0/\text{pow}(2,0) \cdot 180$  }

% Constructing a point  $P_{\{G12745\}}$  which is an image of the point  $T_{\{a\}}$  in a rotation around the point A for the angle  $1/\text{pow}(2,0) \cdot V_{G12669} + 0/\text{pow}(2,0) \cdot 180$ 
rotate  $P_{\{G12745\}}$  A  $angle_{G12748}$   $T_{\{a\}}$ 
cmark_r  $P_{\{G12745\}}$ 
color 200 200 200
drawarc_p A  $T_{\{a\}}$   $angle_{G12748}$ 
color 0 0 0

% Constructing a line AD which passes through point A and point  $P_{\{G12745\}}$ 
line AD A  $P_{\{G12745\}}$ 

color 200 200 200
drawline AD
color 0 0 0

% Choosing randomly a point B on the line  $H_{\{a\}}T_{\{a\}}$ 
online B  $H_{\{a\}}$   $T_{\{a\}}$ 
cmark_b B
color 200 200 200
drawline  $H_{\{a\}}$   $T_{\{a\}}$ 
color 0 0 0

```



```

% Constructing a point  $M_{\{c\}}$  such that  $BM_{\{c\}}/BA=0.5$ 
towards  $M_{\{c\}}$  B A 0.5
cmark_lt  $M_{\{c\}}$ 
color 200 200 200
drawsegment B A
color 0 0 0

% DET: points B and A are not the same
% Constructing a line c which passes through point B and point A
line c B A

color 200 200 200
drawline c
color 0 0 0

% Constructing a line  $m_{\{c\}}$  which is perpendicular to line c and which passes through point  $M_{\{c\}}$ 
perp  $m_{\{c\}}$   $M_{\{c\}}$  c

color 200 200 200
drawline  $m_{\{c\}}$ 
color 0 0 0

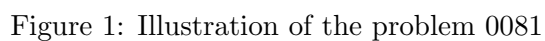
% NDG: lines  $m_{\{c\}}$  and AO are not parallel% DET: lines  $m_{\{c\}}$  and AO are not the same
% Constructing a point O which belongs to line  $m_{\{c\}}$  and line AO
intersec O  $m_{\{c\}}$  AO
cmark_t O

% NDG: points B and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle  $k(O,C)$  intersect% DET: points B and C must be different
% Constructing a point  $P_{\{\backslash\_G13449\}}$  which is a foot of the point O on the line a
foot  $P_{\{\backslash\_G13449\}}$  O a
cmark_r  $P_{\{\backslash\_G13449\}}$ 
color 200 200 200
drawline O  $P_{\{\backslash\_G13449\}}$ 
color 0 0 0

```



```
% Non-degenerate conditions: line a and circle k(O,C) intersect; points B and O are not the same;
  lines m_{c} and AO are not parallel; points A and T_{a} are not the same; points H_{a} and A
  are not the same
% Determination conditions: points B and C must be different; lines m_{c} and AO are not the same;
  points B and A are not the same; points A and T_{a} are not the same; points A and T_{a} are
  not the same; points H_{a} and T_{a} are not the same
```

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_a = \neg H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 5 terms.

**Time Complexity:** Time spent by the prover is 0.088 seconds.

**NDG conditions** Points  $T_a$  and  $A$  are not identical

Line through points  $B$  and  $A$  is not perpendicular to line through points  $A$  and  $P_{G13828}$

Points  $A$  and  $B$  are not identical

Line through points  $A$  and  $H_a$  is not parallel with line through points  $B$  and  $C$

#### 4.1.2 Proving $T_a = \neg T_a$

Proving failed

#### 4.1.3 Proving $A = A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.013 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_a = \neg H_a$

Proving failed

#### 4.2.2 Proving $T_a = \neg T_a$

Proving failed

#### 4.2.3 Proving $A = A$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_a = \neg H_a$

Proving failed

#### 4.3.2 Proving $T_a = \neg T_a$

Proving failed

#### 4.3.3 Proving $A = A$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $H_a = H_a$

Proving failed

### 4.4.2 Proving $T_a = T_a$

Proving failed

### 4.4.3 Proving $A = A$

Proving failed

# Problem 82

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 82: Given a point  $A$ , a point  $H_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H_a$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H_a$  are not the same;
2. Using the point  $A$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $T_b$  are not the same;
3. Using the point  $H_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
4. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same;
5. Using the point  $A$ , the point  $C$ , the point  $T_b$  and the line  $b$ , construct a point  $T'_b$  (rule W19); % NDG: points  $A$  and  $C$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $\text{midpoint}([A, T_b])$  are not the same;
6. Using the point  $T_b$  and the point  $T'_b$ , construct a circle  $k_{over}(T_b, T'_b)$  (rule W09); % NDG: points  $T_b$  and  $T'_b$  are not the same;
7. Using the circle  $k_{over}(T_b, T'_b)$  and the line  $a$ , construct a point  $B_{wc}$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k_{over}(T_b, T'_b)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k_{over}(T_b, T'_b)$  intersect; points  $T_b$  and  $T'_b$  are not the same; points  $A$  and  $C$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $\text{midpoint}([A, T_b])$  are not the same; lines  $a$  and  $b$  are not parallel.

Determination conditions: lines  $a$  and  $b$  are not the same; points  $A$  and  $T_b$  are not the same; points  $A$  and  $H_a$  are not the same.

Rules used: [W02, W03, W04, W09, W10a, W19]

Lemmas used: [D24, D5, D56, D8, GD01, L73, L78]

Solving time: 2572.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point H_{a} 80 40
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_t A
cmark_r H_{a}
cmark_t T_{b}
color 0 0 0
fontsize 8

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% DET: points A and T_{b} are not the same
% Constructing a line b which passes through point A and point T_{b}
line b A T_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
perp a H_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_l C

% NDG: points A and C are not the same; points C and T_{b} are not the same; points C and midpoint
% ([A,T_{b}]) are not the same
```

```

% Constructing a point  $P_{\backslash\_G11512}$  which is an image of the point  $C$  in a rotation around the point
 $T_{\backslash b}$  for the angle 90
rotate P_{\_G11512} T_{\backslash b} 90 C
cmark_r P_{\backslash\_G11512}
color 200 200 200
drawarc_p T_{\backslash b} C 90
color 0 0 0

% Constructing a line  $L_{\backslash\_G11515}$  which passes through point  $T_{\backslash b}$  and point  $P_{\backslash\_G11512}$ 
line L_{\backslash\_G11515} T_{\backslash b} P_{\backslash\_G11512}

color 200 200 200
drawline L_{\backslash\_G11515}
color 0 0 0

% Constructing midpoint  $P_{\backslash\_G11518}$  of the segment  $CP_{\backslash\_G11512}$ 
midpoint P_{\backslash\_G11518} C P_{\backslash\_G11512}
cmark_r P_{\backslash\_G11518}

% Constructing a line  $L_{\backslash\_G11521}$  which passes through point  $A$  and point  $P_{\backslash\_G11518}$ 
line L_{\backslash\_G11521} A P_{\backslash\_G11518}

color 200 200 200
drawline L_{\backslash\_G11521}
color 0 0 0

% Constructing a line  $L_{\backslash\_G11524}$  which passes through point  $C$  and point  $P_{\backslash\_G11518}$ 
line L_{\backslash\_G11524} C P_{\backslash\_G11518}

color 200 200 200
drawline L_{\backslash\_G11524}
color 0 0 0

% Constructing a point  $P_{\backslash\_G11527}$  which belongs to line  $L_{\backslash\_G11515}$  and line  $L_{\backslash\_G11521}$ 
intersec P_{\backslash\_G11527} L_{\backslash\_G11515} L_{\backslash\_G11521}
cmark_r P_{\backslash\_G11527}

% Constructing a point  $P_{\backslash\_G11530}$  which belongs to line  $L_{\backslash\_G11515}$  and line  $L_{\backslash\_G11524}$ 
intersec P_{\backslash\_G11530} L_{\backslash\_G11515} L_{\backslash\_G11524}
cmark_r P_{\backslash\_G11530}

% Constructing a line  $L_{\backslash\_G11533}$  which passes through point  $A$  and point  $P_{\backslash\_G11530}$ 
line L_{\backslash\_G11533} A P_{\backslash\_G11530}

color 200 200 200
drawline L_{\backslash\_G11533}
color 0 0 0

% Constructing a line  $L_{\backslash\_G11536}$  which passes through point  $C$  and point  $P_{\backslash\_G11527}$ 
line L_{\backslash\_G11536} C P_{\backslash\_G11527}

color 200 200 200

```

```

drawline L_{\_G11536}
color 0 0 0

% Constructing a point P_{\_G11539} which belongs to line L_{\_G11533} and line L_{\_G11536}
intersec P_{\_G11539} L_{\_G11533} L_{\_G11536}
cmark_r P_{\_G11539}

% Constructing a line L_{\_G11542} which passes through point P_{\_G11518} and point P_{\_G11539}
line L_{\_G11542} P_{\_G11518} P_{\_G11539}

color 200 200 200
drawline L_{\_G11542}
color 0 0 0

% Constructing a point T'_{b} which belongs to line L_{\_G11542} and line b
intersec T'_{b} L_{\_G11542} b
cmark_r T'_{b}

% NDG: points T_{b} and T'_{b} are not the same
% Constructing midpoint P_{\_G12373} of the segment T_{b}T'_{b}
midpoint P_{\_G12373} T_{b} T'_{b}
cmark_r P_{\_G12373}

% Constructing a circle k_{over}(T_{b},T'_{b}) whose center is at point P_{\_G12373} and which passes
    through point T_{b}
circle k_{over}(T_{b},T'_{b}) P_{\_G12373} T_{b}

color 200 200 200
drawcircle k_{over}(T_{b},T'_{b})
color 0 0 0

% NDG: line a and circle k_{over}(T_{b},T'_{b}) intersect
% Constructing points B_{wc} and B which are in intersection of k_{over}(T_{b},T'_{b}) and a
intersec2 B_{wc} B k_{over}(T_{b},T'_{b}) a
cmark_r B_{wc}
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k_{over}(T_{b},T'_{b}) intersect; points T_{b} and T'_{b}
    are not the same; points A and C are not the same; points C and T_{b} are not the same;
    points C and midpoint([A,T_{b}]) are not the same; lines a and b are not parallel
% Determination conditions: lines a and b are not the same; points A and T_{b} are not the same;
    points A and H_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1



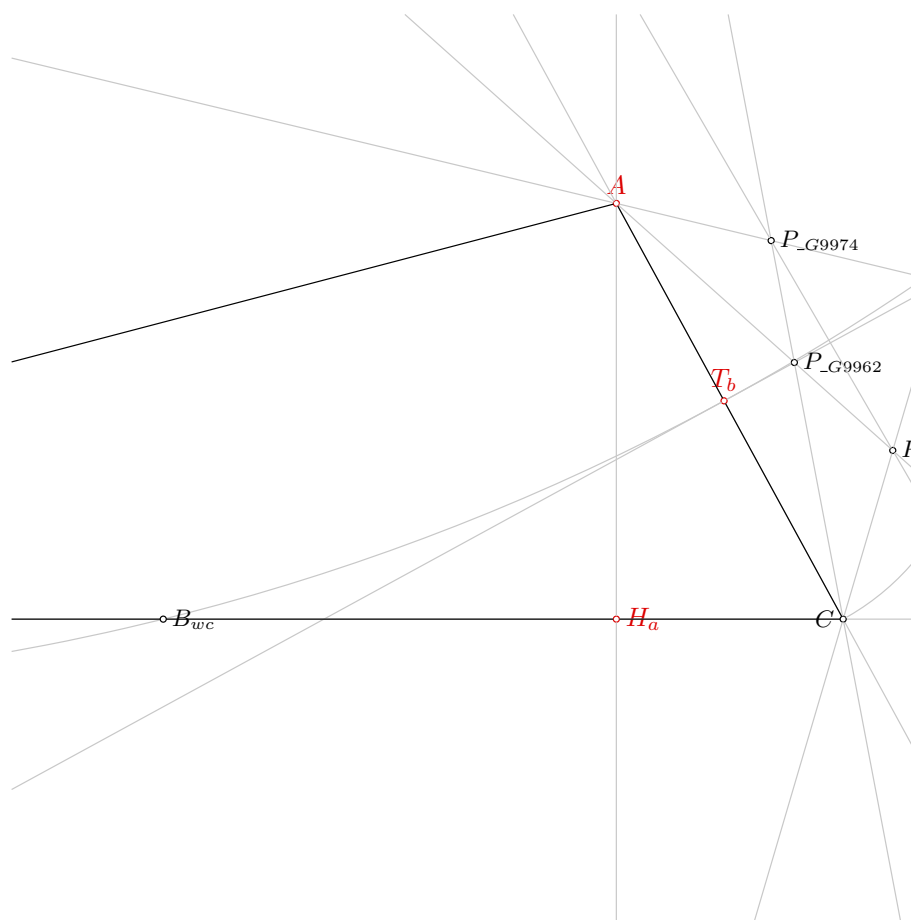


Figure 1: Illustration of the problem 0082

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a = \neg H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.138 seconds.

**NDG conditions** Points  $A$ ,  $T_b$  and  $H_a$  are not collinear

Point  $T_b$  is not the midpoint of segment with endpoints  $A$  and  $C$

Points  $A$  and  $B$  are not identical

Line through points  $A$  and  $H_a$  is not parallel with line through points  $B$  and  $C$

#### 4.1.3 Proving $T_b = \neg T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_a A T_b} \neq S_{T_a^0 A T_b}$  i.e., lines  $H_a T_a^0$  and  $A T_b$  are not parallel (construction based assumption)

$S_{T_b A P_{G11655}} \neq S_{P_{G11649} A P_{G11655}}$  i.e., lines  $T_b P_{G11649}$  and  $A P_{G11655}$  are not parallel (construction based assumption)

$S_{T_b C P_{G11655}} \neq S_{P_{G11649} C P_{G11655}}$  i.e., lines  $T_b P_{G11649}$  and  $C P_{G11655}$  are not parallel (construction based assumption)

$S_{A C P_{G11664}} \neq S_{P_{G11667} C P_{G11664}}$  i.e., lines  $A P_{G11667}$  and  $C P_{G11664}$  are not parallel (construction based assumption)

$S_{P_{G11655} A T_b} \neq S_{P_{G11676} A T_b}$  i.e., lines  $P_{G11655} P_{G11676}$  and  $A T_b$  are not parallel (construction based assumption)

$S_{A B C} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{B A F_{\neg H_a}^1} \neq S_{C A F_{\neg H_a}^1}$  i.e., lines  $B C$  and  $A F_{\neg H_a}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.2.3 Proving $T_b = \neg T_b$

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $A=A$**

Proving failed

#### **4.3.2 Proving $H_a=_H a$**

Proving failed

#### **4.3.3 Proving $T_b=_T b$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $A=A$**

Proving failed

#### **4.4.2 Proving $H_a=_H a$**

Proving failed

#### **4.4.3 Proving $T_b=_T b$**

Proving failed

# Problem 83

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 83: Given a point  $A$ , a point  $H_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H_a$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H_a$  are not the same;
2. Using the point  $A$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $T_c$  are not the same;
3. Using the point  $H_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
4. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same;
5. Using the point  $A$ , the point  $B$ , the point  $T_c$  and the line  $c$ , construct a point  $T'_c$  (rule W19); % NDG: points  $A$  and  $B$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $\text{midpoint}([A, T_c])$  are not the same;
6. Using the point  $T_c$  and the point  $T'_c$ , construct a circle  $k_{over}(T_c, T'_c)$  (rule W09); % NDG: points  $T_c$  and  $T'_c$  are not the same;
7. Using the circle  $k_{over}(T_c, T'_c)$  and the line  $a$ , construct a point  $C_{wb}$  and a point  $C$  (rule W04); % NDG: line  $a$  and circle  $k_{over}(T_c, T'_c)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k_{over}(T_c, T'_c)$  intersect; points  $T_c$  and  $T'_c$  are not the same; points  $A$  and  $B$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $\text{midpoint}([A, T_c])$  are not the same; lines  $a$  and  $c$  are not parallel.

Determination conditions: lines  $a$  and  $c$  are not the same; points  $A$  and  $T_c$  are not the same; points  $A$  and  $H_a$  are not the same.

Rules used: [W02, W03, W04, W09, W10a, W19]

Lemmas used: [D25, D5, D58, D8, GD01, L72, L79]

Solving time: 2600.8 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point H_{a} 80 40
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_t A
cmark_r H_{a}
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% DET: points A and T_{c} are not the same
% Constructing a line c which passes through point A and point T_{c}
line c A T_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
perp a H_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B

% NDG: points A and B are not the same; points B and T_{c} are not the same; points B and midpoint
% ([A,T_{c}]) are not the same
```

```

% Constructing a point  $P_{\{ \_G10819 \}}$  which is an image of the point  $B$  in a rotation around the point
 $T_{\{c\}}$  for the angle 90
rotate  $P_{\{ \_G10819 \}}$   $T_{\{c\}}$  90 B
cmark_r  $P_{\{ \_G10819 \}}$ 
color 200 200 200
drawarc_p  $T_{\{c\}}$  B 90
color 0 0 0

% Constructing a line  $L_{\{ \_G10822 \}}$  which passes through point  $T_{\{c\}}$  and point  $P_{\{ \_G10819 \}}$ 
line  $L_{\{ \_G10822 \}}$   $T_{\{c\}}$   $P_{\{ \_G10819 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G10822 \}}$ 
color 0 0 0

% Constructing midpoint  $P_{\{ \_G10825 \}}$  of the segment  $BP_{\{ \_G10819 \}}$ 
midpoint  $P_{\{ \_G10825 \}}$  B  $P_{\{ \_G10819 \}}$ 
cmark_r  $P_{\{ \_G10825 \}}$ 

% Constructing a line  $L_{\{ \_G10828 \}}$  which passes through point  $A$  and point  $P_{\{ \_G10825 \}}$ 
line  $L_{\{ \_G10828 \}}$  A  $P_{\{ \_G10825 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G10828 \}}$ 
color 0 0 0

% Constructing a line  $L_{\{ \_G10831 \}}$  which passes through point  $B$  and point  $P_{\{ \_G10825 \}}$ 
line  $L_{\{ \_G10831 \}}$  B  $P_{\{ \_G10825 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G10831 \}}$ 
color 0 0 0

% Constructing a point  $P_{\{ \_G10834 \}}$  which belongs to line  $L_{\{ \_G10822 \}}$  and line  $L_{\{ \_G10828 \}}$ 
intersec  $P_{\{ \_G10834 \}}$   $L_{\{ \_G10822 \}}$   $L_{\{ \_G10828 \}}$ 
cmark_r  $P_{\{ \_G10834 \}}$ 

% Constructing a point  $P_{\{ \_G10837 \}}$  which belongs to line  $L_{\{ \_G10822 \}}$  and line  $L_{\{ \_G10831 \}}$ 
intersec  $P_{\{ \_G10837 \}}$   $L_{\{ \_G10822 \}}$   $L_{\{ \_G10831 \}}$ 
cmark_r  $P_{\{ \_G10837 \}}$ 

% Constructing a line  $L_{\{ \_G10840 \}}$  which passes through point  $A$  and point  $P_{\{ \_G10837 \}}$ 
line  $L_{\{ \_G10840 \}}$  A  $P_{\{ \_G10837 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G10840 \}}$ 
color 0 0 0

% Constructing a line  $L_{\{ \_G10843 \}}$  which passes through point  $B$  and point  $P_{\{ \_G10834 \}}$ 
line  $L_{\{ \_G10843 \}}$  B  $P_{\{ \_G10834 \}}$ 

color 200 200 200

```

```

drawline L_{\_G10843}
color 0 0 0

% Constructing a point P_{\_G10846} which belongs to line L_{\_G10840} and line L_{\_G10843}
intersec P_{\_G10846} L_{\_G10840} L_{\_G10843}
cmark_r P_{\_G10846}

% Constructing a line L_{\_G10849} which passes through point P_{\_G10825} and point P_{\_G10846}
line L_{\_G10849} P_{\_G10825} P_{\_G10846}

color 200 200 200
drawline L_{\_G10849}
color 0 0 0

% Constructing a point T'_{c} which belongs to line L_{\_G10849} and line c
intersec T'_{c} L_{\_G10849} c
cmark_r T'_{c}

% NDG: points T_{c} and T'_{c} are not the same
% Constructing midpoint P_{\_G11680} of the segment T_{c}T'_{c}
midpoint P_{\_G11680} T_{c} T'_{c}
cmark_r P_{\_G11680}

% Constructing a circle k_{over}(T_{c},T'_{c}) whose center is at point P_{\_G11680} and which passes
    through point T_{c}
circle k_{over}(T_{c},T'_{c}) P_{\_G11680} T_{c}

color 200 200 200
drawcircle k_{over}(T_{c},T'_{c})
color 0 0 0

% NDG: line a and circle k_{over}(T_{c},T'_{c}) intersect
% Constructing points C_{wb} and C which are in intersection of k_{over}(T_{c},T'_{c}) and a
intersec2 C_{wb} C k_{over}(T_{c},T'_{c}) a
cmark_r C_{wb}
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k_{over}(T_{c},T'_{c}) intersect; points T_{c} and T'_{c}
    are not the same; points A and B are not the same; points B and T_{c} are not the same;
    points B and midpoint([A,T_{c}]) are not the same; lines a and c are not parallel
% Determination conditions: lines a and c are not the same; points A and T_{c} are not the same;
    points A and H_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

Figure 1: Illustration of the problem 0083

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a=H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.138 seconds.

**NDG conditions** Points  $A$  and  $T_c$  are not identical

Point  $T_c$  is not the midpoint of segment with endpoints  $A$  and  $B$

Points  $B$  and  $C$  are not identical

Line through points  $A$  and  $H_a$  is not parallel with line through points  $B$  and  $C$

#### 4.1.3 Proving $T_c=T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_aAT_c} \neq S_{T_a^0AT_c}$  i.e., lines  $H_aT_a^0$  and  $AT_c$  are not parallel (construction based assumption)

$S_{T_cAP_{G11258}} \neq S_{P_{G11252}AP_{G11258}}$  i.e., lines  $T_cP_{G11252}$  and  $AP_{G11258}$  are not parallel (construction based assumption)

$S_{T_cBP_{G11258}} \neq S_{P_{G11252}BP_{G11258}}$  i.e., lines  $T_cP_{G11252}$  and  $BP_{G11258}$  are not parallel (construction based assumption)

$S_{ABP_{G11267}} \neq S_{P_{G11270}BP_{G11267}}$  i.e., lines  $AP_{G11270}$  and  $BP_{G11267}$  are not parallel (construction based assumption)

$S_{P_{G11258}AT_c} \neq S_{P_{G11279}AT_c}$  i.e., lines  $P_{G11258}P_{G11279}$  and  $AT_c$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{H_a}^1} \neq S_{CAF_{H_a}^1}$  i.e., lines  $BC$  and  $AF_{H_a}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a=H_a$

Proving failed



#### **4.2.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $A = A$**

Proving failed

#### **4.3.2 Proving $H_a = \neg H_a$**

Proving failed

#### **4.3.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $A = A$**

Proving failed

#### **4.4.2 Proving $H_a = \neg H_a$**

Proving failed

#### **4.4.3 Proving $T_c = \neg T_c$**

Proving failed

# Problem 84

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 84: Given a point  $A$ , a point  $H_a$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H_a$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H_a$  are not the same;
2. Using the point  $H_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
3. Using the point  $I$  and the line  $a$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $a$ ;
4. Using the circle  $k(I, P_a)$ , the point  $A$  and the point  $I$ , construct a line  $c$  and a line  $b$  (rule W12); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
5. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same;
6. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; lines  $c$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $a$ .

Determination conditions: lines  $a$  and  $b$  are not the same; lines  $c$  and  $a$  are not the same; points  $A$  and  $H_a$  are not the same.

Rules used: [W02,W03,W10a,W11,W12]

Lemmas used: [D27,D5,D8,GD01,L59,L60,L61]

Solving time: 1.0 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point H_{a} 80 40
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_t A
cmark_r H_{a}
cmark_b I
color 0 0 0
fontsize 8

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
perp a H_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: point I is not incident to the line a
% Constructing a point P_{\_G15068} which is a foot of the point I on the line a
foot P_{\_G15068} I a
cmark_r P_{\_G15068}
color 200 200 200
drawline I P_{\_G15068}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
15068}
circle k(I,P_{a}) I P_{\_G15068}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point A is outside the circle k(I,P_{a})
```

```

% Constructing midpoint  $P_{\{ \_G15302 \}}$  of the segment  $AI$ 
midpoint  $P_{\{ \_G15302 \}}$  A I
cmark_r  $P_{\{ \_G15302 \}}$ 

% Constructing a circle  $C_{\{ \_G15305 \}}$  whose center is at point  $P_{\{ \_G15302 \}}$  and which passes through
point A
circle  $C_{\{ \_G15305 \}}$   $P_{\{ \_G15302 \}}$  A

color 200 200 200
drawcircle  $C_{\{ \_G15305 \}}$ 
color 0 0 0

% Constructing points  $P_{\{ \_G15308 \}}$  and  $P_{\{ \_G15311 \}}$  which are in intersection of  $C_{\{ \_G15305 \}}$  and  $k$ 
( $I, P_{\{ a \}}$ )
intersec2  $P_{\{ \_G15308 \}}$   $P_{\{ \_G15311 \}}$   $C_{\{ \_G15305 \}}$   $k(I, P_{\{ a \}})$ 
cmark_r  $P_{\{ \_G15308 \}}$ 
cmark_r  $P_{\{ \_G15311 \}}$ 

% Constructing a line  $c$  which passes through point A and point  $P_{\{ \_G15308 \}}$ 
line c A  $P_{\{ \_G15308 \}}$ 

color 200 200 200
drawline c
color 0 0 0

% Constructing a line  $b$  which passes through point A and point  $P_{\{ \_G15311 \}}$ 
line b A  $P_{\{ \_G15311 \}}$ 

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $c$  and  $a$  are not parallel% DET: lines  $c$  and  $a$  are not the same
% Constructing a point  $B$  which belongs to line  $c$  and line  $a$ 
intersec B c a
cmark_b B

% NDG: lines  $a$  and  $b$  are not parallel% DET: lines  $a$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $a$  and line  $b$ 
intersec C a b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; lines  $c$  and  $a$  are not parallel; point
A is outside the circle  $k(I, P_{\{ a \}})$ ; point  $I$  is not incident to the line  $a$ 

```

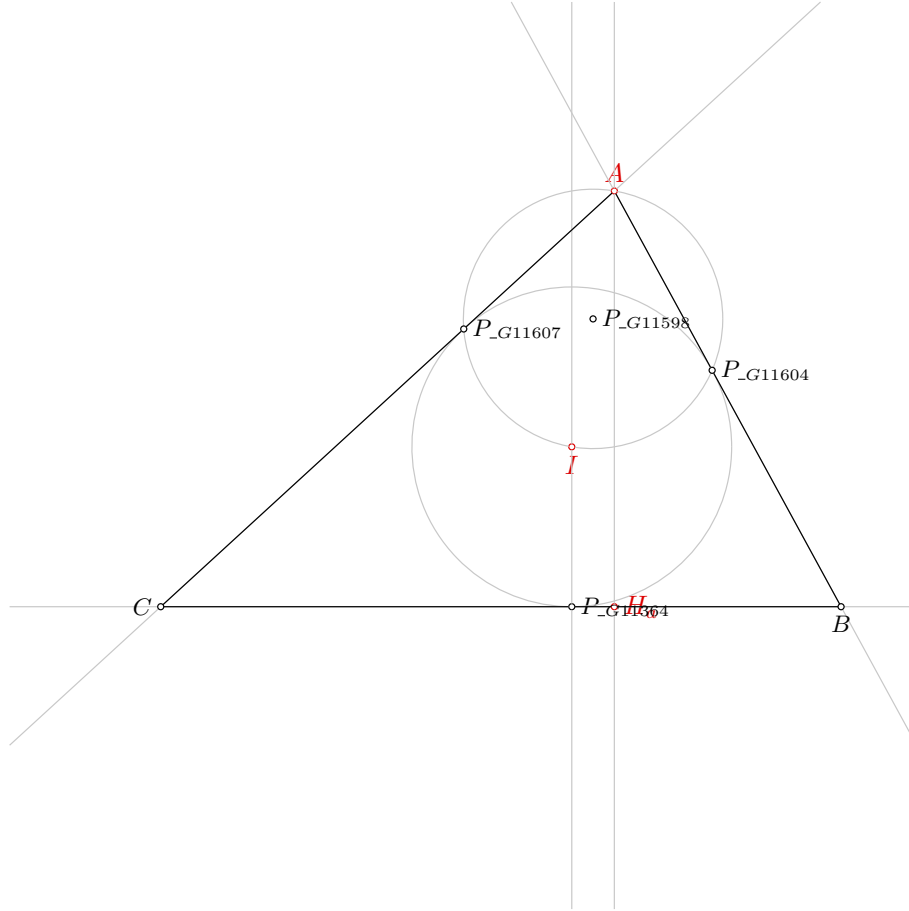


Figure 1: Illustration of the problem 0084

*% Determination conditions: lines a and b are not the same; lines c and a are not the same; points A and H\_{a} are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a = \neg H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.081 seconds.

**NDG conditions** Line through points  $A$  and  $H_a$  is not parallel with line through points  $B$  and  $C$

#### 4.1.3 Proving $I = \neg I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A = A$

NDG conditions are:

$S_{AH_aT_a^0} \neq S_{P_{G12460}H_aT_a^0}$  i.e., lines  $AP_{G12460}$  and  $H_aT_a^0$  are not parallel (construction based assumption)

$S_{H_aAP_{G12463}} \neq S_{T_a^0AP_{G12463}}$  i.e., lines  $H_aT_a^0$  and  $AP_{G12463}$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^1} \neq S_{CAF_{h_a}^1}$  i.e., lines  $BC$  and  $AF_{h_a}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.2.3 Proving $I = \neg I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I = \neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 494 terms.

**Time Complexity:** Time spent by the prover is 0.350 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $H_a=_H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.3 Proving $I=_I$

Proving failed

# Problem 85

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 85: Given a point  $A$ , a point  $H_b$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $H_b$  are not the same;
2. Using the point  $A$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $H_c$  are not the same;
3. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
4. Using the line  $h_b$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $h_b$  and  $c$  are not parallel % DET: lines  $h_b$  and  $c$  are not the same;
5. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
6. Using the line  $h_c$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $h_c$  and  $b$  are not parallel % DET: lines  $h_c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $h_c$  and  $b$  are not parallel; lines  $h_b$  and  $c$  are not parallel.

Determination conditions: lines  $h_c$  and  $b$  are not the same; lines  $h_b$  and  $c$  are not the same; points  $A$  and  $H_c$  are not the same; points  $A$  and  $H_b$  are not the same.

Rules used: [W02,W03,W10b]

Lemmas used: [D10,D6,D7,D9,GD01]

Solving time: 0.9 seconds.



### 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point H_{b} 89.36 77.83
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_t A
cmark_l H_{b}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines h_{b} and c are not parallel% DET: lines h_{b} and c are not the same
% Constructing a point B which belongs to line h_{b} and line c
intersec B h_{b} c
cmark_b B

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c
```

```

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines h_{c} and b are not parallel% DET: lines h_{c} and b are not the same
% Constructing a point C which belongs to line h_{c} and line b
intersec C h_{c} b
cmark_1 C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and b are not parallel; lines h_{b} and c are not parallel
% Determination conditions: lines h_{c} and b are not the same; lines h_{b} and c are not the same;
  points A and H_{c} are not the same; points A and H_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.022 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b = H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.097 seconds.

**NDG conditions** Points  $H_b$  and  $H_c$  are not identical

Points  $A$  and  $C$  are not identical

#### 4.1.3 Proving $H_c = H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.156 seconds.

**NDG conditions** Points  $H_b$  and  $H_c$  are not identical

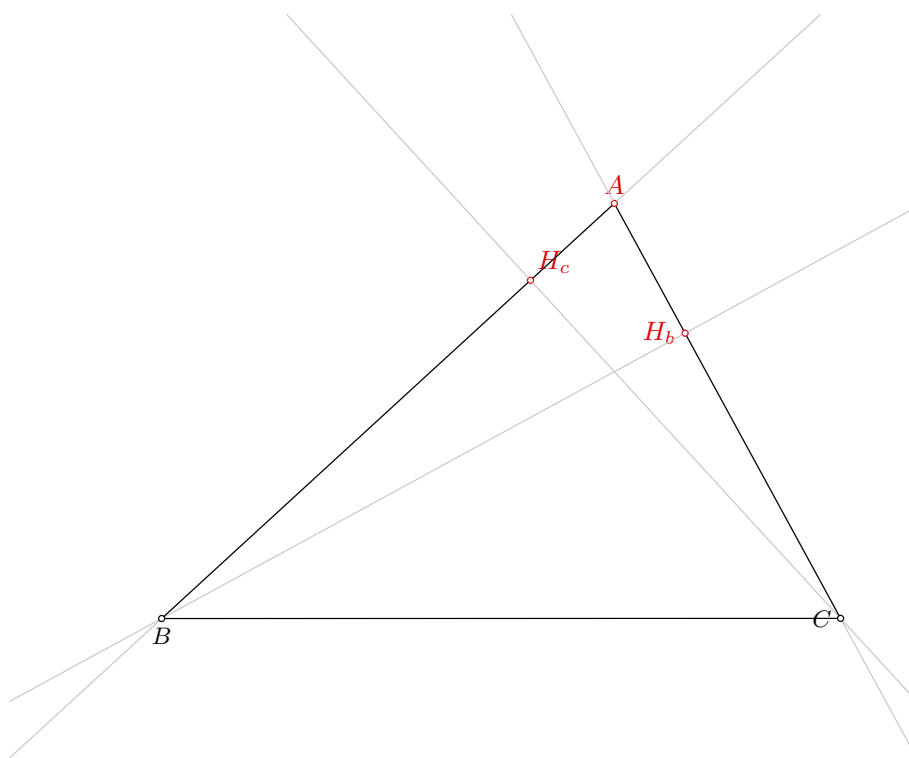


Figure 1: Illustration of the problem 0085

Points  $A$  and  $B$  are not identical

Points  $H_b$ ,  $A$  and  $B$  are not collinear

## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_bAH_c} \neq S_{T_{h_b}^0AH_c}$  i.e., lines  $H_bT_{h_b}^0$  and  $AH_c$  are not parallel (construction based assumption)

$S_{H_cAH_b} \neq S_{T_{h_c}^1AH_b}$  i.e., lines  $H_cT_{h_c}^1$  and  $AH_b$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^2} \neq S_{CBF_{h_b}^2}$  i.e., lines  $AC$  and  $BF_{h_b}^2$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^3} \neq S_{BCF_{h_c}^3}$  i.e., lines  $AB$  and  $CF_{h_c}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_b=_H H_b$

Proving failed

### 4.2.3 Proving $H_c=_H H_c$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_b=_H H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $H_c=_H H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 12 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_b = H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $H_c = H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 11 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

# Problem 86

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 86: Given a point  $H_b$ , a point  $H$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $H_b$  and  $H$  are not the same;
2. Using the point  $H_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
3. Choose freely a point  $A$  on the line  $b$  (rule WOnline2);
4. Using the point  $A$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H$  are not the same;
5. Choose freely a point  $B$  on the line  $h_b$  (rule WOnline1) ;
6. Using the point  $B$  and the point  $A$ , construct a point  $M_c$  (rule W01); ;
7. Using the point  $B$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $B$  and  $M_c$  are not the same;
8. Using the circle  $k(M_c, A)$ , the line  $h_a$ , the point  $M_c$  and the point  $A$ , construct a point  $H_a$  (rule W05); % NDG: line  $h_a$  and circle  $k(M_c, A)$  intersect % DET: points  $A$  and  $H_a$  must be different;
9. Using the point  $H_a$  and the point  $B$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $B$  are not the same;
10. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; line  $h_a$  and circle  $k(M_c, A)$  intersect; points  $B$  and  $M_c$  are not the same.

Determination conditions: lines  $b$  and  $a$  are not the same; points  $H_a$  and  $B$  are not the same; points  $A$  and  $H_a$  must be different; points  $A$  and  $H$  are not the same; points  $H_b$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D20,D3,D5,D6,D8,D9,GD01,GD02,GL04,L40,L42]

Solving time: 24.1 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{b} 89.36 77.83
```

```
point H 80 72.73
```

```
point A 80 95
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_l H_{b}
```

```
cmark_rt H
```

```
cmark_t A
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{b} and H are not the same
```

```
% Constructing a line h_{b} which passes through point H_{b} and point H
```

```
line h_{b} H_{b} H
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```
% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
```

```
perp b H_{b} h_{b}
```

```
color 200 200 200
```

```
drawline b
```

```
color 0 0 0
```

```
% Generating random value V[_G12351]
```

```
random V[_G12351]
```

```
% Calculating value V[_G12372] using formula V[_G12351]*20
```

```
expression V[_G12372] { V[_G12351]*20 }
```

```

% Constructing a point A which is a point for which holds  $H_{\{b\}}A = V[_{G12372}]$  and angle  $HH_{\{b\}}A = 90$ 
turtle A H  $H_{\{b\}}$  90  $V[_{G12372}]$ 
cmark_t A

% DET: points A and H are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point A and point H
line  $h_{\{a\}}$  A H

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% Choosing randomly a point B on the line  $H_{\{b\}}H$ 
online B  $H_{\{b\}}$  H
cmark_b B
color 200 200 200
drawline  $H_{\{b\}}$  H
color 0 0 0

% Constructing a point  $M_{\{c\}}$  such that  $BM_{\{c\}}/BA=0.5$ 
towards  $M_{\{c\}}$  B A 0.5
cmark_lt  $M_{\{c\}}$ 
color 200 200 200
drawsegment B A
color 0 0 0

% NDG: points B and  $M_{\{c\}}$  are not the same
% Constructing a circle  $k(M_{\{c\}},A)$  whose center is at point  $M_{\{c\}}$  and which passes through point B
circle  $k(M_{\{c\}},A)$   $M_{\{c\}}$  B

color 200 200 200
drawcircle  $k(M_{\{c\}},A)$ 
color 0 0 0

% NDG: line  $h_{\{a\}}$  and circle  $k(M_{\{c\}},A)$  intersect% DET: points A and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G12901\}}$  which is a foot of the point  $M_{\{c\}}$  on the line  $h_{\{a\}}$ 
foot  $P_{\{\backslash\_G12901\}}$   $M_{\{c\}}$   $h_{\{a\}}$ 
cmark_r  $P_{\{\backslash\_G12901\}}$ 
color 200 200 200
drawline  $M_{\{c\}}$   $P_{\{\backslash\_G12901\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point A in the symmetry to point/line  $P_{\{\backslash\_G12901\}}$ 
sim  $H_{\{a\}}$   $P_{\{\backslash\_G12901\}}$  A

```



```

cmark_r H_{a}

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and a are not parallel; line h_{a} and circle k(M_{c},A)
% intersect; points B and M_{c} are not the same
% Determination conditions: lines b and a are not the same; points H_{a} and B are not the same;
% points A and H_{a} must be different; points A and H are not the same; points H_{b} and H are
% not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_b = \_H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.092 seconds.

**NDG conditions** Points  $H_b$  and  $A$  are not identical

Points  $H_b$  and  $H$  are not identical

Line through points  $H_a$  and  $B$  is not perpendicular to line through points  $B$  and  $H_b$

Line through points  $H_b$  and  $H$  is not parallel with line through points  $A$  and  $C$

#### 4.1.2 Proving $H = \_H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 31 terms.



Figure 1: Illustration of the problem 0086

**Time Complexity:** Time spent by the prover is 0.475 seconds.

**NDG conditions** Points  $H_b$  and  $A$  are not identical

Points  $H_b$  and  $H$  are not identical

Line through points  $H_a$  and  $B$  is not perpendicular to line through points  $B$  and  $H_b$

Line through points  $H_b$  and  $H$  is not parallel with line through points  $A$  and  $C$

Points  $B$  and  $H$  are not identical

#### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.011 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_b=_H H_b$

Proving failed

#### 4.2.2 Proving $H=_H H$

Proving failed

#### 4.2.3 Proving $A=A$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_b=_H H_b$

Proving failed

#### 4.3.2 Proving $H=_H H$

Proving failed

#### 4.3.3 Proving $A=A$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $H_b=_H H_b$

Proving failed

#### 4.4.2 Proving $H=_H H$

Proving failed

#### 4.4.3 Proving $A=A$

Proving failed

# Problem 87

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 87: Given a point  $A$ , a point  $H_b$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $H_b$  are not the same;
2. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
3. Using the point  $T_a$  and the line  $b$ , construct a circle  $k(T_a, foot[T_a, c])$  (rule W11); % NDG: point  $T_a$  is not incident to the line  $b$ ;
4. Using the circle  $k(T_a, foot[T_a, c])$ , the point  $A$ , the point  $T_a$  and the line  $b$ , construct a line  $c$  (rule W13); % NDG: point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ;
5. Using the line  $c$  and the line  $h_b$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $h_b$  are not parallel % DET: lines  $c$  and  $h_b$  are not the same;
6. Using the point  $T_a$  and the point  $B$ , construct a line  $a$  (rule W02); % DET: points  $T_a$  and  $B$  are not the same;
7. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; lines  $c$  and  $h_b$  are not parallel; point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ; point  $T_a$  is not incident to the line  $b$ .

Determination conditions: lines  $b$  and  $a$  are not the same; points  $T_a$  and  $B$  are not the same; lines  $c$  and  $h_b$  are not the same; points  $A$  and  $H_b$  are not the same.

Rules used: [W02,W03,W10b,W11,W13]

Lemmas used: [D23,D6,D9,GD01,GD02,GL10,GL11,L8]

Solving time: 0.7 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point H_{b} 89.36 77.83
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_t A
cmark_l H_{b}
cmark_rb T_{a}
color 0 0 0
fontsize 8

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: point T_{a} is not incident to the line b
% Constructing a point P_{\_G9340} which is a foot of the point T_{a} on the line b
foot P_{\_G9340} T_{a} b
cmark_r P_{\_G9340}
color 200 200 200
drawline T_{a} P_{\_G9340}
color 0 0 0

% Constructing a circle k(T_{a},foot[T_{a},c]) whose center is at point T_{a} and which passes
    through point P_{\_G9340}
circle k(T_{a},foot[T_{a},c]) T_{a} P_{\_G9340}

color 200 200 200
drawcircle k(T_{a},foot[T_{a},c])
color 0 0 0

% NDG: point A is outside the circle k(T_{a},foot[T_{a},c])
```

```

% Constructing a point  $P_{\{a\}}$  which is a foot of the point  $T_{\{a\}}$  on the line  $b$ 
foot  $P_{\{a\}}$   $T_{\{a\}}$   $b$ 
cmark_r  $P_{\{a\}}$ 
color 200 200 200
drawline  $T_{\{a\}}$   $P_{\{a\}}$ 
color 0 0 0

% Constructing a line  $L_{\{a\}}$  which passes through point  $T_{\{a\}}$  and point  $A$ 
line  $L_{\{a\}}$   $T_{\{a\}}$   $A$ 

color 200 200 200
drawline  $L_{\{a\}}$ 
color 0 0 0

% Constructing a point  $P_{\{b\}}$  which is an image of the point  $P_{\{a\}}$  in the symmetry to
point/line  $L_{\{a\}}$ 
sim  $P_{\{b\}}$   $L_{\{a\}}$   $P_{\{a\}}$ 
cmark_r  $P_{\{b\}}$ 

% Constructing a line  $c$  which passes through point  $A$  and point  $P_{\{b\}}$ 
line  $c$   $A$   $P_{\{b\}}$ 

color 200 200 200
drawline  $c$ 
color 0 0 0

% NDG: lines  $c$  and  $h_{\{b\}}$  are not parallel% DET: lines  $c$  and  $h_{\{b\}}$  are not the same
% Constructing a point  $B$  which belongs to line  $c$  and line  $h_{\{b\}}$ 
intersec  $B$   $c$   $h_{\{b\}}$ 
cmark_b  $B$ 

% DET: points  $T_{\{a\}}$  and  $B$  are not the same
% Constructing a line  $a$  which passes through point  $T_{\{a\}}$  and point  $B$ 
line  $a$   $T_{\{a\}}$   $B$ 

color 200 200 200
drawline  $a$ 
color 0 0 0

% NDG: lines  $b$  and  $a$  are not parallel% DET: lines  $b$  and  $a$  are not the same
% Constructing a point  $C$  which belongs to line  $b$  and line  $a$ 
intersec  $C$   $b$   $a$ 
cmark_l  $C$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

```

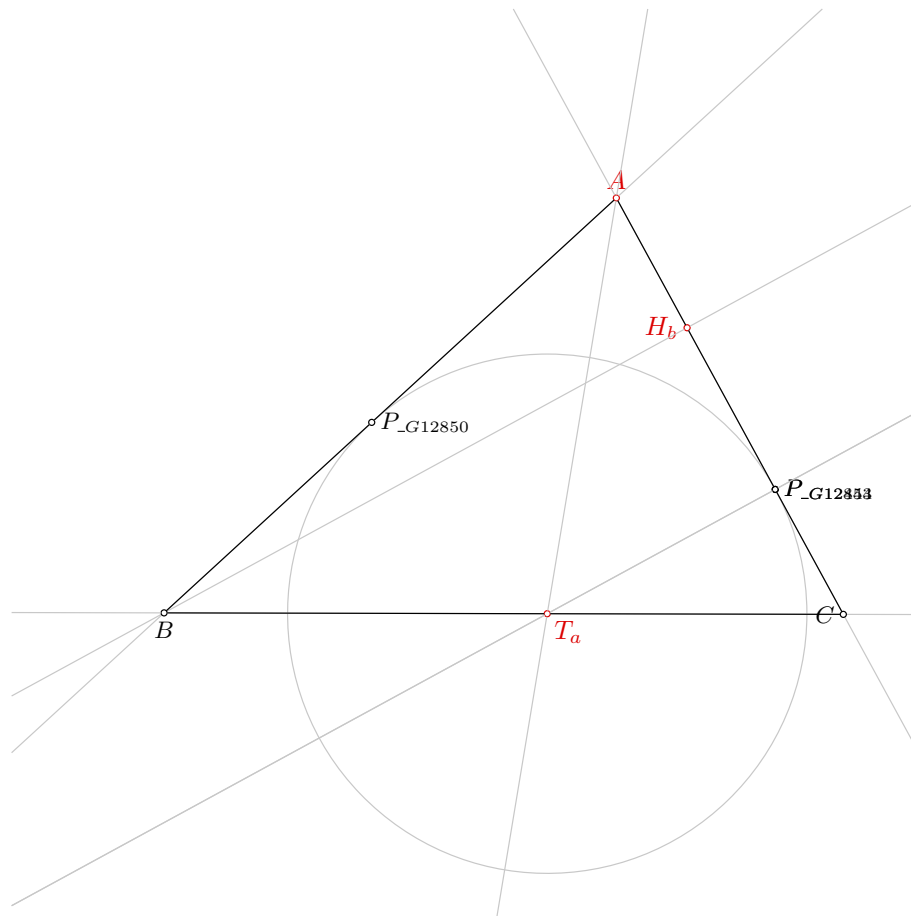


Figure 1: Illustration of the problem 0087

```
% Non-degenerate conditions: lines b and a are not parallel; lines c and h_{b} are not parallel;
    point A is outside the circle k(T_{a},foot[T_{a},c]); point T_{a} is not incident to the line b
% Determination conditions: lines b and a are not the same; points T_{a} and B are not the same;
    lines c and h_{b} are not the same; points A and H_{b} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b = \neg H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.074 seconds.

**NDG conditions** Points  $T_a$ ,  $H_b$  and  $A$  are not collinear

Points  $H_b$  and  $A$  are not identical

Points  $A$ ,  $P_{G13290}$  and  $C$  are not collinear

Line through points  $T_a$  and  $B$  is not parallel with line through points  $A$  and  $P_{G13135}$

Line through points  $H_b$  and  $A$  is not perpendicular to line through points  $A$  and  $P_{G13135}$

#### 4.1.3 Proving $T_a = \neg T_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A = A$

NDG conditions are:

$S_{AH_bT_{h_b}^0} \neq S_{P_{G13919}H_bT_{h_b}^0}$  i.e., lines  $AP_{G13919}$  and  $H_bT_{h_b}^0$  are not parallel (construction based assumption)

$S_{AT_aB} \neq S_{H_bT_aB}$  i.e., lines  $AH_b$  and  $T_aB$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^1} \neq S_{CBF_{h_b}^1}$  i.e., lines  $AC$  and  $BF_{h_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.2.3 Proving $T_a = \neg T_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.3.3 Proving $T_a = \neg T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 67 terms.

**Time Complexity:** Time spent by the prover is 0.040 seconds. There are no ndg conditions.



## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $H_b = \neg H_b$

Proving failed

### 4.4.3 Proving $T_a = \neg T_a$

Proving failed

# Problem 88

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 88: Given a point  $H_b$ , a point  $T_b$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $T_b$  are not the same;
2. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) ;
3. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
4. Choose freely a point  $B$  on the line  $h_b$  (rule WOnline2);
5. Using the point  $B$  and the point  $A$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $A$  are not the same;
6. Using the point  $T_b$  and the line  $c$ , construct a circle  $k(T_b, foot[T_b, c])$  (rule W11); % NDG: point  $T_b$  is not incident to the line  $c$ ;
7. Using the circle  $k(T_b, foot[T_b, c])$ , the point  $B$ , the point  $T_b$  and the line  $c$ , construct a line  $a$  (rule W13); % NDG: point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ;
8. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ; point  $T_b$  is not incident to the line  $c$ .

Determination conditions: lines  $a$  and  $b$  are not the same; points  $B$  and  $A$  are not the same; points  $H_b$  and  $T_b$  are not the same.

Rules used: [W02,W03,W10b,W11,W13,WOnline1,WOnline2]

Lemmas used: [D24,D6,D9,GD01,GD02,GL10,GL11,L9]

Solving time: 15.4 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point H_{b} 89.36 77.83
point T_{b} 94.25 68.88
point A 80 95

color 220 0 0
fontsize 9

cmark_l H_{b}
cmark_t T_{b}
cmark_t A
color 0 0 0
fontsize 8

% DET: points H_{b} and T_{b} are not the same
% Constructing a line b which passes through point H_{b} and point T_{b}
line b H_{b} T_{b}

color 200 200 200
drawline b
color 0 0 0


% Choosing randomly a point A on the line H_{b}T_{b}
online A H_{b} T_{b}
cmark_t A
color 200 200 200
drawline H_{b} T_{b}
color 0 0 0


% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0


% Generating random value V[_G11061]
random V[_G11061]


% Calculating value V[_G11082] using formula V[_G11061]*20
expression V[_G11082] { V[_G11061]*20 }
```

```

% Constructing a point B which is a point for which holds  $H_{\{b\}}B = V[_{G11082}]$  and angle  $AH_{\{b\}}B = 90$ 
turtle B A  $H_{\{b\}}$  90  $V[_{G11082}]$ 
cmark_b B

% DET: points B and A are not the same
% Constructing a line c which passes through point B and point A
line c B A

color 200 200 200
drawline c
color 0 0 0

% NDG: point  $T_{\{b\}}$  is not incident to the line c
% Constructing a point  $P_{\{\_G11326\}}$  which is a foot of the point  $T_{\{b\}}$  on the line c
foot  $P_{\{\_G11326\}}$   $T_{\{b\}}$  c
cmark_r  $P_{\{\_G11326\}}$ 
color 200 200 200
drawline  $T_{\{b\}}$   $P_{\{\_G11326\}}$ 
color 0 0 0

% Constructing a circle  $k(T_{\{b\}}, foot[T_{\{b\}}, c])$  whose center is at point  $T_{\{b\}}$  and which passes
    through point  $P_{\{\_G11326\}}$ 
circle  $k(T_{\{b\}}, foot[T_{\{b\}}, c])$   $T_{\{b\}}$   $P_{\{\_G11326\}}$ 

color 200 200 200
drawcircle  $k(T_{\{b\}}, foot[T_{\{b\}}, c])$ 
color 0 0 0

% NDG: point B is outside the circle  $k(T_{\{b\}}, foot[T_{\{b\}}, c])$ 
% Constructing a point  $P_{\{\_G11717\}}$  which is a foot of the point  $T_{\{b\}}$  on the line c
foot  $P_{\{\_G11717\}}$   $T_{\{b\}}$  c
cmark_r  $P_{\{\_G11717\}}$ 
color 200 200 200
drawline  $T_{\{b\}}$   $P_{\{\_G11717\}}$ 
color 0 0 0

% Constructing a line  $L_{\{\_G11720\}}$  which passes through point  $T_{\{b\}}$  and point B
line  $L_{\{\_G11720\}}$   $T_{\{b\}}$  B

color 200 200 200
drawline  $L_{\{\_G11720\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\_G11723\}}$  which is an image of the point  $P_{\{\_G11717\}}$  in the symmetry to
    point/line  $L_{\{\_G11720\}}$ 
sim  $P_{\{\_G11723\}}$   $L_{\{\_G11720\}}$   $P_{\{\_G11717\}}$ 
cmark_r  $P_{\{\_G11723\}}$ 

% Constructing a line a which passes through point B and point  $P_{\{\_G11723\}}$ 

```

```

line a B P_{\_G11723}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and b are not parallel; point B is outside the circle k(T_{b},
% foot[T_{b},c]); point T_{b} is not incident to the line c
% Determination conditions: lines a and b are not the same; points B and A are not the same; points
% H_{b} and T_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_b = H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.136 seconds.

**NDG conditions** Points  $A$  and  $B$  are not identical

Points  $A$  and  $B$  are not identical

Points  $H_b$ ,  $B$  and  $T_b$  are not collinear

Points  $A$ ,  $B$  and  $T_b$  are not collinear

Line through points  $P_{G13138}$  and  $B$  is not perpendicular to line through points  $B$  and  $H_b$

Line through points  $H_b$  and  $A$  is not perpendicular to line through points  $A$  and  $T_b$

#### 4.1.2 Proving $T_b = T_b$

Proving failed

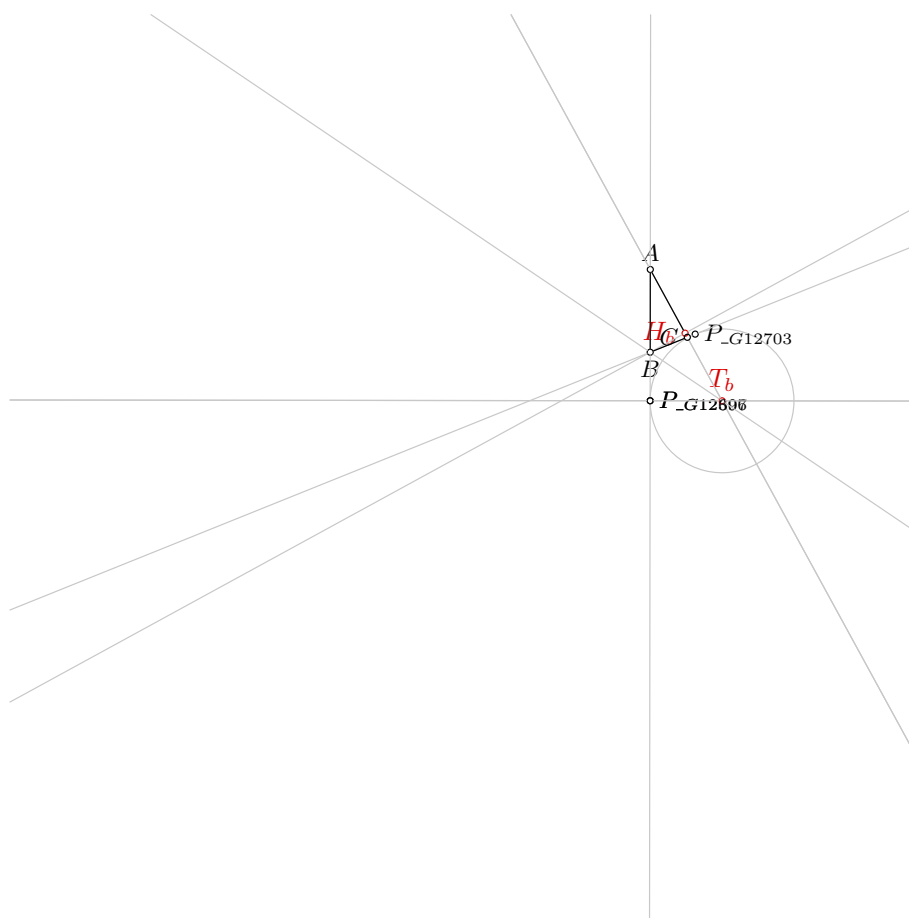


Figure 1: Illustration of the problem 0088

### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.007 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $H_b = \neg H_b$

Proving failed

### 4.2.2 Proving $T_b = \neg T_b$

Proving failed

### 4.2.3 Proving $A=A$

NDG conditions are:

$S_{BH_bT_b} \neq S_{P_{G11768}H_bT_b}$  i.e., lines  $BP_{G11768}$  and  $H_bT_b$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^2_{h_b}} \neq S_{CBF^2_{h_b}}$  i.e., lines  $AC$  and  $BF^2_{h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $H_b = \neg H_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $T_b = \neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $H_b = \neg H_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 19 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.2 Proving $T_b = \neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 19 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.3 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.



# Problem 89

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 89: Given a point  $A$ , a point  $H_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $H_b$  are not the same;
2. Using the point  $A$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $T_c$  are not the same;
3. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
4. Using the line  $h_b$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $h_b$  and  $c$  are not parallel % DET: lines  $h_b$  and  $c$  are not the same;
5. Using the point  $A$ , the point  $B$ , the point  $T_c$  and the line  $c$ , construct a point  $T'_c$  (rule W19); % NDG: points  $A$  and  $B$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $midpoint([A, T_c])$  are not the same;
6. Using the point  $T_c$  and the point  $T'_c$ , construct a circle  $k_{over}(T_c, T'_c)$  (rule W09); % NDG: points  $T_c$  and  $T'_c$  are not the same;
7. Using the circle  $k_{over}(T_c, T'_c)$  and the line  $b$ , construct a point  $C_{wa}$  and a point  $C$  (rule W04); % NDG: line  $b$  and circle  $k_{over}(T_c, T'_c)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k_{over}(T_c, T'_c)$  intersect; points  $T_c$  and  $T'_c$  are not the same; points  $A$  and  $B$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $midpoint([A, T_c])$  are not the same; lines  $h_b$  and  $c$  are not parallel.

Determination conditions: lines  $h_b$  and  $c$  are not the same; points  $A$  and  $T_c$  are not the same; points  $A$  and  $H_b$  are not the same.

Rules used: [W02, W03, W04, W09, W10b, W19]

Lemmas used: [D25, D57, D6, D9, GD01, L72, L79]

Solving time: 2272.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point H_{b} 89.36 77.83
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_t A
cmark_l H_{b}
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points A and T_{c} are not the same
% Constructing a line c which passes through point A and point T_{c}
line c A T_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines h_{b} and c are not parallel% DET: lines h_{b} and c are not the same
% Constructing a point B which belongs to line h_{b} and line c
intersec B h_{b} c
cmark_b B

% NDG: points A and B are not the same; points B and T_{c} are not the same; points B and midpoint
% ([A,T_{c}]) are not the same
```

```

% Constructing a point  $P_{\backslash\_G23770}$  which is an image of the point  $B$  in a rotation around the point
 $T_{\{c\}}$  for the angle 90
rotate  $P_{\backslash\_G23770}$   $T_{\{c\}}$  90 B
cmark_r  $P_{\backslash\_G23770}$ 
color 200 200 200
drawarc_p  $T_{\{c\}}$  B 90
color 0 0 0

% Constructing a line  $L_{\backslash\_G23773}$  which passes through point  $T_{\{c\}}$  and point  $P_{\backslash\_G23770}$ 
line  $L_{\backslash\_G23773}$   $T_{\{c\}}$   $P_{\backslash\_G23770}$ 

color 200 200 200
drawline  $L_{\backslash\_G23773}$ 
color 0 0 0

% Constructing midpoint  $P_{\backslash\_G23776}$  of the segment  $BP_{\backslash\_G23770}$ 
midpoint  $P_{\backslash\_G23776}$  B  $P_{\backslash\_G23770}$ 
cmark_r  $P_{\backslash\_G23776}$ 

% Constructing a line  $L_{\backslash\_G23779}$  which passes through point  $A$  and point  $P_{\backslash\_G23776}$ 
line  $L_{\backslash\_G23779}$  A  $P_{\backslash\_G23776}$ 

color 200 200 200
drawline  $L_{\backslash\_G23779}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G23782}$  which passes through point  $B$  and point  $P_{\backslash\_G23776}$ 
line  $L_{\backslash\_G23782}$  B  $P_{\backslash\_G23776}$ 

color 200 200 200
drawline  $L_{\backslash\_G23782}$ 
color 0 0 0

% Constructing a point  $P_{\backslash\_G23785}$  which belongs to line  $L_{\backslash\_G23773}$  and line  $L_{\backslash\_G23779}$ 
intersec  $P_{\backslash\_G23785}$   $L_{\backslash\_G23773}$   $L_{\backslash\_G23779}$ 
cmark_r  $P_{\backslash\_G23785}$ 

% Constructing a point  $P_{\backslash\_G23788}$  which belongs to line  $L_{\backslash\_G23773}$  and line  $L_{\backslash\_G23782}$ 
intersec  $P_{\backslash\_G23788}$   $L_{\backslash\_G23773}$   $L_{\backslash\_G23782}$ 
cmark_r  $P_{\backslash\_G23788}$ 

% Constructing a line  $L_{\backslash\_G23791}$  which passes through point  $A$  and point  $P_{\backslash\_G23788}$ 
line  $L_{\backslash\_G23791}$  A  $P_{\backslash\_G23788}$ 

color 200 200 200
drawline  $L_{\backslash\_G23791}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G23794}$  which passes through point  $B$  and point  $P_{\backslash\_G23785}$ 
line  $L_{\backslash\_G23794}$  B  $P_{\backslash\_G23785}$ 

color 200 200 200

```

```

drawline L_{\_G23794}
color 0 0 0

% Constructing a point P_{\_G23797} which belongs to line L_{\_G23791} and line L_{\_G23794}
intersec P_{\_G23797} L_{\_G23791} L_{\_G23794}
cmark_r P_{\_G23797}

% Constructing a line L_{\_G23800} which passes through point P_{\_G23776} and point P_{\_G23797}
line L_{\_G23800} P_{\_G23776} P_{\_G23797}

color 200 200 200
drawline L_{\_G23800}
color 0 0 0

% Constructing a point T'_{c} which belongs to line L_{\_G23800} and line c
intersec T'_{c} L_{\_G23800} c
cmark_r T'_{c}

% NDG: points T_{c} and T'_{c} are not the same
% Constructing midpoint P_{\_G24631} of the segment T_{c}T'_{c}
midpoint P_{\_G24631} T_{c} T'_{c}
cmark_r P_{\_G24631}

% Constructing a circle k_{over}(T_{c},T'_{c}) whose center is at point P_{\_G24631} and which passes
    through point T_{c}
circle k_{over}(T_{c},T'_{c}) P_{\_G24631} T_{c}

color 200 200 200
drawcircle k_{over}(T_{c},T'_{c})
color 0 0 0

% NDG: line b and circle k_{over}(T_{c},T'_{c}) intersect
% Constructing points C_{wa} and C which are in intersection of k_{over}(T_{c},T'_{c}) and b
intersec2 C_{wa} C k_{over}(T_{c},T'_{c}) b
cmark_r C_{wa}
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k_{over}(T_{c},T'_{c}) intersect; points T_{c} and T'_{c}
    are not the same; points A and B are not the same; points B and T_{c} are not the same;
    points B and midpoint([A,T_{c}]) are not the same; lines h_{b} and c are not parallel
% Determination conditions: lines h_{b} and c are not the same; points A and T_{c} are not the same
    ; points A and H_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

Figure 1: Illustration of the problem 0089

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b=_H H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 5 terms.

**Time Complexity:** Time spent by the prover is 0.124 seconds.

**NDG conditions** Points  $A$  and  $T_c$  are not identical

Point  $T_c$  is not the midpoint of segment with endpoints  $A$  and  $B$

Points  $H_b$  and  $A$  are not identical

Points  $A$  and  $C$  are not identical

#### 4.1.3 Proving $T_c=_T T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_b A T_c} \neq S_{T_{h_b}^0 A T_c}$  i.e., lines  $H_b T_{h_b}^0$  and  $A T_c$  are not parallel (construction based assumption)

$S_{T_c A P_{G12006}} \neq S_{P_{G12000} A P_{G12006}}$  i.e., lines  $T_c P_{G12000}$  and  $A P_{G12006}$  are not parallel (construction based assumption)

$S_{T_c B P_{G12006}} \neq S_{P_{G12000} B P_{G12006}}$  i.e., lines  $T_c P_{G12000}$  and  $B P_{G12006}$  are not parallel (construction based assumption)

$S_{A B P_{G12015}} \neq S_{P_{G12018} B P_{G12015}}$  i.e., lines  $A P_{G12018}$  and  $B P_{G12015}$  are not parallel (construction based assumption)

$S_{P_{G12006} A T_c} \neq S_{P_{G12027} A T_c}$  i.e., lines  $P_{G12006} P_{G12027}$  and  $A T_c$  are not parallel (construction based assumption)

$S_{B A C} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{A B F_{h_b}^1} \neq S_{C B F_{h_b}^1}$  i.e., lines  $A C$  and  $B F_{h_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_b=_H H_b$

Proving failed

#### **4.2.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $A = A$**

Proving failed

#### **4.3.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.3.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $A = A$**

Proving failed

#### **4.4.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.4.3 Proving $T_c = \neg T_c$**

Proving failed

# Problem 90

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 90: Given a point  $A$ , a point  $H_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $H_b$  are not the same;
2. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
3. Using the point  $I$  and the line  $b$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $b$ ;
4. Using the circle  $k(I, P_a)$ , the point  $A$ , the point  $I$  and the line  $b$ , construct a line  $c$  (rule W13); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
5. Using the line  $c$  and the line  $h_b$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $h_b$  are not parallel % DET: lines  $c$  and  $h_b$  are not the same;
6. Using the circle  $k(I, P_a)$ , the point  $B$ , the point  $I$  and the line  $c$ , construct a line  $a$  (rule W13); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
7. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; point  $B$  is outside the circle  $k(I, P_a)$ ; lines  $c$  and  $h_b$  are not parallel; point  $A$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $b$ .

Determination conditions: lines  $a$  and  $b$  are not the same; lines  $c$  and  $h_b$  are not the same; points  $A$  and  $H_b$  are not the same.

Rules used: [W02,W03,W10b,W11,W13]

Lemmas used: [D27,D6,D9,GD01,L59,L60,L61]

Solving time: 12.5 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point H_{b} 89.36 77.83
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_t A
cmark_l H_{b}
cmark_b I
color 0 0 0
fontsize 8

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: point I is not incident to the line b
% Constructing a point P_{\_G25453} which is a foot of the point I on the line b
foot P_{\_G25453} I b
cmark_r P_{\_G25453}
color 200 200 200
drawline I P_{\_G25453}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
25453}
circle k(I,P_{a}) I P_{\_G25453}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point A is outside the circle k(I,P_{a})
```



```

% Constructing a point  $P_{\{G25660\}}$  which is a foot of the point  $I$  on the line  $b$ 
foot  $P_{\{G25660\}}$  I b
cmark_r  $P_{\{G25660\}}$ 
color 200 200 200
drawline I  $P_{\{G25660\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G25663\}}$  which passes through point  $I$  and point  $A$ 
line  $L_{\{G25663\}}$  I A

color 200 200 200
drawline  $L_{\{G25663\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G25666\}}$  which is an image of the point  $P_{\{G25660\}}$  in the symmetry to
point/line  $L_{\{G25663\}}$ 
sim  $P_{\{G25666\}}$   $L_{\{G25663\}}$   $P_{\{G25660\}}$ 
cmark_r  $P_{\{G25666\}}$ 

% Constructing a line  $c$  which passes through point  $A$  and point  $P_{\{G25666\}}$ 
line c A  $P_{\{G25666\}}$ 

color 200 200 200
drawline c
color 0 0 0

% NDG: lines  $c$  and  $h_{\{b\}}$  are not parallel% DET: lines  $c$  and  $h_{\{b\}}$  are not the same
% Constructing a point  $B$  which belongs to line  $c$  and line  $h_{\{b\}}$ 
intersec B c  $h_{\{b\}}$ 
cmark_b B

% NDG: point  $B$  is outside the circle  $k(I, P_{\{a\}})$ 
% Constructing a point  $P_{\{G26052\}}$  which is a foot of the point  $I$  on the line  $c$ 
foot  $P_{\{G26052\}}$  I c
cmark_r  $P_{\{G26052\}}$ 
color 200 200 200
drawline I  $P_{\{G26052\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G26055\}}$  which passes through point  $I$  and point  $B$ 
line  $L_{\{G26055\}}$  I B

color 200 200 200
drawline  $L_{\{G26055\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G26058\}}$  which is an image of the point  $P_{\{G26052\}}$  in the symmetry to
point/line  $L_{\{G26055\}}$ 
sim  $P_{\{G26058\}}$   $L_{\{G26055\}}$   $P_{\{G26052\}}$ 
cmark_r  $P_{\{G26058\}}$ 

```

```

% Constructing a line a which passes through point B and point P_{\_G26058}
line a B P_{\_G26058}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and b are not parallel; point B is outside the circle k(I,P_{a}
% ); lines c and h_{b} are not parallel; point A is outside the circle k(I,P_{a}); point I is
% not incident to the line b
% Determination conditions: lines a and b are not the same; lines c and h_{b} are not the same;
% points A and H_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b = \neg H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 10 terms.

**Time Complexity:** Time spent by the prover is 0.217 seconds.

**NDG conditions** Points  $H_b$  and  $I$  are not identical

Points  $A$ ,  $I$  and  $P_{G21757}$  are not collinear

Points  $P_{G21860}$ ,  $C$  and  $P_{G21866}$  are not collinear

Points  $A$  and  $P_{G21866}$  are not identical

Points  $B$  and  $I$  are not identical

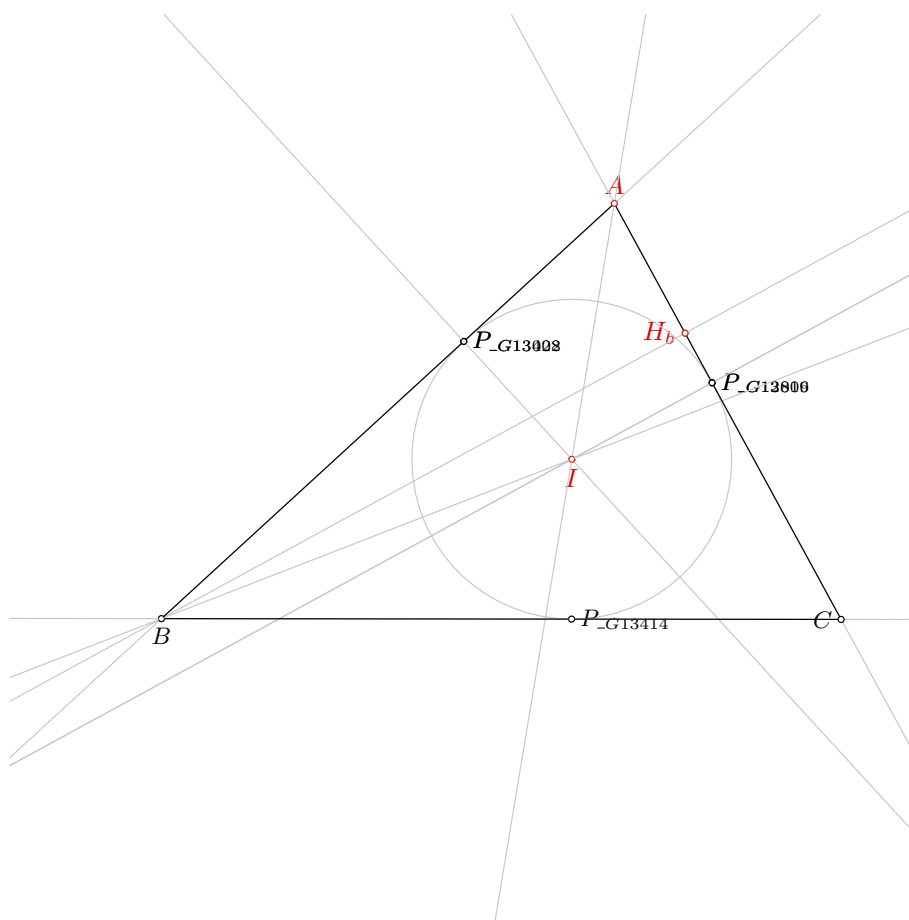


Figure 1: Illustration of the problem 0090

Points  $P_{G21860}$ ,  $B$  and  $C$  are not collinear

Line through points  $P_{G22028}$  and  $B$  is not parallel with line through points  $H_b$  and  $A$

Line through points  $H_b$  and  $A$  is not perpendicular to line through points  $A$  and  $P_{G21860}$

### 4.1.3 Proving $I=_I$

Proving failed

## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{AH_bT_{h_b}^0} \neq S_{P_{G14148}H_bT_{h_b}^0}$  i.e., lines  $AP_{G14148}$  and  $H_bT_{h_b}^0$  are not parallel (construction based assumption)

$S_{BAH_b} \neq S_{P_{G14540}AH_b}$  i.e., lines  $BP_{G14540}$  and  $AH_b$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^1} \neq S_{CBF_{h_b}^1}$  i.e., lines  $AC$  and  $BF_{h_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_b=_H H_b$

Proving failed

### 4.2.3 Proving $I=_I$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_b=_H H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $I=_I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 185 terms.

**Time Complexity:** Time spent by the prover is 0.120 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $I = \neg I$

Proving failed

# Problem 91

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 91: Given a point  $H_c$ , a point  $H$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $H_c$  and  $H$  are not the same;
2. Using the point  $H_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
3. Choose freely a point  $A$  on the line  $c$  (rule WOnline2);
4. Using the point  $A$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H$  are not the same;
5. Choose freely a point  $B$  on the line  $c$  (rule WOnline1) ;
6. Using the point  $B$  and the point  $A$ , construct a point  $M_c$  (rule W01); ;
7. Using the point  $B$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $B$  and  $M_c$  are not the same;
8. Using the circle  $k(M_c, A)$ , the line  $h_a$ , the point  $M_c$  and the point  $A$ , construct a point  $H_a$  (rule W05); % NDG: line  $h_a$  and circle  $k(M_c, A)$  intersect % DET: points  $A$  and  $H_a$  must be different;
9. Using the point  $H_a$  and the point  $B$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $B$  are not the same;
10. Using the line  $h_c$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $h_c$  and  $a$  are not parallel % DET: lines  $h_c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $h_c$  and  $a$  are not parallel; line  $h_a$  and circle  $k(M_c, A)$  intersect; points  $B$  and  $M_c$  are not the same.

Determination conditions: lines  $h_c$  and  $a$  are not the same; points  $H_a$  and  $B$  are not the same; points  $A$  and  $H_a$  must be different; points  $A$  and  $H$  are not the same; points  $H_c$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D10,D20,D3,D5,D7,D8,GD01,GD02,GL04,L3,L40,L42]

Solving time: 23.3 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{c} 68.91 84.83
```

```
point H 80 72.73
```

```
point A 80 95
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H_{c}
```

```
cmark_rt H
```

```
cmark_t A
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{c} and H are not the same
```

```
% Constructing a line h_{c} which passes through point H_{c} and point H
```

```
line h_{c} H_{c} H
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```
% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
```

```
perp c H_{c} h_{c}
```

```
color 200 200 200
```

```
drawline c
```

```
color 0 0 0
```

```
% Generating random value V[_G22715]
```

```
random V[_G22715]
```

```
% Calculating value V[_G22736] using formula V[_G22715]*20
```

```
expression V[_G22736] { V[_G22715]*20 }
```

```

% Constructing a point A which is a point for which holds  $H_{\{c\}}A = V[_G22736]$  and angle  $HH_{\{c\}}A = 90$ 
turtle A H  $H_{\{c\}}$  90  $V[_G22736]$ 
cmark_t A

% DET: points A and H are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point A and point H
line  $h_{\{a\}}$  A H

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% Choosing randomly a point B on the line  $AH_{\{c\}}$ 
online B A  $H_{\{c\}}$ 
cmark_b B
color 200 200 200
drawline A  $H_{\{c\}}$ 
color 0 0 0

% Constructing a point  $M_{\{c\}}$  such that  $BM_{\{c\}}/BA=0.5$ 
towards  $M_{\{c\}}$  B A 0.5
cmark_lt  $M_{\{c\}}$ 
color 200 200 200
drawsegment B A
color 0 0 0

% NDG: points B and  $M_{\{c\}}$  are not the same
% Constructing a circle  $k(M_{\{c\}},A)$  whose center is at point  $M_{\{c\}}$  and which passes through point B
circle  $k(M_{\{c\}},A)$   $M_{\{c\}}$  B

color 200 200 200
drawcircle  $k(M_{\{c\}},A)$ 
color 0 0 0

% NDG: line  $h_{\{a\}}$  and circle  $k(M_{\{c\}},A)$  intersect% DET: points A and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\_G23265\}}$  which is a foot of the point  $M_{\{c\}}$  on the line  $h_{\{a\}}$ 
foot  $P_{\{\_G23265\}}$   $M_{\{c\}}$   $h_{\{a\}}$ 
cmark_r  $P_{\{\_G23265\}}$ 
color 200 200 200
drawline  $M_{\{c\}}$   $P_{\{\_G23265\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point A in the symmetry to point/line  $P_{\{\_G23265\}}$ 
sim  $H_{\{a\}}$   $P_{\{\_G23265\}}$  A

```



```

cmark_r H_{a}

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% NDG: lines h_{c} and a are not parallel% DET: lines h_{c} and a are not the same
% Constructing a point C which belongs to line h_{c} and line a
intersec C h_{c} a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and a are not parallel; line h_{a} and circle k(M_{c},A)
% intersect; points B and M_{c} are not the same
% Determination conditions: lines h_{c} and a are not the same; points H_{a} and B are not the same
% ; points A and H_{a} must be different; points A and H are not the same; points H_{c} and H are
% not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.114 seconds.

**NDG conditions** Points  $A$  and  $H_c$  are not identical

Points  $B$  and  $H$  are not identical

Line through points  $B$  and  $H_a$  is not parallel with line through points  $\neg H_c$  and  $H$

#### 4.1.2 Proving $H = \neg H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.324 seconds.

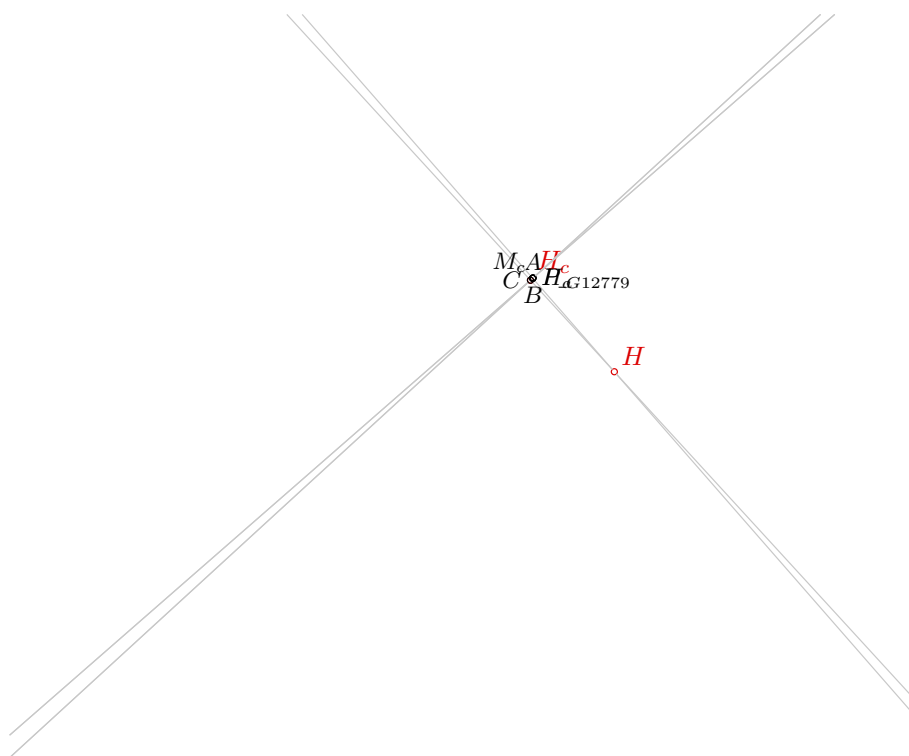


Figure 1: Illustration of the problem 0091

**NDG conditions** Points  $A$  and  $H_c$  are not identical

Points  $H_c$  and  $H$  are not identical

Line through points  $A$  and  $B$  is not perpendicular to line through points  $B$  and  $H_c$

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $C$ ,  $M_c$  and  $H$  are not collinear

### 4.1.3 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.007 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $H_c=_H H_c$

Proving failed

### 4.2.2 Proving $H=_H H$

Proving failed

### 4.2.3 Proving $A=A$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $H_c=_H H_c$

Proving failed

### 4.3.2 Proving $H=_H H$

Proving failed

### 4.3.3 Proving $A=A$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $H_c=_H H_c$

Proving failed

### 4.4.2 Proving $H=_H H$

Proving failed

### 4.4.3 Proving $A=A$

Proving failed

# Problem 92

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 92: Given a point  $A$ , a point  $H_c$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $H_c$  are not the same;
2. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
3. Using the point  $T_a$  and the line  $c$ , construct a circle  $k(T_a, foot[T_a, c])$  (rule W11); % NDG: point  $T_a$  is not incident to the line  $c$ ;
4. Using the circle  $k(T_a, foot[T_a, c])$ , the point  $A$ , the point  $T_a$  and the line  $c$ , construct a line  $b$  (rule W13); % NDG: point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ;
5. Using the line  $b$  and the line  $h_c$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $h_c$  are not parallel % DET: lines  $b$  and  $h_c$  are not the same;
6. Using the point  $T_a$  and the point  $C$ , construct a line  $a$  (rule W02); % DET: points  $T_a$  and  $C$  are not the same;
7. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; lines  $b$  and  $h_c$  are not parallel; point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ; point  $T_a$  is not incident to the line  $c$ .

Determination conditions: lines  $c$  and  $a$  are not the same; points  $T_a$  and  $C$  are not the same; lines  $b$  and  $h_c$  are not the same; points  $A$  and  $H_c$  are not the same.

Rules used: [W02,W03,W10b,W11,W13]

Lemmas used: [D10,D23,D7,GD01,GD02,GL10,GL11,L8]

Solving time: 0.7 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point H_{c} 68.91 84.83
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_t A
cmark_rt H_{c}
cmark_rb T_{a}
color 0 0 0
fontsize 8

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0


% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0


% NDG: point T_{a} is not incident to the line c
% Constructing a point P_{\_G20398} which is a foot of the point T_{a} on the line c
foot P_{\_G20398} T_{a} c
cmark_r P_{\_G20398}
color 200 200 200
drawline T_{a} P_{\_G20398}
color 0 0 0


% Constructing a circle k(T_{a},foot[T_{a},c]) whose center is at point T_{a} and which passes
through point P_{\_G20398}
circle k(T_{a},foot[T_{a},c]) T_{a} P_{\_G20398}

color 200 200 200
drawcircle k(T_{a},foot[T_{a},c])
color 0 0 0


% NDG: point A is outside the circle k(T_{a},foot[T_{a},c])
```

```

% Constructing a point  $P_{\{a\}}$  which is a foot of the point  $T_{\{a\}}$  on the line  $c$ 
foot  $P_{\{a\}}$   $T_{\{a\}}$   $c$ 
cmark_r  $P_{\{a\}}$ 
color 200 200 200
drawline  $T_{\{a\}}$   $P_{\{a\}}$ 
color 0 0 0

% Constructing a line  $L_{\{a\}}$  which passes through point  $T_{\{a\}}$  and point  $A$ 
line  $L_{\{a\}}$   $T_{\{a\}}$   $A$ 

color 200 200 200
drawline  $L_{\{a\}}$ 
color 0 0 0

% Constructing a point  $P_{\{c\}}$  which is an image of the point  $P_{\{a\}}$  in the symmetry to
point/line  $L_{\{a\}}$ 
sim  $P_{\{c\}}$   $L_{\{a\}}$   $P_{\{a\}}$ 
cmark_r  $P_{\{c\}}$ 

% Constructing a line  $b$  which passes through point  $A$  and point  $P_{\{c\}}$ 
line  $b$   $A$   $P_{\{c\}}$ 

color 200 200 200
drawline  $b$ 
color 0 0 0

% NDG: lines  $b$  and  $h_{\{c\}}$  are not parallel% DET: lines  $b$  and  $h_{\{c\}}$  are not the same
% Constructing a point  $C$  which belongs to line  $b$  and line  $h_{\{c\}}$ 
intersec  $C$   $b$   $h_{\{c\}}$ 
cmark_l  $C$ 

% DET: points  $T_{\{a\}}$  and  $C$  are not the same
% Constructing a line  $a$  which passes through point  $T_{\{a\}}$  and point  $C$ 
line  $a$   $T_{\{a\}}$   $C$ 

color 200 200 200
drawline  $a$ 
color 0 0 0

% NDG: lines  $c$  and  $a$  are not parallel% DET: lines  $c$  and  $a$  are not the same
% Constructing a point  $B$  which belongs to line  $c$  and line  $a$ 
intersec  $B$   $c$   $a$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

```

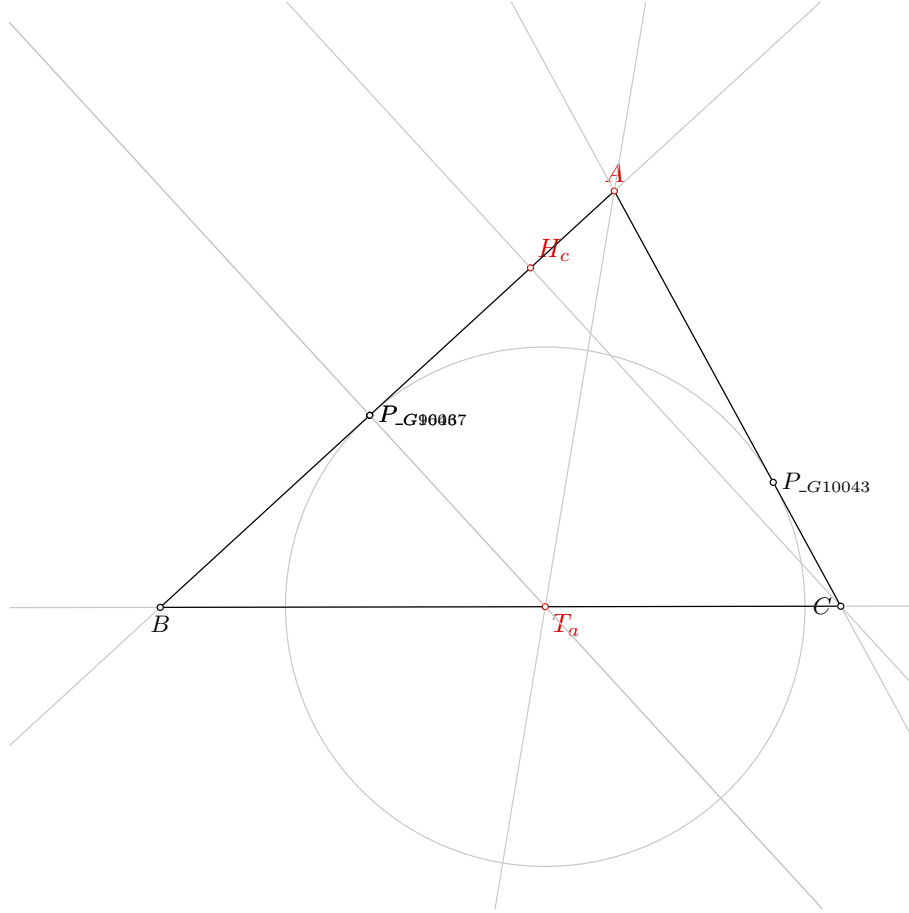


Figure 1: Illustration of the problem 0092

*% Non-degenerate conditions: lines c and a are not parallel; lines b and h\_{c} are not parallel;  
 point A is outside the circle k(T\_{a},foot[T\_{a},c]); point T\_{a} is not incident to the line c  
 % Determination conditions: lines c and a are not the same; points T\_{a} and C are not the same;  
 lines b and h\_{c} are not the same; points A and H\_{c} are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.032 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.136 seconds.

**NDG conditions** Points  $T_a$  and  $A$  are not identical

Points  $T_a$  and  $B$  are not identical

Points  $A$ ,  $P_{G17364}$  and  $P_{G17370}$  are not collinear

Line through points  $C$  and  $T_a$  is not perpendicular to line through points  $T_a$  and  $P_{G17215}$

Points  $A$  and  $B$  are not identical

#### 4.1.3 Proving $T_a = \neg T_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A = A$

NDG conditions are:

$S_{AH_cT_{h_c}^0} \neq S_{P_{G11112}H_cT_{h_c}^0}$  i.e., lines  $AP_{G11112}$  and  $H_cT_{h_c}^0$  are not parallel (construction based assumption)

$S_{AT_aC} \neq S_{H_cT_aC}$  i.e., lines  $AH_c$  and  $T_aC$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^1} \neq S_{BCF_{h_c}^1}$  i.e., lines  $AB$  and  $CF_{h_c}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.2.3 Proving $T_a = \neg T_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.



#### 4.3.3 Proving $T_a = \neg T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 67 terms.

**Time Complexity:** Time spent by the prover is 0.040 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_a = \neg T_a$

Proving failed

# Problem 93

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 93: Given a point  $A$ , a point  $H_c$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $H_c$  are not the same;
2. Using the point  $A$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $T_b$  are not the same;
3. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
4. Using the line  $h_c$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $h_c$  and  $b$  are not parallel % DET: lines  $h_c$  and  $b$  are not the same;
5. Using the point  $A$ , the point  $C$ , the point  $T_b$  and the line  $b$ , construct a point  $T'_b$  (rule W19); % NDG: points  $A$  and  $C$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $\text{midpoint}([A, T_b])$  are not the same;
6. Using the point  $T_b$  and the point  $T'_b$ , construct a circle  $k_{over}(T_b, T'_b)$  (rule W09); % NDG: points  $T_b$  and  $T'_b$  are not the same;
7. Using the circle  $k_{over}(T_b, T'_b)$  and the line  $c$ , construct a point  $B_{wa}$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k_{over}(T_b, T'_b)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k_{over}(T_b, T'_b)$  intersect; points  $T_b$  and  $T'_b$  are not the same; points  $A$  and  $C$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $\text{midpoint}([A, T_b])$  are not the same; lines  $h_c$  and  $b$  are not parallel.

Determination conditions: lines  $h_c$  and  $b$  are not the same; points  $A$  and  $T_b$  are not the same; points  $A$  and  $H_c$  are not the same.

Rules used: [W02,W03,W04,W09,W10b,W19]

Lemmas used: [D10,D24,D55,D7,GD01,L73,L78]

Solving time: 2273.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point H_{c} 68.91 84.83
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_t A
cmark_rt H_{c}
cmark_t T_{b}
color 0 0 0
fontsize 8

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0

% DET: points A and T_{b} are not the same
% Constructing a line b which passes through point A and point T_{b}
line b A T_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines h_{c} and b are not parallel% DET: lines h_{c} and b are not the same
% Constructing a point C which belongs to line h_{c} and line b
intersec C h_{c} b
cmark_l C

% NDG: points A and C are not the same; points C and T_{b} are not the same; points C and midpoint
% ([A,T_{b}]) are not the same
```

```

% Constructing a point  $P_{\backslash\_G25195}$  which is an image of the point  $C$  in a rotation around the point
 $T_{\backslash b}$  for the angle 90
rotate  $P_{\backslash\_G25195}$   $T_{\backslash b}$  90 C
cmark_r  $P_{\backslash\_G25195}$ 
color 200 200 200
drawarc_p  $T_{\backslash b}$  C 90
color 0 0 0

% Constructing a line  $L_{\backslash\_G25198}$  which passes through point  $T_{\backslash b}$  and point  $P_{\backslash\_G25195}$ 
line  $L_{\backslash\_G25198}$   $T_{\backslash b}$   $P_{\backslash\_G25195}$ 

color 200 200 200
drawline  $L_{\backslash\_G25198}$ 
color 0 0 0

% Constructing midpoint  $P_{\backslash\_G25201}$  of the segment  $CP_{\backslash\_G25195}$ 
midpoint  $P_{\backslash\_G25201}$  C  $P_{\backslash\_G25195}$ 
cmark_r  $P_{\backslash\_G25201}$ 

% Constructing a line  $L_{\backslash\_G25204}$  which passes through point  $A$  and point  $P_{\backslash\_G25201}$ 
line  $L_{\backslash\_G25204}$  A  $P_{\backslash\_G25201}$ 

color 200 200 200
drawline  $L_{\backslash\_G25204}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G25207}$  which passes through point  $C$  and point  $P_{\backslash\_G25201}$ 
line  $L_{\backslash\_G25207}$  C  $P_{\backslash\_G25201}$ 

color 200 200 200
drawline  $L_{\backslash\_G25207}$ 
color 0 0 0

% Constructing a point  $P_{\backslash\_G25210}$  which belongs to line  $L_{\backslash\_G25198}$  and line  $L_{\backslash\_G25204}$ 
intersec  $P_{\backslash\_G25210}$   $L_{\backslash\_G25198}$   $L_{\backslash\_G25204}$ 
cmark_r  $P_{\backslash\_G25210}$ 

% Constructing a point  $P_{\backslash\_G25213}$  which belongs to line  $L_{\backslash\_G25198}$  and line  $L_{\backslash\_G25207}$ 
intersec  $P_{\backslash\_G25213}$   $L_{\backslash\_G25198}$   $L_{\backslash\_G25207}$ 
cmark_r  $P_{\backslash\_G25213}$ 

% Constructing a line  $L_{\backslash\_G25216}$  which passes through point  $A$  and point  $P_{\backslash\_G25213}$ 
line  $L_{\backslash\_G25216}$  A  $P_{\backslash\_G25213}$ 

color 200 200 200
drawline  $L_{\backslash\_G25216}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G25219}$  which passes through point  $C$  and point  $P_{\backslash\_G25210}$ 
line  $L_{\backslash\_G25219}$  C  $P_{\backslash\_G25210}$ 

color 200 200 200

```

```

drawline L_{\_G25219}
color 0 0 0

% Constructing a point P_{\_G25222} which belongs to line L_{\_G25216} and line L_{\_G25219}
intersec P_{\_G25222} L_{\_G25216} L_{\_G25219}
cmark_r P_{\_G25222}

% Constructing a line L_{\_G25225} which passes through point P_{\_G25201} and point P_{\_G25222}
line L_{\_G25225} P_{\_G25201} P_{\_G25222}

color 200 200 200
drawline L_{\_G25225}
color 0 0 0

% Constructing a point T'_{b} which belongs to line L_{\_G25225} and line b
intersec T'_{b} L_{\_G25225} b
cmark_r T'_{b}

% NDG: points T_{b} and T'_{b} are not the same
% Constructing midpoint P_{\_G26056} of the segment T_{b}T'_{b}
midpoint P_{\_G26056} T_{b} T'_{b}
cmark_r P_{\_G26056}

% Constructing a circle k_{over}(T_{b},T'_{b}) whose center is at point P_{\_G26056} and which passes
    through point T_{b}
circle k_{over}(T_{b},T'_{b}) P_{\_G26056} T_{b}

color 200 200 200
drawcircle k_{over}(T_{b},T'_{b})
color 0 0 0

% NDG: line c and circle k_{over}(T_{b},T'_{b}) intersect
% Constructing points B_{wa} and B which are in intersection of k_{over}(T_{b},T'_{b}) and c
intersec2 B_{wa} B k_{over}(T_{b},T'_{b}) c
cmark_r B_{wa}
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k_{over}(T_{b},T'_{b}) intersect; points T_{b} and T'_{b}
    are not the same; points A and C are not the same; points C and T_{b} are not the same;
    points C and midpoint([A,T_{b}]) are not the same; lines h_{c} and b are not parallel
% Determination conditions: lines h_{c} and b are not the same; points A and T_{b} are not the same
    ; points A and H_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

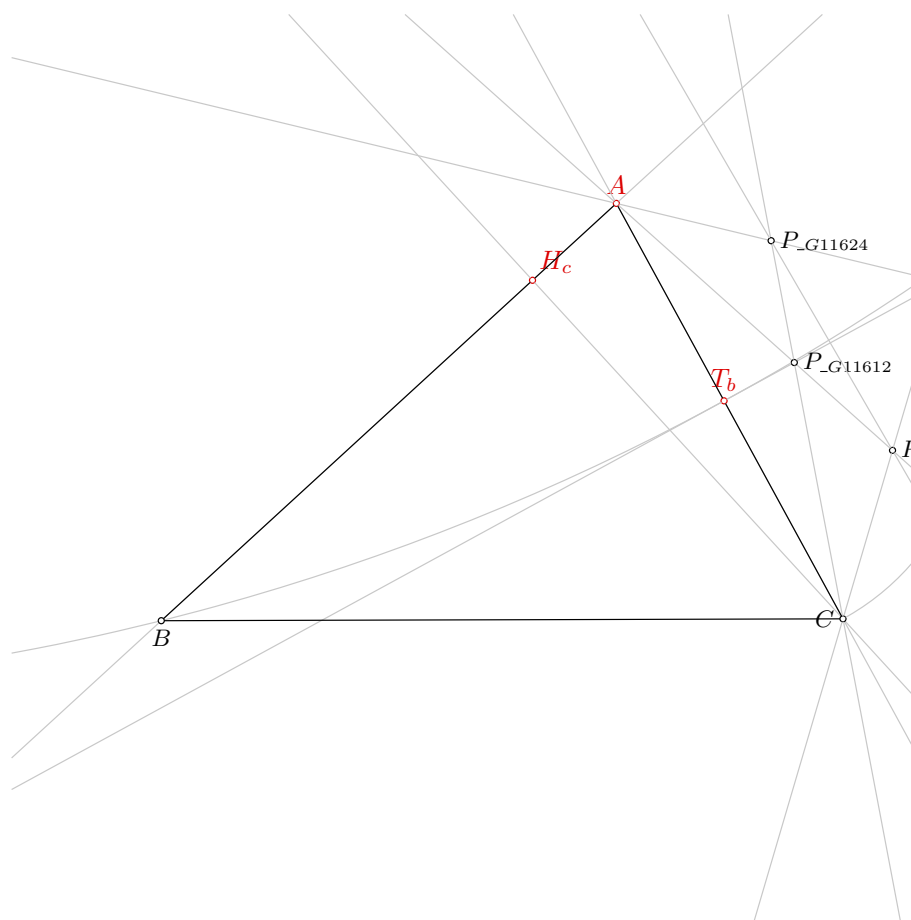


Figure 1: Illustration of the problem 0093

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 5 terms.

**Time Complexity:** Time spent by the prover is 0.133 seconds.

**NDG conditions** Points  $A$  and  $B$  are not identical

Point  $T_b$  is not the midpoint of segment with endpoints  $A$  and  $C$

Points  $A$  and  $B$  are not identical

Points  $A$  and  $B$  are not identical

#### 4.1.3 Proving $T_b = \neg T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_c A T_b} \neq S_{T_{h_c}^0 A T_b}$  i.e., lines  $H_c T_{h_c}^0$  and  $A T_b$  are not parallel (construction based assumption)

$S_{T_b A P_{G13431}} \neq S_{P_{G13425} A P_{G13431}}$  i.e., lines  $T_b P_{G13425}$  and  $A P_{G13431}$  are not parallel (construction based assumption)

$S_{T_b C P_{G13431}} \neq S_{P_{G13425} C P_{G13431}}$  i.e., lines  $T_b P_{G13425}$  and  $C P_{G13431}$  are not parallel (construction based assumption)

$S_{A C P_{G13440}} \neq S_{P_{G13443} C P_{G13440}}$  i.e., lines  $A P_{G13443}$  and  $C P_{G13440}$  are not parallel (construction based assumption)

$S_{P_{G13431} A T_b} \neq S_{P_{G13452} A T_b}$  i.e., lines  $P_{G13431} P_{G13452}$  and  $A T_b$  are not parallel (construction based assumption)

$S_{C A B} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{A C F_{h_c}^1} \neq S_{B C F_{h_c}^1}$  i.e., lines  $A B$  and  $C F_{h_c}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.2.3 Proving $T_b = \neg T_b$

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $A=A$**

Proving failed

#### **4.3.2 Proving $H_c=\neg H_c$**

Proving failed

#### **4.3.3 Proving $T_b=\neg T_b$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $A=A$**

Proving failed

#### **4.4.2 Proving $H_c=\neg H_c$**

Proving failed

#### **4.4.3 Proving $T_b=\neg T_b$**

Proving failed



# Problem 94

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 94: Given a point  $H_c$ , a point  $T_c$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $T_c$  are not the same;
2. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) ;
3. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
4. Choose freely a point  $B$  on the line  $c$  (rule WOnline1) ;
5. Choose freely a point  $C$  on the line  $h_c$  (rule WOnline2).

Non-degenerate conditions: .

Determination conditions: points  $H_c$  and  $T_c$  are not the same.

Rules used: [W02,W10b,WOnline1,WOnline2]

Lemmas used: [D10,D25,D7]

Solving time: 1893.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point H_{c} 68.91 84.83
point T_{c} 55.38 72.43
point A 80 95

color 220 0 0
fontsize 9
```

```

cmark_rt H_{c}
cmark_rt T_{c}
cmark_t A
color 0 0 0
fontsize 8

% DET: points H_{c} and T_{c} are not the same
% Constructing a line c which passes through point H_{c} and point T_{c}
line c H_{c} T_{c}

color 200 200 200
drawline c
color 0 0 0

% Choosing randomly a point A on the line H_{c}T_{c}
online A H_{c} T_{c}
cmark_t A
color 200 200 200
drawline H_{c} T_{c}
color 0 0 0

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0

% Choosing randomly a point B on the line AH_{c}
online B A H_{c}
cmark_b B
color 200 200 200
drawline A H_{c}
color 0 0 0

% Generating random value V[_G17608]
random V[_G17608]

% Calculating value V[_G17629] using formula V[_G17608]*20
expression V[_G17629] { V[_G17608]*20 }

% Constructing a point C which is a point for which holds  $H_{c}C = V[_G17629]$  and angle  $BH_{c}C = 90$ 
turtle C B H_{c} 90 V[_G17629]

```

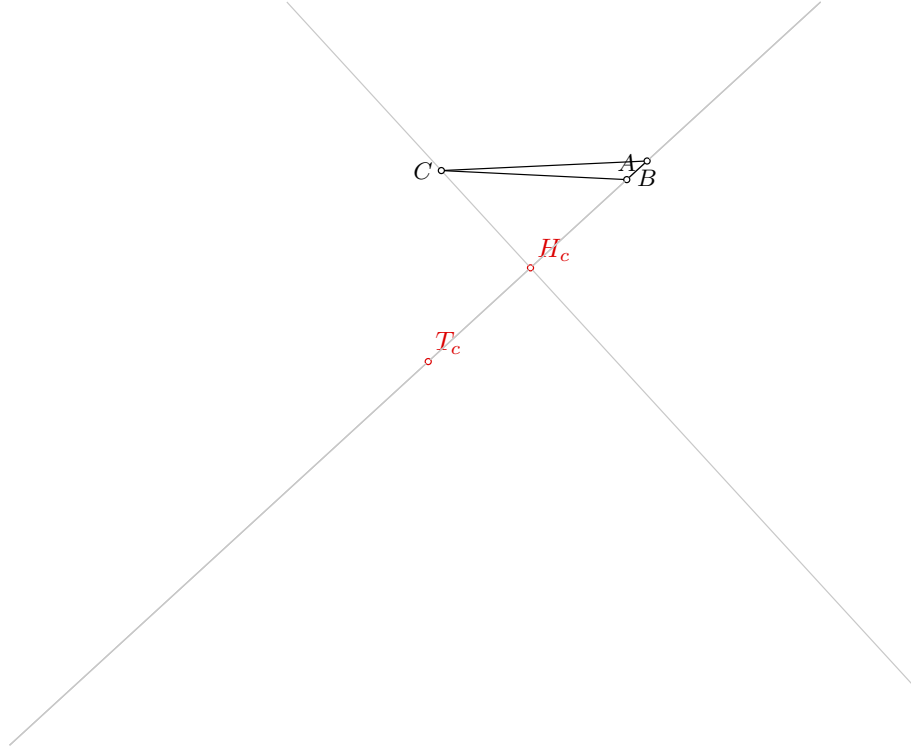


Figure 1: Illustration of the problem 0094

```
cmark_1 C
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions:
```

```
% Determination conditions: points  $H_{\{c\}}$  and  $T_{\{c\}}$  are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.006 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_c = \neg T_c$

Proving failed

#### 4.1.3 Proving $A = A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.02 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_c = \neg H_c$

Proving failed

#### 4.2.2 Proving $T_c = \neg T_c$

Proving failed

#### 4.2.3 Proving $A = A$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^3} \neq S_{BCF_{h_c}^3}$  i.e., lines  $AB$  and  $CF_{h_c}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_c = \neg H_c$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $T_c = \neg T_c$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $H_c = -H_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 31.630 seconds. There are no ndg conditions.

#### 4.4.2 Proving $T_c = -T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 31.710 seconds. There are no ndg conditions.

#### 4.4.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 95

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 95: Given a point  $A$ , a point  $H_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $H_c$  are not the same;
2. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
3. Using the point  $I$  and the line  $c$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $c$ ;
4. Using the circle  $k(I, P_a)$ , the point  $A$ , the point  $I$  and the line  $c$ , construct a line  $b$  (rule W13); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
5. Using the line  $b$  and the line  $h_c$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $h_c$  are not parallel % DET: lines  $b$  and  $h_c$  are not the same;
6. Using the circle  $k(I, P_a)$ , the point  $C$ , the point  $I$  and the line  $b$ , construct a line  $a$  (rule W13); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
7. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; point  $C$  is outside the circle  $k(I, P_a)$ ; lines  $b$  and  $h_c$  are not parallel; point  $A$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $c$ .

Determination conditions: lines  $a$  and  $c$  are not the same; lines  $b$  and  $h_c$  are not the same; points  $A$  and  $H_c$  are not the same.

Rules used: [W02,W03,W10b,W11,W13]

Lemmas used: [D10,D27,D7,GD01,L59,L60,L61]

Solving time: 12.5 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point H_{c} 68.91 84.83
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_t A
cmark_rt H_{c}
cmark_b I
color 0 0 0
fontsize 8

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: point I is not incident to the line c
% Constructing a point P_{\_G19028} which is a foot of the point I on the line c
foot P_{\_G19028} I c
cmark_r P_{\_G19028}
color 200 200 200
drawline I P_{\_G19028}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
19028}
circle k(I,P_{a}) I P_{\_G19028}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point A is outside the circle k(I,P_{a})
```

```

% Constructing a point  $P_{\{G19235\}}$  which is a foot of the point  $I$  on the line  $c$ 
foot  $P_{\{G19235\}}$  I c
cmark_r  $P_{\{G19235\}}$ 
color 200 200 200
drawline I  $P_{\{G19235\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G19238\}}$  which passes through point  $I$  and point  $A$ 
line  $L_{\{G19238\}}$  I A

color 200 200 200
drawline  $L_{\{G19238\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G19241\}}$  which is an image of the point  $P_{\{G19235\}}$  in the symmetry to
point/line  $L_{\{G19238\}}$ 
sim  $P_{\{G19241\}}$   $L_{\{G19238\}}$   $P_{\{G19235\}}$ 
cmark_r  $P_{\{G19241\}}$ 

% Constructing a line  $b$  which passes through point  $A$  and point  $P_{\{G19241\}}$ 
line b A  $P_{\{G19241\}}$ 

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $b$  and  $h_{\{c\}}$  are not parallel% DET: lines  $b$  and  $h_{\{c\}}$  are not the same
% Constructing a point  $C$  which belongs to line  $b$  and line  $h_{\{c\}}$ 
intersec C b  $h_{\{c\}}$ 
cmark_l C

% NDG: point  $C$  is outside the circle  $k(I, P_{\{a\}})$ 
% Constructing a point  $P_{\{G19627\}}$  which is a foot of the point  $I$  on the line  $b$ 
foot  $P_{\{G19627\}}$  I b
cmark_r  $P_{\{G19627\}}$ 
color 200 200 200
drawline I  $P_{\{G19627\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G19630\}}$  which passes through point  $I$  and point  $C$ 
line  $L_{\{G19630\}}$  I C

color 200 200 200
drawline  $L_{\{G19630\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G19633\}}$  which is an image of the point  $P_{\{G19627\}}$  in the symmetry to
point/line  $L_{\{G19630\}}$ 
sim  $P_{\{G19633\}}$   $L_{\{G19630\}}$   $P_{\{G19627\}}$ 
cmark_r  $P_{\{G19633\}}$ 

```



```

% Constructing a line a which passes through point C and point P_{\_G19633}
line a C P_{\_G19633}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and c are not parallel; point C is outside the circle k(I,P_{a}
% ); lines b and h_{c} are not parallel; point A is outside the circle k(I,P_{a}); point I is
% not incident to the line c
% Determination conditions: lines a and c are not the same; lines b and h_{c} are not the same;
% points A and H_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c=H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 10 terms.

**Time Complexity:** Time spent by the prover is 0.214 seconds.

**NDG conditions** Points  $B$ ,  $H_c$  and  $I$  are not collinear

Points  $A$  and  $B$  are not identical

Points  $A$  and  $P_{G15441}$  are not identical

Points  $A$  and  $P_{G15441}$  are not identical

Points  $C$  and  $I$  are not identical

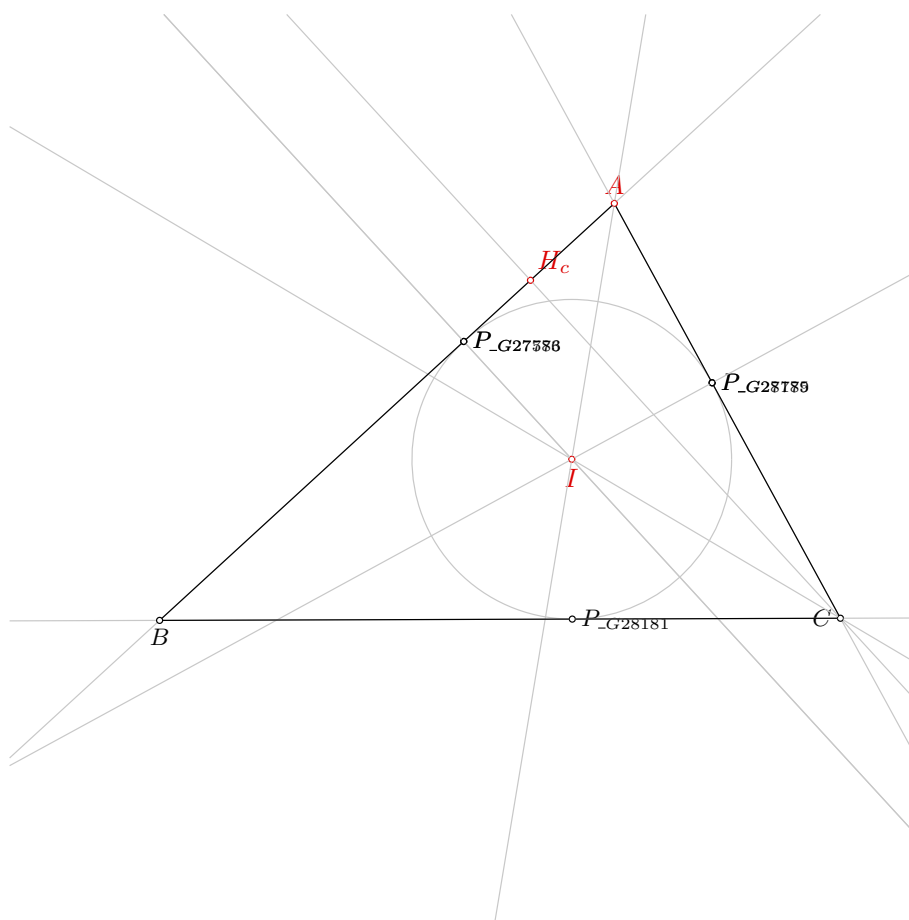


Figure 1: Illustration of the problem 0095

Points  $H_c$  and  $I$  are not identical

Line through points  $A$  and  $H_c$  is not parallel with line through points  $C$  and  $P_{G15603}$

Line through points  $A$  and  $H_c$  is not perpendicular to line through points  $H_c$  and  $B$

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{AH_cT_{h_c}^0} \neq S_{P_{G28915}H_cT_{h_c}^0}$  i.e., lines  $AP_{G28915}$  and  $H_cT_{h_c}^0$  are not parallel (construction based assumption)

$S_{CAH_c} \neq S_{P_{G29307}AH_c}$  i.e., lines  $CP_{G29307}$  and  $AH_c$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^1} \neq S_{BCF_{h_c}^1}$  i.e., lines  $AB$  and  $CF_{h_c}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c=_H H_c$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_c=_H H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I=_I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 23 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $I = \neg I$

Proving failed

## Problem 96

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 96: Given a point  $A$ , a point  $H$  and a point  $T_a$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 97

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 97: Given a point  $A$ , a point  $H$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 98

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 98: Given a point  $A$ , a point  $H$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 99

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 99: Given a point  $A$ , a point  $H$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.



# Problem 100

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 100: Given a point  $A$ , a point  $T_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $T_b$  are not the same;
2. Using the point  $T_a$  and the line  $b$ , construct a circle  $k(T_a, foot[T_a, c])$  (rule W11); % NDG: point  $T_a$  is not incident to the line  $b$ ;
3. Using the circle  $k(T_a, foot[T_a, c])$ , the point  $A$ , the point  $T_a$  and the line  $b$ , construct a line  $c$  (rule W13); % NDG: point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ;
4. Using the point  $T_b$  and the line  $c$ , construct a circle  $k(T_b, foot[T_b, c])$  (rule W11); % NDG: point  $T_b$  is not incident to the line  $c$ ;
5. Using the circle  $k(T_b, foot[T_b, c])$ , the point  $T_a$  and the point  $T_b$ , construct a line  $a1$  and a line  $a$  (rule W12); % NDG: point  $T_a$  is outside the circle  $k(T_b, foot[T_b, c])$ ;
6. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same;
7. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; lines  $a$  and  $b$  are not parallel; point  $T_a$  is outside the circle  $k(T_b, foot[T_b, c])$ ; point  $T_b$  is not incident to the line  $c$ ; point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ; point  $T_a$  is not incident to the line  $b$ .

Determination conditions: lines  $c$  and  $a$  are not the same; lines  $a$  and  $b$  are not the same; points  $A$  and  $T_b$  are not the same.

Rules used: [W02,W03,W11,W12,W13]

Lemmas used: [D23,D24,D32,GD01,GD02,GL10,GL11,L8,L9]

Solving time: 0.9 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point T_{a} 70.86 40
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_t A
cmark_rb T_{a}
cmark_t T_{b}
color 0 0 0
fontsize 8

% DET: points A and T_{b} are not the same
% Constructing a line b which passes through point A and point T_{b}
line b A T_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: point T_{a} is not incident to the line b
% Constructing a point P_{\_G17171} which is a foot of the point T_{a} on the line b
foot P_{\_G17171} T_{a} b
cmark_r P_{\_G17171}
color 200 200 200
drawline T_{a} P_{\_G17171}
color 0 0 0

% Constructing a circle k(T_{a},foot[T_{a},c]) whose center is at point T_{a} and which passes
    through point P_{\_G17171}
circle k(T_{a},foot[T_{a},c]) T_{a} P_{\_G17171}

color 200 200 200
drawcircle k(T_{a},foot[T_{a},c])
color 0 0 0

% NDG: point A is outside the circle k(T_{a},foot[T_{a},c])
% Constructing a point P_{\_G17562} which is a foot of the point T_{a} on the line b
foot P_{\_G17562} T_{a} b
cmark_r P_{\_G17562}
color 200 200 200
drawline T_{a} P_{\_G17562}
color 0 0 0

% Constructing a line L_{\_G17565} which passes through point T_{a} and point A
line L_{\_G17565} T_{a} A
```

```

color 200 200 200
drawline L_{\_G17565}
color 0 0 0

% Constructing a point P_{\_G17568} which is an image of the point P_{\_G17562} in the symmetry to
  point/line L_{\_G17565}
sim P_{\_G17568} L_{\_G17565} P_{\_G17562}
cmark_r P_{\_G17568}

% Constructing a line c which passes through point A and point P_{\_G17568}
line c A P_{\_G17568}

color 200 200 200
drawline c
color 0 0 0

% NDG: point T_{b} is not incident to the line c
% Constructing a point P_{\_G17849} which is a foot of the point T_{b} on the line c
foot P_{\_G17849} T_{b} c
cmark_r P_{\_G17849}
color 200 200 200
drawline T_{b} P_{\_G17849}
color 0 0 0

% Constructing a circle k(T_{b},foot[T_{b},c]) whose center is at point T_{b} and which passes
  through point P_{\_G17849}
circle k(T_{b},foot[T_{b},c]) T_{b} P_{\_G17849}

color 200 200 200
drawcircle k(T_{b},foot[T_{b},c])
color 0 0 0

% NDG: point T_{a} is outside the circle k(T_{b},foot[T_{b},c])
% Constructing midpoint P_{\_G18267} of the segment T_{a}T_{b}
midpoint P_{\_G18267} T_{a} T_{b}
cmark_r P_{\_G18267}

% Constructing a circle C_{\_G18270} whose center is at point P_{\_G18267} and which passes through
  point T_{a}
circle C_{\_G18270} P_{\_G18267} T_{a}

color 200 200 200
drawcircle C_{\_G18270}
color 0 0 0

% Constructing points P_{\_G18273} and P_{\_G18276} which are in intersection of C_{\_G18270} and k
  (T_{b},foot[T_{b},c])
intersec2 P_{\_G18273} P_{\_G18276} C_{\_G18270} k(T_{b},foot[T_{b},c])
cmark_r P_{\_G18273}
cmark_r P_{\_G18276}

```

```

% Constructing a line a1 which passes through point T_{a} and point P_{\_G18273}
line a1 T_{a} P_{\_G18273}

color 200 200 200
drawline a1
color 0 0 0

% Constructing a line a which passes through point T_{a} and point P_{\_G18276}
line a T_{a} P_{\_G18276}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_l C

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and a are not parallel; lines a and b are not parallel; point
% T_{a} is outside the circle k(T_{b},foot[T_{b},c]); point T_{b} is not incident to the line c;
% point A is outside the circle k(T_{a},foot[T_{a},c]); point T_{a} is not incident to the line b
% Determination conditions: lines c and a are not the same; lines a and b are not the same; points
% A and T_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

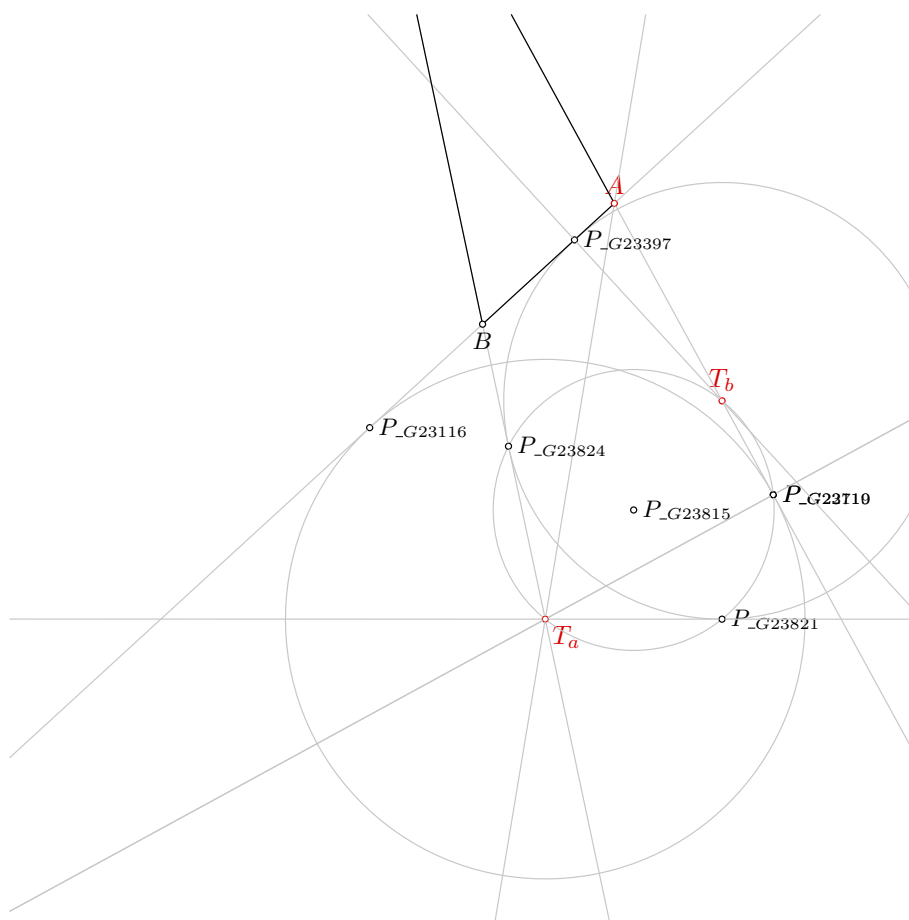


Figure 1: Illustration of the problem 0100

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_a = T_a$

Proving failed

#### 4.1.3 Proving $T_b = T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A = A$

NDG conditions are:

$S_{T_a A T_b} \neq S_{P_{G25696} A T_b}$  i.e., lines  $T_a P_{G25696}$  and  $A T_b$  are not parallel (construction based assumption)  
 $S_{A T_a P_{G25696}} \neq S_{P_{G24988} T_a P_{G25696}}$  i.e., lines  $A P_{G24988}$  and  $T_a P_{G25696}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $T_a = T_a$

Proving failed

#### 4.2.3 Proving $T_b = T_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $T_a = T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 178 terms.

**Time Complexity:** Time spent by the prover is 0.400 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_b = T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 108 terms.

**Time Complexity:** Time spent by the prover is 0.260 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4.2 Proving $T_a = \neg T_a$

Proving failed

### 4.4.3 Proving $T_b = \neg T_b$

Proving failed

# Problem 101

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 101: Given a point  $A$ , a point  $T_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $T_c$  are not the same;
2. Using the point  $T_a$  and the line  $c$ , construct a circle  $k(T_a, foot[T_a, c])$  (rule W11); % NDG: point  $T_a$  is not incident to the line  $c$ ;
3. Using the circle  $k(T_a, foot[T_a, c])$ , the point  $A$ , the point  $T_a$  and the line  $c$ , construct a line  $b$  (rule W13); % NDG: point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ;
4. Using the point  $T_c$  and the line  $b$ , construct a circle  $k(T_c, foot[T_c, b])$  (rule W11); % NDG: point  $T_c$  is not incident to the line  $b$ ;
5. Using the circle  $k(T_c, foot[T_c, b])$ , the point  $T_a$  and the point  $T_c$ , construct a line  $a2$  and a line  $a$  (rule W12); % NDG: point  $T_a$  is outside the circle  $k(T_c, foot[T_c, b])$ ;
6. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same;
7. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; lines  $a$  and  $c$  are not parallel; point  $T_a$  is outside the circle  $k(T_c, foot[T_c, b])$ ; point  $T_c$  is not incident to the line  $b$ ; point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ; point  $T_a$  is not incident to the line  $c$ .

Determination conditions: lines  $b$  and  $a$  are not the same; lines  $a$  and  $c$  are not the same; points  $A$  and  $T_c$  are not the same.

Rules used: [W02,W03,W11,W12,W13]

Lemmas used: [D23,D25,D33,GD01,GD02,GL10,GL11,L10,L8]

Solving time: 0.9 seconds.



## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point T_{a} 70.86 40
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_t A
cmark_rb T_{a}
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points A and T_{c} are not the same
% Constructing a line c which passes through point A and point T_{c}
line c A T_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: point T_{a} is not incident to the line c
% Constructing a point P_{\_G21220} which is a foot of the point T_{a} on the line c
foot P_{\_G21220} T_{a} c
cmark_r P_{\_G21220}
color 200 200 200
drawline T_{a} P_{\_G21220}
color 0 0 0

% Constructing a circle k(T_{a},foot[T_{a},c]) whose center is at point T_{a} and which passes
    through point P_{\_G21220}
circle k(T_{a},foot[T_{a},c]) T_{a} P_{\_G21220}

color 200 200 200
drawcircle k(T_{a},foot[T_{a},c])
color 0 0 0

% NDG: point A is outside the circle k(T_{a},foot[T_{a},c])
% Constructing a point P_{\_G21611} which is a foot of the point T_{a} on the line c
foot P_{\_G21611} T_{a} c
cmark_r P_{\_G21611}
color 200 200 200
drawline T_{a} P_{\_G21611}
color 0 0 0

% Constructing a line L_{\_G21614} which passes through point T_{a} and point A
line L_{\_G21614} T_{a} A
```

```

color 200 200 200
drawline L_{\_G21614}
color 0 0 0

% Constructing a point P_{\_G21617} which is an image of the point P_{\_G21611} in the symmetry to
point/line L_{\_G21614}
sim P_{\_G21617} L_{\_G21614} P_{\_G21611}
cmark_r P_{\_G21617}

% Constructing a line b which passes through point A and point P_{\_G21617}
line b A P_{\_G21617}

color 200 200 200
drawline b
color 0 0 0

% NDG: point T_{c} is not incident to the line b
% Constructing a point P_{\_G21898} which is a foot of the point T_{c} on the line b
foot P_{\_G21898} T_{c} b
cmark_r P_{\_G21898}
color 200 200 200
drawline T_{c} P_{\_G21898}
color 0 0 0

% Constructing a circle k(T_{c},foot[T_{c},b]) whose center is at point T_{c} and which passes
through point P_{\_G21898}
circle k(T_{c},foot[T_{c},b]) T_{c} P_{\_G21898}

color 200 200 200
drawcircle k(T_{c},foot[T_{c},b])
color 0 0 0

% NDG: point T_{a} is outside the circle k(T_{c},foot[T_{c},b])
% Constructing midpoint P_{\_G22316} of the segment T_{a}T_{c}
midpoint P_{\_G22316} T_{a} T_{c}
cmark_r P_{\_G22316}

% Constructing a circle C_{\_G22319} whose center is at point P_{\_G22316} and which passes through
point T_{a}
circle C_{\_G22319} P_{\_G22316} T_{a}

color 200 200 200
drawcircle C_{\_G22319}
color 0 0 0

% Constructing points P_{\_G22322} and P_{\_G22325} which are in intersection of C_{\_G22319} and k
(T_{c},foot[T_{c},b])
intersec2 P_{\_G22322} P_{\_G22325} C_{\_G22319} k(T_{c},foot[T_{c},b])
cmark_r P_{\_G22322}
cmark_r P_{\_G22325}

```

```

% Constructing a line a2 which passes through point T_{a} and point P_{\_G22322}
line a2 T_{a} P_{\_G22322}

color 200 200 200
drawline a2
color 0 0 0

% Constructing a line a which passes through point T_{a} and point P_{\_G22325}
line a T_{a} P_{\_G22325}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and a are not parallel; lines a and c are not parallel; point
  T_{a} is outside the circle k(T_{c},foot[T_{c},b]); point T_{c} is not incident to the line b;
  point A is outside the circle k(T_{a},foot[T_{a},c]); point T_{a} is not incident to the line c
% Determination conditions: lines b and a are not the same; lines a and c are not the same; points
  A and T_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

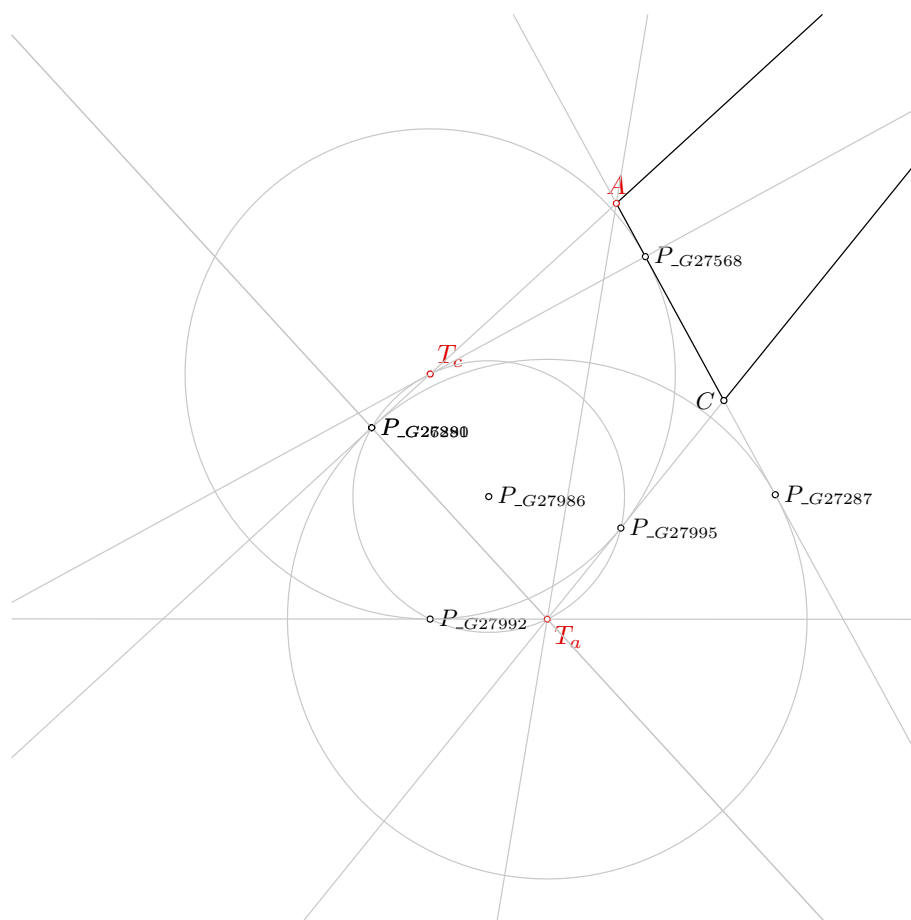


Figure 1: Illustration of the problem 0101

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_a = T_a$

Proving failed

#### 4.1.3 Proving $T_c = T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A = A$

NDG conditions are:

$S_{T_a AT_c} \neq S_{P_{G29867} AT_c}$  i.e., lines  $T_a P_{G29867}$  and  $AT_c$  are not parallel (construction based assumption)

$S_{AT_a P_{G29867}} \neq S_{P_{G29159} T_a P_{G29867}}$  i.e., lines  $AP_{G29159}$  and  $T_a P_{G29867}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $T_a = T_a$

Proving failed

#### 4.2.3 Proving $T_c = T_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $T_a = T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 178 terms.

**Time Complexity:** Time spent by the prover is 0.550 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_c = T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 108 terms.

**Time Complexity:** Time spent by the prover is 0.290 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

### 4.4.2 Proving $T_a = T_a$

Proving failed

### 4.4.3 Proving $T_c = T_c$

Proving failed

# Problem 102

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 102: Given a point  $T_a$ , a point  $I$  and a point  $A$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $T_a$  and the point  $I$ , construct a line  $s_a$  (rule W02); % DET: points  $T_a$  and  $I$  are not the same;
2. Choose freely a point  $A$  on the line  $s_a$  (rule WOnline1) ;
3. Using the point  $A$  and the point  $I$ , construct a line  $m(AI)$  (rule W14); % DET: points  $A$  and  $I$  are not the same;
4. Choose freely a point  $B$  (rule free);
5. Using the point  $B$  and the point  $A$ , construct a point  $M_c$  (rule W01); ;
6. Using the point  $B$  and the point  $A$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $A$  are not the same;
7. Using the point  $B$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $T_a$  are not the same;
8. Using the point  $M_c$  and the line  $c$ , construct a line  $m_c$  (rule W10b); ;
9. Using the line  $m_c$  and the line  $m(AI)$ , construct a point  $N_c$  (rule W03); % NDG: lines  $m_c$  and  $m(AI)$  are not parallel % DET: lines  $m_c$  and  $m(AI)$  are not the same;
10. Using the point  $N_c$  and the point  $I$ , construct a line  $s_c$  (rule W02); % DET: points  $N_c$  and  $I$  are not the same;
11. Using the line  $a$  and the line  $s_c$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $s_c$  are not parallel % DET: lines  $a$  and  $s_c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $s_c$  are not parallel; lines  $m_c$  and  $m(AI)$  are not parallel.

Determination conditions: lines  $a$  and  $s_c$  are not the same; points  $N_c$  and  $I$  are not the same; lines  $m_c$  and  $m(AI)$  are not the same; points  $B$  and  $T_a$  are not the same; points  $B$  and  $A$  are not the same; points  $A$  and  $I$  are not the same; points  $T_a$  and  $I$  are not the same.

Rules used: [W01,W02,W03,W10b,W14,WOnline1,free]

Lemmas used: [D13,D2,D20,D23,D49,GD01,GD02,GL01,GL04,L2,L27,L35,L36,L7]

Solving time: 5.8 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point T_{a} 70.86 40
point I 74.37 61.15
point A 80 95

color 220 0 0
fontsize 9

cmark_rb T_{a}
cmark_b I
cmark_t A
color 0 0 0
fontsize 8

% DET: points T_{a} and I are not the same
% Constructing a line s_{a} which passes through point T_{a} and point I
line s_{a} T_{a} I

color 200 200 200
drawline s_{a}
color 0 0 0

% Choosing randomly a point A on the line T_{a}I
online A T_{a} I
cmark_t A
color 200 200 200
drawline T_{a} I
color 0 0 0

% DET: points A and I are not the same
% Constructing bisector m(AI) of the segment AI
med m(AI) A I

color 200 200 200
drawline m(AI)
color 0 0 0

color 200 200 200
drawsegment A I
```



```
color 0 0 0
```

```
% Constructing a free point B  
point B 20 40
```

```
cmark_b B
```

```
% Constructing a point  $M_{\{c\}}$  such that  $BM_{\{c\}}/BA=0.5$   
towards  $M_{\{c\}}$  B A 0.5  
cmark_lt  $M_{\{c\}}$   
color 200 200 200  
drawsegment B A  
color 0 0 0
```

```
% DET: points B and A are not the same  
% Constructing a line c which passes through point B and point A  
line c B A
```

```
color 200 200 200  
drawline c  
color 0 0 0
```

```
% DET: points B and  $T_{\{a\}}$  are not the same  
% Constructing a line a which passes through point B and point  $T_{\{a\}}$   
line a B  $T_{\{a\}}$ 
```

```
color 200 200 200  
drawline a  
color 0 0 0
```

```
% Constructing a line  $m_{\{c\}}$  which is perpendicular to line c and which passes through point  $M_{\{c\}}$   
perp  $m_{\{c\}}$   $M_{\{c\}}$  c
```

```
color 200 200 200  
drawline  $m_{\{c\}}$   
color 0 0 0
```

```
% NDG: lines  $m_{\{c\}}$  and  $m(AI)$  are not parallel  
% DET: lines  $m_{\{c\}}$  and  $m(AI)$  are not the same  
% Constructing a point  $N_{\{c\}}$  which belongs to line  $m_{\{c\}}$  and line  $m(AI)$   
intersec  $N_{\{c\}}$   $m_{\{c\}}$   $m(AI)$   
cmark_b  $N_{\{c\}}$ 
```

```
% DET: points  $N_{\{c\}}$  and I are not the same
```

```

% Constructing a line s_{c} which passes through point N_{c} and point I
line s_{c} N_{c} I

color 200 200 200
drawline s_{c}
color 0 0 0

% NDG: lines a and s_{c} are not parallel% DET: lines a and s_{c} are not the same
% Constructing a point C which belongs to line a and line s_{c}
intersec C a s_{c}
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and s_{c} are not parallel; lines m_{c} and m(AI) are not
parallel
% Determination conditions: lines a and s_{c} are not the same; points N_{c} and I are not the same
; lines m_{c} and m(AI) are not the same; points B and T_{a} are not the same; points B and A
are not the same; points A and I are not the same; points T_{a} and I are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $T_a = T_a$

Proving failed

#### 4.1.2 Proving $I = I$

Proving failed

#### 4.1.3 Proving $A = A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

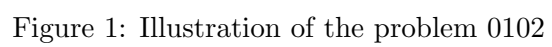
**Time Complexity:** Time spent by the prover is 0.007 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $T_a = T_a$

Proving failed



#### 4.2.2 Proving $I=_I$

Proving failed

#### 4.2.3 Proving $A=A$

NDG conditions are:

$S_{M_cBA} \neq 0$  i.e., points  $M_c$ ,  $B$  and  $A$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_cM_{m(AI)}^1T_{m(AI)}^2} \neq S_{F_{m_c}^3M_{m(AI)}^1T_{m(AI)}^2}$  i.e., lines  $M_cF_{m_c}^3$  and  $M_{m(AI)}^1T_{m(AI)}^2$  are not parallel (construction based assumption)

$S_{BN_cI} \neq S_{T_aN_cI}$  i.e., lines  $BT_a$  and  $N_cI$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $T_a=_T T_a$

Proving failed

#### 4.3.2 Proving $I=_I$

Proving failed

#### 4.3.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $T_a=_T T_a$

Proving failed

#### 4.4.2 Proving $I=_I$

Proving failed

#### 4.4.3 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

# Problem 103

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 103: Given a point  $A$ , a point  $T_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $T_b$  are not the same;
2. Using the point  $A$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $T_c$  are not the same;
3. Using the line  $b$ , the point  $T_c$ , the point  $A$ , the point  $T_b$  and the line  $c$ , construct a line  $s_a$  (rule W17); % NDG: points  $A$  and  $T_b$  are not the same; points  $T_c$  and  $A$  are not the same % DET: points  $A$  and  $T_c$  are not the same;
4. Using the point  $T_c$ , the point  $A$  and the point  $T_b$ , construct a circle  $circle[T_b, T_c, angle[s[b]][s[c]]]$  (rule W20); % NDG: points  $T_c$  and  $A$  are not the same; points  $A$  and  $T_b$  are not the same; points  $T_b$  and  $T_c$  are not the same;
5. Using the circle  $circle[T_b, T_c, angle[s[b]][s[c]]]$  and the line  $s_a$ , construct a point  $A_{sa}$  and a point  $I$  (rule W04); % NDG: line  $s_a$  and circle  $circle[T_b, T_c, angle[s[b]][s[c]]]$  intersect;
6. Using the point  $I$  and the point  $T_b$ , construct a line  $s_b$  (rule W02); % DET: points  $I$  and  $T_b$  are not the same;
7. Using the point  $T_c$  and the point  $I$ , construct a line  $s_c$  (rule W02); % DET: points  $T_c$  and  $I$  are not the same;
8. Using the line  $b$  and the line  $s_c$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $s_c$  are not parallel % DET: lines  $b$  and  $s_c$  are not the same;
9. Using the line  $c$  and the line  $s_b$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $s_b$  are not parallel % DET: lines  $c$  and  $s_b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $s_b$  are not parallel; lines  $b$  and  $s_c$  are not parallel; line  $s_a$  and circle  $circle[T_b, T_c, angle[s[b]][s[c]]]$  intersect; points  $T_c$  and  $A$  are not the same; points  $A$  and  $T_b$  are not the same; points  $T_b$  and  $T_c$  are not the same; points  $A$  and  $T_b$  are not the same; points  $T_c$  and  $A$  are not the same.

Determination conditions: lines  $c$  and  $s_b$  are not the same; lines  $b$  and  $s_c$  are not the same; points  $T_c$  and  $I$  are not the same; points  $I$  and  $T_b$  are not the same; points  $A$  and  $T_c$  are not the same; points  $A$  and  $T_c$  are not the same; points  $A$  and  $T_b$  are not the same.

Rules used: [W02,W03,W04,W17,W20]

Lemmas used: [D2,D24,D25,D62,GD01,L110,L2,L5,L6,L7,L86,L98]

Solving time: 25.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point T_{b} 94.25 68.88
point T_{c} 55.38 72.43
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_t T_{b}
cmark_rt T_{c}
color 0 0 0
fontsize 8
```

```
% DET: points A and T_{b} are not the same
% Constructing a line b which passes through point A and point T_{b}
line b A T_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% DET: points A and T_{c} are not the same
% Constructing a line c which passes through point A and point T_{c}
line c A T_{c}
```

```
color 200 200 200
drawline c
color 0 0 0
```

```
% NDG: points A and T_{b} are not the same; points T_{c} and A are not the same% DET: points A and
T_{c} are not the same
% Constructing an angle V[_G16394] which is equal to the angle T_{c}AT_{b}
angle_o V[_G16394] T_{c} A T_{b}
```

```

% Calculating value angle[_G16473] using formula  $1/\text{pow}(2,1)*V[_G16394]+0/\text{pow}(2,0)*180$ 
expression angle[_G16473] {  $1/\text{pow}(2,1)*V[_G16394]+0/\text{pow}(2,0)*180$  }

% Constructing a point P_{\_G16470} which is an image of the point T_{c} in a rotation around the
point A for the angle  $1/\text{pow}(2,1)*V[_G16394]+0/\text{pow}(2,0)*180$ 
rotate P_{\_G16470} A angle[_G16473] T_{c}
cmark_r P_{\_G16470}
color 200 200 200
drawarc_p A T_{c} angle[_G16473]
color 0 0 0

% Constructing a line s_{a} which passes through point A and point P_{\_G16470}
line s_{a} A P_{\_G16470}

color 200 200 200
drawline s_{a}
color 0 0 0

% NDG: points T_{c} and A are not the same; points A and T_{b} are not the same; points T_{b} and T
_{c} are not the same
% Constructing an angle angle[_G16920] which is equal to the angle T_{c}AT_{b}
angle_o angle[_G16920] T_{c} A T_{b}

% Calculating value angle[_G16931] using formula  $90-(1/\text{pow}(2,1)*\text{angle}[_G16920]+1/\text{pow}(2,1)*180)$ 
expression angle[_G16931] {  $90-(1/\text{pow}(2,1)*\text{angle}[_G16920]+1/\text{pow}(2,1)*180)$  }

% Constructing a line L_{\_G17002} which passes through point T_{b} and point T_{c}
line L_{\_G17002} T_{b} T_{c}

color 200 200 200
drawline L_{\_G17002}
color 0 0 0

% Constructing bisector L_{\_G17005} of the segment T_{b}T_{c}
med L_{\_G17005} T_{b} T_{c}

color 200 200 200
drawline L_{\_G17005}
color 0 0 0

color 200 200 200
drawsegment T_{b} T_{c}
color 0 0 0

% Constructing a point P_{\_G17008} which is an image of the point T_{c} in a rotation around the
point T_{b} for the angle angle[_G16931]
rotate P_{\_G17008} T_{b} angle[_G16931] T_{c}
cmark_r P_{\_G17008}
color 200 200 200

```

```

drawarc_p T_{b} T_{c} angle[_G16931]
color 0 0 0

% Constructing a line L_{\_G17011} which passes through point T_{b} and point P_{\_G17008}
line L_{\_G17011} T_{b} P_{\_G17008}

color 200 200 200
drawline L_{\_G17011}
color 0 0 0

% Constructing a point P_{\_G17014} which belongs to line L_{\_G17005} and line L_{\_G17011}
intersec P_{\_G17014} L_{\_G17005} L_{\_G17011}
cmark_r P_{\_G17014}

% Constructing a circle circle[T_{b},T_{c},angle[s[b]][s[c]]] whose center is at point P_{\_G17014}
and which passes through point T_{b}
circle circle[T_{b},T_{c},angle[s[b]][s[c]]] P_{\_G17014} T_{b}

color 200 200 200
drawcircle circle[T_{b},T_{c},angle[s[b]][s[c]]]
color 0 0 0

% NDG: line s_{a} and circle circle[T_{b},T_{c},angle[s[b]][s[c]]] intersect
% Constructing points A_{sa} and I which are in intersection of circle[T_{b},T_{c},angle[s[b]][s[c]]] and s_{a}
intersec2 A_{sa} I circle[T_{b},T_{c},angle[s[b]][s[c]]] s_{a}
cmark_r A_{sa}
cmark_b I

% DET: points I and T_{b} are not the same
% Constructing a line s_{b} which passes through point I and point T_{b}
line s_{b} I T_{b}

color 200 200 200
drawline s_{b}
color 0 0 0

% DET: points T_{c} and I are not the same
% Constructing a line s_{c} which passes through point T_{c} and point I
line s_{c} T_{c} I

color 200 200 200
drawline s_{c}
color 0 0 0

% NDG: lines b and s_{c} are not parallel% DET: lines b and s_{c} are not the same
% Constructing a point C which belongs to line b and line s_{c}
intersec C b s_{c}

```



```
cmark_l C
```

```
% NDG: lines c and s_{b} are not parallel% DET: lines c and s_{b} are not the same
% Constructing a point B which belongs to line c and line s_{b}
intersec B c s_{b}
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines c and s_{b} are not parallel; lines b and s_{c} are not parallel
; line s_{a} and circle circle[T_{b},T_{c},angle[s[b]][s[c]]] intersect; points T_{c} and A are
not the same; points A and T_{b} are not the same; points T_{b} and T_{c} are not the same;
points A and T_{b} are not the same; points T_{c} and A are not the same
% Determination conditions: lines c and s_{b} are not the same; lines b and s_{c} are not the same;
points T_{c} and I are not the same; points I and T_{b} are not the same; points A and T_{c}
are not the same; points A and T_{c} are not the same; points A and T_{b} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_b = T_b$

Proving failed

#### 4.1.3 Proving $T_c = T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{M_{L_{G18192}}^0 T_b P_{G18195}} \neq S_{T_{L_{G18192}}^1 T_b P_{G18195}}$  i.e., lines  $M_{L_{G18192}}^0 T_{L_{G18192}}^1$  and  $T_b P_{G18195}$  are not parallel (construction based assumption)

$S_{AT_c I} \neq S_{T_b T_c I}$  i.e., lines  $AT_b$  and  $T_c I$  are not parallel (construction based assumption)

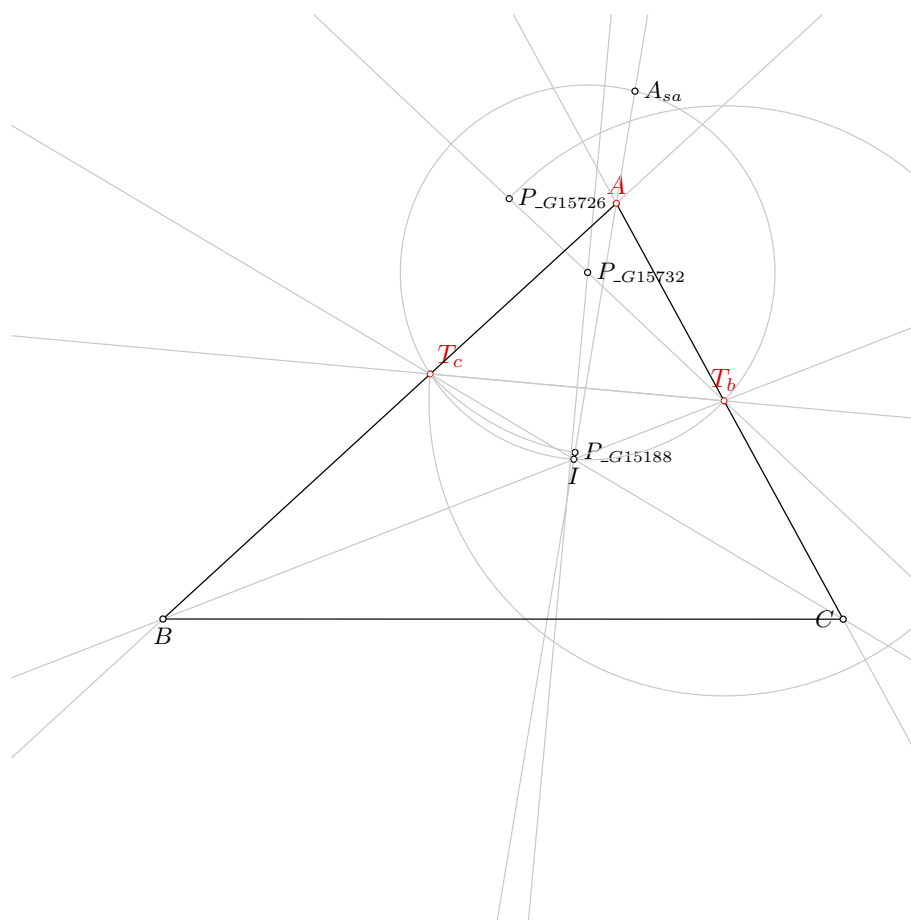


Figure 1: Illustration of the problem 0103

$S_{AIT_b} \neq S_{T_cIT_b}$  i.e., lines  $AT_c$  and  $IT_b$  are not parallel (construction based assumption)  
Total number of proof steps: 1  
Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.2.3 Proving $T_c = \neg T_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $T_b = \neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4235 terms.

**Time Complexity:** Time spent by the prover is 1.820 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_c = \neg T_c$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $A = A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.4.3 Proving $T_c = \neg T_c$

Proving failed

# Problem 104

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 104: Given a point  $A$ , a point  $T_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $T_b$  are not the same;
2. Using the point  $T_b$  and the point  $I$ , construct a line  $s_b$  (rule W02); % DET: points  $T_b$  and  $I$  are not the same;
3. Using the point  $I$  and the line  $b$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $b$ ;
4. Using the circle  $k(I, P_a)$ , the point  $A$ , the point  $I$  and the line  $b$ , construct a line  $c$  (rule W13); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
5. Using the line  $c$  and the line  $s_b$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $s_b$  are not parallel % DET: lines  $c$  and  $s_b$  are not the same;
6. Using the circle  $k(I, P_a)$ , the point  $B$ , the point  $I$  and the line  $c$ , construct a line  $a$  (rule W13); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
7. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; point  $B$  is outside the circle  $k(I, P_a)$ ; lines  $c$  and  $s_b$  are not parallel; point  $A$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $b$ .

Determination conditions: lines  $a$  and  $b$  are not the same; lines  $c$  and  $s_b$  are not the same; points  $T_b$  and  $I$  are not the same; points  $A$  and  $T_b$  are not the same.

Rules used: [W02,W03,W11,W13]

Lemmas used: [D2,D24,D27,GD01,L59,L6,L60,L61]

Solving time: 13.2 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point T_{b} 94.25 68.88
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_t A
cmark_t T_{b}
cmark_b I
color 0 0 0
fontsize 8

% DET: points A and T_{b} are not the same
% Constructing a line b which passes through point A and point T_{b}
line b A T_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points T_{b} and I are not the same
% Constructing a line s_{b} which passes through point T_{b} and point I
line s_{b} T_{b} I

color 200 200 200
drawline s_{b}
color 0 0 0

% NDG: point I is not incident to the line b
% Constructing a point P_{\_G21617} which is a foot of the point I on the line b
foot P_{\_G21617} I b
cmark_r P_{\_G21617}
color 200 200 200
drawline I P_{\_G21617}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
21617}
circle k(I,P_{a}) I P_{\_G21617}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point A is outside the circle k(I,P_{a})
```

```

% Constructing a point  $P_{\{G21824\}}$  which is a foot of the point  $I$  on the line  $b$ 
foot  $P_{\{G21824\}}$  I b
cmark_r  $P_{\{G21824\}}$ 
color 200 200 200
drawline I  $P_{\{G21824\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G21827\}}$  which passes through point  $I$  and point  $A$ 
line  $L_{\{G21827\}}$  I A

color 200 200 200
drawline  $L_{\{G21827\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G21830\}}$  which is an image of the point  $P_{\{G21824\}}$  in the symmetry to
point/line  $L_{\{G21827\}}$ 
sim  $P_{\{G21830\}}$   $L_{\{G21827\}}$   $P_{\{G21824\}}$ 
cmark_r  $P_{\{G21830\}}$ 

% Constructing a line  $c$  which passes through point  $A$  and point  $P_{\{G21830\}}$ 
line c A  $P_{\{G21830\}}$ 

color 200 200 200
drawline c
color 0 0 0

% NDG: lines  $c$  and  $s_{\{b\}}$  are not parallel% DET: lines  $c$  and  $s_{\{b\}}$  are not the same
% Constructing a point  $B$  which belongs to line  $c$  and line  $s_{\{b\}}$ 
intersec B c  $s_{\{b\}}$ 
cmark_b B

% NDG: point  $B$  is outside the circle  $k(I, P_{\{a\}})$ 
% Constructing a point  $P_{\{G22216\}}$  which is a foot of the point  $I$  on the line  $c$ 
foot  $P_{\{G22216\}}$  I c
cmark_r  $P_{\{G22216\}}$ 
color 200 200 200
drawline I  $P_{\{G22216\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G22219\}}$  which passes through point  $I$  and point  $B$ 
line  $L_{\{G22219\}}$  I B

color 200 200 200
drawline  $L_{\{G22219\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G22222\}}$  which is an image of the point  $P_{\{G22216\}}$  in the symmetry to
point/line  $L_{\{G22219\}}$ 
sim  $P_{\{G22222\}}$   $L_{\{G22219\}}$   $P_{\{G22216\}}$ 
cmark_r  $P_{\{G22222\}}$ 

```

```

% Constructing a line a which passes through point B and point P_{\_G22222}
line a B P_{\_G22222}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and b are not parallel; point B is outside the circle k(I,P_{a
}); lines c and s_{b} are not parallel; point A is outside the circle k(I,P_{a}); point I is
not incident to the line b
% Determination conditions: lines a and b are not the same; lines c and s_{b} are not the same;
points T_{b} and I are not the same; points A and T_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_b = \_T_b$

Proving failed

#### 4.1.3 Proving $I = \_I$

Proving failed

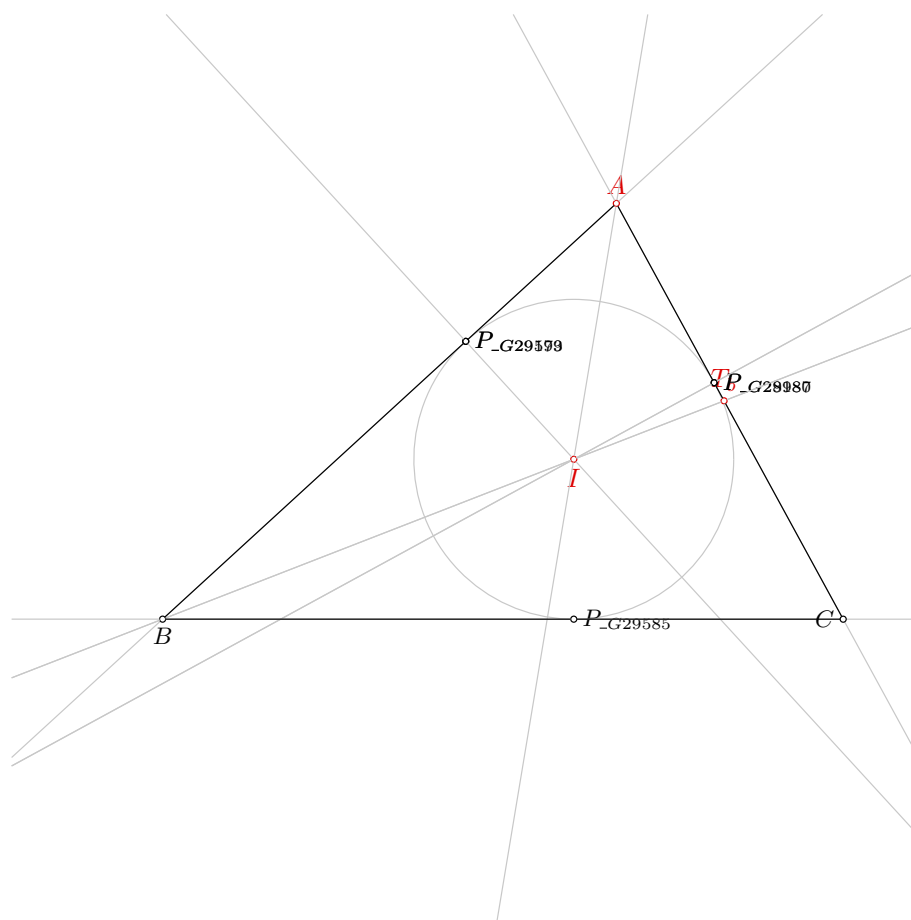


Figure 1: Illustration of the problem 0104



## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{AT_b I} \neq S_{P_{G30331} T_b I}$  i.e., lines  $AP_{G30331}$  and  $T_b I$  are not parallel (construction based assumption)

$S_{BAT_b} \neq S_{P_{G30723} AT_b}$  i.e., lines  $BP_{G30723}$  and  $AT_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $T_b = \neg T_b$

Proving failed

### 4.2.3 Proving $I = \neg I$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $T_b = \neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 16 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.3 Proving $I = \neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 344 terms.

**Time Complexity:** Time spent by the prover is 0.160 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $T_b = \neg T_b$

Proving failed

### 4.4.3 Proving $I = \neg I$

Proving failed

# Problem 105

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 105: Given a point  $A$ , a point  $T_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $A$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $T_c$  are not the same;
2. Using the point  $T_c$  and the point  $I$ , construct a line  $s_c$  (rule W02); % DET: points  $T_c$  and  $I$  are not the same;
3. Using the point  $I$  and the line  $c$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $c$ ;
4. Using the circle  $k(I, P_a)$ , the point  $A$ , the point  $I$  and the line  $c$ , construct a line  $b$  (rule W13); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
5. Using the line  $b$  and the line  $s_c$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $s_c$  are not parallel % DET: lines  $b$  and  $s_c$  are not the same;
6. Using the circle  $k(I, P_a)$ , the point  $C$ , the point  $I$  and the line  $b$ , construct a line  $a$  (rule W13); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
7. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; point  $C$  is outside the circle  $k(I, P_a)$ ; lines  $b$  and  $s_c$  are not parallel; point  $A$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $c$ .

Determination conditions: lines  $a$  and  $c$  are not the same; lines  $b$  and  $s_c$  are not the same; points  $T_c$  and  $I$  are not the same; points  $A$  and  $T_c$  are not the same.

Rules used: [W02,W03,W11,W13]

Lemmas used: [D25,D27,GD01,L2,L59,L60,L61,L7]

Solving time: 13.1 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point T_{c} 55.38 72.43
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_t A
cmark_rt T_{c}
cmark_b I
color 0 0 0
fontsize 8

% DET: points A and T_{c} are not the same
% Constructing a line c which passes through point A and point T_{c}
line c A T_{c}

color 200 200 200
drawline c
color 0 0 0

% DET: points T_{c} and I are not the same
% Constructing a line s_{c} which passes through point T_{c} and point I
line s_{c} T_{c} I

color 200 200 200
drawline s_{c}
color 0 0 0

% NDG: point I is not incident to the line c
% Constructing a point P_{\_G25853} which is a foot of the point I on the line c
foot P_{\_G25853} I c
cmark_r P_{\_G25853}
color 200 200 200
drawline I P_{\_G25853}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
25853}
circle k(I,P_{a}) I P_{\_G25853}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point A is outside the circle k(I,P_{a})
```

```

% Constructing a point  $P_{\{G26060\}}$  which is a foot of the point  $I$  on the line  $c$ 
foot  $P_{\{G26060\}}$  I c
cmark_r  $P_{\{G26060\}}$ 
color 200 200 200
drawline I  $P_{\{G26060\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G26063\}}$  which passes through point  $I$  and point  $A$ 
line  $L_{\{G26063\}}$  I A

color 200 200 200
drawline  $L_{\{G26063\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G26066\}}$  which is an image of the point  $P_{\{G26060\}}$  in the symmetry to
point/line  $L_{\{G26063\}}$ 
sim  $P_{\{G26066\}}$   $L_{\{G26063\}}$   $P_{\{G26060\}}$ 
cmark_r  $P_{\{G26066\}}$ 

% Constructing a line  $b$  which passes through point  $A$  and point  $P_{\{G26066\}}$ 
line b A  $P_{\{G26066\}}$ 

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $b$  and  $s_{\{c\}}$  are not parallel% DET: lines  $b$  and  $s_{\{c\}}$  are not the same
% Constructing a point  $C$  which belongs to line  $b$  and line  $s_{\{c\}}$ 
intersec C b  $s_{\{c\}}$ 
cmark_l C

% NDG: point  $C$  is outside the circle  $k(I, P_{\{a\}})$ 
% Constructing a point  $P_{\{G26452\}}$  which is a foot of the point  $I$  on the line  $b$ 
foot  $P_{\{G26452\}}$  I b
cmark_r  $P_{\{G26452\}}$ 
color 200 200 200
drawline I  $P_{\{G26452\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G26455\}}$  which passes through point  $I$  and point  $C$ 
line  $L_{\{G26455\}}$  I C

color 200 200 200
drawline  $L_{\{G26455\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G26458\}}$  which is an image of the point  $P_{\{G26452\}}$  in the symmetry to
point/line  $L_{\{G26455\}}$ 
sim  $P_{\{G26458\}}$   $L_{\{G26455\}}$   $P_{\{G26452\}}$ 
cmark_r  $P_{\{G26458\}}$ 

```

```

% Constructing a line a which passes through point C and point P_{\_G26458}
line a C P_{\_G26458}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and c are not parallel; point C is outside the circle k(I,P_{a
}); lines b and s_{c} are not parallel; point A is outside the circle k(I,P_{a}); point I is
not incident to the line c
% Determination conditions: lines a and c are not the same; lines b and s_{c} are not the same;
points T_{c} and I are not the same; points A and T_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $A=A$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_c = \_T_c$

Proving failed

#### 4.1.3 Proving $I = \_I$

Proving failed

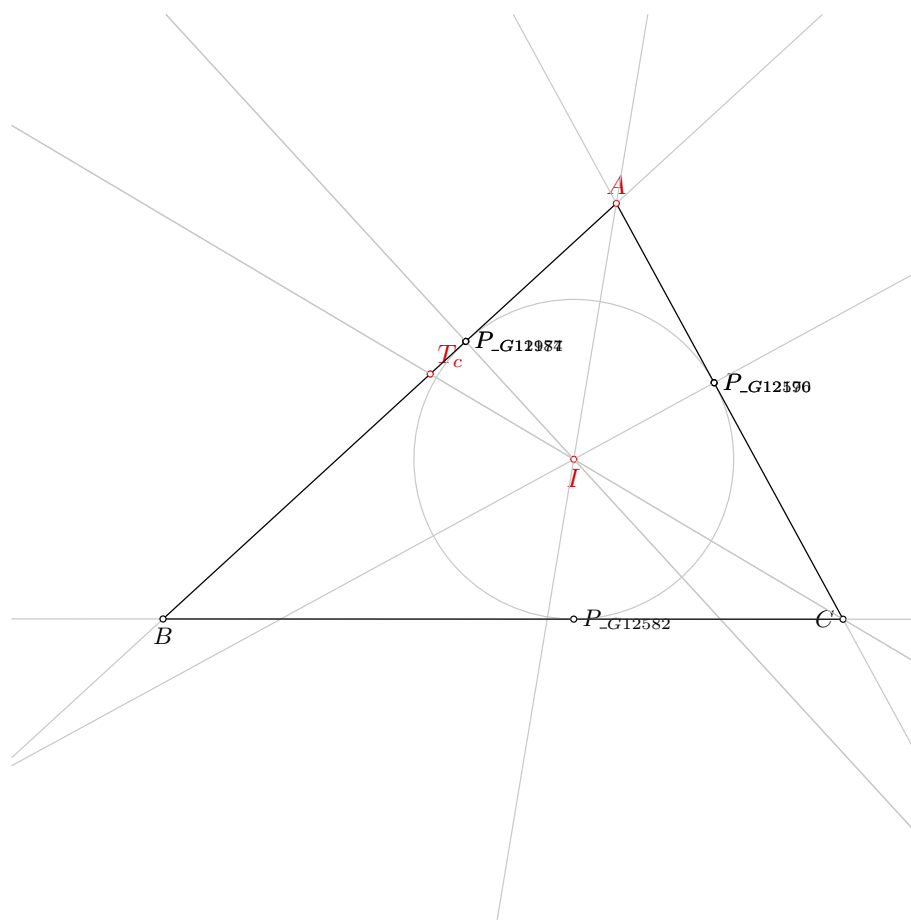


Figure 1: Illustration of the problem 0105

## 4.2 GCLC - Area method

### 4.2.1 Proving $A=A$

NDG conditions are:

$S_{AT_c I} \neq S_{P_{G13328} T_c I}$  i.e., lines  $AP_{G13328}$  and  $T_c I$  are not parallel (construction based assumption)

$S_{CAT_c} \neq S_{P_{G13720} AT_c}$  i.e., lines  $CP_{G13720}$  and  $AT_c$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $T_c = \neg T_c$

Proving failed

### 4.2.3 Proving $I = \neg I$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.2 Proving $T_c = \neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 16 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.3 Proving $I = \neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 36 terms.

**Time Complexity:** Time spent by the prover is 0.030 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $A=A$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4.2 Proving $T_c = \neg T_c$

Proving failed

### 4.4.3 Proving $I = \neg I$

Proving failed

# Problem 106

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 106: Given a point  $C$ , a point  $O$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
2. Choose freely a point  $B$  on the circle  $k(O, C)$  (rule WOncircle);
3. Choose freely a point  $A$  on the circle  $k(O, C)$  (rule WOncircle).

Non-degenerate conditions: points  $C$  and  $O$  are not the same.

Determination conditions: .

Rules used: [W06,WOncircle1]

Lemmas used: [D26]

Solving time: 263.8 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point O 65 51.14
point B 20 40

color 220 0 0
fontsize 9

cmark_l C
cmark_t O
cmark_b B
color 0 0 0
```



```

fontsize 8

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Choosing randomly a point B on the circle with center O through point C
oncircle B O C
cmark_b B
color 200 200 200
drawcircle O C
color 0 0 0

% Choosing randomly a point A on the circle with center O through point B
oncircle A O B
cmark_t A
color 200 200 200
drawcircle O B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: points C and O are not the same
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

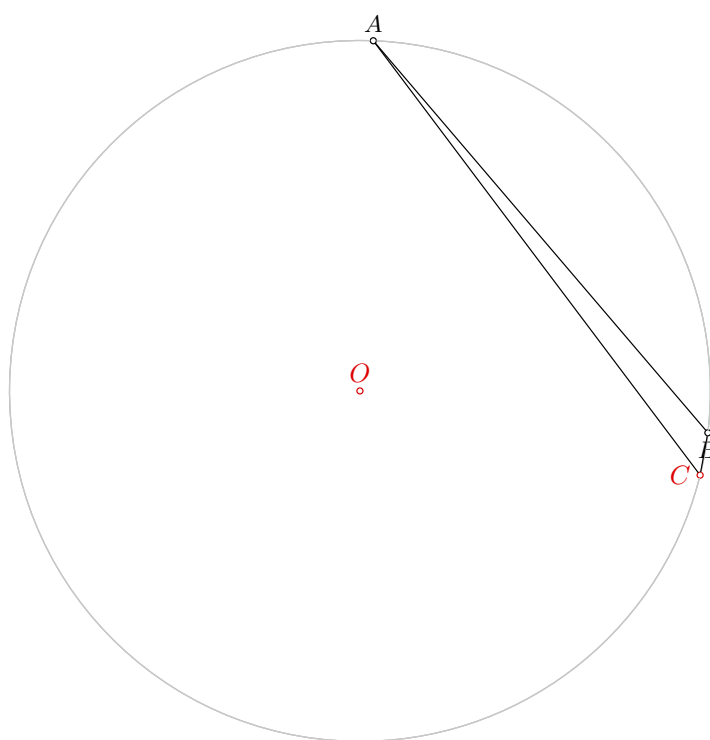


Figure 1: Illustration of the problem 0106

### 4.1.2 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 31 terms.

**Time Complexity:** Time spent by the prover is 0.368 seconds.

**NDG conditions** Points  $A$ ,  $B$  and  $C$  are not collinear

Line through points  $O$  and  $C$  is not perpendicular to line through points  $C$  and  $B$

### 4.1.3 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{_M_aBC} \neq 0$  i.e., points  $_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$  i.e., points  $_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a-M_bF_{_m_b}^1} \neq S_{F_{_m_a}^0-M_bF_{_m_b}^1}$  i.e., lines  $_M_aF_{_m_a}^0$  and  $_M_bF_{_m_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $O=_O$

Proving failed

### 4.2.3 Proving $B=B$

NDG conditions are:

$S_{_M_aBC} \neq 0$  i.e., points  $_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$  i.e., points  $_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a-M_bF_{_m_b}^1} \neq S_{F_{_m_a}^0-M_bF_{_m_b}^1}$  i.e., lines  $_M_aF_{_m_a}^0$  and  $_M_bF_{_m_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $O=_O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 175 terms.

**Time Complexity:** Time spent by the prover is 0.150 seconds. There are no ndg conditions.

#### 4.3.3 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $O=_O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 42 terms.

**Time Complexity:** Time spent by the prover is 0.230 seconds. There are no ndg conditions.

#### 4.4.3 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 107

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 107: Given a point  $C$ , a point  $M_a$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is redundant.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
2. Choose freely a point  $A$  (rule free).

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01,free]

Lemmas used: [D21,GL03,GL04]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point M_{a} 65 40
point B 20 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_r M_{a}
cmark_b B
color 0 0 0
fontsize 8
```

```
% Constructing a point B such that CB/CM_{a}=2
```

```

towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% Constructing a free point A
point A 80 95

cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.033 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a = -M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.053 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.023 seconds.

**NDG conditions** There are no NDG conditions for this theorem

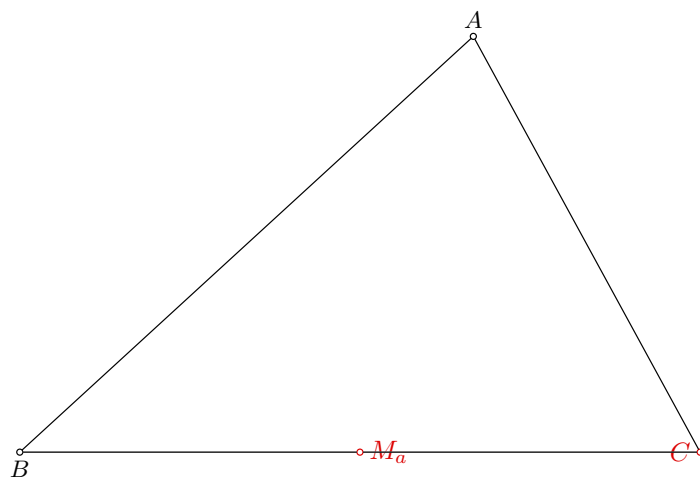


Figure 1: Illustration of the problem 0107

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

There are no ndg conditions.  
Total number of proof steps: 1  
Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_a=-M_a$

There are no ndg conditions.  
Total number of proof steps: 42  
Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $B=B$

There are no ndg conditions.  
Total number of proof steps: 1  
Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_a=-M_a$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $B=B$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $M_a=-M_a$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.



#### 4.4.3 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 108

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 108: Given a point  $B$ , a point  $C$  and a point  $M_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D22, GL03]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
```

```
point C 110 40
```

```
point M_{b} 95 67.5
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_b B
```

```
cmark_l C
```

```
cmark_lt M_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point A such that CA/CM_{b}=2
```

```
towards A C M_{b} 2
```

```
cmark_t A
```

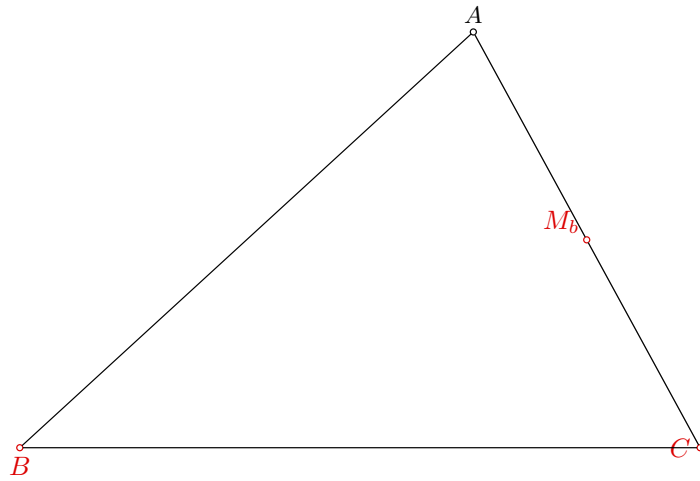


Figure 1: Illustration of the problem 0108

```
color 200 200 200
drawsegment C A
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions:
% Determination conditions:
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.035 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.034 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $M_b=_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.052 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

There are no ndg conditions.

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $C=C$

There are no ndg conditions.

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $M_b=_M_b$

There are no ndg conditions.

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $M_b=\neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 19 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_b=\neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 42 terms.

**Time Complexity:** Time spent by the prover is 0.090 seconds. There are no ndg conditions.

# Problem 109

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 109: Given a point  $B$ , a point  $C$  and a point  $M_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D20,GL03,GL04]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point C 110 40
point M_{c} 50 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_l C
cmark_lt M_{c}
color 0 0 0
fontsize 8
```

```
% Constructing a point A such that BA/BM_{c}=2
towards A B M_{c} 2
cmark_t A
```

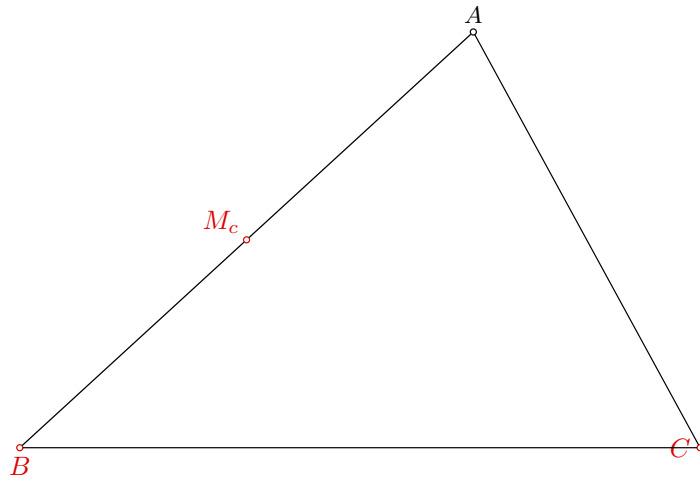


Figure 1: Illustration of the problem 0109

```
color 200 200 200
drawsegment B A
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions:
% Determination conditions:
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.034 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.033 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $M_c=_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.056 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

There are no ndg conditions.

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $C=C$

There are no ndg conditions.

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $M_c=_M_c$

There are no ndg conditions.

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.



#### 4.3.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $M_c=_M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_c=_M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

# Problem 110

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 110: Given a point  $B$ , a point  $C$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $G$ , construct a point  $M_c$  (rule W01); ;
2. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D20,GL03,GL04,L57]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point C 110 40
point G 70 58.33
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_l C
cmark_t G
color 0 0 0
fontsize 8
```

```
% Constructing a point  $M_{\{c\}}$  such that  $CM_{\{c\}}/CG=1.5$ 
```

```

towards M_{c} C G 1.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% Constructing a point A such that BA/BM_{c}=2
towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.024 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $G=_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 23 terms.

**Time Complexity:** Time spent by the prover is 0.397 seconds.

**NDG conditions** Line through points  $A$  and  $\neg M_a$  is not parallel with line through points  $B$  and  $\neg M_b$   
Points  $A$  and  $B$  are not identical

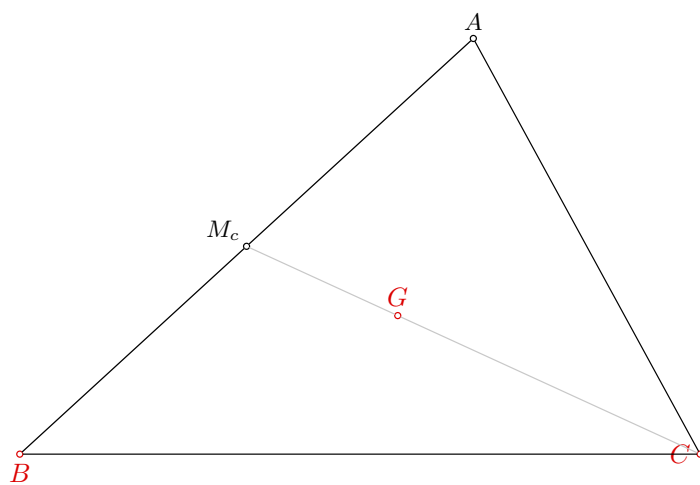


Figure 1: Illustration of the problem 0110

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{AB\_M_b} \neq S_{M_aB\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $C=C$

NDG conditions are:

$S_{AB\_M_b} \neq S_{M_aB\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $G=_G$

NDG conditions are:

$S_{AB\_M_b} \neq S_{M_aB\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (cancellation assumption)

Total number of proof steps: 679

Time spent by the prover: 0.210 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $G=_G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 33 terms.

**Time Complexity:** Time spent by the prover is 0.130 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $G=_G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 305 terms.

**Time Complexity:** Time spent by the prover is 3.550 seconds. There are no ndg conditions.

# Problem 111

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 111: Given a point  $C$ , a point  $H_a$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $H_a$  are not the same;
2. Choose freely a point  $B$  on the line  $a$  (rule WOnline1) ;
3. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
4. Choose freely a point  $A$  on the line  $h_a$  (rule WOnline2).

Non-degenerate conditions: .

Determination conditions: points  $C$  and  $H_a$  are not the same.

Rules used: [W02,W10b,WOnline1,WOnline2]

Lemmas used: [D5,D8,GD01]

Solving time: 154.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{a} 80 40
point B 20 40

color 220 0 0
fontsize 9

cmark_l C
cmark_r H_{a}
```

```

cmark_b B
color 0 0 0
fontsize 8

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0


% Choosing randomly a point B on the line CH_{a}
online B C H_{a}
cmark_b B
color 200 200 200
drawline C H_{a}
color 0 0 0


% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0


% Generating random value V[_G15230]
random V[_G15230]


% Calculating value V[_G15251] using formula V[_G15230]*20
expression V[_G15251] { V[_G15230]*20 }


% Constructing a point A which is a point for which holds  $H_{a}A = V[_G15251]$  and angle  $BH_{a}A = 90$ 
turtle A B H_{a} 90 V[_G15251]
cmark_t A


drawsegment A B
drawsegment A C
drawsegment B C


% Non-degenerate conditions:
% Determination conditions: points C and H_{a} are not the same

```



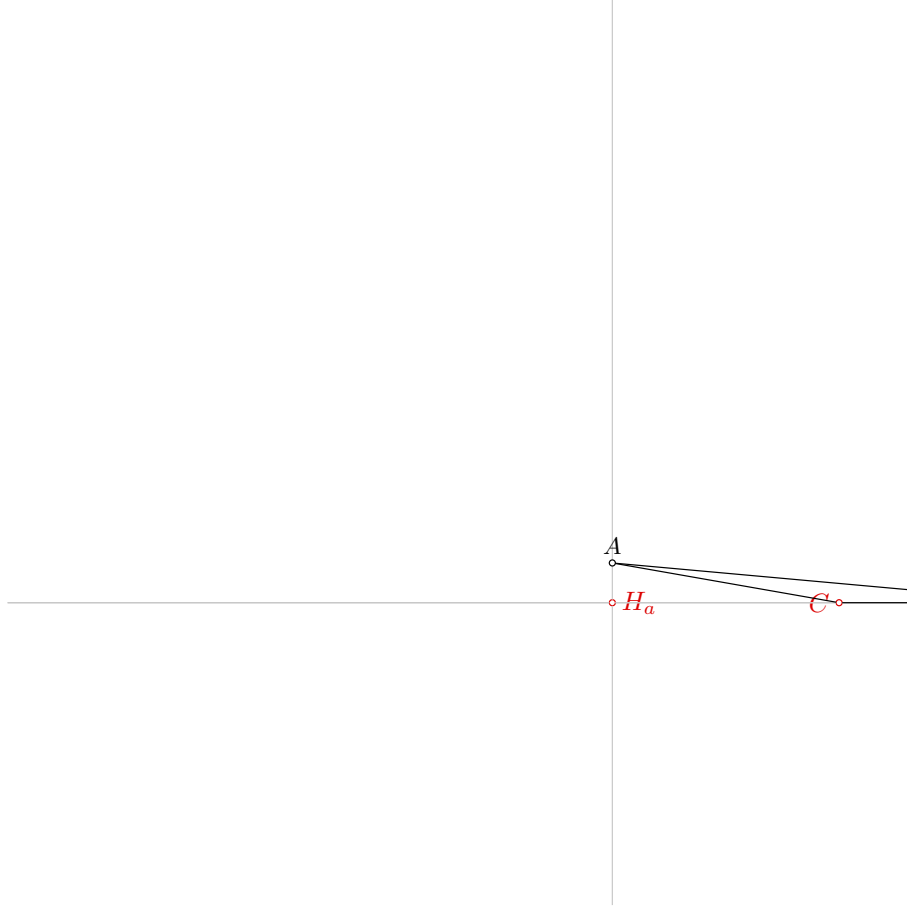


Figure 1: Illustration of the problem 0111

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a=H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.008 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.1.3 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF^2_{\neg h_a}} \neq S_{CAF^2_{\neg h_a}}$  i.e., lines  $BC$  and  $AF^2_{\neg h_a}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_a=\neg H_a$

Proving failed

### 4.2.3 Proving $B=B$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF^2_{\neg h_a}} \neq S_{CAF^2_{\neg h_a}}$  i.e., lines  $BC$  and  $AF^2_{\neg h_a}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_a=\neg H_a$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 31.730 seconds. There are no ndg conditions.

### 4.4.3 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 112

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 112: Given a point  $C$ , a point  $H_b$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $H_b$  are not the same;
2. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
3. Choose freely a point  $B$  on the line  $h_b$  (rule WOnline2);
4. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) .

Non-degenerate conditions: .

Determination conditions: points  $C$  and  $H_b$  are not the same.

Rules used: [W02,W10b,WOnline1,WOnline2]

Lemmas used: [D6,D9,GD01]

Solving time: 865.5 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{b} 89.36 77.83
point B 20 40

color 220 0 0
fontsize 9

cmark_1 C
cmark_1 H_{b}
```

```

cmark_b B
color 0 0 0
fontsize 8

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0

% Generating random value V[_G28276]
random V[_G28276]

% Calculating value V[_G28297] using formula V[_G28276]*20
expression V[_G28297] { V[_G28276]*20 }

% Constructing a point B which is a point for which holds H_{b}B = V[_G28297] and angle CH_{b}B =
90
turtle B C H_{b} 90 V[_G28297]
cmark_b B

% Choosing randomly a point A on the line CH_{b}
online A C H_{b}
cmark_t A
color 200 200 200
drawline C H_{b}
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions: points C and H_{b} are not the same

```

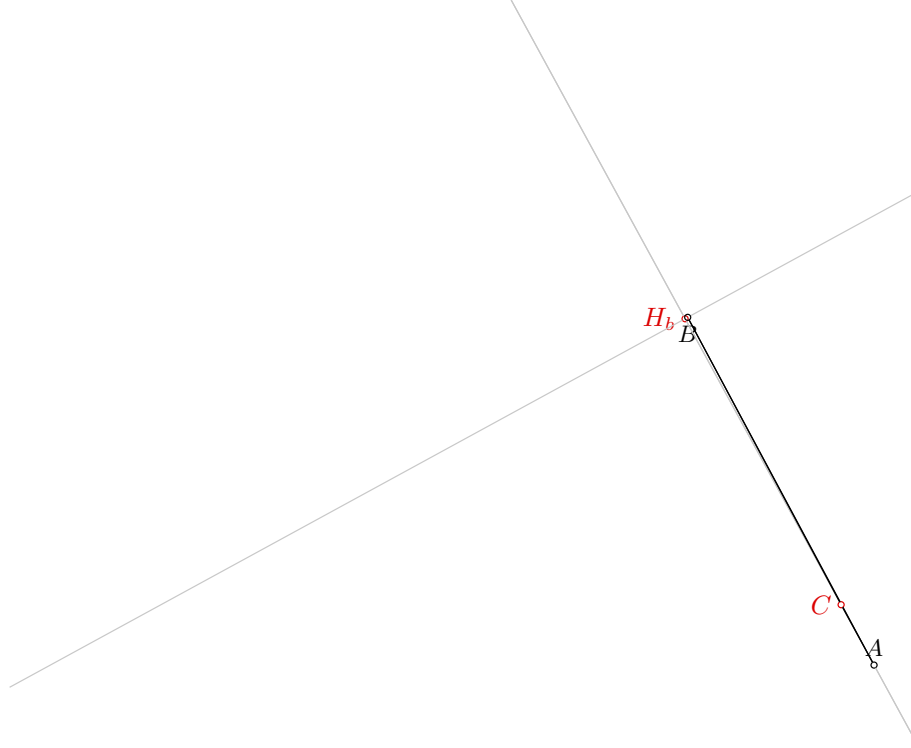


Figure 1: Illustration of the problem 0112

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b=H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.008 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.1.3 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^2_{\neg h_b}} \neq S_{CBF^2_{\neg h_b}}$  i.e., lines  $AC$  and  $BF^2_{\neg h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_b=\neg H_b$

Proving failed

### 4.2.3 Proving $B=B$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_b=\neg H_b$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $B=B$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 31.480 seconds. There are no ndg conditions.

#### 4.4.3 Proving $B = B$

Proving failed



# Problem 113

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 113: Given a point  $C$ , a point  $H_c$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_c$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H_c$  are not the same;
2. Using the point  $H_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
3. Choose freely a point  $B$  on the line  $c$  (rule WOnline2);
4. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) .

Non-degenerate conditions: .

Determination conditions: points  $C$  and  $H_c$  are not the same.

Rules used: [W02,W10a,WOnline1,WOnline2]

Lemmas used: [D10,D7]

Solving time: 823.8 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{c} 68.91 84.83
point B 20 40

color 220 0 0
fontsize 9

cmark_l C
cmark_rt H_{c}
```

```

cmark_b B
color 0 0 0
fontsize 8

% DET: points C and H_{c} are not the same
% Constructing a line h_{c} which passes through point C and point H_{c}
line h_{c} C H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0

% Generating random value V[_G19079]
random V[_G19079]

% Calculating value V[_G19100] using formula V[_G19079]*20
expression V[_G19100] { V[_G19079]*20 }

% Constructing a point B which is a point for which holds  $H_{c}B = V[_G19100]$  and angle  $CH_{c}B = 90$ 
turtle B C H_{c} 90 V[_G19100]
cmark_b B

% Choosing randomly a point A on the line BH_{c}
online A B H_{c}
cmark_t A
color 200 200 200
drawline B H_{c}
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions: points C and H_{c} are not the same

```

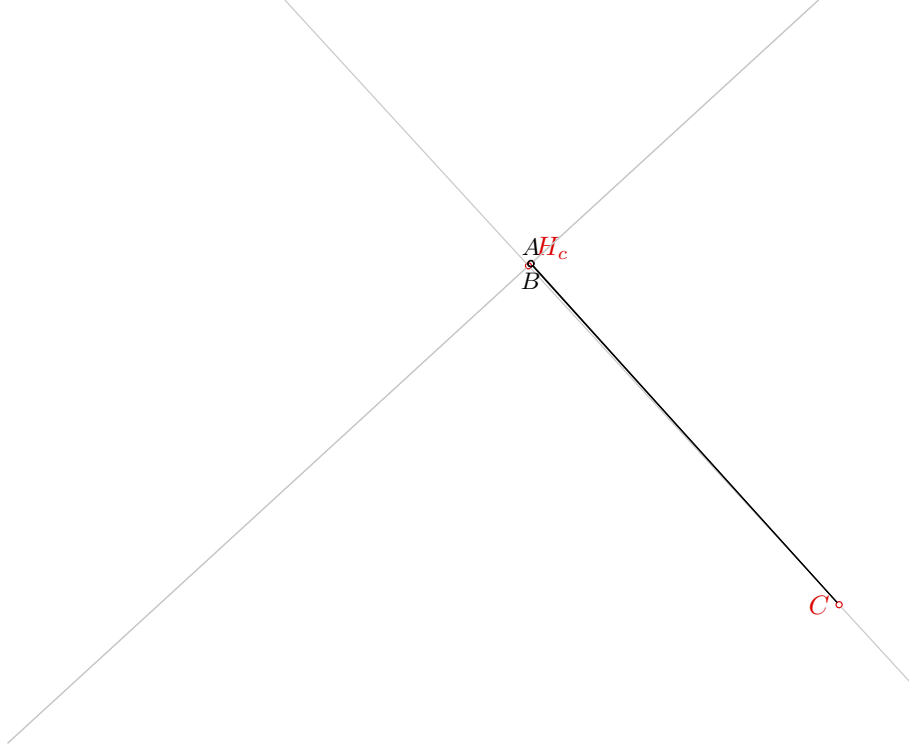


Figure 1: Illustration of the problem 0113

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c=H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.024 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.1.3 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.017 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{-h_c}^2} \neq S_{BCF_{-h_c}^2}$  i.e., lines  $AB$  and  $CF_{-h_c}^2$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_c=_Hc$

Proving failed

### 4.2.3 Proving $B=B$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

Proving failed

### 4.3.2 Proving $H_c=_Hc$

Proving failed

### 4.3.3 Proving $B=B$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

Proving failed

### 4.4.2 Proving $H_c=_Hc$

Proving failed

### 4.4.3 Proving $B=B$

Proving failed

# Problem 114

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 114: Given a point  $B$ , a point  $C$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $C$ , construct a point  $M_a$  (rule W01); ;
2. Using the point  $B$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H$  are not the same;
3. Using the point  $C$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H$  are not the same;
4. Using the point  $B$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $B$  and  $M_a$  are not the same;
5. Using the circle  $k(M_a, B)$ , the line  $h_b$ , the point  $M_a$  and the point  $B$ , construct a point  $H_b$  (rule W05); % NDG: line  $h_b$  and circle  $k(M_a, B)$  intersect % DET: points  $B$  and  $H_b$  must be different;
6. Using the point  $H_b$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $C$  are not the same;
7. Using the circle  $k(M_a, B)$ , the line  $h_c$ , the point  $M_a$  and the point  $C$ , construct a point  $H_c$  (rule W05); % NDG: line  $h_c$  and circle  $k(M_a, B)$  intersect % DET: points  $C$  and  $H_c$  must be different;
8. Using the point  $H_c$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $B$  are not the same;
9. Using the line  $b$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $c$  are not parallel % DET: lines  $b$  and  $c$  are not the same.

Non-degenerate conditions: lines  $b$  and  $c$  are not parallel; line  $h_c$  and circle  $k(M_a, B)$  intersect; line  $h_b$  and circle  $k(M_a, B)$  intersect; points  $B$  and  $M_a$  are not the same.

Determination conditions: lines  $b$  and  $c$  are not the same; points  $H_c$  and  $B$  are not the same; points  $C$  and  $H_c$  must be different; points  $H_b$  and  $C$  are not the same; points  $B$  and  $H_b$  must be different; points  $C$  and  $H$  are not the same; points  $B$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D21,D3,D6,D7,D9,GD01,GD02,L3,L37,L38,L39]

Solving time: 1.8 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
```

```
point C 110 40
```

```
point H 80 72.73
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_b B
```

```
cmark_l C
```

```
cmark_rt H
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point M_{a} such that BM_{a}/BC=0.5
```

```
towards M_{a} B C 0.5
```

```
cmark_r M_{a}
```

```
color 200 200 200
```

```
drawsegment B C
```

```
color 0 0 0
```

```
% DET: points B and H are not the same
```

```
% Constructing a line h_{b} which passes through point B and point H
```

```
line h_{b} B H
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```
% DET: points C and H are not the same
```

```
% Constructing a line h_{c} which passes through point C and point H
```

```
line h_{c} C H
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```

% NDG: points B and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point B
circle k(M_{a},B) M_{a} B

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line h_{b} and circle k(M_{a},B) intersect% DET: points B and H_{b} must be different
% Constructing a point P_{\_G18585} which is a foot of the point M_{a} on the line h_{b}
foot P_{\_G18585} M_{a} h_{b}
cmark_r P_{\_G18585}
color 200 200 200
drawline M_{a} P_{\_G18585}
color 0 0 0

% Constructing a point H_{b} which is an image of the point B in the symmetry to point/line P_{\_G
18585}
sim H_{b} P_{\_G18585} B
cmark_l H_{b}

% DET: points H_{b} and C are not the same
% Constructing a line b which passes through point H_{b} and point C
line b H_{b} C

color 200 200 200
drawline b
color 0 0 0

% NDG: line h_{c} and circle k(M_{a},B) intersect% DET: points C and H_{c} must be different
% Constructing a point P_{\_G18823} which is a foot of the point M_{a} on the line h_{c}
foot P_{\_G18823} M_{a} h_{c}
cmark_r P_{\_G18823}
color 200 200 200
drawline M_{a} P_{\_G18823}
color 0 0 0

% Constructing a point H_{c} which is an image of the point C in the symmetry to point/line P_{\_G
18823}
sim H_{c} P_{\_G18823} C
cmark_rt H_{c}

% DET: points H_{c} and B are not the same
% Constructing a line c which passes through point H_{c} and point B
line c H_{c} B

color 200 200 200
drawline c
color 0 0 0

```

```

% NDG: lines b and c are not parallel% DET: lines b and c are not the same
% Constructing a point A which belongs to line b and line c
intersec A b c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and c are not parallel; line h_{c} and circle k(M_{a},B)
% intersect; line h_{b} and circle k(M_{a},B) intersect; points B and M_{a} are not the same
% Determination conditions: lines b and c are not the same; points H_{c} and B are not the same;
% points C and H_{c} must be different; points H_{b} and C are not the same; points B and H_{b}
% must be different; points C and H are not the same; points B and H are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $H=H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 36 terms.

**Time Complexity:** Time spent by the prover is 1.933 seconds.

**NDG conditions** Points  $B$ ,  $C$  and  $H$  are not collinear

Points  $B$ ,  $C$  and  $H$  are not collinear

Points  $B$ ,  $C$  and  $H$  are not collinear



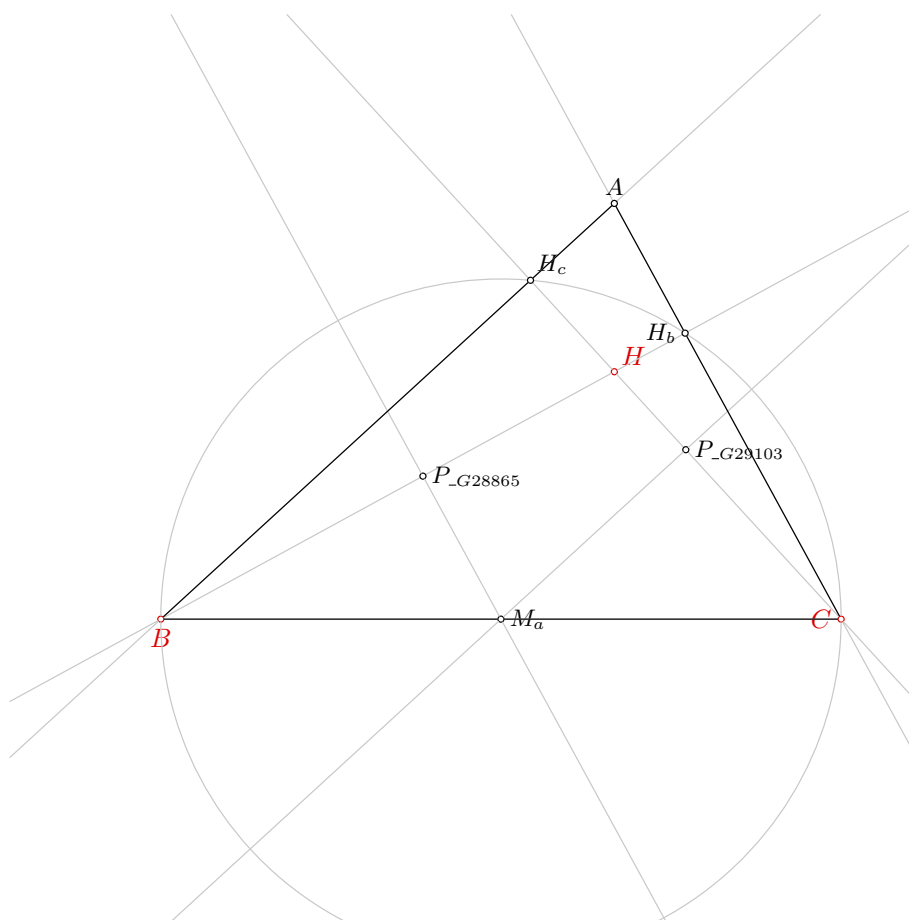


Figure 1: Illustration of the problem 0114

Points  $B$ ,  $C$  and  $H$  are not collinear

Line through points  $H_b$  and  $C$  is not parallel with line through points  $B$  and  $H_c$

Points  $H_b$ ,  $B$  and  $C$  are not collinear

Points  $A$  and  $B$  are not identical

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_bH_cB} \neq S_{CH_cB}$  i.e., lines  $H_bC$  and  $H_cB$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$  i.e., lines  $AF_{-h_a}^0$  and  $BF_{-h_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $C=C$

NDG conditions are:

$S_{H_bH_cB} \neq S_{CH_cB}$  i.e., lines  $H_bC$  and  $H_cB$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$  i.e., lines  $AF_{-h_a}^0$  and  $BF_{-h_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $H=_H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $H=_H$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 9 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.3 Proving $H=_H$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 115

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 115: Given a point  $C$ , a point  $T_a$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $T_a$  are not the same;
2. Choose freely a point  $B$  on the line  $a$  (rule WOnline1) ;
3. Using the point  $B$ , the point  $C$ , the point  $T_a$  and the line  $a$ , construct a point  $T'_a$  (rule W19);  
% NDG: points  $B$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $\text{midpoint}([B, T_a])$  are not the same;
4. Choose freely a point  $A$  on the circle  $k_{over}(T_a, T'_a)$  (rule WOncircle).

Non-degenerate conditions: points  $B$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $\text{midpoint}([B, T_a])$  are not the same.

Determination conditions: points  $C$  and  $T_a$  are not the same.

Rules used: [W02, W19, WOncircle2, WOnline1]

Lemmas used: [D23, GD01, L77]

Solving time: 88.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point T_{a} 70.86 40
point B 20 40

color 220 0 0
fontsize 9
```

```

cmark_l C
cmark_rb T_{a}
cmark_b B
color 0 0 0
fontsize 8

% DET: points C and T_{a} are not the same
% Constructing a line a which passes through point C and point T_{a}
line a C T_{a}

color 200 200 200
drawline a
color 0 0 0

% Choosing randomly a point B on the line CT_{a}
online B C T_{a}
cmark_b B
color 200 200 200
drawline C T_{a}
color 0 0 0

% NDG: points B and C are not the same; points C and T_{a} are not the same; points C and midpoint
% ([B,T_{a}]) are not the same
% Constructing a point P_{\_G15750} which is an image of the point C in a rotation around the point
% T_{a} for the angle 90
rotate P_{\_G15750} T_{a} 90 C
cmark_r P_{\_G15750}
color 200 200 200
drawarc_p T_{a} C 90
color 0 0 0

% Constructing a line L_{\_G15753} which passes through point T_{a} and point P_{\_G15750}
line L_{\_G15753} T_{a} P_{\_G15750}

color 200 200 200
drawline L_{\_G15753}
color 0 0 0

% Constructing midpoint P_{\_G15756} of the segment CP_{\_G15750}
midpoint P_{\_G15756} C P_{\_G15750}
cmark_r P_{\_G15756}

% Constructing a line L_{\_G15759} which passes through point B and point P_{\_G15756}
line L_{\_G15759} B P_{\_G15756}

color 200 200 200
drawline L_{\_G15759}
color 0 0 0

```

```

% Constructing a line  $L_{G15762}$  which passes through point  $C$  and point  $P_{G15756}$ 
line  $L_{G15762}$   $C$   $P_{G15756}$ 

color 200 200 200
drawline  $L_{G15762}$ 
color 0 0 0

% Constructing a point  $P_{G15765}$  which belongs to line  $L_{G15753}$  and line  $L_{G15759}$ 
intersec  $P_{G15765}$   $L_{G15753}$   $L_{G15759}$ 
cmark_r  $P_{G15765}$ 

% Constructing a point  $P_{G15768}$  which belongs to line  $L_{G15753}$  and line  $L_{G15762}$ 
intersec  $P_{G15768}$   $L_{G15753}$   $L_{G15762}$ 
cmark_r  $P_{G15768}$ 

% Constructing a line  $L_{G15771}$  which passes through point  $B$  and point  $P_{G15768}$ 
line  $L_{G15771}$   $B$   $P_{G15768}$ 

color 200 200 200
drawline  $L_{G15771}$ 
color 0 0 0

% Constructing a line  $L_{G15774}$  which passes through point  $C$  and point  $P_{G15765}$ 
line  $L_{G15774}$   $C$   $P_{G15765}$ 

color 200 200 200
drawline  $L_{G15774}$ 
color 0 0 0

% Constructing a point  $P_{G15777}$  which belongs to line  $L_{G15771}$  and line  $L_{G15774}$ 
intersec  $P_{G15777}$   $L_{G15771}$   $L_{G15774}$ 
cmark_r  $P_{G15777}$ 

% Constructing a line  $L_{G15780}$  which passes through point  $P_{G15756}$  and point  $P_{G15777}$ 
line  $L_{G15780}$   $P_{G15756}$   $P_{G15777}$ 

color 200 200 200
drawline  $L_{G15780}$ 
color 0 0 0

% Constructing a point  $T'_a$  which belongs to line  $L_{G15780}$  and line  $a$ 
intersec  $T'_a$   $L_{G15780}$   $a$ 
cmark_r  $T'_a$ 

% Constructing midpoint  $P_{G16605}$  of the segment  $T_aT'_a$ 
midpoint  $P_{G16605}$   $T_a$   $T'_a$ 
cmark_r  $P_{G16605}$ 

% Choosing randomly a point  $A$  on the circle with center  $P_{G16605}$  through point  $T_a$ 
oncircle  $A$   $P_{G16605}$   $T_a$ 

```

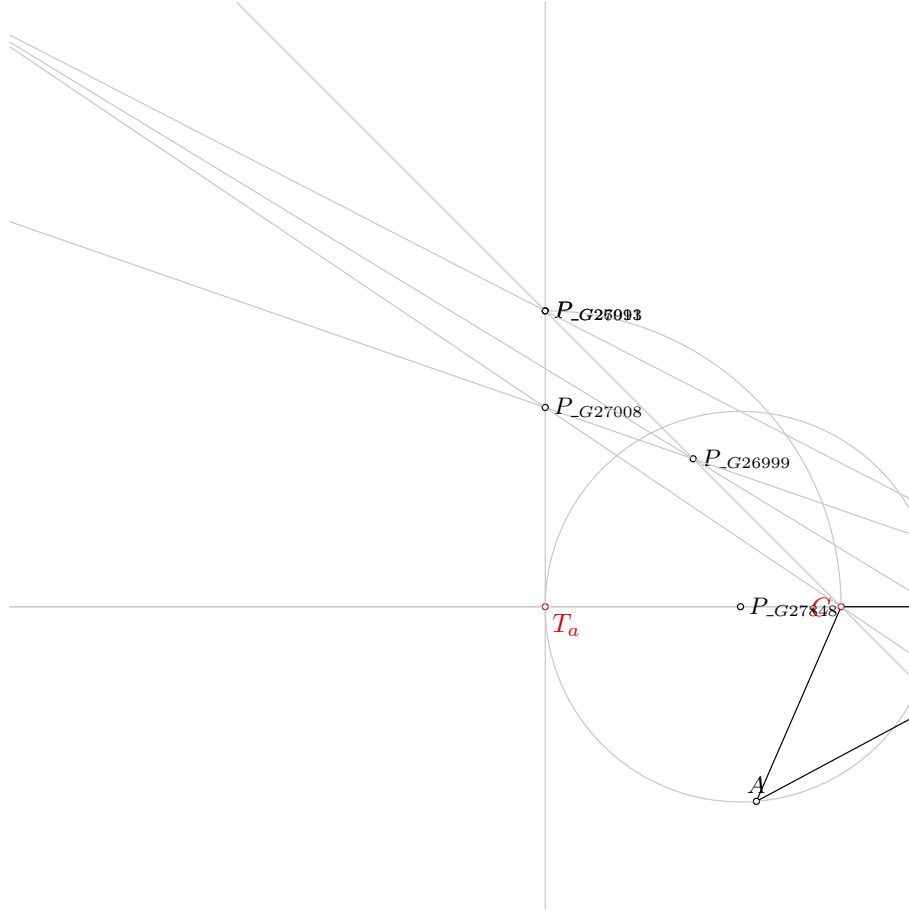


Figure 1: Illustration of the problem 0115

```

cmark_t A
color 200 200 200
drawcircle P_{\_G16605} T_{a}
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions: points B and C are not the same; points C and T_{a} are not the same;
%                             points C and midpoint([B,T_{a}]) are not the same
% Determination conditions: points C and T_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_a=T_a$

Proving failed

#### 4.1.3 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.017 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{T_aBP_{G28475}} \neq S_{P_{G28469}BP_{G28475}}$  i.e., lines  $T_aP_{G28469}$  and  $BP_{G28475}$  are not parallel (construction based assumption)

$S_{T_aCP_{G28475}} \neq S_{P_{G28469}CP_{G28475}}$  i.e., lines  $T_aP_{G28469}$  and  $CP_{G28475}$  are not parallel (construction based assumption)

$S_{BCP_{G28484}} \neq S_{P_{G28487}CP_{G28484}}$  i.e., lines  $BP_{G28487}$  and  $CP_{G28484}$  are not parallel (construction based assumption)

$S_{P_{G28475}CT_a} \neq S_{P_{G28496}CT_a}$  i.e., lines  $P_{G28475}P_{G28496}$  and  $CT_a$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $T_a=T_a$

Proving failed

#### 4.2.3 Proving $B=B$

NDG conditions are:

$S_{T_aBP_{G11459}} \neq S_{P_{G11453}BP_{G11459}}$  i.e., lines  $T_aP_{G11453}$  and  $BP_{G11459}$  are not parallel (construction based assumption)

$S_{T_aCP_{G11459}} \neq S_{P_{G11453}CP_{G11459}}$  i.e., lines  $T_aP_{G11453}$  and  $CP_{G11459}$  are not parallel (construction based assumption)

$S_{BCP_{G11468}} \neq S_{P_{G11471}CP_{G11468}}$  i.e., lines  $BP_{G11471}$  and  $CP_{G11468}$  are not parallel (construction based assumption)

$S_{P_{G11459}CT_a} \neq S_{P_{G11480}CT_a}$  i.e., lines  $P_{G11459}P_{G11480}$  and  $CT_a$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds



### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $C=C$**

Proving failed

#### **4.3.2 Proving $T_a=T_a$**

Proving failed

#### **4.3.3 Proving $B=B$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $C=C$**

Proving failed

#### **4.4.2 Proving $T_a=T_a$**

Proving failed

#### **4.4.3 Proving $B=B$**

Proving failed

# Problem 116

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 116: Given a point  $B$ , a point  $C$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $C$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $C$  are not the same;
2. Using the point  $C$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $T_b$  are not the same;
3. Using the point  $T_b$  and the line  $a$ , construct a circle  $k(T_b, foot[T_b, a])$  (rule W11); % NDG: point  $T_b$  is not incident to the line  $a$ ;
4. Using the circle  $k(T_b, foot[T_b, a])$ , the point  $B$ , the point  $T_b$  and the line  $a$ , construct a line  $c$  (rule W13); % NDG: point  $B$  is outside the circle  $k(T_b, foot[T_b, a])$ ;
5. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; point  $B$  is outside the circle  $k(T_b, foot[T_b, a])$ ; point  $T_b$  is not incident to the line  $a$ .

Determination conditions: lines  $c$  and  $b$  are not the same; points  $C$  and  $T_b$  are not the same; points  $B$  and  $C$  are not the same.

Rules used: [W02,W03,W11,W13]

Lemmas used: [D24,GD01,GD02,GL10,GL11,L9]

Solving time: 1.7 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point C 110 40
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_b B
cmark_l C
cmark_t T_{b}
color 0 0 0
fontsize 8

% DET: points B and C are not the same
% Constructing a line a which passes through point B and point C
line a B C

color 200 200 200
drawline a
color 0 0 0

% DET: points C and T_{b} are not the same
% Constructing a line b which passes through point C and point T_{b}
line b C T_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: point T_{b} is not incident to the line a
% Constructing a point P_{\_G13072} which is a foot of the point T_{b} on the line a
foot P_{\_G13072} T_{b} a
cmark_r P_{\_G13072}
color 200 200 200
drawline T_{b} P_{\_G13072}
color 0 0 0

% Constructing a circle k(T_{b},foot[T_{b},c]) whose center is at point T_{b} and which passes
    through point P_{\_G13072}
circle k(T_{b},foot[T_{b},c]) T_{b} P_{\_G13072}

color 200 200 200
drawcircle k(T_{b},foot[T_{b},c])
color 0 0 0

% NDG: point B is outside the circle k(T_{b},foot[T_{b},c])
```

```

% Constructing a point  $P_{\{ \_G13463 \}}$  which is a foot of the point  $T_{\{b\}}$  on the line  $a$ 
foot  $P_{\{ \_G13463 \}}$   $T_{\{b\}}$   $a$ 
cmark_r  $P_{\{ \_G13463 \}}$ 
color 200 200 200
drawline  $T_{\{b\}}$   $P_{\{ \_G13463 \}}$ 
color 0 0 0

% Constructing a line  $L_{\{ \_G13466 \}}$  which passes through point  $T_{\{b\}}$  and point  $B$ 
line  $L_{\{ \_G13466 \}}$   $T_{\{b\}}$   $B$ 

color 200 200 200
drawline  $L_{\{ \_G13466 \}}$ 
color 0 0 0

% Constructing a point  $P_{\{ \_G13469 \}}$  which is an image of the point  $P_{\{ \_G13463 \}}$  in the symmetry to
point/line  $L_{\{ \_G13466 \}}$ 
sim  $P_{\{ \_G13469 \}}$   $L_{\{ \_G13466 \}}$   $P_{\{ \_G13463 \}}$ 
cmark_r  $P_{\{ \_G13469 \}}$ 

% Constructing a line  $c$  which passes through point  $B$  and point  $P_{\{ \_G13469 \}}$ 
line  $c$   $B$   $P_{\{ \_G13469 \}}$ 

color 200 200 200
drawline  $c$ 
color 0 0 0

% NDG: lines  $c$  and  $b$  are not parallel% DET: lines  $c$  and  $b$  are not the same
% Constructing a point  $A$  which belongs to line  $c$  and line  $b$ 
intersec  $A$   $c$   $b$ 
cmark_t  $A$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; point  $B$  is outside the circle  $k(T_{\{b\}}$ ,
foot[ $T_{\{b\}}$ , $c$ ]); point  $T_{\{b\}}$  is not incident to the line  $a$ 
% Determination conditions: lines  $c$  and  $b$  are not the same; points  $C$  and  $T_{\{b\}}$  are not the same;
points  $B$  and  $C$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

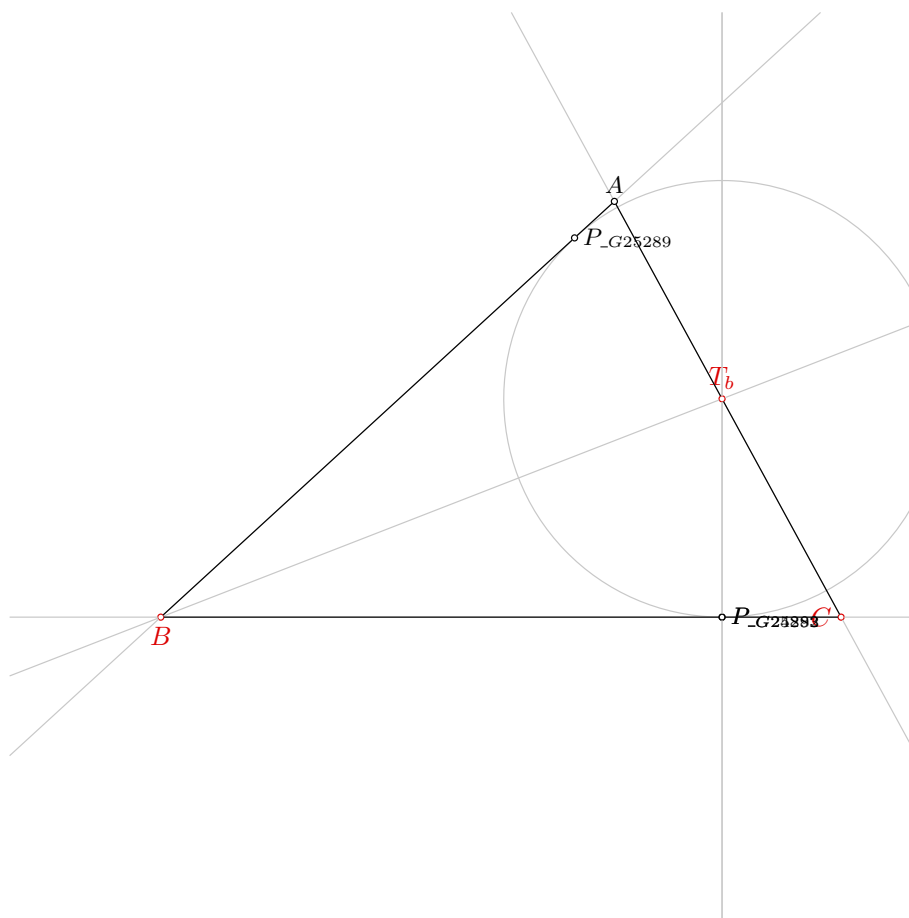


Figure 1: Illustration of the problem 0116

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $T_b=\neg T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{BCT_b} \neq S_{P_{G26241}CT_b}$  i.e., lines  $BP_{G26241}$  and  $CT_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $C=C$

NDG conditions are:

$S_{BCT_b} \neq S_{P_{G27752}CT_b}$  i.e., lines  $BP_{G27752}$  and  $CT_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $T_b=\neg T_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_b = \neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 30 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_b = \neg T_b$

Proving failed

# Problem 117

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 117: Given a point  $B$ , a point  $C$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $C$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $C$  are not the same;
2. Using the point  $B$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $T_c$  are not the same;
3. Using the point  $T_c$  and the line  $a$ , construct a circle  $k(T_c, foot[T_c, a])$  (rule W11); % NDG: point  $T_c$  is not incident to the line  $a$ ;
4. Using the circle  $k(T_c, foot[T_c, a])$ , the point  $C$ , the point  $T_c$  and the line  $a$ , construct a line  $b$  (rule W13); % NDG: point  $C$  is outside the circle  $k(T_c, foot[T_c, a])$ ;
5. Using the line  $b$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $c$  are not parallel % DET: lines  $b$  and  $c$  are not the same.

Non-degenerate conditions: lines  $b$  and  $c$  are not parallel; point  $C$  is outside the circle  $k(T_c, foot[T_c, a])$ ; point  $T_c$  is not incident to the line  $a$ .

Determination conditions: lines  $b$  and  $c$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $C$  are not the same.

Rules used: [W02,W03,W11,W13]

Lemmas used: [D25,GD01,GD02,GL10,GL11,L10]

Solving time: 1.7 seconds.



### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point C 110 40
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_b B
cmark_l C
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points B and C are not the same
% Constructing a line a which passes through point B and point C
line a B C

color 200 200 200
drawline a
color 0 0 0

% DET: points B and T_{c} are not the same
% Constructing a line c which passes through point B and point T_{c}
line c B T_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: point T_{c} is not incident to the line a
% Constructing a point P_{\_G30778} which is a foot of the point T_{c} on the line a
foot P_{\_G30778} T_{c} a
cmark_r P_{\_G30778}
color 200 200 200
drawline T_{c} P_{\_G30778}
color 0 0 0

% Constructing a circle k(T_{c},foot[T_{c},b]) whose center is at point T_{c} and which passes
    through point P_{\_G30778}
circle k(T_{c},foot[T_{c},b]) T_{c} P_{\_G30778}

color 200 200 200
drawcircle k(T_{c},foot[T_{c},b])
color 0 0 0

% NDG: point C is outside the circle k(T_{c},foot[T_{c},b])
```

```

% Constructing a point  $P_{\{\_G31169\}}$  which is a foot of the point  $T_{\{c\}}$  on the line  $a$ 
foot  $P_{\{\_G31169\}}$   $T_{\{c\}}$   $a$ 
cmark_r  $P_{\{\_G31169\}}$ 
color 200 200 200
drawline  $T_{\{c\}}$   $P_{\{\_G31169\}}$ 
color 0 0 0

% Constructing a line  $L_{\{\_G31172\}}$  which passes through point  $T_{\{c\}}$  and point  $C$ 
line  $L_{\{\_G31172\}}$   $T_{\{c\}}$   $C$ 

color 200 200 200
drawline  $L_{\{\_G31172\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\_G31175\}}$  which is an image of the point  $P_{\{\_G31169\}}$  in the symmetry to
point/line  $L_{\{\_G31172\}}$ 
sim  $P_{\{\_G31175\}}$   $L_{\{\_G31172\}}$   $P_{\{\_G31169\}}$ 
cmark_r  $P_{\{\_G31175\}}$ 

% Constructing a line  $b$  which passes through point  $C$  and point  $P_{\{\_G31175\}}$ 
line  $b$   $C$   $P_{\{\_G31175\}}$ 

color 200 200 200
drawline  $b$ 
color 0 0 0

% NDG: lines  $b$  and  $c$  are not parallel% DET: lines  $b$  and  $c$  are not the same
% Constructing a point  $A$  which belongs to line  $b$  and line  $c$ 
intersec  $A$   $b$   $c$ 
cmark_t  $A$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $b$  and  $c$  are not parallel; point  $C$  is outside the circle  $k(T_{\{c\}},$ 
foot[ $T_{\{c\}}$ , $b$ ]); point  $T_{\{c\}}$  is not incident to the line  $a$ 
% Determination conditions: lines  $b$  and  $c$  are not the same; points  $B$  and  $T_{\{c\}}$  are not the same;
points  $B$  and  $C$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

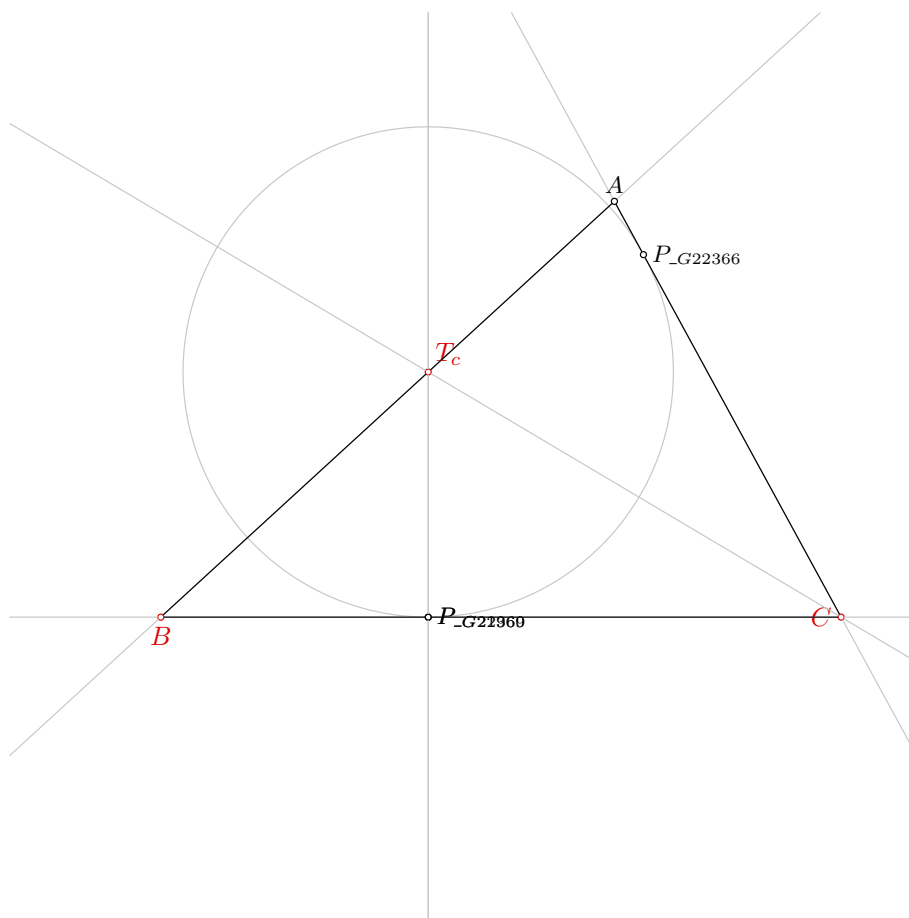


Figure 1: Illustration of the problem 0117

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $T_c=T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{CBT_c} \neq S_{P_{G23318}BT_c}$  i.e., lines  $CP_{G23318}$  and  $BT_c$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $C=C$

NDG conditions are:

$S_{CBT_c} \neq S_{P_{G24829}BT_c}$  i.e., lines  $CP_{G24829}$  and  $BT_c$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $T_c=T_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_c = \neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 16 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_c = \neg T_c$

Proving failed

# Problem 118

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 118: Given a point  $B$ , a point  $C$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $C$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $C$  are not the same;
2. Using the point  $I$  and the line  $a$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $a$ ;
3. Using the circle  $k(I, P_a)$ , the point  $B$ , the point  $I$  and the line  $a$ , construct a line  $c$  (rule W13); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
4. Using the circle  $k(I, P_a)$ , the point  $C$ , the point  $I$  and the line  $a$ , construct a line  $b$  (rule W13); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
5. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; point  $C$  is outside the circle  $k(I, P_a)$ ; point  $B$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $a$ .

Determination conditions: lines  $c$  and  $b$  are not the same; points  $B$  and  $C$  are not the same.

Rules used: [W02,W03,W11,W13]

Lemmas used: [D27,GD01,L59,L60,L61]

Solving time: 17.0 seconds.

### 3.2 Construction in GCLC language

```

dim 120 120

point B 20 40
point C 110 40
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_b B
cmark_l C
cmark_b I
color 0 0 0
fontsize 8

% DET: points B and C are not the same
% Constructing a line a which passes through point B and point C
line a B C

color 200 200 200
drawline a
color 0 0 0

% NDG: point I is not incident to the line a
% Constructing a point P_{\_G30058} which is a foot of the point I on the line a
foot P_{\_G30058} I a
cmark_r P_{\_G30058}
color 200 200 200
drawline I P_{\_G30058}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
30058}
circle k(I,P_{a}) I P_{\_G30058}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point B is outside the circle k(I,P_{a})
% Constructing a point P_{\_G30265} which is a foot of the point I on the line a
foot P_{\_G30265} I a
cmark_r P_{\_G30265}
color 200 200 200
drawline I P_{\_G30265}
color 0 0 0

% Constructing a line L_{\_G30268} which passes through point I and point B
line L_{\_G30268} I B

color 200 200 200
drawline L_{\_G30268}

```

```

color 0 0 0

% Constructing a point  $P_{\backslash\_G30271}$  which is an image of the point  $P_{\backslash\_G30265}$  in the symmetry to
point/line  $L_{\backslash\_G30268}$ 
sim  $P_{\backslash\_G30271}$   $L_{\backslash\_G30268}$   $P_{\backslash\_G30265}$ 
cmark_r  $P_{\backslash\_G30271}$ 

% Constructing a line  $c$  which passes through point  $B$  and point  $P_{\backslash\_G30271}$ 
line c B  $P_{\backslash\_G30271}$ 

color 200 200 200
drawline c
color 0 0 0

% NDG: point  $C$  is outside the circle  $k(I, P_{\backslash\_a})$ 
% Constructing a point  $P_{\backslash\_G30600}$  which is a foot of the point  $I$  on the line  $a$ 
foot  $P_{\backslash\_G30600}$  I a
cmark_r  $P_{\backslash\_G30600}$ 
color 200 200 200
drawline I  $P_{\backslash\_G30600}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G30603}$  which passes through point  $I$  and point  $C$ 
line  $L_{\backslash\_G30603}$  I C

color 200 200 200
drawline  $L_{\backslash\_G30603}$ 
color 0 0 0

% Constructing a point  $P_{\backslash\_G30606}$  which is an image of the point  $P_{\backslash\_G30600}$  in the symmetry to
point/line  $L_{\backslash\_G30603}$ 
sim  $P_{\backslash\_G30606}$   $L_{\backslash\_G30603}$   $P_{\backslash\_G30600}$ 
cmark_r  $P_{\backslash\_G30606}$ 

% Constructing a line  $b$  which passes through point  $C$  and point  $P_{\backslash\_G30606}$ 
line b C  $P_{\backslash\_G30606}$ 

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $c$  and  $b$  are not parallel% DET: lines  $c$  and  $b$  are not the same
% Constructing a point  $A$  which belongs to line  $c$  and line  $b$ 
intersec A c b
cmark_t A

drawsegment A B
drawsegment A C

```



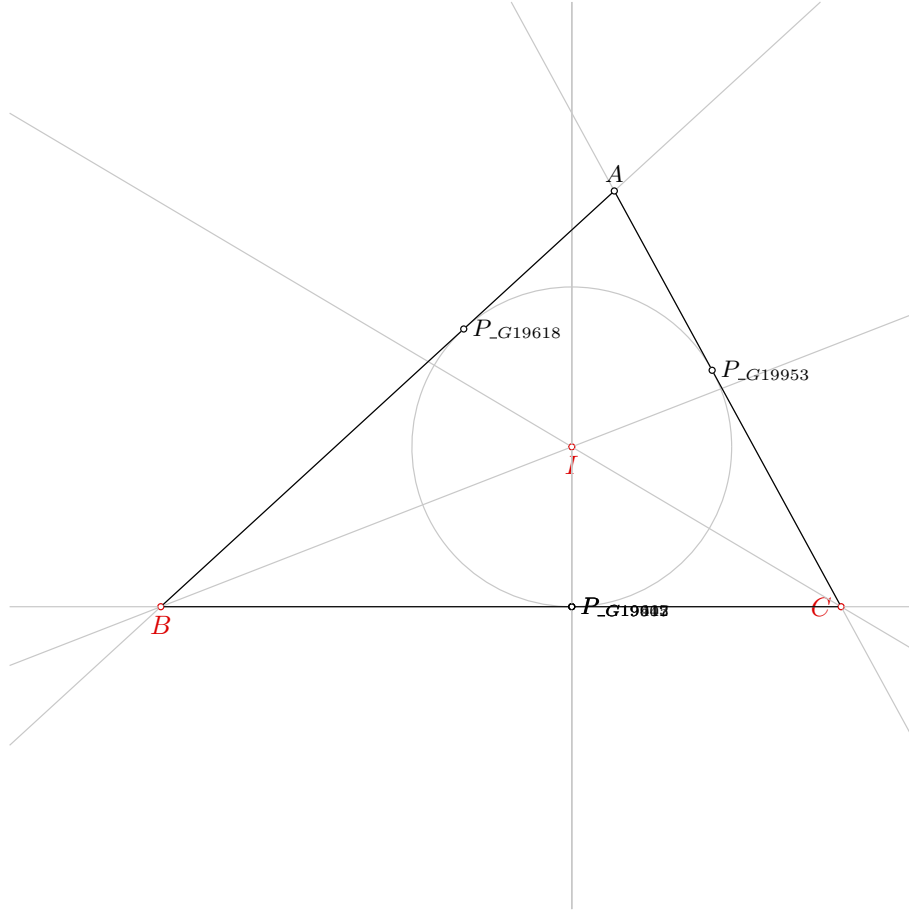


Figure 1: Illustration of the problem 0118

drawsegment B C

*% Non-degenerate conditions: lines c and b are not parallel; point C is outside the circle  $k(I, P_{\{a\}})$ ; point B is outside the circle  $k(I, P_{\{a\}})$ ; point I is not incident to the line a*  
*% Determination conditions: lines c and b are not the same; points B and C are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{BCP_{G20959}} \neq S_{P_{G20624}CP_{G20959}}$  i.e., lines  $BP_{G20624}$  and  $CP_{G20959}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $C=C$

NDG conditions are:

$S_{BCP_{G22788}} \neq S_{P_{G22453}CP_{G22788}}$  i.e., lines  $BP_{G22453}$  and  $CP_{G22788}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I=_I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 105 terms.

**Time Complexity:** Time spent by the prover is 0.040 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.3 Proving $I=_I$

Proving failed

# Problem 119

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 119: Given a point  $O$ , a point  $M_a$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $M_a$ , construct a line  $m_a$  (rule W02); % DET: points  $O$  and  $M_a$  are not the same;
2. Using the point  $M_a$  and the line  $m_a$ , construct a line  $a$  (rule W10a); ;
3. Choose freely a point  $B$  on the line  $a$  (rule WOnline2);
4. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
5. Choose freely a point  $A$  on the circle  $k(O, C)$  (rule WOncircle).

Non-degenerate conditions: .

Determination conditions: points  $O$  and  $M_a$  are not the same.

Rules used: [W01,W02,W10a,WOncircle1,WOnline2]

Lemmas used: [D1,D11,D21,GD01,GL03,GL09]

Solving time: 270.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point O 65 51.14
```

```
point M_{a} 65 40
```

```
point B 20 40
```

```
color 220 0 0
```

```
fontsize 9
```

```

cmark_t 0
cmark_r M_{a}
cmark_b B
color 0 0 0
fontsize 8

% DET: points O and M_{a} are not the same
% Constructing a line m_{a} which passes through point O and point M_{a}
line m_{a} 0 M_{a}

color 200 200 200
drawline m_{a}
color 0 0 0

% Constructing a line a which is perpendicular to line m_{a} and which passes through point M_{a}
perp a M_{a} m_{a}

color 200 200 200
drawline a
color 0 0 0

% Generating random value V[_G25567]
random V[_G25567]

% Calculating value V[_G25588] using formula  $V[_G25567]*20$ 
expression V[_G25588] { V[_G25567]*20 }

% Constructing a point B which is a point for which holds  $M_{a}B = V[_G25588]$  and angle  $OM_{a}B = 90$ 
turtle B 0 M_{a} 90 V[_G25588]
cmark_b B

% Constructing a point C such that  $BC/BM_{a}=2$ 
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% Choosing randomly a point A on the circle with center O through point B
oncircle A 0 B
cmark_t A
color 200 200 200
drawcircle 0 B

```

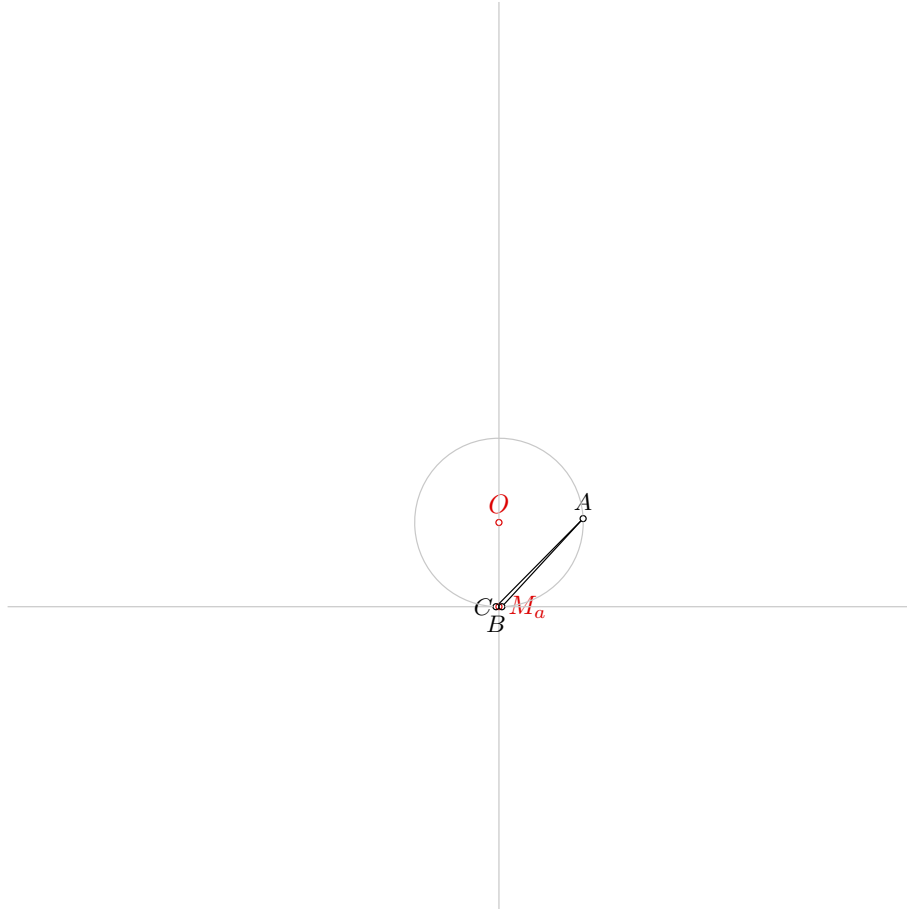


Figure 1: Illustration of the problem 0119

```
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions:
% Determination conditions: points O and M_{a} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 49 terms.

**Time Complexity:** Time spent by the prover is 0.422 seconds.

**NDG conditions** Points  $B$ ,  $C$  and  $O$  are not collinear

Line through points  $O$  and  $M_a$  is not perpendicular to line through points  $M_a$  and  $A$

#### 4.1.2 Proving $M_a=_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.037 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.012 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $O=_O$

Proving failed

#### 4.2.2 Proving $M_a=_M_a$

Proving failed

#### 4.2.3 Proving $B=B$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $O=_O$

Proving failed

#### 4.3.2 Proving $M_a=_M_a$

Proving failed

#### 4.3.3 Proving $B=B$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $O=_O$

Proving failed

### 4.4.2 Proving $M_a=_M a$

Proving failed

### 4.4.3 Proving $B=B$

Proving failed



# Problem 120

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 120: Given a point  $B$ , a point  $O$  and a point  $M_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $M_b$ , construct a point  $G$  (rule W01); ;
2. Using the point  $O$  and the point  $G$ , construct a point  $H$  (rule W01); ;
3. Using the point  $B$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H$  are not the same;
4. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
5. Using the point  $M_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same.

Determination conditions: points  $B$  and  $H$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D22,D26,D3,D9,GD01,GL03,GL04,GL09,L11,L12,L56,L58]

Solving time: 1.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point O 65 51.14
point M_{b} 95 67.5
```

```

color 220 0 0
fontsize 9

cmark_b B
cmark_t 0
cmark_lt M_{b}
color 0 0 0
fontsize 8

% Constructing a line L_{\_G25867} which passes through point B and point M_{b}
line L_{\_G25867} B M_{b}

color 200 200 200
drawline L_{\_G25867}
color 0 0 0

% Constructing a point P_{\_G25968} with coordinates (0,0)
point P_{\_G25968} 0 0
cmark_r P_{\_G25968}

% Constructing a point P_{\_G25892} such that BP_{\_G25892}/BP_{\_G25968}=2
towards P_{\_G25892} B P_{\_G25968} 2
cmark_r P_{\_G25892}
color 200 200 200
drawsegment B P_{\_G25892}
color 0 0 0

% Constructing a point P_{\_G25937} such that BP_{\_G25937}/BP_{\_G25968}=3
towards P_{\_G25937} B P_{\_G25968} 3
cmark_r P_{\_G25937}
color 200 200 200
drawsegment B P_{\_G25937}
color 0 0 0

% Constructing a line L_{\_G25898} which passes through point M_{b} and point P_{\_G25937}
line L_{\_G25898} M_{b} P_{\_G25937}

color 200 200 200
drawline L_{\_G25898}
color 0 0 0

% Constructing a line L_{\_G25861} which contains the point P_{\_G25892} and is parallel to the
line L_{\_G25898}
parallel L_{\_G25861} P_{\_G25892} L_{\_G25898}

color 200 200 200
drawline L_{\_G25861}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G25861} and line L_{\_G25867}
intersec G L_{\_G25861} L_{\_G25867}

```

```

cmark_t G

% Constructing a point H such that  $OH/OG=3$ 
towards H O G 3
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0

% DET: points B and H are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point B and point H
line  $h_{\{b\}}$  B H

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point B
circle  $k(O,C)$  O B

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% Constructing a line b which is perpendicular to line  $h_{\{b\}}$  and which passes through point  $M_{\{b\}}$ 
perp b  $M_{\{b\}}$   $h_{\{b\}}$ 

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle  $k(O,C)$  intersect
% Constructing points C and A which are in intersection of  $k(O,C)$  and b
intersec2 C A  $k(O,C)$  b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle  $k(O,C)$  intersect; points B and O are not the same
% Determination conditions: points B and H are not the same

```

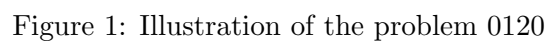


Illustration of the constructed figure is given in Figure 1

## 4 Correctness proof

#### 4.1.1 Proving $B=B$

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 180 terms.

**Time Complexity:** Time spent by the prover is 1.5 seconds.

**NDG conditions** Points  $M_b$ ,  $B$  and  $P_{G22490}$  are not collinear

Line through points  $M_b$  and  $P_{G22490}$  is not parallel with line through points  $B$  and  $O$

Points  $B$  and  $H$  are not identical

Points  $B$ ,  $H$  and  $O$  are not collinear

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $B$  and  $C$  are not identical

#### 4.1.3 Proving $M_b = \neg M_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B = B$

NDG conditions are:

$S_{P_{G14491}BM_b} \neq S_{P_{L_{G14460}}^{0}BM_b}$  i.e., lines  $P_{G14491}P_{L_{G14460}}^0$  and  $BM_b$  are not parallel (construction based assumption)

$S_{M_bBH} \neq 0$  i.e., points  $M_b$ ,  $B$  and  $H$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aBC} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aM_bF_{M_b}^3} \neq S_{F_{M_a}^2M_bF_{M_b}^3}$  i.e., lines  $M_aF_{M_a}^2$  and  $M_bF_{M_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O = \neg O$

Proving failed

#### 4.2.3 Proving $M_b = \neg M_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $O = \neg O$

Proving failed

#### 4.3.3 Proving $M_b = \neg M_b$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4.2 Proving $O=_O$

Proving failed

### 4.4.3 Proving $M_b=_M M_b$

Proving failed

# Problem 121

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 121: Given a point  $O$ , a point  $M_c$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $M_c$ , construct a line  $m_c$  (rule W02); % DET: points  $O$  and  $M_c$  are not the same;
2. Using the point  $M_c$  and the line  $m_c$ , construct a line  $c$  (rule W10a); ;
3. Choose freely a point  $B$  on the line  $c$  (rule WOnline2);
4. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01); ;
5. Choose freely a point  $C$  on the circle  $k(O, C)$  (rule WOncircle).

Non-degenerate conditions: .

Determination conditions: points  $O$  and  $M_c$  are not the same.

Rules used: [W01,W02,W10a,WOncircle1,WOnline2]

Lemmas used: [D13,D20,GD01,GL03,GL04,GL09,L1]

Solving time: 274.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point O 65 51.14
point M_{c} 50 67.5
point B 20 40
```

```
color 220 0 0
fontsize 9
```

```

cmark_t 0
cmark_lt M_{c}
cmark_b B
color 0 0 0
fontsize 8

% DET: points O and M_{c} are not the same
% Constructing a line m_{c} which passes through point O and point M_{c}
line m_{c} 0 M_{c}

color 200 200 200
drawline m_{c}
color 0 0 0

% Constructing a line c which is perpendicular to line m_{c} and which passes through point M_{c}
perp c M_{c} m_{c}

color 200 200 200
drawline c
color 0 0 0

% Generating random value V[_G20427]
random V[_G20427]

% Calculating value V[_G20448] using formula V[_G20427]*20
expression V[_G20448] { V[_G20427]*20 }

% Constructing a point B which is a point for which holds  $M_{c}B = V[_G20448]$  and angle  $OM_{c}B = 90$ 
turtle B 0 M_{c} 90 V[_G20448]
cmark_b B

% Constructing a point A such that  $BA/BM_{c}=2$ 
towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

% Choosing randomly a point C on the circle with center O through point B
oncircle C 0 B
cmark_l C
color 200 200 200
drawcircle 0 B

```



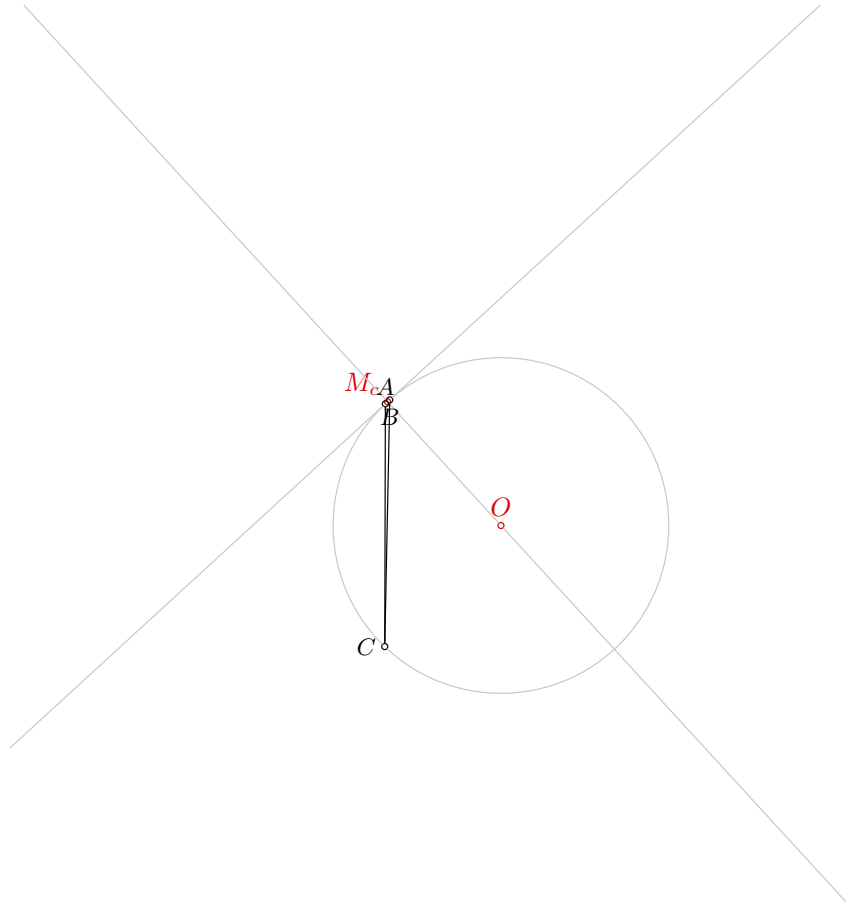


Figure 1: Illustration of the problem 0121

```
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

*% Non-degenerate conditions:*

*% Determination conditions: points O and M\_{c} are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 138 terms.

**Time Complexity:** Time spent by the prover is 0.833 seconds.

**NDG conditions** Line through points  $A$  and  $B$  is not parallel with line through points  $C$  and  $M_c$   
Points  $B$ ,  $C$  and  $M_c$  are not collinear

#### 4.1.2 Proving $M_c=_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.034 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.015 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $O=_O$

Proving failed

#### 4.2.2 Proving $M_c=_M_c$

Proving failed

#### 4.2.3 Proving $B=B$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $O=_O$

Proving failed

#### 4.3.2 Proving $M_c=_M_c$

Proving failed

#### 4.3.3 Proving $B=B$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $O=_O$

Proving failed

### 4.4.2 Proving $M_c=_M M_c$

Proving failed

### 4.4.3 Proving $B=B$

Proving failed

# Problem 122

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 122: Given a point  $B$ , a point  $O$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $G$ , construct a point  $M_b$  (rule W01); ;
2. Using the point  $O$  and the point  $G$ , construct a point  $H$  (rule W01); ;
3. Using the point  $B$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H$  are not the same;
4. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
5. Using the point  $M_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same.

Determination conditions: points  $B$  and  $H$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D22,D26,D3,D9,GD01,GL03,GL04,GL09,L11,L12,L56,L58]

Solving time: 1.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40  
point O 65 51.14  
point G 70 58.33
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_t 0
cmark_t G
color 0 0 0
fontsize 8
```

```
% Constructing a point  $M_{\{b\}}$  such that  $BM_{\{b\}}/BG=1.5$ 
towards M_{b} B G 1.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0
```

```
% Constructing a point H such that  $OH/OG=3$ 
towards H O G 3
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0
```

```
% DET: points B and H are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point B and point H
line h_{b} B H

color 200 200 200
drawline h_{b}
color 0 0 0
```

```
% NDG: points B and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0
```

```
% Constructing a line b which is perpendicular to line  $h_{\{b\}}$  and which passes through point  $M_{\{b\}}$ 
perp b M_{b} h_{b}

color 200 200 200
drawline b
color 0 0 0
```

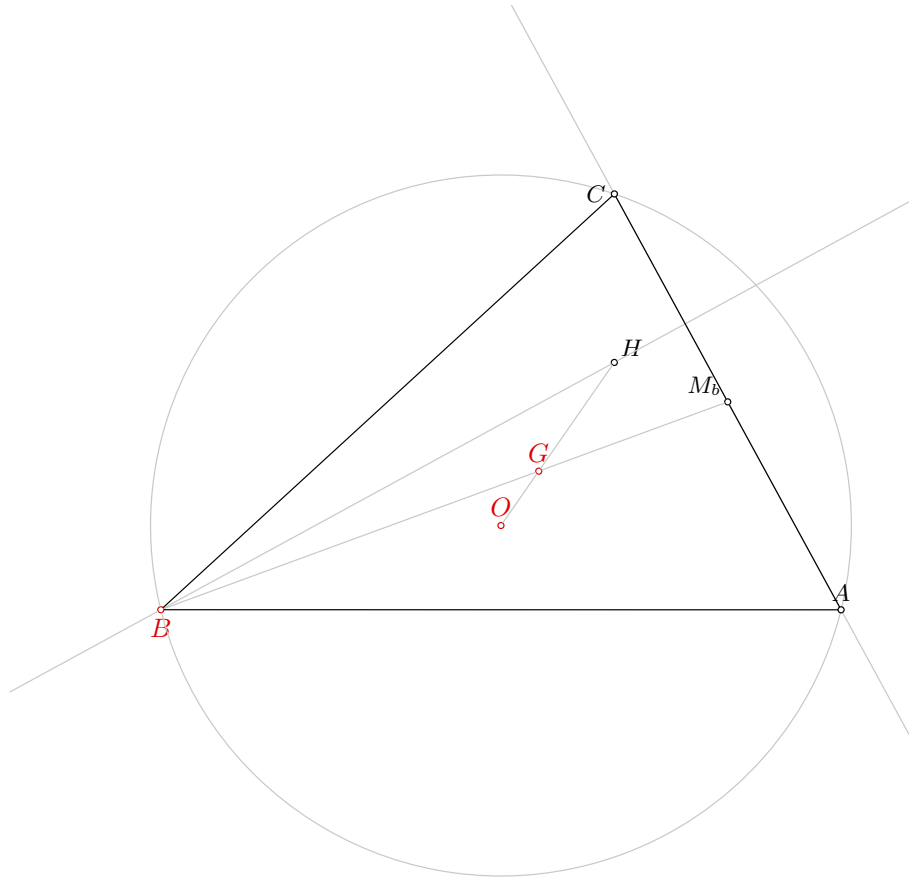


Figure 1: Illustration of the problem 0122

```
% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same
% Determination conditions: points B and H are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 847 terms.

**Time Complexity:** Time spent by the prover is 2.219 seconds.

**NDG conditions** Points  $B$  and  $H$  are not identical

Points  $B$ ,  $H$  and  $O$  are not collinear

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $B$  and  $C$  are not identical

#### 4.1.3 Proving $G=_G$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{M_bBH} \neq 0$  i.e., points  $M_b$ ,  $B$  and  $H$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_aBC} \neq 0$  i.e., points  $_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$  i.e., points  $_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_aM_bF_{_m_b}^2} \neq S_{F_{_m_a}^1M_bF_{_m_b}^2}$  i.e., lines  $_M_aF_{_m_a}^1$  and  $_M_bF_{_m_b}^2$  are not parallel (construction based assumption)

$S_{ABM_b} \neq S_{_M_aB_M_b}$  i.e., lines  $A_M_a$  and  $B_M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $G=_G$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $O=_O$

Proving failed

#### 4.3.3 Proving $G=_G$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.2 Proving $O=_O$

Proving failed

#### 4.4.3 Proving $G=_G$

Proving failed



# Problem 123

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 123: Given a point  $B$ , a point  $O$  and a point  $H_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $H_a$  are not the same;
2. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
3. Using the circle  $k(O, C)$ , the line  $a$ , the point  $O$  and the point  $B$ , construct a point  $C$  (rule W05); % NDG: line  $a$  and circle  $k(O, C)$  intersect % DET: points  $B$  and  $C$  must be different;
4. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
5. Using the circle  $k(O, C)$  and the line  $h_a$ , construct a point  $A_k$  and a point  $A$  (rule W04); % NDG: line  $h_a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $h_a$  and circle  $k(O, C)$  intersect; line  $a$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same.

Determination conditions: points  $B$  and  $C$  must be different; points  $B$  and  $H_a$  are not the same.

Rules used: [W02,W04,W05,W06,W10b]

Lemmas used: [D26,D38,D5,D8,GD01,L11,L12]

Solving time: 4.5 seconds.

### 3.2 Construction in GCLC language

```

dim 120 120

point B 20 40
point O 65 51.14
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_b B
cmark_t O
cmark_r H_{a}
color 0 0 0
fontsize 8

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect% DET: points B and C must be different
% Constructing a point P_{\_G18889} which is a foot of the point O on the line a
foot P_{\_G18889} O a
cmark_r P_{\_G18889}
color 200 200 200
drawline O P_{\_G18889}
color 0 0 0

% Constructing a point C which is an image of the point B in the symmetry to point/line P_{\_G
18889}
sim C P_{\_G18889} B
cmark_l C

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

```

```

% NDG: line  $h_{\{a\}}$  and circle  $k(O,C)$  intersect
% Constructing points  $A_{\{k\}}$  and  $A$  which are in intersection of  $k(O,C)$  and  $h_{\{a\}}$ 
intersec2 A_{k} A k(O,C) h_{a}
cmark_r A_{k}
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line  $h_{\{a\}}$  and circle  $k(O,C)$  intersect; line  $a$  and circle  $k(O,C)$ 
% intersect; points  $B$  and  $O$  are not the same
% Determination conditions: points  $B$  and  $C$  must be different; points  $B$  and  $H_{\{a\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 85 terms.

**Time Complexity:** Time spent by the prover is 0.803 seconds.

**NDG conditions** Points  $B$ ,  $O$  and  $H_a$  are not collinear

Points  $B$ ,  $O$  and  $H_a$  are not collinear

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $B$  and  $C$  are not identical

#### 4.1.3 Proving $H_a=H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 17 terms.

**Time Complexity:** Time spent by the prover is 0.367 seconds.

**NDG conditions** Points  $B$ ,  $O$  and  $H_a$  are not collinear

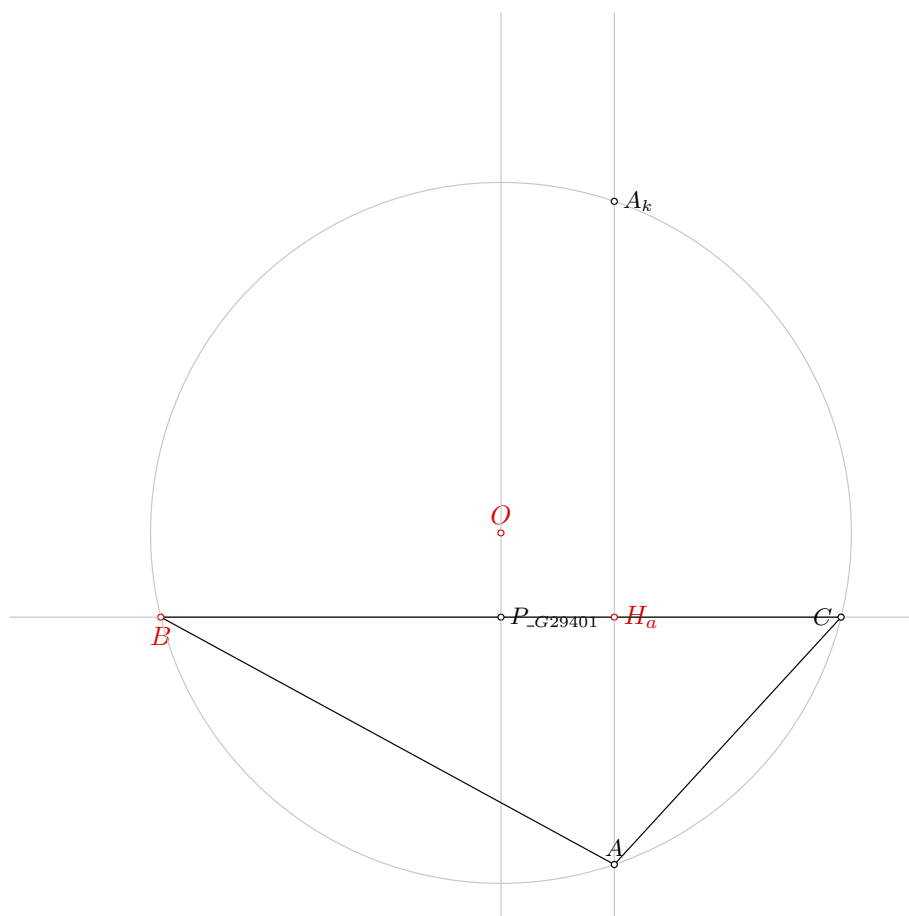


Figure 1: Illustration of the problem 0123

Points  $B$ ,  $O$  and  $H_a$  are not collinear

Points  $B$  and  $C$  are not identical

Points  $B$  and  $C$  are not identical

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{M_aBC} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aM_bF_{m_b}^2} \neq S_{F_{m_a}^1M_bF_{m_b}^2}$  i.e., lines  $M_aF_{m_a}^1$  and  $M_bF_{m_b}^2$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^3} \neq S_{CAF_{h_a}^3}$  i.e., lines  $BC$  and  $AF_{h_a}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $O=_O$

Proving failed

### 4.2.3 Proving $H_a=_H_a$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

Proving failed

### 4.3.2 Proving $O=_O$

Proving failed

### 4.3.3 Proving $H_a=_H_a$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

Proving failed

### 4.4.2 Proving $O=_O$

Proving failed

### 4.4.3 Proving $H_a=_H_a$

Proving failed

# Problem 124

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 124: Given a point  $B$ , a point  $O$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_b$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H_b$  are not the same;
2. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
3. Using the point  $H_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
4. Using the circle  $k(O, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same.

Determination conditions: points  $B$  and  $H_b$  are not the same.

Rules used: [W02,W04,W06,W10a]

Lemmas used: [D26,D6,D9,GD01,L11,L12]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point O 65 51.14
point H_{b} 89.36 77.83
```

```
color 220 0 0
fontsize 9
```

```

cmark_b B
cmark_t 0
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points B and H_{b} are not the same
% Constructing a line h_{b} which passes through point B and point H_{b}
line h_{b} B H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
perp b H_{b} h_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same
% Determination conditions: points B and H_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

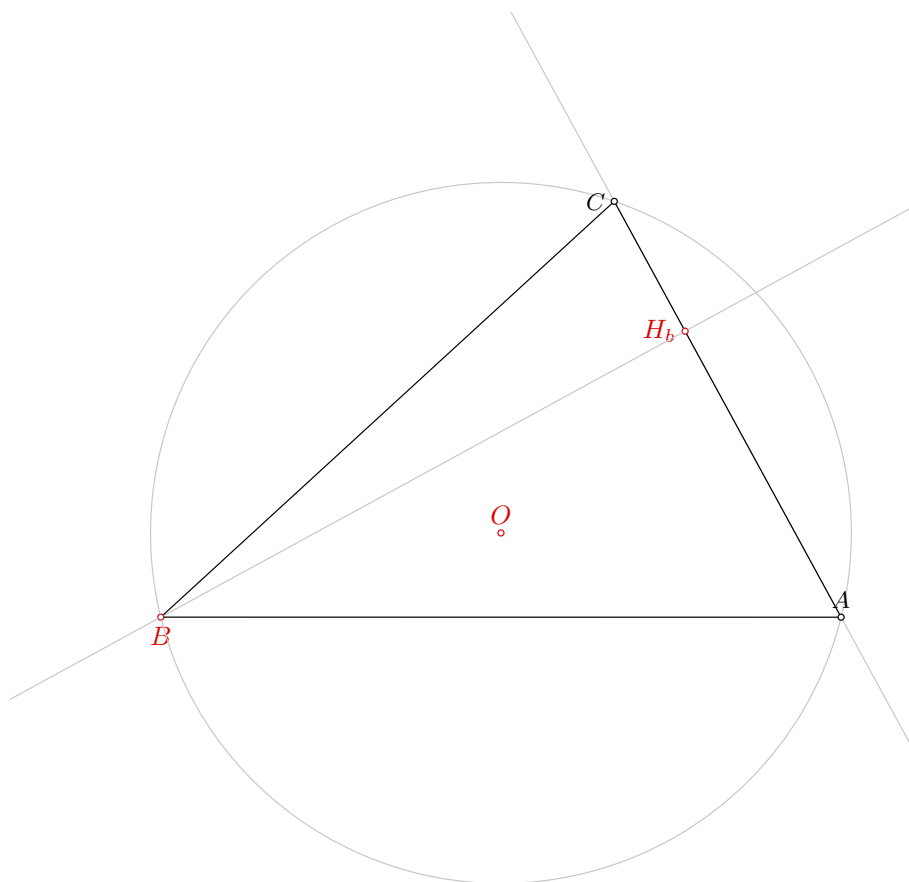


Figure 1: Illustration of the problem 0124



## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 85 terms.

**Time Complexity:** Time spent by the prover is 0.861 seconds.

**NDG conditions** Points  $H_b$ ,  $B$  and  $O$  are not collinear

Points  $H_b$ ,  $B$  and  $O$  are not collinear

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $B$  and  $C$  are not identical

#### 4.1.3 Proving $H_b=_H H_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{M_aBC} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aM_bF_{m_b}^2} \neq S_{F_{m_a}^1M_bF_{m_b}^2}$  i.e., lines  $M_aF_{m_a}^1$  and  $M_bF_{m_b}^2$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^3} \neq S_{CBF_{h_b}^3}$  i.e., lines  $AC$  and  $BF_{h_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $H_b=_H H_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $O=_O$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 16774 terms.

**Time Complexity:** Time spent by the prover is 47.510 seconds. There are no ndg conditions.

#### 4.3.3 Proving $H_b=_H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 5989 terms.

**Time Complexity:** Time spent by the prover is 4.690 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $O=_O$

Proving failed

#### 4.4.3 Proving $H_b=_H_b$

Proving failed

# Problem 125

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 125: Given a point  $B$ , a point  $O$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $H_c$  are not the same;
2. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
3. Using the circle  $k(O, C)$ , the line  $c$ , the point  $O$  and the point  $B$ , construct a point  $A$  (rule W05); % NDG: line  $c$  and circle  $k(O, C)$  intersect % DET: points  $B$  and  $A$  must be different;
4. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
5. Using the circle  $k(O, C)$  and the line  $h_c$ , construct a point  $C$  and a point  $C_k$  (rule W04); % NDG: line  $h_c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $h_c$  and circle  $k(O, C)$  intersect; line  $c$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same.

Determination conditions: points  $B$  and  $A$  must be different; points  $B$  and  $H_c$  are not the same.

Rules used: [W02,W04,W05,W06,W10b]

Lemmas used: [D10,D26,D40,D7,GD01,L11,L12]

Solving time: 4.5 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
```

```
point O 65 51.14
```

```

point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_b B
cmark_t 0
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line c and circle k(O,C) intersect% DET: points B and A must be different
% Constructing a point P_{\_G30464} which is a foot of the point O on the line c
foot P_{\_G30464} O c
cmark_r P_{\_G30464}
color 200 200 200
drawline O P_{\_G30464}
color 0 0 0

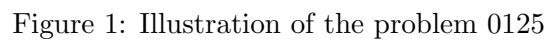
% Constructing a point A which is an image of the point B in the symmetry to point/line P_{\_G
30464}
sim A P_{\_G30464} B
cmark_t A

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: line h_{c} and circle k(O,C) intersect

```



```
% Non-degenerate conditions: line  $h_{\{c\}}$  and circle  $k(D,C)$  intersect; line  $c$  and circle  $k(D,C)$ 
intersect; points  $B$  and  $O$  are not the same
% Determination conditions: points  $B$  and  $A$  must be different; points  $B$  and  $H_{\{c\}}$  are not the same
```

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 85 terms.

**Time Complexity:** Time spent by the prover is 1.064 seconds.

**NDG conditions** Points  $H_c$  and  $O$  are not identical

Points  $H_c$  and  $O$  are not identical

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $B$  and  $C$  are not identical

#### 4.1.3 Proving $H_c=_Hc$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 17 terms.

**Time Complexity:** Time spent by the prover is 0.363 seconds.

**NDG conditions** Points  $H_c$  and  $O$  are not identical

Points  $H_c$  and  $O$  are not identical

Points  $A$  and  $B$  are not identical

Points  $A$  and  $B$  are not identical

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{M_aBC} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aM_bF_{M_b}^2} \neq S_{F_{M_a}^1M_bF_{M_b}^2}$  i.e., lines  $M_aF_{M_a}^1$  and  $M_bF_{M_b}^2$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{H_c}^3} \neq S_{BCF_{H_c}^3}$  i.e., lines  $AB$  and  $CF_{H_c}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $H_c=_Hc$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

Proving failed

#### 4.3.2 Proving $O=_O$

Proving failed

#### 4.3.3 Proving $H_c=_Hc$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B=B$

Proving failed

#### 4.4.2 Proving $O=_O$

Proving failed

#### 4.4.3 Proving $H_c=_Hc$

Proving failed

# Problem 126

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 126: Given a point  $B$ , a point  $O$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $H$ , construct a point  $G$  (rule W01); ;
2. Using the point  $B$  and the point  $G$ , construct a point  $M_b$  (rule W01); ;
3. Using the point  $B$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H$  are not the same;
4. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
5. Using the point  $M_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same.

Determination conditions: points  $B$  and  $H$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D22,D26,D3,D9,GD01,GL03,GL04,GL09,L11,L12,L56,L58]

Solving time: 1.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40  
point O 65 51.14  
point H 80 72.73
```



```

color 220 0 0
fontsize 9

cmark_b B
cmark_t 0
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a line  $L_{\backslash\_G13809}$  which passes through point  $O$  and point  $H$ 
line  $L_{\backslash\_G13809}$  O H

color 200 200 200
drawline  $L_{\backslash\_G13809}$ 
color 0 0 0

% Constructing a point  $P_{\backslash\_G13910}$  with coordinates  $(0,0)$ 
point  $P_{\backslash\_G13910}$  0 0
cmark_r  $P_{\backslash\_G13910}$ 

% Constructing a point  $P_{\backslash\_G13834}$  such that  $OP_{\backslash\_G13834}/OP_{\backslash\_G13910}=1$ 
towards  $P_{\backslash\_G13834}$  0  $P_{\backslash\_G13910}$  1
cmark_r  $P_{\backslash\_G13834}$ 
color 200 200 200
drawsegment 0  $P_{\backslash\_G13834}$ 
color 0 0 0

% Constructing a point  $P_{\backslash\_G13879}$  such that  $OP_{\backslash\_G13879}/OP_{\backslash\_G13910}=3$ 
towards  $P_{\backslash\_G13879}$  0  $P_{\backslash\_G13910}$  3
cmark_r  $P_{\backslash\_G13879}$ 
color 200 200 200
drawsegment 0  $P_{\backslash\_G13879}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G13840}$  which passes through point  $H$  and point  $P_{\backslash\_G13879}$ 
line  $L_{\backslash\_G13840}$  H  $P_{\backslash\_G13879}$ 

color 200 200 200
drawline  $L_{\backslash\_G13840}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G13803}$  which contains the point  $P_{\backslash\_G13834}$  and is parallel to the
line  $L_{\backslash\_G13840}$ 
parallel  $L_{\backslash\_G13803}$   $P_{\backslash\_G13834}$   $L_{\backslash\_G13840}$ 

color 200 200 200
drawline  $L_{\backslash\_G13803}$ 
color 0 0 0

% Constructing a point  $G$  which belongs to line  $L_{\backslash\_G13803}$  and line  $L_{\backslash\_G13809}$ 
intersec G  $L_{\backslash\_G13803}$   $L_{\backslash\_G13809}$ 

```

```

cmark_t G

% Constructing a point  $M_{\{b\}}$  such that  $BM_{\{b\}}/BG=1.5$ 
towards  $M_{\{b\}}$  B G 1.5
cmark_lt  $M_{\{b\}}$ 
color 200 200 200
drawsegment B  $M_{\{b\}}$ 
color 0 0 0

% DET: points B and H are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point B and point H
line  $h_{\{b\}}$  B H

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point B
circle  $k(O,C)$  O B

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% Constructing a line b which is perpendicular to line  $h_{\{b\}}$  and which passes through point  $M_{\{b\}}$ 
perp b  $M_{\{b\}}$   $h_{\{b\}}$ 

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle  $k(O,C)$  intersect
% Constructing points C and A which are in intersection of  $k(O,C)$  and b
intersec2 C A  $k(O,C)$  b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle  $k(O,C)$  intersect; points B and O are not the same
% Determination conditions: points B and H are not the same

```

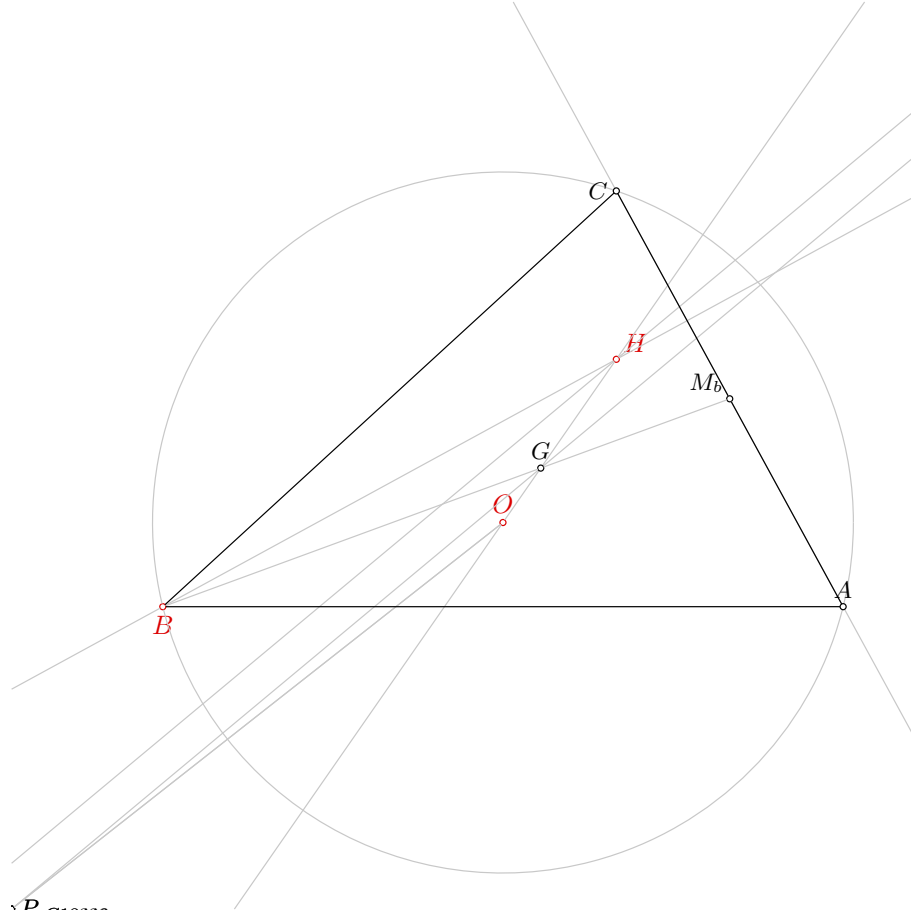


Figure 1: Illustration of the problem 0126

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 146 terms.

**Time Complexity:** Time spent by the prover is 1.331 seconds.

**NDG conditions** Points  $P_{G28699}$ ,  $H$  and  $O$  are not collinear

Line through points  $B$  and  $O$  is not parallel with line through points  $P_{G28699}$  and  $H$

Points  $B$ ,  $H$  and  $O$  are not collinear

Points  $B$ ,  $H$  and  $O$  are not collinear

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $B$  and  $C$  are not identical

#### 4.1.3 Proving $H=_H$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{P_{G19388}OH} \neq S_{P_{L_{G19357}}^{0}OH}$  i.e., lines  $P_{G19388}P_{L_{G19357}}^0$  and  $OH$  are not parallel (construction based assumption)

$S_{M_bBH} \neq 0$  i.e., points  $M_b$ ,  $B$  and  $H$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aBC} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aM_bF_{M_b}^3} \neq S_{F_{M_a}^2M_bF_{M_b}^3}$  i.e., lines  $M_aF_{M_a}^2$  and  $M_bF_{M_b}^3$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^5} \neq S_{F_{h_a}^4BF_{h_b}^5}$  i.e., lines  $AF_{h_a}^4$  and  $BF_{h_b}^5$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $H=_H$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $O=_O$

Proving failed

#### 4.3.3 Proving $H=_H$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4.2 Proving $O=_O$

Proving failed

### 4.4.3 Proving $H=_H$

Proving failed

# Problem 127

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 127: Given a point  $B$ , a point  $O$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $T_a$  are not the same;
2. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
3. Using the circle  $k(O, C)$ , the line  $a$ , the point  $O$  and the point  $B$ , construct a point  $C$  (rule W05); % NDG: line  $a$  and circle  $k(O, C)$  intersect % DET: points  $B$  and  $C$  must be different;
4. Using the point  $C$  and the point  $B$ , construct a point  $M_a$  (rule W01); ;
5. Using the point  $O$  and the point  $M_a$ , construct a line  $m_a$  (rule W02); % DET: points  $O$  and  $M_a$  are not the same;
6. Using the circle  $k(O, C)$  and the line  $m_a$ , construct a point  $N_a$  and a point  $N_{ak}$  (rule W04); % NDG: line  $m_a$  and circle  $k(O, C)$  intersect;
7. Using the point  $N_a$  and the point  $T_a$ , construct a line  $s_a$  (rule W02); % DET: points  $N_a$  and  $T_a$  are not the same;
8. Using the circle  $k(O, C)$ , the line  $s_a$ , the point  $O$  and the point  $N_a$ , construct a point  $A$  (rule W05); % NDG: line  $s_a$  and circle  $k(O, C)$  intersect % DET: points  $N_a$  and  $A$  must be different.

Non-degenerate conditions: line  $s_a$  and circle  $k(O, C)$  intersect; line  $m_a$  and circle  $k(O, C)$  intersect; line  $a$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same.

Determination conditions: points  $N_a$  and  $A$  must be different; points  $N_a$  and  $T_a$  are not the same; points  $O$  and  $M_a$  are not the same; points  $B$  and  $C$  must be different; points  $B$  and  $T_a$  are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D1,D11,D21,D23,D26,D47,D50,GD01,GL04,L11,L12,L25,L5]

Solving time: 2.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point O 65 51.14
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_b B
cmark_t O
cmark_rb T_{a}
color 0 0 0
fontsize 8

% DET: points B and T_{a} are not the same
% Constructing a line a which passes through point B and point T_{a}
line a B T_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect% DET: points B and C must be different
% Constructing a point P_{\_G17007} which is a foot of the point O on the line a
foot P_{\_G17007} O a
cmark_r P_{\_G17007}
color 200 200 200
drawline O P_{\_G17007}
color 0 0 0

% Constructing a point C which is an image of the point B in the symmetry to point/line P_{\_G17007}
```

```

sim C P_{\_G17007} B
cmark_l C

% Constructing a point  $M_{\{a\}}$  such that  $CM_{\{a\}}/CB=0.5$ 
towards M_{a} C B 0.5
cmark_r M_{a}
color 200 200 200
drawsegment C B
color 0 0 0

% DET: points  $O$  and  $M_{\{a\}}$  are not the same
% Constructing a line  $m_{\{a\}}$  which passes through point  $O$  and point  $M_{\{a\}}$ 
line m_{a} O M_{a}

color 200 200 200
drawline m_{a}
color 0 0 0

% NDG: line  $m_{\{a\}}$  and circle  $k(O,C)$  intersect
% Constructing points  $N_{\{a\}}$  and  $N_{\{ak\}}$  which are in intersection of  $k(O,C)$  and  $m_{\{a\}}$ 
intersec2 N_{a} N_{ak} k(O,C) m_{a}
cmark_b N_{a}
cmark_r N_{ak}

% DET: points  $N_{\{a\}}$  and  $T_{\{a\}}$  are not the same
% Constructing a line  $s_{\{a\}}$  which passes through point  $N_{\{a\}}$  and point  $T_{\{a\}}$ 
line s_{a} N_{a} T_{a}

color 200 200 200
drawline s_{a}
color 0 0 0

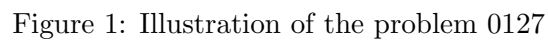
% NDG: line  $s_{\{a\}}$  and circle  $k(O,C)$  intersect% DET: points  $N_{\{a\}}$  and  $A$  must be different
% Constructing a point  $P_{\{\_G17417\}}$  which is a foot of the point  $O$  on the line  $s_{\{a\}}$ 
foot P_{\_G17417} O s_{a}
cmark_r P_{\_G17417}
color 200 200 200
drawline O P_{\_G17417}
color 0 0 0

% Constructing a point  $A$  which is an image of the point  $N_{\{a\}}$  in the symmetry to point/line  $P_{\{\_G17417\}}$ 
sim A P_{\_G17417} N_{a}
cmark_t A

drawsegment A B

```





```
% Non-degenerate conditions: line s_{a} and circle k(O,C) intersect; line m_{a} and circle k(O,C)
% intersect; line a and circle k(O,C) intersect; points B and O are not the same
% Determination conditions: points N_{a} and A must be different; points N_{a} and T_{a} are not
% the same; points O and M_{a} are not the same; points B and C must be different; points B and T
% _{a} are not the same
```

Illustration of the constructed figure is given in Figure 1

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## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.019 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

Proving failed

#### 4.1.3 Proving $T_a=T_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{_M_aBC} \neq 0$  i.e., points  $_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$  i.e., points  $_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a _M_b F_{_m_b}^1} \neq S_{F_{_m_a}^0 _M_b F_{_m_b}^1}$  i.e., lines  $_M_a F_{_m_a}^0$  and  $_M_b F_{_m_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $T_a=T_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

Proving failed

#### 4.3.2 Proving $O=_O$

Proving failed

#### 4.3.3 Proving $T_a=T_a$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B=B$

Proving failed

#### 4.4.2 Proving $O=_O$

Proving failed

#### 4.4.3 Proving $T_a=_T T_a$

Proving failed

# Problem 128

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 128: Given a point  $B$ , a point  $O$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $T_b$ , construct a line  $s_b$  (rule W02); % DET: points  $B$  and  $T_b$  are not the same;
2. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
3. Using the circle  $k(O, C)$ , the line  $s_b$ , the point  $O$  and the point  $B$ , construct a point  $N_b$  (rule W05); % NDG: line  $s_b$  and circle  $k(O, C)$  intersect % DET: points  $B$  and  $N_b$  must be different;
4. Using the point  $N_b$  and the point  $O$ , construct a line  $m_b$  (rule W02); % DET: points  $N_b$  and  $O$  are not the same;
5. Using the point  $T_b$  and the line  $m_b$ , construct a line  $b$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; line  $s_b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same.

Determination conditions: points  $N_b$  and  $O$  are not the same; points  $B$  and  $N_b$  must be different; points  $B$  and  $T_b$  are not the same.

Rules used: [W02,W04,W05,W06,W10a]

Lemmas used: [D1,D12,D24,D26,D48,GD01,L11,L12,L26,L6]

Solving time: 1.0 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point O 65 51.14
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_b B
cmark_t O
cmark_t T_{b}
color 0 0 0
fontsize 8

% DET: points B and T_{b} are not the same
% Constructing a line s_{b} which passes through point B and point T_{b}
line s_{b} B T_{b}

color 200 200 200
drawline s_{b}
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line s_{b} and circle k(O,C) intersect% DET: points B and N_{b} must be different
% Constructing a point P_{\_G17425} which is a foot of the point O on the line s_{b}
foot P_{\_G17425} O s_{b}
cmark_r P_{\_G17425}
color 200 200 200
drawline O P_{\_G17425}
color 0 0 0

% Constructing a point N_{b} which is an image of the point B in the symmetry to point/line P_{\_G17425}
sim N_{b} P_{\_G17425} B
cmark_rb N_{b}

% DET: points N_{b} and O are not the same
% Constructing a line m_{b} which passes through point N_{b} and point O
line m_{b} N_{b} O

color 200 200 200
```

```

drawline m_{b}
color 0 0 0

% Constructing a line b which is perpendicular to line m_{b} and which passes through point T_{b}
perp b T_{b} m_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k(O,C) intersect; line s_{b} and circle k(O,C)
% intersect; points B and O are not the same
% Determination conditions: points N_{b} and O are not the same; points B and N_{b} must be
% different; points B and T_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

Proving failed

#### 4.1.3 Proving $T_b=_T_b$

Proving failed

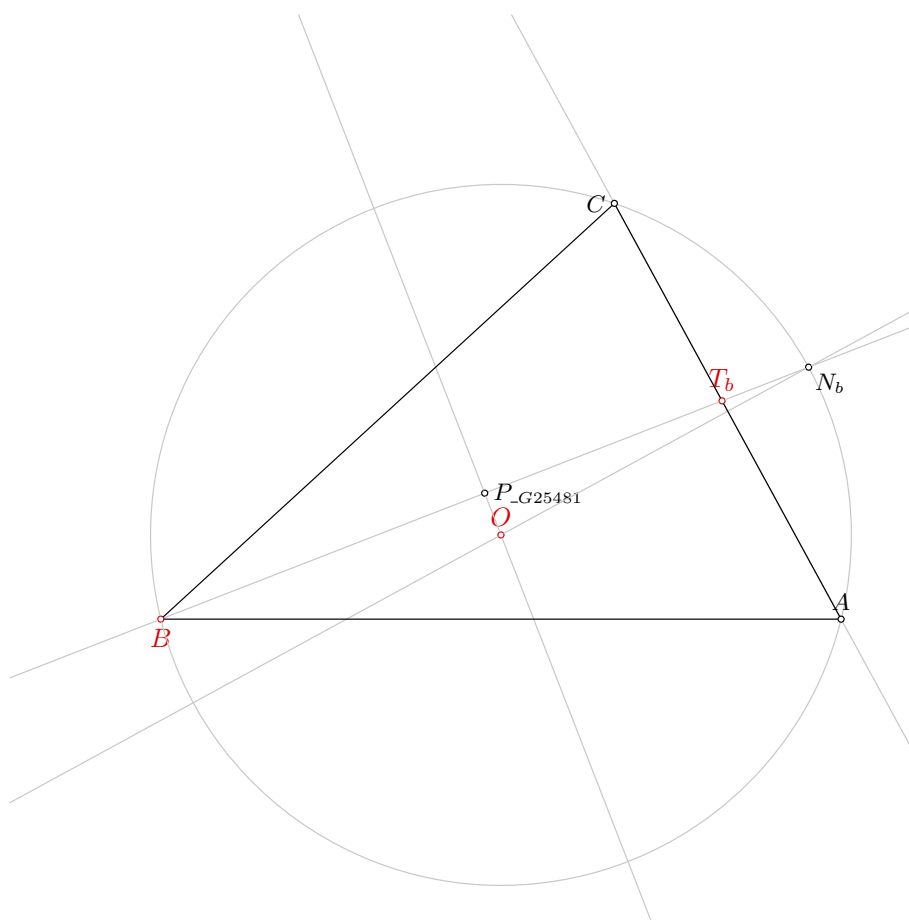


Figure 1: Illustration of the problem 0128

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{T_b N_b O} \neq 0$  i.e., points  $T_b$ ,  $N_b$  and  $O$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_a BC} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_b AC} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_a M_b F_{m_b}^2} \neq S_{F_{m_a}^1 M_b F_{m_b}^2}$  i.e., lines  $M_a F_{m_a}^1$  and  $M_b F_{m_b}^2$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $O=_O$

Proving failed

### 4.2.3 Proving $T_b=_T T_b$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.2 Proving $O=_O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 16774 terms.

**Time Complexity:** Time spent by the prover is 26.460 seconds. There are no ndg conditions.

### 4.3.3 Proving $T_b=_T T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6468 terms.

**Time Complexity:** Time spent by the prover is 5.110 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4.2 Proving $O=_O$

Proving failed

### 4.4.3 Proving $T_b=_T T_b$

Proving failed



# Problem 129

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 129: Given a point  $B$ , a point  $O$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $T_c$  are not the same;
2. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
3. Using the circle  $k(O, C)$ , the line  $c$ , the point  $O$  and the point  $B$ , construct a point  $A$  (rule W05); % NDG: line  $c$  and circle  $k(O, C)$  intersect % DET: points  $B$  and  $A$  must be different;
4. Using the point  $A$  and the point  $B$ , construct a point  $M_c$  (rule W01); ;
5. Using the point  $O$  and the point  $M_c$ , construct a line  $m_c$  (rule W02); % DET: points  $O$  and  $M_c$  are not the same;
6. Using the circle  $k(O, C)$  and the line  $m_c$ , construct a point  $N_c$  and a point  $N_{ck}$  (rule W04); % NDG: line  $m_c$  and circle  $k(O, C)$  intersect;
7. Using the point  $N_c$  and the point  $T_c$ , construct a line  $s_c$  (rule W02); % DET: points  $N_c$  and  $T_c$  are not the same;
8. Using the circle  $k(O, C)$ , the line  $s_c$ , the point  $O$  and the point  $N_c$ , construct a point  $C$  (rule W05); % NDG: line  $s_c$  and circle  $k(O, C)$  intersect % DET: points  $N_c$  and  $C$  must be different.

Non-degenerate conditions: line  $s_c$  and circle  $k(O, C)$  intersect; line  $m_c$  and circle  $k(O, C)$  intersect; line  $c$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same.

Determination conditions: points  $N_c$  and  $C$  must be different; points  $N_c$  and  $T_c$  are not the same; points  $O$  and  $M_c$  are not the same; points  $B$  and  $A$  must be different; points  $B$  and  $T_c$  are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D13,D20,D25,D26,D49,D52,GD01,L1,L11,L12,L27,L7]

Solving time: 2.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point O 65 51.14
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_b B
cmark_t O
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points B and T_{c} are not the same
% Constructing a line c which passes through point B and point T_{c}
line c B T_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line c and circle k(O,C) intersect% DET: points B and A must be different
% Constructing a point P_{\_G20483} which is a foot of the point O on the line c
foot P_{\_G20483} O c
cmark_r P_{\_G20483}
color 200 200 200
drawline O P_{\_G20483}
color 0 0 0

% Constructing a point A which is an image of the point B in the symmetry to point/line P_{\_G
20483}
```

```

sim A P_{\_G20483} B
cmark_t A

% Constructing a point  $M_{\{c\}}$  such that  $AM_{\{c\}}/AB=0.5$ 
towards M_{c} A B 0.5
cmark_lt M_{c}
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points  $O$  and  $M_{\{c\}}$  are not the same
% Constructing a line  $m_{\{c\}}$  which passes through point  $O$  and point  $M_{\{c\}}$ 
line m_{c} O M_{c}

color 200 200 200
drawline m_{c}
color 0 0 0

% NDG: line  $m_{\{c\}}$  and circle  $k(O,C)$  intersect
% Constructing points  $N_{\{c\}}$  and  $N_{\{ck\}}$  which are in intersection of  $k(O,C)$  and  $m_{\{c\}}$ 
intersec2 N_{c} N_{ck} k(O,C) m_{c}
cmark_b N_{c}
cmark_r N_{ck}

% DET: points  $N_{\{c\}}$  and  $T_{\{c\}}$  are not the same
% Constructing a line  $s_{\{c\}}$  which passes through point  $N_{\{c\}}$  and point  $T_{\{c\}}$ 
line s_{c} N_{c} T_{c}

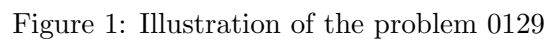
color 200 200 200
drawline s_{c}
color 0 0 0

% NDG: line  $s_{\{c\}}$  and circle  $k(O,C)$  intersect% DET: points  $N_{\{c\}}$  and  $C$  must be different
% Constructing a point  $P_{\{\_G20893\}}$  which is a foot of the point  $O$  on the line  $s_{\{c\}}$ 
foot P_{\_G20893} O s_{c}
cmark_r P_{\_G20893}
color 200 200 200
drawline O P_{\_G20893}
color 0 0 0

% Constructing a point  $C$  which is an image of the point  $N_{\{c\}}$  in the symmetry to point/line  $P_{\{\_G20893\}}$ 
sim C P_{\_G20893} N_{c}
cmark_l C

drawsegment A B

```



```
% Non-degenerate conditions: line s_{fc} and circle k(O,C) intersect; line m_{fc} and circle k(O,C)
% intersect; line c and circle k(O,C) intersect; points B and O are not the same
% Determination conditions: points N_{fc} and C must be different; points N_{fc} and T_{fc} are not
% the same; points O and M_{fc} are not the same; points B and A must be different; points B and T
% _{fc} are not the same
```

Illustration of the constructed figure is given in Figure 1

## 700

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.033 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

Proving failed

#### 4.1.3 Proving $T_c=_Tc$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{_M_aBC} \neq 0$  i.e., points  $_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$  i.e., points  $_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a_M_bF_{_m_b}^1} \neq S_{F_{_m_a}^0_M_bF_{_m_b}^1}$  i.e., lines  $_M_aF_{_m_a}^0$  and  $_M_bF_{_m_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $T_c=_Tc$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

Proving failed

#### 4.3.2 Proving $O=_O$

Proving failed

#### 4.3.3 Proving $T_c=_Tc$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B=B$

Proving failed

#### 4.4.2 Proving $O=_O$

Proving failed

#### 4.4.3 Proving $T_c=_T T_c$

Proving failed

# Problem 130

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 130: Given a point  $B$ , a point  $O$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $I$ , construct a line  $s_b$  (rule W02); % DET: points  $B$  and  $I$  are not the same;
2. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
3. Using the circle  $k(O, C)$ , the line  $s_b$ , the point  $O$  and the point  $B$ , construct a point  $N_b$  (rule W05); % NDG: line  $s_b$  and circle  $k(O, C)$  intersect % DET: points  $B$  and  $N_b$  must be different;
4. Using the point  $I$  and the point  $N_b$ , construct a circle  $k(N_b, A)$  (rule W06); % NDG: points  $I$  and  $N_b$  are not the same;
5. Using the circle  $k(O, C)$  and the circle  $k(N_b, A)$ , construct a point  $C$  and a point  $A$  (rule W07); % NDG: circles  $k(O, C)$  and  $k(N_b, A)$  intersect % DET: circles  $k(O, C)$  and  $k(N_b, A)$  are not the same.

Non-degenerate conditions: circles  $k(O, C)$  and  $k(N_b, A)$  intersect; points  $I$  and  $N_b$  are not the same; line  $s_b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same.

Determination conditions: circles  $k(O, C)$  and  $k(N_b, A)$  are not the same; points  $B$  and  $N_b$  must be different; points  $B$  and  $I$  are not the same.

Rules used: [W02,W05,W06,W07]

Lemmas used: [D2,D26,D48,GD02,L11,L12,L33,L34,L6]

Solving time: 0.3 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point O 65 51.14
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_b B
cmark_t O
cmark_b I
color 0 0 0
fontsize 8

% DET: points B and I are not the same
% Constructing a line  $s_{\{b\}}$  which passes through point B and point I
line  $s_{\{b\}}$  B I

color 200 200 200
drawline  $s_{\{b\}}$ 
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point B
circle  $k(O,C)$  O B

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $s_{\{b\}}$  and circle  $k(O,C)$  intersect% DET: points B and  $N_{\{b\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G22869\}}$  which is a foot of the point O on the line  $s_{\{b\}}$ 
foot  $P_{\{\backslash\_G22869\}}$  O  $s_{\{b\}}$ 
cmark_r  $P_{\{\backslash\_G22869\}}$ 
color 200 200 200
drawline O  $P_{\{\backslash\_G22869\}}$ 
color 0 0 0

% Constructing a point  $N_{\{b\}}$  which is an image of the point B in the symmetry to point/line  $P_{\{\backslash\_G22869\}}$ 
sim  $N_{\{b\}}$   $P_{\{\backslash\_G22869\}}$  B
cmark_rb  $N_{\{b\}}$ 

% NDG: points I and  $N_{\{b\}}$  are not the same
% Constructing a circle  $k(N_{\{b\}},A)$  whose center is at point  $N_{\{b\}}$  and which passes through point I
circle  $k(N_{\{b\}},A)$   $N_{\{b\}}$  I

color 200 200 200
```



```
drawcircle k(N_{b},A)
color 0 0 0
```

```
% NDG: circles k(O,C) and k(N_{b},A) intersect% DET: circles k(O,C) and k(N_{b},A) are not the same
% Constructing points C and A which are in intersection of k(O,C) and k(N_{b},A)
intersec2 C A k(O,C) k(N_{b},A)
cmark_l C
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: circles k(O,C) and k(N_{b},A) intersect; points I and N_{b} are not
the same; line s_{b} and circle k(O,C) intersect; points B and O are not the same
% Determination conditions: circles k(O,C) and k(N_{b},A) are not the same; points B and N_{b} must
be different; points B and I are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

Proving failed

#### 4.1.3 Proving $I=_I$

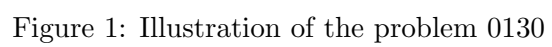
Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{M_aBC} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)  
 $S_{M_bAC} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)  
 $S_{M_aM_bF_{-m_b}^1} \neq S_{F_{-m_a}^0M_bF_{-m_b}^1}$  i.e., lines  $M_aF_{-m_a}^0$  and  $M_bF_{-m_b}^1$  are not parallel (construction based assumption)



Total number of proof steps: 1  
Time spent by the prover: 0.000 seconds

#### **4.2.2 Proving $O=_O$**

Proving failed

#### **4.2.3 Proving $I=_I$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $B=B$**

Proving failed

#### **4.3.2 Proving $O=_O$**

Proving failed

#### **4.3.3 Proving $I=_I$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $B=B$**

Proving failed

#### **4.4.2 Proving $O=_O$**

Proving failed

#### **4.4.3 Proving $I=_I$**

Proving failed

# Problem 131

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 131: Given a point  $B$ , a point  $M_a$  and a point  $M_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
2. Using the point  $B$  and the point  $M_b$ , construct a point  $G$  (rule W01); ;
3. Using the point  $M_a$  and the point  $G$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D21,GL03,GL04,L55,L56]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point M_{a} 65 40
point M_{b} 95 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_r M_{a}
cmark_lt M_{b}
color 0 0 0
fontsize 8
```

```

% Constructing a point C such that  $BC/BM_{\{a\}}=2$ 
towards C B  $M_{\{a\}}$  2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% Constructing a line  $L_{\{G14947\}}$  which passes through point B and point  $M_{\{b\}}$ 
line  $L_{\{G14947\}}$  B  $M_{\{b\}}$ 

color 200 200 200
drawline  $L_{\{G14947\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G15048\}}$  with coordinates (0,0)
point  $P_{\{G15048\}}$  0 0
cmark_r  $P_{\{G15048\}}$ 

% Constructing a point  $P_{\{G14972\}}$  such that  $BP_{\{G14972\}}/BP_{\{G15048\}}=2$ 
towards  $P_{\{G14972\}}$  B  $P_{\{G15048\}}$  2
cmark_r  $P_{\{G14972\}}$ 
color 200 200 200
drawsegment B  $P_{\{G14972\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G15017\}}$  such that  $BP_{\{G15017\}}/BP_{\{G15048\}}=3$ 
towards  $P_{\{G15017\}}$  B  $P_{\{G15048\}}$  3
cmark_r  $P_{\{G15017\}}$ 
color 200 200 200
drawsegment B  $P_{\{G15017\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G14978\}}$  which passes through point  $M_{\{b\}}$  and point  $P_{\{G15017\}}$ 
line  $L_{\{G14978\}}$   $M_{\{b\}}$   $P_{\{G15017\}}$ 

color 200 200 200
drawline  $L_{\{G14978\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G14941\}}$  which contains the point  $P_{\{G14972\}}$  and is parallel to the
line  $L_{\{G14978\}}$ 
parallel  $L_{\{G14941\}}$   $P_{\{G14972\}}$   $L_{\{G14978\}}$ 

color 200 200 200
drawline  $L_{\{G14941\}}$ 
color 0 0 0

% Constructing a point G which belongs to line  $L_{\{G14941\}}$  and line  $L_{\{G14947\}}$ 
intersec G  $L_{\{G14941\}}$   $L_{\{G14947\}}$ 
cmark_t G

```

```

% Constructing a point A such that  $M_{\{a\}A}/M_{\{a\}G}=3$ 
towards A M_{a} G 3
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a=_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.053 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $M_b=_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 91 terms.

**Time Complexity:** Time spent by the prover is 0.699 seconds.

**NDG conditions** Points  $M_b$ ,  $B$  and  $P_{G14405}$  are not collinear

Line through points  $M_b$  and  $P_{G14405}$  is not parallel with line through points  $B$  and  $M_a$

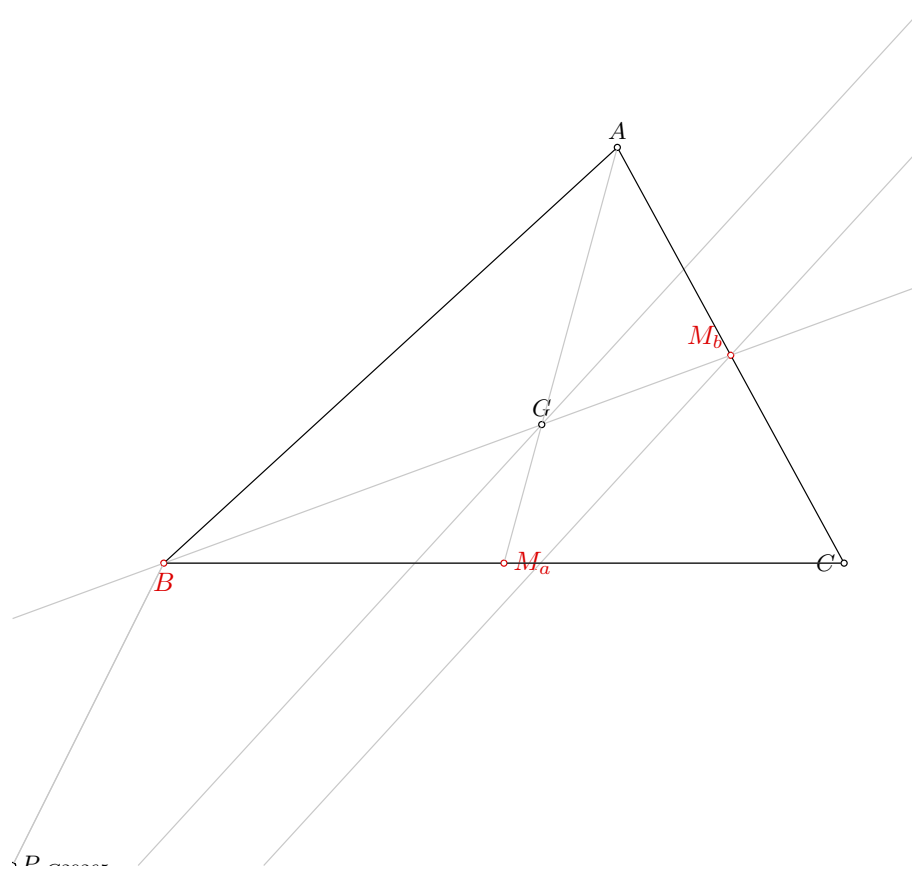


Figure 1: Illustration of the problem 0131

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{P_{-G30068}BM_b} \neq S_{P_{L_{-G30037}}^0}^{BM_b}$  i.e., lines  $P_{-G30068}P_{L_{-G30037}}^0$  and  $BM_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_a=_M M_a$

NDG conditions are:

$S_{P_{-G31028}BM_b} \neq S_{P_{L_{-G30997}}^0}^{BM_b}$  i.e., lines  $P_{-G31028}P_{L_{-G30997}}^0$  and  $BM_b$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $M_b=_M M_b$

NDG conditions are:

$S_{P_{-G12417}BM_b} \neq S_{P_{L_{-G12386}}^0}^{BM_b}$  i.e., lines  $P_{-G12417}P_{L_{-G12386}}^0$  and  $BM_b$  are not parallel (construction based assumption)

$S_{BM_bP_{-G12493}} \neq 0$  i.e., points  $B$ ,  $M_b$  and  $P_{-G12493}$  are not collinear (cancellation assumption)

Total number of proof steps: 4022

Time spent by the prover: 5.530 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_a=_M M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.3 Proving $M_b=_M M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3932 terms.

**Time Complexity:** Time spent by the prover is 6.690 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.



#### 4.4.2 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 107 terms.

**Time Complexity:** Time spent by the prover is 0.620 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 355 terms.

**Time Complexity:** Time spent by the prover is 0.630 seconds. There are no ndg conditions.

# Problem 132

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 132: Given a point  $B$ , a point  $M_a$  and a point  $M_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
2. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D20,D21,GL03,GL04]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point M_{a} 65 40
point M_{c} 50 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_r M_{a}
cmark_lt M_{c}
color 0 0 0
fontsize 8
```

```
% Constructing a point C such that BC/BM_{a}=2
```

```

towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% Constructing a point A such that BA/BM_{c}=2
towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.033 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a=_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.05 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $M_c=_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.062 seconds.

**NDG conditions** There are no NDG conditions for this theorem

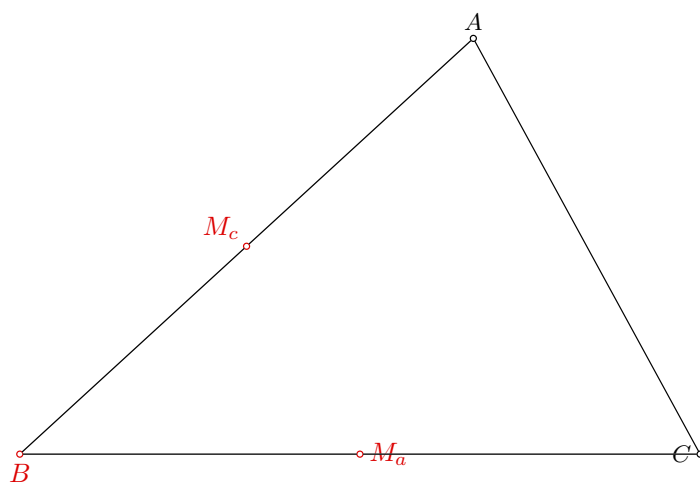


Figure 1: Illustration of the problem 0132

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

There are no ndg conditions.  
Total number of proof steps: 1  
Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_a=_M M_a$

There are no ndg conditions.  
Total number of proof steps: 41  
Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $M_c=_M M_c$

There are no ndg conditions.  
Total number of proof steps: 42  
Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_a=_M M_a$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $M_c=_M M_c$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $M_a=_M M_a$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.  
**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_c = M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

# Problem 133

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 133: Given a point  $B$ , a point  $M_a$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
2. Using the point  $M_a$  and the point  $G$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D21,GL03,GL04,L55]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point M_{a} 65 40
point G 70 58.33
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_r M_{a}
cmark_t G
color 0 0 0
fontsize 8
```

```
% Constructing a point C such that BC/BM_{a}=2
```

```

towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% Constructing a point A such that M_{a}A/M_{a}G=3
towards A M_{a} G 3
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a=_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.057 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $G=_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 25 terms.

**Time Complexity:** Time spent by the prover is 0.385 seconds.

**NDG conditions** Line through points  $A$  and  $_M_a$  is not parallel with line through points  $B$  and  $_M_b$   
Points  $A$  and  $B$  are not identical



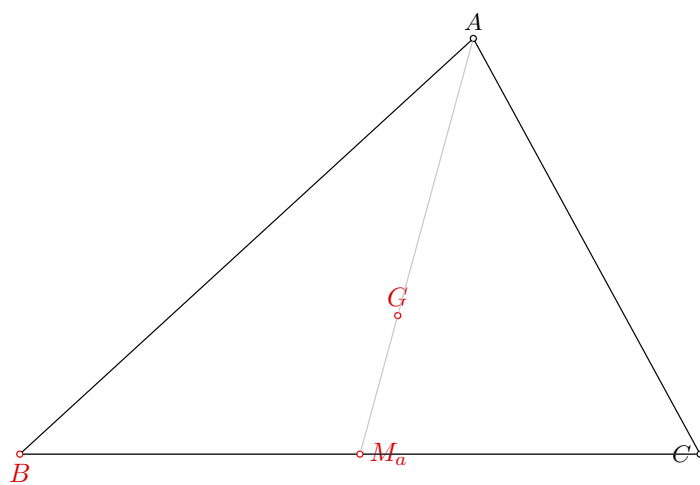


Figure 1: Illustration of the problem 0133

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{AB\_M_b} \neq S_{\_M_a B\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_a=_M_a$

NDG conditions are:

$S_{AB\_M_b} \neq S_{\_M_a B\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $G=_G$

NDG conditions are:

$S_{AB\_M_b} \neq S_{\_M_a B\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (cancellation assumption)

Total number of proof steps: 669

Time spent by the prover: 0.130 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_a=_M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.3 Proving $G=_G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 68 terms.

**Time Complexity:** Time spent by the prover is 0.180 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 77 terms.

**Time Complexity:** Time spent by the prover is 0.830 seconds. There are no ndg conditions.

#### 4.4.3 Proving $G = \neg G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 77 terms.

**Time Complexity:** Time spent by the prover is 0.900 seconds. There are no ndg conditions.

# Problem 134

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 134: Given a point  $M_a$ , a point  $H_a$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $H_a$  are not the same;
2. Choose freely a point  $B$  on the line  $a$  (rule WOnline1) ;
3. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
4. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
5. Choose freely a point  $A$  on the line  $h_a$  (rule WOnline2).

Non-degenerate conditions: .

Determination conditions: points  $M_a$  and  $H_a$  are not the same.

Rules used: [W01,W02,W10b,WOnline1,WOnline2]

Lemmas used: [D21,D5,D8,GD01,GL03,GL09]

Solving time: 157.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40  
point H_{a} 80 40  
point B 20 40
```

```
color 220 0 0  
fontsize 9
```

```

cmark_r M_{a}
cmark_r H_{a}
cmark_b B
color 0 0 0
fontsize 8

% DET: points M_{a} and H_{a} are not the same
% Constructing a line a which passes through point M_{a} and point H_{a}
line a M_{a} H_{a}

color 200 200 200
drawline a
color 0 0 0

% Choosing randomly a point B on the line M_{a}H_{a}
online B M_{a} H_{a}
cmark_b B
color 200 200 200
drawline M_{a} H_{a}
color 0 0 0

% Constructing a point C such that BC/BM_{a}=2
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% Generating random value V[_G26179]
random V[_G26179]

% Calculating value V[_G26200] using formula V[_G26179]*20
expression V[_G26200] { V[_G26179]*20 }

% Constructing a point A which is a point for which holds H_{a}A = V[_G26200] and angle BH_{a}A =
90
turtle A B H_{a} 90 V[_G26200]

```

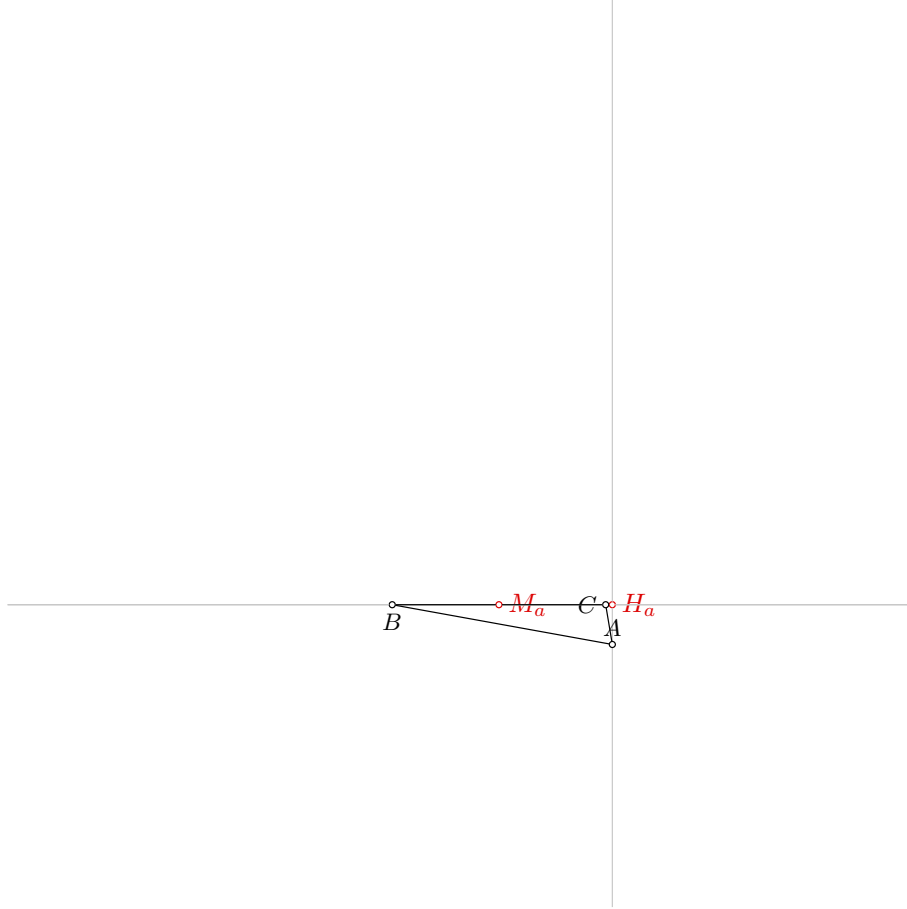


Figure 1: Illustration of the problem 0134

```
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions:
```

```
% Determination conditions: points  $M_{\{a\}}$  and  $H_{\{a\}}$  are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = \neg M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.032 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a = \neg H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.114 seconds.

**NDG conditions** Line through points  $C$  and  $B$  is not perpendicular to line through points  $B$  and  $M_a$

#### 4.1.3 Proving $B = B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.015 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF^2_{\neg h_a}} \neq S_{CAF^2_{\neg h_a}}$  i.e., lines  $BC$  and  $AF^2_{\neg h_a}$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.2.3 Proving $B = B$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF^2_{\neg h_a}} \neq S_{CAF^2_{\neg h_a}}$  i.e., lines  $BC$  and  $AF^2_{\neg h_a}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 2 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.



# Problem 135

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 135: Given a point  $M_a$ , a point  $H_b$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_b$  and  $M_a$  are not the same;
2. Choose freely a point  $B$  on the circle  $k(M_a, B)$  (rule WOncircle);
3. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
4. Using the point  $H_b$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $C$  are not the same;
5. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) .

Non-degenerate conditions: points  $H_b$  and  $M_a$  are not the same.

Determination conditions: points  $H_b$  and  $C$  are not the same.

Rules used: [W01,W02,W06,WOncircle1,WOnline1]

Lemmas used: [D21,D6,GD01,GD02,GL03,L38]

Solving time: 957.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{a} 65 40
point H_{b} 89.36 77.83
point B 20 40

color 220 0 0
fontsize 9
```

```

cmark_r M_{a}
cmark_l H_{b}
cmark_b B
color 0 0 0
fontsize 8

% NDG: points H_{b} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{b}
circle k(M_{a},B) M_{a} H_{b}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% Choosing randomly a point B on the circle with center M_{a} through point H_{b}
oncircle B M_{a} H_{b}
cmark_b B
color 200 200 200
drawcircle M_{a} H_{b}
color 0 0 0

% Constructing a point C such that BC/BM_{a}=2
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% DET: points H_{b} and C are not the same
% Constructing a line b which passes through point H_{b} and point C
line b H_{b} C

color 200 200 200
drawline b
color 0 0 0

% Choosing randomly a point A on the line H_{b}C
online A H_{b} C
cmark_t A
color 200 200 200
drawline H_{b} C
color 0 0 0

```

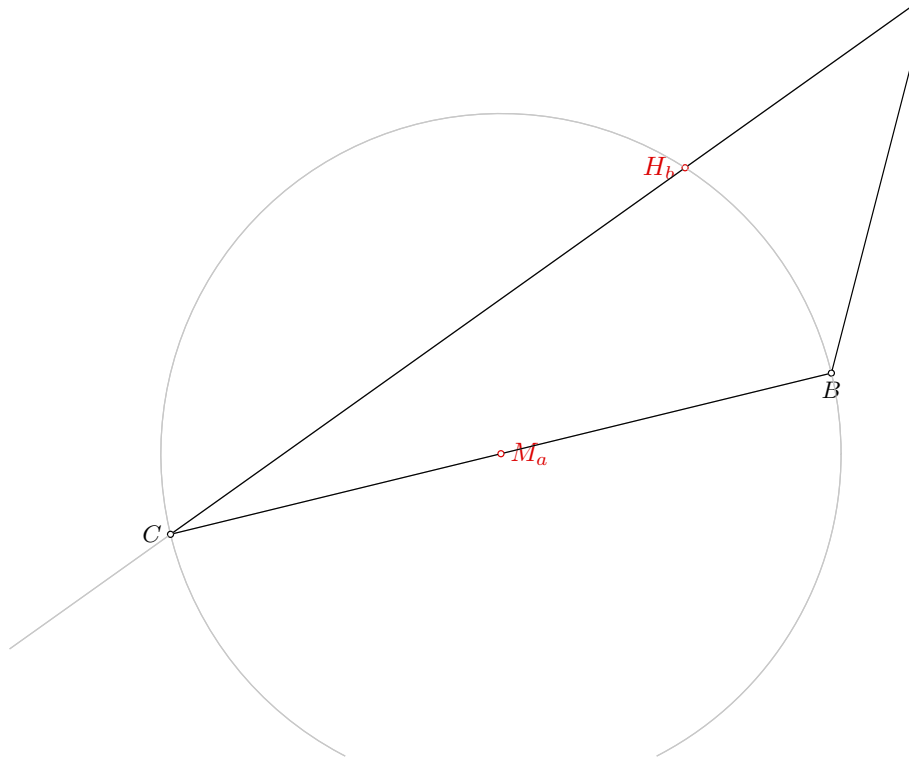


Figure 1: Illustration of the problem 0135

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: points  $H_{\{b\}}$  and  $M_{\{a\}}$  are not the same
% Determination conditions: points  $H_{\{b\}}$  and  $C$  are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = -M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.118 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.1.3 Proving $B = B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{h_b}} \neq S_{CBF^1_{h_b}}$  i.e., lines  $AC$  and  $BF^1_{h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.2.3 Proving $B = B$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{h_b}} \neq S_{CBF^1_{h_b}}$  i.e., lines  $AC$  and  $BF^1_{h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 17 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 280 terms.

**Time Complexity:** Time spent by the prover is 0.270 seconds. There are no ndg conditions.

#### 4.3.3 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_a = M_a$

Proving failed

#### 4.4.2 Proving $H_b = H_b$

Proving failed

#### 4.4.3 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 136

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 136: Given a point  $M_a$ , a point  $H_c$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_c$  and  $M_a$  are not the same;
2. Choose freely a point  $B$  on the circle  $k(M_a, B)$  (rule WOncircle);
3. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
4. Using the point  $B$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $H_c$  are not the same;
5. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) .

Non-degenerate conditions: points  $H_c$  and  $M_a$  are not the same.

Determination conditions: points  $B$  and  $H_c$  are not the same.

Rules used: [W01,W02,W06,WOncircle1,WOnline1]

Lemmas used: [D21,D7,GD01,GD02,GL03,L39]

Solving time: 815.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{a} 65 40
point H_{c} 68.91 84.83
point B 20 40

color 220 0 0
fontsize 9
```

```

cmark_r M_{a}
cmark_rt H_{c}
cmark_b B
color 0 0 0
fontsize 8

% NDG: points H_{c} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{c}
circle k(M_{a},B) M_{a} H_{c}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% Choosing randomly a point B on the circle with center M_{a} through point H_{c}
oncircle B M_{a} H_{c}
cmark_b B
color 200 200 200
drawcircle M_{a} H_{c}
color 0 0 0

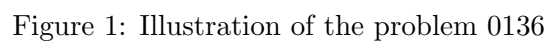
% Constructing a point C such that BC/BM_{a}=2
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

color 200 200 200
drawline c
color 0 0 0

% Choosing randomly a point A on the line BH_{c}
online A B H_{c}
cmark_t A
color 200 200 200
drawline B H_{c}
color 0 0 0

```



```
% Non-degenerate conditions: points  $H_{\{c\}}$  and  $M_{\{a\}}$  are not the same
% Determination conditions: points  $B$  and  $H_{\{c\}}$  are not the same
```

Illustration of the constructed figure is given in Figure 1

## 4 Correctness proof

### 4.1.1 Proving $M_a = -M_a$

736



**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.117 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 47 terms.

**Time Complexity:** Time spent by the prover is 0.662 seconds.

**NDG conditions** Points  $H_c$ ,  $B$  and  $M_a$  are not collinear

Points  $A$  and  $B$  are not identical

Points  $A$  and  $B$  are not identical

#### 4.1.3 Proving $B = B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^1} \neq S_{BCF_{h_c}^1}$  i.e., lines  $AB$  and  $CF_{h_c}^1$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.2.3 Proving $B = B$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^1} \neq S_{BCF_{h_c}^1}$  i.e., lines  $AB$  and  $CF_{h_c}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 17 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 280 terms.

**Time Complexity:** Time spent by the prover is 0.190 seconds. There are no ndg conditions.

#### 4.3.3 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 312 terms.

**Time Complexity:** Time spent by the prover is 2.040 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 312 terms.

**Time Complexity:** Time spent by the prover is 1.840 seconds. There are no ndg conditions.

#### 4.4.3 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 137

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 137: Given a point  $B$ , a point  $M_a$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
2. Using the point  $B$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H$  are not the same;
3. Using the point  $H$  and the point  $C$ , construct a line  $h_c$  (rule W02); % DET: points  $H$  and  $C$  are not the same;
4. Using the point  $B$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $B$  and  $M_a$  are not the same;
5. Using the circle  $k(M_a, B)$ , the line  $h_b$ , the point  $M_a$  and the point  $B$ , construct a point  $H_b$  (rule W05); % NDG: line  $h_b$  and circle  $k(M_a, B)$  intersect % DET: points  $B$  and  $H_b$  must be different;
6. Using the point  $H_b$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $C$  are not the same;
7. Using the circle  $k(M_a, B)$ , the line  $h_c$ , the point  $M_a$  and the point  $C$ , construct a point  $H_c$  (rule W05); % NDG: line  $h_c$  and circle  $k(M_a, B)$  intersect % DET: points  $C$  and  $H_c$  must be different;
8. Using the point  $H_c$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $B$  are not the same;
9. Using the line  $b$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $c$  are not parallel % DET: lines  $b$  and  $c$  are not the same.

Non-degenerate conditions: lines  $b$  and  $c$  are not parallel; line  $h_c$  and circle  $k(M_a, B)$  intersect; line  $h_b$  and circle  $k(M_a, B)$  intersect; points  $B$  and  $M_a$  are not the same.

Determination conditions: lines  $b$  and  $c$  are not the same; points  $H_c$  and  $B$  are not the same; points  $C$  and  $H_c$  must be different; points  $H_b$  and  $C$  are not the same; points  $B$  and  $H_b$  must be different; points  $H$  and  $C$  are not the same; points  $B$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D21,D3,D6,D7,D9,GD01,GD02,GL03,L3,L37,L38,L39]

Solving time: 1.9 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
```

```
point M_{a} 65 40
```

```
point H 80 72.73
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_b B
```

```
cmark_r M_{a}
```

```
cmark_rt H
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point C such that BC/BM_{a}=2
```

```
towards C B M_{a} 2
```

```
cmark_l C
```

```
color 200 200 200
```

```
drawsegment B C
```

```
color 0 0 0
```

```
% DET: points B and H are not the same
```

```
% Constructing a line h_{b} which passes through point B and point H
```

```
line h_{b} B H
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```
% DET: points H and C are not the same
```

```
% Constructing a line h_{c} which passes through point H and point C
```

```
line h_{c} H C
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```

% NDG: points B and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point B
circle k(M_{a},B) M_{a} B

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line h_{b} and circle k(M_{a},B) intersect% DET: points B and H_{b} must be different
% Constructing a point P_{\G21163} which is a foot of the point M_{a} on the line h_{b}
foot P_{\G21163} M_{a} h_{b}
cmark_r P_{\G21163}
color 200 200 200
drawline M_{a} P_{\G21163}
color 0 0 0

% Constructing a point H_{b} which is an image of the point B in the symmetry to point/line P_{\G
21163}
sim H_{b} P_{\G21163} B
cmark_l H_{b}

% DET: points H_{b} and C are not the same
% Constructing a line b which passes through point H_{b} and point C
line b H_{b} C

color 200 200 200
drawline b
color 0 0 0

% NDG: line h_{c} and circle k(M_{a},B) intersect% DET: points C and H_{c} must be different
% Constructing a point P_{\G21401} which is a foot of the point M_{a} on the line h_{c}
foot P_{\G21401} M_{a} h_{c}
cmark_r P_{\G21401}
color 200 200 200
drawline M_{a} P_{\G21401}
color 0 0 0

% Constructing a point H_{c} which is an image of the point C in the symmetry to point/line P_{\G
21401}
sim H_{c} P_{\G21401} C
cmark_rt H_{c}

% DET: points H_{c} and B are not the same
% Constructing a line c which passes through point H_{c} and point B
line c H_{c} B

color 200 200 200
drawline c
color 0 0 0

```

```

% NDG: lines b and c are not parallel% DET: lines b and c are not the same
% Constructing a point A which belongs to line b and line c
intersec A b c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and c are not parallel; line h_{c} and circle k(M_{a},B)
% intersect; line h_{b} and circle k(M_{a},B) intersect; points B and M_{a} are not the same
% Determination conditions: lines b and c are not the same; points H_{c} and B are not the same;
% points C and H_{c} must be different; points H_{b} and C are not the same; points B and H_{b}
% must be different; points H and C are not the same; points B and H are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a=_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $H=_H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 88 terms.

**Time Complexity:** Time spent by the prover is 2.173 seconds.

**NDG conditions** Points  $B$  and  $H$  are not identical

Points  $B$  and  $H$  are not identical

Points  $C$  and  $H$  are not identical

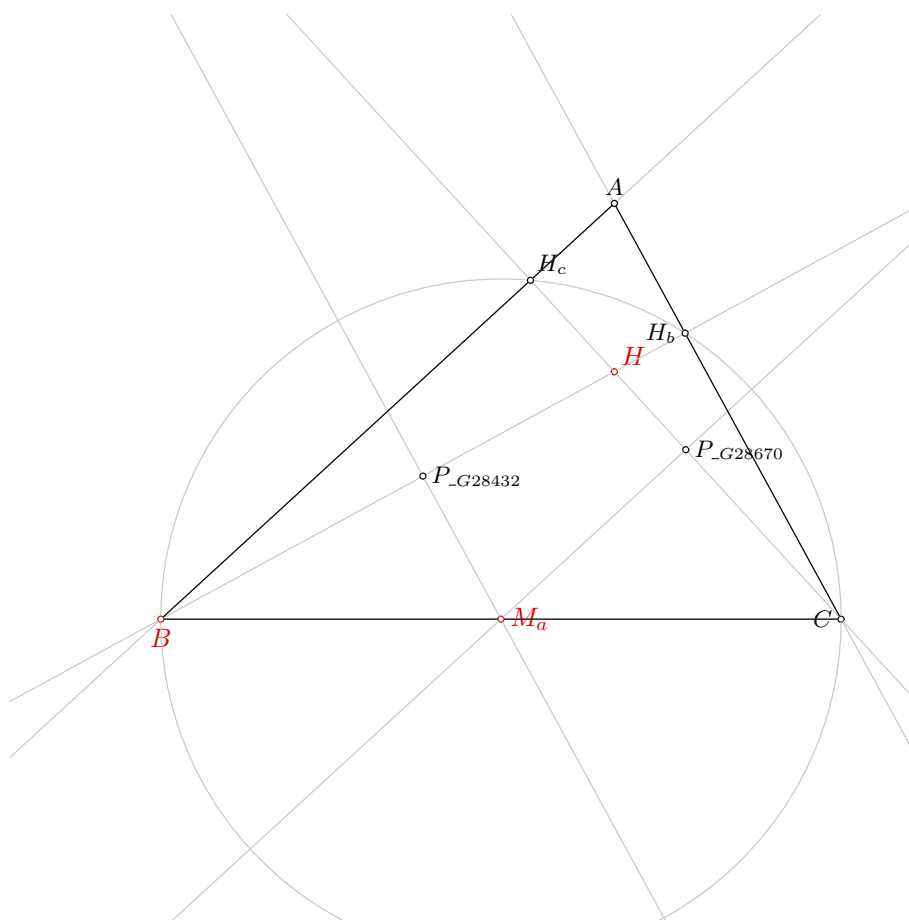


Figure 1: Illustration of the problem 0137

Line through points  $B$  and  $C$  is not perpendicular to line through points  $C$  and  $M_a$

Line through points  $H_b$  and  $C$  is not parallel with line through points  $B$  and  $H_c$

Points  $H_b$  and  $B$  are not identical

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $B$  and  $C$  are not identical

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_bH_cB} \neq S_{CH_cB}$  i.e., lines  $H_bC$  and  $H_cB$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-H_b}^1} \neq S_{F_{-H_a}^0BF_{-H_b}^1}$  i.e., lines  $AF_{-H_a}^0$  and  $BF_{-H_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_a=_M M_a$

Proving failed

### 4.2.3 Proving $H=_H H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_a=_M M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $H=_H H$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 9 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.



#### 4.4.2 Proving $M_a = -M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $H = -H$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

# Problem 138

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 138: Given a point  $M_a$ , a point  $T_a$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $T_a$  are not the same;
2. Choose freely a point  $B$  on the line  $a$  (rule WOnline1) ;
3. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
4. Using the point  $B$ , the point  $C$ , the point  $T_a$  and the line  $a$ , construct a point  $T'_a$  (rule W19);  
% NDG: points  $B$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $midpoint([B, T_a])$  are not the same;
5. Choose freely a point  $A$  on the circle  $k_{over}(T_a, T'_a)$  (rule WOncircle).

Non-degenerate conditions: points  $B$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $midpoint([B, T_a])$  are not the same.

Determination conditions: points  $M_a$  and  $T_a$  are not the same.

Rules used: [W01, W02, W19, WOncircle2, WOnline1]

Lemmas used: [D21, D23, GD01, GL03, GL09, L77]

Solving time: 88.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40  
point T_{a} 70.86 40  
point B 20 40
```

```

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_rb T_{a}
cmark_b B
color 0 0 0
fontsize 8

% DET: points M_{a} and T_{a} are not the same
% Constructing a line a which passes through point M_{a} and point T_{a}
line a M_{a} T_{a}

color 200 200 200
drawline a
color 0 0 0


% Choosing randomly a point B on the line M_{a}T_{a}
online B M_{a} T_{a}
cmark_b B
color 200 200 200
drawline M_{a} T_{a}
color 0 0 0


% Constructing a point C such that  $BC/BM_{a}=2$ 
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0


% NDG: points B and C are not the same; points C and T_{a} are not the same; points C and midpoint
% ([B,T_{a}]) are not the same
% Constructing a point P_{\_G21736} which is an image of the point C in a rotation around the point
% T_{a} for the angle 90
rotate P_{\_G21736} T_{a} 90 C
cmark_r P_{\_G21736}
color 200 200 200
drawarc_p T_{a} C 90
color 0 0 0


% Constructing a line L_{\_G21739} which passes through point T_{a} and point P_{\_G21736}
line L_{\_G21739} T_{a} P_{\_G21736}

color 200 200 200
drawline L_{\_G21739}
color 0 0 0


% Constructing midpoint P_{\_G21742} of the segment CP_{\_G21736}

```

```

midpoint P_{\_G21742} C P_{\_G21736}
cmark_r P_{\_G21742}

% Constructing a line L_{\_G21745} which passes through point B and point P_{\_G21742}
line L_{\_G21745} B P_{\_G21742}

color 200 200 200
drawline L_{\_G21745}
color 0 0 0

% Constructing a line L_{\_G21748} which passes through point C and point P_{\_G21742}
line L_{\_G21748} C P_{\_G21742}

color 200 200 200
drawline L_{\_G21748}
color 0 0 0

% Constructing a point P_{\_G21751} which belongs to line L_{\_G21739} and line L_{\_G21745}
intersec P_{\_G21751} L_{\_G21739} L_{\_G21745}
cmark_r P_{\_G21751}

% Constructing a point P_{\_G21754} which belongs to line L_{\_G21739} and line L_{\_G21748}
intersec P_{\_G21754} L_{\_G21739} L_{\_G21748}
cmark_r P_{\_G21754}

% Constructing a line L_{\_G21757} which passes through point B and point P_{\_G21754}
line L_{\_G21757} B P_{\_G21754}

color 200 200 200
drawline L_{\_G21757}
color 0 0 0

% Constructing a line L_{\_G21760} which passes through point C and point P_{\_G21751}
line L_{\_G21760} C P_{\_G21751}

color 200 200 200
drawline L_{\_G21760}
color 0 0 0

% Constructing a point P_{\_G21763} which belongs to line L_{\_G21757} and line L_{\_G21760}
intersec P_{\_G21763} L_{\_G21757} L_{\_G21760}
cmark_r P_{\_G21763}

% Constructing a line L_{\_G21766} which passes through point P_{\_G21742} and point P_{\_G21763}
line L_{\_G21766} P_{\_G21742} P_{\_G21763}

color 200 200 200
drawline L_{\_G21766}
color 0 0 0

% Constructing a point T'_{a} which belongs to line L_{\_G21766} and line a

```

```

intersec T'_{a} L_{\_G21766} a
cmark_r T'_{a}

```

```

% Constructing midpoint P_{\_G22591} of the segment T_{a}T'_{a}
midpoint P_{\_G22591} T_{a} T'_{a}
cmark_r P_{\_G22591}

```

```

% Choosing randomly a point A on the circle with center P_{\_G22591} through point T_{a}
oncircle A P_{\_G22591} T_{a}
cmark_t A
color 200 200 200
drawcircle P_{\_G22591} T_{a}
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

*% Non-degenerate conditions: points B and C are not the same; points C and T\_{a} are not the same;  
points C and midpoint([B,T\_{a}]) are not the same  
% Determination conditions: points M\_{a} and T\_{a} are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = \_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_a = \_T_a$

Proving failed

#### 4.1.3 Proving $B = B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.018 seconds.

**NDG conditions** There are no NDG conditions for this theorem

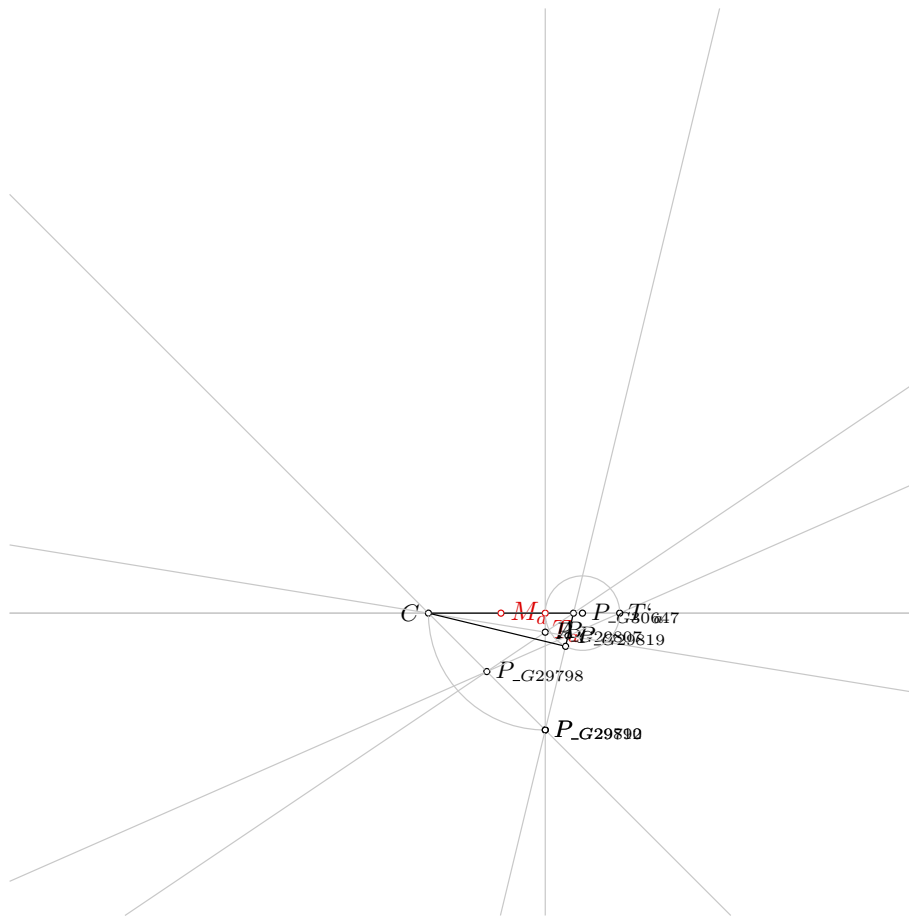


Figure 1: Illustration of the problem 0138

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_a = \neg M_a$

Proving failed

### 4.2.2 Proving $T_a = \neg T_a$

Proving failed

### 4.2.3 Proving $B = B$

NDG conditions are:

$S_{T_a B P_{G17147}} \neq S_{P_{G17141} B P_{G17147}}$  i.e., lines  $T_a P_{G17141}$  and  $B P_{G17147}$  are not parallel (construction based assumption)

$S_{T_a C P_{G17147}} \neq S_{P_{G17141} C P_{G17147}}$  i.e., lines  $T_a P_{G17141}$  and  $C P_{G17147}$  are not parallel (construction based assumption)

$S_{B C P_{G17156}} \neq S_{P_{G17159} C P_{G17156}}$  i.e., lines  $B P_{G17159}$  and  $C P_{G17156}$  are not parallel (construction based assumption)

$S_{P_{G17147} M_a T_a} \neq S_{P_{G17168} M_a T_a}$  i.e., lines  $P_{G17147} P_{G17168}$  and  $M_a T_a$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_a = \neg M_a$

Proving failed

### 4.3.2 Proving $T_a = \neg T_a$

Proving failed

### 4.3.3 Proving $B = B$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_a = \neg M_a$

Proving failed

### 4.4.2 Proving $T_a = \neg T_a$

Proving failed

### 4.4.3 Proving $B = B$

Proving failed

# Problem 139

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 139: Given a point  $B$ , a point  $M_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
2. Using the point  $B$  and the point  $M_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $M_a$  are not the same;
3. Using the point  $T_b$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $T_b$  and  $C$  are not the same;
4. Using the point  $T_b$  and the line  $a$ , construct a circle  $k(T_b, foot[T_b, a])$  (rule W11); % NDG: point  $T_b$  is not incident to the line  $a$ ;
5. Using the circle  $k(T_b, foot[T_b, a])$ , the point  $B$ , the point  $T_b$  and the line  $a$ , construct a line  $c$  (rule W13); % NDG: point  $B$  is outside the circle  $k(T_b, foot[T_b, a])$ ;
6. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; point  $B$  is outside the circle  $k(T_b, foot[T_b, a])$ ; point  $T_b$  is not incident to the line  $a$ .

Determination conditions: lines  $c$  and  $b$  are not the same; points  $T_b$  and  $C$  are not the same; points  $B$  and  $M_a$  are not the same.

Rules used: [W01,W02,W03,W11,W13]

Lemmas used: [D21,D24,GD01,GD02,GL03,GL09,GL10,GL11,L9]

Solving time: 1.8 seconds.



## 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point M_{a} 65 40
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_b B
cmark_r M_{a}
cmark_t T_{b}
color 0 0 0
fontsize 8

% Constructing a point C such that  $BC/BM_{\{a\}}=2$ 
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% DET: points B and  $M_{\{a\}}$  are not the same
% Constructing a line a which passes through point B and point  $M_{\{a\}}$ 
line a B M_{a}

color 200 200 200
drawline a
color 0 0 0

% DET: points  $T_{\{b\}}$  and C are not the same
% Constructing a line b which passes through point  $T_{\{b\}}$  and point C
line b T_{b} C

color 200 200 200
drawline b
color 0 0 0

% NDG: point  $T_{\{b\}}$  is not incident to the line a
% Constructing a point  $P_{\{\backslash\_G23006\}}$  which is a foot of the point  $T_{\{b\}}$  on the line a
foot P_{\backslash\_G23006} T_{b} a
cmark_r P_{\backslash\_G23006}
color 200 200 200
drawline T_{b} P_{\backslash\_G23006}
color 0 0 0

% Constructing a circle  $k(T_{\{b\}}, foot[T_{\{b\}}, c])$  whose center is at point  $T_{\{b\}}$  and which passes
    through point  $P_{\{\backslash\_G23006\}}$ 
```

```

circle k(T_{b},foot[T_{b},c]) T_{b} P_{\_G23006}

color 200 200 200
drawcircle k(T_{b},foot[T_{b},c])
color 0 0 0

% NDG: point B is outside the circle k(T_{b},foot[T_{b},c])
% Constructing a point P_{\_G23397} which is a foot of the point T_{b} on the line a
foot P_{\_G23397} T_{b} a
cmark_r P_{\_G23397}
color 200 200 200
drawline T_{b} P_{\_G23397}
color 0 0 0

% Constructing a line L_{\_G23400} which passes through point T_{b} and point B
line L_{\_G23400} T_{b} B

color 200 200 200
drawline L_{\_G23400}
color 0 0 0

% Constructing a point P_{\_G23403} which is an image of the point P_{\_G23397} in the symmetry to
point/line L_{\_G23400}
sim P_{\_G23403} L_{\_G23400} P_{\_G23397}
cmark_r P_{\_G23403}

% Constructing a line c which passes through point B and point P_{\_G23403}
line c B P_{\_G23403}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and b are not parallel; point B is outside the circle k(T_{b},
foot[T_{b},c]); point T_{b} is not incident to the line a
% Determination conditions: lines c and b are not the same; points T_{b} and C are not the same;
points B and M_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

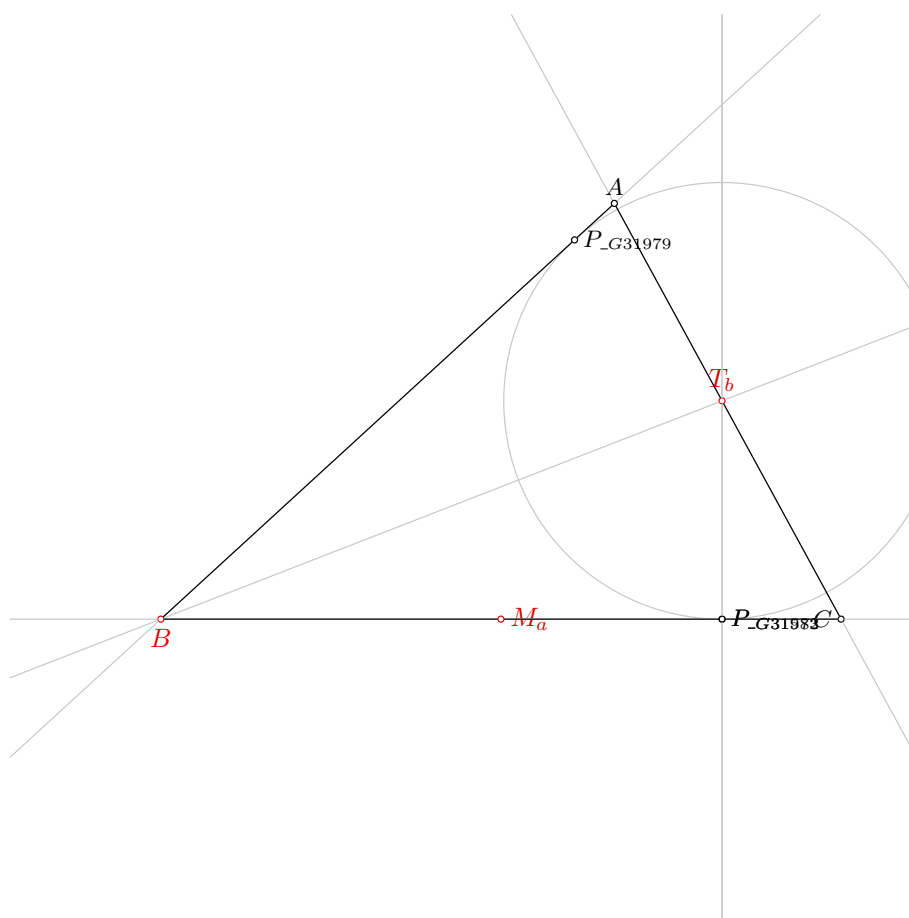


Figure 1: Illustration of the problem 0139

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a=_M M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $T_b=_T T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{BT_bC} \neq S_{P_{G15082}T_bC}$  i.e., lines  $BP_{G15082}$  and  $T_bC$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_a=_M M_a$

Proving failed

#### 4.2.3 Proving $T_b=_T T_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_a=_M M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 5 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_b = \neg T_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 14 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_b = \neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 140

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 140: Given a point  $B$ , a point  $M_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
2. Using the point  $B$  and the point  $M_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $M_a$  are not the same;
3. Using the point  $B$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $T_c$  are not the same;
4. Using the point  $T_c$  and the line  $a$ , construct a circle  $k(T_c, foot[T_c, b])$  (rule W11); % NDG: point  $T_c$  is not incident to the line  $a$ ;
5. Using the circle  $k(T_c, foot[T_c, b])$ , the point  $C$ , the point  $T_c$  and the line  $a$ , construct a line  $b$  (rule W13); % NDG: point  $C$  is outside the circle  $k(T_c, foot[T_c, b])$ ;
6. Using the line  $b$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $c$  are not parallel % DET: lines  $b$  and  $c$  are not the same.

Non-degenerate conditions: lines  $b$  and  $c$  are not parallel; point  $C$  is outside the circle  $k(T_c, foot[T_c, b])$ ; point  $T_c$  is not incident to the line  $a$ .

Determination conditions: lines  $b$  and  $c$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $M_a$  are not the same.

Rules used: [W01,W02,W03,W11,W13]

Lemmas used: [D21,D25,GD01,GD02,GL03,GL09,GL10,GL11,L10]

Solving time: 1.8 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point M_{a} 65 40
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_b B
cmark_r M_{a}
cmark_rt T_{c}
color 0 0 0
fontsize 8

% Constructing a point C such that BC/BM_{a}=2
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% DET: points B and M_{a} are not the same
% Constructing a line a which passes through point B and point M_{a}
line a B M_{a}

color 200 200 200
drawline a
color 0 0 0

% DET: points B and T_{c} are not the same
% Constructing a line c which passes through point B and point T_{c}
line c B T_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: point T_{c} is not incident to the line a
% Constructing a point P_{\_G24095} which is a foot of the point T_{c} on the line a
foot P_{\_G24095} T_{c} a
cmark_r P_{\_G24095}
color 200 200 200
drawline T_{c} P_{\_G24095}
color 0 0 0

% Constructing a circle k(T_{c},foot[T_{c},b]) whose center is at point T_{c} and which passes
through point P_{\_G24095}
```

```

circle k(T_{c},foot[T_{c},b]) T_{c} P_{\_G24095}

color 200 200 200
drawcircle k(T_{c},foot[T_{c},b])
color 0 0 0

% NDG: point C is outside the circle k(T_{c},foot[T_{c},b])
% Constructing a point P_{\_G24486} which is a foot of the point T_{c} on the line a
foot P_{\_G24486} T_{c} a
cmark_r P_{\_G24486}
color 200 200 200
drawline T_{c} P_{\_G24486}
color 0 0 0

% Constructing a line L_{\_G24489} which passes through point T_{c} and point C
line L_{\_G24489} T_{c} C

color 200 200 200
drawline L_{\_G24489}
color 0 0 0

% Constructing a point P_{\_G24492} which is an image of the point P_{\_G24486} in the symmetry to
point/line L_{\_G24489}
sim P_{\_G24492} L_{\_G24489} P_{\_G24486}
cmark_r P_{\_G24492}

% Constructing a line b which passes through point C and point P_{\_G24492}
line b C P_{\_G24492}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines b and c are not parallel% DET: lines b and c are not the same
% Constructing a point A which belongs to line b and line c
intersec A b c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and c are not parallel; point C is outside the circle k(T_{c},
foot[T_{c},b]); point T_{c} is not incident to the line a
% Determination conditions: lines b and c are not the same; points B and T_{c} are not the same;
points B and M_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1



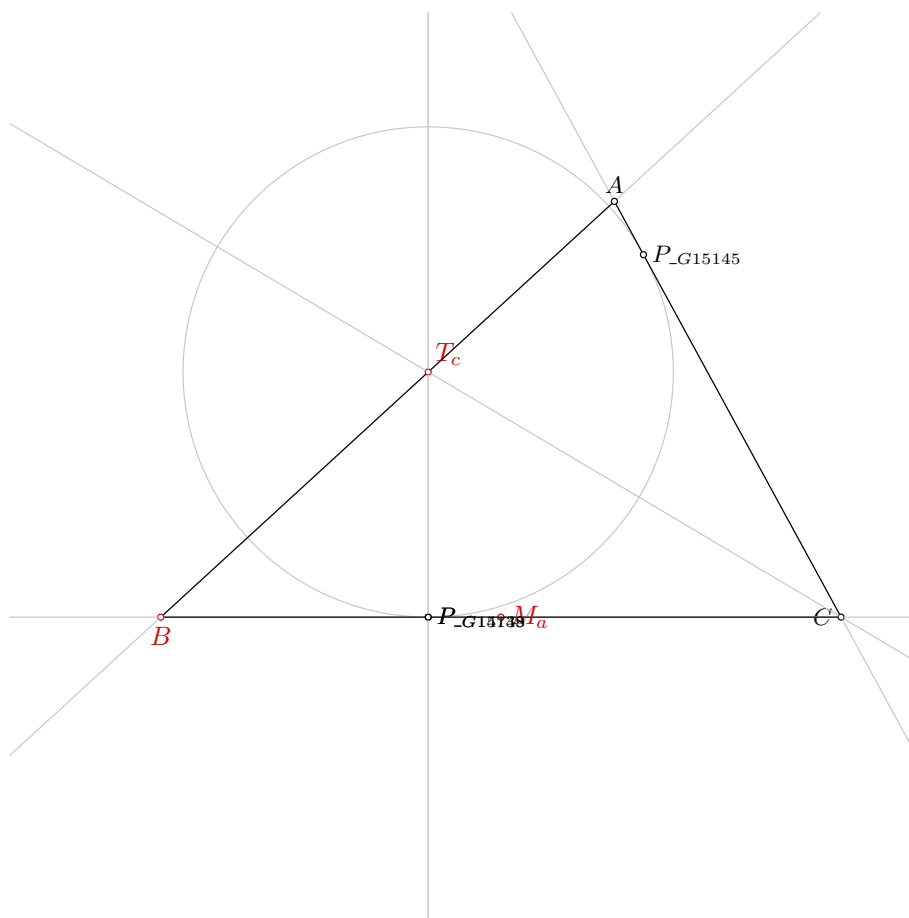


Figure 1: Illustration of the problem 0140

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a=_M a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $T_c=_T c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{CBT_c} \neq S_{P_{G16171}BT_c}$  i.e., lines  $CP_{G16171}$  and  $BT_c$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_a=_M a$

Proving failed

#### 4.2.3 Proving $T_c=_T c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_a=_M a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_c = \neg T_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 10 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 7 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_c = \neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 7 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

# Problem 141

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 141: Given a point  $B$ , a point  $M_a$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
2. Using the point  $B$  and the point  $M_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $M_a$  are not the same;
3. Using the point  $I$  and the line  $a$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $a$ ;
4. Using the circle  $k(I, P_a)$ , the point  $B$ , the point  $I$  and the line  $a$ , construct a line  $c$  (rule W13); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
5. Using the circle  $k(I, P_a)$ , the point  $C$ , the point  $I$  and the line  $a$ , construct a line  $b$  (rule W13); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
6. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; point  $C$  is outside the circle  $k(I, P_a)$ ; point  $B$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $a$ .

Determination conditions: lines  $c$  and  $b$  are not the same; points  $B$  and  $M_a$  are not the same.

Rules used: [W01,W02,W03,W11,W13]

Lemmas used: [D21,D27,GD01,GL03,GL09,L59,L60,L61]

Solving time: 17.2 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point M_{a} 65 40
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_b B
cmark_r M_{a}
cmark_b I
color 0 0 0
fontsize 8

% Constructing a point C such that  $BC/BM_{\{a\}}=2$ 
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% DET: points B and  $M_{\{a\}}$  are not the same
% Constructing a line a which passes through point B and point  $M_{\{a\}}$ 
line a B M_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: point I is not incident to the line a
% Constructing a point  $P_{\{\backslash\_G29343\}}$  which is a foot of the point I on the line a
foot P_{\backslash\_G29343} I a
cmark_r P_{\backslash\_G29343}
color 200 200 200
drawline I P_{\backslash\_G29343}
color 0 0 0

% Constructing a circle  $k(I, P_{\{a\}})$  whose center is at point I and which passes through point  $P_{\{\backslash\_G29343\}}$ 
circle k(I, P_{a}) I P_{\backslash\_G29343}

color 200 200 200
drawcircle k(I, P_{a})
color 0 0 0

% NDG: point B is outside the circle  $k(I, P_{\{a\}})$ 
% Constructing a point  $P_{\{\backslash\_G29550\}}$  which is a foot of the point I on the line a
```

```

foot P_{\_G29550} I a
cmark_r P_{\_G29550}
color 200 200 200
drawline I P_{\_G29550}
color 0 0 0

% Constructing a line L_{\_G29553} which passes through point I and point B
line L_{\_G29553} I B

color 200 200 200
drawline L_{\_G29553}
color 0 0 0

% Constructing a point P_{\_G29556} which is an image of the point P_{\_G29550} in the symmetry to
point/line L_{\_G29553}
sim P_{\_G29556} L_{\_G29553} P_{\_G29550}
cmark_r P_{\_G29556}

% Constructing a line c which passes through point B and point P_{\_G29556}
line c B P_{\_G29556}

color 200 200 200
drawline c
color 0 0 0

% NDG: point C is outside the circle k(I,P_{a})
% Constructing a point P_{\_G29885} which is a foot of the point I on the line a
foot P_{\_G29885} I a
cmark_r P_{\_G29885}
color 200 200 200
drawline I P_{\_G29885}
color 0 0 0

% Constructing a line L_{\_G29888} which passes through point I and point C
line L_{\_G29888} I C

color 200 200 200
drawline L_{\_G29888}
color 0 0 0

% Constructing a point P_{\_G29891} which is an image of the point P_{\_G29885} in the symmetry to
point/line L_{\_G29888}
sim P_{\_G29891} L_{\_G29888} P_{\_G29885}
cmark_r P_{\_G29891}

% Constructing a line b which passes through point C and point P_{\_G29891}
line b C P_{\_G29891}

color 200 200 200
drawline b
color 0 0 0

```

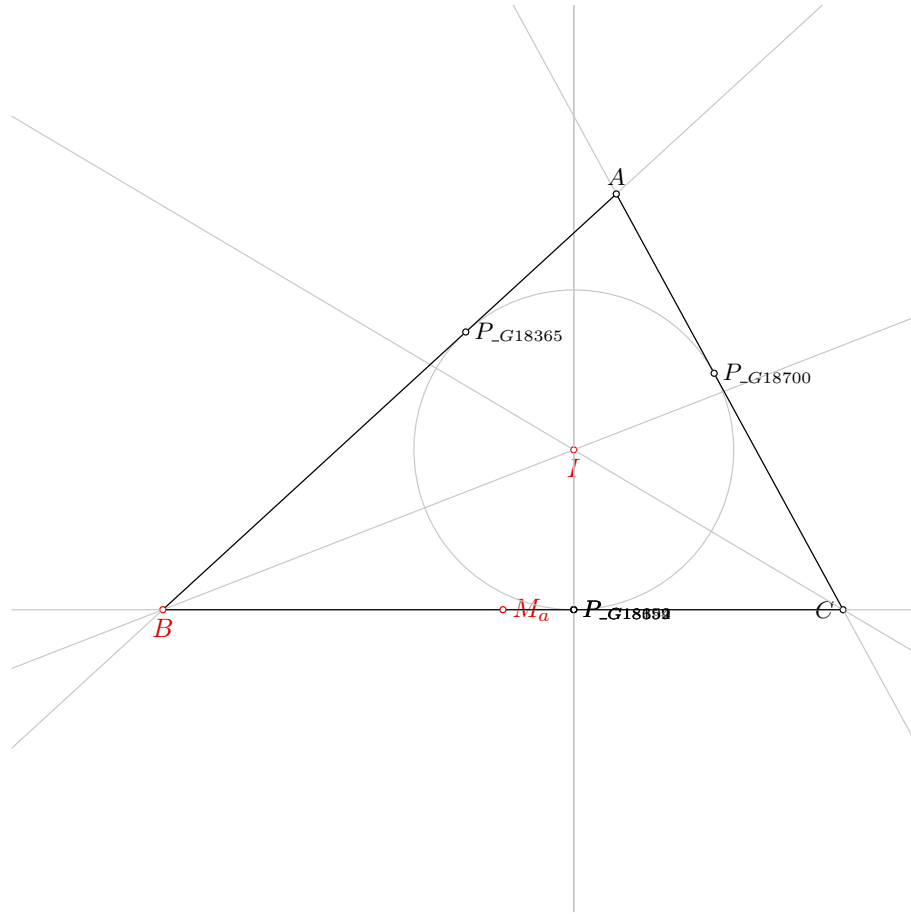


Figure 1: Illustration of the problem 0141

```
% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines c and b are not parallel; point C is outside the circle k(I,P_{a}
}); point B is outside the circle k(I,P_{a}); point I is not incident to the line a
% Determination conditions: lines c and b are not the same; points B and M_{a} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a=_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.033 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{BCP_{G19780}} \neq S_{P_{G19445}CP_{G19780}}$  i.e., lines  $BP_{G19445}$  and  $CP_{G19780}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_a=_M_a$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.



#### 4.3.2 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 26 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I = \neg I$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 65 terms.

**Time Complexity:** Time spent by the prover is 0.060 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 15 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.3 Proving $I = \neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 15 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

# Problem 142

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 142: Given a point  $B$ , a point  $M_b$  and a point  $M_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01); ;
2. Using the point  $M_b$  and the point  $A$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D20,D22,GL03,GL04]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point M_{b} 95 67.5
point M_{c} 50 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_lt M_{b}
cmark_lt M_{c}
color 0 0 0
fontsize 8
```

```
% Constructing a point A such that BA/BM_{c}=2
```

```

towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

% Constructing a point C such that M_{b}C/M_{b}A=-1
towards C M_{b} A -1
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.019 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b=_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.165 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $M_c=_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.06 seconds.

**NDG conditions** There are no NDG conditions for this theorem

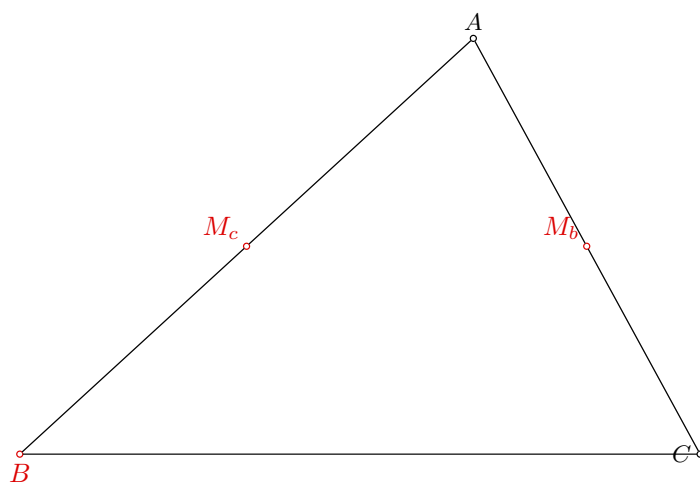


Figure 1: Illustration of the problem 0142

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

There are no ndg conditions.  
Total number of proof steps: 1  
Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_b=_M M_b$

There are no ndg conditions.  
Total number of proof steps: 44  
Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $M_c=_M M_c$

There are no ndg conditions.  
Total number of proof steps: 42  
Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_b=_M M_b$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 19 terms.  
**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

### 4.3.3 Proving $M_c=_M M_c$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.  
**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $M_b=_M M_b$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 67 terms.  
**Time Complexity:** Time spent by the prover is 0.580 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_c = M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 67 terms.

**Time Complexity:** Time spent by the prover is 0.590 seconds. There are no ndg conditions.

# Problem 143

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 143: Given a point  $M_b$ , a point  $G$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is redundant.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $G$ , construct a point  $B$  (rule W01); ;
2. Choose freely a point  $A$  (rule free);
3. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01,free]

Lemmas used: [D22,GL03,GL04,L56]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{b} 95 67.5
```

```
point G 70 58.33
```

```
point B 20 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_lt M_{b}
```

```
cmark_t G
```

```
cmark_b B
```

```
color 0 0 0
```

```
fontsize 8
```

```

% Constructing a point B such that  $M_{\{b\}B}/M_{\{b\}G}=3$ 
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

```

```

% Constructing a free point A
point A 80 95

```

```

cmark_t A

```

```

% Constructing a point C such that  $AC/AM_{\{b\}}=2$ 
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = \_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.054 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $G = \_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 174 terms.



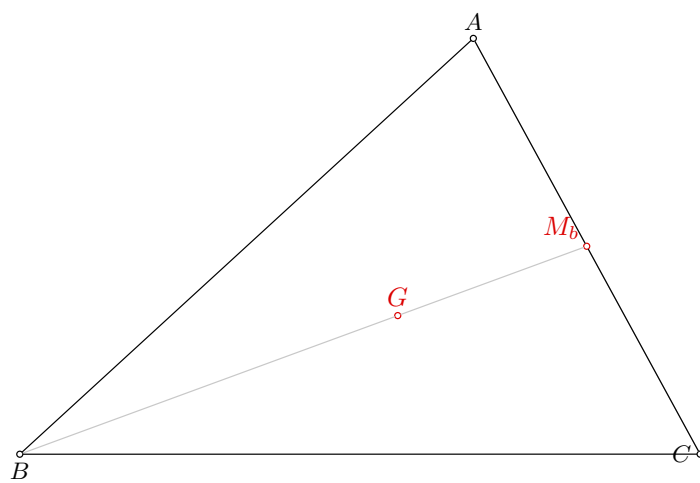


Figure 1: Illustration of the problem 0143

**Time Complexity:** Time spent by the prover is 1.114 seconds.

**NDG conditions** Line through points  $A$  and  $M_a$  is not parallel with line through points  $B$  and  $M_b$   
Line through points  $M_b$  and  $G$  is not parallel with line through points  $A$  and  $M_a$

### 4.1.3 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_b=M_b$

NDG conditions are:

$S_{AB-M_b} \neq S_{M_aB-M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $G=G$

NDG conditions are:

$S_{AB-M_b} \neq S_{M_aB-M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (cancellation assumption)

Total number of proof steps: 615

Time spent by the prover: 0.200 seconds

### 4.2.3 Proving $B=B$

NDG conditions are:

$S_{AB-M_b} \neq S_{M_aB-M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_b=M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 12 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.2 Proving $G=G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 745 terms.

**Time Complexity:** Time spent by the prover is 0.630 seconds. There are no ndg conditions.

### 4.3.3 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 341 terms.

**Time Complexity:** Time spent by the prover is 24.250 seconds. There are no ndg conditions.

### 4.4.2 Proving $G = \neg G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 341 terms.

**Time Complexity:** Time spent by the prover is 24.160 seconds. There are no ndg conditions.

### 4.4.3 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 144

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 144: Given a point  $B$ , a point  $M_b$  and a point  $H_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $H_a$  are not the same;
2. Using the point  $H_a$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_a$  and  $M_b$  are not the same;
3. Using the circle  $k(M_b, C)$ , the line  $a$ , the point  $M_b$  and the point  $H_a$ , construct a point  $C$  (rule W05); % NDG: line  $a$  and circle  $k(M_b, C)$  intersect % DET: points  $H_a$  and  $C$  must be different;
4. Using the point  $M_b$  and the point  $C$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: line  $a$  and circle  $k(M_b, C)$  intersect; points  $H_a$  and  $M_b$  are not the same.

Determination conditions: points  $H_a$  and  $C$  must be different; points  $B$  and  $H_a$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D22,D5,GD01,GD02,GL03,GL04,L44]

Solving time: 0.2 seconds.

### 3.2 Construction in GCLC language

dim 120 120

```
point B 20 40
point M_{b} 95 67.5
point H_{a} 80 40
```

```

color 220 0 0
fontsize 9

cmark_b B
cmark_lt M_{b}
cmark_r H_{a}
color 0 0 0
fontsize 8

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points H_{a} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{a}
circle k(M_{b},C) M_{b} H_{a}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line a and circle k(M_{b},C) intersect% DET: points H_{a} and C must be different
% Constructing a point P_{\_G22089} which is a foot of the point M_{b} on the line a
foot P_{\_G22089} M_{b} a
cmark_r P_{\_G22089}
color 200 200 200
drawline M_{b} P_{\_G22089}
color 0 0 0

% Constructing a point C which is an image of the point H_{a} in the symmetry to point/line P_{\_G22089}
sim C P_{\_G22089} H_{a}
cmark_l C

% Constructing a point A such that M_{b}A/M_{b}C=-1
towards A M_{b} C -1
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

drawsegment A B

```

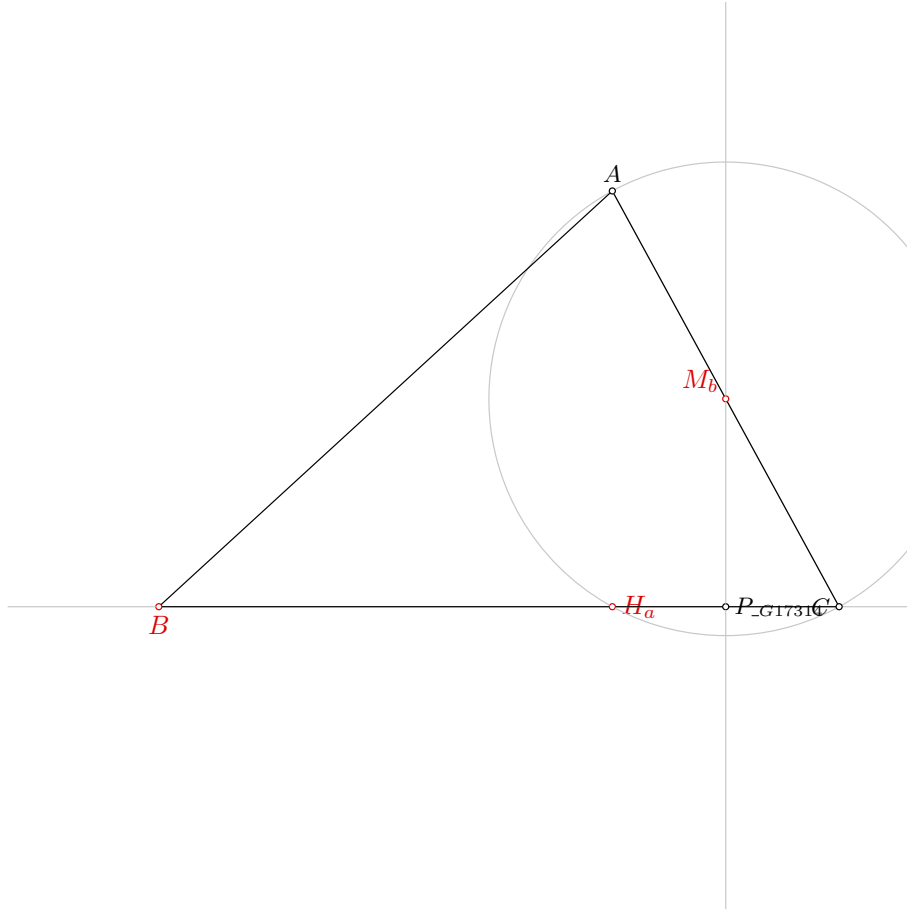


Figure 1: Illustration of the problem 0144

```
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(M_{b},C) intersect; points H_{a} and M_{b} are not
the same
% Determination conditions: points H_{a} and C must be different; points B and H_{a} are not the
same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.019 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b = \neg M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.168 seconds.

**NDG conditions** Points  $B$  and  $H_a$  are not identical

Points  $B$  and  $H_a$  are not identical

#### 4.1.3 Proving $H_a = \neg H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 39 terms.

**Time Complexity:** Time spent by the prover is 0.528 seconds.

**NDG conditions** Points  $B$  and  $H_a$  are not identical

Points  $B$  and  $H_a$  are not identical

Points  $B$  and  $C$  are not identical

Points  $B$  and  $C$  are not identical

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B = B$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{\neg H_a}^0} \neq S_{CAF_{\neg H_a}^0}$  i.e., lines  $BC$  and  $AF_{\neg H_a}^0$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{\neg H_a}^0} \neq S_{CAF_{\neg H_a}^0}$  i.e., lines  $BC$  and  $AF_{\neg H_a}^0$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $H_a = \neg H_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B = B$

Proving failed

#### 4.3.2 Proving $M_b = \neg M_b$

Proving failed

### 4.3.3 Proving $H_a = \neg H_a$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B = B$

Proving failed

### 4.4.2 Proving $M_b = \neg M_b$

Proving failed

### 4.4.3 Proving $H_a = \neg H_a$

Proving failed



# Problem 145

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 145: Given a point  $M_b$ , a point  $H_b$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $H_b$  are not the same;
2. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
3. Choose freely a point  $B$  on the line  $h_b$  (rule WOnline2);
4. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) ;
5. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: points  $M_b$  and  $H_b$  are not the same.

Rules used: [W01,W02,W10b,WOnline1,WOnline2]

Lemmas used: [D22,D6,D9,GD01,GL03,GL04,GL09]

Solving time: 146.5 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point H_{b} 89.36 77.83
point B 20 40

color 220 0 0
fontsize 9
```

```

cmark_lt M_{b}
cmark_l H_{b}
cmark_b B
color 0 0 0
fontsize 8

% DET: points M_{b} and H_{b} are not the same
% Constructing a line b which passes through point M_{b} and point H_{b}
line b M_{b} H_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0

% Generating random value V[_G17346]
random V[_G17346]

% Calculating value V[_G17367] using formula V[_G17346]*20
expression V[_G17367] { V[_G17346]*20 }

% Constructing a point B which is a point for which holds  $H_{b}B = V[_G17367]$  and angle  $M_{b}H_{b}B = 90$ 
turtle B M_{b} H_{b} 90 V[_G17367]
cmark_b B

% Choosing randomly a point A on the line M_{b}H_{b}
online A M_{b} H_{b}
cmark_t A
color 200 200 200
drawline M_{b} H_{b}
color 0 0 0

% Constructing a point C such that  $AC/AM_{b}=2$ 
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C

```

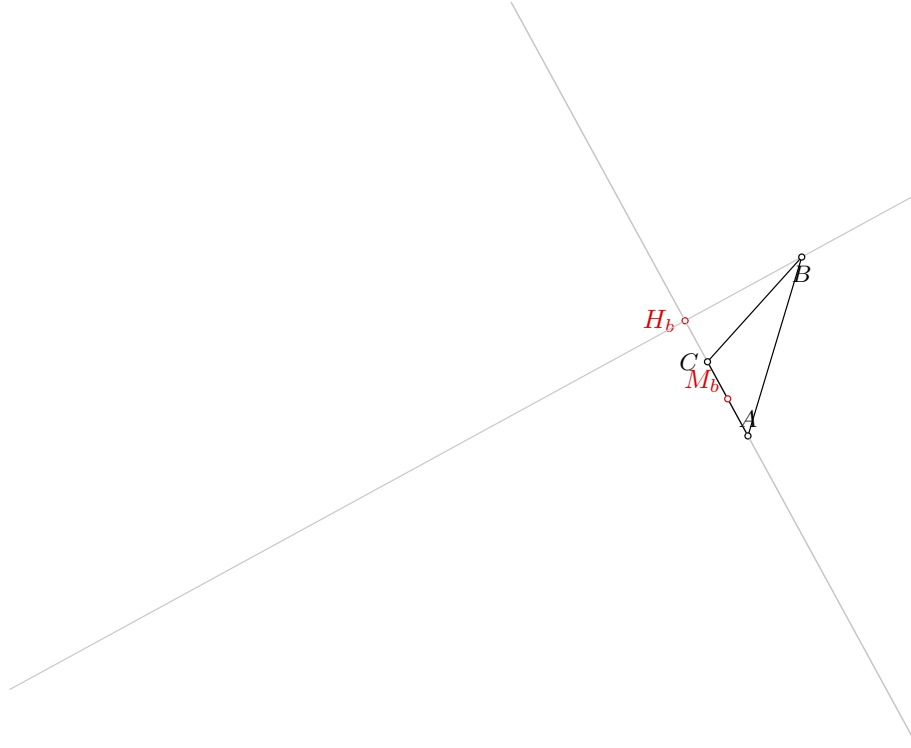


Figure 1: Illustration of the problem 0145

```
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

*% Non-degenerate conditions:*

*% Determination conditions: points  $M_{\{b\}}$  and  $H_{\{b\}}$  are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = \neg M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.036 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b = \neg H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.091 seconds.

**NDG conditions** Line through points  $C$  and  $A$  is not perpendicular to line through points  $A$  and  $M_b$

#### 4.1.3 Proving $B = B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.017 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^2_{\neg h_b}} \neq S_{CBF^2_{\neg h_b}}$  i.e., lines  $AC$  and  $BF^2_{\neg h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.2.3 Proving $B = B$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $B = B$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 2 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $B = B$

Proving failed

# Problem 146

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 146: Given a point  $B$ , a point  $M_b$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $H_c$  are not the same;
2. Using the point  $H_c$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_c$  and  $M_b$  are not the same;
3. Using the circle  $k(M_b, C)$ , the line  $c$ , the point  $M_b$  and the point  $H_c$ , construct a point  $A$  (rule W05); % NDG: line  $c$  and circle  $k(M_b, C)$  intersect % DET: points  $H_c$  and  $A$  must be different;
4. Using the point  $M_b$  and the point  $A$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: line  $c$  and circle  $k(M_b, C)$  intersect; points  $H_c$  and  $M_b$  are not the same.

Determination conditions: points  $H_c$  and  $A$  must be different; points  $B$  and  $H_c$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D22,D7,GD01,GD02,GL03,GL04,L43,L45]

Solving time: 0.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point M_{b} 95 67.5
point H_{c} 68.91 84.83
```

```

color 220 0 0
fontsize 9

cmark_b B
cmark_lt M_{b}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points H_{c} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{c}
circle k(M_{b},C) M_{b} H_{c}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

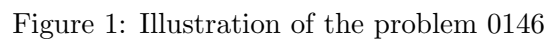
% NDG: line c and circle k(M_{b},C) intersect% DET: points H_{c} and A must be different
% Constructing a point P_{\_G29328} which is a foot of the point M_{b} on the line c
foot P_{\_G29328} M_{b} c
cmark_r P_{\_G29328}
color 200 200 200
drawline M_{b} P_{\_G29328}
color 0 0 0

% Constructing a point A which is an image of the point H_{c} in the symmetry to point/line P_{\_G29328}
sim A P_{\_G29328} H_{c}
cmark_t A

% Constructing a point C such that M_{b}C/M_{b}A=-1
towards C M_{b} A -1
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

drawsegment A B

```



```
% Non-degenerate conditions: line c and circle k(M_{b},C) intersect; points H_{c} and M_{b} are not
the same
% Determination conditions: points H_{c} and A must be different; points B and H_{c} are not the
same
```

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

#### 4.1 OGP - Wu method

### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.



**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b = \neg M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.167 seconds.

**NDG conditions** Points  $B$  and  $H_c$  are not identical

Points  $B$  and  $H_c$  are not identical

#### 4.1.3 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 39 terms.

**Time Complexity:** Time spent by the prover is 0.522 seconds.

**NDG conditions** Points  $B$  and  $H_c$  are not identical

Points  $B$  and  $H_c$  are not identical

Points  $A$  and  $B$  are not identical

Points  $A$  and  $B$  are not identical

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B = B$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{H_c}^0} \neq S_{BCF_{H_c}^0}$  i.e., lines  $AB$  and  $CF_{H_c}^0$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{H_c}^0} \neq S_{BCF_{H_c}^0}$  i.e., lines  $AB$  and  $CF_{H_c}^0$  are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $H_c = \neg H_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B = B$

Proving failed

#### 4.3.2 Proving $M_b = \neg M_b$

Proving failed

### 4.3.3 Proving $H_c = \neg H_c$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B = B$

Proving failed

### 4.4.2 Proving $M_b = \neg M_b$

Proving failed

### 4.4.3 Proving $H_c = \neg H_c$

Proving failed

# Problem 147

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 147: Given a point  $B$ , a point  $M_b$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $M_b$ , construct a point  $G$  (rule W01); ;
2. Using the point  $H$  and the point  $G$ , construct a point  $O$  (rule W01); ;
3. Using the point  $B$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H$  are not the same;
4. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
5. Using the point  $M_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same.

Determination conditions: points  $B$  and  $H$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D22,D26,D3,D9,GD01,GL03,GL09,L11,L12,L56,L58]

Solving time: 1.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point M_{b} 95 67.5
point H 80 72.73
```

```

color 220 0 0
fontsize 9

cmark_b B
cmark_lt M_{b}
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a line L_{\_G29079} which passes through point B and point M_{b}
line L_{\_G29079} B M_{b}

color 200 200 200
drawline L_{\_G29079}
color 0 0 0

% Constructing a point P_{\_G29180} with coordinates (0,0)
point P_{\_G29180} 0 0
cmark_r P_{\_G29180}

% Constructing a point P_{\_G29104} such that BP_{\_G29104}/BP_{\_G29180}=2
towards P_{\_G29104} B P_{\_G29180} 2
cmark_r P_{\_G29104}
color 200 200 200
drawsegment B P_{\_G29104}
color 0 0 0

% Constructing a point P_{\_G29149} such that BP_{\_G29149}/BP_{\_G29180}=3
towards P_{\_G29149} B P_{\_G29180} 3
cmark_r P_{\_G29149}
color 200 200 200
drawsegment B P_{\_G29149}
color 0 0 0

% Constructing a line L_{\_G29110} which passes through point M_{b} and point P_{\_G29149}
line L_{\_G29110} M_{b} P_{\_G29149}

color 200 200 200
drawline L_{\_G29110}
color 0 0 0

% Constructing a line L_{\_G29073} which contains the point P_{\_G29104} and is parallel to the
line L_{\_G29110}
parallel L_{\_G29073} P_{\_G29104} L_{\_G29110}

color 200 200 200
drawline L_{\_G29073}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G29073} and line L_{\_G29079}
intersec G L_{\_G29073} L_{\_G29079}

```

```

cmark_t G

% Constructing a point O such that HO/HG=1.5
towards O H G 1.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% DET: points B and H are not the same
% Constructing a line h_{b} which passes through point B and point H
line h_{b} B H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Constructing a line b which is perpendicular to line h_{b} and which passes through point M_{b}
perp b M_{b} h_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same
% Determination conditions: points B and H are not the same

```

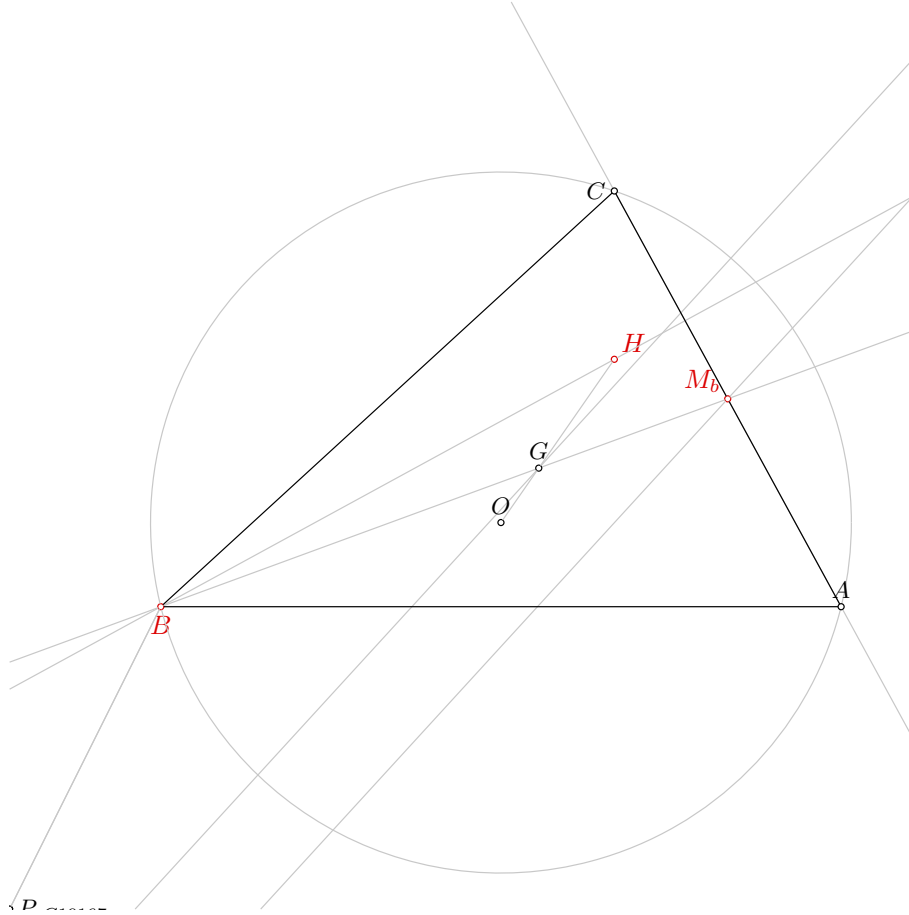


Figure 1: Illustration of the problem 0147

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.034 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b=M_b$

Proving failed

### 4.1.3 Proving $H=_H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 182 terms.

**Time Complexity:** Time spent by the prover is 4.318 seconds.

**NDG conditions** Points  $P_{G27651}$  and  $B$  are not identical

Points  $M_b$ ,  $B$  and  $H$  are not collinear

Points  $M_b$ ,  $B$  and  $H$  are not collinear

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $B$  and  $C$  are not identical

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{P_{G20133}BM_b} \neq S_{P_{L_{G20102}}^{0}BM_b}$  i.e., lines  $P_{G20133}P_{L_{G20102}}^0$  and  $BM_b$  are not parallel (construction based assumption)

$S_{M_bBH} \neq 0$  i.e., points  $M_b$ ,  $B$  and  $H$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{F_{-h_a}^2BF_{-h_b}^3}$  i.e., lines  $AF_{-h_a}^2$  and  $BF_{-h_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_b=_M M_b$

Proving failed

### 4.2.3 Proving $H=_H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_b=_M M_b$

Proving failed

### 4.3.3 Proving $H=_H$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $M_b=_M M_b$

Proving failed

### 4.4.3 Proving $H=_H H$

Proving failed



## Problem 148

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 148: Given a point  $B$ , a point  $M_b$  and a point  $T_a$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 149

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 149: Given a point  $B$ , a point  $M_b$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $T_b$ , construct a line  $s_b$  (rule W02); % DET: points  $B$  and  $T_b$  are not the same;
2. Using the point  $M_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $T_b$  are not the same;
3. Using the point  $M_b$  and the line  $b$ , construct a line  $m_b$  (rule W10b); ;
4. Using the line  $m_b$  and the line  $s_b$ , construct a point  $N_b$  (rule W03); % NDG: lines  $m_b$  and  $s_b$  are not parallel % DET: lines  $m_b$  and  $s_b$  are not the same;
5. Using the point  $B$  and the point  $N_b$ , construct a line  $m(BN_b)$  (rule W14); % DET: points  $B$  and  $N_b$  are not the same;
6. Using the line  $m(BN_b)$  and the line  $m_b$ , construct a point  $O$  (rule W03); % NDG: lines  $m(BN_b)$  and  $m_b$  are not parallel % DET: lines  $m(BN_b)$  and  $m_b$  are not the same;
7. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
8. Using the circle  $k(O, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same; lines  $m(BN_b)$  and  $m_b$  are not parallel; lines  $m_b$  and  $s_b$  are not parallel.

Determination conditions: lines  $m(BN_b)$  and  $m_b$  are not the same; points  $B$  and  $N_b$  are not the same; lines  $m_b$  and  $s_b$  are not the same; points  $M_b$  and  $T_b$  are not the same; points  $B$  and  $T_b$  are not the same.

Rules used: [W02,W03,W04,W06,W10b,W14]  
 Lemmas used: [D1,D12,D22,D24,D26,D48,GD01,GL01,GL09,L11,L12,L26,L6]  
 Solving time: 1.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point M_{b} 95 67.5
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_b B
cmark_lt M_{b}
cmark_t T_{b}
color 0 0 0
fontsize 8

% DET: points B and T_{b} are not the same
% Constructing a line s_{b} which passes through point B and point T_{b}
line s_{b} B T_{b}

color 200 200 200
drawline s_{b}
color 0 0 0

% DET: points M_{b} and T_{b} are not the same
% Constructing a line b which passes through point M_{b} and point T_{b}
line b M_{b} T_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line m_{b} which is perpendicular to line b and which passes through point M_{b}
perp m_{b} M_{b} b

color 200 200 200
drawline m_{b}
color 0 0 0

% NDG: lines m_{b} and s_{b} are not parallel% DET: lines m_{b} and s_{b} are not the same
% Constructing a point N_{b} which belongs to line m_{b} and line s_{b}
intersec N_{b} m_{b} s_{b}
cmark_rb N_{b}
```

```

% DET: points B and N_{b} are not the same
% Constructing bisector m(BN_{b}) of the segment BN_{b}
med m(BN_{b}) B N_{b}

color 200 200 200
drawline m(BN_{b})
color 0 0 0

color 200 200 200
drawsegment B N_{b}
color 0 0 0

% NDG: lines m(BN_{b}) and m_{b} are not parallel% DET: lines m(BN_{b}) and m_{b} are not the same
% Constructing a point O which belongs to line m(BN_{b}) and line m_{b}
intersec O m(BN_{b}) m_{b}
cmark_t O

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;
% lines m(BN_{b}) and m_{b} are not parallel; lines m_{b} and s_{b} are not parallel
% Determination conditions: lines m(BN_{b}) and m_{b} are not the same; points B and N_{b} are not
% the same; lines m_{b} and s_{b} are not the same; points M_{b} and T_{b} are not the same;
% points B and T_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

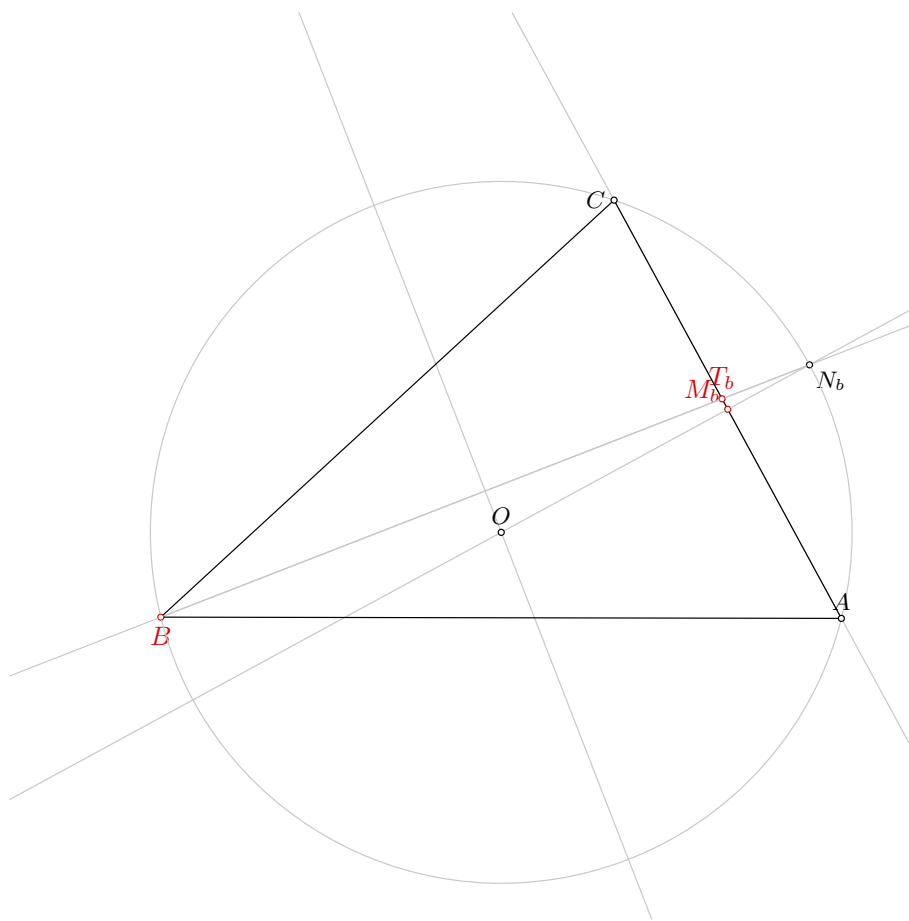


Figure 1: Illustration of the problem 0149

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b=_M M_b$

Proving failed

#### 4.1.3 Proving $T_b=_T T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{M_b B T_b} \neq S_{T_{m_b}^0 B T_b}$  i.e., lines  $M_b T_{m_b}^0$  and  $B T_b$  are not parallel (construction based assumption)

$S_{M_{m(BN_b)}^1 M_b T_{m_b}^0} \neq S_{T_{m(BN_b)}^2 M_b T_{m_b}^0}$  i.e., lines  $M_{m(BN_b)}^1 T_{m(BN_b)}^2$  and  $M_b T_{m_b}^0$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_b=_M M_b$

Proving failed

#### 4.2.3 Proving $T_b=_T T_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_b=_M M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 7807 terms.

**Time Complexity:** Time spent by the prover is 7.690 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_b=_T T_b$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $M_b=_M M_b$

Proving failed

### 4.4.3 Proving $T_b=_T T_b$

Proving failed

## Problem 150

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 150: Given a point  $B$ , a point  $M_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.



# Problem 151

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 151: Given a point  $B$ , a point  $M_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $I$ , construct a line  $IM_b$  (rule W02); % DET: points  $M_b$  and  $I$  are not the same;
2. Using the point  $I$  and the point  $M_b$ , construct a circle  $k_{over}(I, M_b)$  (rule W09); % NDG: points  $I$  and  $M_b$  are not the same;
3. Using the point  $B$  and the line  $IM_b$ , construct a line  $BP'_b$  (rule W16); ;
4. Using the point  $M_b$ , the line  $BP'_b$  and the point  $B$ , construct a line  $h_{M_b, -1/1}(BP'_b)$  (rule W15); ;
5. Using the circle  $k_{over}(I, M_b)$  and the line  $h_{M_b, -1/1}(BP'_b)$ , construct a point  $B_{fo}$  and a point  $P_b$  (rule W04); % NDG: line  $h_{M_b, -1/1}(BP'_b)$  and circle  $k_{over}(I, M_b)$  intersect;
6. Using the point  $P_b$  and the point  $I$ , construct a circle  $k(I, P_a)$  (rule W06); % NDG: points  $P_b$  and  $I$  are not the same;
7. Using the circle  $k(I, P_a)$ , the point  $B$  and the point  $I$ , construct a line  $c$  and a line  $a$  (rule W12); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
8. Using the circle  $k(I, P_a)$ , the point  $M_b$  and the point  $I$ , construct a line  $x2$  and a line  $b$  (rule W12); % NDG: point  $M_b$  is outside the circle  $k(I, P_a)$ ;
9. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same;
10. Using the point  $M_b$  and the point  $A$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; point  $M_b$  is outside the circle  $k(I, P_a)$ ; point  $B$  is outside the circle  $k(I, P_a)$ ; points  $P_b$  and  $I$  are not the same; line  $h_{M_b, -1/1}(BP'_b)$  and circle  $k_{over}(I, M_b)$  intersect; points  $I$  and  $M_b$  are not the same.

Determination conditions: lines  $c$  and  $b$  are not the same; points  $M_b$  and  $I$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W09,W12,W15,W16]

Lemmas used: [D22,D27,D60,D86,GD01,GL02,GL03,GL04,GL09,L13,L59,L60,L61,L63,L75]

Solving time: 2.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point M_{b} 95 67.5
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_b B
cmark_lt M_{b}
cmark_b I
color 0 0 0
fontsize 8

% DET: points M_{b} and I are not the same
% Constructing a line IM_{b} which passes through point M_{b} and point I
line IM_{b} M_{b} I

color 200 200 200
drawline IM_{b}
color 0 0 0

% NDG: points I and M_{b} are not the same
% Constructing midpoint P_{\_G18188} of the segment IM_{b}
midpoint P_{\_G18188} I M_{b}
cmark_r P_{\_G18188}

% Constructing a circle k_{over}(I, M_{b}) whose center is at point P_{\_G18188} and which passes
    through point I
circle k_{over}(I, M_{b}) P_{\_G18188} I

color 200 200 200
drawcircle k_{over}(I, M_{b})
color 0 0 0

% Constructing a line BP'_{b} which contains the point B and is parallel to the line IM_{b}
parallel BP'_{b} B IM_{b}
```

```

color 200 200 200
drawline BP'_{b}
color 0 0 0

% Constructing a point  $P_{\backslash\_G18489}$  such that  $M_{\{b\}}P_{\backslash\_G18489}/M_{\{b\}}B=-1$ 
towards P_{\backslash\_G18489} M_{\{b\}} B -1
cmark_r P_{\backslash\_G18489}
color 200 200 200
drawsegment B P_{\backslash\_G18489}
color 0 0 0

% Constructing a line  $h_{\{M_{\{b\}}, -1/1\}}(BP'_{\{b\}})$  which contains the point  $P_{\backslash\_G18489}$  and is parallel
to the line  $BP'_{\{b\}}$ 
parallel h_{\{M_{\{b\}}, -1/1\}}(BP'_{\{b\}}) P_{\backslash\_G18489} BP'_{\{b\}}

color 200 200 200
drawline h_{\{M_{\{b\}}, -1/1\}}(BP'_{\{b\}})
color 0 0 0

% NDG: line  $h_{\{M_{\{b\}}, -1/1\}}(BP'_{\{b\}})$  and circle  $k_{\text{over}}(I, M_{\{b\}})$  intersect
% Constructing points  $B_{\{fo\}}$  and  $P_{\{b\}}$  which are in intersection of  $k_{\text{over}}(I, M_{\{b\}})$  and  $h_{\{M_{\{b\}}, -1/1\}}(BP'_{\{b\}})$ 
intersec2 B_{fo} P_{b} k_{\text{over}}(I, M_{\{b\}}) h_{\{M_{\{b\}}, -1/1\}}(BP'_{\{b\}})
cmark_r B_{fo}
cmark_r P_{b}

% NDG: points  $P_{\{b\}}$  and  $I$  are not the same
% Constructing a circle  $k(I, P_{\{a\}})$  whose center is at point  $I$  and which passes through point  $P_{\{b\}}$ 
circle k(I, P_{a}) I P_{b}

color 200 200 200
drawcircle k(I, P_{a})
color 0 0 0

% NDG: point  $B$  is outside the circle  $k(I, P_{\{a\}})$ 
% Constructing midpoint  $P_{\backslash\_G19238}$  of the segment  $BI$ 
midpoint P_{\backslash\_G19238} B I
cmark_r P_{\backslash\_G19238}

% Constructing a circle  $C_{\backslash\_G19241}$  whose center is at point  $P_{\backslash\_G19238}$  and which passes through
point  $B$ 
circle C_{\backslash\_G19241} P_{\backslash\_G19238} B

color 200 200 200
drawcircle C_{\backslash\_G19241}
color 0 0 0

```

```

% Constructing points  $P_{\setminus G19244}$  and  $P_{\setminus G19247}$  which are in intersection of  $C_{\setminus G19241}$  and  $k(I, P_{\setminus a})$ 
intersec2  $P_{\setminus G19244}$   $P_{\setminus G19247}$   $C_{\setminus G19241}$   $k(I, P_{\setminus a})$ 
cmark_r  $P_{\setminus G19244}$ 
cmark_r  $P_{\setminus G19247}$ 

% Constructing a line  $c$  which passes through point  $B$  and point  $P_{\setminus G19244}$ 
line c B  $P_{\setminus G19244}$ 

color 200 200 200
drawline c
color 0 0 0

% Constructing a line  $a$  which passes through point  $B$  and point  $P_{\setminus G19247}$ 
line a B  $P_{\setminus G19247}$ 

color 200 200 200
drawline a
color 0 0 0

% NDG: point  $M_{\setminus b}$  is outside the circle  $k(I, P_{\setminus a})$ 
% Constructing midpoint  $P_{\setminus G19650}$  of the segment  $M_{\setminus b}I$ 
midpoint  $P_{\setminus G19650}$   $M_{\setminus b}$  I
cmark_r  $P_{\setminus G19650}$ 

% Constructing a circle  $C_{\setminus G19653}$  whose center is at point  $P_{\setminus G19650}$  and which passes through
point  $M_{\setminus b}$ 
circle  $C_{\setminus G19653}$   $P_{\setminus G19650}$   $M_{\setminus b}$ 

color 200 200 200
drawcircle  $C_{\setminus G19653}$ 
color 0 0 0

% Constructing points  $P_{\setminus G19656}$  and  $P_{\setminus G19659}$  which are in intersection of  $C_{\setminus G19653}$  and  $k(I, P_{\setminus a})$ 
intersec2  $P_{\setminus G19656}$   $P_{\setminus G19659}$   $C_{\setminus G19653}$   $k(I, P_{\setminus a})$ 
cmark_r  $P_{\setminus G19656}$ 
cmark_r  $P_{\setminus G19659}$ 

% Constructing a line  $x2$  which passes through point  $M_{\setminus b}$  and point  $P_{\setminus G19656}$ 
line x2  $M_{\setminus b}$   $P_{\setminus G19656}$ 

color 200 200 200
drawline x2
color 0 0 0

% Constructing a line  $b$  which passes through point  $M_{\setminus b}$  and point  $P_{\setminus G19659}$ 
line b  $M_{\setminus b}$   $P_{\setminus G19659}$ 

color 200 200 200
drawline b

```

```
color 0 0 0
```

```
% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A
```

```
% Constructing a point C such that  $M_{\{b\}}C/M_{\{b\}}A=-1$ 
towards C  $M_{\{b\}}$  A -1
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines c and b are not parallel; point  $M_{\{b\}}$  is outside the circle  $k(I, P_{\{a\}})$ ; point B is outside the circle  $k(I, P_{\{a\}})$ ; points  $P_{\{b\}}$  and I are not the same; line  $h_{\{M_{\{b\}}, -1/1\}}(BP'_{\{b\}})$  and circle  $k_{\text{over}}(I, M_{\{b\}})$  intersect; points I and  $M_{\{b\}}$  are not the same
% Determination conditions: lines c and b are not the same; points  $M_{\{b\}}$  and I are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b = -M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 10 terms.

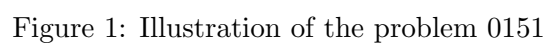
**Time Complexity:** Time spent by the prover is 0.259 seconds.

**NDG conditions** Points  $M_b$ , I and  $P_{G29698}$  are not collinear

Points tempPoint-17 $BP'_b$  and B are not identical

Points  $M_b$ , tempPoint-17 $BP'_b$  and B are not collinear

Points  $M_b$  and B are not identical



#### 4.1.3 Proving $I=J$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{BM_bP_{G20494}} \neq S_{P_{G20079}M_bP_{G20494}}$  i.e., lines  $BP_{G20079}$  and  $M_bP_{G20494}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_b=M_b$

Proving failed

#### 4.2.3 Proving $I=J$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.030 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_b=M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 138 terms.

**Time Complexity:** Time spent by the prover is 0.290 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I=J$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 2325 terms.

**Time Complexity:** Time spent by the prover is 7.820 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.030 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_b=M_b$

Proving failed

#### 4.4.3 Proving $I=J$

Proving failed

# Problem 152

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 152: Given a point  $B$ , a point  $M_c$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01); ;
2. Using the point  $M_c$  and the point  $G$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D20, GL03, GL04, L57]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point M_{c} 50 67.5
point G 70 58.33
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_lt M_{c}
cmark_t G
color 0 0 0
fontsize 8
```

```
% Constructing a point A such that BA/BM_{c}=2
```



```

towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

% Constructing a point C such that M_{c}C/M_{c}G=3
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.024 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c=_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.054 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $G=_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 23 terms.

**Time Complexity:** Time spent by the prover is 0.419 seconds.

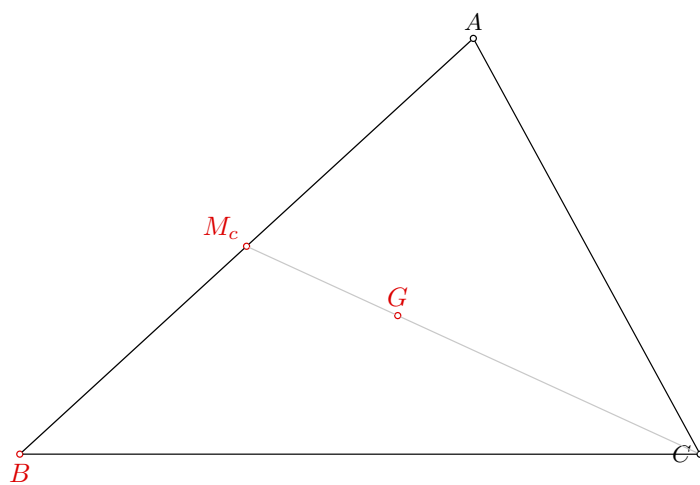


Figure 1: Illustration of the problem 0152

**NDG conditions** Line through points  $A$  and  $M_a$  is not parallel with line through points  $B$  and  $M_b$   
Points  $A$ ,  $B$  and  $M_a$  are not collinear

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{AB-M_b} \neq S_{M_aB-M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_c=M_c$

NDG conditions are:

$S_{AB-M_b} \neq S_{M_aB-M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $G=G$

NDG conditions are:

$S_{AB-M_b} \neq S_{M_aB-M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (cancellation assumption)

Total number of proof steps: 636

Time spent by the prover: 0.200 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_c=M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $G=G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 113 terms.

**Time Complexity:** Time spent by the prover is 0.190 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_c = \neg M_c$

Proving failed

#### 4.4.3 Proving $G = \neg G$

Proving failed

# Problem 153

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 153: Given a point  $M_c$ , a point  $H_a$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_a$  and  $M_c$  are not the same;
2. Choose freely a point  $B$  on the circle  $k(M_c, A)$  (rule WOncircle);
3. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01); ;
4. Using the point  $B$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $H_a$  are not the same;
5. Choose freely a point  $C$  on the line  $a$  (rule WOnline1) .

Non-degenerate conditions: points  $H_a$  and  $M_c$  are not the same.

Determination conditions: points  $B$  and  $H_a$  are not the same.

Rules used: [W01,W02,W06,WOncircle1,WOnline1]

Lemmas used: [D20,D5,GD01,GD02,GL03,GL04,L42]

Solving time: 823.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point H_{a} 80 40
point B 20 40

color 220 0 0
fontsize 9
```

```

cmark_lt M_{c}
cmark_r H_{a}
cmark_b B
color 0 0 0
fontsize 8

% NDG: points H_{a} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{a}
circle k(M_{c},A) M_{c} H_{a}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% Choosing randomly a point B on the circle with center M_{c} through point H_{a}
oncircle B M_{c} H_{a}
cmark_b B
color 200 200 200
drawcircle M_{c} H_{a}
color 0 0 0

% Constructing a point A such that BA/BM_{c}=2
towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% Choosing randomly a point C on the line BH_{a}
online C B H_{a}
cmark_l C
color 200 200 200
drawline B H_{a}
color 0 0 0

```

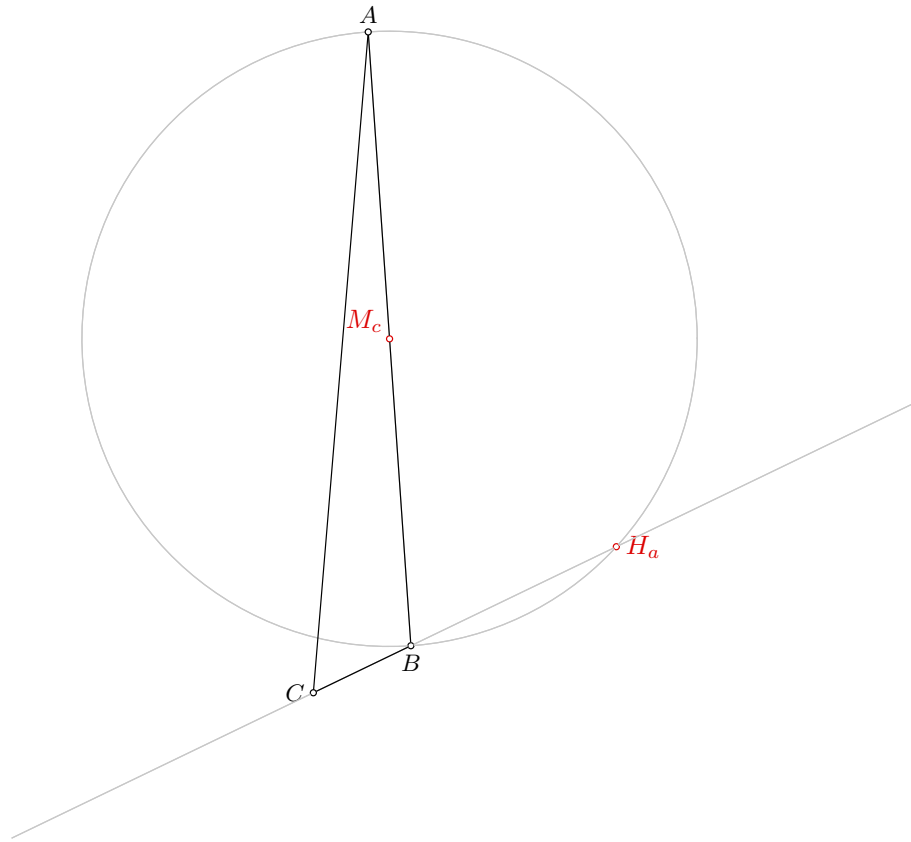


Figure 1: Illustration of the problem 0153

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: points  $H_{\{a\}}$  and  $M_{\{c\}}$  are not the same
% Determination conditions: points B and  $H_{\{a\}}$  are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.093 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a = \neg H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 47 terms.

**Time Complexity:** Time spent by the prover is 0.561 seconds.

**NDG conditions** Points  $B$ ,  $M_c$  and  $H_a$  are not collinear

Points  $B$  and  $C$  are not identical

Points  $B$ ,  $C$  and  $M_c$  are not collinear

#### 4.1.3 Proving $B = B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{\neg h_a}^1} \neq S_{CAF_{\neg h_a}^1}$  i.e., lines  $BC$  and  $AF_{\neg h_a}^1$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.2.3 Proving $B = B$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{\neg h_a}^1} \neq S_{CAF_{\neg h_a}^1}$  i.e., lines  $BC$  and  $AF_{\neg h_a}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 17 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.



#### 4.3.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 280 terms.

**Time Complexity:** Time spent by the prover is 0.280 seconds. There are no ndg conditions.

#### 4.3.3 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 312 terms.

**Time Complexity:** Time spent by the prover is 1.970 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 312 terms.

**Time Complexity:** Time spent by the prover is 1.850 seconds. There are no ndg conditions.

#### 4.4.3 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 154

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 154: Given a point  $M_c$ , a point  $H_b$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_b$  and  $M_c$  are not the same;
2. Choose freely a point  $B$  on the circle  $k(M_c, A)$  (rule WOncircle);
3. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01); ;
4. Using the point  $H_b$  and the point  $A$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $A$  are not the same;
5. Choose freely a point  $C$  on the line  $b$  (rule WOnline1) .

Non-degenerate conditions: points  $H_b$  and  $M_c$  are not the same.

Determination conditions: points  $H_b$  and  $A$  are not the same.

Rules used: [W01,W02,W06,WOncircle1,WOnline1]

Lemmas used: [D20,D6,GD01,GD02,GL03,GL04,L41]

Solving time: 929.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point H_{b} 89.36 77.83
point B 20 40

color 220 0 0
fontsize 9
```

```

cmark_lt M_{c}
cmark_l H_{b}
cmark_b B
color 0 0 0
fontsize 8

% NDG: points H_{b} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{b}
circle k(M_{c},A) M_{c} H_{b}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% Choosing randomly a point B on the circle with center M_{c} through point H_{b}
oncircle B M_{c} H_{b}
cmark_b B
color 200 200 200
drawcircle M_{c} H_{b}
color 0 0 0

% Constructing a point A such that BA/BM_{c}=2
towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

% DET: points H_{b} and A are not the same
% Constructing a line b which passes through point H_{b} and point A
line b H_{b} A

color 200 200 200
drawline b
color 0 0 0

% Choosing randomly a point C on the line H_{b}A
online C H_{b} A
cmark_l C
color 200 200 200
drawline H_{b} A
color 0 0 0

```

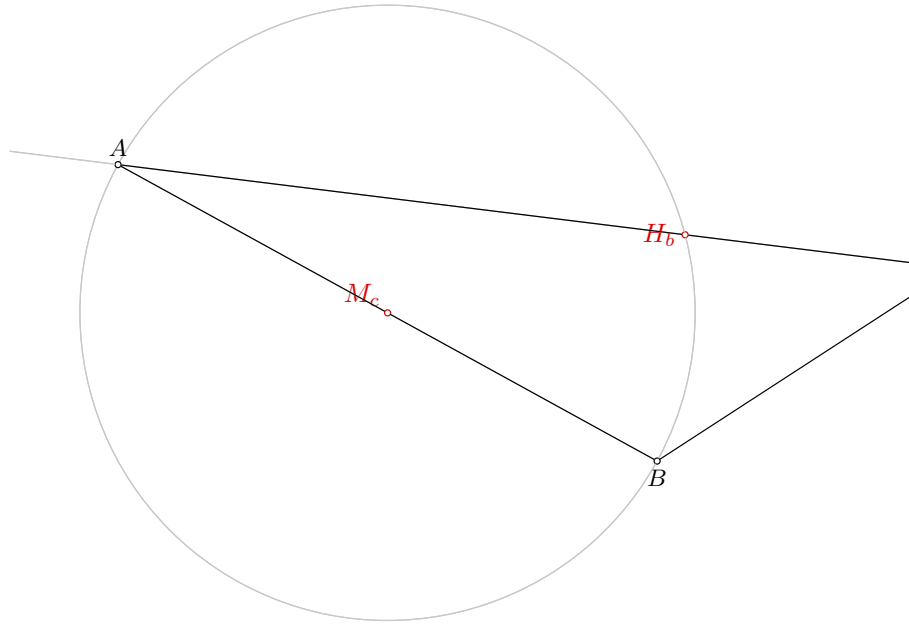


Figure 1: Illustration of the problem 0154

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: points  $H_{\{b\}}$  and  $M_{\{c\}}$  are not the same
% Determination conditions: points  $H_{\{b\}}$  and A are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.099 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.1.3 Proving $B = B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.024 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{h_b}} \neq S_{CBF^1_{h_b}}$  i.e., lines  $AC$  and  $BF^1_{h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.2.3 Proving $B = B$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{h_b}} \neq S_{CBF^1_{h_b}}$  i.e., lines  $AC$  and  $BF^1_{h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 17 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 280 terms.

**Time Complexity:** Time spent by the prover is 0.270 seconds. There are no ndg conditions.

#### 4.3.3 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_c = M_c$

Proving failed

#### 4.4.2 Proving $H_b = H_b$

Proving failed

#### 4.4.3 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 155

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 155: Given a point  $M_c$ , a point  $H_c$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $H_c$  are not the same;
2. Choose freely a point  $B$  on the line  $c$  (rule WOnline1) ;
3. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01); ;
4. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
5. Choose freely a point  $C$  on the line  $h_c$  (rule WOnline2).

Non-degenerate conditions: .

Determination conditions: points  $M_c$  and  $H_c$  are not the same.

Rules used: [W01,W02,W10b,WOnline1,WOnline2]

Lemmas used: [D10,D20,D7,GD01,GL03,GL04,GL09]

Solving time: 155.5 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point H_{c} 68.91 84.83
point B 20 40

color 220 0 0
fontsize 9
```

```

cmark_lt M_{c}
cmark_rt H_{c}
cmark_b B
color 0 0 0
fontsize 8

% DET: points M_{c} and H_{c} are not the same
% Constructing a line c which passes through point M_{c} and point H_{c}
line c M_{c} H_{c}

color 200 200 200
drawline c
color 0 0 0

% Choosing randomly a point B on the line M_{c}H_{c}
online B M_{c} H_{c}
cmark_b B
color 200 200 200
drawline M_{c} H_{c}
color 0 0 0

% Constructing a point A such that BA/BM_{c}=2
towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0

% Generating random value V[_G21597]
random V[_G21597]

% Calculating value V[_G21618] using formula V[_G21597]*20
expression V[_G21618] { V[_G21597]*20 }

% Constructing a point C which is a point for which holds H_{c}C = V[_G21618] and angle BH_{c}C =
90
turtle C B H_{c} 90 V[_G21618]

```



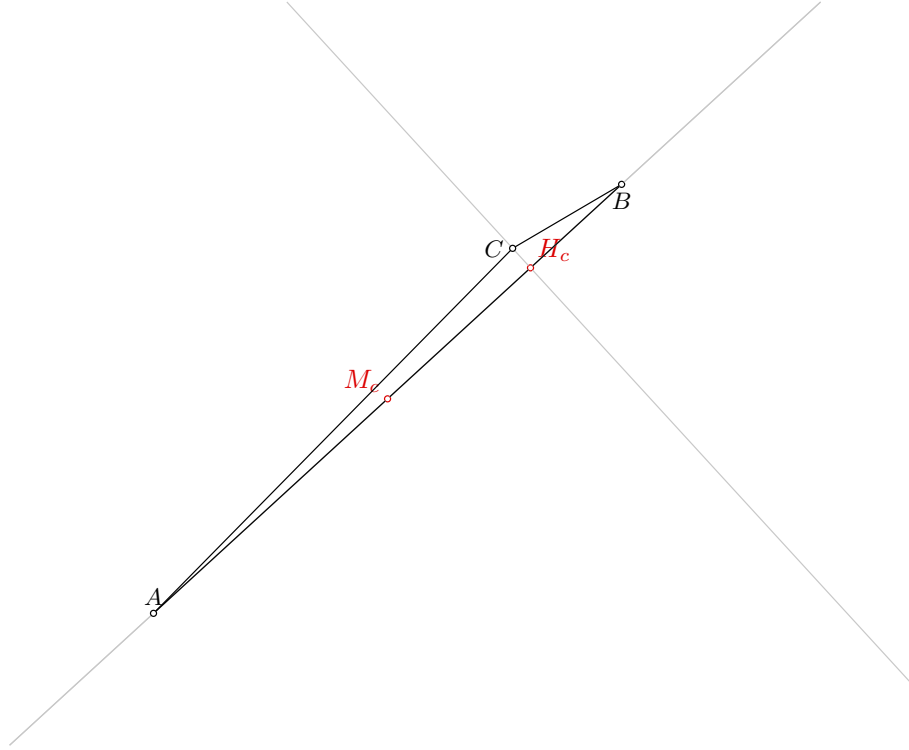


Figure 1: Illustration of the problem 0155

cmark\_1 C

drawsegment A B  
drawsegment A C  
drawsegment B C

*% Non-degenerate conditions:*  
*% Determination conditions: points  $M_{\{c\}}$  and  $H_{\{c\}}$  are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = \neg M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.103 seconds.

**NDG conditions** Line through points  $A$  and  $H_c$  is not perpendicular to line through points  $H_c$  and  $B$

#### 4.1.3 Proving $B = B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF^2_{H_c}} \neq S_{BCF^2_{H_c}}$  i.e., lines  $AB$  and  $CF^2_{H_c}$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.2.3 Proving $B = B$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF^2_{H_c}} \neq S_{BCF^2_{H_c}}$  i.e., lines  $AB$  and  $CF^2_{H_c}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 2 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 156

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 156: Given a point  $B$ , a point  $M_c$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01); ;
2. Using the point  $B$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H$  are not the same;
3. Using the point  $H$  and the point  $A$ , construct a line  $h_a$  (rule W02); % DET: points  $H$  and  $A$  are not the same;
4. Using the point  $B$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $B$  and  $M_c$  are not the same;
5. Using the circle  $k(M_c, A)$ , the line  $h_b$ , the point  $M_c$  and the point  $B$ , construct a point  $H_b$  (rule W05); % NDG: line  $h_b$  and circle  $k(M_c, A)$  intersect % DET: points  $B$  and  $H_b$  must be different;
6. Using the point  $H_b$  and the point  $A$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $A$  are not the same;
7. Using the circle  $k(M_c, A)$ , the line  $h_a$ , the point  $M_c$  and the point  $A$ , construct a point  $H_a$  (rule W05); % NDG: line  $h_a$  and circle  $k(M_c, A)$  intersect % DET: points  $A$  and  $H_a$  must be different;
8. Using the point  $H_a$  and the point  $B$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $B$  are not the same;
9. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; line  $h_a$  and circle  $k(M_c, A)$  intersect; line  $h_b$  and circle  $k(M_c, A)$  intersect; points  $B$  and  $M_c$  are not the same.

Determination conditions: lines  $b$  and  $a$  are not the same; points  $H_a$  and  $B$  are not the same; points  $A$  and  $H_a$  must be different; points  $H_b$  and  $A$  are not the same; points  $B$  and  $H_b$  must be different; points  $H$  and  $A$  are not the same; points  $B$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D20,D3,D5,D6,D8,D9,GD01,GD02,GL03,GL04,L40,L41,L42]

Solving time: 1.9 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point M_{c} 50 67.5
point H 80 72.73

color 220 0 0
fontsize 9

cmark_b B
cmark_lt M_{c}
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a point A such that BA/BM_{c}=2
towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

% DET: points B and H are not the same
% Constructing a line h_{b} which passes through point B and point H
line h_{b} B H

color 200 200 200
drawline h_{b}
color 0 0 0

% DET: points H and A are not the same
% Constructing a line h_{a} which passes through point H and point A
line h_{a} H A

color 200 200 200
drawline h_{a}
color 0 0 0
```

```

% NDG: points B and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point B
circle k(M_{c},A) M_{c} B

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line h_{b} and circle k(M_{c},A) intersect% DET: points B and H_{b} must be different
% Constructing a point P_{\G25891} which is a foot of the point M_{c} on the line h_{b}
foot P_{\G25891} M_{c} h_{b}
cmark_r P_{\G25891}
color 200 200 200
drawline M_{c} P_{\G25891}
color 0 0 0

% Constructing a point H_{b} which is an image of the point B in the symmetry to point/line P_{\G
25891}
sim H_{b} P_{\G25891} B
cmark_l H_{b}

% DET: points H_{b} and A are not the same
% Constructing a line b which passes through point H_{b} and point A
line b H_{b} A

color 200 200 200
drawline b
color 0 0 0

% NDG: line h_{a} and circle k(M_{c},A) intersect% DET: points A and H_{a} must be different
% Constructing a point P_{\G26129} which is a foot of the point M_{c} on the line h_{a}
foot P_{\G26129} M_{c} h_{a}
cmark_r P_{\G26129}
color 200 200 200
drawline M_{c} P_{\G26129}
color 0 0 0

% Constructing a point H_{a} which is an image of the point A in the symmetry to point/line P_{\G
26129}
sim H_{a} P_{\G26129} A
cmark_r H_{a}

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

```

```

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_1 C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and a are not parallel; line h_{a} and circle k(M_{c},A)
% intersect; line h_{b} and circle k(M_{c},A) intersect; points B and M_{c} are not the same
% Determination conditions: lines b and a are not the same; points H_{a} and B are not the same;
% points A and H_{a} must be different; points H_{b} and A are not the same; points B and H_{b}
% must be different; points H and A are not the same; points B and H are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c=M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.033 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $H=H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 227 terms.

**Time Complexity:** Time spent by the prover is 8.461 seconds.

**NDG conditions** Points  $B$  and  $H$  are not identical

Points  $B$  and  $H$  are not identical

Points  $A$  and  $H$  are not identical

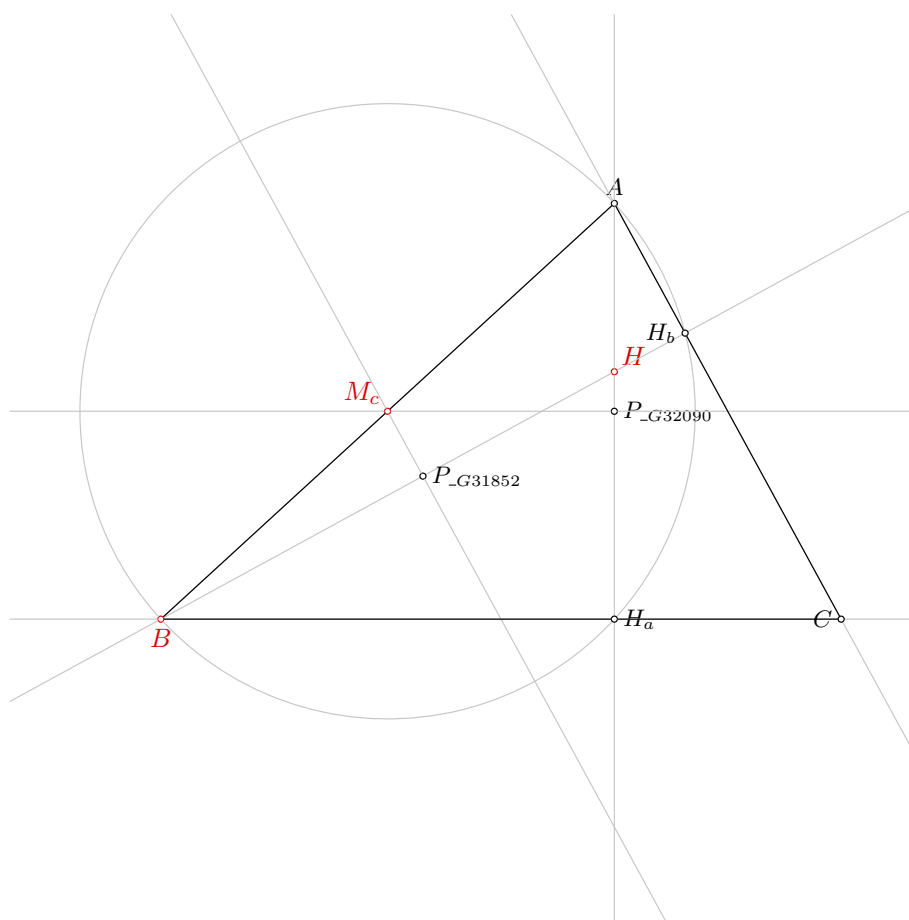


Figure 1: Illustration of the problem 0156



Line through points  $A$  and  $B$  is not perpendicular to line through points  $B$  and  $H$

Line through points  $H_b$  and  $A$  is not parallel with line through points  $B$  and  $H_a$

Points  $H_b$ ,  $A$  and  $B$  are not collinear

Line through points  $A$  and  $B$  is not parallel with line through points  $C$  and  $M_c$

Points  $B$ ,  $C$  and  $M_c$  are not collinear

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_bH_aB} \neq S_{AH_aB}$  i.e., lines  $H_bA$  and  $H_aB$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0BF_{-h_b}^1}$  i.e., lines  $AF_{-h_a}^0$  and  $BF_{-h_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_c=M_c$

Proving failed

### 4.2.3 Proving $H=H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_c=M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $H=H$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 21 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_c = M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.3 Proving $H = H$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

# Problem 157

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 157: Given a point  $B$ , a point  $M_c$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01); ;
2. Using the point  $B$  and the point  $M_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $M_c$  are not the same;
3. Using the point  $B$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $T_a$  are not the same;
4. Using the point  $T_a$  and the line  $c$ , construct a circle  $k(T_a, foot[T_a, c])$  (rule W11); % NDG: point  $T_a$  is not incident to the line  $c$ ;
5. Using the circle  $k(T_a, foot[T_a, c])$ , the point  $A$ , the point  $T_a$  and the line  $c$ , construct a line  $b$  (rule W13); % NDG: point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ;
6. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ; point  $T_a$  is not incident to the line  $c$ .

Determination conditions: lines  $b$  and  $a$  are not the same; points  $B$  and  $T_a$  are not the same; points  $B$  and  $M_c$  are not the same.

Rules used: [W01,W02,W03,W11,W13]

Lemmas used: [D20,D23,GD01,GD02,GL03,GL04,GL09,GL10,GL11,L8]

Solving time: 1.8 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point M_{c} 50 67.5
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_b B
cmark_lt M_{c}
cmark_rb T_{a}
color 0 0 0
fontsize 8

% Constructing a point A such that BA/BM_{c}=2
towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

% DET: points B and M_{c} are not the same
% Constructing a line c which passes through point B and point M_{c}
line c B M_{c}

color 200 200 200
drawline c
color 0 0 0

% DET: points B and T_{a} are not the same
% Constructing a line a which passes through point B and point T_{a}
line a B T_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: point T_{a} is not incident to the line c
% Constructing a point P_{\_G28538} which is a foot of the point T_{a} on the line c
foot P_{\_G28538} T_{a} c
cmark_r P_{\_G28538}
color 200 200 200
drawline T_{a} P_{\_G28538}
color 0 0 0

% Constructing a circle k(T_{a},foot[T_{a},c]) whose center is at point T_{a} and which passes
    through point P_{\_G28538}
```

```

circle k(T_{a},foot[T_{a},c]) T_{a} P_{\_G28538}

color 200 200 200
drawcircle k(T_{a},foot[T_{a},c])
color 0 0 0

% NDG: point A is outside the circle k(T_{a},foot[T_{a},c])
% Constructing a point P_{\_G28929} which is a foot of the point T_{a} on the line c
foot P_{\_G28929} T_{a} c
cmark_r P_{\_G28929}
color 200 200 200
drawline T_{a} P_{\_G28929}
color 0 0 0

% Constructing a line L_{\_G28932} which passes through point T_{a} and point A
line L_{\_G28932} T_{a} A

color 200 200 200
drawline L_{\_G28932}
color 0 0 0

% Constructing a point P_{\_G28935} which is an image of the point P_{\_G28929} in the symmetry to
point/line L_{\_G28932}
sim P_{\_G28935} L_{\_G28932} P_{\_G28929}
cmark_r P_{\_G28935}

% Constructing a line b which passes through point A and point P_{\_G28935}
line b A P_{\_G28935}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and a are not parallel; point A is outside the circle k(T_{a},
foot[T_{a},c]); point T_{a} is not incident to the line c
% Determination conditions: lines b and a are not the same; points B and T_{a} are not the same;
points B and M_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

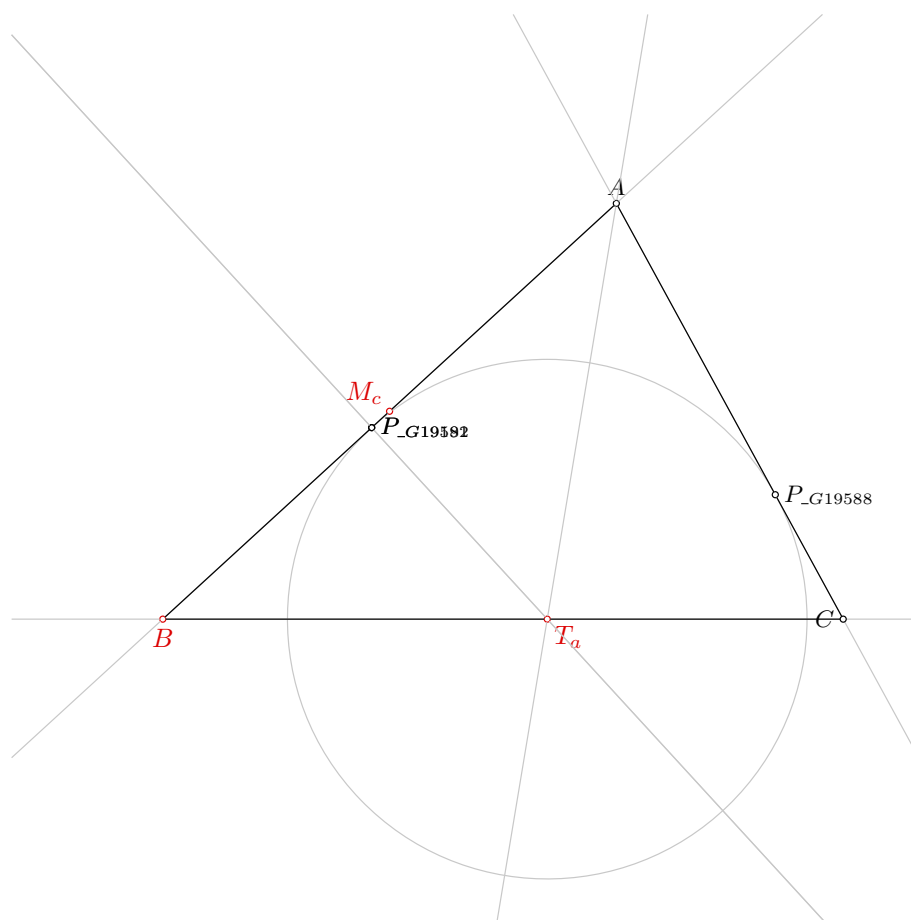


Figure 1: Illustration of the problem 0157

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.032 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c=_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.036 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $T_a=_T_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{ABT_a} \neq S_{P_{G20614}BT_a}$  i.e., lines  $AP_{G20614}$  and  $BT_a$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_c=_M_c$

Proving failed

#### 4.2.3 Proving $T_a=_T_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_c=_M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_a = \neg T_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 10 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 7 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_a = \neg T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 7 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.



# Problem 158

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 158: Given a point  $B$ , a point  $M_c$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01); ;
2. Using the point  $B$  and the point  $M_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $M_c$  are not the same;
3. Using the point  $T_b$  and the point  $A$ , construct a line  $b$  (rule W02); % DET: points  $T_b$  and  $A$  are not the same;
4. Using the point  $T_b$  and the line  $c$ , construct a circle  $k(T_b, foot[T_b, c])$  (rule W11); % NDG: point  $T_b$  is not incident to the line  $c$ ;
5. Using the circle  $k(T_b, foot[T_b, c])$ , the point  $B$ , the point  $T_b$  and the line  $c$ , construct a line  $a$  (rule W13); % NDG: point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ;
6. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ; point  $T_b$  is not incident to the line  $c$ .

Determination conditions: lines  $a$  and  $b$  are not the same; points  $T_b$  and  $A$  are not the same; points  $B$  and  $M_c$  are not the same.

Rules used: [W01,W02,W03,W11,W13]

Lemmas used: [D20,D24,GD01,GD02,GL03,GL04,GL09,GL10,GL11,L9]

Solving time: 1.8 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point M_{c} 50 67.5
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_b B
cmark_lt M_{c}
cmark_t T_{b}
color 0 0 0
fontsize 8

% Constructing a point A such that  $BA/BM_{\{c\}}=2$ 
towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

% DET: points B and  $M_{\{c\}}$  are not the same
% Constructing a line c which passes through point B and point  $M_{\{c\}}$ 
line c B M_{c}

color 200 200 200
drawline c
color 0 0 0

% DET: points  $T_{\{b\}}$  and A are not the same
% Constructing a line b which passes through point  $T_{\{b\}}$  and point A
line b T_{b} A

color 200 200 200
drawline b
color 0 0 0

% NDG: point  $T_{\{b\}}$  is not incident to the line c
% Constructing a point  $P_{\{\backslash\_G30425\}}$  which is a foot of the point  $T_{\{b\}}$  on the line c
foot P_{\backslash\_G30425} T_{b} c
cmark_r P_{\backslash\_G30425}
color 200 200 200
drawline T_{b} P_{\backslash\_G30425}
color 0 0 0

% Constructing a circle  $k(T_{\{b\}}, foot[T_{\{b\}}, c])$  whose center is at point  $T_{\{b\}}$  and which passes
    through point  $P_{\{\backslash\_G30425\}}$ 
```

```

circle k(T_{b},foot[T_{b},c]) T_{b} P_{\_G30425}

color 200 200 200
drawcircle k(T_{b},foot[T_{b},c])
color 0 0 0

% NDG: point B is outside the circle k(T_{b},foot[T_{b},c])
% Constructing a point P_{\_G30816} which is a foot of the point T_{b} on the line c
foot P_{\_G30816} T_{b} c
cmark_r P_{\_G30816}
color 200 200 200
drawline T_{b} P_{\_G30816}
color 0 0 0

% Constructing a line L_{\_G30819} which passes through point T_{b} and point B
line L_{\_G30819} T_{b} B

color 200 200 200
drawline L_{\_G30819}
color 0 0 0

% Constructing a point P_{\_G30822} which is an image of the point P_{\_G30816} in the symmetry to
point/line L_{\_G30819}
sim P_{\_G30822} L_{\_G30819} P_{\_G30816}
cmark_r P_{\_G30822}

% Constructing a line a which passes through point B and point P_{\_G30822}
line a B P_{\_G30822}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and b are not parallel; point B is outside the circle k(T_{b},
foot[T_{b},c]); point T_{b} is not incident to the line c
% Determination conditions: lines a and b are not the same; points T_{b} and A are not the same;
points B and M_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

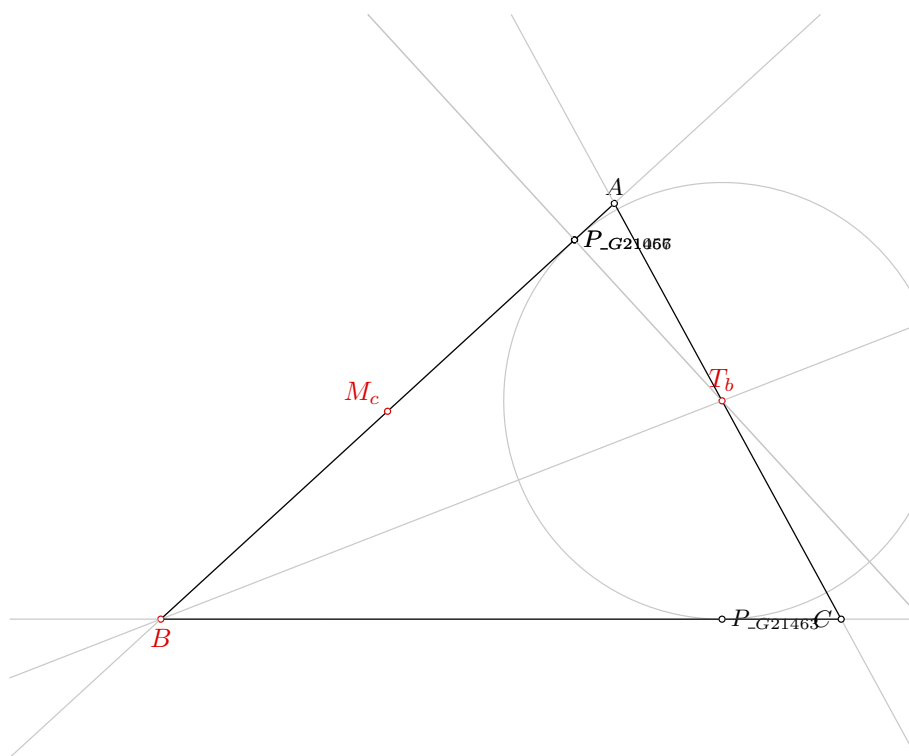


Figure 1: Illustration of the problem 0158

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c=M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $T_b=T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{BT_bA} \neq S_{P_{G22492}T_bA}$  i.e., lines  $BP_{G22492}$  and  $T_bA$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_c=M_c$

Proving failed

#### 4.2.3 Proving $T_b=T_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_c=M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 5 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_b = \neg T_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 14 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_b = \neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 159

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 159: Given a point  $M_c$ , a point  $T_c$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $T_c$  are not the same;
2. Choose freely a point  $B$  on the line  $c$  (rule WOnline1) ;
3. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01); ;
4. Using the point  $A$ , the point  $B$ , the point  $T_c$  and the line  $c$ , construct a point  $T'_c$  (rule W19);  
% NDG: points  $A$  and  $B$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $midpoint([A, T_c])$  are not the same;
5. Choose freely a point  $C$  on the circle  $k_{over}(T_c, T'_c)$  (rule WOncircle).

Non-degenerate conditions: points  $A$  and  $B$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $midpoint([A, T_c])$  are not the same.

Determination conditions: points  $M_c$  and  $T_c$  are not the same.

Rules used: [W01,W02,W19,WOncircle2,WOnline1]

Lemmas used: [D20,D25,GD01,GL03,GL04,GL09,L79]

Solving time: 88.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{c} 50 67.5  
point T_{c} 55.38 72.43  
point B 20 40
```

```

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_rt T_{c}
cmark_b B
color 0 0 0
fontsize 8

% DET: points M_{c} and T_{c} are not the same
% Constructing a line c which passes through point M_{c} and point T_{c}
line c M_{c} T_{c}

color 200 200 200
drawline c
color 0 0 0


% Choosing randomly a point B on the line M_{c}T_{c}
online B M_{c} T_{c}
cmark_b B
color 200 200 200
drawline M_{c} T_{c}
color 0 0 0


% Constructing a point A such that BA/BM_{c}=2
towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0


% NDG: points A and B are not the same; points B and T_{c} are not the same; points B and midpoint
      ([A,T_{c}]) are not the same
% Constructing a point P_{\_G16286} which is an image of the point B in a rotation around the point
      T_{c} for the angle 90
rotate P_{\_G16286} T_{c} 90 B
cmark_r P_{\_G16286}
color 200 200 200
drawarc_p T_{c} B 90
color 0 0 0


% Constructing a line L_{\_G16289} which passes through point T_{c} and point P_{\_G16286}
line L_{\_G16289} T_{c} P_{\_G16286}

color 200 200 200
drawline L_{\_G16289}
color 0 0 0


% Constructing midpoint P_{\_G16292} of the segment BP_{\_G16286}

```



```

midpoint P_{\_G16292} B P_{\_G16286}
cmark_r P_{\_G16292}

% Constructing a line L_{\_G16295} which passes through point A and point P_{\_G16292}
line L_{\_G16295} A P_{\_G16292}

color 200 200 200
drawline L_{\_G16295}
color 0 0 0

% Constructing a line L_{\_G16298} which passes through point B and point P_{\_G16292}
line L_{\_G16298} B P_{\_G16292}

color 200 200 200
drawline L_{\_G16298}
color 0 0 0

% Constructing a point P_{\_G16301} which belongs to line L_{\_G16289} and line L_{\_G16295}
intersec P_{\_G16301} L_{\_G16289} L_{\_G16295}
cmark_r P_{\_G16301}

% Constructing a point P_{\_G16304} which belongs to line L_{\_G16289} and line L_{\_G16298}
intersec P_{\_G16304} L_{\_G16289} L_{\_G16298}
cmark_r P_{\_G16304}

% Constructing a line L_{\_G16307} which passes through point A and point P_{\_G16304}
line L_{\_G16307} A P_{\_G16304}

color 200 200 200
drawline L_{\_G16307}
color 0 0 0

% Constructing a line L_{\_G16310} which passes through point B and point P_{\_G16301}
line L_{\_G16310} B P_{\_G16301}

color 200 200 200
drawline L_{\_G16310}
color 0 0 0

% Constructing a point P_{\_G16313} which belongs to line L_{\_G16307} and line L_{\_G16310}
intersec P_{\_G16313} L_{\_G16307} L_{\_G16310}
cmark_r P_{\_G16313}

% Constructing a line L_{\_G16316} which passes through point P_{\_G16292} and point P_{\_G16313}
line L_{\_G16316} P_{\_G16292} P_{\_G16313}

color 200 200 200
drawline L_{\_G16316}
color 0 0 0

% Constructing a point T'_{c} which belongs to line L_{\_G16316} and line c

```

```

intersec T'_{c} L_{\_G16316} c
cmark_r T'_{c}

% Constructing midpoint P_{\_G17141} of the segment T_{c}T'_{c}
midpoint P_{\_G17141} T_{c} T'_{c}
cmark_r P_{\_G17141}

% Choosing randomly a point C on the circle with center P_{\_G17141} through point T_{c}
oncircle C P_{\_G17141} T_{c}
cmark_l C
color 200 200 200
drawcircle P_{\_G17141} T_{c}
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: points A and B are not the same; points B and T_{c} are not the same;
% points B and midpoint([A,T_{c}]) are not the same
% Determination conditions: points M_{c} and T_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = \_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_c = \_T_c$

Proving failed

#### 4.1.3 Proving $B = B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.014 seconds.

**NDG conditions** There are no NDG conditions for this theorem

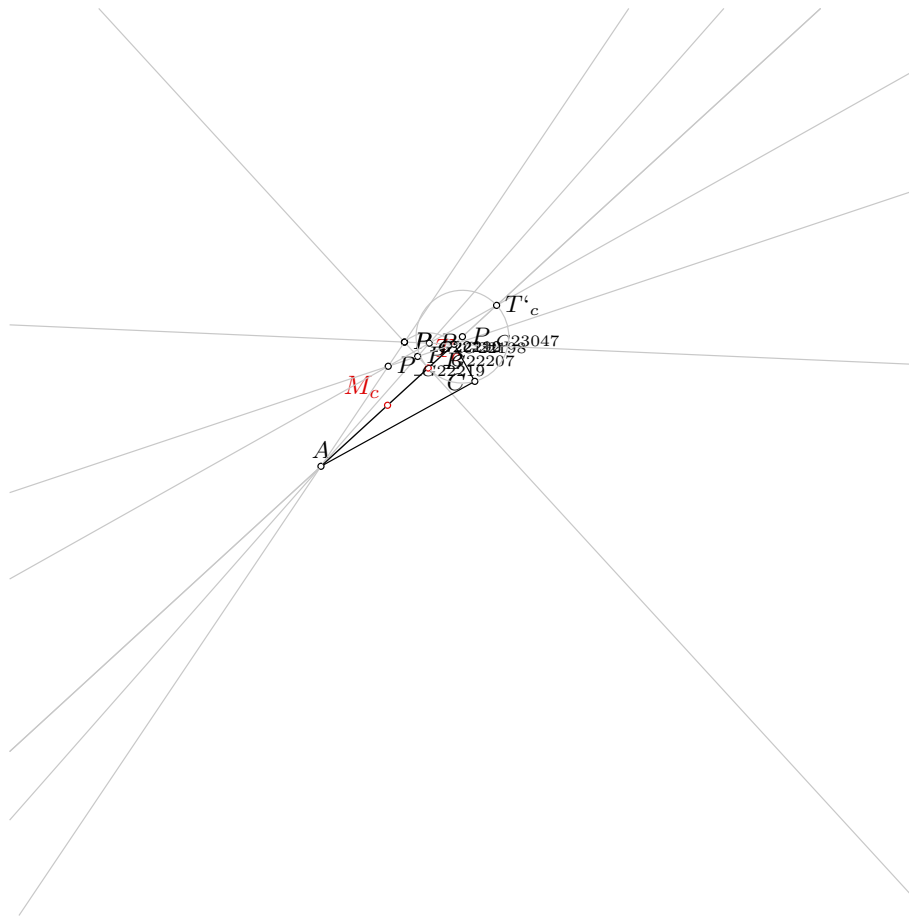


Figure 1: Illustration of the problem 0159

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_c = M_c$

Proving failed

### 4.2.2 Proving $T_c = T_c$

Proving failed

### 4.2.3 Proving $B = B$

NDG conditions are:

$S_{T_cAP_{G27936}} \neq S_{P_{G27930}AP_{G27936}}$  i.e., lines  $T_cP_{G27930}$  and  $AP_{G27936}$  are not parallel (construction based assumption)

$S_{T_cBP_{G27936}} \neq S_{P_{G27930}BP_{G27936}}$  i.e., lines  $T_cP_{G27930}$  and  $BP_{G27936}$  are not parallel (construction based assumption)

$S_{ABP_{G27945}} \neq S_{P_{G27948}BP_{G27945}}$  i.e., lines  $AP_{G27948}$  and  $BP_{G27945}$  are not parallel (construction based assumption)

$S_{P_{G27936}M_cT_c} \neq S_{P_{G27957}M_cT_c}$  i.e., lines  $P_{G27936}P_{G27957}$  and  $M_cT_c$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_c = M_c$

Proving failed

### 4.3.2 Proving $T_c = T_c$

Proving failed

### 4.3.3 Proving $B = B$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_c = M_c$

Proving failed

### 4.4.2 Proving $T_c = T_c$

Proving failed

### 4.4.3 Proving $B = B$

Proving failed

# Problem 160

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 160: Given a point  $B$ , a point  $M_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01); ;
2. Using the point  $B$  and the point  $M_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $M_c$  are not the same;
3. Using the point  $I$  and the line  $c$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $c$ ;
4. Using the circle  $k(I, P_a)$ , the point  $B$ , the point  $I$  and the line  $c$ , construct a line  $a$  (rule W13); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
5. Using the circle  $k(I, P_a)$ , the point  $A$ , the point  $I$  and the line  $c$ , construct a line  $b$  (rule W13); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
6. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; point  $A$  is outside the circle  $k(I, P_a)$ ; point  $B$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $c$ .

Determination conditions: lines  $a$  and  $b$  are not the same; points  $B$  and  $M_c$  are not the same.

Rules used: [W01,W02,W03,W11,W13]

Lemmas used: [D20,D27,GD01,GL03,GL04,GL09,L59,L60,L61]

Solving time: 17.2 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point M_{c} 50 67.5
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_b B
cmark_lt M_{c}
cmark_b I
color 0 0 0
fontsize 8

% Constructing a point A such that BA/BM_{c}=2
towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

% DET: points B and M_{c} are not the same
% Constructing a line c which passes through point B and point M_{c}
line c B M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: point I is not incident to the line c
% Constructing a point P_{\_G24576} which is a foot of the point I on the line c
foot P_{\_G24576} I c
cmark_r P_{\_G24576}
color 200 200 200
drawline I P_{\_G24576}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
24576}
circle k(I,P_{a}) I P_{\_G24576}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point B is outside the circle k(I,P_{a})
% Constructing a point P_{\_G24783} which is a foot of the point I on the line c
```

```

foot P_{\_G24783} I c
cmark_r P_{\_G24783}
color 200 200 200
drawline I P_{\_G24783}
color 0 0 0

% Constructing a line L_{\_G24786} which passes through point I and point B
line L_{\_G24786} I B

color 200 200 200
drawline L_{\_G24786}
color 0 0 0

% Constructing a point P_{\_G24789} which is an image of the point P_{\_G24783} in the symmetry to
point/line L_{\_G24786}
sim P_{\_G24789} L_{\_G24786} P_{\_G24783}
cmark_r P_{\_G24789}

% Constructing a line a which passes through point B and point P_{\_G24789}
line a B P_{\_G24789}

color 200 200 200
drawline a
color 0 0 0

% NDG: point A is outside the circle k(I,P_{a})
% Constructing a point P_{\_G25118} which is a foot of the point I on the line c
foot P_{\_G25118} I c
cmark_r P_{\_G25118}
color 200 200 200
drawline I P_{\_G25118}
color 0 0 0

% Constructing a line L_{\_G25121} which passes through point I and point A
line L_{\_G25121} I A

color 200 200 200
drawline L_{\_G25121}
color 0 0 0

% Constructing a point P_{\_G25124} which is an image of the point P_{\_G25118} in the symmetry to
point/line L_{\_G25121}
sim P_{\_G25124} L_{\_G25121} P_{\_G25118}
cmark_r P_{\_G25124}

% Constructing a line b which passes through point A and point P_{\_G25124}
line b A P_{\_G25124}

color 200 200 200
drawline b
color 0 0 0

```

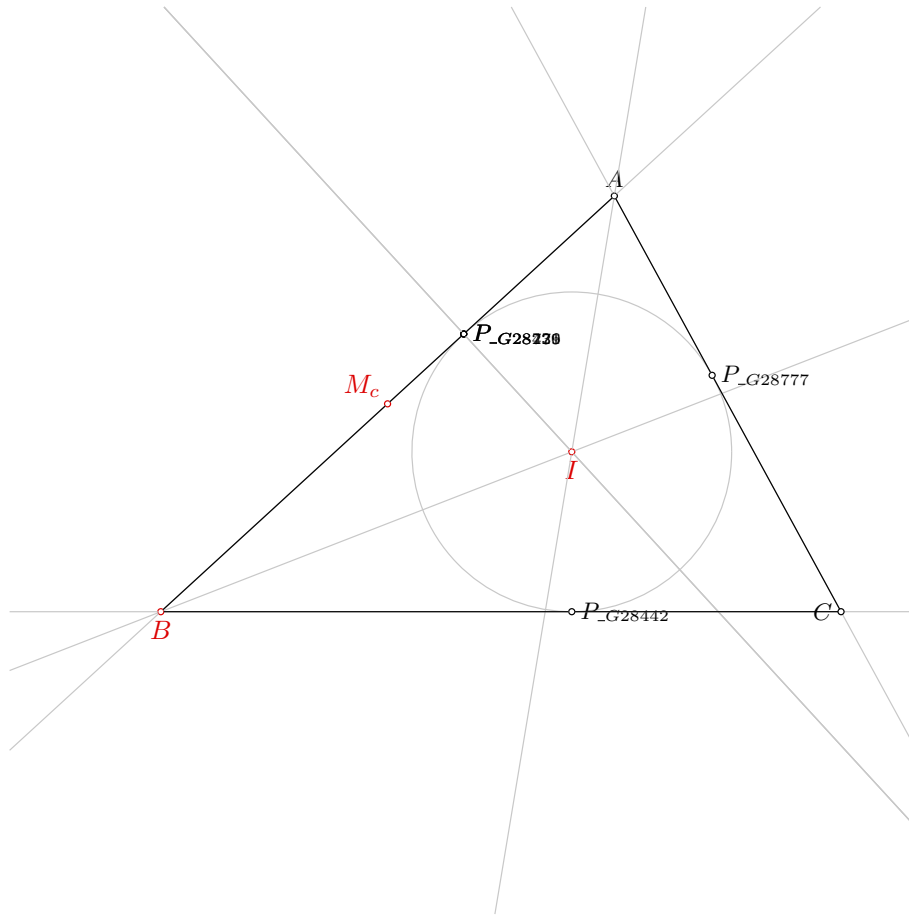


Figure 1: Illustration of the problem 0160

```
% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_1 C
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines a and b are not parallel; point A is outside the circle k(I,P_{a}
}); point B is outside the circle k(I,P_{a}); point I is not incident to the line c
% Determination conditions: lines a and b are not the same; points B and M_{c} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1



### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c=M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.036 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $I=I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{BAP_{G29860}} \neq S_{P_{G29525}AP_{G29860}}$  i.e., lines  $BP_{G29525}$  and  $AP_{G29860}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_c=M_c$

Proving failed

#### 4.2.3 Proving $I=I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 10 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I = \neg I$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 15 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $I = \neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

# Problem 161

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 161: Given a point  $B$ , a point  $G$  and a point  $H_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $G$ , construct a point  $M_b$  (rule W01); ;
2. Using the point  $B$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $H_a$  are not the same;
3. Using the point  $H_a$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_a$  and  $M_b$  are not the same;
4. Using the circle  $k(M_b, C)$ , the line  $a$ , the point  $M_b$  and the point  $H_a$ , construct a point  $C$  (rule W05); % NDG: line  $a$  and circle  $k(M_b, C)$  intersect % DET: points  $H_a$  and  $C$  must be different;
5. Using the point  $G$  and the point  $C$ , construct a point  $M_c$  (rule W01); ;
6. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: line  $a$  and circle  $k(M_b, C)$  intersect; points  $H_a$  and  $M_b$  are not the same.

Determination conditions: points  $H_a$  and  $C$  must be different; points  $B$  and  $H_a$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D20,D5,GD01,GD02,GL03,GL04,L44,L56,L57]

Solving time: 0.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point G 70 58.33
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_b B
cmark_t G
cmark_r H_{a}
color 0 0 0
fontsize 8

% Constructing a point M_{b} such that BM_{b}/BG=1.5
towards M_{b} B G 1.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points H_{a} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{a}
circle k(M_{b},C) M_{b} H_{a}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line a and circle k(M_{b},C) intersect% DET: points H_{a} and C must be different
% Constructing a point P_{\_G27680} which is a foot of the point M_{b} on the line a
foot P_{\_G27680} M_{b} a
cmark_r P_{\_G27680}
color 200 200 200
drawline M_{b} P_{\_G27680}
color 0 0 0
```

```

% Constructing a point C which is an image of the point H_{a} in the symmetry to point/line P_{\_G
27680}
sim C P_{\_G27680} H_{a}
cmark_l C

% Constructing a point M_{c} such that GM_{c}/GC=-0.5
towards M_{c} G C -0.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% Constructing a point A such that BA/BM_{c}=2
towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(M_{b},C) intersect; points H_{a} and M_{b} are not
the same
% Determination conditions: points H_{a} and C must be different; points B and H_{a} are not the
same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $G=_G$

Proving failed

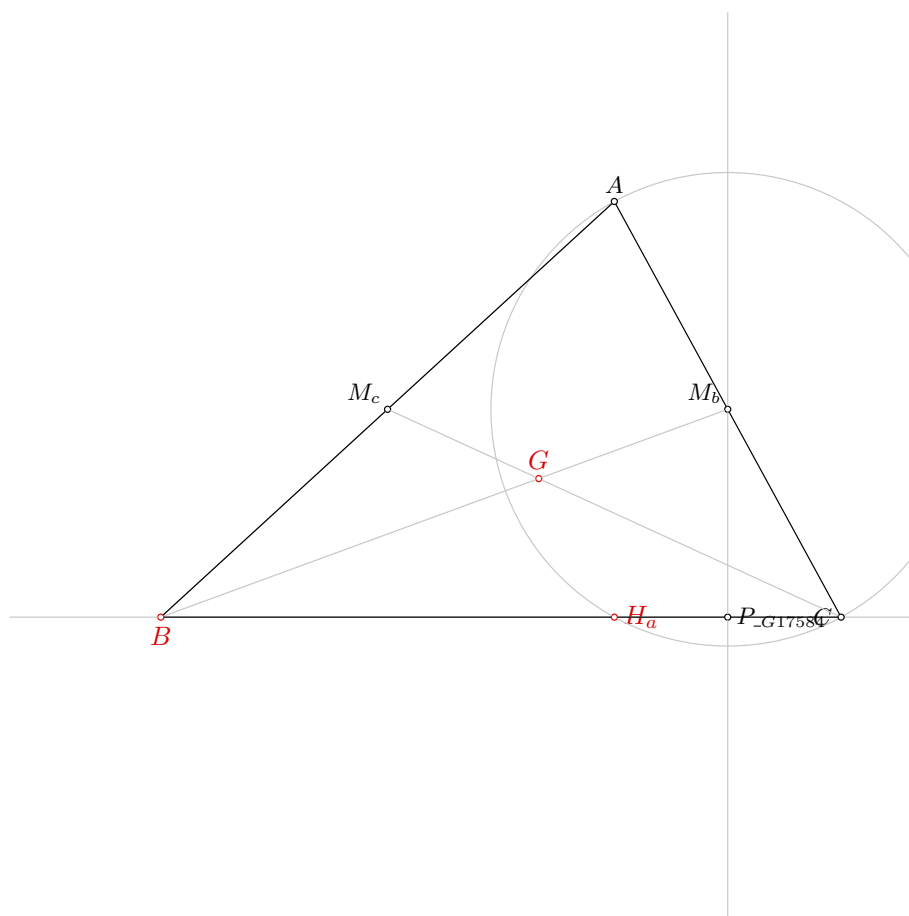


Figure 1: Illustration of the problem 0161

### 4.1.3 Proving $H_a = H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 615 terms.

**Time Complexity:** Time spent by the prover is 19.977 seconds.

**NDG conditions** Points  $G$  and  $H_a$  are not identical

Points  $G$  and  $H_a$  are not identical

Points  $C$ ,  $G$  and  $P_{G25697}$  are not collinear

Line through points  $B$  and  $G$  is not parallel with line through points  $C$  and  $P_{G25697}$

Points  $B$  and  $C$  are not identical

Points  $B$ ,  $C$  and  $G$  are not collinear

## 4.2 GCLC - Area method

### 4.2.1 Proving $B = B$

NDG conditions are:

$S_{AB}M_b \neq S_{M_aB}M_b$  i.e., lines  $A_Ma$  and  $B_Mb$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF^0_{h_a}} \neq S_{CAF^0_{h_a}}$  i.e., lines  $BC$  and  $AF^0_{h_a}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $G = G$

Proving failed

### 4.2.3 Proving $H_a = H_a$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B = B$

Proving failed

### 4.3.2 Proving $G = G$

Proving failed

### 4.3.3 Proving $H_a = H_a$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B = B$

Proving failed

### 4.4.2 Proving $G = G$

Proving failed

### 4.4.3 Proving $H_a = H_a$

Proving failed



# Problem 162

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 162: Given a point  $B$ , a point  $H_b$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_b$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H_b$  are not the same;
2. Using the point  $H_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
3. Using the point  $B$ , the line  $b$  and the point  $H_b$ , construct a line  $h_{B,2/3}(b)$  (rule W15); ;
4. Choose freely a point  $G$  on the line  $h_{B,2/3}(b)$  (rule WOnline4);
5. Using the point  $G$  and the point  $B$ , construct a point  $M_b$  (rule W01); ;
6. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) ;
7. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: points  $B$  and  $H_b$  are not the same.

Rules used: [W01,W02,W10a,W15,WOnline1,WOnline4]

Lemmas used: [D22,D6,D9,GL03,GL04,L56]

Solving time: 192.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point H_{b} 89.36 77.83
point G 70 58.33
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_l H_{b}
cmark_t G
color 0 0 0
fontsize 8
```

```
% DET: points B and H_{b} are not the same
% Constructing a line h_{b} which passes through point B and point H_{b}
line h_{b} B H_{b}
```

```
color 200 200 200
drawline h_{b}
color 0 0 0
```

```
% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
perp b H_{b} h_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% Constructing a point P_{\_G30047} such that BP_{\_G30047}/BH_{b}=0.6666666666666666
towards P_{\_G30047} B H_{b} 0.6666666666666666
cmark_r P_{\_G30047}
color 200 200 200
drawsegment B H_{b}
color 0 0 0
```

```
% Constructing a line h_{B,2/3}(b) which contains the point P_{\_G30047} and is parallel to the
line b
parallel h_{B,2/3}(b) P_{\_G30047} b
```

```
color 200 200 200
drawline h_{B,2/3}(b)
color 0 0 0
```

```
% Constructing a point P_{\_G30354} such that BP_{\_G30354}/BH_{b}=0.6666666666666666
towards P_{\_G30354} B H_{b} 0.6666666666666666
cmark_r P_{\_G30354}
color 200 200 200
drawsegment B H_{b}
color 0 0 0
```

```

% Generating random value V[_G30293]
random V[_G30293]

% Calculating value V[_G30314] using formula V[_G30293]*20
expression V[_G30314] { V[_G30293]*20 }

% Constructing a point G which is a point for which holds  $P_{\backslash\_G30354}G = V[_G30314]$  and angle BP
   $\backslash\_G30354}G = 90$ 
turtle G B P_{\_G30354} 90 V[_G30314]
cmark_t G

% Constructing a point  $M_{\{b\}}$  such that  $GM_{\{b\}}/GB=-0.5$ 
towards M_{b} G B -0.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% Choosing randomly a point A on the line  $H_{\{b\}}M_{\{b\}}$ 
online A H_{b} M_{b}
cmark_t A
color 200 200 200
drawline H_{b} M_{b}
color 0 0 0

% Constructing a point C such that  $AC/AM_{\{b\}}=2$ 
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions: points B and  $H_{\{b\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

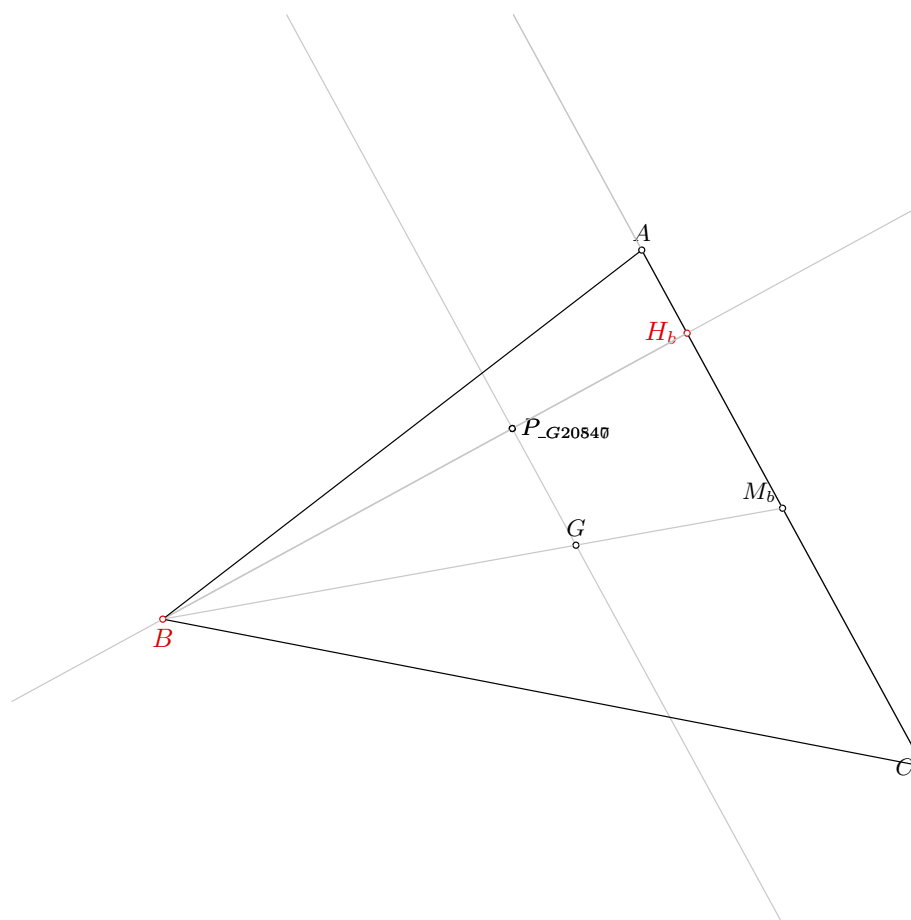


Figure 1: Illustration of the problem 0162

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.032 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b=_Hb$

##### Construction steps:

- Free point  $B$
- Free point  $H_b$
- Line  $h_b$  through two points  $B$  and  $H_b$
- Line  $b$  through point  $H_b$  perpendicular to line  $h_b$
- Segment division point  $P_{G27765}$  of segment  $BH_b$  with division coefficient 1.9999999999999998
- Line  $h_{B,2/3}(b)$  through point  $P_{G27765}$  parallel with line  $b$
- Segment division point  $P_{G27897}$  of segment  $BH_b$  with division coefficient -0.6666666666666666
- Line  $L_{G27900}$  through point  $P_{G27897}$  parallel with line  $h_{B,2/3}(b)$
- Random point  $G$  from line  $L_{G27900}$
- Line  $L_{G28126}$  through two points  $G$  and  $B$
- Free point  $P_{G28129}$
- Segment division point  $P_{G28132}$  of segment  $GP_{G28129}$  with division coefficient -0.5
- Segment division point  $P_{G28135}$  of segment  $GP_{G28129}$  with division coefficient -2.0
- Line  $L_{G28138}$  through two points  $B$  and  $P_{G28135}$
- Line  $L_{G28141}$  through point  $P_{G28132}$  parallel with line  $L_{G28138}$
- Intersection point  $M_b$  of point sets  $L_{G28141}$  and  $L_{G28126}$
- Random point  $A$  from line  $b$
- Segment division point  $C$  of segment  $AM_b$  with division coefficient -2.0
- Line  $_b$  through two points  $A$  and  $C$
- Line  $_h b$  through point  $B$  perpendicular to line  $_b$
- Intersection point  $_H b$  of point sets  $_b$  and  $_h b$

##### Theorem statement:

- Points  $H_b$  and  $_H b$  are identical

**Info:** Attempting to add the construction of new random point tempPoint-171 $h_{B,2/3}(b)$  necessary for completion of construction of line  $L_{G27900}$

**Warning:** Generated new random point tempPoint-171 $h_{B,2/3}(b)$  on line  $h_{B,2/3}(b)$  in order to complete the construction of parallel line  $L_{G27900}$

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $B$  has been assigned following coordinates: (0, 0)

### 4.1.3 Proving $G=_G$

Proving failed

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^3} \neq S_{CBF_{h_b}^3}$  i.e., lines  $AC$  and  $BF_{h_b}^3$  are not parallel (construction based assumption)

$S_{AB_{M_b}} \neq S_{M_aB_{M_b}}$  i.e., lines  $A_{M_a}$  and  $B_{M_b}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_b=_H H_b$

Proving failed

### 4.2.3 Proving $G=_G$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

Proving failed

### 4.3.2 Proving $H_b=_H H_b$

Proving failed

### 4.3.3 Proving $G=_G$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

Proving failed

### 4.4.2 Proving $H_b=_H H_b$

Proving failed

### 4.4.3 Proving $G=_G$

Proving failed

# Problem 163

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 163: Given a point  $B$ , a point  $G$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $G$ , construct a point  $M_b$  (rule W01); ;
2. Using the point  $B$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $H_c$  are not the same;
3. Using the point  $H_c$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_c$  and  $M_b$  are not the same;
4. Using the circle  $k(M_b, C)$ , the line  $c$ , the point  $M_b$  and the point  $H_c$ , construct a point  $A$  (rule W05); % NDG: line  $c$  and circle  $k(M_b, C)$  intersect % DET: points  $H_c$  and  $A$  must be different;
5. Using the point  $G$  and the point  $A$ , construct a point  $M_a$  (rule W01); ;
6. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: line  $c$  and circle  $k(M_b, C)$  intersect; points  $H_c$  and  $M_b$  are not the same.

Determination conditions: points  $H_c$  and  $A$  must be different; points  $B$  and  $H_c$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D21,D7,GD01,GD02,GL03,GL04,L43,L45,L55,L56]

Solving time: 0.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point G 70 58.33
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_b B
cmark_t G
cmark_rt H_{c}
color 0 0 0
fontsize 8

% Constructing a point M_{b} such that BM_{b}/BG=1.5
towards M_{b} B G 1.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points H_{c} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{c}
circle k(M_{b},C) M_{b} H_{c}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line c and circle k(M_{b},C) intersect% DET: points H_{c} and A must be different
% Constructing a point P_{\_G31897} which is a foot of the point M_{b} on the line c
foot P_{\_G31897} M_{b} c
cmark_r P_{\_G31897}
color 200 200 200
drawline M_{b} P_{\_G31897}
color 0 0 0
```



```

% Constructing a point A which is an image of the point H_{c} in the symmetry to point/line P_{\_G
31897}
sim A P_{\_G31897} H_{c}
cmark_t A

% Constructing a point M_{a} such that GM_{a}/GA=-0.5
towards M_{a} G A -0.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% Constructing a point C such that BC/BM_{a}=2
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k(M_{b},C) intersect; points H_{c} and M_{b} are not
the same
% Determination conditions: points H_{c} and A must be different; points B and H_{c} are not the
same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

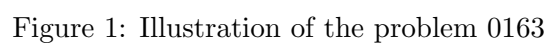
**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $G=_G$

Proving failed



### 4.1.3 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 615 terms.

**Time Complexity:** Time spent by the prover is 19.84 seconds.

**NDG conditions** Points  $H_c$  and  $G$  are not identical

Points  $H_c$  and  $G$  are not identical

Points  $A$ ,  $G$  and  $P_{G29908}$  are not collinear

Line through points  $A$  and  $P_{G29908}$  is not parallel with line through points  $B$  and  $G$

Points  $A$  and  $B$  are not identical

Points  $M_b$ ,  $A$  and  $B$  are not collinear

## 4.2 GCLC - Area method

### 4.2.1 Proving $B = B$

NDG conditions are:

$S_{AB \neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A \neg M_a$  and  $B \neg M_b$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{\neg h_c}^0} \neq S_{BCF_{\neg h_c}^0}$  i.e., lines  $AB$  and  $CF_{\neg h_c}^0$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $G = \neg G$

NDG conditions are:

$S_{AB \neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A \neg M_a$  and  $B \neg M_b$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{\neg h_c}^0} \neq S_{BCF_{\neg h_c}^0}$  i.e., lines  $AB$  and  $CF_{\neg h_c}^0$  are not parallel (construction based assumption)

Total number of proof steps: 595

Time spent by the prover: 0.200 seconds

### 4.2.3 Proving $H_c = \neg H_c$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B = B$

Proving failed

### 4.3.2 Proving $G = \neg G$

Proving failed

### 4.3.3 Proving $H_c = \neg H_c$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B = B$

Proving failed

#### 4.4.2 Proving $G=_G$

Proving failed

#### 4.4.3 Proving $H_c=_H H_c$

Proving failed

# Problem 164

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 164: Given a point  $B$ , a point  $G$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $G$ , construct a point  $M_b$  (rule W01); ;
2. Using the point  $G$  and the point  $H$ , construct a point  $O$  (rule W01); ;
3. Using the point  $B$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H$  are not the same;
4. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
5. Using the point  $M_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same.

Determination conditions: points  $B$  and  $H$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D22,D26,D3,D9,GD01,GL03,GL04,GL09,L11,L12,L56,L58]

Solving time: 1.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40  
point G 70 58.33  
point H 80 72.73
```

```

color 220 0 0
fontsize 9

```

```

cmark_b B
cmark_t G
cmark_rt H
color 0 0 0
fontsize 8

```

```

% Constructing a point  $M_{\{b\}}$  such that  $BM_{\{b\}}/BG=1.5$ 
towards  $M_{\{b\}}$  B G 1.5
cmark_lt  $M_{\{b\}}$ 
color 200 200 200
drawsegment B  $M_{\{b\}}$ 
color 0 0 0

```

```

% Constructing a point  $O$  such that  $GO/GH=-0.5$ 
towards  $O$  G H -0.5
cmark_t  $O$ 
color 200 200 200
drawsegment H  $O$ 
color 0 0 0

```

```

% DET: points  $B$  and  $H$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $B$  and point  $H$ 
line  $h_{\{b\}}$  B H

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

```

```

% NDG: points  $B$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $B$ 
circle  $k(O,C)$   $O$  B

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

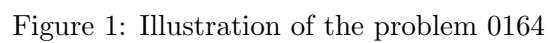
```

```

% Constructing a line  $b$  which is perpendicular to line  $h_{\{b\}}$  and which passes through point  $M_{\{b\}}$ 
perp b  $M_{\{b\}}$   $h_{\{b\}}$ 

color 200 200 200
drawline b
color 0 0 0

```



### 3.3 Illustration

887

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $G=_G$

Proving failed

#### 4.1.3 Proving $H=_H$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{M_bBH} \neq 0$  i.e., points  $M_b$ ,  $B$  and  $H$  are not collinear (foot is not the point itself; construction based assumption)

$S_{AB-M_b} \neq S_{M_aB-M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^2} \neq S_{F_{-h_a}^1BF_{-h_b}^2}$  i.e., lines  $AF_{-h_a}^1$  and  $BF_{-h_b}^2$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $G=_G$

Proving failed

#### 4.2.3 Proving $H=_H$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $G=_G$

Proving failed



### 4.3.3 Proving $H=_H$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $G=_G$

Proving failed

### 4.4.3 Proving $H=_H$

Proving failed

## Problem 165

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 165: Given a point  $B$ , a point  $G$  and a point  $T_a$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 166

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 166: Given a point  $B$ , a point  $G$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $G$ , construct a point  $M_b$  (rule W01); ;
2. Using the point  $B$  and the point  $T_b$ , construct a line  $s_b$  (rule W02); % DET: points  $B$  and  $T_b$  are not the same;
3. Using the point  $T_b$  and the point  $M_b$ , construct a line  $b$  (rule W02); % DET: points  $T_b$  and  $M_b$  are not the same;
4. Using the point  $M_b$  and the line  $b$ , construct a line  $m_b$  (rule W10b); ;
5. Using the line  $m_b$  and the line  $s_b$ , construct a point  $N_b$  (rule W03); % NDG: lines  $m_b$  and  $s_b$  are not parallel % DET: lines  $m_b$  and  $s_b$  are not the same;
6. Using the point  $B$  and the point  $N_b$ , construct a line  $m(BN_b)$  (rule W14); % DET: points  $B$  and  $N_b$  are not the same;
7. Using the line  $m(BN_b)$  and the line  $m_b$ , construct a point  $O$  (rule W03); % NDG: lines  $m(BN_b)$  and  $m_b$  are not parallel % DET: lines  $m(BN_b)$  and  $m_b$  are not the same;
8. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
9. Using the circle  $k(O, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same; lines  $m(BN_b)$  and  $m_b$  are not parallel; lines  $m_b$  and  $s_b$  are not parallel.

Determination conditions: lines  $m(BN_b)$  and  $m_b$  are not the same; points  $B$  and  $N_b$  are not the same; lines  $m_b$  and  $s_b$  are not the same; points  $T_b$  and  $M_b$  are not the same; points  $B$  and  $T_b$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14]

Lemmas used: [D1,D12,D22,D24,D26,D48,GD01,GL01,GL03,GL09,L11,L12,L26,L56,L6]

Solving time: 1.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point G 70 58.33
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_b B
cmark_t G
cmark_t T_{b}
color 0 0 0
fontsize 8

% Constructing a point M_{b} such that BM_{b}/BG=1.5
towards M_{b} B G 1.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% DET: points B and T_{b} are not the same
% Constructing a line s_{b} which passes through point B and point T_{b}
line s_{b} B T_{b}

color 200 200 200
drawline s_{b}
color 0 0 0

% DET: points T_{b} and M_{b} are not the same
% Constructing a line b which passes through point T_{b} and point M_{b}
line b T_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0
```

```

% Constructing a line  $m_{\{b\}}$  which is perpendicular to line  $b$  and which passes through point  $M_{\{b\}}$ 
perp  $m_{\{b\}}$   $M_{\{b\}}$   $b$ 

color 200 200 200
drawline  $m_{\{b\}}$ 
color 0 0 0

% NDG: lines  $m_{\{b\}}$  and  $s_{\{b\}}$  are not parallel% DET: lines  $m_{\{b\}}$  and  $s_{\{b\}}$  are not the same
% Constructing a point  $N_{\{b\}}$  which belongs to line  $m_{\{b\}}$  and line  $s_{\{b\}}$ 
intersec  $N_{\{b\}}$   $m_{\{b\}}$   $s_{\{b\}}$ 
cmark_rb  $N_{\{b\}}$ 

% DET: points  $B$  and  $N_{\{b\}}$  are not the same
% Constructing bisector  $m(BN_{\{b\}})$  of the segment  $BN_{\{b\}}$ 
med  $m(BN_{\{b\}})$   $B$   $N_{\{b\}}$ 

color 200 200 200
drawline  $m(BN_{\{b\}})$ 
color 0 0 0

color 200 200 200
drawsegment  $B$   $N_{\{b\}}$ 
color 0 0 0

% NDG: lines  $m(BN_{\{b\}})$  and  $m_{\{b\}}$  are not parallel% DET: lines  $m(BN_{\{b\}})$  and  $m_{\{b\}}$  are not the same
% Constructing a point  $O$  which belongs to line  $m(BN_{\{b\}})$  and line  $m_{\{b\}}$ 
intersec  $O$   $m(BN_{\{b\}})$   $m_{\{b\}}$ 
cmark_t  $O$ 

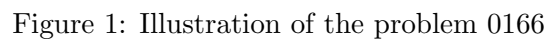
% NDG: points  $B$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $B$ 
circle  $k(O,C)$   $O$   $B$ 

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $b$  and circle  $k(O,C)$  intersect
% Constructing points  $C$  and  $A$  which are in intersection of  $k(O,C)$  and  $b$ 
intersec2  $C$   $A$   $k(O,C)$   $b$ 
cmark_l  $C$ 
cmark_t  $A$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

```



```
% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;
  lines m(BN_{b}) and m_{b} are not parallel; lines m_{b} and s_{b} are not parallel
% Determination conditions: lines m(BN_{b}) and m_{b} are not the same; points B and N_{b} are not
  the same; lines m_{b} and s_{b} are not the same; points T_{b} and M_{b} are not the same;
  points B and T_{b} are not the same
```

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

#### 4.1 OGP - Wu method

### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.019 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $G=_G$

Proving failed

#### 4.1.3 Proving $T_b=_T T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{M_b B T_b} \neq S_{T_{m_b}^0 B T_b}$  i.e., lines  $M_b T_{m_b}^0$  and  $B T_b$  are not parallel (construction based assumption)

$S_{M_{m(BN_b)}^1 M_b T_{m_b}^0} \neq S_{T_{m(BN_b)}^2 M_b T_{m_b}^0}$  i.e., lines  $M_{m(BN_b)}^1 T_{m(BN_b)}^2$  and  $M_b T_{m_b}^0$  are not parallel (construction based assumption)

$S_{AB-M_b} \neq S_{-M_a B-M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $G=_G$

Proving failed

#### 4.2.3 Proving $T_b=_T T_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

#### 4.3.2 Proving $G=_G$

Proving failed

#### 4.3.3 Proving $T_b=_T T_b$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

#### 4.4.2 Proving $G=_G$

Proving failed

#### 4.4.3 Proving $T_b=_T T_b$

Proving failed



## Problem 167

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 167: Given a point  $B$ , a point  $G$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 168

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 168: Given a point  $B$ , a point  $G$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $G$ , construct a point  $M_b$  (rule W01); ;
2. Using the point  $I$  and the point  $M_b$ , construct a line  $IM_b$  (rule W02); % DET: points  $I$  and  $M_b$  are not the same;
3. Using the point  $I$  and the point  $M_b$ , construct a circle  $k_{over}(I, M_b)$  (rule W09); % NDG: points  $I$  and  $M_b$  are not the same;
4. Using the point  $B$  and the line  $IM_b$ , construct a line  $BP'_b$  (rule W16); ;
5. Using the point  $M_b$ , the line  $BP'_b$  and the point  $B$ , construct a line  $h_{M_b, -1/1}(BP'_b)$  (rule W15); ;
6. Using the circle  $k_{over}(I, M_b)$  and the line  $h_{M_b, -1/1}(BP'_b)$ , construct a point  $B_{fo}$  and a point  $P_b$  (rule W04); % NDG: line  $h_{M_b, -1/1}(BP'_b)$  and circle  $k_{over}(I, M_b)$  intersect;
7. Using the point  $P_b$  and the point  $I$ , construct a circle  $k(I, P_a)$  (rule W06); % NDG: points  $P_b$  and  $I$  are not the same;
8. Using the circle  $k(I, P_a)$ , the point  $B$  and the point  $I$ , construct a line  $c$  and a line  $a$  (rule W12); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
9. Using the circle  $k(I, P_a)$ , the point  $M_b$  and the point  $I$ , construct a line  $x2$  and a line  $b$  (rule W12); % NDG: point  $M_b$  is outside the circle  $k(I, P_a)$ ;
10. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same;
11. Using the point  $G$  and the point  $A$ , construct a point  $M_a$  (rule W01); ;

12. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; point  $M_b$  is outside the circle  $k(I, P_a)$ ; point  $B$  is outside the circle  $k(I, P_a)$ ; points  $P_b$  and  $I$  are not the same; line  $h_{M_b, -1/1}(BP'_b)$  and circle  $k_{over}(I, M_b)$  intersect; points  $I$  and  $M_b$  are not the same.

Determination conditions: lines  $c$  and  $b$  are not the same; points  $I$  and  $M_b$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W09,W12,W15,W16]

Lemmas used: [D21,D22,D27,D60,D86,GD01,GL02,GL03,GL04,GL09,L13,L55,L56,L59,L60,L61,L63,L75]

Solving time: 3.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point G 70 58.33
point I 74.37 61.15
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_t G
cmark_b I
color 0 0 0
fontsize 8
```

```
% Constructing a point M_{b} such that BM_{b}/BG=1.5
towards M_{b} B G 1.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0
```

```
% DET: points I and M_{b} are not the same
% Constructing a line IM_{b} which passes through point I and point M_{b}
line IM_{b} I M_{b}
```

```
color 200 200 200
drawline IM_{b}
color 0 0 0
```

```
% NDG: points I and M_{b} are not the same
% Constructing midpoint P_{\_G29025} of the segment IM_{b}
midpoint P_{\_G29025} I M_{b}
cmark_r P_{\_G29025}
```

```
% Constructing a circle k_{over}(I,M_{b}) whose center is at point P_{\_G29025} and which passes
through point I
circle k_{over}(I,M_{b}) P_{\_G29025} I
```

```

color 200 200 200
drawcircle k_over(I,M_{b})
color 0 0 0

% Constructing a line BP'_{b} which contains the point B and is parallel to the line IM_{b}
parallel BP'_{b} B IM_{b}

color 200 200 200
drawline BP'_{b}
color 0 0 0

% Constructing a point P_{\_G29326} such that M_{b}P_{\_G29326}/M_{b}B=-1
towards P_{\_G29326} M_{b} B -1
cmark_r P_{\_G29326}
color 200 200 200
drawsegment B P_{\_G29326}
color 0 0 0

% Constructing a line h_{M_{b},-1/1}(BP'_{b}) which contains the point P_{\_G29326} and is parallel
to the line BP'_{b}
parallel h_{M_{b},-1/1}(BP'_{b}) P_{\_G29326} BP'_{b}

color 200 200 200
drawline h_{M_{b},-1/1}(BP'_{b})
color 0 0 0

% NDG: line h_{M_{b},-1/1}(BP'_{b}) and circle k_over(I,M_{b}) intersect
% Constructing points B_{fo} and P_{b} which are in intersection of k_over(I,M_{b}) and h_{M_{b},-1/1}(BP'_{b})
intersec2 B_{fo} P_{b} k_over(I,M_{b}) h_{M_{b},-1/1}(BP'_{b})
cmark_r B_{fo}
cmark_r P_{b}

% NDG: points P_{b} and I are not the same
% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{b}
circle k(I,P_{a}) I P_{b}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point B is outside the circle k(I,P_{a})
% Constructing midpoint P_{\_G30075} of the segment BI
midpoint P_{\_G30075} B I

```

```

cmark_r P_{\_G30075}

% Constructing a circle C_{\_G30078} whose center is at point P_{\_G30075} and which passes through
point B
circle C_{\_G30078} P_{\_G30075} B

color 200 200 200
drawcircle C_{\_G30078}
color 0 0 0

% Constructing points P_{\_G30081} and P_{\_G30084} which are in intersection of C_{\_G30078} and k
(I,P_{a})
intersec2 P_{\_G30081} P_{\_G30084} C_{\_G30078} k(I,P_{a})
cmark_r P_{\_G30081}
cmark_r P_{\_G30084}

% Constructing a line c which passes through point B and point P_{\_G30081}
line c B P_{\_G30081}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line a which passes through point B and point P_{\_G30084}
line a B P_{\_G30084}

color 200 200 200
drawline a
color 0 0 0

% NDG: point M_{b} is outside the circle k(I,P_{a})
% Constructing midpoint P_{\_G30487} of the segment M_{b}I
midpoint P_{\_G30487} M_{b} I
cmark_r P_{\_G30487}

% Constructing a circle C_{\_G30490} whose center is at point P_{\_G30487} and which passes through
point M_{b}
circle C_{\_G30490} P_{\_G30487} M_{b}

color 200 200 200
drawcircle C_{\_G30490}
color 0 0 0

% Constructing points P_{\_G30493} and P_{\_G30496} which are in intersection of C_{\_G30490} and k
(I,P_{a})
intersec2 P_{\_G30493} P_{\_G30496} C_{\_G30490} k(I,P_{a})
cmark_r P_{\_G30493}
cmark_r P_{\_G30496}

% Constructing a line x2 which passes through point M_{b} and point P_{\_G30493}
line x2 M_{b} P_{\_G30493}

```

```

color 200 200 200
drawline x2
color 0 0 0

% Constructing a line b which passes through point M_{b} and point P_{\_G30496}
line b M_{b} P_{\_G30496}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A

% Constructing a point M_{a} such that GM_{a}/GA=-0.5
towards M_{a} G A -0.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% Constructing a point C such that BC/BM_{a}=2
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and b are not parallel; point M_{b} is outside the circle k(I,
P_{a}); point B is outside the circle k(I,P_{a}); points P_{b} and I are not the same; line h_{
M_{b},-1/1}(BP'_{b}) and circle k_{over}(I,M_{b}) intersect; points I and M_{b} are not the same
% Determination conditions: lines c and b are not the same; points I and M_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

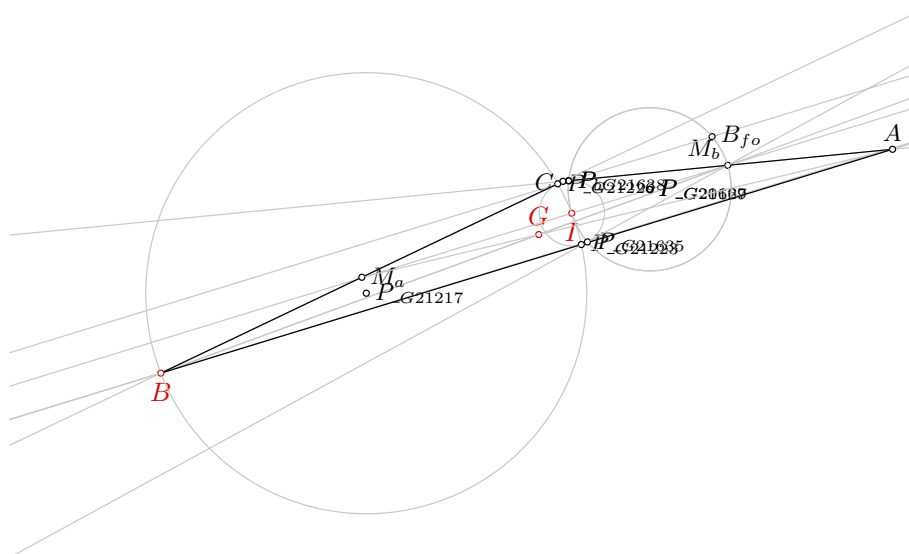


Figure 1: Illustration of the problem 0168

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.024 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $G=_G$

Proving failed

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{BM_bP_{G23971}} \neq S_{P_{G23556}M_bP_{G23971}}$  i.e., lines  $BP_{G23556}$  and  $M_bP_{G23971}$  are not parallel (construction based assumption)

$S_{AB_{-}M_b} \neq S_{_{-}M_aB_{-}M_b}$  i.e., lines  $A_{-}M_a$  and  $B_{-}M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $G=_G$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.050 seconds. There are no ndg conditions.

#### 4.3.2 Proving $G=_G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 143 terms.

**Time Complexity:** Time spent by the prover is 0.840 seconds. There are no ndg conditions.



### 4.3.3 Proving $I=_I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6385 terms.

**Time Complexity:** Time spent by the prover is 51.780 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.050 seconds. There are no ndg conditions.

### 4.4.2 Proving $G=_G$

Proving failed

### 4.4.3 Proving $I=_I$

Proving failed

# Problem 169

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 169: Given a point  $B$ , a point  $H_a$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $H_a$  are not the same;
2. Using the point  $B$  and the point  $H_b$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H_b$  are not the same;
3. Using the point  $H_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
4. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same;
5. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
6. Using the line  $b$  and the line  $h_a$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $h_a$  are not parallel % DET: lines  $b$  and  $h_a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $h_a$  are not parallel; lines  $b$  and  $a$  are not parallel.

Determination conditions: lines  $b$  and  $h_a$  are not the same; lines  $b$  and  $a$  are not the same; points  $B$  and  $H_b$  are not the same; points  $B$  and  $H_a$  are not the same.

Rules used: [W02,W03,W10a,W10b]

Lemmas used: [D5,D6,D8,D9,GD01]

Solving time: 1.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point H_{a} 80 40
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_b B
cmark_r H_{a}
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% DET: points B and H_{b} are not the same
% Constructing a line h_{b} which passes through point B and point H_{b}
line h_{b} B H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
perp b H_{b} h_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a
```

```

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: lines b and h_{a} are not parallel% DET: lines b and h_{a} are not the same
% Constructing a point A which belongs to line b and line h_{a}
intersec A b h_{a}
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and h_{a} are not parallel; lines b and a are not parallel
% Determination conditions: lines b and h_{a} are not the same; lines b and a are not the same;
  points B and H_{b} are not the same; points B and H_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a=H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.075 seconds.

**NDG conditions** Points  $H_b$ ,  $B$  and  $C$  are not collinear

Points  $H_b$ ,  $B$  and  $C$  are not collinear

Point  $B$  is not on circle with center  $H_a$  and point from it  $C$

#### 4.1.3 Proving $H_b=H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 20 terms.

**Time Complexity:** Time spent by the prover is 0.351 seconds.

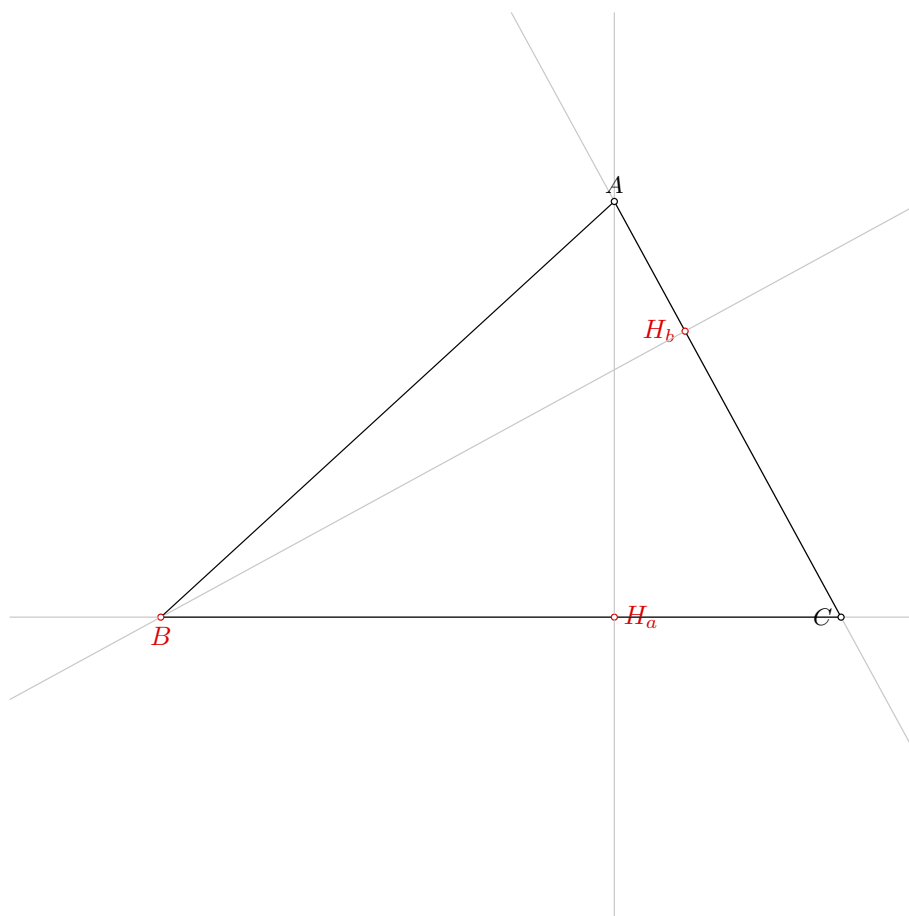


Figure 1: Illustration of the problem 0169

**NDG conditions** Points  $H_b$ ,  $B$  and  $C$  are not collinear  
 Points  $H_b$ ,  $B$  and  $C$  are not collinear  
 Point  $H_a$  is not the midpoint of segment with endpoints  $A$  and  $C$   
 Points  $A$  and  $H_a$  are not identical

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_b B H_a} \neq S_{T_b^0 B H_a}$  i.e., lines  $H_b T_b^0$  and  $B H_a$  are not parallel (construction based assumption)  
 $S_{H_b H_a T_{h_a}^1} \neq S_{T_b^0 H_a T_{h_a}^1}$  i.e., lines  $H_b T_b^0$  and  $H_a T_{h_a}^1$  are not parallel (construction based assumption)  
 $S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)  
 $S_{BAF_{h_a}^2} \neq S_{CAF_{h_a}^2}$  i.e., lines  $BC$  and  $AF_{h_a}^2$  are not parallel (construction based assumption)  
 $S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)  
 $S_{ABF_{h_b}^3} \neq S_{CBF_{h_b}^3}$  i.e., lines  $AC$  and  $BF_{h_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_a = \neg H_a$

Proving failed

### 4.2.3 Proving $H_b = \neg H_b$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $H_b = \neg H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 91 terms.

**Time Complexity:** Time spent by the prover is 0.130 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_a = H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $H_b = H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 176 terms.

**Time Complexity:** Time spent by the prover is 0.690 seconds. There are no ndg conditions.

# Problem 170

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 170: Given a point  $B$ , a point  $H_a$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $H_a$  are not the same;
2. Using the point  $B$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $H_c$  are not the same;
3. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
4. Using the line  $h_a$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $h_a$  and  $c$  are not parallel % DET: lines  $h_a$  and  $c$  are not the same;
5. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
6. Using the line  $h_c$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $h_c$  and  $a$  are not parallel % DET: lines  $h_c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $h_c$  and  $a$  are not parallel; lines  $h_a$  and  $c$  are not parallel.

Determination conditions: lines  $h_c$  and  $a$  are not the same; lines  $h_a$  and  $c$  are not the same; points  $B$  and  $H_c$  are not the same; points  $B$  and  $H_a$  are not the same.

Rules used: [W02,W03,W10b]

Lemmas used: [D10,D5,D7,D8,GD01]

Solving time: 0.9 seconds.



### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point H_{a} 80 40
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_b B
cmark_r H_{a}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: lines h_{a} and c are not parallel% DET: lines h_{a} and c are not the same
% Constructing a point A which belongs to line h_{a} and line c
intersec A h_{a} c
cmark_t A

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c
```

```

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines h_{c} and a are not parallel% DET: lines h_{c} and a are not the same
% Constructing a point C which belongs to line h_{c} and line a
intersec C h_{c} a
cmark_1 C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and a are not parallel; lines h_{a} and c are not parallel
% Determination conditions: lines h_{c} and a are not the same; lines h_{a} and c are not the same;
  points B and H_{c} are not the same; points B and H_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a=H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.097 seconds.

**NDG conditions** Points  $B$  and  $H_c$  are not identical

Point  $B$  is not on circle with center  $H_a$  and point from it  $C$

#### 4.1.3 Proving $H_c=H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.16 seconds.

**NDG conditions** Points  $B$  and  $H_c$  are not identical

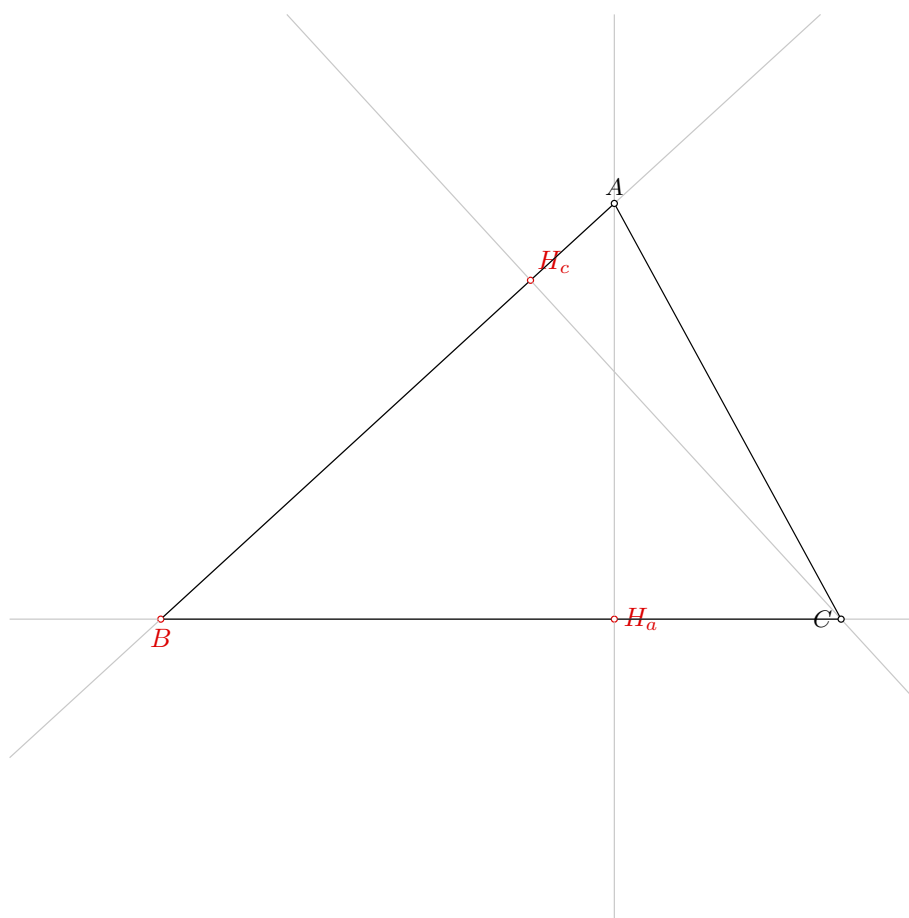


Figure 1: Illustration of the problem 0170

Points  $A$  and  $B$  are not identical

Points  $A$  and  $H_a$  are not identical

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_aBH_c} \neq S_{T_{h_a}^0BH_c}$  i.e., lines  $H_aT_{h_a}^0$  and  $BH_c$  are not parallel (construction based assumption)

$S_{H_cBH_a} \neq S_{T_{h_c}^1BH_a}$  i.e., lines  $H_cT_{h_c}^1$  and  $BH_a$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^2} \neq S_{CAF_{h_a}^2}$  i.e., lines  $BC$  and  $AF_{h_a}^2$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^3} \neq S_{BCF_{h_c}^3}$  i.e., lines  $AB$  and  $CF_{h_c}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_a=H_a$

Proving failed

### 4.2.3 Proving $H_c=H_c$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_a=H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $H_c=H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 12 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_a = H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $H_c = H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 11 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

# Problem 171

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 171: Given a point  $H_a$ , a point  $H$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $H_a$  and  $H$  are not the same;
2. Using the point  $H_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
3. Choose freely a point  $B$  on the line  $a$  (rule WOnline2);
4. Using the point  $B$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H$  are not the same;
5. Choose freely a point  $A$  on the line  $h_a$  (rule WOnline1) ;
6. Using the point  $A$  and the point  $B$ , construct a point  $M_c$  (rule W01); ;
7. Using the point  $A$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $A$  and  $M_c$  are not the same;
8. Using the circle  $k(M_c, A)$ , the line  $h_b$ , the point  $M_c$  and the point  $B$ , construct a point  $H_b$  (rule W05); % NDG: line  $h_b$  and circle  $k(M_c, A)$  intersect % DET: points  $B$  and  $H_b$  must be different;
9. Using the point  $H_b$  and the point  $A$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $A$  are not the same;
10. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; line  $h_b$  and circle  $k(M_c, A)$  intersect; points  $A$  and  $M_c$  are not the same.

Determination conditions: lines  $a$  and  $b$  are not the same; points  $H_b$  and  $A$  are not the same; points  $B$  and  $H_b$  must be different; points  $B$  and  $H$  are not the same; points  $H_a$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D20,D3,D5,D6,D8,D9,GD01,GD02,L40,L41]

Solving time: 24.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{a} 80 40
```

```
point H 80 72.73
```

```
point B 20 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r H_{a}
```

```
cmark_rt H
```

```
cmark_b B
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{a} and H are not the same
```

```
% Constructing a line h_{a} which passes through point H_{a} and point H
```

```
line h_{a} H_{a} H
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```
% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
```

```
perp a H_{a} h_{a}
```

```
color 200 200 200
```

```
drawline a
```

```
color 0 0 0
```

```
% Generating random value V[_G31900]
```

```
random V[_G31900]
```

```
% Calculating value V[_G31921] using formula V[_G31900]*20
```

```
expression V[_G31921] { V[_G31900]*20 }
```

```

% Constructing a point B which is a point for which holds  $H_{\{a\}}B = V[_{G31921}]$  and angle  $HH_{\{a\}}B = 90$ 
turtle B H  $H_{\{a\}}$  90  $V[_{G31921}]$ 
cmark_b B

% DET: points B and H are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point B and point H
line  $h_{\{b\}}$  B H

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% Choosing randomly a point A on the line  $H_{\{a\}}H$ 
online A  $H_{\{a\}}$  H
cmark_t A
color 200 200 200
drawline  $H_{\{a\}}$  H
color 0 0 0

% Constructing a point  $M_{\{c\}}$  such that  $AM_{\{c\}}/AB=0.5$ 
towards  $M_{\{c\}}$  A B 0.5
cmark_lt  $M_{\{c\}}$ 
color 200 200 200
drawsegment A B
color 0 0 0

% NDG: points A and  $M_{\{c\}}$  are not the same
% Constructing a circle  $k(M_{\{c\}},A)$  whose center is at point  $M_{\{c\}}$  and which passes through point A
circle  $k(M_{\{c\}},A)$   $M_{\{c\}}$  A

color 200 200 200
drawcircle  $k(M_{\{c\}},A)$ 
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(M_{\{c\}},A)$  intersect% DET: points B and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G18665\}}$  which is a foot of the point  $M_{\{c\}}$  on the line  $h_{\{b\}}$ 
foot  $P_{\{\backslash\_G18665\}}$   $M_{\{c\}}$   $h_{\{b\}}$ 
cmark_r  $P_{\{\backslash\_G18665\}}$ 
color 200 200 200
drawline  $M_{\{c\}}$   $P_{\{\backslash\_G18665\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point B in the symmetry to point/line  $P_{\{\backslash\_G18665\}}$ 
sim  $H_{\{b\}}$   $P_{\{\backslash\_G18665\}}$  B

```



```

cmark_l H_{b}

% DET: points H_{b} and A are not the same
% Constructing a line b which passes through point H_{b} and point A
line b H_{b} A

color 200 200 200
drawline b
color 0 0 0

% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and b are not parallel; line h_{b} and circle k(M_{c},A)
% intersect; points A and M_{c} are not the same
% Determination conditions: lines a and b are not the same; points H_{b} and A are not the same;
% points B and H_{b} must be different; points B and H are not the same; points H_{a} and H are
% not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_a = \_H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.155 seconds.

**NDG conditions** Points  $B$  and  $H$  are not identical

Points  $B$  and  $H$  are not identical

Line through points  $H_b$  and  $H$  is not perpendicular to line through points  $H$  and  $A$

Points  $B$ ,  $C$  and  $H$  are not collinear

#### 4.1.2 Proving $H = \_H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 31 terms.

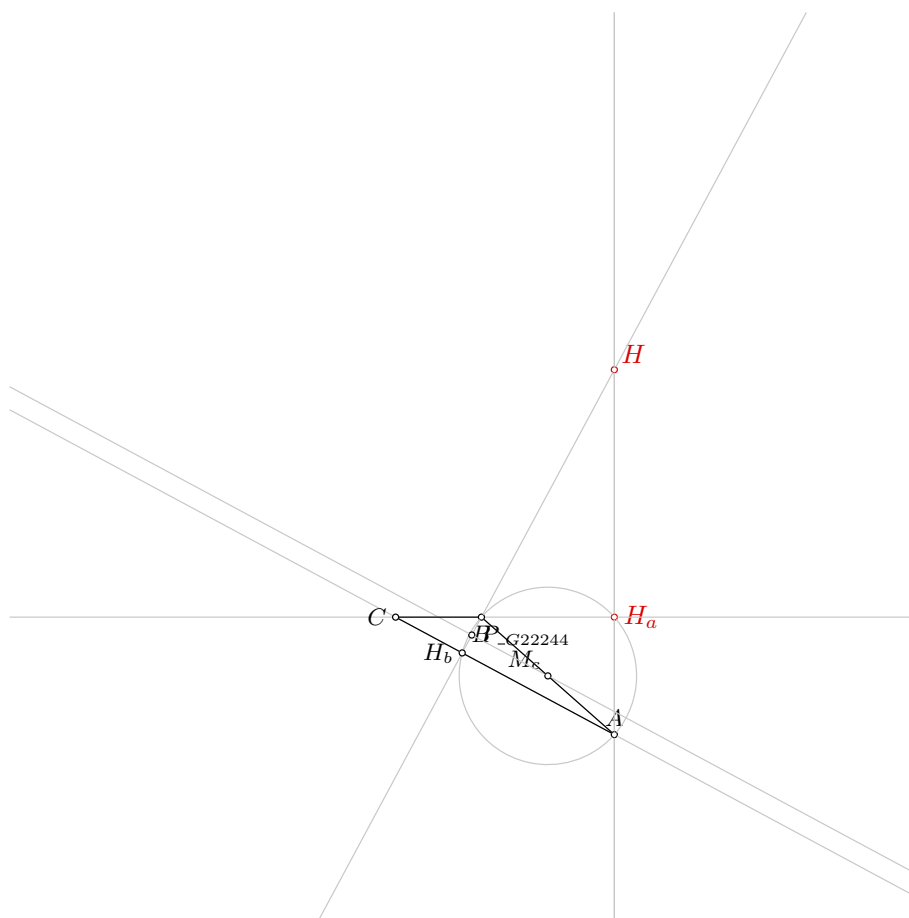


Figure 1: Illustration of the problem 0171

**Time Complexity:** Time spent by the prover is 0.471 seconds.

**NDG conditions** Points  $B$  and  $H$  are not identical

Points  $B$  and  $H$  are not identical

Line through points  $H_b$  and  $H$  is not perpendicular to line through points  $H$  and  $A$

Points  $B$ ,  $C$  and  $H$  are not collinear

Points  $A$  and  $B$  are not identical

#### 4.1.3 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.007 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_a=_H a$

Proving failed

#### 4.2.2 Proving $H=_H H$

Proving failed

#### 4.2.3 Proving $B=B$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_a=_H a$

Proving failed

#### 4.3.2 Proving $H=_H H$

Proving failed

#### 4.3.3 Proving $B=B$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $H_a=_H a$

Proving failed

#### 4.4.2 Proving $H=_H H$

Proving failed

#### 4.4.3 Proving $B=B$

Proving failed

# Problem 172

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 172: Given a point  $H_a$ , a point  $T_a$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $T_a$  are not the same;
2. Choose freely a point  $B$  on the line  $a$  (rule WOnline1) ;
3. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
4. Choose freely a point  $A$  on the line  $h_a$  (rule WOnline2);
5. Using the point  $A$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $B$  are not the same;
6. Using the point  $T_a$  and the line  $c$ , construct a circle  $k(T_a, foot[T_a, c])$  (rule W11); % NDG: point  $T_a$  is not incident to the line  $c$ ;
7. Using the circle  $k(T_a, foot[T_a, c])$ , the point  $A$ , the point  $T_a$  and the line  $c$ , construct a line  $b$  (rule W13); % NDG: point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ;
8. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ; point  $T_a$  is not incident to the line  $c$ .

Determination conditions: lines  $b$  and  $a$  are not the same; points  $A$  and  $B$  are not the same; points  $H_a$  and  $T_a$  are not the same.

Rules used: [W02,W03,W10b,W11,W13,WOnline1,WOnline2]

Lemmas used: [D23,D5,D8,GD01,GD02,GL10,GL11,L8]

Solving time: 15.4 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point H_{a} 80 40
point T_{a} 70.86 40
point B 20 40

color 220 0 0
fontsize 9

cmark_r H_{a}
cmark_rb T_{a}
cmark_b B
color 0 0 0
fontsize 8

% DET: points H_{a} and T_{a} are not the same
% Constructing a line a which passes through point H_{a} and point T_{a}
line a H_{a} T_{a}

color 200 200 200
drawline a
color 0 0 0


% Choosing randomly a point B on the line H_{a}T_{a}
online B H_{a} T_{a}
cmark_b B
color 200 200 200
drawline H_{a} T_{a}
color 0 0 0


% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0


% Generating random value V[_G26543]
random V[_G26543]


% Calculating value V[_G26564] using formula V[_G26543]*20
expression V[_G26564] { V[_G26543]*20 }
```

```

% Constructing a point A which is a point for which holds  $H_{\{a\}}A = V_{G26564}$  and angle  $BH_{\{a\}}A = 90$ 
turtle A B  $H_{\{a\}}$  90  $V_{G26564}$ 
cmark_t A

% DET: points A and B are not the same
% Constructing a line c which passes through point A and point B
line c A B

color 200 200 200
drawline c
color 0 0 0

% NDG: point  $T_{\{a\}}$  is not incident to the line c
% Constructing a point  $P_{G26808}$  which is a foot of the point  $T_{\{a\}}$  on the line c
foot  $P_{G26808}$   $T_{\{a\}}$  c
cmark_r  $P_{G26808}$ 
color 200 200 200
drawline  $T_{\{a\}}$   $P_{G26808}$ 
color 0 0 0

% Constructing a circle  $k(T_{\{a\}}, foot[T_{\{a\}}, c])$  whose center is at point  $T_{\{a\}}$  and which passes
    through point  $P_{G26808}$ 
circle  $k(T_{\{a\}}, foot[T_{\{a\}}, c])$   $T_{\{a\}}$   $P_{G26808}$ 

color 200 200 200
drawcircle  $k(T_{\{a\}}, foot[T_{\{a\}}, c])$ 
color 0 0 0

% NDG: point A is outside the circle  $k(T_{\{a\}}, foot[T_{\{a\}}, c])$ 
% Constructing a point  $P_{G27199}$  which is a foot of the point  $T_{\{a\}}$  on the line c
foot  $P_{G27199}$   $T_{\{a\}}$  c
cmark_r  $P_{G27199}$ 
color 200 200 200
drawline  $T_{\{a\}}$   $P_{G27199}$ 
color 0 0 0

% Constructing a line  $L_{G27202}$  which passes through point  $T_{\{a\}}$  and point A
line  $L_{G27202}$   $T_{\{a\}}$  A

color 200 200 200
drawline  $L_{G27202}$ 
color 0 0 0

% Constructing a point  $P_{G27205}$  which is an image of the point  $P_{G27199}$  in the symmetry to
    point/line  $L_{G27202}$ 
sim  $P_{G27205}$   $L_{G27202}$   $P_{G27199}$ 
cmark_r  $P_{G27205}$ 

% Constructing a line b which passes through point A and point  $P_{G27205}$ 

```

```

line b A P_{\_G27205}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and a are not parallel; point A is outside the circle k(T_{a},
    foot[T_{a},c]); point T_{a} is not incident to the line c
% Determination conditions: lines b and a are not the same; points A and B are not the same; points
    H_{a} and T_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_a = H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.307 seconds.

**NDG conditions** Points  $T_a$  and  $A$  are not identical

Points  $A$  and  $B$  are not identical

Points  $T_a$  and  $A$  are not identical

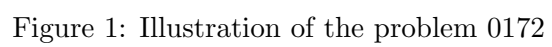
Point  $H_a$  is not on circle with center  $T_a$  and point from it  $C$

Line through points  $A$  and  $P_{G22822}$  is not parallel with line through points  $B$  and  $H_a$

Line through points  $A$  and  $H_a$  is not parallel with line through points  $B$  and  $C$

#### 4.1.2 Proving $T_a = T_a$

Proving failed





### 4.1.3 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.013 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $H_a = \neg H_a$

Proving failed

### 4.2.2 Proving $T_a = \neg T_a$

Proving failed

### 4.2.3 Proving $B=B$

NDG conditions are:

$S_{AH_aT_a} \neq S_{P_{G21452}H_aT_a}$  i.e., lines  $AP_{G21452}$  and  $H_aT_a$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^2} \neq S_{CAF_{h_a}^2}$  i.e., lines  $BC$  and  $AF_{h_a}^2$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $H_a = \neg H_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $T_a = \neg T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $H_a = \neg H_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 19 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.2 Proving $T_a = T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 19 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

#### 4.4.3 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 173

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 173: Given a point  $B$ , a point  $H_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $H_a$  are not the same;
2. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
3. Using the point  $T_b$  and the line  $a$ , construct a circle  $k(T_b, foot[T_b, c])$  (rule W11); % NDG: point  $T_b$  is not incident to the line  $a$ ;
4. Using the circle  $k(T_b, foot[T_b, c])$ , the point  $B$ , the point  $T_b$  and the line  $a$ , construct a line  $c$  (rule W13); % NDG: point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ;
5. Using the line  $c$  and the line  $h_a$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $h_a$  are not parallel % DET: lines  $c$  and  $h_a$  are not the same;
6. Using the point  $T_b$  and the point  $A$ , construct a line  $b$  (rule W02); % DET: points  $T_b$  and  $A$  are not the same;
7. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; lines  $c$  and  $h_a$  are not parallel; point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ; point  $T_b$  is not incident to the line  $a$ .

Determination conditions: lines  $a$  and  $b$  are not the same; points  $T_b$  and  $A$  are not the same; lines  $c$  and  $h_a$  are not the same; points  $B$  and  $H_a$  are not the same.

Rules used: [W02,W03,W10b,W11,W13]

Lemmas used: [D24,D5,D8,GD01,GD02,GL10,GL11,L9]

Solving time: 0.6 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point H_{a} 80 40
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_b B
cmark_r H_{a}
cmark_t T_{b}
color 0 0 0
fontsize 8

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: point T_{b} is not incident to the line a
% Constructing a point P_{\_G31245} which is a foot of the point T_{b} on the line a
foot P_{\_G31245} T_{b} a
cmark_r P_{\_G31245}
color 200 200 200
drawline T_{b} P_{\_G31245}
color 0 0 0

% Constructing a circle k(T_{b},foot[T_{b},c]) whose center is at point T_{b} and which passes
    through point P_{\_G31245}
circle k(T_{b},foot[T_{b},c]) T_{b} P_{\_G31245}

color 200 200 200
drawcircle k(T_{b},foot[T_{b},c])
color 0 0 0

% NDG: point B is outside the circle k(T_{b},foot[T_{b},c])
```

```

% Constructing a point  $P_{\{b\}}$  which is a foot of the point  $T_{\{b\}}$  on the line  $a$ 
foot  $P_{\{b\}}$   $T_{\{b\}}$   $a$ 
cmark_r  $P_{\{b\}}$ 
color 200 200 200
drawline  $T_{\{b\}}$   $P_{\{b\}}$ 
color 0 0 0

% Constructing a line  $L_{\{b\}}$  which passes through point  $T_{\{b\}}$  and point  $B$ 
line  $L_{\{b\}}$   $T_{\{b\}}$   $B$ 

color 200 200 200
drawline  $L_{\{b\}}$ 
color 0 0 0

% Constructing a point  $P_{\{a\}}$  which is an image of the point  $P_{\{b\}}$  in the symmetry to
point/line  $L_{\{b\}}$ 
sim  $P_{\{a\}}$   $L_{\{b\}}$   $P_{\{b\}}$ 
cmark_r  $P_{\{a\}}$ 

% Constructing a line  $c$  which passes through point  $B$  and point  $P_{\{a\}}$ 
line  $c$   $B$   $P_{\{a\}}$ 

color 200 200 200
drawline  $c$ 
color 0 0 0

% NDG: lines  $c$  and  $h_{\{a\}}$  are not parallel% DET: lines  $c$  and  $h_{\{a\}}$  are not the same
% Constructing a point  $A$  which belongs to line  $c$  and line  $h_{\{a\}}$ 
intersec  $A$   $c$   $h_{\{a\}}$ 
cmark_t  $A$ 

% DET: points  $T_{\{b\}}$  and  $A$  are not the same
% Constructing a line  $b$  which passes through point  $T_{\{b\}}$  and point  $A$ 
line  $b$   $T_{\{b\}}$   $A$ 

color 200 200 200
drawline  $b$ 
color 0 0 0

% NDG: lines  $a$  and  $b$  are not parallel% DET: lines  $a$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $a$  and line  $b$ 
intersec  $C$   $a$   $b$ 
cmark_l  $C$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

```

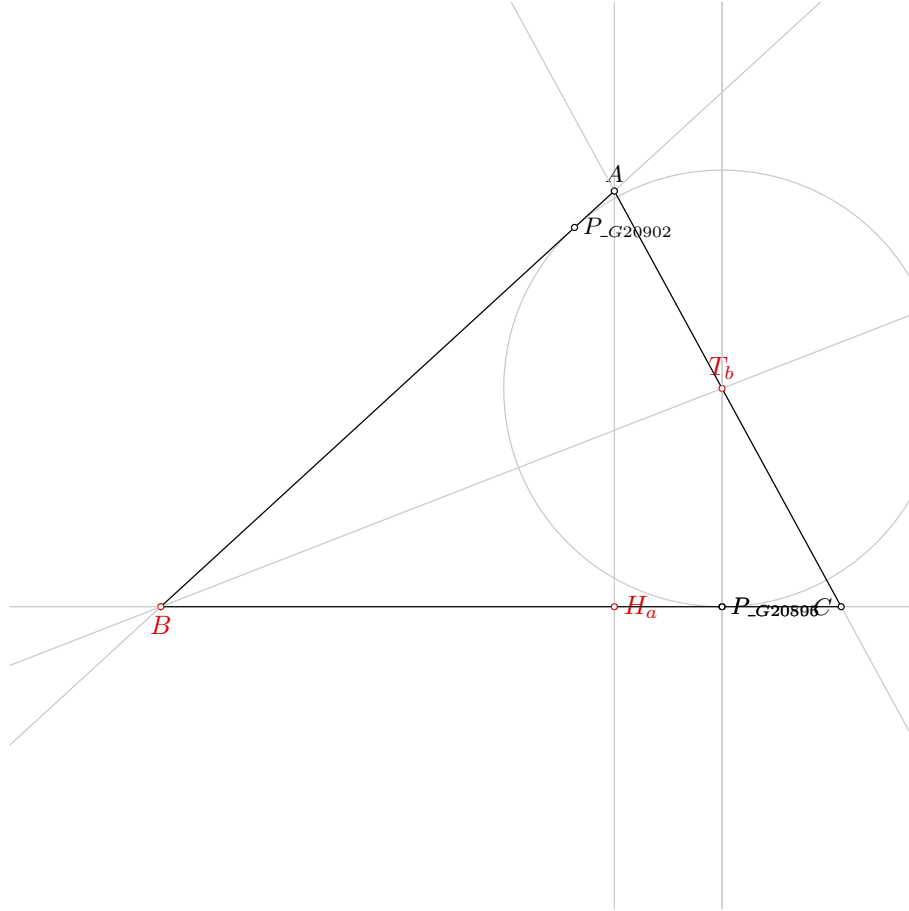


Figure 1: Illustration of the problem 0173

*% Non-degenerate conditions: lines a and b are not parallel; lines c and h\_{a} are not parallel;  
point B is outside the circle k(T\_{b},foot[T\_{b},c]); point T\_{b} is not incident to the line a  
% Determination conditions: lines a and b are not the same; points T\_{b} and A are not the same;  
lines c and h\_{a} are not the same; points B and H\_{a} are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.032 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a = H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.161 seconds.

**NDG conditions** Points  $B$ ,  $C$  and  $T_b$  are not collinear

Points  $P_{G28214}$  and  $B$  are not identical

Points  $P_{G28220}$ ,  $C$  and  $H_a$  are not collinear

Line through points  $A$  and  $T_b$  is not parallel with line through points  $B$  and  $H_a$

Point  $B$  is not on circle with center  $H_a$  and point from it  $C$

#### 4.1.3 Proving $T_b = T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B = B$

NDG conditions are:

$S_{BH_aT_{h_a}^0} \neq S_{P_{G21968}H_aT_{h_a}^0}$  i.e., lines  $BP_{G21968}$  and  $H_aT_{h_a}^0$  are not parallel (construction based assumption)

$S_{BT_bA} \neq S_{H_aT_bA}$  i.e., lines  $BH_a$  and  $T_bA$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^1} \neq S_{CAF_{h_a}^1}$  i.e., lines  $BC$  and  $AF_{h_a}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a = H_a$

Proving failed

#### 4.2.3 Proving $T_b = T_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_a = H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_b = \neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 67 terms.

**Time Complexity:** Time spent by the prover is 0.050 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_b = \neg T_b$

Proving failed



# Problem 174

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 174: Given a point  $B$ , a point  $H_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $H_a$  are not the same;
2. Using the point  $B$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $T_c$  are not the same;
3. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
4. Using the line  $h_a$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $h_a$  and  $c$  are not parallel % DET: lines  $h_a$  and  $c$  are not the same;
5. Using the point  $A$ , the point  $B$ , the point  $T_c$  and the line  $c$ , construct a point  $T'_c$  (rule W19); % NDG: points  $A$  and  $B$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $\text{midpoint}([A, T_c])$  are not the same;
6. Using the point  $T_c$  and the point  $T'_c$ , construct a circle  $k_{over}(T_c, T'_c)$  (rule W09); % NDG: points  $T_c$  and  $T'_c$  are not the same;
7. Using the circle  $k_{over}(T_c, T'_c)$  and the line  $a$ , construct a point  $C_{wb}$  and a point  $C$  (rule W04); % NDG: line  $a$  and circle  $k_{over}(T_c, T'_c)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k_{over}(T_c, T'_c)$  intersect; points  $T_c$  and  $T'_c$  are not the same; points  $A$  and  $B$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $\text{midpoint}([A, T_c])$  are not the same; lines  $h_a$  and  $c$  are not parallel.

Determination conditions: lines  $h_a$  and  $c$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $H_a$  are not the same.

Rules used: [W02, W03, W04, W09, W10b, W19]

Lemmas used: [D25, D5, D58, D8, GD01, L72, L79]

Solving time: 2255.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point H_{a} 80 40
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_b B
cmark_r H_{a}
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% DET: points B and T_{c} are not the same
% Constructing a line c which passes through point B and point T_{c}
line c B T_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: lines h_{a} and c are not parallel% DET: lines h_{a} and c are not the same
% Constructing a point A which belongs to line h_{a} and line c
intersec A h_{a} c
cmark_t A

% NDG: points A and B are not the same; points B and T_{c} are not the same; points B and midpoint
% ([A,T_{c}]) are not the same
```

```

% Constructing a point  $P_{\setminus\_G29308}$  which is an image of the point  $B$  in a rotation around the point
 $T_{\setminus\_c}$  for the angle 90
rotate  $P_{\setminus\_G29308}$   $T_{\setminus\_c}$  90 B
cmark_r  $P_{\setminus\_G29308}$ 
color 200 200 200
drawarc_p  $T_{\setminus\_c}$  B 90
color 0 0 0

% Constructing a line  $L_{\setminus\_G29311}$  which passes through point  $T_{\setminus\_c}$  and point  $P_{\setminus\_G29308}$ 
line  $L_{\setminus\_G29311}$   $T_{\setminus\_c}$   $P_{\setminus\_G29308}$ 

color 200 200 200
drawline  $L_{\setminus\_G29311}$ 
color 0 0 0

% Constructing midpoint  $P_{\setminus\_G29314}$  of the segment  $BP_{\setminus\_G29308}$ 
midpoint  $P_{\setminus\_G29314}$  B  $P_{\setminus\_G29308}$ 
cmark_r  $P_{\setminus\_G29314}$ 

% Constructing a line  $L_{\setminus\_G29317}$  which passes through point  $A$  and point  $P_{\setminus\_G29314}$ 
line  $L_{\setminus\_G29317}$  A  $P_{\setminus\_G29314}$ 

color 200 200 200
drawline  $L_{\setminus\_G29317}$ 
color 0 0 0

% Constructing a line  $L_{\setminus\_G29320}$  which passes through point  $B$  and point  $P_{\setminus\_G29314}$ 
line  $L_{\setminus\_G29320}$  B  $P_{\setminus\_G29314}$ 

color 200 200 200
drawline  $L_{\setminus\_G29320}$ 
color 0 0 0

% Constructing a point  $P_{\setminus\_G29323}$  which belongs to line  $L_{\setminus\_G29311}$  and line  $L_{\setminus\_G29317}$ 
intersec  $P_{\setminus\_G29323}$   $L_{\setminus\_G29311}$   $L_{\setminus\_G29317}$ 
cmark_r  $P_{\setminus\_G29323}$ 

% Constructing a point  $P_{\setminus\_G29326}$  which belongs to line  $L_{\setminus\_G29311}$  and line  $L_{\setminus\_G29320}$ 
intersec  $P_{\setminus\_G29326}$   $L_{\setminus\_G29311}$   $L_{\setminus\_G29320}$ 
cmark_r  $P_{\setminus\_G29326}$ 

% Constructing a line  $L_{\setminus\_G29329}$  which passes through point  $A$  and point  $P_{\setminus\_G29326}$ 
line  $L_{\setminus\_G29329}$  A  $P_{\setminus\_G29326}$ 

color 200 200 200
drawline  $L_{\setminus\_G29329}$ 
color 0 0 0

% Constructing a line  $L_{\setminus\_G29332}$  which passes through point  $B$  and point  $P_{\setminus\_G29323}$ 
line  $L_{\setminus\_G29332}$  B  $P_{\setminus\_G29323}$ 

color 200 200 200

```

```

drawline L_{\_G29332}
color 0 0 0

% Constructing a point P_{\_G29335} which belongs to line L_{\_G29329} and line L_{\_G29332}
intersec P_{\_G29335} L_{\_G29329} L_{\_G29332}
cmark_r P_{\_G29335}

% Constructing a line L_{\_G29338} which passes through point P_{\_G29314} and point P_{\_G29335}
line L_{\_G29338} P_{\_G29314} P_{\_G29335}

color 200 200 200
drawline L_{\_G29338}
color 0 0 0

% Constructing a point T'_{c} which belongs to line L_{\_G29338} and line c
intersec T'_{c} L_{\_G29338} c
cmark_r T'_{c}

% NDG: points T_{c} and T'_{c} are not the same
% Constructing midpoint P_{\_G30169} of the segment T_{c}T'_{c}
midpoint P_{\_G30169} T_{c} T'_{c}
cmark_r P_{\_G30169}

% Constructing a circle k_{over}(T_{c},T'_{c}) whose center is at point P_{\_G30169} and which passes
    through point T_{c}
circle k_{over}(T_{c},T'_{c}) P_{\_G30169} T_{c}

color 200 200 200
drawcircle k_{over}(T_{c},T'_{c})
color 0 0 0

% NDG: line a and circle k_{over}(T_{c},T'_{c}) intersect
% Constructing points C_{wb} and C which are in intersection of k_{over}(T_{c},T'_{c}) and a
intersec2 C_{wb} C k_{over}(T_{c},T'_{c}) a
cmark_r C_{wb}
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k_{over}(T_{c},T'_{c}) intersect; points T_{c} and T'_{c}
    are not the same; points A and B are not the same; points B and T_{c} are not the same;
    points B and midpoint([A,T_{c}]) are not the same; lines h_{a} and c are not parallel
% Determination conditions: lines h_{a} and c are not the same; points B and T_{c} are not the same
    ; points B and H_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

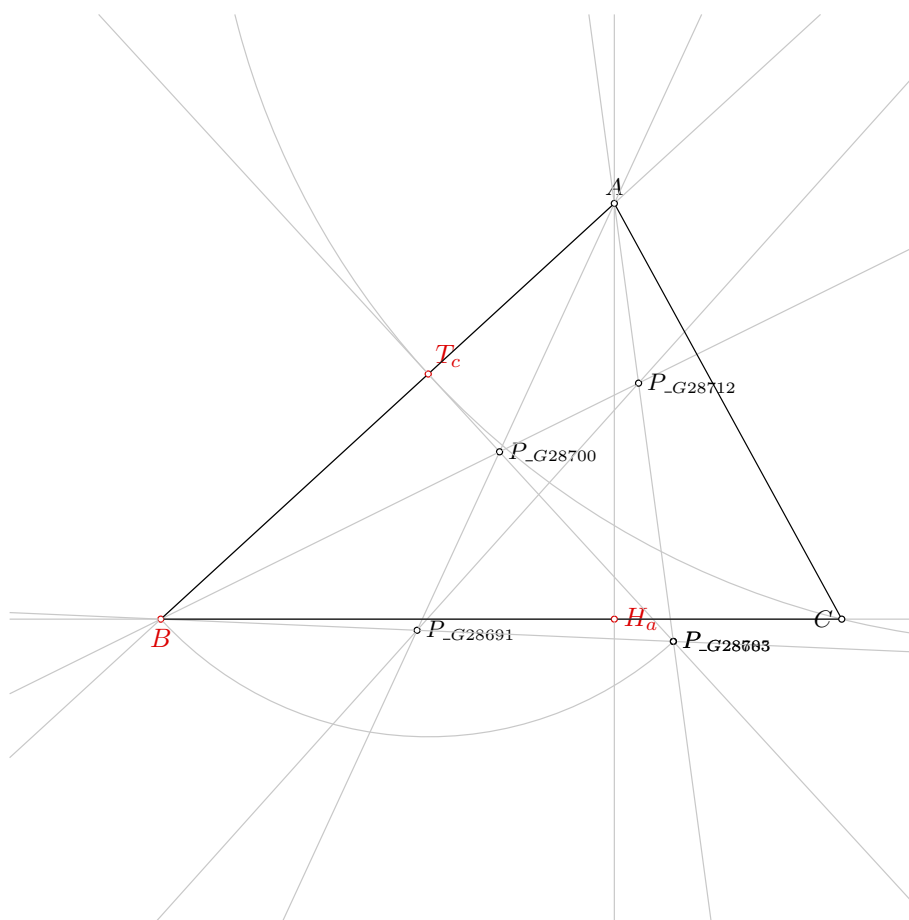


Figure 1: Illustration of the problem 0174

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a=_H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 5 terms.

**Time Complexity:** Time spent by the prover is 0.138 seconds.

**NDG conditions** Points  $B$  and  $C$  are not identical

Point  $T_c$  is not the midpoint of segment with endpoints  $A$  and  $B$

Points  $B$  and  $C$  are not identical

Point  $B$  is not on circle with center  $H_a$  and point from it  $C$

#### 4.1.3 Proving $T_c=_T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_aBT_c} \neq S_{T_{h_a}^0BT_c}$  i.e., lines  $H_aT_{h_a}^0$  and  $BT_c$  are not parallel (construction based assumption)

$S_{T_cAP_{G30513}} \neq S_{P_{G30507}AP_{G30513}}$  i.e., lines  $T_cP_{G30507}$  and  $AP_{G30513}$  are not parallel (construction based assumption)

$S_{T_cBP_{G30513}} \neq S_{P_{G30507}BP_{G30513}}$  i.e., lines  $T_cP_{G30507}$  and  $BP_{G30513}$  are not parallel (construction based assumption)

$S_{ABP_{G30522}} \neq S_{P_{G30525}BP_{G30522}}$  i.e., lines  $AP_{G30525}$  and  $BP_{G30522}$  are not parallel (construction based assumption)

$S_{P_{G30513}BT_c} \neq S_{P_{G30534}BT_c}$  i.e., lines  $P_{G30513}P_{G30534}$  and  $BT_c$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^1} \neq S_{CAF_{h_a}^1}$  i.e., lines  $BC$  and  $AF_{h_a}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a=_H_a$

Proving failed

#### 4.2.3 Proving $T_c=_T_c$

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $B=B$**

Proving failed

#### **4.3.2 Proving $H_a=_H H_a$**

Proving failed

#### **4.3.3 Proving $T_c=_T T_c$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $B=B$**

Proving failed

#### **4.4.2 Proving $H_a=_H H_a$**

Proving failed

#### **4.4.3 Proving $T_c=_T T_c$**

Proving failed

# Problem 175

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 175: Given a point  $B$ , a point  $H_a$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $H_a$  are not the same;
2. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
3. Using the point  $I$  and the line  $a$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $a$ ;
4. Using the circle  $k(I, P_a)$ , the point  $B$ , the point  $I$  and the line  $a$ , construct a line  $c$  (rule W13); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
5. Using the line  $c$  and the line  $h_a$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $h_a$  are not parallel % DET: lines  $c$  and  $h_a$  are not the same;
6. Using the circle  $k(I, P_a)$ , the point  $A$ , the point  $I$  and the line  $c$ , construct a line  $b$  (rule W13); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
7. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(I, P_a)$ ; lines  $c$  and  $h_a$  are not parallel; point  $B$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $a$ .

Determination conditions: lines  $b$  and  $a$  are not the same; lines  $c$  and  $h_a$  are not the same; points  $B$  and  $H_a$  are not the same.

Rules used: [W02,W03,W10b,W11,W13]

Lemmas used: [D27,D5,D8,GD01,L59,L60,L61]

Solving time: 12.5 seconds.



## 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point H_{a} 80 40
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_b B
cmark_r H_{a}
cmark_b I
color 0 0 0
fontsize 8

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: point I is not incident to the line a
% Constructing a point P_{\_G24976} which is a foot of the point I on the line a
foot P_{\_G24976} I a
cmark_r P_{\_G24976}
color 200 200 200
drawline I P_{\_G24976}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
24976}
circle k(I,P_{a}) I P_{\_G24976}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point B is outside the circle k(I,P_{a})
```

```

% Constructing a point  $P_{\{G25183\}}$  which is a foot of the point  $I$  on the line  $a$ 
foot  $P_{\{G25183\}}$  I a
cmark_r  $P_{\{G25183\}}$ 
color 200 200 200
drawline I  $P_{\{G25183\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G25186\}}$  which passes through point  $I$  and point  $B$ 
line  $L_{\{G25186\}}$  I B

color 200 200 200
drawline  $L_{\{G25186\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G25189\}}$  which is an image of the point  $P_{\{G25183\}}$  in the symmetry to
point/line  $L_{\{G25186\}}$ 
sim  $P_{\{G25189\}}$   $L_{\{G25186\}}$   $P_{\{G25183\}}$ 
cmark_r  $P_{\{G25189\}}$ 

% Constructing a line  $c$  which passes through point  $B$  and point  $P_{\{G25189\}}$ 
line c B  $P_{\{G25189\}}$ 

color 200 200 200
drawline c
color 0 0 0

% NDG: lines  $c$  and  $h_{\{a\}}$  are not parallel% DET: lines  $c$  and  $h_{\{a\}}$  are not the same
% Constructing a point  $A$  which belongs to line  $c$  and line  $h_{\{a\}}$ 
intersec A c  $h_{\{a\}}$ 
cmark_t A

% NDG: point  $A$  is outside the circle  $k(I, P_{\{a\}})$ 
% Constructing a point  $P_{\{G25575\}}$  which is a foot of the point  $I$  on the line  $c$ 
foot  $P_{\{G25575\}}$  I c
cmark_r  $P_{\{G25575\}}$ 
color 200 200 200
drawline I  $P_{\{G25575\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G25578\}}$  which passes through point  $I$  and point  $A$ 
line  $L_{\{G25578\}}$  I A

color 200 200 200
drawline  $L_{\{G25578\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G25581\}}$  which is an image of the point  $P_{\{G25575\}}$  in the symmetry to
point/line  $L_{\{G25578\}}$ 
sim  $P_{\{G25581\}}$   $L_{\{G25578\}}$   $P_{\{G25575\}}$ 
cmark_r  $P_{\{G25581\}}$ 

```

```

% Constructing a line b which passes through point A and point P_{\_G25581}
line b A P_{\_G25581}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and a are not parallel; point A is outside the circle k(I,P_{a}
% ); lines c and h_{a} are not parallel; point B is outside the circle k(I,P_{a}); point I is
% not incident to the line a
% Determination conditions: lines b and a are not the same; lines c and h_{a} are not the same;
% points B and H_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a = \neg H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 10 terms.

**Time Complexity:** Time spent by the prover is 0.213 seconds.

**NDG conditions** Points  $B$  and  $I$  are not identical

Points  $B$  and  $P_{G21286}$  are not identical

Points  $P_{G21395}$ ,  $P_{G21286}$  and  $H_a$  are not collinear

Points  $P_{G21395}$  and  $B$  are not identical

Points  $A$  and  $I$  are not identical

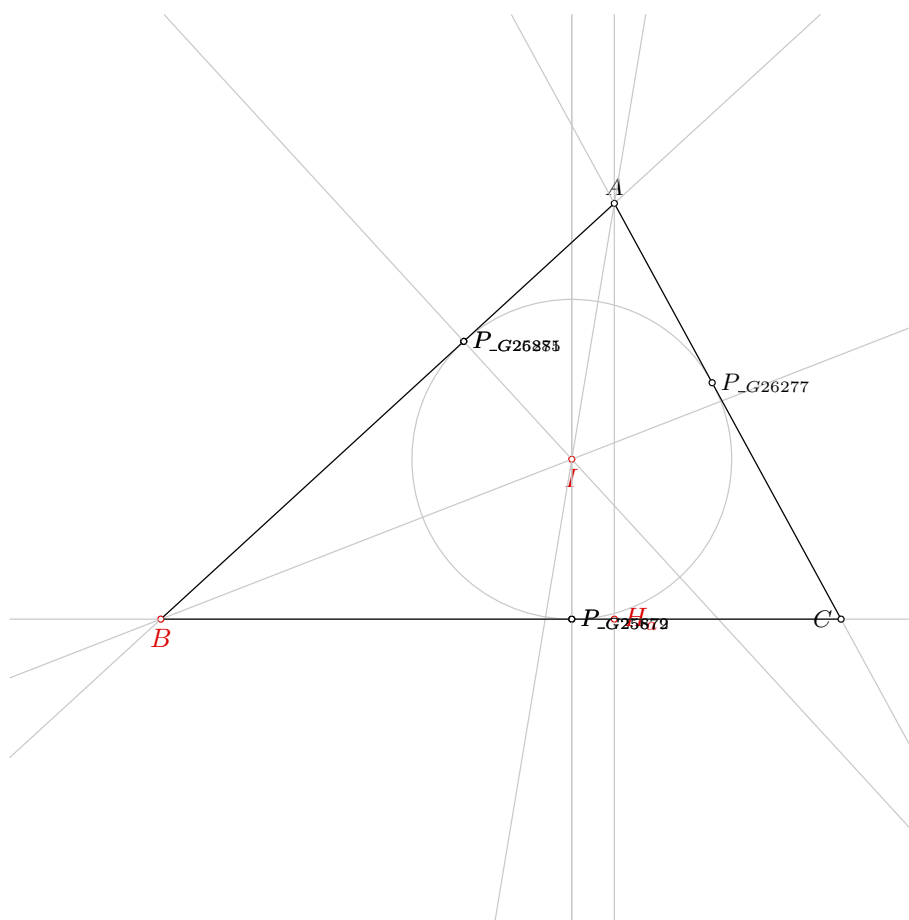


Figure 1: Illustration of the problem 0175

Points  $A$  and  $C$  are not identical

Line through points  $A$  and  $P_{G21557}$  is not parallel with line through points  $B$  and  $H_a$

Point  $B$  is not on circle with center  $H_a$  and point from it  $C$

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{BH_aT_{h_a}^0} \neq S_{P_{G27005}H_aT_{h_a}^0}$  i.e., lines  $BP_{G27005}$  and  $H_aT_{h_a}^0$  are not parallel (construction based assumption)

$S_{ABH_a} \neq S_{P_{G27397}BH_a}$  i.e., lines  $AP_{G27397}$  and  $BH_a$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^1} \neq S_{CAF_{h_a}^1}$  i.e., lines  $BC$  and  $AF_{h_a}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a=_H H_a$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_a=_H H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I=_I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 105 terms.

**Time Complexity:** Time spent by the prover is 0.070 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $I = \neg I$

Proving failed

# Problem 176

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 176: Given a point  $B$ , a point  $H_b$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_b$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H_b$  are not the same;
2. Using the point  $B$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $H_c$  are not the same;
3. Using the point  $H_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
4. Using the line  $b$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $c$  are not parallel % DET: lines  $b$  and  $c$  are not the same;
5. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
6. Using the line  $b$  and the line  $h_c$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $h_c$  are not parallel % DET: lines  $b$  and  $h_c$  are not the same.

Non-degenerate conditions: lines  $b$  and  $h_c$  are not parallel; lines  $b$  and  $c$  are not parallel.

Determination conditions: lines  $b$  and  $h_c$  are not the same; lines  $b$  and  $c$  are not the same; points  $B$  and  $H_c$  are not the same; points  $B$  and  $H_b$  are not the same.

Rules used: [W02,W03,W10a,W10b]

Lemmas used: [D10,D6,D7,D9,GD01]

Solving time: 1.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point H_{b} 89.36 77.83
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_b B
cmark_l H_{b}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points B and H_{b} are not the same
% Constructing a line h_{b} which passes through point B and point H_{b}
line h_{b} B H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
perp b H_{b} h_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines b and c are not parallel% DET: lines b and c are not the same
% Constructing a point A which belongs to line b and line c
intersec A b c
cmark_t A

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c
```



```

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines b and h_{c} are not parallel% DET: lines b and h_{c} are not the same
% Constructing a point C which belongs to line b and line h_{c}
intersec C b h_{c}
cmark_1 C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and h_{c} are not parallel; lines b and c are not parallel
% Determination conditions: lines b and h_{c} are not the same; lines b and c are not the same;
  points B and H_{c} are not the same; points B and H_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b=H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.103 seconds.

**NDG conditions** Points  $B$  and  $H_c$  are not identical

Points  $B$  and  $H_c$  are not identical

Points  $A$ ,  $B$  and  $C$  are not collinear

#### 4.1.3 Proving $H_c=H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 11 terms.

**Time Complexity:** Time spent by the prover is 0.194 seconds.

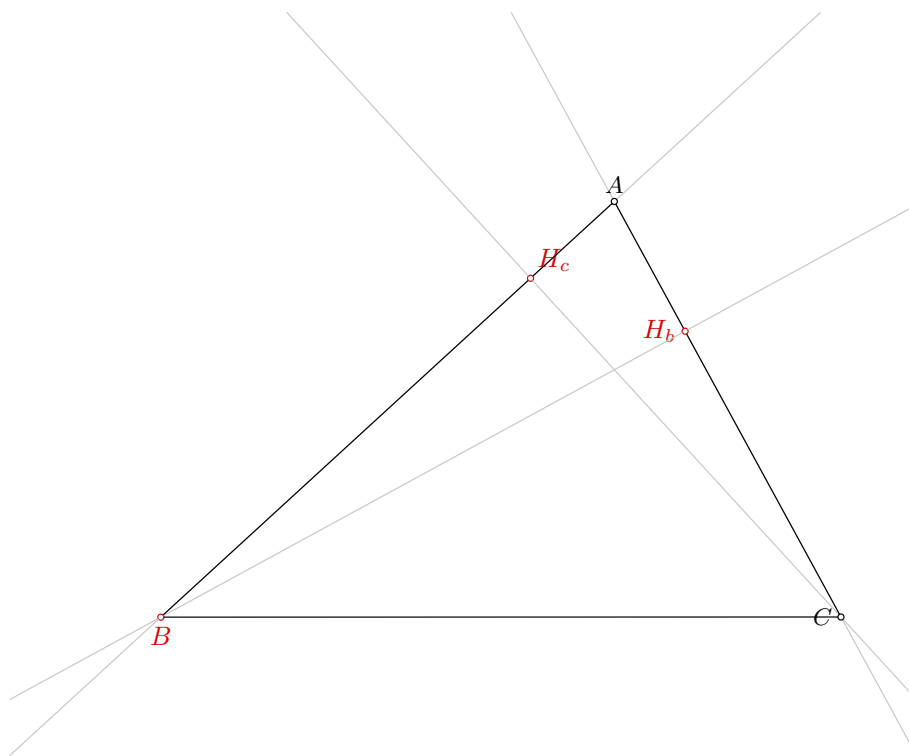


Figure 1: Illustration of the problem 0176

**NDG conditions** Points  $B$  and  $H_c$  are not identical

Points  $B$  and  $H_c$  are not identical

Points  $A$  and  $B$  are not identical

Points  $H_b$ ,  $A$  and  $B$  are not collinear

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_bBH_c} \neq S_{T_b^0BH_c}$  i.e., lines  $H_bT_b^0$  and  $BH_c$  are not parallel (construction based assumption)

$S_{H_bH_cT_{h_c}^1} \neq S_{T_b^0H_cT_{h_c}^1}$  i.e., lines  $H_bT_b^0$  and  $H_cT_{h_c}^1$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^2} \neq S_{CBF_{h_b}^2}$  i.e., lines  $AC$  and  $BF_{h_b}^2$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^3} \neq S_{BCF_{h_c}^3}$  i.e., lines  $AB$  and  $CF_{h_c}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_b=_H H_b$

Proving failed

### 4.2.3 Proving $H_c=_H H_c$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_b=_H H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $H_c=_H H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 23 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_b = H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $H_c = H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 15 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

# Problem 177

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 177: Given a point  $H_b$ , a point  $H$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $H_b$  and  $H$  are not the same;
2. Choose freely a point  $B$  on the line  $h_b$  (rule WOnline1) ;
3. Using the point  $H_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
4. Choose freely a point  $A$  on the line  $b$  (rule WOnline2);
5. Using the point  $A$  and the point  $B$ , construct a point  $M_c$  (rule W01); ;
6. Using the point  $A$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H$  are not the same;
7. Using the point  $A$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $A$  and  $M_c$  are not the same;
8. Using the circle  $k(M_c, A)$ , the line  $h_a$ , the point  $M_c$  and the point  $A$ , construct a point  $H_a$  (rule W05); % NDG: line  $h_a$  and circle  $k(M_c, A)$  intersect % DET: points  $A$  and  $H_a$  must be different;
9. Using the point  $H_a$  and the point  $B$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $B$  are not the same;
10. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; line  $h_a$  and circle  $k(M_c, A)$  intersect; points  $A$  and  $M_c$  are not the same.

Determination conditions: lines  $b$  and  $a$  are not the same; points  $H_a$  and  $B$  are not the same; points  $A$  and  $H_a$  must be different; points  $A$  and  $H$  are not the same; points  $H_b$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D20,D3,D5,D6,D8,D9,GD01,GD02,L42]

Solving time: 9.7 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{b} 89.36 77.83
```

```
point H 80 72.73
```

```
point B 20 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_l H_{b}
```

```
cmark_rt H
```

```
cmark_b B
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{b} and H are not the same
```

```
% Constructing a line h_{b} which passes through point H_{b} and point H
```

```
line h_{b} H_{b} H
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```
% Choosing randomly a point B on the line H_{b}H
```

```
online B H_{b} H
```

```
cmark_b B
```

```
color 200 200 200
```

```
drawline H_{b} H
```

```
color 0 0 0
```

```
% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
```

```
perp b H_{b} h_{b}
```

```
color 200 200 200
```

```
drawline b
```

```
color 0 0 0
```

```

% Generating random value V[_G31418]
random V[_G31418]

% Calculating value V[_G31439] using formula V[_G31418]*20
expression V[_G31439] { V[_G31418]*20 }

% Constructing a point A which is a point for which holds  $H_{\{b\}}A = V[_G31439]$  and angle  $BH_{\{b\}}A = 90$ 
turtle A B H_{b} 90 V[_G31439]
cmark_t A

% Constructing a point  $M_{\{c\}}$  such that  $AM_{\{c\}}/AB=0.5$ 
towards M_{c} A B 0.5
cmark_lt M_{c}
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points A and H are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points A and  $M_{\{c\}}$  are not the same
% Constructing a circle  $k(M_{\{c\}},A)$  whose center is at point  $M_{\{c\}}$  and which passes through point A
circle k(M_{c},A) M_{c} A

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line  $h_{\{a\}}$  and circle  $k(M_{\{c\}},A)$  intersect% DET: points A and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G31911\}}$  which is a foot of the point  $M_{\{c\}}$  on the line  $h_{\{a\}}$ 
foot P_{\_G31911} M_{c} h_{a}
cmark_r P_{\_G31911}
color 200 200 200
drawline M_{c} P_{\_G31911}
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point A in the symmetry to point/line  $P_{\{\backslash\_G31911\}}$ 
sim H_{a} P_{\_G31911} A

```

```

cmark_r H_{a}

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and a are not parallel; line h_{a} and circle k(M_{c},A)
% intersect; points A and M_{c} are not the same
% Determination conditions: lines b and a are not the same; points H_{a} and B are not the same;
% points A and H_{a} must be different; points A and H are not the same; points H_{b} and H are
% not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_b = \_H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.151 seconds.

**NDG conditions** Points  $H_b$  and  $A$  are not identical

Points  $H_b$  and  $H$  are not identical

Line through points  $H_a$  and  $B$  is not perpendicular to line through points  $B$  and  $H_b$

Line through points  $H_b$  and  $H$  is not parallel with line through points  $A$  and  $C$

#### 4.1.2 Proving $H = \_H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 31 terms.



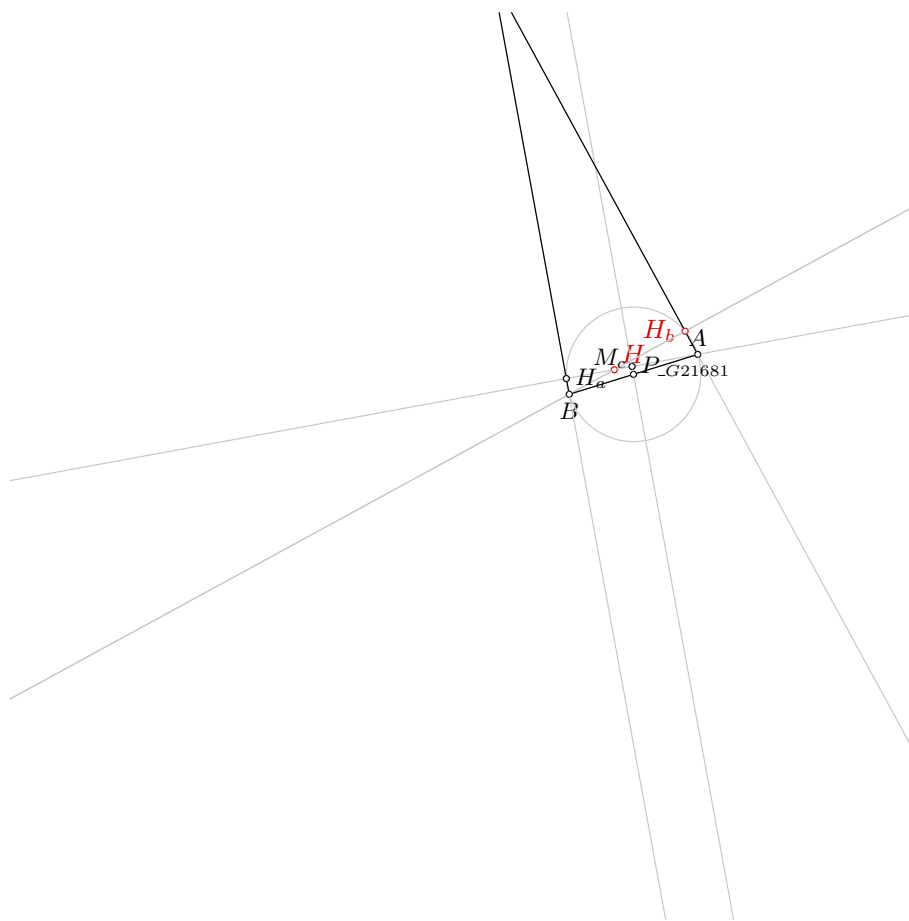


Figure 1: Illustration of the problem 0177

**Time Complexity:** Time spent by the prover is 0.471 seconds.

**NDG conditions** Points  $H_b$  and  $A$  are not identical

Points  $H_b$  and  $H$  are not identical

Line through points  $H_a$  and  $B$  is not perpendicular to line through points  $B$  and  $H_b$

Line through points  $H_b$  and  $H$  is not parallel with line through points  $A$  and  $C$

Points  $B$  and  $H$  are not identical

#### 4.1.3 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.007 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_b=_H H_b$

Proving failed

#### 4.2.2 Proving $H=_H H$

Proving failed

#### 4.2.3 Proving $B=B$

NDG conditions are:

$S_{H_b H_a B} \neq S_{T_b^1 H_a B}$  i.e., lines  $H_b T_b^1$  and  $H_a B$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-H_b}^2} \neq S_{CBF_{-H_b}^2}$  i.e., lines  $AC$  and  $BF_{-H_b}^2$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-H_b}^2} \neq S_{F_{-H_a}^3 BF_{-H_b}^2}$  i.e., lines  $AF_{-H_a}^3$  and  $BF_{-H_b}^2$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_b=_H H_b$

Proving failed

#### 4.3.2 Proving $H=_H H$

Proving failed

#### 4.3.3 Proving $B=B$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $H_b=_H H_b$

Proving failed

#### 4.4.2 Proving $H=_H$

Proving failed

#### 4.4.3 Proving $B=B$

Proving failed

# Problem 178

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 178: Given a point  $B$ , a point  $H_b$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_b$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H_b$  are not the same;
2. Using the point  $B$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $T_a$  are not the same;
3. Using the point  $H_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
4. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same;
5. Using the point  $B$ , the point  $C$ , the point  $T_a$  and the line  $a$ , construct a point  $T'_a$  (rule W19); % NDG: points  $B$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $midpoint([B, T_a])$  are not the same;
6. Using the point  $T_a$  and the point  $T'_a$ , construct a circle  $k_{over}(T_a, T'_a)$  (rule W09); % NDG: points  $T_a$  and  $T'_a$  are not the same;
7. Using the circle  $k_{over}(T_a, T'_a)$  and the line  $b$ , construct a point  $A_{wc}$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k_{over}(T_a, T'_a)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k_{over}(T_a, T'_a)$  intersect; points  $T_a$  and  $T'_a$  are not the same; points  $B$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $midpoint([B, T_a])$  are not the same; lines  $b$  and  $a$  are not parallel.

Determination conditions: lines  $b$  and  $a$  are not the same; points  $B$  and  $T_a$  are not the same; points  $B$  and  $H_b$  are not the same.

Rules used: [W02, W03, W04, W09, W10a, W19]

Lemmas used: [D23, D54, D6, D9, GD01, L71, L77]

Solving time: 2593.8 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point H_{b} 89.36 77.83
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_b B
cmark_l H_{b}
cmark_rb T_{a}
color 0 0 0
fontsize 8

% DET: points B and H_{b} are not the same
% Constructing a line h_{b} which passes through point B and point H_{b}
line h_{b} B H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% DET: points B and T_{a} are not the same
% Constructing a line a which passes through point B and point T_{a}
line a B T_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
perp b H_{b} h_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

% NDG: points B and C are not the same; points C and T_{a} are not the same; points C and midpoint
% ([B,T_{a}]) are not the same
```

```

% Constructing a point  $P_{\backslash\_G30067}$  which is an image of the point  $C$  in a rotation around the point
 $T_{\{a\}}$  for the angle 90
rotate  $P_{\backslash\_G30067}$   $T_{\{a\}}$  90 C
cmark_r  $P_{\backslash\_G30067}$ 
color 200 200 200
drawarc_p  $T_{\{a\}}$  C 90
color 0 0 0

% Constructing a line  $L_{\backslash\_G30070}$  which passes through point  $T_{\{a\}}$  and point  $P_{\backslash\_G30067}$ 
line  $L_{\backslash\_G30070}$   $T_{\{a\}}$   $P_{\backslash\_G30067}$ 

color 200 200 200
drawline  $L_{\backslash\_G30070}$ 
color 0 0 0

% Constructing midpoint  $P_{\backslash\_G30073}$  of the segment  $CP_{\backslash\_G30067}$ 
midpoint  $P_{\backslash\_G30073}$  C  $P_{\backslash\_G30067}$ 
cmark_r  $P_{\backslash\_G30073}$ 

% Constructing a line  $L_{\backslash\_G30076}$  which passes through point  $B$  and point  $P_{\backslash\_G30073}$ 
line  $L_{\backslash\_G30076}$  B  $P_{\backslash\_G30073}$ 

color 200 200 200
drawline  $L_{\backslash\_G30076}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G30079}$  which passes through point  $C$  and point  $P_{\backslash\_G30073}$ 
line  $L_{\backslash\_G30079}$  C  $P_{\backslash\_G30073}$ 

color 200 200 200
drawline  $L_{\backslash\_G30079}$ 
color 0 0 0

% Constructing a point  $P_{\backslash\_G30082}$  which belongs to line  $L_{\backslash\_G30070}$  and line  $L_{\backslash\_G30076}$ 
intersec  $P_{\backslash\_G30082}$   $L_{\backslash\_G30070}$   $L_{\backslash\_G30076}$ 
cmark_r  $P_{\backslash\_G30082}$ 

% Constructing a point  $P_{\backslash\_G30085}$  which belongs to line  $L_{\backslash\_G30070}$  and line  $L_{\backslash\_G30079}$ 
intersec  $P_{\backslash\_G30085}$   $L_{\backslash\_G30070}$   $L_{\backslash\_G30079}$ 
cmark_r  $P_{\backslash\_G30085}$ 

% Constructing a line  $L_{\backslash\_G30088}$  which passes through point  $B$  and point  $P_{\backslash\_G30085}$ 
line  $L_{\backslash\_G30088}$  B  $P_{\backslash\_G30085}$ 

color 200 200 200
drawline  $L_{\backslash\_G30088}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G30091}$  which passes through point  $C$  and point  $P_{\backslash\_G30082}$ 
line  $L_{\backslash\_G30091}$  C  $P_{\backslash\_G30082}$ 

color 200 200 200

```

```

drawline L_{\_G30091}
color 0 0 0

% Constructing a point P_{\_G30094} which belongs to line L_{\_G30088} and line L_{\_G30091}
intersec P_{\_G30094} L_{\_G30088} L_{\_G30091}
cmark_r P_{\_G30094}

% Constructing a line L_{\_G30097} which passes through point P_{\_G30073} and point P_{\_G30094}
line L_{\_G30097} P_{\_G30073} P_{\_G30094}

color 200 200 200
drawline L_{\_G30097}
color 0 0 0

% Constructing a point T'_{a} which belongs to line L_{\_G30097} and line a
intersec T'_{a} L_{\_G30097} a
cmark_r T'_{a}

% NDG: points T_{a} and T'_{a} are not the same
% Constructing midpoint P_{\_G30928} of the segment T_{a}T'_{a}
midpoint P_{\_G30928} T_{a} T'_{a}
cmark_r P_{\_G30928}

% Constructing a circle k_{over}(T_{a},T'_{a}) whose center is at point P_{\_G30928} and which passes
    through point T_{a}
circle k_{over}(T_{a},T'_{a}) P_{\_G30928} T_{a}

color 200 200 200
drawcircle k_{over}(T_{a},T'_{a})
color 0 0 0

% NDG: line b and circle k_{over}(T_{a},T'_{a}) intersect
% Constructing points A_{wc} and A which are in intersection of k_{over}(T_{a},T'_{a}) and b
intersec2 A_{wc} A k_{over}(T_{a},T'_{a}) b
cmark_r A_{wc}
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k_{over}(T_{a},T'_{a}) intersect; points T_{a} and T'_{a}
    are not the same; points B and C are not the same; points C and T_{a} are not the same;
    points C and midpoint([B,T_{a}]) are not the same; lines b and a are not parallel
% Determination conditions: lines b and a are not the same; points B and T_{a} are not the same;
    points B and H_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

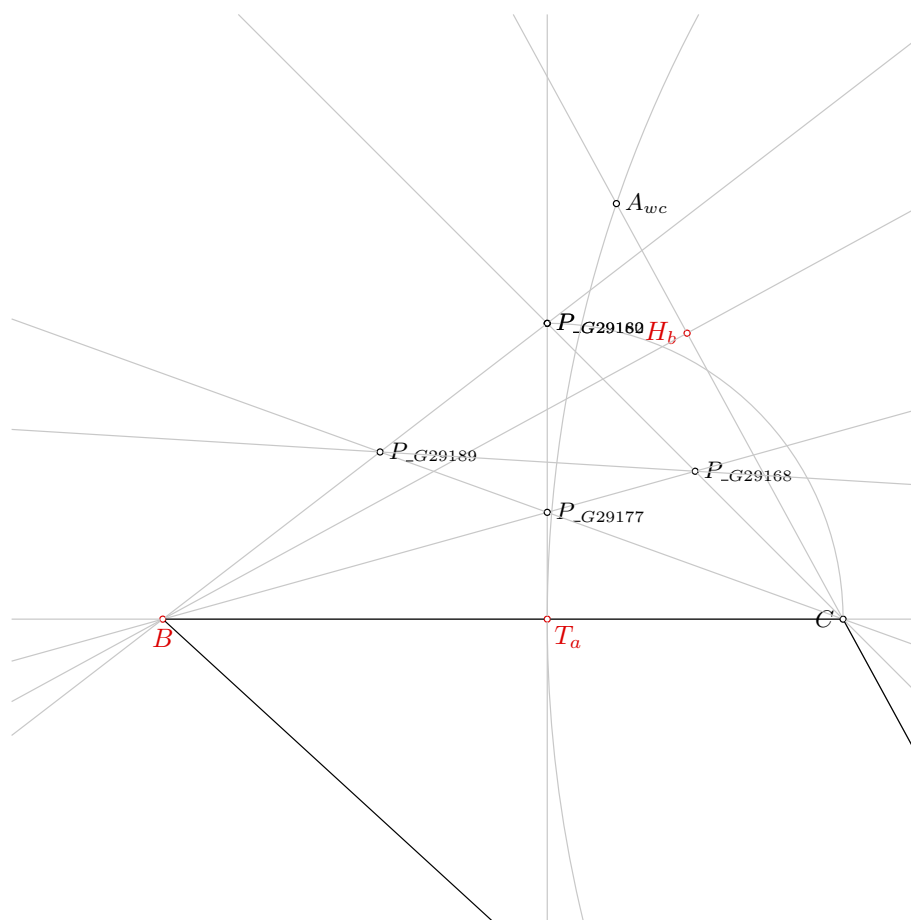


Figure 1: Illustration of the problem 0178



### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b=_H H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.138 seconds.

**NDG conditions** Points  $T_a$  and  $H_b$  are not identical

Point  $T_a$  is not the midpoint of segment with endpoints  $B$  and  $C$

Points  $T_a$ ,  $H_b$  and  $A$  are not collinear

Points  $A$ ,  $B$  and  $C$  are not collinear

#### 4.1.3 Proving $T_a=_T T_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_b B T_a} \neq S_{T_b^0 B T_a}$  i.e., lines  $H_b T_b^0$  and  $B T_a$  are not parallel (construction based assumption)

$S_{T_a B P_{G30993}} \neq S_{P_{G30987} B P_{G30993}}$  i.e., lines  $T_a P_{G30987}$  and  $B P_{G30993}$  are not parallel (construction based assumption)

$S_{T_a C P_{G30993}} \neq S_{P_{G30987} C P_{G30993}}$  i.e., lines  $T_a P_{G30987}$  and  $C P_{G30993}$  are not parallel (construction based assumption)

$S_{B C P_{G31002}} \neq S_{P_{G31005} C P_{G31002}}$  i.e., lines  $B P_{G31005}$  and  $C P_{G31002}$  are not parallel (construction based assumption)

$S_{P_{G30993} B T_a} \neq S_{P_{G31014} B T_a}$  i.e., lines  $P_{G30993} P_{G31014}$  and  $B T_a$  are not parallel (construction based assumption)

$S_{B A C} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{A B F_{H_b}^1} \neq S_{C B F_{H_b}^1}$  i.e., lines  $A C$  and  $B F_{H_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_b=_H H_b$

Proving failed

#### 4.2.3 Proving $T_a=_T T_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

Proving failed

#### 4.3.2 Proving $H_b=_H H_b$

Proving failed

#### 4.3.3 Proving $T_a=_T T_a$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B=B$

Proving failed

#### 4.4.2 Proving $H_b=_H H_b$

Proving failed

#### 4.4.3 Proving $T_a=_T T_a$

Proving failed

# Problem 179

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 179: Given a point  $H_b$ , a point  $T_b$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $T_b$  are not the same;
2. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
3. Choose freely a point  $B$  on the line  $h_b$  (rule WOnline2);
4. Using the point  $B$  and the point  $T_b$ , construct a line  $s_b$  (rule W02); % DET: points  $B$  and  $T_b$  are not the same;
5. Using the point  $H_b$ , the point  $B$ , the point  $T_b$ , the line  $s_b$  and the line  $h_b$ , construct a line  $BO$  (rule W17); % NDG: points  $B$  and  $T_b$  are not the same; points  $H_b$  and  $B$  are not the same % DET: points  $B$  and  $T_b$  are not the same;
6. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) ;
7. Using the point  $A$  and the point  $B$ , construct a point  $M_c$  (rule W01); ;
8. Using the point  $A$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $B$  are not the same;
9. Using the point  $M_c$  and the line  $c$ , construct a line  $m_c$  (rule W10b); ;
10. Using the line  $m_c$  and the line  $BO$ , construct a point  $O$  (rule W03); % NDG: lines  $m_c$  and  $BO$  are not parallel % DET: lines  $m_c$  and  $BO$  are not the same;
11. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;

12. Using the circle  $k(O, C)$ , the line  $b$ , the point  $O$  and the point  $A$ , construct a point  $C$  (rule W05); % NDG: line  $b$  and circle  $k(O, C)$  intersect % DET: points  $A$  and  $C$  must be different.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same; lines  $m_c$  and  $BO$  are not parallel; points  $B$  and  $T_b$  are not the same; points  $H_b$  and  $B$  are not the same.

Determination conditions: points  $A$  and  $C$  must be different; lines  $m_c$  and  $BO$  are not the same; points  $A$  and  $B$  are not the same; points  $B$  and  $T_b$  are not the same; points  $B$  and  $T_b$  are not the same; points  $H_b$  and  $T_b$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W10b,W17,WOnline1,WOnline2]

Lemmas used: [D13,D20,D24,D26,D6,D9,GD01,GL17,L1,L105,L11,L6]

Solving time: 20.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point H_{b} 89.36 77.83
point T_{b} 94.25 68.88
point B 20 40

color 220 0 0
fontsize 9

cmark_l H_{b}
cmark_t T_{b}
cmark_b B
color 0 0 0
fontsize 8

% DET: points H_{b} and T_{b} are not the same
% Constructing a line b which passes through point H_{b} and point T_{b}
line b H_{b} T_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0

% Generating random value V[_G27888]
random V[_G27888]
```

```

% Calculating value  $V_{G27909}$  using formula  $V_{G27888} \cdot 20$ 
expression  $V_{G27909}$  {  $V_{G27888} \cdot 20$  }

% Constructing a point B which is a point for which holds  $H_{\{b\}}B = V_{G27909}$  and angle  $T_{\{b\}}H_{\{b\}}B = 90$ 
turtle B  $T_{\{b\}}$   $H_{\{b\}}$  90  $V_{G27909}$ 
cmark_b B

% DET: points B and  $T_{\{b\}}$  are not the same
% Constructing a line  $s_{\{b\}}$  which passes through point B and point  $T_{\{b\}}$ 
line  $s_{\{b\}}$  B  $T_{\{b\}}$ 

color 200 200 200
drawline  $s_{\{b\}}$ 
color 0 0 0

% NDG: points B and  $T_{\{b\}}$  are not the same; points  $H_{\{b\}}$  and B are not the same% DET: points B and  $T_{\{b\}}$  are not the same
% Constructing an angle  $V_{G28140}$  which is equal to the angle  $H_{\{b\}}BT_{\{b\}}$ 
angle_o  $V_{G28140}$   $H_{\{b\}}$  B  $T_{\{b\}}$ 

% Calculating value  $angle_{G28219}$  using formula  $1/\text{pow}(2,0) \cdot V_{G28140} + 0/\text{pow}(2,0) \cdot 180$ 
expression  $angle_{G28219}$  {  $1/\text{pow}(2,0) \cdot V_{G28140} + 0/\text{pow}(2,0) \cdot 180$  }

% Constructing a point  $P_{\{G28216\}}$  which is an image of the point  $T_{\{b\}}$  in a rotation around the point B for the angle  $1/\text{pow}(2,0) \cdot V_{G28140} + 0/\text{pow}(2,0) \cdot 180$ 
rotate  $P_{\{G28216\}}$  B  $angle_{G28219}$   $T_{\{b\}}$ 
cmark_r  $P_{\{G28216\}}$ 
color 200 200 200
drawarc_p B  $T_{\{b\}}$   $angle_{G28219}$ 
color 0 0 0

% Constructing a line  $BO$  which passes through point B and point  $P_{\{G28216\}}$ 
line BO B  $P_{\{G28216\}}$ 

color 200 200 200
drawline BO
color 0 0 0

% Choosing randomly a point A on the line  $H_{\{b\}}T_{\{b\}}$ 
online A  $H_{\{b\}}$   $T_{\{b\}}$ 
cmark_t A
color 200 200 200
drawline  $H_{\{b\}}$   $T_{\{b\}}$ 
color 0 0 0

```

```

% Constructing a point  $M_{\{c\}}$  such that  $AM_{\{c\}}/AB=0.5$ 
towards  $M_{\{c\}}$  A B 0.5
cmark_lt  $M_{\{c\}}$ 
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points A and B are not the same
% Constructing a line c which passes through point A and point B
line c A B

color 200 200 200
drawline c
color 0 0 0

% Constructing a line  $m_{\{c\}}$  which is perpendicular to line c and which passes through point  $M_{\{c\}}$ 
perp  $m_{\{c\}}$   $M_{\{c\}}$  c

color 200 200 200
drawline  $m_{\{c\}}$ 
color 0 0 0

% NDG: lines  $m_{\{c\}}$  and BO are not parallel% DET: lines  $m_{\{c\}}$  and BO are not the same
% Constructing a point O which belongs to line  $m_{\{c\}}$  and line BO
intersec O  $m_{\{c\}}$  BO
cmark_t O

% NDG: points A and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point A
circle  $k(O,C)$  O A

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line b and circle  $k(O,C)$  intersect% DET: points A and C must be different
% Constructing a point  $P_{\{\backslash\_G28920\}}$  which is a foot of the point O on the line b
foot  $P_{\{\backslash\_G28920\}}$  O b
cmark_r  $P_{\{\backslash\_G28920\}}$ 
color 200 200 200
drawline O  $P_{\{\backslash\_G28920\}}$ 
color 0 0 0

```

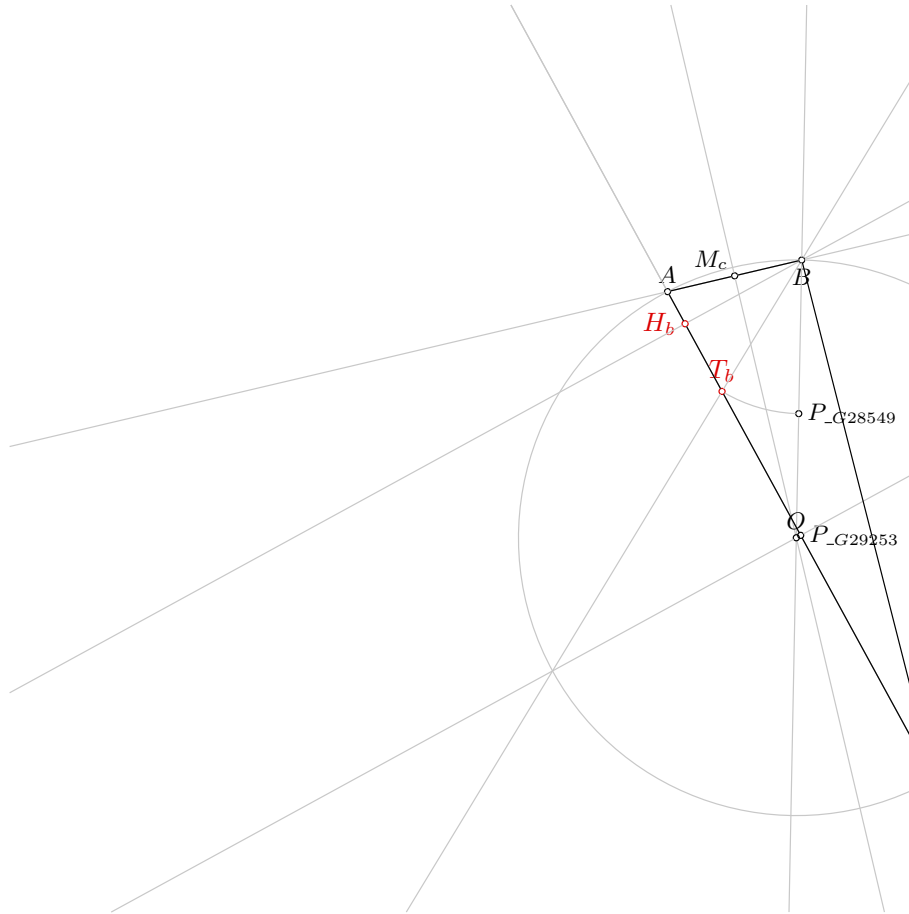


Figure 1: Illustration of the problem 0179

*% Constructing a point C which is an image of the point A in the symmetry to point/line P\_{\\_G*  
*28920}*

`sim C P_{\_G28920} A`  
`cmark_l C`

`drawsegment A B`  
`drawsegment A C`  
`drawsegment B C`

*% Non-degenerate conditions: line b and circle k(O,C) intersect; points A and O are not the same;*  
*lines m\_{c} and BO are not parallel; points B and T\_{b} are not the same; points H\_{b} and B*  
*are not the same*

*% Determination conditions: points A and C must be different; lines m\_{c} and BO are not the same;*  
*points A and B are not the same; points B and T\_{b} are not the same; points B and T\_{b} are*  
*not the same; points H\_{b} and T\_{b} are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_b = \neg H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 5 terms.

**Time Complexity:** Time spent by the prover is 0.157 seconds.

**NDG conditions** Points  $B$  and  $T_b$  are not identical

Line through points  $P_{G23417}$  and  $B$  is not perpendicular to line through points  $B$  and  $A$

Points  $A$  and  $B$  are not identical

Line through points  $A$  and  $B$  is not perpendicular to line through points  $B$  and  $C$

#### 4.1.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.1.3 Proving $B = B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.009 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_b = \neg H_b$

Proving failed

#### 4.2.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.2.3 Proving $B = B$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_b = \neg H_b$

Proving failed

#### 4.3.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.3.3 Proving $B = B$

Proving failed



## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $H_b = \neg H_b$

Proving failed

### 4.4.2 Proving $T_b = \neg T_b$

Proving failed

### 4.4.3 Proving $B = B$

Proving failed

# Problem 180

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 180: Given a point  $B$ , a point  $H_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_b$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H_b$  are not the same;
2. Using the point  $B$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $T_c$  are not the same;
3. Using the point  $H_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
4. Using the line  $b$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $c$  are not parallel % DET: lines  $b$  and  $c$  are not the same;
5. Using the point  $A$ , the point  $B$ , the point  $T_c$  and the line  $c$ , construct a point  $T'_c$  (rule W19); % NDG: points  $A$  and  $B$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $\text{midpoint}([A, T_c])$  are not the same;
6. Using the point  $T_c$  and the point  $T'_c$ , construct a circle  $k_{over}(T_c, T'_c)$  (rule W09); % NDG: points  $T_c$  and  $T'_c$  are not the same;
7. Using the circle  $k_{over}(T_c, T'_c)$  and the line  $b$ , construct a point  $C_{wa}$  and a point  $C$  (rule W04); % NDG: line  $b$  and circle  $k_{over}(T_c, T'_c)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k_{over}(T_c, T'_c)$  intersect; points  $T_c$  and  $T'_c$  are not the same; points  $A$  and  $B$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $\text{midpoint}([A, T_c])$  are not the same; lines  $b$  and  $c$  are not parallel.

Determination conditions: lines  $b$  and  $c$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $H_b$  are not the same.

Rules used: [W02,W03,W04,W09,W10a,W19]

Lemmas used: [D25,D57,D6,D9,GD01,L72,L79]

Solving time: 2578.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point H_{b} 89.36 77.83
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_b B
cmark_l H_{b}
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points B and H_{b} are not the same
% Constructing a line h_{b} which passes through point B and point H_{b}
line h_{b} B H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% DET: points B and T_{c} are not the same
% Constructing a line c which passes through point B and point T_{c}
line c B T_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
perp b H_{b} h_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines b and c are not parallel% DET: lines b and c are not the same
% Constructing a point A which belongs to line b and line c
intersec A b c
cmark_t A

% NDG: points A and B are not the same; points B and T_{c} are not the same; points B and midpoint
% ([A,T_{c}]) are not the same
```

```

% Constructing a point P_{\_G27759} which is an image of the point B in a rotation around the point
  T_{c} for the angle 90
rotate P_{\_G27759} T_{c} 90 B
cmark_r P_{\_G27759}
color 200 200 200
drawarc_p T_{c} B 90
color 0 0 0

% Constructing a line L_{\_G27762} which passes through point T_{c} and point P_{\_G27759}
line L_{\_G27762} T_{c} P_{\_G27759}

color 200 200 200
drawline L_{\_G27762}
color 0 0 0

% Constructing midpoint P_{\_G27765} of the segment BP_{\_G27759}
midpoint P_{\_G27765} B P_{\_G27759}
cmark_r P_{\_G27765}

% Constructing a line L_{\_G27768} which passes through point A and point P_{\_G27765}
line L_{\_G27768} A P_{\_G27765}

color 200 200 200
drawline L_{\_G27768}
color 0 0 0

% Constructing a line L_{\_G27771} which passes through point B and point P_{\_G27765}
line L_{\_G27771} B P_{\_G27765}

color 200 200 200
drawline L_{\_G27771}
color 0 0 0

% Constructing a point P_{\_G27774} which belongs to line L_{\_G27762} and line L_{\_G27768}
intersec P_{\_G27774} L_{\_G27762} L_{\_G27768}
cmark_r P_{\_G27774}

% Constructing a point P_{\_G27777} which belongs to line L_{\_G27762} and line L_{\_G27771}
intersec P_{\_G27777} L_{\_G27762} L_{\_G27771}
cmark_r P_{\_G27777}

% Constructing a line L_{\_G27780} which passes through point A and point P_{\_G27777}
line L_{\_G27780} A P_{\_G27777}

color 200 200 200
drawline L_{\_G27780}
color 0 0 0

% Constructing a line L_{\_G27783} which passes through point B and point P_{\_G27774}
line L_{\_G27783} B P_{\_G27774}

color 200 200 200

```

```

drawline L_{\_G27783}
color 0 0 0

% Constructing a point P_{\_G27786} which belongs to line L_{\_G27780} and line L_{\_G27783}
intersec P_{\_G27786} L_{\_G27780} L_{\_G27783}
cmark_r P_{\_G27786}

% Constructing a line L_{\_G27789} which passes through point P_{\_G27765} and point P_{\_G27786}
line L_{\_G27789} P_{\_G27765} P_{\_G27786}

color 200 200 200
drawline L_{\_G27789}
color 0 0 0

% Constructing a point T'_{c} which belongs to line L_{\_G27789} and line c
intersec T'_{c} L_{\_G27789} c
cmark_r T'_{c}

% NDG: points T_{c} and T'_{c} are not the same
% Constructing midpoint P_{\_G28620} of the segment T_{c}T'_{c}
midpoint P_{\_G28620} T_{c} T'_{c}
cmark_r P_{\_G28620}

% Constructing a circle k_{over}(T_{c},T'_{c}) whose center is at point P_{\_G28620} and which passes
    through point T_{c}
circle k_{over}(T_{c},T'_{c}) P_{\_G28620} T_{c}

color 200 200 200
drawcircle k_{over}(T_{c},T'_{c})
color 0 0 0

% NDG: line b and circle k_{over}(T_{c},T'_{c}) intersect
% Constructing points C_{wa} and C which are in intersection of k_{over}(T_{c},T'_{c}) and b
intersec2 C_{wa} C k_{over}(T_{c},T'_{c}) b
cmark_r C_{wa}
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k_{over}(T_{c},T'_{c}) intersect; points T_{c} and T'_{c}
    are not the same; points A and B are not the same; points B and T_{c} are not the same;
    points B and midpoint([A,T_{c}]) are not the same; lines b and c are not parallel
% Determination conditions: lines b and c are not the same; points B and T_{c} are not the same;
    points B and H_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

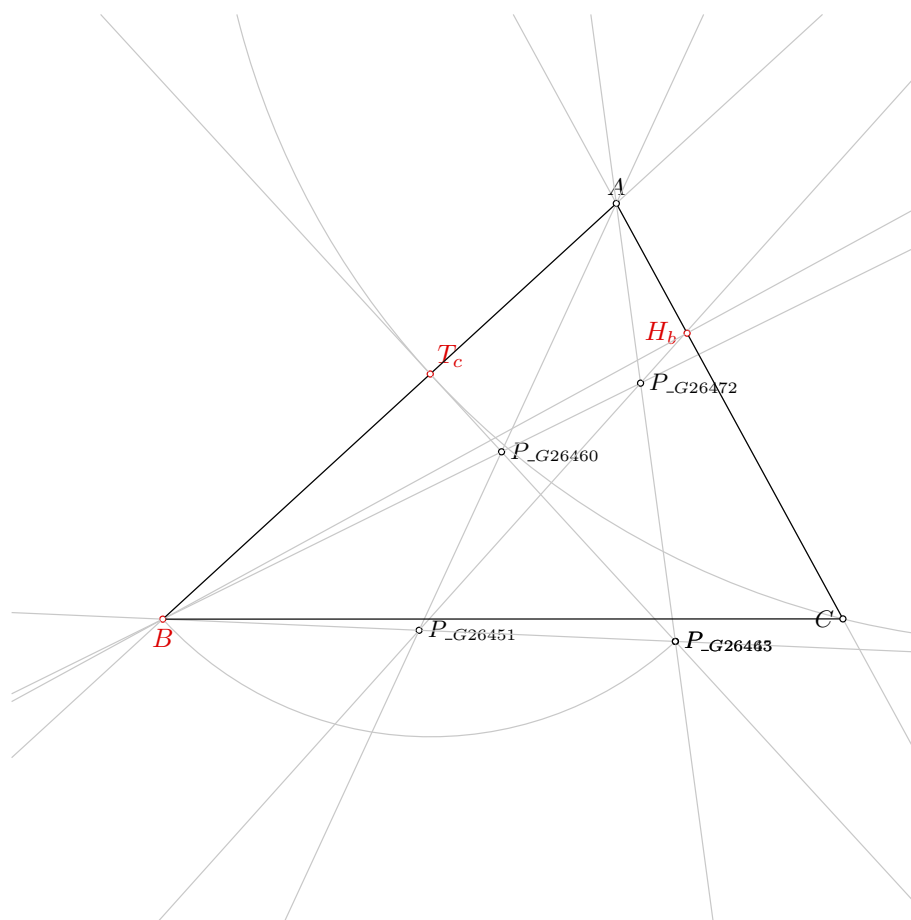


Figure 1: Illustration of the problem 0180

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b=_H H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.138 seconds.

**NDG conditions** Points  $B$  and  $T_c$  are not identical

Point  $T_c$  is not the midpoint of segment with endpoints  $A$  and  $B$

Points  $A$  and  $T_c$  are not identical

Points  $A$ ,  $B$  and  $C$  are not collinear

#### 4.1.3 Proving $T_c=_T T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_b B T_c} \neq S_{T_b^0 B T_c}$  i.e., lines  $H_b T_b^0$  and  $B T_c$  are not parallel (construction based assumption)

$S_{T_c A P_{G28279}} \neq S_{P_{G28273} A P_{G28279}}$  i.e., lines  $T_c P_{G28273}$  and  $A P_{G28279}$  are not parallel (construction based assumption)

$S_{T_c B P_{G28279}} \neq S_{P_{G28273} B P_{G28279}}$  i.e., lines  $T_c P_{G28273}$  and  $B P_{G28279}$  are not parallel (construction based assumption)

$S_{A B P_{G28288}} \neq S_{P_{G28291} B P_{G28288}}$  i.e., lines  $A P_{G28291}$  and  $B P_{G28288}$  are not parallel (construction based assumption)

$S_{P_{G28279} B T_c} \neq S_{P_{G28300} B T_c}$  i.e., lines  $P_{G28279} P_{G28300}$  and  $B T_c$  are not parallel (construction based assumption)

$S_{B A C} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{A B F_{H_b}^1} \neq S_{C B F_{H_b}^1}$  i.e., lines  $A C$  and  $B F_{H_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_b=_H H_b$

Proving failed

#### 4.2.3 Proving $T_c=_T T_c$

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $B=B$**

Proving failed

#### **4.3.2 Proving $H_b=\neg H_b$**

Proving failed

#### **4.3.3 Proving $T_c=\neg T_c$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $B=B$**

Proving failed

#### **4.4.2 Proving $H_b=\neg H_b$**

Proving failed

#### **4.4.3 Proving $T_c=\neg T_c$**

Proving failed



# Problem 181

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 181: Given a point  $B$ , a point  $H_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_b$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H_b$  are not the same;
2. Using the point  $H_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
3. Using the point  $I$  and the line  $b$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $b$ ;
4. Using the circle  $k(I, P_a)$ , the point  $B$  and the point  $I$ , construct a line  $c$  and a line  $a$  (rule W12); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
5. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same;
6. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; lines  $c$  and  $b$  are not parallel; point  $B$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $b$ .

Determination conditions: lines  $b$  and  $a$  are not the same; lines  $c$  and  $b$  are not the same; points  $B$  and  $H_b$  are not the same.

Rules used: [W02,W03,W10a,W11,W12]

Lemmas used: [D27,D6,D9,GD01,L59,L60,L61]

Solving time: 1.0 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point H_{b} 89.36 77.83
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_b B
cmark_l H_{b}
cmark_b I
color 0 0 0
fontsize 8

% DET: points B and H_{b} are not the same
% Constructing a line h_{b} which passes through point B and point H_{b}
line h_{b} B H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
perp b H_{b} h_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: point I is not incident to the line b
% Constructing a point P_{\_G20440} which is a foot of the point I on the line b
foot P_{\_G20440} I b
cmark_r P_{\_G20440}
color 200 200 200
drawline I P_{\_G20440}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
20440}
circle k(I,P_{a}) I P_{\_G20440}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point B is outside the circle k(I,P_{a})
```

```

% Constructing midpoint  $P_{\{G20674\}}$  of the segment  $BI$ 
midpoint  $P_{\{G20674\}}$  B I
cmark_r  $P_{\{G20674\}}$ 

% Constructing a circle  $C_{\{G20677\}}$  whose center is at point  $P_{\{G20674\}}$  and which passes through
point B
circle  $C_{\{G20677\}}$   $P_{\{G20674\}}$  B

color 200 200 200
drawcircle  $C_{\{G20677\}}$ 
color 0 0 0

% Constructing points  $P_{\{G20680\}}$  and  $P_{\{G20683\}}$  which are in intersection of  $C_{\{G20677\}}$  and  $k$ 
( $I, P_{\{a\}}$ )
intersec2  $P_{\{G20680\}}$   $P_{\{G20683\}}$   $C_{\{G20677\}}$   $k(I, P_{\{a\}})$ 
cmark_r  $P_{\{G20680\}}$ 
cmark_r  $P_{\{G20683\}}$ 

% Constructing a line  $c$  which passes through point  $B$  and point  $P_{\{G20680\}}$ 
line c B  $P_{\{G20680\}}$ 

color 200 200 200
drawline c
color 0 0 0

% Constructing a line  $a$  which passes through point  $B$  and point  $P_{\{G20683\}}$ 
line a B  $P_{\{G20683\}}$ 

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $c$  and  $b$  are not parallel% DET: lines  $c$  and  $b$  are not the same
% Constructing a point  $A$  which belongs to line  $c$  and line  $b$ 
intersec A c b
cmark_t A

% NDG: lines  $b$  and  $a$  are not parallel% DET: lines  $b$  and  $a$  are not the same
% Constructing a point  $C$  which belongs to line  $b$  and line  $a$ 
intersec C b a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; lines  $c$  and  $b$  are not parallel; point
 $B$  is outside the circle  $k(I, P_{\{a\}})$ ; point  $I$  is not incident to the line  $b$ 

```

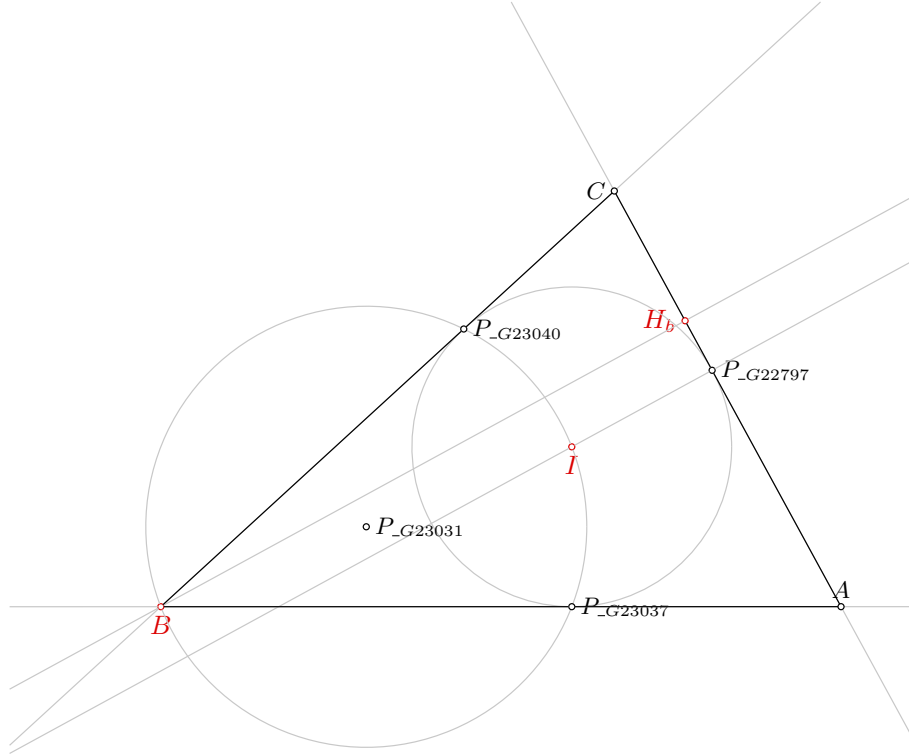


Figure 1: Illustration of the problem 0181

*% Determination conditions: lines b and a are not the same; lines c and b are not the same; points B and H\_{b} are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.032 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b = \neg H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.081 seconds.

**NDG conditions** Points  $A$ ,  $B$  and  $C$  are not collinear

#### 4.1.3 Proving $I = \neg I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B = B$

NDG conditions are:

$S_{BH_bT_b^0} \neq S_{P_{G23899}H_bT_b^0}$  i.e., lines  $BP_{G23899}$  and  $H_bT_b^0$  are not parallel (construction based assumption)

$S_{H_bBP_{G23902}} \neq S_{T_b^0BP_{G23902}}$  i.e., lines  $H_bT_b^0$  and  $BP_{G23902}$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^1} \neq S_{CBF_{\neg h_b}^1}$  i.e., lines  $AC$  and  $BF_{\neg h_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.2.3 Proving $I = \neg I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I = \neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 516 terms.

**Time Complexity:** Time spent by the prover is 0.320 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $H_b=H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.3 Proving $I=I$

Proving failed

# Problem 182

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 182: Given a point  $H_c$ , a point  $H$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $H_c$  and  $H$  are not the same;
2. Using the point  $H_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
3. Choose freely a point  $B$  on the line  $c$  (rule WOnline2);
4. Using the point  $B$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H$  are not the same;
5. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) ;
6. Using the point  $A$  and the point  $B$ , construct a point  $M_c$  (rule W01); ;
7. Using the point  $A$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $A$  and  $M_c$  are not the same;
8. Using the circle  $k(M_c, A)$ , the line  $h_b$ , the point  $M_c$  and the point  $B$ , construct a point  $H_b$  (rule W05); % NDG: line  $h_b$  and circle  $k(M_c, A)$  intersect % DET: points  $B$  and  $H_b$  must be different;
9. Using the point  $H_b$  and the point  $A$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $A$  are not the same;
10. Using the line  $h_c$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $h_c$  and  $b$  are not parallel % DET: lines  $h_c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $h_c$  and  $b$  are not parallel; line  $h_b$  and circle  $k(M_c, A)$  intersect; points  $A$  and  $M_c$  are not the same.

Determination conditions: lines  $h_c$  and  $b$  are not the same; points  $H_b$  and  $A$  are not the same; points  $B$  and  $H_b$  must be different; points  $B$  and  $H$  are not the same; points  $H_c$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D10,D20,D3,D6,D7,D9,GD01,GD02,L3,L40,L41]

Solving time: 23.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{c} 68.91 84.83
```

```
point H 80 72.73
```

```
point B 20 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H_{c}
```

```
cmark_rt H
```

```
cmark_b B
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{c} and H are not the same
```

```
% Constructing a line h_{c} which passes through point H_{c} and point H
```

```
line h_{c} H_{c} H
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```
% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
```

```
perp c H_{c} h_{c}
```

```
color 200 200 200
```

```
drawline c
```

```
color 0 0 0
```

```
% Generating random value V[_G29077]
```

```
random V[_G29077]
```

```
% Calculating value V[_G29098] using formula V[_G29077]*20
```

```
expression V[_G29098] { V[_G29077]*20 }
```



```

% Constructing a point B which is a point for which holds  $H_{\{c\}}B = V[_{G29098}]$  and angle  $HH_{\{c\}}B = 90$ 
turtle B H  $H_{\{c\}}$  90  $V[_{G29098}]$ 
cmark_b B

% DET: points B and H are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point B and point H
line  $h_{\{b\}}$  B H

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% Choosing randomly a point A on the line  $BH_{\{c\}}$ 
online A B  $H_{\{c\}}$ 
cmark_t A
color 200 200 200
drawline B  $H_{\{c\}}$ 
color 0 0 0

% Constructing a point  $M_{\{c\}}$  such that  $AM_{\{c\}}/AB=0.5$ 
towards  $M_{\{c\}}$  A B 0.5
cmark_lt  $M_{\{c\}}$ 
color 200 200 200
drawsegment A B
color 0 0 0

% NDG: points A and  $M_{\{c\}}$  are not the same
% Constructing a circle  $k(M_{\{c\}},A)$  whose center is at point  $M_{\{c\}}$  and which passes through point A
circle  $k(M_{\{c\}},A)$   $M_{\{c\}}$  A

color 200 200 200
drawcircle  $k(M_{\{c\}},A)$ 
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(M_{\{c\}},A)$  intersect% DET: points B and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G29627\}}$  which is a foot of the point  $M_{\{c\}}$  on the line  $h_{\{b\}}$ 
foot  $P_{\{\backslash\_G29627\}}$   $M_{\{c\}}$   $h_{\{b\}}$ 
cmark_r  $P_{\{\backslash\_G29627\}}$ 
color 200 200 200
drawline  $M_{\{c\}}$   $P_{\{\backslash\_G29627\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point B in the symmetry to point/line  $P_{\{\backslash\_G29627\}}$ 
sim  $H_{\{b\}}$   $P_{\{\backslash\_G29627\}}$  B

```

```

cmark_l H_{b}

% DET: points H_{b} and A are not the same
% Constructing a line b which passes through point H_{b} and point A
line b H_{b} A

color 200 200 200
drawline b
color 0 0 0

% NDG: lines h_{c} and b are not parallel% DET: lines h_{c} and b are not the same
% Constructing a point C which belongs to line h_{c} and line b
intersec C h_{c} b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and b are not parallel; line h_{b} and circle k(M_{c},A)
% intersect; points A and M_{c} are not the same
% Determination conditions: lines h_{c} and b are not the same; points H_{b} and A are not the same
% ; points B and H_{b} must be different; points B and H are not the same; points H_{c} and H are
% not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_c = H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.116 seconds.

**NDG conditions** Points  $H_c$ ,  $B$  and  $H$  are not collinear

Points  $B$  and  $H$  are not identical

Line through points  $H_b$  and  $B$  is not perpendicular to line through points  $B$  and  $A$

#### 4.1.2 Proving $H = H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.304 seconds.

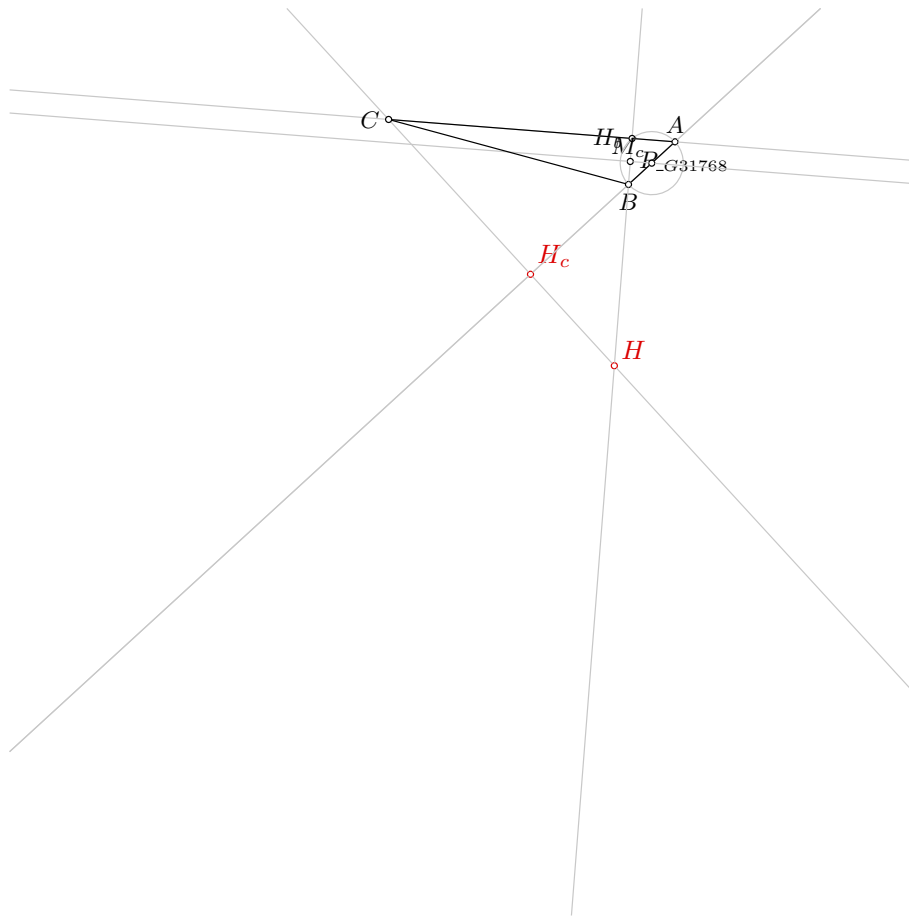


Figure 1: Illustration of the problem 0182

**NDG conditions** Points  $H_c$ ,  $B$  and  $H$  are not collinear

Points  $H_c$  and  $H$  are not identical

Line through points  $H_c$  and  $A$  is not perpendicular to line through points  $A$  and  $H_b$

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $C$ ,  $M_c$  and  $H$  are not collinear

### 4.1.3 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.007 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $H_c=_H H_c$

Proving failed

### 4.2.2 Proving $H=_H H$

Proving failed

### 4.2.3 Proving $B=B$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $H_c=_H H_c$

Proving failed

### 4.3.2 Proving $H=_H H$

Proving failed

### 4.3.3 Proving $B=B$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $H_c=_H H_c$

Proving failed

### 4.4.2 Proving $H=_H H$

Proving failed

### 4.4.3 Proving $B=B$

Proving failed

# Problem 183

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 183: Given a point  $B$ , a point  $H_c$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $H_c$  are not the same;
2. Using the point  $B$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $T_a$  are not the same;
3. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
4. Using the line  $h_c$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $h_c$  and  $a$  are not parallel % DET: lines  $h_c$  and  $a$  are not the same;
5. Using the point  $B$ , the point  $C$ , the point  $T_a$  and the line  $a$ , construct a point  $T'_a$  (rule W19); % NDG: points  $B$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $\text{midpoint}([B, T_a])$  are not the same;
6. Using the point  $T_a$  and the point  $T'_a$ , construct a circle  $k_{over}(T_a, T'_a)$  (rule W09); % NDG: points  $T_a$  and  $T'_a$  are not the same;
7. Using the circle  $k_{over}(T_a, T'_a)$  and the line  $c$ , construct a point  $A_{wb}$  and a point  $A$  (rule W04); % NDG: line  $c$  and circle  $k_{over}(T_a, T'_a)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k_{over}(T_a, T'_a)$  intersect; points  $T_a$  and  $T'_a$  are not the same; points  $B$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $\text{midpoint}([B, T_a])$  are not the same; lines  $h_c$  and  $a$  are not parallel.

Determination conditions: lines  $h_c$  and  $a$  are not the same; points  $B$  and  $T_a$  are not the same; points  $B$  and  $H_c$  are not the same.

Rules used: [W02,W03,W04,W09,W10b,W19]

Lemmas used: [D10,D23,D53,D7,GD01,L71,L77]

Solving time: 2284.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point H_{c} 68.91 84.83
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_b B
cmark_rt H_{c}
cmark_rb T_{a}
color 0 0 0
fontsize 8

% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

color 200 200 200
drawline c
color 0 0 0

% DET: points B and T_{a} are not the same
% Constructing a line a which passes through point B and point T_{a}
line a B T_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines h_{c} and a are not parallel% DET: lines h_{c} and a are not the same
% Constructing a point C which belongs to line h_{c} and line a
intersec C h_{c} a
cmark_l C

% NDG: points B and C are not the same; points C and T_{a} are not the same; points C and midpoint
% ([B,T_{a}]) are not the same
```

```

% Constructing a point  $P_{\setminus G30394}$  which is an image of the point  $C$  in a rotation around the point
 $T_{\setminus a}$  for the angle 90
rotate  $P_{\setminus G30394}$   $T_{\setminus a}$  90 C
cmark_r  $P_{\setminus G30394}$ 
color 200 200 200
drawarc_p  $T_{\setminus a}$  C 90
color 0 0 0

% Constructing a line  $L_{\setminus G30397}$  which passes through point  $T_{\setminus a}$  and point  $P_{\setminus G30394}$ 
line  $L_{\setminus G30397}$   $T_{\setminus a}$   $P_{\setminus G30394}$ 

color 200 200 200
drawline  $L_{\setminus G30397}$ 
color 0 0 0

% Constructing midpoint  $P_{\setminus G30400}$  of the segment  $CP_{\setminus G30394}$ 
midpoint  $P_{\setminus G30400}$  C  $P_{\setminus G30394}$ 
cmark_r  $P_{\setminus G30400}$ 

% Constructing a line  $L_{\setminus G30403}$  which passes through point  $B$  and point  $P_{\setminus G30400}$ 
line  $L_{\setminus G30403}$  B  $P_{\setminus G30400}$ 

color 200 200 200
drawline  $L_{\setminus G30403}$ 
color 0 0 0

% Constructing a line  $L_{\setminus G30406}$  which passes through point  $C$  and point  $P_{\setminus G30400}$ 
line  $L_{\setminus G30406}$  C  $P_{\setminus G30400}$ 

color 200 200 200
drawline  $L_{\setminus G30406}$ 
color 0 0 0

% Constructing a point  $P_{\setminus G30409}$  which belongs to line  $L_{\setminus G30397}$  and line  $L_{\setminus G30403}$ 
intersec  $P_{\setminus G30409}$   $L_{\setminus G30397}$   $L_{\setminus G30403}$ 
cmark_r  $P_{\setminus G30409}$ 

% Constructing a point  $P_{\setminus G30412}$  which belongs to line  $L_{\setminus G30397}$  and line  $L_{\setminus G30406}$ 
intersec  $P_{\setminus G30412}$   $L_{\setminus G30397}$   $L_{\setminus G30406}$ 
cmark_r  $P_{\setminus G30412}$ 

% Constructing a line  $L_{\setminus G30415}$  which passes through point  $B$  and point  $P_{\setminus G30412}$ 
line  $L_{\setminus G30415}$  B  $P_{\setminus G30412}$ 

color 200 200 200
drawline  $L_{\setminus G30415}$ 
color 0 0 0

% Constructing a line  $L_{\setminus G30418}$  which passes through point  $C$  and point  $P_{\setminus G30409}$ 
line  $L_{\setminus G30418}$  C  $P_{\setminus G30409}$ 

color 200 200 200

```

```

drawline L_{\_G30418}
color 0 0 0

% Constructing a point P_{\_G30421} which belongs to line L_{\_G30415} and line L_{\_G30418}
intersec P_{\_G30421} L_{\_G30415} L_{\_G30418}
cmark_r P_{\_G30421}

% Constructing a line L_{\_G30424} which passes through point P_{\_G30400} and point P_{\_G30421}
line L_{\_G30424} P_{\_G30400} P_{\_G30421}

color 200 200 200
drawline L_{\_G30424}
color 0 0 0

% Constructing a point T'_{a} which belongs to line L_{\_G30424} and line a
intersec T'_{a} L_{\_G30424} a
cmark_r T'_{a}

% NDG: points T_{a} and T'_{a} are not the same
% Constructing midpoint P_{\_G31255} of the segment T_{a}T'_{a}
midpoint P_{\_G31255} T_{a} T'_{a}
cmark_r P_{\_G31255}

% Constructing a circle k_{over}(T_{a},T'_{a}) whose center is at point P_{\_G31255} and which passes
    through point T_{a}
circle k_{over}(T_{a},T'_{a}) P_{\_G31255} T_{a}

color 200 200 200
drawcircle k_{over}(T_{a},T'_{a})
color 0 0 0

% NDG: line c and circle k_{over}(T_{a},T'_{a}) intersect
% Constructing points A_{wb} and A which are in intersection of k_{over}(T_{a},T'_{a}) and c
intersec2 A_{wb} A k_{over}(T_{a},T'_{a}) c
cmark_r A_{wb}
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k_{over}(T_{a},T'_{a}) intersect; points T_{a} and T'_{a}
    are not the same; points B and C are not the same; points C and T_{a} are not the same;
    points C and midpoint([B,T_{a}]) are not the same; lines h_{c} and a are not parallel
% Determination conditions: lines h_{c} and a are not the same; points B and T_{a} are not the same
    ; points B and H_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1



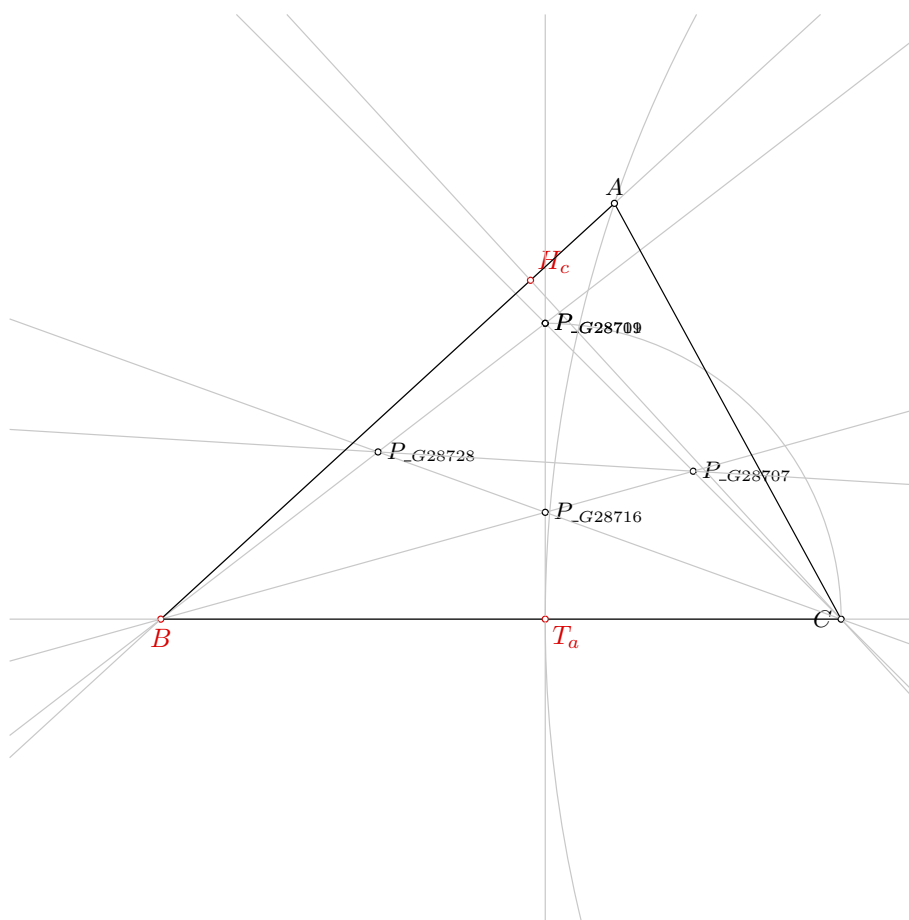


Figure 1: Illustration of the problem 0183

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c=_Hc$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 5 terms.

**Time Complexity:** Time spent by the prover is 0.136 seconds.

**NDG conditions** Points  $T_a$ ,  $A$  and  $B$  are not collinear

Point  $T_a$  is not the midpoint of segment with endpoints  $B$  and  $C$

Points  $T_a$  and  $A$  are not identical

Points  $A$  and  $B$  are not identical

#### 4.1.3 Proving $T_a=_T a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_c B T_a} \neq S_{T_{h_c}^0 B T_a}$  i.e., lines  $H_c T_{h_c}^0$  and  $B T_a$  are not parallel (construction based assumption)

$S_{T_a B P_{G30532}} \neq S_{P_{G30526} B P_{G30532}}$  i.e., lines  $T_a P_{G30526}$  and  $B P_{G30532}$  are not parallel (construction based assumption)

$S_{T_a C P_{G30532}} \neq S_{P_{G30526} C P_{G30532}}$  i.e., lines  $T_a P_{G30526}$  and  $C P_{G30532}$  are not parallel (construction based assumption)

$S_{B C P_{G30541}} \neq S_{P_{G30544} C P_{G30541}}$  i.e., lines  $B P_{G30544}$  and  $C P_{G30541}$  are not parallel (construction based assumption)

$S_{P_{G30532} B T_a} \neq S_{P_{G30553} B T_a}$  i.e., lines  $P_{G30532} P_{G30553}$  and  $B T_a$  are not parallel (construction based assumption)

$S_{C A B} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{A C F_{h_c}^1} \neq S_{B C F_{h_c}^1}$  i.e., lines  $A B$  and  $C F_{h_c}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c=_Hc$

Proving failed

#### 4.2.3 Proving $T_a=_T a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

Proving failed

#### 4.3.2 Proving $H_c=_H H_c$

Proving failed

#### 4.3.3 Proving $T_a=_T T_a$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B=B$

Proving failed

#### 4.4.2 Proving $H_c=_H H_c$

Proving failed

#### 4.4.3 Proving $T_a=_T T_a$

Proving failed

# Problem 184

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 184: Given a point  $B$ , a point  $H_c$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $H_c$  are not the same;
2. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
3. Using the point  $T_b$  and the line  $c$ , construct a circle  $k(T_b, foot[T_b, c])$  (rule W11); % NDG: point  $T_b$  is not incident to the line  $c$ ;
4. Using the circle  $k(T_b, foot[T_b, c])$ , the point  $B$ , the point  $T_b$  and the line  $c$ , construct a line  $a$  (rule W13); % NDG: point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ;
5. Using the line  $a$  and the line  $h_c$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $h_c$  are not parallel % DET: lines  $a$  and  $h_c$  are not the same;
6. Using the point  $T_b$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $T_b$  and  $C$  are not the same;
7. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; lines  $a$  and  $h_c$  are not parallel; point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ; point  $T_b$  is not incident to the line  $c$ .

Determination conditions: lines  $c$  and  $b$  are not the same; points  $T_b$  and  $C$  are not the same; lines  $a$  and  $h_c$  are not the same; points  $B$  and  $H_c$  are not the same.

Rules used: [W02,W03,W10b,W11,W13]

Lemmas used: [D10,D24,D7,GD01,GD02,GL10,GL11,L9]

Solving time: 0.7 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point H_{c} 68.91 84.83
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_b B
cmark_rt H_{c}
cmark_t T_{b}
color 0 0 0
fontsize 8

% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: point T_{b} is not incident to the line c
% Constructing a point P_{\_G23807} which is a foot of the point T_{b} on the line c
foot P_{\_G23807} T_{b} c
cmark_r P_{\_G23807}
color 200 200 200
drawline T_{b} P_{\_G23807}
color 0 0 0

% Constructing a circle k(T_{b},foot[T_{b},c]) whose center is at point T_{b} and which passes
    through point P_{\_G23807}
circle k(T_{b},foot[T_{b},c]) T_{b} P_{\_G23807}

color 200 200 200
drawcircle k(T_{b},foot[T_{b},c])
color 0 0 0

% NDG: point B is outside the circle k(T_{b},foot[T_{b},c])
```

```

% Constructing a point  $P_{\{G24198\}}$  which is a foot of the point  $T_{\{b\}}$  on the line  $c$ 
foot  $P_{\{G24198\}}$   $T_{\{b\}}$   $c$ 
cmark_r  $P_{\{G24198\}}$ 
color 200 200 200
drawline  $T_{\{b\}}$   $P_{\{G24198\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G24201\}}$  which passes through point  $T_{\{b\}}$  and point  $B$ 
line  $L_{\{G24201\}}$   $T_{\{b\}}$   $B$ 

color 200 200 200
drawline  $L_{\{G24201\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G24204\}}$  which is an image of the point  $P_{\{G24198\}}$  in the symmetry to
point/line  $L_{\{G24201\}}$ 
sim  $P_{\{G24204\}}$   $L_{\{G24201\}}$   $P_{\{G24198\}}$ 
cmark_r  $P_{\{G24204\}}$ 

% Constructing a line  $a$  which passes through point  $B$  and point  $P_{\{G24204\}}$ 
line  $a$   $B$   $P_{\{G24204\}}$ 

color 200 200 200
drawline  $a$ 
color 0 0 0

% NDG: lines  $a$  and  $h_{\{c\}}$  are not parallel% DET: lines  $a$  and  $h_{\{c\}}$  are not the same
% Constructing a point  $C$  which belongs to line  $a$  and line  $h_{\{c\}}$ 
intersec  $C$   $a$   $h_{\{c\}}$ 
cmark_l  $C$ 

% DET: points  $T_{\{b\}}$  and  $C$  are not the same
% Constructing a line  $b$  which passes through point  $T_{\{b\}}$  and point  $C$ 
line  $b$   $T_{\{b\}}$   $C$ 

color 200 200 200
drawline  $b$ 
color 0 0 0

% NDG: lines  $c$  and  $b$  are not parallel% DET: lines  $c$  and  $b$  are not the same
% Constructing a point  $A$  which belongs to line  $c$  and line  $b$ 
intersec  $A$   $c$   $b$ 
cmark_t  $A$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

```

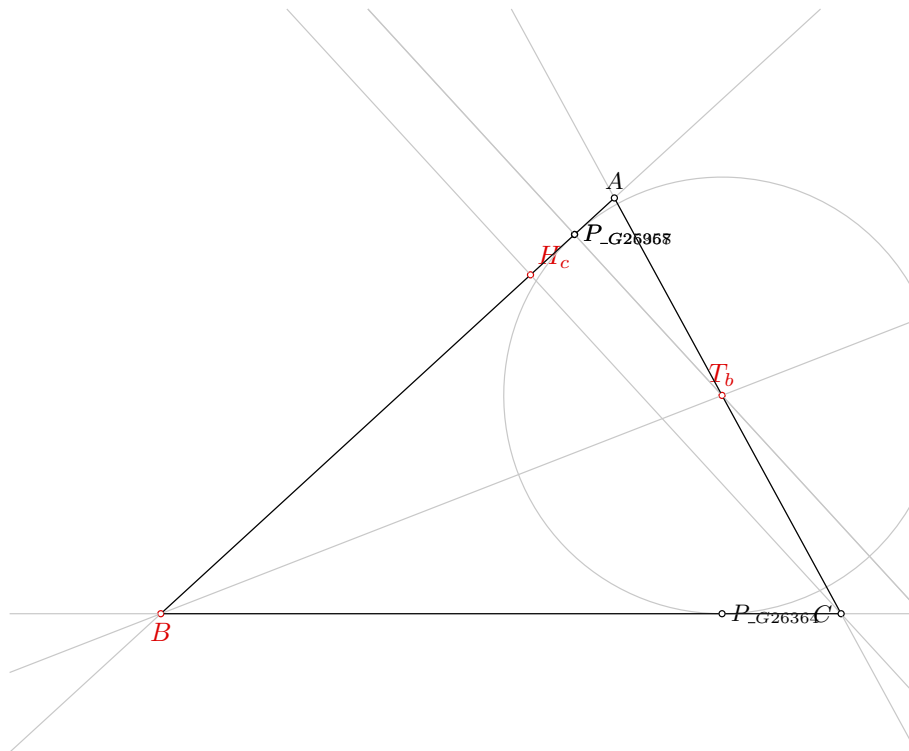


Figure 1: Illustration of the problem 0184

```
% Non-degenerate conditions: lines c and b are not parallel; lines a and h_{c} are not parallel;
    point B is outside the circle k(T_{b}, foot[T_{b}, c]); point T_{b} is not incident to the line c
% Determination conditions: lines c and b are not the same; points T_{b} and C are not the same;
    lines a and h_{c} are not the same; points B and H_{c} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.033 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.131 seconds.

**NDG conditions** Point  $T_b$  is not on circle with center  $P_{G20770}$  and point from it  $P_{G20621}$

Point  $T_b$  is not on circle with center  $P_{G20770}$  and point from it  $P_{G20621}$

Points  $P_{G20621}$ ,  $H_c$  and  $P_{G20776}$  are not collinear

Line through points  $H_c$  and  $B$  is not parallel with line through points  $C$  and  $T_b$

Points  $A$  and  $B$  are not identical

#### 4.1.3 Proving $T_b = \neg T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B = B$

NDG conditions are:

$S_{BH_cT_{h_c}^0} \neq S_{P_{G27436}H_cT_{h_c}^0}$  i.e., lines  $BP_{G27436}$  and  $H_cT_{h_c}^0$  are not parallel (construction based assumption)

$S_{BT_bC} \neq S_{H_cT_bC}$  i.e., lines  $BH_c$  and  $T_bC$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^1} \neq S_{BCF_{h_c}^1}$  i.e., lines  $AB$  and  $CF_{h_c}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.2.3 Proving $T_b = \neg T_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.



#### 4.3.3 Proving $T_b = \neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 67 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_b = \neg T_b$

Proving failed

# Problem 185

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 185: Given a point  $H_c$ , a point  $T_c$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $T_c$  are not the same;
2. Choose freely a point  $B$  on the line  $c$  (rule WOnline1) ;
3. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
4. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) ;
5. Choose freely a point  $C$  on the line  $h_c$  (rule WOnline2).

Non-degenerate conditions: .

Determination conditions: points  $H_c$  and  $T_c$  are not the same.

Rules used: [W02,W10b,WOnline1,WOnline2]

Lemmas used: [D10,D25,D7]

Solving time: 1903.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point H_{c} 68.91 84.83
point T_{c} 55.38 72.43
point B 20 40

color 220 0 0
fontsize 9
```

```

cmark_rt H_{c}
cmark_rt T_{c}
cmark_b B
color 0 0 0
fontsize 8

% DET: points H_{c} and T_{c} are not the same
% Constructing a line c which passes through point H_{c} and point T_{c}
line c H_{c} T_{c}

color 200 200 200
drawline c
color 0 0 0

% Choosing randomly a point B on the line H_{c}T_{c}
online B H_{c} T_{c}
cmark_b B
color 200 200 200
drawline H_{c} T_{c}
color 0 0 0

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0

% Choosing randomly a point A on the line BH_{c}
online A B H_{c}
cmark_t A
color 200 200 200
drawline B H_{c}
color 0 0 0

% Generating random value V[_G26514]
random V[_G26514]

% Calculating value V[_G26535] using formula V[_G26514]*20
expression V[_G26535] { V[_G26514]*20 }

% Constructing a point C which is a point for which holds  $H_{c}C = V[_G26535]$  and angle  $AH_{c}C = 90$ 
turtle C A H_{c} 90 V[_G26535]

```

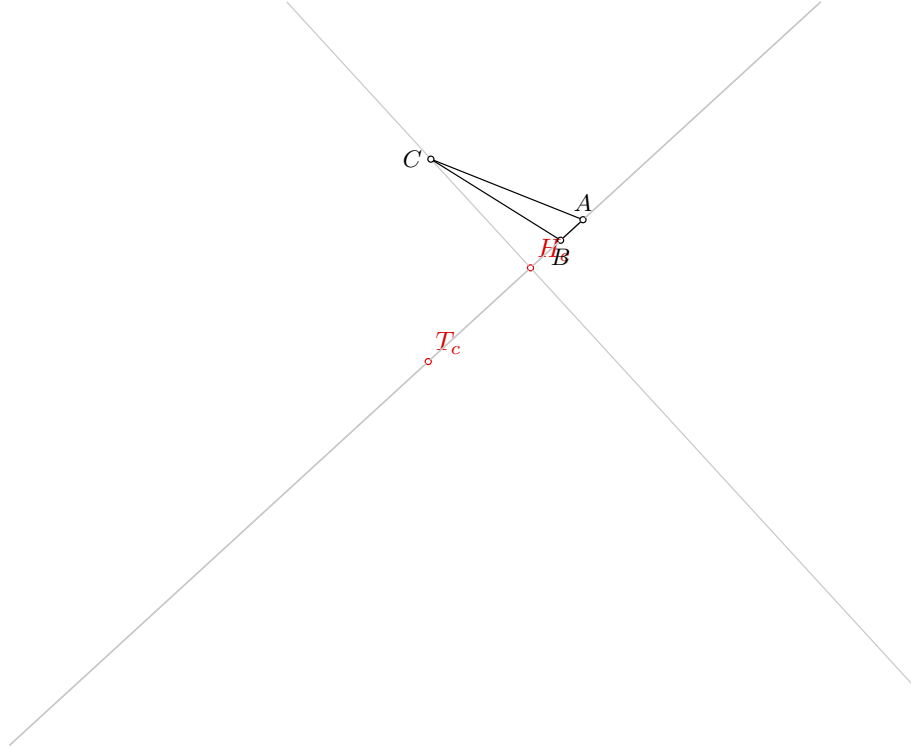


Figure 1: Illustration of the problem 0185

cmark\_1 C

drawsegment A B  
drawsegment A C  
drawsegment B C

*% Non-degenerate conditions:*

*% Determination conditions: points  $H_{\{c\}}$  and  $T_{\{c\}}$  are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.008 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_c = \neg T_c$

Proving failed

#### 4.1.3 Proving $B = B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_c = \neg H_c$

Proving failed

#### 4.2.2 Proving $T_c = \neg T_c$

Proving failed

#### 4.2.3 Proving $B = B$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^3} \neq S_{BCF_{h_c}^3}$  i.e., lines  $AB$  and  $CF_{h_c}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_c = \neg H_c$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $T_c = \neg T_c$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $H_c = \neg H_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 31.570 seconds. There are no ndg conditions.

#### 4.4.2 Proving $T_c = \neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 31.450 seconds. There are no ndg conditions.

#### 4.4.3 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 186

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 186: Given a point  $B$ , a point  $H_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $H_c$  are not the same;
2. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
3. Using the point  $I$  and the line  $c$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $c$ ;
4. Using the circle  $k(I, P_a)$ , the point  $B$ , the point  $I$  and the line  $c$ , construct a line  $a$  (rule W13); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
5. Using the line  $a$  and the line  $h_c$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $h_c$  are not parallel % DET: lines  $a$  and  $h_c$  are not the same;
6. Using the circle  $k(I, P_a)$ , the point  $C$ , the point  $I$  and the line  $a$ , construct a line  $b$  (rule W13); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
7. Using the line  $b$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $c$  are not parallel % DET: lines  $b$  and  $c$  are not the same.

Non-degenerate conditions: lines  $b$  and  $c$  are not parallel; point  $C$  is outside the circle  $k(I, P_a)$ ; lines  $a$  and  $h_c$  are not parallel; point  $B$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $c$ .

Determination conditions: lines  $b$  and  $c$  are not the same; lines  $a$  and  $h_c$  are not the same; points  $B$  and  $H_c$  are not the same.

Rules used: [W02,W03,W10b,W11,W13]

Lemmas used: [D10,D27,D7,GD01,L59,L60,L61]

Solving time: 12.5 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point H_{c} 68.91 84.83
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_b B
cmark_rt H_{c}
cmark_b I
color 0 0 0
fontsize 8

% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: point I is not incident to the line c
% Constructing a point P_{\_G23675} which is a foot of the point I on the line c
foot P_{\_G23675} I c
cmark_r P_{\_G23675}
color 200 200 200
drawline I P_{\_G23675}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
23675}
circle k(I,P_{a}) I P_{\_G23675}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point B is outside the circle k(I,P_{a})
```



```

% Constructing a point  $P_{\{G23882\}}$  which is a foot of the point  $I$  on the line  $c$ 
foot  $P_{\{G23882\}}$  I c
cmark_r  $P_{\{G23882\}}$ 
color 200 200 200
drawline I  $P_{\{G23882\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G23885\}}$  which passes through point  $I$  and point  $B$ 
line  $L_{\{G23885\}}$  I B

color 200 200 200
drawline  $L_{\{G23885\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G23888\}}$  which is an image of the point  $P_{\{G23882\}}$  in the symmetry to
point/line  $L_{\{G23885\}}$ 
sim  $P_{\{G23888\}}$   $L_{\{G23885\}}$   $P_{\{G23882\}}$ 
cmark_r  $P_{\{G23888\}}$ 

% Constructing a line  $a$  which passes through point  $B$  and point  $P_{\{G23888\}}$ 
line a B  $P_{\{G23888\}}$ 

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $a$  and  $h_{\{c\}}$  are not parallel% DET: lines  $a$  and  $h_{\{c\}}$  are not the same
% Constructing a point  $C$  which belongs to line  $a$  and line  $h_{\{c\}}$ 
intersec C a  $h_{\{c\}}$ 
cmark_l C

% NDG: point  $C$  is outside the circle  $k(I, P_{\{a\}})$ 
% Constructing a point  $P_{\{G24274\}}$  which is a foot of the point  $I$  on the line  $a$ 
foot  $P_{\{G24274\}}$  I a
cmark_r  $P_{\{G24274\}}$ 
color 200 200 200
drawline I  $P_{\{G24274\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G24277\}}$  which passes through point  $I$  and point  $C$ 
line  $L_{\{G24277\}}$  I C

color 200 200 200
drawline  $L_{\{G24277\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G24280\}}$  which is an image of the point  $P_{\{G24274\}}$  in the symmetry to
point/line  $L_{\{G24277\}}$ 
sim  $P_{\{G24280\}}$   $L_{\{G24277\}}$   $P_{\{G24274\}}$ 
cmark_r  $P_{\{G24280\}}$ 

```

```

% Constructing a line b which passes through point C and point P_{G24280}
line b C P_{G24280}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines b and c are not parallel% DET: lines b and c are not the same
% Constructing a point A which belongs to line b and line c
intersec A b c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and c are not parallel; point C is outside the circle k(I,P_{a}
% ); lines a and h_{c} are not parallel; point B is outside the circle k(I,P_{a}); point I is
% not incident to the line c
% Determination conditions: lines b and c are not the same; lines a and h_{c} are not the same;
% points B and H_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c=H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 10 terms.

**Time Complexity:** Time spent by the prover is 0.22 seconds.

**NDG conditions** Points  $H_c$ ,  $B$  and  $I$  are not collinear

Points  $B$  and  $I$  are not identical

Points  $A$  and  $P_{G20088}$  are not identical

Points  $B$  and  $P_{G20088}$  are not identical

Points  $C$  and  $I$  are not identical

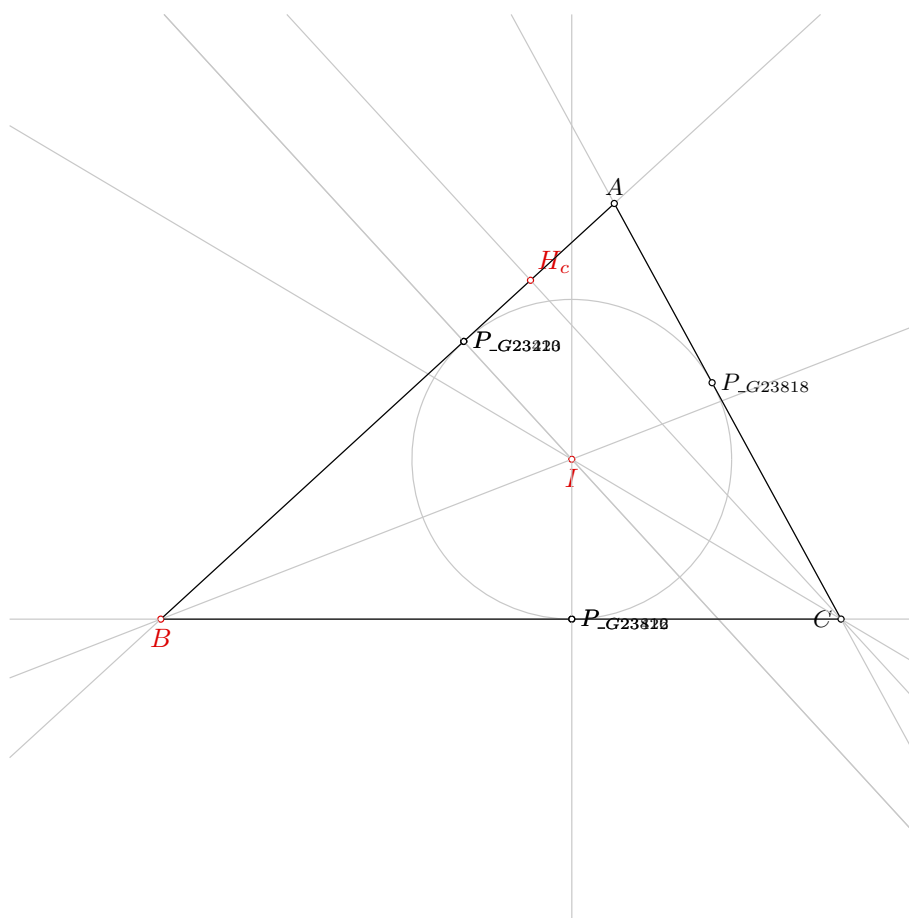


Figure 1: Illustration of the problem 0186

Points  $A$  and  $B$  are not identical

Line through points  $H_c$  and  $B$  is not parallel with line through points  $C$  and  $P_{G20250}$

Points  $A$  and  $B$  are not identical

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{BH_cT_{h_c}^0} \neq S_{P_{G24552}H_cT_{h_c}^0}$  i.e., lines  $BP_{G24552}$  and  $H_cT_{h_c}^0$  are not parallel (construction based assumption)

$S_{CBH_c} \neq S_{P_{G24944}BH_c}$  i.e., lines  $CP_{G24944}$  and  $BH_c$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^1} \neq S_{BCF_{h_c}^1}$  i.e., lines  $AB$  and  $CF_{h_c}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c=_H H_c$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_c=_H H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I=_I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 23 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $I = \neg I$

Proving failed

# Problem 187

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 187: Given a point  $B$ , a point  $H$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 188

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 188: Given a point  $B$ , a point  $H$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 189

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 189: Given a point  $B$ , a point  $H$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.



## Problem 190

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 190: Given a point  $B$ , a point  $H$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 191

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 191: Given a point  $B$ , a point  $T_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $T_a$  are not the same;
2. Using the point  $T_b$  and the line  $a$ , construct a circle  $k(T_b, foot[T_b, c])$  (rule W11); % NDG: point  $T_b$  is not incident to the line  $a$ ;
3. Using the circle  $k(T_b, foot[T_b, c])$ , the point  $B$ , the point  $T_b$  and the line  $a$ , construct a line  $c$  (rule W13); % NDG: point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ;
4. Using the point  $T_a$  and the line  $c$ , construct a circle  $k(T_a, foot[T_a, c])$  (rule W11); % NDG: point  $T_a$  is not incident to the line  $c$ ;
5. Using the circle  $k(T_a, foot[T_a, c])$ , the point  $T_b$  and the point  $T_a$ , construct a line  $b1$  and a line  $b$  (rule W12); % NDG: point  $T_b$  is outside the circle  $k(T_a, foot[T_a, c])$ ;
6. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same;
7. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; lines  $b$  and  $a$  are not parallel; point  $T_b$  is outside the circle  $k(T_a, foot[T_a, c])$ ; point  $T_a$  is not incident to the line  $c$ ; point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ; point  $T_b$  is not incident to the line  $a$ .

Determination conditions: lines  $c$  and  $b$  are not the same; lines  $b$  and  $a$  are not the same; points  $B$  and  $T_a$  are not the same.

Rules used: [W02,W03,W11,W12,W13]

Lemmas used: [D23,D24,D34,GD01,GD02,GL10,GL11,L8,L9]

Solving time: 0.9 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point T_{a} 70.86 40
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_b B
cmark_rb T_{a}
cmark_t T_{b}
color 0 0 0
fontsize 8

% DET: points B and T_{a} are not the same
% Constructing a line a which passes through point B and point T_{a}
line a B T_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: point T_{b} is not incident to the line a
% Constructing a point P_{\_G32205} which is a foot of the point T_{b} on the line a
foot P_{\_G19374} T_{b} a
cmark_r P_{\_G19374}
color 200 200 200
drawline T_{b} P_{\_G19374}
color 0 0 0

% Constructing a circle k(T_{b},foot[T_{b},c]) whose center is at point T_{b} and which passes
    through point P_{\_G32205}
circle k(T_{b},foot[T_{b},c]) T_{b} P_{\_G19374}

color 200 200 200
drawcircle k(T_{b},foot[T_{b},c])
color 0 0 0

% NDG: point B is outside the circle k(T_{b},foot[T_{b},c])
% Constructing a point P_{\_G19732} which is a foot of the point T_{b} on the line a
foot P_{\_G19732} T_{b} a
cmark_r P_{\_G19732}
color 200 200 200
drawline T_{b} P_{\_G19732}
color 0 0 0

% Constructing a line L_{\_G19735} which passes through point T_{b} and point B
line L_{\_G19735} T_{b} B
```

```

color 200 200 200
drawline L_{\_G19735}
color 0 0 0

% Constructing a point P_{\_G19738} which is an image of the point P_{\_G19732} in the symmetry to
point/line L_{\_G19735}
sim P_{\_G19738} L_{\_G19735} P_{\_G19732}
cmark_r P_{\_G19738}

% Constructing a line c which passes through point B and point P_{\_G19738}
line c B P_{\_G19738}

color 200 200 200
drawline c
color 0 0 0

% NDG: point T_{a} is not incident to the line c
% Constructing a point P_{\_G20019} which is a foot of the point T_{a} on the line c
foot P_{\_G20019} T_{a} c
cmark_r P_{\_G20019}
color 200 200 200
drawline T_{a} P_{\_G20019}
color 0 0 0

% Constructing a circle k(T_{a},foot[T_{a},c]) whose center is at point T_{a} and which passes
through point P_{\_G20019}
circle k(T_{a},foot[T_{a},c]) T_{a} P_{\_G20019}

color 200 200 200
drawcircle k(T_{a},foot[T_{a},c])
color 0 0 0

% NDG: point T_{b} is outside the circle k(T_{a},foot[T_{a},c])
% Constructing midpoint P_{\_G20437} of the segment T_{b}T_{a}
midpoint P_{\_G20437} T_{b} T_{a}
cmark_r P_{\_G20437}

% Constructing a circle C_{\_G20440} whose center is at point P_{\_G20437} and which passes through
point T_{b}
circle C_{\_G20440} P_{\_G20437} T_{b}

color 200 200 200
drawcircle C_{\_G20440}
color 0 0 0

% Constructing points P_{\_G20443} and P_{\_G20446} which are in intersection of C_{\_G20440} and k
(T_{a},foot[T_{a},c])
intersec2 P_{\_G20443} P_{\_G20446} C_{\_G20440} k(T_{a},foot[T_{a},c])
cmark_r P_{\_G20443}
cmark_r P_{\_G20446}

```

```

% Constructing a line b1 which passes through point T_{b} and point P_{\_G20443}
line b1 T_{b} P_{\_G20443}

color 200 200 200
drawline b1
color 0 0 0

% Constructing a line b which passes through point T_{b} and point P_{\_G20446}
line b T_{b} P_{\_G20446}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and b are not parallel; lines b and a are not parallel; point
  T_{b} is outside the circle k(T_{a},foot[T_{a},c]); point T_{a} is not incident to the line c;
  point B is outside the circle k(T_{b},foot[T_{b},c]); point T_{b} is not incident to the line a
% Determination conditions: lines c and b are not the same; lines b and a are not the same; points
  B and T_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

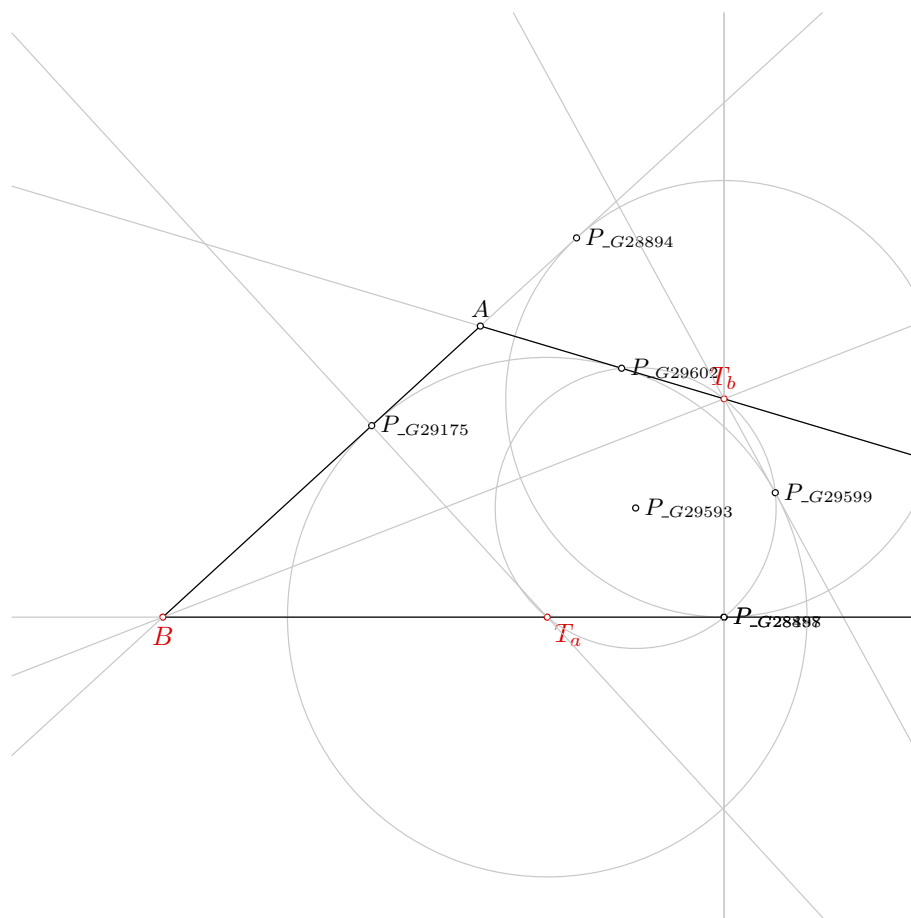


Figure 1: Illustration of the problem 0191

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_a = T_a$

Proving failed

#### 4.1.3 Proving $T_b = T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B = B$

NDG conditions are:

$S_{T_b B T_a} \neq S_{P_{G31474} B T_a}$  i.e., lines  $T_b P_{G31474}$  and  $B T_a$  are not parallel (construction based assumption)  
 $S_{B T_b P_{G31474}} \neq S_{P_{G30766} T_b P_{G31474}}$  i.e., lines  $B P_{G30766}$  and  $T_b P_{G31474}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $T_a = T_a$

Proving failed

#### 4.2.3 Proving $T_b = T_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $T_a = T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 20 terms.

**Time Complexity:** Time spent by the prover is 0.040 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_b = T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 30 terms.

**Time Complexity:** Time spent by the prover is 0.080 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $T_a=T_a$

Proving failed

### 4.4.3 Proving $T_b=T_b$

Proving failed



## Problem 192

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 192: Given a point  $B$ , a point  $T_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 193

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 193: Given a point  $B$ , a point  $T_a$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $T_a$  are not the same;
2. Using the point  $T_a$  and the point  $I$ , construct a line  $s_a$  (rule W02); % DET: points  $T_a$  and  $I$  are not the same;
3. Using the point  $I$  and the line  $a$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $a$ ;
4. Using the circle  $k(I, P_a)$ , the point  $B$ , the point  $I$  and the line  $a$ , construct a line  $c$  (rule W13); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
5. Using the line  $c$  and the line  $s_a$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $s_a$  are not parallel % DET: lines  $c$  and  $s_a$  are not the same;
6. Using the circle  $k(I, P_a)$ , the point  $A$ , the point  $I$  and the line  $c$ , construct a line  $b$  (rule W13); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
7. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(I, P_a)$ ; lines  $c$  and  $s_a$  are not parallel; point  $B$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $a$ .

Determination conditions: lines  $b$  and  $a$  are not the same; lines  $c$  and  $s_a$  are not the same; points  $T_a$  and  $I$  are not the same; points  $B$  and  $T_a$  are not the same.

Rules used: [W02,W03,W11,W13]

Lemmas used: [D2,D23,D27,GD01,L5,L59,L60,L61]

Solving time: 13.1 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point T_{a} 70.86 40
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_b B
cmark_rb T_{a}
cmark_b I
color 0 0 0
fontsize 8

% DET: points B and T_{a} are not the same
% Constructing a line a which passes through point B and point T_{a}
line a B T_{a}

color 200 200 200
drawline a
color 0 0 0

% DET: points T_{a} and I are not the same
% Constructing a line s_{a} which passes through point T_{a} and point I
line s_{a} T_{a} I

color 200 200 200
drawline s_{a}
color 0 0 0

% NDG: point I is not incident to the line a
% Constructing a point P_{\_G25567} which is a foot of the point I on the line a
foot P_{\_G25567} I a
cmark_r P_{\_G25567}
color 200 200 200
drawline I P_{\_G25567}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
25567}
circle k(I,P_{a}) I P_{\_G25567}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point B is outside the circle k(I,P_{a})
```

```

% Constructing a point  $P_{\{G25774\}}$  which is a foot of the point  $I$  on the line  $a$ 
foot  $P_{\{G25774\}}$  I a
cmark_r  $P_{\{G25774\}}$ 
color 200 200 200
drawline I  $P_{\{G25774\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G25777\}}$  which passes through point  $I$  and point  $B$ 
line  $L_{\{G25777\}}$  I B

color 200 200 200
drawline  $L_{\{G25777\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G25780\}}$  which is an image of the point  $P_{\{G25774\}}$  in the symmetry to
point/line  $L_{\{G25777\}}$ 
sim  $P_{\{G25780\}}$   $L_{\{G25777\}}$   $P_{\{G25774\}}$ 
cmark_r  $P_{\{G25780\}}$ 

% Constructing a line  $c$  which passes through point  $B$  and point  $P_{\{G25780\}}$ 
line c B  $P_{\{G25780\}}$ 

color 200 200 200
drawline c
color 0 0 0

% NDG: lines  $c$  and  $s_{\{a\}}$  are not parallel% DET: lines  $c$  and  $s_{\{a\}}$  are not the same
% Constructing a point  $A$  which belongs to line  $c$  and line  $s_{\{a\}}$ 
intersec A c  $s_{\{a\}}$ 
cmark_t A

% NDG: point  $A$  is outside the circle  $k(I, P_{\{a\}})$ 
% Constructing a point  $P_{\{G26166\}}$  which is a foot of the point  $I$  on the line  $c$ 
foot  $P_{\{G26166\}}$  I c
cmark_r  $P_{\{G26166\}}$ 
color 200 200 200
drawline I  $P_{\{G26166\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G26169\}}$  which passes through point  $I$  and point  $A$ 
line  $L_{\{G26169\}}$  I A

color 200 200 200
drawline  $L_{\{G26169\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G26172\}}$  which is an image of the point  $P_{\{G26166\}}$  in the symmetry to
point/line  $L_{\{G26169\}}$ 
sim  $P_{\{G26172\}}$   $L_{\{G26169\}}$   $P_{\{G26166\}}$ 
cmark_r  $P_{\{G26172\}}$ 

```

```

% Constructing a line b which passes through point A and point P_{\_G26172}
line b A P_{\_G26172}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and a are not parallel; point A is outside the circle k(I,P_{a
}); lines c and s_{a} are not parallel; point B is outside the circle k(I,P_{a}); point I is
not incident to the line a
% Determination conditions: lines b and a are not the same; lines c and s_{a} are not the same;
points T_{a} and I are not the same; points B and T_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_a=T_a$

Proving failed

#### 4.1.3 Proving $I=I$

Proving failed

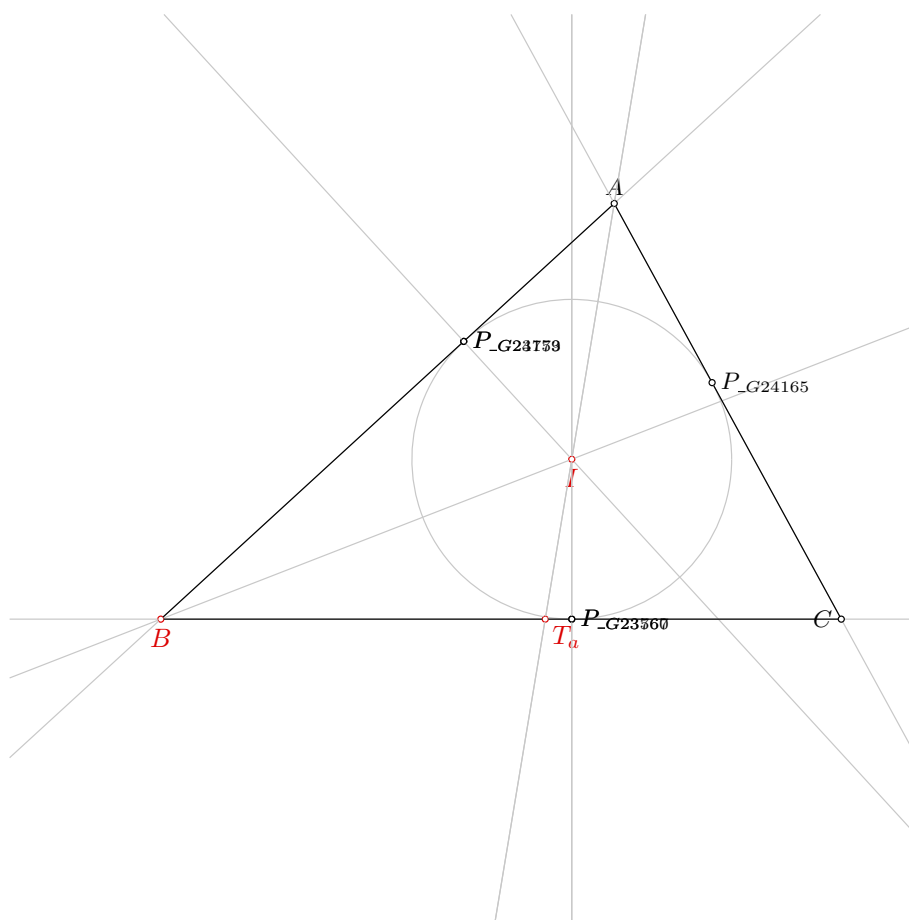


Figure 1: Illustration of the problem 0193

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{BT_aI} \neq S_{P_{G24908}T_aI}$  i.e., lines  $BP_{G24908}$  and  $T_aI$  are not parallel (construction based assumption)

$S_{ABT_a} \neq S_{P_{G25300}BT_a}$  i.e., lines  $AP_{G25300}$  and  $BT_a$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $T_a=_T T_a$

Proving failed

### 4.2.3 Proving $I=_I I$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $T_a=_T T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 16 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.3 Proving $I=_I I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 344 terms.

**Time Complexity:** Time spent by the prover is 0.160 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $T_a=_T T_a$

Proving failed

### 4.4.3 Proving $I=_I I$

Proving failed

# Problem 194

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 194: Given a point  $B$ , a point  $T_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $T_c$  are not the same;
2. Using the point  $T_b$  and the line  $c$ , construct a circle  $k(T_b, foot[T_b, c])$  (rule W11); % NDG: point  $T_b$  is not incident to the line  $c$ ;
3. Using the circle  $k(T_b, foot[T_b, c])$ , the point  $B$ , the point  $T_b$  and the line  $c$ , construct a line  $a$  (rule W13); % NDG: point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ;
4. Using the point  $T_c$  and the line  $a$ , construct a circle  $k(T_c, foot[T_c, a])$  (rule W11); % NDG: point  $T_c$  is not incident to the line  $a$ ;
5. Using the circle  $k(T_c, foot[T_c, a])$ , the point  $T_b$  and the point  $T_c$ , construct a line  $b_2$  and a line  $b$  (rule W12); % NDG: point  $T_b$  is outside the circle  $k(T_c, foot[T_c, a])$ ;
6. Using the line  $b$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $c$  are not parallel % DET: lines  $b$  and  $c$  are not the same;
7. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; lines  $b$  and  $c$  are not parallel; point  $T_b$  is outside the circle  $k(T_c, foot[T_c, a])$ ; point  $T_c$  is not incident to the line  $a$ ; point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ; point  $T_b$  is not incident to the line  $c$ .

Determination conditions: lines  $a$  and  $b$  are not the same; lines  $b$  and  $c$  are not the same; points  $B$  and  $T_c$  are not the same.

Rules used: [W02,W03,W11,W12,W13]

Lemmas used: [D24,D25,D35,GD01,GD02,GL10,GL11,L10,L9]

Solving time: 0.9 seconds.



## 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point T_{b} 94.25 68.88
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_b B
cmark_t T_{b}
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points B and T_{c} are not the same
% Constructing a line c which passes through point B and point T_{c}
line c B T_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: point T_{b} is not incident to the line c
% Constructing a point P_{\_G26394} which is a foot of the point T_{b} on the line c
foot P_{\_G26394} T_{b} c
cmark_r P_{\_G26394}
color 200 200 200
drawline T_{b} P_{\_G26394}
color 0 0 0

% Constructing a circle k(T_{b},foot[T_{b},c]) whose center is at point T_{b} and which passes
    through point P_{\_G26394}
circle k(T_{b},foot[T_{b},c]) T_{b} P_{\_G26394}

color 200 200 200
drawcircle k(T_{b},foot[T_{b},c])
color 0 0 0

% NDG: point B is outside the circle k(T_{b},foot[T_{b},c])
% Constructing a point P_{\_G26785} which is a foot of the point T_{b} on the line c
foot P_{\_G26785} T_{b} c
cmark_r P_{\_G26785}
color 200 200 200
drawline T_{b} P_{\_G26785}
color 0 0 0

% Constructing a line L_{\_G26788} which passes through point T_{b} and point B
line L_{\_G26788} T_{b} B
```

```

color 200 200 200
drawline L_{\_G26788}
color 0 0 0

% Constructing a point P_{\_G26791} which is an image of the point P_{\_G26785} in the symmetry to
point/line L_{\_G26788}
sim P_{\_G26791} L_{\_G26788} P_{\_G26785}
cmark_r P_{\_G26791}

% Constructing a line a which passes through point B and point P_{\_G26791}
line a B P_{\_G26791}

color 200 200 200
drawline a
color 0 0 0

% NDG: point T_{c} is not incident to the line a
% Constructing a point P_{\_G27072} which is a foot of the point T_{c} on the line a
foot P_{\_G27072} T_{c} a
cmark_r P_{\_G27072}
color 200 200 200
drawline T_{c} P_{\_G27072}
color 0 0 0

% Constructing a circle k(T_{c},foot[T_{c},b]) whose center is at point T_{c} and which passes
through point P_{\_G27072}
circle k(T_{c},foot[T_{c},b]) T_{c} P_{\_G27072}

color 200 200 200
drawcircle k(T_{c},foot[T_{c},b])
color 0 0 0

% NDG: point T_{b} is outside the circle k(T_{c},foot[T_{c},b])
% Constructing midpoint P_{\_G27490} of the segment T_{b}T_{c}
midpoint P_{\_G27490} T_{b} T_{c}
cmark_r P_{\_G27490}

% Constructing a circle C_{\_G27493} whose center is at point P_{\_G27490} and which passes through
point T_{b}
circle C_{\_G27493} P_{\_G27490} T_{b}

color 200 200 200
drawcircle C_{\_G27493}
color 0 0 0

% Constructing points P_{\_G27496} and P_{\_G27499} which are in intersection of C_{\_G27493} and k
(T_{c},foot[T_{c},b])
intersec2 P_{\_G27496} P_{\_G27499} C_{\_G27493} k(T_{c},foot[T_{c},b])
cmark_r P_{\_G27496}
cmark_r P_{\_G27499}

```

```
% Constructing a line b2 which passes through point T_{b} and point P_{\_G27496}
line b2 T_{b} P_{\_G27496}
```

```
color 200 200 200
drawline b2
color 0 0 0
```

```
% Constructing a line b which passes through point T_{b} and point P_{\_G27499}
line b T_{b} P_{\_G27499}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% NDG: lines b and c are not parallel% DET: lines b and c are not the same
% Constructing a point A which belongs to line b and line c
intersec A b c
cmark_t A
```

```
% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_l C
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines a and b are not parallel; lines b and c are not parallel; point
T_{b} is outside the circle k(T_{c},foot[T_{c},b]); point T_{c} is not incident to the line a;
point B is outside the circle k(T_{b},foot[T_{b},c]); point T_{b} is not incident to the line c
% Determination conditions: lines a and b are not the same; lines b and c are not the same; points
B and T_{c} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

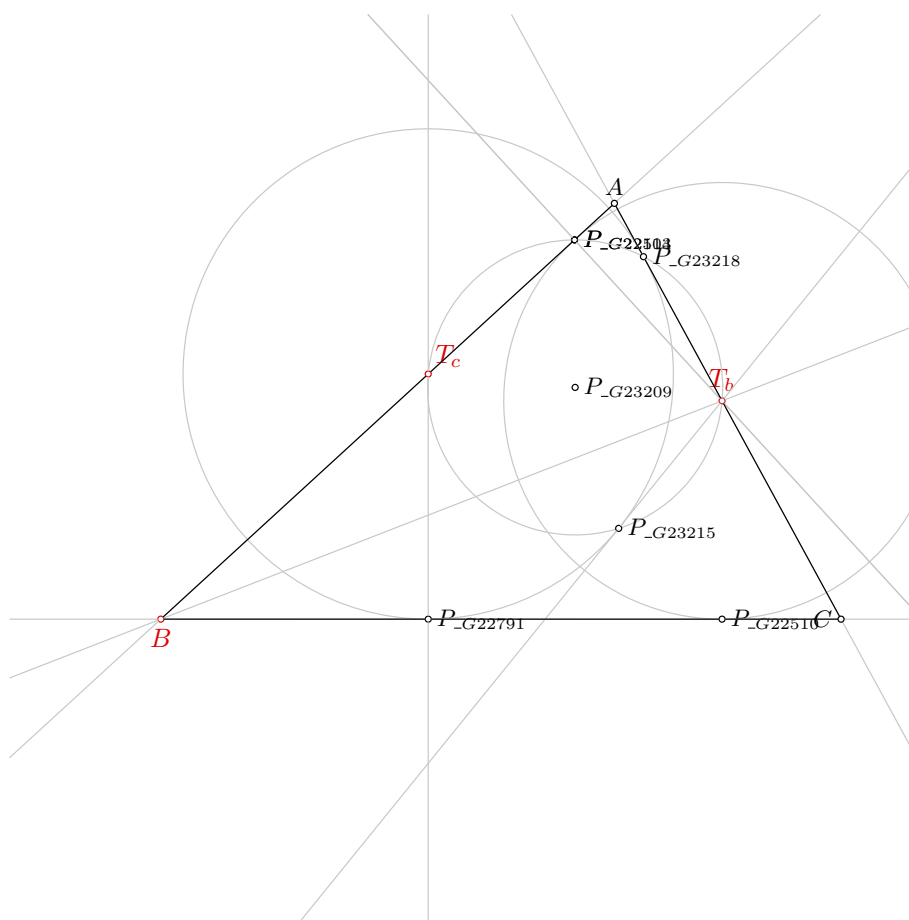


Figure 1: Illustration of the problem 0194

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.1.3 Proving $T_c = \neg T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $B = B$

NDG conditions are:

$S_{T_b B T_c} \neq S_{P_{G25093} B T_c}$  i.e., lines  $T_b P_{G25093}$  and  $B T_c$  are not parallel (construction based assumption)  
 $S_{B T_b P_{G25093}} \neq S_{P_{G24385} T_b P_{G25093}}$  i.e., lines  $B P_{G24385}$  and  $T_b P_{G25093}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.2.3 Proving $T_c = \neg T_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $B = B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

#### 4.3.2 Proving $T_b = \neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 178 terms.

**Time Complexity:** Time spent by the prover is 0.490 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_c = \neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 108 terms.

**Time Complexity:** Time spent by the prover is 0.290 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

### 4.4.2 Proving $T_b=\neg T_b$

Proving failed

### 4.4.3 Proving $T_c=\neg T_c$

Proving failed

# Problem 195

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 195: Given a point  $T_b$ , a point  $I$  and a point  $B$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $T_b$  and the point  $I$ , construct a line  $s_b$  (rule W02); % DET: points  $T_b$  and  $I$  are not the same;
2. Choose freely a point  $B$  on the line  $s_b$  (rule WOnline1) ;
3. Using the point  $B$  and the point  $I$ , construct a line  $m(BI)$  (rule W14); % DET: points  $B$  and  $I$  are not the same;
4. Choose freely a point  $A$  (rule free);
5. Using the point  $A$  and the point  $B$ , construct a point  $M_c$  (rule W01); ;
6. Using the point  $A$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $B$  are not the same;
7. Using the point  $A$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $T_b$  are not the same;
8. Using the point  $M_c$  and the line  $c$ , construct a line  $m_c$  (rule W10b); ;
9. Using the line  $m_c$  and the line  $m(BI)$ , construct a point  $N_c$  (rule W03); % NDG: lines  $m_c$  and  $m(BI)$  are not parallel % DET: lines  $m_c$  and  $m(BI)$  are not the same;
10. Using the point  $N_c$  and the point  $I$ , construct a line  $s_c$  (rule W02); % DET: points  $N_c$  and  $I$  are not the same;
11. Using the line  $b$  and the line  $s_c$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $s_c$  are not parallel % DET: lines  $b$  and  $s_c$  are not the same.

Non-degenerate conditions: lines  $b$  and  $s_c$  are not parallel; lines  $m_c$  and  $m(BI)$  are not parallel.

Determination conditions: lines  $b$  and  $s_c$  are not the same; points  $N_c$  and  $I$  are not the same; lines  $m_c$  and  $m(BI)$  are not the same; points  $A$  and  $T_b$  are not the same; points  $A$  and  $B$  are not the same; points  $B$  and  $I$  are not the same; points  $T_b$  and  $I$  are not the same.

Rules used: [W01,W02,W03,W10b,W14,WOnline1,free]

Lemmas used: [D13,D2,D20,D24,D49,GD01,GD02,GL01,L2,L27,L36,L7]

Solving time: 5.8 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point T_{b} 94.25 68.88
point I 74.37 61.15
point B 20 40

color 220 0 0
fontsize 9

cmark_t T_{b}
cmark_b I
cmark_b B
color 0 0 0
fontsize 8

% DET: points T_{b} and I are not the same
% Constructing a line s_{b} which passes through point T_{b} and point I
line s_{b} T_{b} I

color 200 200 200
drawline s_{b}
color 0 0 0

% Choosing randomly a point B on the line T_{b}I
online B T_{b} I
cmark_b B
color 200 200 200
drawline T_{b} I
color 0 0 0

% DET: points B and I are not the same
% Constructing bisector m(BI) of the segment BI
med m(BI) B I

color 200 200 200
drawline m(BI)
color 0 0 0

color 200 200 200
drawsegment B I
```



```
color 0 0 0
```

```
% Constructing a free point A  
point A 80 95
```

```
cmark_t A
```

```
% Constructing a point  $M_{\{c\}}$  such that  $AM_{\{c\}}/AB=0.5$   
towards  $M_{\{c\}}$  A B 0.5  
cmark_lt  $M_{\{c\}}$   
color 200 200 200  
drawsegment A B  
color 0 0 0
```

```
% DET: points A and B are not the same  
% Constructing a line c which passes through point A and point B  
line c A B
```

```
color 200 200 200  
drawline c  
color 0 0 0
```

```
% DET: points A and  $T_{\{b\}}$  are not the same  
% Constructing a line b which passes through point A and point  $T_{\{b\}}$   
line b A  $T_{\{b\}}$ 
```

```
color 200 200 200  
drawline b  
color 0 0 0
```

```
% Constructing a line  $m_{\{c\}}$  which is perpendicular to line c and which passes through point  $M_{\{c\}}$   
perp  $m_{\{c\}}$   $M_{\{c\}}$  c
```

```
color 200 200 200  
drawline  $m_{\{c\}}$   
color 0 0 0
```

```
% NDG: lines  $m_{\{c\}}$  and  $m(BI)$  are not parallel  
% DET: lines  $m_{\{c\}}$  and  $m(BI)$  are not the same  
% Constructing a point  $N_{\{c\}}$  which belongs to line  $m_{\{c\}}$  and line  $m(BI)$   
intersec  $N_{\{c\}}$   $m_{\{c\}}$   $m(BI)$   
cmark_b  $N_{\{c\}}$ 
```

```
% DET: points  $N_{\{c\}}$  and I are not the same
```

```

% Constructing a line s_{c} which passes through point N_{c} and point I
line s_{c} N_{c} I

color 200 200 200
drawline s_{c}
color 0 0 0

% NDG: lines b and s_{c} are not parallel% DET: lines b and s_{c} are not the same
% Constructing a point C which belongs to line b and line s_{c}
intersec C b s_{c}
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and s_{c} are not parallel; lines m_{c} and m(BI) are not
parallel
% Determination conditions: lines b and s_{c} are not the same; points N_{c} and I are not the same
; lines m_{c} and m(BI) are not the same; points A and T_{b} are not the same; points A and B
are not the same; points B and I are not the same; points T_{b} and I are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $T_b = \_T_b$

Proving failed

#### 4.1.2 Proving $I = \_I$

Proving failed

#### 4.1.3 Proving $B = B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.013 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $T_b = \_T_b$

Proving failed

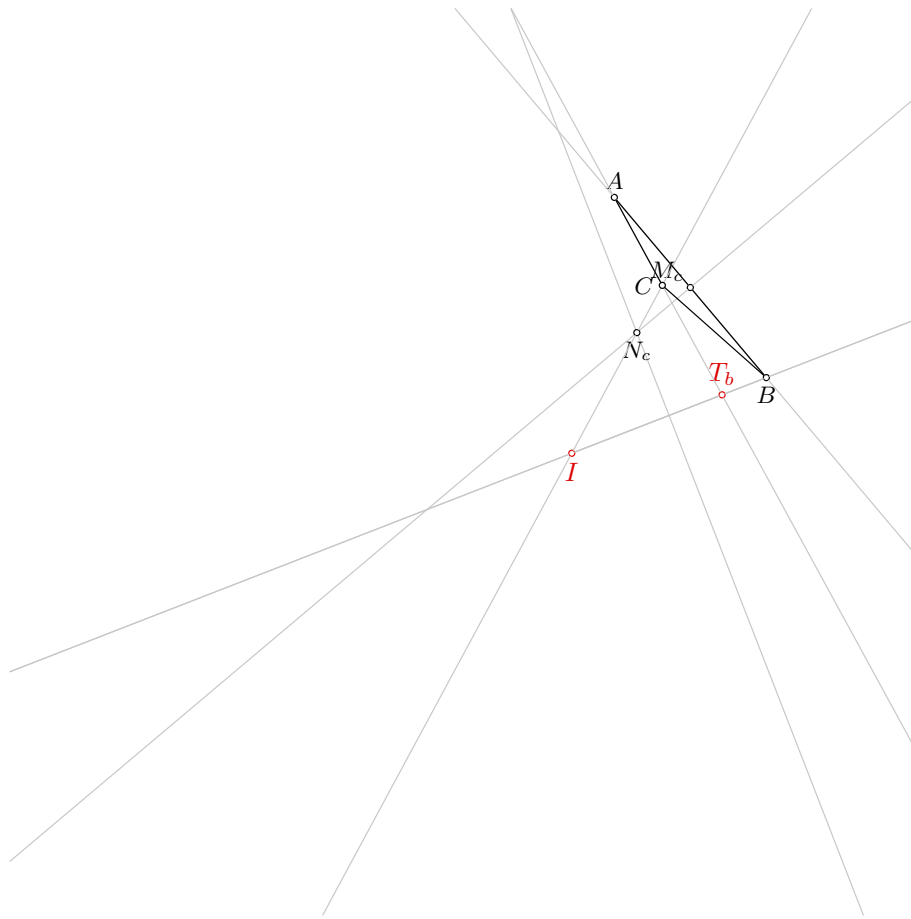


Figure 1: Illustration of the problem 0195

#### 4.2.2 Proving $I=_I$

Proving failed

#### 4.2.3 Proving $B=B$

NDG conditions are:

$S_{M_cAB} \neq 0$  i.e., points  $M_c$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_cM_{m(BI)}^1T_{m(BI)}^2} \neq S_{F_{m_c}^3M_{m(BI)}^1T_{m(BI)}^2}$  i.e., lines  $M_cF_{m_c}^3$  and  $M_{m(BI)}^1T_{m(BI)}^2$  are not parallel (construction based assumption)

$S_{AN_cI} \neq S_{T_bN_cI}$  i.e., lines  $AT_b$  and  $N_cI$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $T_b=_T T_b$

Proving failed

#### 4.3.2 Proving $I=_I$

Proving failed

#### 4.3.3 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $T_b=_T T_b$

Proving failed

#### 4.4.2 Proving $I=_I$

Proving failed

#### 4.4.3 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

# Problem 196

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 196: Given a point  $B$ , a point  $T_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $B$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $T_c$  are not the same;
2. Using the point  $T_c$  and the point  $I$ , construct a line  $s_c$  (rule W02); % DET: points  $T_c$  and  $I$  are not the same;
3. Using the point  $I$  and the line  $c$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $c$ ;
4. Using the circle  $k(I, P_a)$ , the point  $B$ , the point  $I$  and the line  $c$ , construct a line  $a$  (rule W13); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
5. Using the line  $a$  and the line  $s_c$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $s_c$  are not parallel % DET: lines  $a$  and  $s_c$  are not the same;
6. Using the circle  $k(I, P_a)$ , the point  $C$ , the point  $I$  and the line  $a$ , construct a line  $b$  (rule W13); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
7. Using the line  $b$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $c$  are not parallel % DET: lines  $b$  and  $c$  are not the same.

Non-degenerate conditions: lines  $b$  and  $c$  are not parallel; point  $C$  is outside the circle  $k(I, P_a)$ ; lines  $a$  and  $s_c$  are not parallel; point  $B$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $c$ .

Determination conditions: lines  $b$  and  $c$  are not the same; lines  $a$  and  $s_c$  are not the same; points  $T_c$  and  $I$  are not the same; points  $B$  and  $T_c$  are not the same.

Rules used: [W02,W03,W11,W13]

Lemmas used: [D25,D27,GD01,L2,L59,L60,L61,L7]

Solving time: 13.2 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point T_{c} 55.38 72.43
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_b B
cmark_rt T_{c}
cmark_b I
color 0 0 0
fontsize 8

% DET: points B and T_{c} are not the same
% Constructing a line c which passes through point B and point T_{c}
line c B T_{c}

color 200 200 200
drawline c
color 0 0 0

% DET: points T_{c} and I are not the same
% Constructing a line s_{c} which passes through point T_{c} and point I
line s_{c} T_{c} I

color 200 200 200
drawline s_{c}
color 0 0 0

% NDG: point I is not incident to the line c
% Constructing a point P_{\_G26489} which is a foot of the point I on the line c
foot P_{\_G26489} I c
cmark_r P_{\_G26489}
color 200 200 200
drawline I P_{\_G26489}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
26489}
circle k(I,P_{a}) I P_{\_G26489}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point B is outside the circle k(I,P_{a})
```

```

% Constructing a point  $P_{\{G26696\}}$  which is a foot of the point  $I$  on the line  $c$ 
foot  $P_{\{G26696\}}$  I c
cmark_r  $P_{\{G26696\}}$ 
color 200 200 200
drawline I  $P_{\{G26696\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G26699\}}$  which passes through point  $I$  and point  $B$ 
line  $L_{\{G26699\}}$  I B

color 200 200 200
drawline  $L_{\{G26699\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G26702\}}$  which is an image of the point  $P_{\{G26696\}}$  in the symmetry to
point/line  $L_{\{G26699\}}$ 
sim  $P_{\{G26702\}}$   $L_{\{G26699\}}$   $P_{\{G26696\}}$ 
cmark_r  $P_{\{G26702\}}$ 

% Constructing a line  $a$  which passes through point  $B$  and point  $P_{\{G26702\}}$ 
line a B  $P_{\{G26702\}}$ 

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $a$  and  $s_{\{c\}}$  are not parallel% DET: lines  $a$  and  $s_{\{c\}}$  are not the same
% Constructing a point  $C$  which belongs to line  $a$  and line  $s_{\{c\}}$ 
intersec C a  $s_{\{c\}}$ 
cmark_l C

% NDG: point  $C$  is outside the circle  $k(I, P_{\{a\}})$ 
% Constructing a point  $P_{\{G27088\}}$  which is a foot of the point  $I$  on the line  $a$ 
foot  $P_{\{G27088\}}$  I a
cmark_r  $P_{\{G27088\}}$ 
color 200 200 200
drawline I  $P_{\{G27088\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G27091\}}$  which passes through point  $I$  and point  $C$ 
line  $L_{\{G27091\}}$  I C

color 200 200 200
drawline  $L_{\{G27091\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G27094\}}$  which is an image of the point  $P_{\{G27088\}}$  in the symmetry to
point/line  $L_{\{G27091\}}$ 
sim  $P_{\{G27094\}}$   $L_{\{G27091\}}$   $P_{\{G27088\}}$ 
cmark_r  $P_{\{G27094\}}$ 

```

```

% Constructing a line b which passes through point C and point P_{\_G27094}
line b C P_{\_G27094}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines b and c are not parallel% DET: lines b and c are not the same
% Constructing a point A which belongs to line b and line c
intersec A b c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and c are not parallel; point C is outside the circle k(I,P_{a
}); lines a and s_{c} are not parallel; point B is outside the circle k(I,P_{a}); point I is
not incident to the line c
% Determination conditions: lines b and c are not the same; lines a and s_{c} are not the same;
points T_{c} and I are not the same; points B and T_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $B=B$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_c = \_T_c$

Proving failed

#### 4.1.3 Proving $I = \_I$

Proving failed



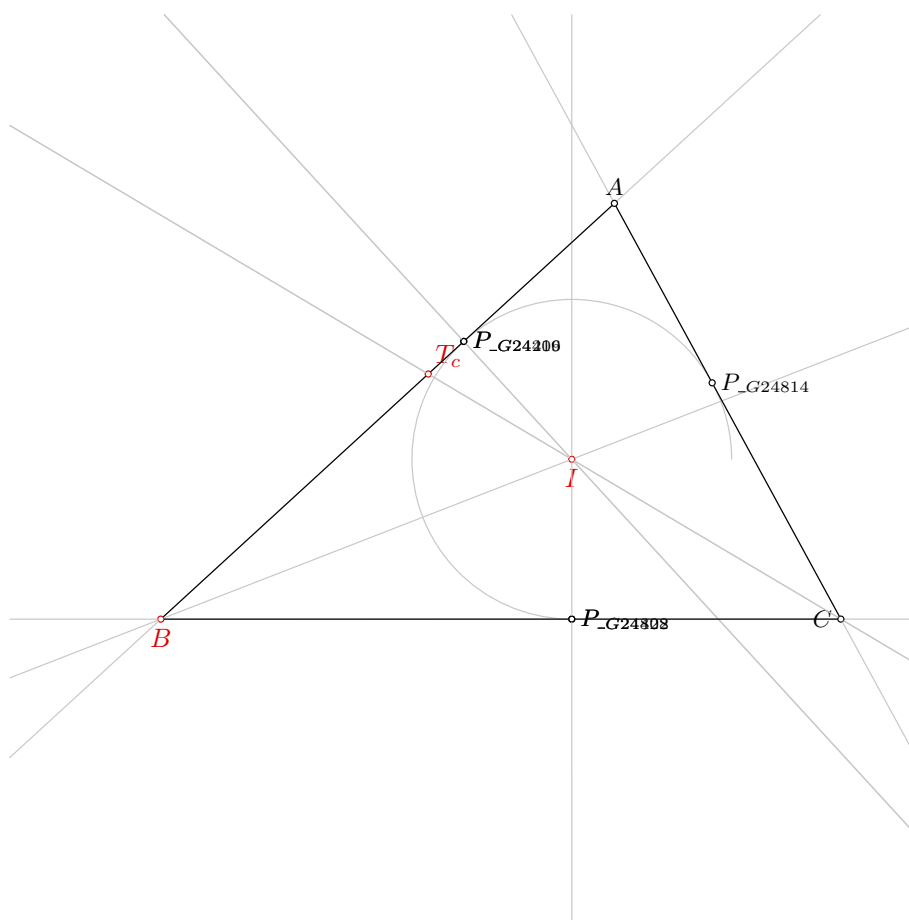


Figure 1: Illustration of the problem 0196

## 4.2 GCLC - Area method

### 4.2.1 Proving $B=B$

NDG conditions are:

$S_{BT_cI} \neq S_{P_{G25560}T_cI}$  i.e., lines  $BP_{G25560}$  and  $T_cI$  are not parallel (construction based assumption)

$S_{CBT_c} \neq S_{P_{G25952}BT_c}$  i.e., lines  $CP_{G25952}$  and  $BT_c$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $T_c=\neg T_c$

Proving failed

### 4.2.3 Proving $I=\neg I$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $T_c=\neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 16 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

### 4.3.3 Proving $I=\neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 36 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $B=B$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $T_c=\neg T_c$

Proving failed

### 4.4.3 Proving $I=\neg I$

Proving failed

# Problem 197

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 197: Given a point  $O$ , a point  $M_a$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $M_a$ , construct a line  $m_a$  (rule W02); % DET: points  $O$  and  $M_a$  are not the same;
2. Using the point  $M_a$  and the line  $m_a$ , construct a line  $a$  (rule W10a); ;
3. Choose freely a point  $C$  on the line  $a$  (rule WOnline2);
4. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
5. Choose freely a point  $A$  on the circle  $k(O, C)$  (rule WOncircle).

Non-degenerate conditions: .

Determination conditions: points  $O$  and  $M_a$  are not the same.

Rules used: [W01,W02,W10a,WOncircle1,WOnline2]

Lemmas used: [D1,D11,D21,GD01,GL03,GL04,GL09]

Solving time: 269.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point O 65 51.14
point M_{a} 65 40
point C 110 40
```

```
color 220 0 0
fontsize 9
```

```

cmark_t 0
cmark_r M_{a}
cmark_l C
color 0 0 0
fontsize 8

% DET: points O and M_{a} are not the same
% Constructing a line m_{a} which passes through point O and point M_{a}
line m_{a} 0 M_{a}

color 200 200 200
drawline m_{a}
color 0 0 0

% Constructing a line a which is perpendicular to line m_{a} and which passes through point M_{a}
perp a M_{a} m_{a}

color 200 200 200
drawline a
color 0 0 0

% Generating random value V[_G31252]
random V[_G31252]

% Calculating value V[_G31273] using formula  $V[_G31252]*20$ 
expression V[_G31273] { V[_G31252]*20 }

% Constructing a point C which is a point for which holds  $M_{a}C = V[_G31273]$  and angle  $OM_{a}C = 90$ 
turtle C 0 M_{a} 90 V[_G31273]
cmark_l C

% Constructing a point B such that  $CB/CM_{a}=2$ 
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% Choosing randomly a point A on the circle with center O through point C
oncircle A 0 C
cmark_t A
color 200 200 200
drawcircle 0 C

```

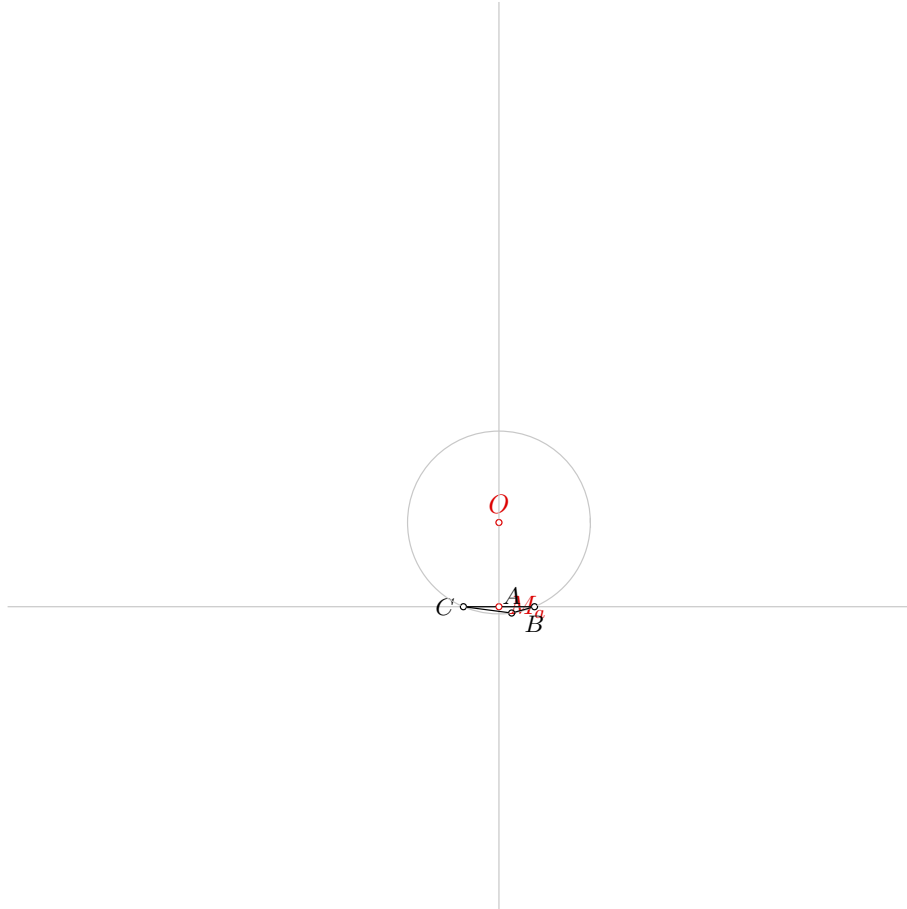


Figure 1: Illustration of the problem 0197

```
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions:
% Determination conditions: points O and M_{a} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 39 terms.

**Time Complexity:** Time spent by the prover is 0.365 seconds.

**NDG conditions** Points  $B$ ,  $C$  and  $O$  are not collinear

Line through points  $O$  and  $M_a$  is not perpendicular to line through points  $M_a$  and  $A$

#### 4.1.2 Proving $M_a=_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.024 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.02 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $O=_O$

Proving failed

#### 4.2.2 Proving $M_a=_M_a$

Proving failed

#### 4.2.3 Proving $C=C$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $O=_O$

Proving failed

#### 4.3.2 Proving $M_a=_M_a$

Proving failed

#### 4.3.3 Proving $C=C$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $O=_O$

Proving failed

### 4.4.2 Proving $M_a=_M a$

Proving failed

### 4.4.3 Proving $C=C$

Proving failed

# Problem 198

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 198: Given a point  $O$ , a point  $M_b$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $M_b$ , construct a line  $m_b$  (rule W02); % DET: points  $O$  and  $M_b$  are not the same;
2. Using the point  $M_b$  and the line  $m_b$ , construct a line  $b$  (rule W10a); ;
3. Choose freely a point  $C$  on the line  $b$  (rule WOnline2);
4. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); ;
5. Choose freely a point  $B$  on the circle  $k(O, C)$  (rule WOncircle).

Non-degenerate conditions: .

Determination conditions: points  $O$  and  $M_b$  are not the same.

Rules used: [W01,W02,W10a,WOncircle1,WOnline2]

Lemmas used: [D1,D12,D22,GD01,GL03,GL09]

Solving time: 271.7 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point O 65 51.14
point M_{b} 95 67.5
point C 110 40
```

```
color 220 0 0
fontsize 9
```



```

cmark_t O
cmark_lt M_{b}
cmark_l C
color 0 0 0
fontsize 8

% DET: points O and M_{b} are not the same
% Constructing a line m_{b} which passes through point O and point M_{b}
line m_{b} O M_{b}

color 200 200 200
drawline m_{b}
color 0 0 0

% Constructing a line b which is perpendicular to line m_{b} and which passes through point M_{b}
perp b M_{b} m_{b}

color 200 200 200
drawline b
color 0 0 0

% Generating random value V[_G24769]
random V[_G24769]

% Calculating value V[_G24790] using formula  $V[_G24769]*20$ 
expression V[_G24790] { V[_G24769]*20 }

% Constructing a point C which is a point for which holds  $M_{b}C = V[_G24790]$  and angle  $OM_{b}C = 90$ 
turtle C O M_{b} 90 V[_G24790]
cmark_l C

% Constructing a point A such that  $CA/CM_{b}=2$ 
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

% Choosing randomly a point B on the circle with center O through point C
oncircle B O C
cmark_b B
color 200 200 200
drawcircle O C

```

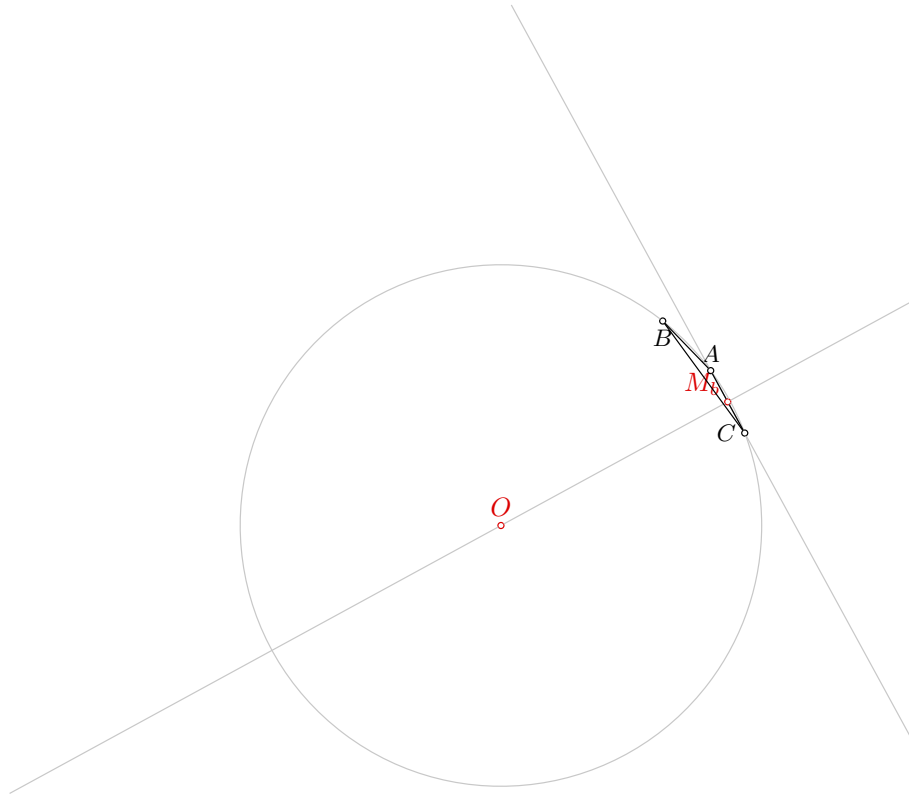


Figure 1: Illustration of the problem 0198

```
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions:
% Determination conditions: points O and M_{b} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 39 terms.

**Time Complexity:** Time spent by the prover is 0.356 seconds.

**NDG conditions** Line through points  $M_b$  and  $C$  is not perpendicular to line through points  $C$  and  $A$   
Line through points  $M_b$  and  $A$  is not perpendicular to line through points  $A$  and  $O$

#### 4.1.2 Proving $M_b=_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.041 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.009 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $O=_O$

Proving failed

#### 4.2.2 Proving $M_b=_M_b$

Proving failed

#### 4.2.3 Proving $C=C$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $O=_O$

Proving failed

#### 4.3.2 Proving $M_b=_M_b$

Proving failed

#### 4.3.3 Proving $C=C$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $O=_O$

Proving failed

### 4.4.2 Proving $M_b=_M M_b$

Proving failed

### 4.4.3 Proving $C=C$

Proving failed

# Problem 199

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 199: Given a point  $C$ , a point  $O$  and a point  $M_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_c$ , construct a point  $G$  (rule W01); ;
2. Using the point  $O$  and the point  $G$ , construct a point  $H$  (rule W01); ;
3. Using the point  $C$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H$  are not the same;
4. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
5. Using the point  $M_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same.

Determination conditions: points  $C$  and  $H$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D10,D20,D26,GD01,GL03,GL04,GL09,L11,L12,L3,L57,L58]

Solving time: 1.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
```

```
point O 65 51.14
```

```
point M_{c} 50 67.5
```

```

color 220 0 0
fontsize 9

cmark_l C
cmark_t 0
cmark_lt M_{c}
color 0 0 0
fontsize 8

% Constructing a line L_{\_G20250} which passes through point C and point M_{c}
line L_{\_G20250} C M_{c}

color 200 200 200
drawline L_{\_G20250}
color 0 0 0

% Constructing a point P_{\_G20351} with coordinates (0,0)
point P_{\_G20351} 0 0
cmark_r P_{\_G20351}

% Constructing a point P_{\_G20275} such that CP_{\_G20275}/CP_{\_G20351}=2
towards P_{\_G20275} C P_{\_G20351} 2
cmark_r P_{\_G20275}
color 200 200 200
drawsegment C P_{\_G20275}
color 0 0 0

% Constructing a point P_{\_G20320} such that CP_{\_G20320}/CP_{\_G20351}=3
towards P_{\_G20320} C P_{\_G20351} 3
cmark_r P_{\_G20320}
color 200 200 200
drawsegment C P_{\_G20320}
color 0 0 0

% Constructing a line L_{\_G20281} which passes through point M_{c} and point P_{\_G20320}
line L_{\_G20281} M_{c} P_{\_G20320}

color 200 200 200
drawline L_{\_G20281}
color 0 0 0

% Constructing a line L_{\_G20244} which contains the point P_{\_G20275} and is parallel to the
line L_{\_G20281}
parallel L_{\_G20244} P_{\_G20275} L_{\_G20281}

color 200 200 200
drawline L_{\_G20244}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G20244} and line L_{\_G20250}
intersec G L_{\_G20244} L_{\_G20250}

```

```

cmark_t G

% Constructing a point H such that  $OH/OG=3$ 
towards H O G 3
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0

% DET: points C and H are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point C and point H
line  $h_{\{c\}}$  C H

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point C
circle  $k(O,C)$  O C

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% Constructing a line c which is perpendicular to line  $h_{\{c\}}$  and which passes through point  $M_{\{c\}}$ 
perp c  $M_{\{c\}}$   $h_{\{c\}}$ 

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle  $k(O,C)$  intersect
% Constructing points A and B which are in intersection of  $k(O,C)$  and c
intersec2 A B  $k(O,C)$  c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle  $k(O,C)$  intersect; points C and O are not the same
% Determination conditions: points C and H are not the same

```

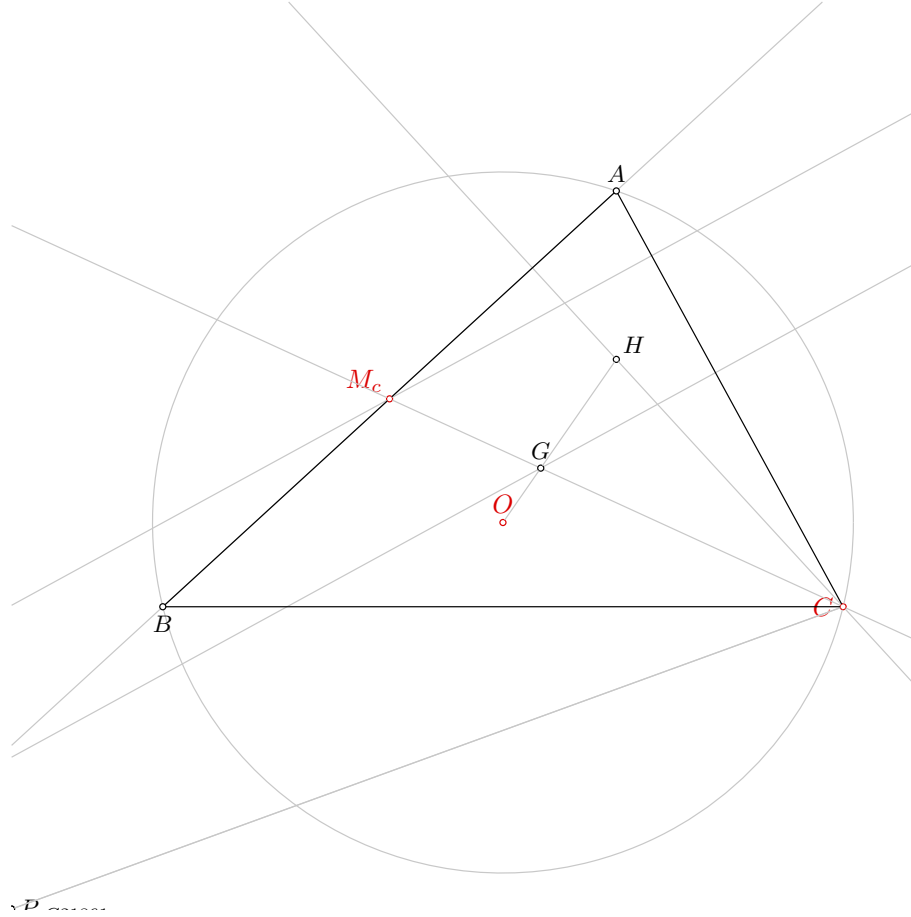


Figure 1: Illustration of the problem 0199

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.033 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

**Status:** Theorem has been proved.



**Space Complexity:** The biggest polynomial obtained during prover execution contains 414 terms.

**Time Complexity:** Time spent by the prover is 1.968 seconds.

**NDG conditions** Points  $C$ ,  $M_c$  and  $P_{G29693}$  are not collinear

Line through points  $C$  and  $O$  is not parallel with line through points  $M_c$  and  $P_{G29693}$

Points  $C$  and  $H$  are not identical

Points  $C$  and  $H$  are not identical

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $B$  and  $C$  are not identical

#### 4.1.3 Proving $M_c = \neg M_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C = C$

NDG conditions are:

$S_{P_{G22818}CM_c} \neq S_{P_{L_{G22787}}^0 CM_c}$  i.e., lines  $P_{G22818}P_{L_{G22787}}^0$  and  $CM_c$  are not parallel (construction based assumption)

$S_{M_cCH} \neq 0$  i.e., points  $M_c$ ,  $C$  and  $H$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aBC} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aM_bF_{M_b}^3} \neq S_{F_{M_a}^2M_bF_{M_b}^3}$  i.e., lines  $M_aF_{M_a}^2$  and  $M_bF_{M_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O = \neg O$

Proving failed

#### 4.2.3 Proving $M_c = \neg M_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

#### 4.3.2 Proving $O = \neg O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 11043 terms.

**Time Complexity:** Time spent by the prover is 45.150 seconds. There are no ndg conditions.

#### 4.3.3 Proving $M_c = \neg M_c$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4.2 Proving $O=_O$

Proving failed

### 4.4.3 Proving $M_c=_M_c$

Proving failed

# Problem 200

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 200: Given a point  $C$ , a point  $O$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $G$ , construct a point  $M_c$  (rule W01); ;
2. Using the point  $O$  and the point  $G$ , construct a point  $H$  (rule W01); ;
3. Using the point  $C$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H$  are not the same;
4. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
5. Using the point  $M_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same.

Determination conditions: points  $C$  and  $H$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D10,D20,D26,GD01,GL03,GL04,GL09,L11,L12,L3,L57,L58]

Solving time: 1.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point O 65 51.14
point G 70 58.33
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_t O
cmark_t G
color 0 0 0
fontsize 8
```

```
% Constructing a point M_{c} such that CM_{c}/CG=1.5
towards M_{c} C G 1.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0
```

```
% Constructing a point H such that OH/OG=3
towards H O G 3
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0
```

```
% DET: points C and H are not the same
% Constructing a line h_{c} which passes through point C and point H
line h_{c} C H

color 200 200 200
drawline h_{c}
color 0 0 0
```

```
% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0
```

```
% Constructing a line c which is perpendicular to line h_{c} and which passes through point M_{c}
perp c M_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0
```

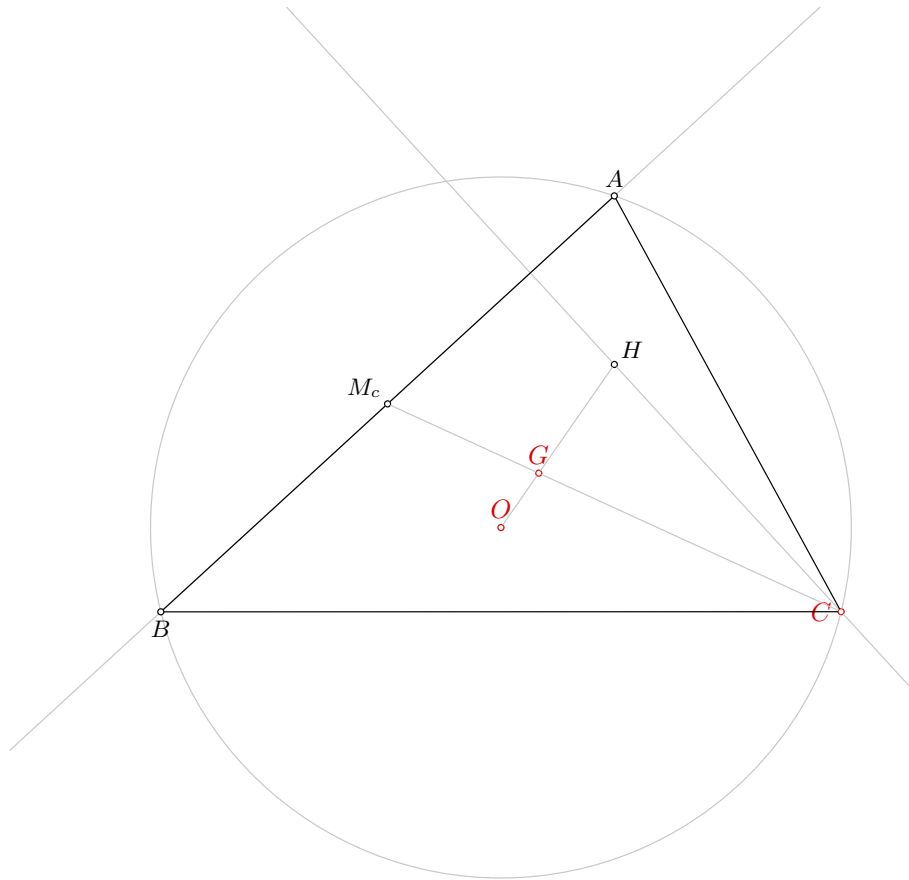


Figure 1: Illustration of the problem 0200

```
% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same
% Determination conditions: points C and H are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 1081 terms.

**Time Complexity:** Time spent by the prover is 3.114 seconds.

**NDG conditions** Points  $C$  and  $H$  are not identical

Points  $C$  and  $H$  are not identical

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $B$  and  $C$  are not identical

#### 4.1.3 Proving $G=_G$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{M_cCH} \neq 0$  i.e., points  $M_c$ ,  $C$  and  $H$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_aBC} \neq 0$  i.e., points  $_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$  i.e., points  $_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_aM_bF_{_m_b}^2} \neq S_{F_{_m_a}^1M_bF_{_m_b}^2}$  i.e., lines  $_M_aF_{_m_a}^1$  and  $_M_bF_{_m_b}^2$  are not parallel (construction based assumption)

$S_{ABM_b} \neq S_{_M_aB_M_b}$  i.e., lines  $A_M_a$  and  $B_M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $G=_G$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $O=_O$

Proving failed

#### 4.3.3 Proving $G=_G$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $O=_O$

Proving failed

#### 4.4.3 Proving $G=_G$

Proving failed

# Problem 201

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 201: Given a point  $C$ , a point  $O$  and a point  $H_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $H_a$  are not the same;
2. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
3. Using the circle  $k(O, C)$ , the line  $a$ , the point  $O$  and the point  $C$ , construct a point  $B$  (rule W05); % NDG: line  $a$  and circle  $k(O, C)$  intersect % DET: points  $C$  and  $B$  must be different;
4. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
5. Using the circle  $k(O, C)$  and the line  $h_a$ , construct a point  $A_k$  and a point  $A$  (rule W04); % NDG: line  $h_a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $h_a$  and circle  $k(O, C)$  intersect; line  $a$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same.

Determination conditions: points  $C$  and  $B$  must be different; points  $C$  and  $H_a$  are not the same.

Rules used: [W02,W04,W05,W06,W10b]

Lemmas used: [D26,D38,D5,D8,GD01,L11,L12]

Solving time: 4.5 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
```

```
point O 65 51.14
```



```

point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_l C
cmark_t 0
cmark_r H_{a}
color 0 0 0
fontsize 8

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect% DET: points C and B must be different
% Constructing a point P_{\_G24223} which is a foot of the point O on the line a
foot P_{\_G24223} O a
cmark_r P_{\_G24223}
color 200 200 200
drawline O P_{\_G24223}
color 0 0 0

% Constructing a point B which is an image of the point C in the symmetry to point/line P_{\_G
24223}
sim B P_{\_G24223} C
cmark_b B

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{a} and circle k(O,C) intersect

```

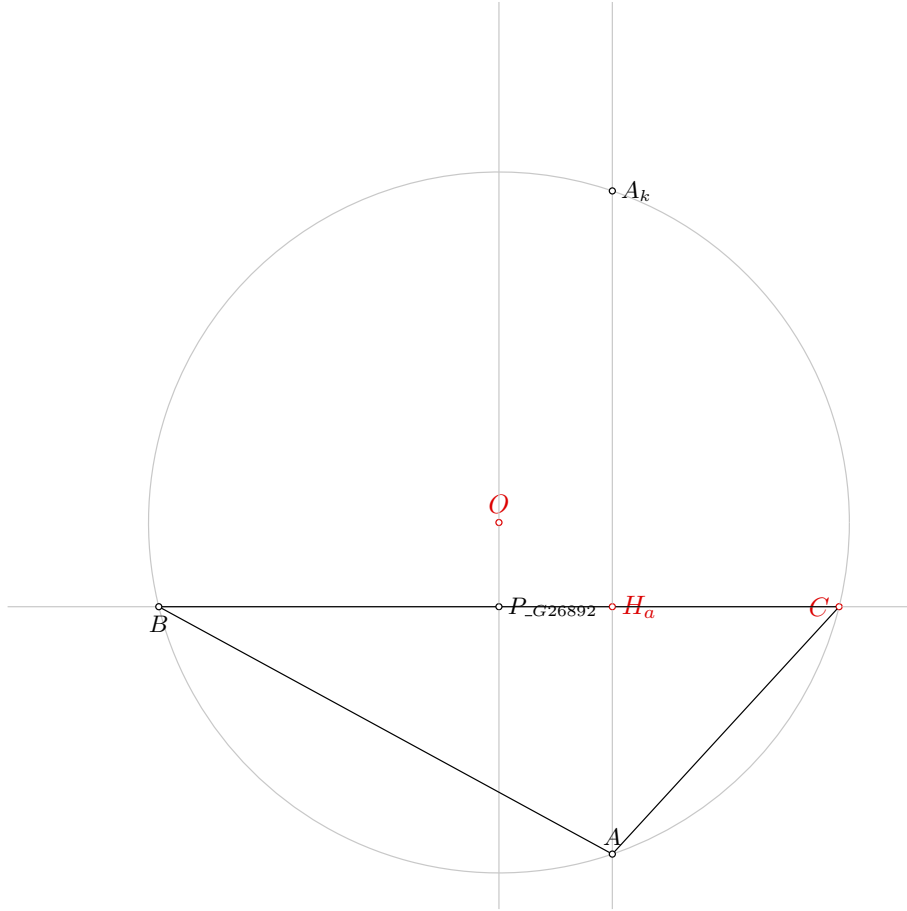


Figure 1: Illustration of the problem 0201

```
% Constructing points A_{k} and A which are in intersection of k(O,C) and h_{a}
intersec2 A_{k} A k(O,C) h_{a}
cmark_r A_{k}
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line h_{a} and circle k(O,C) intersect; line a and circle k(O,C)
    intersect; points C and O are not the same
% Determination conditions: points C and B must be different; points C and H_{a} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 37 terms.

**Time Complexity:** Time spent by the prover is 0.652 seconds.

**NDG conditions** Points  $C$ ,  $O$  and  $H_a$  are not collinear

Points  $C$ ,  $O$  and  $H_a$  are not collinear

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $B$  and  $C$  are not identical

#### 4.1.3 Proving $H_a=_H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 17 terms.

**Time Complexity:** Time spent by the prover is 0.357 seconds.

**NDG conditions** Points  $C$ ,  $O$  and  $H_a$  are not collinear

Points  $C$ ,  $O$  and  $H_a$  are not collinear

Points  $B$  and  $C$  are not identical

Points  $B$  and  $C$  are not identical

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{_M_aBC} \neq 0$  i.e., points  $_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$  i.e., points  $_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a_M_bF_{_m_b}^2} \neq S_{F_{_m_a}^1_M_bF_{_m_b}^2}$  i.e., lines  $_M_aF_{_m_a}^1$  and  $_M_bF_{_m_b}^2$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{_h_a}^3} \neq S_{CAF_{_h_a}^3}$  i.e., lines  $BC$  and  $AF_{_h_a}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $H_a=_H_a$

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $C=C$**

Proving failed

#### **4.3.2 Proving $O=_O$**

Proving failed

#### **4.3.3 Proving $H_a=_H_a$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $C=C$**

Proving failed

#### **4.4.2 Proving $O=_O$**

Proving failed

#### **4.4.3 Proving $H_a=_H_a$**

Proving failed

# Problem 202

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 202: Given a point  $C$ , a point  $O$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $H_b$  are not the same;
2. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
3. Using the circle  $k(O, C)$ , the line  $b$ , the point  $O$  and the point  $C$ , construct a point  $A$  (rule W05); % NDG: line  $b$  and circle  $k(O, C)$  intersect % DET: points  $C$  and  $A$  must be different;
4. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
5. Using the circle  $k(O, C)$  and the line  $h_b$ , construct a point  $B_k$  and a point  $B$  (rule W04); % NDG: line  $h_b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $h_b$  and circle  $k(O, C)$  intersect; line  $b$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same.

Determination conditions: points  $C$  and  $A$  must be different; points  $C$  and  $H_b$  are not the same.

Rules used: [W02,W04,W05,W06,W10b]

Lemmas used: [D26,D39,D6,D9,GD01,L11,L12]

Solving time: 4.5 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
```

```
point O 65 51.14
```

```

point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_l C
cmark_t 0
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line b and circle k(O,C) intersect% DET: points C and A must be different
% Constructing a point P_{\_G31370} which is a foot of the point O on the line b
foot P_{\_G31370} O b
cmark_r P_{\_G31370}
color 200 200 200
drawline O P_{\_G31370}
color 0 0 0

% Constructing a point A which is an image of the point C in the symmetry to point/line P_{\_G
31370}
sim A P_{\_G31370} C
cmark_t A

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{b} and circle k(O,C) intersect

```

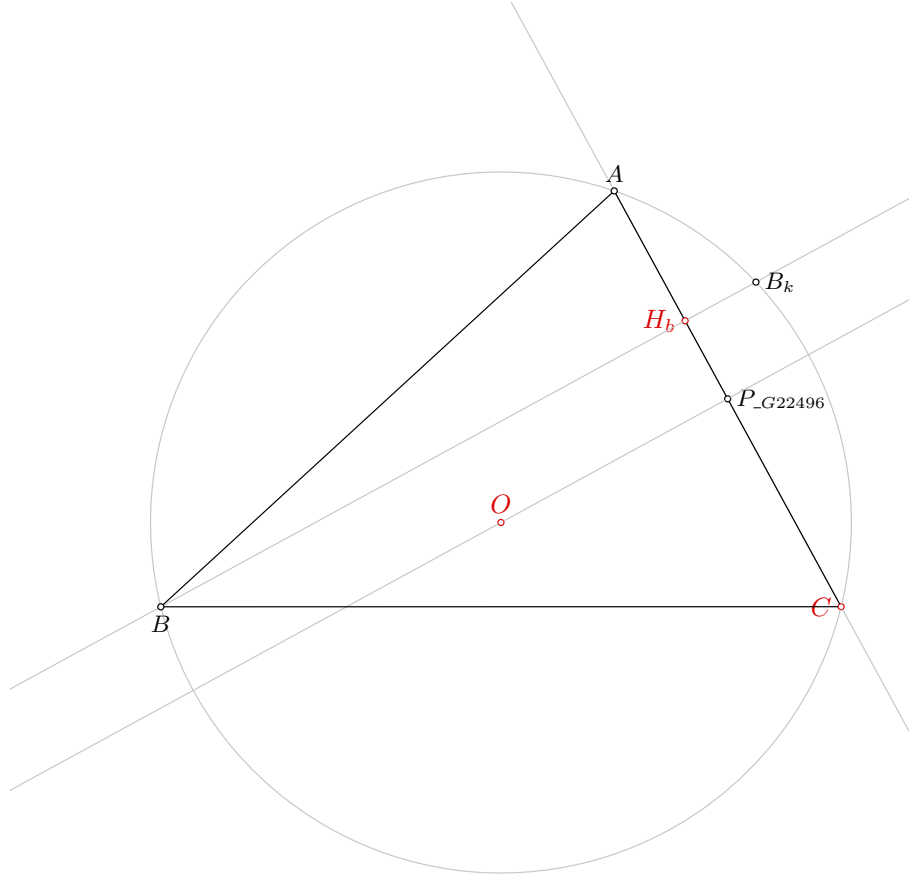


Figure 1: Illustration of the problem 0202

```
% Constructing points B_{k} and B which are in intersection of k(O,C) and h_{b}
intersec2 B_{k} B k(O,C) h_{b}
cmark_r B_{k}
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line h_{b} and circle k(O,C) intersect; line b and circle k(O,C)
intersect; points C and O are not the same
```

```
% Determination conditions: points C and A must be different; points C and H_{b} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 56 terms.

**Time Complexity:** Time spent by the prover is 0.983 seconds.

**NDG conditions** Points  $H_b$  and  $O$  are not identical

Points  $H_b$  and  $O$  are not identical

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $B$  and  $C$  are not identical

#### 4.1.3 Proving $H_b=_H H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 17 terms.

**Time Complexity:** Time spent by the prover is 0.386 seconds.

**NDG conditions** Points  $H_b$  and  $O$  are not identical

Points  $H_b$  and  $O$  are not identical

Points  $A$  and  $C$  are not identical

Points  $A$  and  $C$  are not identical

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{M_a BC} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_b AC} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_a M_b F_{M_a}^2} \neq S_{F_{M_a}^1 M_b F_{M_b}^2}$  i.e., lines  $M_a F_{M_a}^1$  and  $M_b F_{M_b}^2$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{H_b}^3} \neq S_{CBF_{H_b}^3}$  i.e., lines  $AC$  and  $BF_{H_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $H_b=_H H_b$

Proving failed



### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

Proving failed

#### 4.3.2 Proving $O=_O$

Proving failed

#### 4.3.3 Proving $H_b=_Hb$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C=C$

Proving failed

#### 4.4.2 Proving $O=_O$

Proving failed

#### 4.4.3 Proving $H_b=_Hb$

Proving failed

# Problem 203

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 203: Given a point  $C$ , a point  $O$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_c$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H_c$  are not the same;
2. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
3. Using the point  $H_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
4. Using the circle  $k(O, C)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same.

Determination conditions: points  $C$  and  $H_c$  are not the same.

Rules used: [W02,W04,W06,W10a]

Lemmas used: [D10,D26,D7,GD01,L11,L12]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
```

```
point O 65 51.14
```

```
point H_{c} 68.91 84.83
```

```
color 220 0 0
```

```
fontsize 9
```

```

cmark_l C
cmark_t O
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points C and H_{c} are not the same
% Constructing a line h_{c} which passes through point C and point H_{c}
line h_{c} C H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same
% Determination conditions: points C and H_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

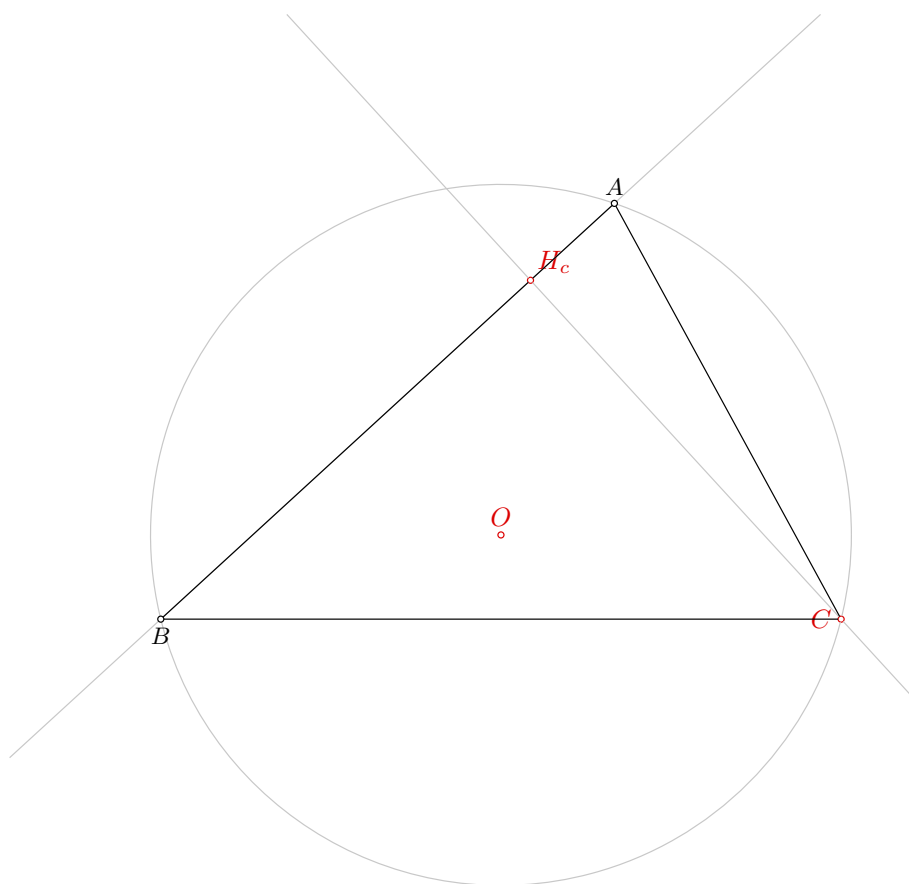


Figure 1: Illustration of the problem 0203

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 56 terms.

**Time Complexity:** Time spent by the prover is 0.994 seconds.

**NDG conditions** Points  $H_c$  and  $O$  are not identical

Points  $H_c$  and  $O$  are not identical

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $B$  and  $C$  are not identical

#### 4.1.3 Proving $H_c=_Hc$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 135 terms.

**Time Complexity:** Time spent by the prover is 1.132 seconds.

**NDG conditions** Points  $H_c$  and  $O$  are not identical

Points  $H_c$  and  $O$  are not identical

Points  $A$  and  $B$  are not identical

Line through points  $A$  and  $B$  is not parallel with line through points  $C$  and  $O$

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{_M_aBC} \neq 0$  i.e., points  $_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$  i.e., points  $_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a_M_bF_{_m_b}^2} \neq S_{F_{_m_a}^1_M_bF_{_m_b}^2}$  i.e., lines  $_M_aF_{_m_a}^1$  and  $_M_bF_{_m_b}^2$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{_h_c}^3} \neq S_{BCF_{_h_c}^3}$  i.e., lines  $AB$  and  $CF_{_h_c}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $H_c=_Hc$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $O=_O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 5192 terms.

**Time Complexity:** Time spent by the prover is 12.190 seconds. There are no ndg conditions.

#### 4.3.3 Proving $H_c=_H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 5989 terms.

**Time Complexity:** Time spent by the prover is 4.700 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $O=_O$

Proving failed

#### 4.4.3 Proving $H_c=_H_c$

Proving failed

# Problem 204

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 204: Given a point  $C$ , a point  $O$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $H$ , construct a point  $G$  (rule W01); ;
2. Using the point  $C$  and the point  $G$ , construct a point  $M_c$  (rule W01); ;
3. Using the point  $C$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H$  are not the same;
4. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
5. Using the point  $M_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same.

Determination conditions: points  $C$  and  $H$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D10,D20,D26,GD01,GL03,GL04,GL09,L11,L12,L3,L57,L58]

Solving time: 1.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point O 65 51.14
point H 80 72.73
```

```

color 220 0 0
fontsize 9

cmark_l C
cmark_t 0
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a line  $L_{\setminus G23730}$  which passes through point  $O$  and point  $H$ 
line  $L_{\setminus G23730}$  O H

color 200 200 200
drawline  $L_{\setminus G23730}$ 
color 0 0 0

% Constructing a point  $P_{\setminus G23831}$  with coordinates  $(0,0)$ 
point  $P_{\setminus G23831}$  0 0
cmark_r  $P_{\setminus G23831}$ 

% Constructing a point  $P_{\setminus G23755}$  such that  $OP_{\setminus G23755}/OP_{\setminus G23831}=1$ 
towards  $P_{\setminus G23755}$  0  $P_{\setminus G23831}$  1
cmark_r  $P_{\setminus G23755}$ 
color 200 200 200
drawsegment 0  $P_{\setminus G23755}$ 
color 0 0 0

% Constructing a point  $P_{\setminus G23800}$  such that  $OP_{\setminus G23800}/OP_{\setminus G23831}=3$ 
towards  $P_{\setminus G23800}$  0  $P_{\setminus G23831}$  3
cmark_r  $P_{\setminus G23800}$ 
color 200 200 200
drawsegment 0  $P_{\setminus G23800}$ 
color 0 0 0

% Constructing a line  $L_{\setminus G23761}$  which passes through point  $H$  and point  $P_{\setminus G23800}$ 
line  $L_{\setminus G23761}$  H  $P_{\setminus G23800}$ 

color 200 200 200
drawline  $L_{\setminus G23761}$ 
color 0 0 0

% Constructing a line  $L_{\setminus G23724}$  which contains the point  $P_{\setminus G23755}$  and is parallel to the
line  $L_{\setminus G23761}$ 
parallel  $L_{\setminus G23724}$   $P_{\setminus G23755}$   $L_{\setminus G23761}$ 

color 200 200 200
drawline  $L_{\setminus G23724}$ 
color 0 0 0

% Constructing a point  $G$  which belongs to line  $L_{\setminus G23724}$  and line  $L_{\setminus G23730}$ 
intersec G  $L_{\setminus G23724}$   $L_{\setminus G23730}$ 

```



```

cmark_t G

% Constructing a point  $M_{\{c\}}$  such that  $CM_{\{c\}}/CG=1.5$ 
towards M_{c} C G 1.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% DET: points C and H are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point C and point H
line h_{c} C H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Constructing a line c which is perpendicular to line  $h_{\{c\}}$  and which passes through point  $M_{\{c\}}$ 
perp c M_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle  $k(O,C)$  intersect
% Constructing points A and B which are in intersection of  $k(O,C)$  and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle  $k(O,C)$  intersect; points C and O are not the same
% Determination conditions: points C and H are not the same

```

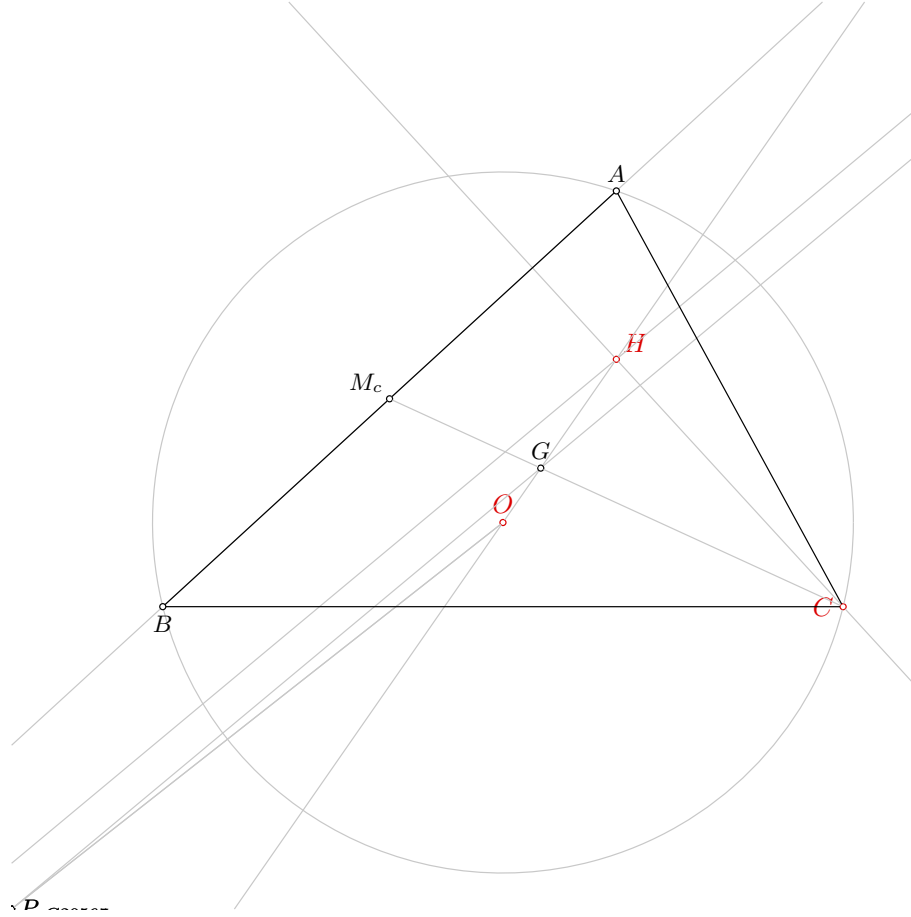


Figure 1: Illustration of the problem 0204

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 273 terms.

**Time Complexity:** Time spent by the prover is 1.771 seconds.

**NDG conditions** Points  $P_{G30745}$ ,  $H$  and  $O$  are not collinear

Line through points  $P_{G30745}$  and  $H$  is not parallel with line through points  $C$  and  $O$

Points  $C$  and  $H$  are not identical

Points  $C$  and  $H$  are not identical

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $B$  and  $C$  are not identical

#### 4.1.3 Proving $H=_H$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{P_{G21434}OH} \neq S_{P_{L_{G21403}}^0 OH}$  i.e., lines  $P_{G21434}P_{L_{G21403}}^0$  and  $OH$  are not parallel (construction based assumption)

$S_{M_cCH} \neq 0$  i.e., points  $M_c$ ,  $C$  and  $H$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aBC} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aM_bF_{M_a}^3} \neq S_{F_{M_a}^2M_bF_{M_b}^3}$  i.e., lines  $M_aF_{M_a}^2$  and  $M_bF_{M_b}^3$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{M_b}^5} \neq S_{F_{M_a}^4BF_{M_b}^5}$  i.e., lines  $AF_{M_a}^4$  and  $BF_{M_b}^5$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $H=_H$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $O=_O$

Proving failed

#### 4.3.3 Proving $H=_H$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4.2 Proving $O=_O$

Proving failed

### 4.4.3 Proving $H=_H$

Proving failed

# Problem 205

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 205: Given a point  $C$ , a point  $O$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $T_a$  are not the same;
2. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
3. Using the circle  $k(O, C)$ , the line  $a$ , the point  $O$  and the point  $C$ , construct a point  $B$  (rule W05); % NDG: line  $a$  and circle  $k(O, C)$  intersect % DET: points  $C$  and  $B$  must be different;
4. Using the point  $B$  and the point  $C$ , construct a point  $M_a$  (rule W01); ;
5. Using the point  $O$  and the point  $M_a$ , construct a line  $m_a$  (rule W02); % DET: points  $O$  and  $M_a$  are not the same;
6. Using the circle  $k(O, C)$  and the line  $m_a$ , construct a point  $N_a$  and a point  $N_{ak}$  (rule W04); % NDG: line  $m_a$  and circle  $k(O, C)$  intersect;
7. Using the point  $N_a$  and the point  $T_a$ , construct a line  $s_a$  (rule W02); % DET: points  $N_a$  and  $T_a$  are not the same;
8. Using the circle  $k(O, C)$ , the line  $s_a$ , the point  $O$  and the point  $N_a$ , construct a point  $A$  (rule W05); % NDG: line  $s_a$  and circle  $k(O, C)$  intersect % DET: points  $N_a$  and  $A$  must be different.

Non-degenerate conditions: line  $s_a$  and circle  $k(O, C)$  intersect; line  $m_a$  and circle  $k(O, C)$  intersect; line  $a$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same.

Determination conditions: points  $N_a$  and  $A$  must be different; points  $N_a$  and  $T_a$  are not the same; points  $O$  and  $M_a$  are not the same; points  $C$  and  $B$  must be different; points  $C$  and  $T_a$  are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D1,D11,D21,D23,D26,D47,D50,GD01,L11,L12,L25,L5]

Solving time: 1.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point O 65 51.14
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_l C
cmark_t 0
cmark_rb T_{a}
color 0 0 0
fontsize 8

% DET: points C and T_{a} are not the same
% Constructing a line a which passes through point C and point T_{a}
line a C T_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect% DET: points C and B must be different
% Constructing a point P_{\_G24981} which is a foot of the point O on the line a
foot P_{\_G24981} O a
cmark_r P_{\_G24981}
color 200 200 200
drawline O P_{\_G24981}
color 0 0 0

% Constructing a point B which is an image of the point C in the symmetry to point/line P_{\_G
24981}
```

```

sim B P_{\_G24981} C
cmark_b B

% Constructing a point  $M_{\{a\}}$  such that  $BM_{\{a\}}/BC=0.5$ 
towards M_{a} B C 0.5
cmark_r M_{a}
color 200 200 200
drawsegment B C
color 0 0 0

% DET: points  $O$  and  $M_{\{a\}}$  are not the same
% Constructing a line  $m_{\{a\}}$  which passes through point  $O$  and point  $M_{\{a\}}$ 
line m_{a} O M_{a}

color 200 200 200
drawline m_{a}
color 0 0 0

% NDG: line  $m_{\{a\}}$  and circle  $k(O,C)$  intersect
% Constructing points  $N_{\{a\}}$  and  $N_{\{ak\}}$  which are in intersection of  $k(O,C)$  and  $m_{\{a\}}$ 
intersec2 N_{a} N_{ak} k(O,C) m_{a}
cmark_b N_{a}
cmark_r N_{ak}

% DET: points  $N_{\{a\}}$  and  $T_{\{a\}}$  are not the same
% Constructing a line  $s_{\{a\}}$  which passes through point  $N_{\{a\}}$  and point  $T_{\{a\}}$ 
line s_{a} N_{a} T_{a}

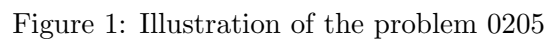
color 200 200 200
drawline s_{a}
color 0 0 0

% NDG: line  $s_{\{a\}}$  and circle  $k(O,C)$  intersect% DET: points  $N_{\{a\}}$  and  $A$  must be different
% Constructing a point  $P_{\{\_G25391\}}$  which is a foot of the point  $O$  on the line  $s_{\{a\}}$ 
foot P_{\_G25391} O s_{a}
cmark_r P_{\_G25391}
color 200 200 200
drawline O P_{\_G25391}
color 0 0 0

% Constructing a point  $A$  which is an image of the point  $N_{\{a\}}$  in the symmetry to point/line  $P_{\{\_G25391\}}$ 
sim A P_{\_G25391} N_{a}
cmark_t A

drawsegment A B

```



```
% Non-degenerate conditions: line s_{a} and circle k(O,C) intersect; line m_{a} and circle k(O,C)
% intersect; line a and circle k(O,C) intersect; points C and O are not the same
% Determination conditions: points N_{a} and A must be different; points N_{a} and T_{a} are not
% the same; points O and M_{a} are not the same; points C and B must be different; points C and T
% _{a} are not the same
```

Illustration of the constructed figure is given in Figure 1

## 1104



## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

Proving failed

#### 4.1.3 Proving $T_a=T_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{_M_aBC} \neq 0$  i.e., points  $_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$  i.e., points  $_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a _M_b F_{_m_b}^1} \neq S_{F_{_m_a}^0 _M_b F_{_m_b}^1}$  i.e., lines  $_M_a F_{_m_a}^0$  and  $_M_b F_{_m_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $T_a=T_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

Proving failed

#### 4.3.2 Proving $O=_O$

Proving failed

#### 4.3.3 Proving $T_a=T_a$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C=C$

Proving failed

#### 4.4.2 Proving $O=_O$

Proving failed

#### 4.4.3 Proving $T_a=_T T_a$

Proving failed

# Problem 206

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 206: Given a point  $C$ , a point  $O$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $T_b$  are not the same;
2. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
3. Using the circle  $k(O, C)$ , the line  $b$ , the point  $O$  and the point  $C$ , construct a point  $A$  (rule W05); % NDG: line  $b$  and circle  $k(O, C)$  intersect % DET: points  $C$  and  $A$  must be different;
4. Using the point  $A$  and the point  $C$ , construct a point  $M_b$  (rule W01); ;
5. Using the point  $O$  and the point  $M_b$ , construct a line  $m_b$  (rule W02); % DET: points  $O$  and  $M_b$  are not the same;
6. Using the circle  $k(O, C)$  and the line  $m_b$ , construct a point  $N_b$  and a point  $N_{bk}$  (rule W04); % NDG: line  $m_b$  and circle  $k(O, C)$  intersect;
7. Using the point  $N_b$  and the point  $T_b$ , construct a line  $s_b$  (rule W02); % DET: points  $N_b$  and  $T_b$  are not the same;
8. Using the circle  $k(O, C)$ , the line  $s_b$ , the point  $O$  and the point  $N_b$ , construct a point  $B$  (rule W05); % NDG: line  $s_b$  and circle  $k(O, C)$  intersect % DET: points  $N_b$  and  $B$  must be different.

Non-degenerate conditions: line  $s_b$  and circle  $k(O, C)$  intersect; line  $m_b$  and circle  $k(O, C)$  intersect; line  $b$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same.

Determination conditions: points  $N_b$  and  $B$  must be different; points  $N_b$  and  $T_b$  are not the same; points  $O$  and  $M_b$  are not the same; points  $C$  and  $A$  must be different; points  $C$  and  $T_b$  are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D1,D12,D22,D24,D26,D48,D51,GD01,GL04,L11,L12,L26,L6]

Solving time: 2.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point O 65 51.14
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_l C
cmark_t O
cmark_t T_{b}
color 0 0 0
fontsize 8

% DET: points C and T_{b} are not the same
% Constructing a line b which passes through point C and point T_{b}
line b C T_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line b and circle k(O,C) intersect% DET: points C and A must be different
% Constructing a point P_{\_G25119} which is a foot of the point O on the line b
foot P_{\_G25119} O b
cmark_r P_{\_G25119}
color 200 200 200
drawline O P_{\_G25119}
color 0 0 0

% Constructing a point A which is an image of the point C in the symmetry to point/line P_{\_G
25119}
```

```

sim A P_{\_G25119} C
cmark_t A

% Constructing a point  $M_{\{b\}}$  such that  $AM_{\{b\}}/AC=0.5$ 
towards M_{b} A C 0.5
cmark_lt M_{b}
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points  $O$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $m_{\{b\}}$  which passes through point  $O$  and point  $M_{\{b\}}$ 
line m_{b} O M_{b}

color 200 200 200
drawline m_{b}
color 0 0 0

% NDG: line  $m_{\{b\}}$  and circle  $k(O,C)$  intersect
% Constructing points  $N_{\{b\}}$  and  $N_{\{bk\}}$  which are in intersection of  $k(O,C)$  and  $m_{\{b\}}$ 
intersec2 N_{b} N_{bk} k(O,C) m_{b}
cmark_rb N_{b}
cmark_r N_{bk}

% DET: points  $N_{\{b\}}$  and  $T_{\{b\}}$  are not the same
% Constructing a line  $s_{\{b\}}$  which passes through point  $N_{\{b\}}$  and point  $T_{\{b\}}$ 
line s_{b} N_{b} T_{b}

color 200 200 200
drawline s_{b}
color 0 0 0

% NDG: line  $s_{\{b\}}$  and circle  $k(O,C)$  intersect% DET: points  $N_{\{b\}}$  and  $B$  must be different
% Constructing a point  $P_{\{\_G25529\}}$  which is a foot of the point  $O$  on the line  $s_{\{b\}}$ 
foot P_{\_G25529} O s_{b}
cmark_r P_{\_G25529}
color 200 200 200
drawline O P_{\_G25529}
color 0 0 0

% Constructing a point  $B$  which is an image of the point  $N_{\{b\}}$  in the symmetry to point/line  $P_{\{\_G25529\}}$ 
sim B P_{\_G25529} N_{b}
cmark_b B

drawsegment A B

```

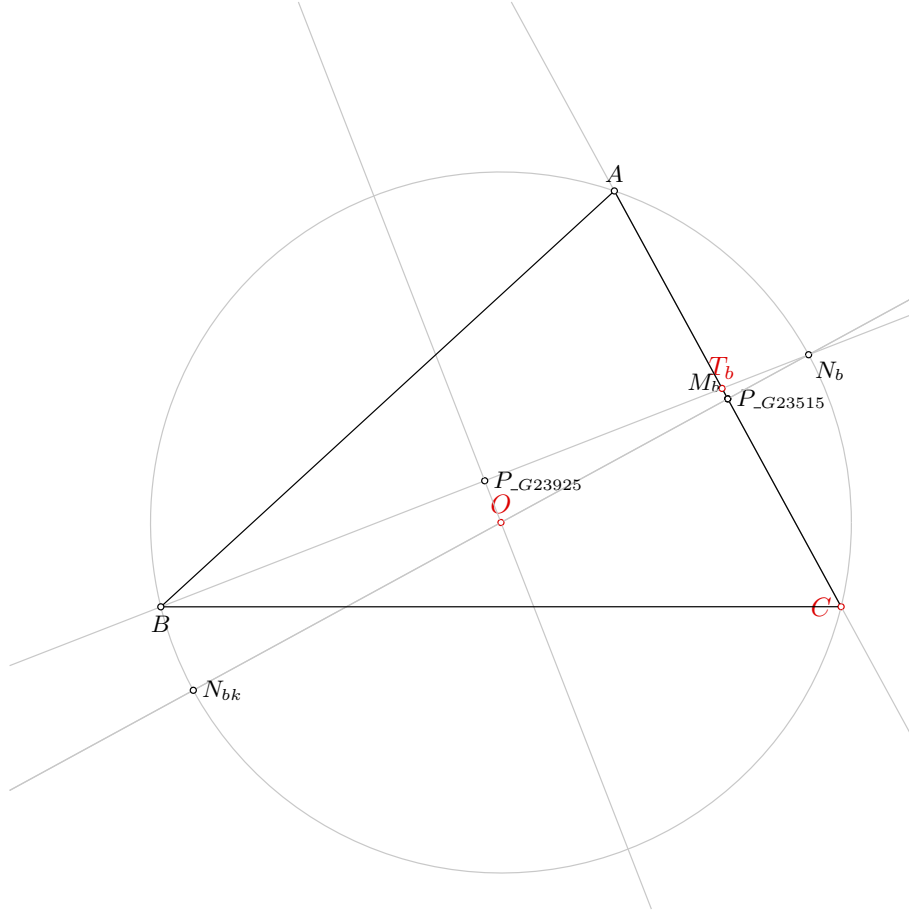


Figure 1: Illustration of the problem 0206

```
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line s_{b} and circle k(O,C) intersect; line m_{b} and circle k(O,C)
% intersect; line b and circle k(O,C) intersect; points C and O are not the same
% Determination conditions: points N_{b} and B must be different; points N_{b} and T_{b} are not
% the same; points O and M_{b} are not the same; points C and A must be different; points C and T
% _{b} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.033 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

Proving failed

#### 4.1.3 Proving $T_b=_Tb$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{_M_aBC} \neq 0$  i.e., points  $_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$  i.e., points  $_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a _M_b F_{_m_b}^1} \neq S_{F_{_m_a}^0 _M_b F_{_m_b}^1}$  i.e., lines  $_M_a F_{_m_a}^0$  and  $_M_b F_{_m_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $O=_O$

Proving failed

#### 4.2.3 Proving $T_b=_Tb$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

Proving failed

#### 4.3.2 Proving $O=_O$

Proving failed

#### 4.3.3 Proving $T_b=_Tb$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C=C$

Proving failed

#### 4.4.2 Proving $O=_O$

Proving failed

#### 4.4.3 Proving $T_b=_T T_b$

Proving failed



# Problem 207

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 207: Given a point  $C$ , a point  $O$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $T_c$ , construct a line  $s_c$  (rule W02); % DET: points  $C$  and  $T_c$  are not the same;
2. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
3. Using the circle  $k(O, C)$ , the line  $s_c$ , the point  $O$  and the point  $C$ , construct a point  $N_c$  (rule W05); % NDG: line  $s_c$  and circle  $k(O, C)$  intersect % DET: points  $C$  and  $N_c$  must be different;
4. Using the point  $N_c$  and the point  $O$ , construct a line  $m_c$  (rule W02); % DET: points  $N_c$  and  $O$  are not the same;
5. Using the point  $T_c$  and the line  $m_c$ , construct a line  $c$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; line  $s_c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same.

Determination conditions: points  $N_c$  and  $O$  are not the same; points  $C$  and  $N_c$  must be different; points  $C$  and  $T_c$  are not the same.

Rules used: [W02,W04,W05,W06,W10a]

Lemmas used: [D13,D25,D26,D49,GD01,L1,L11,L12,L27,L7]

Solving time: 1.1 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point O 65 51.14
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_l C
cmark_t O
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points C and T_{c} are not the same
% Constructing a line s_{c} which passes through point C and point T_{c}
line s_{c} C T_{c}

color 200 200 200
drawline s_{c}
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line s_{c} and circle k(O,C) intersect% DET: points C and N_{c} must be different
% Constructing a point P_{\_G22822} which is a foot of the point O on the line s_{c}
foot P_{\_G22822} O s_{c}
cmark_r P_{\_G22822}
color 200 200 200
drawline O P_{\_G22822}
color 0 0 0

% Constructing a point N_{c} which is an image of the point C in the symmetry to point/line P_{\_G
22822}
sim N_{c} P_{\_G22822} C
cmark_b N_{c}

% DET: points N_{c} and O are not the same
% Constructing a line m_{c} which passes through point N_{c} and point O
line m_{c} N_{c} O

color 200 200 200
```

```

drawline m_{c}
color 0 0 0

% Constructing a line c which is perpendicular to line m_{c} and which passes through point T_{c}
perp c T_{c} m_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k(O,C) intersect; line s_{c} and circle k(O,C)
% intersect; points C and O are not the same
% Determination conditions: points N_{c} and O are not the same; points C and N_{c} must be
% different; points C and T_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=O$

Proving failed

#### 4.1.3 Proving $T_c=T_c$

Proving failed

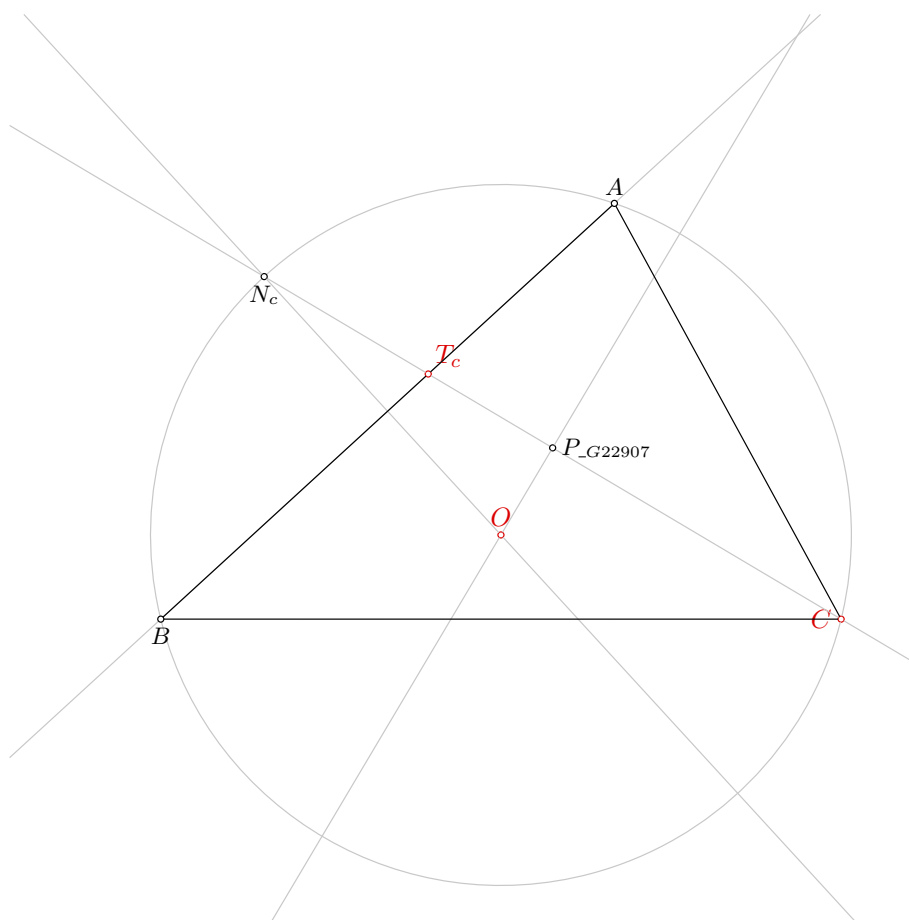


Figure 1: Illustration of the problem 0207

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{T_c N_c O} \neq 0$  i.e., points  $T_c$ ,  $N_c$  and  $O$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_a B C} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_b A C} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_a M_b F_{m_b}^2} \neq S_{F_{m_a}^1 M_b F_{m_b}^2}$  i.e., lines  $M_a F_{m_a}^1$  and  $M_b F_{m_b}^2$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $O=_O$

Proving failed

### 4.2.3 Proving $T_c=_T T_c$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.2 Proving $O=_O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 5192 terms.

**Time Complexity:** Time spent by the prover is 5.960 seconds. There are no ndg conditions.

### 4.3.3 Proving $T_c=_T T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6468 terms.

**Time Complexity:** Time spent by the prover is 5.060 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $O=_O$

Proving failed

### 4.4.3 Proving $T_c=_T T_c$

Proving failed

# Problem 208

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 208: Given a point  $C$ , a point  $O$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $I$ , construct a line  $s_c$  (rule W02); % DET: points  $C$  and  $I$  are not the same;
2. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
3. Using the circle  $k(O, C)$ , the line  $s_c$ , the point  $O$  and the point  $C$ , construct a point  $N_c$  (rule W05); % NDG: line  $s_c$  and circle  $k(O, C)$  intersect % DET: points  $C$  and  $N_c$  must be different;
4. Using the point  $I$  and the point  $N_c$ , construct a circle  $k(N_c, B)$  (rule W06); % NDG: points  $I$  and  $N_c$  are not the same;
5. Using the circle  $k(O, C)$  and the circle  $k(N_c, B)$ , construct a point  $A$  and a point  $B$  (rule W07); % NDG: circles  $k(O, C)$  and  $k(N_c, B)$  intersect % DET: circles  $k(O, C)$  and  $k(N_c, B)$  are not the same.

Non-degenerate conditions: circles  $k(O, C)$  and  $k(N_c, B)$  intersect; points  $I$  and  $N_c$  are not the same; line  $s_c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same.

Determination conditions: circles  $k(O, C)$  and  $k(N_c, B)$  are not the same; points  $C$  and  $N_c$  must be different; points  $C$  and  $I$  are not the same.

Rules used: [W02,W05,W06,W07]

Lemmas used: [D26,D49,GD02,L11,L12,L2,L35,L36,L7]

Solving time: 0.3 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point O 65 51.14
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_l C
cmark_t O
cmark_b I
color 0 0 0
fontsize 8

% DET: points C and I are not the same
% Constructing a line s_{c} which passes through point C and point I
line s_{c} C I

color 200 200 200
drawline s_{c}
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line s_{c} and circle k(O,C) intersect% DET: points C and N_{c} must be different
% Constructing a point P_{\_G22511} which is a foot of the point O on the line s_{c}
foot P_{\_G22511} O s_{c}
cmark_r P_{\_G22511}
color 200 200 200
drawline O P_{\_G22511}
color 0 0 0

% Constructing a point N_{c} which is an image of the point C in the symmetry to point/line P_{\_G
22511}
sim N_{c} P_{\_G22511} C
cmark_b N_{c}

% NDG: points I and N_{c} are not the same
% Constructing a circle k(N_{c},B) whose center is at point N_{c} and which passes through point I
circle k(N_{c},B) N_{c} I

color 200 200 200
```

```
drawcircle k(N_{c},B)
color 0 0 0
```

```
% NDG: circles k(O,C) and k(N_{c},B) intersect% DET: circles k(O,C) and k(N_{c},B) are not the same
% Constructing points A and B which are in intersection of k(O,C) and k(N_{c},B)
intersec2 A B k(O,C) k(N_{c},B)
cmark_t A
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: circles k(O,C) and k(N_{c},B) intersect; points I and N_{c} are not
the same; line s_{c} and circle k(O,C) intersect; points C and O are not the same
% Determination conditions: circles k(O,C) and k(N_{c},B) are not the same; points C and N_{c} must
be different; points C and I are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $O=_O$

Proving failed

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{M_aBC} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)  
 $S_{M_bAC} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)  
 $S_{M_aM_bF_{-m_b}^1} \neq S_{F_{-m_a}^0M_bF_{-m_b}^1}$  i.e., lines  $M_aF_{-m_a}^0$  and  $M_bF_{-m_b}^1$  are not parallel (construction based assumption)



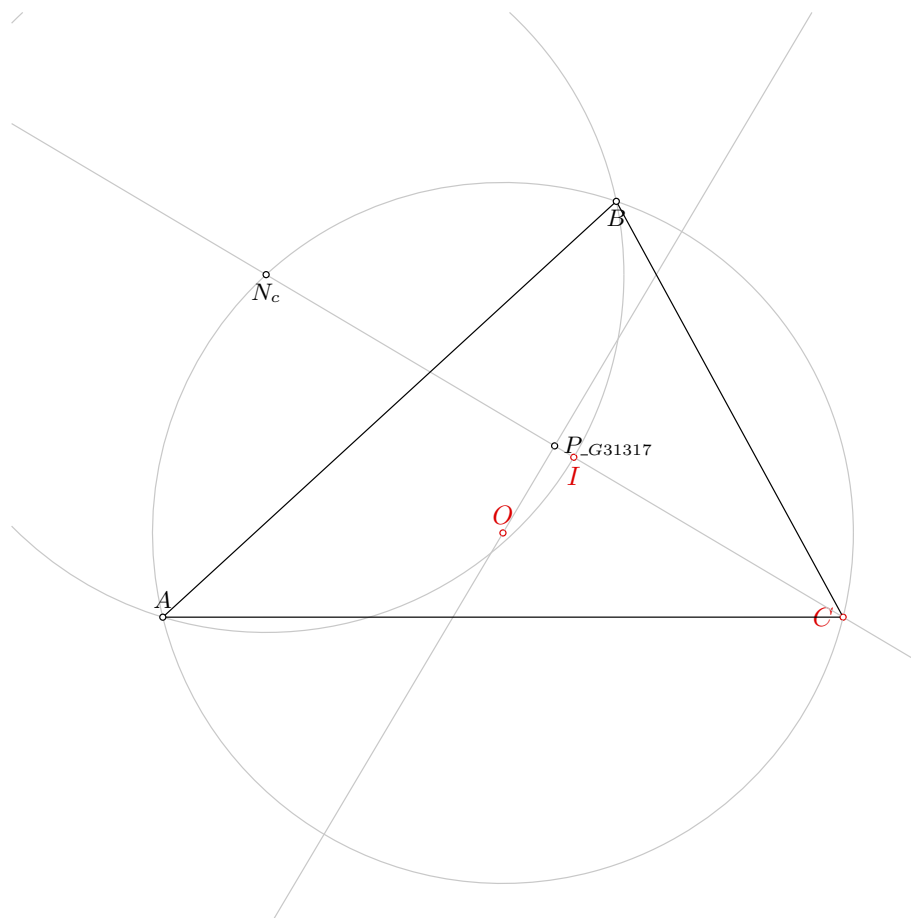


Figure 1: Illustration of the problem 0208

Total number of proof steps: 1  
Time spent by the prover: 0.000 seconds

#### **4.2.2 Proving $O=_O$**

Proving failed

#### **4.2.3 Proving $I=_I$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $C=C$**

Proving failed

#### **4.3.2 Proving $O=_O$**

Proving failed

#### **4.3.3 Proving $I=_I$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $C=C$**

Proving failed

#### **4.4.2 Proving $O=_O$**

Proving failed

#### **4.4.3 Proving $I=_I$**

Proving failed

# Problem 209

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 209: Given a point  $C$ , a point  $M_a$  and a point  $M_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
2. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D21,D22,GL03,GL04]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point M_{a} 65 40
point M_{b} 95 67.5

color 220 0 0
fontsize 9

cmark_l C
cmark_r M_{a}
cmark_lt M_{b}
color 0 0 0
fontsize 8
```

*% Constructing a point B such that  $CB/CM_{\{a\}}=2$*

```

towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% Constructing a point A such that CA/CM_{b}=2
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.035 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a=_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.054 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $M_b=_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.056 seconds.

**NDG conditions** There are no NDG conditions for this theorem

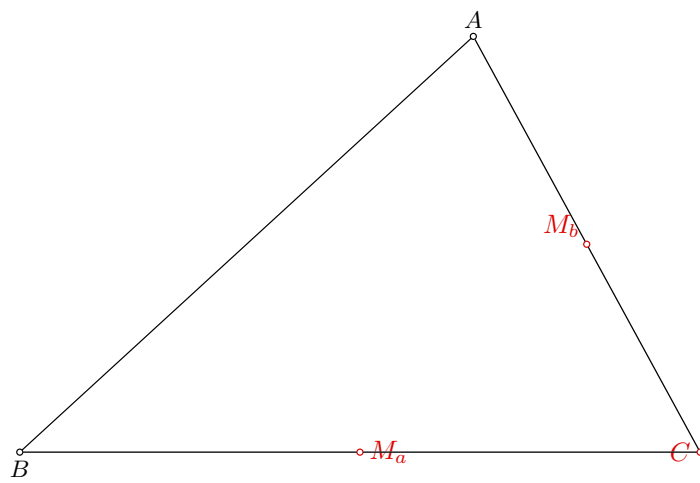


Figure 1: Illustration of the problem 0209

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

There are no ndg conditions.  
Total number of proof steps: 1  
Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_a=_M M_a$

There are no ndg conditions.  
Total number of proof steps: 42  
Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $M_b=_M M_b$

There are no ndg conditions.  
Total number of proof steps: 41  
Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_a=_M M_a$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $M_b=_M M_b$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.  
**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.  
**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $M_a=_M M_a$

**Status:** The conjecture has been proved.  
**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.  
**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_b = -M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

# Problem 210

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 210: Given a point  $C$ , a point  $M_a$  and a point  $M_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
2. Using the point  $C$  and the point  $M_c$ , construct a point  $G$  (rule W01); ;
3. Using the point  $M_a$  and the point  $G$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D21,GL03,GL04,L55,L57]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point M_{a} 65 40
point M_{c} 50 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_r M_{a}
cmark_lt M_{c}
color 0 0 0
fontsize 8
```



```

% Constructing a point B such that  $CB/CM_{\{a\}}=2$ 
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% Constructing a line  $L_{\{G30441\}}$  which passes through point C and point  $M_{\{c\}}$ 
line L_{G30441} C M_{c}

color 200 200 200
drawline L_{G30441}
color 0 0 0

% Constructing a point  $P_{\{G30542\}}$  with coordinates (0,0)
point P_{G30542} 0 0
cmark_r P_{G30542}

% Constructing a point  $P_{\{G30466\}}$  such that  $CP_{\{G30466\}}/CP_{\{G30542\}}=2$ 
towards P_{G30466} C P_{G30542} 2
cmark_r P_{G30466}
color 200 200 200
drawsegment C P_{G30466}
color 0 0 0

% Constructing a point  $P_{\{G30511\}}$  such that  $CP_{\{G30511\}}/CP_{\{G30542\}}=3$ 
towards P_{G30511} C P_{G30542} 3
cmark_r P_{G30511}
color 200 200 200
drawsegment C P_{G30511}
color 0 0 0

% Constructing a line  $L_{\{G30472\}}$  which passes through point  $M_{\{c\}}$  and point  $P_{\{G30511\}}$ 
line L_{G30472} M_{c} P_{G30511}

color 200 200 200
drawline L_{G30472}
color 0 0 0

% Constructing a line  $L_{\{G30435\}}$  which contains the point  $P_{\{G30466\}}$  and is parallel to the
line  $L_{\{G30472\}}$ 
parallel L_{G30435} P_{G30466} L_{G30472}

color 200 200 200
drawline L_{G30435}
color 0 0 0

% Constructing a point G which belongs to line  $L_{\{G30435\}}$  and line  $L_{\{G30441\}}$ 
intersec G L_{G30435} L_{G30441}
cmark_t G

```

```

% Constructing a point A such that  $M_{\{a\}A}/M_{\{a\}G}=3$ 
towards A  $M_{\{a\}}$  G 3
cmark_t A
color 200 200 200
drawsegment  $M_{\{a\}}$  A
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a=_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.055 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $M_c=_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 91 terms.

**Time Complexity:** Time spent by the prover is 0.74 seconds.

**NDG conditions** Points  $C$ ,  $M_c$  and  $P_{G29899}$  are not collinear

Line through points  $C$  and  $M_a$  is not parallel with line through points  $M_c$  and  $P_{G29899}$

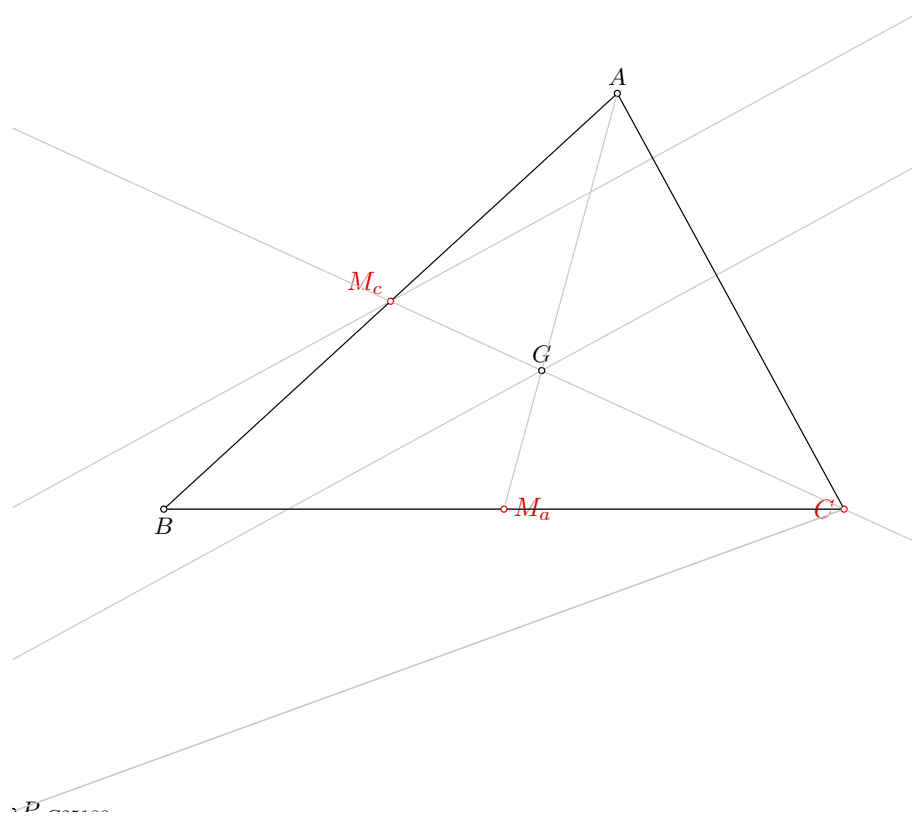


Figure 1: Illustration of the problem 0210

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{P_{-G25991}CM_c} \neq S_{P_{L_{-G25960}}^0}^{CM_c}$  i.e., lines  $P_{-G25991}P_{L_{-G25960}}^0$  and  $CM_c$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_a=_M_a$

NDG conditions are:

$S_{P_{-G26951}CM_c} \neq S_{P_{L_{-G26920}}^0}^{CM_c}$  i.e., lines  $P_{-G26951}P_{L_{-G26920}}^0$  and  $CM_c$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $M_c=_M_c$

NDG conditions are:

$S_{P_{-G27911}CM_c} \neq S_{P_{L_{-G27880}}^0}^{CM_c}$  i.e., lines  $P_{-G27911}P_{L_{-G27880}}^0$  and  $CM_c$  are not parallel (construction based assumption)

$S_{P_{-G27987}M_cC} \neq 0$  i.e., points  $P_{-G27987}$ ,  $M_c$  and  $C$  are not collinear (cancellation assumption)

Total number of proof steps: 4688

Time spent by the prover: 8.230 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_a=_M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $M_c=_M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3932 terms.

**Time Complexity:** Time spent by the prover is 6.480 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_a = -M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 107 terms.

**Time Complexity:** Time spent by the prover is 0.580 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_c = -M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 355 terms.

**Time Complexity:** Time spent by the prover is 0.610 seconds. There are no ndg conditions.

# Problem 211

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 211: Given a point  $C$ , a point  $M_a$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
2. Using the point  $M_a$  and the point  $G$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D21,GL03,GL04,L55]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point M_{a} 65 40
point G 70 58.33
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_r M_{a}
cmark_t G
color 0 0 0
fontsize 8
```

```
% Constructing a point B such that CB/CM_{a}=2
```

```

towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% Constructing a point A such that M_{a}A/M_{a}G=3
towards A M_{a} G 3
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a=_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.054 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $G=_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 45 terms.

**Time Complexity:** Time spent by the prover is 0.433 seconds.

**NDG conditions** Line through points  $A$  and  $_M_a$  is not parallel with line through points  $B$  and  $_M_b$   
Points  $A$  and  $C$  are not identical

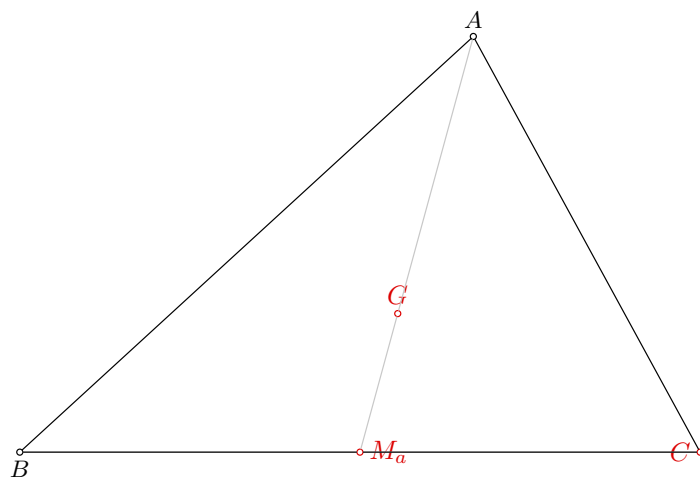


Figure 1: Illustration of the problem 0211



## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{AB\_M_b} \neq S_{M_aB\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_a=_M_a$

NDG conditions are:

$S_{AB\_M_b} \neq S_{M_aB\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $G=_G$

NDG conditions are:

$S_{AB\_M_b} \neq S_{M_aB\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (cancellation assumption)

Total number of proof steps: 673

Time spent by the prover: 0.050 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_a=_M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.3 Proving $G=_G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 55 terms.

**Time Complexity:** Time spent by the prover is 0.120 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 163 terms.

**Time Complexity:** Time spent by the prover is 0.940 seconds. There are no ndg conditions.

#### 4.4.3 Proving $G = \neg G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 163 terms.

**Time Complexity:** Time spent by the prover is 1.150 seconds. There are no ndg conditions.

# Problem 212

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 212: Given a point  $M_a$ , a point  $H_a$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $H_a$  are not the same;
2. Choose freely a point  $C$  on the line  $a$  (rule WOnline1) ;
3. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
4. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
5. Choose freely a point  $A$  on the line  $h_a$  (rule WOnline2).

Non-degenerate conditions: .

Determination conditions: points  $M_a$  and  $H_a$  are not the same.

Rules used: [W01,W02,W10b,WOnline1,WOnline2]

Lemmas used: [D21,D5,D8,GD01,GL03,GL04,GL09]

Solving time: 154.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40
point H_{a} 80 40
point C 110 40
```

```
color 220 0 0
fontsize 9
```

```

cmark_r M_{a}
cmark_r H_{a}
cmark_l C
color 0 0 0
fontsize 8

% DET: points M_{a} and H_{a} are not the same
% Constructing a line a which passes through point M_{a} and point H_{a}
line a M_{a} H_{a}

color 200 200 200
drawline a
color 0 0 0

% Choosing randomly a point C on the line M_{a}H_{a}
online C M_{a} H_{a}
cmark_l C
color 200 200 200
drawline M_{a} H_{a}
color 0 0 0

% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% Generating random value V[_G31650]
random V[_G31650]

% Calculating value V[_G31671] using formula V[_G31650]*20
expression V[_G31671] { V[_G31650]*20 }

% Constructing a point A which is a point for which holds H_{a}A = V[_G31671] and angle CH_{a}A =
90
turtle A C H_{a} 90 V[_G31671]

```

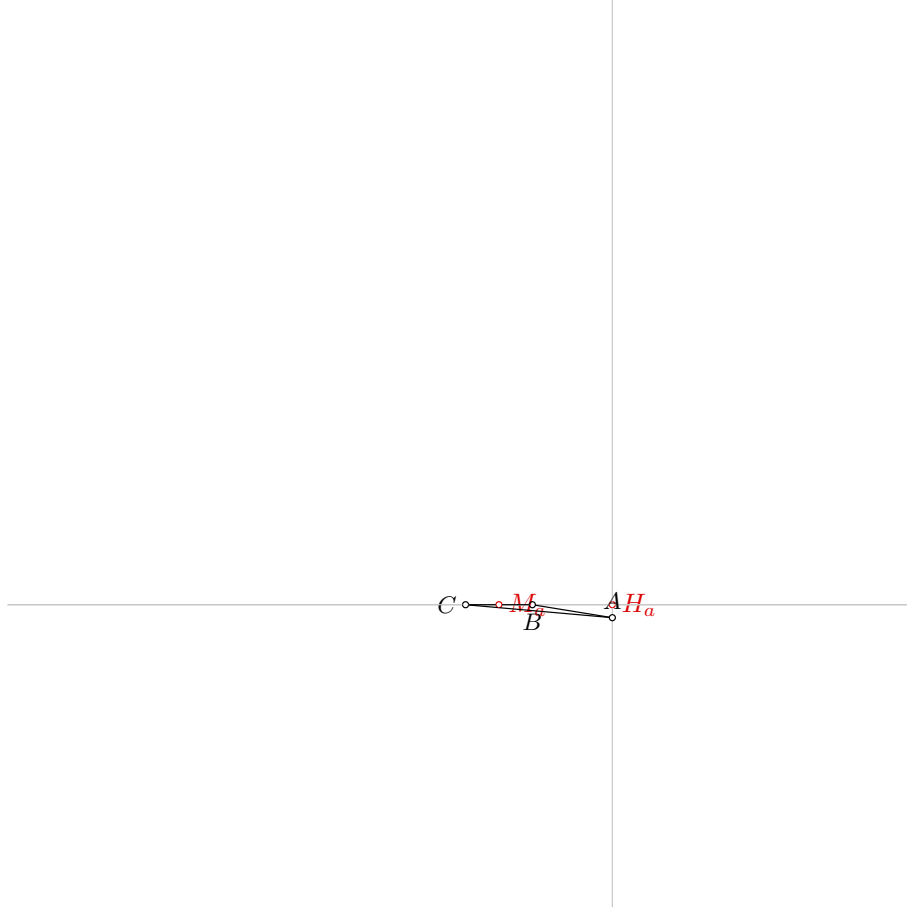


Figure 1: Illustration of the problem 0212

```
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions:
```

```
% Determination conditions: points  $M_{\{a\}}$  and  $H_{\{a\}}$  are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = \neg M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.032 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a = \neg H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.075 seconds.

**NDG conditions** Line through points  $M_a$  and  $C$  is not perpendicular to line through points  $C$  and  $B$

#### 4.1.3 Proving $C = C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.014 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF^2_{\neg h_a}} \neq S_{CAF^2_{\neg h_a}}$  i.e., lines  $BC$  and  $AF^2_{\neg h_a}$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.2.3 Proving $C = C$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF^2_{\neg h_a}} \neq S_{CAF^2_{\neg h_a}}$  i.e., lines  $BC$  and  $AF^2_{\neg h_a}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 2 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 213

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 213: Given a point  $M_a$ , a point  $H_b$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_b$  and  $M_a$  are not the same;
2. Choose freely a point  $C$  on the circle  $k(M_a, B)$  (rule WOncircle);
3. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
4. Using the point  $C$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $H_b$  are not the same;
5. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) .

Non-degenerate conditions: points  $H_b$  and  $M_a$  are not the same.

Determination conditions: points  $C$  and  $H_b$  are not the same.

Rules used: [W01,W02,W06,WOncircle1,WOnline1]

Lemmas used: [D21,D6,GD01,GD02,GL03,GL04,L38]

Solving time: 815.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{a} 65 40
point H_{b} 89.36 77.83
point C 110 40

color 220 0 0
fontsize 9
```



```

cmark_r M_{a}
cmark_l H_{b}
cmark_l C
color 0 0 0
fontsize 8

% NDG: points H_{b} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{b}
circle k(M_{a},B) M_{a} H_{b}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% Choosing randomly a point C on the circle with center M_{a} through point H_{b}
oncircle C M_{a} H_{b}
cmark_l C
color 200 200 200
drawcircle M_{a} H_{b}
color 0 0 0

% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% Choosing randomly a point A on the line CH_{b}
online A C H_{b}
cmark_t A
color 200 200 200
drawline C H_{b}
color 0 0 0

```

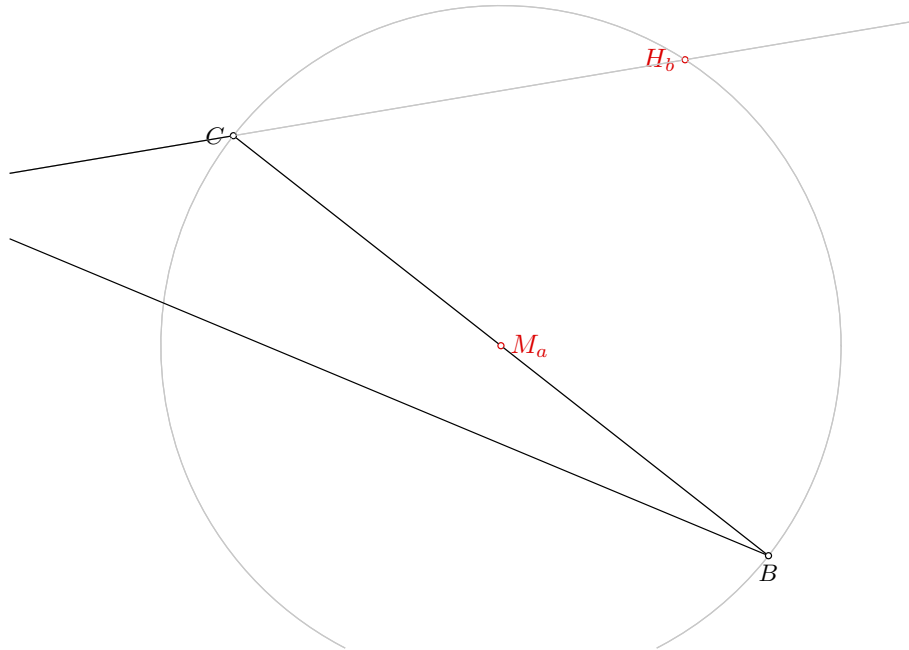


Figure 1: Illustration of the problem 0213

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: points  $H_{\{b\}}$  and  $M_{\{a\}}$  are not the same
% Determination conditions: points  $C$  and  $H_{\{b\}}$  are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = -M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.116 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b = \neg H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 47 terms.

**Time Complexity:** Time spent by the prover is 0.681 seconds.

**NDG conditions** Points  $C$  and  $M_a$  are not identical

Points  $A$  and  $C$  are not identical

Points  $H_b$  and  $A$  are not identical

#### 4.1.3 Proving $C = C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.035 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg H_b}^1} \neq S_{CBF_{\neg H_b}^1}$  i.e., lines  $AC$  and  $BF_{\neg H_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.2.3 Proving $C = C$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg H_b}^1} \neq S_{CBF_{\neg H_b}^1}$  i.e., lines  $AC$  and  $BF_{\neg H_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 17 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 280 terms.

**Time Complexity:** Time spent by the prover is 0.280 seconds. There are no ndg conditions.

#### 4.3.3 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 312 terms.

**Time Complexity:** Time spent by the prover is 2.010 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 312 terms.

**Time Complexity:** Time spent by the prover is 1.970 seconds. There are no ndg conditions.

#### 4.4.3 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 214

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 214: Given a point  $M_a$ , a point  $H_c$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_c$  and  $M_a$  are not the same;
2. Choose freely a point  $C$  on the circle  $k(M_a, B)$  (rule WOncircle);
3. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
4. Using the point  $H_c$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $B$  are not the same;
5. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) .

Non-degenerate conditions: points  $H_c$  and  $M_a$  are not the same.

Determination conditions: points  $H_c$  and  $B$  are not the same.

Rules used: [W01,W02,W06,WOncircle1,WOnline1]

Lemmas used: [D21,D7,GD01,GD02,GL03,GL04,L39]

Solving time: 929.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{a} 65 40
point H_{c} 68.91 84.83
point C 110 40

color 220 0 0
fontsize 9
```

```

cmark_r M_{a}
cmark_rt H_{c}
cmark_l C
color 0 0 0
fontsize 8

% NDG: points H_{c} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{c}
circle k(M_{a},B) M_{a} H_{c}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% Choosing randomly a point C on the circle with center M_{a} through point H_{c}
oncircle C M_{a} H_{c}
cmark_l C
color 200 200 200
drawcircle M_{a} H_{c}
color 0 0 0

% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% DET: points H_{c} and B are not the same
% Constructing a line c which passes through point H_{c} and point B
line c H_{c} B

color 200 200 200
drawline c
color 0 0 0

% Choosing randomly a point A on the line H_{c}B
online A H_{c} B
cmark_t A
color 200 200 200
drawline H_{c} B
color 0 0 0

```

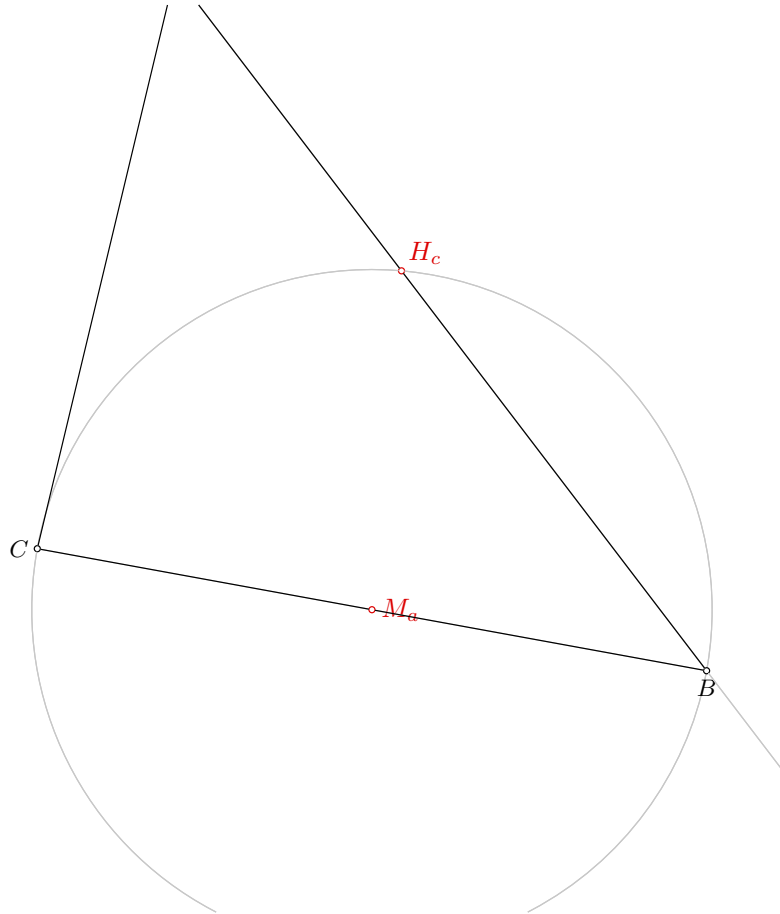


Figure 1: Illustration of the problem 0214

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: points  $H_{\{c\}}$  and  $M_{\{a\}}$  are not the same
% Determination conditions: points  $H_{\{c\}}$  and  $B$  are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.111 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.1.3 Proving $C = C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^1} \neq S_{BCF_{h_c}^1}$  i.e., lines  $AB$  and  $CF_{h_c}^1$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.2.3 Proving $C = C$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^1} \neq S_{BCF_{h_c}^1}$  i.e., lines  $AB$  and  $CF_{h_c}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 17 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 280 terms.

**Time Complexity:** Time spent by the prover is 0.470 seconds. There are no ndg conditions.



### 4.3.3 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_a=_M M_a$

Proving failed

### 4.4.2 Proving $H_c=_H H_c$

Proving failed

### 4.4.3 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 215

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 215: Given a point  $C$ , a point  $M_a$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
2. Using the point  $C$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H$  are not the same;
3. Using the point  $H$  and the point  $B$ , construct a line  $h_b$  (rule W02); % DET: points  $H$  and  $B$  are not the same;
4. Using the point  $C$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $C$  and  $M_a$  are not the same;
5. Using the circle  $k(M_a, B)$ , the line  $h_c$ , the point  $M_a$  and the point  $C$ , construct a point  $H_c$  (rule W05); % NDG: line  $h_c$  and circle  $k(M_a, B)$  intersect % DET: points  $C$  and  $H_c$  must be different;
6. Using the point  $H_c$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $B$  are not the same;
7. Using the circle  $k(M_a, B)$ , the line  $h_b$ , the point  $M_a$  and the point  $B$ , construct a point  $H_b$  (rule W05); % NDG: line  $h_b$  and circle  $k(M_a, B)$  intersect % DET: points  $B$  and  $H_b$  must be different;
8. Using the point  $H_b$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $C$  are not the same;
9. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; line  $h_b$  and circle  $k(M_a, B)$  intersect; line  $h_c$  and circle  $k(M_a, B)$  intersect; points  $C$  and  $M_a$  are not the same.

Determination conditions: lines  $c$  and  $b$  are not the same; points  $H_b$  and  $C$  are not the same; points  $B$  and  $H_b$  must be different; points  $H_c$  and  $B$  are not the same; points  $C$  and  $H_c$  must be different; points  $H$  and  $B$  are not the same; points  $C$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D21,D3,D6,D7,D9,GD01,GD02,GL03,GL04,L3,L37,L38,L39]

Solving time: 1.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point M_{a} 65 40
point H 80 72.73

color 220 0 0
fontsize 9

cmark_l C
cmark_r M_{a}
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% DET: points C and H are not the same
% Constructing a line h_{c} which passes through point C and point H
line h_{c} C H

color 200 200 200
drawline h_{c}
color 0 0 0

% DET: points H and B are not the same
% Constructing a line h_{b} which passes through point H and point B
line h_{b} H B

color 200 200 200
drawline h_{b}
color 0 0 0
```

```

% NDG: points C and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point C
circle k(M_{a},B) M_{a} C

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line h_{c} and circle k(M_{a},B) intersect% DET: points C and H_{c} must be different
% Constructing a point P_{\G60407} which is a foot of the point M_{a} on the line h_{c}
foot P_{\G60407} M_{a} h_{c}
cmark_r P_{\G60407}
color 200 200 200
drawline M_{a} P_{\G60407}
color 0 0 0

% Constructing a point H_{c} which is an image of the point C in the symmetry to point/line P_{\G
60407}
sim H_{c} P_{\G60407} C
cmark_rt H_{c}

% DET: points H_{c} and B are not the same
% Constructing a line c which passes through point H_{c} and point B
line c H_{c} B

color 200 200 200
drawline c
color 0 0 0

% NDG: line h_{b} and circle k(M_{a},B) intersect% DET: points B and H_{b} must be different
% Constructing a point P_{\G60645} which is a foot of the point M_{a} on the line h_{b}
foot P_{\G60645} M_{a} h_{b}
cmark_r P_{\G60645}
color 200 200 200
drawline M_{a} P_{\G60645}
color 0 0 0

% Constructing a point H_{b} which is an image of the point B in the symmetry to point/line P_{\G
60645}
sim H_{b} P_{\G60645} B
cmark_l H_{b}

% DET: points H_{b} and C are not the same
% Constructing a line b which passes through point H_{b} and point C
line b H_{b} C

color 200 200 200
drawline b
color 0 0 0

```

```

% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and b are not parallel; line h_{b} and circle k(M_{a},B)
% intersect; line h_{c} and circle k(M_{a},B) intersect; points C and M_{a} are not the same
% Determination conditions: lines c and b are not the same; points H_{b} and C are not the same;
% points B and H_{b} must be different; points H_{c} and B are not the same; points C and H_{c}
% must be different; points H and B are not the same; points C and H are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a=_M a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $H=_H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 88 terms.

**Time Complexity:** Time spent by the prover is 2.163 seconds.

**NDG conditions** Points  $B$ ,  $C$  and  $H$  are not collinear

Points  $B$ ,  $C$  and  $H$  are not collinear

Points  $B$  and  $H$  are not identical

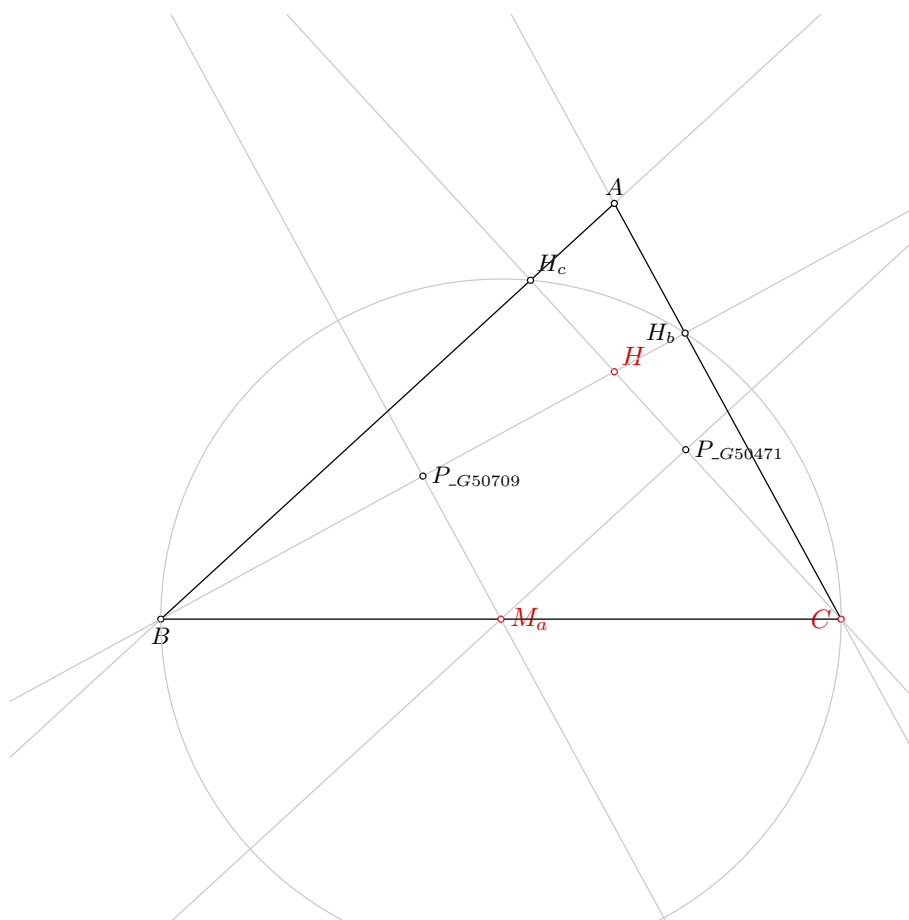


Figure 1: Illustration of the problem 0215

Line through points  $B$  and  $C$  is not perpendicular to line through points  $C$  and  $H$

Line through points  $H_b$  and  $C$  is not parallel with line through points  $H_c$  and  $B$

Points  $B$ ,  $H_c$  and  $M_a$  are not collinear

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $B$  and  $C$  are not identical

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_c H_b C} \neq S_{B H_b C}$  i.e., lines  $H_c B$  and  $H_b C$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$  i.e., lines  $AF_{-h_a}^0$  and  $BF_{-h_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_a=_M M_a$

Proving failed

### 4.2.3 Proving $H=_H H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_a=_M M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $H=_H H$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 9 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_a = M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $H = H$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.



# Problem 216

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 216: Given a point  $M_a$ , a point  $T_a$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $T_a$  are not the same;
2. Choose freely a point  $C$  on the line  $a$  (rule WOnline1) ;
3. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
4. Using the point  $B$ , the point  $C$ , the point  $T_a$  and the line  $a$ , construct a point  $T'_a$  (rule W19);  
% NDG: points  $B$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $midpoint([B, T_a])$  are not the same;
5. Choose freely a point  $A$  on the circle  $k_{over}(T_a, T'_a)$  (rule WOncircle).

Non-degenerate conditions: points  $B$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $midpoint([B, T_a])$  are not the same.

Determination conditions: points  $M_a$  and  $T_a$  are not the same.

Rules used: [W01, W02, W19, WOncircle2, WOnline1]

Lemmas used: [D21, D23, GD01, GL03, GL04, GL09, L77]

Solving time: 88.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40
point T_{a} 70.86 40
point C 110 40
```

```

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_rb T_{a}
cmark_l C
color 0 0 0
fontsize 8

% DET: points M_{a} and T_{a} are not the same
% Constructing a line a which passes through point M_{a} and point T_{a}
line a M_{a} T_{a}

color 200 200 200
drawline a
color 0 0 0

% Choosing randomly a point C on the line M_{a}T_{a}
online C M_{a} T_{a}
cmark_l C
color 200 200 200
drawline M_{a} T_{a}
color 0 0 0

% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% NDG: points B and C are not the same; points C and T_{a} are not the same; points C and midpoint
% ([B,T_{a}]) are not the same
% Constructing a point P_{\_G36584} which is an image of the point C in a rotation around the point
% T_{a} for the angle 90
rotate P_{\_G36584} T_{a} 90 C
cmark_r P_{\_G36584}
color 200 200 200
drawarc_p T_{a} C 90
color 0 0 0

% Constructing a line L_{\_G36587} which passes through point T_{a} and point P_{\_G36584}
line L_{\_G36587} T_{a} P_{\_G36584}

color 200 200 200
drawline L_{\_G36587}
color 0 0 0

% Constructing midpoint P_{\_G36590} of the segment CP_{\_G36584}

```

```

midpoint P_{\_G36590} C P_{\_G36584}
cmark_r P_{\_G36590}

% Constructing a line L_{\_G36593} which passes through point B and point P_{\_G36590}
line L_{\_G36593} B P_{\_G36590}

color 200 200 200
drawline L_{\_G36593}
color 0 0 0

% Constructing a line L_{\_G36596} which passes through point C and point P_{\_G36590}
line L_{\_G36596} C P_{\_G36590}

color 200 200 200
drawline L_{\_G36596}
color 0 0 0

% Constructing a point P_{\_G36599} which belongs to line L_{\_G36587} and line L_{\_G36593}
intersec P_{\_G36599} L_{\_G36587} L_{\_G36593}
cmark_r P_{\_G36599}

% Constructing a point P_{\_G36602} which belongs to line L_{\_G36587} and line L_{\_G36596}
intersec P_{\_G36602} L_{\_G36587} L_{\_G36596}
cmark_r P_{\_G36602}

% Constructing a line L_{\_G36605} which passes through point B and point P_{\_G36602}
line L_{\_G36605} B P_{\_G36602}

color 200 200 200
drawline L_{\_G36605}
color 0 0 0

% Constructing a line L_{\_G36608} which passes through point C and point P_{\_G36599}
line L_{\_G36608} C P_{\_G36599}

color 200 200 200
drawline L_{\_G36608}
color 0 0 0

% Constructing a point P_{\_G36611} which belongs to line L_{\_G36605} and line L_{\_G36608}
intersec P_{\_G36611} L_{\_G36605} L_{\_G36608}
cmark_r P_{\_G36611}

% Constructing a line L_{\_G36614} which passes through point P_{\_G36590} and point P_{\_G36611}
line L_{\_G36614} P_{\_G36590} P_{\_G36611}

color 200 200 200
drawline L_{\_G36614}
color 0 0 0

% Constructing a point T'_{a} which belongs to line L_{\_G36614} and line a

```

```

intersec T'_{a} L_{\_G36614} a
cmark_r T'_{a}

```

```

% Constructing midpoint P_{\_G37439} of the segment T_{a}T'_{a}
midpoint P_{\_G37439} T_{a} T'_{a}
cmark_r P_{\_G37439}

```

```

% Choosing randomly a point A on the circle with center P_{\_G37439} through point T_{a}
oncircle A P_{\_G37439} T_{a}
cmark_t A
color 200 200 200
drawcircle P_{\_G37439} T_{a}
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

*% Non-degenerate conditions: points B and C are not the same; points C and T\_{a} are not the same;  
points C and midpoint([B,T\_{a}]) are not the same  
% Determination conditions: points M\_{a} and T\_{a} are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = \_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_a = \_T_a$

Proving failed

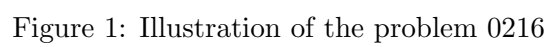
#### 4.1.3 Proving $C = C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.017 seconds.

**NDG conditions** There are no NDG conditions for this theorem



## 4.2 GCLC - Area method

### 4.2.1 Proving $M_a = \neg M_a$

Proving failed

### 4.2.2 Proving $T_a = \neg T_a$

Proving failed

### 4.2.3 Proving $C = C$

NDG conditions are:

$S_{T_a BP_{G31995}} \neq S_{P_{G31989} BP_{G31995}}$  i.e., lines  $T_a P_{G31989}$  and  $BP_{G31995}$  are not parallel (construction based assumption)

$S_{T_a CP_{G31995}} \neq S_{P_{G31989} CP_{G31995}}$  i.e., lines  $T_a P_{G31989}$  and  $CP_{G31995}$  are not parallel (construction based assumption)

$S_{BCP_{G32004}} \neq S_{P_{G32007} CP_{G32004}}$  i.e., lines  $BP_{G32007}$  and  $CP_{G32004}$  are not parallel (construction based assumption)

$S_{P_{G31995} M_a T_a} \neq S_{P_{G32016} M_a T_a}$  i.e., lines  $P_{G31995} P_{G32016}$  and  $M_a T_a$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_a = \neg M_a$

Proving failed

### 4.3.2 Proving $T_a = \neg T_a$

Proving failed

### 4.3.3 Proving $C = C$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_a = \neg M_a$

Proving failed

### 4.4.2 Proving $T_a = \neg T_a$

Proving failed

### 4.4.3 Proving $C = C$

Proving failed

# Problem 217

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 217: Given a point  $C$ , a point  $M_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
2. Using the point  $C$  and the point  $M_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $M_a$  are not the same;
3. Using the point  $C$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $T_b$  are not the same;
4. Using the point  $T_b$  and the line  $a$ , construct a circle  $k(T_b, foot[T_b, c])$  (rule W11); % NDG: point  $T_b$  is not incident to the line  $a$ ;
5. Using the circle  $k(T_b, foot[T_b, c])$ , the point  $B$ , the point  $T_b$  and the line  $a$ , construct a line  $c$  (rule W13); % NDG: point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ;
6. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ; point  $T_b$  is not incident to the line  $a$ .

Determination conditions: lines  $c$  and  $b$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $M_a$  are not the same.

Rules used: [W01,W02,W03,W11,W13]

Lemmas used: [D21,D24,GD01,GD02,GL03,GL04,GL09,GL10,GL11,L9]

Solving time: 1.8 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point M_{a} 65 40
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_l C
cmark_r M_{a}
cmark_t T_{b}
color 0 0 0
fontsize 8

% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% DET: points C and M_{a} are not the same
% Constructing a line a which passes through point C and point M_{a}
line a C M_{a}

color 200 200 200
drawline a
color 0 0 0

% DET: points C and T_{b} are not the same
% Constructing a line b which passes through point C and point T_{b}
line b C T_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: point T_{b} is not incident to the line a
% Constructing a point P_{\G56197} which is a foot of the point T_{b} on the line a
foot P_{\G56197} T_{b} a
cmark_r P_{\G56197}
color 200 200 200
drawline T_{b} P_{\G56197}
color 0 0 0

% Constructing a circle k(T_{b},foot[T_{b},c]) whose center is at point T_{b} and which passes
    through point P_{\G56197}
```



```

circle k(T_{b},foot[T_{b},c]) T_{b} P_{\_G56197}

color 200 200 200
drawcircle k(T_{b},foot[T_{b},c])
color 0 0 0

% NDG: point B is outside the circle k(T_{b},foot[T_{b},c])
% Constructing a point P_{\_G56588} which is a foot of the point T_{b} on the line a
foot P_{\_G56588} T_{b} a
cmark_r P_{\_G56588}
color 200 200 200
drawline T_{b} P_{\_G56588}
color 0 0 0

% Constructing a line L_{\_G56591} which passes through point T_{b} and point B
line L_{\_G56591} T_{b} B

color 200 200 200
drawline L_{\_G56591}
color 0 0 0

% Constructing a point P_{\_G56594} which is an image of the point P_{\_G56588} in the symmetry to
point/line L_{\_G56591}
sim P_{\_G56594} L_{\_G56591} P_{\_G56588}
cmark_r P_{\_G56594}

% Constructing a line c which passes through point B and point P_{\_G56594}
line c B P_{\_G56594}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and b are not parallel; point B is outside the circle k(T_{b},
foot[T_{b},c]); point T_{b} is not incident to the line a
% Determination conditions: lines c and b are not the same; points C and T_{b} are not the same;
points C and M_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

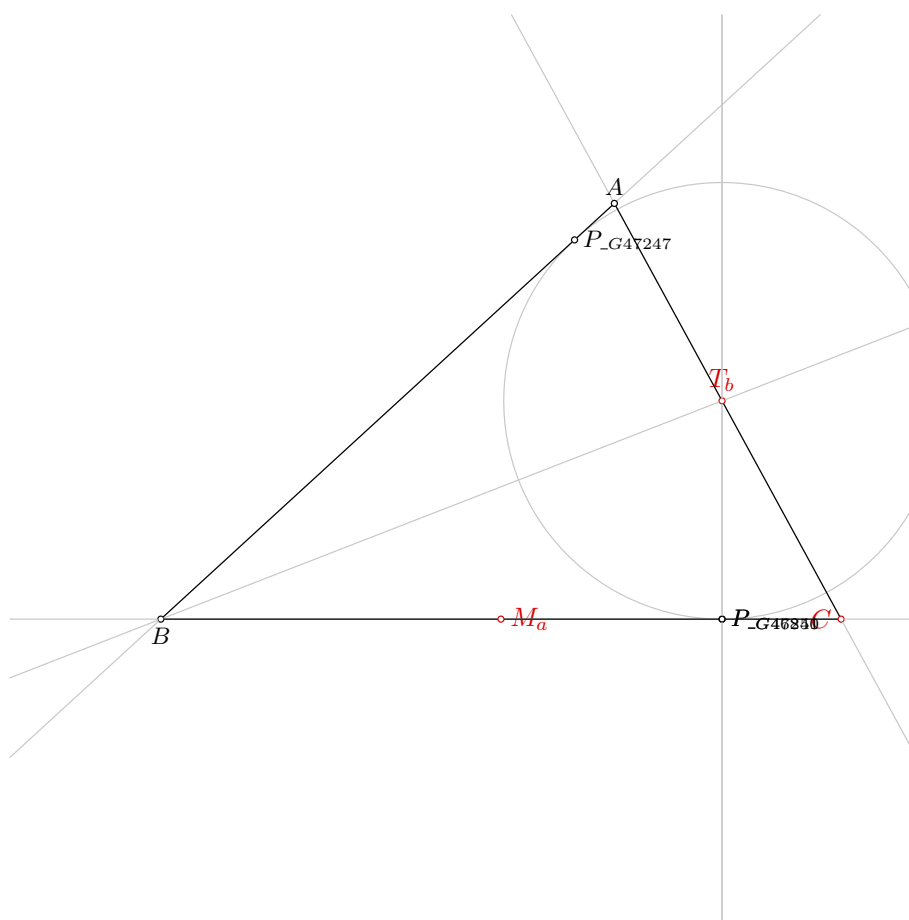


Figure 1: Illustration of the problem 0217

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a=_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.032 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $T_b=_T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{BCT_b} \neq S_{P_{G48273}CT_b}$  i.e., lines  $BP_{G48273}$  and  $CT_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_a=_M_a$

Proving failed

#### 4.2.3 Proving $T_b=_T_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_a=_M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_b = \neg T_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 10 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 7 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_b = \neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 7 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 218

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 218: Given a point  $C$ , a point  $M_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
2. Using the point  $C$  and the point  $M_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $M_a$  are not the same;
3. Using the point  $T_c$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $T_c$  and  $B$  are not the same;
4. Using the point  $T_c$  and the line  $a$ , construct a circle  $k(T_c, foot[T_c, b])$  (rule W11); % NDG: point  $T_c$  is not incident to the line  $a$ ;
5. Using the circle  $k(T_c, foot[T_c, b])$ , the point  $C$ , the point  $T_c$  and the line  $a$ , construct a line  $b$  (rule W13); % NDG: point  $C$  is outside the circle  $k(T_c, foot[T_c, b])$ ;
6. Using the line  $b$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $c$  are not parallel % DET: lines  $b$  and  $c$  are not the same.

Non-degenerate conditions: lines  $b$  and  $c$  are not parallel; point  $C$  is outside the circle  $k(T_c, foot[T_c, b])$ ; point  $T_c$  is not incident to the line  $a$ .

Determination conditions: lines  $b$  and  $c$  are not the same; points  $T_c$  and  $B$  are not the same; points  $C$  and  $M_a$  are not the same.

Rules used: [W01,W02,W03,W11,W13]

Lemmas used: [D21,D25,GD01,GD02,GL03,GL04,GL09,GL10,GL11,L10]

Solving time: 1.8 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point M_{a} 65 40
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_l C
cmark_r M_{a}
cmark_rt T_{c}
color 0 0 0
fontsize 8

% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% DET: points C and M_{a} are not the same
% Constructing a line a which passes through point C and point M_{a}
line a C M_{a}

color 200 200 200
drawline a
color 0 0 0

% DET: points T_{c} and B are not the same
% Constructing a line c which passes through point T_{c} and point B
line c T_{c} B

color 200 200 200
drawline c
color 0 0 0

% NDG: point T_{c} is not incident to the line a
% Constructing a point P_{\_G32757} which is a foot of the point T_{c} on the line a
foot P_{\_G32757} T_{c} a
cmark_r P_{\_G32757}
color 200 200 200
drawline T_{c} P_{\_G32757}
color 0 0 0

% Constructing a circle k(T_{c},foot[T_{c},b]) whose center is at point T_{c} and which passes
    through point P_{\_G32757}
```

```

circle k(T_{c},foot[T_{c},b]) T_{c} P_{\_G32757}

color 200 200 200
drawcircle k(T_{c},foot[T_{c},b])
color 0 0 0

% NDG: point C is outside the circle k(T_{c},foot[T_{c},b])
% Constructing a point P_{\_G33148} which is a foot of the point T_{c} on the line a
foot P_{\_G33148} T_{c} a
cmark_r P_{\_G33148}
color 200 200 200
drawline T_{c} P_{\_G33148}
color 0 0 0

% Constructing a line L_{\_G33151} which passes through point T_{c} and point C
line L_{\_G33151} T_{c} C

color 200 200 200
drawline L_{\_G33151}
color 0 0 0

% Constructing a point P_{\_G33154} which is an image of the point P_{\_G33148} in the symmetry to
point/line L_{\_G33151}
sim P_{\_G33154} L_{\_G33151} P_{\_G33148}
cmark_r P_{\_G33154}

% Constructing a line b which passes through point C and point P_{\_G33154}
line b C P_{\_G33154}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines b and c are not parallel% DET: lines b and c are not the same
% Constructing a point A which belongs to line b and line c
intersec A b c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and c are not parallel; point C is outside the circle k(T_{c},
foot[T_{c},b]); point T_{c} is not incident to the line a
% Determination conditions: lines b and c are not the same; points T_{c} and B are not the same;
points C and M_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

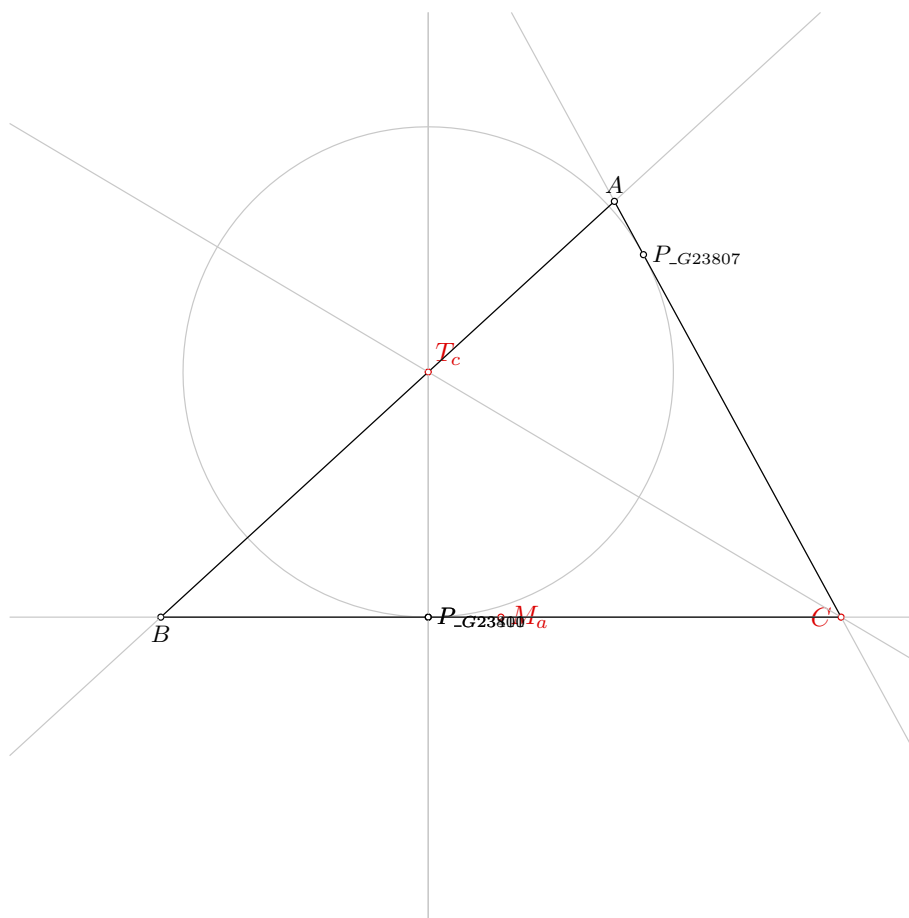


Figure 1: Illustration of the problem 0218



### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.032 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a=_M a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $T_c=_T c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$SC_{T_c B} \neq SP_{G24833 T_c B}$  i.e., lines  $CP_{G24833}$  and  $T_c B$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_a=_M a$

Proving failed

#### 4.2.3 Proving $T_c=_T c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_a=_M a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 5 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_c = \neg T_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 14 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_c = \neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

# Problem 219

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 219: Given a point  $C$ , a point  $M_a$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
2. Using the point  $C$  and the point  $M_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $M_a$  are not the same;
3. Using the point  $I$  and the line  $a$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $a$ ;
4. Using the circle  $k(I, P_a)$ , the point  $C$ , the point  $I$  and the line  $a$ , construct a line  $b$  (rule W13); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
5. Using the circle  $k(I, P_a)$ , the point  $B$ , the point  $I$  and the line  $a$ , construct a line  $c$  (rule W13); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
6. Using the line  $b$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $c$  are not parallel % DET: lines  $b$  and  $c$  are not the same.

Non-degenerate conditions: lines  $b$  and  $c$  are not parallel; point  $B$  is outside the circle  $k(I, P_a)$ ; point  $C$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $a$ .

Determination conditions: lines  $b$  and  $c$  are not the same; points  $C$  and  $M_a$  are not the same.

Rules used: [W01,W02,W03,W11,W13]

Lemmas used: [D21,D27,GD01,GL03,GL04,GL09,L59,L60,L61]

Solving time: 17.3 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point M_{a} 65 40
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_l C
cmark_r M_{a}
cmark_b I
color 0 0 0
fontsize 8

% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% DET: points C and M_{a} are not the same
% Constructing a line a which passes through point C and point M_{a}
line a C M_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: point I is not incident to the line a
% Constructing a point P_{\_G55673} which is a foot of the point I on the line a
foot P_{\_G55673} I a
cmark_r P_{\_G55673}
color 200 200 200
drawline I P_{\_G55673}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
55673}
circle k(I,P_{a}) I P_{\_G55673}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point C is outside the circle k(I,P_{a})
% Constructing a point P_{\_G55880} which is a foot of the point I on the line a
```

```

foot P_{\_G55880} I a
cmark_r P_{\_G55880}
color 200 200 200
drawline I P_{\_G55880}
color 0 0 0

% Constructing a line L_{\_G55883} which passes through point I and point C
line L_{\_G55883} I C

color 200 200 200
drawline L_{\_G55883}
color 0 0 0

% Constructing a point P_{\_G55886} which is an image of the point P_{\_G55880} in the symmetry to
point/line L_{\_G55883}
sim P_{\_G55886} L_{\_G55883} P_{\_G55880}
cmark_r P_{\_G55886}

% Constructing a line b which passes through point C and point P_{\_G55886}
line b C P_{\_G55886}

color 200 200 200
drawline b
color 0 0 0

% NDG: point B is outside the circle k(I,P_{a})
% Constructing a point P_{\_G56215} which is a foot of the point I on the line a
foot P_{\_G56215} I a
cmark_r P_{\_G56215}
color 200 200 200
drawline I P_{\_G56215}
color 0 0 0

% Constructing a line L_{\_G56218} which passes through point I and point B
line L_{\_G56218} I B

color 200 200 200
drawline L_{\_G56218}
color 0 0 0

% Constructing a point P_{\_G56221} which is an image of the point P_{\_G56215} in the symmetry to
point/line L_{\_G56218}
sim P_{\_G56221} L_{\_G56218} P_{\_G56215}
cmark_r P_{\_G56221}

% Constructing a line c which passes through point B and point P_{\_G56221}
line c B P_{\_G56221}

color 200 200 200
drawline c
color 0 0 0

```

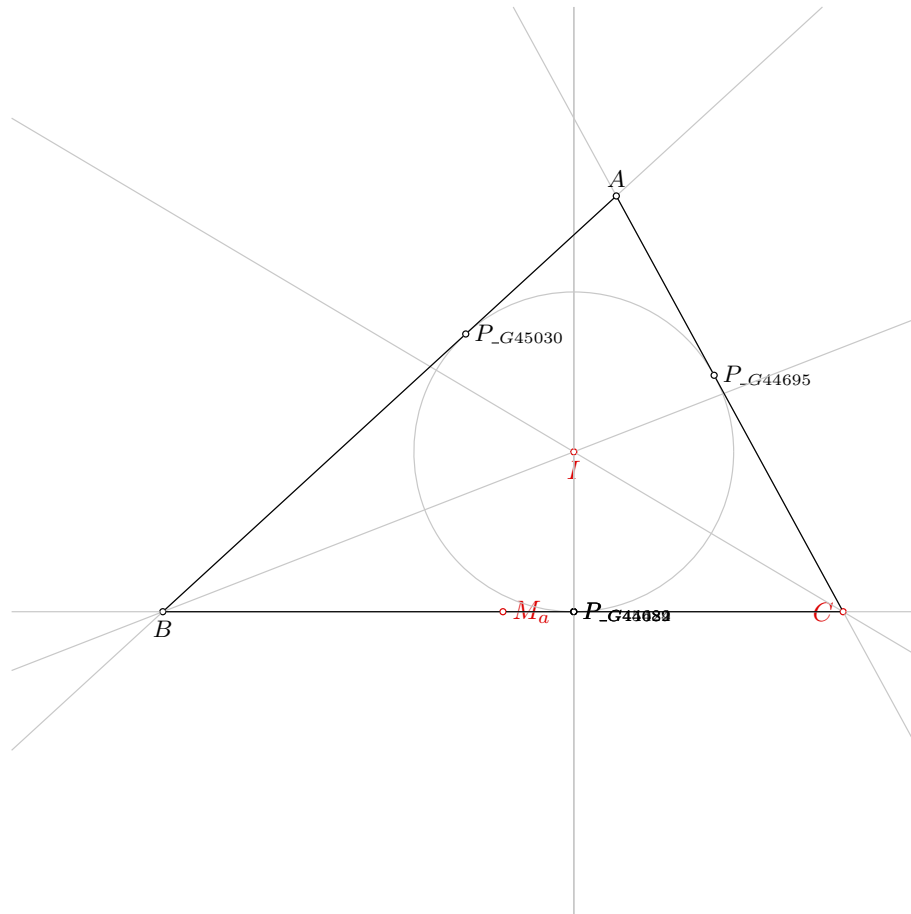


Figure 1: Illustration of the problem 0219

```
% NDG: lines b and c are not parallel% DET: lines b and c are not the same
% Constructing a point A which belongs to line b and line c
intersec A b c
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines b and c are not parallel; point B is outside the circle k(I,P_{a}
}); point C is outside the circle k(I,P_{a}); point I is not incident to the line a
% Determination conditions: lines b and c are not the same; points C and M_{a} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.02 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_a=_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.021 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{CBP\_G46110} \neq S_{P\_G45775BP\_G46110}$  i.e., lines  $CP\_G45775$  and  $BP\_G46110$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_a=_M_a$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 58 terms.

**Time Complexity:** Time spent by the prover is 0.050 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I = \neg I$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 265 terms.

**Time Complexity:** Time spent by the prover is 0.420 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 20 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.3 Proving $I = \neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 20 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.



# Problem 220

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 220: Given a point  $C$ , a point  $M_b$  and a point  $M_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); ;
2. Using the point  $C$  and the point  $M_c$ , construct a point  $G$  (rule W01); ;
3. Using the point  $M_b$  and the point  $G$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D22,GL03,GL04,L56,L57]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point M_{b} 95 67.5
point M_{c} 50 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_lt M_{b}
cmark_lt M_{c}
color 0 0 0
fontsize 8
```

```

% Constructing a point A such that  $CA/CM_{\{b\}}=2$ 
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

% Constructing a line  $L_{\{G25677\}}$  which passes through point C and point  $M_{\{c\}}$ 
line L_{G25677} C M_{c}

color 200 200 200
drawline L_{G25677}
color 0 0 0

% Constructing a point  $P_{\{G25778\}}$  with coordinates (0,0)
point P_{G25778} 0 0
cmark_r P_{G25778}

% Constructing a point  $P_{\{G25702\}}$  such that  $CP_{\{G25702\}}/CP_{\{G25778\}}=2$ 
towards P_{G25702} C P_{G25778} 2
cmark_r P_{G25702}
color 200 200 200
drawsegment C P_{G25702}
color 0 0 0

% Constructing a point  $P_{\{G25747\}}$  such that  $CP_{\{G25747\}}/CP_{\{G25778\}}=3$ 
towards P_{G25747} C P_{G25778} 3
cmark_r P_{G25747}
color 200 200 200
drawsegment C P_{G25747}
color 0 0 0

% Constructing a line  $L_{\{G25708\}}$  which passes through point  $M_{\{c\}}$  and point  $P_{\{G25747\}}$ 
line L_{G25708} M_{c} P_{G25747}

color 200 200 200
drawline L_{G25708}
color 0 0 0

% Constructing a line  $L_{\{G25671\}}$  which contains the point  $P_{\{G25702\}}$  and is parallel to the
line  $L_{\{G25708\}}$ 
parallel L_{G25671} P_{G25702} L_{G25708}

color 200 200 200
drawline L_{G25671}
color 0 0 0

% Constructing a point G which belongs to line  $L_{\{G25671\}}$  and line  $L_{\{G25677\}}$ 
intersec G L_{G25671} L_{G25677}
cmark_t G

```

```

% Constructing a point B such that  $M_{\{b\}B}/M_{\{b\}G}=3$ 
towards B  $M_{\{b\}}$  G 3
cmark_b B
color 200 200 200
drawsegment  $M_{\{b\}}$  B
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.024 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b=_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.052 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $M_c=_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 91 terms.

**Time Complexity:** Time spent by the prover is 0.409 seconds.

**NDG conditions** Points  $C$ ,  $M_c$  and  $P_{G25132}$  are not collinear

Line through points  $M_b$  and  $C$  is not parallel with line through points  $M_c$  and  $P_{G25132}$

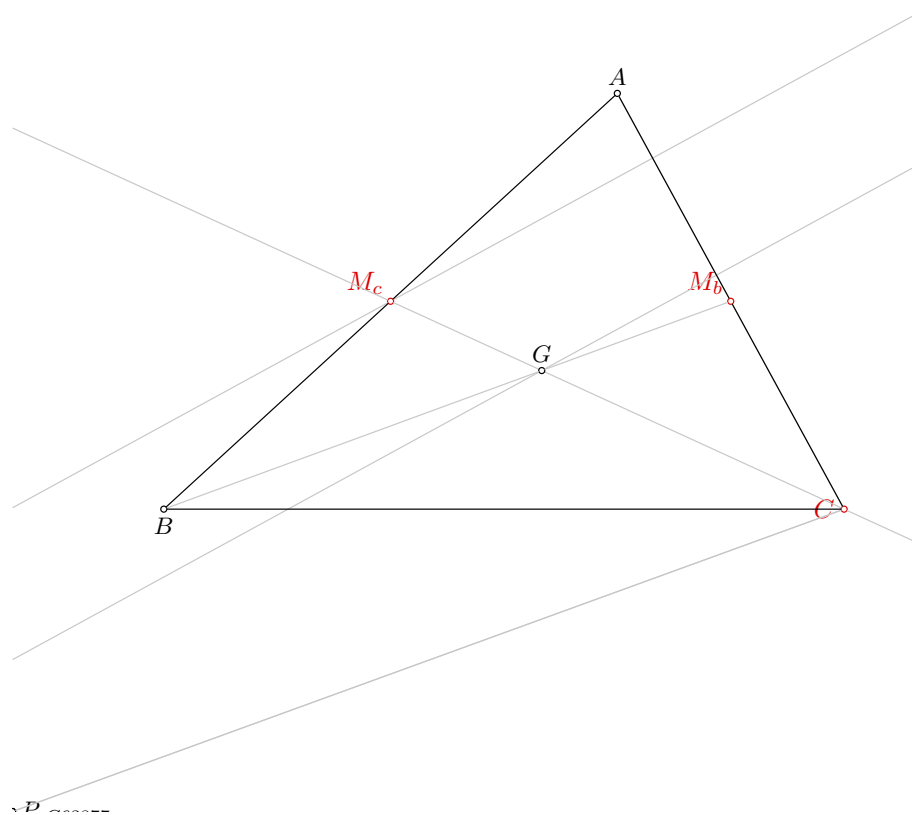


Figure 1: Illustration of the problem 0220

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{P_{G21390}CM_c} \neq S_{P_{L_{G21388}}^0}^{CM_c}$  i.e., lines  $P_{G21390}P_{L_{G21388}}^0$  and  $CM_c$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_b=_M_b$

NDG conditions are:

$S_{P_{G22181}CM_c} \neq S_{P_{L_{G22150}}^0}^{CM_c}$  i.e., lines  $P_{G22181}P_{L_{G22150}}^0$  and  $CM_c$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $M_c=_M_c$

NDG conditions are:

$S_{P_{G23144}CM_c} \neq S_{P_{L_{G23113}}^0}^{CM_c}$  i.e., lines  $P_{G23144}P_{L_{G23113}}^0$  and  $CM_c$  are not parallel (construction based assumption)

$S_{CM_cP_{G23220}} \neq 0$  i.e., points  $C$ ,  $M_c$  and  $P_{G23220}$  are not collinear (cancellation assumption)

Total number of proof steps: 4022

Time spent by the prover: 5.520 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_b=_M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.3 Proving $M_c=_M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3932 terms.

**Time Complexity:** Time spent by the prover is 6.460 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_b = M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 107 terms.

**Time Complexity:** Time spent by the prover is 0.680 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_c = M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 355 terms.

**Time Complexity:** Time spent by the prover is 0.800 seconds. There are no ndg conditions.

# Problem 221

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 221: Given a point  $C$ , a point  $M_b$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); ;
2. Using the point  $M_b$  and the point  $G$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D22,GL03,GL04,L56]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point M_{b} 95 67.5
point G 70 58.33
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_lt M_{b}
cmark_t G
color 0 0 0
fontsize 8
```

```
% Constructing a point A such that CA/CM_{b}=2
```

```

towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

% Constructing a point B such that M_{b}B/M_{b}G=3
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b=_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.055 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $G=_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 45 terms.

**Time Complexity:** Time spent by the prover is 0.478 seconds.

**NDG conditions** Line through points  $A$  and  $_M_a$  is not parallel with line through points  $B$  and  $_M_b$   
Points  $A$  and  $_M_a$  are not identical



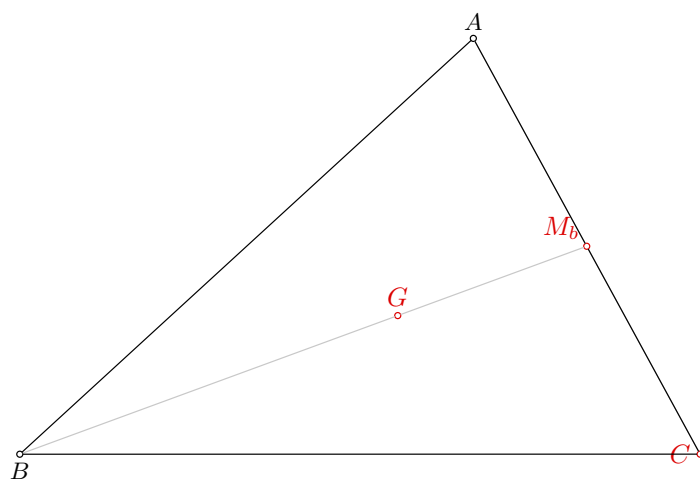


Figure 1: Illustration of the problem 0221

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{AB\_M_b} \neq S_{\_M_a B\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_b=_M_b$

NDG conditions are:

$S_{AB\_M_b} \neq S_{\_M_a B\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $G=_G$

NDG conditions are:

$S_{AB\_M_b} \neq S_{\_M_a B\_M_b}$  i.e., lines  $A\_M_a$  and  $B\_M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (cancellation assumption)

Total number of proof steps: 673

Time spent by the prover: 0.080 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_b=_M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $G=_G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 55 terms.

**Time Complexity:** Time spent by the prover is 0.090 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_b = M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 365 terms.

**Time Complexity:** Time spent by the prover is 2.130 seconds. There are no ndg conditions.

#### 4.4.3 Proving $G = G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 365 terms.

**Time Complexity:** Time spent by the prover is 2.240 seconds. There are no ndg conditions.

# Problem 222

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 222: Given a point  $M_b$ , a point  $H_a$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_a$  and  $M_b$  are not the same;
2. Choose freely a point  $C$  on the circle  $k(M_b, C)$  (rule WOncircle);
3. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); ;
4. Using the point  $C$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $H_a$  are not the same;
5. Choose freely a point  $B$  on the line  $a$  (rule WOnline1) .

Non-degenerate conditions: points  $H_a$  and  $M_b$  are not the same.

Determination conditions: points  $C$  and  $H_a$  are not the same.

Rules used: [W01,W02,W06,WOncircle1,WOnline1]

Lemmas used: [D22,D5,GD01,GD02,GL03,L44]

Solving time: 823.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point H_{a} 80 40
point C 110 40

color 220 0 0
fontsize 9
```

```

cmark_lt M_{b}
cmark_r H_{a}
cmark_l C
color 0 0 0
fontsize 8

% NDG: points H_{a} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{a}
circle k(M_{b},C) M_{b} H_{a}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% Choosing randomly a point C on the circle with center M_{b} through point H_{a}
oncircle C M_{b} H_{a}
cmark_l C
color 200 200 200
drawcircle M_{b} H_{a}
color 0 0 0

% Constructing a point A such that CA/CM_{b}=2
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% Choosing randomly a point B on the line CH_{a}
online B C H_{a}
cmark_b B
color 200 200 200
drawline C H_{a}
color 0 0 0

```

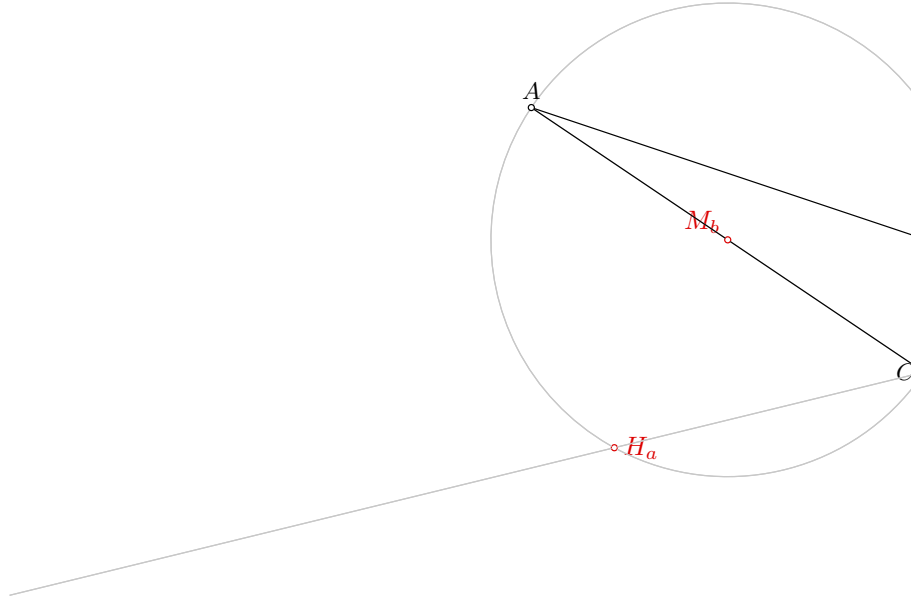


Figure 1: Illustration of the problem 0222

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: points  $H_{\{a\}}$  and  $M_{\{b\}}$  are not the same
% Determination conditions: points  $C$  and  $H_{\{a\}}$  are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.109 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a = \neg H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 47 terms.

**Time Complexity:** Time spent by the prover is 0.631 seconds.

**NDG conditions** Points  $C$  and  $H_a$  are not identical

Points  $B$  and  $C$  are not identical

Points  $B$ ,  $C$  and  $H_a$  are not collinear

#### 4.1.3 Proving $C = C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.022 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{\neg h_a}^1} \neq S_{CAF_{\neg h_a}^1}$  i.e., lines  $BC$  and  $AF_{\neg h_a}^1$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.2.3 Proving $C = C$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{\neg h_a}^1} \neq S_{CAF_{\neg h_a}^1}$  i.e., lines  $BC$  and  $AF_{\neg h_a}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 17 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 280 terms.

**Time Complexity:** Time spent by the prover is 0.110 seconds. There are no ndg conditions.

#### 4.3.3 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 312 terms.

**Time Complexity:** Time spent by the prover is 2.080 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 312 terms.

**Time Complexity:** Time spent by the prover is 1.980 seconds. There are no ndg conditions.

#### 4.4.3 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.



# Problem 223

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 223: Given a point  $M_b$ , a point  $H_b$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $H_b$  are not the same;
2. Choose freely a point  $C$  on the line  $b$  (rule WOnline1) ;
3. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); ;
4. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
5. Choose freely a point  $B$  on the line  $h_b$  (rule WOnline2).

Non-degenerate conditions: .

Determination conditions: points  $M_b$  and  $H_b$  are not the same.

Rules used: [W01,W02,W10b,WOnline1,WOnline2]

Lemmas used: [D22,D6,D9,GD01,GL03,GL09]

Solving time: 155.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point H_{b} 89.36 77.83
point C 110 40

color 220 0 0
fontsize 9
```

```

cmark_lt M_{b}
cmark_l H_{b}
cmark_l C
color 0 0 0
fontsize 8

% DET: points M_{b} and H_{b} are not the same
% Constructing a line b which passes through point M_{b} and point H_{b}
line b M_{b} H_{b}

color 200 200 200
drawline b
color 0 0 0

% Choosing randomly a point C on the line M_{b}H_{b}
online C M_{b} H_{b}
cmark_l C
color 200 200 200
drawline M_{b} H_{b}
color 0 0 0

% Constructing a point A such that CA/CM_{b}=2
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0

% Generating random value V[_G23894]
random V[_G23894]

% Calculating value V[_G23915] using formula V[_G23894]*20
expression V[_G23915] { V[_G23894]*20 }

% Constructing a point B which is a point for which holds H_{b}B = V[_G23915] and angle CH_{b}B =
90
turtle B C H_{b} 90 V[_G23915]

```

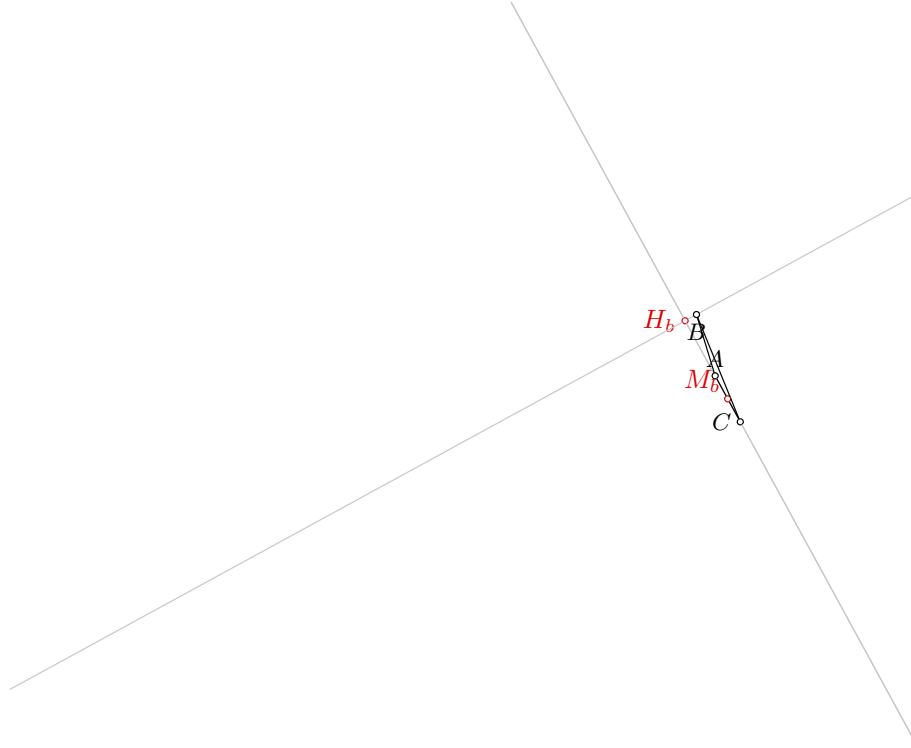


Figure 1: Illustration of the problem 0223

```
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

*% Non-degenerate conditions:*

*% Determination conditions: points  $M_{\{b\}}$  and  $H_{\{b\}}$  are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = \neg M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b = \neg H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.095 seconds.

**NDG conditions** Line through points  $M_b$  and  $C$  is not perpendicular to line through points  $C$  and  $A$

#### 4.1.3 Proving $C = C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.015 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^2_{h_b}} \neq S_{CBF^2_{h_b}}$  i.e., lines  $AC$  and  $BF^2_{h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.2.3 Proving $C = C$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^2_{h_b}} \neq S_{CBF^2_{h_b}}$  i.e., lines  $AC$  and  $BF^2_{h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 2 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 224

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 224: Given a point  $M_b$ , a point  $H_c$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_c$  and  $M_b$  are not the same;
2. Choose freely a point  $C$  on the circle  $k(M_b, C)$  (rule WOncircle);
3. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); ;
4. Using the point  $H_c$  and the point  $A$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $A$  are not the same;
5. Choose freely a point  $B$  on the line  $c$  (rule WOnline1) .

Non-degenerate conditions: points  $H_c$  and  $M_b$  are not the same.

Determination conditions: points  $H_c$  and  $A$  are not the same.

Rules used: [W01,W02,W06,WOncircle1,WOnline1]

Lemmas used: [D22,D7,GD01,GD02,GL03,L45]

Solving time: 952.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point H_{c} 68.91 84.83
point C 110 40

color 220 0 0
fontsize 9
```

```

cmark_lt M_{b}
cmark_rt H_{c}
cmark_l C
color 0 0 0
fontsize 8

% NDG: points H_{c} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{c}
circle k(M_{b},C) M_{b} H_{c}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% Choosing randomly a point C on the circle with center M_{b} through point H_{c}
oncircle C M_{b} H_{c}
cmark_l C
color 200 200 200
drawcircle M_{b} H_{c}
color 0 0 0

% Constructing a point A such that CA/CM_{b}=2
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

% DET: points H_{c} and A are not the same
% Constructing a line c which passes through point H_{c} and point A
line c H_{c} A

color 200 200 200
drawline c
color 0 0 0

% Choosing randomly a point B on the line H_{c}A
online B H_{c} A
cmark_b B
color 200 200 200
drawline H_{c} A
color 0 0 0

```

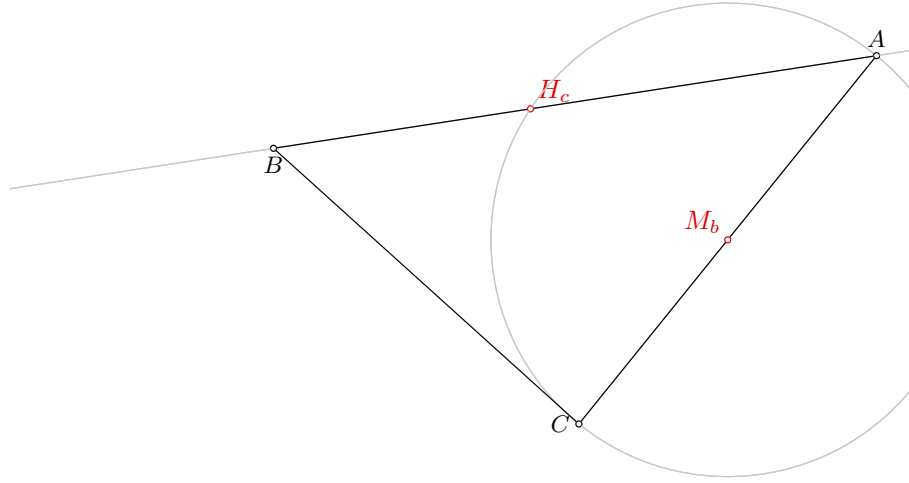


Figure 1: Illustration of the problem 0224

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: points  $H_{\{c\}}$  and  $M_{\{b\}}$  are not the same
% Determination conditions: points  $H_{\{c\}}$  and A are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = M_b$

**Status:** Theorem has been proved.



**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.081 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.1.3 Proving $C = C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^1} \neq S_{BCF_{h_c}^1}$  i.e., lines  $AB$  and  $CF_{h_c}^1$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.2.3 Proving $C = C$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^1} \neq S_{BCF_{h_c}^1}$  i.e., lines  $AB$  and  $CF_{h_c}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 17 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 280 terms.

**Time Complexity:** Time spent by the prover is 0.500 seconds. There are no ndg conditions.

### 4.3.3 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_b=_M M_b$

Proving failed

### 4.4.2 Proving $H_c=_H H_c$

Proving failed

### 4.4.3 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 225

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 225: Given a point  $C$ , a point  $M_b$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); ;
2. Using the point  $C$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H$  are not the same;
3. Using the point  $H$  and the point  $A$ , construct a line  $h_a$  (rule W02); % DET: points  $H$  and  $A$  are not the same;
4. Using the point  $C$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $C$  and  $M_b$  are not the same;
5. Using the circle  $k(M_b, C)$ , the line  $h_c$ , the point  $M_b$  and the point  $C$ , construct a point  $H_c$  (rule W05); % NDG: line  $h_c$  and circle  $k(M_b, C)$  intersect % DET: points  $C$  and  $H_c$  must be different;
6. Using the point  $H_c$  and the point  $A$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $A$  are not the same;
7. Using the circle  $k(M_b, C)$ , the line  $h_a$ , the point  $M_b$  and the point  $A$ , construct a point  $H_a$  (rule W05); % NDG: line  $h_a$  and circle  $k(M_b, C)$  intersect % DET: points  $A$  and  $H_a$  must be different;
8. Using the point  $H_a$  and the point  $C$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $C$  are not the same;
9. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; line  $h_a$  and circle  $k(M_b, C)$  intersect; line  $h_c$  and circle  $k(M_b, C)$  intersect; points  $C$  and  $M_b$  are not the same.

Determination conditions: lines  $c$  and  $a$  are not the same; points  $H_a$  and  $C$  are not the same; points  $A$  and  $H_a$  must be different; points  $H_c$  and  $A$  are not the same; points  $C$  and  $H_c$  must be different; points  $H$  and  $A$  are not the same; points  $C$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D22,D3,D5,D7,D8,GD01,GD02,GL03,L3,L43,L44,L45]

Solving time: 1.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point M_{b} 95 67.5
point H 80 72.73

color 220 0 0
fontsize 9

cmark_l C
cmark_lt M_{b}
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a point A such that CA/CM_{b}=2
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

% DET: points C and H are not the same
% Constructing a line h_{c} which passes through point C and point H
line h_{c} C H

color 200 200 200
drawline h_{c}
color 0 0 0

% DET: points H and A are not the same
% Constructing a line h_{a} which passes through point H and point A
line h_{a} H A

color 200 200 200
drawline h_{a}
color 0 0 0
```

```

% NDG: points C and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point C
circle k(M_{b},C) M_{b} C

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line h_{c} and circle k(M_{b},C) intersect% DET: points C and H_{c} must be different
% Constructing a point P_{\G58977} which is a foot of the point M_{b} on the line h_{c}
foot P_{\G58977} M_{b} h_{c}
cmark_r P_{\G58977}
color 200 200 200
drawline M_{b} P_{\G58977}
color 0 0 0

% Constructing a point H_{c} which is an image of the point C in the symmetry to point/line P_{\G
58977}
sim H_{c} P_{\G58977} C
cmark_rt H_{c}

% DET: points H_{c} and A are not the same
% Constructing a line c which passes through point H_{c} and point A
line c H_{c} A

color 200 200 200
drawline c
color 0 0 0

% NDG: line h_{a} and circle k(M_{b},C) intersect% DET: points A and H_{a} must be different
% Constructing a point P_{\G59215} which is a foot of the point M_{b} on the line h_{a}
foot P_{\G59215} M_{b} h_{a}
cmark_r P_{\G59215}
color 200 200 200
drawline M_{b} P_{\G59215}
color 0 0 0

% Constructing a point H_{a} which is an image of the point A in the symmetry to point/line P_{\G
59215}
sim H_{a} P_{\G59215} A
cmark_r H_{a}

% DET: points H_{a} and C are not the same
% Constructing a line a which passes through point H_{a} and point C
line a H_{a} C

color 200 200 200
drawline a
color 0 0 0

```

```

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and a are not parallel; line h_{a} and circle k(M_{b},C)
% intersect; line h_{c} and circle k(M_{b},C) intersect; points C and M_{b} are not the same
% Determination conditions: lines c and a are not the same; points H_{a} and C are not the same;
% points A and H_{a} must be different; points H_{c} and A are not the same; points C and H_{c}
% must be different; points H and A are not the same; points C and H are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.019 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b=_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $H=_H$

**Status:** Theorem has been proved.

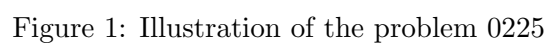
**Space Complexity:** The biggest polynomial obtained during prover execution contains 227 terms.

**Time Complexity:** Time spent by the prover is 8.342 seconds.

**NDG conditions** Points  $M_b$  and  $H$  are not identical

Points  $M_b$  and  $H$  are not identical

Points  $A$  and  $H$  are not identical



Line through points  $M_b$  and  $A$  is not perpendicular to line through points  $A$  and  $C$

Line through points  $A$  and  $H_c$  is not parallel with line through points  $C$  and  $H_a$

Points  $M_b$ ,  $H_c$  and  $C$  are not collinear

Line through points  $M_b$  and  $B$  is not parallel with line through points  $A$  and  $C$

Points  $M_b$ ,  $A$  and  $B$  are not collinear

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_c H_a C} \neq S_{A H_a C}$  i.e., lines  $H_c A$  and  $H_a C$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$  i.e., lines  $AF_{-h_a}^0$  and  $BF_{-h_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_b=_M M_b$

Proving failed

### 4.2.3 Proving $H=_H H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_b=_M M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $H=_H H$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 9 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.



#### 4.4.2 Proving $M_b = M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $H = H$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

# Problem 226

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 226: Given a point  $C$ , a point  $M_b$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); ;
2. Using the point  $C$  and the point  $M_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $M_b$  are not the same;
3. Using the point  $C$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $T_a$  are not the same;
4. Using the point  $T_a$  and the line  $b$ , construct a circle  $k(T_a, foot[T_a, c])$  (rule W11); % NDG: point  $T_a$  is not incident to the line  $b$ ;
5. Using the circle  $k(T_a, foot[T_a, c])$ , the point  $A$ , the point  $T_a$  and the line  $b$ , construct a line  $c$  (rule W13); % NDG: point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ;
6. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ; point  $T_a$  is not incident to the line  $b$ .

Determination conditions: lines  $c$  and  $a$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $M_b$  are not the same.

Rules used: [W01,W02,W03,W11,W13]

Lemmas used: [D22,D23,GD01,GD02,GL03,GL09,GL10,GL11,L8]

Solving time: 1.9 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point M_{b} 95 67.5
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_l C
cmark_lt M_{b}
cmark_rb T_{a}
color 0 0 0
fontsize 8

% Constructing a point A such that CA/CM_{b}=2
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

% DET: points C and M_{b} are not the same
% Constructing a line b which passes through point C and point M_{b}
line b C M_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points C and T_{a} are not the same
% Constructing a line a which passes through point C and point T_{a}
line a C T_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: point T_{a} is not incident to the line b
% Constructing a point P_{\G36119} which is a foot of the point T_{a} on the line b
foot P_{\G36119} T_{a} b
cmark_r P_{\G36119}
color 200 200 200
drawline T_{a} P_{\G36119}
color 0 0 0

% Constructing a circle k(T_{a},foot[T_{a},c]) whose center is at point T_{a} and which passes
through point P_{\G36119}
```

```

circle k(T_{a},foot[T_{a},c]) T_{a} P_{\_G36119}

color 200 200 200
drawcircle k(T_{a},foot[T_{a},c])
color 0 0 0

% NDG: point A is outside the circle k(T_{a},foot[T_{a},c])
% Constructing a point P_{\_G36510} which is a foot of the point T_{a} on the line b
foot P_{\_G36510} T_{a} b
cmark_r P_{\_G36510}
color 200 200 200
drawline T_{a} P_{\_G36510}
color 0 0 0

% Constructing a line L_{\_G36513} which passes through point T_{a} and point A
line L_{\_G36513} T_{a} A

color 200 200 200
drawline L_{\_G36513}
color 0 0 0

% Constructing a point P_{\_G36516} which is an image of the point P_{\_G36510} in the symmetry to
point/line L_{\_G36513}
sim P_{\_G36516} L_{\_G36513} P_{\_G36510}
cmark_r P_{\_G36516}

% Constructing a line c which passes through point A and point P_{\_G36516}
line c A P_{\_G36516}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and a are not parallel; point A is outside the circle k(T_{a},
foot[T_{a},c]); point T_{a} is not incident to the line b
% Determination conditions: lines c and a are not the same; points C and T_{a} are not the same;
points C and M_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

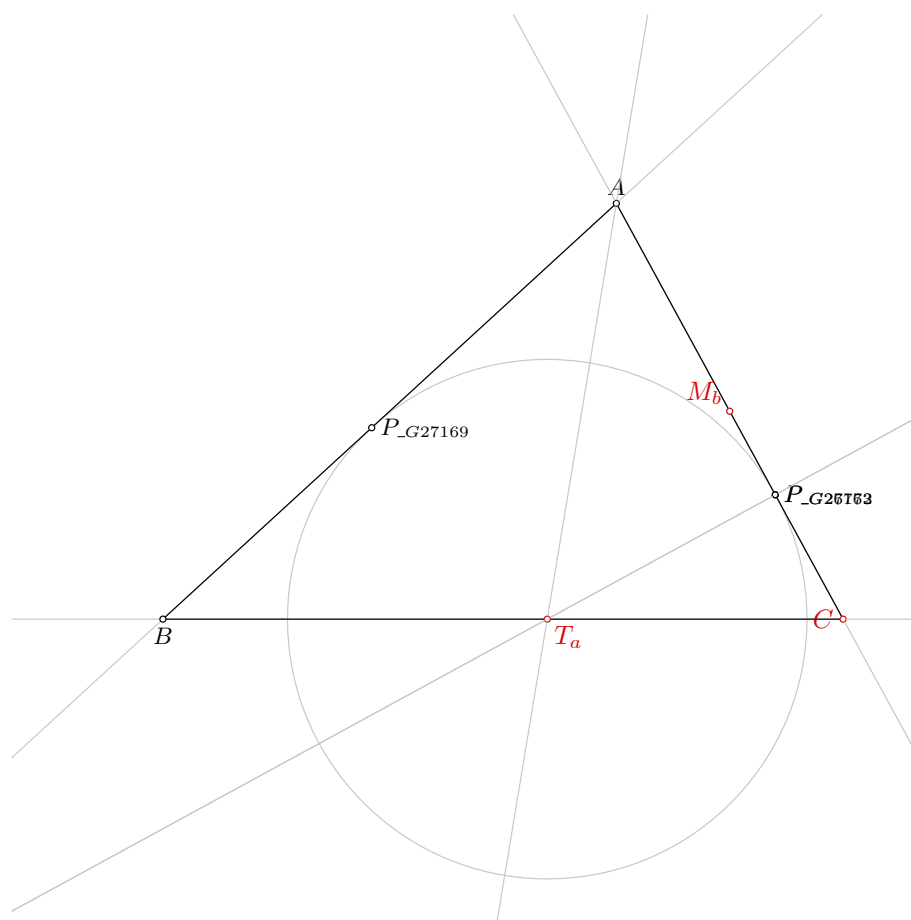


Figure 1: Illustration of the problem 0226

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b=_M M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.033 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $T_a=_T T_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{ACT_a} \neq S_{P_{G28195}CT_a}$  i.e., lines  $AP_{G28195}$  and  $CT_a$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_b=_M M_b$

Proving failed

#### 4.2.3 Proving $T_a=_T T_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_b=_M M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_a = \neg T_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 10 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 7 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_a = \neg T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 7 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 227

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 227: Given a point  $M_b$ , a point  $T_b$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $T_b$  are not the same;
2. Choose freely a point  $C$  on the line  $b$  (rule WOnline1) ;
3. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); ;
4. Using the point  $A$ , the point  $C$ , the point  $T_b$  and the line  $b$ , construct a point  $T'_b$  (rule W19);  
% NDG: points  $A$  and  $C$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $midpoint([A, T_b])$  are not the same;
5. Choose freely a point  $B$  on the circle  $k_{over}(T_b, T'_b)$  (rule WOncircle).

Non-degenerate conditions: points  $A$  and  $C$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $midpoint([A, T_b])$  are not the same.

Determination conditions: points  $M_b$  and  $T_b$  are not the same.

Rules used: [W01,W02,W19,WOncircle2,WOnline1]

Lemmas used: [D22,D24,GD01,GL03,GL09,L78]

Solving time: 89.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{b} 95 67.5  
point T_{b} 94.25 68.88  
point C 110 40
```



```

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_t T_{b}
cmark_l C
color 0 0 0
fontsize 8

% DET: points M_{b} and T_{b} are not the same
% Constructing a line b which passes through point M_{b} and point T_{b}
line b M_{b} T_{b}

color 200 200 200
drawline b
color 0 0 0

% Choosing randomly a point C on the line M_{b}T_{b}
online C M_{b} T_{b}
cmark_l C
color 200 200 200
drawline M_{b} T_{b}
color 0 0 0

% Constructing a point A such that CA/CM_{b}=2
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

% NDG: points A and C are not the same; points C and T_{b} are not the same; points C and midpoint
% ([A,T_{b}]) are not the same
% Constructing a point P_{\_G55679} which is an image of the point C in a rotation around the point
% T_{b} for the angle 90
rotate P_{\_G55679} T_{b} 90 C
cmark_r P_{\_G55679}
color 200 200 200
drawarc_p T_{b} C 90
color 0 0 0

% Constructing a line L_{\_G55682} which passes through point T_{b} and point P_{\_G55679}
line L_{\_G55682} T_{b} P_{\_G55679}

color 200 200 200
drawline L_{\_G55682}
color 0 0 0

% Constructing midpoint P_{\_G55685} of the segment CP_{\_G55679}

```

```

midpoint P_{\_G55685} C P_{\_G55679}
cmark_r P_{\_G55685}

% Constructing a line L_{\_G55688} which passes through point A and point P_{\_G55685}
line L_{\_G55688} A P_{\_G55685}

color 200 200 200
drawline L_{\_G55688}
color 0 0 0

% Constructing a line L_{\_G55691} which passes through point C and point P_{\_G55685}
line L_{\_G55691} C P_{\_G55685}

color 200 200 200
drawline L_{\_G55691}
color 0 0 0

% Constructing a point P_{\_G55694} which belongs to line L_{\_G55682} and line L_{\_G55688}
intersec P_{\_G55694} L_{\_G55682} L_{\_G55688}
cmark_r P_{\_G55694}

% Constructing a point P_{\_G55697} which belongs to line L_{\_G55682} and line L_{\_G55691}
intersec P_{\_G55697} L_{\_G55682} L_{\_G55691}
cmark_r P_{\_G55697}

% Constructing a line L_{\_G55700} which passes through point A and point P_{\_G55697}
line L_{\_G55700} A P_{\_G55697}

color 200 200 200
drawline L_{\_G55700}
color 0 0 0

% Constructing a line L_{\_G55703} which passes through point C and point P_{\_G55694}
line L_{\_G55703} C P_{\_G55694}

color 200 200 200
drawline L_{\_G55703}
color 0 0 0

% Constructing a point P_{\_G55706} which belongs to line L_{\_G55700} and line L_{\_G55703}
intersec P_{\_G55706} L_{\_G55700} L_{\_G55703}
cmark_r P_{\_G55706}

% Constructing a line L_{\_G55709} which passes through point P_{\_G55685} and point P_{\_G55706}
line L_{\_G55709} P_{\_G55685} P_{\_G55706}

color 200 200 200
drawline L_{\_G55709}
color 0 0 0

% Constructing a point T'_{b} which belongs to line L_{\_G55709} and line b

```

```

intersec T'_{b} L_{\_G55709} b
cmark_r T'_{b}

% Constructing midpoint P_{\_G56534} of the segment T_{b}T'_{b}
midpoint P_{\_G56534} T_{b} T'_{b}
cmark_r P_{\_G56534}

% Choosing randomly a point B on the circle with center P_{\_G56534} through point T_{b}
oncircle B P_{\_G56534} T_{b}
cmark_b B
color 200 200 200
drawcircle P_{\_G56534} T_{b}
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: points A and C are not the same; points C and T_{b} are not the same;
% points C and midpoint([A,T_{b}]) are not the same
% Determination conditions: points M_{b} and T_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_b = T_b$

Proving failed

#### 4.1.3 Proving $C = C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.013 seconds.

**NDG conditions** There are no NDG conditions for this theorem



Figure 1: Illustration of the problem 0227

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_b = \neg M_b$

Proving failed

### 4.2.2 Proving $T_b = \neg T_b$

Proving failed

### 4.2.3 Proving $C = C$

NDG conditions are:

$S_{T_bAP\_G51084} \neq S_{P\_G51078AP\_G51084}$  i.e., lines  $T_bP\_G51078$  and  $AP\_G51084$  are not parallel (construction based assumption)

$S_{T_bCP\_G51084} \neq S_{P\_G51078CP\_G51084}$  i.e., lines  $T_bP\_G51078$  and  $CP\_G51084$  are not parallel (construction based assumption)

$S_{ACP\_G51093} \neq S_{P\_G51096CP\_G51093}$  i.e., lines  $AP\_G51096$  and  $CP\_G51093$  are not parallel (construction based assumption)

$S_{P\_G51084M_bT_b} \neq S_{P\_G51105M_bT_b}$  i.e., lines  $P\_G51084P\_G51105$  and  $M_bT_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_b = \neg M_b$

Proving failed

### 4.3.2 Proving $T_b = \neg T_b$

Proving failed

### 4.3.3 Proving $C = C$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_b = \neg M_b$

Proving failed

### 4.4.2 Proving $T_b = \neg T_b$

Proving failed

### 4.4.3 Proving $C = C$

Proving failed

# Problem 228

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 228: Given a point  $C$ , a point  $M_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); ;
2. Using the point  $C$  and the point  $M_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $M_b$  are not the same;
3. Using the point  $T_c$  and the point  $A$ , construct a line  $c$  (rule W02); % DET: points  $T_c$  and  $A$  are not the same;
4. Using the point  $T_c$  and the line  $b$ , construct a circle  $k(T_c, foot[T_c, b])$  (rule W11); % NDG: point  $T_c$  is not incident to the line  $b$ ;
5. Using the circle  $k(T_c, foot[T_c, b])$ , the point  $C$ , the point  $T_c$  and the line  $b$ , construct a line  $a$  (rule W13); % NDG: point  $C$  is outside the circle  $k(T_c, foot[T_c, b])$ ;
6. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; point  $C$  is outside the circle  $k(T_c, foot[T_c, b])$ ; point  $T_c$  is not incident to the line  $b$ .

Determination conditions: lines  $a$  and  $c$  are not the same; points  $T_c$  and  $A$  are not the same; points  $C$  and  $M_b$  are not the same.

Rules used: [W01,W02,W03,W11,W13]

Lemmas used: [D22,D25,GD01,GD02,GL03,GL09,GL10,GL11,L10]

Solving time: 1.8 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point M_{b} 95 67.5
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_l C
cmark_lt M_{b}
cmark_rt T_{c}
color 0 0 0
fontsize 8

% Constructing a point A such that CA/CM_{b}=2
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

% DET: points C and M_{b} are not the same
% Constructing a line b which passes through point C and point M_{b}
line b C M_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points T_{c} and A are not the same
% Constructing a line c which passes through point T_{c} and point A
line c T_{c} A

color 200 200 200
drawline c
color 0 0 0

% NDG: point T_{c} is not incident to the line b
% Constructing a point P_{\_G33753} which is a foot of the point T_{c} on the line b
foot P_{\_G33753} T_{c} b
cmark_r P_{\_G33753}
color 200 200 200
drawline T_{c} P_{\_G33753}
color 0 0 0

% Constructing a circle k(T_{c},foot[T_{c},b]) whose center is at point T_{c} and which passes
    through point P_{\_G33753}
```

```

circle k(T_{c},foot[T_{c},b]) T_{c} P_{\_G33753}

color 200 200 200
drawcircle k(T_{c},foot[T_{c},b])
color 0 0 0

% NDG: point C is outside the circle k(T_{c},foot[T_{c},b])
% Constructing a point P_{\_G34144} which is a foot of the point T_{c} on the line b
foot P_{\_G34144} T_{c} b
cmark_r P_{\_G34144}
color 200 200 200
drawline T_{c} P_{\_G34144}
color 0 0 0

% Constructing a line L_{\_G34147} which passes through point T_{c} and point C
line L_{\_G34147} T_{c} C

color 200 200 200
drawline L_{\_G34147}
color 0 0 0

% Constructing a point P_{\_G34150} which is an image of the point P_{\_G34144} in the symmetry to
point/line L_{\_G34147}
sim P_{\_G34150} L_{\_G34147} P_{\_G34144}
cmark_r P_{\_G34150}

% Constructing a line a which passes through point C and point P_{\_G34150}
line a C P_{\_G34150}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and c are not parallel; point C is outside the circle k(T_{c},
foot[T_{c},b]); point T_{c} is not incident to the line b
% Determination conditions: lines a and c are not the same; points T_{c} and A are not the same;
points C and M_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1



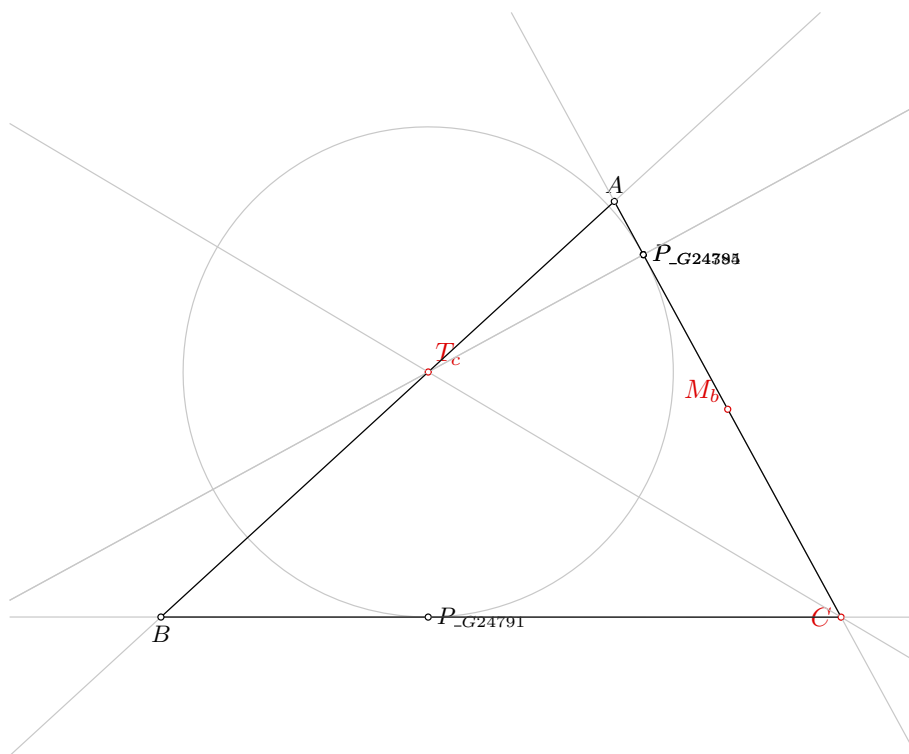


Figure 1: Illustration of the problem 0228

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b=_M M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.029 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $T_c=_T T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$SC_{T_c A} \neq SP_{G25820 T_c A}$  i.e., lines  $CP_{G25820}$  and  $T_c A$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_b=_M M_b$

Proving failed

#### 4.2.3 Proving $T_c=_T T_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_b=_M M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 5 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_c = \neg T_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 14 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_c = \neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 229

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 229: Given a point  $C$ , a point  $M_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); ;
2. Using the point  $C$  and the point  $M_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $M_b$  are not the same;
3. Using the point  $I$  and the line  $b$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $b$ ;
4. Using the circle  $k(I, P_a)$ , the point  $C$ , the point  $I$  and the line  $b$ , construct a line  $a$  (rule W13); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
5. Using the circle  $k(I, P_a)$ , the point  $A$ , the point  $I$  and the line  $b$ , construct a line  $c$  (rule W13); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
6. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; point  $A$  is outside the circle  $k(I, P_a)$ ; point  $C$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $b$ .

Determination conditions: lines  $a$  and  $c$  are not the same; points  $C$  and  $M_b$  are not the same.

Rules used: [W01,W02,W03,W11,W13]

Lemmas used: [D22,D27,GD01,GL03,GL09,L59,L60,L61]

Solving time: 17.2 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point M_{b} 95 67.5
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_l C
cmark_lt M_{b}
cmark_b I
color 0 0 0
fontsize 8

% Constructing a point A such that CA/CM_{b}=2
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

% DET: points C and M_{b} are not the same
% Constructing a line b which passes through point C and point M_{b}
line b C M_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: point I is not incident to the line b
% Constructing a point P_{\_G56660} which is a foot of the point I on the line b
foot P_{\_G56660} I b
cmark_r P_{\_G56660}
color 200 200 200
drawline I P_{\_G56660}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
56660}
circle k(I,P_{a}) I P_{\_G56660}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point C is outside the circle k(I,P_{a})
% Constructing a point P_{\_G56867} which is a foot of the point I on the line b
```

```

foot P_{\_G56867} I b
cmark_r P_{\_G56867}
color 200 200 200
drawline I P_{\_G56867}
color 0 0 0

% Constructing a line L_{\_G56870} which passes through point I and point C
line L_{\_G56870} I C

color 200 200 200
drawline L_{\_G56870}
color 0 0 0

% Constructing a point P_{\_G56873} which is an image of the point P_{\_G56867} in the symmetry to
point/line L_{\_G56870}
sim P_{\_G56873} L_{\_G56870} P_{\_G56867}
cmark_r P_{\_G56873}

% Constructing a line a which passes through point C and point P_{\_G56873}
line a C P_{\_G56873}

color 200 200 200
drawline a
color 0 0 0

% NDG: point A is outside the circle k(I,P_{a})
% Constructing a point P_{\_G57202} which is a foot of the point I on the line b
foot P_{\_G57202} I b
cmark_r P_{\_G57202}
color 200 200 200
drawline I P_{\_G57202}
color 0 0 0

% Constructing a line L_{\_G57205} which passes through point I and point A
line L_{\_G57205} I A

color 200 200 200
drawline L_{\_G57205}
color 0 0 0

% Constructing a point P_{\_G57208} which is an image of the point P_{\_G57202} in the symmetry to
point/line L_{\_G57205}
sim P_{\_G57208} L_{\_G57205} P_{\_G57202}
cmark_r P_{\_G57208}

% Constructing a line c which passes through point A and point P_{\_G57208}
line c A P_{\_G57208}

color 200 200 200
drawline c
color 0 0 0

```

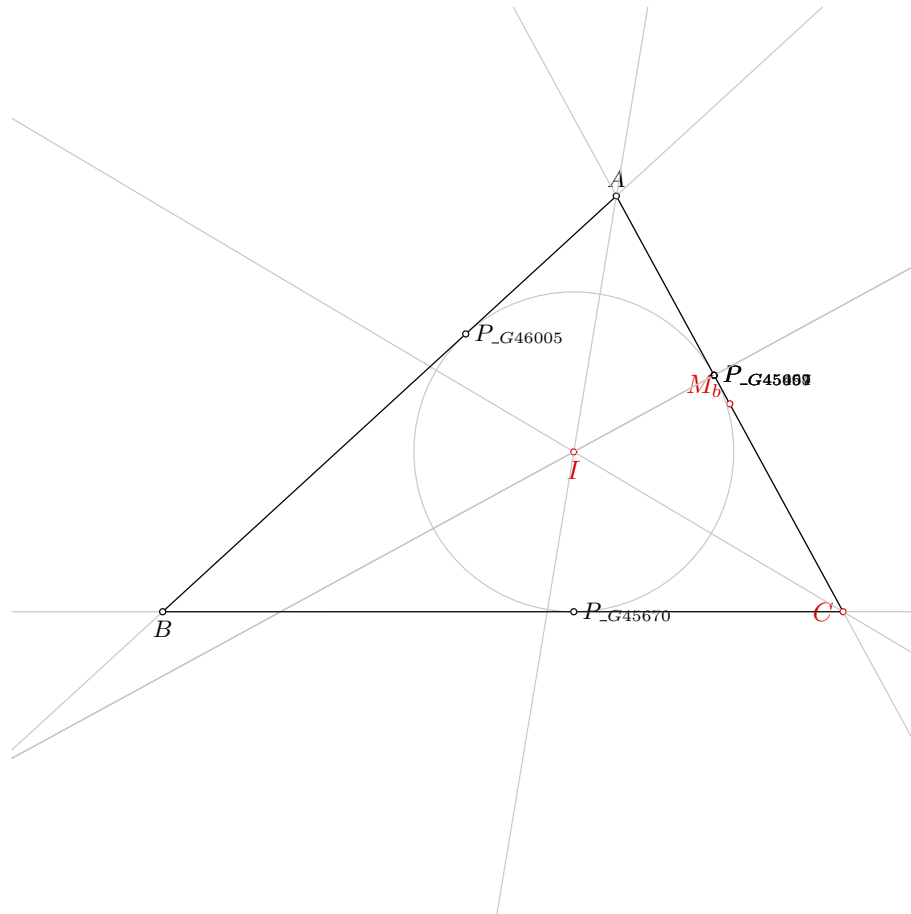


Figure 1: Illustration of the problem 0229

```
% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines a and c are not parallel; point A is outside the circle k(I,P_{a}
}); point C is outside the circle k(I,P_{a}); point I is not incident to the line b
% Determination conditions: lines a and c are not the same; points C and M_{b} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.03 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b=_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.035 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{CAP\_G47088} \neq S_{P\_G46753AP\_G47088}$  i.e., lines  $CP\_G46753$  and  $AP\_G47088$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_b=_M_b$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.



#### 4.3.2 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 74 terms.

**Time Complexity:** Time spent by the prover is 0.080 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I = \neg I$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 315 terms.

**Time Complexity:** Time spent by the prover is 0.460 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 20 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.3 Proving $I = \neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 20 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 230

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 230: Given a point  $M_c$ , a point  $G$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is redundant.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $G$ , construct a point  $C$  (rule W01); ;
2. Choose freely a point  $A$  (rule free);
3. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01,free]

Lemmas used: [D20,GL03,GL04,L57]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{c} 50 67.5
```

```
point G 70 58.33
```

```
point C 110 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_lt M_{c}
```

```
cmark_t G
```

```
cmark_l C
```

```
color 0 0 0
```

```
fontsize 8
```

```

% Constructing a point C such that  $M_{\{c\}}C/M_{\{c\}}G=3$ 
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

```

```

% Constructing a free point A
point A 80 95

```

```

cmark_t A

```

```

% Constructing a point B such that  $AB/AM_{\{c\}}=2$ 
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = \_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.054 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $G = \_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 378 terms.

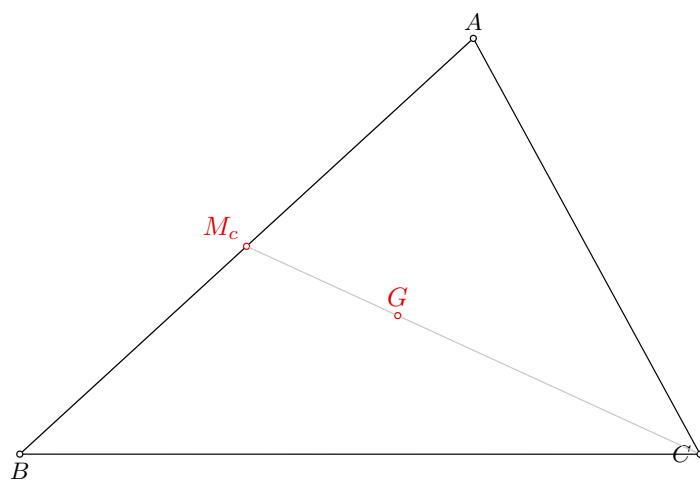


Figure 1: Illustration of the problem 0230

**Time Complexity:** Time spent by the prover is 1.353 seconds.

**NDG conditions** Line through points  $A$  and  $M_a$  is not parallel with line through points  $B$  and  $M_b$   
Line through points  $A$  and  $M_a$  is not parallel with line through points  $M_c$  and  $G$

### 4.1.3 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.015 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_c=M_c$

NDG conditions are:

$S_{AB-M_b} \neq S_{M_aB-M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $G=G$

NDG conditions are:

$S_{AB-M_b} \neq S_{M_aB-M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (cancellation assumption)

Total number of proof steps: 678

Time spent by the prover: 0.200 seconds

### 4.2.3 Proving $C=C$

NDG conditions are:

$S_{AB-M_b} \neq S_{M_aB-M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_c=M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 16 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.2 Proving $G=G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 807 terms.

**Time Complexity:** Time spent by the prover is 1.500 seconds. There are no ndg conditions.

### 4.3.3 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_c = M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 713 terms.

**Time Complexity:** Time spent by the prover is 46.470 seconds. There are no ndg conditions.

### 4.4.2 Proving $G = G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 713 terms.

**Time Complexity:** Time spent by the prover is 46.910 seconds. There are no ndg conditions.

### 4.4.3 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 231

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 231: Given a point  $C$ , a point  $M_c$  and a point  $H_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $H_a$  are not the same;
2. Using the point  $H_a$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_a$  and  $M_c$  are not the same;
3. Using the circle  $k(M_c, A)$ , the line  $a$ , the point  $M_c$  and the point  $H_a$ , construct a point  $B$  (rule W05); % NDG: line  $a$  and circle  $k(M_c, A)$  intersect % DET: points  $H_a$  and  $B$  must be different;
4. Using the point  $M_c$  and the point  $B$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: line  $a$  and circle  $k(M_c, A)$  intersect; points  $H_a$  and  $M_c$  are not the same.

Determination conditions: points  $H_a$  and  $B$  must be different; points  $C$  and  $H_a$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D20,D5,GD01,GD02,GL03,GL04,L40,L42]

Solving time: 0.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point M_{c} 50 67.5
point H_{a} 80 40
```

```

color 220 0 0
fontsize 9

cmark_l C
cmark_lt M_{c}
cmark_r H_{a}
color 0 0 0
fontsize 8

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points H_{a} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{a}
circle k(M_{c},A) M_{c} H_{a}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line a and circle k(M_{c},A) intersect% DET: points H_{a} and B must be different
% Constructing a point P_{\_G40312} which is a foot of the point M_{c} on the line a
foot P_{\_G40312} M_{c} a
cmark_r P_{\_G40312}
color 200 200 200
drawline M_{c} P_{\_G40312}
color 0 0 0

% Constructing a point B which is an image of the point H_{a} in the symmetry to point/line P_{\_G40312}
sim B P_{\_G40312} H_{a}
cmark_b B

% Constructing a point A such that M_{c}A/M_{c}B=-1
towards A M_{c} B -1
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

drawsegment A B

```



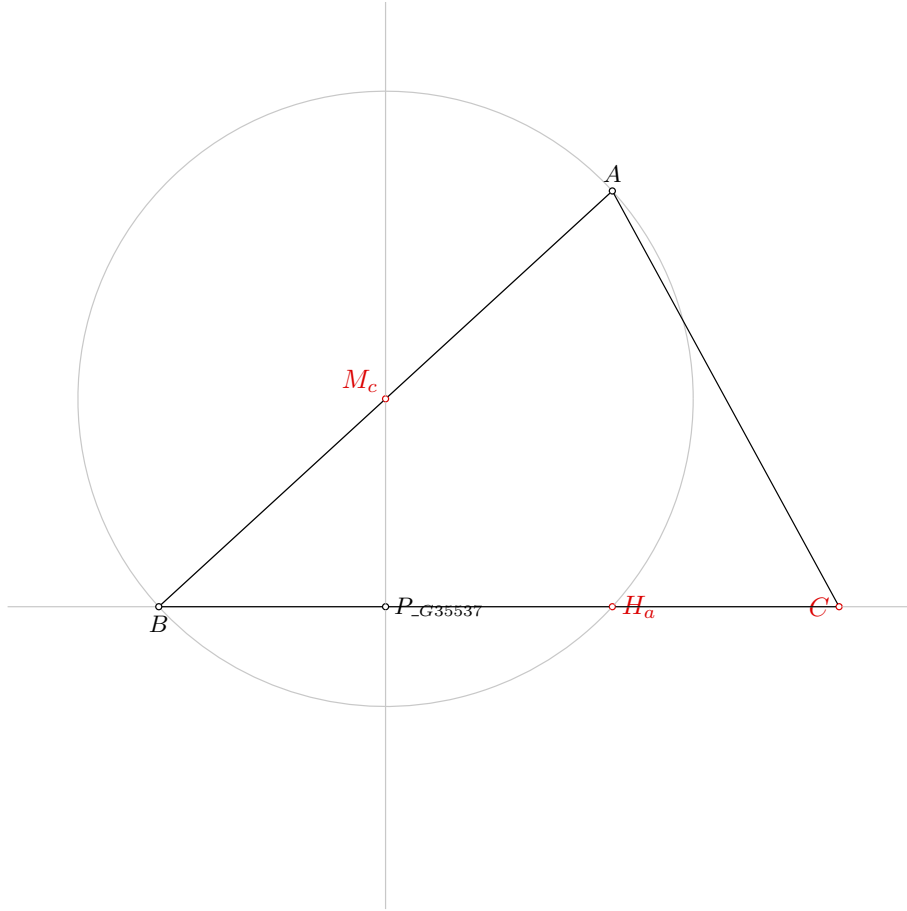


Figure 1: Illustration of the problem 0231

```
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(M_{c},A) intersect; points H_{a} and M_{c} are not
the same
% Determination conditions: points H_{a} and B must be different; points C and H_{a} are not the
same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c = \neg M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.166 seconds.

**NDG conditions** Points  $C$  and  $H_a$  are not identical

Points  $C$  and  $H_a$  are not identical

#### 4.1.3 Proving $H_a = \neg H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 39 terms.

**Time Complexity:** Time spent by the prover is 0.525 seconds.

**NDG conditions** Points  $C$  and  $H_a$  are not identical

Points  $C$  and  $H_a$  are not identical

Points  $B$  and  $C$  are not identical

Points  $B$ ,  $C$  and  $M_c$  are not collinear

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C = C$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{\neg H_a}^0} \neq S_{CAF_{\neg H_a}^0}$  i.e., lines  $BC$  and  $AF_{\neg H_a}^0$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{\neg H_a}^0} \neq S_{CAF_{\neg H_a}^0}$  i.e., lines  $BC$  and  $AF_{\neg H_a}^0$  are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $H_a = \neg H_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C = C$

Proving failed

#### 4.3.2 Proving $M_c = \neg M_c$

Proving failed

### 4.3.3 Proving $H_a = H_a$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C = C$

Proving failed

### 4.4.2 Proving $M_c = M_c$

Proving failed

### 4.4.3 Proving $H_a = H_a$

Proving failed

# Problem 232

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 232: Given a point  $C$ , a point  $M_c$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $H_b$  are not the same;
2. Using the point  $H_b$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_b$  and  $M_c$  are not the same;
3. Using the circle  $k(M_c, A)$ , the line  $b$ , the point  $M_c$  and the point  $H_b$ , construct a point  $A$  (rule W05); % NDG: line  $b$  and circle  $k(M_c, A)$  intersect % DET: points  $H_b$  and  $A$  must be different;
4. Using the point  $M_c$  and the point  $A$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: line  $b$  and circle  $k(M_c, A)$  intersect; points  $H_b$  and  $M_c$  are not the same.

Determination conditions: points  $H_b$  and  $A$  must be different; points  $C$  and  $H_b$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D20,D6,GD01,GD02,GL03,GL04,L41]

Solving time: 0.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point M_{c} 50 67.5
point H_{b} 89.36 77.83
```

```

color 220 0 0
fontsize 9

cmark_l C
cmark_lt M_{c}
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points H_{b} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{b}
circle k(M_{c},A) M_{c} H_{b}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line b and circle k(M_{c},A) intersect% DET: points H_{b} and A must be different
% Constructing a point P_{\_G52582} which is a foot of the point M_{c} on the line b
foot P_{\_G52582} M_{c} b
cmark_r P_{\_G52582}
color 200 200 200
drawline M_{c} P_{\_G52582}
color 0 0 0

% Constructing a point A which is an image of the point H_{b} in the symmetry to point/line P_{\_G52582}
sim A P_{\_G52582} H_{b}
cmark_t A

% Constructing a point B such that M_{c}B/M_{c}A=-1
towards B M_{c} A -1
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

drawsegment A B

```

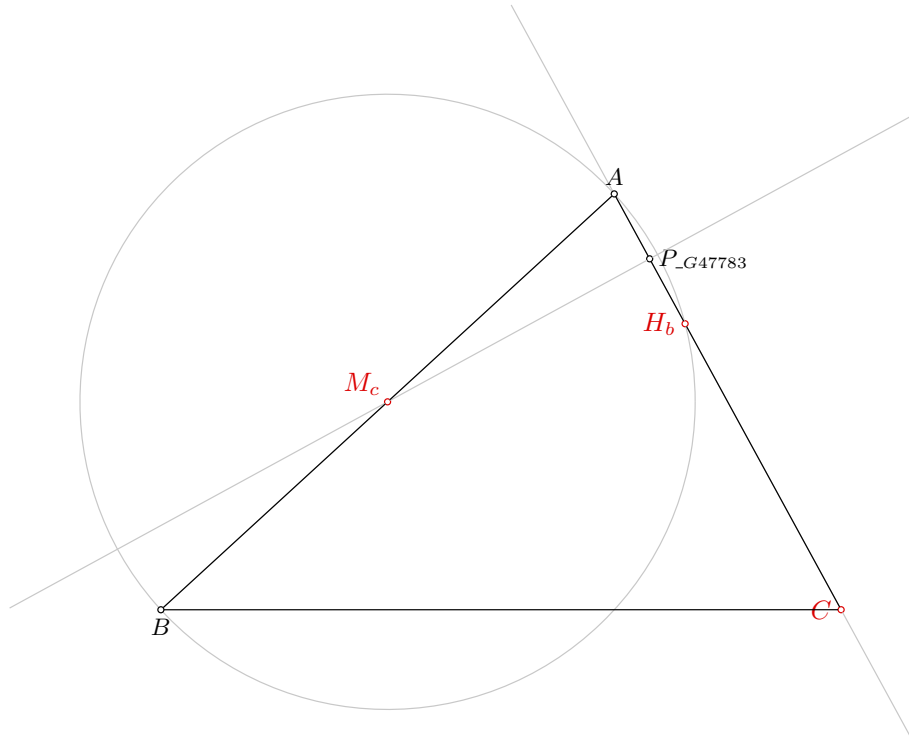


Figure 1: Illustration of the problem 0232

```
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line b and circle k(M_{c},A) intersect; points H_{b} and M_{c} are not
the same
% Determination conditions: points H_{b} and A must be different; points C and H_{b} are not the
same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c = \neg M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.166 seconds.

**NDG conditions** Points  $H_b$  and  $M_c$  are not identical

Points  $H_b$  and  $M_c$  are not identical

#### 4.1.3 Proving $H_b = \neg H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 39 terms.

**Time Complexity:** Time spent by the prover is 0.507 seconds.

**NDG conditions** Points  $H_b$  and  $M_c$  are not identical

Points  $H_b$  and  $M_c$  are not identical

Points  $A$  and  $C$  are not identical

Points  $A$ ,  $C$  and  $M_c$  are not collinear

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C = C$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^0_{\neg H_b}} \neq S_{CBF^0_{\neg H_b}}$  i.e., lines  $AC$  and  $BF^0_{\neg H_b}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^0_{\neg H_b}} \neq S_{CBF^0_{\neg H_b}}$  i.e., lines  $AC$  and  $BF^0_{\neg H_b}$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $H_b = \neg H_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C = C$

Proving failed

#### 4.3.2 Proving $M_c = \neg M_c$

Proving failed

#### **4.3.3 Proving $H_b = \neg H_b$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $C = C$**

Proving failed

#### **4.4.2 Proving $M_c = \neg M_c$**

Proving failed

#### **4.4.3 Proving $H_b = \neg H_b$**

Proving failed



# Problem 233

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 233: Given a point  $M_c$ , a point  $H_c$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $H_c$  are not the same;
2. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
3. Choose freely a point  $C$  on the line  $h_c$  (rule WOnline2);
4. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) ;
5. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: points  $M_c$  and  $H_c$  are not the same.

Rules used: [W01,W02,W10b,WOnline1,WOnline2]

Lemmas used: [D10,D20,D7,GD01,GL03,GL09]

Solving time: 148.5 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point H_{c} 68.91 84.83
point C 110 40

color 220 0 0
fontsize 9
```

```

cmark_lt M_{c}
cmark_rt H_{c}
cmark_l C
color 0 0 0
fontsize 8

% DET: points M_{c} and H_{c} are not the same
% Constructing a line c which passes through point M_{c} and point H_{c}
line c M_{c} H_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0

% Generating random value V[_G24276]
random V[_G24276]

% Calculating value V[_G24297] using formula  $V[_G24276]*20$ 
expression V[_G24297] { V[_G24276]*20 }

% Constructing a point C which is a point for which holds  $H_{c}C = V[_G24297]$  and angle  $M_{c}H_{c}C = 90$ 
turtle C M_{c} H_{c} 90 V[_G24297]
cmark_l C

% Choosing randomly a point A on the line M_{c}H_{c}
online A M_{c} H_{c}
cmark_t A
color 200 200 200
drawline M_{c} H_{c}
color 0 0 0

% Constructing a point B such that  $AB/AM_{c}=2$ 
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B

```

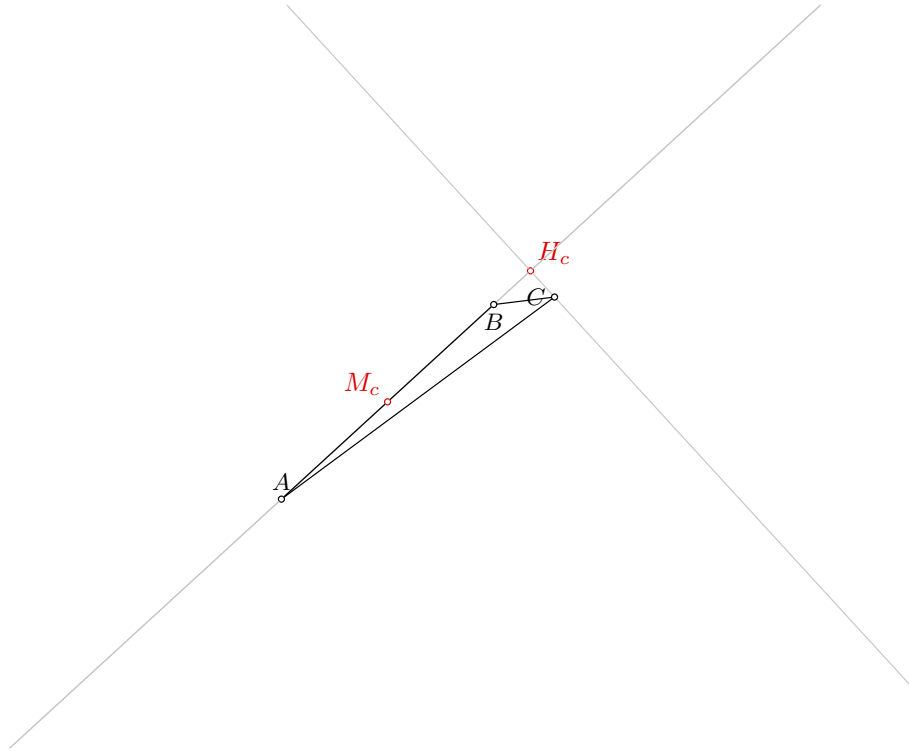


Figure 1: Illustration of the problem 0233

```
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions:
```

```
% Determination conditions: points  $M_{\{c\}}$  and  $H_{\{c\}}$  are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = \neg M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.032 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.099 seconds.

**NDG conditions** Line through points  $A$  and  $H_c$  is not perpendicular to line through points  $H_c$  and  $B$

#### 4.1.3 Proving $C = C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.008 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{H_c}^2} \neq S_{BCF_{H_c}^2}$  i.e., lines  $AB$  and  $CF_{H_c}^2$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.2.3 Proving $C = C$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $C = C$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 2 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $C = C$

Proving failed

# Problem 234

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 234: Given a point  $C$ , a point  $M_c$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $M_c$ , construct a point  $G$  (rule W01); ;
2. Using the point  $H$  and the point  $G$ , construct a point  $O$  (rule W01); ;
3. Using the point  $C$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H$  are not the same;
4. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
5. Using the point  $M_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same.

Determination conditions: points  $C$  and  $H$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D10,D20,D26,GD01,GL03,GL09,L11,L12,L3,L57,L58]

Solving time: 1.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point M_{c} 50 67.5
point H 80 72.73
```

```

color 220 0 0
fontsize 9

cmark_l C
cmark_lt M_{c}
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a line L_{\_G42906} which passes through point C and point M_{c}
line L_{\_G42906} C M_{c}

color 200 200 200
drawline L_{\_G42906}
color 0 0 0

% Constructing a point P_{\_G43007} with coordinates (0,0)
point P_{\_G43007} 0 0
cmark_r P_{\_G43007}

% Constructing a point P_{\_G42931} such that CP_{\_G42931}/CP_{\_G43007}=2
towards P_{\_G42931} C P_{\_G43007} 2
cmark_r P_{\_G42931}
color 200 200 200
drawsegment C P_{\_G42931}
color 0 0 0

% Constructing a point P_{\_G42976} such that CP_{\_G42976}/CP_{\_G43007}=3
towards P_{\_G42976} C P_{\_G43007} 3
cmark_r P_{\_G42976}
color 200 200 200
drawsegment C P_{\_G42976}
color 0 0 0

% Constructing a line L_{\_G42937} which passes through point M_{c} and point P_{\_G42976}
line L_{\_G42937} M_{c} P_{\_G42976}

color 200 200 200
drawline L_{\_G42937}
color 0 0 0

% Constructing a line L_{\_G42900} which contains the point P_{\_G42931} and is parallel to the
line L_{\_G42937}
parallel L_{\_G42900} P_{\_G42931} L_{\_G42937}

color 200 200 200
drawline L_{\_G42900}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G42900} and line L_{\_G42906}
intersec G L_{\_G42900} L_{\_G42906}

```

```

cmark_t G

% Constructing a point O such that HO/HG=1.5
towards O H G 1.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% DET: points C and H are not the same
% Constructing a line h_{c} which passes through point C and point H
line h_{c} C H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Constructing a line c which is perpendicular to line h_{c} and which passes through point M_{c}
perp c M_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same
% Determination conditions: points C and H are not the same

```



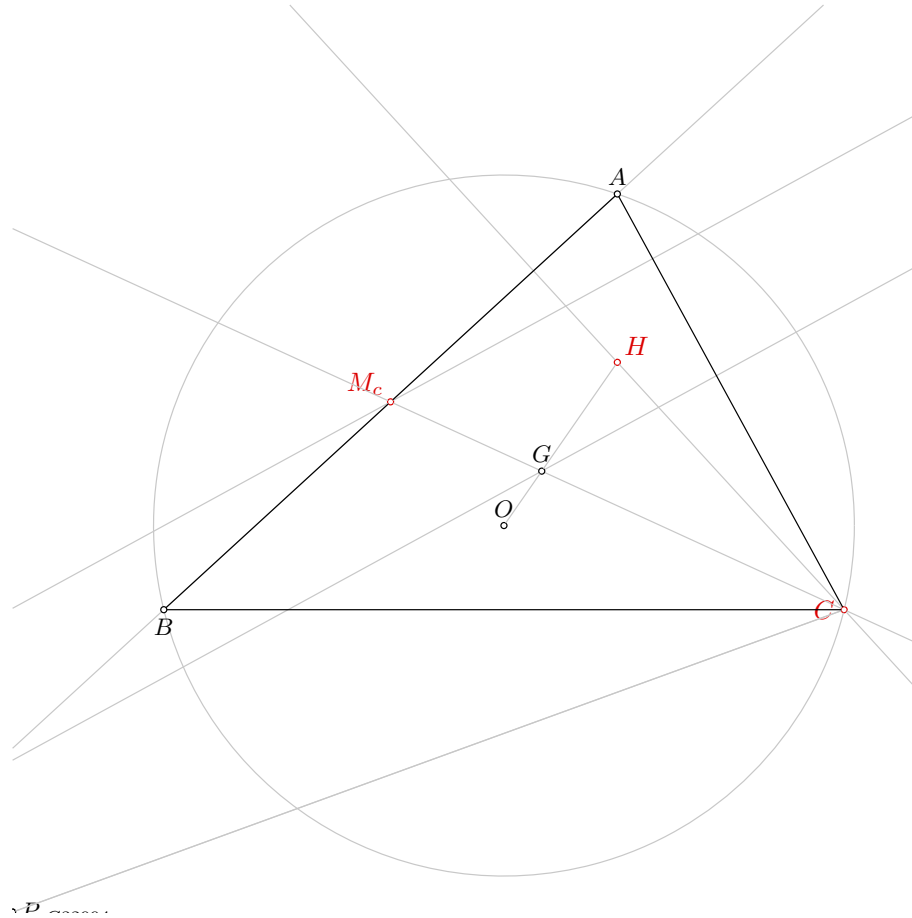


Figure 1: Illustration of the problem 0234

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.034 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c=M_c$

Proving failed

### 4.1.3 Proving $H=_H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 332 terms.

**Time Complexity:** Time spent by the prover is 4.825 seconds.

**NDG conditions** Points  $P_{G41478}$  and  $G$  are not identical

Points  $M_c$  and  $H$  are not identical

Points  $M_c$  and  $H$  are not identical

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $B$ ,  $C$  and  $M_c$  are not collinear

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{P_{G33960}CM_c} \neq S_{P_{L_{G33929}}^0 CM_c}$  i.e., lines  $P_{G33960}P_{L_{G33929}}^0$  and  $CM_c$  are not parallel (construction based assumption)

$S_{M_cCH} \neq 0$  i.e., points  $M_c$ ,  $C$  and  $H$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{F_{-h_a}^2 BF_{-h_b}^3}$  i.e., lines  $AF_{-h_a}^2$  and  $BF_{-h_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_c=_M_c$

Proving failed

### 4.2.3 Proving $H=_H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_c=_M_c$

Proving failed

### 4.3.3 Proving $H=_H$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $M_c=_M_c$

Proving failed

### 4.4.3 Proving $H=_H$

Proving failed

## Problem 235

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 235: Given a point  $C$ , a point  $M_c$  and a point  $T_a$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 236

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 236: Given a point  $C$ , a point  $M_c$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 237

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 237: Given a point  $C$ , a point  $M_c$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $T_c$ , construct a line  $s_c$  (rule W02); % DET: points  $C$  and  $T_c$  are not the same;
2. Using the point  $M_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $T_c$  are not the same;
3. Using the point  $M_c$  and the line  $c$ , construct a line  $m_c$  (rule W10b); ;
4. Using the line  $m_c$  and the line  $s_c$ , construct a point  $N_c$  (rule W03); % NDG: lines  $m_c$  and  $s_c$  are not parallel % DET: lines  $m_c$  and  $s_c$  are not the same;
5. Using the point  $C$  and the point  $N_c$ , construct a line  $m(CN_c)$  (rule W14); % DET: points  $C$  and  $N_c$  are not the same;
6. Using the line  $m(CN_c)$  and the line  $m_c$ , construct a point  $O$  (rule W03); % NDG: lines  $m(CN_c)$  and  $m_c$  are not parallel % DET: lines  $m(CN_c)$  and  $m_c$  are not the same;
7. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
8. Using the circle  $k(O, C)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same; lines  $m(CN_c)$  and  $m_c$  are not parallel; lines  $m_c$  and  $s_c$  are not parallel.

Determination conditions: lines  $m(CN_c)$  and  $m_c$  are not the same; points  $C$  and  $N_c$  are not the same; lines  $m_c$  and  $s_c$  are not the same; points  $M_c$  and  $T_c$  are not the same; points  $C$  and  $T_c$  are not the same.

Rules used: [W02,W03,W04,W06,W10b,W14]  
 Lemmas used: [D13,D20,D25,D26,D49,GD01,GL01,GL09,L1,L11,L12,L27,L7]  
 Solving time: 1.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point M_{c} 50 67.5
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_l C
cmark_lt M_{c}
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points C and T_{c} are not the same
% Constructing a line s_{c} which passes through point C and point T_{c}
line s_{c} C T_{c}

color 200 200 200
drawline s_{c}
color 0 0 0

% DET: points M_{c} and T_{c} are not the same
% Constructing a line c which passes through point M_{c} and point T_{c}
line c M_{c} T_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line m_{c} which is perpendicular to line c and which passes through point M_{c}
perp m_{c} M_{c} c

color 200 200 200
drawline m_{c}
color 0 0 0

% NDG: lines m_{c} and s_{c} are not parallel% DET: lines m_{c} and s_{c} are not the same
% Constructing a point N_{c} which belongs to line m_{c} and line s_{c}
intersec N_{c} m_{c} s_{c}
cmark_b N_{c}
```

```

% DET: points C and N_{c} are not the same
% Constructing bisector m(CN_{c}) of the segment CN_{c}
med m(CN_{c}) C N_{c}

color 200 200 200
drawline m(CN_{c})
color 0 0 0

color 200 200 200
drawsegment C N_{c}
color 0 0 0

% NDG: lines m(CN_{c}) and m_{c} are not parallel% DET: lines m(CN_{c}) and m_{c} are not the same
% Constructing a point O which belongs to line m(CN_{c}) and line m_{c}
intersec O m(CN_{c}) m_{c}
cmark_t O

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
% lines m(CN_{c}) and m_{c} are not parallel; lines m_{c} and s_{c} are not parallel
% Determination conditions: lines m(CN_{c}) and m_{c} are not the same; points C and N_{c} are not
% the same; lines m_{c} and s_{c} are not the same; points M_{c} and T_{c} are not the same;
% points C and T_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format



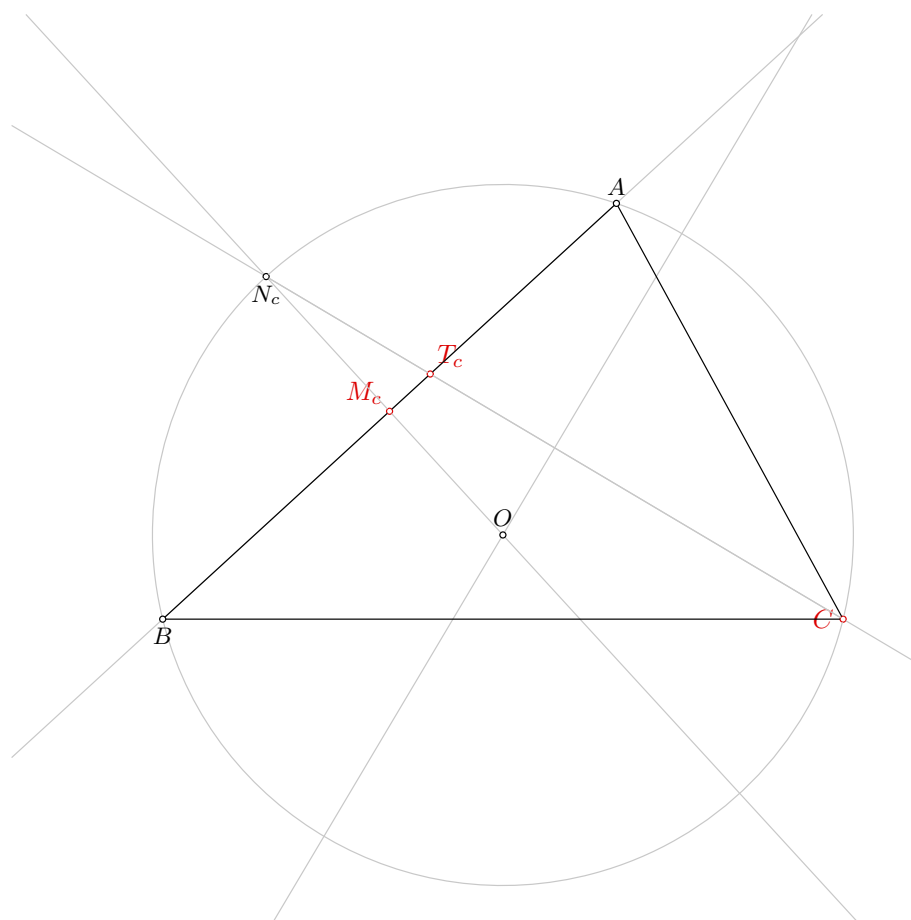


Figure 1: Illustration of the problem 0237

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c=_M M_c$

Proving failed

#### 4.1.3 Proving $T_c=_T T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{M_c C T_c} \neq S_{T_{m_c}^0 C T_c}$  i.e., lines  $M_c T_{m_c}^0$  and  $C T_c$  are not parallel (construction based assumption)

$S_{M_{m(CN_c)}^1 M_c T_{m_c}^0} \neq S_{T_{m(CN_c)}^2 M_c T_{m_c}^0}$  i.e., lines  $M_{m(CN_c)}^1 T_{m(CN_c)}^2$  and  $M_c T_{m_c}^0$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_c=_M M_c$

Proving failed

#### 4.2.3 Proving $T_c=_T T_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_c=_M M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 7807 terms.

**Time Complexity:** Time spent by the prover is 7.760 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_c=_T T_c$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4.2 Proving $M_c=_M_c$

Proving failed

### 4.4.3 Proving $T_c=_T_c$

Proving failed

# Problem 238

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 238: Given a point  $C$ , a point  $M_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $I$ , construct a line  $IM_c$  (rule W02); % DET: points  $M_c$  and  $I$  are not the same;
2. Using the point  $I$  and the point  $M_c$ , construct a circle  $k_{over}(I, M_c)$  (rule W09); % NDG: points  $I$  and  $M_c$  are not the same;
3. Using the point  $C$  and the line  $IM_c$ , construct a line  $CP'_c$  (rule W16); ;
4. Using the point  $M_c$ , the line  $CP'_c$  and the point  $C$ , construct a line  $h_{M_c, -1/1}(CP'_c)$  (rule W15); ;
5. Using the circle  $k_{over}(I, M_c)$  and the line  $h_{M_c, -1/1}(CP'_c)$ , construct a point  $C_{fo}$  and a point  $P_c$  (rule W04); % NDG: line  $h_{M_c, -1/1}(CP'_c)$  and circle  $k_{over}(I, M_c)$  intersect;
6. Using the point  $P_c$  and the point  $I$ , construct a circle  $k(I, P_a)$  (rule W06); % NDG: points  $P_c$  and  $I$  are not the same;
7. Using the circle  $k(I, P_a)$ , the point  $C$  and the point  $I$ , construct a line  $a$  and a line  $b$  (rule W12); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
8. Using the circle  $k(I, P_a)$ , the point  $M_c$  and the point  $I$ , construct a line  $x3$  and a line  $c$  (rule W12); % NDG: point  $M_c$  is outside the circle  $k(I, P_a)$ ;
9. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same;
10. Using the point  $M_c$  and the point  $B$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; point  $M_c$  is outside the circle  $k(I, P_a)$ ; point  $C$  is outside the circle  $k(I, P_a)$ ; points  $P_c$  and  $I$  are not the same; line  $h_{M_c, -1/1}(CP'_c)$  and circle  $k_{over}(I, M_c)$  intersect; points  $I$  and  $M_c$  are not the same.

Determination conditions: lines  $a$  and  $c$  are not the same; points  $M_c$  and  $I$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W09,W12,W15,W16]

Lemmas used: [D20,D27,D61,D87,GD01,GL02,GL03,GL04,GL09,L14,L59,L60,L61,L64,L76]

Solving time: 2.5 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point M_{c} 50 67.5
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_l C
cmark_lt M_{c}
cmark_b I
color 0 0 0
fontsize 8

% DET: points M_{c} and I are not the same
% Constructing a line IM_{c} which passes through point M_{c} and point I
line IM_{c} M_{c} I

color 200 200 200
drawline IM_{c}
color 0 0 0

% NDG: points I and M_{c} are not the same
% Constructing midpoint P_{\_G60882} of the segment IM_{c}
midpoint P_{\_G60882} I M_{c}
cmark_r P_{\_G60882}

% Constructing a circle k_{over}(I,M_{c}) whose center is at point P_{\_G60882} and which passes
    through point I
circle k_{over}(I,M_{c}) P_{\_G60882} I

color 200 200 200
drawcircle k_{over}(I,M_{c})
color 0 0 0

% Constructing a line CP'_{c} which contains the point C and is parallel to the line IM_{c}
parallel CP'_{c} C IM_{c}
```

```

color 200 200 200
drawline CP'_{c}
color 0 0 0

% Constructing a point  $P_{\backslash\_G61183}$  such that  $M_{\{c\}}P_{\backslash\_G61183}/M_{\{c\}}C=-1$ 
towards P_{\backslash\_G61183} M_{\{c\}} C -1
cmark_r P_{\backslash\_G61183}
color 200 200 200
drawsegment C P_{\backslash\_G61183}
color 0 0 0

% Constructing a line  $h_{\{M_{\{c\}},-1/1\}}(CP'_{\{c\}})$  which contains the point  $P_{\backslash\_G61183}$  and is parallel
to the line  $CP'_{\{c\}}$ 
parallel h_{\{M_{\{c\}},-1/1\}}(CP'_{\{c\}}) P_{\backslash\_G61183} CP'_{\{c\}}

color 200 200 200
drawline h_{\{M_{\{c\}},-1/1\}}(CP'_{\{c\}})
color 0 0 0

% NDG: line  $h_{\{M_{\{c\}},-1/1\}}(CP'_{\{c\}})$  and circle  $k_{\text{over}}(I,M_{\{c\}})$  intersect
% Constructing points  $C_{\{fo\}}$  and  $P_{\{c\}}$  which are in intersection of  $k_{\text{over}}(I,M_{\{c\}})$  and  $h_{\{M_{\{c\}},-1/1\}}(CP'_{\{c\}})$ 
intersec2 C_{fo} P_{\{c\}} k_{\text{over}}(I,M_{\{c\}}) h_{\{M_{\{c\}},-1/1\}}(CP'_{\{c\}})
cmark_r C_{fo}
cmark_r P_{\{c\}}

% NDG: points  $P_{\{c\}}$  and  $I$  are not the same
% Constructing a circle  $k(I,P_{\{a\}})$  whose center is at point  $I$  and which passes through point  $P_{\{c\}}$ 
circle k(I,P_{\{a\}}) I P_{\{c\}}

color 200 200 200
drawcircle k(I,P_{\{a\}})
color 0 0 0

% NDG: point  $C$  is outside the circle  $k(I,P_{\{a\}})$ 
% Constructing midpoint  $P_{\backslash\_G61932}$  of the segment  $CI$ 
midpoint P_{\backslash\_G61932} C I
cmark_r P_{\backslash\_G61932}

% Constructing a circle  $C_{\backslash\_G61935}$  whose center is at point  $P_{\backslash\_G61932}$  and which passes through
point  $C$ 
circle C_{\backslash\_G61935} P_{\backslash\_G61932} C

color 200 200 200
drawcircle C_{\backslash\_G61935}
color 0 0 0

```

```

% Constructing points  $P_{\{G61938\}}$  and  $P_{\{G61941\}}$  which are in intersection of  $C_{\{G61935\}}$  and  $k(I, P_{\{a\}})$ 
intersec2  $P_{\{G61938\}}$   $P_{\{G61941\}}$   $C_{\{G61935\}}$   $k(I, P_{\{a\}})$ 
cmark_r  $P_{\{G61938\}}$ 
cmark_r  $P_{\{G61941\}}$ 

% Constructing a line  $a$  which passes through point  $C$  and point  $P_{\{G61938\}}$ 
line a C  $P_{\{G61938\}}$ 

color 200 200 200
drawline a
color 0 0 0

% Constructing a line  $b$  which passes through point  $C$  and point  $P_{\{G61941\}}$ 
line b C  $P_{\{G61941\}}$ 

color 200 200 200
drawline b
color 0 0 0

% NDG: point  $M_{\{c\}}$  is outside the circle  $k(I, P_{\{a\}})$ 
% Constructing midpoint  $P_{\{G62344\}}$  of the segment  $M_{\{c\}}I$ 
midpoint  $P_{\{G62344\}}$   $M_{\{c\}}$  I
cmark_r  $P_{\{G62344\}}$ 

% Constructing a circle  $C_{\{G62347\}}$  whose center is at point  $P_{\{G62344\}}$  and which passes through
point  $M_{\{c\}}$ 
circle  $C_{\{G62347\}}$   $P_{\{G62344\}}$   $M_{\{c\}}$ 

color 200 200 200
drawcircle  $C_{\{G62347\}}$ 
color 0 0 0

% Constructing points  $P_{\{G62350\}}$  and  $P_{\{G62353\}}$  which are in intersection of  $C_{\{G62347\}}$  and  $k(I, P_{\{a\}})$ 
intersec2  $P_{\{G62350\}}$   $P_{\{G62353\}}$   $C_{\{G62347\}}$   $k(I, P_{\{a\}})$ 
cmark_r  $P_{\{G62350\}}$ 
cmark_r  $P_{\{G62353\}}$ 

% Constructing a line  $x3$  which passes through point  $M_{\{c\}}$  and point  $P_{\{G62350\}}$ 
line x3  $M_{\{c\}}$   $P_{\{G62350\}}$ 

color 200 200 200
drawline x3
color 0 0 0

% Constructing a line  $c$  which passes through point  $M_{\{c\}}$  and point  $P_{\{G62353\}}$ 
line c  $M_{\{c\}}$   $P_{\{G62353\}}$ 

color 200 200 200
drawline c

```

```
color 0 0 0
```

```
% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B
```

```
% Constructing a point A such that  $M_{\{c\}}A/M_{\{c\}}B=-1$ 
towards A  $M_{\{c\}}$  B -1
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines a and c are not parallel; point  $M_{\{c\}}$  is outside the circle  $k(I, P_{\{a\}})$ ; point C is outside the circle  $k(I, P_{\{c\}})$ ; points  $P_{\{c\}}$  and I are not the same; line  $h_{\{M_{\{c\}}, -1/1\}}(CP'_{\{c\}})$  and circle  $k_{\text{over}}(I, M_{\{c\}})$  intersect; points I and  $M_{\{c\}}$  are not the same
% Determination conditions: lines a and c are not the same; points  $M_{\{c\}}$  and I are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c = -M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 10 terms.

**Time Complexity:** Time spent by the prover is 0.254 seconds.

**NDG conditions** Points  $C$  and  $I$  are not identical

Points  $C$  and tempPoint-601 $CP'_c$  are not identical

Points  $C$ ,  $P_{G56424}$  and tempPoint-601 $CP'_c$  are not collinear

Points  $C$  and  $M_c$  are not identical



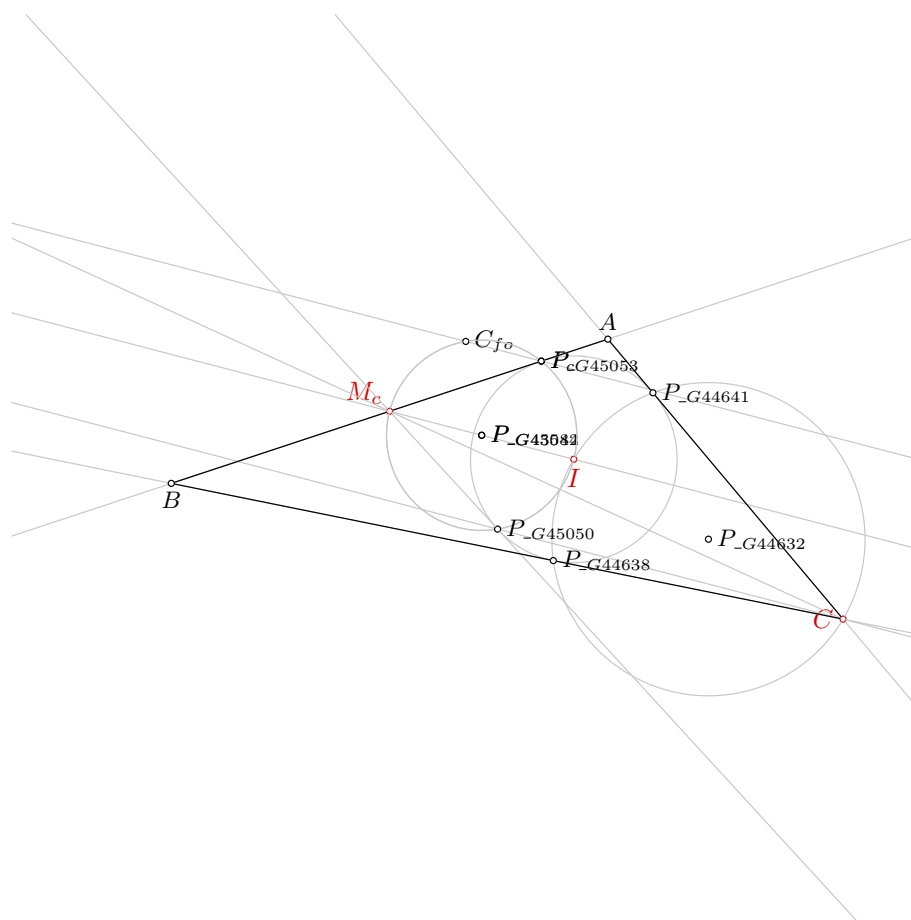


Figure 1: Illustration of the problem 0238

#### 4.1.3 Proving $I=_J$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{CM_cP_{G47220}} \neq S_{P_{G46805}M_cP_{G47220}}$  i.e., lines  $CP_{G46805}$  and  $M_cP_{G47220}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_c=_M_c$

Proving failed

#### 4.2.3 Proving $I=_J$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.040 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_c=_M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 540 terms.

**Time Complexity:** Time spent by the prover is 0.890 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I=_J$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.050 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_c=_M_c$

Proving failed

#### 4.4.3 Proving $I=_J$

Proving failed

# Problem 239

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 239: Given a point  $C$ , a point  $G$  and a point  $H_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $G$ , construct a point  $M_c$  (rule W01); ;
2. Using the point  $C$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $H_a$  are not the same;
3. Using the point  $H_a$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_a$  and  $M_c$  are not the same;
4. Using the circle  $k(M_c, A)$ , the line  $a$ , the point  $M_c$  and the point  $H_a$ , construct a point  $B$  (rule W05); % NDG: line  $a$  and circle  $k(M_c, A)$  intersect % DET: points  $H_a$  and  $B$  must be different;
5. Using the point  $G$  and the point  $B$ , construct a point  $M_b$  (rule W01); ;
6. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: line  $a$  and circle  $k(M_c, A)$  intersect; points  $H_a$  and  $M_c$  are not the same.

Determination conditions: points  $H_a$  and  $B$  must be different; points  $C$  and  $H_a$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D22,D5,GD01,GD02,GL03,GL04,L40,L42,L56,L57]

Solving time: 0.2 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point G 70 58.33
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_l C
cmark_t G
cmark_r H_{a}
color 0 0 0
fontsize 8

% Constructing a point M_{c} such that CM_{c}/CG=1.5
towards M_{c} C G 1.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points H_{a} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{a}
circle k(M_{c},A) M_{c} H_{a}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line a and circle k(M_{c},A) intersect% DET: points H_{a} and B must be different
% Constructing a point P_{\_G40585} which is a foot of the point M_{c} on the line a
foot P_{\_G40585} M_{c} a
cmark_r P_{\_G40585}
color 200 200 200
drawline M_{c} P_{\_G40585}
color 0 0 0
```

```

% Constructing a point B which is an image of the point H_{a} in the symmetry to point/line P_{\_G
40585}
sim B P_{\_G40585} H_{a}
cmark_b B

% Constructing a point M_{b} such that GM_{b}/GB=-0.5
towards M_{b} G B -0.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% Constructing a point A such that CA/CM_{b}=2
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(M_{c},A) intersect; points H_{a} and M_{c} are not
the same
% Determination conditions: points H_{a} and B must be different; points C and H_{a} are not the
same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $G=_G$

Proving failed

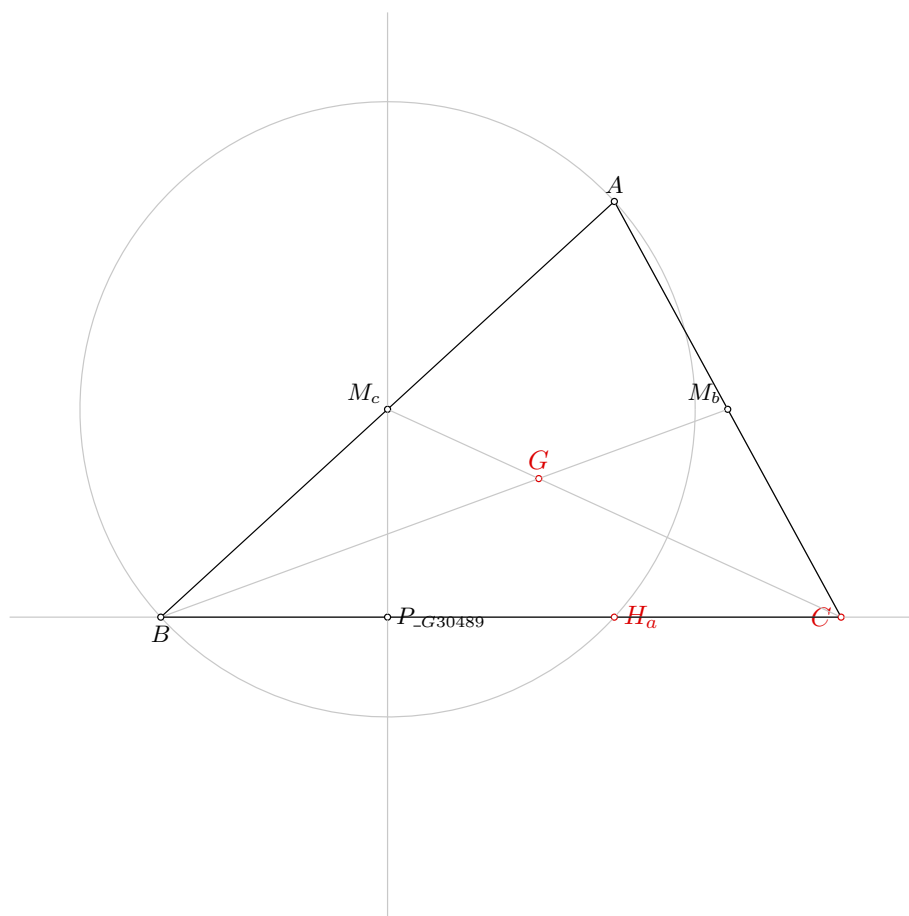


Figure 1: Illustration of the problem 0239

### 4.1.3 Proving $H_a = H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 615 terms.

**Time Complexity:** Time spent by the prover is 20.027 seconds.

**NDG conditions** Points  $G$  and  $H_a$  are not identical

Points  $G$  and  $H_a$  are not identical

Points  $P_{G38602}$ ,  $B$  and  $G$  are not collinear

Line through points  $P_{G38602}$  and  $B$  is not parallel with line through points  $C$  and  $G$

Points  $B$  and  $C$  are not identical

Points  $B$ ,  $C$  and  $M_c$  are not collinear

## 4.2 GCLC - Area method

### 4.2.1 Proving $C = C$

NDG conditions are:

$S_{AB_{M_b}} \neq S_{M_a B_{M_b}}$  i.e., lines  $A_{M_a}$  and  $B_{M_b}$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{H_a}^0} \neq S_{CAF_{H_a}^0}$  i.e., lines  $BC$  and  $AF_{H_a}^0$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $G = G$

Proving failed

### 4.2.3 Proving $H_a = H_a$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C = C$

Proving failed

### 4.3.2 Proving $G = G$

Proving failed

### 4.3.3 Proving $H_a = H_a$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C = C$

Proving failed

### 4.4.2 Proving $G = G$

Proving failed

### 4.4.3 Proving $H_a = H_a$

Proving failed



# Problem 240

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 240: Given a point  $C$ , a point  $G$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $G$ , construct a point  $M_c$  (rule W01); ;
2. Using the point  $C$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $H_b$  are not the same;
3. Using the point  $H_b$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_b$  and  $M_c$  are not the same;
4. Using the circle  $k(M_c, A)$ , the line  $b$ , the point  $M_c$  and the point  $H_b$ , construct a point  $A$  (rule W05); % NDG: line  $b$  and circle  $k(M_c, A)$  intersect % DET: points  $H_b$  and  $A$  must be different;
5. Using the point  $G$  and the point  $A$ , construct a point  $M_a$  (rule W01); ;
6. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: line  $b$  and circle  $k(M_c, A)$  intersect; points  $H_b$  and  $M_c$  are not the same.

Determination conditions: points  $H_b$  and  $A$  must be different; points  $C$  and  $H_b$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D21,D6,GD01,GD02,GL03,GL04,L41,L55,L57]

Solving time: 0.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point G 70 58.33
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_l C
cmark_t G
cmark_l H_{b}
color 0 0 0
fontsize 8

% Constructing a point M_{c} such that CM_{c}/CG=1.5
towards M_{c} C G 1.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points H_{b} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{b}
circle k(M_{c},A) M_{c} H_{b}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line b and circle k(M_{c},A) intersect% DET: points H_{b} and A must be different
% Constructing a point P_{\_G60601} which is a foot of the point M_{c} on the line b
foot P_{\_G60601} M_{c} b
cmark_r P_{\_G60601}
color 200 200 200
drawline M_{c} P_{\_G60601}
color 0 0 0
```

```

% Constructing a point A which is an image of the point H_{b} in the symmetry to point/line P_{\_G
60601}
sim A P_{\_G60601} H_{b}
cmark_t A

% Constructing a point M_{a} such that GM_{a}/GA=-0.5
towards M_{a} G A -0.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k(M_{c},A) intersect; points H_{b} and M_{c} are not
the same
% Determination conditions: points H_{b} and A must be different; points C and H_{b} are not the
same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.022 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $G=_G$

Proving failed

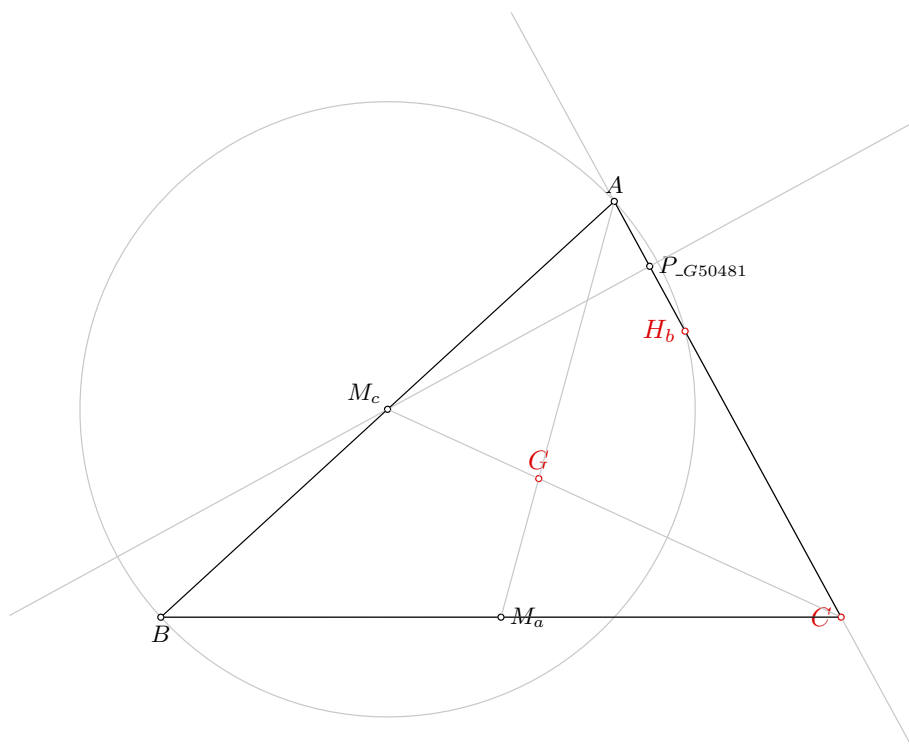


Figure 1: Illustration of the problem 0240

### 4.1.3 Proving $H_b = \neg H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 615 terms.

**Time Complexity:** Time spent by the prover is 20.183 seconds.

**NDG conditions** Points  $H_b$  and  $G$  are not identical

Points  $H_b$  and  $G$  are not identical

Points  $A$ ,  $P_{G58612}$  and  $G$  are not collinear

Line through points  $A$  and  $P_{G58612}$  is not parallel with line through points  $C$  and  $G$

Points  $A$  and  $C$  are not identical

Points  $A$  and  $C$  are not identical

## 4.2 GCLC - Area method

### 4.2.1 Proving $C = C$

NDG conditions are:

$S_{AB \neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A \neg M_a$  and  $B \neg M_b$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^0} \neq S_{CBF_{\neg h_b}^0}$  i.e., lines  $AC$  and  $BF_{\neg h_b}^0$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $G = \neg G$

NDG conditions are:

$S_{AB \neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A \neg M_a$  and  $B \neg M_b$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^0} \neq S_{CBF_{\neg h_b}^0}$  i.e., lines  $AC$  and  $BF_{\neg h_b}^0$  are not parallel (construction based assumption)

Total number of proof steps: 606

Time spent by the prover: 0.200 seconds

### 4.2.3 Proving $H_b = \neg H_b$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C = C$

Proving failed

### 4.3.2 Proving $G = \neg G$

Proving failed

### 4.3.3 Proving $H_b = \neg H_b$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C = C$

Proving failed

#### 4.4.2 Proving $G=_G$

Proving failed

#### 4.4.3 Proving $H_b=_H H_b$

Proving failed

# Problem 241

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 241: Given a point  $C$ , a point  $H_c$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_c$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H_c$  are not the same;
2. Using the point  $H_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
3. Using the point  $C$ , the line  $c$  and the point  $H_c$ , construct a line  $h_{C,2/3}(c)$  (rule W15); ;
4. Choose freely a point  $G$  on the line  $h_{C,2/3}(c)$  (rule WOnline4);
5. Using the point  $G$  and the point  $C$ , construct a point  $M_c$  (rule W01); ;
6. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) ;
7. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: points  $C$  and  $H_c$  are not the same.

Rules used: [W01,W02,W10a,W15,WOnline1,WOnline4]

Lemmas used: [D10,D20,D7,GL03,GL04,L57]

Solving time: 189.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point H_{c} 68.91 84.83
point G 70 58.33
```

```

color 220 0 0
fontsize 9

cmark_l C
cmark_rt H_{c}
cmark_t G
color 0 0 0
fontsize 8

% DET: points C and H_{c} are not the same
% Constructing a line h_{c} which passes through point C and point H_{c}
line h_{c} C H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a point P_{\_G40184} such that CP_{\_G40184}/CH_{c}=0.6666666666666666
towards P_{\_G40184} C H_{c} 0.6666666666666666
cmark_r P_{\_G40184}
color 200 200 200
drawsegment C H_{c}
color 0 0 0

% Constructing a line h_{C,2/3}(c) which contains the point P_{\_G40184} and is parallel to the
line c
parallel h_{C,2/3}(c) P_{\_G40184} c

color 200 200 200
drawline h_{C,2/3}(c)
color 0 0 0

% Constructing a point P_{\_G40491} such that CP_{\_G40491}/CH_{c}=0.6666666666666666
towards P_{\_G40491} C H_{c} 0.6666666666666666
cmark_r P_{\_G40491}
color 200 200 200
drawsegment C H_{c}
color 0 0 0

```



```

% Generating random value V[_G40430]
random V[_G40430]

% Calculating value V[_G40451] using formula V[_G40430]*20
expression V[_G40451] { V[_G40430]*20 }

% Constructing a point G which is a point for which holds  $P_{\setminus G40491}G = V[_G40451]$  and angle  $CP_{\setminus G40491}G = 90$ 
turtle G C P[_G40491] 90 V[_G40451]
cmark_t G

% Constructing a point  $M_{\{c\}}$  such that  $GM_{\{c\}}/GC=-0.5$ 
towards M_{c} G C -0.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% Choosing randomly a point A on the line  $H_{\{c\}}M_{\{c\}}$ 
online A H_{c} M_{c}
cmark_t A
color 200 200 200
drawline H_{c} M_{c}
color 0 0 0

% Constructing a point B such that  $AB/AM_{\{c\}}=2$ 
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions: points C and  $H_{\{c\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

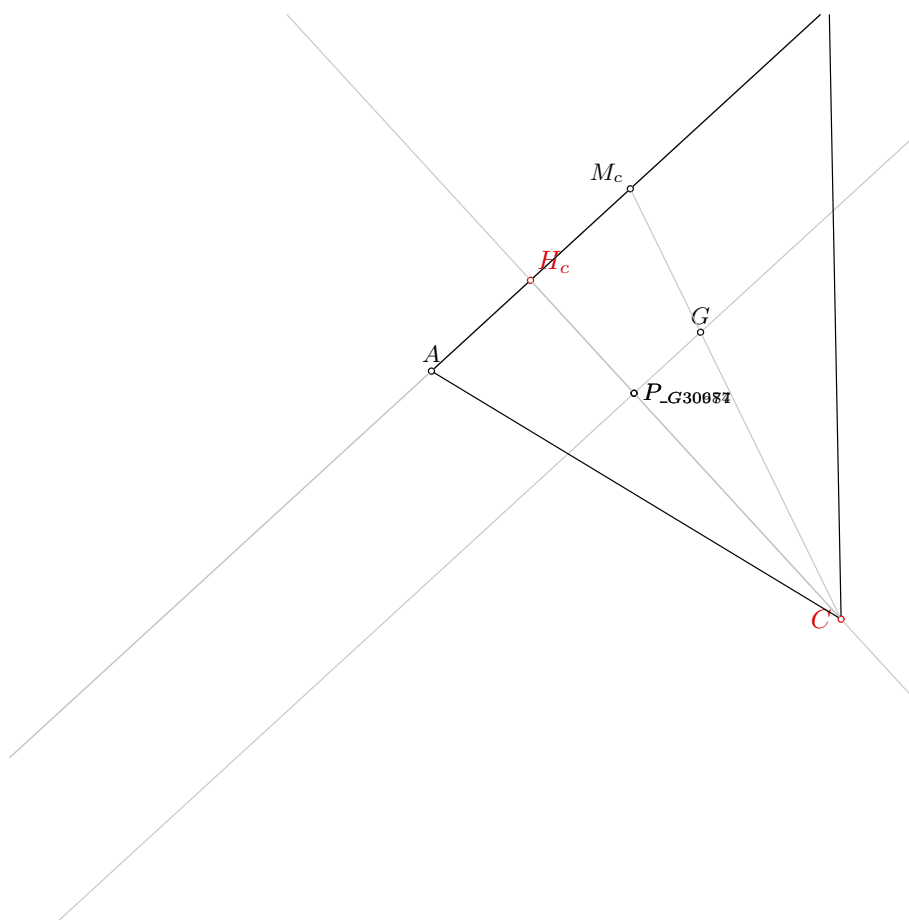


Figure 1: Illustration of the problem 0241

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.032 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c=_Hc$

##### Construction steps:

- Free point  $C$
- Free point  $H_c$
- Line  $h_c$  through two points  $C$  and  $H_c$
- Line  $c$  through point  $H_c$  perpendicular to line  $h_c$
- Segment division point  $P_{G37902}$  of segment  $CH_c$  with division coefficient 1.999999999999998
- Line  $h_{C,2/3}(c)$  through point  $P_{G37902}$  parallel with line  $c$
- Segment division point  $P_{G38034}$  of segment  $CH_c$  with division coefficient -0.6666666666666666
- Line  $L_{G38037}$  through point  $P_{G38034}$  parallel with line  $h_{C,2/3}(c)$
- Random point  $G$  from line  $L_{G38037}$
- Line  $L_{G38263}$  through two points  $G$  and  $C$
- Free point  $P_{G38266}$
- Segment division point  $P_{G38269}$  of segment  $GP_{G38266}$  with division coefficient -0.5
- Segment division point  $P_{G38272}$  of segment  $GP_{G38266}$  with division coefficient -2.0
- Line  $L_{G38275}$  through two points  $C$  and  $P_{G38272}$
- Line  $L_{G38278}$  through point  $P_{G38269}$  parallel with line  $L_{G38275}$
- Intersection point  $M_c$  of point sets  $L_{G38278}$  and  $L_{G38263}$
- Random point  $A$  from line  $c$
- Segment division point  $B$  of segment  $AM_c$  with division coefficient -2.0
- Line  $_c$  through two points  $A$  and  $B$
- Line  $_h c$  through point  $C$  perpendicular to line  $_c$
- Intersection point  $_H c$  of point sets  $_c$  and  $_h c$

##### Theorem statement:

- Points  $H_c$  and  $_H c$  are identical

**Info:** Attempting to add the construction of new random point tempPoint-686 $h_{C,2/3}(c)$  necessary for completion of construction of line  $L_{G38037}$

**Warning:** Generated new random point tempPoint-686 $h_{C,2/3}(c)$  on line  $h_{C,2/3}(c)$  in order to complete the construction of parallel line  $L_{G38037}$

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $C$  has been assigned following coordinates: (0, 0)

### 4.1.3 Proving $G=_G$

Proving failed

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^3} \neq S_{BCF_{h_c}^3}$  i.e., lines  $AB$  and  $CF_{h_c}^3$  are not parallel (construction based assumption)

$S_{AB_{M_b}} \neq S_{M_aB_{M_b}}$  i.e., lines  $AM_a$  and  $BM_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_c=_Hc$

Proving failed

### 4.2.3 Proving $G=_G$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

Proving failed

### 4.3.2 Proving $H_c=_Hc$

Proving failed

### 4.3.3 Proving $G=_G$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

Proving failed

### 4.4.2 Proving $H_c=_Hc$

Proving failed

### 4.4.3 Proving $G=_G$

Proving failed

# Problem 242

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 242: Given a point  $C$ , a point  $G$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $G$ , construct a point  $M_c$  (rule W01); ;
2. Using the point  $G$  and the point  $H$ , construct a point  $O$  (rule W01); ;
3. Using the point  $C$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H$  are not the same;
4. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
5. Using the point  $M_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same.

Determination conditions: points  $C$  and  $H$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D10,D20,D26,GD01,GL03,GL04,GL09,L11,L12,L3,L57,L58]

Solving time: 1.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point G 70 58.33
point H 80 72.73
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_t G
cmark_rt H
color 0 0 0
fontsize 8
```

```
% Constructing a point M_{c} such that CM_{c}/CG=1.5
towards M_{c} C G 1.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0
```

```
% Constructing a point O such that GO/GH=-0.5
towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0
```

```
% DET: points C and H are not the same
% Constructing a line h_{c} which passes through point C and point H
line h_{c} C H

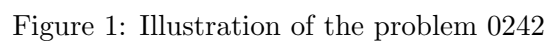
color 200 200 200
drawline h_{c}
color 0 0 0
```

```
% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0
```

```
% Constructing a line c which is perpendicular to line h_{c} and which passes through point M_{c}
perp c M_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0
```



### 3.3 Illustration

1303

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.034 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $G=_G$

Proving failed

#### 4.1.3 Proving $H=_H$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{M_cCH} \neq 0$  i.e., points  $M_c$ ,  $C$  and  $H$  are not collinear (foot is not the point itself; construction based assumption)

$S_{AB-M_b} \neq S_{M_aB-M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^2} \neq S_{F_{-h_a}^1BF_{-h_b}^2}$  i.e., lines  $AF_{-h_a}^1$  and  $BF_{-h_b}^2$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $G=_G$

Proving failed

#### 4.2.3 Proving $H=_H$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $G=_G$

Proving failed



### 4.3.3 Proving $H=_H$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $G=_G$

Proving failed

### 4.4.3 Proving $H=_H$

Proving failed

## Problem 243

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 243: Given a point  $C$ , a point  $G$  and a point  $T_a$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 244

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 244: Given a point  $C$ , a point  $G$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 245

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 245: Given a point  $C$ , a point  $G$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $G$ , construct a point  $M_c$  (rule W01); ;
2. Using the point  $C$  and the point  $T_c$ , construct a line  $s_c$  (rule W02); % DET: points  $C$  and  $T_c$  are not the same;
3. Using the point  $T_c$  and the point  $M_c$ , construct a line  $c$  (rule W02); % DET: points  $T_c$  and  $M_c$  are not the same;
4. Using the point  $M_c$  and the line  $c$ , construct a line  $m_c$  (rule W10b); ;
5. Using the line  $m_c$  and the line  $s_c$ , construct a point  $N_c$  (rule W03); % NDG: lines  $m_c$  and  $s_c$  are not parallel % DET: lines  $m_c$  and  $s_c$  are not the same;
6. Using the point  $C$  and the point  $N_c$ , construct a line  $m(CN_c)$  (rule W14); % DET: points  $C$  and  $N_c$  are not the same;
7. Using the line  $m(CN_c)$  and the line  $m_c$ , construct a point  $O$  (rule W03); % NDG: lines  $m(CN_c)$  and  $m_c$  are not parallel % DET: lines  $m(CN_c)$  and  $m_c$  are not the same;
8. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
9. Using the circle  $k(O, C)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same; lines  $m(CN_c)$  and  $m_c$  are not parallel; lines  $m_c$  and  $s_c$  are not parallel.

Determination conditions: lines  $m(CN_c)$  and  $m_c$  are not the same; points  $C$  and  $N_c$  are not the same; lines  $m_c$  and  $s_c$  are not the same; points  $T_c$  and  $M_c$  are not the same; points  $C$  and  $T_c$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14]

Lemmas used: [D13,D20,D25,D26,D49,GD01,GL01,GL03,GL09,L1,L11,L12,L27,L57,L7]

Solving time: 1.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point G 70 58.33
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_l C
cmark_t G
cmark_rt T_{c}
color 0 0 0
fontsize 8

% Constructing a point M_{c} such that CM_{c}/CG=1.5
towards M_{c} C G 1.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% DET: points C and T_{c} are not the same
% Constructing a line s_{c} which passes through point C and point T_{c}
line s_{c} C T_{c}

color 200 200 200
drawline s_{c}
color 0 0 0

% DET: points T_{c} and M_{c} are not the same
% Constructing a line c which passes through point T_{c} and point M_{c}
line c T_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0
```

```

% Constructing a line  $m_{\{c\}}$  which is perpendicular to line  $c$  and which passes through point  $M_{\{c\}}$ 
perp  $m_{\{c\}}$   $M_{\{c\}}$   $c$ 

color 200 200 200
drawline  $m_{\{c\}}$ 
color 0 0 0

% NDG: lines  $m_{\{c\}}$  and  $s_{\{c\}}$  are not parallel% DET: lines  $m_{\{c\}}$  and  $s_{\{c\}}$  are not the same
% Constructing a point  $N_{\{c\}}$  which belongs to line  $m_{\{c\}}$  and line  $s_{\{c\}}$ 
intersec  $N_{\{c\}}$   $m_{\{c\}}$   $s_{\{c\}}$ 
cmark_b  $N_{\{c\}}$ 

% DET: points  $C$  and  $N_{\{c\}}$  are not the same
% Constructing bisector  $m(CN_{\{c\}})$  of the segment  $CN_{\{c\}}$ 
med  $m(CN_{\{c\}})$   $C$   $N_{\{c\}}$ 

color 200 200 200
drawline  $m(CN_{\{c\}})$ 
color 0 0 0

color 200 200 200
drawsegment  $C$   $N_{\{c\}}$ 
color 0 0 0

% NDG: lines  $m(CN_{\{c\}})$  and  $m_{\{c\}}$  are not parallel% DET: lines  $m(CN_{\{c\}})$  and  $m_{\{c\}}$  are not the same
% Constructing a point  $O$  which belongs to line  $m(CN_{\{c\}})$  and line  $m_{\{c\}}$ 
intersec  $O$   $m(CN_{\{c\}})$   $m_{\{c\}}$ 
cmark_t  $O$ 

% NDG: points  $C$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $C$ 
circle  $k(O,C)$   $O$   $C$ 

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $c$  and circle  $k(O,C)$  intersect
% Constructing points  $A$  and  $B$  which are in intersection of  $k(O,C)$  and  $c$ 
intersec2  $A$   $B$   $k(O,C)$   $c$ 
cmark_t  $A$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

```

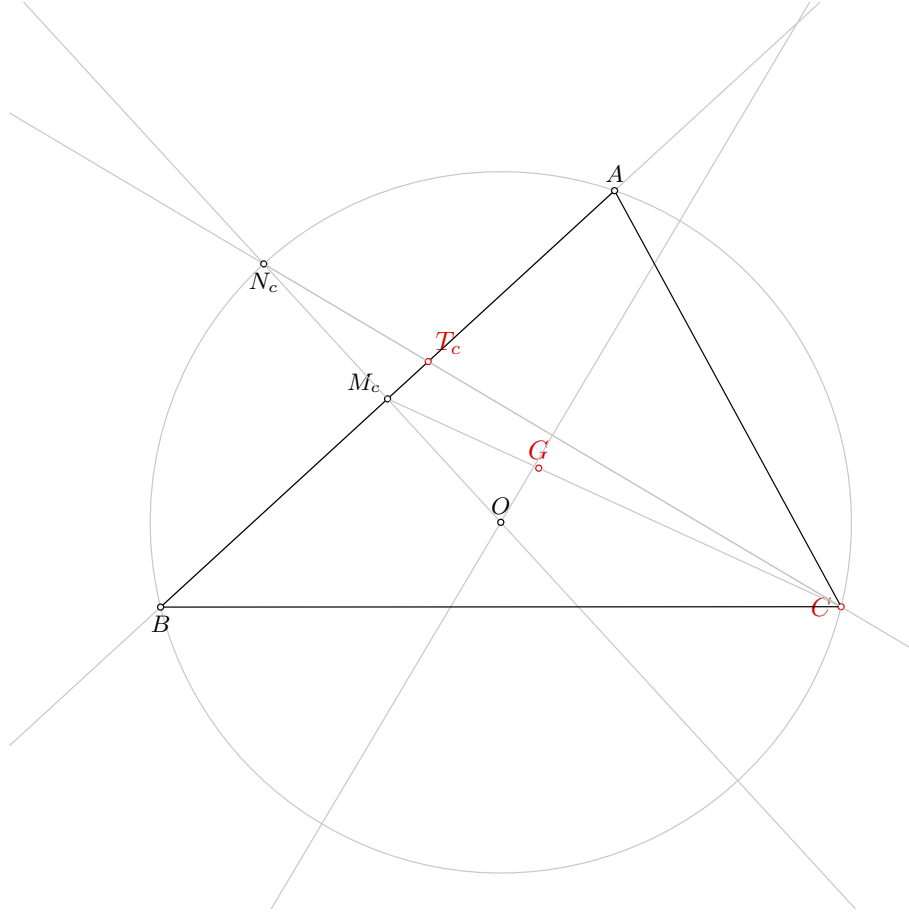


Figure 1: Illustration of the problem 0245

*% Non-degenerate conditions: line  $c$  and circle  $k(O,C)$  intersect; points  $C$  and  $O$  are not the same;  
 lines  $m(CN_{\{c\}})$  and  $m_{\{c\}}$  are not parallel; lines  $m_{\{c\}}$  and  $s_{\{c\}}$  are not parallel  
 % Determination conditions: lines  $m(CN_{\{c\}})$  and  $m_{\{c\}}$  are not the same; points  $C$  and  $N_{\{c\}}$  are not  
 the same; lines  $m_{\{c\}}$  and  $s_{\{c\}}$  are not the same; points  $T_{\{c\}}$  and  $M_{\{c\}}$  are not the same;  
 points  $C$  and  $T_{\{c\}}$  are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $G=_G$

Proving failed

#### 4.1.3 Proving $T_c=_T T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{M_c C T_c} \neq S_{T_{m_c}^0 C T_c}$  i.e., lines  $M_c T_{m_c}^0$  and  $C T_c$  are not parallel (construction based assumption)

$S_{M_{m(CN_c)}^1 M_c T_{m_c}^0} \neq S_{T_{m(CN_c)}^2 M_c T_{m_c}^0}$  i.e., lines  $M_{m(CN_c)}^1 T_{m(CN_c)}^2$  and  $M_c T_{m_c}^0$  are not parallel (construction based assumption)

$S_{AB-M_b} \neq S_{-M_a B-M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $G=_G$

Proving failed

#### 4.2.3 Proving $T_c=_T T_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $G=_G$

Proving failed

#### 4.3.3 Proving $T_c=_T T_c$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.



#### 4.4.2 Proving $G=_G$

Proving failed

#### 4.4.3 Proving $T_c=_T T_c$

Proving failed

# Problem 246

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 246: Given a point  $C$ , a point  $G$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $G$ , construct a point  $M_c$  (rule W01); ;
2. Using the point  $I$  and the point  $M_c$ , construct a line  $IM_c$  (rule W02); % DET: points  $I$  and  $M_c$  are not the same;
3. Using the point  $I$  and the point  $M_c$ , construct a circle  $k_{over}(I, M_c)$  (rule W09); % NDG: points  $I$  and  $M_c$  are not the same;
4. Using the point  $C$  and the line  $IM_c$ , construct a line  $CP'_c$  (rule W16); ;
5. Using the point  $M_c$ , the line  $CP'_c$  and the point  $C$ , construct a line  $h_{M_c, -1/1}(CP'_c)$  (rule W15); ;
6. Using the circle  $k_{over}(I, M_c)$  and the line  $h_{M_c, -1/1}(CP'_c)$ , construct a point  $C_{fo}$  and a point  $P_c$  (rule W04); % NDG: line  $h_{M_c, -1/1}(CP'_c)$  and circle  $k_{over}(I, M_c)$  intersect;
7. Using the point  $P_c$  and the point  $I$ , construct a circle  $k(I, P_a)$  (rule W06); % NDG: points  $P_c$  and  $I$  are not the same;
8. Using the circle  $k(I, P_a)$ , the point  $C$  and the point  $I$ , construct a line  $a$  and a line  $b$  (rule W12); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
9. Using the circle  $k(I, P_a)$ , the point  $M_c$  and the point  $I$ , construct a line  $x3$  and a line  $c$  (rule W12); % NDG: point  $M_c$  is outside the circle  $k(I, P_a)$ ;
10. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same;
11. Using the point  $G$  and the point  $B$ , construct a point  $M_b$  (rule W01); ;

12. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; point  $M_c$  is outside the circle  $k(I, P_a)$ ; point  $C$  is outside the circle  $k(I, P_a)$ ; points  $P_c$  and  $I$  are not the same; line  $h_{M_c, -1/1}(CP_c)$  and circle  $k_{over}(I, M_c)$  intersect; points  $I$  and  $M_c$  are not the same.

Determination conditions: lines  $a$  and  $c$  are not the same; points  $I$  and  $M_c$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W09,W12,W15,W16]

Lemmas used: [D20,D22,D27,D61,D87,GD01,GL02,GL03,GL04,GL09,L14,L56,L57,L59,L60,L61,L64,L76]

Solving time: 3.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point G 70 58.33
point I 74.37 61.15
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_t G
cmark_b I
color 0 0 0
fontsize 8
```

```
% Constructing a point M_{c} such that CM_{c}/CG=1.5
towards M_{c} C G 1.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0
```

```
% DET: points I and M_{c} are not the same
% Constructing a line IM_{c} which passes through point I and point M_{c}
line IM_{c} I M_{c}
```

```
color 200 200 200
drawline IM_{c}
color 0 0 0
```

```
% NDG: points I and M_{c} are not the same
% Constructing midpoint P_{\_G53236} of the segment IM_{c}
midpoint P_{\_G53236} I M_{c}
cmark_r P_{\_G53236}
```

```
% Constructing a circle k_{over}(I,M_{c}) whose center is at point P_{\_G53236} and which passes
through point I
circle k_{over}(I,M_{c}) P_{\_G53236} I
```

```

color 200 200 200
drawcircle k_over(I,M_{c})
color 0 0 0

% Constructing a line CP'_{c} which contains the point C and is parallel to the line IM_{c}
parallel CP'_{c} C IM_{c}

color 200 200 200
drawline CP'_{c}
color 0 0 0

% Constructing a point P_{\_G53537} such that M_{c}P_{\_G53537}/M_{c}C=-1
towards P_{\_G53537} M_{c} C -1
cmark_r P_{\_G53537}
color 200 200 200
drawsegment C P_{\_G53537}
color 0 0 0

% Constructing a line h_{M_{c},-1/1}(CP'_{c}) which contains the point P_{\_G53537} and is parallel
to the line CP'_{c}
parallel h_{M_{c},-1/1}(CP'_{c}) P_{\_G53537} CP'_{c}

color 200 200 200
drawline h_{M_{c},-1/1}(CP'_{c})
color 0 0 0

% NDG: line h_{M_{c},-1/1}(CP'_{c}) and circle k_over(I,M_{c}) intersect
% Constructing points C_{fo} and P_{c} which are in intersection of k_over(I,M_{c}) and h_{M_{c},-1/1}(CP'_{c})
intersec2 C_{fo} P_{c} k_over(I,M_{c}) h_{M_{c},-1/1}(CP'_{c})
cmark_r C_{fo}
cmark_r P_{c}

% NDG: points P_{c} and I are not the same
% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{c}
circle k(I,P_{a}) I P_{c}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point C is outside the circle k(I,P_{a})
% Constructing midpoint P_{\_G54286} of the segment CI
midpoint P_{\_G54286} C I

```

```

cmark_r P_{\_G54286}

% Constructing a circle C_{\_G54289} whose center is at point P_{\_G54286} and which passes through
point C
circle C_{\_G54289} P_{\_G54286} C

color 200 200 200
drawcircle C_{\_G54289}
color 0 0 0

% Constructing points P_{\_G54292} and P_{\_G54295} which are in intersection of C_{\_G54289} and k
(I,P_{a})
intersec2 P_{\_G54292} P_{\_G54295} C_{\_G54289} k(I,P_{a})
cmark_r P_{\_G54292}
cmark_r P_{\_G54295}

% Constructing a line a which passes through point C and point P_{\_G54292}
line a C P_{\_G54292}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line b which passes through point C and point P_{\_G54295}
line b C P_{\_G54295}

color 200 200 200
drawline b
color 0 0 0

% NDG: point M_{c} is outside the circle k(I,P_{a})
% Constructing midpoint P_{\_G54698} of the segment M_{c}I
midpoint P_{\_G54698} M_{c} I
cmark_r P_{\_G54698}

% Constructing a circle C_{\_G54701} whose center is at point P_{\_G54698} and which passes through
point M_{c}
circle C_{\_G54701} P_{\_G54698} M_{c}

color 200 200 200
drawcircle C_{\_G54701}
color 0 0 0

% Constructing points P_{\_G54704} and P_{\_G54707} which are in intersection of C_{\_G54701} and k
(I,P_{a})
intersec2 P_{\_G54704} P_{\_G54707} C_{\_G54701} k(I,P_{a})
cmark_r P_{\_G54704}
cmark_r P_{\_G54707}

% Constructing a line x3 which passes through point M_{c} and point P_{\_G54704}
line x3 M_{c} P_{\_G54704}

```

```

color 200 200 200
drawline x3
color 0 0 0

% Constructing a line c which passes through point M_{c} and point P_{\_G54707}
line c M_{c} P_{\_G54707}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B

% Constructing a point M_{b} such that GM_{b}/GB=-0.5
towards M_{b} G B -0.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% Constructing a point A such that CA/CM_{b}=2
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and c are not parallel; point M_{c} is outside the circle k(I,
P_{a}); point C is outside the circle k(I,P_{a}); points P_{c} and I are not the same; line h_{
M_{c},-1/1}(CP'_{c}) and circle k_{over}(I,M_{c}) intersect; points I and M_{c} are not the same
% Determination conditions: lines a and c are not the same; points I and M_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

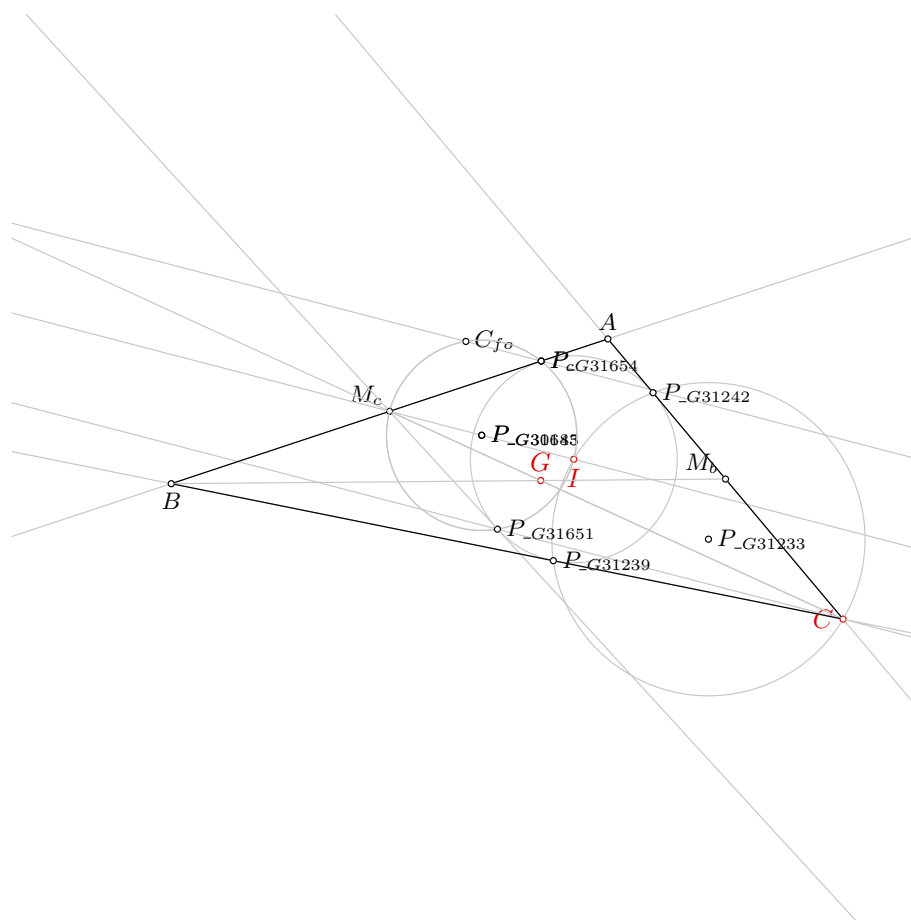


Figure 1: Illustration of the problem 0246

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.025 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $G=_G$

Proving failed

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{CM_cP_{G33987}} \neq S_{P_{G33572}M_cP_{G33987}}$  i.e., lines  $CP_{G33572}$  and  $M_cP_{G33987}$  are not parallel (construction based assumption)

$S_{AB_{-}M_b} \neq S_{_{-}M_aB_{-}M_b}$  i.e., lines  $A_{-}M_a$  and  $B_{-}M_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $G=_G$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.070 seconds. There are no ndg conditions.

#### 4.3.2 Proving $G=_G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 540 terms.

**Time Complexity:** Time spent by the prover is 1.600 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I=_I$

Proving failed



## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.070 seconds. There are no ndg conditions.

### 4.4.2 Proving $G=_G$

Proving failed

### 4.4.3 Proving $I=_I$

Proving failed

# Problem 247

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 247: Given a point  $C$ , a point  $H_a$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $H_a$  are not the same;
2. Using the point  $C$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $H_b$  are not the same;
3. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
4. Using the line  $h_a$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $h_a$  and  $b$  are not parallel % DET: lines  $h_a$  and  $b$  are not the same;
5. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
6. Using the line  $h_b$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $h_b$  and  $a$  are not parallel % DET: lines  $h_b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $h_b$  and  $a$  are not parallel; lines  $h_a$  and  $b$  are not parallel.

Determination conditions: lines  $h_b$  and  $a$  are not the same; lines  $h_a$  and  $b$  are not the same; points  $C$  and  $H_b$  are not the same; points  $C$  and  $H_a$  are not the same.

Rules used: [W02,W03,W10b]

Lemmas used: [D5,D6,D8,D9,GD01]

Solving time: 0.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{a} 80 40
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_l C
cmark_r H_{a}
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: lines h_{a} and b are not parallel% DET: lines h_{a} and b are not the same
% Constructing a point A which belongs to line h_{a} and line b
intersec A h_{a} b
cmark_t A

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b
```

```

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines h_{b} and a are not parallel% DET: lines h_{b} and a are not the same
% Constructing a point B which belongs to line h_{b} and line a
intersec B h_{b} a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{b} and a are not parallel; lines h_{a} and b are not parallel
% Determination conditions: lines h_{b} and a are not the same; lines h_{a} and b are not the same;
  points C and H_{b} are not the same; points C and H_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a=H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.093 seconds.

**NDG conditions** Points  $H_b$  and  $H_a$  are not identical

Point  $B$  is not on circle with center  $H_a$  and point from it  $C$

#### 4.1.3 Proving $H_b=H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.16 seconds.

**NDG conditions** Points  $H_b$  and  $H_a$  are not identical

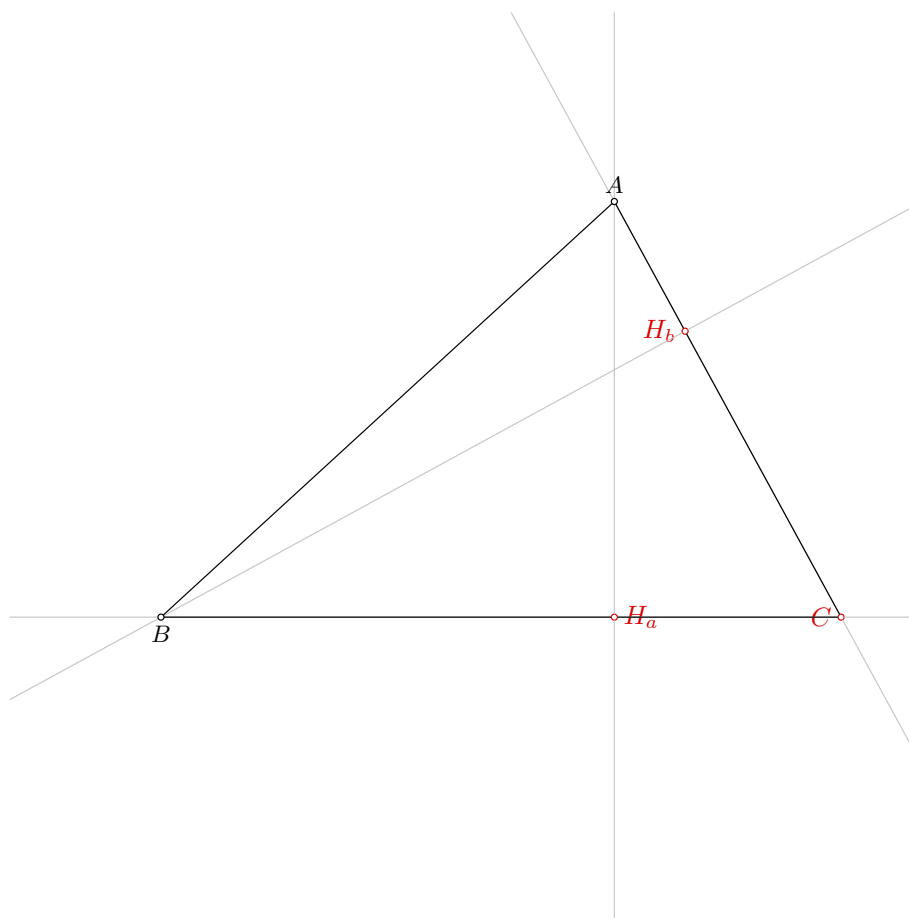


Figure 1: Illustration of the problem 0247

Point  $H_a$  is not the midpoint of segment with endpoints  $A$  and  $C$

Points  $A$  and  $H_a$  are not identical

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_aCH_b} \neq S_{T_{h_a}^0CH_b}$  i.e., lines  $H_aT_{h_a}^0$  and  $CH_b$  are not parallel (construction based assumption)

$S_{H_bCH_a} \neq S_{T_{h_b}^1CH_a}$  i.e., lines  $H_bT_{h_b}^1$  and  $CH_a$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^2} \neq S_{CAF_{h_a}^2}$  i.e., lines  $BC$  and  $AF_{h_a}^2$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^3} \neq S_{CBF_{h_b}^3}$  i.e., lines  $AC$  and  $BF_{h_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_a=H_a$

Proving failed

### 4.2.3 Proving $H_b=H_b$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_a=H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $H_b=H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 12 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_a = H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $H_b = H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 11 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

# Problem 248

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 248: Given a point  $C$ , a point  $H_a$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $H_a$  are not the same;
2. Using the point  $C$  and the point  $H_c$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H_c$  are not the same;
3. Using the point  $H_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
4. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same;
5. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
6. Using the line  $c$  and the line  $h_a$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $h_a$  are not parallel % DET: lines  $c$  and  $h_a$  are not the same.

Non-degenerate conditions: lines  $c$  and  $h_a$  are not parallel; lines  $c$  and  $a$  are not parallel.

Determination conditions: lines  $c$  and  $h_a$  are not the same; lines  $c$  and  $a$  are not the same; points  $C$  and  $H_c$  are not the same; points  $C$  and  $H_a$  are not the same.

Rules used: [W02,W03,W10a,W10b]

Lemmas used: [D10,D5,D7,D8,GD01]

Solving time: 1.2 seconds.



### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{a} 80 40
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_l C
cmark_r H_{a}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% DET: points C and H_{c} are not the same
% Constructing a line h_{c} which passes through point C and point H_{c}
line h_{c} C H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a
```

```

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: lines c and h_{a} are not parallel% DET: lines c and h_{a} are not the same
% Constructing a point A which belongs to line c and line h_{a}
intersec A c h_{a}
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and h_{a} are not parallel; lines c and a are not parallel
% Determination conditions: lines c and h_{a} are not the same; lines c and a are not the same;
  points C and H_{c} are not the same; points C and H_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a=H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.07 seconds.

**NDG conditions** Points  $H_c$ ,  $C$  and  $H_a$  are not collinear

Points  $H_c$ ,  $C$  and  $H_a$  are not collinear

Point  $B$  is not on circle with center  $H_a$  and point from it  $C$

#### 4.1.3 Proving $H_c=H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 20 terms.

**Time Complexity:** Time spent by the prover is 0.372 seconds.

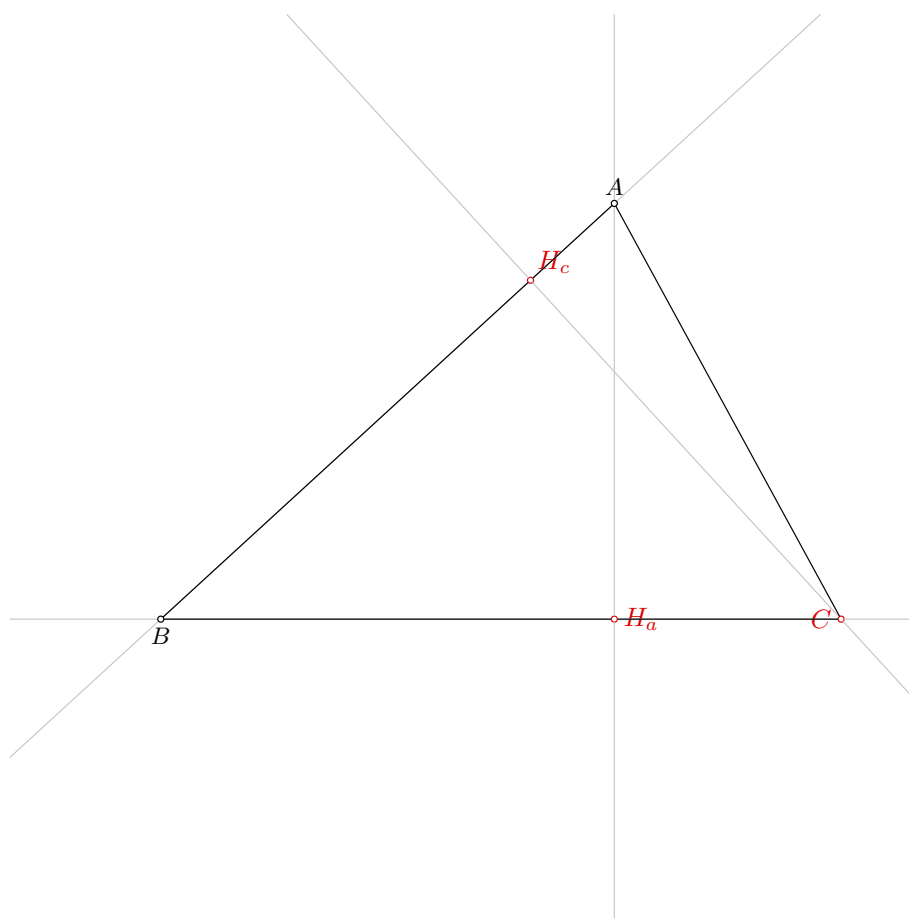


Figure 1: Illustration of the problem 0248

**NDG conditions** Points  $H_c$ ,  $C$  and  $H_a$  are not collinear

Points  $H_c$ ,  $C$  and  $H_a$  are not collinear

Points  $A$  and  $B$  are not identical

Points  $A$  and  $H_a$  are not identical

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_cCH_a} \neq S_{T_c^0CH_a}$  i.e., lines  $H_cT_c^0$  and  $CH_a$  are not parallel (construction based assumption)

$S_{H_cH_aT_{h_a}^1} \neq S_{T_c^0H_aT_{h_a}^1}$  i.e., lines  $H_cT_c^0$  and  $H_aT_{h_a}^1$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^2} \neq S_{CAF_{h_a}^2}$  i.e., lines  $BC$  and  $AF_{h_a}^2$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^3} \neq S_{BCF_{h_c}^3}$  i.e., lines  $AB$  and  $CF_{h_c}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_a=_H H_a$

Proving failed

### 4.2.3 Proving $H_c=_H H_c$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_a=_H H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $H_c=_H H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 91 terms.

**Time Complexity:** Time spent by the prover is 0.050 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_a = H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $H_c = H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 176 terms.

**Time Complexity:** Time spent by the prover is 0.720 seconds. There are no ndg conditions.

# Problem 249

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 249: Given a point  $H_a$ , a point  $H$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $H_a$  and  $H$  are not the same;
2. Using the point  $H_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
3. Choose freely a point  $C$  on the line  $a$  (rule WOnline2);
4. Using the point  $C$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H$  are not the same;
5. Choose freely a point  $A$  on the line  $h_a$  (rule WOnline1) ;
6. Using the point  $A$  and the point  $C$ , construct a point  $M_b$  (rule W01); ;
7. Using the point  $A$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $A$  and  $M_b$  are not the same;
8. Using the circle  $k(M_b, C)$ , the line  $h_c$ , the point  $M_b$  and the point  $C$ , construct a point  $H_c$  (rule W05); % NDG: line  $h_c$  and circle  $k(M_b, C)$  intersect % DET: points  $C$  and  $H_c$  must be different;
9. Using the point  $H_c$  and the point  $A$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $A$  are not the same;
10. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; line  $h_c$  and circle  $k(M_b, C)$  intersect; points  $A$  and  $M_b$  are not the same.

Determination conditions: lines  $a$  and  $c$  are not the same; points  $H_c$  and  $A$  are not the same; points  $C$  and  $H_c$  must be different; points  $C$  and  $H$  are not the same; points  $H_a$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D10,D22,D3,D5,D7,D8,GD01,GD02,GL04,L3,L43,L45]

Solving time: 23.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{a} 80 40
```

```
point H 80 72.73
```

```
point C 110 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r H_{a}
```

```
cmark_rt H
```

```
cmark_l C
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{a} and H are not the same
```

```
% Constructing a line h_{a} which passes through point H_{a} and point H
```

```
line h_{a} H_{a} H
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```
% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
```

```
perp a H_{a} h_{a}
```

```
color 200 200 200
```

```
drawline a
```

```
color 0 0 0
```

```
% Generating random value V[_G28854]
```

```
random V[_G28854]
```

```
% Calculating value V[_G28875] using formula V[_G28854]*20
```

```
expression V[_G28875] { V[_G28854]*20 }
```

```

% Constructing a point C which is a point for which holds  $H_{\{a\}}C = V[_G28875]$  and angle  $HH_{\{a\}}C = 90$ 
turtle C H  $H_{\{a\}}$  90  $V[_G28875]$ 
cmark_l C

% DET: points C and H are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point C and point H
line  $h_{\{c\}}$  C H

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% Choosing randomly a point A on the line  $H_{\{a\}}H$ 
online A  $H_{\{a\}}$  H
cmark_t A
color 200 200 200
drawline  $H_{\{a\}}$  H
color 0 0 0

% Constructing a point  $M_{\{b\}}$  such that  $AM_{\{b\}}/AC=0.5$ 
towards  $M_{\{b\}}$  A C 0.5
cmark_lt  $M_{\{b\}}$ 
color 200 200 200
drawsegment A C
color 0 0 0

% NDG: points A and  $M_{\{b\}}$  are not the same
% Constructing a circle  $k(M_{\{b\}},C)$  whose center is at point  $M_{\{b\}}$  and which passes through point A
circle  $k(M_{\{b\}},C)$   $M_{\{b\}}$  A

color 200 200 200
drawcircle  $k(M_{\{b\}},C)$ 
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(M_{\{b\}},C)$  intersect% DET: points C and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G29404\}}$  which is a foot of the point  $M_{\{b\}}$  on the line  $h_{\{c\}}$ 
foot  $P_{\{\backslash\_G29404\}}$   $M_{\{b\}}$   $h_{\{c\}}$ 
cmark_r  $P_{\{\backslash\_G29404\}}$ 
color 200 200 200
drawline  $M_{\{b\}}$   $P_{\{\backslash\_G29404\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point C in the symmetry to point/line  $P_{\{\backslash\_G29404\}}$ 
sim  $H_{\{c\}}$   $P_{\{\backslash\_G29404\}}$  C

```



```

cmark_rt H_{c}

% DET: points H_{c} and A are not the same
% Constructing a line c which passes through point H_{c} and point A
line c H_{c} A

color 200 200 200
drawline c
color 0 0 0

% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and c are not parallel; line h_{c} and circle k(M_{b},C)
% intersect; points A and M_{b} are not the same
% Determination conditions: lines a and c are not the same; points H_{c} and A are not the same;
% points C and H_{c} must be different; points C and H are not the same; points H_{a} and H are
% not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_a = \_H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.158 seconds.

**NDG conditions** Points  $B$ ,  $C$  and  $H$  are not collinear

Points  $B$  and  $H$  are not identical

Line through points  $A$  and  $H$  is not perpendicular to line through points  $H$  and  $H_c$

Points  $B$ ,  $C$  and  $H$  are not collinear

#### 4.1.2 Proving $H = \_H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 31 terms.

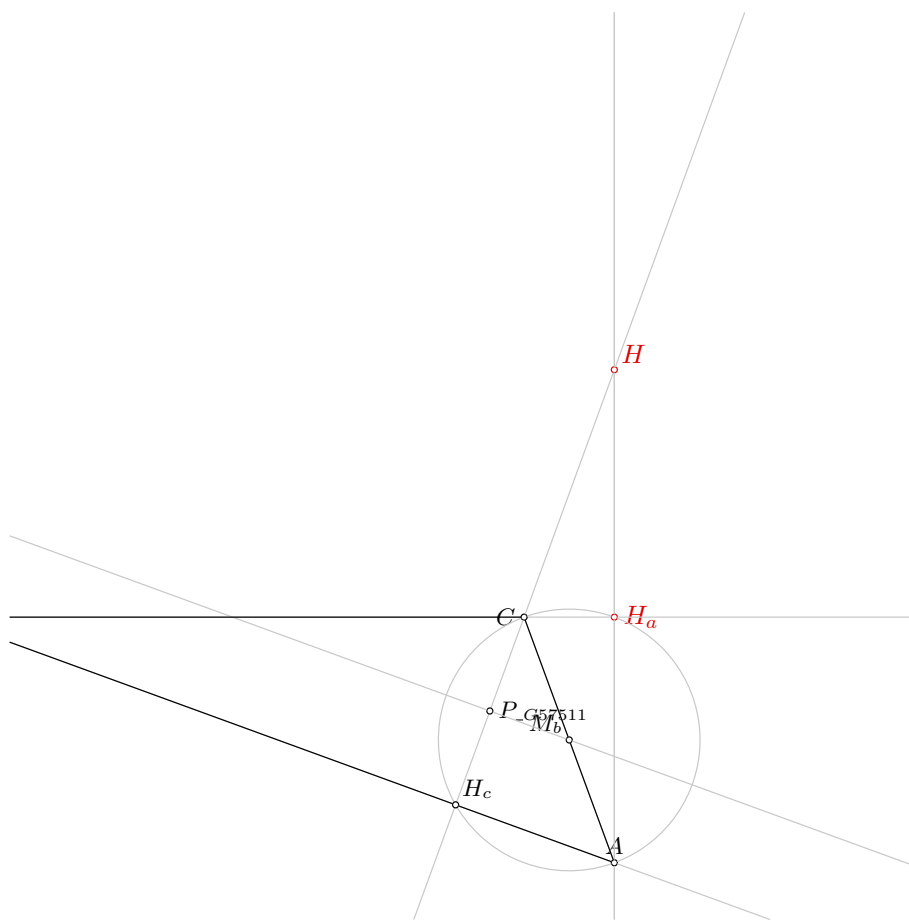


Figure 1: Illustration of the problem 0249

**Time Complexity:** Time spent by the prover is 0.458 seconds.

**NDG conditions** Points  $B$ ,  $C$  and  $H$  are not collinear

Points  $B$  and  $H$  are not identical

Line through points  $A$  and  $H$  is not perpendicular to line through points  $H$  and  $H_c$

Points  $B$ ,  $C$  and  $H$  are not collinear

Points  $A$  and  $B$  are not identical

#### 4.1.3 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.007 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_a=_H a$

Proving failed

#### 4.2.2 Proving $H=_H H$

Proving failed

#### 4.2.3 Proving $C=C$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_a=_H a$

Proving failed

#### 4.3.2 Proving $H=_H H$

Proving failed

#### 4.3.3 Proving $C=C$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $H_a=_H a$

Proving failed

#### 4.4.2 Proving $H=_H H$

Proving failed

#### 4.4.3 Proving $C=C$

Proving failed

# Problem 250

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 250: Given a point  $H_a$ , a point  $T_a$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $T_a$  are not the same;
2. Choose freely a point  $C$  on the line  $a$  (rule WOnline1) ;
3. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
4. Choose freely a point  $A$  on the line  $h_a$  (rule WOnline2);
5. Using the point  $A$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $C$  are not the same;
6. Using the point  $T_a$  and the line  $b$ , construct a circle  $k(T_a, foot[T_a, c])$  (rule W11); % NDG: point  $T_a$  is not incident to the line  $b$ ;
7. Using the circle  $k(T_a, foot[T_a, c])$ , the point  $A$ , the point  $T_a$  and the line  $b$ , construct a line  $c$  (rule W13); % NDG: point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ;
8. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ; point  $T_a$  is not incident to the line  $b$ .

Determination conditions: lines  $c$  and  $a$  are not the same; points  $A$  and  $C$  are not the same; points  $H_a$  and  $T_a$  are not the same.

Rules used: [W02,W03,W10b,W11,W13,WOnline1,WOnline2]

Lemmas used: [D23,D5,D8,GD01,GD02,GL10,GL11,L8]

Solving time: 15.4 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point H_{a} 80 40
point T_{a} 70.86 40
point C 110 40

color 220 0 0
fontsize 9

cmark_r H_{a}
cmark_rb T_{a}
cmark_l C
color 0 0 0
fontsize 8

% DET: points H_{a} and T_{a} are not the same
% Constructing a line a which passes through point H_{a} and point T_{a}
line a H_{a} T_{a}

color 200 200 200
drawline a
color 0 0 0

% Choosing randomly a point C on the line H_{a}T_{a}
online C H_{a} T_{a}
cmark_l C
color 200 200 200
drawline H_{a} T_{a}
color 0 0 0

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% Generating random value V[_G52652]
random V[_G52652]

% Calculating value V[_G52673] using formula V[_G52652]*20
expression V[_G52673] { V[_G52652]*20 }
```

```

% Constructing a point A which is a point for which holds  $H_{\{a\}}A = V[_G52673]$  and angle  $CH_{\{a\}}A = 90$ 
turtle A C  $H_{\{a\}}$  90  $V[_G52673]$ 
cmark_t A

% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C

color 200 200 200
drawline b
color 0 0 0

% NDG: point  $T_{\{a\}}$  is not incident to the line b
% Constructing a point  $P_{\{\_G52917\}}$  which is a foot of the point  $T_{\{a\}}$  on the line b
foot  $P_{\{\_G52917\}}$   $T_{\{a\}}$  b
cmark_r  $P_{\{\_G52917\}}$ 
color 200 200 200
drawline  $T_{\{a\}}$   $P_{\{\_G52917\}}$ 
color 0 0 0

% Constructing a circle  $k(T_{\{a\}}, foot[T_{\{a\}}, c])$  whose center is at point  $T_{\{a\}}$  and which passes
    through point  $P_{\{\_G52917\}}$ 
circle  $k(T_{\{a\}}, foot[T_{\{a\}}, c])$   $T_{\{a\}}$   $P_{\{\_G52917\}}$ 

color 200 200 200
drawcircle  $k(T_{\{a\}}, foot[T_{\{a\}}, c])$ 
color 0 0 0

% NDG: point A is outside the circle  $k(T_{\{a\}}, foot[T_{\{a\}}, c])$ 
% Constructing a point  $P_{\{\_G53308\}}$  which is a foot of the point  $T_{\{a\}}$  on the line b
foot  $P_{\{\_G53308\}}$   $T_{\{a\}}$  b
cmark_r  $P_{\{\_G53308\}}$ 
color 200 200 200
drawline  $T_{\{a\}}$   $P_{\{\_G53308\}}$ 
color 0 0 0

% Constructing a line  $L_{\{\_G53311\}}$  which passes through point  $T_{\{a\}}$  and point A
line  $L_{\{\_G53311\}}$   $T_{\{a\}}$  A

color 200 200 200
drawline  $L_{\{\_G53311\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\_G53314\}}$  which is an image of the point  $P_{\{\_G53308\}}$  in the symmetry to
    point/line  $L_{\{\_G53311\}}$ 
sim  $P_{\{\_G53314\}}$   $L_{\{\_G53311\}}$   $P_{\{\_G53308\}}$ 
cmark_r  $P_{\{\_G53314\}}$ 

% Constructing a line c which passes through point A and point  $P_{\{\_G53314\}}$ 

```

```

line c A P_{\_G53314}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and a are not parallel; point A is outside the circle k(T_{a},
    foot[T_{a},c]); point T_{a} is not incident to the line b
% Determination conditions: lines c and a are not the same; points A and C are not the same; points
    H_{a} and T_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_a = H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.219 seconds.

**NDG conditions** Points  $A$ ,  $C$  and  $H_a$  are not collinear

Point  $H_a$  is not on circle with center  $T_a$  and point from it  $C$

Points  $T_a$  and  $A$  are not identical

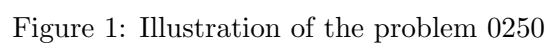
Point  $H_a$  is not on circle with center  $T_a$  and point from it  $C$

Line through points  $P_{G48931}$  and  $A$  is not perpendicular to line through points  $A$  and  $H_a$

Line through points  $A$  and  $H_a$  is not parallel with line through points  $B$  and  $C$

#### 4.1.2 Proving $T_a = T_a$

Proving failed





### 4.1.3 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.007 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $H_a = \neg H_a$

Proving failed

### 4.2.2 Proving $T_a = \neg T_a$

Proving failed

### 4.2.3 Proving $C=C$

NDG conditions are:

$S_{AH_aT_a} \neq S_{P_{G47561}H_aT_a}$  i.e., lines  $AP_{G47561}$  and  $H_aT_a$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^2} \neq S_{CAF_{h_a}^2}$  i.e., lines  $BC$  and  $AF_{h_a}^2$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $H_a = \neg H_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $T_a = \neg T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $H_a = \neg H_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 19 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.2 Proving $T_a = T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 19 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 251

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 251: Given a point  $C$ , a point  $H_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $H_a$  are not the same;
2. Using the point  $C$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $T_b$  are not the same;
3. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
4. Using the line  $h_a$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $h_a$  and  $b$  are not parallel % DET: lines  $h_a$  and  $b$  are not the same;
5. Using the point  $A$ , the point  $C$ , the point  $T_b$  and the line  $b$ , construct a point  $T'_b$  (rule W19); % NDG: points  $A$  and  $C$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $\text{midpoint}([A, T_b])$  are not the same;
6. Using the point  $T_b$  and the point  $T'_b$ , construct a circle  $k_{over}(T_b, T'_b)$  (rule W09); % NDG: points  $T_b$  and  $T'_b$  are not the same;
7. Using the circle  $k_{over}(T_b, T'_b)$  and the line  $a$ , construct a point  $B_{wc}$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k_{over}(T_b, T'_b)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k_{over}(T_b, T'_b)$  intersect; points  $T_b$  and  $T'_b$  are not the same; points  $A$  and  $C$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $\text{midpoint}([A, T_b])$  are not the same; lines  $h_a$  and  $b$  are not parallel.

Determination conditions: lines  $h_a$  and  $b$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $H_a$  are not the same.

Rules used: [W02, W03, W04, W09, W10b, W19]

Lemmas used: [D24, D5, D56, D8, GD01, L73, L78]

Solving time: 2282.5 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{a} 80 40
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_l C
cmark_r H_{a}
cmark_t T_{b}
color 0 0 0
fontsize 8

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% DET: points C and T_{b} are not the same
% Constructing a line b which passes through point C and point T_{b}
line b C T_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: lines h_{a} and b are not parallel% DET: lines h_{a} and b are not the same
% Constructing a point A which belongs to line h_{a} and line b
intersec A h_{a} b
cmark_t A

% NDG: points A and C are not the same; points C and T_{b} are not the same; points C and midpoint
% ([A,T_{b}]) are not the same
```

```

% Constructing a point  $P_{\{b\}}$  which is an image of the point  $C$  in a rotation around the point
 $T_{\{b\}}$  for the angle 90
rotate P_{\_G42039} T_{\{b\}} 90 C
cmark_r P_{\_G42039}
color 200 200 200
drawarc_p T_{\{b\}} C 90
color 0 0 0

% Constructing a line  $L_{\{b\}}$  which passes through point  $T_{\{b\}}$  and point  $P_{\{b\}}$ 
line L_{\_G42042} T_{\{b\}} P_{\_G42039}

color 200 200 200
drawline L_{\_G42042}
color 0 0 0

% Constructing midpoint  $P_{\{c\}}$  of the segment  $CP_{\{b\}}$ 
midpoint P_{\_G42045} C P_{\_G42039}
cmark_r P_{\_G42045}

% Constructing a line  $L_{\{c\}}$  which passes through point  $A$  and point  $P_{\{c\}}$ 
line L_{\_G42048} A P_{\_G42045}

color 200 200 200
drawline L_{\_G42048}
color 0 0 0

% Constructing a line  $L_{\{d\}}$  which passes through point  $C$  and point  $P_{\{c\}}$ 
line L_{\_G42051} C P_{\_G42045}

color 200 200 200
drawline L_{\_G42051}
color 0 0 0

% Constructing a point  $P_{\{d\}}$  which belongs to line  $L_{\{b\}}$  and line  $L_{\{c\}}$ 
intersec P_{\_G42054} L_{\_G42042} L_{\_G42048}
cmark_r P_{\_G42054}

% Constructing a point  $P_{\{e\}}$  which belongs to line  $L_{\{b\}}$  and line  $L_{\{d\}}$ 
intersec P_{\_G42057} L_{\_G42042} L_{\_G42051}
cmark_r P_{\_G42057}

% Constructing a line  $L_{\{e\}}$  which passes through point  $A$  and point  $P_{\{e\}}$ 
line L_{\_G42060} A P_{\_G42057}

color 200 200 200
drawline L_{\_G42060}
color 0 0 0

% Constructing a line  $L_{\{f\}}$  which passes through point  $C$  and point  $P_{\{d\}}$ 
line L_{\_G42063} C P_{\_G42054}

color 200 200 200

```

```

drawline L_{\_G42063}
color 0 0 0

% Constructing a point P_{\_G42066} which belongs to line L_{\_G42060} and line L_{\_G42063}
intersec P_{\_G42066} L_{\_G42060} L_{\_G42063}
cmark_r P_{\_G42066}

% Constructing a line L_{\_G42069} which passes through point P_{\_G42045} and point P_{\_G42066}
line L_{\_G42069} P_{\_G42045} P_{\_G42066}

color 200 200 200
drawline L_{\_G42069}
color 0 0 0

% Constructing a point T'_{b} which belongs to line L_{\_G42069} and line b
intersec T'_{b} L_{\_G42069} b
cmark_r T'_{b}

% NDG: points T_{b} and T'_{b} are not the same
% Constructing midpoint P_{\_G42900} of the segment T_{b}T'_{b}
midpoint P_{\_G42900} T_{b} T'_{b}
cmark_r P_{\_G42900}

% Constructing a circle k_{over}(T_{b},T'_{b}) whose center is at point P_{\_G42900} and which passes
    through point T_{b}
circle k_{over}(T_{b},T'_{b}) P_{\_G42900} T_{b}

color 200 200 200
drawcircle k_{over}(T_{b},T'_{b})
color 0 0 0

% NDG: line a and circle k_{over}(T_{b},T'_{b}) intersect
% Constructing points B_{wc} and B which are in intersection of k_{over}(T_{b},T'_{b}) and a
intersec2 B_{wc} B k_{over}(T_{b},T'_{b}) a
cmark_r B_{wc}
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k_{over}(T_{b},T'_{b}) intersect; points T_{b} and T'_{b}
    are not the same; points A and C are not the same; points C and T_{b} are not the same;
    points C and midpoint([A,T_{b}]) are not the same; lines h_{a} and b are not parallel
% Determination conditions: lines h_{a} and b are not the same; points C and T_{b} are not the same
    ; points C and H_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

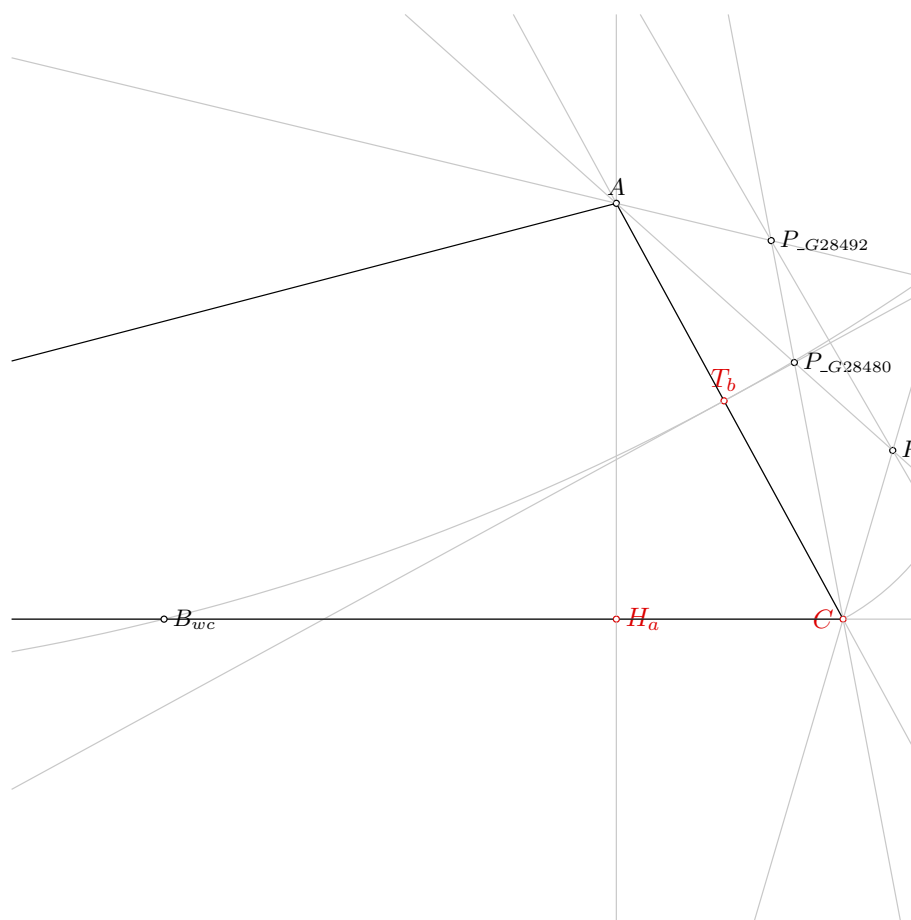


Figure 1: Illustration of the problem 0251

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.024 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a=_H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 5 terms.

**Time Complexity:** Time spent by the prover is 0.144 seconds.

**NDG conditions** Points  $C$ ,  $T_b$  and  $H_a$  are not collinear

Point  $T_b$  is not the midpoint of segment with endpoints  $A$  and  $C$

Points  $A$  and  $B$  are not identical

Point  $B$  is not on circle with center  $H_a$  and point from it  $C$

#### 4.1.3 Proving $T_b=_T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_aCT_b} \neq S_{T_{h_a}^0CT_b}$  i.e., lines  $H_aT_{h_a}^0$  and  $CT_b$  are not parallel (construction based assumption)

$S_{T_bAP_{G30293}} \neq S_{P_{G30287}AP_{G30293}}$  i.e., lines  $T_bP_{G30287}$  and  $AP_{G30293}$  are not parallel (construction based assumption)

$S_{T_bCP_{G30293}} \neq S_{P_{G30287}CP_{G30293}}$  i.e., lines  $T_bP_{G30287}$  and  $CP_{G30293}$  are not parallel (construction based assumption)

$S_{ACP_{G30302}} \neq S_{P_{G30305}CP_{G30302}}$  i.e., lines  $AP_{G30305}$  and  $CP_{G30302}$  are not parallel (construction based assumption)

$S_{P_{G30293}CT_b} \neq S_{P_{G30314}CT_b}$  i.e., lines  $P_{G30293}P_{G30314}$  and  $CT_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^1} \neq S_{CAF_{h_a}^1}$  i.e., lines  $BC$  and  $AF_{h_a}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a=_H_a$

Proving failed

#### 4.2.3 Proving $T_b=_T_b$

Proving failed



### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $C=C$**

Proving failed

#### **4.3.2 Proving $H_a=_H H_a$**

Proving failed

#### **4.3.3 Proving $T_b=_T T_b$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $C=C$**

Proving failed

#### **4.4.2 Proving $H_a=_H H_a$**

Proving failed

#### **4.4.3 Proving $T_b=_T T_b$**

Proving failed

# Problem 252

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 252: Given a point  $C$ , a point  $H_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $H_a$  are not the same;
2. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
3. Using the point  $T_c$  and the line  $a$ , construct a circle  $k(T_c, foot[T_c, a])$  (rule W11); % NDG: point  $T_c$  is not incident to the line  $a$ ;
4. Using the circle  $k(T_c, foot[T_c, a])$ , the point  $C$ , the point  $T_c$  and the line  $a$ , construct a line  $b$  (rule W13); % NDG: point  $C$  is outside the circle  $k(T_c, foot[T_c, a])$ ;
5. Using the line  $b$  and the line  $h_a$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $h_a$  are not parallel % DET: lines  $b$  and  $h_a$  are not the same;
6. Using the point  $T_c$  and the point  $A$ , construct a line  $c$  (rule W02); % DET: points  $T_c$  and  $A$  are not the same;
7. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; lines  $b$  and  $h_a$  are not parallel; point  $C$  is outside the circle  $k(T_c, foot[T_c, a])$ ; point  $T_c$  is not incident to the line  $a$ .

Determination conditions: lines  $a$  and  $c$  are not the same; points  $T_c$  and  $A$  are not the same; lines  $b$  and  $h_a$  are not the same; points  $C$  and  $H_a$  are not the same.

Rules used: [W02,W03,W10b,W11,W13]

Lemmas used: [D25,D5,D8,GD01,GD02,GL10,GL11,L10]

Solving time: 0.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{a} 80 40
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_l C
cmark_r H_{a}
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: point T_{c} is not incident to the line a
% Constructing a point P_{\_G63520} which is a foot of the point T_{c} on the line a
foot P_{\_G63520} T_{c} a
cmark_r P_{\_G63520}
color 200 200 200
drawline T_{c} P_{\_G63520}
color 0 0 0

% Constructing a circle k(T_{c},foot[T_{c},b]) whose center is at point T_{c} and which passes
    through point P_{\_G63520}
circle k(T_{c},foot[T_{c},b]) T_{c} P_{\_G63520}

color 200 200 200
drawcircle k(T_{c},foot[T_{c},b])
color 0 0 0

% NDG: point C is outside the circle k(T_{c},foot[T_{c},b])
```

```

% Constructing a point  $P_{\{c\}}$  which is a foot of the point  $T_{\{c\}}$  on the line a
foot  $P_{\{c\}}$   $T_{\{c\}}$  a
cmark_r  $P_{\{c\}}$ 
color 200 200 200
drawline  $T_{\{c\}}$   $P_{\{c\}}$ 
color 0 0 0

% Constructing a line  $L_{\{c\}}$  which passes through point  $T_{\{c\}}$  and point C
line  $L_{\{c\}}$   $T_{\{c\}}$  C

color 200 200 200
drawline  $L_{\{c\}}$ 
color 0 0 0

% Constructing a point  $P_{\{c\}}$  which is an image of the point  $P_{\{a\}}$  in the symmetry to
point/line  $L_{\{c\}}$ 
sim  $P_{\{c\}}$   $L_{\{c\}}$   $P_{\{a\}}$ 
cmark_r  $P_{\{c\}}$ 

% Constructing a line b which passes through point C and point  $P_{\{c\}}$ 
line b C  $P_{\{c\}}$ 

color 200 200 200
drawline b
color 0 0 0

% NDG: lines b and  $h_{\{a\}}$  are not parallel% DET: lines b and  $h_{\{a\}}$  are not the same
% Constructing a point A which belongs to line b and line  $h_{\{a\}}$ 
intersec A b  $h_{\{a\}}$ 
cmark_t A

% DET: points  $T_{\{c\}}$  and A are not the same
% Constructing a line c which passes through point  $T_{\{c\}}$  and point A
line c  $T_{\{c\}}$  A

color 200 200 200
drawline c
color 0 0 0

% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```

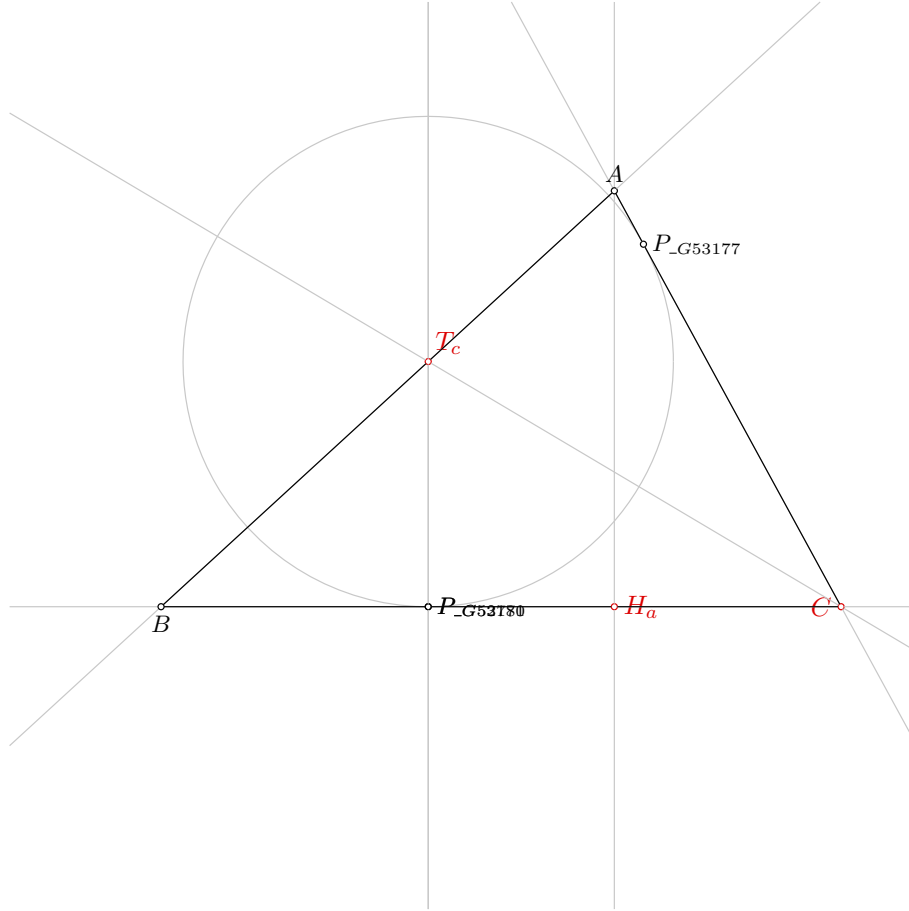


Figure 1: Illustration of the problem 0252

*% Non-degenerate conditions: lines a and c are not parallel; lines b and h\_{a} are not parallel;  
point C is outside the circle k(T\_{c},foot[T\_{c},b]); point T\_{c} is not incident to the line a  
% Determination conditions: lines a and c are not the same; points T\_{c} and A are not the same;  
lines b and h\_{a} are not the same; points C and H\_{a} are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.015 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a = H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.139 seconds.

**NDG conditions** Points  $B$  and  $C$  are not identical

Points  $B$  and  $C$  are not identical

Points  $C$ ,  $P_{G60495}$  and  $H_a$  are not collinear

Line through points  $P_{G60489}$  and  $T_c$  is not perpendicular to line through points  $T_c$  and  $A$

Points  $P_{G60340}$  and  $B$  are not identical

#### 4.1.3 Proving $T_c = T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C = C$

NDG conditions are:

$S_{CH_aT_{h_a}^0} \neq S_{P_{G54243}H_aT_{h_a}^0}$  i.e., lines  $CP_{G54243}$  and  $H_aT_{h_a}^0$  are not parallel (construction based assumption)

$S_{CT_cA} \neq S_{H_aT_cA}$  i.e., lines  $CH_a$  and  $T_cA$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^1} \neq S_{CAF_{h_a}^1}$  i.e., lines  $BC$  and  $AF_{h_a}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a = H_a$

Proving failed

#### 4.2.3 Proving $T_c = T_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_a = H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_c = \neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 67 terms.

**Time Complexity:** Time spent by the prover is 0.040 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_c = \neg T_c$

Proving failed

# Problem 253

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 253: Given a point  $C$ , a point  $H_a$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $H_a$  are not the same;
2. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
3. Using the point  $I$  and the line  $a$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $a$ ;
4. Using the circle  $k(I, P_a)$ , the point  $C$ , the point  $I$  and the line  $a$ , construct a line  $b$  (rule W13); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
5. Using the line  $b$  and the line  $h_a$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $h_a$  are not parallel % DET: lines  $b$  and  $h_a$  are not the same;
6. Using the circle  $k(I, P_a)$ , the point  $A$ , the point  $I$  and the line  $b$ , construct a line  $c$  (rule W13); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
7. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(I, P_a)$ ; lines  $b$  and  $h_a$  are not parallel; point  $C$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $a$ .

Determination conditions: lines  $c$  and  $a$  are not the same; lines  $b$  and  $h_a$  are not the same; points  $C$  and  $H_a$  are not the same.

Rules used: [W02,W03,W10b,W11,W13]

Lemmas used: [D27,D5,D8,GD01,L59,L60,L61]

Solving time: 12.5 seconds.



## 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{a} 80 40
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_l C
cmark_r H_{a}
cmark_b I
color 0 0 0
fontsize 8

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: point I is not incident to the line a
% Constructing a point P_{\_G48248} which is a foot of the point I on the line a
foot P_{\_G48248} I a
cmark_r P_{\_G48248}
color 200 200 200
drawline I P_{\_G48248}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
48248}
circle k(I,P_{a}) I P_{\_G48248}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point C is outside the circle k(I,P_{a})
```

```

% Constructing a point  $P_{\{G48455\}}$  which is a foot of the point  $I$  on the line  $a$ 
foot  $P_{\{G48455\}}$  I a
cmark_r  $P_{\{G48455\}}$ 
color 200 200 200
drawline I  $P_{\{G48455\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G48458\}}$  which passes through point  $I$  and point  $C$ 
line  $L_{\{G48458\}}$  I C

color 200 200 200
drawline  $L_{\{G48458\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G48461\}}$  which is an image of the point  $P_{\{G48455\}}$  in the symmetry to
point/line  $L_{\{G48458\}}$ 
sim  $P_{\{G48461\}}$   $L_{\{G48458\}}$   $P_{\{G48455\}}$ 
cmark_r  $P_{\{G48461\}}$ 

% Constructing a line  $b$  which passes through point  $C$  and point  $P_{\{G48461\}}$ 
line b C  $P_{\{G48461\}}$ 

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $b$  and  $h_{\{a\}}$  are not parallel% DET: lines  $b$  and  $h_{\{a\}}$  are not the same
% Constructing a point  $A$  which belongs to line  $b$  and line  $h_{\{a\}}$ 
intersec A b  $h_{\{a\}}$ 
cmark_t A

% NDG: point  $A$  is outside the circle  $k(I, P_{\{a\}})$ 
% Constructing a point  $P_{\{G48847\}}$  which is a foot of the point  $I$  on the line  $b$ 
foot  $P_{\{G48847\}}$  I b
cmark_r  $P_{\{G48847\}}$ 
color 200 200 200
drawline I  $P_{\{G48847\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G48850\}}$  which passes through point  $I$  and point  $A$ 
line  $L_{\{G48850\}}$  I A

color 200 200 200
drawline  $L_{\{G48850\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G48853\}}$  which is an image of the point  $P_{\{G48847\}}$  in the symmetry to
point/line  $L_{\{G48850\}}$ 
sim  $P_{\{G48853\}}$   $L_{\{G48850\}}$   $P_{\{G48847\}}$ 
cmark_r  $P_{\{G48853\}}$ 

```

```

% Constructing a line c which passes through point A and point P_{G48853}
line c A P_{G48853}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and a are not parallel; point A is outside the circle k(I,P_{a}
% ); lines b and h_{a} are not parallel; point C is outside the circle k(I,P_{a}); point I is
% not incident to the line a
% Determination conditions: lines c and a are not the same; lines b and h_{a} are not the same;
% points C and H_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.015 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a=H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 10 terms.

**Time Complexity:** Time spent by the prover is 0.214 seconds.

**NDG conditions** Points  $I$  and  $H_a$  are not identical

Points  $I$  and  $H_a$  are not identical

Points  $P_{G44667}$ ,  $C$  and  $H_a$  are not collinear

Points  $P_{G44667}$  and  $C$  are not identical

Points  $A$  and  $I$  are not identical

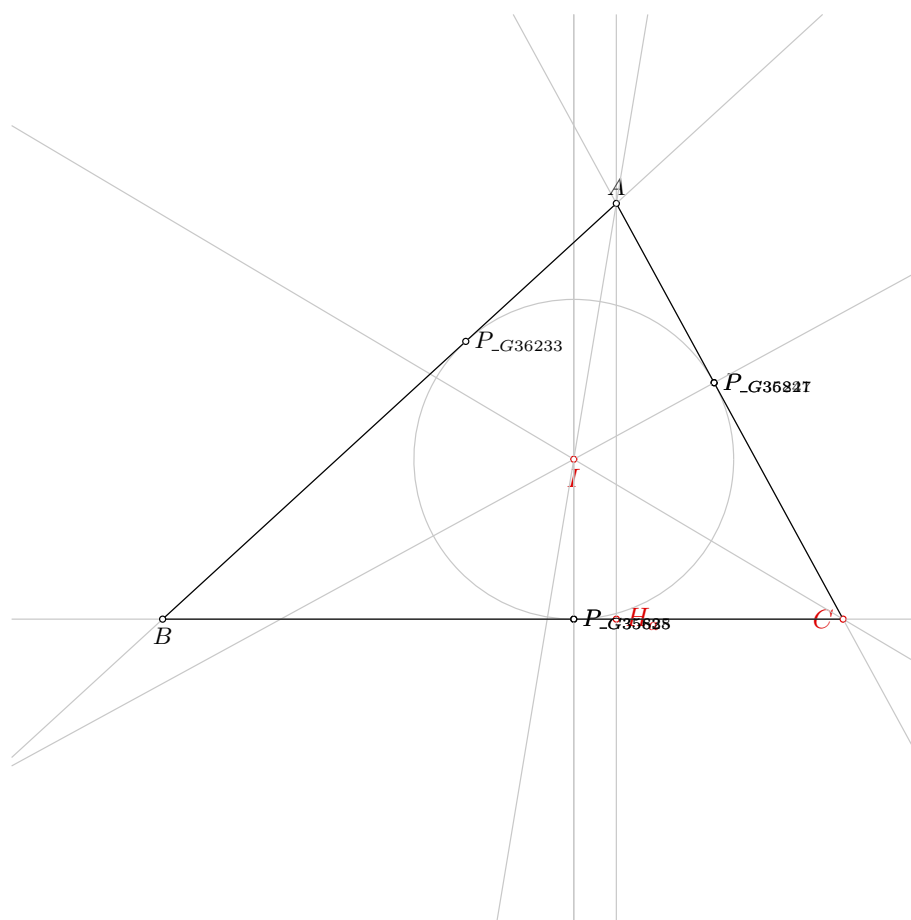


Figure 1: Illustration of the problem 0253

Points  $I$  and  $P_{G44661}$  are not identical

Line through points  $A$  and  $P_{G44829}$  is not parallel with line through points  $C$  and  $H_a$

Point  $B$  is not on circle with center  $H_a$  and point from it  $C$

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{CH_aT_{h_a}^0} \neq S_{P_{G36961}H_aT_{h_a}^0}$  i.e., lines  $CP_{G36961}$  and  $H_aT_{h_a}^0$  are not parallel (construction based assumption)

$S_{ACH_a} \neq S_{P_{G37353}CH_a}$  i.e., lines  $AP_{G37353}$  and  $CH_a$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^1} \neq S_{CAF_{h_a}^1}$  i.e., lines  $BC$  and  $AF_{h_a}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a=_H H_a$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_a=_H H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I=_I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 425 terms.

**Time Complexity:** Time spent by the prover is 0.290 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $I = \neg I$

Proving failed

# Problem 254

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 254: Given a point  $C$ , a point  $H_b$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $H_b$  are not the same;
2. Using the point  $C$  and the point  $H_c$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H_c$  are not the same;
3. Using the point  $H_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
4. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same;
5. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
6. Using the line  $c$  and the line  $h_b$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $h_b$  are not parallel % DET: lines  $c$  and  $h_b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $h_b$  are not parallel; lines  $c$  and  $b$  are not parallel.

Determination conditions: lines  $c$  and  $h_b$  are not the same; lines  $c$  and  $b$  are not the same; points  $C$  and  $H_c$  are not the same; points  $C$  and  $H_b$  are not the same.

Rules used: [W02,W03,W10a,W10b]

Lemmas used: [D10,D6,D7,D9,GD01]

Solving time: 1.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{b} 89.36 77.83
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_l C
cmark_l H_{b}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points C and H_{c} are not the same
% Constructing a line h_{c} which passes through point C and point H_{c}
line h_{c} C H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b
```



```

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines c and h_{b} are not parallel% DET: lines c and h_{b} are not the same
% Constructing a point B which belongs to line c and line h_{b}
intersec B c h_{b}
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and h_{b} are not parallel; lines c and b are not parallel
% Determination conditions: lines c and h_{b} are not the same; lines c and b are not the same;
  points C and H_{c} are not the same; points C and H_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b=_H H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.074 seconds.

**NDG conditions** Points  $H_b$  and  $H_c$  are not identical

Points  $H_b$  and  $H_c$  are not identical

Points  $A$  and  $C$  are not identical

#### 4.1.3 Proving $H_c=_H H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 20 terms.

**Time Complexity:** Time spent by the prover is 0.355 seconds.

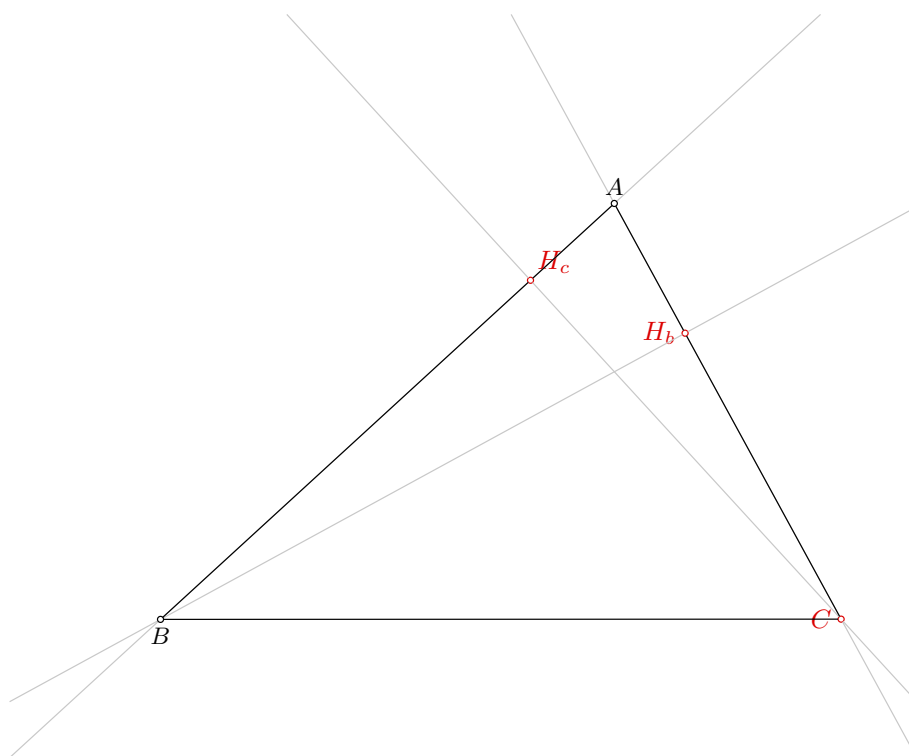


Figure 1: Illustration of the problem 0254

**NDG conditions** Points  $H_b$  and  $H_c$  are not identical

Points  $H_b$  and  $H_c$  are not identical

Points  $A$  and  $B$  are not identical

Points  $H_b$ ,  $B$  and  $C$  are not collinear

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_cCH_b} \neq S_{T_c^0CH_b}$  i.e., lines  $H_cT_c^0$  and  $CH_b$  are not parallel (construction based assumption)

$S_{H_cH_bT_{h_b}^1} \neq S_{T_c^0H_bT_{h_b}^1}$  i.e., lines  $H_cT_c^0$  and  $H_bT_{h_b}^1$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^2} \neq S_{CBF_{h_b}^2}$  i.e., lines  $AC$  and  $BF_{h_b}^2$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^3} \neq S_{BCF_{h_c}^3}$  i.e., lines  $AB$  and  $CF_{h_c}^3$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $H_b=_H H_b$

Proving failed

### 4.2.3 Proving $H_c=_H H_c$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_b=_H H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $H_c=_H H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 91 terms.

**Time Complexity:** Time spent by the prover is 0.130 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_b = H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $H_c = H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 176 terms.

**Time Complexity:** Time spent by the prover is 0.740 seconds. There are no ndg conditions.

# Problem 255

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 255: Given a point  $H_b$ , a point  $H$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $H_b$  and  $H$  are not the same;
2. Using the point  $H_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
3. Choose freely a point  $C$  on the line  $b$  (rule WOnline2);
4. Using the point  $C$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H$  are not the same;
5. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) ;
6. Using the point  $A$  and the point  $C$ , construct a point  $M_b$  (rule W01); ;
7. Using the point  $A$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $A$  and  $M_b$  are not the same;
8. Using the circle  $k(M_b, C)$ , the line  $h_c$ , the point  $M_b$  and the point  $C$ , construct a point  $H_c$  (rule W05); % NDG: line  $h_c$  and circle  $k(M_b, C)$  intersect % DET: points  $C$  and  $H_c$  must be different;
9. Using the point  $H_c$  and the point  $A$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $A$  are not the same;
10. Using the line  $h_b$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $h_b$  and  $c$  are not parallel % DET: lines  $h_b$  and  $c$  are not the same.

Non-degenerate conditions: lines  $h_b$  and  $c$  are not parallel; line  $h_c$  and circle  $k(M_b, C)$  intersect; points  $A$  and  $M_b$  are not the same.

Determination conditions: lines  $h_b$  and  $c$  are not the same; points  $H_c$  and  $A$  are not the same; points  $C$  and  $H_c$  must be different; points  $C$  and  $H$  are not the same; points  $H_b$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D10,D22,D3,D6,D7,D9,GD01,GD02,GL04,L3,L43,L45]

Solving time: 23.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{b} 89.36 77.83
```

```
point H 80 72.73
```

```
point C 110 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_l H_{b}
```

```
cmark_rt H
```

```
cmark_l C
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{b} and H are not the same
```

```
% Constructing a line h_{b} which passes through point H_{b} and point H
```

```
line h_{b} H_{b} H
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```
% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
```

```
perp b H_{b} h_{b}
```

```
color 200 200 200
```

```
drawline b
```

```
color 0 0 0
```

```
% Generating random value V[_G47419]
```

```
random V[_G47419]
```

```
% Calculating value V[_G47440] using formula V[_G47419]*20
```

```
expression V[_G47440] { V[_G47419]*20 }
```

```

% Constructing a point C which is a point for which holds  $H_{\{b\}}C = V[_G47440]$  and angle  $HH_{\{b\}}C = 90$ 
turtle C H  $H_{\{b\}}$  90  $V[_G47440]$ 
cmark_l C

% DET: points C and H are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point C and point H
line  $h_{\{c\}}$  C H

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% Choosing randomly a point A on the line  $CH_{\{b\}}$ 
online A C  $H_{\{b\}}$ 
cmark_t A
color 200 200 200
drawline C  $H_{\{b\}}$ 
color 0 0 0

% Constructing a point  $M_{\{b\}}$  such that  $AM_{\{b\}}/AC=0.5$ 
towards  $M_{\{b\}}$  A C 0.5
cmark_lt  $M_{\{b\}}$ 
color 200 200 200
drawsegment A C
color 0 0 0

% NDG: points A and  $M_{\{b\}}$  are not the same
% Constructing a circle  $k(M_{\{b\}},C)$  whose center is at point  $M_{\{b\}}$  and which passes through point A
circle  $k(M_{\{b\}},C)$   $M_{\{b\}}$  A

color 200 200 200
drawcircle  $k(M_{\{b\}},C)$ 
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(M_{\{b\}},C)$  intersect% DET: points C and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{G47969\}}$  which is a foot of the point  $M_{\{b\}}$  on the line  $h_{\{c\}}$ 
foot  $P_{\{G47969\}}$   $M_{\{b\}}$   $h_{\{c\}}$ 
cmark_r  $P_{\{G47969\}}$ 
color 200 200 200
drawline  $M_{\{b\}}$   $P_{\{G47969\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point C in the symmetry to point/line  $P_{\{G47969\}}$ 
sim  $H_{\{c\}}$   $P_{\{G47969\}}$  C

```

```

cmark_rt H_{c}

% DET: points H_{c} and A are not the same
% Constructing a line c which passes through point H_{c} and point A
line c H_{c} A

color 200 200 200
drawline c
color 0 0 0

% NDG: lines h_{b} and c are not parallel% DET: lines h_{b} and c are not the same
% Constructing a point B which belongs to line h_{b} and line c
intersec B h_{b} c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{b} and c are not parallel; line h_{c} and circle k(M_{b},C)
% intersect; points A and M_{b} are not the same
% Determination conditions: lines h_{b} and c are not the same; points H_{c} and A are not the same
% ; points C and H_{c} must be different; points C and H are not the same; points H_{b} and H are
% not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_b = H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.074 seconds.

**NDG conditions** Points  $H_b$  and  $C$  are not identical

Points  $B$  and  $H$  are not identical

Line through points  $H_b$  and  $A$  is not perpendicular to line through points  $A$  and  $C$

#### 4.1.2 Proving $H = H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.184 seconds.



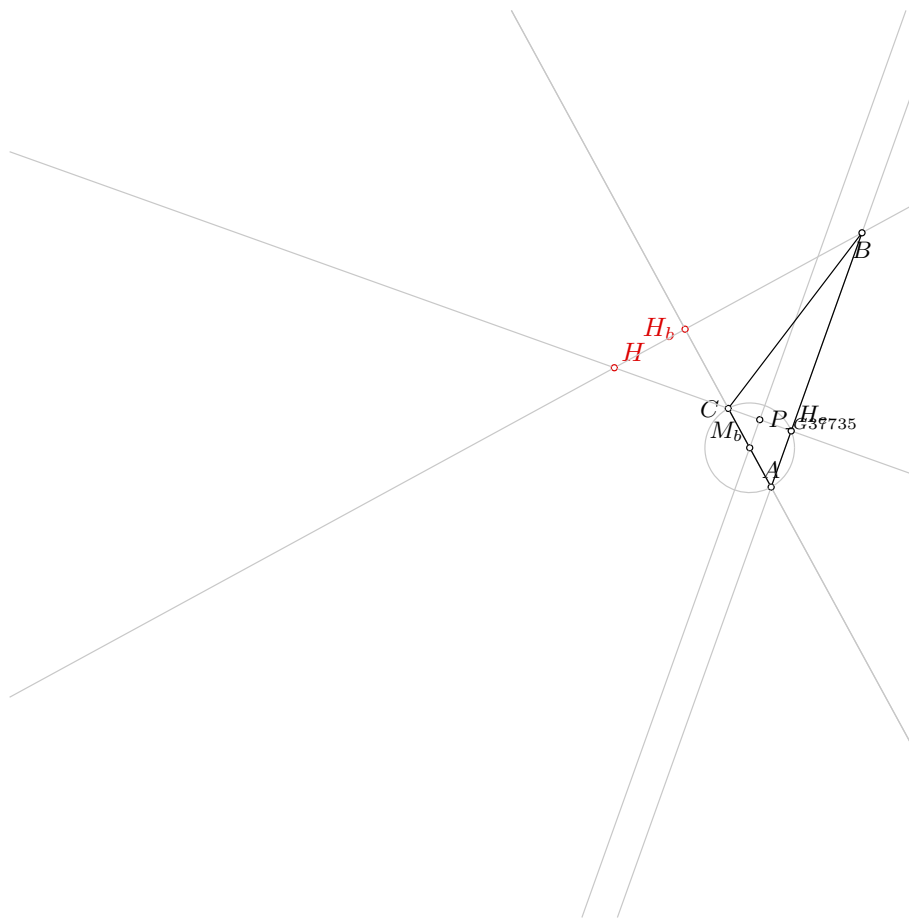


Figure 1: Illustration of the problem 0255

**NDG conditions** Points  $C$  and  $H$  are not identical

Points  $B$  and  $H$  are not identical

Line through points  $H_b$  and  $A$  is not perpendicular to line through points  $A$  and  $C$

Points  $H_b$ ,  $B$  and  $C$  are not collinear

#### 4.1.3 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_b=_H H_b$

Proving failed

#### 4.2.2 Proving $H=_H H$

Proving failed

#### 4.2.3 Proving $C=C$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_b=_H H_b$

Proving failed

#### 4.3.2 Proving $H=_H H$

Proving failed

#### 4.3.3 Proving $C=C$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $H_b=_H H_b$

Proving failed

#### 4.4.2 Proving $H=_H H$

Proving failed

#### 4.4.3 Proving $C=C$

Proving failed

# Problem 256

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 256: Given a point  $C$ , a point  $H_b$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $H_b$  are not the same;
2. Using the point  $C$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $T_a$  are not the same;
3. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
4. Using the line  $h_b$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $h_b$  and  $a$  are not parallel % DET: lines  $h_b$  and  $a$  are not the same;
5. Using the point  $B$ , the point  $C$ , the point  $T_a$  and the line  $a$ , construct a point  $T'_a$  (rule W19); % NDG: points  $B$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $midpoint([B, T_a])$  are not the same;
6. Using the point  $T_a$  and the point  $T'_a$ , construct a circle  $k_{over}(T_a, T'_a)$  (rule W09); % NDG: points  $T_a$  and  $T'_a$  are not the same;
7. Using the circle  $k_{over}(T_a, T'_a)$  and the line  $b$ , construct a point  $A_{wc}$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k_{over}(T_a, T'_a)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k_{over}(T_a, T'_a)$  intersect; points  $T_a$  and  $T'_a$  are not the same; points  $B$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $midpoint([B, T_a])$  are not the same; lines  $h_b$  and  $a$  are not parallel.

Determination conditions: lines  $h_b$  and  $a$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $H_b$  are not the same.

Rules used: [W02,W03,W04,W09,W10b,W19]

Lemmas used: [D23,D54,D6,D9,GD01,L71,L77]

Solving time: 2288.8 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{b} 89.36 77.83
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_l C
cmark_l H_{b}
cmark_rb T_{a}
color 0 0 0
fontsize 8

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points C and T_{a} are not the same
% Constructing a line a which passes through point C and point T_{a}
line a C T_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines h_{b} and a are not parallel% DET: lines h_{b} and a are not the same
% Constructing a point B which belongs to line h_{b} and line a
intersec B h_{b} a
cmark_b B

% NDG: points B and C are not the same; points C and T_{a} are not the same; points C and midpoint
% ([B,T_{a}]) are not the same
```

```

% Constructing a point  $P_{\backslash\_G39145}$  which is an image of the point  $C$  in a rotation around the point
 $T_{\{a\}}$  for the angle 90
rotate  $P_{\backslash\_G39145}$   $T_{\{a\}}$  90  $C$ 
cmark_r  $P_{\backslash\_G39145}$ 
color 200 200 200
drawarc_p  $T_{\{a\}}$   $C$  90
color 0 0 0

% Constructing a line  $L_{\backslash\_G39148}$  which passes through point  $T_{\{a\}}$  and point  $P_{\backslash\_G39145}$ 
line  $L_{\backslash\_G39148}$   $T_{\{a\}}$   $P_{\backslash\_G39145}$ 

color 200 200 200
drawline  $L_{\backslash\_G39148}$ 
color 0 0 0

% Constructing midpoint  $P_{\backslash\_G39151}$  of the segment  $CP_{\backslash\_G39145}$ 
midpoint  $P_{\backslash\_G39151}$   $C$   $P_{\backslash\_G39145}$ 
cmark_r  $P_{\backslash\_G39151}$ 

% Constructing a line  $L_{\backslash\_G39154}$  which passes through point  $B$  and point  $P_{\backslash\_G39151}$ 
line  $L_{\backslash\_G39154}$   $B$   $P_{\backslash\_G39151}$ 

color 200 200 200
drawline  $L_{\backslash\_G39154}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G39157}$  which passes through point  $C$  and point  $P_{\backslash\_G39151}$ 
line  $L_{\backslash\_G39157}$   $C$   $P_{\backslash\_G39151}$ 

color 200 200 200
drawline  $L_{\backslash\_G39157}$ 
color 0 0 0

% Constructing a point  $P_{\backslash\_G39160}$  which belongs to line  $L_{\backslash\_G39148}$  and line  $L_{\backslash\_G39154}$ 
intersec  $P_{\backslash\_G39160}$   $L_{\backslash\_G39148}$   $L_{\backslash\_G39154}$ 
cmark_r  $P_{\backslash\_G39160}$ 

% Constructing a point  $P_{\backslash\_G39163}$  which belongs to line  $L_{\backslash\_G39148}$  and line  $L_{\backslash\_G39157}$ 
intersec  $P_{\backslash\_G39163}$   $L_{\backslash\_G39148}$   $L_{\backslash\_G39157}$ 
cmark_r  $P_{\backslash\_G39163}$ 

% Constructing a line  $L_{\backslash\_G39166}$  which passes through point  $B$  and point  $P_{\backslash\_G39163}$ 
line  $L_{\backslash\_G39166}$   $B$   $P_{\backslash\_G39163}$ 

color 200 200 200
drawline  $L_{\backslash\_G39166}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G39169}$  which passes through point  $C$  and point  $P_{\backslash\_G39160}$ 
line  $L_{\backslash\_G39169}$   $C$   $P_{\backslash\_G39160}$ 

color 200 200 200

```

```

drawline L_{\_G39169}
color 0 0 0

% Constructing a point P_{\_G39172} which belongs to line L_{\_G39166} and line L_{\_G39169}
intersec P_{\_G39172} L_{\_G39166} L_{\_G39169}
cmark_r P_{\_G39172}

% Constructing a line L_{\_G39175} which passes through point P_{\_G39151} and point P_{\_G39172}
line L_{\_G39175} P_{\_G39151} P_{\_G39172}

color 200 200 200
drawline L_{\_G39175}
color 0 0 0

% Constructing a point T'_{a} which belongs to line L_{\_G39175} and line a
intersec T'_{a} L_{\_G39175} a
cmark_r T'_{a}

% NDG: points T_{a} and T'_{a} are not the same
% Constructing midpoint P_{\_G40006} of the segment T_{a}T'_{a}
midpoint P_{\_G40006} T_{a} T'_{a}
cmark_r P_{\_G40006}

% Constructing a circle k_{over}(T_{a},T'_{a}) whose center is at point P_{\_G40006} and which passes
    through point T_{a}
circle k_{over}(T_{a},T'_{a}) P_{\_G40006} T_{a}

color 200 200 200
drawcircle k_{over}(T_{a},T'_{a})
color 0 0 0

% NDG: line b and circle k_{over}(T_{a},T'_{a}) intersect
% Constructing points A_{wc} and A which are in intersection of k_{over}(T_{a},T'_{a}) and b
intersec2 A_{wc} A k_{over}(T_{a},T'_{a}) b
cmark_r A_{wc}
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k_{over}(T_{a},T'_{a}) intersect; points T_{a} and T'_{a}
    are not the same; points B and C are not the same; points C and T_{a} are not the same;
    points C and midpoint([B,T_{a}]) are not the same; lines h_{b} and a are not parallel
% Determination conditions: lines h_{b} and a are not the same; points C and T_{a} are not the same
    ; points C and H_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

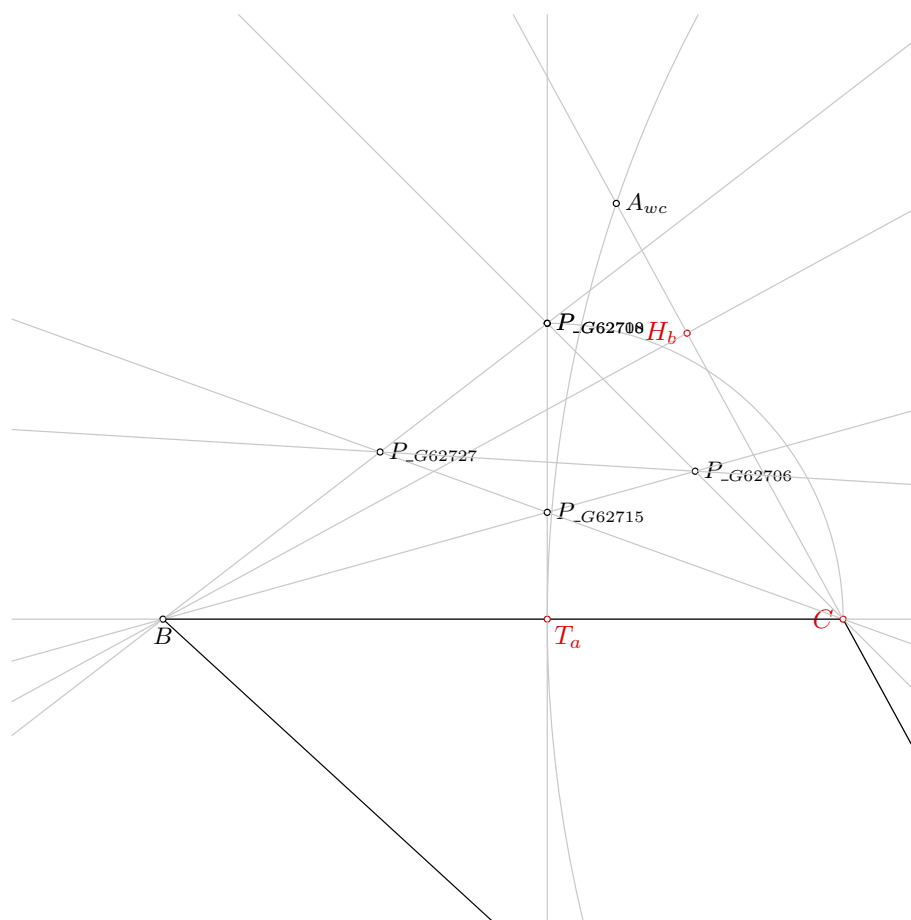


Figure 1: Illustration of the problem 0256

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.054 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b=_H H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 5 terms.

**Time Complexity:** Time spent by the prover is 0.145 seconds.

**NDG conditions** Points  $T_a$ ,  $H_b$  and  $C$  are not collinear

Point  $T_a$  is not the midpoint of segment with endpoints  $B$  and  $C$

Points  $T_a$ ,  $H_b$  and  $A$  are not collinear

Points  $A$  and  $C$  are not identical

#### 4.1.3 Proving $T_a=_T T_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

Proving failed

#### 4.2.2 Proving $H_b=_H H_b$

Proving failed

#### 4.2.3 Proving $T_a=_T T_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

Proving failed

#### 4.3.2 Proving $H_b=_H H_b$

Proving failed

#### 4.3.3 Proving $T_a=_T T_a$

Proving failed



## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

Proving failed

### 4.4.2 Proving $H_b=_H H_b$

Proving failed

### 4.4.3 Proving $T_a=_T T_a$

Proving failed

# Problem 257

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 257: Given a point  $H_b$ , a point  $T_b$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $T_b$  are not the same;
2. Choose freely a point  $C$  on the line  $b$  (rule WOnline1) ;
3. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
4. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) ;
5. Choose freely a point  $B$  on the line  $h_b$  (rule WOnline2).

Non-degenerate conditions: .

Determination conditions: points  $H_b$  and  $T_b$  are not the same.

Rules used: [W02,W10b,WOnline1,WOnline2]

Lemmas used: [D24,D6,D9]

Solving time: 1906.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point H_{b} 89.36 77.83
point T_{b} 94.25 68.88
point C 110 40

color 220 0 0
fontsize 9
```

```

cmark_l H_{b}
cmark_t T_{b}
cmark_l C
color 0 0 0
fontsize 8

% DET: points H_{b} and T_{b} are not the same
% Constructing a line b which passes through point H_{b} and point T_{b}
line b H_{b} T_{b}

color 200 200 200
drawline b
color 0 0 0

% Choosing randomly a point C on the line H_{b}T_{b}
online C H_{b} T_{b}
cmark_l C
color 200 200 200
drawline H_{b} T_{b}
color 0 0 0

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0

% Choosing randomly a point A on the line CH_{b}
online A C H_{b}
cmark_t A
color 200 200 200
drawline C H_{b}
color 0 0 0

% Generating random value V[_G56363]
random V[_G56363]

% Calculating value V[_G56384] using formula V[_G56363]*20
expression V[_G56384] { V[_G56363]*20 }

% Constructing a point B which is a point for which holds  $H_{b}B = V[_G56384]$  and angle  $AH_{b}B = 90$ 
turtle B A H_{b} 90 V[_G56384]

```

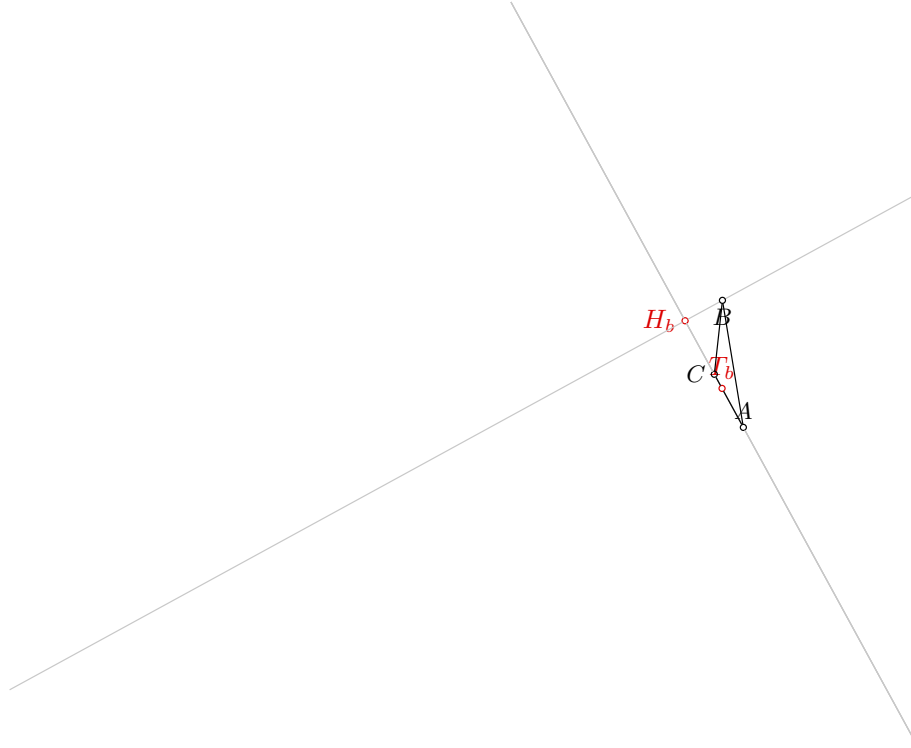


Figure 1: Illustration of the problem 0257

```
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

*% Non-degenerate conditions:*

*% Determination conditions: points  $H_{\{b\}}$  and  $T_{\{b\}}$  are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_b = \neg H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.012 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.1.3 Proving $C = C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.016 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_b = \neg H_b$

Proving failed

#### 4.2.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.2.3 Proving $C = C$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^3_{\neg h_b}} \neq S_{CBF^3_{\neg h_b}}$  i.e., lines  $AC$  and  $BF^3_{\neg h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_b = \neg H_b$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $T_b = \neg T_b$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $H_b=\neg H_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 31.450 seconds. There are no ndg conditions.

#### 4.4.2 Proving $T_b=\neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 31.530 seconds. There are no ndg conditions.

#### 4.4.3 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 258

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 258: Given a point  $C$ , a point  $H_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $H_b$  are not the same;
2. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
3. Using the point  $T_c$  and the line  $b$ , construct a circle  $k(T_c, foot[T_c, b])$  (rule W11); % NDG: point  $T_c$  is not incident to the line  $b$ ;
4. Using the circle  $k(T_c, foot[T_c, b])$ , the point  $C$ , the point  $T_c$  and the line  $b$ , construct a line  $a$  (rule W13); % NDG: point  $C$  is outside the circle  $k(T_c, foot[T_c, b])$ ;
5. Using the line  $a$  and the line  $h_b$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $h_b$  are not parallel % DET: lines  $a$  and  $h_b$  are not the same;
6. Using the point  $T_c$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $T_c$  and  $B$  are not the same;
7. Using the line  $b$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $c$  are not parallel % DET: lines  $b$  and  $c$  are not the same.

Non-degenerate conditions: lines  $b$  and  $c$  are not parallel; lines  $a$  and  $h_b$  are not parallel; point  $C$  is outside the circle  $k(T_c, foot[T_c, b])$ ; point  $T_c$  is not incident to the line  $b$ .

Determination conditions: lines  $b$  and  $c$  are not the same; points  $T_c$  and  $B$  are not the same; lines  $a$  and  $h_b$  are not the same; points  $C$  and  $H_b$  are not the same.

Rules used: [W02,W03,W10b,W11,W13]

Lemmas used: [D25,D6,D9,GD01,GD02,GL10,GL11,L10]

Solving time: 0.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{b} 89.36 77.83
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_l C
cmark_l H_{b}
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: point T_{c} is not incident to the line b
% Constructing a point P_{\_G38141} which is a foot of the point T_{c} on the line b
foot P_{\_G38141} T_{c} b
cmark_r P_{\_G38141}
color 200 200 200
drawline T_{c} P_{\_G38141}
color 0 0 0

% Constructing a circle k(T_{c},foot[T_{c},b]) whose center is at point T_{c} and which passes
through point P_{\_G38141}
circle k(T_{c},foot[T_{c},b]) T_{c} P_{\_G38141}

color 200 200 200
drawcircle k(T_{c},foot[T_{c},b])
color 0 0 0

% NDG: point C is outside the circle k(T_{c},foot[T_{c},b])
```



```

% Constructing a point  $P_{\{ \_G38532 \}}$  which is a foot of the point  $T_{\{c\}}$  on the line  $b$ 
foot  $P_{\{ \_G38532 \}}$   $T_{\{c\}}$   $b$ 
cmark_r  $P_{\{ \_G38532 \}}$ 
color 200 200 200
drawline  $T_{\{c\}}$   $P_{\{ \_G38532 \}}$ 
color 0 0 0

% Constructing a line  $L_{\{ \_G38535 \}}$  which passes through point  $T_{\{c\}}$  and point  $C$ 
line  $L_{\{ \_G38535 \}}$   $T_{\{c\}}$   $C$ 

color 200 200 200
drawline  $L_{\{ \_G38535 \}}$ 
color 0 0 0

% Constructing a point  $P_{\{ \_G38538 \}}$  which is an image of the point  $P_{\{ \_G38532 \}}$  in the symmetry to
point/line  $L_{\{ \_G38535 \}}$ 
sim  $P_{\{ \_G38538 \}}$   $L_{\{ \_G38535 \}}$   $P_{\{ \_G38532 \}}$ 
cmark_r  $P_{\{ \_G38538 \}}$ 

% Constructing a line  $a$  which passes through point  $C$  and point  $P_{\{ \_G38538 \}}$ 
line  $a$   $C$   $P_{\{ \_G38538 \}}$ 

color 200 200 200
drawline  $a$ 
color 0 0 0

% NDG: lines  $a$  and  $h_{\{b\}}$  are not parallel% DET: lines  $a$  and  $h_{\{b\}}$  are not the same
% Constructing a point  $B$  which belongs to line  $a$  and line  $h_{\{b\}}$ 
intersec  $B$   $a$   $h_{\{b\}}$ 
cmark_b  $B$ 

% DET: points  $T_{\{c\}}$  and  $B$  are not the same
% Constructing a line  $c$  which passes through point  $T_{\{c\}}$  and point  $B$ 
line  $c$   $T_{\{c\}}$   $B$ 

color 200 200 200
drawline  $c$ 
color 0 0 0

% NDG: lines  $b$  and  $c$  are not parallel% DET: lines  $b$  and  $c$  are not the same
% Constructing a point  $A$  which belongs to line  $b$  and line  $c$ 
intersec  $A$   $b$   $c$ 
cmark_t  $A$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

```

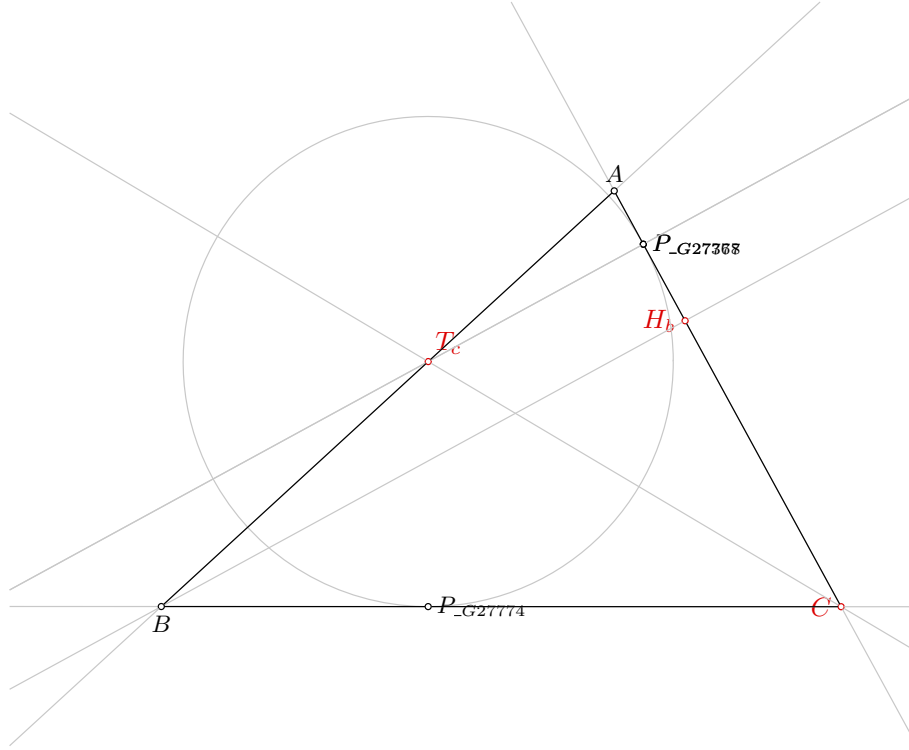


Figure 1: Illustration of the problem 0258

*% Non-degenerate conditions: lines b and c are not parallel; lines a and h\_{b} are not parallel;  
 point C is outside the circle k(T\_{c},foot[T\_{c},b]); point T\_{c} is not incident to the line b  
 % Determination conditions: lines b and c are not the same; points T\_{c} and B are not the same;  
 lines a and h\_{b} are not the same; points C and H\_{b} are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.033 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b = \neg H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.14 seconds.

**NDG conditions** Points  $A$  and  $T_c$  are not identical

Points  $A$  and  $P_{G34955}$  are not identical

Points  $A$ ,  $C$  and  $P_{G35110}$  are not collinear

Line through points  $P_{G35104}$  and  $C$  is not parallel with line through points  $B$  and  $T_c$

Line through points  $H_b$  and  $A$  is not perpendicular to line through points  $A$  and  $P_{G34955}$

#### 4.1.3 Proving $T_c = \neg T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C = C$

NDG conditions are:

$S_{CH_bT_{h_b}^0} \neq S_{P_{G28846}H_bT_{h_b}^0}$  i.e., lines  $CP_{G28846}$  and  $H_bT_{h_b}^0$  are not parallel (construction based assumption)

$S_{CT_cB} \neq S_{H_bT_cB}$  i.e., lines  $CH_b$  and  $T_cB$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^1} \neq S_{CBF_{h_b}^1}$  i.e., lines  $AC$  and  $BF_{h_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.2.3 Proving $T_c = \neg T_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_c = \neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 67 terms.

**Time Complexity:** Time spent by the prover is 0.040 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $T_c = \neg T_c$

Proving failed

# Problem 259

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 259: Given a point  $C$ , a point  $H_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $H_b$  are not the same;
2. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
3. Using the point  $I$  and the line  $b$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $b$ ;
4. Using the circle  $k(I, P_a)$ , the point  $C$ , the point  $I$  and the line  $b$ , construct a line  $a$  (rule W13); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
5. Using the line  $a$  and the line  $h_b$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $h_b$  are not parallel % DET: lines  $a$  and  $h_b$  are not the same;
6. Using the circle  $k(I, P_a)$ , the point  $B$ , the point  $I$  and the line  $a$ , construct a line  $c$  (rule W13); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
7. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; point  $B$  is outside the circle  $k(I, P_a)$ ; lines  $a$  and  $h_b$  are not parallel; point  $C$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $b$ .

Determination conditions: lines  $c$  and  $b$  are not the same; lines  $a$  and  $h_b$  are not the same; points  $C$  and  $H_b$  are not the same.

Rules used: [W02,W03,W10b,W11,W13]

Lemmas used: [D27,D6,D9,GD01,L59,L60,L61]

Solving time: 12.5 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{b} 89.36 77.83
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_l C
cmark_l H_{b}
cmark_b I
color 0 0 0
fontsize 8

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: point I is not incident to the line b
% Constructing a point P_{\_G63108} which is a foot of the point I on the line b
foot P_{\_G63108} I b
cmark_r P_{\_G63108}
color 200 200 200
drawline I P_{\_G63108}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
63108}
circle k(I,P_{a}) I P_{\_G63108}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point C is outside the circle k(I,P_{a})
```

```

% Constructing a point  $P_{\{G63315\}}$  which is a foot of the point  $I$  on the line  $b$ 
foot  $P_{\{G63315\}}$  I b
cmark_r  $P_{\{G63315\}}$ 
color 200 200 200
drawline I  $P_{\{G63315\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G63318\}}$  which passes through point  $I$  and point  $C$ 
line  $L_{\{G63318\}}$  I C

color 200 200 200
drawline  $L_{\{G63318\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G63321\}}$  which is an image of the point  $P_{\{G63315\}}$  in the symmetry to
point/line  $L_{\{G63318\}}$ 
sim  $P_{\{G63321\}}$   $L_{\{G63318\}}$   $P_{\{G63315\}}$ 
cmark_r  $P_{\{G63321\}}$ 

% Constructing a line  $a$  which passes through point  $C$  and point  $P_{\{G63321\}}$ 
line a C  $P_{\{G63321\}}$ 

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $a$  and  $h_{\{b\}}$  are not parallel% DET: lines  $a$  and  $h_{\{b\}}$  are not the same
% Constructing a point  $B$  which belongs to line  $a$  and line  $h_{\{b\}}$ 
intersec B a  $h_{\{b\}}$ 
cmark_b B

% NDG: point  $B$  is outside the circle  $k(I, P_{\{a\}})$ 
% Constructing a point  $P_{\{G63707\}}$  which is a foot of the point  $I$  on the line  $a$ 
foot  $P_{\{G63707\}}$  I a
cmark_r  $P_{\{G63707\}}$ 
color 200 200 200
drawline I  $P_{\{G63707\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G63710\}}$  which passes through point  $I$  and point  $B$ 
line  $L_{\{G63710\}}$  I B

color 200 200 200
drawline  $L_{\{G63710\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G63713\}}$  which is an image of the point  $P_{\{G63707\}}$  in the symmetry to
point/line  $L_{\{G63710\}}$ 
sim  $P_{\{G63713\}}$   $L_{\{G63710\}}$   $P_{\{G63707\}}$ 
cmark_r  $P_{\{G63713\}}$ 

```

```

% Constructing a line c which passes through point B and point P_{\_G63713}
line c B P_{\_G63713}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and b are not parallel; point B is outside the circle k(I,P_{a
}); lines a and h_{b} are not parallel; point C is outside the circle k(I,P_{a}); point I is
not incident to the line b
% Determination conditions: lines c and b are not the same; lines a and h_{b} are not the same;
points C and H_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.036 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b=H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 10 terms.

**Time Complexity:** Time spent by the prover is 0.223 seconds.

**NDG conditions** Points  $H_b$  and  $I$  are not identical

Point  $P_{G59412}$  is not on circle with center  $H_b$  and point from it  $A$

Points  $H_b$ ,  $C$  and  $P_{G59515}$  are not collinear

Points  $C$  and  $P_{G59521}$  are not identical

Points  $B$  and  $I$  are not identical



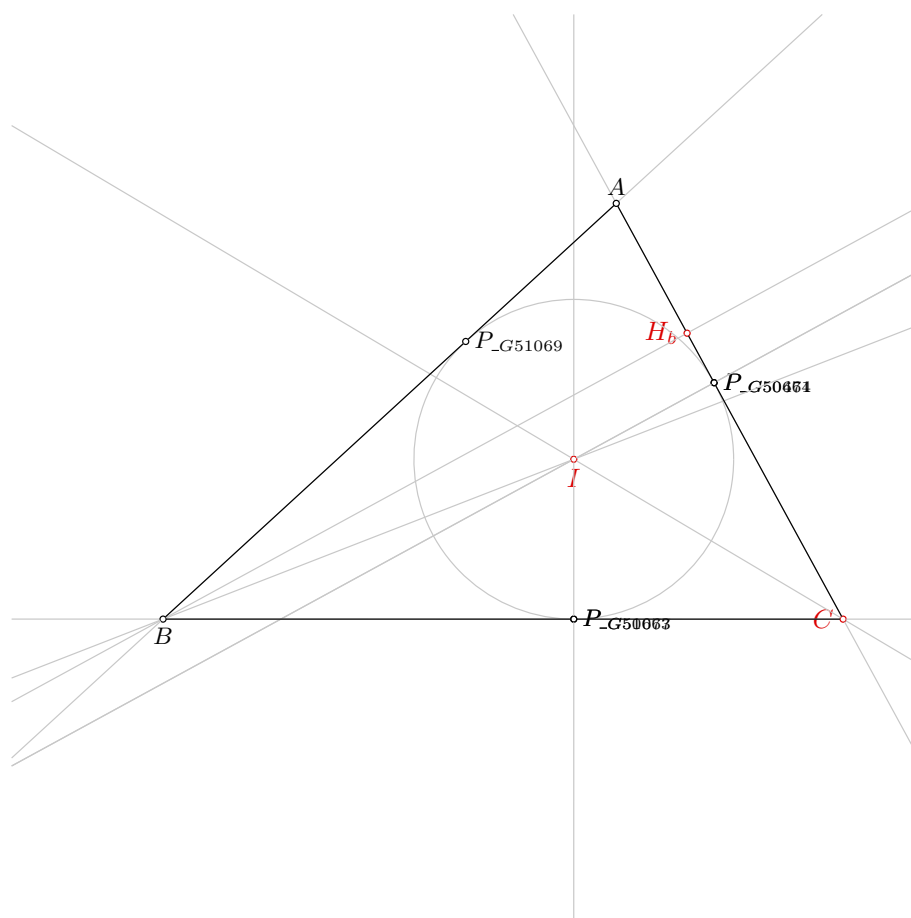


Figure 1: Illustration of the problem 0259

Point  $P_{G59412}$  is not on circle with center  $H_b$  and point from it  $A$   
Line through points  $H_b$  and  $C$  is not parallel with line through points  $P_{G59683}$  and  $B$   
Points  $A$  and  $C$  are not identical

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{CH_bT_{h_b}^0} \neq S_{P_{G51803}H_bT_{h_b}^0}$  i.e., lines  $CP_{G51803}$  and  $H_bT_{h_b}^0$  are not parallel (construction based assumption)  
 $S_{BCH_b} \neq S_{P_{G52195}CH_b}$  i.e., lines  $BP_{G52195}$  and  $CH_b$  are not parallel (construction based assumption)  
 $S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)  
 $S_{ABF_{h_b}^1} \neq S_{CBF_{h_b}^1}$  i.e., lines  $AC$  and  $BF_{h_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_b=_H H_b$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_b=_H H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I=_I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 505 terms.

**Time Complexity:** Time spent by the prover is 0.370 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.3 Proving $I = \neg I$

Proving failed

# Problem 260

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 260: Given a point  $H_c$ , a point  $H$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $H_c$  and  $H$  are not the same;
2. Choose freely a point  $C$  on the line  $h_c$  (rule WOnline1) ;
3. Using the point  $H_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
4. Choose freely a point  $A$  on the line  $c$  (rule WOnline2);
5. Using the point  $A$  and the point  $C$ , construct a point  $M_b$  (rule W01); ;
6. Using the point  $A$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H$  are not the same;
7. Using the point  $A$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $A$  and  $M_b$  are not the same;
8. Using the circle  $k(M_b, C)$ , the line  $h_a$ , the point  $M_b$  and the point  $A$ , construct a point  $H_a$  (rule W05); % NDG: line  $h_a$  and circle  $k(M_b, C)$  intersect % DET: points  $A$  and  $H_a$  must be different;
9. Using the point  $H_a$  and the point  $C$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $C$  are not the same;
10. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; line  $h_a$  and circle  $k(M_b, C)$  intersect; points  $A$  and  $M_b$  are not the same.

Determination conditions: lines  $c$  and  $a$  are not the same; points  $H_a$  and  $C$  are not the same; points  $A$  and  $H_a$  must be different; points  $A$  and  $H$  are not the same; points  $H_c$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D10,D22,D3,D5,D7,D8,GD01,GD02,GL04,L3,L43,L44]

Solving time: 9.7 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{c} 68.91 84.83
```

```
point H 80 72.73
```

```
point C 110 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H_{c}
```

```
cmark_rt H
```

```
cmark_l C
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{c} and H are not the same
```

```
% Constructing a line h_{c} which passes through point H_{c} and point H
```

```
line h_{c} H_{c} H
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```
% Choosing randomly a point C on the line H_{c}H
```

```
online C H_{c} H
```

```
cmark_l C
```

```
color 200 200 200
```

```
drawline H_{c} H
```

```
color 0 0 0
```

```
% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
```

```
perp c H_{c} h_{c}
```

```
color 200 200 200
```

```
drawline c
```

```
color 0 0 0
```

```

% Generating random value V[_G47010]
random V[_G47010]

% Calculating value V[_G47031] using formula V[_G47010]*20
expression V[_G47031] { V[_G47010]*20 }

% Constructing a point A which is a point for which holds  $H_{\{c\}A} = V[_G47031]$  and angle  $CH_{\{c\}A} = 90$ 
turtle A C H_{c} 90 V[_G47031]
cmark_t A

% Constructing a point  $M_{\{b\}}$  such that  $AM_{\{b\}}/AC=0.5$ 
towards M_{b} A C 0.5
cmark_lt M_{b}
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points A and H are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points A and  $M_{\{b\}}$  are not the same
% Constructing a circle  $k(M_{\{b\}},C)$  whose center is at point  $M_{\{b\}}$  and which passes through point A
circle k(M_{b},C) M_{b} A

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line  $h_{\{a\}}$  and circle  $k(M_{\{b\}},C)$  intersect% DET: points A and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G47503\}}$  which is a foot of the point  $M_{\{b\}}$  on the line  $h_{\{a\}}$ 
foot P_{\_G47503} M_{b} h_{a}
cmark_r P_{\_G47503}
color 200 200 200
drawline M_{b} P_{\_G47503}
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point A in the symmetry to point/line  $P_{\{\backslash\_G47503\}}$ 
sim H_{a} P_{\_G47503} A

```

```

cmark_r H_{a}

% DET: points H_{a} and C are not the same
% Constructing a line a which passes through point H_{a} and point C
line a H_{a} C

color 200 200 200
drawline a
color 0 0 0

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and a are not parallel; line h_{a} and circle k(M_{b},C)
% intersect; points A and M_{b} are not the same
% Determination conditions: lines c and a are not the same; points H_{a} and C are not the same;
% points A and H_{a} must be different; points A and H are not the same; points H_{c} and H are
% not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_c = \_H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.152 seconds.

**NDG conditions** Points  $A$  and  $H_c$  are not identical

Points  $H_c$  and  $H$  are not identical

Line through points  $H_c$  and  $C$  is not perpendicular to line through points  $C$  and  $H$

Points  $A$ ,  $B$  and  $H$  are not collinear

#### 4.1.2 Proving $H = \_H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 31 terms.

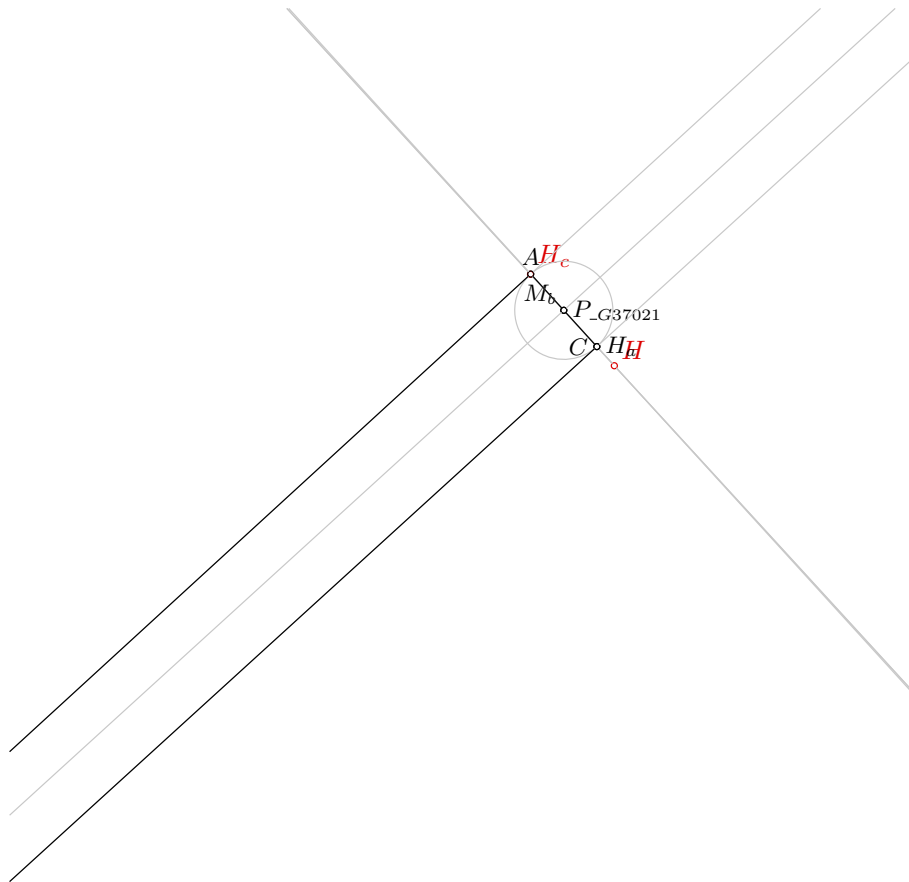


Figure 1: Illustration of the problem 0260



**Time Complexity:** Time spent by the prover is 0.497 seconds.

**NDG conditions** Points  $A$  and  $H_c$  are not identical

Points  $H_c$  and  $H$  are not identical

Line through points  $H_c$  and  $C$  is not perpendicular to line through points  $C$  and  $H$

Point  $H$  is not on circle with center  $H_c$  and point from it  $C$

Points  $A$ ,  $B$  and  $H$  are not collinear

### 4.1.3 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.009 seconds.

**NDG conditions** There are no NDG conditions for this theorem

## 4.2 GCLC - Area method

### 4.2.1 Proving $H_c=_H H_c$

Proving failed

### 4.2.2 Proving $H=_H H$

Proving failed

### 4.2.3 Proving $C=C$

NDG conditions are:

$S_{H_c H_a C} \neq S_{T_c^1 H_a C}$  i.e., lines  $H_c T_c^1$  and  $H_a C$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{-H_c}^2} \neq S_{BCF_{-H_c}^2}$  i.e., lines  $AB$  and  $CF_{-H_c}^2$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-H_b}^4} \neq S_{F_{-H_a}^3 BF_{-H_b}^4}$  i.e., lines  $AF_{-H_a}^3$  and  $BF_{-H_b}^4$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $H_c=_H H_c$

Proving failed

### 4.3.2 Proving $H=_H H$

Proving failed

### 4.3.3 Proving $C=C$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $H_c=_H H_c$

Proving failed

#### 4.4.2 Proving $H=_H$

Proving failed

#### 4.4.3 Proving $C=C$

Proving failed

# Problem 261

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 261: Given a point  $C$ , a point  $H_c$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_c$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H_c$  are not the same;
2. Using the point  $C$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $T_a$  are not the same;
3. Using the point  $H_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
4. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same;
5. Using the point  $B$ , the point  $C$ , the point  $T_a$  and the line  $a$ , construct a point  $T'_a$  (rule W19); % NDG: points  $B$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $midpoint([B, T_a])$  are not the same;
6. Using the point  $T_a$  and the point  $T'_a$ , construct a circle  $k_{over}(T_a, T'_a)$  (rule W09); % NDG: points  $T_a$  and  $T'_a$  are not the same;
7. Using the circle  $k_{over}(T_a, T'_a)$  and the line  $c$ , construct a point  $A_{wb}$  and a point  $A$  (rule W04); % NDG: line  $c$  and circle  $k_{over}(T_a, T'_a)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k_{over}(T_a, T'_a)$  intersect; points  $T_a$  and  $T'_a$  are not the same; points  $B$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $midpoint([B, T_a])$  are not the same; lines  $c$  and  $a$  are not parallel.

Determination conditions: lines  $c$  and  $a$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $H_c$  are not the same.

Rules used: [W02, W03, W04, W09, W10a, W19]

Lemmas used: [D10, D23, D53, D7, GD01, L71, L77]

Solving time: 2576.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{c} 68.91 84.83
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_l C
cmark_rt H_{c}
cmark_rb T_{a}
color 0 0 0
fontsize 8

% DET: points C and H_{c} are not the same
% Constructing a line h_{c} which passes through point C and point H_{c}
line h_{c} C H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% DET: points C and T_{a} are not the same
% Constructing a line a which passes through point C and point T_{a}
line a C T_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

% NDG: points B and C are not the same; points C and T_{a} are not the same; points C and midpoint
% ([B,T_{a}]) are not the same
```

```

% Constructing a point  $P_{\backslash\_G38773}$  which is an image of the point  $C$  in a rotation around the point
 $T_{\{a\}}$  for the angle 90
rotate  $P_{\backslash\_G38773}$   $T_{\{a\}}$  90  $C$ 
cmark_r  $P_{\backslash\_G38773}$ 
color 200 200 200
drawarc_p  $T_{\{a\}}$   $C$  90
color 0 0 0

% Constructing a line  $L_{\backslash\_G38776}$  which passes through point  $T_{\{a\}}$  and point  $P_{\backslash\_G38773}$ 
line  $L_{\backslash\_G38776}$   $T_{\{a\}}$   $P_{\backslash\_G38773}$ 

color 200 200 200
drawline  $L_{\backslash\_G38776}$ 
color 0 0 0

% Constructing midpoint  $P_{\backslash\_G38779}$  of the segment  $CP_{\backslash\_G38773}$ 
midpoint  $P_{\backslash\_G38779}$   $C$   $P_{\backslash\_G38773}$ 
cmark_r  $P_{\backslash\_G38779}$ 

% Constructing a line  $L_{\backslash\_G38782}$  which passes through point  $B$  and point  $P_{\backslash\_G38779}$ 
line  $L_{\backslash\_G38782}$   $B$   $P_{\backslash\_G38779}$ 

color 200 200 200
drawline  $L_{\backslash\_G38782}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G38785}$  which passes through point  $C$  and point  $P_{\backslash\_G38779}$ 
line  $L_{\backslash\_G38785}$   $C$   $P_{\backslash\_G38779}$ 

color 200 200 200
drawline  $L_{\backslash\_G38785}$ 
color 0 0 0

% Constructing a point  $P_{\backslash\_G38788}$  which belongs to line  $L_{\backslash\_G38776}$  and line  $L_{\backslash\_G38782}$ 
intersec  $P_{\backslash\_G38788}$   $L_{\backslash\_G38776}$   $L_{\backslash\_G38782}$ 
cmark_r  $P_{\backslash\_G38788}$ 

% Constructing a point  $P_{\backslash\_G38791}$  which belongs to line  $L_{\backslash\_G38776}$  and line  $L_{\backslash\_G38785}$ 
intersec  $P_{\backslash\_G38791}$   $L_{\backslash\_G38776}$   $L_{\backslash\_G38785}$ 
cmark_r  $P_{\backslash\_G38791}$ 

% Constructing a line  $L_{\backslash\_G38794}$  which passes through point  $B$  and point  $P_{\backslash\_G38791}$ 
line  $L_{\backslash\_G38794}$   $B$   $P_{\backslash\_G38791}$ 

color 200 200 200
drawline  $L_{\backslash\_G38794}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G38797}$  which passes through point  $C$  and point  $P_{\backslash\_G38788}$ 
line  $L_{\backslash\_G38797}$   $C$   $P_{\backslash\_G38788}$ 

color 200 200 200

```

```

drawline L_{\_G38797}
color 0 0 0

% Constructing a point P_{\_G38800} which belongs to line L_{\_G38794} and line L_{\_G38797}
intersec P_{\_G38800} L_{\_G38794} L_{\_G38797}
cmark_r P_{\_G38800}

% Constructing a line L_{\_G38803} which passes through point P_{\_G38779} and point P_{\_G38800}
line L_{\_G38803} P_{\_G38779} P_{\_G38800}

color 200 200 200
drawline L_{\_G38803}
color 0 0 0

% Constructing a point T'_{a} which belongs to line L_{\_G38803} and line a
intersec T'_{a} L_{\_G38803} a
cmark_r T'_{a}

% NDG: points T_{a} and T'_{a} are not the same
% Constructing midpoint P_{\_G39634} of the segment T_{a}T'_{a}
midpoint P_{\_G39634} T_{a} T'_{a}
cmark_r P_{\_G39634}

% Constructing a circle k_{over}(T_{a},T'_{a}) whose center is at point P_{\_G39634} and which passes
    through point T_{a}
circle k_{over}(T_{a},T'_{a}) P_{\_G39634} T_{a}

color 200 200 200
drawcircle k_{over}(T_{a},T'_{a})
color 0 0 0

% NDG: line c and circle k_{over}(T_{a},T'_{a}) intersect
% Constructing points A_{wb} and A which are in intersection of k_{over}(T_{a},T'_{a}) and c
intersec2 A_{wb} A k_{over}(T_{a},T'_{a}) c
cmark_r A_{wb}
cmark_t A

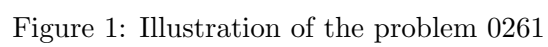
drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k_{over}(T_{a},T'_{a}) intersect; points T_{a} and T'_{a}
    are not the same; points B and C are not the same; points C and T_{a} are not the same;
    points C and midpoint([B,T_{a}]) are not the same; lines c and a are not parallel
% Determination conditions: lines c and a are not the same; points C and T_{a} are not the same;
    points C and H_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1



### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.026 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c=_Hc$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.151 seconds.

**NDG conditions** Points  $T_a$ ,  $H_c$  and  $C$  are not collinear

Point  $T_a$  is not the midpoint of segment with endpoints  $B$  and  $C$

Points  $T_a$  and  $A$  are not identical

Points  $A$ ,  $B$  and  $C$  are not collinear

#### 4.1.3 Proving $T_a=_T a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_cCT_a} \neq S_{T_c^0CT_a}$  i.e., lines  $H_cT_c^0$  and  $CT_a$  are not parallel (construction based assumption)

$S_{T_aBP_{G63885}} \neq S_{P_{G63879}BP_{G63885}}$  i.e., lines  $T_aP_{G63879}$  and  $BP_{G63885}$  are not parallel (construction based assumption)

$S_{T_aCP_{G63885}} \neq S_{P_{G63879}CP_{G63885}}$  i.e., lines  $T_aP_{G63879}$  and  $CP_{G63885}$  are not parallel (construction based assumption)

$S_{BCP_{G63894}} \neq S_{P_{G63897}CP_{G63894}}$  i.e., lines  $BP_{G63897}$  and  $CP_{G63894}$  are not parallel (construction based assumption)

$S_{P_{G63885}CT_a} \neq S_{P_{G63906}CT_a}$  i.e., lines  $P_{G63885}P_{G63906}$  and  $CT_a$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{-h_c}^1} \neq S_{BCF_{-h_c}^1}$  i.e., lines  $AB$  and  $CF_{-h_c}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c=_Hc$

Proving failed

#### 4.2.3 Proving $T_a=_T a$

Proving failed



### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C=C$

Proving failed

#### 4.3.2 Proving $H_c=_Hc$

Proving failed

#### 4.3.3 Proving $T_a=_T a$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $C=C$

Proving failed

#### 4.4.2 Proving $H_c=_Hc$

Proving failed

#### 4.4.3 Proving $T_a=_T a$

Proving failed

# Problem 262

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 262: Given a point  $C$ , a point  $H_c$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_c$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H_c$  are not the same;
2. Using the point  $C$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $T_b$  are not the same;
3. Using the point  $H_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
4. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same;
5. Using the point  $A$ , the point  $C$ , the point  $T_b$  and the line  $b$ , construct a point  $T'_b$  (rule W19); % NDG: points  $A$  and  $C$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $\text{midpoint}([A, T_b])$  are not the same;
6. Using the point  $T_b$  and the point  $T'_b$ , construct a circle  $k_{over}(T_b, T'_b)$  (rule W09); % NDG: points  $T_b$  and  $T'_b$  are not the same;
7. Using the circle  $k_{over}(T_b, T'_b)$  and the line  $c$ , construct a point  $B_{wa}$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k_{over}(T_b, T'_b)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k_{over}(T_b, T'_b)$  intersect; points  $T_b$  and  $T'_b$  are not the same; points  $A$  and  $C$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $\text{midpoint}([A, T_b])$  are not the same; lines  $c$  and  $b$  are not parallel.

Determination conditions: lines  $c$  and  $b$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $H_c$  are not the same.

Rules used: [W02,W03,W04,W09,W10a,W19]

Lemmas used: [D10,D24,D55,D7,GD01,L73,L78]

Solving time: 2601.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{c} 68.91 84.83
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_l C
cmark_rt H_{c}
cmark_t T_{b}
color 0 0 0
fontsize 8

% DET: points C and H_{c} are not the same
% Constructing a line h_{c} which passes through point C and point H_{c}
line h_{c} C H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% DET: points C and T_{b} are not the same
% Constructing a line b which passes through point C and point T_{b}
line b C T_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A

% NDG: points A and C are not the same; points C and T_{b} are not the same; points C and midpoint
% ([A,T_{b}]) are not the same
```

```

% Constructing a point  $P_{\backslash\_G30443}$  which is an image of the point  $C$  in a rotation around the point
 $T_{\backslash b}$  for the angle 90
rotate  $P_{\backslash\_G30443}$   $T_{\backslash b}$  90 C
cmark_r  $P_{\backslash\_G30443}$ 
color 200 200 200
drawarc_p  $T_{\backslash b}$  C 90
color 0 0 0

% Constructing a line  $L_{\backslash\_G30446}$  which passes through point  $T_{\backslash b}$  and point  $P_{\backslash\_G30443}$ 
line  $L_{\backslash\_G30446}$   $T_{\backslash b}$   $P_{\backslash\_G30443}$ 

color 200 200 200
drawline  $L_{\backslash\_G30446}$ 
color 0 0 0

% Constructing midpoint  $P_{\backslash\_G30449}$  of the segment  $CP_{\backslash\_G30443}$ 
midpoint  $P_{\backslash\_G30449}$  C  $P_{\backslash\_G30443}$ 
cmark_r  $P_{\backslash\_G30449}$ 

% Constructing a line  $L_{\backslash\_G30452}$  which passes through point  $A$  and point  $P_{\backslash\_G30449}$ 
line  $L_{\backslash\_G30452}$  A  $P_{\backslash\_G30449}$ 

color 200 200 200
drawline  $L_{\backslash\_G30452}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G30455}$  which passes through point  $C$  and point  $P_{\backslash\_G30449}$ 
line  $L_{\backslash\_G30455}$  C  $P_{\backslash\_G30449}$ 

color 200 200 200
drawline  $L_{\backslash\_G30455}$ 
color 0 0 0

% Constructing a point  $P_{\backslash\_G30458}$  which belongs to line  $L_{\backslash\_G30446}$  and line  $L_{\backslash\_G30452}$ 
intersec  $P_{\backslash\_G30458}$   $L_{\backslash\_G30446}$   $L_{\backslash\_G30452}$ 
cmark_r  $P_{\backslash\_G30458}$ 

% Constructing a point  $P_{\backslash\_G30461}$  which belongs to line  $L_{\backslash\_G30446}$  and line  $L_{\backslash\_G30455}$ 
intersec  $P_{\backslash\_G30461}$   $L_{\backslash\_G30446}$   $L_{\backslash\_G30455}$ 
cmark_r  $P_{\backslash\_G30461}$ 

% Constructing a line  $L_{\backslash\_G30464}$  which passes through point  $A$  and point  $P_{\backslash\_G30461}$ 
line  $L_{\backslash\_G30464}$  A  $P_{\backslash\_G30461}$ 

color 200 200 200
drawline  $L_{\backslash\_G30464}$ 
color 0 0 0

% Constructing a line  $L_{\backslash\_G30467}$  which passes through point  $C$  and point  $P_{\backslash\_G30458}$ 
line  $L_{\backslash\_G30467}$  C  $P_{\backslash\_G30458}$ 

color 200 200 200

```

```

drawline L_{\_G30467}
color 0 0 0

% Constructing a point P_{\_G30470} which belongs to line L_{\_G30464} and line L_{\_G30467}
intersec P_{\_G30470} L_{\_G30464} L_{\_G30467}
cmark_r P_{\_G30470}

% Constructing a line L_{\_G30473} which passes through point P_{\_G30449} and point P_{\_G30470}
line L_{\_G30473} P_{\_G30449} P_{\_G30470}

color 200 200 200
drawline L_{\_G30473}
color 0 0 0

% Constructing a point T'_{b} which belongs to line L_{\_G30473} and line b
intersec T'_{b} L_{\_G30473} b
cmark_r T'_{b}

% NDG: points T_{b} and T'_{b} are not the same
% Constructing midpoint P_{\_G31304} of the segment T_{b}T'_{b}
midpoint P_{\_G31304} T_{b} T'_{b}
cmark_r P_{\_G31304}

% Constructing a circle k_{over}(T_{b},T'_{b}) whose center is at point P_{\_G31304} and which passes
    through point T_{b}
circle k_{over}(T_{b},T'_{b}) P_{\_G31304} T_{b}

color 200 200 200
drawcircle k_{over}(T_{b},T'_{b})
color 0 0 0

% NDG: line c and circle k_{over}(T_{b},T'_{b}) intersect
% Constructing points B_{wa} and B which are in intersection of k_{over}(T_{b},T'_{b}) and c
intersec2 B_{wa} B k_{over}(T_{b},T'_{b}) c
cmark_r B_{wa}
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k_{over}(T_{b},T'_{b}) intersect; points T_{b} and T'_{b}
    are not the same; points A and C are not the same; points C and T_{b} are not the same;
    points C and midpoint([A,T_{b}]) are not the same; lines c and b are not parallel
% Determination conditions: lines c and b are not the same; points C and T_{b} are not the same;
    points C and H_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

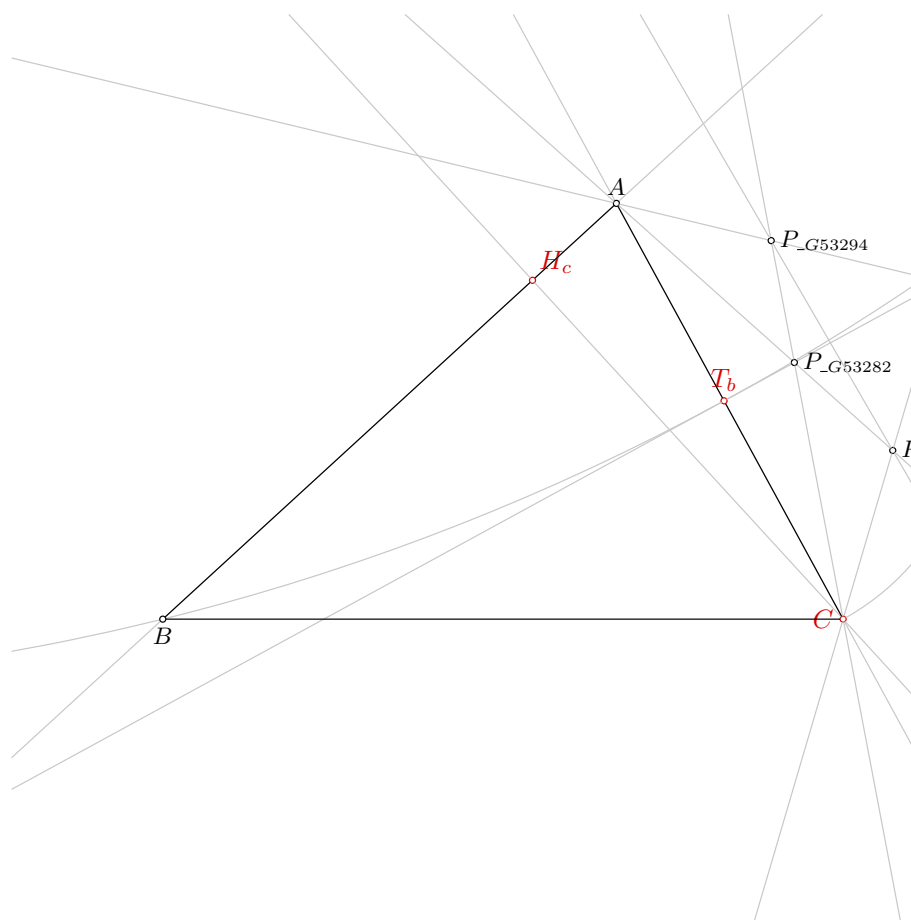


Figure 1: Illustration of the problem 0262

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c=_Hc$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.145 seconds.

**NDG conditions** Points  $H_c$ ,  $C$  and  $T_b$  are not collinear

Point  $T_b$  is not the midpoint of segment with endpoints  $A$  and  $C$

Points  $H_c$  and  $B$  are not identical

Points  $A$ ,  $B$  and  $C$  are not collinear

#### 4.1.3 Proving $T_b=_Tb$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_cCT_b} \neq S_{T_c^0CT_b}$  i.e., lines  $H_cT_c^0$  and  $CT_b$  are not parallel (construction based assumption)

$S_{T_bAP_{G55101}} \neq S_{P_{G55095}AP_{G55101}}$  i.e., lines  $T_bP_{G55095}$  and  $AP_{G55101}$  are not parallel (construction based assumption)

$S_{T_bCP_{G55101}} \neq S_{P_{G55095}CP_{G55101}}$  i.e., lines  $T_bP_{G55095}$  and  $CP_{G55101}$  are not parallel (construction based assumption)

$S_{ACP_{G55110}} \neq S_{P_{G55113}CP_{G55110}}$  i.e., lines  $AP_{G55113}$  and  $CP_{G55110}$  are not parallel (construction based assumption)

$S_{P_{G55101}CT_b} \neq S_{P_{G55122}CT_b}$  i.e., lines  $P_{G55101}P_{G55122}$  and  $CT_b$  are not parallel (construction based assumption)

$SC_{AB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{-h_c}^1} \neq S_{BCF_{-h_c}^1}$  i.e., lines  $AB$  and  $CF_{-h_c}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c=_Hc$

Proving failed

#### 4.2.3 Proving $T_b=_Tb$

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $C=C$**

Proving failed

#### **4.3.2 Proving $H_c=\neg H_c$**

Proving failed

#### **4.3.3 Proving $T_b=\neg T_b$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $C=C$**

Proving failed

#### **4.4.2 Proving $H_c=\neg H_c$**

Proving failed

#### **4.4.3 Proving $T_b=\neg T_b$**

Proving failed



# Problem 263

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 263: Given a point  $H_c$ , a point  $T_c$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $T_c$  are not the same;
2. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
3. Choose freely a point  $C$  on the line  $h_c$  (rule WOnline2);
4. Using the point  $C$  and the point  $T_c$ , construct a line  $s_c$  (rule W02); % DET: points  $C$  and  $T_c$  are not the same;
5. Using the point  $H_c$ , the point  $C$ , the point  $T_c$ , the line  $s_c$  and the line  $h_c$ , construct a line  $CO$  (rule W17); % NDG: points  $C$  and  $T_c$  are not the same; points  $H_c$  and  $C$  are not the same  
% DET: points  $C$  and  $T_c$  are not the same;
6. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) ;
7. Using the point  $A$  and the point  $C$ , construct a point  $M_b$  (rule W01); ;
8. Using the point  $A$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $C$  are not the same;
9. Using the point  $M_b$  and the line  $b$ , construct a line  $m_b$  (rule W10b); ;
10. Using the line  $m_b$  and the line  $CO$ , construct a point  $O$  (rule W03); % NDG: lines  $m_b$  and  $CO$  are not parallel % DET: lines  $m_b$  and  $CO$  are not the same;
11. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;

12. Using the circle  $k(O, C)$ , the line  $c$ , the point  $O$  and the point  $A$ , construct a point  $B$  (rule W05); % NDG: line  $c$  and circle  $k(O, C)$  intersect % DET: points  $A$  and  $B$  must be different.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same; lines  $m_b$  and  $CO$  are not parallel; points  $C$  and  $T_c$  are not the same; points  $H_c$  and  $C$  are not the same.

Determination conditions: points  $A$  and  $B$  must be different; lines  $m_b$  and  $CO$  are not the same; points  $A$  and  $C$  are not the same; points  $C$  and  $T_c$  are not the same; points  $C$  and  $T_c$  are not the same; points  $H_c$  and  $T_c$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W10b,W17,WOnline1,WOnline2]

Lemmas used: [D1,D10,D12,D22,D25,D26,D7,GD01,GL04,GL17,L106,L11,L12,L7]

Solving time: 20.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point H_{c} 68.91 84.83
point T_{c} 55.38 72.43
point C 110 40

color 220 0 0
fontsize 9

cmark_rt H_{c}
cmark_rt T_{c}
cmark_l C
color 0 0 0
fontsize 8

% DET: points H_{c} and T_{c} are not the same
% Constructing a line c which passes through point H_{c} and point T_{c}
line c H_{c} T_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0

% Generating random value V[_G56578]
random V[_G56578]
```

```

% Calculating value  $V_{G56599}$  using formula  $V_{G56578} * 20$ 
expression  $V_{G56599}$  {  $V_{G56578} * 20$  }

% Constructing a point  $C$  which is a point for which holds  $H_{\{c\}}C = V_{G56599}$  and angle  $T_{\{c\}}H_{\{c\}}C = 90$ 
turtle C  $T_{\{c\}}$   $H_{\{c\}}$  90  $V_{G56599}$ 
cmark_l C

% DET: points  $C$  and  $T_{\{c\}}$  are not the same
% Constructing a line  $s_{\{c\}}$  which passes through point  $C$  and point  $T_{\{c\}}$ 
line  $s_{\{c\}}$  C  $T_{\{c\}}$ 

color 200 200 200
drawline  $s_{\{c\}}$ 
color 0 0 0

% NDG: points  $C$  and  $T_{\{c\}}$  are not the same; points  $H_{\{c\}}$  and  $C$  are not the same% DET: points  $C$  and  $T_{\{c\}}$  are not the same
% Constructing an angle  $V_{G56830}$  which is equal to the angle  $H_{\{c\}}CT_{\{c\}}$ 
angle_o  $V_{G56830}$   $H_{\{c\}}$  C  $T_{\{c\}}$ 

% Calculating value  $angle_{G56909}$  using formula  $1/\text{pow}(2,0)*V_{G56830}+0/\text{pow}(2,0)*180$ 
expression  $angle_{G56909}$  {  $1/\text{pow}(2,0)*V_{G56830}+0/\text{pow}(2,0)*180$  }

% Constructing a point  $P_{\{G56906\}}$  which is an image of the point  $T_{\{c\}}$  in a rotation around the point  $C$  for the angle  $1/\text{pow}(2,0)*V_{G56830}+0/\text{pow}(2,0)*180$ 
rotate  $P_{\{G56906\}}$  C  $angle_{G56909}$   $T_{\{c\}}$ 
cmark_r  $P_{\{G56906\}}$ 
color 200 200 200
drawarc_p C  $T_{\{c\}}$   $angle_{G56909}$ 
color 0 0 0

% Constructing a line  $CD$  which passes through point  $C$  and point  $P_{\{G56906\}}$ 
line CD C  $P_{\{G56906\}}$ 

color 200 200 200
drawline CD
color 0 0 0

% Choosing randomly a point  $A$  on the line  $H_{\{c\}}T_{\{c\}}$ 
online A  $H_{\{c\}}$   $T_{\{c\}}$ 
cmark_t A
color 200 200 200
drawline  $H_{\{c\}}$   $T_{\{c\}}$ 
color 0 0 0

```

```

% Constructing a point  $M_{\{b\}}$  such that  $AM_{\{b\}}/AC=0.5$ 
towards  $M_{\{b\}}$  A C 0.5
cmark_lt  $M_{\{b\}}$ 
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C

color 200 200 200
drawline b
color 0 0 0

% Constructing a line  $m_{\{b\}}$  which is perpendicular to line b and which passes through point  $M_{\{b\}}$ 
perp  $m_{\{b\}}$   $M_{\{b\}}$  b

color 200 200 200
drawline  $m_{\{b\}}$ 
color 0 0 0

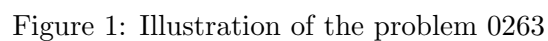
% NDG: lines  $m_{\{b\}}$  and CO are not parallel% DET: lines  $m_{\{b\}}$  and CO are not the same
% Constructing a point O which belongs to line  $m_{\{b\}}$  and line CO
intersec O  $m_{\{b\}}$  CO
cmark_t O

% NDG: points A and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point A
circle  $k(O,C)$  O A

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line c and circle  $k(O,C)$  intersect% DET: points A and B must be different
% Constructing a point  $P_{\{\backslash\_G57610\}}$  which is a foot of the point O on the line c
foot  $P_{\{\backslash\_G57610\}}$  O c
cmark_r  $P_{\{\backslash\_G57610\}}$ 
color 200 200 200
drawline O  $P_{\{\backslash\_G57610\}}$ 
color 0 0 0

```



% Determination conditions: points A and B must be different; lines  $m_{\{b\}}$  and  $CD$  are not the same;  
 points A and C are not the same; points C and  $T_{\{c\}}$  are not the same; points C and  $T_{\{c\}}$  are  
 not the same; points  $H_{\{c\}}$  and  $T_{\{c\}}$  are not the same

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 5 terms.

**Time Complexity:** Time spent by the prover is 0.149 seconds.

**NDG conditions** Points  $H_c$  and  $C$  are not identical

Line through points  $P_{G52107}$  and  $C$  is not perpendicular to line through points  $C$  and  $A$

Points  $A$  and  $C$  are not identical

Line through points  $A$  and  $H_c$  is not perpendicular to line through points  $H_c$  and  $B$

#### 4.1.2 Proving $T_c = \neg T_c$

Proving failed

#### 4.1.3 Proving $C = C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.011 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_c = \neg H_c$

Proving failed

#### 4.2.2 Proving $T_c = \neg T_c$

Proving failed

#### 4.2.3 Proving $C = C$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_c = \neg H_c$

Proving failed

#### 4.3.2 Proving $T_c = \neg T_c$

Proving failed

#### 4.3.3 Proving $C = C$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $H_c = \neg H_c$

Proving failed

### 4.4.2 Proving $T_c = \neg T_c$

Proving failed

### 4.4.3 Proving $C = C$

Proving failed

# Problem 264

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 264: Given a point  $C$ , a point  $H_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $H_c$ , construct a line  $h_c$  (rule W02); % DET: points  $C$  and  $H_c$  are not the same;
2. Using the point  $H_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
3. Using the point  $I$  and the line  $c$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $c$ ;
4. Using the circle  $k(I, P_a)$ , the point  $C$  and the point  $I$ , construct a line  $a$  and a line  $b$  (rule W12); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
5. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same;
6. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; lines  $a$  and  $c$  are not parallel; point  $C$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $c$ .

Determination conditions: lines  $c$  and  $b$  are not the same; lines  $a$  and  $c$  are not the same; points  $C$  and  $H_c$  are not the same.

Rules used: [W02,W03,W10a,W11,W12]

Lemmas used: [D10,D27,D7,GD01,L59,L60,L61]

Solving time: 1.0 seconds.



## 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{c} 68.91 84.83
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_l C
cmark_rt H_{c}
cmark_b I
color 0 0 0
fontsize 8

% DET: points C and H_{c} are not the same
% Constructing a line h_{c} which passes through point C and point H_{c}
line h_{c} C H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: point I is not incident to the line c
% Constructing a point P_{\_G40065} which is a foot of the point I on the line c
foot P_{\_G40065} I c
cmark_r P_{\_G40065}
color 200 200 200
drawline I P_{\_G40065}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G40065}
circle k(I,P_{a}) I P_{\_G40065}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point C is outside the circle k(I,P_{a})
```

```

% Constructing midpoint  $P_{\{G40299\}}$  of the segment  $CI$ 
midpoint  $P_{\{G40299\}}$  C I
cmark_r  $P_{\{G40299\}}$ 

% Constructing a circle  $C_{\{G40302\}}$  whose center is at point  $P_{\{G40299\}}$  and which passes through
point C
circle  $C_{\{G40302\}}$   $P_{\{G40299\}}$  C

color 200 200 200
drawcircle  $C_{\{G40302\}}$ 
color 0 0 0

% Constructing points  $P_{\{G40305\}}$  and  $P_{\{G40308\}}$  which are in intersection of  $C_{\{G40302\}}$  and  $k$ 
 $(I, P_{\{a\}})$ 
intersec2  $P_{\{G40305\}}$   $P_{\{G40308\}}$   $C_{\{G40302\}}$   $k(I, P_{\{a\}})$ 
cmark_r  $P_{\{G40305\}}$ 
cmark_r  $P_{\{G40308\}}$ 

% Constructing a line  $a$  which passes through point  $C$  and point  $P_{\{G40305\}}$ 
line a C  $P_{\{G40305\}}$ 

color 200 200 200
drawline a
color 0 0 0

% Constructing a line  $b$  which passes through point  $C$  and point  $P_{\{G40308\}}$ 
line b C  $P_{\{G40308\}}$ 

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $a$  and  $c$  are not parallel% DET: lines  $a$  and  $c$  are not the same
% Constructing a point  $B$  which belongs to line  $a$  and line  $c$ 
intersec B a c
cmark_b B

% NDG: lines  $c$  and  $b$  are not parallel% DET: lines  $c$  and  $b$  are not the same
% Constructing a point  $A$  which belongs to line  $c$  and line  $b$ 
intersec A c b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; lines  $a$  and  $c$  are not parallel; point
 $C$  is outside the circle  $k(I, P_{\{a\}})$ ; point  $I$  is not incident to the line  $c$ 

```

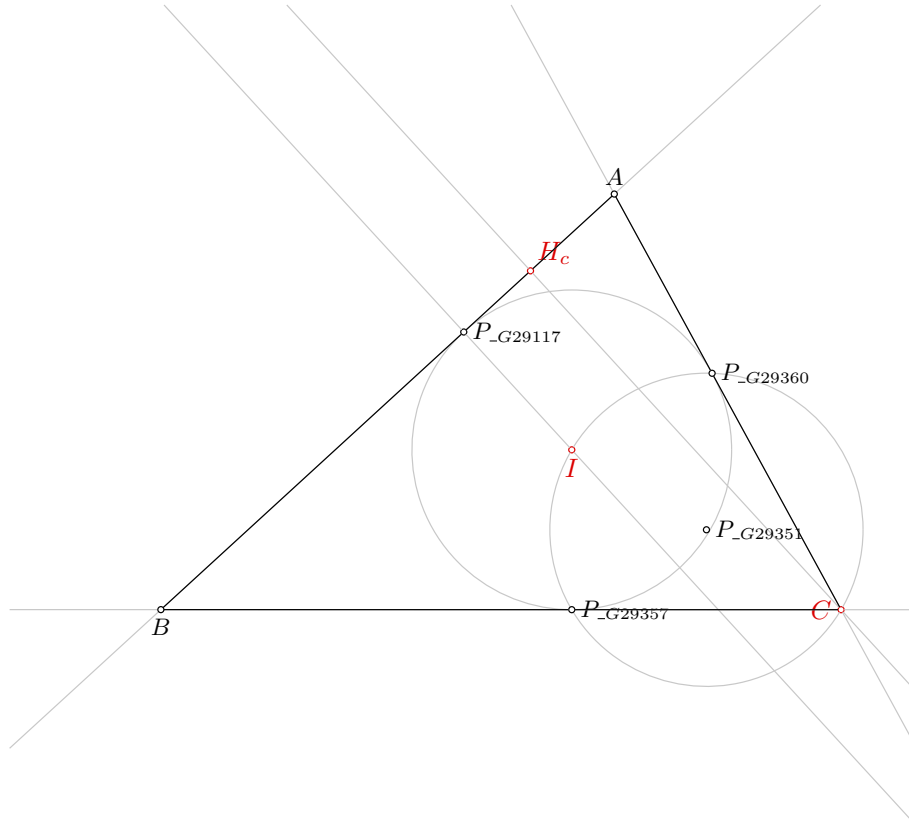


Figure 1: Illustration of the problem 0264

*% Determination conditions: lines c and b are not the same; lines a and c are not the same; points C and H\_{c} are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.085 seconds.

**NDG conditions** Points  $A$ ,  $B$  and  $C$  are not collinear

#### 4.1.3 Proving $I = \neg I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C = C$

NDG conditions are:

$S_{CH_cT_c^0} \neq S_{P_{G30219}H_cT_c^0}$  i.e., lines  $CP_{G30219}$  and  $H_cT_c^0$  are not parallel (construction based assumption)

$S_{H_cCP_{G30222}} \neq S_{T_c^0CP_{G30222}}$  i.e., lines  $H_cT_c^0$  and  $CP_{G30222}$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^1} \neq S_{BCF_{h_c}^1}$  i.e., lines  $AB$  and  $CF_{h_c}^1$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.2.3 Proving $I = \neg I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I = \neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 234 terms.

**Time Complexity:** Time spent by the prover is 0.240 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $H_c=_Hc$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.4.3 Proving $I=_I$

Proving failed

## Problem 265

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 265: Given a point  $C$ , a point  $H$  and a point  $T_a$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 266

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 266: Given a point  $C$ , a point  $H$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 267

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 267: Given a point  $C$ , a point  $H$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.



## Problem 268

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 268: Given a point  $C$ , a point  $H$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 269

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 269: Given a point  $C$ , a point  $T_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 270

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 270: Given a point  $C$ , a point  $T_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $T_a$  are not the same;
2. Using the point  $T_c$  and the line  $a$ , construct a circle  $k(T_c, foot[T_c, a])$  (rule W11); % NDG: point  $T_c$  is not incident to the line  $a$ ;
3. Using the circle  $k(T_c, foot[T_c, a])$ , the point  $C$ , the point  $T_c$  and the line  $a$ , construct a line  $b$  (rule W13); % NDG: point  $C$  is outside the circle  $k(T_c, foot[T_c, a])$ ;
4. Using the point  $T_a$  and the line  $b$ , construct a circle  $k(T_a, foot[T_a, b])$  (rule W11); % NDG: point  $T_a$  is not incident to the line  $b$ ;
5. Using the circle  $k(T_a, foot[T_a, b])$ , the point  $T_c$  and the point  $T_a$ , construct a line  $c1$  and a line  $c$  (rule W12); % NDG: point  $T_c$  is outside the circle  $k(T_a, foot[T_a, b])$ ;
6. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same;
7. Using the line  $b$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $c$  are not parallel % DET: lines  $b$  and  $c$  are not the same.

Non-degenerate conditions: lines  $b$  and  $c$  are not parallel; lines  $c$  and  $a$  are not parallel; point  $T_c$  is outside the circle  $k(T_a, foot[T_a, b])$ ; point  $T_a$  is not incident to the line  $b$ ; point  $C$  is outside the circle  $k(T_c, foot[T_c, a])$ ; point  $T_c$  is not incident to the line  $a$ .

Determination conditions: lines  $b$  and  $c$  are not the same; lines  $c$  and  $a$  are not the same; points  $C$  and  $T_a$  are not the same.

Rules used: [W02,W03,W11,W12,W13]

Lemmas used: [D23,D25,D36,GD01,GD02,GL10,GL11,L10,L8]

Solving time: 0.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point T_{a} 70.86 40
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_l C
cmark_rb T_{a}
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points C and T_{a} are not the same
% Constructing a line a which passes through point C and point T_{a}
line a C T_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: point T_{c} is not incident to the line a
% Constructing a point P_{\_G59480} which is a foot of the point T_{c} on the line a
foot P_{\_G59480} T_{c} a
cmark_r P_{\_G59480}
color 200 200 200
drawline T_{c} P_{\_G59480}
color 0 0 0

% Constructing a circle k(T_{c},foot[T_{c},b]) whose center is at point T_{c} and which passes
    through point P_{\_G59480}
circle k(T_{c},foot[T_{c},b]) T_{c} P_{\_G59480}

color 200 200 200
drawcircle k(T_{c},foot[T_{c},b])
color 0 0 0

% NDG: point C is outside the circle k(T_{c},foot[T_{c},b])
% Constructing a point P_{\_G59871} which is a foot of the point T_{c} on the line a
foot P_{\_G59871} T_{c} a
cmark_r P_{\_G59871}
color 200 200 200
drawline T_{c} P_{\_G59871}
color 0 0 0

% Constructing a line L_{\_G59874} which passes through point T_{c} and point C
line L_{\_G59874} T_{c} C
```

```

color 200 200 200
drawline L_{\_G59874}
color 0 0 0

% Constructing a point P_{\_G59877} which is an image of the point P_{\_G59871} in the symmetry to
point/line L_{\_G59874}
sim P_{\_G59877} L_{\_G59874} P_{\_G59871}
cmark_r P_{\_G59877}

% Constructing a line b which passes through point C and point P_{\_G59877}
line b C P_{\_G59877}

color 200 200 200
drawline b
color 0 0 0

% NDG: point T_{a} is not incident to the line b
% Constructing a point P_{\_G60158} which is a foot of the point T_{a} on the line b
foot P_{\_G60158} T_{a} b
cmark_r P_{\_G60158}
color 200 200 200
drawline T_{a} P_{\_G60158}
color 0 0 0

% Constructing a circle k(T_{a},foot[T_{a},c]) whose center is at point T_{a} and which passes
through point P_{\_G60158}
circle k(T_{a},foot[T_{a},c]) T_{a} P_{\_G60158}

color 200 200 200
drawcircle k(T_{a},foot[T_{a},c])
color 0 0 0

% NDG: point T_{c} is outside the circle k(T_{a},foot[T_{a},c])
% Constructing midpoint P_{\_G60576} of the segment T_{c}T_{a}
midpoint P_{\_G60576} T_{c} T_{a}
cmark_r P_{\_G60576}

% Constructing a circle C_{\_G60579} whose center is at point P_{\_G60576} and which passes through
point T_{c}
circle C_{\_G60579} P_{\_G60576} T_{c}

color 200 200 200
drawcircle C_{\_G60579}
color 0 0 0

% Constructing points P_{\_G60582} and P_{\_G60585} which are in intersection of C_{\_G60579} and k
(T_{a},foot[T_{a},c])
intersec2 P_{\_G60582} P_{\_G60585} C_{\_G60579} k(T_{a},foot[T_{a},c])
cmark_r P_{\_G60582}
cmark_r P_{\_G60585}

```

```

% Constructing a line c1 which passes through point T_{c} and point P_{\_G60582}
line c1 T_{c} P_{\_G60582}

color 200 200 200
drawline c1
color 0 0 0

% Constructing a line c which passes through point T_{c} and point P_{\_G60585}
line c T_{c} P_{\_G60585}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

% NDG: lines b and c are not parallel% DET: lines b and c are not the same
% Constructing a point A which belongs to line b and line c
intersec A b c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and c are not parallel; lines c and a are not parallel; point
  T_{c} is outside the circle k(T_{a},foot[T_{a},c]); point T_{a} is not incident to the line b;
  point C is outside the circle k(T_{c},foot[T_{c},b]); point T_{c} is not incident to the line a
% Determination conditions: lines b and c are not the same; lines c and a are not the same; points
  C and T_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

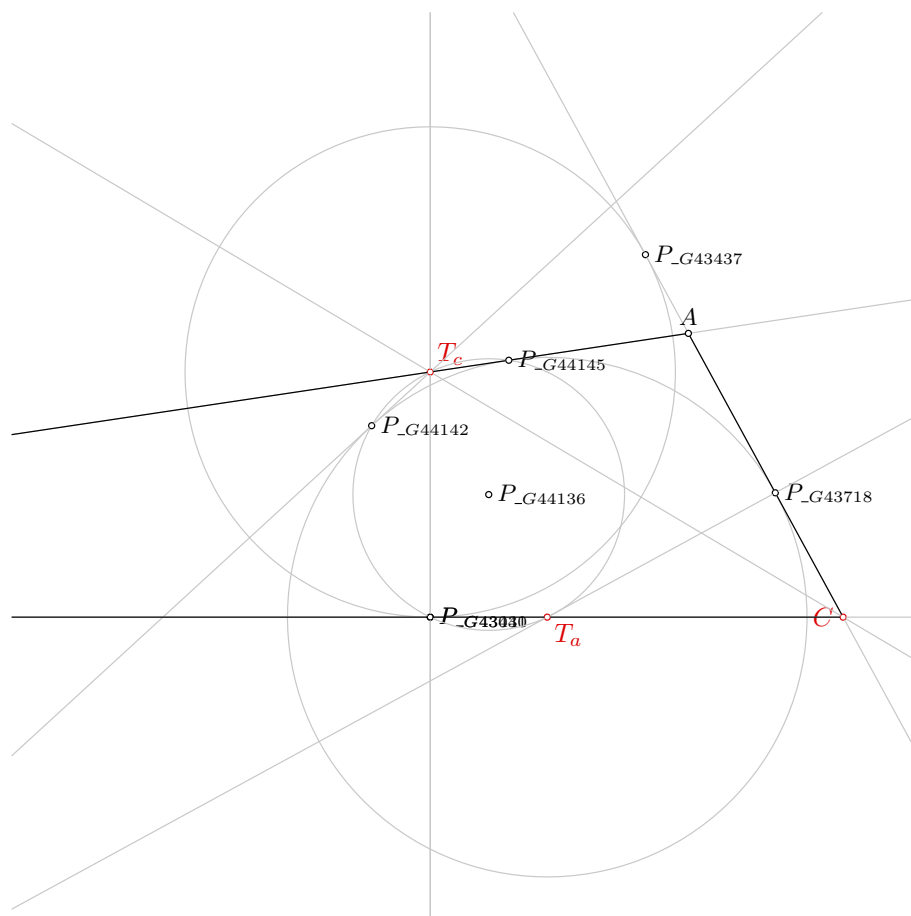


Figure 1: Illustration of the problem 0270

**Time Complexity:** Time spent by the prover is 0.03 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_a = T_a$

Proving failed

#### 4.1.3 Proving $T_c = T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C = C$

NDG conditions are:

$S_{T_c C T_a} \neq S_{P_{G46017} C T_a}$  i.e., lines  $T_c P_{G46017}$  and  $C T_a$  are not parallel (construction based assumption)  
 $S_{C T_c P_{G46017}} \neq S_{P_{G45309} T_c P_{G46017}}$  i.e., lines  $C P_{G45309}$  and  $T_c P_{G46017}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $T_a = T_a$

Proving failed

#### 4.2.3 Proving $T_c = T_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $T_a = T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 20 terms.

**Time Complexity:** Time spent by the prover is 0.040 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_c = T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 30 terms.

**Time Complexity:** Time spent by the prover is 0.080 seconds. There are no ndg conditions.



## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $T_a=T_a$

Proving failed

### 4.4.3 Proving $T_c=T_c$

Proving failed

# Problem 271

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 271: Given a point  $C$ , a point  $T_a$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $T_a$  are not the same;
2. Using the point  $T_a$  and the point  $I$ , construct a line  $s_a$  (rule W02); % DET: points  $T_a$  and  $I$  are not the same;
3. Using the point  $I$  and the line  $a$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $a$ ;
4. Using the circle  $k(I, P_a)$ , the point  $C$ , the point  $I$  and the line  $a$ , construct a line  $b$  (rule W13); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
5. Using the line  $b$  and the line  $s_a$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $s_a$  are not parallel % DET: lines  $b$  and  $s_a$  are not the same;
6. Using the circle  $k(I, P_a)$ , the point  $A$ , the point  $I$  and the line  $b$ , construct a line  $c$  (rule W13); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
7. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(I, P_a)$ ; lines  $b$  and  $s_a$  are not parallel; point  $C$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $a$ .

Determination conditions: lines  $c$  and  $a$  are not the same; lines  $b$  and  $s_a$  are not the same; points  $T_a$  and  $I$  are not the same; points  $C$  and  $T_a$  are not the same.

Rules used: [W02,W03,W11,W13]

Lemmas used: [D2,D23,D27,GD01,L5,L59,L60,L61]

Solving time: 13.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point T_{a} 70.86 40
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_l C
cmark_rb T_{a}
cmark_b I
color 0 0 0
fontsize 8

% DET: points C and T_{a} are not the same
% Constructing a line a which passes through point C and point T_{a}
line a C T_{a}

color 200 200 200
drawline a
color 0 0 0

% DET: points T_{a} and I are not the same
% Constructing a line s_{a} which passes through point T_{a} and point I
line s_{a} T_{a} I

color 200 200 200
drawline s_{a}
color 0 0 0

% NDG: point I is not incident to the line a
% Constructing a point P_{\_G47451} which is a foot of the point I on the line a
foot P_{\_G47451} I a
cmark_r P_{\_G47451}
color 200 200 200
drawline I P_{\_G47451}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
47451}
circle k(I,P_{a}) I P_{\_G47451}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point C is outside the circle k(I,P_{a})
```

```

% Constructing a point  $P_{\{G47658\}}$  which is a foot of the point  $I$  on the line  $a$ 
foot  $P_{\{G47658\}}$  I a
cmark_r  $P_{\{G47658\}}$ 
color 200 200 200
drawline I  $P_{\{G47658\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G47661\}}$  which passes through point  $I$  and point  $C$ 
line  $L_{\{G47661\}}$  I C

color 200 200 200
drawline  $L_{\{G47661\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G47664\}}$  which is an image of the point  $P_{\{G47658\}}$  in the symmetry to
point/line  $L_{\{G47661\}}$ 
sim  $P_{\{G47664\}}$   $L_{\{G47661\}}$   $P_{\{G47658\}}$ 
cmark_r  $P_{\{G47664\}}$ 

% Constructing a line  $b$  which passes through point  $C$  and point  $P_{\{G47664\}}$ 
line b C  $P_{\{G47664\}}$ 

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $b$  and  $s_{\{a\}}$  are not parallel% DET: lines  $b$  and  $s_{\{a\}}$  are not the same
% Constructing a point  $A$  which belongs to line  $b$  and line  $s_{\{a\}}$ 
intersec A b  $s_{\{a\}}$ 
cmark_t A

% NDG: point  $A$  is outside the circle  $k(I, P_{\{a\}})$ 
% Constructing a point  $P_{\{G48050\}}$  which is a foot of the point  $I$  on the line  $b$ 
foot  $P_{\{G48050\}}$  I b
cmark_r  $P_{\{G48050\}}$ 
color 200 200 200
drawline I  $P_{\{G48050\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G48053\}}$  which passes through point  $I$  and point  $A$ 
line  $L_{\{G48053\}}$  I A

color 200 200 200
drawline  $L_{\{G48053\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G48056\}}$  which is an image of the point  $P_{\{G48050\}}$  in the symmetry to
point/line  $L_{\{G48053\}}$ 
sim  $P_{\{G48056\}}$   $L_{\{G48053\}}$   $P_{\{G48050\}}$ 
cmark_r  $P_{\{G48056\}}$ 

```

```

% Constructing a line c which passes through point A and point P_{\_G48056}
line c A P_{\_G48056}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and a are not parallel; point A is outside the circle k(I,P_{a
}); lines b and s_{a} are not parallel; point C is outside the circle k(I,P_{a}); point I is
not incident to the line a
% Determination conditions: lines c and a are not the same; lines b and s_{a} are not the same;
points T_{a} and I are not the same; points C and T_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_a=T_a$

Proving failed

#### 4.1.3 Proving $I=I$

Proving failed

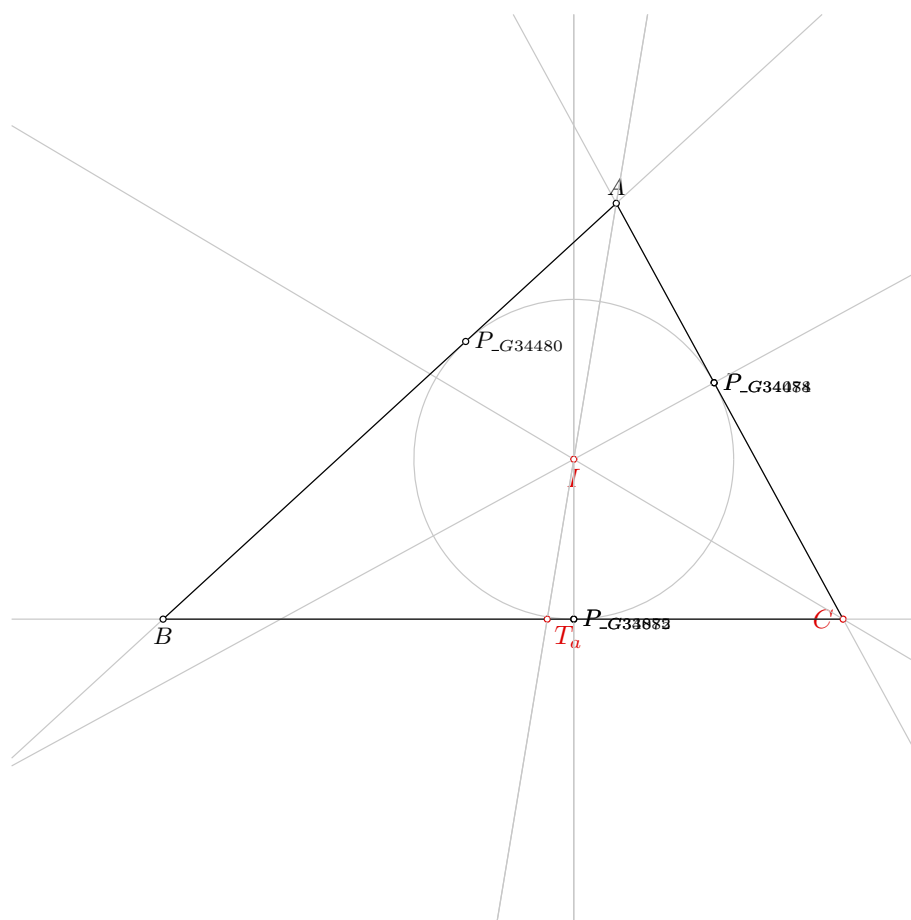


Figure 1: Illustration of the problem 0271

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{CT_a I} \neq S_{P_{G35223} T_a I}$  i.e., lines  $CP_{G35223}$  and  $T_a I$  are not parallel (construction based assumption)

$S_{ACT_a} \neq S_{P_{G35615} CT_a}$  i.e., lines  $AP_{G35615}$  and  $CT_a$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $T_a = \neg T_a$

Proving failed

### 4.2.3 Proving $I = \neg I$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.2 Proving $T_a = \neg T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 16 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $I = \neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 457 terms.

**Time Complexity:** Time spent by the prover is 0.230 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $T_a = \neg T_a$

Proving failed

### 4.4.3 Proving $I = \neg I$

Proving failed

# Problem 272

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 272: Given a point  $C$ , a point  $T_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $T_b$  are not the same;
2. Using the point  $T_c$  and the line  $b$ , construct a circle  $k(T_c, foot[T_c, b])$  (rule W11); % NDG: point  $T_c$  is not incident to the line  $b$ ;
3. Using the circle  $k(T_c, foot[T_c, b])$ , the point  $C$ , the point  $T_c$  and the line  $b$ , construct a line  $a$  (rule W13); % NDG: point  $C$  is outside the circle  $k(T_c, foot[T_c, b])$ ;
4. Using the point  $T_b$  and the line  $a$ , construct a circle  $k(T_b, foot[T_b, a])$  (rule W11); % NDG: point  $T_b$  is not incident to the line  $a$ ;
5. Using the circle  $k(T_b, foot[T_b, a])$ , the point  $T_c$  and the point  $T_b$ , construct a line  $c_2$  and a line  $c$  (rule W12); % NDG: point  $T_c$  is outside the circle  $k(T_b, foot[T_b, a])$ ;
6. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same;
7. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; lines  $c$  and  $b$  are not parallel; point  $T_c$  is outside the circle  $k(T_b, foot[T_b, a])$ ; point  $T_b$  is not incident to the line  $a$ ; point  $C$  is outside the circle  $k(T_c, foot[T_c, b])$ ; point  $T_c$  is not incident to the line  $b$ .

Determination conditions: lines  $a$  and  $c$  are not the same; lines  $c$  and  $b$  are not the same; points  $C$  and  $T_b$  are not the same.

Rules used: [W02,W03,W11,W12,W13]

Lemmas used: [D24,D25,D37,GD01,GD02,GL10,GL11,L10,L9]

Solving time: 0.9 seconds.



## 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point T_{b} 94.25 68.88
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_l C
cmark_t T_{b}
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points C and T_{b} are not the same
% Constructing a line b which passes through point C and point T_{b}
line b C T_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: point T_{c} is not incident to the line b
% Constructing a point P_{\_G37875} which is a foot of the point T_{c} on the line b
foot P_{\_G37875} T_{c} b
cmark_r P_{\_G37875}
color 200 200 200
drawline T_{c} P_{\_G37875}
color 0 0 0

% Constructing a circle k(T_{c},foot[T_{c},b]) whose center is at point T_{c} and which passes
    through point P_{\_G37875}
circle k(T_{c},foot[T_{c},b]) T_{c} P_{\_G37875}

color 200 200 200
drawcircle k(T_{c},foot[T_{c},b])
color 0 0 0

% NDG: point C is outside the circle k(T_{c},foot[T_{c},b])
% Constructing a point P_{\_G38266} which is a foot of the point T_{c} on the line b
foot P_{\_G38266} T_{c} b
cmark_r P_{\_G38266}
color 200 200 200
drawline T_{c} P_{\_G38266}
color 0 0 0

% Constructing a line L_{\_G38269} which passes through point T_{c} and point C
line L_{\_G38269} T_{c} C
```

```

color 200 200 200
drawline L_{\_G38269}
color 0 0 0

% Constructing a point P_{\_G38272} which is an image of the point P_{\_G38266} in the symmetry to
point/line L_{\_G38269}
sim P_{\_G38272} L_{\_G38269} P_{\_G38266}
cmark_r P_{\_G38272}

% Constructing a line a which passes through point C and point P_{\_G38272}
line a C P_{\_G38272}

color 200 200 200
drawline a
color 0 0 0

% NDG: point T_{b} is not incident to the line a
% Constructing a point P_{\_G38553} which is a foot of the point T_{b} on the line a
foot P_{\_G38553} T_{b} a
cmark_r P_{\_G38553}
color 200 200 200
drawline T_{b} P_{\_G38553}
color 0 0 0

% Constructing a circle k(T_{b},foot[T_{b},c]) whose center is at point T_{b} and which passes
through point P_{\_G38553}
circle k(T_{b},foot[T_{b},c]) T_{b} P_{\_G38553}

color 200 200 200
drawcircle k(T_{b},foot[T_{b},c])
color 0 0 0

% NDG: point T_{c} is outside the circle k(T_{b},foot[T_{b},c])
% Constructing midpoint P_{\_G38971} of the segment T_{c}T_{b}
midpoint P_{\_G38971} T_{c} T_{b}
cmark_r P_{\_G38971}

% Constructing a circle C_{\_G38974} whose center is at point P_{\_G38971} and which passes through
point T_{c}
circle C_{\_G38974} P_{\_G38971} T_{c}

color 200 200 200
drawcircle C_{\_G38974}
color 0 0 0

% Constructing points P_{\_G38977} and P_{\_G38980} which are in intersection of C_{\_G38974} and k
(T_{b},foot[T_{b},c])
intersec2 P_{\_G38977} P_{\_G38980} C_{\_G38974} k(T_{b},foot[T_{b},c])
cmark_r P_{\_G38977}
cmark_r P_{\_G38980}

```

```

% Constructing a line c2 which passes through point T_{c} and point P_{\_G38977}
line c2 T_{c} P_{\_G38977}

color 200 200 200
drawline c2
color 0 0 0

% Constructing a line c which passes through point T_{c} and point P_{\_G38980}
line c T_{c} P_{\_G38980}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A

% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and c are not parallel; lines c and b are not parallel; point
  T_{c} is outside the circle k(T_{b},foot[T_{b},c]); point T_{b} is not incident to the line a;
  point C is outside the circle k(T_{c},foot[T_{c},b]); point T_{c} is not incident to the line b
% Determination conditions: lines a and c are not the same; lines c and b are not the same; points
  C and T_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

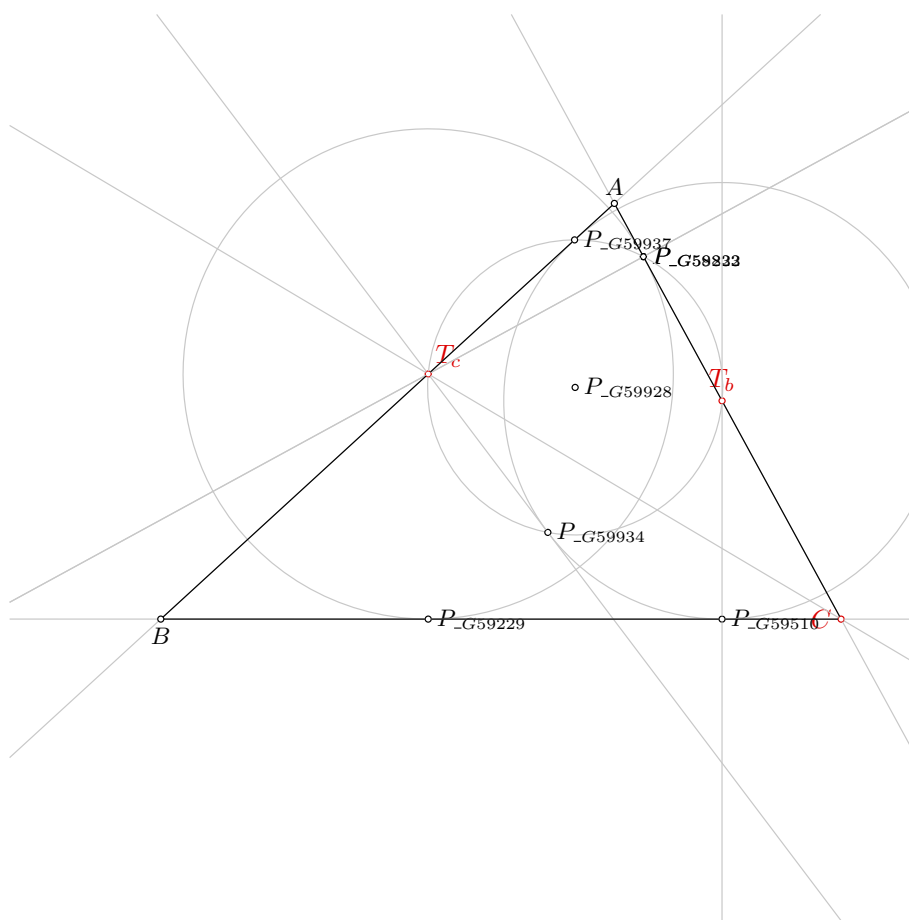


Figure 1: Illustration of the problem 0272

**Time Complexity:** Time spent by the prover is 0.032 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.1.3 Proving $T_c = \neg T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $C = C$

NDG conditions are:

$S_{T_c C T_b} \neq S_{P_{G61812} C T_b}$  i.e., lines  $T_c P_{G61812}$  and  $C T_b$  are not parallel (construction based assumption)

$S_{C T_c P_{G61812}} \neq S_{P_{G61104} T_c P_{G61812}}$  i.e., lines  $C P_{G61104}$  and  $T_c P_{G61812}$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.2.3 Proving $T_c = \neg T_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $C = C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $T_b = \neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 20 terms.

**Time Complexity:** Time spent by the prover is 0.050 seconds. There are no ndg conditions.

#### 4.3.3 Proving $T_c = \neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 30 terms.

**Time Complexity:** Time spent by the prover is 0.090 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $T_b=\neg T_b$

Proving failed

### 4.4.3 Proving $T_c=\neg T_c$

Proving failed

# Problem 273

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 273: Given a point  $C$ , a point  $T_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $C$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $T_b$  are not the same;
2. Using the point  $T_b$  and the point  $I$ , construct a line  $s_b$  (rule W02); % DET: points  $T_b$  and  $I$  are not the same;
3. Using the point  $I$  and the line  $b$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $b$ ;
4. Using the circle  $k(I, P_a)$ , the point  $C$ , the point  $I$  and the line  $b$ , construct a line  $a$  (rule W13); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
5. Using the line  $a$  and the line  $s_b$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $s_b$  are not parallel % DET: lines  $a$  and  $s_b$  are not the same;
6. Using the circle  $k(I, P_a)$ , the point  $B$ , the point  $I$  and the line  $a$ , construct a line  $c$  (rule W13); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
7. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; point  $B$  is outside the circle  $k(I, P_a)$ ; lines  $a$  and  $s_b$  are not parallel; point  $C$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $b$ .

Determination conditions: lines  $c$  and  $b$  are not the same; lines  $a$  and  $s_b$  are not the same; points  $T_b$  and  $I$  are not the same; points  $C$  and  $T_b$  are not the same.

Rules used: [W02,W03,W11,W13]

Lemmas used: [D2,D24,D27,GD01,L59,L6,L60,L61]

Solving time: 13.1 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point T_{b} 94.25 68.88
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_l C
cmark_t T_{b}
cmark_b I
color 0 0 0
fontsize 8

% DET: points C and T_{b} are not the same
% Constructing a line b which passes through point C and point T_{b}
line b C T_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points T_{b} and I are not the same
% Constructing a line s_{b} which passes through point T_{b} and point I
line s_{b} T_{b} I

color 200 200 200
drawline s_{b}
color 0 0 0

% NDG: point I is not incident to the line b
% Constructing a point P_{\_G64878} which is a foot of the point I on the line b
foot P_{\_G64878} I b
cmark_r P_{\_G64878}
color 200 200 200
drawline I P_{\_G64878}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
64878}
circle k(I,P_{a}) I P_{\_G64878}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point C is outside the circle k(I,P_{a})
```



```

% Constructing a point  $P_{\{G28568\}}$  which is a foot of the point  $I$  on the line  $b$ 
foot  $P_{\{G28568\}}$  I b
cmark_r  $P_{\{G28568\}}$ 
color 200 200 200
drawline I  $P_{\{G28568\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G28571\}}$  which passes through point  $I$  and point  $C$ 
line  $L_{\{G28571\}}$  I C

color 200 200 200
drawline  $L_{\{G28571\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G28574\}}$  which is an image of the point  $P_{\{G28568\}}$  in the symmetry to
point/line  $L_{\{G28571\}}$ 
sim  $P_{\{G28574\}}$   $L_{\{G28571\}}$   $P_{\{G28568\}}$ 
cmark_r  $P_{\{G28574\}}$ 

% Constructing a line  $a$  which passes through point  $C$  and point  $P_{\{G28574\}}$ 
line a C  $P_{\{G28574\}}$ 

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $a$  and  $s_{\{b\}}$  are not parallel% DET: lines  $a$  and  $s_{\{b\}}$  are not the same
% Constructing a point  $B$  which belongs to line  $a$  and line  $s_{\{b\}}$ 
intersec B a  $s_{\{b\}}$ 
cmark_b B

% NDG: point  $B$  is outside the circle  $k(I, P_{\{a\}})$ 
% Constructing a point  $P_{\{G28960\}}$  which is a foot of the point  $I$  on the line  $a$ 
foot  $P_{\{G28960\}}$  I a
cmark_r  $P_{\{G28960\}}$ 
color 200 200 200
drawline I  $P_{\{G28960\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G28963\}}$  which passes through point  $I$  and point  $B$ 
line  $L_{\{G28963\}}$  I B

color 200 200 200
drawline  $L_{\{G28963\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G28966\}}$  which is an image of the point  $P_{\{G28960\}}$  in the symmetry to
point/line  $L_{\{G28963\}}$ 
sim  $P_{\{G28966\}}$   $L_{\{G28963\}}$   $P_{\{G28960\}}$ 
cmark_r  $P_{\{G28966\}}$ 

```

```

% Constructing a line c which passes through point B and point P_{\_G28966}
line c B P_{\_G28966}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and b are not parallel; point B is outside the circle k(I,P_{a
}); lines a and s_{b} are not parallel; point C is outside the circle k(I,P_{a}); point I is
not incident to the line b
% Determination conditions: lines c and b are not the same; lines a and s_{b} are not the same;
points T_{b} and I are not the same; points C and T_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $C=C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.027 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_b=_T T_b$

Proving failed

#### 4.1.3 Proving $I=_I I$

Proving failed

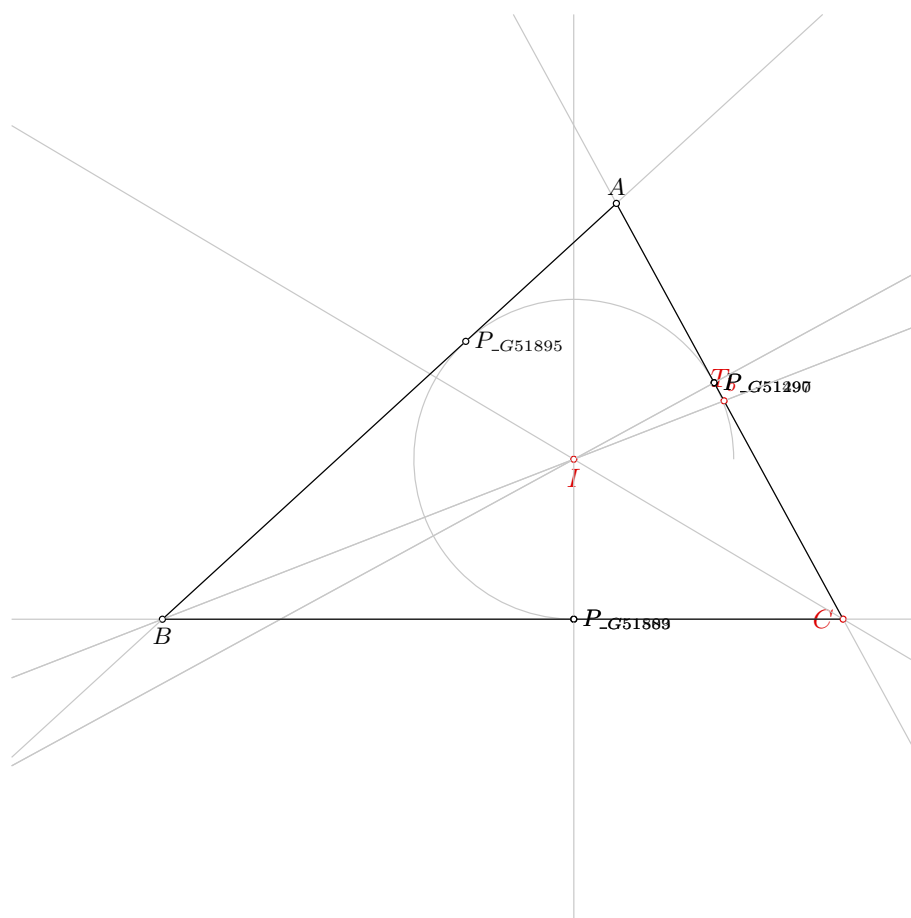


Figure 1: Illustration of the problem 0273

## 4.2 GCLC - Area method

### 4.2.1 Proving $C=C$

NDG conditions are:

$S_{CT_bI} \neq S_{P_{G52641}T_bI}$  i.e., lines  $CP_{G52641}$  and  $T_bI$  are not parallel (construction based assumption)

$S_{BCT_b} \neq S_{P_{G53033}CT_b}$  i.e., lines  $BP_{G53033}$  and  $CT_b$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $T_b=\neg T_b$

Proving failed

### 4.2.3 Proving $I=\neg I$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.2 Proving $T_b=\neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 16 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

### 4.3.3 Proving $I=\neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 457 terms.

**Time Complexity:** Time spent by the prover is 0.240 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $T_b=\neg T_b$

Proving failed

### 4.4.3 Proving $I=\neg I$

Proving failed

# Problem 274

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 274: Given a point  $T_c$ , a point  $I$  and a point  $C$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $T_c$  and the point  $I$ , construct a line  $s_c$  (rule W02); % DET: points  $T_c$  and  $I$  are not the same;
2. Choose freely a point  $C$  on the line  $s_c$  (rule WOnline1) ;
3. Using the point  $C$  and the point  $I$ , construct a line  $m(CI)$  (rule W14); % DET: points  $C$  and  $I$  are not the same;
4. Choose freely a point  $A$  (rule free);
5. Using the point  $A$  and the point  $C$ , construct a point  $M_b$  (rule W01); ;
6. Using the point  $A$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $C$  are not the same;
7. Using the point  $A$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $T_c$  are not the same;
8. Using the point  $M_b$  and the line  $b$ , construct a line  $m_b$  (rule W10b); ;
9. Using the line  $m_b$  and the line  $m(CI)$ , construct a point  $N_b$  (rule W03); % NDG: lines  $m_b$  and  $m(CI)$  are not parallel % DET: lines  $m_b$  and  $m(CI)$  are not the same;
10. Using the point  $N_b$  and the point  $I$ , construct a line  $s_b$  (rule W02); % DET: points  $N_b$  and  $I$  are not the same;
11. Using the line  $c$  and the line  $s_b$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $s_b$  are not parallel % DET: lines  $c$  and  $s_b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $s_b$  are not parallel; lines  $m_b$  and  $m(CI)$  are not parallel.

Determination conditions: lines  $c$  and  $s_b$  are not the same; points  $N_b$  and  $I$  are not the same; lines  $m_b$  and  $m(CI)$  are not the same; points  $A$  and  $T_c$  are not the same; points  $A$  and  $C$  are not the same; points  $C$  and  $I$  are not the same; points  $T_c$  and  $I$  are not the same.

Rules used: [W01,W02,W03,W10b,W14,WOnline1,free]

Lemmas used: [D12,D2,D22,D25,D48,GD01,GD02,GL01,GL04,L2,L26,L33,L34,L6]

Solving time: 5.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point T_{c} 55.38 72.43
point I 74.37 61.15
point C 110 40

color 220 0 0
fontsize 9

cmark_rt T_{c}
cmark_b I
cmark_l C
color 0 0 0
fontsize 8

% DET: points T_{c} and I are not the same
% Constructing a line s_{c} which passes through point T_{c} and point I
line s_{c} T_{c} I

color 200 200 200
drawline s_{c}
color 0 0 0

% Choosing randomly a point C on the line T_{c}I
online C T_{c} I
cmark_l C
color 200 200 200
drawline T_{c} I
color 0 0 0

% DET: points C and I are not the same
% Constructing bisector m(CI) of the segment CI
med m(CI) C I

color 200 200 200
drawline m(CI)
color 0 0 0

color 200 200 200
drawsegment C I
```

```
color 0 0 0
```

```
% Constructing a free point A  
point A 80 95
```

```
cmark_t A
```

```
% Constructing a point  $M_{\{b\}}$  such that  $AM_{\{b\}}/AC=0.5$   
towards  $M_{\{b\}}$  A C 0.5  
cmark_lt  $M_{\{b\}}$   
color 200 200 200  
drawsegment A C  
color 0 0 0
```

```
% DET: points A and C are not the same  
% Constructing a line b which passes through point A and point C  
line b A C
```

```
color 200 200 200  
drawline b  
color 0 0 0
```

```
% DET: points A and  $T_{\{c\}}$  are not the same  
% Constructing a line c which passes through point A and point  $T_{\{c\}}$   
line c A  $T_{\{c\}}$ 
```

```
color 200 200 200  
drawline c  
color 0 0 0
```

```
% Constructing a line  $m_{\{b\}}$  which is perpendicular to line b and which passes through point  $M_{\{b\}}$   
perp  $m_{\{b\}}$   $M_{\{b\}}$  b
```

```
color 200 200 200  
drawline  $m_{\{b\}}$   
color 0 0 0
```

```
% NDG: lines  $m_{\{b\}}$  and  $m(CI)$  are not parallel  
% DET: lines  $m_{\{b\}}$  and  $m(CI)$  are not the same  
% Constructing a point  $N_{\{b\}}$  which belongs to line  $m_{\{b\}}$  and line  $m(CI)$   
intersec  $N_{\{b\}}$   $m_{\{b\}}$   $m(CI)$   
cmark_rb  $N_{\{b\}}$ 
```

```
% DET: points  $N_{\{b\}}$  and I are not the same
```

```

% Constructing a line  $s_{\{b\}}$  which passes through point  $N_{\{b\}}$  and point  $I$ 
line  $s_{\{b\}}$   $N_{\{b\}}$   $I$ 

color 200 200 200
drawline  $s_{\{b\}}$ 
color 0 0 0

% NDG: lines  $c$  and  $s_{\{b\}}$  are not parallel% DET: lines  $c$  and  $s_{\{b\}}$  are not the same
% Constructing a point  $B$  which belongs to line  $c$  and line  $s_{\{b\}}$ 
intersec  $B$   $c$   $s_{\{b\}}$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $c$  and  $s_{\{b\}}$  are not parallel; lines  $m_{\{b\}}$  and  $m(CI)$  are not
parallel
% Determination conditions: lines  $c$  and  $s_{\{b\}}$  are not the same; points  $N_{\{b\}}$  and  $I$  are not the same
; lines  $m_{\{b\}}$  and  $m(CI)$  are not the same; points  $A$  and  $T_{\{c\}}$  are not the same; points  $A$  and  $C$ 
are not the same; points  $C$  and  $I$  are not the same; points  $T_{\{c\}}$  and  $I$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $T_c = T_c$

Proving failed

#### 4.1.2 Proving $I = I$

Proving failed

#### 4.1.3 Proving $C = C$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 0 terms.

**Time Complexity:** Time spent by the prover is 0.007 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.2 GCLC - Area method

#### 4.2.1 Proving $T_c = T_c$

Proving failed



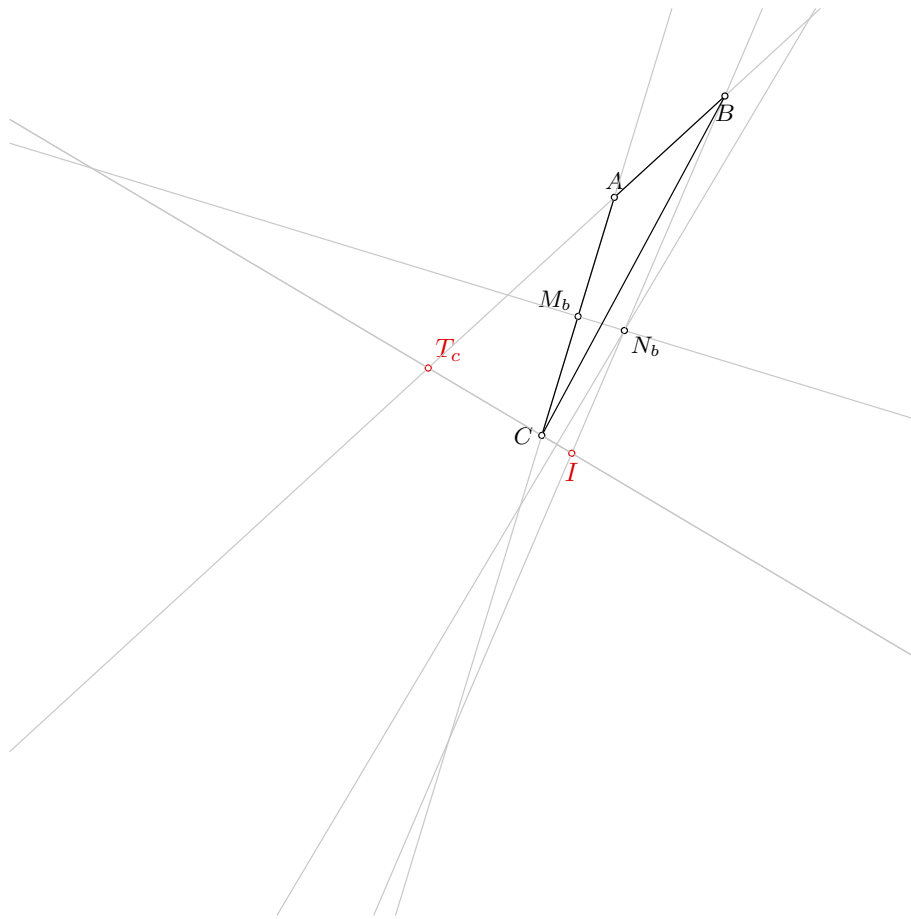


Figure 1: Illustration of the problem 0274

#### 4.2.2 Proving $I=J$

Proving failed

#### 4.2.3 Proving $C=C$

NDG conditions are:

$S_{M_bAC} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bM_{m(CI)}^1T_{m(CI)}^2} \neq S_{F_{m_b}^3M_{m(CI)}^1T_{m(CI)}^2}$  i.e., lines  $M_bF_{m_b}^3$  and  $M_{m(CI)}^1T_{m(CI)}^2$  are not parallel (construction based assumption)

$S_{AN_bI} \neq S_{T_cN_bI}$  i.e., lines  $AT_c$  and  $N_bI$  are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $T_c=T_c$

Proving failed

#### 4.3.2 Proving $I=J$

Proving failed

#### 4.3.3 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.050 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $T_c=T_c$

Proving failed

#### 4.4.2 Proving $I=J$

Proving failed

#### 4.4.3 Proving $C=C$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.050 seconds. There are no ndg conditions.

# Problem 275

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 275: Given a point  $O$ , a point  $M_a$  and a point  $M_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $M_a$ , construct a line  $m_a$  (rule W02); % DET: points  $O$  and  $M_a$  are not the same;
2. Using the point  $O$  and the point  $M_b$ , construct a line  $m_b$  (rule W02); % DET: points  $O$  and  $M_b$  are not the same;
3. Using the point  $M_a$  and the line  $m_a$ , construct a line  $a$  (rule W10a); ;
4. Using the point  $M_b$  and the line  $m_b$ , construct a line  $b$  (rule W10a); ;
5. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same;
6. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
7. Using the point  $M_b$  and the point  $C$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel.

Determination conditions: lines  $b$  and  $a$  are not the same; points  $O$  and  $M_b$  are not the same; points  $O$  and  $M_a$  are not the same.

Rules used: [W01,W02,W03,W10a]

Lemmas used: [D1,D11,D12,D21,D22,GD01,GL03,GL04,GL09]

Solving time: 0.3 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point O 65 51.14
point M_{a} 65 40
point M_{b} 95 67.5

color 220 0 0
fontsize 9

cmark_t O
cmark_r M_{a}
cmark_lt M_{b}
color 0 0 0
fontsize 8

% DET: points O and M_{a} are not the same
% Constructing a line m_{a} which passes through point O and point M_{a}
line m_{a} O M_{a}

color 200 200 200
drawline m_{a}
color 0 0 0

% DET: points O and M_{b} are not the same
% Constructing a line m_{b} which passes through point O and point M_{b}
line m_{b} O M_{b}

color 200 200 200
drawline m_{b}
color 0 0 0

% Constructing a line a which is perpendicular to line m_{a} and which passes through point M_{a}
perp a M_{a} m_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line b which is perpendicular to line m_{b} and which passes through point M_{b}
perp b M_{b} m_{b}

color 200 200 200
drawline b
color 0 0 0
```

```

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

% Constructing a point B such that  $CB/CM_{\{a\}}=2$ 
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% Constructing a point A such that  $M_{\{b\}}A/M_{\{b\}}C=-1$ 
towards A M_{b} C -1
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and a are not parallel
% Determination conditions: lines b and a are not the same; points O and  $M_{\{b\}}$  are not the same;
  points O and  $M_{\{a\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 79 terms.

**Time Complexity:** Time spent by the prover is 0.56 seconds.

**NDG conditions** Points  $M_b$  and  $O$  are not identical

Points  $B$ ,  $C$  and  $O$  are not collinear

Line through points  $O$  and  $M_a$  is not perpendicular to line through points  $M_a$  and  $A$

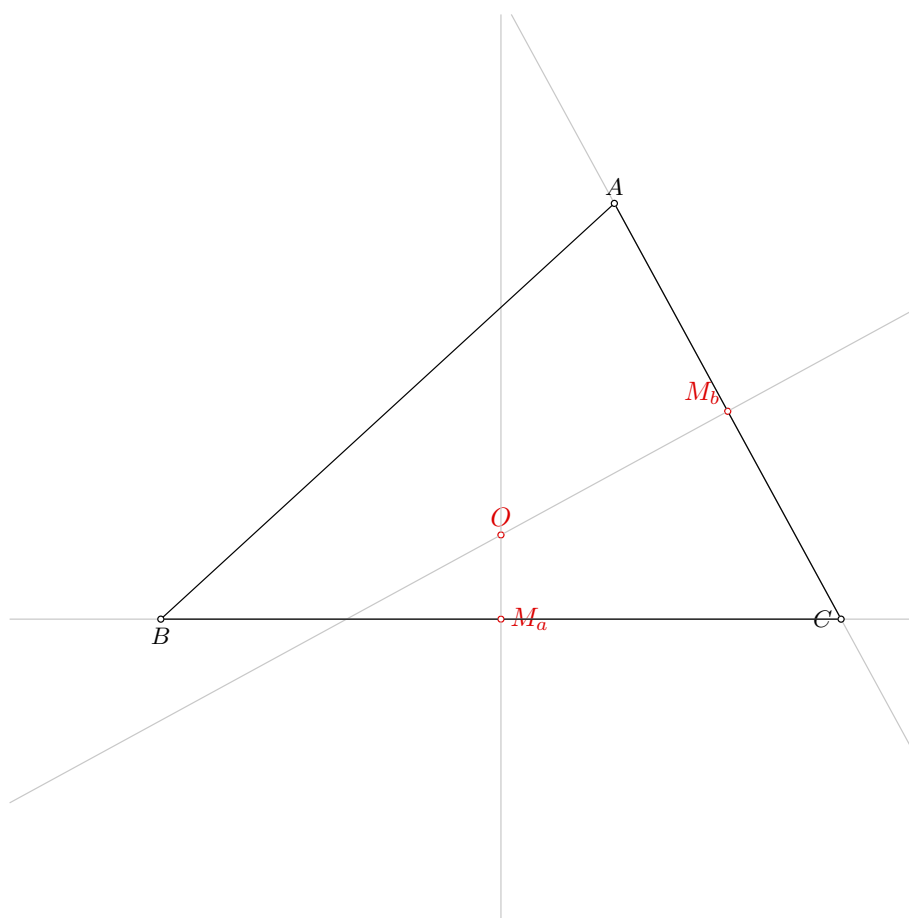


Figure 1: Illustration of the problem 0275

### 4.1.2 Proving $M_a = \neg M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.056 seconds.

**NDG conditions** Points  $M_b$  and  $O$  are not identical

### 4.1.3 Proving $M_b = \neg M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 8 terms.

**Time Complexity:** Time spent by the prover is 0.122 seconds.

**NDG conditions** Points  $M_b$  and  $O$  are not identical

## 4.2 GCLC - Area method

### 4.2.1 Proving $O = \neg O$

Proving failed

### 4.2.2 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{M_b M_a T_a^0} \neq S_{T_b^1 M_a T_a^0}$  i.e., lines  $M_b T_b^1$  and  $M_a T_a^0$  are not parallel (construction based assumption)

$S_{\neg M_a BC} \neq 0$  i.e., points  $\neg M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_b AC} \neq 0$  i.e., points  $\neg M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_a \neg M_b F_{\neg m_b}^3} \neq S_{F_{\neg m_a}^2 \neg M_b F_{\neg m_b}^3}$  i.e., lines  $\neg M_a F_{\neg m_a}^2$  and  $\neg M_b F_{\neg m_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{M_b M_a T_a^0} \neq S_{T_b^1 M_a T_a^0}$  i.e., lines  $M_b T_b^1$  and  $M_a T_a^0$  are not parallel (construction based assumption)

$S_{\neg M_a BC} \neq 0$  i.e., points  $\neg M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_b AC} \neq 0$  i.e., points  $\neg M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_a \neg M_b F_{\neg m_b}^3} \neq S_{F_{\neg m_a}^2 \neg M_b F_{\neg m_b}^3}$  i.e., lines  $\neg M_a F_{\neg m_a}^2$  and  $\neg M_b F_{\neg m_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $O = \neg O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 258 terms.

**Time Complexity:** Time spent by the prover is 0.300 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 30 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $O = \neg O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 172 terms.

**Time Complexity:** Time spent by the prover is 0.390 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 124 terms.

**Time Complexity:** Time spent by the prover is 0.400 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 124 terms.

**Time Complexity:** Time spent by the prover is 0.420 seconds. There are no ndg conditions.



# Problem 276

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 276: Given a point  $O$ , a point  $M_a$  and a point  $M_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $M_a$ , construct a line  $m_a$  (rule W02); % DET: points  $O$  and  $M_a$  are not the same;
2. Using the point  $O$  and the point  $M_c$ , construct a line  $m_c$  (rule W02); % DET: points  $O$  and  $M_c$  are not the same;
3. Using the point  $M_a$  and the line  $m_a$ , construct a line  $a$  (rule W10a); ;
4. Using the point  $M_c$  and the line  $m_c$ , construct a line  $c$  (rule W10a); ;
5. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same;
6. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
7. Using the point  $M_c$  and the point  $B$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel.

Determination conditions: lines  $c$  and  $a$  are not the same; points  $O$  and  $M_c$  are not the same; points  $O$  and  $M_a$  are not the same.

Rules used: [W01,W02,W03,W10a]

Lemmas used: [D1,D11,D13,D20,D21,GD01,GL03,GL04,GL09,L1]

Solving time: 0.3 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point O 65 51.14
point M_{a} 65 40
point M_{c} 50 67.5

color 220 0 0
fontsize 9

cmark_t O
cmark_r M_{a}
cmark_lt M_{c}
color 0 0 0
fontsize 8

% DET: points O and M_{a} are not the same
% Constructing a line m_{a} which passes through point O and point M_{a}
line m_{a} O M_{a}

color 200 200 200
drawline m_{a}
color 0 0 0

% DET: points O and M_{c} are not the same
% Constructing a line m_{c} which passes through point O and point M_{c}
line m_{c} O M_{c}

color 200 200 200
drawline m_{c}
color 0 0 0

% Constructing a line a which is perpendicular to line m_{a} and which passes through point M_{a}
perp a M_{a} m_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line c which is perpendicular to line m_{c} and which passes through point M_{c}
perp c M_{c} m_{c}

color 200 200 200
drawline c
color 0 0 0
```

```

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

% Constructing a point C such that  $BC/BM_{\{a\}}=2$ 
towards C B  $M_{\{a\}}$  2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% Constructing a point A such that  $M_{\{c\}}A/M_{\{c\}}B=-1$ 
towards A  $M_{\{c\}}$  B -1
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and a are not parallel
% Determination conditions: lines c and a are not the same; points O and  $M_{\{c\}}$  are not the same;
% points O and  $M_{\{a\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 72 terms.

**Time Complexity:** Time spent by the prover is 0.574 seconds.

**NDG conditions** Points  $M_c$  and  $O$  are not identical

Points  $B$ ,  $C$  and  $O$  are not collinear

Line through points  $O$  and  $M_a$  is not perpendicular to line through points  $M_a$  and  $A$

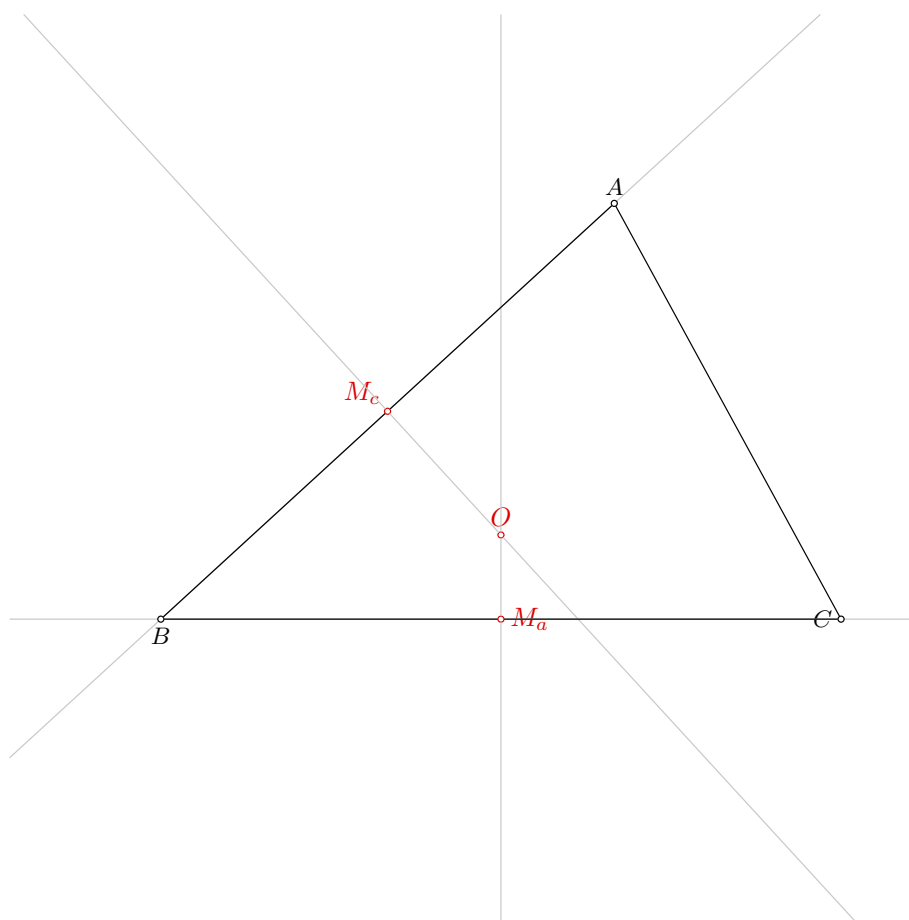


Figure 1: Illustration of the problem 0276

### 4.1.2 Proving $M_a = \_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.045 seconds.

**NDG conditions** Points  $M_c$  and  $O$  are not identical

### 4.1.3 Proving $M_c = \_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 8 terms.

**Time Complexity:** Time spent by the prover is 0.146 seconds.

**NDG conditions** Points  $M_c$  and  $O$  are not identical

## 4.2 GCLC - Area method

### 4.2.1 Proving $O = \_O$

Proving failed

### 4.2.2 Proving $M_a = \_M_a$

NDG conditions are:

$S_{M_c M_a T_a^0} \neq S_{T_c^1 M_a T_a^0}$  i.e., lines  $M_c T_c^1$  and  $M_a T_a^0$  are not parallel (construction based assumption)

$S_{\_M_a B C} \neq 0$  i.e., points  $\_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{\_M_b A C} \neq 0$  i.e., points  $\_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{\_M_a \_M_b F_{\_m_b}^3} \neq S_{F_{\_m_a}^2 \_M_b F_{\_m_b}^3}$  i.e., lines  $\_M_a F_{\_m_a}^2$  and  $\_M_b F_{\_m_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $M_c = \_M_c$

NDG conditions are:

$S_{M_c M_a T_a^0} \neq S_{T_c^1 M_a T_a^0}$  i.e., lines  $M_c T_c^1$  and  $M_a T_a^0$  are not parallel (construction based assumption)

$S_{\_M_a B C} \neq 0$  i.e., points  $\_M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{\_M_b A C} \neq 0$  i.e., points  $\_M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{\_M_a \_M_b F_{\_m_b}^3} \neq S_{F_{\_m_a}^2 \_M_b F_{\_m_b}^3}$  i.e., lines  $\_M_a F_{\_m_a}^2$  and  $\_M_b F_{\_m_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $O = \_O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 361 terms.

**Time Complexity:** Time spent by the prover is 0.270 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_a = \_M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.3 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 28 terms.

**Time Complexity:** Time spent by the prover is 0.030 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $O = \neg O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 172 terms.

**Time Complexity:** Time spent by the prover is 0.380 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 172 terms.

**Time Complexity:** Time spent by the prover is 0.540 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 172 terms.

**Time Complexity:** Time spent by the prover is 0.520 seconds. There are no ndg conditions.

# Problem 277

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 277: Given a point  $O$ , a point  $M_a$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $G$ , construct a point  $A$  (rule W01); ;
2. Using the point  $O$  and the point  $M_a$ , construct a line  $m_a$  (rule W02); % DET: points  $O$  and  $M_a$  are not the same;
3. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
4. Using the point  $M_a$  and the line  $m_a$ , construct a line  $a$  (rule W10a); ;
5. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $O$  and  $M_a$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D1,D11,D21,D26,GD01,GL03,GL04,GL09,L11,L12,L55]

Solving time: 1.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point O 65 51.14
```

```
point M_{a} 65 40
```

```
point G 70 58.33
```

```
color 220 0 0
```

```

fontsize 9

cmark_t 0
cmark_r M_{a}
cmark_t G
color 0 0 0
fontsize 8

% Constructing a point A such that  $M_{\{a\}}A/M_{\{a\}}G=3$ 
towards A M_{a} G 3
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

% DET: points O and M_{a} are not the same
% Constructing a line m_{a} which passes through point O and point M_{a}
line m_{a} O M_{a}

color 200 200 200
drawline m_{a}
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Constructing a line a which is perpendicular to line m_{a} and which passes through point M_{a}
perp a M_{a} m_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B

```



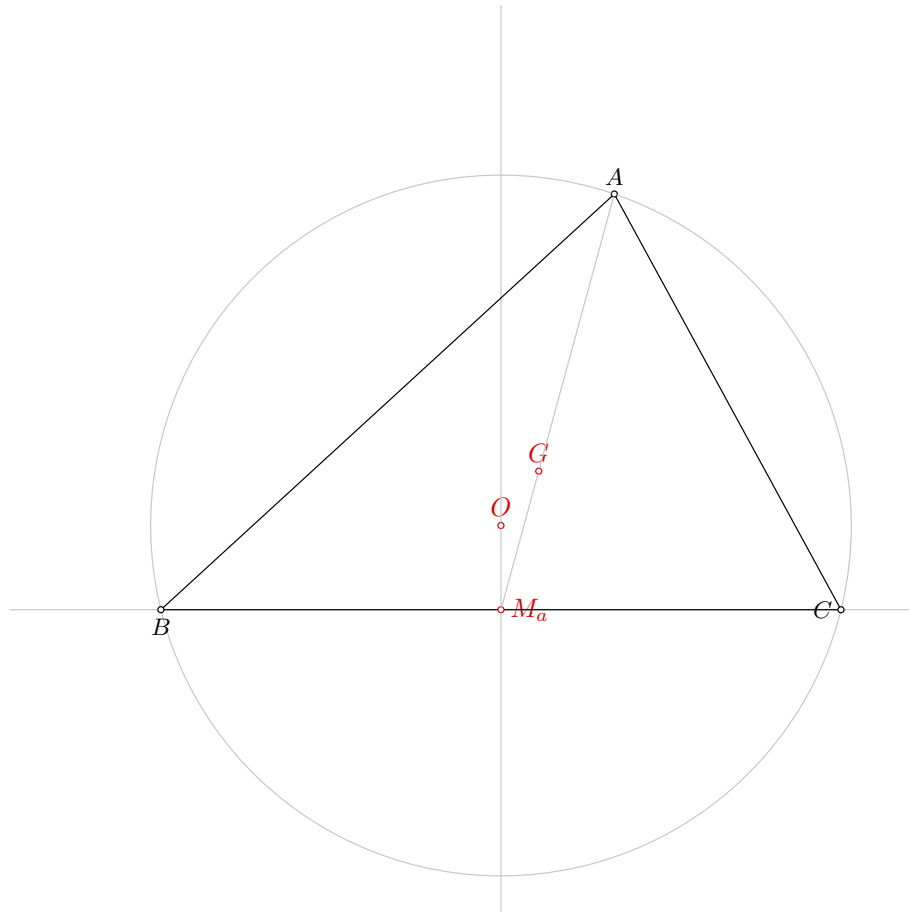


Figure 1: Illustration of the problem 0277

```
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same
% Determination conditions: points O and M_{a} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

Construction steps:

- Free point  $O$

- Free point  $M_a$
- Free point  $G$
- Segment division point  $A$  of segment  $M_aG$  with division coefficient -1.5
- Line  $m_a$  through two points  $O$  and  $M_a$
- Circle  $k(O, C)$  with center  $O$  and one point  $A$
- Line  $a$  through point  $M_a$  perpendicular to line  $m_a$
- Intersection point  $C$  of point sets  $k(O, C)$  and  $a$
- Intersection point  $B$  of point sets  $k(O, C)$  and  $a$
- Segment division point  $\_M_a$  of segment  $BC$  with division coefficient 1.0
- Line  $\_b$  through two points  $A$  and  $C$
- Segment division point  $\_M_b$  of segment  $CA$  with division coefficient 1.0
- Line  $\_a$  through two points  $B$  and  $C$
- Line  $\_m_a$  through point  $\_M_a$  perpendicular to line  $\_a$
- Line  $\_m_b$  through point  $\_M_b$  perpendicular to line  $\_b$
- Intersection point  $\_O$  of point sets  $\_m_a$  and  $\_m_b$
- Line  $\_t_a$  through two points  $A$  and  $\_M_a$
- Line  $\_t_b$  through two points  $B$  and  $\_M_b$
- Intersection point  $\_G$  of point sets  $\_t_a$  and  $\_t_b$

### Theorem statement:

- Points  $O$  and  $\_O$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $O$  has been assigned following coordinates:  $(0, 0)$

#### 4.1.2 Proving $M_a = \_M_a$

Proving failed

#### 4.1.3 Proving $G = \_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 147 terms.

**Time Complexity:** Time spent by the prover is 0.622 seconds.

**NDG conditions** Points  $A$ ,  $C$  and  $\_M_b$  are not collinear

Point  $A$  is not on angle bisector of angle with vertex  $B$  and two points from different rays  $O$  and  $\_M_a$

### 4.2 GCLC - Area method

#### 4.2.1 Proving $O = \_O$

Proving failed

#### 4.2.2 Proving $M_a = \_M_a$

Proving failed

#### 4.2.3 Proving $G = \_G$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $O=_O$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 121 terms.

**Time Complexity:** Time spent by the prover is 0.190 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_a=_M_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 14 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.3 Proving $G=_G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 1060 terms.

**Time Complexity:** Time spent by the prover is 2.170 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $O=_O$

Proving failed

#### 4.4.2 Proving $M_a=_M_a$

Proving failed

#### 4.4.3 Proving $G=_G$

Proving failed

# Problem 278

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 278: Given a point  $M_a$ , a point  $H_a$  and a point  $O$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $H_a$  are not the same;
2. Using the point  $M_a$  and the line  $a$ , construct a line  $m_a$  (rule W10b); ;
3. Choose freely a point  $O$  on the line  $m_a$  (rule WOnline2);
4. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
5. Choose freely a point  $A$  on the line  $h_a$  (rule WOnline2);
6. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
7. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $M_a$  and  $H_a$  are not the same.

Rules used: [W02,W04,W06,W10b,WOnline2]

Lemmas used: [D11,D21,D26,D5,D8,GD01,GL09,L11,L12]

Solving time: 21.8 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point M_{a} 65 40
point H_{a} 80 40
point O 65 51.14

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_r H_{a}
cmark_t O
color 0 0 0
fontsize 8

% DET: points M_{a} and H_{a} are not the same
% Constructing a line a which passes through point M_{a} and point H_{a}
line a M_{a} H_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line m_{a} which is perpendicular to line a and which passes through point M_{a}
perp m_{a} M_{a} a

color 200 200 200
drawline m_{a}
color 0 0 0

% Generating random value V[_G56990]
random V[_G56990]

% Calculating value V[_G57011] using formula  $V[_G56990]*20$ 
expression V[_G57011] { V[_G56990]*20 }

% Constructing a point O which is a point for which holds  $M_{a}O = V[_G57011]$  and angle  $H_{a}M_{a}O = 90$ 
turtle O H_{a} M_{a} 90 V[_G57011]
cmark_t O

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a
```

```

color 200 200 200
drawline h_{a}
color 0 0 0

% Generating random value V[_G57209]
random V[_G57209]

% Calculating value V[_G57230] using formula V[_G57209]*20
expression V[_G57230] { V[_G57209]*20 }

% Constructing a point A which is a point for which holds  $H_{\{a\}}A = V[_G57230]$  and angle  $M_{\{a\}}H_{\{a\}}A = 90$ 
turtle A M_{a} H_{a} 90 V[_G57230]
cmark_t A

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same
% Determination conditions: points  $M_{\{a\}}$  and  $H_{\{a\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

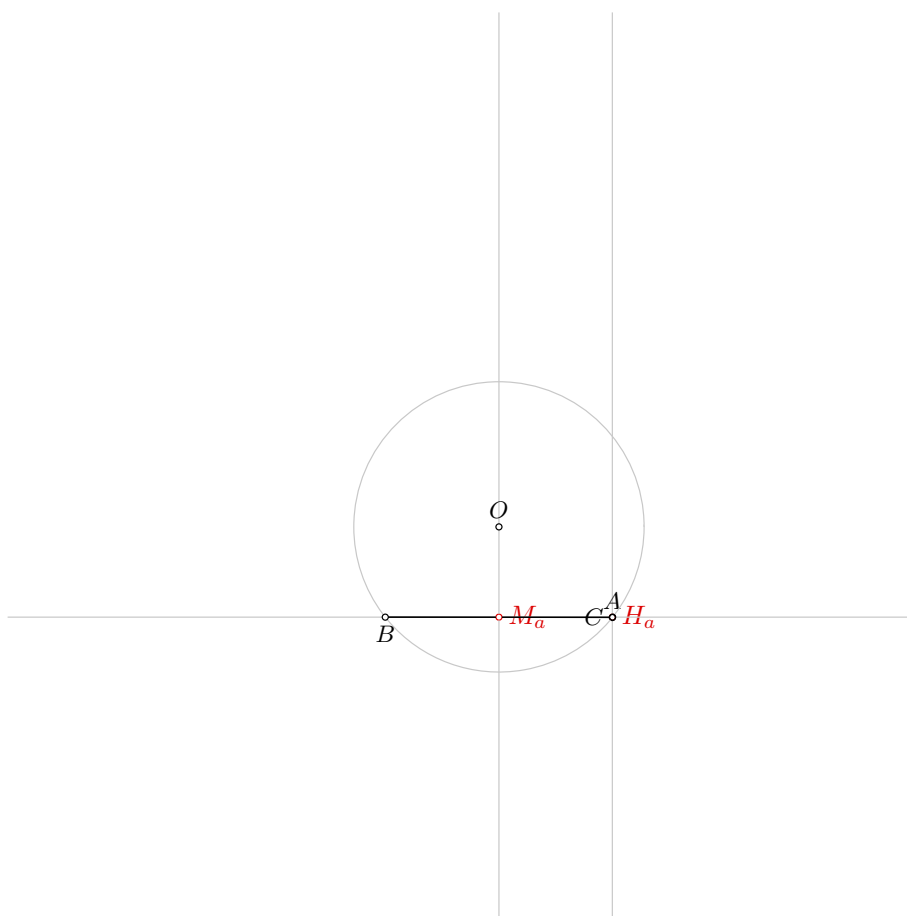


Figure 1: Illustration of the problem 0278

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.1.2 Proving $H_a = \neg H_a$

**Construction steps:**

- Free point  $M_a$
- Free point  $H_a$
- Line  $a$  through two points  $M_a$  and  $H_a$
- Line  $m_a$  through point  $M_a$  perpendicular to line  $a$
- Random point  $O$  from line  $m_a$
- Line  $h_a$  through point  $H_a$  perpendicular to line  $a$
- Random point  $A$  from line  $h_a$
- Circle  $k(O, C)$  with center  $O$  and one point  $A$
- Intersection point  $C$  of point sets  $k(O, C)$  and  $a$
- Intersection point  $B$  of point sets  $k(O, C)$  and  $a$
- Segment division point  $\neg M_a$  of segment  $BC$  with division coefficient 1.0
- Line  $\neg a$  through two points  $B$  and  $C$
- Line  $\neg h_a$  through point  $A$  perpendicular to line  $\neg a$
- Intersection point  $\neg H_a$  of point sets  $\neg a$  and  $\neg h_a$

**Theorem statement:**

- Points  $H_a$  and  $\neg H_a$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $M_a$  has been assigned following coordinates:  $(0, 0)$

#### 4.1.3 Proving $O = \neg O$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.2.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.2.3 Proving $O = \neg O$

Proving failed



### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.3.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.3.3 Proving $O = \neg O$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.4.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.4.3 Proving $O = \neg O$

Proving failed

# Problem 279

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 279: Given a point  $O$ , a point  $M_a$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $M_a$ , construct a line  $m_a$  (rule W02); % DET: points  $O$  and  $M_a$  are not the same;
2. Using the point  $H_b$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_b$  and  $M_a$  are not the same;
3. Using the point  $M_a$  and the line  $m_a$ , construct a line  $a$  (rule W10a); ;
4. Using the circle  $k(M_a, B)$  and the line  $a$ , construct a point  $B$  and a point  $C$  (rule W04); % NDG: line  $a$  and circle  $k(M_a, B)$  intersect;
5. Using the point  $H_b$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $C$  are not the same;
6. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
7. Using the circle  $k(O, C)$ , the line  $b$ , the point  $O$  and the point  $C$ , construct a point  $A$  (rule W05); % NDG: line  $b$  and circle  $k(O, C)$  intersect % DET: points  $C$  and  $A$  must be different.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same; line  $a$  and circle  $k(M_a, B)$  intersect; points  $H_b$  and  $M_a$  are not the same.

Determination conditions: points  $C$  and  $A$  must be different; points  $H_b$  and  $C$  are not the same; points  $O$  and  $M_a$  are not the same.

Rules used: [W02,W04,W05,W06,W10a]

Lemmas used: [D1,D11,D21,D26,D6,GD01,GD02,GL09,L11,L12,L37,L38]

Solving time: 2.4 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point O 65 51.14
point M_{a} 65 40
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_t O
cmark_r M_{a}
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points O and M_{a} are not the same
% Constructing a line m_{a} which passes through point O and point M_{a}
line m_{a} O M_{a}

color 200 200 200
drawline m_{a}
color 0 0 0

% NDG: points H_{b} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{b}
circle k(M_{a},B) M_{a} H_{b}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% Constructing a line a which is perpendicular to line m_{a} and which passes through point M_{a}
perp a M_{a} m_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(M_{a},B) intersect
% Constructing points B and C which are in intersection of k(M_{a},B) and a
intersec2 B C k(M_{a},B) a
cmark_b B
cmark_l C

% DET: points H_{b} and C are not the same
```

```

% Constructing a line b which passes through point  $H_{\{b\}}$  and point C
line b  $H_{\{b\}}$  C

color 200 200 200
drawline b
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line b and circle  $k(O,C)$  intersect% DET: points C and A must be different
% Constructing a point  $P_{\{\_G43049\}}$  which is a foot of the point O on the line b
foot  $P_{\{\_G43049\}}$  O b
cmark_r  $P_{\{\_G43049\}}$ 
color 200 200 200
drawline O  $P_{\{\_G43049\}}$ 
color 0 0 0

% Constructing a point A which is an image of the point C in the symmetry to point/line  $P_{\{\_G43049\}}$ 
sim A  $P_{\{\_G43049\}}$  C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle  $k(O,C)$  intersect; points B and O are not the same;
% line a and circle  $k(M_{\{a\}},B)$  intersect; points  $H_{\{b\}}$  and  $M_{\{a\}}$  are not the same
% Determination conditions: points C and A must be different; points  $H_{\{b\}}$  and C are not the same;
% points O and  $M_{\{a\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

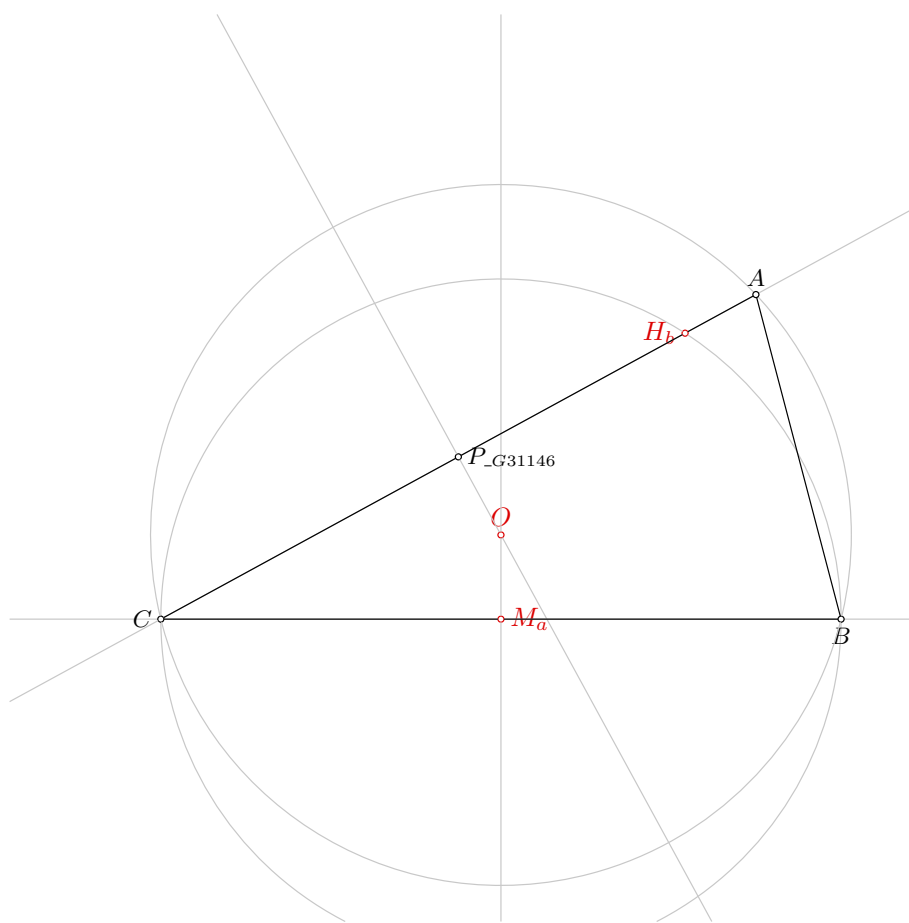


Figure 1: Illustration of the problem 0279

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

Construction steps:

- Free point  $O$
- Free point  $M_a$
- Free point  $H_b$
- Line  $m_a$  through two points  $O$  and  $M_a$
- Circle  $k(M_a, B)$  with center  $M_a$  and one point  $H_b$
- Line  $a$  through point  $M_a$  perpendicular to line  $m_a$
- Intersection point  $B$  of point sets  $k(M_a, B)$  and  $a$
- Intersection point  $C$  of point sets  $k(M_a, B)$  and  $a$
- Line  $b$  through two points  $H_b$  and  $C$
- Circle  $k(O, C)$  with center  $O$  and one point  $B$
- Line footPointPerpLine788 through point  $O$  perpendicular to line  $b$
- Intersection point  $P_{G37450}$  of point sets footPointPerpLine788 and  $b$
- Cental symmetric point  $A$  of point  $C$  with respect to center of symmetry  $P_{G37450}$
- Segment division point  $_M a$  of segment  $BC$  with division coefficient 1.0
- Line  $_b$  through two points  $A$  and  $C$
- Segment division point  $_M b$  of segment  $CA$  with division coefficient 1.0
- Line  $_a$  through two points  $B$  and  $C$
- Line  $_m a$  through point  $_M a$  perpendicular to line  $_a$
- Line  $_m b$  through point  $_M b$  perpendicular to line  $_b$
- Intersection point  $_O$  of point sets  $_m a$  and  $_m b$
- Line  $_h b$  through point  $B$  perpendicular to line  $_b$
- Intersection point  $_H b$  of point sets  $_b$  and  $_h b$

**Theorem statement:**

- Points  $O$  and  $_O$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $O$  has been assigned following coordinates:  $(0, 0)$

#### 4.1.2 Proving $M_a=_M a$

Proving failed

#### 4.1.3 Proving $H_b=_H b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $O=_O$

Proving failed

#### **4.2.2 Proving $M_a = \neg M_a$**

Proving failed

#### **4.2.3 Proving $H_b = \neg H_b$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $O = \neg O$**

Proving failed

#### **4.3.2 Proving $M_a = \neg M_a$**

Proving failed

#### **4.3.3 Proving $H_b = \neg H_b$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $O = \neg O$**

Proving failed

#### **4.4.2 Proving $M_a = \neg M_a$**

Proving failed

#### **4.4.3 Proving $H_b = \neg H_b$**

Proving failed

# Problem 280

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 280: Given a point  $O$ , a point  $M_a$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $M_a$ , construct a line  $m_a$  (rule W02); % DET: points  $O$  and  $M_a$  are not the same;
2. Using the point  $H_c$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_c$  and  $M_a$  are not the same;
3. Using the point  $M_a$  and the line  $m_a$ , construct a line  $a$  (rule W10a); ;
4. Using the circle  $k(M_a, B)$  and the line  $a$ , construct a point  $B$  and a point  $C$  (rule W04); % NDG: line  $a$  and circle  $k(M_a, B)$  intersect;
5. Using the point  $H_c$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $B$  are not the same;
6. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
7. Using the circle  $k(O, C)$ , the line  $c$ , the point  $O$  and the point  $B$ , construct a point  $A$  (rule W05); % NDG: line  $c$  and circle  $k(O, C)$  intersect % DET: points  $B$  and  $A$  must be different.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same; line  $a$  and circle  $k(M_a, B)$  intersect; points  $H_c$  and  $M_a$  are not the same.

Determination conditions: points  $B$  and  $A$  must be different; points  $H_c$  and  $B$  are not the same; points  $O$  and  $M_a$  are not the same.

Rules used: [W02,W04,W05,W06,W10a]

Lemmas used: [D1,D11,D21,D26,D7,GD01,GD02,GL09,L11,L12,L37,L39]

Solving time: 2.4 seconds.



## 3.2 Construction in GCLC language

```
dim 120 120

point O 65 51.14
point M_{a} 65 40
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_t O
cmark_r M_{a}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points O and M_{a} are not the same
% Constructing a line m_{a} which passes through point O and point M_{a}
line m_{a} O M_{a}

color 200 200 200
drawline m_{a}
color 0 0 0

% NDG: points H_{c} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{c}
circle k(M_{a},B) M_{a} H_{c}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% Constructing a line a which is perpendicular to line m_{a} and which passes through point M_{a}
perp a M_{a} m_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(M_{a},B) intersect
% Constructing points B and C which are in intersection of k(M_{a},B) and a
intersec2 B C k(M_{a},B) a
cmark_b B
cmark_l C

% DET: points H_{c} and B are not the same
```

```

% Constructing a line c which passes through point H_{c} and point B
line c H_{c} B

color 200 200 200
drawline c
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line c and circle k(O,C) intersect% DET: points B and A must be different
% Constructing a point P_{\_G29367} which is a foot of the point O on the line c
foot P_{\_G29367} O c
cmark_r P_{\_G29367}
color 200 200 200
drawline O P_{\_G29367}
color 0 0 0

% Constructing a point A which is an image of the point B in the symmetry to point/line P_{\_G
29367}
sim A P_{\_G29367} B
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k(O,C) intersect; points B and O are not the same;
% line a and circle k(M_{a},B) intersect; points H_{c} and M_{a} are not the same
% Determination conditions: points B and A must be different; points H_{c} and B are not the same;
% points O and M_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

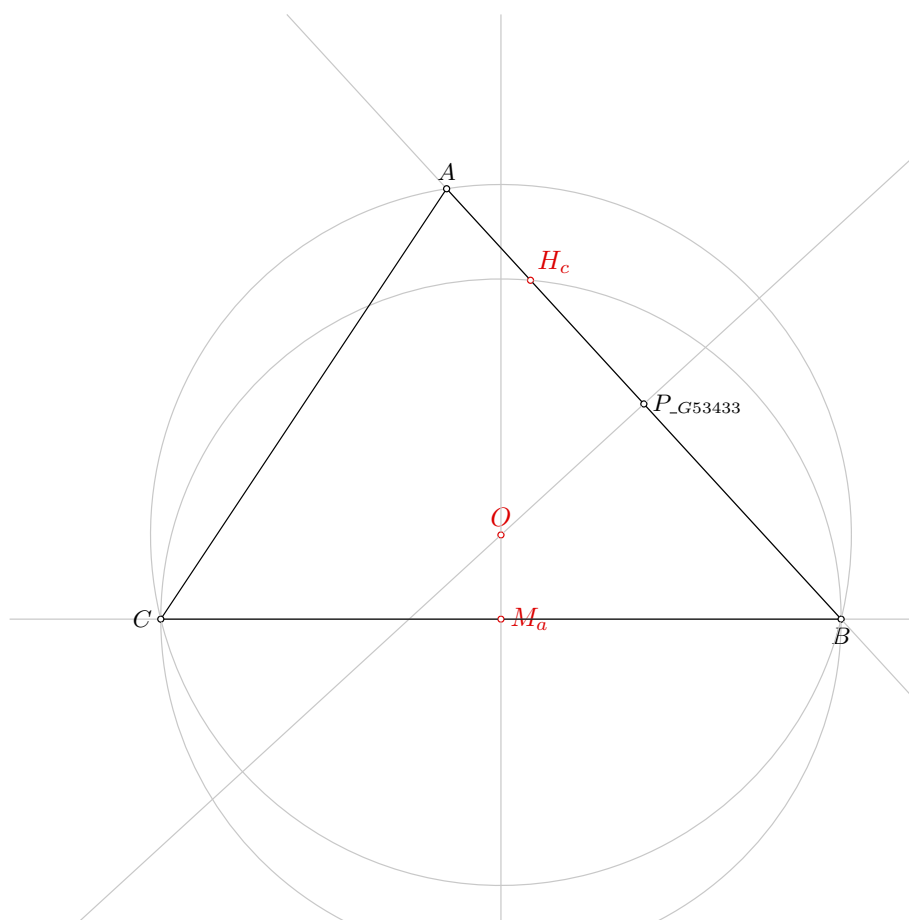


Figure 1: Illustration of the problem 0280

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

Construction steps:

- Free point  $O$
- Free point  $M_a$
- Free point  $H_c$
- Line  $m_a$  through two points  $O$  and  $M_a$
- Circle  $k(M_a, B)$  with center  $M_a$  and one point  $H_c$
- Line  $a$  through point  $M_a$  perpendicular to line  $m_a$
- Intersection point  $B$  of point sets  $k(M_a, B)$  and  $a$
- Intersection point  $C$  of point sets  $k(M_a, B)$  and  $a$
- Line  $c$  through two points  $H_c$  and  $B$
- Circle  $k(O, C)$  with center  $O$  and one point  $B$
- Line footPointPerpLine636 through point  $O$  perpendicular to line  $c$
- Intersection point  $P_{G59908}$  of point sets footPointPerpLine636 and  $c$
- Cental symmetric point  $A$  of point  $B$  with respect to center of symmetry  $P_{G59908}$
- Segment division point  $_M_a$  of segment  $BC$  with division coefficient 1.0
- Line  $_b$  through two points  $A$  and  $C$
- Segment division point  $_M_b$  of segment  $CA$  with division coefficient 1.0
- Line  $_a$  through two points  $B$  and  $C$
- Line  $_m_a$  through point  $_M_a$  perpendicular to line  $_a$
- Line  $_m_b$  through point  $_M_b$  perpendicular to line  $_b$
- Intersection point  $_O$  of point sets  $_m_a$  and  $_m_b$
- Line  $_c$  through two points  $A$  and  $B$
- Line  $_h_c$  through point  $C$  perpendicular to line  $_c$
- Intersection point  $_H_c$  of point sets  $_c$  and  $_h_c$

**Theorem statement:**

- Points  $O$  and  $_O$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $O$  has been assigned following coordinates:  $(0, 0)$

#### 4.1.2 Proving $M_a=_M_a$

Proving failed

#### 4.1.3 Proving $H_c=_H_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $O=_O$

Proving failed

#### **4.2.2 Proving $M_a = \neg M_a$**

Proving failed

#### **4.2.3 Proving $H_c = \neg H_c$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $O = \neg O$**

Proving failed

#### **4.3.2 Proving $M_a = \neg M_a$**

Proving failed

#### **4.3.3 Proving $H_c = \neg H_c$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $O = \neg O$**

Proving failed

#### **4.4.2 Proving $M_a = \neg M_a$**

Proving failed

#### **4.4.3 Proving $H_c = \neg H_c$**

Proving failed

# Problem 281

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 281: Given a point  $O$ , a point  $M_a$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $H$ , construct a point  $G$  (rule W01); ;
2. Using the point  $M_a$  and the point  $G$ , construct a point  $A$  (rule W01); ;
3. Using the point  $O$  and the point  $M_a$ , construct a line  $m_a$  (rule W02); % DET: points  $O$  and  $M_a$  are not the same;
4. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
5. Using the point  $M_a$  and the line  $m_a$ , construct a line  $a$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $O$  and  $M_a$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D1,D11,D21,D26,GD01,GL03,GL04,GL09,L11,L12,L55,L58]

Solving time: 1.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point O 65 51.14
```

```
point M_{a} 65 40
```

```
point H 80 72.73
```

```

color 220 0 0
fontsize 9

cmark_t 0
cmark_r M_{a}
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a line L_{\_G56115} which passes through point O and point H
line L_{\_G56115} O H

color 200 200 200
drawline L_{\_G56115}
color 0 0 0

% Constructing a point P_{\_G56216} with coordinates (0,0)
point P_{\_G56216} 0 0
cmark_r P_{\_G56216}

% Constructing a point P_{\_G56140} such that  $OP_{\_G56140}/OP_{\_G56216}=1$ 
towards P_{\_G56140} 0 P_{\_G56216} 1
cmark_r P_{\_G56140}
color 200 200 200
drawsegment 0 P_{\_G56140}
color 0 0 0

% Constructing a point P_{\_G56185} such that  $OP_{\_G56185}/OP_{\_G56216}=3$ 
towards P_{\_G56185} 0 P_{\_G56216} 3
cmark_r P_{\_G56185}
color 200 200 200
drawsegment 0 P_{\_G56185}
color 0 0 0

% Constructing a line L_{\_G56146} which passes through point H and point P_{\_G56185}
line L_{\_G56146} H P_{\_G56185}

color 200 200 200
drawline L_{\_G56146}
color 0 0 0

% Constructing a line L_{\_G56109} which contains the point P_{\_G56140} and is parallel to the
line L_{\_G56146}
parallel L_{\_G56109} P_{\_G56140} L_{\_G56146}

color 200 200 200
drawline L_{\_G56109}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G56109} and line L_{\_G56115}
intersec G L_{\_G56109} L_{\_G56115}

```

```

cmark_t G

% Constructing a point A such that  $M_{\{a\}}A/M_{\{a\}}G=3$ 
towards A M_{a} G 3
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

% DET: points O and M_{a} are not the same
% Constructing a line m_{a} which passes through point O and point M_{a}
line m_{a} O M_{a}

color 200 200 200
drawline m_{a}
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Constructing a line a which is perpendicular to line m_{a} and which passes through point M_{a}
perp a M_{a} m_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same
% Determination conditions: points O and M_{a} are not the same

```



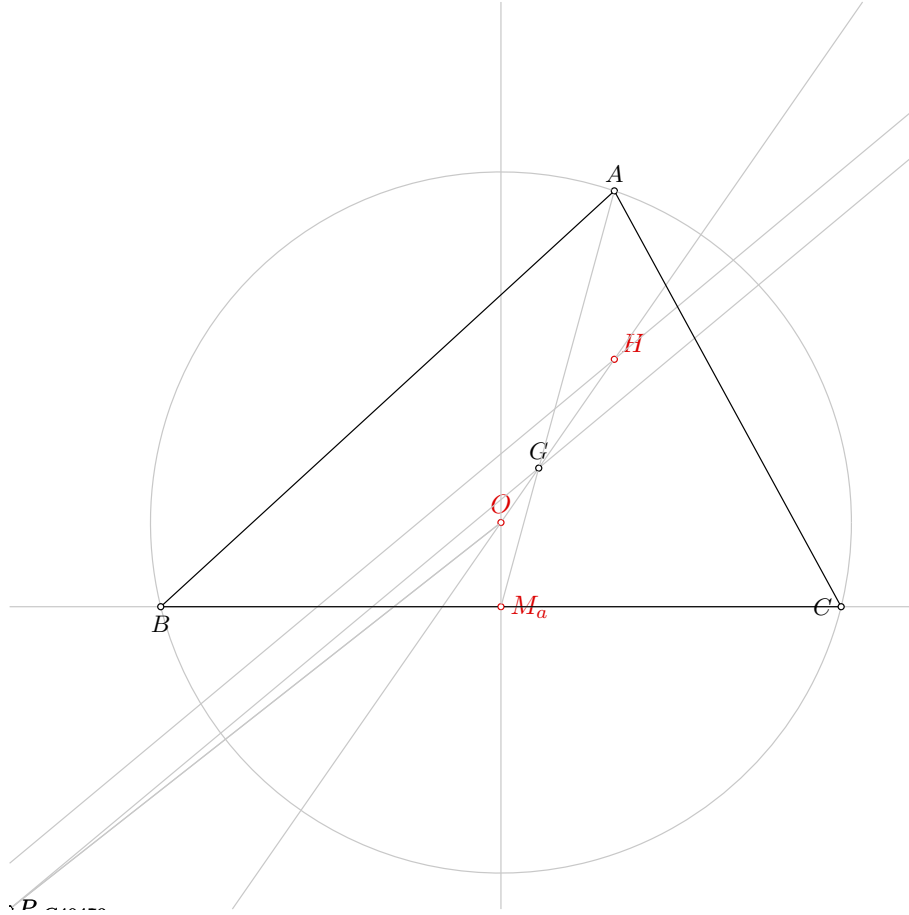


Figure 1: Illustration of the problem 0281

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

Construction steps:

- Free point  $O$
- Free point  $M_a$
- Free point  $H$
- Line  $L_{G49526}$  through two points  $O$  and  $H$
- Free point  $P_{G49529}$

- Segment division point  $P_{G49532}$  of segment  $P_{G49529}P_{G49529}$  with division coefficient 1.0
- Segment division point  $P_{G49535}$  of segment  $OP_{G49529}$  with division coefficient -1.5
- Line  $L_{G49538}$  through two points  $H$  and  $P_{G49535}$
- Line  $L_{G49541}$  through point  $P_{G49532}$  parallel with line  $L_{G49538}$
- Intersection point  $G$  of point sets  $L_{G49541}$  and  $L_{G49526}$
- Segment division point  $A$  of segment  $M_aG$  with division coefficient -1.5
- Line  $m_a$  through two points  $O$  and  $M_a$
- Circle  $k(O, C)$  with center  $O$  and one point  $A$
- Line  $a$  through point  $M_a$  perpendicular to line  $m_a$
- Intersection point  $C$  of point sets  $k(O, C)$  and  $a$
- Intersection point  $B$  of point sets  $k(O, C)$  and  $a$
- Segment division point  $\_M_a$  of segment  $BC$  with division coefficient 1.0
- Line  $\_b$  through two points  $A$  and  $C$
- Segment division point  $\_M_b$  of segment  $CA$  with division coefficient 1.0
- Line  $\_a$  through two points  $B$  and  $C$
- Line  $\_m_a$  through point  $\_M_a$  perpendicular to line  $\_a$
- Line  $\_m_b$  through point  $\_M_b$  perpendicular to line  $\_b$
- Intersection point  $\_O$  of point sets  $\_m_a$  and  $\_m_b$
- Line  $\_h_a$  through point  $A$  perpendicular to line  $\_a$
- Line  $\_h_b$  through point  $B$  perpendicular to line  $\_b$
- Intersection point  $\_H$  of point sets  $\_h_a$  and  $\_h_b$

### Theorem statement:

- Points  $O$  and  $\_O$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $O$  has been assigned following coordinates:  $(0, 0)$

#### 4.1.2 Proving $M_a = \_M_a$

Proving failed

#### 4.1.3 Proving $H = \_H$

### Construction steps:

- Free point  $O$
- Free point  $M_a$
- Free point  $H$
- Line  $L_{G53706}$  through two points  $O$  and  $H$
- Free point  $P_{G53709}$
- Segment division point  $P_{G53712}$  of segment  $P_{G53709}P_{G53709}$  with division coefficient 1.0
- Segment division point  $P_{G53715}$  of segment  $OP_{G53709}$  with division coefficient -1.5
- Line  $L_{G53718}$  through two points  $H$  and  $P_{G53715}$
- Line  $L_{G53721}$  through point  $P_{G53712}$  parallel with line  $L_{G53718}$
- Intersection point  $G$  of point sets  $L_{G53721}$  and  $L_{G53706}$

- Segment division point  $A$  of segment  $M_aG$  with division coefficient -1.5
- Line  $m_a$  through two points  $O$  and  $M_a$
- Circle  $k(O, C)$  with center  $O$  and one point  $A$
- Line  $a$  through point  $M_a$  perpendicular to line  $m_a$
- Intersection point  $C$  of point sets  $k(O, C)$  and  $a$
- Intersection point  $B$  of point sets  $k(O, C)$  and  $a$
- Segment division point  $\_M_a$  of segment  $BC$  with division coefficient 1.0
- Line  $\_b$  through two points  $A$  and  $C$
- Segment division point  $\_M_b$  of segment  $CA$  with division coefficient 1.0
- Line  $\_a$  through two points  $B$  and  $C$
- Line  $\_m_a$  through point  $\_M_a$  perpendicular to line  $\_a$
- Line  $\_m_b$  through point  $\_M_b$  perpendicular to line  $\_b$
- Intersection point  $\_O$  of point sets  $\_m_a$  and  $\_m_b$
- Line  $\_h_a$  through point  $A$  perpendicular to line  $\_a$
- Line  $\_h_b$  through point  $B$  perpendicular to line  $\_b$
- Intersection point  $\_H$  of point sets  $\_h_a$  and  $\_h_b$

### Theorem statement:

- Points  $H$  and  $\_H$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $O$  has been assigned following coordinates:  $(0, 0)$

## 4.2 GCLC - Area method

### 4.2.1 Proving $O=\_O$

Proving failed

### 4.2.2 Proving $M_a=\_M_a$

Proving failed

### 4.2.3 Proving $H=\_H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $O=\_O$

Proving failed

### 4.3.2 Proving $M_a=\_M_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 268 terms.

**Time Complexity:** Time spent by the prover is 0.490 seconds. There are no ndg conditions.

### 4.3.3 Proving $H=_H$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 11621 terms.

**Time Complexity:** Time spent by the prover is 51.100 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $O=_O$

Proving failed

### 4.4.2 Proving $M_a=_M_a$

Proving failed

### 4.4.3 Proving $H=_H$

Proving failed

# Problem 282

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 282: Given a point  $M_a$ , a point  $T_a$  and a point  $O$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $T_a$  are not the same;
2. Using the point  $M_a$  and the line  $a$ , construct a line  $m_a$  (rule W10b); ;
3. Choose freely a point  $O$  on the line  $m_a$  (rule WOnline2);
4. Choose freely a point  $B$  on the line  $a$  (rule WOnline1) ;
5. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
6. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
7. Using the circle  $k(O, C)$  and the line  $m_a$ , construct a point  $N_a$  and a point  $N_{ak}$  (rule W04); % NDG: line  $m_a$  and circle  $k(O, C)$  intersect;
8. Using the point  $N_a$  and the point  $T_a$ , construct a line  $s_a$  (rule W02); % DET: points  $N_a$  and  $T_a$  are not the same;
9. Using the circle  $k(O, C)$ , the line  $s_a$ , the point  $O$  and the point  $N_a$ , construct a point  $A$  (rule W05); % NDG: line  $s_a$  and circle  $k(O, C)$  intersect % DET: points  $N_a$  and  $A$  must be different.

Non-degenerate conditions: line  $s_a$  and circle  $k(O, C)$  intersect; line  $m_a$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same.

Determination conditions: points  $N_a$  and  $A$  must be different; points  $N_a$  and  $T_a$  are not the same; points  $M_a$  and  $T_a$  are not the same.

Rules used: [W01,W02,W04,W05,W06,W10b,WOnline1,WOnline2]  
 Lemmas used: [D11,D21,D23,D26,D47,D50,GD01,GL03,GL09,L11,L12,L25,L5]  
 Solving time: 7.8 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{a} 65 40
point T_{a} 70.86 40
point O 65 51.14

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_rb T_{a}
cmark_t O
color 0 0 0
fontsize 8

% DET: points M_{a} and T_{a} are not the same
% Constructing a line a which passes through point M_{a} and point T_{a}
line a M_{a} T_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line m_{a} which is perpendicular to line a and which passes through point M_{a}
perp m_{a} M_{a} a

color 200 200 200
drawline m_{a}
color 0 0 0

% Generating random value V[_G40669]
random V[_G40669]

% Calculating value V[_G40690] using formula V[_G40669]*20
expression V[_G40690] { V[_G40669]*20 }

% Constructing a point O which is a point for which holds M_{a}O = V[_G40690] and angle T_{a}M_{a}O
= 90
turtle O T_{a} M_{a} 90 V[_G40690]
cmark_t O
```

```

% Choosing randomly a point B on the line M_{a}T_{a}
online B M_{a} T_{a}
cmark_b B
color 200 200 200
drawline M_{a} T_{a}
color 0 0 0

% Constructing a point C such that BC/BM_{a}=2
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line m_{a} and circle k(O,C) intersect
% Constructing points N_{a} and N_{ak} which are in intersection of k(O,C) and m_{a}
intersec2 N_{a} N_{ak} k(O,C) m_{a}
cmark_b N_{a}
cmark_r N_{ak}

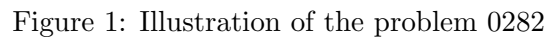
% DET: points N_{a} and T_{a} are not the same
% Constructing a line s_{a} which passes through point N_{a} and point T_{a}
line s_{a} N_{a} T_{a}

color 200 200 200
drawline s_{a}
color 0 0 0

% NDG: line s_{a} and circle k(O,C) intersect% DET: points N_{a} and A must be different
% Constructing a point P_{\_G41182} which is a foot of the point O on the line s_{a}
foot P_{\_G41182} O s_{a}
cmark_r P_{\_G41182}
color 200 200 200
drawline O P_{\_G41182}
color 0 0 0

% Constructing a point A which is an image of the point N_{a} in the symmetry to point/line P_{\_G
41182}

```



```
% Non-degenerate conditions: line s_{a} and circle k(D,C) intersect; line m_{a} and circle k(D,C)
    intersect; points B and O are not the same
% Determination conditions: points N_{a} and A must be different; points N_{a} and T_{a} are not
    the same; points M_{a} and T_{a} are not the same
```

Illustration of the constructed figure is given in Figure 1

## 1520



## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = \neg M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_a = \neg T_a$

Proving failed

#### 4.1.3 Proving $O = \neg O$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.2.2 Proving $T_a = \neg T_a$

Proving failed

#### 4.2.3 Proving $O = \neg O$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.3.2 Proving $T_a = \neg T_a$

Proving failed

#### 4.3.3 Proving $O = \neg O$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.4.2 Proving $T_a = \neg T_a$

Proving failed

### 4.4.3 Proving $O=_O$

Proving failed

## Problem 283

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 283: Given a point  $O$ , a point  $M_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 284

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 284: Given a point  $O$ , a point  $M_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 285

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 285: Given a point  $O$ , a point  $M_a$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 286

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 286: Given a point  $O$ , a point  $M_b$  and a point  $M_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $M_b$ , construct a line  $m_b$  (rule W02); % DET: points  $O$  and  $M_b$  are not the same;
2. Using the point  $O$  and the point  $M_c$ , construct a line  $m_c$  (rule W02); % DET: points  $O$  and  $M_c$  are not the same;
3. Using the point  $M_b$  and the line  $m_b$ , construct a line  $b$  (rule W10a); ;
4. Using the point  $M_c$  and the line  $m_c$ , construct a line  $c$  (rule W10a); ;
5. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same;
6. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
7. Using the point  $M_c$  and the point  $A$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel.

Determination conditions: lines  $c$  and  $b$  are not the same; points  $O$  and  $M_c$  are not the same; points  $O$  and  $M_b$  are not the same.

Rules used: [W01,W02,W03,W10a]

Lemmas used: [D1,D12,D13,D20,D22,GD01,GL03,GL04,GL09,L1]

Solving time: 0.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point O 65 51.14
point M_{b} 95 67.5
point M_{c} 50 67.5

color 220 0 0
fontsize 9

cmark_t O
cmark_lt M_{b}
cmark_lt M_{c}
color 0 0 0
fontsize 8

% DET: points O and M_{b} are not the same
% Constructing a line m_{b} which passes through point O and point M_{b}
line m_{b} O M_{b}

color 200 200 200
drawline m_{b}
color 0 0 0

% DET: points O and M_{c} are not the same
% Constructing a line m_{c} which passes through point O and point M_{c}
line m_{c} O M_{c}

color 200 200 200
drawline m_{c}
color 0 0 0

% Constructing a line b which is perpendicular to line m_{b} and which passes through point M_{b}
perp b M_{b} m_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line c which is perpendicular to line m_{c} and which passes through point M_{c}
perp c M_{c} m_{c}

color 200 200 200
drawline c
color 0 0 0
```

```

% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A

% Constructing a point C such that  $AC/AM_{\{b\}}=2$ 
towards C A  $M_{\{b\}}$  2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% Constructing a point B such that  $M_{\{c\}}B/M_{\{c\}}A=-1$ 
towards B  $M_{\{c\}}$  A -1
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and b are not parallel
% Determination conditions: lines c and b are not the same; points O and  $M_{\{c\}}$  are not the same;
% points O and  $M_{\{b\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 72 terms.

**Time Complexity:** Time spent by the prover is 0.553 seconds.

**NDG conditions** Points  $M_c$  and  $O$  are not identical

Line through points  $M_b$  and  $O$  is not parallel with line through points  $A$  and  $C$

Line through points  $M_b$  and  $A$  is not perpendicular to line through points  $A$  and  $O$



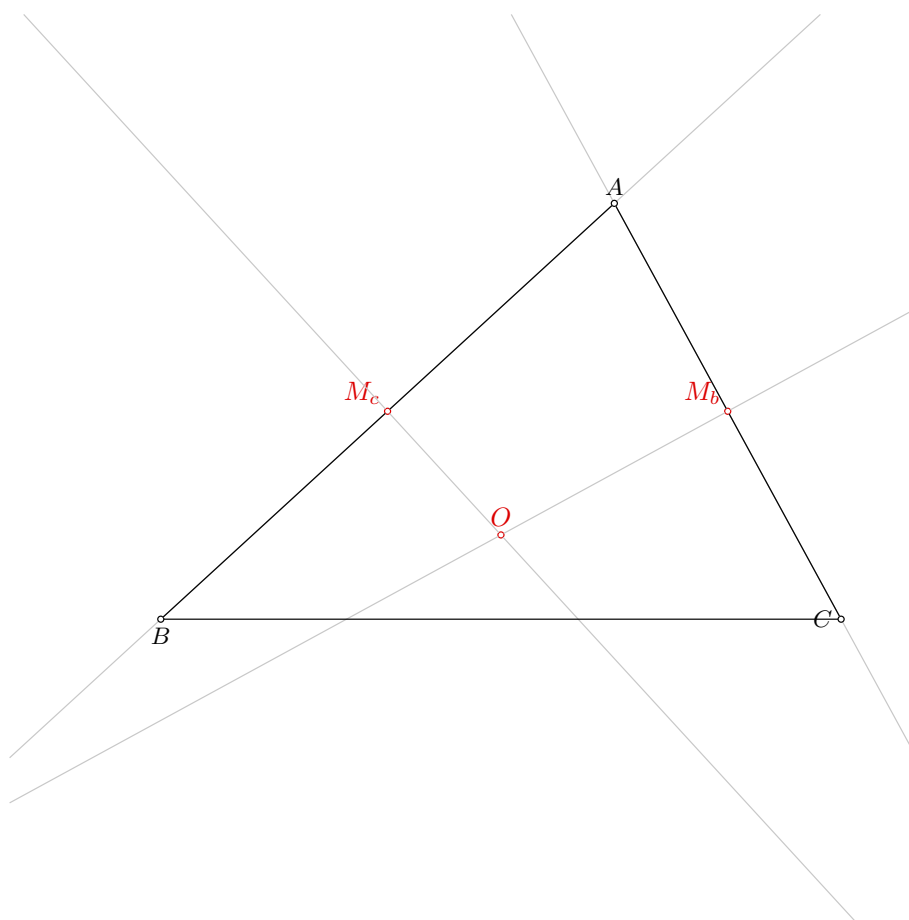


Figure 1: Illustration of the problem 0286

#### 4.1.2 Proving $M_b = M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.047 seconds.

**NDG conditions** Points  $M_c$  and  $O$  are not identical

#### 4.1.3 Proving $M_c = M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 8 terms.

**Time Complexity:** Time spent by the prover is 0.149 seconds.

**NDG conditions** Points  $M_c$  and  $O$  are not identical

### 4.2 GCLC - Area method

#### 4.2.1 Proving $O = O$

Proving failed

#### 4.2.2 Proving $M_b = M_b$

NDG conditions are:

$S_{M_c M_b T_b^0} \neq S_{T_c^1 M_b T_b^0}$  i.e., lines  $M_c T_c^1$  and  $M_b T_b^0$  are not parallel (construction based assumption)

$S_{M_a B C} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_b A C} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_a M_b F_{-m_b}^3} \neq S_{F_{-m_a}^2 M_b F_{-m_b}^3}$  i.e., lines  $M_a F_{-m_a}^2$  and  $M_b F_{-m_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $M_c = M_c$

NDG conditions are:

$S_{M_c M_b T_b^0} \neq S_{T_c^1 M_b T_b^0}$  i.e., lines  $M_c T_c^1$  and  $M_b T_b^0$  are not parallel (construction based assumption)

$S_{M_a B C} \neq 0$  i.e., points  $M_a$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_b A C} \neq 0$  i.e., points  $M_b$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{M_a M_b F_{-m_b}^3} \neq S_{F_{-m_a}^2 M_b F_{-m_b}^3}$  i.e., lines  $M_a F_{-m_a}^2$  and  $M_b F_{-m_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $O = O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 144 terms.

**Time Complexity:** Time spent by the prover is 0.070 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_b = M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.3 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 30 terms.

**Time Complexity:** Time spent by the prover is 0.030 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $O = \neg O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 101 terms.

**Time Complexity:** Time spent by the prover is 0.270 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 101 terms.

**Time Complexity:** Time spent by the prover is 0.520 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 101 terms.

**Time Complexity:** Time spent by the prover is 0.540 seconds. There are no ndg conditions.

# Problem 287

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 287: Given a point  $O$ , a point  $M_b$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $G$ , construct a point  $B$  (rule W01); ;
2. Using the point  $O$  and the point  $M_b$ , construct a line  $m_b$  (rule W02); % DET: points  $O$  and  $M_b$  are not the same;
3. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
4. Using the point  $M_b$  and the line  $m_b$ , construct a line  $b$  (rule W10a); ;
5. Using the circle  $k(O, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same.

Determination conditions: points  $O$  and  $M_b$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D1,D12,D22,D26,GD01,GL03,GL04,GL09,L11,L12,L56]

Solving time: 1.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point O 65 51.14
```

```
point M_{b} 95 67.5
```

```
point G 70 58.33
```

```
color 220 0 0
```

```

fontsize 9

cmark_t 0
cmark_lt M_{b}
cmark_t G
color 0 0 0
fontsize 8

% Constructing a point B such that  $M_{\{b\}}B/M_{\{b\}}G=3$ 
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

% DET: points O and  $M_{\{b\}}$  are not the same
% Constructing a line  $m_{\{b\}}$  which passes through point O and point  $M_{\{b\}}$ 
line m_{b} O M_{b}

color 200 200 200
drawline m_{b}
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Constructing a line b which is perpendicular to line  $m_{\{b\}}$  and which passes through point  $M_{\{b\}}$ 
perp b M_{b} m_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle  $k(O,C)$  intersect
% Constructing points C and A which are in intersection of  $k(O,C)$  and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A

drawsegment A B

```

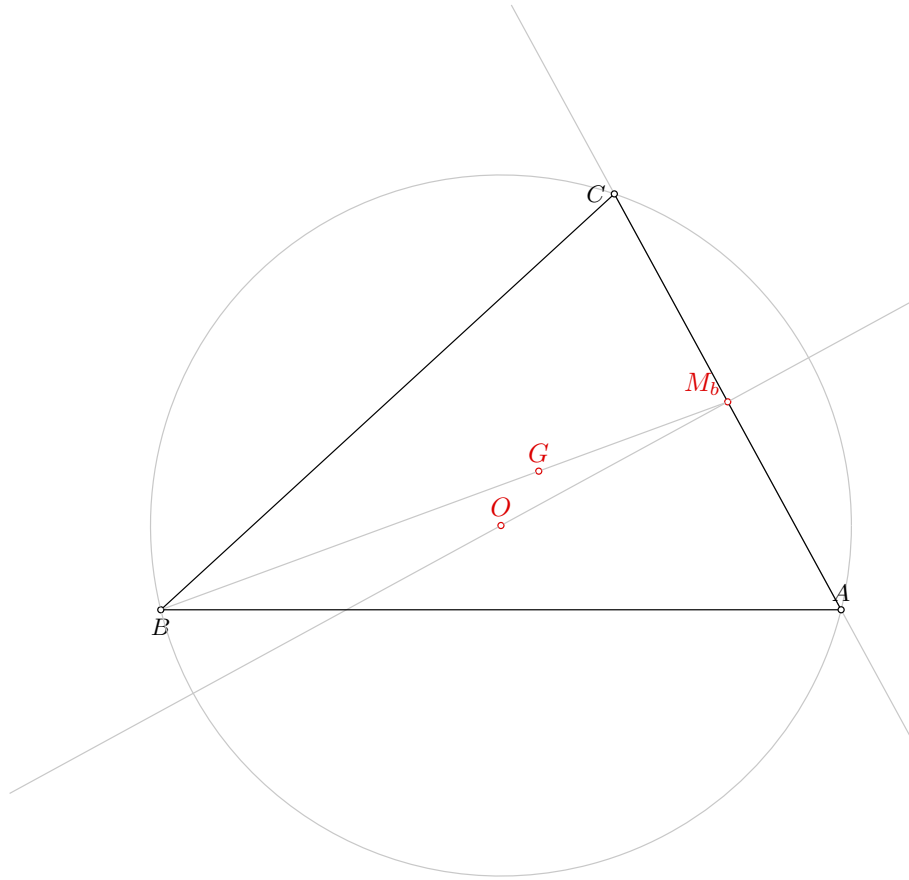


Figure 1: Illustration of the problem 0287

```
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same
% Determination conditions: points O and M_{b} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

Construction steps:

- Free point  $O$

- Free point  $M_b$
- Free point  $G$
- Segment division point  $B$  of segment  $M_bG$  with division coefficient -1.5
- Line  $m_b$  through two points  $O$  and  $M_b$
- Circle  $k(O, C)$  with center  $O$  and one point  $B$
- Line  $b$  through point  $M_b$  perpendicular to line  $m_b$
- Intersection point  $C$  of point sets  $k(O, C)$  and  $b$
- Intersection point  $A$  of point sets  $k(O, C)$  and  $b$
- Segment division point  $\_M_b$  of segment  $CA$  with division coefficient 1.0
- Line  $\_b$  through two points  $A$  and  $C$
- Line  $\_a$  through two points  $B$  and  $C$
- Segment division point  $\_M_a$  of segment  $BC$  with division coefficient 1.0
- Line  $\_m_a$  through point  $\_M_a$  perpendicular to line  $\_a$
- Line  $\_m_b$  through point  $\_M_b$  perpendicular to line  $\_b$
- Intersection point  $\_O$  of point sets  $\_m_a$  and  $\_m_b$
- Line  $\_t_a$  through two points  $A$  and  $\_M_a$
- Line  $\_t_b$  through two points  $B$  and  $\_M_b$
- Intersection point  $\_G$  of point sets  $\_t_a$  and  $\_t_b$

### Theorem statement:

- Points  $O$  and  $\_O$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $O$  has been assigned following coordinates:  $(0, 0)$

#### 4.1.2 Proving $M_b = \_M_b$

Proving failed

#### 4.1.3 Proving $G = \_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 147 terms.

**Time Complexity:** Time spent by the prover is 0.626 seconds.

**NDG conditions** Points  $B$ ,  $C$  and  $\_M_a$  are not collinear

Point  $\_M_a$  is not on angle bisector of angle with vertex  $A$  and two points from different rays  $M_b$  and  $\_M_b$

### 4.2 GCLC - Area method

#### 4.2.1 Proving $O = \_O$

Proving failed

#### 4.2.2 Proving $M_b = \_M_b$

Proving failed

#### 4.2.3 Proving $G = \_G$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $O=_O$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 75 terms.

**Time Complexity:** Time spent by the prover is 0.050 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_b=_M_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 14 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.3 Proving $G=_G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 594 terms.

**Time Complexity:** Time spent by the prover is 0.970 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $O=_O$

Proving failed

#### 4.4.2 Proving $M_b=_M_b$

Proving failed

#### 4.4.3 Proving $G=_G$

Proving failed



# Problem 288

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 288: Given a point  $O$ , a point  $M_b$  and a point  $H_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $M_b$ , construct a line  $m_b$  (rule W02); % DET: points  $O$  and  $M_b$  are not the same;
2. Using the point  $H_a$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_a$  and  $M_b$  are not the same;
3. Using the point  $M_b$  and the line  $m_b$ , construct a line  $b$  (rule W10a); ;
4. Using the circle  $k(M_b, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(M_b, C)$  intersect;
5. Using the point  $H_a$  and the point  $C$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $C$  are not the same;
6. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
7. Using the circle  $k(O, C)$ , the line  $a$ , the point  $O$  and the point  $C$ , construct a point  $B$  (rule W05); % NDG: line  $a$  and circle  $k(O, C)$  intersect % DET: points  $C$  and  $B$  must be different.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same; line  $b$  and circle  $k(M_b, C)$  intersect; points  $H_a$  and  $M_b$  are not the same.

Determination conditions: points  $C$  and  $B$  must be different; points  $H_a$  and  $C$  are not the same; points  $O$  and  $M_b$  are not the same.

Rules used: [W02,W04,W05,W06,W10a]

Lemmas used: [D1,D12,D22,D26,D5,GD01,GD02,GL09,L12,L43,L44]

Solving time: 2.4 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point O 65 51.14
point M_{b} 95 67.5
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_t O
cmark_lt M_{b}
cmark_r H_{a}
color 0 0 0
fontsize 8

% DET: points O and M_{b} are not the same
% Constructing a line m_{b} which passes through point O and point M_{b}
line m_{b} O M_{b}

color 200 200 200
drawline m_{b}
color 0 0 0

% NDG: points H_{a} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{a}
circle k(M_{b},C) M_{b} H_{a}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% Constructing a line b which is perpendicular to line m_{b} and which passes through point M_{b}
perp b M_{b} m_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(M_{b},C) intersect
% Constructing points C and A which are in intersection of k(M_{b},C) and b
intersec2 C A k(M_{b},C) b
cmark_l C
cmark_t A

% DET: points H_{a} and C are not the same
```

```

% Constructing a line a which passes through point H_{a} and point C
line a H_{a} C

color 200 200 200
drawline a
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect% DET: points C and B must be different
% Constructing a point P_{\_G48454} which is a foot of the point O on the line a
foot P_{\_G48454} O a
cmark_r P_{\_G48454}
color 200 200 200
drawline O P_{\_G48454}
color 0 0 0

% Constructing a point B which is an image of the point C in the symmetry to point/line P_{\_G
48454}
sim B P_{\_G48454} C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; points C and O are not the same;
line b and circle k(M_{b},C) intersect; points H_{a} and M_{b} are not the same
% Determination conditions: points C and B must be different; points H_{a} and C are not the same;
points O and M_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

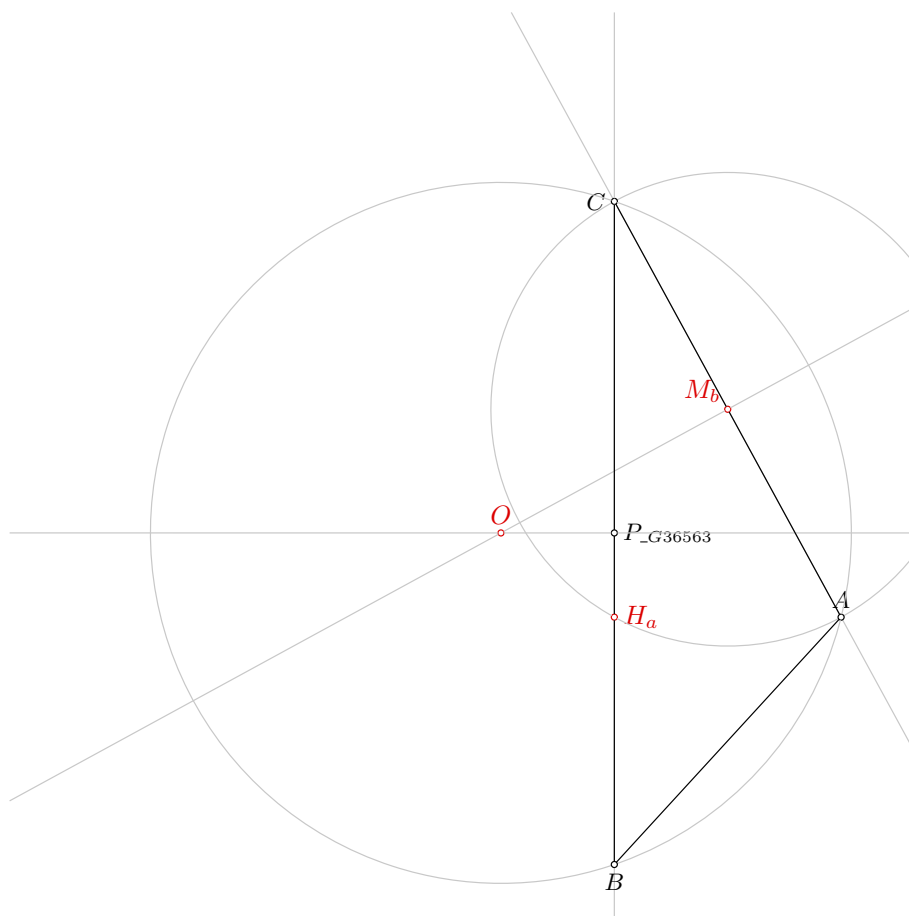


Figure 1: Illustration of the problem 0288

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

Construction steps:

- Free point  $O$
- Free point  $M_b$
- Free point  $H_a$
- Line  $m_b$  through two points  $O$  and  $M_b$
- Circle  $k(M_b, C)$  with center  $M_b$  and one point  $H_a$
- Line  $b$  through point  $M_b$  perpendicular to line  $m_b$
- Intersection point  $C$  of point sets  $k(M_b, C)$  and  $b$
- Intersection point  $A$  of point sets  $k(M_b, C)$  and  $b$
- Line  $a$  through two points  $H_a$  and  $C$
- Circle  $k(O, C)$  with center  $O$  and one point  $C$
- Line footPointPerpLine309 through point  $O$  perpendicular to line  $a$
- Intersection point  $P_{G42858}$  of point sets footPointPerpLine309 and  $a$
- Cental symmetric point  $B$  of point  $C$  with respect to center of symmetry  $P_{G42858}$
- Segment division point  $_M_b$  of segment  $CA$  with division coefficient 1.0
- Line  $_b$  through two points  $A$  and  $C$
- Line  $_a$  through two points  $B$  and  $C$
- Segment division point  $_M_a$  of segment  $BC$  with division coefficient 1.0
- Line  $_m_a$  through point  $_M_a$  perpendicular to line  $_a$
- Line  $_m_b$  through point  $_M_b$  perpendicular to line  $_b$
- Intersection point  $_O$  of point sets  $_m_a$  and  $_m_b$
- Line  $_h_a$  through point  $A$  perpendicular to line  $_a$
- Intersection point  $_H_a$  of point sets  $_a$  and  $_h_a$

**Theorem statement:**

- Points  $O$  and  $_O$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $O$  has been assigned following coordinates:  $(0, 0)$

#### 4.1.2 Proving $M_b=_M_b$

Proving failed

#### 4.1.3 Proving $H_a=_H_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $O=_O$

Proving failed

#### **4.2.2 Proving $M_b = \neg M_b$**

Proving failed

#### **4.2.3 Proving $H_a = \neg H_a$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $O = \neg O$**

Proving failed

#### **4.3.2 Proving $M_b = \neg M_b$**

Proving failed

#### **4.3.3 Proving $H_a = \neg H_a$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $O = \neg O$**

Proving failed

#### **4.4.2 Proving $M_b = \neg M_b$**

Proving failed

#### **4.4.3 Proving $H_a = \neg H_a$**

Proving failed

# Problem 289

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 289: Given a point  $M_b$ , a point  $H_b$  and a point  $O$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $H_b$  are not the same;
2. Using the point  $M_b$  and the line  $b$ , construct a line  $m_b$  (rule W10b); ;
3. Choose freely a point  $O$  on the line  $m_b$  (rule WOnline2);
4. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
5. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) ;
6. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
7. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
8. Using the circle  $k(O, C)$  and the line  $h_b$ , construct a point  $B_k$  and a point  $B$  (rule W04); % NDG: line  $h_b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $h_b$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $M_b$  and  $H_b$  are not the same.

Rules used: [W01,W02,W04,W06,W10b,WOnline1,WOnline2]

Lemmas used: [D12,D22,D26,D39,D6,D9,GD01,GL03,GL04,GL09,L11,L12]

Solving time: 16.7 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point H_{b} 89.36 77.83
point O 65 51.14

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_l H_{b}
cmark_t O
color 0 0 0
fontsize 8

% DET: points M_{b} and H_{b} are not the same
% Constructing a line b which passes through point M_{b} and point H_{b}
line b M_{b} H_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line m_{b} which is perpendicular to line b and which passes through point M_{b}
perp m_{b} M_{b} b

color 200 200 200
drawline m_{b}
color 0 0 0

% Generating random value V[_G30939]
random V[_G30939]

% Calculating value V[_G30960] using formula  $V[_G30939]*20$ 
expression V[_G30960] { V[_G30939]*20 }

% Constructing a point O which is a point for which holds  $M_{b}O = V[_G30960]$  and angle  $H_{b}M_{b}O = 90$ 
turtle O H_{b} M_{b} 90 V[_G30960]
cmark_t O

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b
```



```

color 200 200 200
drawline h_{b}
color 0 0 0

% Choosing randomly a point A on the line M_{b}H_{b}
online A M_{b} H_{b}
cmark_t A
color 200 200 200
drawline M_{b} H_{b}
color 0 0 0

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line h_{b} and circle k(O,C) intersect
% Constructing points B_{k} and B which are in intersection of k(O,C) and h_{b}
intersec2 B_{k} B k(O,C) h_{b}
cmark_r B_{k}
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line h_{b} and circle k(O,C) intersect; points A and O are not the
same
% Determination conditions: points M_{b} and H_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

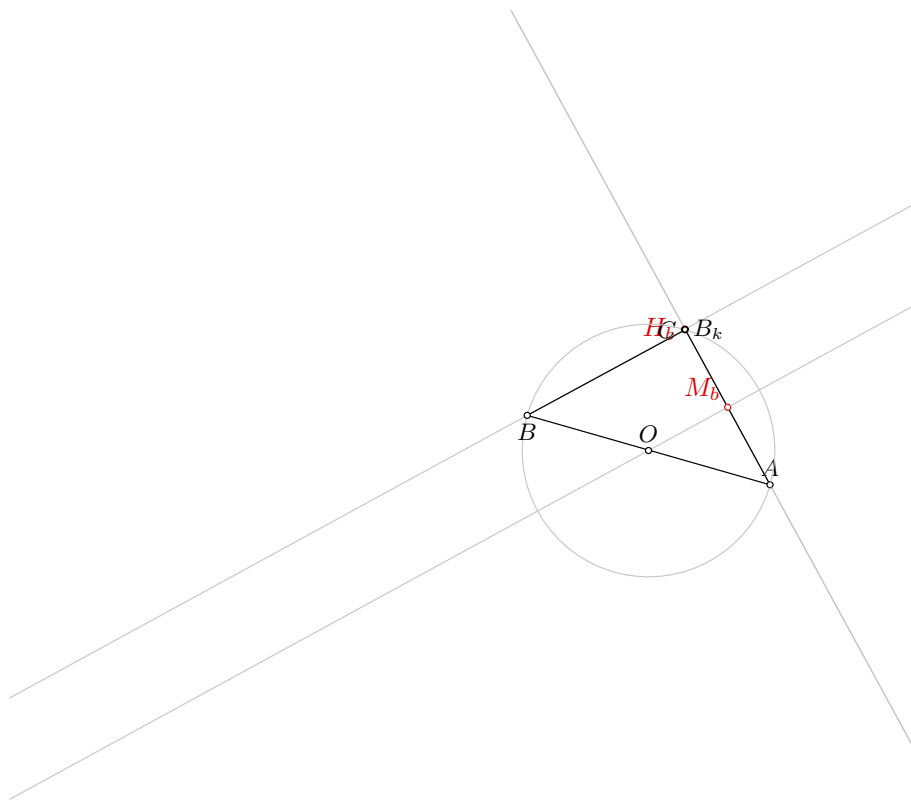


Figure 1: Illustration of the problem 0289

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = \neg M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.062 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b = \neg H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.088 seconds.

**NDG conditions** Line through points  $C$  and  $A$  is not perpendicular to line through points  $A$  and  $M_b$

#### 4.1.3 Proving $O = \neg O$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b = \neg M_b$

Proving failed

#### 4.2.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.2.3 Proving $O = \neg O$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = \neg M_b$

Proving failed

#### 4.3.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.3.3 Proving $O = \neg O$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_b = \neg M_b$

Proving failed

#### 4.4.2 Proving $H_b = H_b$

Proving failed

#### 4.4.3 Proving $O = O$

Proving failed

# Problem 290

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 290: Given a point  $O$ , a point  $M_b$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $M_b$ , construct a line  $m_b$  (rule W02); % DET: points  $O$  and  $M_b$  are not the same;
2. Using the point  $H_c$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_c$  and  $M_b$  are not the same;
3. Using the point  $M_b$  and the line  $m_b$ , construct a line  $b$  (rule W10a); ;
4. Using the circle  $k(M_b, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(M_b, C)$  intersect;
5. Using the point  $H_c$  and the point  $A$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $A$  are not the same;
6. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
7. Using the circle  $k(O, C)$ , the line  $c$ , the point  $O$  and the point  $A$ , construct a point  $B$  (rule W05); % NDG: line  $c$  and circle  $k(O, C)$  intersect % DET: points  $A$  and  $B$  must be different.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same; line  $b$  and circle  $k(M_b, C)$  intersect; points  $H_c$  and  $M_b$  are not the same.

Determination conditions: points  $A$  and  $B$  must be different; points  $H_c$  and  $A$  are not the same; points  $O$  and  $M_b$  are not the same.

Rules used: [W02,W04,W05,W06,W10a]

Lemmas used: [D1,D12,D22,D26,D7,GD01,GD02,GL09,L11,L12,L43,L45]

Solving time: 2.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point O 65 51.14
point M_{b} 95 67.5
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_t O
cmark_lt M_{b}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points O and M_{b} are not the same
% Constructing a line m_{b} which passes through point O and point M_{b}
line m_{b} O M_{b}

color 200 200 200
drawline m_{b}
color 0 0 0

% NDG: points H_{c} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{c}
circle k(M_{b},C) M_{b} H_{c}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% Constructing a line b which is perpendicular to line m_{b} and which passes through point M_{b}
perp b M_{b} m_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(M_{b},C) intersect
% Constructing points C and A which are in intersection of k(M_{b},C) and b
intersec2 C A k(M_{b},C) b
cmark_l C
cmark_t A

% DET: points H_{c} and A are not the same
```

```

% Constructing a line c which passes through point H_{c} and point A
line c H_{c} A

color 200 200 200
drawline c
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line c and circle k(O,C) intersect% DET: points A and B must be different
% Constructing a point P_{\_G54175} which is a foot of the point O on the line c
foot P_{\_G54175} O c
cmark_r P_{\_G54175}
color 200 200 200
drawline O P_{\_G54175}
color 0 0 0

% Constructing a point B which is an image of the point A in the symmetry to point/line P_{\_G
54175}
sim B P_{\_G54175} A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
line b and circle k(M_{b},C) intersect; points H_{c} and M_{b} are not the same
% Determination conditions: points A and B must be different; points H_{c} and A are not the same;
points O and M_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

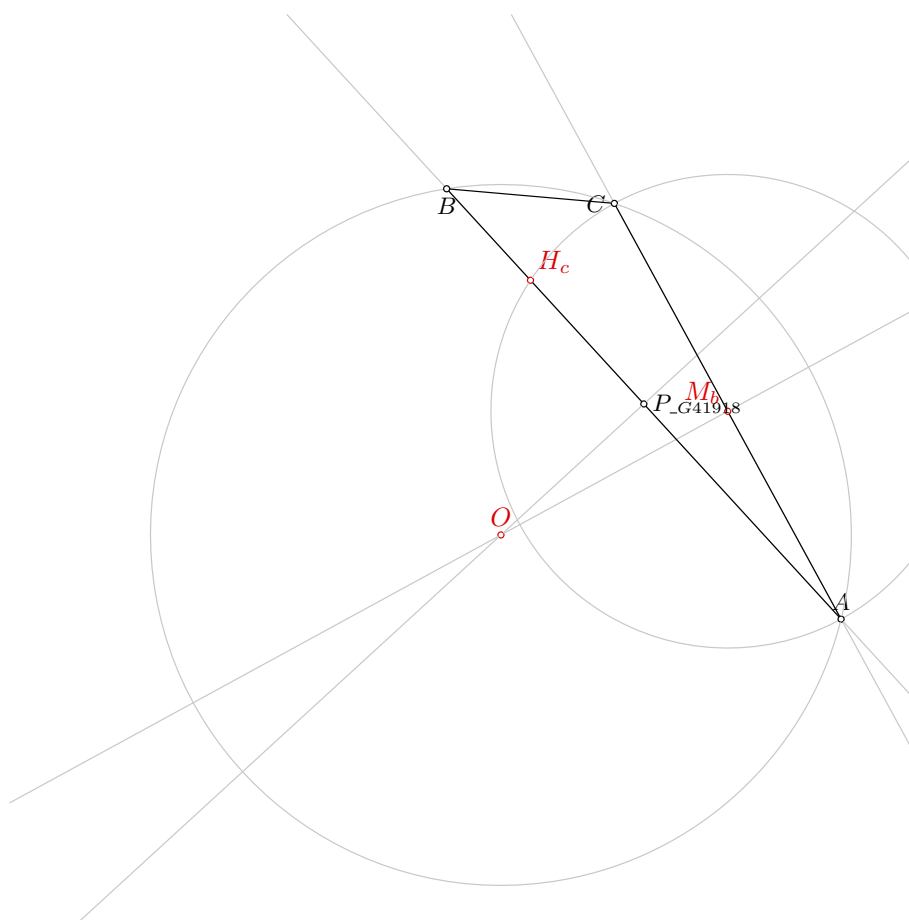


Figure 1: Illustration of the problem 0290



## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

Construction steps:

- Free point  $O$
- Free point  $M_b$
- Free point  $H_c$
- Line  $m_b$  through two points  $O$  and  $M_b$
- Circle  $k(M_b, C)$  with center  $M_b$  and one point  $H_c$
- Line  $b$  through point  $M_b$  perpendicular to line  $m_b$
- Intersection point  $C$  of point sets  $k(M_b, C)$  and  $b$
- Intersection point  $A$  of point sets  $k(M_b, C)$  and  $b$
- Line  $c$  through two points  $H_c$  and  $A$
- Circle  $k(O, C)$  with center  $O$  and one point  $C$
- Line footPointPerpLine517 through point  $O$  perpendicular to line  $c$
- Intersection point  $P_{G48402}$  of point sets footPointPerpLine517 and  $c$
- Cental symmetric point  $B$  of point  $A$  with respect to center of symmetry  $P_{G48402}$
- Segment division point  $_M_b$  of segment  $CA$  with division coefficient 1.0
- Line  $_b$  through two points  $A$  and  $C$
- Line  $_a$  through two points  $B$  and  $C$
- Segment division point  $_M_a$  of segment  $BC$  with division coefficient 1.0
- Line  $_m_a$  through point  $_M_a$  perpendicular to line  $_a$
- Line  $_m_b$  through point  $_M_b$  perpendicular to line  $_b$
- Intersection point  $_O$  of point sets  $_m_a$  and  $_m_b$
- Line  $_c$  through two points  $A$  and  $B$
- Line  $_h_c$  through point  $C$  perpendicular to line  $_c$
- Intersection point  $_H_c$  of point sets  $_c$  and  $_h_c$

**Theorem statement:**

- Points  $O$  and  $_O$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $O$  has been assigned following coordinates:  $(0, 0)$

#### 4.1.2 Proving $M_b=_M_b$

Proving failed

#### 4.1.3 Proving $H_c=_H_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $O=_O$

Proving failed

#### **4.2.2 Proving $M_b = M_b$**

Proving failed

#### **4.2.3 Proving $H_c = H_c$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $O = O$**

Proving failed

#### **4.3.2 Proving $M_b = M_b$**

Proving failed

#### **4.3.3 Proving $H_c = H_c$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $O = O$**

Proving failed

#### **4.4.2 Proving $M_b = M_b$**

Proving failed

#### **4.4.3 Proving $H_c = H_c$**

Proving failed

# Problem 291

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 291: Given a point  $O$ , a point  $M_b$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $H$ , construct a point  $G$  (rule W01); ;
2. Using the point  $M_b$  and the point  $G$ , construct a point  $B$  (rule W01); ;
3. Using the point  $O$  and the point  $M_b$ , construct a line  $m_b$  (rule W02); % DET: points  $O$  and  $M_b$  are not the same;
4. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
5. Using the point  $M_b$  and the line  $m_b$ , construct a line  $b$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same.

Determination conditions: points  $O$  and  $M_b$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D1,D12,D22,D26,GD01,GL03,GL04,GL09,L11,L12,L56,L58]

Solving time: 1.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point O 65 51.14  
point M_{b} 95 67.5  
point H 80 72.73
```

```

color 220 0 0
fontsize 9

cmark_t 0
cmark_lt M_{b}
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a line L_{\_G45436} which passes through point O and point H
line L_{\_G45436} O H

color 200 200 200
drawline L_{\_G45436}
color 0 0 0

% Constructing a point P_{\_G45537} with coordinates (0,0)
point P_{\_G45537} 0 0
cmark_r P_{\_G45537}

% Constructing a point P_{\_G45461} such that  $OP_{\_G45461}/OP_{\_G45537}=1$ 
towards P_{\_G45461} 0 P_{\_G45537} 1
cmark_r P_{\_G45461}
color 200 200 200
drawsegment 0 P_{\_G45461}
color 0 0 0

% Constructing a point P_{\_G45506} such that  $OP_{\_G45506}/OP_{\_G45537}=3$ 
towards P_{\_G45506} 0 P_{\_G45537} 3
cmark_r P_{\_G45506}
color 200 200 200
drawsegment 0 P_{\_G45506}
color 0 0 0

% Constructing a line L_{\_G45467} which passes through point H and point P_{\_G45506}
line L_{\_G45467} H P_{\_G45506}

color 200 200 200
drawline L_{\_G45467}
color 0 0 0

% Constructing a line L_{\_G45430} which contains the point P_{\_G45461} and is parallel to the
line L_{\_G45467}
parallel L_{\_G45430} P_{\_G45461} L_{\_G45467}

color 200 200 200
drawline L_{\_G45430}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G45430} and line L_{\_G45436}
intersec G L_{\_G45430} L_{\_G45436}

```

```

cmark_t G

% Constructing a point B such that  $M_{\{b\}}B/M_{\{b\}}G=3$ 
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

% DET: points O and M_{b} are not the same
% Constructing a line m_{b} which passes through point O and point M_{b}
line m_{b} O M_{b}

color 200 200 200
drawline m_{b}
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Constructing a line b which is perpendicular to line m_{b} and which passes through point M_{b}
perp b M_{b} m_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same
% Determination conditions: points O and M_{b} are not the same

```

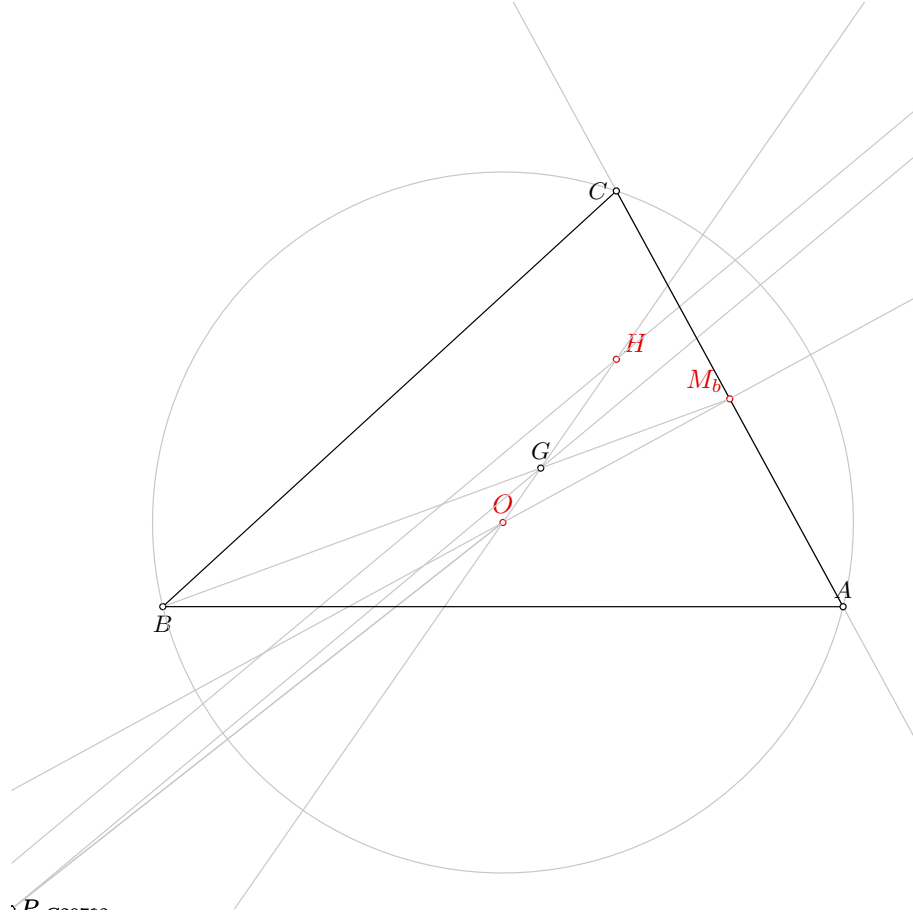


Figure 1: Illustration of the problem 0291

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O = O$

Construction steps:

- Free point  $O$
- Free point  $M_b$
- Free point  $H$
- Line  $L_{G38844}$  through two points  $O$  and  $H$
- Free point  $P_{G38847}$

- Segment division point  $P_{G38850}$  of segment  $P_{G38847}P_{G38847}$  with division coefficient 1.0
- Segment division point  $P_{G38853}$  of segment  $OP_{G38847}$  with division coefficient -1.5
- Line  $L_{G38856}$  through two points  $H$  and  $P_{G38853}$
- Line  $L_{G38859}$  through point  $P_{G38850}$  parallel with line  $L_{G38856}$
- Intersection point  $G$  of point sets  $L_{G38859}$  and  $L_{G38844}$
- Segment division point  $B$  of segment  $M_bG$  with division coefficient -1.5
- Line  $m_b$  through two points  $O$  and  $M_b$
- Circle  $k(O, C)$  with center  $O$  and one point  $B$
- Line  $b$  through point  $M_b$  perpendicular to line  $m_b$
- Intersection point  $C$  of point sets  $k(O, C)$  and  $b$
- Intersection point  $A$  of point sets  $k(O, C)$  and  $b$
- Segment division point  $\_M_b$  of segment  $CA$  with division coefficient 1.0
- Line  $\_b$  through two points  $A$  and  $C$
- Line  $\_a$  through two points  $B$  and  $C$
- Segment division point  $\_M_a$  of segment  $BC$  with division coefficient 1.0
- Line  $\_m_a$  through point  $\_M_a$  perpendicular to line  $\_a$
- Line  $\_m_b$  through point  $\_M_b$  perpendicular to line  $\_b$
- Intersection point  $\_O$  of point sets  $\_m_a$  and  $\_m_b$
- Line  $\_h_a$  through point  $A$  perpendicular to line  $\_a$
- Line  $\_h_b$  through point  $B$  perpendicular to line  $\_b$
- Intersection point  $\_H$  of point sets  $\_h_a$  and  $\_h_b$

### Theorem statement:

- Points  $O$  and  $\_O$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $O$  has been assigned following coordinates:  $(0, 0)$

#### 4.1.2 Proving $M_b = \_M_b$

Proving failed

#### 4.1.3 Proving $H = \_H$

### Construction steps:

- Free point  $O$
- Free point  $M_b$
- Free point  $H$
- Line  $L_{G43024}$  through two points  $O$  and  $H$
- Free point  $P_{G43027}$
- Segment division point  $P_{G43030}$  of segment  $P_{G43027}P_{G43027}$  with division coefficient 1.0
- Segment division point  $P_{G43033}$  of segment  $OP_{G43027}$  with division coefficient -1.5
- Line  $L_{G43036}$  through two points  $H$  and  $P_{G43033}$
- Line  $L_{G43039}$  through point  $P_{G43030}$  parallel with line  $L_{G43036}$
- Intersection point  $G$  of point sets  $L_{G43039}$  and  $L_{G43024}$

- Segment division point  $B$  of segment  $M_bG$  with division coefficient -1.5
- Line  $m_b$  through two points  $O$  and  $M_b$
- Circle  $k(O, C)$  with center  $O$  and one point  $B$
- Line  $b$  through point  $M_b$  perpendicular to line  $m_b$
- Intersection point  $C$  of point sets  $k(O, C)$  and  $b$
- Intersection point  $A$  of point sets  $k(O, C)$  and  $b$
- Segment division point  $\_M_b$  of segment  $CA$  with division coefficient 1.0
- Line  $\_b$  through two points  $A$  and  $C$
- Line  $\_a$  through two points  $B$  and  $C$
- Segment division point  $\_M_a$  of segment  $BC$  with division coefficient 1.0
- Line  $\_m_a$  through point  $\_M_a$  perpendicular to line  $\_a$
- Line  $\_m_b$  through point  $\_M_b$  perpendicular to line  $\_b$
- Intersection point  $\_O$  of point sets  $\_m_a$  and  $\_m_b$
- Line  $\_h_a$  through point  $A$  perpendicular to line  $\_a$
- Line  $\_h_b$  through point  $B$  perpendicular to line  $\_b$
- Intersection point  $\_H$  of point sets  $\_h_a$  and  $\_h_b$

### Theorem statement:

- Points  $H$  and  $\_H$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $O$  has been assigned following coordinates:  $(0, 0)$

## 4.2 GCLC - Area method

### 4.2.1 Proving $O=\_O$

Proving failed

### 4.2.2 Proving $M_b=\_M_b$

Proving failed

### 4.2.3 Proving $H=\_H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $O=\_O$

Proving failed

### 4.3.2 Proving $M_b=\_M_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 268 terms.

**Time Complexity:** Time spent by the prover is 0.410 seconds. There are no ndg conditions.



### 4.3.3 Proving $H=_H$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 11621 terms.

**Time Complexity:** Time spent by the prover is 51.250 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $O=_O$

Proving failed

### 4.4.2 Proving $M_b=_M_b$

Proving failed

### 4.4.3 Proving $H=_H$

Proving failed

## Problem 292

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 292: Given a point  $O$ , a point  $M_b$  and a point  $T_a$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 293

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 293: Given a point  $M_b$ , a point  $T_b$  and a point  $O$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $T_b$  are not the same;
2. Using the point  $M_b$  and the line  $b$ , construct a line  $m_b$  (rule W10b); ;
3. Choose freely a point  $O$  on the line  $m_b$  (rule WOnline2);
4. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) ;
5. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
6. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
7. Using the circle  $k(O, C)$  and the line  $m_b$ , construct a point  $N_b$  and a point  $N_{bk}$  (rule W04); % NDG: line  $m_b$  and circle  $k(O, C)$  intersect;
8. Using the point  $N_b$  and the point  $T_b$ , construct a line  $s_b$  (rule W02); % DET: points  $N_b$  and  $T_b$  are not the same;
9. Using the circle  $k(O, C)$ , the line  $s_b$ , the point  $O$  and the point  $N_b$ , construct a point  $B$  (rule W05); % NDG: line  $s_b$  and circle  $k(O, C)$  intersect % DET: points  $N_b$  and  $B$  must be different.

Non-degenerate conditions: line  $s_b$  and circle  $k(O, C)$  intersect; line  $m_b$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $N_b$  and  $B$  must be different; points  $N_b$  and  $T_b$  are not the same; points  $M_b$  and  $T_b$  are not the same.

Rules used: [W01,W02,W04,W05,W06,W10b,WOnline1,WOnline2]  
 Lemmas used: [D12,D22,D24,D26,D48,D51,GD01,GL03,GL04,GL09,L11,L12,L26,L6]  
 Solving time: 7.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point T_{b} 94.25 68.88
point O 65 51.14

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_t T_{b}
cmark_t O
color 0 0 0
fontsize 8

% DET: points M_{b} and T_{b} are not the same
% Constructing a line b which passes through point M_{b} and point T_{b}
line b M_{b} T_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line m_{b} which is perpendicular to line b and which passes through point M_{b}
perp m_{b} M_{b} b

color 200 200 200
drawline m_{b}
color 0 0 0

% Generating random value V[_G38258]
random V[_G38258]

% Calculating value V[_G38279] using formula V[_G38258]*20
expression V[_G38279] { V[_G38258]*20 }

% Constructing a point O which is a point for which holds M_{b}O = V[_G38279] and angle T_{b}M_{b}O
= 90
turtle O T_{b} M_{b} 90 V[_G38279]
cmark_t O
```

```

% Choosing randomly a point A on the line  $M_{\{b\}}T_{\{b\}}$ 
online A  $M_{\{b\}}$   $T_{\{b\}}$ 
cmark_t A
color 200 200 200
drawline  $M_{\{b\}}$   $T_{\{b\}}$ 
color 0 0 0

% Constructing a point C such that  $AC/AM_{\{b\}}=2$ 
towards C A  $M_{\{b\}}$  2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line  $m_{\{b\}}$  and circle  $k(O,C)$  intersect
% Constructing points  $N_{\{b\}}$  and  $N_{\{bk\}}$  which are in intersection of  $k(O,C)$  and  $m_{\{b\}}$ 
intersec2  $N_{\{b\}}$   $N_{\{bk\}}$  k(O,C)  $m_{\{b\}}$ 
cmark_rb  $N_{\{b\}}$ 
cmark_r  $N_{\{bk\}}$ 

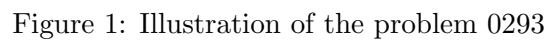
% DET: points  $N_{\{b\}}$  and  $T_{\{b\}}$  are not the same
% Constructing a line  $s_{\{b\}}$  which passes through point  $N_{\{b\}}$  and point  $T_{\{b\}}$ 
line  $s_{\{b\}}$   $N_{\{b\}}$   $T_{\{b\}}$ 

color 200 200 200
drawline  $s_{\{b\}}$ 
color 0 0 0

% NDG: line  $s_{\{b\}}$  and circle  $k(O,C)$  intersect% DET: points  $N_{\{b\}}$  and B must be different
% Constructing a point  $P_{\{\backslash\_G38771\}}$  which is a foot of the point O on the line  $s_{\{b\}}$ 
foot  $P_{\{\backslash\_G38771\}}$  O  $s_{\{b\}}$ 
cmark_r  $P_{\{\backslash\_G38771\}}$ 
color 200 200 200
drawline O  $P_{\{\backslash\_G38771\}}$ 
color 0 0 0

% Constructing a point B which is an image of the point  $N_{\{b\}}$  in the symmetry to point/line  $P_{\{\backslash\_G38771\}}$ 

```



```
% Non-degenerate conditions: line s_{b} and circle k(O,C) intersect; line m_{b} and circle k(O,C)
    intersect; points A and O are not the same
% Determination conditions: points N_{b} and B must be different; points N_{b} and T_{b} are not
    the same; points M_{b} and T_{b} are not the same
```

Illustration of the constructed figure is given in Figure 1

## 1566

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = \neg M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.1.3 Proving $O = \neg O$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b = \neg M_b$

Proving failed

#### 4.2.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.2.3 Proving $O = \neg O$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = \neg M_b$

Proving failed

#### 4.3.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.3.3 Proving $O = \neg O$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_b = \neg M_b$

Proving failed

#### 4.4.2 Proving $T_b = \neg T_b$

Proving failed

### 4.4.3 Proving $O=_O$

Proving failed



## Problem 294

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 294: Given a point  $O$ , a point  $M_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 295

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 295: Given a point  $O$ , a point  $M_b$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 296

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 296: Given a point  $O$ , a point  $M_c$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $G$ , construct a point  $C$  (rule W01); ;
2. Using the point  $O$  and the point  $M_c$ , construct a line  $m_c$  (rule W02); % DET: points  $O$  and  $M_c$  are not the same;
3. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
4. Using the point  $M_c$  and the line  $m_c$ , construct a line  $c$  (rule W10a); ;
5. Using the circle  $k(O, C)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same.

Determination conditions: points  $O$  and  $M_c$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D13,D20,D26,GD01,GL03,GL04,GL09,L1,L11,L12,L57]

Solving time: 1.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point O 65 51.14
```

```
point M_{c} 50 67.5
```

```
point G 70 58.33
```

```
color 220 0 0
```

```

fontsize 9

cmark_t 0
cmark_lt M_{c}
cmark_t G
color 0 0 0
fontsize 8

% Constructing a point C such that  $M_{\{c\}}C/M_{\{c\}}G=3$ 
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

% DET: points O and  $M_{\{c\}}$  are not the same
% Constructing a line  $m_{\{c\}}$  which passes through point O and point  $M_{\{c\}}$ 
line m_{c} O M_{c}

color 200 200 200
drawline m_{c}
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Constructing a line c which is perpendicular to line  $m_{\{c\}}$  and which passes through point  $M_{\{c\}}$ 
perp c M_{c} m_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle  $k(O,C)$  intersect
% Constructing points A and B which are in intersection of  $k(O,C)$  and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B

```

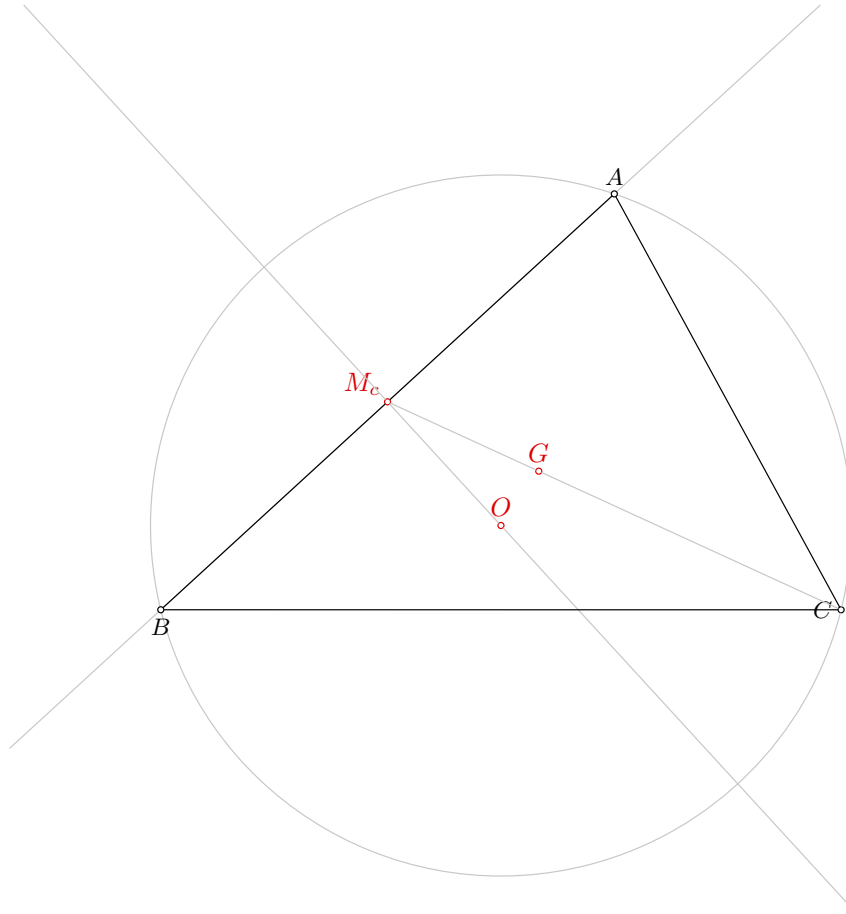


Figure 1: Illustration of the problem 0296

```
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same
% Determination conditions: points O and M_{c} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

Proving failed

#### 4.1.2 Proving $M_c = \neg M_c$

Proving failed

#### 4.1.3 Proving $G = \neg G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 159 terms.

**Time Complexity:** Time spent by the prover is 0.996 seconds.

**NDG conditions** Points  $A$ ,  $\neg M_a$  and  $\neg M_b$  are not collinear

Point  $\neg M_a$  is not on angle bisector of angle with vertex  $A$  and two points from different rays  $O$  and  $B$

### 4.2 GCLC - Area method

#### 4.2.1 Proving $O = \neg O$

Proving failed

#### 4.2.2 Proving $M_c = \neg M_c$

Proving failed

#### 4.2.3 Proving $G = \neg G$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $O = \neg O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 922 terms.

**Time Complexity:** Time spent by the prover is 1.710 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_c = \neg M_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 18 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

#### 4.3.3 Proving $G = \neg G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 1633 terms.

**Time Complexity:** Time spent by the prover is 4.700 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $O = \neg O$

Proving failed

#### 4.4.2 Proving $M_c = \neg M_c$

Proving failed

### 4.4.3 Proving $G=_G$

Proving failed

# Problem 297

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 297: Given a point  $O$ , a point  $M_c$  and a point  $H_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $M_c$ , construct a line  $m_c$  (rule W02); % DET: points  $O$  and  $M_c$  are not the same;
2. Using the point  $H_a$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_a$  and  $M_c$  are not the same;
3. Using the point  $M_c$  and the line  $m_c$ , construct a line  $c$  (rule W10a); ;
4. Using the circle  $k(M_c, A)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(M_c, A)$  intersect;
5. Using the point  $H_a$  and the point  $B$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $B$  are not the same;
6. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
7. Using the circle  $k(O, C)$ , the line  $a$ , the point  $O$  and the point  $B$ , construct a point  $C$  (rule W05); % NDG: line  $a$  and circle  $k(O, C)$  intersect % DET: points  $B$  and  $C$  must be different.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same; line  $c$  and circle  $k(M_c, A)$  intersect; points  $H_a$  and  $M_c$  are not the same.

Determination conditions: points  $B$  and  $C$  must be different; points  $H_a$  and  $B$  are not the same; points  $O$  and  $M_c$  are not the same.

Rules used: [W02,W04,W05,W06,W10a]

Lemmas used: [D13,D20,D26,D5,GD01,GD02,GL09,L1,L11,L12,L40,L42]

Solving time: 2.4 seconds.



## 3.2 Construction in GCLC language

```
dim 120 120

point O 65 51.14
point M_{c} 50 67.5
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_t O
cmark_lt M_{c}
cmark_r H_{a}
color 0 0 0
fontsize 8

% DET: points O and M_{c} are not the same
% Constructing a line m_{c} which passes through point O and point M_{c}
line m_{c} O M_{c}

color 200 200 200
drawline m_{c}
color 0 0 0

% NDG: points H_{a} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{a}
circle k(M_{c},A) M_{c} H_{a}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% Constructing a line c which is perpendicular to line m_{c} and which passes through point M_{c}
perp c M_{c} m_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(M_{c},A) intersect
% Constructing points A and B which are in intersection of k(M_{c},A) and c
intersec2 A B k(M_{c},A) c
cmark_t A
cmark_b B

% DET: points H_{a} and B are not the same
```

```

% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect% DET: points B and C must be different
% Constructing a point P_{\_G57796} which is a foot of the point O on the line a
foot P_{\_G57796} O a
cmark_r P_{\_G57796}
color 200 200 200
drawline O P_{\_G57796}
color 0 0 0

% Constructing a point C which is an image of the point B in the symmetry to point/line P_{\_G
57796}
sim C P_{\_G57796} B
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% line c and circle k(M_{c},A) intersect; points H_{a} and M_{c} are not the same
% Determination conditions: points B and C must be different; points H_{a} and B are not the same;
% points O and M_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O =_O$

Proving failed

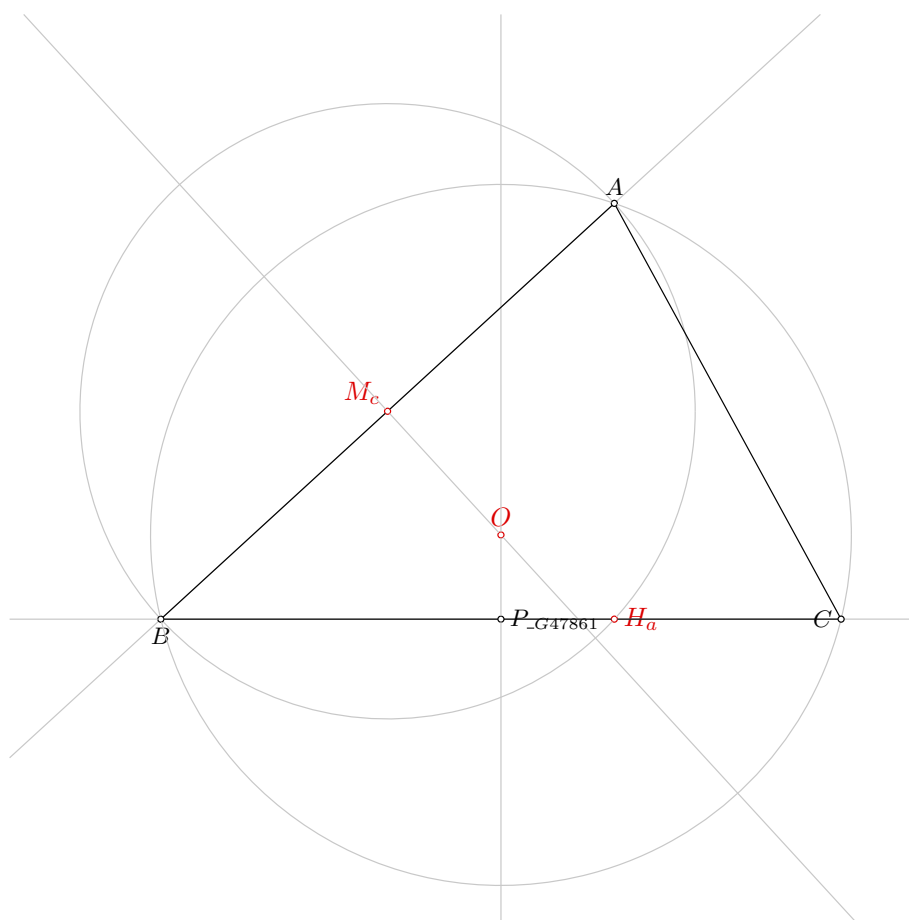


Figure 1: Illustration of the problem 0297

#### **4.1.2 Proving $M_c = M_c$**

Proving failed

#### **4.1.3 Proving $H_a = H_a$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $O = O$**

Proving failed

#### **4.2.2 Proving $M_c = M_c$**

Proving failed

#### **4.2.3 Proving $H_a = H_a$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $O = O$**

Proving failed

#### **4.3.2 Proving $M_c = M_c$**

Proving failed

#### **4.3.3 Proving $H_a = H_a$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $O = O$**

Proving failed

#### **4.4.2 Proving $M_c = M_c$**

Proving failed

#### **4.4.3 Proving $H_a = H_a$**

Proving failed

# Problem 298

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 298: Given a point  $O$ , a point  $M_c$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $M_c$ , construct a line  $m_c$  (rule W02); % DET: points  $O$  and  $M_c$  are not the same;
2. Using the point  $H_b$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_b$  and  $M_c$  are not the same;
3. Using the point  $M_c$  and the line  $m_c$ , construct a line  $c$  (rule W10a); ;
4. Using the circle  $k(M_c, A)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(M_c, A)$  intersect;
5. Using the point  $H_b$  and the point  $A$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $A$  are not the same;
6. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
7. Using the circle  $k(O, C)$ , the line  $b$ , the point  $O$  and the point  $A$ , construct a point  $C$  (rule W05); % NDG: line  $b$  and circle  $k(O, C)$  intersect % DET: points  $A$  and  $C$  must be different.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same; line  $c$  and circle  $k(M_c, A)$  intersect; points  $H_b$  and  $M_c$  are not the same.

Determination conditions: points  $A$  and  $C$  must be different; points  $H_b$  and  $A$  are not the same; points  $O$  and  $M_c$  are not the same.

Rules used: [W02,W04,W05,W06,W10a]

Lemmas used: [D13,D20,D26,D6,GD01,GD02,GL09,L1,L11,L40,L41]

Solving time: 2.4 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point O 65 51.14
point M_{c} 50 67.5
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_t O
cmark_lt M_{c}
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points O and M_{c} are not the same
% Constructing a line m_{c} which passes through point O and point M_{c}
line m_{c} O M_{c}

color 200 200 200
drawline m_{c}
color 0 0 0

% NDG: points H_{b} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{b}
circle k(M_{c},A) M_{c} H_{b}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% Constructing a line c which is perpendicular to line m_{c} and which passes through point M_{c}
perp c M_{c} m_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(M_{c},A) intersect
% Constructing points A and B which are in intersection of k(M_{c},A) and c
intersec2 A B k(M_{c},A) c
cmark_t A
cmark_b B

% DET: points H_{b} and A are not the same
```

```

% Constructing a line b which passes through point H_{b} and point A
line b H_{b} A

color 200 200 200
drawline b
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line b and circle k(O,C) intersect% DET: points A and C must be different
% Constructing a point P_{\_G44005} which is a foot of the point O on the line b
foot P_{\_G44005} O b
cmark_r P_{\_G44005}
color 200 200 200
drawline O P_{\_G44005}
color 0 0 0

% Constructing a point C which is an image of the point A in the symmetry to point/line P_{\_G
44005}
sim C P_{\_G44005} A
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k(O,C) intersect; points A and O are not the same;
% line c and circle k(M_{c},A) intersect; points H_{b} and M_{c} are not the same
% Determination conditions: points A and C must be different; points H_{b} and A are not the same;
% points O and M_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O =_O$

Proving failed

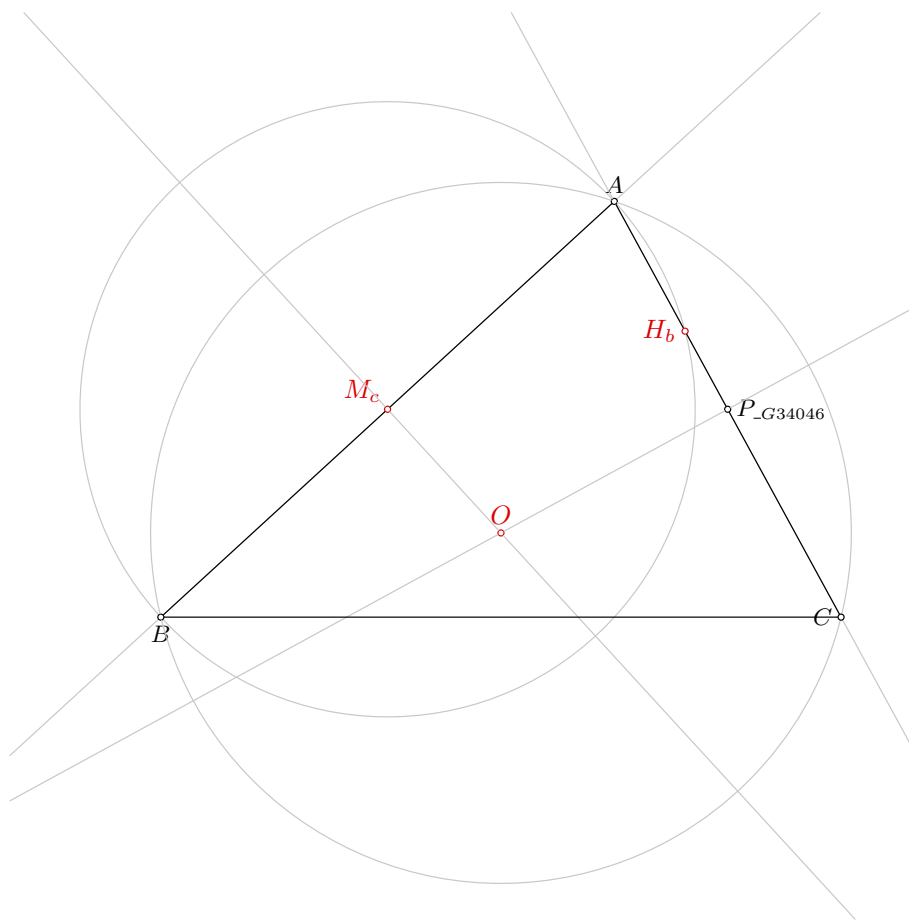


Figure 1: Illustration of the problem 0298



#### **4.1.2 Proving $M_c = \neg M_c$**

Proving failed

#### **4.1.3 Proving $H_b = \neg H_b$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $O = \neg O$**

Proving failed

#### **4.2.2 Proving $M_c = \neg M_c$**

Proving failed

#### **4.2.3 Proving $H_b = \neg H_b$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $O = \neg O$**

Proving failed

#### **4.3.2 Proving $M_c = \neg M_c$**

Proving failed

#### **4.3.3 Proving $H_b = \neg H_b$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $O = \neg O$**

Proving failed

#### **4.4.2 Proving $M_c = \neg M_c$**

Proving failed

#### **4.4.3 Proving $H_b = \neg H_b$**

Proving failed

# Problem 299

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 299: Given a point  $M_c$ , a point  $H_c$  and a point  $O$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $H_c$  are not the same;
2. Using the point  $M_c$  and the line  $c$ , construct a line  $m_c$  (rule W10b); ;
3. Choose freely a point  $O$  on the line  $m_c$  (rule WOnline2);
4. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
5. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) ;
6. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); ;
7. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
8. Using the circle  $k(O, C)$  and the line  $h_c$ , construct a point  $C$  and a point  $C_k$  (rule W04); % NDG: line  $h_c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $h_c$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $M_c$  and  $H_c$  are not the same.

Rules used: [W01,W02,W04,W06,W10b,WOnline1,WOnline2]

Lemmas used: [D10,D13,D20,D26,D40,D7,GD01,GL03,GL09,L11]

Solving time: 16.6 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point H_{c} 68.91 84.83
point O 65 51.14

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_rt H_{c}
cmark_t O
color 0 0 0
fontsize 8

% DET: points M_{c} and H_{c} are not the same
% Constructing a line c which passes through point M_{c} and point H_{c}
line c M_{c} H_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line m_{c} which is perpendicular to line c and which passes through point M_{c}
perp m_{c} M_{c} c

color 200 200 200
drawline m_{c}
color 0 0 0

% Generating random value V[_G64064]
random V[_G64064]

% Calculating value V[_G64085] using formula V[_G64064]*20
expression V[_G64085] { V[_G64064]*20 }

% Constructing a point O which is a point for which holds M_{c}O = V[_G64085] and angle H_{c}M_{c}O
      = 90
turtle O H_{c} M_{c} 90 V[_G64085]
cmark_t O

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c
```

Figure 1: Illustration of the problem 0299

```

color 200 200 200
drawline h_{c}
color 0 0 0

% Choosing randomly a point A on the line M_{c}H_{c}
online A M_{c} H_{c}
cmark_t A
color 200 200 200
drawline M_{c} H_{c}
color 0 0 0

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line h_{c} and circle k(O,C) intersect
% Constructing points C and C_{k} which are in intersection of k(O,C) and h_{c}
intersec2 C C_{k} k(O,C) h_{c}
cmark_l C
cmark_r C_{k}

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line h_{c} and circle k(O,C) intersect; points A and O are not the
same
% Determination conditions: points M_{c} and H_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = \neg M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.082 seconds.

**NDG conditions** Line through points  $A$  and  $H_c$  is not perpendicular to line through points  $H_c$  and  $B$

#### 4.1.3 Proving $O = \neg O$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_c = \neg M_c$

Proving failed

#### 4.2.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.2.3 Proving $O = \neg O$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c = \neg M_c$

Proving failed

#### 4.3.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $O = \neg O$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_c = \neg M_c$

Proving failed

### 4.4.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.3 Proving $O = \neg O$

Proving failed

# Problem 300

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 300: Given a point  $O$ , a point  $M_c$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $H$ , construct a point  $G$  (rule W01); ;
2. Using the point  $M_c$  and the point  $G$ , construct a point  $C$  (rule W01); ;
3. Using the point  $O$  and the point  $M_c$ , construct a line  $m_c$  (rule W02); % DET: points  $O$  and  $M_c$  are not the same;
4. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
5. Using the point  $M_c$  and the line  $m_c$ , construct a line  $c$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same.

Determination conditions: points  $O$  and  $M_c$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D13,D20,D26,GD01,GL03,GL04,GL09,L1,L11,L12,L57,L58]

Solving time: 1.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point O 65 51.14  
point M_{c} 50 67.5  
point H 80 72.73
```

```

color 220 0 0
fontsize 9

cmark_t 0
cmark_lt M_{c}
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a line L_{\_G53344} which passes through point O and point H
line L_{\_G53344} 0 H

color 200 200 200
drawline L_{\_G53344}
color 0 0 0

% Constructing a point P_{\_G53445} with coordinates (0,0)
point P_{\_G53445} 0 0
cmark_r P_{\_G53445}

% Constructing a point P_{\_G53369} such that  $OP_{\_G53369}/OP_{\_G53445}=1$ 
towards P_{\_G53369} 0 P_{\_G53445} 1
cmark_r P_{\_G53369}
color 200 200 200
drawsegment 0 P_{\_G53369}
color 0 0 0

% Constructing a point P_{\_G53414} such that  $OP_{\_G53414}/OP_{\_G53445}=3$ 
towards P_{\_G53414} 0 P_{\_G53445} 3
cmark_r P_{\_G53414}
color 200 200 200
drawsegment 0 P_{\_G53414}
color 0 0 0

% Constructing a line L_{\_G53375} which passes through point H and point P_{\_G53414}
line L_{\_G53375} H P_{\_G53414}

color 200 200 200
drawline L_{\_G53375}
color 0 0 0

% Constructing a line L_{\_G53338} which contains the point P_{\_G53369} and is parallel to the
line L_{\_G53375}
parallel L_{\_G53338} P_{\_G53369} L_{\_G53375}

color 200 200 200
drawline L_{\_G53338}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G53338} and line L_{\_G53344}
intersec G L_{\_G53338} L_{\_G53344}

```



```

cmark_t G

% Constructing a point C such that  $M_{\{c\}}C/M_{\{c\}}G=3$ 
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

% DET: points O and M_{c} are not the same
% Constructing a line m_{c} which passes through point O and point M_{c}
line m_{c} O M_{c}

color 200 200 200
drawline m_{c}
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Constructing a line c which is perpendicular to line m_{c} and which passes through point M_{c}
perp c M_{c} m_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same
% Determination conditions: points O and M_{c} are not the same

```

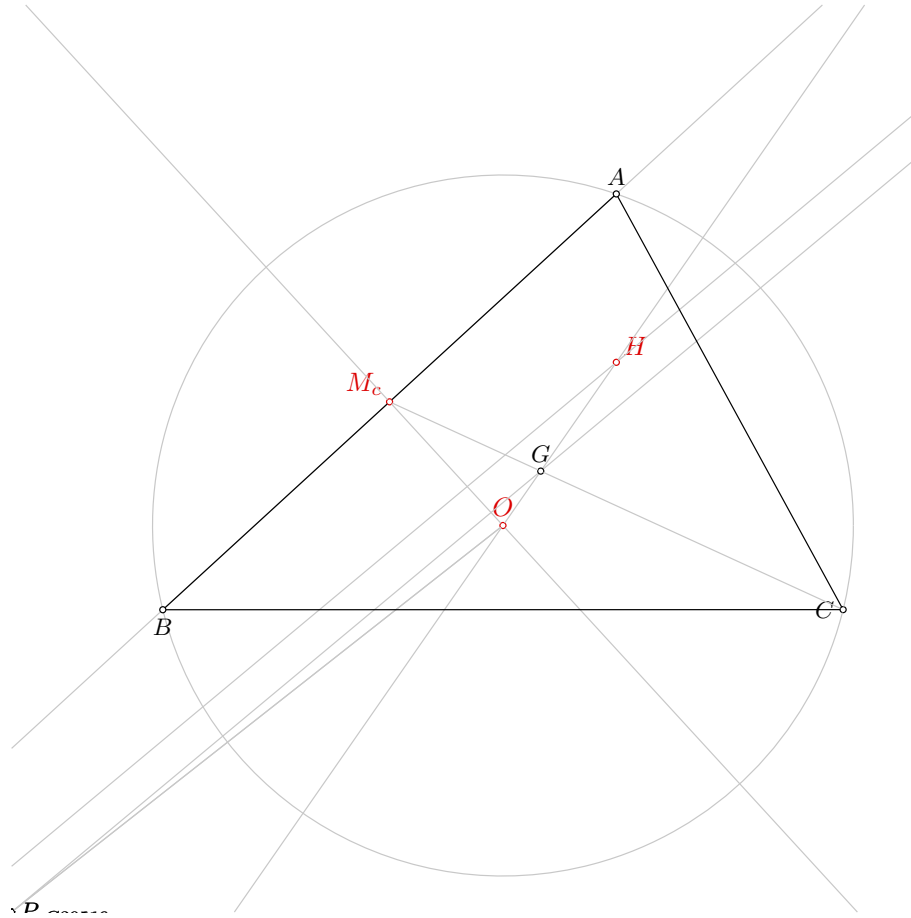


Figure 1: Illustration of the problem 0300

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

Proving failed

#### 4.1.2 Proving $M_c=_M_c$

Proving failed

#### 4.1.3 Proving $H=_H$

Proving failed

## 4.2 GCLC - Area method

### 4.2.1 Proving $O=_O$

Proving failed

### 4.2.2 Proving $M_c=_M_c$

Proving failed

### 4.2.3 Proving $H=_H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $O=_O$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 922 terms.

**Time Complexity:** Time spent by the prover is 1.850 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_c=_M_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 268 terms.

**Time Complexity:** Time spent by the prover is 0.440 seconds. There are no ndg conditions.

### 4.3.3 Proving $H=_H$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $O=_O$

Proving failed

### 4.4.2 Proving $M_c=_M_c$

Proving failed

### 4.4.3 Proving $H=_H$

Proving failed

# Problem 301

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 301: Given a point  $O$ , a point  $M_c$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 302

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 302: Given a point  $O$ , a point  $M_c$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 303

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 303: Given a point  $M_c$ , a point  $T_c$  and a point  $O$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $T_c$  are not the same;
2. Using the point  $M_c$  and the line  $c$ , construct a line  $m_c$  (rule W10b); ;
3. Choose freely a point  $O$  on the line  $m_c$  (rule WOnline2);
4. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) ;
5. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); ;
6. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
7. Using the circle  $k(O, C)$  and the line  $m_c$ , construct a point  $N_c$  and a point  $N_{ck}$  (rule W04); % NDG: line  $m_c$  and circle  $k(O, C)$  intersect;
8. Using the point  $N_c$  and the point  $T_c$ , construct a line  $s_c$  (rule W02); % DET: points  $N_c$  and  $T_c$  are not the same;
9. Using the circle  $k(O, C)$ , the line  $s_c$ , the point  $O$  and the point  $N_c$ , construct a point  $C$  (rule W05); % NDG: line  $s_c$  and circle  $k(O, C)$  intersect % DET: points  $N_c$  and  $C$  must be different.

Non-degenerate conditions: line  $s_c$  and circle  $k(O, C)$  intersect; line  $m_c$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $N_c$  and  $C$  must be different; points  $N_c$  and  $T_c$  are not the same; points  $M_c$  and  $T_c$  are not the same.

Rules used: [W01,W02,W04,W05,W06,W10b,WOnline1,WOnline2]  
 Lemmas used: [D13,D20,D25,D26,D49,D52,GD01,GL03,GL09,L11,L27,L7]  
 Solving time: 7.5 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point T_{c} 55.38 72.43
point O 65 51.14

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_rt T_{c}
cmark_t O
color 0 0 0
fontsize 8

% DET: points M_{c} and T_{c} are not the same
% Constructing a line c which passes through point M_{c} and point T_{c}
line c M_{c} T_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line m_{c} which is perpendicular to line c and which passes through point M_{c}
perp m_{c} M_{c} c

color 200 200 200
drawline m_{c}
color 0 0 0

% Generating random value V[_G54970]
random V[_G54970]

% Calculating value V[_G54991] using formula V[_G54970]*20
expression V[_G54991] { V[_G54970]*20 }

% Constructing a point O which is a point for which holds M_{c}O = V[_G54991] and angle T_{c}M_{c}O
= 90
turtle O T_{c} M_{c} 90 V[_G54991]
cmark_t O
```

```

% Choosing randomly a point A on the line M_{c}T_{c}
online A M_{c} T_{c}
cmark_t A
color 200 200 200
drawline M_{c} T_{c}
color 0 0 0

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line m_{c} and circle k(O,C) intersect
% Constructing points N_{c} and N_{ck} which are in intersection of k(O,C) and m_{c}
intersec2 N_{c} N_{ck} k(O,C) m_{c}
cmark_b N_{c}
cmark_r N_{ck}

% DET: points N_{c} and T_{c} are not the same
% Constructing a line s_{c} which passes through point N_{c} and point T_{c}
line s_{c} N_{c} T_{c}

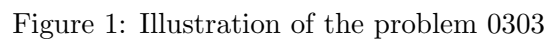
color 200 200 200
drawline s_{c}
color 0 0 0

% NDG: line s_{c} and circle k(O,C) intersect% DET: points N_{c} and C must be different
% Constructing a point P_{\G55483} which is a foot of the point O on the line s_{c}
foot P_{\G55483} O s_{c}
cmark_r P_{\G55483}
color 200 200 200
drawline O P_{\G55483}
color 0 0 0

% Constructing a point C which is an image of the point N_{c} in the symmetry to point/line P_{\G
55483}

```





```
% Non-degenerate conditions: line s_{c} and circle k(O,C) intersect; line m_{c} and circle k(O,C)
    intersect; points A and O are not the same
% Determination conditions: points N_{c} and C must be different; points N_{c} and T_{c} are not
    the same; points M_{c} and T_{c} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = \neg M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.031 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $T_c = \neg T_c$

Proving failed

#### 4.1.3 Proving $O = \neg O$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_c = \neg M_c$

Proving failed

#### 4.2.2 Proving $T_c = \neg T_c$

Proving failed

#### 4.2.3 Proving $O = \neg O$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c = \neg M_c$

Proving failed

#### 4.3.2 Proving $T_c = \neg T_c$

Proving failed

#### 4.3.3 Proving $O = \neg O$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_c = \neg M_c$

Proving failed

#### 4.4.2 Proving $T_c = \neg T_c$

Proving failed

### 4.4.3 Proving $O=_O$

Proving failed

## Problem 304

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 304: Given a point  $O$ , a point  $M_c$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 305

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 305: Given a point  $O$ , a point  $G$  and a point  $H_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $G$ , construct a point  $H$  (rule W01); ;
2. Using the point  $H_a$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $H_a$  and  $H$  are not the same;
3. Using the point  $H_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
4. Using the point  $O$  and the line  $a$ , construct a line  $m_a$  (rule W10b); ;
5. Using the line  $m_a$  and the line  $a$ , construct a point  $M_a$  (rule W03); % NDG: lines  $m_a$  and  $a$  are not parallel % DET: lines  $m_a$  and  $a$  are not the same;
6. Using the point  $M_a$  and the point  $G$ , construct a point  $A$  (rule W01); ;
7. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
8. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same; lines  $m_a$  and  $a$  are not parallel.

Determination conditions: lines  $m_a$  and  $a$  are not the same; points  $H_a$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W10b]

Lemmas used: [D1,D11,D21,D26,D3,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L55,L58]

Solving time: 0.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point O 65 51.14
point G 70 58.33
point H_{a} 80 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_t O
cmark_t G
cmark_r H_{a}
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that OH/OG=3
towards H O G 3
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0
```

```
% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H
```

```
color 200 200 200
drawline h_{a}
color 0 0 0
```

```
% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
perp a H_{a} h_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% Constructing a line m_{a} which is perpendicular to line a and which passes through point O
perp m_{a} O a
```

```
color 200 200 200
drawline m_{a}
color 0 0 0
```

```

% NDG: lines  $m_{\{a\}}$  and  $a$  are not parallel% DET: lines  $m_{\{a\}}$  and  $a$  are not the same
% Constructing a point  $M_{\{a\}}$  which belongs to line  $m_{\{a\}}$  and line  $a$ 
intersec  $M_{\{a\}}$   $m_{\{a\}}$   $a$ 
cmark_r  $M_{\{a\}}$ 

% Constructing a point  $A$  such that  $M_{\{a\}}A/M_{\{a\}}G=3$ 
towards  $A$   $M_{\{a\}}$   $G$  3
cmark_t  $A$ 
color 200 200 200
drawsegment  $M_{\{a\}}$   $A$ 
color 0 0 0

% NDG: points  $A$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $A$ 
circle  $k(O,C)$   $O$   $A$ 

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $a$  and circle  $k(O,C)$  intersect
% Constructing points  $C$  and  $B$  which are in intersection of  $k(O,C)$  and  $a$ 
intersec2  $C$   $B$   $k(O,C)$   $a$ 
cmark_l  $C$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: line  $a$  and circle  $k(O,C)$  intersect; points  $A$  and  $O$  are not the same;
% lines  $m_{\{a\}}$  and  $a$  are not parallel
% Determination conditions: lines  $m_{\{a\}}$  and  $a$  are not the same; points  $H_{\{a\}}$  and  $H$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

Proving failed

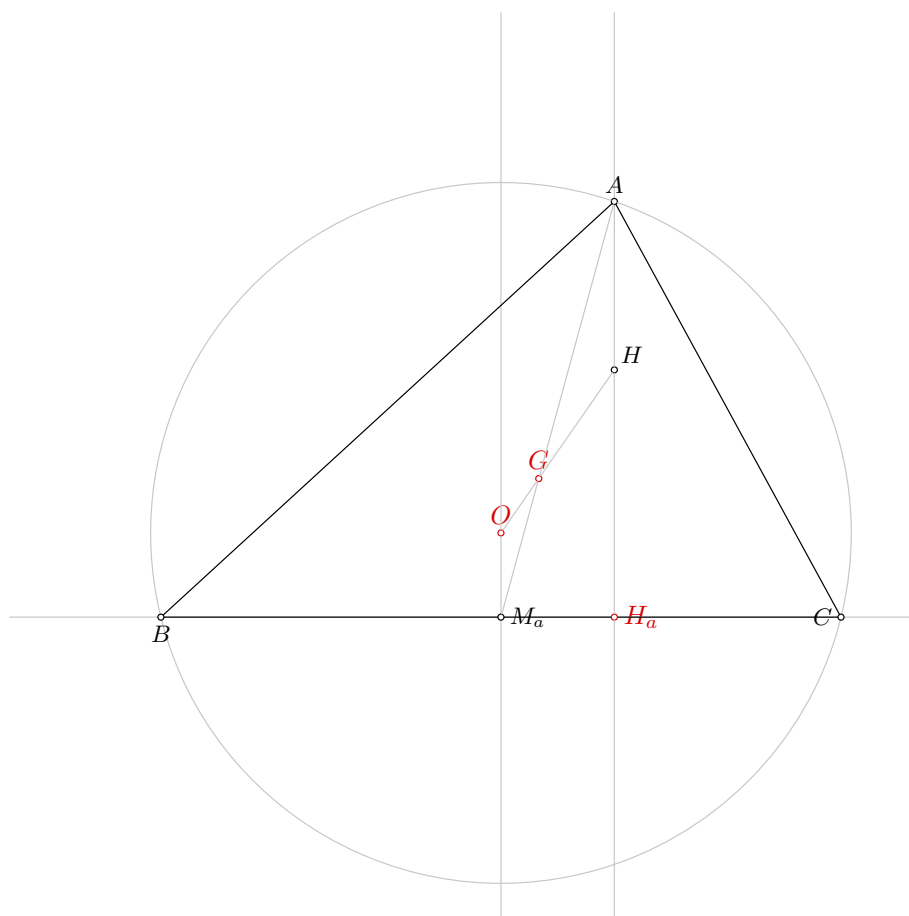


Figure 1: Illustration of the problem 0305



#### **4.1.2 Proving $G=_G$**

Proving failed

#### **4.1.3 Proving $H_a=_H H_a$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $O=_O$**

Proving failed

#### **4.2.2 Proving $G=_G$**

Proving failed

#### **4.2.3 Proving $H_a=_H H_a$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $O=_O$**

Proving failed

#### **4.3.2 Proving $G=_G$**

Proving failed

#### **4.3.3 Proving $H_a=_H H_a$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $O=_O$**

Proving failed

#### **4.4.2 Proving $G=_G$**

Proving failed

#### **4.4.3 Proving $H_a=_H H_a$**

Proving failed

# Problem 306

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 306: Given a point  $O$ , a point  $G$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $G$ , construct a point  $H$  (rule W01); ;
2. Using the point  $H_b$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $H_b$  and  $H$  are not the same;
3. Using the point  $H_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
4. Using the point  $O$  and the line  $b$ , construct a line  $m_b$  (rule W10b); ;
5. Using the line  $m_b$  and the line  $b$ , construct a point  $M_b$  (rule W03); % NDG: lines  $m_b$  and  $b$  are not parallel % DET: lines  $m_b$  and  $b$  are not the same;
6. Using the point  $M_b$  and the point  $G$ , construct a point  $B$  (rule W01); ;
7. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
8. Using the circle  $k(O, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same; lines  $m_b$  and  $b$  are not parallel.

Determination conditions: lines  $m_b$  and  $b$  are not the same; points  $H_b$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W10b]

Lemmas used: [D1,D12,D22,D26,D3,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L56,L58]

Solving time: 0.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point O 65 51.14
point G 70 58.33
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_t O
cmark_t G
cmark_l H_{b}
color 0 0 0
fontsize 8

% Constructing a point H such that OH/OG=3
towards H O G 3
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
perp b H_{b} h_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line m_{b} which is perpendicular to line b and which passes through point O
perp m_{b} O b

color 200 200 200
drawline m_{b}
color 0 0 0
```

```

% NDG: lines  $m_{\{b\}}$  and  $b$  are not parallel% DET: lines  $m_{\{b\}}$  and  $b$  are not the same
% Constructing a point  $M_{\{b\}}$  which belongs to line  $m_{\{b\}}$  and line  $b$ 
intersec  $M_{\{b\}}$   $m_{\{b\}}$   $b$ 
cmark_lt  $M_{\{b\}}$ 

% Constructing a point  $B$  such that  $M_{\{b\}}B/M_{\{b\}}G=3$ 
towards B  $M_{\{b\}}$  G 3
cmark_b B
color 200 200 200
drawsegment  $M_{\{b\}}$  B
color 0 0 0

% NDG: points  $B$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $B$ 
circle  $k(O,C)$  O B

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $b$  and circle  $k(O,C)$  intersect
% Constructing points  $C$  and  $A$  which are in intersection of  $k(O,C)$  and  $b$ 
intersec2 C A  $k(O,C)$   $b$ 
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line  $b$  and circle  $k(O,C)$  intersect; points  $B$  and  $O$  are not the same;
% lines  $m_{\{b\}}$  and  $b$  are not parallel
% Determination conditions: lines  $m_{\{b\}}$  and  $b$  are not the same; points  $H_{\{b\}}$  and  $H$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

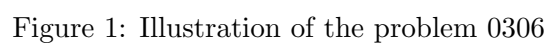
### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

Proving failed



#### **4.1.2 Proving $G=_G$**

Proving failed

#### **4.1.3 Proving $H_b=_H H_b$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $O=_O$**

Proving failed

#### **4.2.2 Proving $G=_G$**

Proving failed

#### **4.2.3 Proving $H_b=_H H_b$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $O=_O$**

Proving failed

#### **4.3.2 Proving $G=_G$**

Proving failed

#### **4.3.3 Proving $H_b=_H H_b$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $O=_O$**

Proving failed

#### **4.4.2 Proving $G=_G$**

Proving failed

#### **4.4.3 Proving $H_b=_H H_b$**

Proving failed

# Problem 307

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 307: Given a point  $O$ , a point  $G$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $G$ , construct a point  $H$  (rule W01); ;
2. Using the point  $H_c$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $H_c$  and  $H$  are not the same;
3. Using the point  $H_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
4. Using the point  $O$  and the line  $c$ , construct a line  $m_c$  (rule W10b); ;
5. Using the line  $m_c$  and the line  $c$ , construct a point  $M_c$  (rule W03); % NDG: lines  $m_c$  and  $c$  are not parallel % DET: lines  $m_c$  and  $c$  are not the same;
6. Using the point  $M_c$  and the point  $G$ , construct a point  $C$  (rule W01); ;
7. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
8. Using the circle  $k(O, C)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same; lines  $m_c$  and  $c$  are not parallel.

Determination conditions: lines  $m_c$  and  $c$  are not the same; points  $H_c$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W10b]

Lemmas used: [D10,D13,D20,D26,D7,GD01,GL03,GL04,GL09,L1,L11,L12,L3,L57,L58]

Solving time: 0.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point O 65 51.14
point G 70 58.33
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_t O
cmark_t G
cmark_rt H_{c}
color 0 0 0
fontsize 8

% Constructing a point H such that OH/OG=3
towards H O G 3
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line m_{c} which is perpendicular to line c and which passes through point O
perp m_{c} O c

color 200 200 200
drawline m_{c}
color 0 0 0
```



```

% NDG: lines  $m_{\{c\}}$  and  $c$  are not parallel% DET: lines  $m_{\{c\}}$  and  $c$  are not the same
% Constructing a point  $M_{\{c\}}$  which belongs to line  $m_{\{c\}}$  and line  $c$ 
intersec  $M_{\{c\}}$   $m_{\{c\}}$   $c$ 
cmark_lt  $M_{\{c\}}$ 

% Constructing a point  $C$  such that  $M_{\{c\}}C/M_{\{c\}}G=3$ 
towards C  $M_{\{c\}}$  G 3
cmark_l C
color 200 200 200
drawsegment  $M_{\{c\}}$  C
color 0 0 0

% NDG: points  $C$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $C$ 
circle k( $O,C$ )  $O$  C

color 200 200 200
drawcircle k( $O,C$ )
color 0 0 0

% NDG: line  $c$  and circle  $k(O,C)$  intersect
% Constructing points  $A$  and  $B$  which are in intersection of  $k(O,C)$  and  $c$ 
intersec2 A B k( $O,C$ )  $c$ 
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line  $c$  and circle  $k(O,C)$  intersect; points  $C$  and  $O$  are not the same;
% lines  $m_{\{c\}}$  and  $c$  are not parallel
% Determination conditions: lines  $m_{\{c\}}$  and  $c$  are not the same; points  $H_{\{c\}}$  and  $H$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

Proving failed

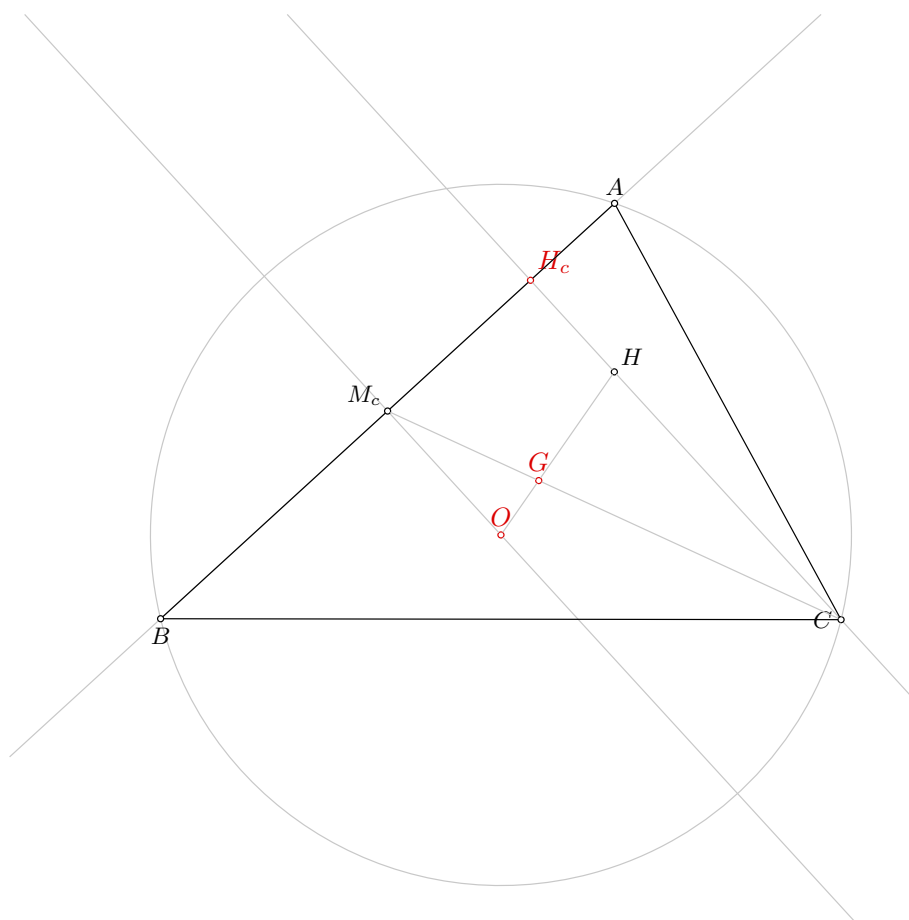


Figure 1: Illustration of the problem 0307

#### **4.1.2 Proving $G=_G$**

Proving failed

#### **4.1.3 Proving $H_c=_Hc$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $O=_O$**

Proving failed

#### **4.2.2 Proving $G=_G$**

Proving failed

#### **4.2.3 Proving $H_c=_Hc$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $O=_O$**

Proving failed

#### **4.3.2 Proving $G=_G$**

Proving failed

#### **4.3.3 Proving $H_c=_Hc$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $O=_O$**

Proving failed

#### **4.4.2 Proving $G=_G$**

Proving failed

#### **4.4.3 Proving $H_c=_Hc$**

Proving failed

# Problem 308

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 308: Given a point  $G$ , a point  $H$  and a point  $O$ , construct the triangle  $ABC$ .

## 2 Status

Problem is redundant.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $G$  and the point  $H$ , construct a point  $O$  (rule W01); ;
2. Choose freely a point  $A$  (rule free);
3. Using the point  $A$  and the point  $G$ , construct a point  $M_a$  (rule W01); ;
4. Using the point  $A$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H$  are not the same;
5. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
6. Using the point  $M_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
7. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $A$  and  $H$  are not the same.

Rules used: [W01,W02,W04,W06,W10a,free]

Lemmas used: [D21,D26,D3,D8,GD01,GL03,GL04,GL09,L11,L12,L55,L58]

Solving time: 1.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point G 70 58.33
point H 80 72.73
point O 65 51.14

color 220 0 0
fontsize 9

cmark_t G
cmark_rt H
cmark_t O
color 0 0 0
fontsize 8

% Constructing a point O such that GO/GH=-0.5
towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% Constructing a free point A
point A 80 95

cmark_t A

% Constructing a point M_{a} such that AM_{a}/AG=1.5
towards M_{a} A G 1.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A
```

```

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Constructing a line a which is perpendicular to line h_{a} and which passes through point M_{a}
perp a M_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same
% Determination conditions: points A and H are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $G=_G$

Proving failed

#### 4.1.2 Proving $H=_H$

Proving failed

#### 4.1.3 Proving $O=_O$

Proving failed

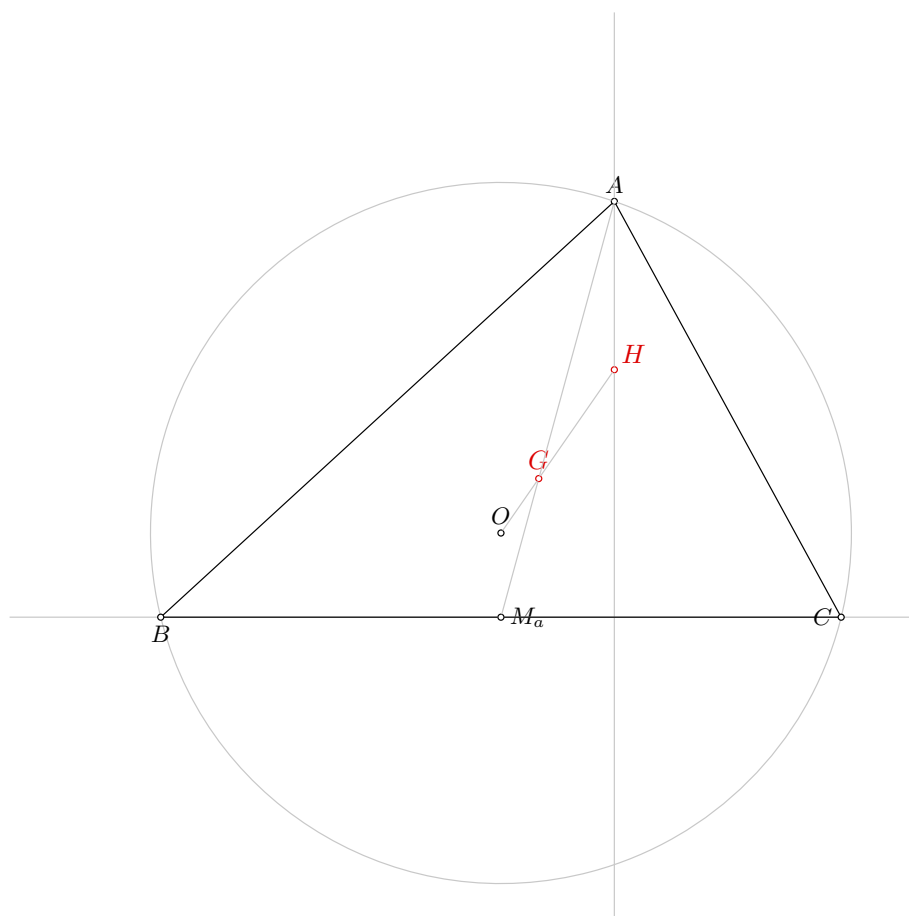


Figure 1: Illustration of the problem 0308

## 4.2 GCLC - Area method

### 4.2.1 Proving $G=_G$

Proving failed

### 4.2.2 Proving $H=_H$

Proving failed

### 4.2.3 Proving $O=_O$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $G=_G$

Proving failed

### 4.3.2 Proving $H=_H$

Proving failed

### 4.3.3 Proving $O=_O$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $G=_G$

Proving failed

### 4.4.2 Proving $H=_H$

Proving failed

### 4.4.3 Proving $O=_O$

Proving failed



## Problem 309

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 309: Given a point  $O$ , a point  $G$  and a point  $T_a$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 310

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 310: Given a point  $O$ , a point  $G$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 311

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 311: Given a point  $O$ , a point  $G$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 312

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 312: Given a point  $O$ , a point  $G$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 313

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 313: Given a point  $O$ , a point  $H_a$  and a point  $H_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 314

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 314: Given a point  $O$ , a point  $H_a$  and a point  $H_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 315

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 315: Given a point  $O$ , a point  $H_a$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $H$ , construct a point  $G$  (rule W01); ;
2. Using the point  $H_a$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $H_a$  and  $H$  are not the same;
3. Using the point  $H_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
4. Using the point  $O$  and the line  $a$ , construct a line  $m_a$  (rule W10b); ;
5. Using the line  $m_a$  and the line  $a$ , construct a point  $M_a$  (rule W03); % NDG: lines  $m_a$  and  $a$  are not parallel % DET: lines  $m_a$  and  $a$  are not the same;
6. Using the point  $M_a$  and the point  $G$ , construct a point  $A$  (rule W01); ;
7. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
8. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same; lines  $m_a$  and  $a$  are not parallel.

Determination conditions: lines  $m_a$  and  $a$  are not the same; points  $H_a$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W10b]

Lemmas used: [D1,D11,D21,D26,D3,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L55,L58]

Solving time: 0.5 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point O 65 51.14
point H_{a} 80 40
point H 80 72.73

color 220 0 0
fontsize 9

cmark_t O
cmark_r H_{a}
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a line L_{\_G43387} which passes through point O and point H
line L_{\_G43387} O H

color 200 200 200
drawline L_{\_G43387}
color 0 0 0

% Constructing a point P_{\_G43488} with coordinates (0,0)
point P_{\_G43488} 0 0
cmark_r P_{\_G43488}

% Constructing a point P_{\_G43412} such that OP_{\_G43412}/OP_{\_G43488}=1
towards P_{\_G43412} O P_{\_G43488} 1
cmark_r P_{\_G43412}
color 200 200 200
drawsegment O P_{\_G43412}
color 0 0 0

% Constructing a point P_{\_G43457} such that OP_{\_G43457}/OP_{\_G43488}=3
towards P_{\_G43457} O P_{\_G43488} 3
cmark_r P_{\_G43457}
color 200 200 200
drawsegment O P_{\_G43457}
color 0 0 0

% Constructing a line L_{\_G43418} which passes through point H and point P_{\_G43457}
line L_{\_G43418} H P_{\_G43457}

color 200 200 200
drawline L_{\_G43418}
color 0 0 0

% Constructing a line L_{\_G43381} which contains the point P_{\_G43412} and is parallel to the
line L_{\_G43418}
parallel L_{\_G43381} P_{\_G43412} L_{\_G43418}
```



```

color 200 200 200
drawline L_{\_G43381}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G43381} and line L_{\_G43387}
intersec G L_{\_G43381} L_{\_G43387}
cmark_t G

% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
perp a H_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line m_{a} which is perpendicular to line a and which passes through point O
perp m_{a} O a

color 200 200 200
drawline m_{a}
color 0 0 0

% NDG: lines m_{a} and a are not parallel% DET: lines m_{a} and a are not the same
% Constructing a point M_{a} which belongs to line m_{a} and line a
intersec M_{a} m_{a} a
cmark_r M_{a}

% Constructing a point A such that M_{a}A/M_{a}G=3
towards A M_{a} G 3
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

% NDG: points A and O are not the same

```

```

% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% lines m_{a} and a are not parallel
% Determination conditions: lines m_{a} and a are not the same; points H_{a} and H are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

Proving failed

#### 4.1.2 Proving $H_a=_H_a$

Proving failed

#### 4.1.3 Proving $H=_H$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $O=_O$

Proving failed

#### 4.2.2 Proving $H_a=_H_a$

Proving failed

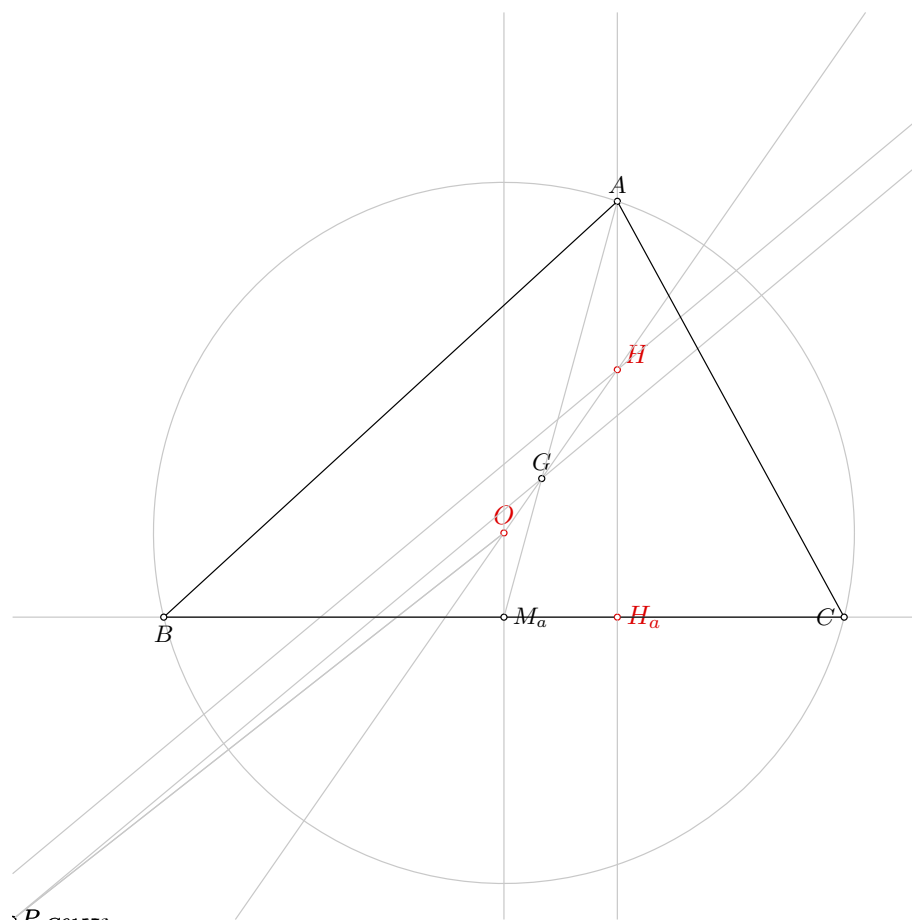


Figure 1: Illustration of the problem 0315

#### **4.2.3 Proving $H=_H$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $O=_O$**

Proving failed

#### **4.3.2 Proving $H_a=_H_a$**

Proving failed

#### **4.3.3 Proving $H=_H$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $O=_O$**

Proving failed

#### **4.4.2 Proving $H_a=_H_a$**

Proving failed

#### **4.4.3 Proving $H=_H$**

Proving failed

## Problem 316

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 316: Given a point  $O$ , a point  $H_a$  and a point  $T_a$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 317

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 317: Given a point  $O$ , a point  $H_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 318

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 318: Given a point  $O$ , a point  $H_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 319

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 319: Given a point  $O$ , a point  $H_a$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.



## Problem 320

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 320: Given a point  $O$ , a point  $H_b$  and a point  $H_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 321

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 321: Given a point  $O$ , a point  $H_b$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $H$ , construct a point  $G$  (rule W01); ;
2. Using the point  $H_b$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $H_b$  and  $H$  are not the same;
3. Using the point  $H_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
4. Using the point  $O$  and the line  $b$ , construct a line  $m_b$  (rule W10b); ;
5. Using the line  $m_b$  and the line  $b$ , construct a point  $M_b$  (rule W03); % NDG: lines  $m_b$  and  $b$  are not parallel % DET: lines  $m_b$  and  $b$  are not the same;
6. Using the point  $M_b$  and the point  $G$ , construct a point  $B$  (rule W01); ;
7. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
8. Using the circle  $k(O, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same; lines  $m_b$  and  $b$  are not parallel.

Determination conditions: lines  $m_b$  and  $b$  are not the same; points  $H_b$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W10b]

Lemmas used: [D1,D12,D22,D26,D3,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L56,L58]

Solving time: 0.5 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point O 65 51.14
point H_{b} 89.36 77.83
point H 80 72.73

color 220 0 0
fontsize 9

cmark_t O
cmark_l H_{b}
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a line L_{\_G31495} which passes through point O and point H
line L_{\_G31495} O H

color 200 200 200
drawline L_{\_G31495}
color 0 0 0

% Constructing a point P_{\_G31596} with coordinates (0,0)
point P_{\_G31596} 0 0
cmark_r P_{\_G31596}

% Constructing a point P_{\_G31520} such that OP_{\_G31520}/OP_{\_G31596}=1
towards P_{\_G31520} O P_{\_G31596} 1
cmark_r P_{\_G31520}
color 200 200 200
drawsegment O P_{\_G31520}
color 0 0 0

% Constructing a point P_{\_G31565} such that OP_{\_G31565}/OP_{\_G31596}=3
towards P_{\_G31565} O P_{\_G31596} 3
cmark_r P_{\_G31565}
color 200 200 200
drawsegment O P_{\_G31565}
color 0 0 0

% Constructing a line L_{\_G31526} which passes through point H and point P_{\_G31565}
line L_{\_G31526} H P_{\_G31565}

color 200 200 200
drawline L_{\_G31526}
color 0 0 0

% Constructing a line L_{\_G31489} which contains the point P_{\_G31520} and is parallel to the
line L_{\_G31526}
parallel L_{\_G31489} P_{\_G31520} L_{\_G31526}
```

```

color 200 200 200
drawline L_{\_G31489}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G31489} and line L_{\_G31495}
intersec G L_{\_G31489} L_{\_G31495}
cmark_t G

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
perp b H_{b} h_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line m_{b} which is perpendicular to line b and which passes through point O
perp m_{b} O b

color 200 200 200
drawline m_{b}
color 0 0 0

% NDG: lines m_{b} and b are not parallel% DET: lines m_{b} and b are not the same
% Constructing a point M_{b} which belongs to line m_{b} and line b
intersec M_{b} m_{b} b
cmark_lt M_{b}

% Constructing a point B such that M_{b}B/M_{b}G=3
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

% NDG: points B and O are not the same

```

```

% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;
% lines m_{b} and b are not parallel
% Determination conditions: lines m_{b} and b are not the same; points H_{b} and H are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 515 terms.

**Time Complexity:** Time spent by the prover is 13.985 seconds.

**NDG conditions** Points  $H$ ,  $P_{G59166}$  and  $O$  are not collinear

Line through points  $H_b$  and  $O$  is not parallel with line through points  $H$  and  $P_{G59166}$

Points  $H_b$ ,  $H$  and  $O$  are not collinear

Points  $H_b$ ,  $H$  and  $O$  are not collinear

Points  $A$ ,  $B$  and  $C$  are not collinear

Line through points  $H_b$  and  $B$  is not perpendicular to line through points  $B$  and  $O$

#### 4.1.2 Proving $H_b=_H$

Proving failed

#### 4.1.3 Proving $H=_H$

Proving failed

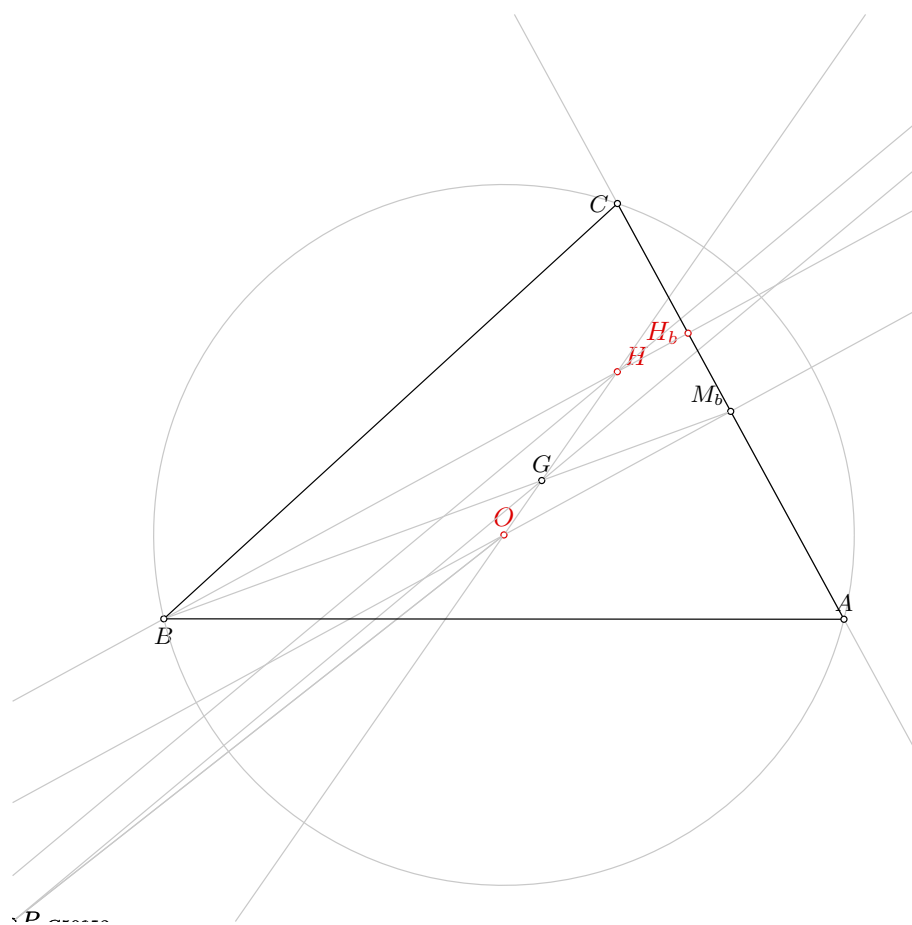


Figure 1: Illustration of the problem 0321

## 4.2 GCLC - Area method

### 4.2.1 Proving $O=_O$

Proving failed

### 4.2.2 Proving $H_b=_H H_b$

Proving failed

### 4.2.3 Proving $H=_H H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $O=_O O$

Proving failed

### 4.3.2 Proving $H_b=_H H_b$

Proving failed

### 4.3.3 Proving $H=_H H$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $O=_O O$

Proving failed

### 4.4.2 Proving $H_b=_H H_b$

Proving failed

### 4.4.3 Proving $H=_H H$

Proving failed

## Problem 322

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 322: Given a point  $O$ , a point  $H_b$  and a point  $T_a$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.



## Problem 323

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 323: Given a point  $O$ , a point  $H_b$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 324

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 324: Given a point  $O$ , a point  $H_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 325

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 325: Given a point  $O$ , a point  $H_b$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 326

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 326: Given a point  $O$ , a point  $H_c$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $O$  and the point  $H$ , construct a point  $G$  (rule W01); ;
2. Using the point  $H_c$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $H_c$  and  $H$  are not the same;
3. Using the point  $H_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
4. Using the point  $O$  and the line  $c$ , construct a line  $m_c$  (rule W10b); ;
5. Using the line  $m_c$  and the line  $c$ , construct a point  $M_c$  (rule W03); % NDG: lines  $m_c$  and  $c$  are not parallel % DET: lines  $m_c$  and  $c$  are not the same;
6. Using the point  $M_c$  and the point  $G$ , construct a point  $C$  (rule W01); ;
7. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
8. Using the circle  $k(O, C)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same; lines  $m_c$  and  $c$  are not parallel.

Determination conditions: lines  $m_c$  and  $c$  are not the same; points  $H_c$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W10b]

Lemmas used: [D10,D13,D20,D26,D7,GD01,GL03,GL04,GL09,L1,L11,L12,L3,L57,L58]

Solving time: 0.6 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point O 65 51.14
point H_{c} 68.91 84.83
point H 80 72.73

color 220 0 0
fontsize 9

cmark_t O
cmark_rt H_{c}
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a line L_{\_G48829} which passes through point O and point H
line L_{\_G48829} O H

color 200 200 200
drawline L_{\_G48829}
color 0 0 0

% Constructing a point P_{\_G48930} with coordinates (0,0)
point P_{\_G48930} 0 0
cmark_r P_{\_G48930}

% Constructing a point P_{\_G48854} such that OP_{\_G48854}/OP_{\_G48930}=1
towards P_{\_G48854} O P_{\_G48930} 1
cmark_r P_{\_G48854}
color 200 200 200
drawsegment O P_{\_G48854}
color 0 0 0

% Constructing a point P_{\_G48899} such that OP_{\_G48899}/OP_{\_G48930}=3
towards P_{\_G48899} O P_{\_G48930} 3
cmark_r P_{\_G48899}
color 200 200 200
drawsegment O P_{\_G48899}
color 0 0 0

% Constructing a line L_{\_G48860} which passes through point H and point P_{\_G48899}
line L_{\_G48860} H P_{\_G48899}

color 200 200 200
drawline L_{\_G48860}
color 0 0 0

% Constructing a line L_{\_G48823} which contains the point P_{\_G48854} and is parallel to the
line L_{\_G48860}
parallel L_{\_G48823} P_{\_G48854} L_{\_G48860}
```

```

color 200 200 200
drawline L_{\_G48823}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G48823} and line L_{\_G48829}
intersec G L_{\_G48823} L_{\_G48829}
cmark_t G

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line m_{c} which is perpendicular to line c and which passes through point O
perp m_{c} O c

color 200 200 200
drawline m_{c}
color 0 0 0

% NDG: lines m_{c} and c are not parallel% DET: lines m_{c} and c are not the same
% Constructing a point M_{c} which belongs to line m_{c} and line c
intersec M_{c} m_{c} c
cmark_lt M_{c}

% Constructing a point C such that M_{c}C/M_{c}G=3
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

% NDG: points C and O are not the same

```

```

% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
% lines m_{c} and c are not parallel
% Determination conditions: lines m_{c} and c are not the same; points H_{c} and H are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $O=_O$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 562 terms.

**Time Complexity:** Time spent by the prover is 22.185 seconds.

**NDG conditions** Points  $H$ ,  $O$  and  $P_{G42134}$  are not collinear

Line through points  $H_c$  and  $O$  is not parallel with line through points  $H$  and  $P_{G42134}$

Points  $H_c$  and  $H$  are not identical

Points  $H_c$  and  $H$  are not identical

Points  $A$ ,  $B$  and  $C$  are not collinear

Line through points  $H_c$  and  $B$  is not perpendicular to line through points  $B$  and  $O$

#### 4.1.2 Proving $H_c=_H$

Proving failed

#### 4.1.3 Proving $H=_H$

Proving failed

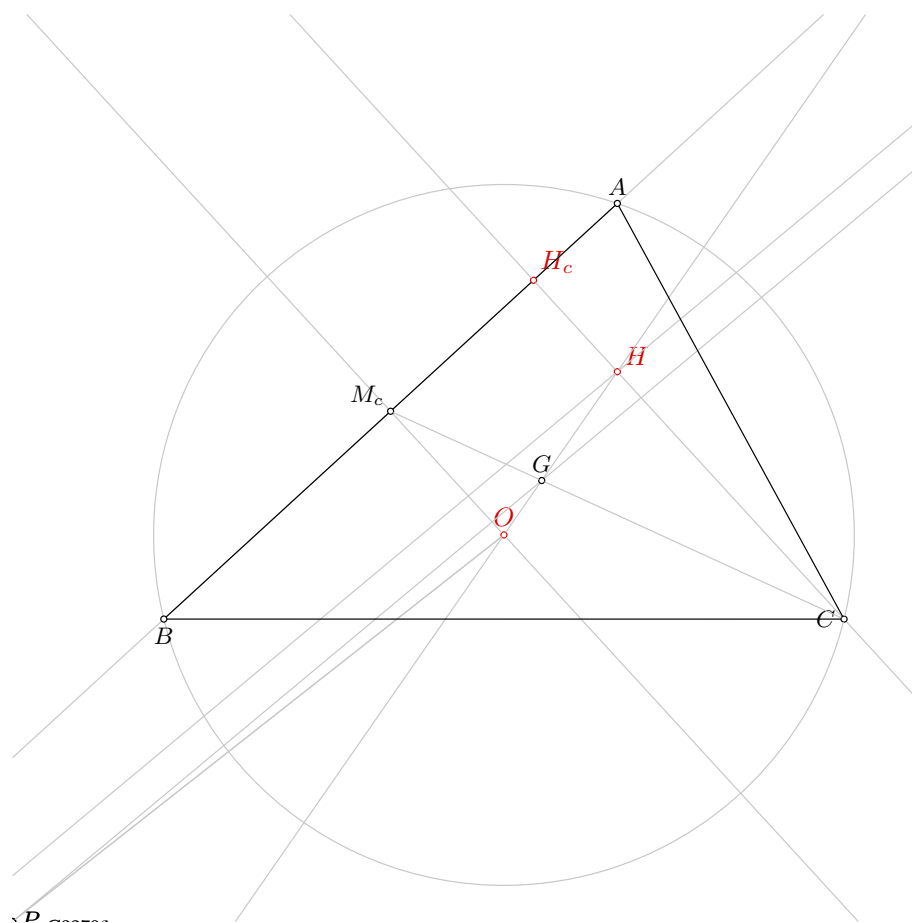


Figure 1: Illustration of the problem 0326



## 4.2 GCLC - Area method

### 4.2.1 Proving $O=_O$

Proving failed

### 4.2.2 Proving $H_c=_Hc$

Proving failed

### 4.2.3 Proving $H=_H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $O=_O$

Proving failed

### 4.3.2 Proving $H_c=_Hc$

Proving failed

### 4.3.3 Proving $H=_H$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $O=_O$

Proving failed

### 4.4.2 Proving $H_c=_Hc$

Proving failed

### 4.4.3 Proving $H=_H$

Proving failed

## Problem 327

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 327: Given a point  $O$ , a point  $H_c$  and a point  $T_a$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 328

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 328: Given a point  $O$ , a point  $H_c$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 329

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 329: Given a point  $O$ , a point  $H_c$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 330

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 330: Given a point  $O$ , a point  $H_c$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 331

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 331: Given a point  $O$ , a point  $H$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 332

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 332: Given a point  $O$ , a point  $H$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 333

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 333: Given a point  $O$ , a point  $H$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.



## Problem 334

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 334: Given a point  $O$ , a point  $H$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 335

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 335: Given a point  $O$ , a point  $T_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 336

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 336: Given a point  $O$ , a point  $T_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 337

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 337: Given a point  $O$ , a point  $T_a$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 338

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 338: Given a point  $O$ , a point  $T_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 339

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 339: Given a point  $O$ , a point  $T_b$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 340

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 340: Given a point  $O$ , a point  $T_c$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 341

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 341: Given a point  $M_a$ , a point  $M_b$  and a point  $M_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $M_c$ , construct a line  $M_cM_a$  (rule W02); % DET: points  $M_a$  and  $M_c$  are not the same;
2. Using the point  $M_b$  and the point  $M_c$ , construct a line  $M_cM_b$  (rule W02); % DET: points  $M_b$  and  $M_c$  are not the same;
3. Using the point  $M_a$  and the line  $M_cM_b$ , construct a line  $a$  (rule W16); ;
4. Using the point  $M_b$  and the line  $M_cM_a$ , construct a line  $b$  (rule W16); ;
5. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same;
6. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
7. Using the point  $M_b$  and the point  $C$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel.

Determination conditions: lines  $b$  and  $a$  are not the same; points  $M_b$  and  $M_c$  are not the same; points  $M_a$  and  $M_c$  are not the same.

Rules used: [W01,W02,W03,W16]

Lemmas used: [D21,D22,GD01,GL03,GL04,GL09]

Solving time: 1.8 seconds.



## 3.2 Construction in GCLC language

```
dim 120 120

point M_{a} 65 40
point M_{b} 95 67.5
point M_{c} 50 67.5

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_lt M_{b}
cmark_lt M_{c}
color 0 0 0
fontsize 8

% DET: points M_{a} and M_{c} are not the same
% Constructing a line M_{c}M_{a} which passes through point M_{a} and point M_{c}
line M_{c}M_{a} M_{a} M_{c}

color 200 200 200
drawline M_{c}M_{a}
color 0 0 0

% DET: points M_{b} and M_{c} are not the same
% Constructing a line M_{c}M_{b} which passes through point M_{b} and point M_{c}
line M_{c}M_{b} M_{b} M_{c}

color 200 200 200
drawline M_{c}M_{b}
color 0 0 0

% Constructing a line a which contains the point M_{a} and is parallel to the line M_{c}M_{b}
parallel a M_{a} M_{c}M_{b}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line b which contains the point M_{b} and is parallel to the line M_{c}M_{a}
parallel b M_{b} M_{c}M_{a}

color 200 200 200
drawline b
color 0 0 0
```

```

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% Constructing a point A such that M_{b}A/M_{b}C=-1
towards A M_{b} C -1
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and a are not parallel
% Determination conditions: lines b and a are not the same; points M_{b} and M_{c} are not the same
; points M_{a} and M_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.171 seconds.

**NDG conditions** Points  $M_c$  and  $M_a$  are not identical

Points  $M_b$  and  $M_a$  are not identical

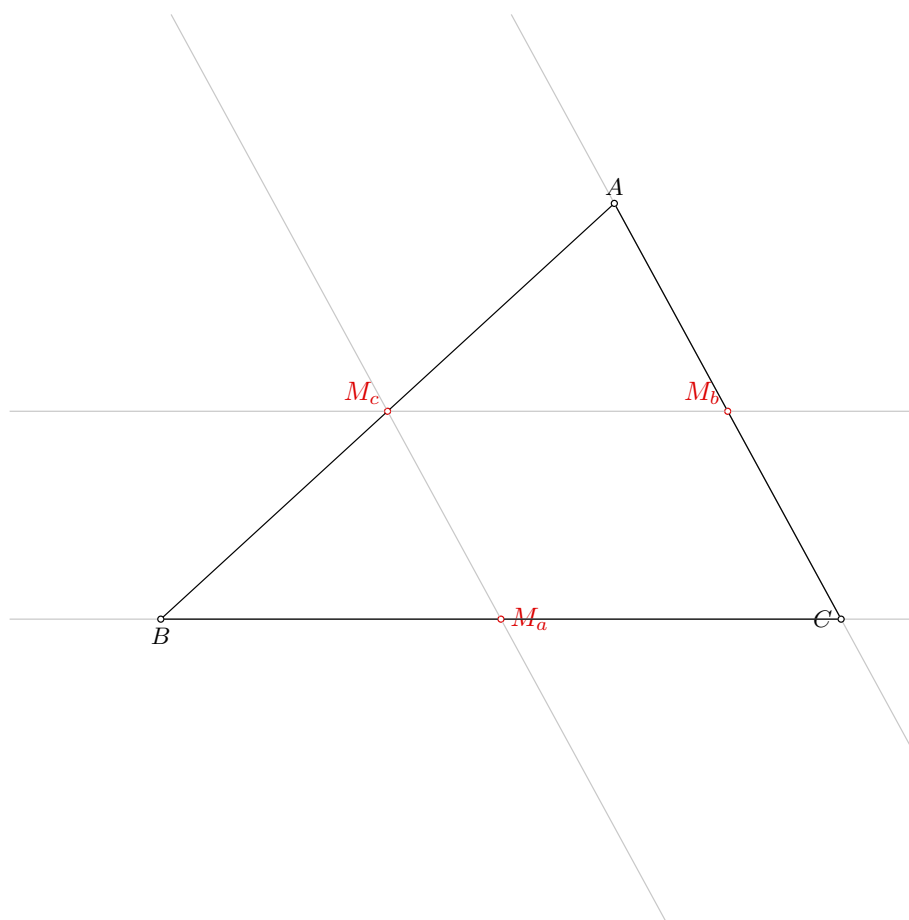


Figure 1: Illustration of the problem 0341

#### 4.1.2 Proving $M_b = M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.186 seconds.

**NDG conditions** Points  $M_c$  and  $M_a$  are not identical

Points  $M_b$  and  $M_a$  are not identical

#### 4.1.3 Proving $M_c = M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 11 terms.

**Time Complexity:** Time spent by the prover is 0.327 seconds.

**NDG conditions** Points  $M_c$  and  $M_a$  are not identical

Points  $M_b$  and  $M_a$  are not identical

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = M_a$

NDG conditions are:

$S_{M_b M_a P_a^0} \neq S_{P_b^1 M_a P_a^0}$  i.e., lines  $M_b P_b^1$  and  $M_a P_a^0$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $M_b = M_b$

NDG conditions are:

$S_{M_b M_a P_a^0} \neq S_{P_b^1 M_a P_a^0}$  i.e., lines  $M_b P_b^1$  and  $M_a P_a^0$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.3 Proving $M_c = M_c$

NDG conditions are:

$S_{M_b M_a P_a^0} \neq S_{P_b^1 M_a P_a^0}$  i.e., lines  $M_b P_b^1$  and  $M_a P_a^0$  are not parallel (construction based assumption)

$M_b M_a \neq 0$  (cancellation assumption)

$S_{M_b M_a M_c} \neq 0$  i.e., points  $M_b$ ,  $M_a$  and  $M_c$  are not collinear (cancellation assumption)

Total number of proof steps: 1291

Time spent by the prover: 0.480 seconds

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 12 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

#### 4.3.2 Proving $M_b = M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 19 terms.

**Time Complexity:** Time spent by the prover is 0.030 seconds. There are no ndg conditions.

#### 4.3.3 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 117 terms.

**Time Complexity:** Time spent by the prover is 0.130 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 405 terms.

**Time Complexity:** Time spent by the prover is 17.470 seconds. There are no ndg conditions.

#### 4.4.2 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 405 terms.

**Time Complexity:** Time spent by the prover is 17.560 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 405 terms.

**Time Complexity:** Time spent by the prover is 17.440 seconds. There are no ndg conditions.

# Problem 342

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 342: Given a point  $M_a$ , a point  $M_b$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $G$ , construct a point  $A$  (rule W01); ;
2. Using the point  $M_b$  and the point  $G$ , construct a point  $B$  (rule W01); ;
3. Using the point  $M_a$  and the point  $B$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D21,GL03,GL04,L55,L56]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40  
point M_{b} 95 67.5  
point G 70 58.33
```

```
color 220 0 0  
fontsize 9
```

```
cmark_r M_{a}  
cmark_lt M_{b}  
cmark_t G  
color 0 0 0  
fontsize 8
```

```

% Constructing a point A such that  $M_{\{a\}A}/M_{\{a\}G}=3$ 
towards A M_{a} G 3
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

```

```

% Constructing a point B such that  $M_{\{b\}B}/M_{\{b\}G}=3$ 
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

```

```

% Constructing a point C such that  $M_{\{a\}C}/M_{\{a\}B}=-1$ 
towards C M_{a} B -1
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = -M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.143 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b = -M_b$

**Status:** Theorem has been proved.

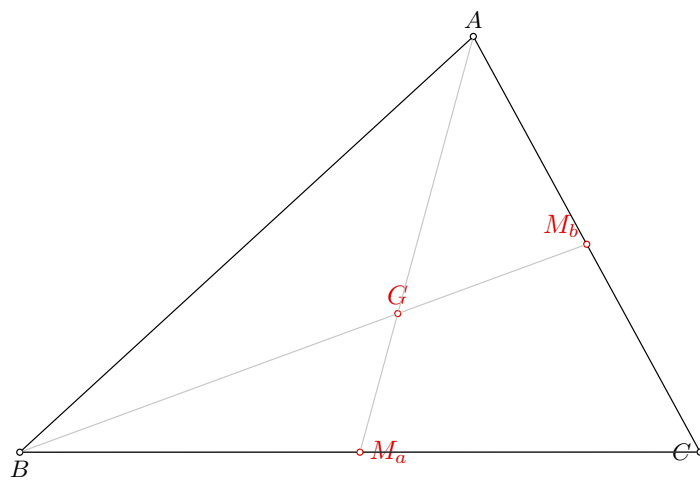


Figure 1: Illustration of the problem 0342



**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.216 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.1.3 Proving $G = \neg G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 843 terms.

**Time Complexity:** Time spent by the prover is 2.143 seconds.

**NDG conditions** Line through points  $A$  and  $\neg M_a$  is not parallel with line through points  $B$  and  $\neg M_b$

Line through points  $M_b$  and  $M_a$  is not parallel with line through points  $A$  and  $\neg M_a$

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{AB\neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A\neg M_a$  and  $B\neg M_b$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{AB\neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A\neg M_a$  and  $B\neg M_b$  are not parallel (construction based assumption)

Total number of proof steps: 235

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $G = \neg G$

NDG conditions are:

$S_{AB\neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A\neg M_a$  and  $B\neg M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (cancellation assumption)

Total number of proof steps: 612

Time spent by the prover: 0.120 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 16 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 84 terms.

**Time Complexity:** Time spent by the prover is 0.030 seconds. There are no ndg conditions.

### 4.3.3 Proving $G=_G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 5369 terms.

**Time Complexity:** Time spent by the prover is 55.930 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_a=_M M_a$

Proving failed

### 4.4.2 Proving $M_b=_M M_b$

Proving failed

### 4.4.3 Proving $G=_G$

Proving failed

# Problem 343

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 343: Given a point  $M_a$ , a point  $M_b$  and a point  $H_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $H_a$  are not the same;
2. Using the point  $H_a$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_a$  and  $M_b$  are not the same;
3. Using the circle  $k(M_b, C)$ , the line  $a$ , the point  $M_b$  and the point  $H_a$ , construct a point  $C$  (rule W05); % NDG: line  $a$  and circle  $k(M_b, C)$  intersect % DET: points  $H_a$  and  $C$  must be different;
4. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
5. Using the point  $M_b$  and the point  $C$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: line  $a$  and circle  $k(M_b, C)$  intersect; points  $H_a$  and  $M_b$  are not the same.

Determination conditions: points  $H_a$  and  $C$  must be different; points  $M_a$  and  $H_a$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D21,D22,D5,GD01,GD02,GL03,GL04,GL09,L44]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```

dim 120 120

point M_{a} 65 40
point M_{b} 95 67.5
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_lt M_{b}
cmark_r H_{a}
color 0 0 0
fontsize 8

% DET: points M_{a} and H_{a} are not the same
% Constructing a line a which passes through point M_{a} and point H_{a}
line a M_{a} H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points H_{a} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{a}
circle k(M_{b},C) M_{b} H_{a}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line a and circle k(M_{b},C) intersect% DET: points H_{a} and C must be different
% Constructing a point P_{\_G44992} which is a foot of the point M_{b} on the line a
foot P_{\_G44992} M_{b} a
cmark_r P_{\_G44992}
color 200 200 200
drawline M_{b} P_{\_G44992}
color 0 0 0

% Constructing a point C which is an image of the point H_{a} in the symmetry to point/line P_{\_G44992}
sim C P_{\_G44992} H_{a}
cmark_l C

% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B

```

```
color 0 0 0
```

```
% Constructing a point A such that  $M_{\{b\}A/M_{\{b\}C}=-1$ 
towards A  $M_{\{b\}}$  C -1
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle  $k(M_{\{b\}}, C)$  intersect; points  $H_{\{a\}}$  and  $M_{\{b\}}$  are not
the same
% Determination conditions: points  $H_{\{a\}}$  and C must be different; points  $M_{\{a\}}$  and  $H_{\{a\}}$  are not
the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.152 seconds.

**NDG conditions** Points  $M_b$  and  $H_a$  are not identical

Points  $M_b$  and  $H_a$  are not identical

#### 4.1.2 Proving $M_b = M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.161 seconds.

**NDG conditions** Points  $M_b$  and  $H_a$  are not identical

Points  $M_b$  and  $H_a$  are not identical

#### 4.1.3 Proving $H_a = H_a$

Proving failed

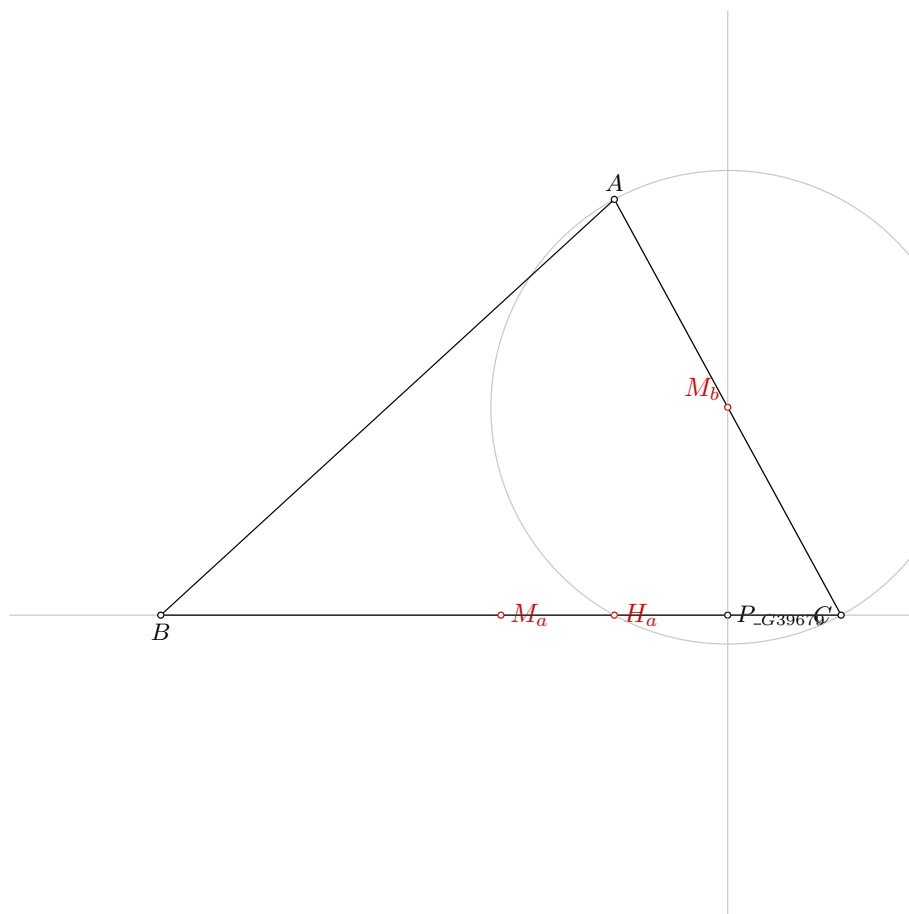


Figure 1: Illustration of the problem 0343

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF^0_{\neg h_a}} \neq S_{CAF^0_{\neg h_a}}$  i.e., lines  $BC$  and  $AF^0_{\neg h_a}$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF^0_{\neg h_a}} \neq S_{CAF^0_{\neg h_a}}$  i.e., lines  $BC$  and  $AF^0_{\neg h_a}$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $H_a = \neg H_a$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_a = \neg M_a$

Proving failed

### 4.3.2 Proving $M_b = \neg M_b$

Proving failed

### 4.3.3 Proving $H_a = \neg H_a$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_a = \neg M_a$

Proving failed

### 4.4.2 Proving $M_b = \neg M_b$

Proving failed

### 4.4.3 Proving $H_a = \neg H_a$

Proving failed

# Problem 344

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 344: Given a point  $M_a$ , a point  $M_b$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $H_b$  are not the same;
2. Using the point  $H_b$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_b$  and  $M_a$  are not the same;
3. Using the circle  $k(M_a, B)$ , the line  $b$ , the point  $M_a$  and the point  $H_b$ , construct a point  $C$  (rule W05); % NDG: line  $b$  and circle  $k(M_a, B)$  intersect % DET: points  $H_b$  and  $C$  must be different;
4. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
5. Using the point  $M_b$  and the point  $C$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: line  $b$  and circle  $k(M_a, B)$  intersect; points  $H_b$  and  $M_a$  are not the same.

Determination conditions: points  $H_b$  and  $C$  must be different; points  $M_b$  and  $H_b$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D21,D22,D6,GD01,GD02,GL03,GL04,GL09,L37,L38]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language



```

dim 120 120

point M_{a} 65 40
point M_{b} 95 67.5
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_lt M_{b}
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points M_{b} and H_{b} are not the same
% Constructing a line b which passes through point M_{b} and point H_{b}
line b M_{b} H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points H_{b} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{b}
circle k(M_{a},B) M_{a} H_{b}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line b and circle k(M_{a},B) intersect% DET: points H_{b} and C must be different
% Constructing a point P_{\_G58783} which is a foot of the point M_{a} on the line b
foot P_{\_G58783} M_{a} b
cmark_r P_{\_G58783}
color 200 200 200
drawline M_{a} P_{\_G58783}
color 0 0 0

% Constructing a point C which is an image of the point H_{b} in the symmetry to point/line P_{\_G58783}
sim C P_{\_G58783} H_{b}
cmark_l C

% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B

```

```
color 0 0 0
```

```
% Constructing a point A such that  $M_{\{b\}A/M_{\{b\}C}=-1$ 
towards A  $M_{\{b\}}$  C -1
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line  $b$  and circle  $k(M_{\{a\}}, B)$  intersect; points  $H_{\{b\}}$  and  $M_{\{a\}}$  are not
the same
% Determination conditions: points  $H_{\{b\}}$  and  $C$  must be different; points  $M_{\{b\}}$  and  $H_{\{b\}}$  are not
the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = _M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.159 seconds.

**NDG conditions** Points  $M_b$ ,  $H_b$  and  $M_a$  are not collinear

Points  $M_b$ ,  $H_b$  and  $M_a$  are not collinear

#### 4.1.2 Proving $M_b = _M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.163 seconds.

**NDG conditions** Points  $M_b$ ,  $H_b$  and  $M_a$  are not collinear

Points  $M_b$ ,  $H_b$  and  $M_a$  are not collinear

#### 4.1.3 Proving $H_b = _H_b$

Proving failed

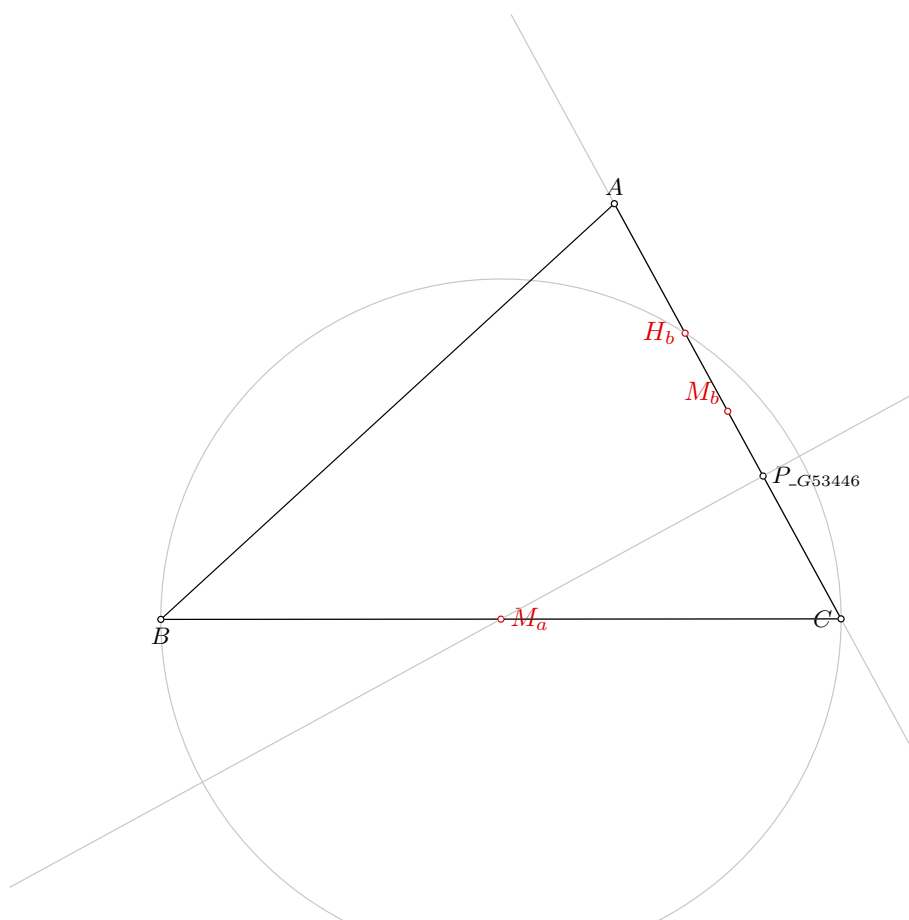


Figure 1: Illustration of the problem 0344

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^0_{\neg h_b}} \neq S_{CBF^0_{\neg h_b}}$  i.e., lines  $AC$  and  $BF^0_{\neg h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^0_{\neg h_b}} \neq S_{CBF^0_{\neg h_b}}$  i.e., lines  $AC$  and  $BF^0_{\neg h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $H_b = \neg H_b$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_a = \neg M_a$

Proving failed

### 4.3.2 Proving $M_b = \neg M_b$

Proving failed

### 4.3.3 Proving $H_b = \neg H_b$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_a = \neg M_a$

Proving failed

### 4.4.2 Proving $M_b = \neg M_b$

Proving failed

### 4.4.3 Proving $H_b = \neg H_b$

Proving failed

# Problem 345

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 345: Given a point  $M_a$ , a point  $M_b$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_c$  and  $M_a$  are not the same;
2. Using the point  $H_c$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_c$  and  $M_b$  are not the same;
3. Using the circle  $k(M_a, B)$ , the circle  $k(M_b, C)$ , the point  $H_c$ , the point  $M_a$  and the point  $M_b$ , construct a point  $C$  (rule W08); % NDG: circles  $k(M_a, B)$  and  $k(M_b, C)$  intersect % DET: circles  $k(M_a, B)$  and  $k(M_b, C)$  are not the same; points  $H_c$  and  $C$  must be different;
4. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
5. Using the point  $M_b$  and the point  $C$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: circles  $k(M_a, B)$  and  $k(M_b, C)$  intersect; points  $H_c$  and  $M_b$  are not the same; points  $H_c$  and  $M_a$  are not the same.

Determination conditions: circles  $k(M_a, B)$  and  $k(M_b, C)$  are not the same; points  $H_c$  and  $C$  must be different.

Rules used: [W01,W06,W08]

Lemmas used: [D21,D22,GD02,GL03,GL04,L37,L39,L45]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```

dim 120 120

point M_{a} 65 40
point M_{b} 95 67.5
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_lt M_{b}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% NDG: points H_{c} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{c}
circle k(M_{a},B) M_{a} H_{c}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: points H_{c} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{c}
circle k(M_{b},C) M_{b} H_{c}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: circles k(M_{a},B) and k(M_{b},C) intersect% DET: circles k(M_{a},B) and k(M_{b},C) are not
the same; points H_{c} and C must be different
% Constructing a line L_{\_G39809} which passes through point M_{a} and point M_{b}
line L_{\_G39809} M_{a} M_{b}

color 200 200 200
drawline L_{\_G39809}
color 0 0 0

% Constructing a point C which is an image of the point H_{c} in the symmetry to point/line L_{\_G
39809}
sim C L_{\_G39809} H_{c}
cmark_l C

% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2

```

```

cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% Constructing a point A such that  $M_{\{b\}A}/M_{\{b\}C}=-1$ 
towards A M_{b} C -1
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: circles  $k(M_{\{a\}},B)$  and  $k(M_{\{b\}},C)$  intersect; points  $H_{\{c\}}$  and  $M_{\{b\}}$ 
% are not the same; points  $H_{\{c\}}$  and  $M_{\{a\}}$  are not the same
% Determination conditions: circles  $k(M_{\{a\}},B)$  and  $k(M_{\{b\}},C)$  are not the same; points  $H_{\{c\}}$  and  $C$ 
% must be different

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.098 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_b = M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.115 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $H_c = H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 134 terms.

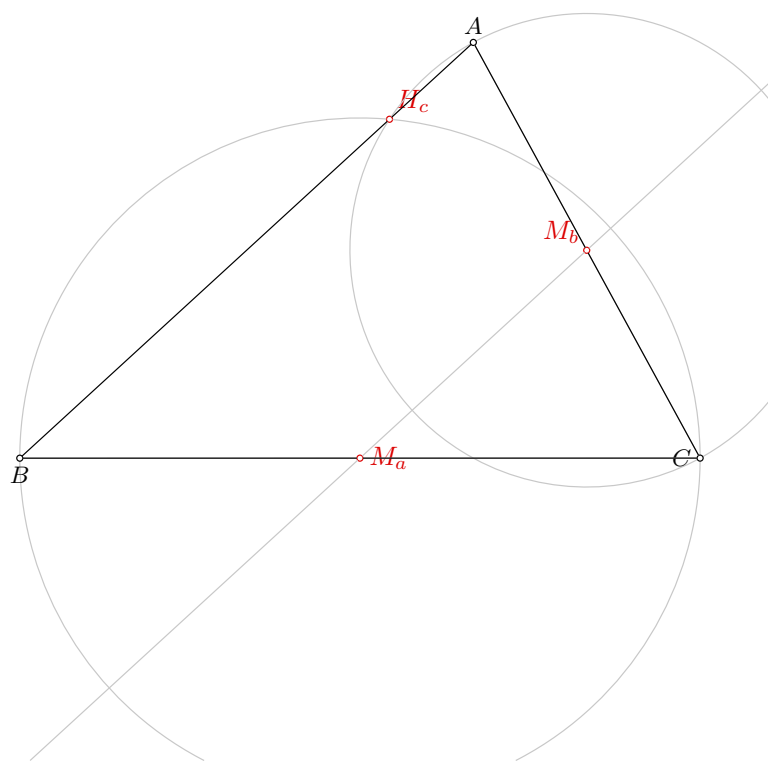


Figure 1: Illustration of the problem 0345



**Time Complexity:** Time spent by the prover is 0.627 seconds.

**NDG conditions** Points  $A$  and  $B$  are not identical

Line through points  $M_b$  and  $M_a$  is not parallel with line through points  $A$  and  $B$

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_a = M_a$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{hc}^0} \neq S_{BCF_{hc}^0}$  i.e., lines  $AB$  and  $CF_{hc}^0$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_b = M_b$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{hc}^0} \neq S_{BCF_{hc}^0}$  i.e., lines  $AB$  and  $CF_{hc}^0$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $H_c = H_c$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_a = M_a$

Proving failed

### 4.3.2 Proving $M_b = M_b$

Proving failed

### 4.3.3 Proving $H_c = H_c$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_a = M_a$

Proving failed

### 4.4.2 Proving $M_b = M_b$

Proving failed

### 4.4.3 Proving $H_c = H_c$

Proving failed

## Problem 346

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 346: Given a point  $M_a$ , a point  $M_b$  and a point  $H$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 347

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 347: Given a point  $M_a$ , a point  $M_b$  and a point  $T_a$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 348

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 348: Given a point  $M_a$ , a point  $M_b$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 349

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 349: Given a point  $M_a$ , a point  $M_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 350

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 350: Given a point  $M_a$ , a point  $M_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 351

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 351: Given a point  $M_a$ , a point  $M_c$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $G$ , construct a point  $A$  (rule W01); ;
2. Using the point  $M_c$  and the point  $G$ , construct a point  $C$  (rule W01); ;
3. Using the point  $M_a$  and the point  $C$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D21,GL03,GL04,L55,L57]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40  
point M_{c} 50 67.5  
point G 70 58.33
```

```
color 220 0 0  
fontsize 9
```

```
cmark_r M_{a}  
cmark_lt M_{c}  
cmark_t G  
color 0 0 0  
fontsize 8
```

```

% Constructing a point A such that  $M_{\{a\}A/M_{\{a\}}G=3$ 
towards A M_{a} G 3
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

```

```

% Constructing a point C such that  $M_{\{c\}C/M_{\{c\}}G=3$ 
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

```

```

% Constructing a point B such that  $M_{\{a\}B/M_{\{a\}}C=-1$ 
towards B M_{a} C -1
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = \_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.143 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c = \_M_c$

**Status:** Theorem has been proved.



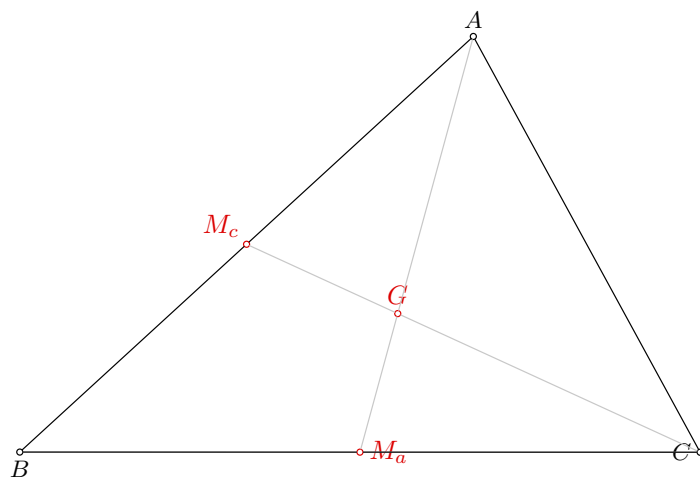


Figure 1: Illustration of the problem 0351

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.213 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.1.3 Proving $G=_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 843 terms.

**Time Complexity:** Time spent by the prover is 2.012 seconds.

**NDG conditions** Line through points  $A$  and  $\neg M_a$  is not parallel with line through points  $B$  and  $\neg M_b$

Line through points  $A$  and  $\neg M_a$  is not parallel with line through points  $M_c$  and  $M_a$

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_a=_M M_a$

NDG conditions are:

$S_{AB\neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A\neg M_a$  and  $B\neg M_b$  are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_c=_M M_c$

NDG conditions are:

$S_{AB\neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A\neg M_a$  and  $B\neg M_b$  are not parallel (construction based assumption)

Total number of proof steps: 127

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $G=_G$

NDG conditions are:

$S_{AB\neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A\neg M_a$  and  $B\neg M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (cancellation assumption)

Total number of proof steps: 622

Time spent by the prover: 0.120 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_a=_M M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 16 terms.

**Time Complexity:** Time spent by the prover is 0.030 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_c=_M M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 84 terms.

**Time Complexity:** Time spent by the prover is 0.080 seconds. There are no ndg conditions.

### 4.3.3 Proving $G=_G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 5369 terms.

**Time Complexity:** Time spent by the prover is 57.870 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_a=_M M_a$

Proving failed

### 4.4.2 Proving $M_c=_M M_c$

Proving failed

### 4.4.3 Proving $G=_G$

Proving failed

# Problem 352

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 352: Given a point  $M_a$ , a point  $M_c$  and a point  $H_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $H_a$  are not the same;
2. Using the point  $H_a$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_a$  and  $M_c$  are not the same;
3. Using the circle  $k(M_c, A)$ , the line  $a$ , the point  $M_c$  and the point  $H_a$ , construct a point  $B$  (rule W05); % NDG: line  $a$  and circle  $k(M_c, A)$  intersect % DET: points  $H_a$  and  $B$  must be different;
4. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
5. Using the point  $M_c$  and the point  $B$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: line  $a$  and circle  $k(M_c, A)$  intersect; points  $H_a$  and  $M_c$  are not the same.

Determination conditions: points  $H_a$  and  $B$  must be different; points  $M_a$  and  $H_a$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D20,D21,D5,GD01,GD02,GL03,GL04,GL09,L40,L42]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```

dim 120 120

point M_{a} 65 40
point M_{c} 50 67.5
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_lt M_{c}
cmark_r H_{a}
color 0 0 0
fontsize 8

% DET: points M_{a} and H_{a} are not the same
% Constructing a line a which passes through point M_{a} and point H_{a}
line a M_{a} H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points H_{a} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{a}
circle k(M_{c},A) M_{c} H_{a}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line a and circle k(M_{c},A) intersect% DET: points H_{a} and B must be different
% Constructing a point P_{\G62110} which is a foot of the point M_{c} on the line a
foot P_{\G62110} M_{c} a
cmark_r P_{\G62110}
color 200 200 200
drawline M_{c} P_{\G62110}
color 0 0 0

% Constructing a point B which is an image of the point H_{a} in the symmetry to point/line P_{\G62110}
sim B P_{\G62110} H_{a}
cmark_b B

% Constructing a point C such that BC/BM_{a}=2
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C

```

```
color 0 0 0
```

```
% Constructing a point A such that  $M_{\{c\}}A/M_{\{c\}}B=-1$ 
towards A  $M_{\{c\}}$  B -1
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle  $k(M_{\{c\}}, A)$  intersect; points  $H_{\{a\}}$  and  $M_{\{c\}}$  are not
the same
% Determination conditions: points  $H_{\{a\}}$  and B must be different; points  $M_{\{a\}}$  and  $H_{\{a\}}$  are not
the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = -M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.152 seconds.

**NDG conditions** Points  $M_a$  and  $H_a$  are not identical

Points  $M_a$  and  $H_a$  are not identical

#### 4.1.2 Proving $M_c = -M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.168 seconds.

**NDG conditions** Points  $M_a$  and  $H_a$  are not identical

Points  $M_a$  and  $H_a$  are not identical

#### 4.1.3 Proving $H_a = -H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 323 terms.

**Time Complexity:** Time spent by the prover is 1.045 seconds.

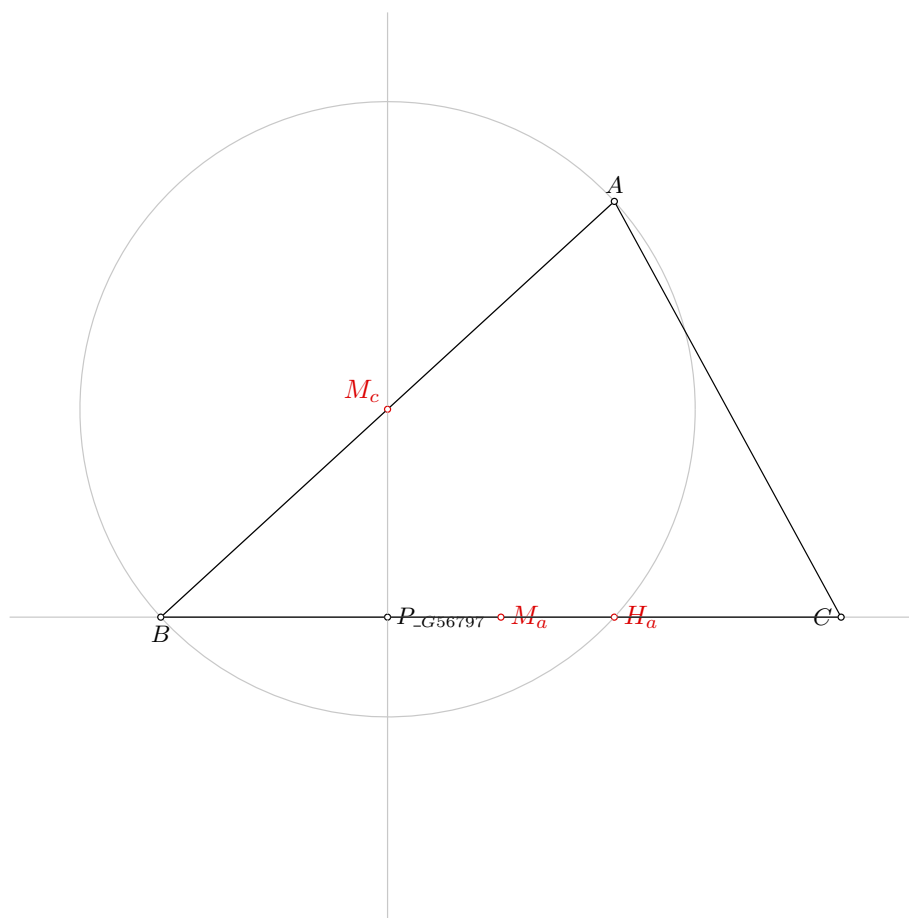


Figure 1: Illustration of the problem 0352

**NDG conditions** Points  $M_a$  and  $H_a$  are not identical

Points  $M_a$  and  $H_a$  are not identical

Points  $B$  and  $C$  are not identical

Line through points  $B$  and  $C$  is not parallel with line through points  $M_c$  and  $M_a$

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_a = M_a$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^0} \neq S_{CAF_{h_a}^0}$  i.e., lines  $BC$  and  $AF_{h_a}^0$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_c = M_c$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{h_a}^0} \neq S_{CAF_{h_a}^0}$  i.e., lines  $BC$  and  $AF_{h_a}^0$  are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $H_a = H_a$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_a = M_a$

Proving failed

### 4.3.2 Proving $M_c = M_c$

Proving failed

### 4.3.3 Proving $H_a = H_a$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_a = M_a$

Proving failed

### 4.4.2 Proving $M_c = M_c$

Proving failed

### 4.4.3 Proving $H_a = H_a$

Proving failed



# Problem 353

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 353: Given a point  $M_a$ , a point  $M_c$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_b$  and  $M_a$  are not the same;
2. Using the point  $H_b$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_b$  and  $M_c$  are not the same;
3. Using the circle  $k(M_a, B)$ , the circle  $k(M_c, A)$ , the point  $H_b$ , the point  $M_a$  and the point  $M_c$ , construct a point  $B$  (rule W08); % NDG: circles  $k(M_a, B)$  and  $k(M_c, A)$  intersect % DET: circles  $k(M_a, B)$  and  $k(M_c, A)$  are not the same; points  $H_b$  and  $B$  must be different;
4. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
5. Using the point  $M_c$  and the point  $B$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: circles  $k(M_a, B)$  and  $k(M_c, A)$  intersect; points  $H_b$  and  $M_c$  are not the same; points  $H_b$  and  $M_a$  are not the same.

Determination conditions: circles  $k(M_a, B)$  and  $k(M_c, A)$  are not the same; points  $H_b$  and  $B$  must be different.

Rules used: [W01,W06,W08]

Lemmas used: [D20,D21,GD02,GL03,GL04,L38,L40,L41]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```

dim 120 120

point M_{a} 65 40
point M_{c} 50 67.5
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_lt M_{c}
cmark_l H_{b}
color 0 0 0
fontsize 8

% NDG: points H_{b} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{b}
circle k(M_{a},B) M_{a} H_{b}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: points H_{b} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{b}
circle k(M_{c},A) M_{c} H_{b}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: circles k(M_{a},B) and k(M_{c},A) intersect% DET: circles k(M_{a},B) and k(M_{c},A) are not
the same; points H_{b} and B must be different
% Constructing a line L_{\_G43683} which passes through point M_{a} and point M_{c}
line L_{\_G43683} M_{a} M_{c}

color 200 200 200
drawline L_{\_G43683}
color 0 0 0

% Constructing a point B which is an image of the point H_{b} in the symmetry to point/line L_{\_G
43683}
sim B L_{\_G43683} H_{b}
cmark_b B

% Constructing a point C such that BC/BM_{a}=2
towards C B M_{a} 2

```

```

cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% Constructing a point A such that  $M_{\{c\}A}/M_{\{c\}B}=-1$ 
towards A M_{c} B -1
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: circles  $k(M_{\{a\}},B)$  and  $k(M_{\{c\}},A)$  intersect; points  $H_{\{b\}}$  and  $M_{\{c\}}$ 
% are not the same; points  $H_{\{b\}}$  and  $M_{\{a\}}$  are not the same
% Determination conditions: circles  $k(M_{\{a\}},B)$  and  $k(M_{\{c\}},A)$  are not the same; points  $H_{\{b\}}$  and  $B$ 
% must be different

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.099 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c = M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.107 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $H_b = H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 134 terms.

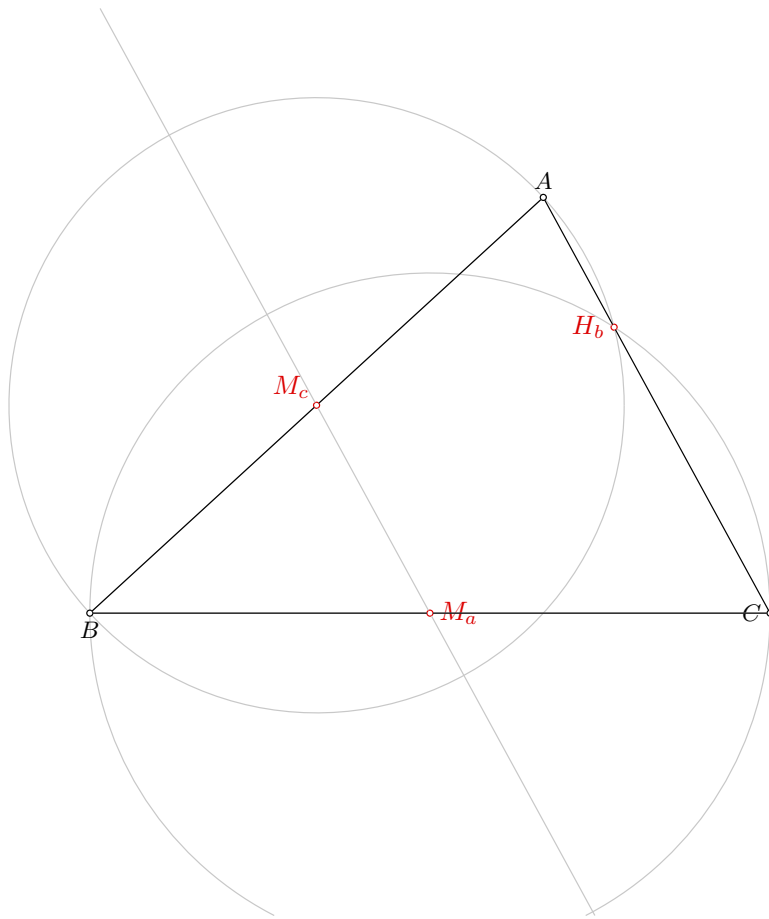


Figure 1: Illustration of the problem 0353

**Time Complexity:** Time spent by the prover is 0.607 seconds.

**NDG conditions** Points  $A$  and  $C$  are not identical

Line through points  $A$  and  $C$  is not parallel with line through points  $M_c$  and  $M_a$

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^0} \neq S_{CBF_{\neg h_b}^0}$  i.e., lines  $AC$  and  $BF_{\neg h_b}^0$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^0} \neq S_{CBF_{\neg h_b}^0}$  i.e., lines  $AC$  and  $BF_{\neg h_b}^0$  are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $H_b = \neg H_b$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_a = \neg M_a$

Proving failed

### 4.3.2 Proving $M_c = \neg M_c$

Proving failed

### 4.3.3 Proving $H_b = \neg H_b$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_a = \neg M_a$

Proving failed

### 4.4.2 Proving $M_c = \neg M_c$

Proving failed

### 4.4.3 Proving $H_b = \neg H_b$

Proving failed

# Problem 354

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 354: Given a point  $M_a$ , a point  $M_c$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $H_c$  are not the same;
2. Using the point  $H_c$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_c$  and  $M_a$  are not the same;
3. Using the circle  $k(M_a, B)$ , the line  $c$ , the point  $M_a$  and the point  $H_c$ , construct a point  $B$  (rule W05); % NDG: line  $c$  and circle  $k(M_a, B)$  intersect % DET: points  $H_c$  and  $B$  must be different;
4. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
5. Using the point  $M_c$  and the point  $B$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: line  $c$  and circle  $k(M_a, B)$  intersect; points  $H_c$  and  $M_a$  are not the same.

Determination conditions: points  $H_c$  and  $B$  must be different; points  $M_c$  and  $H_c$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D20,D21,D7,GD01,GD02,GL03,GL04,GL09,L39]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```

dim 120 120

point M_{a} 65 40
point M_{c} 50 67.5
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_lt M_{c}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points M_{c} and H_{c} are not the same
% Constructing a line c which passes through point M_{c} and point H_{c}
line c M_{c} H_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points H_{c} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{c}
circle k(M_{a},B) M_{a} H_{c}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line c and circle k(M_{a},B) intersect% DET: points H_{c} and B must be different
% Constructing a point P_{\_G57762} which is a foot of the point M_{a} on the line c
foot P_{\_G57762} M_{a} c
cmark_r P_{\_G57762}
color 200 200 200
drawline M_{a} P_{\_G57762}
color 0 0 0

% Constructing a point B which is an image of the point H_{c} in the symmetry to point/line P_{\_G57762}
sim B P_{\_G57762} H_{c}
cmark_b B

% Constructing a point C such that BC/BM_{a}=2
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C

```

```
color 0 0 0
```

```
% Constructing a point A such that  $M_{\{c\}}A/M_{\{c\}}B=-1$ 
towards A  $M_{\{c\}}$  B -1
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line  $c$  and circle  $k(M_{\{a\}}, B)$  intersect; points  $H_{\{c\}}$  and  $M_{\{a\}}$  are not
the same
% Determination conditions: points  $H_{\{c\}}$  and  $B$  must be different; points  $M_{\{c\}}$  and  $H_{\{c\}}$  are not
the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = -M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.135 seconds.

**NDG conditions** Points  $H_c$  and  $M_c$  are not identical

Points  $H_c$  and  $M_c$  are not identical

#### 4.1.2 Proving $M_c = -M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.163 seconds.

**NDG conditions** Points  $H_c$  and  $M_c$  are not identical

Points  $H_c$  and  $M_c$  are not identical

#### 4.1.3 Proving $H_c = -H_c$

Proving failed



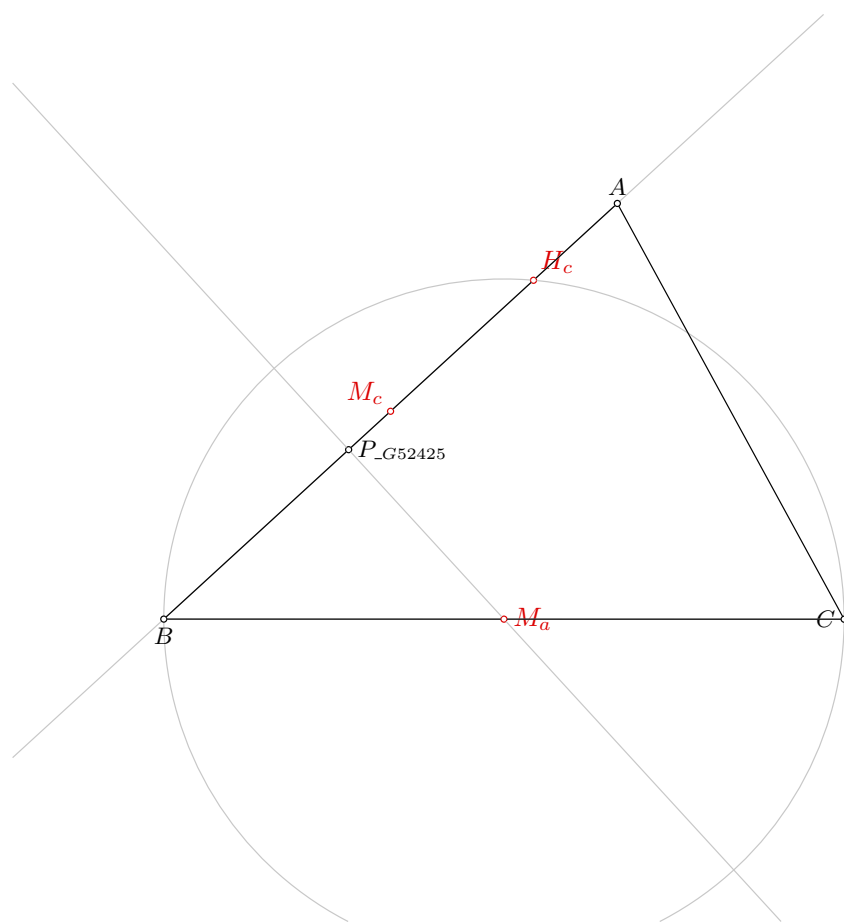


Figure 1: Illustration of the problem 0354

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_a = M_a$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{hc}^0} \neq S_{BCF_{hc}^0}$  i.e., lines  $AB$  and  $CF_{hc}^0$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_c = M_c$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{hc}^0} \neq S_{BCF_{hc}^0}$  i.e., lines  $AB$  and  $CF_{hc}^0$  are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $H_c = H_c$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_a = M_a$

Proving failed

### 4.3.2 Proving $M_c = M_c$

Proving failed

### 4.3.3 Proving $H_c = H_c$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_a = M_a$

Proving failed

### 4.4.2 Proving $M_c = M_c$

Proving failed

### 4.4.3 Proving $H_c = H_c$

Proving failed

# Problem 355

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 355: Given a point  $M_a$ , a point  $M_c$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 356

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 356: Given a point  $M_a$ , a point  $M_c$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 357

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 357: Given a point  $M_a$ , a point  $M_c$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 358

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 358: Given a point  $M_a$ , a point  $M_c$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 359

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 359: Given a point  $M_a$ , a point  $M_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 360

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 360: Given a point  $M_a$ , a point  $H_a$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $H_a$  are not the same;
2. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
3. Using the point  $M_a$ , the line  $h_a$  and the point  $H_a$ , construct a line  $h_{M_a,1/3}(h_a)$  (rule W15); ;
4. Choose freely a point  $G$  on the line  $h_{M_a,1/3}(h_a)$  (rule WOnline4);
5. Using the point  $G$  and the point  $M_a$ , construct a point  $A$  (rule W01); ;
6. Choose freely a point  $B$  on the line  $a$  (rule WOnline1) ;
7. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: points  $M_a$  and  $H_a$  are not the same.

Rules used: [W01,W02,W10b,W15,WOnline1,WOnline4]

Lemmas used: [D21,D5,D8,GD01,GL03,GL04,GL09,L55]

Solving time: 189.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40
point H_{a} 80 40
point G 70 58.33
```



```
color 220 0 0
fontsize 9
```

```
cmark_r M_{a}
cmark_r H_{a}
cmark_t G
color 0 0 0
fontsize 8
```

```
% DET: points M_{a} and H_{a} are not the same
% Constructing a line a which passes through point M_{a} and point H_{a}
line a M_{a} H_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a
```

```
color 200 200 200
drawline h_{a}
color 0 0 0
```

```
% Constructing a point P_{\_G44802} such that M_{a}P_{\_G44802}/M_{a}H_{a}=0.3333333333333333
towards P_{\_G44802} M_{a} H_{a} 0.3333333333333333
cmark_r P_{\_G44802}
color 200 200 200
drawsegment M_{a} H_{a}
color 0 0 0
```

```
% Constructing a line h_{M_{a},1/3}(h_{a}) which contains the point P_{\_G44802} and is parallel to
the line h_{a}
parallel h_{M_{a},1/3}(h_{a}) P_{\_G44802} h_{a}
```

```
color 200 200 200
drawline h_{M_{a},1/3}(h_{a})
color 0 0 0
```

```
% Constructing a point P_{\_G45124} such that M_{a}P_{\_G45124}/M_{a}H_{a}=0.3333333333333333
towards P_{\_G45124} M_{a} H_{a} 0.3333333333333333
cmark_r P_{\_G45124}
color 200 200 200
drawsegment M_{a} H_{a}
color 0 0 0
```

```

% Generating random value V[_G45063]
random V[_G45063]

% Calculating value V[_G45084] using formula V[_G45063]*20
expression V[_G45084] { V[_G45063]*20 }

% Constructing a point G which is a point for which holds  $P_{\backslash\_G45124}G = V[_G45084]$  and angle  $M_{\{a\}}$ 
 $P_{\backslash\_G45124}G = 90$ 
turtle G M_{a} P_{\backslash\_G45124} 90 V[_G45084]
cmark_t G

% Constructing a point A such that  $GA/GM_{\{a\}}=-2$ 
towards A G M_{a} -2
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

% Choosing randomly a point B on the line  $M_{\{a\}}H_{\{a\}}$ 
online B M_{a} H_{a}
cmark_b B
color 200 200 200
drawline M_{a} H_{a}
color 0 0 0

% Constructing a point C such that  $BC/BM_{\{a\}}=2$ 
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions: points  $M_{\{a\}}$  and  $H_{\{a\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

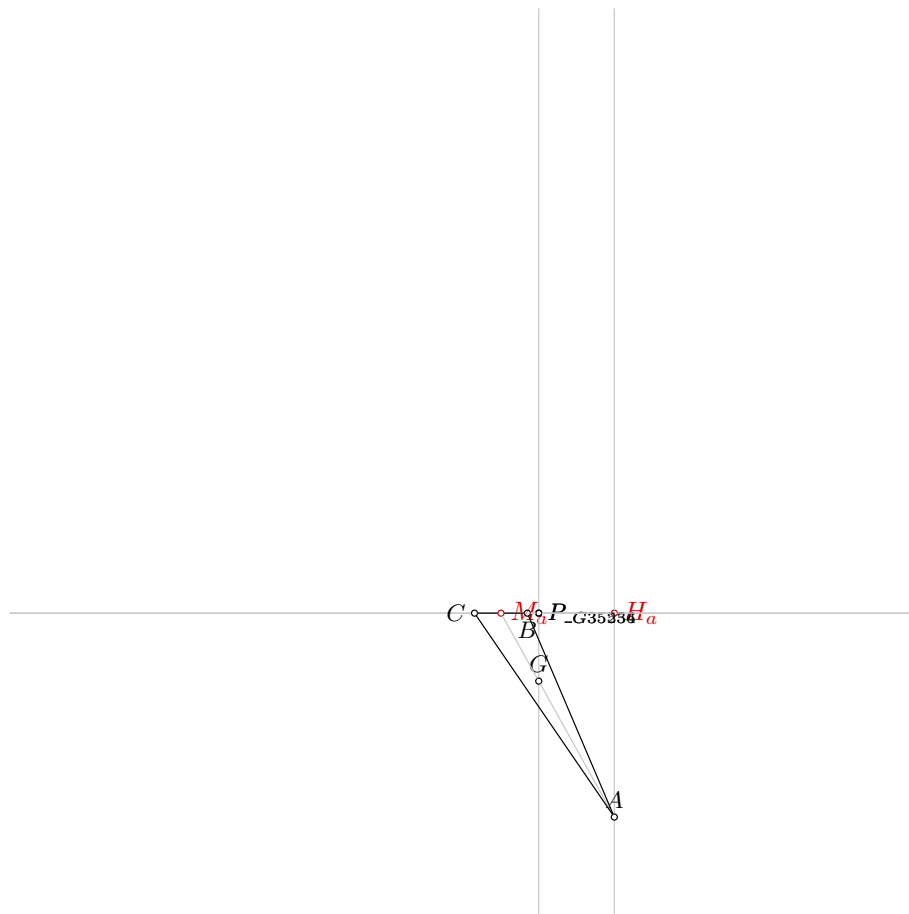


Figure 1: Illustration of the problem 0360

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = \neg M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_a = \neg H_a$

**Status:** Theorem has been disproved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 18 terms.

**Time Complexity:** Time spent by the prover is 0.295 seconds.

**NDG conditions** Points  $G$ ,  $M_a$  and  $P_{G42836}$  are not collinear

Points  $P_{G42842}$  and  $B$  are not identical

Line through points  $B$  and  $M_a$  is not perpendicular to line through points  $M_a$  and  $C$

#### 4.1.3 Proving $G = \neg G$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{\neg h_a}^3} \neq S_{CAF_{\neg h_a}^3}$  i.e., lines  $BC$  and  $AF_{\neg h_a}^3$  are not parallel (construction based assumption)

$S_{AB_{\neg M_b}} \neq S_{\neg M_a B_{\neg M_b}}$  i.e., lines  $A_{\neg M_a}$  and  $B_{\neg M_b}$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.2.3 Proving $G = \neg G$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.3.2 Proving $H_a = \neg H_a$

Proving failed

### 4.3.3 Proving $G=_G$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_a=_M M_a$

Proving failed

### 4.4.2 Proving $H_a=_H H_a$

Proving failed

### 4.4.3 Proving $G=_G$

Proving failed

# Problem 361

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 361: Given a point  $M_a$ , a point  $G$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $G$ , construct a point  $A$  (rule W01); ;
2. Using the point  $H_b$  and the point  $A$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $A$  are not the same;
3. Using the point  $H_b$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_b$  and  $M_a$  are not the same;
4. Using the circle  $k(M_a, B)$ , the line  $b$ , the point  $M_a$  and the point  $H_b$ , construct a point  $C$  (rule W05); % NDG: line  $b$  and circle  $k(M_a, B)$  intersect % DET: points  $H_b$  and  $C$  must be different;
5. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: line  $b$  and circle  $k(M_a, B)$  intersect; points  $H_b$  and  $M_a$  are not the same.

Determination conditions: points  $H_b$  and  $C$  must be different; points  $H_b$  and  $A$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D21,D6,GD01,GD02,GL03,GL04,L37,L38,L55]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```

dim 120 120

point M_{a} 65 40
point G 70 58.33
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_t G
cmark_l H_{b}
color 0 0 0
fontsize 8

% Constructing a point A such that  $M_{\{a\}}A/M_{\{a\}}G=3$ 
towards A M_{a} G 3
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

% DET: points  $H_{\{b\}}$  and  $A$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $A$ 
line b H_{b} A

color 200 200 200
drawline b
color 0 0 0

% NDG: points  $H_{\{b\}}$  and  $M_{\{a\}}$  are not the same
% Constructing a circle  $k(M_{\{a\}}, B)$  whose center is at point  $M_{\{a\}}$  and which passes through point  $H_{\{b\}}$ 
circle k(M_{a}, B) M_{a} H_{b}

color 200 200 200
drawcircle k(M_{a}, B)
color 0 0 0

% NDG: line  $b$  and circle  $k(M_{\{a\}}, B)$  intersect% DET: points  $H_{\{b\}}$  and  $C$  must be different
% Constructing a point  $P_{\{\backslash\_G62010\}}$  which is a foot of the point  $M_{\{a\}}$  on the line  $b$ 
foot P_{\_G62010} M_{a} b
cmark_r P_{\_G62010}
color 200 200 200
drawline M_{a} P_{\_G62010}
color 0 0 0

% Constructing a point  $C$  which is an image of the point  $H_{\{b\}}$  in the symmetry to point/line  $P_{\{\backslash\_G62010\}}$ 
sim C P_{\_G62010} H_{b}

```

```
cmark_l C
```

```
% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line b and circle k(M_{a},B) intersect; points H_{b} and M_{a} are not
the same
% Determination conditions: points H_{b} and C must be different; points H_{b} and A are not the
same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = \_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.192 seconds.

**NDG conditions** Points  $H_b$  and  $A$  are not identical

Line through points  $H_b$  and  $G$  is not perpendicular to line through points  $G$  and  $A$

#### 4.1.2 Proving $G = \_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 145 terms.

**Time Complexity:** Time spent by the prover is 0.878 seconds.

**NDG conditions** Points  $H_b$  and  $A$  are not identical

Line through points  $H_b$  and  $G$  is not perpendicular to line through points  $G$  and  $A$

Line through points  $A$  and  $\_M_a$  is not parallel with line through points  $B$  and  $\_M_b$

Points  $\_M_a$ ,  $G$  and  $M_a$  are not collinear



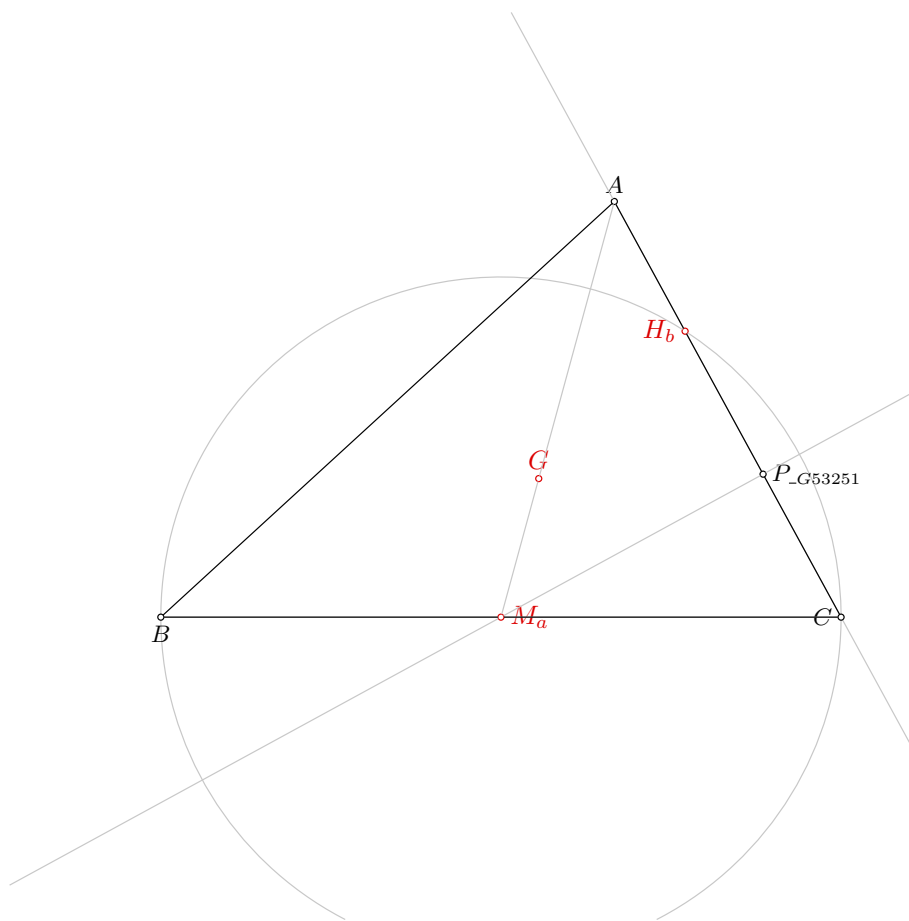


Figure 1: Illustration of the problem 0361

#### 4.1.3 Proving $H_b = \neg H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 174 terms.

**Time Complexity:** Time spent by the prover is 1.138 seconds.

**NDG conditions** Points  $H_b$  and  $A$  are not identical

Line through points  $H_b$  and  $G$  is not perpendicular to line through points  $G$  and  $A$

Points  $A$  and  $C$  are not identical

Points  $A$ ,  $C$  and  $G$  are not collinear

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{AB \neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A \neg M_a$  and  $B \neg M_b$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg H_b}^0} \neq S_{CBF_{\neg H_b}^0}$  i.e., lines  $AC$  and  $BF_{\neg H_b}^0$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $G = \neg G$

Proving failed

#### 4.2.3 Proving $H_b = \neg H_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.3.2 Proving $G = \neg G$

Proving failed

#### 4.3.3 Proving $H_b = \neg H_b$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.4.2 Proving $G = \neg G$

Proving failed

#### 4.4.3 Proving $H_b = \neg H_b$

Proving failed

# Problem 362

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 362: Given a point  $M_a$ , a point  $G$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $G$ , construct a point  $A$  (rule W01); ;
2. Using the point  $H_c$  and the point  $A$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $A$  are not the same;
3. Using the point  $H_c$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_c$  and  $M_a$  are not the same;
4. Using the circle  $k(M_a, B)$ , the line  $c$ , the point  $M_a$  and the point  $H_c$ , construct a point  $B$  (rule W05); % NDG: line  $c$  and circle  $k(M_a, B)$  intersect % DET: points  $H_c$  and  $B$  must be different;
5. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: line  $c$  and circle  $k(M_a, B)$  intersect; points  $H_c$  and  $M_a$  are not the same.

Determination conditions: points  $H_c$  and  $B$  must be different; points  $H_c$  and  $A$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D21,D7,GD01,GD02,GL03,GL04,L39,L55]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```

dim 120 120

point M_{a} 65 40
point G 70 58.33
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_t G
cmark_rt H_{c}
color 0 0 0
fontsize 8

% Constructing a point A such that  $M_{\{a\}}A/M_{\{a\}}G=3$ 
towards A M_{a} G 3
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

% DET: points  $H_{\{c\}}$  and A are not the same
% Constructing a line c which passes through point  $H_{\{c\}}$  and point A
line c H_{c} A

color 200 200 200
drawline c
color 0 0 0

% NDG: points  $H_{\{c\}}$  and  $M_{\{a\}}$  are not the same
% Constructing a circle  $k(M_{\{a\}},B)$  whose center is at point  $M_{\{a\}}$  and which passes through point  $H_{\{c\}}$ 
circle k(M_{a},B) M_{a} H_{c}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line c and circle  $k(M_{\{a\}},B)$  intersect% DET: points  $H_{\{c\}}$  and B must be different
% Constructing a point  $P_{\{\backslash\_G45134\}}$  which is a foot of the point  $M_{\{a\}}$  on the line c
foot P_{\_G45134} M_{a} c
cmark_r P_{\_G45134}
color 200 200 200
drawline M_{a} P_{\_G45134}
color 0 0 0

% Constructing a point B which is an image of the point  $H_{\{c\}}$  in the symmetry to point/line  $P_{\{\backslash\_G45134\}}$ 
sim B P_{\_G45134} H_{c}

```

```
cmark_b B
```

```
% Constructing a point C such that BC/BM_{a}=2
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line c and circle k(M_{a},B) intersect; points H_{c} and M_{a} are not
the same
% Determination conditions: points H_{c} and B must be different; points H_{c} and A are not the
same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = \_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.101 seconds.

**NDG conditions** Points  $A$  and  $H_c$  are not identical

Line through points  $A$  and  $G$  is not perpendicular to line through points  $G$  and  $H_c$

#### 4.1.2 Proving $G = \_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 145 terms.

**Time Complexity:** Time spent by the prover is 0.894 seconds.

**NDG conditions** Points  $A$  and  $H_c$  are not identical

Line through points  $A$  and  $G$  is not perpendicular to line through points  $G$  and  $H_c$

Line through points  $A$  and  $\_M_a$  is not parallel with line through points  $B$  and  $\_M_b$

Points  $\_M_a$ ,  $G$  and  $M_a$  are not collinear

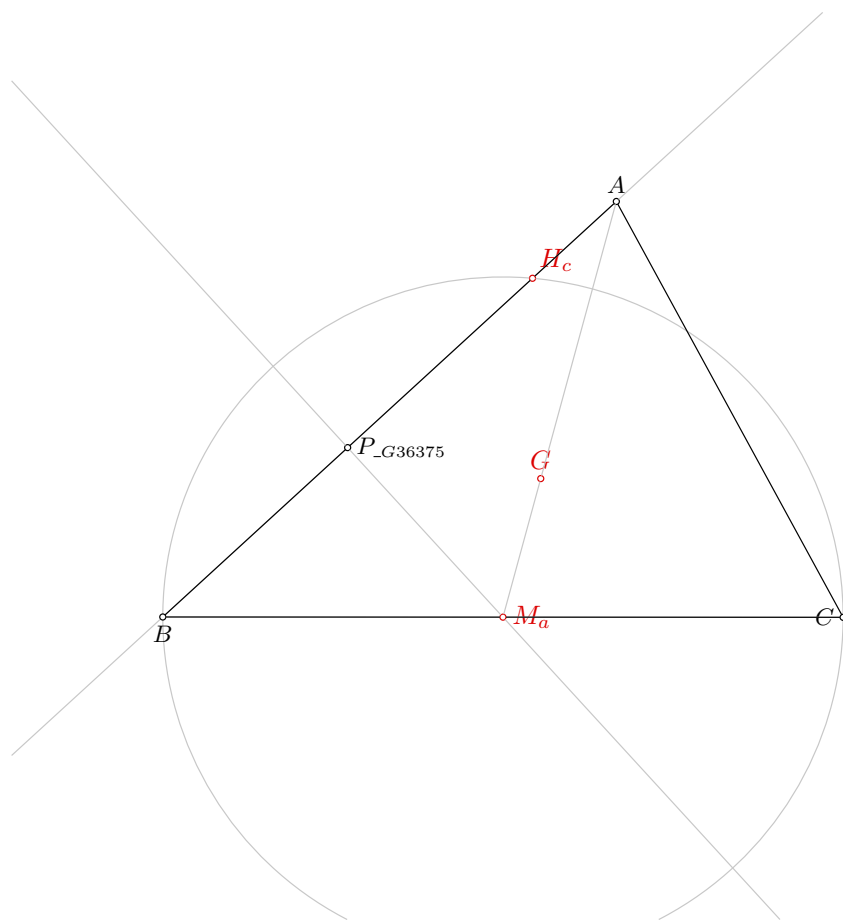


Figure 1: Illustration of the problem 0362

#### 4.1.3 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 174 terms.

**Time Complexity:** Time spent by the prover is 1.134 seconds.

**NDG conditions** Points  $A$  and  $H_c$  are not identical

Line through points  $A$  and  $G$  is not perpendicular to line through points  $G$  and  $H_c$

Points  $A$  and  $B$  are not identical

Points  $A$  and  $B$  are not identical

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{AB \neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A \neg M_a$  and  $B \neg M_b$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{\neg H_c}^0} \neq S_{BCF_{\neg H_c}^0}$  i.e., lines  $AB$  and  $CF_{\neg H_c}^0$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $G = \neg G$

Proving failed

#### 4.2.3 Proving $H_c = \neg H_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.3.2 Proving $G = \neg G$

Proving failed

#### 4.3.3 Proving $H_c = \neg H_c$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.4.2 Proving $G = \neg G$

Proving failed

#### 4.4.3 Proving $H_c = \neg H_c$

Proving failed

# Problem 363

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 363: Given a point  $M_a$ , a point  $G$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $G$ , construct a point  $A$  (rule W01); ;
2. Using the point  $G$  and the point  $H$ , construct a point  $O$  (rule W01); ;
3. Using the point  $M_a$  and the point  $O$ , construct a line  $m_a$  (rule W02); % DET: points  $M_a$  and  $O$  are not the same;
4. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
5. Using the point  $M_a$  and the line  $m_a$ , construct a line  $a$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $M_a$  and  $O$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D1,D11,D21,D26,GD01,GL03,GL04,GL09,L11,L12,L55,L58]

Solving time: 1.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40
point G 70 58.33
point H 80 72.73
```



```
color 220 0 0
fontsize 9
```

```
cmark_r M_{a}
cmark_t G
cmark_rt H
color 0 0 0
fontsize 8
```

```
% Constructing a point A such that  $M_{\{a\}}A/M_{\{a\}}G=3$ 
towards A M_{a} G 3
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0
```

```
% Constructing a point O such that  $GO/GH=-0.5$ 
towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0
```

```
% DET: points  $M_{\{a\}}$  and O are not the same
% Constructing a line  $m_{\{a\}}$  which passes through point  $M_{\{a\}}$  and point O
line m_{a} M_{a} O

color 200 200 200
drawline m_{a}
color 0 0 0
```

```
% NDG: points A and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0
```

```
% Constructing a line a which is perpendicular to line  $m_{\{a\}}$  and which passes through point  $M_{\{a\}}$ 
perp a M_{a} m_{a}

color 200 200 200
drawline a
color 0 0 0
```

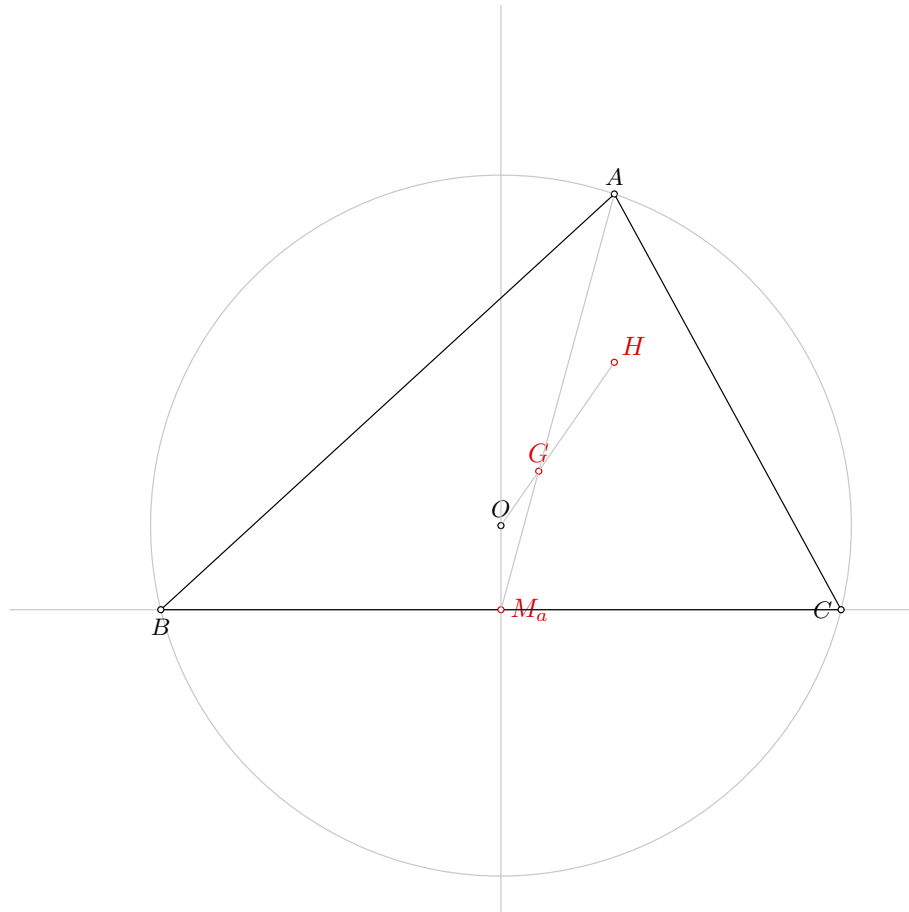


Figure 1: Illustration of the problem 0363

```
% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same
% Determination conditions: points  $M_{\{a\}}$  and O are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.1.2 Proving $G = \neg G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 672 terms.

**Time Complexity:** Time spent by the prover is 1.857 seconds.

**NDG conditions** Points  $G$ ,  $H$  and  $P_{G63661}$  are not collinear

Line through points  $G$  and  $M_a$  is not parallel with line through points  $H$  and  $P_{G63661}$

Points  $M_a$  and  $O$  are not identical

Points  $A$ ,  $M_a$  and  $O$  are not collinear

Points  $C$  and  $M_a$  are not identical

Points  $A$ ,  $C$  and  $G$  are not collinear

Line through points  $A$  and  $\neg M_a$  is not parallel with line through points  $B$  and  $\neg M_b$

Points  $\neg M_a$ ,  $G$  and  $M_a$  are not collinear

#### 4.1.3 Proving $H = \neg H$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.2.2 Proving $G = \neg G$

Proving failed

#### 4.2.3 Proving $H = \neg H$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 60 terms.

**Time Complexity:** Time spent by the prover is 0.110 seconds. There are no ndg conditions.

#### 4.3.2 Proving $G = \neg G$

Proving failed

### 4.3.3 Proving $H=_H$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_a=_M M_a$

Proving failed

### 4.4.2 Proving $G=_G$

Proving failed

### 4.4.3 Proving $H=_H$

Proving failed

# Problem 364

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 364: Given a point  $M_a$ , a point  $G$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $G$ , construct a point  $A$  (rule W01); ;
2. Using the point  $M_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $T_a$  are not the same;
3. Using the point  $T_a$  and the point  $A$ , construct a line  $s_a$  (rule W02); % DET: points  $T_a$  and  $A$  are not the same;
4. Using the point  $M_a$  and the line  $a$ , construct a line  $m_a$  (rule W10b); ;
5. Using the line  $m_a$  and the line  $s_a$ , construct a point  $N_a$  (rule W03); % NDG: lines  $m_a$  and  $s_a$  are not parallel % DET: lines  $m_a$  and  $s_a$  are not the same;
6. Using the point  $A$  and the point  $N_a$ , construct a line  $m(AN_a)$  (rule W14); % DET: points  $A$  and  $N_a$  are not the same;
7. Using the line  $m(AN_a)$  and the line  $m_a$ , construct a point  $O$  (rule W03); % NDG: lines  $m(AN_a)$  and  $m_a$  are not parallel % DET: lines  $m(AN_a)$  and  $m_a$  are not the same;
8. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
9. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same; lines  $m(AN_a)$  and  $m_a$  are not parallel; lines  $m_a$  and  $s_a$  are not parallel.

Determination conditions: lines  $m(AN_a)$  and  $m_a$  are not the same; points  $A$  and  $N_a$  are not the same; lines  $m_a$  and  $s_a$  are not the same; points  $T_a$  and  $A$  are not the same; points  $M_a$  and  $T_a$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14]

Lemmas used: [D1,D11,D21,D23,D26,D47,GD01,GL01,GL03,GL04,GL09,L11,L12,L25,L5,L55]

Solving time: 1.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{a} 65 40
point G 70 58.33
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_t G
cmark_rb T_{a}
color 0 0 0
fontsize 8

% Constructing a point A such that M_{a}A/M_{a}G=3
towards A M_{a} G 3
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

% DET: points M_{a} and T_{a} are not the same
% Constructing a line a which passes through point M_{a} and point T_{a}
line a M_{a} T_{a}

color 200 200 200
drawline a
color 0 0 0

% DET: points T_{a} and A are not the same
% Constructing a line s_{a} which passes through point T_{a} and point A
line s_{a} T_{a} A

color 200 200 200
drawline s_{a}
color 0 0 0
```

```

% Constructing a line  $m_{\{a\}}$  which is perpendicular to line  $a$  and which passes through point  $M_{\{a\}}$ 
perp  $m_{\{a\}}$   $M_{\{a\}}$   $a$ 

color 200 200 200
drawline  $m_{\{a\}}$ 
color 0 0 0

% NDG: lines  $m_{\{a\}}$  and  $s_{\{a\}}$  are not parallel% DET: lines  $m_{\{a\}}$  and  $s_{\{a\}}$  are not the same
% Constructing a point  $N_{\{a\}}$  which belongs to line  $m_{\{a\}}$  and line  $s_{\{a\}}$ 
intersec  $N_{\{a\}}$   $m_{\{a\}}$   $s_{\{a\}}$ 
cmark_b  $N_{\{a\}}$ 

% DET: points  $A$  and  $N_{\{a\}}$  are not the same
% Constructing bisector  $m(AN_{\{a\}})$  of the segment  $AN_{\{a\}}$ 
med  $m(AN_{\{a\}})$   $A$   $N_{\{a\}}$ 

color 200 200 200
drawline  $m(AN_{\{a\}})$ 
color 0 0 0

color 200 200 200
drawsegment  $A$   $N_{\{a\}}$ 
color 0 0 0

% NDG: lines  $m(AN_{\{a\}})$  and  $m_{\{a\}}$  are not parallel% DET: lines  $m(AN_{\{a\}})$  and  $m_{\{a\}}$  are not the same
% Constructing a point  $O$  which belongs to line  $m(AN_{\{a\}})$  and line  $m_{\{a\}}$ 
intersec  $O$   $m(AN_{\{a\}})$   $m_{\{a\}}$ 
cmark_t  $O$ 

% NDG: points  $A$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $A$ 
circle  $k(O,C)$   $O$   $A$ 

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $a$  and circle  $k(O,C)$  intersect
% Constructing points  $C$  and  $B$  which are in intersection of  $k(O,C)$  and  $a$ 
intersec2  $C$   $B$   $k(O,C)$   $a$ 
cmark_l  $C$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

```

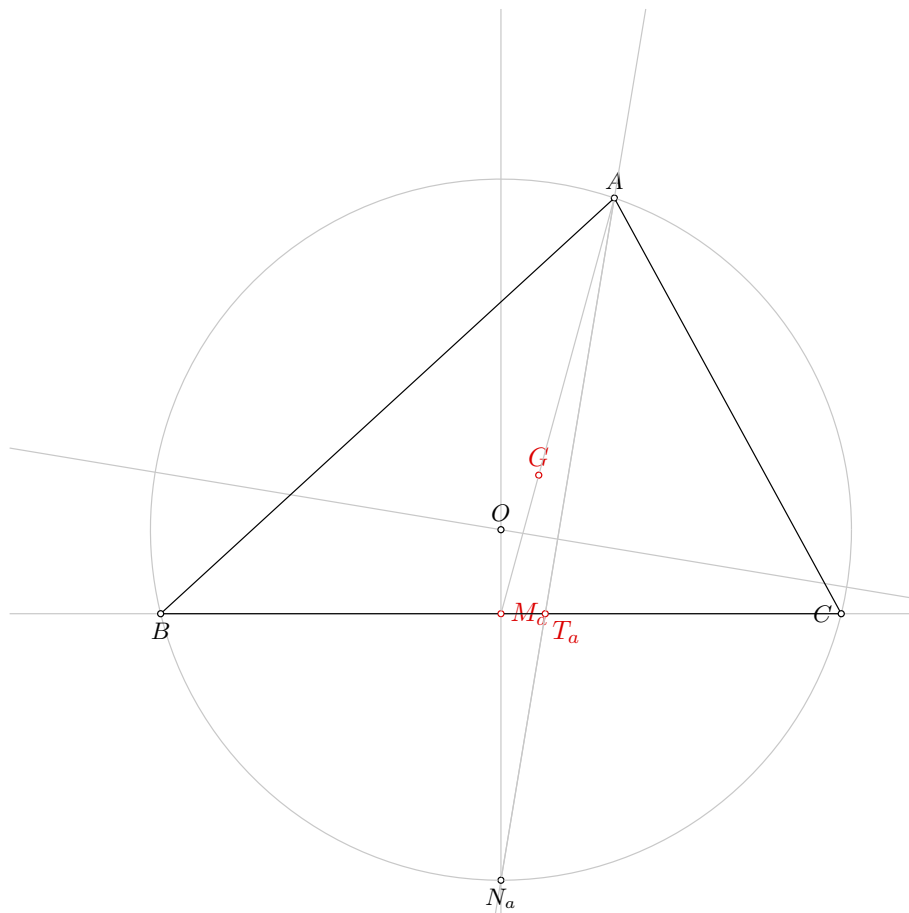


Figure 1: Illustration of the problem 0364

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
  lines m(AN_{a}) and m_{a} are not parallel; lines m_{a} and s_{a} are not parallel
% Determination conditions: lines m(AN_{a}) and m_{a} are not the same; points A and N_{a} are not
  the same; lines m_{a} and s_{a} are not the same; points T_{a} and A are not the same; points M
  _{a} and T_{a} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

#### 4.1 OGP - Wu method

### 4.1.1 Proving $M_a = -M_a$

Proving failed



#### 4.1.2 Proving $G = \neg G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2660 terms.

**Time Complexity:** Time spent by the prover is 17.995 seconds.

**NDG conditions** Line through points  $A$  and  $T_a$  is not perpendicular to line through points  $T_a$  and  $M_a$

Points  $T_a$ ,  $G$  and  $M_a$  are not collinear

Line through points  $T_a$  and  $M_a$  is not parallel with line through points  $N_a$  and  $A$

Line through points  $N_a$  and  $G$  is not perpendicular to line through points  $G$  and  $A$

Points  $T_a$ ,  $G$  and  $M_a$  are not collinear

Points  $T_a$ ,  $G$  and  $M_a$  are not collinear

Line through points  $A$  and  $\neg M_a$  is not parallel with line through points  $B$  and  $\neg M_b$

Points  $\neg M_a$ ,  $G$  and  $M_a$  are not collinear

#### 4.1.3 Proving $T_a = \neg T_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.2.2 Proving $G = \neg G$

Proving failed

#### 4.2.3 Proving $T_a = \neg T_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 1355 terms.

**Time Complexity:** Time spent by the prover is 1.450 seconds. There are no ndg conditions.

#### 4.3.2 Proving $G = \neg G$

Proving failed

#### 4.3.3 Proving $T_a = \neg T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 9188 terms.

**Time Complexity:** Time spent by the prover is 19.710 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_a = \_M_a$

Proving failed

### 4.4.2 Proving $G = \_G$

Proving failed

### 4.4.3 Proving $T_a = \_T_a$

Proving failed

# Problem 365

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 365: Given a point  $M_a$ , a point  $G$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 366

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 366: Given a point  $M_a$ , a point  $G$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 367

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 367: Given a point  $M_a$ , a point  $G$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $G$ , construct a point  $A$  (rule W01); ;
2. Using the point  $M_a$  and the point  $I$ , construct a line  $IM_a$  (rule W02); % DET: points  $M_a$  and  $I$  are not the same;
3. Using the point  $I$  and the point  $A$ , construct a line  $s_a$  (rule W02); % DET: points  $I$  and  $A$  are not the same;
4. Using the point  $I$  and the point  $M_a$ , construct a circle  $k_{over}(I, M_a)$  (rule W09); % NDG: points  $I$  and  $M_a$  are not the same;
5. Using the point  $A$  and the line  $IM_a$ , construct a line  $AP'_a$  (rule W16); ;
6. Using the point  $M_a$ , the line  $AP'_a$  and the point  $A$ , construct a line  $h_{M_a, -1/1}(AP'_a)$  (rule W15); ;
7. Using the circle  $k_{over}(I, M_a)$  and the line  $h_{M_a, -1/1}(AP'_a)$ , construct a point  $A_{fo}$  and a point  $P_a$  (rule W04); % NDG: line  $h_{M_a, -1/1}(AP'_a)$  and circle  $k_{over}(I, M_a)$  intersect;
8. Using the point  $P_a$  and the point  $I$ , construct a circle  $k(I, P_a)$  (rule W06); % NDG: points  $P_a$  and  $I$  are not the same;
9. Using the circle  $k(I, P_a)$ , the point  $M_a$  and the point  $I$ , construct a line  $x1$  and a line  $a$  (rule W12); % NDG: point  $M_a$  is outside the circle  $k(I, P_a)$ ;
10. Using the point  $M_a$  and the line  $a$ , construct a line  $m_a$  (rule W10b); ;
11. Using the line  $m_a$  and the line  $s_a$ , construct a point  $N_a$  (rule W03); % NDG: lines  $m_a$  and  $s_a$  are not parallel % DET: lines  $m_a$  and  $s_a$  are not the same;

12. Using the point  $I$  and the point  $N_a$ , construct a circle  $k(N_a, C)$  (rule W06); % NDG: points  $I$  and  $N_a$  are not the same;
13. Using the circle  $k(N_a, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(N_a, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(N_a, C)$  intersect; points  $I$  and  $N_a$  are not the same; lines  $m_a$  and  $s_a$  are not parallel; point  $M_a$  is outside the circle  $k(I, P_a)$ ; points  $P_a$  and  $I$  are not the same; line  $h_{M_a, -1/1}(AP'_a)$  and circle  $k_{over}(I, M_a)$  intersect; points  $I$  and  $M_a$  are not the same.

Determination conditions: lines  $m_a$  and  $s_a$  are not the same; points  $I$  and  $A$  are not the same; points  $M_a$  and  $I$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W09,W10b,W12,W15,W16]

Lemmas used: [D11,D2,D21,D27,D47,D59,D85,GD01,GD02,GL02,GL03,GL04,GL09,L25,L31,L32,L5,L55,L60,

Solving time: 2.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40
point G 70 58.33
point I 74.37 61.15
```

```
color 220 0 0
fontsize 9
```

```
cmark_r M_{a}
cmark_t G
cmark_b I
color 0 0 0
fontsize 8
```

```
% Constructing a point A such that M_{a}A/M_{a}G=3
towards A M_{a} G 3
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0
```

```
% DET: points M_{a} and I are not the same
% Constructing a line IM_{a} which passes through point M_{a} and point I
line IM_{a} M_{a} I
```

```
color 200 200 200
drawline IM_{a}
color 0 0 0
```

```
% DET: points I and A are not the same
```

```

% Constructing a line  $s_{\{a\}}$  which passes through point  $I$  and point  $A$ 
line  $s_{\{a\}}$  I A

color 200 200 200
drawline  $s_{\{a\}}$ 
color 0 0 0

% NDG: points  $I$  and  $M_{\{a\}}$  are not the same
% Constructing midpoint  $P_{\{\backslash\_G41994\}}$  of the segment  $IM_{\{a\}}$ 
midpoint  $P_{\{\backslash\_G41994\}}$  I  $M_{\{a\}}$ 
cmark_r  $P_{\{\backslash\_G41994\}}$ 

% Constructing a circle  $k_{\text{over}}(I, M_{\{a\}})$  whose center is at point  $P_{\{\backslash\_G41994\}}$  and which passes
    through point  $I$ 
circle  $k_{\text{over}}(I, M_{\{a\}})$   $P_{\{\backslash\_G41994\}}$  I

color 200 200 200
drawcircle  $k_{\text{over}}(I, M_{\{a\}})$ 
color 0 0 0

% Constructing a line  $AP'_{\{a\}}$  which contains the point  $A$  and is parallel to the line  $IM_{\{a\}}$ 
parallel  $AP'_{\{a\}}$  A  $IM_{\{a\}}$ 

color 200 200 200
drawline  $AP'_{\{a\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\backslash\_G42295\}}$  such that  $M_{\{a\}}P_{\{\backslash\_G42295\}}/M_{\{a\}}A=-1$ 
towards  $P_{\{\backslash\_G42295\}}$   $M_{\{a\}}$  A -1
cmark_r  $P_{\{\backslash\_G42295\}}$ 
color 200 200 200
drawsegment A  $P_{\{\backslash\_G42295\}}$ 
color 0 0 0

% Constructing a line  $h_{\{M_{\{a\}}, -1/1\}}(AP'_{\{a\}})$  which contains the point  $P_{\{\backslash\_G42295\}}$  and is parallel
    to the line  $AP'_{\{a\}}$ 
parallel  $h_{\{M_{\{a\}}, -1/1\}}(AP'_{\{a\}})$   $P_{\{\backslash\_G42295\}}$   $AP'_{\{a\}}$ 

color 200 200 200
drawline  $h_{\{M_{\{a\}}, -1/1\}}(AP'_{\{a\}})$ 
color 0 0 0

% NDG: line  $h_{\{M_{\{a\}}, -1/1\}}(AP'_{\{a\}})$  and circle  $k_{\text{over}}(I, M_{\{a\}})$  intersect
% Constructing points  $A_{\{fo\}}$  and  $P_{\{a\}}$  which are in intersection of  $k_{\text{over}}(I, M_{\{a\}})$  and  $h_{\{M_{\{a\}}, -1/1\}}(AP'_{\{a\}})$ 
intersec2  $A_{\{fo\}}$   $P_{\{a\}}$   $k_{\text{over}}(I, M_{\{a\}})$   $h_{\{M_{\{a\}}, -1/1\}}(AP'_{\{a\}})$ 

```

```

cmark_r A_{fo}
cmark_r P_{a}

% NDG: points  $P_{a}$  and  $I$  are not the same
% Constructing a circle  $k(I, P_{a})$  whose center is at point  $I$  and which passes through point  $P_{a}$ 
circle k(I, P_{a}) I P_{a}

color 200 200 200
drawcircle k(I, P_{a})
color 0 0 0

% NDG: point  $M_{a}$  is outside the circle  $k(I, P_{a})$ 
% Constructing midpoint  $P_{\backslash\_G43044}$  of the segment  $M_{a}I$ 
midpoint P_{\backslash\_G43044} M_{a} I
cmark_r P_{\backslash\_G43044}

% Constructing a circle  $C_{\backslash\_G43047}$  whose center is at point  $P_{\backslash\_G43044}$  and which passes through
point  $M_{a}$ 
circle C_{\backslash\_G43047} P_{\backslash\_G43044} M_{a}

color 200 200 200
drawcircle C_{\backslash\_G43047}
color 0 0 0

% Constructing points  $P_{\backslash\_G43050}$  and  $P_{\backslash\_G43053}$  which are in intersection of  $C_{\backslash\_G43047}$  and  $k$ 
( $I, P_{a}$ )
intersec2 P_{\backslash\_G43050} P_{\backslash\_G43053} C_{\backslash\_G43047} k(I, P_{a})
cmark_r P_{\backslash\_G43050}
cmark_r P_{\backslash\_G43053}

% Constructing a line  $x1$  which passes through point  $M_{a}$  and point  $P_{\backslash\_G43050}$ 
line x1 M_{a} P_{\backslash\_G43050}

color 200 200 200
drawline x1
color 0 0 0

% Constructing a line  $a$  which passes through point  $M_{a}$  and point  $P_{\backslash\_G43053}$ 
line a M_{a} P_{\backslash\_G43053}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line  $m_{a}$  which is perpendicular to line  $a$  and which passes through point  $M_{a}$ 
perp m_{a} M_{a} a

color 200 200 200

```



```

drawline m_{a}
color 0 0 0

% NDG: lines m_{a} and s_{a} are not parallel% DET: lines m_{a} and s_{a} are not the same
% Constructing a point N_{a} which belongs to line m_{a} and line s_{a}
intersec N_{a} m_{a} s_{a}
cmark_b N_{a}

% NDG: points I and N_{a} are not the same
% Constructing a circle k(N_{a},C) whose center is at point N_{a} and which passes through point I
circle k(N_{a},C) N_{a} I

color 200 200 200
drawcircle k(N_{a},C)
color 0 0 0

% NDG: line a and circle k(N_{a},C) intersect
% Constructing points C and B which are in intersection of k(N_{a},C) and a
intersec2 C B k(N_{a},C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(N_{a},C) intersect; points I and N_{a} are not the
% same; lines m_{a} and s_{a} are not parallel; point M_{a} is outside the circle k(I,P_{a});
% points P_{a} and I are not the same; line h_{M_{a},-1/1}(AP'_{a}) and circle k_{over}(I,M_{a})
% intersect; points I and M_{a} are not the same
% Determination conditions: lines m_{a} and s_{a} are not the same; points I and A are not the same
% ; points M_{a} and I are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = -M_a$

Proving failed

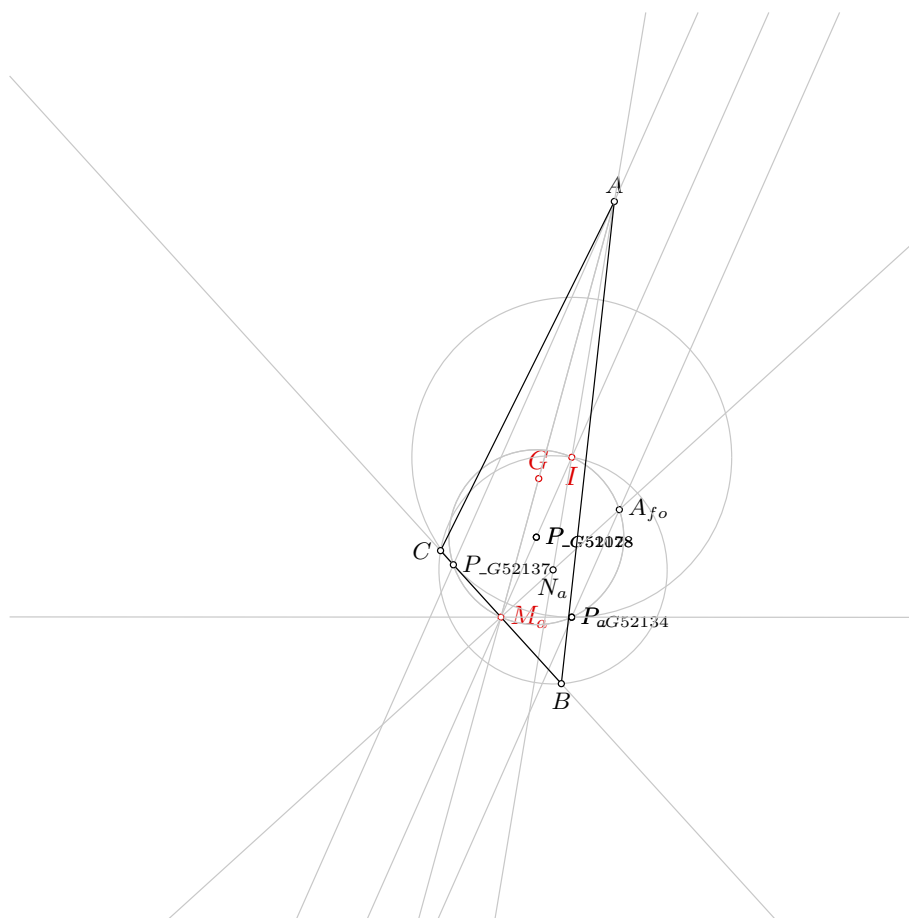


Figure 1: Illustration of the problem 0367

#### 4.1.2 Proving $G=_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 5329 terms.

**Time Complexity:** Time spent by the prover is 8.993 seconds.

**NDG conditions** Points  $A, I$  and  $G$  are not collinear

Points  $A$  and tempPoint-404AP<sub>a</sub> are not identical

Points  $M_a$  and tempPoint-404AP<sub>a</sub> are not identical

Points  $A, I$  and  $G$  are not collinear

Points  $A, I$  and  $G$  are not collinear

Line through points  $P_{G36169}$  and  $A$  is not perpendicular to line through points  $A$  and  $M_a$

Points  $P_{G36169}$  and  $M_a$  are not identical

Points  $P_{G36169}$  and  $M_a$  are not identical

Points  $P_{G36169}$  and  $M_a$  are not identical

Line through points  $A$  and  $_M_a$  is not parallel with line through points  $B$  and  $_M_b$

Points  $_M_a, G$  and  $M_a$  are not collinear

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a=_M_a$

Proving failed

#### 4.2.2 Proving $G=_G$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a=_M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 1448 terms.

**Time Complexity:** Time spent by the prover is 4.060 seconds. There are no ndg conditions.

#### 4.3.2 Proving $G=_G$

Proving failed

#### 4.3.3 Proving $I=_I$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_a =_M M_a$

Proving failed

### 4.4.2 Proving $G =_G G$

Proving failed

### 4.4.3 Proving $I =_I I$

Proving failed

# Problem 368

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 368: Given a point  $M_a$ , a point  $H_a$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $H_a$  are not the same;
2. Using the point  $H_b$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_b$  and  $M_a$  are not the same;
3. Using the circle  $k(M_a, B)$  and the line  $a$ , construct a point  $B$  and a point  $C$  (rule W04); % NDG: line  $a$  and circle  $k(M_a, B)$  intersect;
4. Using the point  $H_b$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $C$  are not the same;
5. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
6. Using the line  $b$  and the line  $h_a$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $h_a$  are not parallel % DET: lines  $b$  and  $h_a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $h_a$  are not parallel; line  $a$  and circle  $k(M_a, B)$  intersect; points  $H_b$  and  $M_a$  are not the same.

Determination conditions: lines  $b$  and  $h_a$  are not the same; points  $H_b$  and  $C$  are not the same; points  $M_a$  and  $H_a$  are not the same.

Rules used: [W02,W03,W04,W06,W10b]

Lemmas used: [D21,D5,D6,D8,GD01,GD02,GL09,L37,L38]

Solving time: 1.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{a} 65 40
point H_{a} 80 40
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_r H_{a}
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points M_{a} and H_{a} are not the same
% Constructing a line a which passes through point M_{a} and point H_{a}
line a M_{a} H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points H_{b} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{b}
circle k(M_{a},B) M_{a} H_{b}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line a and circle k(M_{a},B) intersect
% Constructing points B and C which are in intersection of k(M_{a},B) and a
intersec2 B C k(M_{a},B) a
cmark_b B
cmark_l C

% DET: points H_{b} and C are not the same
% Constructing a line b which passes through point H_{b} and point C
line b H_{b} C

color 200 200 200
drawline b
color 0 0 0
```

```

% Constructing a line  $h_{\{a\}}$  which is perpendicular to line  $a$  and which passes through point  $H_{\{a\}}$ 
perp  $h_{\{a\}}$   $H_{\{a\}}$   $a$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: lines  $b$  and  $h_{\{a\}}$  are not parallel% DET: lines  $b$  and  $h_{\{a\}}$  are not the same
% Constructing a point  $A$  which belongs to line  $b$  and line  $h_{\{a\}}$ 
intersec  $A$   $b$   $h_{\{a\}}$ 
cmark_t  $A$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $b$  and  $h_{\{a\}}$  are not parallel; line  $a$  and circle  $k(M_{\{a\}}, B)$ 
% intersect; points  $H_{\{b\}}$  and  $M_{\{a\}}$  are not the same
% Determination conditions: lines  $b$  and  $h_{\{a\}}$  are not the same; points  $H_{\{b\}}$  and  $C$  are not the same
% ; points  $M_{\{a\}}$  and  $H_{\{a\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.1.2 Proving $H_a = \neg H_a$

Construction steps:

- Free point  $M_a$
- Free point  $H_a$
- Free point  $H_b$
- Line  $a$  through two points  $M_a$  and  $H_a$
- Circle  $k(M_a, B)$  with center  $M_a$  and one point  $H_b$
- Intersection point  $B$  of point sets  $k(M_a, B)$  and  $a$
- Intersection point  $C$  of point sets  $k(M_a, B)$  and  $a$
- Line  $b$  through two points  $H_b$  and  $C$
- Line  $h_a$  through point  $H_a$  perpendicular to line  $a$
- Intersection point  $A$  of point sets  $b$  and  $h_a$

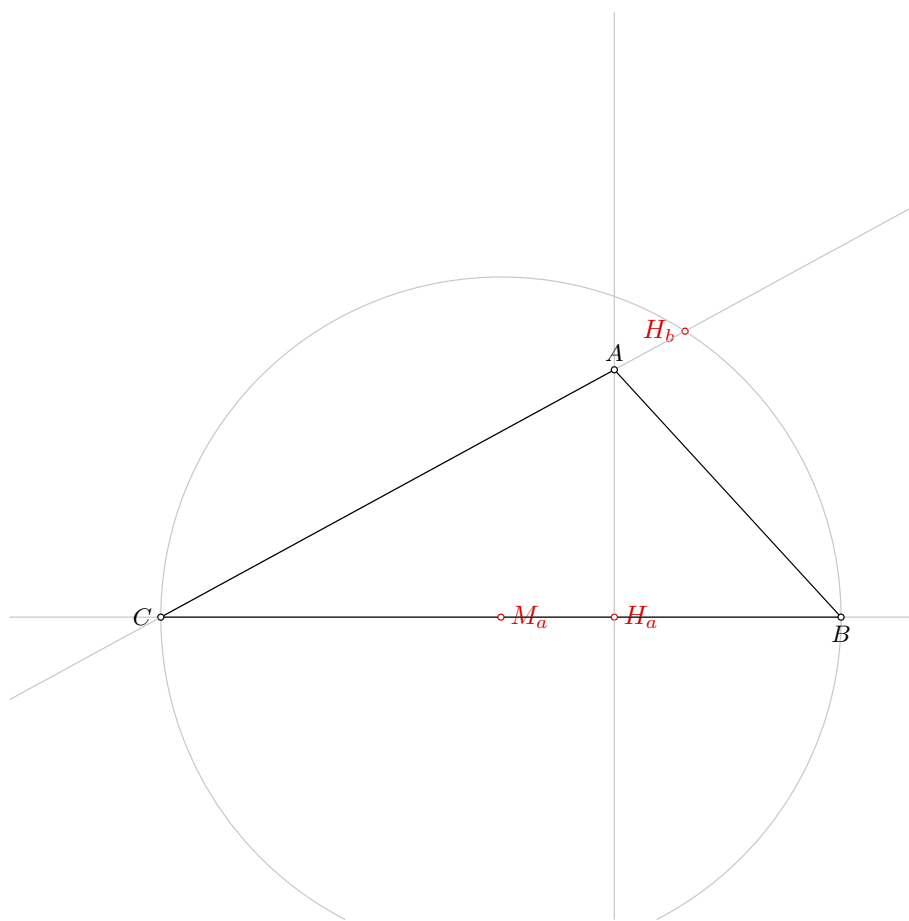


Figure 1: Illustration of the problem 0368



- Segment division point  $M_a$  of segment  $BC$  with division coefficient 1.0
- Line  $a$  through two points  $B$  and  $C$
- Line  $h_a$  through point  $A$  perpendicular to line  $a$
- Intersection point  $H_a$  of point sets  $a$  and  $h_a$
- Line  $b$  through two points  $A$  and  $C$
- Line  $h_b$  through point  $B$  perpendicular to line  $b$
- Intersection point  $H_b$  of point sets  $b$  and  $h_b$

### Theorem statement:

- Points  $H_a$  and  $H_b$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $M_a$  has been assigned following coordinates: (0, 0)

#### 4.1.3 Proving $H_b = H_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = M_a$

Proving failed

#### 4.2.2 Proving $H_a = H_a$

Proving failed

#### 4.2.3 Proving $H_b = H_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = M_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_a = H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $H_b = H_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 224 terms.

**Time Complexity:** Time spent by the prover is 0.370 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_a = \neg M_a$

Proving failed

### 4.4.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.3 Proving $H_b = \neg H_b$

Proving failed

# Problem 369

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 369: Given a point  $M_a$ , a point  $H_a$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $H_a$  are not the same;
2. Using the point  $H_c$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_c$  and  $M_a$  are not the same;
3. Using the circle  $k(M_a, B)$  and the line  $a$ , construct a point  $B$  and a point  $C$  (rule W04); % NDG: line  $a$  and circle  $k(M_a, B)$  intersect;
4. Using the point  $H_c$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $B$  are not the same;
5. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
6. Using the line  $h_a$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $h_a$  and  $c$  are not parallel % DET: lines  $h_a$  and  $c$  are not the same.

Non-degenerate conditions: lines  $h_a$  and  $c$  are not parallel; line  $a$  and circle  $k(M_a, B)$  intersect; points  $H_c$  and  $M_a$  are not the same.

Determination conditions: lines  $h_a$  and  $c$  are not the same; points  $H_c$  and  $B$  are not the same; points  $M_a$  and  $H_a$  are not the same.

Rules used: [W02,W03,W04,W06,W10b]

Lemmas used: [D21,D5,D7,D8,GD01,GD02,GL09,L37,L39]

Solving time: 1.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{a} 65 40
point H_{a} 80 40
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_r H_{a}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points M_{a} and H_{a} are not the same
% Constructing a line a which passes through point M_{a} and point H_{a}
line a M_{a} H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points H_{c} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{c}
circle k(M_{a},B) M_{a} H_{c}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line a and circle k(M_{a},B) intersect
% Constructing points B and C which are in intersection of k(M_{a},B) and a
intersec2 B C k(M_{a},B) a
cmark_b B
cmark_l C

% DET: points H_{c} and B are not the same
% Constructing a line c which passes through point H_{c} and point B
line c H_{c} B

color 200 200 200
drawline c
color 0 0 0
```

```

% Constructing a line  $h_{\{a\}}$  which is perpendicular to line  $a$  and which passes through point  $H_{\{a\}}$ 
perp  $h_{\{a\}}$   $H_{\{a\}}$   $a$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: lines  $h_{\{a\}}$  and  $c$  are not parallel% DET: lines  $h_{\{a\}}$  and  $c$  are not the same
% Constructing a point  $A$  which belongs to line  $h_{\{a\}}$  and line  $c$ 
intersec  $A$   $h_{\{a\}}$   $c$ 
cmark_t  $A$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $h_{\{a\}}$  and  $c$  are not parallel; line  $a$  and circle  $k(M_{\{a\}}, B)$ 
% intersect; points  $H_{\{c\}}$  and  $M_{\{a\}}$  are not the same
% Determination conditions: lines  $h_{\{a\}}$  and  $c$  are not the same; points  $H_{\{c\}}$  and  $B$  are not the same
% ; points  $M_{\{a\}}$  and  $H_{\{a\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.1.2 Proving $H_a = \neg H_a$

Construction steps:

- Free point  $M_a$
- Free point  $H_a$
- Free point  $H_c$
- Line  $a$  through two points  $M_a$  and  $H_a$
- Circle  $k(M_a, B)$  with center  $M_a$  and one point  $H_c$
- Intersection point  $B$  of point sets  $k(M_a, B)$  and  $a$
- Intersection point  $C$  of point sets  $k(M_a, B)$  and  $a$
- Line  $c$  through two points  $H_c$  and  $B$
- Line  $h_a$  through point  $H_a$  perpendicular to line  $a$
- Intersection point  $A$  of point sets  $h_a$  and  $c$

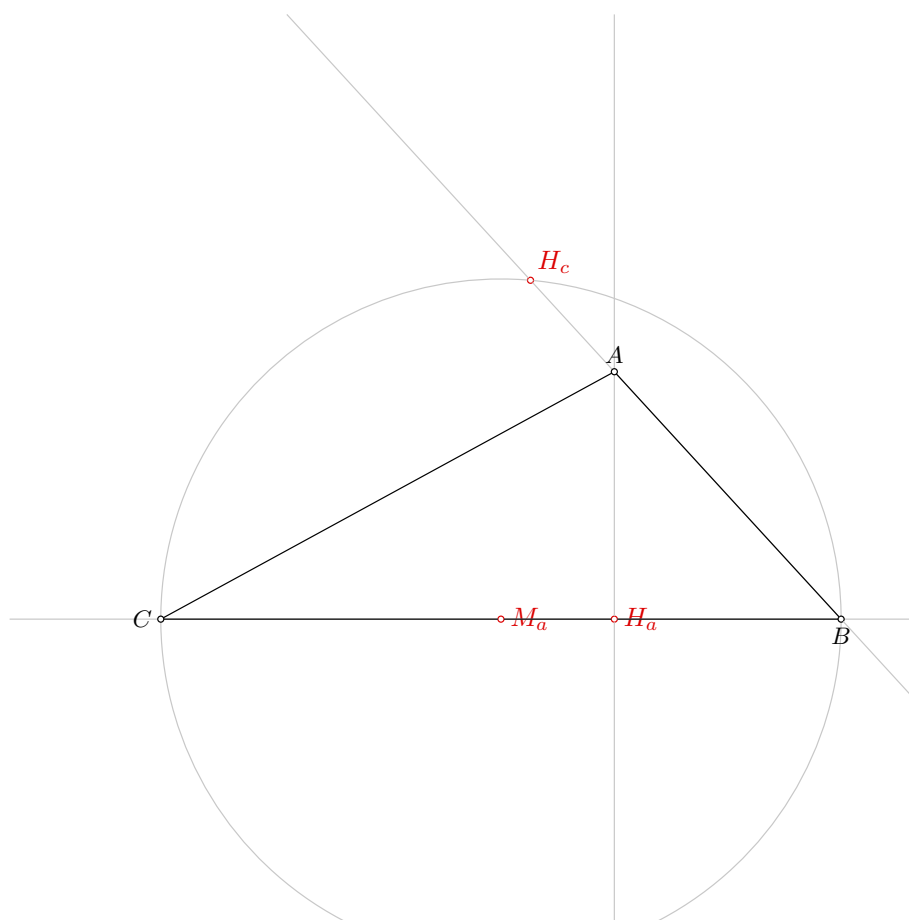


Figure 1: Illustration of the problem 0369

- Segment division point  $M_a$  of segment  $BC$  with division coefficient 1.0
- Line  $a$  through two points  $B$  and  $C$
- Line  $h_a$  through point  $A$  perpendicular to line  $a$
- Intersection point  $H_a$  of point sets  $a$  and  $h_a$
- Line  $c$  through two points  $A$  and  $B$
- Line  $h_c$  through point  $C$  perpendicular to line  $c$
- Intersection point  $H_c$  of point sets  $c$  and  $h_c$

### Theorem statement:

- Points  $H_a$  and  $H_c$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $M_a$  has been assigned following coordinates:  $(0, 0)$

#### 4.1.3 Proving $H_c = H_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = M_a$

Proving failed

#### 4.2.2 Proving $H_a = H_a$

Proving failed

#### 4.2.3 Proving $H_c = H_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = M_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_a = H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $H_c = H_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 224 terms.

**Time Complexity:** Time spent by the prover is 0.340 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_a = \neg M_a$

Proving failed

### 4.4.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.3 Proving $H_c = \neg H_c$

Proving failed



# Problem 370

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 370: Given a point  $H_a$ , a point  $H$  and a point  $M_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $H_a$  and  $H$  are not the same;
2. Using the point  $H_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
3. Choose freely a point  $M_a$  on the line  $a$  (rule WOnline2);
4. Choose freely a point  $A$  on the line  $h_a$  (rule WOnline1) ;
5. Using the point  $A$  and the point  $M_a$ , construct a point  $G$  (rule W01); ;
6. Using the point  $H$  and the point  $G$ , construct a point  $O$  (rule W01); ;
7. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
8. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same.

Determination conditions: points  $H_a$  and  $H$  are not the same.

Rules used: [W01,W02,W04,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D26,D3,D5,D8,GD01,GL03,L11,L12,L55,L58]

Solving time: 30.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point H_{a} 80 40
point H 80 72.73
point M_{a} 65 40

color 220 0 0
fontsize 9

cmark_r H_{a}
cmark_rt H
cmark_r M_{a}
color 0 0 0
fontsize 8

% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
perp a H_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0

% Generating random value V[_G64852]
random V[_G64852]

% Calculating value V[_G64873] using formula V[_G64852]*20
expression V[_G64873] { V[_G64852]*20 }

% Constructing a point M_{a} which is a point for which holds  $H_{a}M_{a} = V[_G64873]$  and angle  $HH_{a}M_{a} = 90$ 
turtle M_{a} H H_{a} 90 V[_G64873]
cmark_r M_{a}

% Choosing randomly a point A on the line H_{a}H
online A H_{a} H
cmark_t A
```

```

color 200 200 200
drawline H_{a} H
color 0 0 0

% Constructing a line L_{\_G33983} which passes through point A and point M_{a}
line L_{\_G33983} A M_{a}

color 200 200 200
drawline L_{\_G33983}
color 0 0 0

% Constructing a point P_{\_G34084} with coordinates (0,0)
point P_{\_G34084} 0 0
cmark_r P_{\_G34084}

% Constructing a point P_{\_G34008} such that AP_{\_G34008}/AP_{\_G34084}=2
towards P_{\_G34008} A P_{\_G34084} 2
cmark_r P_{\_G34008}
color 200 200 200
drawsegment A P_{\_G34008}
color 0 0 0

% Constructing a point P_{\_G34053} such that AP_{\_G34053}/AP_{\_G34084}=3
towards P_{\_G34053} A P_{\_G34084} 3
cmark_r P_{\_G34053}
color 200 200 200
drawsegment A P_{\_G34053}
color 0 0 0

% Constructing a line L_{\_G34014} which passes through point M_{a} and point P_{\_G34053}
line L_{\_G34014} M_{a} P_{\_G34053}

color 200 200 200
drawline L_{\_G34014}
color 0 0 0

% Constructing a line L_{\_G33977} which contains the point P_{\_G34008} and is parallel to the
line L_{\_G34014}
parallel L_{\_G33977} P_{\_G34008} L_{\_G34014}

color 200 200 200
drawline L_{\_G33977}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G33977} and line L_{\_G33983}
intersec G L_{\_G33977} L_{\_G33983}
cmark_t G

% Constructing a point O such that HO/HG=1.5

```

```

towards O H G 1.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same
% Determination conditions: points  $H_{\{a\}}$  and H are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_a = \_H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 8 terms.

**Time Complexity:** Time spent by the prover is 0.175 seconds.

**NDG conditions** Points  $A$ ,  $P_{G60820}$  and  $M_a$  are not collinear

Line through points  $P_{G60820}$  and  $M_a$  is not perpendicular to line through points  $M_a$  and  $H_a$

Line through points  $B$  and  $H$  is not perpendicular to line through points  $H$  and  $C$

#### 4.1.2 Proving $H = \_H$

**Status:** Theorem has been proved.

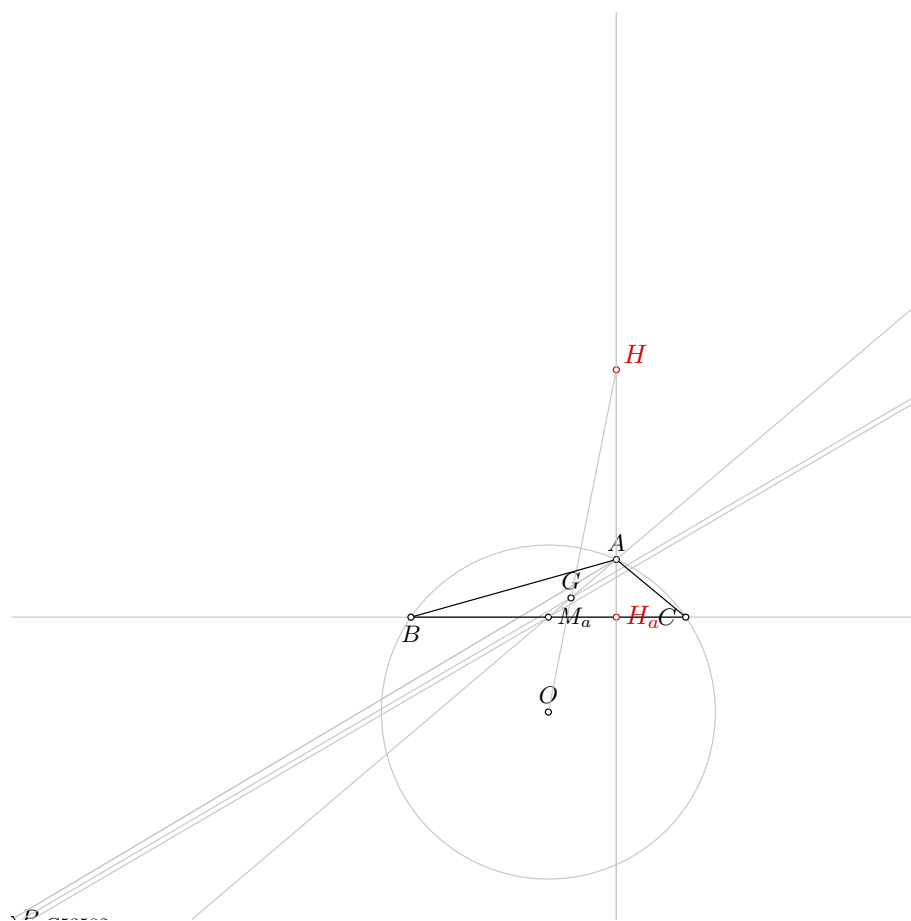


Figure 1: Illustration of the problem 0370

**Space Complexity:** The biggest polynomial obtained during prover execution contains 405 terms.

**Time Complexity:** Time spent by the prover is 1.566 seconds.

**NDG conditions** Points  $A$ ,  $P_{G62039}$  and  $M_a$  are not collinear

Line through points  $H_a$  and  $M_a$  is not perpendicular to line through points  $M_a$  and  $P_{G62039}$

Line through points  $B$  and  $H$  is not perpendicular to line through points  $H$  and  $C$

Points  $A$  and  $B$  are not identical

#### 4.1.3 Proving $M_a = \neg M_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_a = \neg H_a$

Proving failed

#### 4.2.2 Proving $H = \neg H$

Proving failed

#### 4.2.3 Proving $M_a = \neg M_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H = \neg H$

Proving failed

#### 4.3.3 Proving $M_a = \neg M_a$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.2 Proving $H = \neg H$

Proving failed

#### 4.4.3 Proving $M_a = \neg M_a$

Proving failed

# Problem 371

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 371: Given a point  $H_a$ , a point  $T_a$  and a point  $M_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $T_a$  are not the same;
2. Choose freely a point  $M_a$  on the line  $a$  (rule WOnline1) ;
3. Using the point  $M_a$  and the line  $a$ , construct a line  $m_a$  (rule W10b); ;
4. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
5. Choose freely a point  $A$  on the line  $h_a$  (rule WOnline2);
6. Using the point  $A$  and the point  $T_a$ , construct a line  $s_a$  (rule W02); % DET: points  $A$  and  $T_a$  are not the same;
7. Using the line  $m_a$  and the line  $s_a$ , construct a point  $N_a$  (rule W03); % NDG: lines  $m_a$  and  $s_a$  are not parallel % DET: lines  $m_a$  and  $s_a$  are not the same;
8. Using the point  $A$  and the point  $N_a$ , construct a line  $m(AN_a)$  (rule W14); % DET: points  $A$  and  $N_a$  are not the same;
9. Using the line  $m(AN_a)$  and the line  $m_a$ , construct a point  $O$  (rule W03); % NDG: lines  $m(AN_a)$  and  $m_a$  are not parallel % DET: lines  $m(AN_a)$  and  $m_a$  are not the same;
10. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
11. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same; lines  $m(AN_a)$  and  $m_a$  are not parallel; lines  $m_a$  and  $s_a$  are not parallel.

Determination conditions: lines  $m(AN_a)$  and  $m_a$  are not the same; points  $A$  and  $N_a$  are not the same; lines  $m_a$  and  $s_a$  are not the same; points  $A$  and  $T_a$  are not the same; points  $H_a$  and  $T_a$  are not the same.

Rules used: [W02,W03,W04,W06,W10b,W14,WOnline1,WOnline2]

Lemmas used: [D1,D11,D23,D26,D47,D5,D8,GD01,GL01,L11,L12,L25,L5]

Solving time: 18.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{a} 80 40
point T_{a} 70.86 40
point M_{a} 65 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_r H_{a}
cmark_rb T_{a}
cmark_r M_{a}
color 0 0 0
fontsize 8
```

```
% DET: points H_{a} and T_{a} are not the same
% Constructing a line a which passes through point H_{a} and point T_{a}
line a H_{a} T_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% Choosing randomly a point M_{a} on the line H_{a}T_{a}
online M_{a} H_{a} T_{a}
cmark_r M_{a}
color 200 200 200
drawline H_{a} T_{a}
color 0 0 0
```

```
% Constructing a line m_{a} which is perpendicular to line a and which passes through point M_{a}
perp m_{a} M_{a} a
```

```
color 200 200 200
drawline m_{a}
color 0 0 0
```



```

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% Generating random value V[_G56146]
random V[_G56146]

% Calculating value V[_G56167] using formula V[_G56146]*20
expression V[_G56167] { V[_G56146]*20 }

% Constructing a point A which is a point for which holds  $H_{a}A = V[_G56167]$  and angle  $M_{a}H_{a}A = 90$ 
turtle A M_{a} H_{a} 90 V[_G56167]
cmark_t A

% DET: points A and T_{a} are not the same
% Constructing a line s_{a} which passes through point A and point T_{a}
line s_{a} A T_{a}

color 200 200 200
drawline s_{a}
color 0 0 0

% NDG: lines m_{a} and s_{a} are not parallel% DET: lines m_{a} and s_{a} are not the same
% Constructing a point N_{a} which belongs to line m_{a} and line s_{a}
intersec N_{a} m_{a} s_{a}
cmark_b N_{a}

% DET: points A and N_{a} are not the same
% Constructing bisector m(AN_{a}) of the segment AN_{a}
med m(AN_{a}) A N_{a}

color 200 200 200
drawline m(AN_{a})
color 0 0 0

color 200 200 200
drawsegment A N_{a}
color 0 0 0

% NDG: lines m(AN_{a}) and m_{a} are not parallel% DET: lines m(AN_{a}) and m_{a} are not the same

```

Figure 1: Illustration of the problem 0371

```

% Constructing a point O which belongs to line m(AN_{a}) and line m_{a}
intersec O m(AN_{a}) m_{a}
cmark_t O

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% lines m(AN_{a}) and m_{a} are not parallel; lines m_{a} and s_{a} are not parallel
% Determination conditions: lines m(AN_{a}) and m_{a} are not the same; points A and N_{a} are not
% the same; lines m_{a} and s_{a} are not the same; points A and T_{a} are not the same; points H
% _{a} and T_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_a = \neg H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.125 seconds.

**NDG conditions** Point  $H_a$  is not on circle with center  $T_a$  and point from it  $C$

Line through points  $T_a$  and  $H_a$  is not parallel with line through points  $N_a$  and  $A$

Line through points  $T_a$  and  $A$  is not parallel with line through points  $B$  and  $C$

#### 4.1.2 Proving $T_a = \neg T_a$

Proving failed

#### 4.1.3 Proving $M_a = \neg M_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_a = \neg H_a$

Proving failed

#### 4.2.2 Proving $T_a = \neg T_a$

Proving failed

#### 4.2.3 Proving $M_a = \neg M_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_a = \neg H_a$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $T_a = \neg T_a$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $M_a = \neg M_a$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $H_a = \neg H_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $T_a = \neg T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.3 Proving $M_a = \neg M_a$

Proving failed

## Problem 372

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 372: Given a point  $M_a$ , a point  $H_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 373

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 373: Given a point  $M_a$ , a point  $H_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 374

*Generated automatically by ArgoTriCS  
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## 1 Problem

Problem 374: Given a point  $M_a$ , a point  $H_a$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $H_a$  are not the same;
2. Using the point  $M_a$  and the point  $I$ , construct a line  $IM_a$  (rule W02); % DET: points  $M_a$  and  $I$  are not the same;
3. Using the point  $I$  and the point  $M_a$ , construct a circle  $k_{over}(I, M_a)$  (rule W09); % NDG: points  $I$  and  $M_a$  are not the same;
4. Using the point  $M_a$  and the line  $a$ , construct a line  $m_a$  (rule W10b); ;
5. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
6. Using the point  $I$  and the line  $a$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $a$ ;
7. Using the circle  $k(I, P_a)$  and the circle  $k_{over}(I, M_a)$ , construct a point  $P_a$  and a point  $A_{fi}$  (rule W07); % NDG: circles  $k(I, P_a)$  and  $k_{over}(I, M_a)$  intersect % DET: circles  $k(I, P_a)$  and  $k_{over}(I, M_a)$  are not the same;
8. Using the point  $P_a$  and the point  $M_a$ , construct a point  $P'_a$  (rule W01); ;
9. Using the point  $P'_a$  and the line  $IM_a$ , construct a line  $AP'_a$  (rule W16); ;
10. Using the line  $AP'_a$  and the line  $h_a$ , construct a point  $A$  (rule W03); % NDG: lines  $AP'_a$  and  $h_a$  are not parallel % DET: lines  $AP'_a$  and  $h_a$  are not the same;
11. Using the point  $I$  and the point  $A$ , construct a line  $s_a$  (rule W02); % DET: points  $I$  and  $A$  are not the same;

12. Using the line  $m_a$  and the line  $s_a$ , construct a point  $N_a$  (rule W03); % NDG: lines  $m_a$  and  $s_a$  are not parallel % DET: lines  $m_a$  and  $s_a$  are not the same;
13. Using the point  $I$  and the point  $N_a$ , construct a circle  $k(N_a, C)$  (rule W06); % NDG: points  $I$  and  $N_a$  are not the same;
14. Using the circle  $k(N_a, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(N_a, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(N_a, C)$  intersect; points  $I$  and  $N_a$  are not the same; lines  $m_a$  and  $s_a$  are not parallel; lines  $AP'_a$  and  $h_a$  are not parallel; circles  $k(I, P_a)$  and  $k_{over}(I, M_a)$  intersect; point  $I$  is not incident to the line  $a$ ; points  $I$  and  $M_a$  are not the same.

Determination conditions: lines  $m_a$  and  $s_a$  are not the same; points  $I$  and  $A$  are not the same; lines  $AP'_a$  and  $h_a$  are not the same; circles  $k(I, P_a)$  and  $k_{over}(I, M_a)$  are not the same; points  $M_a$  and  $I$  are not the same; points  $M_a$  and  $H_a$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W07,W09,W10b,W11,W16]

Lemmas used: [D11,D2,D21,D27,D47,D5,D65,D8,GD01,GD02,GL03,GL09,L25,L31,L32,L5,L60,L62,L74]

Solving time: 5.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40
point H_{a} 80 40
point I 74.37 61.15
```

```
color 220 0 0
fontsize 9
```

```
cmark_r M_{a}
cmark_r H_{a}
cmark_b I
color 0 0 0
fontsize 8
```

```
% DET: points M_{a} and H_{a} are not the same
% Constructing a line a which passes through point M_{a} and point H_{a}
line a M_{a} H_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% DET: points M_{a} and I are not the same
% Constructing a line IM_{a} which passes through point M_{a} and point I
line IM_{a} M_{a} I
```

```
color 200 200 200
drawline IM_{a}
```

```
color 0 0 0
```

```
% NDG: points I and M_{a} are not the same  
% Constructing midpoint P_{\G35793} of the segment IM_{a}  
midpoint P_{\G35793} I M_{a}  
cmark_r P_{\G35793}
```

```
% Constructing a circle k_{over(I,M_{a})} whose center is at point P_{\G35793} and which passes  
through point I  
circle k_{over(I,M_{a})} P_{\G35793} I
```

```
color 200 200 200  
drawcircle k_{over(I,M_{a})}  
color 0 0 0
```

```
% Constructing a line m_{a} which is perpendicular to line a and which passes through point M_{a}  
perp m_{a} M_{a} a
```

```
color 200 200 200  
drawline m_{a}  
color 0 0 0
```

```
% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}  
perp h_{a} H_{a} a
```

```
color 200 200 200  
drawline h_{a}  
color 0 0 0
```

```
% NDG: point I is not incident to the line a  
% Constructing a point P_{\G36091} which is a foot of the point I on the line a  
foot P_{\G36091} I a  
cmark_r P_{\G36091}  
color 200 200 200  
drawline I P_{\G36091}  
color 0 0 0
```

```
% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\G  
36091}  
circle k(I,P_{a}) I P_{\G36091}
```

```
color 200 200 200  
drawcircle k(I,P_{a})  
color 0 0 0
```



```

% NDG: circles  $k(I, P_{\{a\}})$  and  $k_{\text{over}}(I, M_{\{a\}})$  intersect% DET: circles  $k(I, P_{\{a\}})$  and  $k_{\text{over}}(I, M_{\{a\}})$ 
%) are not the same
% Constructing points  $P_{\{a\}}$  and  $A_{\{fi\}}$  which are in intersection of  $k(I, P_{\{a\}})$  and  $k_{\text{over}}(I, M_{\{a\}})$ 
intersec2 P_{\{a\}} A_{\{fi\}} k(I, P_{\{a\}}) k_{\text{over}}(I, M_{\{a\}})
cmark_r P_{\{a\}}
cmark_r A_{\{fi\}}

% Constructing a point  $P'_{\{a\}}$  such that  $P_{\{a\}}P'_{\{a\}}/P_{\{a\}}M_{\{a\}}=2$ 
towards P'_{\{a\}} P_{\{a\}} M_{\{a\}} 2
cmark_r P'_{\{a\}}
color 200 200 200
drawsegment P_{\{a\}} P'_{\{a\}}
color 0 0 0

% Constructing a line  $AP'_{\{a\}}$  which contains the point  $P'_{\{a\}}$  and is parallel to the line  $IM_{\{a\}}$ 
parallel AP'_{\{a\}} P'_{\{a\}} IM_{\{a\}}

color 200 200 200
drawline AP'_{\{a\}}
color 0 0 0

% NDG: lines  $AP'_{\{a\}}$  and  $h_{\{a\}}$  are not parallel% DET: lines  $AP'_{\{a\}}$  and  $h_{\{a\}}$  are not the same
% Constructing a point  $A$  which belongs to line  $AP'_{\{a\}}$  and line  $h_{\{a\}}$ 
intersec A AP'_{\{a\}} h_{\{a\}}
cmark_t A

% DET: points  $I$  and  $A$  are not the same
% Constructing a line  $s_{\{a\}}$  which passes through point  $I$  and point  $A$ 
line s_{\{a\}} I A

color 200 200 200
drawline s_{\{a\}}
color 0 0 0

% NDG: lines  $m_{\{a\}}$  and  $s_{\{a\}}$  are not parallel% DET: lines  $m_{\{a\}}$  and  $s_{\{a\}}$  are not the same
% Constructing a point  $N_{\{a\}}$  which belongs to line  $m_{\{a\}}$  and line  $s_{\{a\}}$ 
intersec N_{\{a\}} m_{\{a\}} s_{\{a\}}
cmark_b N_{\{a\}}

% NDG: points  $I$  and  $N_{\{a\}}$  are not the same
% Constructing a circle  $k(N_{\{a\}}, C)$  whose center is at point  $N_{\{a\}}$  and which passes through point  $I$ 
circle k(N_{\{a\}}, C) N_{\{a\}} I

color 200 200 200
drawcircle k(N_{\{a\}}, C)

```

```
color 0 0 0
```

```
% NDG: line a and circle k(N_{a},C) intersect
% Constructing points C and B which are in intersection of k(N_{a},C) and a
intersec2 C B k(N_{a},C) a
cmark_l C
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(N_{a},C) intersect; points I and N_{a} are not the
same; lines m_{a} and s_{a} are not parallel; lines AP'_{a} and h_{a} are not parallel;
circles k(I,P_{a}) and k_{over}(I,M_{a}) intersect; point I is not incident to the line a; points
I and M_{a} are not the same
% Determination conditions: lines m_{a} and s_{a} are not the same; points I and A are not the same
; lines AP'_{a} and h_{a} are not the same; circles k(I,P_{a}) and k_{over}(I,M_{a}) are not the
same; points M_{a} and I are not the same; points M_{a} and H_{a} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

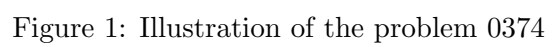
#### 4.1.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.1.2 Proving $H_a = \neg H_a$

**Construction steps:**

- Free point  $M_a$
- Free point  $H_a$
- Free point  $I$
- Line  $a$  through two points  $M_a$  and  $H_a$
- Line  $IM_a$  through two points  $M_a$  and  $I$
- Midpoint  $P_{G60452}$  of segment  $IM_a$
- Circle  $k_{over}(I, M_a)$  with center  $P_{G60452}$  and one point  $I$
- Line  $m_a$  through point  $M_a$  perpendicular to line  $a$
- Line  $h_a$  through point  $H_a$  perpendicular to line  $a$
- Line footPointPerpLine54 through point  $I$  perpendicular to line  $a$
- Intersection point  $P_{G60578}$  of point sets footPointPerpLine54 and  $a$



- Circle  $k(I, P_a)$  with center  $I$  and one point  $P_{G60578}$
- Intersection point  $P_a$  of point sets  $k(I, P_a)$  and  $k_{over}(I, M_a)$
- Intersection point  $A_{fi}$  of point sets  $k(I, P_a)$  and  $k_{over}(I, M_a)$
- Segment division point  $P'_a$  of segment  $P_a M_a$  with division coefficient -2.0
- Line  $AP'_a$  through point  $P'_a$  parallel with line  $IM_a$
- Intersection point  $A$  of point sets  $AP'_a$  and  $h_a$
- Line  $s_a$  through two points  $I$  and  $A$
- Intersection point  $N_a$  of point sets  $m_a$  and  $s_a$
- Circle  $k(N_a, C)$  with center  $N_a$  and one point  $I$
- Intersection point  $C$  of point sets  $k(N_a, C)$  and  $a$
- Intersection point  $B$  of point sets  $k(N_a, C)$  and  $a$
- Segment division point  $_M_a$  of segment  $BC$  with division coefficient 1.0
- Line  $_a$  through two points  $B$  and  $C$
- Line  $_h_a$  through point  $A$  perpendicular to line  $_a$
- Intersection point  $_H_a$  of point sets  $_a$  and  $_h_a$
- Line  $_c$  through two points  $A$  and  $B$
- Line  $_b$  through two points  $A$  and  $C$
- Angle bisector  $_s_a$  of angle  $BAC$
- Angle bisector  $_s_b$  of angle  $CBA$
- Intersection point  $_I$  of point sets  $_s_a$  and  $_s_b$

### Theorem statement:

- Points  $H_a$  and  $_H_a$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $M_a$  has been assigned following coordinates: (0, 0)

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a=_M_a$

Proving failed

#### 4.2.2 Proving $H_a=_H_a$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a=_M_a$

Proving failed

#### **4.3.2 Proving $H_a = \neg H_a$**

Proving failed

#### **4.3.3 Proving $I = \neg I$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $M_a = \neg M_a$**

Proving failed

#### **4.4.2 Proving $H_a = \neg H_a$**

Proving failed

#### **4.4.3 Proving $I = \neg I$**

Proving failed

# Problem 375

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 375: Given a point  $M_a$ , a point  $H_c$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_c$  and  $M_a$  are not the same;
2. Choose freely a point  $H_b$  on the circle  $k(M_a, B)$  (rule WOncircle);
3. Choose freely a point  $B$  on the circle  $k(M_a, B)$  (rule WOncircle);
4. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
5. Using the point  $B$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $H_c$  are not the same;
6. Using the point  $H_b$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $C$  are not the same;
7. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; points  $H_c$  and  $M_a$  are not the same.

Determination conditions: lines  $c$  and  $b$  are not the same; points  $H_b$  and  $C$  are not the same; points  $B$  and  $H_c$  are not the same.

Rules used: [W01,W02,W03,W06,WOncircle1]

Lemmas used: [D21,D6,D7,GD01,GD02,GL03,L39]

Solving time: 2.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{a} 65 40
point H_{c} 68.91 84.83
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_rt H_{c}
cmark_l H_{b}
color 0 0 0
fontsize 8

% NDG: points H_{c} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{c}
circle k(M_{a},B) M_{a} H_{c}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% Choosing randomly a point H_{b} on the circle with center M_{a} through point H_{c}
oncircle H_{b} M_{a} H_{c}
cmark_l H_{b}
color 200 200 200
drawcircle M_{a} H_{c}
color 0 0 0

% Choosing randomly a point B on the circle with center M_{a} through point H_{b}
oncircle B M_{a} H_{b}
cmark_b B
color 200 200 200
drawcircle M_{a} H_{b}
color 0 0 0

% Constructing a point C such that BC/BM_{a}=2
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% DET: points B and H_{c} are not the same
```

```

% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

color 200 200 200
drawline c
color 0 0 0

% DET: points H_{b} and C are not the same
% Constructing a line b which passes through point H_{b} and point C
line b H_{b} C

color 200 200 200
drawline b
color 0 0 0

% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and b are not parallel; points H_{c} and M_{a} are not the
same
% Determination conditions: lines c and b are not the same; points H_{b} and C are not the same;
points B and H_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = \_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.122 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c = \_H_c$

Proving failed



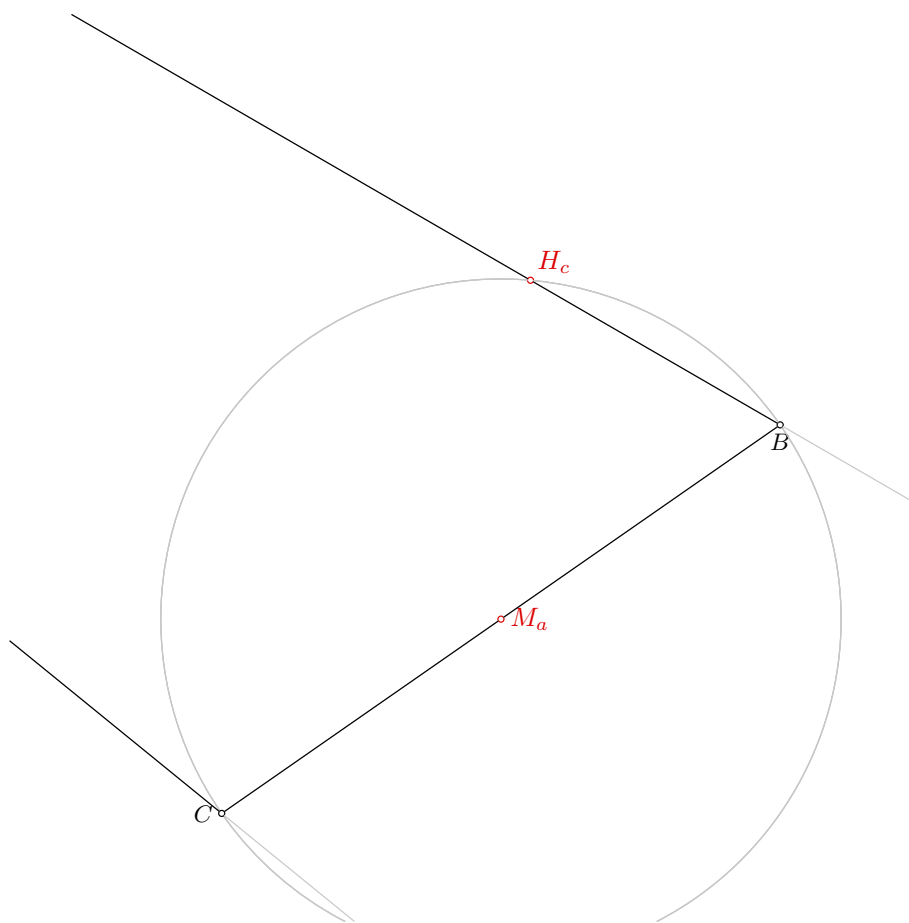


Figure 1: Illustration of the problem 0375

#### 4.1.3 Proving $H_b = \neg H_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{BH_bC} \neq S_{H_cH_bC}$  i.e., lines  $BH_c$  and  $H_bC$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{\neg H_c}^0} \neq S_{BCF_{\neg H_c}^0}$  i.e., lines  $AB$  and  $CF_{\neg H_c}^0$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg H_b}^1} \neq S_{CBF_{\neg H_b}^1}$  i.e., lines  $AC$  and  $BF_{\neg H_b}^1$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.2.3 Proving $H_b = \neg H_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 17 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.3.3 Proving $H_b = \neg H_b$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.4.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.4.3 Proving $H_b = \neg H_b$

Proving failed

# Problem 376

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 376: Given a point  $M_a$ , a point  $H_b$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $H_b$  and  $H$  are not the same;
2. Using the point  $H_b$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_b$  and  $M_a$  are not the same;
3. Using the circle  $k(M_a, B)$ , the line  $h_b$ , the point  $M_a$  and the point  $H_b$ , construct a point  $B$  (rule W05); % NDG: line  $h_b$  and circle  $k(M_a, B)$  intersect % DET: points  $H_b$  and  $B$  must be different;
4. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
5. Using the point  $H_b$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $C$  are not the same;
6. Using the point  $H$  and the point  $C$ , construct a line  $h_c$  (rule W02); % DET: points  $H$  and  $C$  are not the same;
7. Using the circle  $k(M_a, B)$ , the line  $h_c$ , the point  $M_a$  and the point  $C$ , construct a point  $H_c$  (rule W05); % NDG: line  $h_c$  and circle  $k(M_a, B)$  intersect % DET: points  $C$  and  $H_c$  must be different;
8. Using the point  $B$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $H_c$  are not the same;
9. Using the line  $b$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $c$  are not parallel % DET: lines  $b$  and  $c$  are not the same.

Non-degenerate conditions: lines  $b$  and  $c$  are not parallel; line  $h_c$  and circle  $k(M_a, B)$  intersect; line  $h_b$  and circle  $k(M_a, B)$  intersect; points  $H_b$  and  $M_a$  are not the same.

Determination conditions: lines  $b$  and  $c$  are not the same; points  $B$  and  $H_c$  are not the same; points  $C$  and  $H_c$  must be different; points  $H$  and  $C$  are not the same; points  $H_b$  and  $C$  are not the same; points  $H_b$  and  $B$  must be different; points  $H_b$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D21,D3,D6,D7,D9,GD01,GD02,GL03,L3,L37,L38,L39]

Solving time: 1.6 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point M_{a} 65 40
point H_{b} 89.36 77.83
point H 80 72.73

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_l H_{b}
cmark_rt H
color 0 0 0
fontsize 8

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points H_{b} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{b}
circle k(M_{a},B) M_{a} H_{b}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line h_{b} and circle k(M_{a},B) intersect% DET: points H_{b} and B must be different
% Constructing a point P_{\_G43774} which is a foot of the point M_{a} on the line h_{b}
foot P_{\_G43774} M_{a} h_{b}
cmark_r P_{\_G43774}
color 200 200 200
drawline M_{a} P_{\_G43774}
color 0 0 0
```

```

% Constructing a point B which is an image of the point H_{b} in the symmetry to point/line P_{\_G
43774}
sim B P_{\_G43774} H_{b}
cmark_b B

% Constructing a point C such that BC/BM_{a}=2
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% DET: points H_{b} and C are not the same
% Constructing a line b which passes through point H_{b} and point C
line b H_{b} C

color 200 200 200
drawline b
color 0 0 0

% DET: points H and C are not the same
% Constructing a line h_{c} which passes through point H and point C
line h_{c} H C

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: line h_{c} and circle k(M_{a},B) intersect% DET: points C and H_{c} must be different
% Constructing a point P_{\_G44137} which is a foot of the point M_{a} on the line h_{c}
foot P_{\_G44137} M_{a} h_{c}
cmark_r P_{\_G44137}
color 200 200 200
drawline M_{a} P_{\_G44137}
color 0 0 0

% Constructing a point H_{c} which is an image of the point C in the symmetry to point/line P_{\_G
44137}
sim H_{c} P_{\_G44137} C
cmark_rt H_{c}

% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

color 200 200 200
drawline c

```

```
color 0 0 0
```

```
% NDG: lines b and c are not parallel% DET: lines b and c are not the same
% Constructing a point A which belongs to line b and line c
intersec A b c
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines b and c are not parallel; line h_{c} and circle k(M_{a},B)
    intersect; line h_{b} and circle k(M_{a},B) intersect; points H_{b} and M_{a} are not the same
% Determination conditions: lines b and c are not the same; points B and H_{c} are not the same;
    points C and H_{c} must be different; points H and C are not the same; points H_{b} and C are
    not the same; points H_{b} and B must be different; points H_{b} and H are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = \_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.168 seconds.

**NDG conditions** Points  $H_b$  and  $H$  are not identical

Points  $H_b$  and  $H$  are not identical

#### 4.1.2 Proving $H_b = \_H_b$

Proving failed

#### 4.1.3 Proving $H = \_H$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = \_M_a$

NDG conditions are:

$S_{H_b B H_c} \neq S_{C B H_c}$  i.e., lines  $H_b C$  and  $B H_c$  are not parallel (construction based assumption)

$S_{B A C} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{A B F_{-H_b}^0} \neq S_{C B F_{-H_b}^0}$  i.e., lines  $A C$  and  $B F_{-H_b}^0$  are not parallel (construction based assumption)

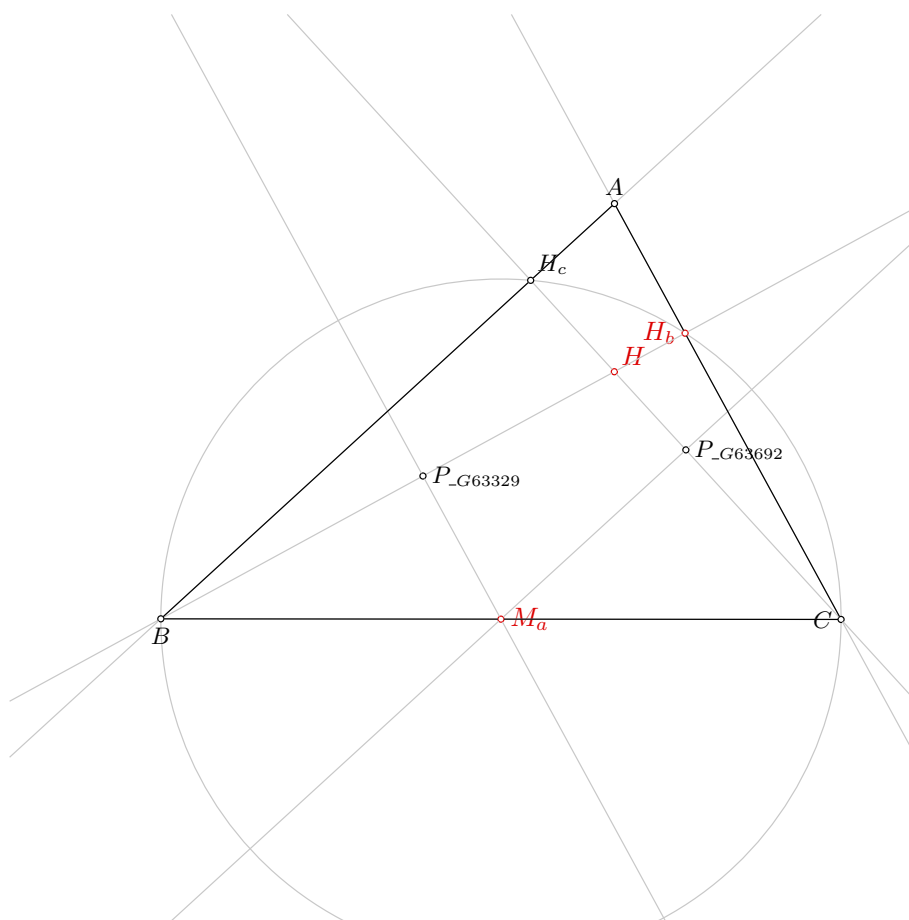


Figure 1: Illustration of the problem 0376

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)  
 $S_{ABF_{-h_b}^0} \neq S_{F_{-h_a}^1 BF_{-h_b}^0}$  i.e., lines  $AF_{-h_a}^1$  and  $BF_{-h_b}^0$  are not parallel (construction based assumption)  
 Total number of proof steps: 41  
 Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.2.3 Proving $H = \neg H$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.3.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.3.3 Proving $H = \neg H$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.4.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.4.3 Proving $H = \neg H$

Proving failed



# Problem 377

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 377: Given a point  $M_a$ , a point  $H_b$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $T_a$  are not the same;
2. Using the point  $H_b$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_b$  and  $M_a$  are not the same;
3. Using the circle  $k(M_a, B)$  and the line  $a$ , construct a point  $B$  and a point  $C$  (rule W04); % NDG: line  $a$  and circle  $k(M_a, B)$  intersect;
4. Using the point  $H_b$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $C$  are not the same;
5. Using the point  $B$ , the point  $C$ , the point  $T_a$  and the line  $a$ , construct a point  $T'_a$  (rule W19); % NDG: points  $B$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $\text{midpoint}([B, T_a])$  are not the same;
6. Using the point  $T_a$  and the point  $T'_a$ , construct a circle  $k_{over}(T_a, T'_a)$  (rule W09); % NDG: points  $T_a$  and  $T'_a$  are not the same;
7. Using the circle  $k_{over}(T_a, T'_a)$  and the line  $b$ , construct a point  $A_{wc}$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k_{over}(T_a, T'_a)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k_{over}(T_a, T'_a)$  intersect; points  $T_a$  and  $T'_a$  are not the same; points  $B$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $\text{midpoint}([B, T_a])$  are not the same; line  $a$  and circle  $k(M_a, B)$  intersect; points  $H_b$  and  $M_a$  are not the same.

Determination conditions: points  $H_b$  and  $C$  are not the same; points  $M_a$  and  $T_a$  are not the same.

Rules used: [W02,W04,W06,W09,W19]

Lemmas used: [D21,D23,D54,D6,GD01,GD02,GL09,L37,L38,L71,L77]

Solving time: 2717.8 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{a} 65 40
point H_{b} 89.36 77.83
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_l H_{b}
cmark_rb T_{a}
color 0 0 0
fontsize 8

% DET: points M_{a} and T_{a} are not the same
% Constructing a line a which passes through point M_{a} and point T_{a}
line a M_{a} T_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points H_{b} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{b}
circle k(M_{a},B) M_{a} H_{b}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line a and circle k(M_{a},B) intersect
% Constructing points B and C which are in intersection of k(M_{a},B) and a
intersec2 B C k(M_{a},B) a
cmark_b B
cmark_l C

% DET: points H_{b} and C are not the same
% Constructing a line b which passes through point H_{b} and point C
line b H_{b} C

color 200 200 200
drawline b
```

```
color 0 0 0
```

```
% NDG: points B and C are not the same; points C and T_{a} are not the same; points C and midpoint
      ([B,T_{a}]) are not the same
% Constructing a point P_{\_G42979} which is an image of the point C in a rotation around the point
      T_{a} for the angle 90
rotate P_{\_G42979} T_{a} 90 C
cmark_r P_{\_G42979}
color 200 200 200
drawarc_p T_{a} C 90
color 0 0 0
```

```
% Constructing a line L_{\_G42982} which passes through point T_{a} and point P_{\_G42979}
line L_{\_G42982} T_{a} P_{\_G42979}
```

```
color 200 200 200
drawline L_{\_G42982}
color 0 0 0
```

```
% Constructing midpoint P_{\_G42985} of the segment CP_{\_G42979}
midpoint P_{\_G42985} C P_{\_G42979}
cmark_r P_{\_G42985}
```

```
% Constructing a line L_{\_G42988} which passes through point B and point P_{\_G42985}
line L_{\_G42988} B P_{\_G42985}
```

```
color 200 200 200
drawline L_{\_G42988}
color 0 0 0
```

```
% Constructing a line L_{\_G42991} which passes through point C and point P_{\_G42985}
line L_{\_G42991} C P_{\_G42985}
```

```
color 200 200 200
drawline L_{\_G42991}
color 0 0 0
```

```
% Constructing a point P_{\_G42994} which belongs to line L_{\_G42982} and line L_{\_G42988}
intersec P_{\_G42994} L_{\_G42982} L_{\_G42988}
cmark_r P_{\_G42994}
```

```
% Constructing a point P_{\_G42997} which belongs to line L_{\_G42982} and line L_{\_G42991}
intersec P_{\_G42997} L_{\_G42982} L_{\_G42991}
cmark_r P_{\_G42997}
```

```
% Constructing a line L_{\_G43000} which passes through point B and point P_{\_G42997}
line L_{\_G43000} B P_{\_G42997}
```

```
color 200 200 200
drawline L_{\_G43000}
color 0 0 0
```

```

% Constructing a line  $L_{G43003}$  which passes through point C and point  $P_{G42994}$ 
line  $L_{G43003}$  C  $P_{G42994}$ 

color 200 200 200
drawline  $L_{G43003}$ 
color 0 0 0

% Constructing a point  $P_{G43006}$  which belongs to line  $L_{G43000}$  and line  $L_{G43003}$ 
intersec  $P_{G43006}$   $L_{G43000}$   $L_{G43003}$ 
cmark_r  $P_{G43006}$ 

% Constructing a line  $L_{G43009}$  which passes through point  $P_{G42985}$  and point  $P_{G43006}$ 
line  $L_{G43009}$   $P_{G42985}$   $P_{G43006}$ 

color 200 200 200
drawline  $L_{G43009}$ 
color 0 0 0

% Constructing a point  $T'_a$  which belongs to line  $L_{G43009}$  and line a
intersec  $T'_a$   $L_{G43009}$  a
cmark_r  $T'_a$ 

% NDG: points  $T_a$  and  $T'_a$  are not the same
% Constructing midpoint  $P_{G43840}$  of the segment  $T_aT'_a$ 
midpoint  $P_{G43840}$   $T_a$   $T'_a$ 
cmark_r  $P_{G43840}$ 

% Constructing a circle  $k_{over}(T_a, T'_a)$  whose center is at point  $P_{G43840}$  and which passes
    through point  $T_a$ 
circle  $k_{over}(T_a, T'_a)$   $P_{G43840}$   $T_a$ 

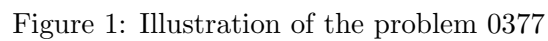
color 200 200 200
drawcircle  $k_{over}(T_a, T'_a)$ 
color 0 0 0

% NDG: line b and circle  $k_{over}(T_a, T'_a)$  intersect
% Constructing points  $A_{wc}$  and A which are in intersection of  $k_{over}(T_a, T'_a)$  and b
intersec2  $A_{wc}$  A  $k_{over}(T_a, T'_a)$  b
cmark_r  $A_{wc}$ 
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle  $k_{over}(T_a, T'_a)$  intersect; points  $T_a$  and  $T'_a$ 
    are not the same; points B and C are not the same; points C and  $T_a$  are not the same;

```



### 3.3 Illustration

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

## 4.1 OGP - Wu method

### 4.1.1 Proving $M_a = -M_a$

1813

#### **4.1.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.1.3 Proving $T_a = \neg T_a$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $M_a = \neg M_a$**

Proving failed

#### **4.2.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.2.3 Proving $T_a = \neg T_a$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $M_a = \neg M_a$**

Proving failed

#### **4.3.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.3.3 Proving $T_a = \neg T_a$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $M_a = \neg M_a$**

Proving failed

#### **4.4.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.4.3 Proving $T_a = \neg T_a$**

Proving failed

# Problem 378

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 378: Given a point  $M_a$ , a point  $H_b$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $T_b$  are not the same;
2. Using the point  $H_b$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_b$  and  $M_a$  are not the same;
3. Using the circle  $k(M_a, B)$ , the line  $b$ , the point  $M_a$  and the point  $H_b$ , construct a point  $C$  (rule W05); % NDG: line  $b$  and circle  $k(M_a, B)$  intersect % DET: points  $H_b$  and  $C$  must be different;
4. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
5. Using the point  $M_a$  and the point  $C$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $C$  are not the same;
6. Using the point  $T_b$  and the line  $a$ , construct a circle  $k(T_b, foot[T_b, c])$  (rule W11); % NDG: point  $T_b$  is not incident to the line  $a$ ;
7. Using the circle  $k(T_b, foot[T_b, c])$ , the point  $B$ , the point  $T_b$  and the line  $a$ , construct a line  $c$  (rule W13); % NDG: point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ;
8. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ; point  $T_b$  is not incident to the line  $a$ ; line  $b$  and circle  $k(M_a, B)$  intersect; points  $H_b$  and  $M_a$  are not the same.

Determination conditions: lines  $c$  and  $b$  are not the same; points  $M_a$  and  $C$  are not the same; points  $H_b$  and  $C$  must be different; points  $H_b$  and  $T_b$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W11,W13]

Lemmas used: [D21,D24,D6,GD01,GD02,GL03,GL04,GL09,GL10,GL11,L37,L38,L9]

Solving time: 1.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{a} 65 40
point H_{b} 89.36 77.83
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_l H_{b}
cmark_t T_{b}
color 0 0 0
fontsize 8

% DET: points H_{b} and T_{b} are not the same
% Constructing a line b which passes through point H_{b} and point T_{b}
line b H_{b} T_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points H_{b} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{b}
circle k(M_{a},B) M_{a} H_{b}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line b and circle k(M_{a},B) intersect% DET: points H_{b} and C must be different
% Constructing a point P_{\G36977} which is a foot of the point M_{a} on the line b
foot P_{\G36977} M_{a} b
cmark_r P_{\G36977}
color 200 200 200
drawline M_{a} P_{\G36977}
color 0 0 0

% Constructing a point C which is an image of the point H_{b} in the symmetry to point/line P_{\G36977}
```



```

sim C P_{\_G36977} H_{b}
cmark_l C

% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% DET: points M_{a} and C are not the same
% Constructing a line a which passes through point M_{a} and point C
line a M_{a} C

color 200 200 200
drawline a
color 0 0 0

% NDG: point T_{b} is not incident to the line a
% Constructing a point P_{\_G37261} which is a foot of the point T_{b} on the line a
foot P_{\_G37261} T_{b} a
cmark_r P_{\_G37261}
color 200 200 200
drawline T_{b} P_{\_G37261}
color 0 0 0

% Constructing a circle k(T_{b},foot[T_{b},c]) whose center is at point T_{b} and which passes
    through point P_{\_G37261}
circle k(T_{b},foot[T_{b},c]) T_{b} P_{\_G37261}

color 200 200 200
drawcircle k(T_{b},foot[T_{b},c])
color 0 0 0

% NDG: point B is outside the circle k(T_{b},foot[T_{b},c])
% Constructing a point P_{\_G37652} which is a foot of the point T_{b} on the line a
foot P_{\_G37652} T_{b} a
cmark_r P_{\_G37652}
color 200 200 200
drawline T_{b} P_{\_G37652}
color 0 0 0

% Constructing a line L_{\_G37655} which passes through point T_{b} and point B
line L_{\_G37655} T_{b} B

color 200 200 200
drawline L_{\_G37655}
color 0 0 0

```

```

% Constructing a point P_{\_G37658} which is an image of the point P_{\_G37652} in the symmetry to
    point/line L_{\_G37655}
sim P_{\_G37658} L_{\_G37655} P_{\_G37652}
cmark_r P_{\_G37658}

% Constructing a line c which passes through point B and point P_{\_G37658}
line c B P_{\_G37658}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and b are not parallel; point B is outside the circle k(T_{b},
    foot[T_{b},c]); point T_{b} is not incident to the line a; line b and circle k(M_{a},B)
    intersect; points H_{b} and M_{a} are not the same
% Determination conditions: lines c and b are not the same; points M_{a} and C are not the same;
    points H_{b} and C must be different; points H_{b} and T_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.164 seconds.

**NDG conditions** Points  $H_b$ ,  $T_b$  and  $M_a$  are not collinear

Points  $H_b$ ,  $T_b$  and  $M_a$  are not collinear

#### 4.1.2 Proving $H_b = H_b$

Proving failed

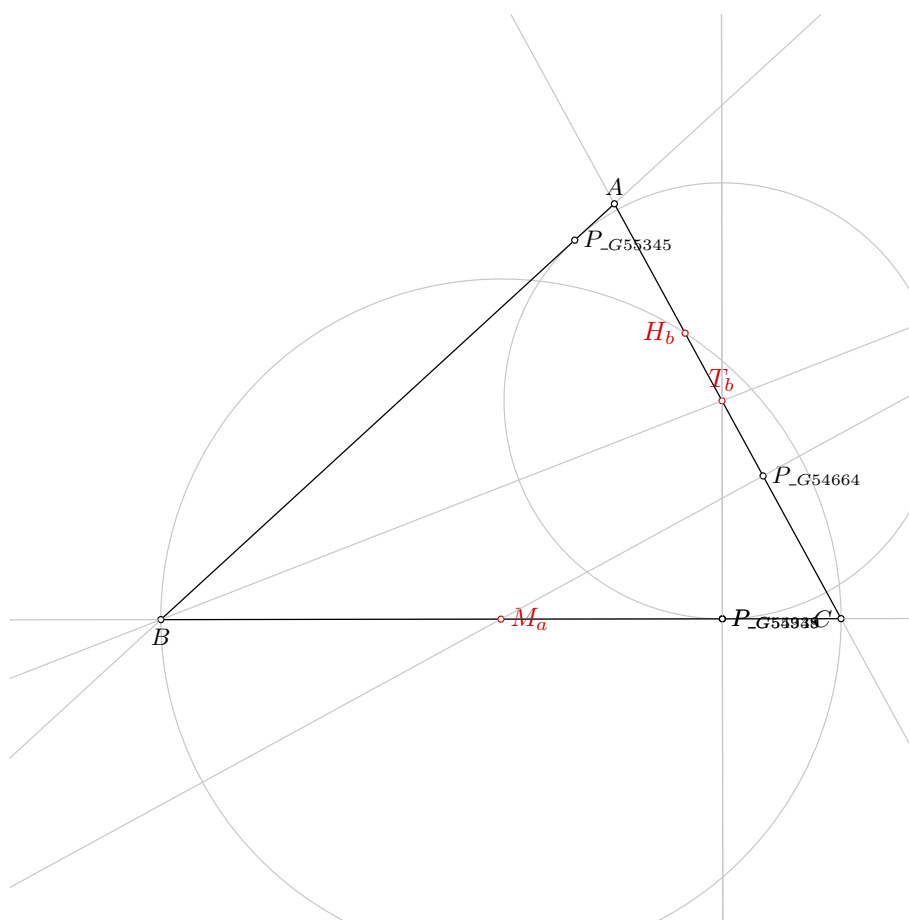


Figure 1: Illustration of the problem 0378

#### **4.1.3 Proving $T_b = \neg T_b$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $M_a = \neg M_a$**

Proving failed

#### **4.2.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.2.3 Proving $T_b = \neg T_b$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $M_a = \neg M_a$**

Proving failed

#### **4.3.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.3.3 Proving $T_b = \neg T_b$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $M_a = \neg M_a$**

Proving failed

#### **4.4.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.4.3 Proving $T_b = \neg T_b$**

Proving failed

## Problem 379

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 379: Given a point  $M_a$ , a point  $H_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 380

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 380: Given a point  $M_a$ , a point  $H_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 381

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 381: Given a point  $M_a$ , a point  $H_c$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $H_c$  and  $H$  are not the same;
2. Using the point  $H_c$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_c$  and  $M_a$  are not the same;
3. Using the circle  $k(M_a, B)$ , the line  $h_c$ , the point  $M_a$  and the point  $H_c$ , construct a point  $C$  (rule W05); % NDG: line  $h_c$  and circle  $k(M_a, B)$  intersect % DET: points  $H_c$  and  $C$  must be different;
4. Using the point  $C$  and the point  $M_a$ , construct a point  $B$  (rule W01); ;
5. Using the point  $H_c$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $B$  are not the same;
6. Using the point  $H$  and the point  $B$ , construct a line  $h_b$  (rule W02); % DET: points  $H$  and  $B$  are not the same;
7. Using the circle  $k(M_a, B)$ , the line  $h_b$ , the point  $M_a$  and the point  $B$ , construct a point  $H_b$  (rule W05); % NDG: line  $h_b$  and circle  $k(M_a, B)$  intersect % DET: points  $B$  and  $H_b$  must be different;
8. Using the point  $C$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $H_b$  are not the same;
9. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; line  $h_b$  and circle  $k(M_a, B)$  intersect; line  $h_c$  and circle  $k(M_a, B)$  intersect; points  $H_c$  and  $M_a$  are not the same.

Determination conditions: lines  $c$  and  $b$  are not the same; points  $C$  and  $H_b$  are not the same; points  $B$  and  $H_b$  must be different; points  $H$  and  $B$  are not the same; points  $H_c$  and  $B$  are not the same; points  $H_c$  and  $C$  must be different; points  $H_c$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D21,D3,D6,D7,D9,GD01,GD02,GL03,GL04,L3,L37,L38,L39]

Solving time: 1.6 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point M_{a} 65 40
point H_{c} 68.91 84.83
point H 80 72.73

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_rt H_{c}
cmark_rt H
color 0 0 0
fontsize 8

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points H_{c} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H
_{c}
circle k(M_{a},B) M_{a} H_{c}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line h_{c} and circle k(M_{a},B) intersect% DET: points H_{c} and C must be different
% Constructing a point P_{\_G41296} which is a foot of the point M_{a} on the line h_{c}
foot P_{\_G41296} M_{a} h_{c}
cmark_r P_{\_G41296}
color 200 200 200
drawline M_{a} P_{\_G41296}
color 0 0 0
```



```

% Constructing a point C which is an image of the point H_{c} in the symmetry to point/line P_{\_G
41296}
sim C P_{\_G41296} H_{c}
cmark_l C

% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% DET: points H_{c} and B are not the same
% Constructing a line c which passes through point H_{c} and point B
line c H_{c} B

color 200 200 200
drawline c
color 0 0 0

% DET: points H and B are not the same
% Constructing a line h_{b} which passes through point H and point B
line h_{b} H B

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{b} and circle k(M_{a},B) intersect% DET: points B and H_{b} must be different
% Constructing a point P_{\_G41659} which is a foot of the point M_{a} on the line h_{b}
foot P_{\_G41659} M_{a} h_{b}
cmark_r P_{\_G41659}
color 200 200 200
drawline M_{a} P_{\_G41659}
color 0 0 0

% Constructing a point H_{b} which is an image of the point B in the symmetry to point/line P_{\_G
41659}
sim H_{b} P_{\_G41659} B
cmark_l H_{b}

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b

```

```
color 0 0 0
```

```
% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines c and b are not parallel; line h_{b} and circle k(M_{a},B)
    intersect; line h_{c} and circle k(M_{a},B) intersect; points H_{c} and M_{a} are not the same
% Determination conditions: lines c and b are not the same; points C and H_{b} are not the same;
    points B and H_{b} must be different; points H and B are not the same; points H_{c} and B are
    not the same; points H_{c} and C must be different; points H_{c} and H are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = \_M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.166 seconds.

**NDG conditions** Points  $H_c$ ,  $H$  and  $M_a$  are not collinear

Points  $H_c$ ,  $H$  and  $M_a$  are not collinear

#### 4.1.2 Proving $H_c = \_H_c$

Proving failed

#### 4.1.3 Proving $H = \_H$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a = \_M_a$

NDG conditions are:

$S_{H_cCH_b} \neq S_{BCH_b}$  i.e., lines  $H_cB$  and  $CH_b$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{-h_c}^0} \neq S_{BCF_{-h_c}^0}$  i.e., lines  $AB$  and  $CF_{-h_c}^0$  are not parallel (construction based assumption)

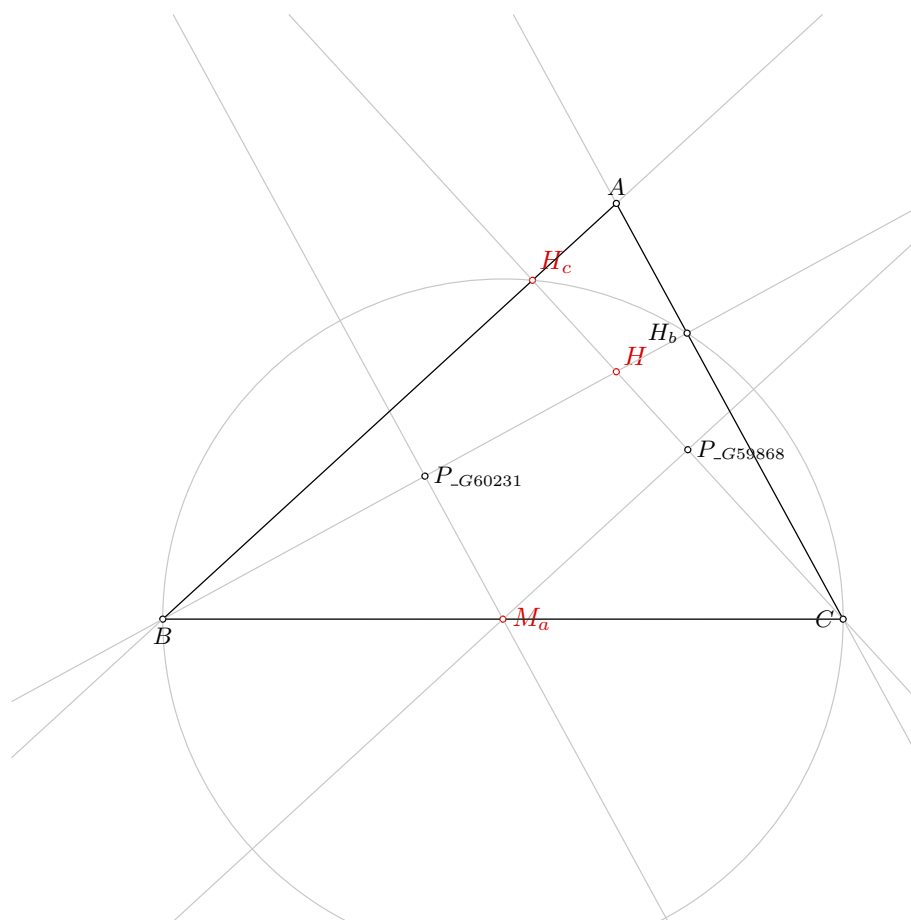


Figure 1: Illustration of the problem 0381

$S_{ABC} \neq 0$  i.e., points  $A, B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B, A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^2_{\perp h_b}} \neq S_{F^1_{\perp h_a}BF^2_{\perp h_b}}$  i.e., lines  $AF^1_{\perp h_a}$  and  $BF^2_{\perp h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c = \perp H_c$

Proving failed

#### 4.2.3 Proving $H = \perp H$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = \perp M_a$

Proving failed

#### 4.3.2 Proving $H_c = \perp H_c$

Proving failed

#### 4.3.3 Proving $H = \perp H$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_a = \perp M_a$

Proving failed

#### 4.4.2 Proving $H_c = \perp H_c$

Proving failed

#### 4.4.3 Proving $H = \perp H$

Proving failed

# Problem 382

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 382: Given a point  $M_a$ , a point  $H_c$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $T_a$  are not the same;
2. Using the point  $H_c$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_c$  and  $M_a$  are not the same;
3. Using the circle  $k(M_a, B)$  and the line  $a$ , construct a point  $B$  and a point  $C$  (rule W04); % NDG: line  $a$  and circle  $k(M_a, B)$  intersect;
4. Using the point  $H_c$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $B$  are not the same;
5. Using the point  $B$ , the point  $C$ , the point  $T_a$  and the line  $a$ , construct a point  $T'_a$  (rule W19); % NDG: points  $B$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $\text{midpoint}([B, T_a])$  are not the same;
6. Using the point  $T_a$  and the point  $T'_a$ , construct a circle  $k_{over}(T_a, T'_a)$  (rule W09); % NDG: points  $T_a$  and  $T'_a$  are not the same;
7. Using the circle  $k_{over}(T_a, T'_a)$  and the line  $c$ , construct a point  $A_{wb}$  and a point  $A$  (rule W04); % NDG: line  $c$  and circle  $k_{over}(T_a, T'_a)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k_{over}(T_a, T'_a)$  intersect; points  $T_a$  and  $T'_a$  are not the same; points  $B$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; points  $C$  and  $\text{midpoint}([B, T_a])$  are not the same; line  $a$  and circle  $k(M_a, B)$  intersect; points  $H_c$  and  $M_a$  are not the same.

Determination conditions: points  $H_c$  and  $B$  are not the same; points  $M_a$  and  $T_a$  are not the same.

Rules used: [W02,W04,W06,W09,W19]

Lemmas used: [D21,D23,D53,D7,GD01,GD02,GL09,L37,L39,L71,L77]

Solving time: 2572.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40
point H_{c} 68.91 84.83
point T_{a} 70.86 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_r M_{a}
cmark_rt H_{c}
cmark_rb T_{a}
color 0 0 0
fontsize 8
```

```
% DET: points M_{a} and T_{a} are not the same
% Constructing a line a which passes through point M_{a} and point T_{a}
line a M_{a} T_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% NDG: points H_{c} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{c}
circle k(M_{a},B) M_{a} H_{c}
```

```
color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0
```

```
% NDG: line a and circle k(M_{a},B) intersect
% Constructing points B and C which are in intersection of k(M_{a},B) and a
intersec2 B C k(M_{a},B) a
cmark_b B
cmark_l C
```

```
% DET: points H_{c} and B are not the same
% Constructing a line c which passes through point H_{c} and point B
line c H_{c} B
```

```
color 200 200 200
drawline c
```

```
color 0 0 0
```

```
% NDG: points B and C are not the same; points C and T_{a} are not the same; points C and midpoint
      ([B,T_{a}]) are not the same
% Constructing a point P_{\_G43222} which is an image of the point C in a rotation around the point
      T_{a} for the angle 90
rotate P_{\_G43222} T_{a} 90 C
cmark_r P_{\_G43222}
color 200 200 200
drawarc_p T_{a} C 90
color 0 0 0
```

```
% Constructing a line L_{\_G43225} which passes through point T_{a} and point P_{\_G43222}
line L_{\_G43225} T_{a} P_{\_G43222}

color 200 200 200
drawline L_{\_G43225}
color 0 0 0
```

```
% Constructing midpoint P_{\_G43228} of the segment CP_{\_G43222}
midpoint P_{\_G43228} C P_{\_G43222}
cmark_r P_{\_G43228}
```

```
% Constructing a line L_{\_G43231} which passes through point B and point P_{\_G43228}
line L_{\_G43231} B P_{\_G43228}

color 200 200 200
drawline L_{\_G43231}
color 0 0 0
```

```
% Constructing a line L_{\_G43234} which passes through point C and point P_{\_G43228}
line L_{\_G43234} C P_{\_G43228}

color 200 200 200
drawline L_{\_G43234}
color 0 0 0
```

```
% Constructing a point P_{\_G43237} which belongs to line L_{\_G43225} and line L_{\_G43231}
intersec P_{\_G43237} L_{\_G43225} L_{\_G43231}
cmark_r P_{\_G43237}
```

```
% Constructing a point P_{\_G43240} which belongs to line L_{\_G43225} and line L_{\_G43234}
intersec P_{\_G43240} L_{\_G43225} L_{\_G43234}
cmark_r P_{\_G43240}
```

```
% Constructing a line L_{\_G43243} which passes through point B and point P_{\_G43240}
line L_{\_G43243} B P_{\_G43240}

color 200 200 200
drawline L_{\_G43243}
color 0 0 0
```

```

% Constructing a line  $L_{\{ \_G43246 \}}$  which passes through point C and point  $P_{\{ \_G43237 \}}$ 
line  $L_{\{ \_G43246 \}}$  C  $P_{\{ \_G43237 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G43246 \}}$ 
color 0 0 0

% Constructing a point  $P_{\{ \_G43249 \}}$  which belongs to line  $L_{\{ \_G43243 \}}$  and line  $L_{\{ \_G43246 \}}$ 
intersec  $P_{\{ \_G43249 \}}$   $L_{\{ \_G43243 \}}$   $L_{\{ \_G43246 \}}$ 
cmark_r  $P_{\{ \_G43249 \}}$ 

% Constructing a line  $L_{\{ \_G43252 \}}$  which passes through point  $P_{\{ \_G43228 \}}$  and point  $P_{\{ \_G43249 \}}$ 
line  $L_{\{ \_G43252 \}}$   $P_{\{ \_G43228 \}}$   $P_{\{ \_G43249 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G43252 \}}$ 
color 0 0 0

% Constructing a point  $T'_{\{ a \}}$  which belongs to line  $L_{\{ \_G43252 \}}$  and line a
intersec  $T'_{\{ a \}}$   $L_{\{ \_G43252 \}}$  a
cmark_r  $T'_{\{ a \}}$ 

% NDG: points  $T_{\{ a \}}$  and  $T'_{\{ a \}}$  are not the same
% Constructing midpoint  $P_{\{ \_G44083 \}}$  of the segment  $T_{\{ a \}}T'_{\{ a \}}$ 
midpoint  $P_{\{ \_G44083 \}}$   $T_{\{ a \}}$   $T'_{\{ a \}}$ 
cmark_r  $P_{\{ \_G44083 \}}$ 

% Constructing a circle  $k_{\text{over}}(T_{\{ a \}}, T'_{\{ a \}})$  whose center is at point  $P_{\{ \_G44083 \}}$  and which passes
    through point  $T_{\{ a \}}$ 
circle  $k_{\text{over}}(T_{\{ a \}}, T'_{\{ a \}})$   $P_{\{ \_G44083 \}}$   $T_{\{ a \}}$ 

color 200 200 200
drawcircle  $k_{\text{over}}(T_{\{ a \}}, T'_{\{ a \}})$ 
color 0 0 0

% NDG: line c and circle  $k_{\text{over}}(T_{\{ a \}}, T'_{\{ a \}})$  intersect
% Constructing points  $A_{\{ wb \}}$  and A which are in intersection of  $k_{\text{over}}(T_{\{ a \}}, T'_{\{ a \}})$  and c
intersec2  $A_{\{ wb \}}$   $k_{\text{over}}(T_{\{ a \}}, T'_{\{ a \}})$  c
cmark_r  $A_{\{ wb \}}$ 
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle  $k_{\text{over}}(T_{\{ a \}}, T'_{\{ a \}})$  intersect; points  $T_{\{ a \}}$  and  $T'_{\{ a \}}$ 
    are not the same; points B and C are not the same; points C and  $T_{\{ a \}}$  are not the same;

```



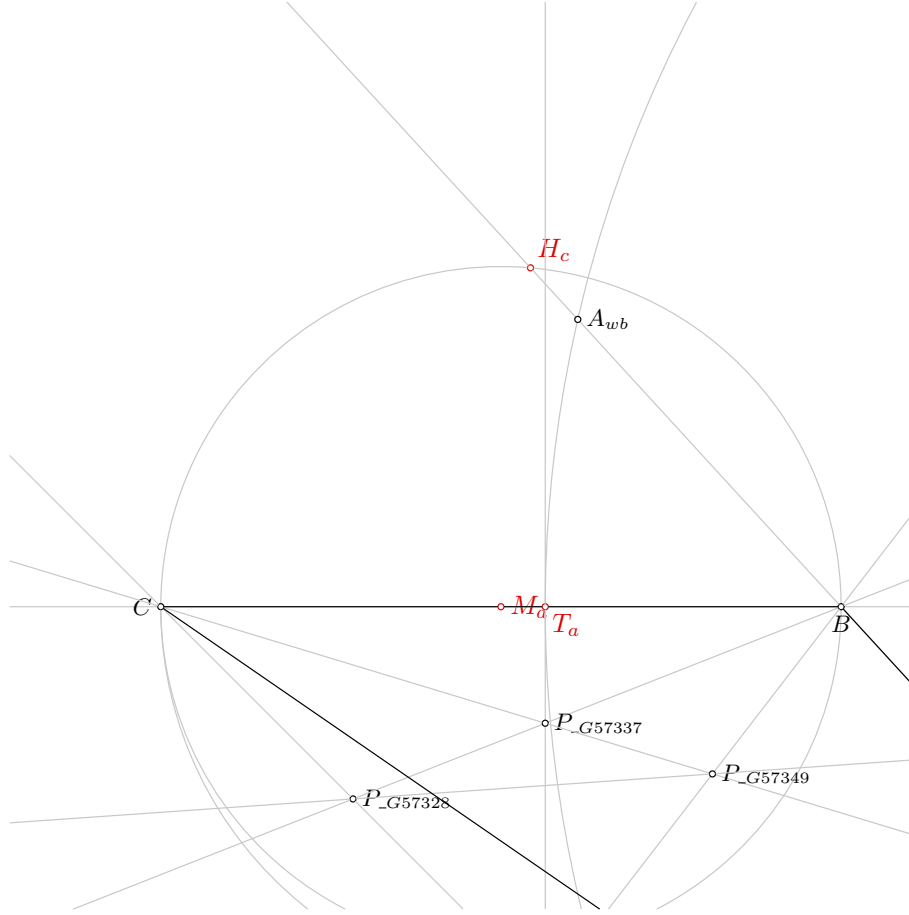


Figure 1: Illustration of the problem 0382

*points C and midpoint([B,T\_{a}]) are not the same; line a and circle k(M\_{a},B) intersect;  
points H\_{c} and M\_{a} are not the same  
% Determination conditions: points H\_{c} and B are not the same; points M\_{a} and T\_{a} are not the  
same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = M_a$

Proving failed

#### **4.1.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.1.3 Proving $T_a = \neg T_a$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $M_a = \neg M_a$**

Proving failed

#### **4.2.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.2.3 Proving $T_a = \neg T_a$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $M_a = \neg M_a$**

Proving failed

#### **4.3.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.3.3 Proving $T_a = \neg T_a$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $M_a = \neg M_a$**

Proving failed

#### **4.4.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.4.3 Proving $T_a = \neg T_a$**

Proving failed

## Problem 383

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 383: Given a point  $M_a$ , a point  $H_c$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 384

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 384: Given a point  $M_a$ , a point  $H_c$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $T_c$  are not the same;
2. Using the point  $H_c$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_c$  and  $M_a$  are not the same;
3. Using the circle  $k(M_a, B)$ , the line  $c$ , the point  $M_a$  and the point  $H_c$ , construct a point  $B$  (rule W05); % NDG: line  $c$  and circle  $k(M_a, B)$  intersect % DET: points  $H_c$  and  $B$  must be different;
4. Using the point  $B$  and the point  $M_a$ , construct a point  $C$  (rule W01); ;
5. Using the point  $M_a$  and the point  $B$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $B$  are not the same;
6. Using the point  $T_c$  and the line  $a$ , construct a circle  $k(T_c, foot[T_c, b])$  (rule W11); % NDG: point  $T_c$  is not incident to the line  $a$ ;
7. Using the circle  $k(T_c, foot[T_c, b])$ , the point  $C$ , the point  $T_c$  and the line  $a$ , construct a line  $b$  (rule W13); % NDG: point  $C$  is outside the circle  $k(T_c, foot[T_c, b])$ ;
8. Using the line  $b$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $c$  are not parallel % DET: lines  $b$  and  $c$  are not the same.

Non-degenerate conditions: lines  $b$  and  $c$  are not parallel; point  $C$  is outside the circle  $k(T_c, foot[T_c, b])$ ; point  $T_c$  is not incident to the line  $a$ ; line  $c$  and circle  $k(M_a, B)$  intersect; points  $H_c$  and  $M_a$  are not the same.

Determination conditions: lines  $b$  and  $c$  are not the same; points  $M_a$  and  $B$  are not the same; points  $H_c$  and  $B$  must be different; points  $H_c$  and  $T_c$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W11,W13]

Lemmas used: [D21,D25,D7,GD01,GD02,GL03,GL09,GL10,GL11,L10,L39]

Solving time: 1.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{a} 65 40
point H_{c} 68.91 84.83
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_r M_{a}
cmark_rt H_{c}
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points H_{c} and T_{c} are not the same
% Constructing a line c which passes through point H_{c} and point T_{c}
line c H_{c} T_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points H_{c} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{c}
circle k(M_{a},B) M_{a} H_{c}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line c and circle k(M_{a},B) intersect% DET: points H_{c} and B must be different
% Constructing a point P_{\_G43736} which is a foot of the point M_{a} on the line c
foot P_{\_G43736} M_{a} c
cmark_r P_{\_G43736}
color 200 200 200
drawline M_{a} P_{\_G43736}
color 0 0 0

% Constructing a point B which is an image of the point H_{c} in the symmetry to point/line P_{\_G43736}
```

```

sim B P_{\_G43736} H_{c}
cmark_b B

% Constructing a point C such that  $BC/BM_{\{a\}}=2$ 
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% DET: points  $M_{\{a\}}$  and  $B$  are not the same
% Constructing a line  $a$  which passes through point  $M_{\{a\}}$  and point  $B$ 
line a M_{a} B

color 200 200 200
drawline a
color 0 0 0

% NDG: point  $T_{\{c\}}$  is not incident to the line  $a$ 
% Constructing a point  $P_{\{\_G44020\}}$  which is a foot of the point  $T_{\{c\}}$  on the line  $a$ 
foot P_{\_G44020} T_{c} a
cmark_r P_{\_G44020}
color 200 200 200
drawline T_{c} P_{\_G44020}
color 0 0 0

% Constructing a circle  $k(T_{\{c\}}, foot[T_{\{c\}}, b])$  whose center is at point  $T_{\{c\}}$  and which passes
    through point  $P_{\{\_G44020\}}$ 
circle k(T_{c}, foot[T_{c}, b]) T_{c} P_{\_G44020}

color 200 200 200
drawcircle k(T_{c}, foot[T_{c}, b])
color 0 0 0

% NDG: point  $C$  is outside the circle  $k(T_{\{c\}}, foot[T_{\{c\}}, b])$ 
% Constructing a point  $P_{\{\_G44411\}}$  which is a foot of the point  $T_{\{c\}}$  on the line  $a$ 
foot P_{\_G44411} T_{c} a
cmark_r P_{\_G44411}
color 200 200 200
drawline T_{c} P_{\_G44411}
color 0 0 0

% Constructing a line  $L_{\{\_G44414\}}$  which passes through point  $T_{\{c\}}$  and point  $C$ 
line L_{\_G44414} T_{c} C

color 200 200 200
drawline L_{\_G44414}
color 0 0 0

```

```

% Constructing a point  $P_{\{G44417\}}$  which is an image of the point  $P_{\{G44411\}}$  in the symmetry to
    point/line  $L_{\{G44414\}}$ 
sim  $P_{\{G44417\}}$   $L_{\{G44414\}}$   $P_{\{G44411\}}$ 
cmark_r  $P_{\{G44417\}}$ 

% Constructing a line  $b$  which passes through point  $C$  and point  $P_{\{G44417\}}$ 
line b C  $P_{\{G44417\}}$ 

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $b$  and  $c$  are not parallel% DET: lines  $b$  and  $c$  are not the same
% Constructing a point  $A$  which belongs to line  $b$  and line  $c$ 
intersec A b c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $b$  and  $c$  are not parallel; point  $C$  is outside the circle  $k(T_{\{c\}},$ 
     $foot[T_{\{c\}},b])$ ; point  $T_{\{c\}}$  is not incident to the line  $a$ ; line  $c$  and circle  $k(M_{\{a\}},B)$ 
    intersect; points  $H_{\{c\}}$  and  $M_{\{a\}}$  are not the same
% Determination conditions: lines  $b$  and  $c$  are not the same; points  $M_{\{a\}}$  and  $B$  are not the same;
    points  $H_{\{c\}}$  and  $B$  must be different; points  $H_{\{c\}}$  and  $T_{\{c\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_a = M_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

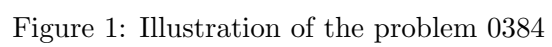
**Time Complexity:** Time spent by the prover is 0.168 seconds.

**NDG conditions** Points  $H_c$ ,  $T_c$  and  $M_a$  are not collinear

Points  $H_c$ ,  $T_c$  and  $M_a$  are not collinear

#### 4.1.2 Proving $H_c = H_c$

Proving failed





#### **4.1.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $M_a = \neg M_a$**

Proving failed

#### **4.2.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.2.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $M_a = \neg M_a$**

Proving failed

#### **4.3.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.3.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $M_a = \neg M_a$**

Proving failed

#### **4.4.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.4.3 Proving $T_c = \neg T_c$**

Proving failed

# Problem 385

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 385: Given a point  $M_a$ , a point  $H_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 386

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 386: Given a point  $M_a$ , a point  $H$  and a point  $T_a$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 387

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 387: Given a point  $M_a$ , a point  $H$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 388

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 388: Given a point  $M_a$ , a point  $H$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 389

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 389: Given a point  $M_a$ , a point  $H$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 390

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 390: Given a point  $M_a$ , a point  $T_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 391

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 391: Given a point  $M_a$ , a point  $T_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.



# Problem 392

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 392: Given a point  $M_a$ , a point  $T_a$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $M_a$  and  $T_a$  are not the same;
2. Using the point  $T_a$  and the point  $I$ , construct a line  $s_a$  (rule W02); % DET: points  $T_a$  and  $I$  are not the same;
3. Using the point  $M_a$  and the line  $a$ , construct a line  $m_a$  (rule W10b); ;
4. Using the line  $m_a$  and the line  $s_a$ , construct a point  $N_a$  (rule W03); % NDG: lines  $m_a$  and  $s_a$  are not parallel % DET: lines  $m_a$  and  $s_a$  are not the same;
5. Using the point  $I$  and the point  $N_a$ , construct a circle  $k(N_a, C)$  (rule W06); % NDG: points  $I$  and  $N_a$  are not the same;
6. Using the circle  $k(N_a, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(N_a, C)$  intersect;
7. Using the point  $I$  and the line  $a$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $a$ ;
8. Using the circle  $k(I, P_a)$ , the point  $C$ , the point  $I$  and the line  $a$ , construct a line  $b$  (rule W13); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
9. Using the line  $b$  and the line  $s_a$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $s_a$  are not parallel % DET: lines  $b$  and  $s_a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $s_a$  are not parallel; point  $C$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $a$ ; line  $a$  and circle  $k(N_a, C)$  intersect; points  $I$  and  $N_a$  are not the same; lines  $m_a$  and  $s_a$  are not parallel.

Determination conditions: lines  $b$  and  $s_a$  are not the same; lines  $m_a$  and  $s_a$  are not the same; points  $T_a$  and  $I$  are not the same; points  $M_a$  and  $T_a$  are not the same.

Rules used: [W02,W03,W04,W06,W10b,W11,W13]

Lemmas used: [D11,D2,D21,D23,D27,D47,GD01,GD02,GL09,L25,L31,L32,L5,L60,L61]

Solving time: 13.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40
point T_{a} 70.86 40
point I 74.37 61.15
```

```
color 220 0 0
fontsize 9
```

```
cmark_r M_{a}
cmark_rb T_{a}
cmark_b I
color 0 0 0
fontsize 8
```

```
% DET: points M_{a} and T_{a} are not the same
% Constructing a line a which passes through point M_{a} and point T_{a}
line a M_{a} T_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% DET: points T_{a} and I are not the same
% Constructing a line s_{a} which passes through point T_{a} and point I
line s_{a} T_{a} I
```

```
color 200 200 200
drawline s_{a}
color 0 0 0
```

```
% Constructing a line m_{a} which is perpendicular to line a and which passes through point M_{a}
perp m_{a} M_{a} a
```

```
color 200 200 200
drawline m_{a}
color 0 0 0
```

```

% NDG: lines  $m_{\{a\}}$  and  $s_{\{a\}}$  are not parallel% DET: lines  $m_{\{a\}}$  and  $s_{\{a\}}$  are not the same
% Constructing a point  $N_{\{a\}}$  which belongs to line  $m_{\{a\}}$  and line  $s_{\{a\}}$ 
intersec  $N_{\{a\}}$   $m_{\{a\}}$   $s_{\{a\}}$ 
cmark_b  $N_{\{a\}}$ 

% NDG: points  $I$  and  $N_{\{a\}}$  are not the same
% Constructing a circle  $k(N_{\{a\}},C)$  whose center is at point  $N_{\{a\}}$  and which passes through point  $I$ 
circle  $k(N_{\{a\}},C)$   $N_{\{a\}}$   $I$ 

color 200 200 200
drawcircle  $k(N_{\{a\}},C)$ 
color 0 0 0

% NDG: line  $a$  and circle  $k(N_{\{a\}},C)$  intersect
% Constructing points  $C$  and  $B$  which are in intersection of  $k(N_{\{a\}},C)$  and  $a$ 
intersec2  $C$   $B$   $k(N_{\{a\}},C)$   $a$ 
cmark_l  $C$ 
cmark_b  $B$ 

% NDG: point  $I$  is not incident to the line  $a$ 
% Constructing a point  $P_{\{\backslash\_G53130\}}$  which is a foot of the point  $I$  on the line  $a$ 
foot  $P_{\{\backslash\_G53130\}}$   $I$   $a$ 
cmark_r  $P_{\{\backslash\_G53130\}}$ 
color 200 200 200
drawline  $I$   $P_{\{\backslash\_G53130\}}$ 
color 0 0 0

% Constructing a circle  $k(I,P_{\{a\}})$  whose center is at point  $I$  and which passes through point  $P_{\{\backslash\_G53130\}}$ 
circle  $k(I,P_{\{a\}})$   $I$   $P_{\{\backslash\_G53130\}}$ 

color 200 200 200
drawcircle  $k(I,P_{\{a\}})$ 
color 0 0 0

% NDG: point  $C$  is outside the circle  $k(I,P_{\{a\}})$ 
% Constructing a point  $P_{\{\backslash\_G53337\}}$  which is a foot of the point  $I$  on the line  $a$ 
foot  $P_{\{\backslash\_G53337\}}$   $I$   $a$ 
cmark_r  $P_{\{\backslash\_G53337\}}$ 
color 200 200 200
drawline  $I$   $P_{\{\backslash\_G53337\}}$ 
color 0 0 0

% Constructing a line  $L_{\{\backslash\_G53340\}}$  which passes through point  $I$  and point  $C$ 
line  $L_{\{\backslash\_G53340\}}$   $I$   $C$ 

color 200 200 200
drawline  $L_{\{\backslash\_G53340\}}$ 

```

```

color 0 0 0

% Constructing a point  $P_{G53343}$  which is an image of the point  $P_{G53337}$  in the symmetry to
point/line  $L_{G53340}$ 
sim  $P_{G53343}$   $L_{G53340}$   $P_{G53337}$ 
cmark_r  $P_{G53343}$ 

% Constructing a line  $b$  which passes through point  $C$  and point  $P_{G53343}$ 
line b C  $P_{G53343}$ 

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $b$  and  $s_a$  are not parallel% DET: lines  $b$  and  $s_a$  are not the same
% Constructing a point  $A$  which belongs to line  $b$  and line  $s_a$ 
intersec A b  $s_a$ 
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $b$  and  $s_a$  are not parallel; point  $C$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $a$ ; line  $a$  and circle  $k(N_a, C)$  intersect; points  $I$  and  $N_a$  are not the same; lines  $m_a$  and  $s_a$  are not parallel
% Determination conditions: lines  $b$  and  $s_a$  are not the same; lines  $m_a$  and  $s_a$  are not the same; points  $T_a$  and  $I$  are not the same; points  $M_a$  and  $T_a$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

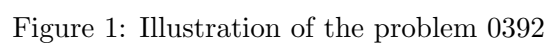
#### 4.1.1 Proving $M_a = M_a$

Proving failed

#### 4.1.2 Proving $T_a = T_a$

Construction steps:

- Free point  $M_a$
- Free point  $T_a$
- Free point  $I$



- Line  $a$  through two points  $M_a$  and  $T_a$
- Line  $s_a$  through two points  $T_a$  and  $I$
- Line  $m_a$  through point  $M_a$  perpendicular to line  $a$
- Intersection point  $N_a$  of point sets  $m_a$  and  $s_a$
- Circle  $k(N_a, C)$  with center  $N_a$  and one point  $I$
- Intersection point  $C$  of point sets  $k(N_a, C)$  and  $a$
- Intersection point  $B$  of point sets  $k(N_a, C)$  and  $a$
- Line footPointPerpLine111 through point  $I$  perpendicular to line  $a$
- Intersection point  $P_{G48679}$  of point sets footPointPerpLine111 and  $a$
- Circle  $k(I, P_a)$  with center  $I$  and one point  $P_{G48679}$
- Line footPointPerpLine629 through point  $I$  perpendicular to line  $a$
- Intersection point  $P_{G48782}$  of point sets footPointPerpLine629 and  $a$
- Line  $L_{G48785}$  through two points  $I$  and  $C$
- Line reflexivePointPerpLine835 through point  $P_{G48782}$  perpendicular to line  $L_{G48785}$
- Intersection point reflexivePointFootPoint706 of point sets reflexivePointPerpLine835 and  $L_{G48785}$
- Cental symmetric point  $P_{G48788}$  of point  $P_{G48782}$  with respect to center of symmetry reflexivePointFootPoint706
- Line  $b$  through two points  $C$  and  $P_{G48788}$
- Intersection point  $A$  of point sets  $b$  and  $s_a$
- Segment division point  $_M_a$  of segment  $BC$  with division coefficient 1.0
- Line  $_b$  through two points  $A$  and  $C$
- Line  $_c$  through two points  $A$  and  $B$
- Angle bisector  $_s_a$  of angle  $BAC$
- Line  $_a$  through two points  $B$  and  $C$
- Intersection point  $_T_a$  of point sets  $_s_a$  and  $_a$
- Angle bisector  $_s_b$  of angle  $CBA$
- Intersection point  $_I$  of point sets  $_s_a$  and  $_s_b$

### Theorem statement:

- Points  $T_a$  and  $_T_a$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $M_a$  has been assigned following coordinates: (0, 0)

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_a=_M_a$

Proving failed

#### 4.2.2 Proving $T_a=_T_a$

Proving failed

#### 4.2.3 Proving $I = \neg I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_a = \neg M_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 22 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

#### 4.3.2 Proving $T_a = \neg T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 28 terms.

**Time Complexity:** Time spent by the prover is 0.030 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I = \neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 2054 terms.

**Time Complexity:** Time spent by the prover is 1.010 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_a = \neg M_a$

Proving failed

#### 4.4.2 Proving $T_a = \neg T_a$

Proving failed

#### 4.4.3 Proving $I = \neg I$

Proving failed

## Problem 393

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 393: Given a point  $M_a$ , a point  $T_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.



## Problem 394

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 394: Given a point  $M_a$ , a point  $T_b$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 395

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 395: Given a point  $M_a$ , a point  $T_c$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 396

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 396: Given a point  $M_b$ , a point  $M_c$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $G$ , construct a point  $B$  (rule W01); ;
2. Using the point  $M_c$  and the point  $G$ , construct a point  $C$  (rule W01); ;
3. Using the point  $M_b$  and the point  $C$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: .

Rules used: [W01]

Lemmas used: [D22,GL03,GL04,L56,L57]

Solving time: 0.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{b} 95 67.5  
point M_{c} 50 67.5  
point G 70 58.33
```

```
color 220 0 0  
fontsize 9
```

```
cmark_lt M_{b}  
cmark_lt M_{c}  
cmark_t G  
color 0 0 0  
fontsize 8
```

```

% Constructing a point B such that  $M_{\{b\}B}/M_{\{b\}G}=3$ 
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

```

```

% Constructing a point C such that  $M_{\{c\}C}/M_{\{c\}G}=3$ 
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

```

```

% Constructing a point A such that  $M_{\{b\}A}/M_{\{b\}C}=-1$ 
towards A M_{b} C -1
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions:
% Determination conditions:

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = -M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.151 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c = -M_c$

**Status:** Theorem has been proved.

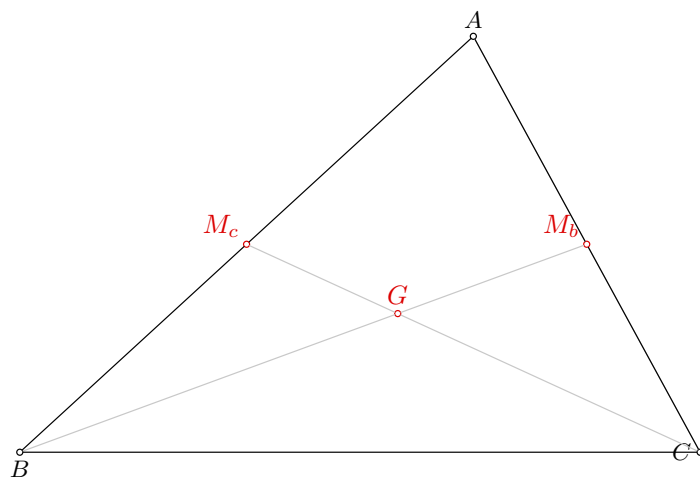


Figure 1: Illustration of the problem 0396

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.222 seconds.

**NDG conditions** There are no NDG conditions for this theorem

### 4.1.3 Proving $G=_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 1377 terms.

**Time Complexity:** Time spent by the prover is 2.688 seconds.

**NDG conditions** Line through points  $A$  and  $_M a$  is not parallel with line through points  $B$  and  $_M b$   
Line through points  $M_b$  and  $M_c$  is not parallel with line through points  $A$  and  $_M a$

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_b=_M b$

NDG conditions are:

$S_{AB\_M_b} \neq S_{_M a B\_M_b}$  i.e., lines  $A\_M a$  and  $B\_M b$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_c=_M c$

NDG conditions are:

$S_{AB\_M_b} \neq S_{_M a B\_M_b}$  i.e., lines  $A\_M a$  and  $B\_M b$  are not parallel (construction based assumption)

Total number of proof steps: 235

Time spent by the prover: 0.010 seconds

### 4.2.3 Proving $G=_G$

NDG conditions are:

$S_{AB\_M_b} \neq S_{_M a B\_M_b}$  i.e., lines  $A\_M a$  and  $B\_M b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (cancellation assumption)

Total number of proof steps: 791

Time spent by the prover: 0.220 seconds

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_b=_M b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 16 terms.

**Time Complexity:** Time spent by the prover is 0.030 seconds. There are no ndg conditions.

### 4.3.2 Proving $M_c=_M c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 84 terms.

**Time Complexity:** Time spent by the prover is 0.080 seconds. There are no ndg conditions.

### 4.3.3 Proving $G=_G$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3566 terms.

**Time Complexity:** Time spent by the prover is 29.010 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_b=_M M_b$

Proving failed

### 4.4.2 Proving $M_c=_M M_c$

Proving failed

### 4.4.3 Proving $G=_G$

Proving failed

# Problem 397

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 397: Given a point  $M_b$ , a point  $M_c$  and a point  $H_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_a$  and  $M_b$  are not the same;
2. Using the point  $H_a$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_a$  and  $M_c$  are not the same;
3. Using the circle  $k(M_b, C)$ , the circle  $k(M_c, A)$ , the point  $H_a$ , the point  $M_b$  and the point  $M_c$ , construct a point  $A$  (rule W08); % NDG: circles  $k(M_b, C)$  and  $k(M_c, A)$  intersect % DET: circles  $k(M_b, C)$  and  $k(M_c, A)$  are not the same; points  $H_a$  and  $A$  must be different;
4. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
5. Using the point  $M_c$  and the point  $A$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: circles  $k(M_b, C)$  and  $k(M_c, A)$  intersect; points  $H_a$  and  $M_c$  are not the same; points  $H_a$  and  $M_b$  are not the same.

Determination conditions: circles  $k(M_b, C)$  and  $k(M_c, A)$  are not the same; points  $H_a$  and  $A$  must be different.

Rules used: [W01,W06,W08]

Lemmas used: [D20,D22,GD02,GL03,GL04,L42,L43,L44]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language



```

dim 120 120

point M_{b} 95 67.5
point M_{c} 50 67.5
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_lt M_{c}
cmark_r H_{a}
color 0 0 0
fontsize 8

% NDG: points H_{a} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{a}
circle k(M_{b},C) M_{b} H_{a}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: points H_{a} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{a}
circle k(M_{c},A) M_{c} H_{a}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: circles k(M_{b},C) and k(M_{c},A) intersect% DET: circles k(M_{b},C) and k(M_{c},A) are not
the same; points H_{a} and A must be different
% Constructing a line L_{\_G39958} which passes through point M_{b} and point M_{c}
line L_{\_G39958} M_{b} M_{c}

color 200 200 200
drawline L_{\_G39958}
color 0 0 0

% Constructing a point A which is an image of the point H_{a} in the symmetry to point/line L_{\_G
39958}
sim A L_{\_G39958} H_{a}
cmark_t A

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2

```

```

cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% Constructing a point B such that  $M_{\{c\}B}/M_{\{c\}A}=-1$ 
towards B M_{c} A -1
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: circles  $k(M_{\{b\}},C)$  and  $k(M_{\{c\}},A)$  intersect; points  $H_{\{a\}}$  and  $M_{\{c\}}$ 
% are not the same; points  $H_{\{a\}}$  and  $M_{\{b\}}$  are not the same
% Determination conditions: circles  $k(M_{\{b\}},C)$  and  $k(M_{\{c\}},A)$  are not the same; points  $H_{\{a\}}$  and  $A$ 
% must be different

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = \_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.097 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $M_c = \_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.091 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.3 Proving $H_a = \_H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 134 terms.

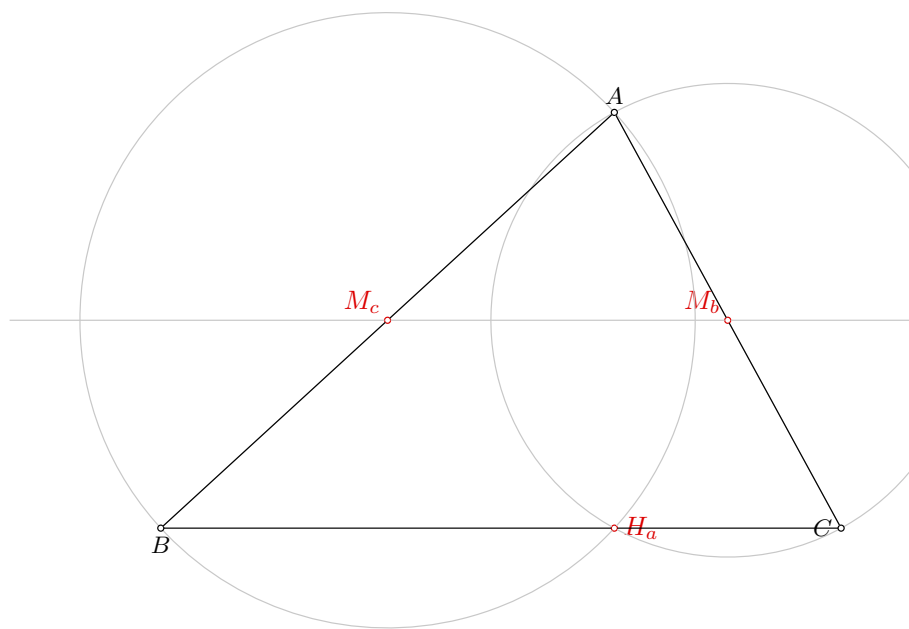


Figure 1: Illustration of the problem 0397

**Time Complexity:** Time spent by the prover is 0.69 seconds.

**NDG conditions** Points  $B$  and  $C$  are not identical

Line through points  $M_b$  and  $M_c$  is not parallel with line through points  $B$  and  $C$

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_b = M_b$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{ha}^0} \neq S_{CAF_{ha}^0}$  i.e., lines  $BC$  and  $AF_{ha}^0$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_c = M_c$

NDG conditions are:

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{ha}^0} \neq S_{CAF_{ha}^0}$  i.e., lines  $BC$  and  $AF_{ha}^0$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $H_a = H_a$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_b = M_b$

Proving failed

### 4.3.2 Proving $M_c = M_c$

Proving failed

### 4.3.3 Proving $H_a = H_a$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_b = M_b$

Proving failed

### 4.4.2 Proving $M_c = M_c$

Proving failed

### 4.4.3 Proving $H_a = H_a$

Proving failed

# Problem 398

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 398: Given a point  $M_b$ , a point  $M_c$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $H_b$  are not the same;
2. Using the point  $H_b$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_b$  and  $M_c$  are not the same;
3. Using the circle  $k(M_c, A)$ , the line  $b$ , the point  $M_c$  and the point  $H_b$ , construct a point  $A$  (rule W05); % NDG: line  $b$  and circle  $k(M_c, A)$  intersect % DET: points  $H_b$  and  $A$  must be different;
4. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
5. Using the point  $M_c$  and the point  $A$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: line  $b$  and circle  $k(M_c, A)$  intersect; points  $H_b$  and  $M_c$  are not the same.

Determination conditions: points  $H_b$  and  $A$  must be different; points  $M_b$  and  $H_b$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D20,D22,D6,GD01,GD02,GL03,GL04,GL09,L41]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```

dim 120 120

point M_{b} 95 67.5
point M_{c} 50 67.5
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_lt M_{c}
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points M_{b} and H_{b} are not the same
% Constructing a line b which passes through point M_{b} and point H_{b}
line b M_{b} H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points H_{b} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{b}
circle k(M_{c},A) M_{c} H_{b}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line b and circle k(M_{c},A) intersect% DET: points H_{b} and A must be different
% Constructing a point P_{\_G54130} which is a foot of the point M_{c} on the line b
foot P_{\_G54130} M_{c} b
cmark_r P_{\_G54130}
color 200 200 200
drawline M_{c} P_{\_G54130}
color 0 0 0

% Constructing a point A which is an image of the point H_{b} in the symmetry to point/line P_{\_G54130}
sim A P_{\_G54130} H_{b}
cmark_t A

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C

```

```
color 0 0 0
```

```
% Constructing a point B such that  $M_{\{c\}B}/M_{\{c\}A}=-1$ 
towards B  $M_{\{c\}}$  A -1
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line  $b$  and circle  $k(M_{\{c\}}, A)$  intersect; points  $H_{\{b\}}$  and  $M_{\{c\}}$  are not
the same
% Determination conditions: points  $H_{\{b\}}$  and  $A$  must be different; points  $M_{\{b\}}$  and  $H_{\{b\}}$  are not
the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = -M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.115 seconds.

**NDG conditions** Points  $H_b$  and  $M_c$  are not identical

Points  $H_b$  and  $M_c$  are not identical

#### 4.1.2 Proving $M_c = -M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.146 seconds.

**NDG conditions** Points  $H_b$  and  $M_c$  are not identical

Points  $H_b$  and  $M_c$  are not identical

#### 4.1.3 Proving $H_b = -H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 323 terms.

**Time Complexity:** Time spent by the prover is 1.096 seconds.

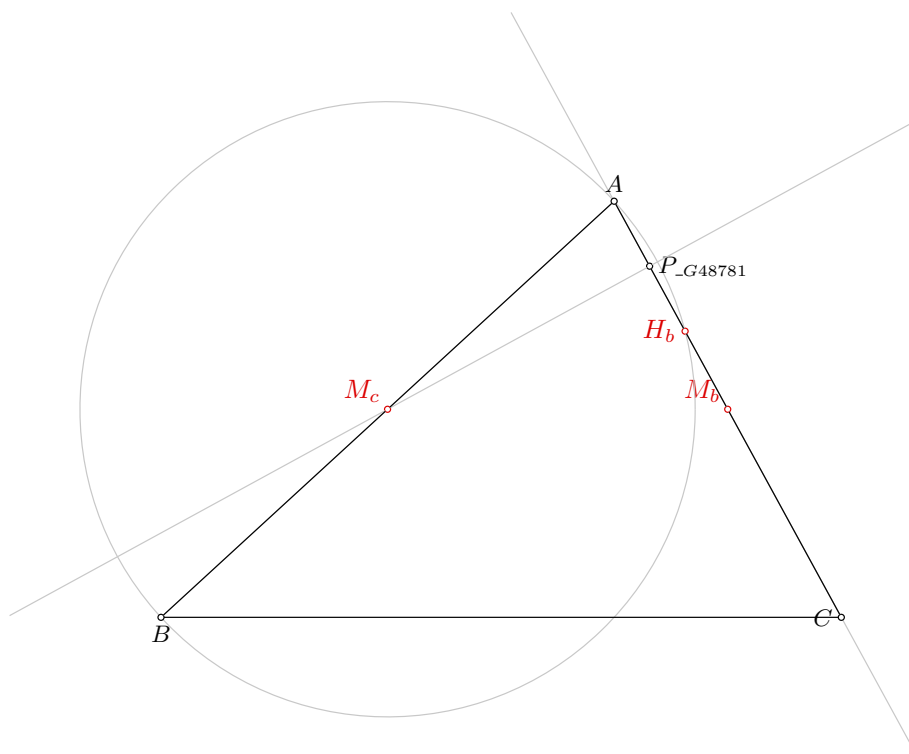


Figure 1: Illustration of the problem 0398



**NDG conditions** Points  $H_b$  and  $M_c$  are not identical

Points  $H_b$  and  $M_c$  are not identical

Points  $A$  and  $C$  are not identical

Line through points  $M_b$  and  $M_c$  is not parallel with line through points  $A$  and  $C$

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_b = M_b$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{H_b}^0} \neq S_{CBF_{H_b}^0}$  i.e., lines  $AC$  and  $BF_{H_b}^0$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_c = M_c$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{H_b}^0} \neq S_{CBF_{H_b}^0}$  i.e., lines  $AC$  and  $BF_{H_b}^0$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $H_b = H_b$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_b = M_b$

Proving failed

### 4.3.2 Proving $M_c = M_c$

Proving failed

### 4.3.3 Proving $H_b = H_b$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_b = M_b$

Proving failed

### 4.4.2 Proving $M_c = M_c$

Proving failed

### 4.4.3 Proving $H_b = H_b$

Proving failed

# Problem 399

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 399: Given a point  $M_b$ , a point  $M_c$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $H_c$  are not the same;
2. Using the point  $H_c$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_c$  and  $M_b$  are not the same;
3. Using the circle  $k(M_b, C)$ , the line  $c$ , the point  $M_b$  and the point  $H_c$ , construct a point  $A$  (rule W05); % NDG: line  $c$  and circle  $k(M_b, C)$  intersect % DET: points  $H_c$  and  $A$  must be different;
4. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
5. Using the point  $M_c$  and the point  $A$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: line  $c$  and circle  $k(M_b, C)$  intersect; points  $H_c$  and  $M_b$  are not the same.

Determination conditions: points  $H_c$  and  $A$  must be different; points  $M_c$  and  $H_c$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D20,D22,D7,GD01,GD02,GL03,GL04,GL09,L43,L45]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```

dim 120 120

point M_{b} 95 67.5
point M_{c} 50 67.5
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_lt M_{c}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points M_{c} and H_{c} are not the same
% Constructing a line c which passes through point M_{c} and point H_{c}
line c M_{c} H_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points H_{c} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{c}
circle k(M_{b},C) M_{b} H_{c}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line c and circle k(M_{b},C) intersect% DET: points H_{c} and A must be different
% Constructing a point P_{\G38381} which is a foot of the point M_{b} on the line c
foot P_{\G38381} M_{b} c
cmark_r P_{\G38381}
color 200 200 200
drawline M_{b} P_{\G38381}
color 0 0 0

% Constructing a point A which is an image of the point H_{c} in the symmetry to point/line P_{\G38381}
sim A P_{\G38381} H_{c}
cmark_t A

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C

```

```
color 0 0 0
```

```
% Constructing a point B such that  $M_{\{c\}B}/M_{\{c\}A}=-1$ 
towards B  $M_{\{c\}}$  A -1
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line  $c$  and circle  $k(M_{\{b\}}, C)$  intersect; points  $H_{\{c\}}$  and  $M_{\{b\}}$  are not
the same
% Determination conditions: points  $H_{\{c\}}$  and  $A$  must be different; points  $M_{\{c\}}$  and  $H_{\{c\}}$  are not
the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = -M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.157 seconds.

**NDG conditions** Points  $M_b$ ,  $H_c$  and  $M_c$  are not collinear

Points  $M_b$ ,  $H_c$  and  $M_c$  are not collinear

#### 4.1.2 Proving $M_c = -M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 9 terms.

**Time Complexity:** Time spent by the prover is 0.162 seconds.

**NDG conditions** Points  $M_b$ ,  $H_c$  and  $M_c$  are not collinear

Points  $M_b$ ,  $H_c$  and  $M_c$  are not collinear

#### 4.1.3 Proving $H_c = -H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 323 terms.

**Time Complexity:** Time spent by the prover is 1.205 seconds.

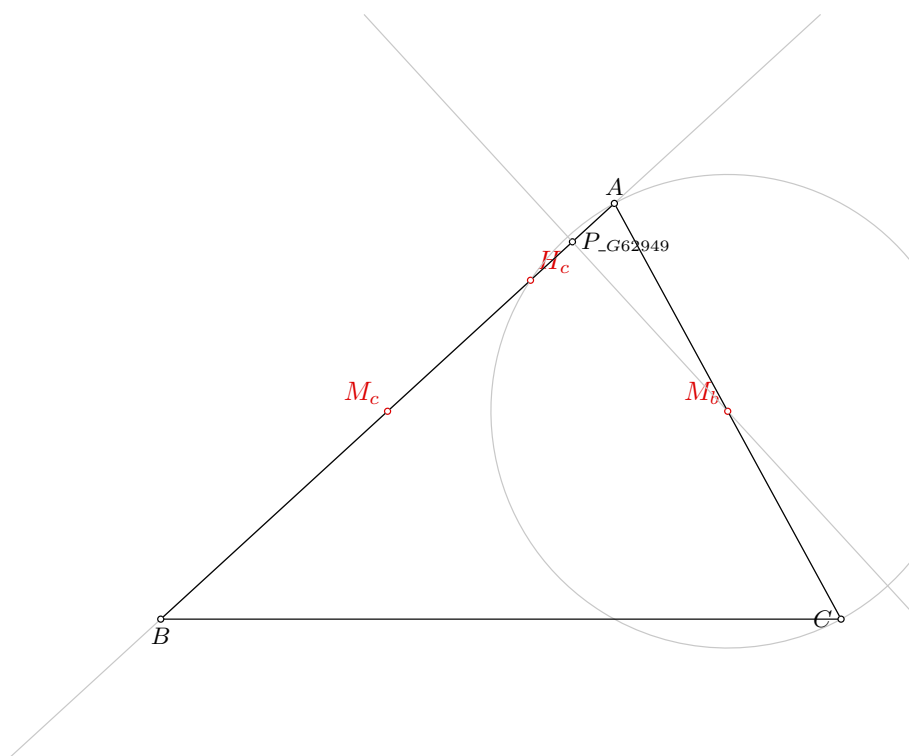


Figure 1: Illustration of the problem 0399

**NDG conditions** Points  $M_b$ ,  $H_c$  and  $M_c$  are not collinear

Points  $M_b$ ,  $H_c$  and  $M_c$  are not collinear

Points  $A$  and  $B$  are not identical

Line through points  $M_b$  and  $M_c$  is not parallel with line through points  $A$  and  $B$

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{\neg h_c}^0} \neq S_{BCF_{\neg h_c}^0}$  i.e., lines  $AB$  and  $CF_{\neg h_c}^0$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{\neg h_c}^0} \neq S_{BCF_{\neg h_c}^0}$  i.e., lines  $AB$  and  $CF_{\neg h_c}^0$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.3 Proving $H_c = \neg H_c$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_b = \neg M_b$

Proving failed

### 4.3.2 Proving $M_c = \neg M_c$

Proving failed

### 4.3.3 Proving $H_c = \neg H_c$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_b = \neg M_b$

Proving failed

### 4.4.2 Proving $M_c = \neg M_c$

Proving failed

### 4.4.3 Proving $H_c = \neg H_c$

Proving failed

# Problem 400

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 400: Given a point  $M_b$ , a point  $M_c$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 401

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 401: Given a point  $M_b$ , a point  $M_c$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.



## Problem 402

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 402: Given a point  $M_b$ , a point  $M_c$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 403

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 403: Given a point  $M_b$ , a point  $M_c$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 404

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 404: Given a point  $M_b$ , a point  $M_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 405

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 405: Given a point  $M_b$ , a point  $G$  and a point  $H_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $G$ , construct a point  $B$  (rule W01); ;
2. Using the point  $H_a$  and the point  $B$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $B$  are not the same;
3. Using the point  $H_a$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_a$  and  $M_b$  are not the same;
4. Using the circle  $k(M_b, C)$ , the line  $a$ , the point  $M_b$  and the point  $H_a$ , construct a point  $C$  (rule W05); % NDG: line  $a$  and circle  $k(M_b, C)$  intersect % DET: points  $H_a$  and  $C$  must be different;
5. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: line  $a$  and circle  $k(M_b, C)$  intersect; points  $H_a$  and  $M_b$  are not the same.

Determination conditions: points  $H_a$  and  $C$  must be different; points  $H_a$  and  $B$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D22,D5,GD01,GD02,GL03,GL04,L44,L56]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```

dim 120 120

point M_{b} 95 67.5
point G 70 58.33
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_t G
cmark_r H_{a}
color 0 0 0
fontsize 8

% Constructing a point B such that  $M_{\{b\}}B/M_{\{b\}}G=3$ 
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

% DET: points  $H_{\{a\}}$  and  $B$  are not the same
% Constructing a line a which passes through point  $H_{\{a\}}$  and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% NDG: points  $H_{\{a\}}$  and  $M_{\{b\}}$  are not the same
% Constructing a circle  $k(M_{\{b\}},C)$  whose center is at point  $M_{\{b\}}$  and which passes through point  $H_{\{a\}}$ 
circle k(M_{b},C) M_{b} H_{a}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line a and circle  $k(M_{\{b\}},C)$  intersect% DET: points  $H_{\{a\}}$  and  $C$  must be different
% Constructing a point  $P_{\{\backslash\_G58565\}}$  which is a foot of the point  $M_{\{b\}}$  on the line a
foot P_{\_G58565} M_{b} a
cmark_r P_{\_G58565}
color 200 200 200
drawline M_{b} P_{\_G58565}
color 0 0 0

% Constructing a point C which is an image of the point  $H_{\{a\}}$  in the symmetry to point/line  $P_{\{\backslash\_G58565\}}$ 
sim C P_{\_G58565} H_{a}

```

```
cmark_l C
```

```
% Constructing a point A such that CA/CM_{b}=2
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(M_{b},C) intersect; points H_{a} and M_{b} are not
the same
% Determination conditions: points H_{a} and C must be different; points H_{a} and B are not the
same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = \_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.182 seconds.

**NDG conditions** Points  $B$  and  $H_a$  are not identical

Line through points  $M_b$  and  $B$  is not perpendicular to line through points  $B$  and  $G$

#### 4.1.2 Proving $G = \_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 295 terms.

**Time Complexity:** Time spent by the prover is 1.092 seconds.

**NDG conditions** Points  $B$  and  $H_a$  are not identical

Line through points  $M_b$  and  $B$  is not perpendicular to line through points  $B$  and  $G$

Line through points  $A$  and  $\_M_a$  is not parallel with line through points  $B$  and  $\_M_b$

Line through points  $M_b$  and  $G$  is not parallel with line through points  $A$  and  $\_M_a$

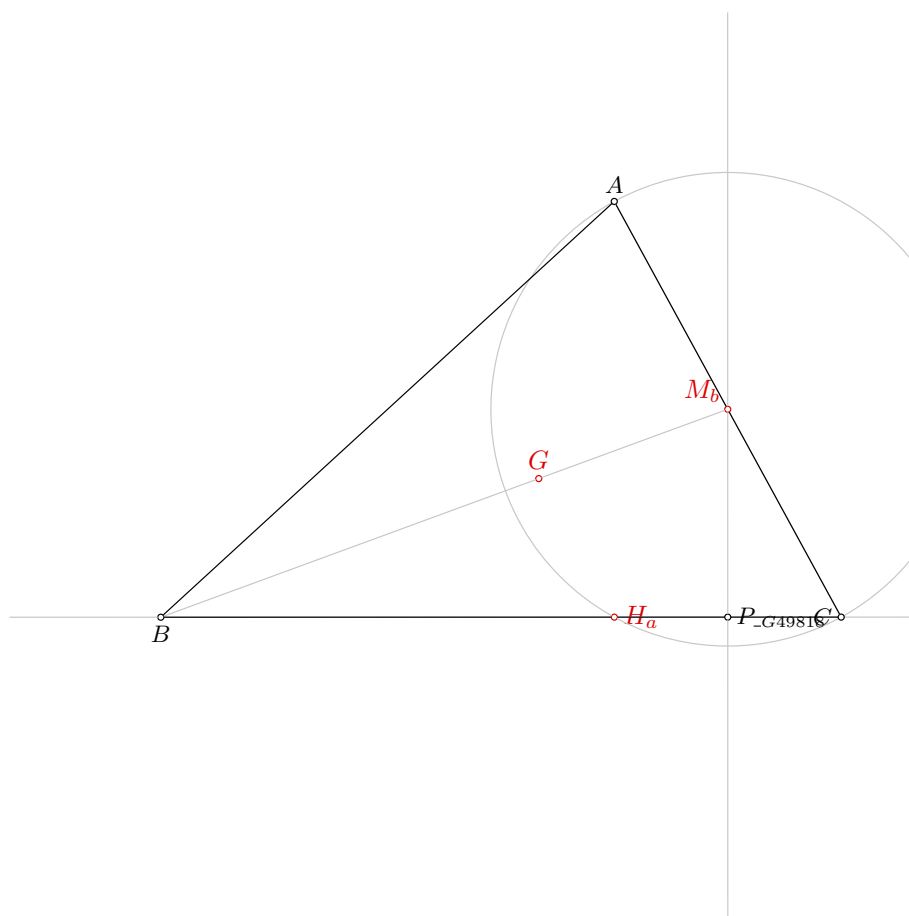


Figure 1: Illustration of the problem 0405

### 4.1.3 Proving $H_a = \neg H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 174 terms.

**Time Complexity:** Time spent by the prover is 1.221 seconds.

**NDG conditions** Points  $B$  and  $H_a$  are not identical

Line through points  $M_b$  and  $B$  is not perpendicular to line through points  $B$  and  $G$

Points  $B$  and  $C$  are not identical

Points  $B$ ,  $C$  and  $G$  are not collinear

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{AB \neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A \neg M_a$  and  $B \neg M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{\neg H_a}^0} \neq S_{CAF_{\neg H_a}^0}$  i.e., lines  $BC$  and  $AF_{\neg H_a}^0$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $G = \neg G$

Proving failed

### 4.2.3 Proving $H_a = \neg H_a$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_b = \neg M_b$

Proving failed

### 4.3.2 Proving $G = \neg G$

Proving failed

### 4.3.3 Proving $H_a = \neg H_a$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_b = \neg M_b$

Proving failed

### 4.4.2 Proving $G = \neg G$

Proving failed

### 4.4.3 Proving $H_a = \neg H_a$

Proving failed



# Problem 406

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 406: Given a point  $M_b$ , a point  $H_b$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $H_b$  are not the same;
2. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
3. Using the point  $M_b$ , the line  $h_b$  and the point  $H_b$ , construct a line  $h_{M_b,1/3}(h_b)$  (rule W15); ;
4. Choose freely a point  $G$  on the line  $h_{M_b,1/3}(h_b)$  (rule WOnline4);
5. Using the point  $G$  and the point  $M_b$ , construct a point  $B$  (rule W01); ;
6. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) ;
7. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: points  $M_b$  and  $H_b$  are not the same.

Rules used: [W01,W02,W10b,W15,WOnline1,WOnline4]

Lemmas used: [D22,D6,D9,GD01,GL03,GL04,GL09,L56]

Solving time: 190.2 seconds.

### 3.2 Construction in GCLC language

dim 120 120

```
point M_{b} 95 67.5
point H_{b} 89.36 77.83
point G 70 58.33
```

```
color 220 0 0
fontsize 9
```

```
cmark_lt M_{b}
cmark_l H_{b}
cmark_t G
color 0 0 0
fontsize 8
```

```
% DET: points M_{b} and H_{b} are not the same
% Constructing a line b which passes through point M_{b} and point H_{b}
line b M_{b} H_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b
```

```
color 200 200 200
drawline h_{b}
color 0 0 0
```

```
% Constructing a point P_{\_G47960} such that M_{b}P_{\_G47960}/M_{b}H_{b}=0.3333333333333333
towards P_{\_G47960} M_{b} H_{b} 0.3333333333333333
cmark_r P_{\_G47960}
color 200 200 200
drawsegment M_{b} H_{b}
color 0 0 0
```

```
% Constructing a line h_{M_{b},1/3}(h_{b}) which contains the point P_{\_G47960} and is parallel to
the line h_{b}
parallel h_{M_{b},1/3}(h_{b}) P_{\_G47960} h_{b}
```

```
color 200 200 200
drawline h_{M_{b},1/3}(h_{b})
color 0 0 0
```

```
% Constructing a point P_{\_G48282} such that M_{b}P_{\_G48282}/M_{b}H_{b}=0.3333333333333333
towards P_{\_G48282} M_{b} H_{b} 0.3333333333333333
cmark_r P_{\_G48282}
color 200 200 200
drawsegment M_{b} H_{b}
color 0 0 0
```

```

% Generating random value V[_G48221]
random V[_G48221]

% Calculating value V[_G48242] using formula V[_G48221]*20
expression V[_G48242] { V[_G48221]*20 }

% Constructing a point G which is a point for which holds  $P_{\setminus\_G48282}G = V[_G48242]$  and angle  $M_{\{b\}}$ 
 $P_{\setminus\_G48282}G = 90$ 
turtle G  $M_{\{b\}}$   $P_{\setminus\_G48282}$  90 V[_G48242]
cmark_t G

% Constructing a point B such that  $GB/GM_{\{b\}}=-2$ 
towards B G  $M_{\{b\}}$  -2
cmark_b B
color 200 200 200
drawsegment  $M_{\{b\}}$  B
color 0 0 0

% Choosing randomly a point A on the line  $M_{\{b\}}H_{\{b\}}$ 
online A  $M_{\{b\}}$   $H_{\{b\}}$ 
cmark_t A
color 200 200 200
drawline  $M_{\{b\}}$   $H_{\{b\}}$ 
color 0 0 0

% Constructing a point C such that  $AC/AM_{\{b\}}=2$ 
towards C A  $M_{\{b\}}$  2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions: points  $M_{\{b\}}$  and  $H_{\{b\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

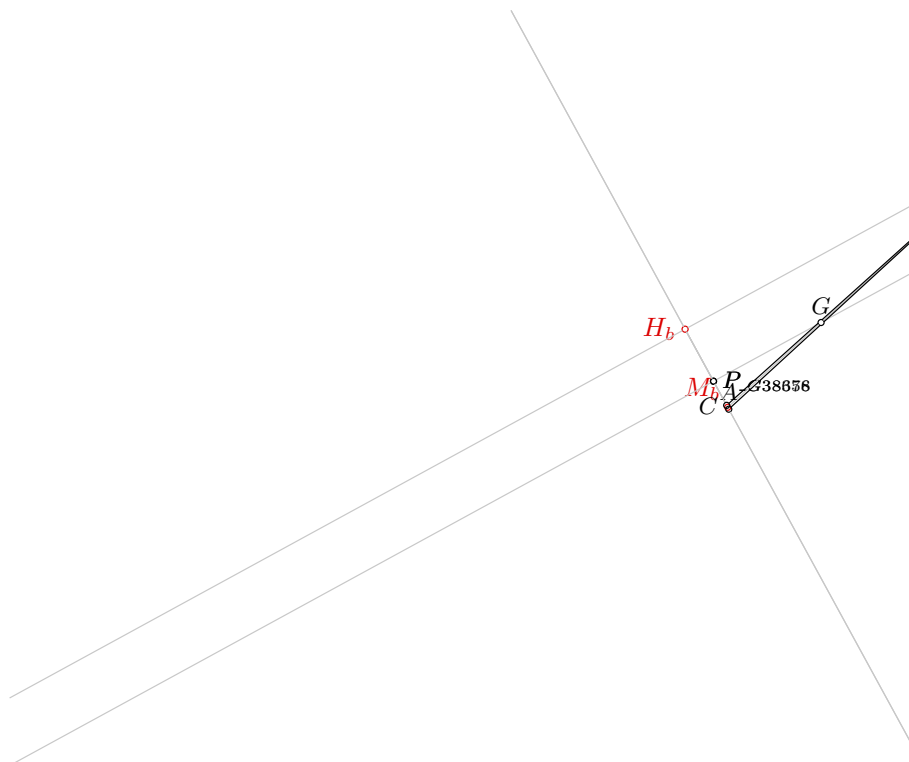


Figure 1: Illustration of the problem 0406

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = \neg M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b = \neg H_b$

**Status:** Theorem has been disproved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 18 terms.

**Time Complexity:** Time spent by the prover is 0.298 seconds.

**NDG conditions** Points  $M_b$ ,  $G$  and  $P_{G45985}$  are not collinear

Points  $P_{G45748}$  and  $A$  are not identical

Line through points  $C$  and  $A$  is not perpendicular to line through points  $A$  and  $M_b$

#### 4.1.3 Proving $G = \neg G$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{CBF_{\neg h_b}^3}$  i.e., lines  $AC$  and  $BF_{\neg h_b}^3$  are not parallel (construction based assumption)

$S_{AB_{\neg M_b}} \neq S_{M_a B_{\neg M_b}}$  i.e., lines  $A_{\neg M_a}$  and  $B_{\neg M_b}$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.2.3 Proving $G = \neg G$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = \neg M_b$

Proving failed

#### 4.3.2 Proving $H_b = \neg H_b$

Proving failed

### 4.3.3 Proving $G=_G$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_b=_M M_b$

Proving failed

### 4.4.2 Proving $H_b=_H H_b$

Proving failed

### 4.4.3 Proving $G=_G$

Proving failed

# Problem 407

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 407: Given a point  $M_b$ , a point  $G$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $G$ , construct a point  $B$  (rule W01); ;
2. Using the point  $H_c$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $B$  are not the same;
3. Using the point  $H_c$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_c$  and  $M_b$  are not the same;
4. Using the circle  $k(M_b, C)$ , the line  $c$ , the point  $M_b$  and the point  $H_c$ , construct a point  $A$  (rule W05); % NDG: line  $c$  and circle  $k(M_b, C)$  intersect % DET: points  $H_c$  and  $A$  must be different;
5. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); .

Non-degenerate conditions: line  $c$  and circle  $k(M_b, C)$  intersect; points  $H_c$  and  $M_b$  are not the same.

Determination conditions: points  $H_c$  and  $A$  must be different; points  $H_c$  and  $B$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D22,D7,GD01,GD02,GL03,GL04,L43,L45,L56]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```

dim 120 120

point M_{b} 95 67.5
point G 70 58.33
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_t G
cmark_rt H_{c}
color 0 0 0
fontsize 8

% Constructing a point B such that  $M_{\{b\}}B/M_{\{b\}}G=3$ 
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

% DET: points  $H_{\{c\}}$  and B are not the same
% Constructing a line c which passes through point  $H_{\{c\}}$  and point B
line c H_{c} B

color 200 200 200
drawline c
color 0 0 0

% NDG: points  $H_{\{c\}}$  and  $M_{\{b\}}$  are not the same
% Constructing a circle  $k(M_{\{b\}}, C)$  whose center is at point  $M_{\{b\}}$  and which passes through point  $H_{\{c\}}$ 
circle k(M_{b}, C) M_{b} H_{c}

color 200 200 200
drawcircle k(M_{b}, C)
color 0 0 0

% NDG: line c and circle  $k(M_{\{b\}}, C)$  intersect% DET: points  $H_{\{c\}}$  and A must be different
% Constructing a point  $P_{\{\backslash\_G35473\}}$  which is a foot of the point  $M_{\{b\}}$  on the line c
foot P_{\_G35473} M_{b} c
cmark_r P_{\_G35473}
color 200 200 200
drawline M_{b} P_{\_G35473}
color 0 0 0

% Constructing a point A which is an image of the point  $H_{\{c\}}$  in the symmetry to point/line  $P_{\{\backslash\_G35473\}}$ 
sim A P_{\_G35473} H_{c}

```



```
cmark_t A
```

```
% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line c and circle k(M_{b},C) intersect; points H_{c} and M_{b} are not
the same
% Determination conditions: points H_{c} and A must be different; points H_{c} and B are not the
same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = \_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.232 seconds.

**NDG conditions** Points  $H_c$  and  $B$  are not identical

Line through points  $M_b$  and  $B$  is not perpendicular to line through points  $B$  and  $G$

#### 4.1.2 Proving $G = \_G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 295 terms.

**Time Complexity:** Time spent by the prover is 1.051 seconds.

**NDG conditions** Points  $H_c$  and  $B$  are not identical

Line through points  $M_b$  and  $B$  is not perpendicular to line through points  $B$  and  $G$

Line through points  $A$  and  $\_M_a$  is not parallel with line through points  $B$  and  $\_M_b$

Line through points  $M_b$  and  $G$  is not parallel with line through points  $A$  and  $\_M_a$

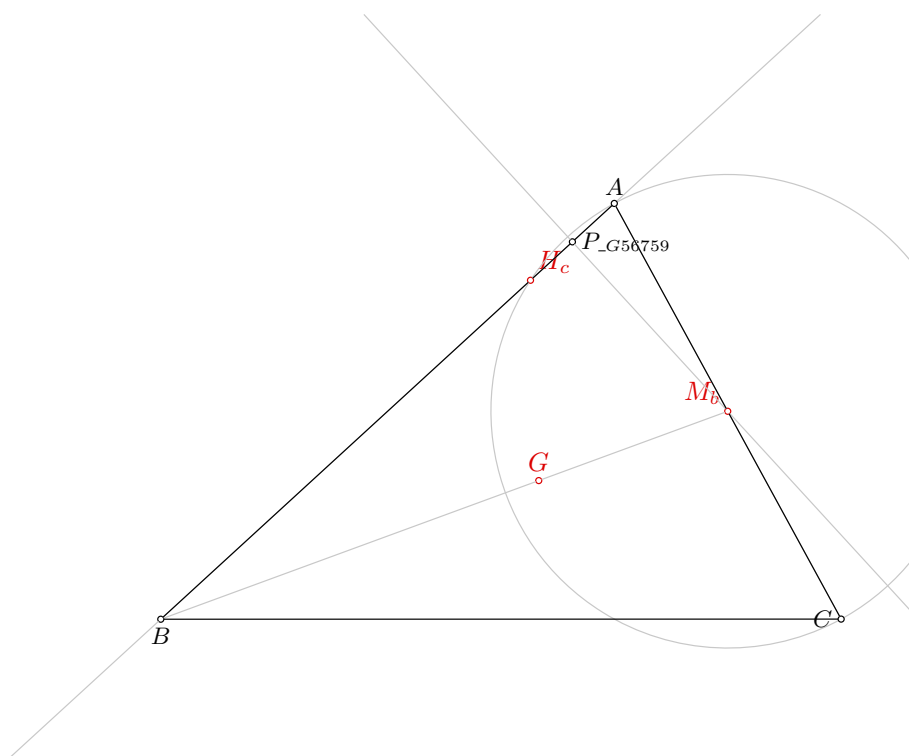


Figure 1: Illustration of the problem 0407

#### 4.1.3 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 174 terms.

**Time Complexity:** Time spent by the prover is 1.153 seconds.

**NDG conditions** Points  $H_c$  and  $B$  are not identical

Line through points  $M_b$  and  $B$  is not perpendicular to line through points  $B$  and  $G$

Points  $A$  and  $B$  are not identical

Points  $M_b$ ,  $A$  and  $B$  are not collinear

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{AB \neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A \neg M_a$  and  $B \neg M_b$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{\neg H_c}^0} \neq S_{BCF_{\neg H_c}^0}$  i.e., lines  $AB$  and  $CF_{\neg H_c}^0$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $G = \neg G$

Proving failed

#### 4.2.3 Proving $H_c = \neg H_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = \neg M_b$

Proving failed

#### 4.3.2 Proving $G = \neg G$

Proving failed

#### 4.3.3 Proving $H_c = \neg H_c$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_b = \neg M_b$

Proving failed

#### 4.4.2 Proving $G = \neg G$

Proving failed

#### 4.4.3 Proving $H_c = \neg H_c$

Proving failed

# Problem 408

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 408: Given a point  $M_b$ , a point  $G$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $G$ , construct a point  $B$  (rule W01); ;
2. Using the point  $G$  and the point  $H$ , construct a point  $O$  (rule W01); ;
3. Using the point  $M_b$  and the point  $O$ , construct a line  $m_b$  (rule W02); % DET: points  $M_b$  and  $O$  are not the same;
4. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
5. Using the point  $M_b$  and the line  $m_b$ , construct a line  $b$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same.

Determination conditions: points  $M_b$  and  $O$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D1,D12,D22,D26,GD01,GL03,GL04,GL09,L11,L12,L56,L58]

Solving time: 1.4 seconds.

### 3.2 Construction in GCLC language

dim 120 120

point M\_{b} 95 67.5  
point G 70 58.33  
point H 80 72.73

```
color 220 0 0
fontsize 9
```

```
cmark_lt M_{b}
cmark_t G
cmark_rt H
color 0 0 0
fontsize 8
```

```
% Constructing a point B such that  $M_{\{b\}}B/M_{\{b\}}G=3$ 
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0
```

```
% Constructing a point O such that  $GO/GH=-0.5$ 
towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0
```

```
% DET: points  $M_{\{b\}}$  and O are not the same
% Constructing a line  $m_{\{b\}}$  which passes through point  $M_{\{b\}}$  and point O
line m_{b} M_{b} O

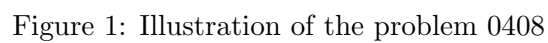
color 200 200 200
drawline m_{b}
color 0 0 0
```

```
% NDG: points B and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0
```

```
% Constructing a line b which is perpendicular to line  $m_{\{b\}}$  and which passes through point  $M_{\{b\}}$ 
perp b M_{b} m_{b}

color 200 200 200
drawline b
color 0 0 0
```



% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same  
 % Determination conditions: points M\_{b} and O are not the same

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = \neg M_b$

Proving failed

#### 4.1.2 Proving $G = \neg G$

Proving failed

#### 4.1.3 Proving $H = \neg H$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b = \neg M_b$

Proving failed

#### 4.2.2 Proving $G = \neg G$

Proving failed

#### 4.2.3 Proving $H = \neg H$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 60 terms.

**Time Complexity:** Time spent by the prover is 0.100 seconds. There are no ndg conditions.

#### 4.3.2 Proving $G = \neg G$

Proving failed

#### 4.3.3 Proving $H = \neg H$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_b = \neg M_b$

Proving failed

#### 4.4.2 Proving $G = \neg G$

Proving failed

### 4.4.3 Proving $H=_H$

Proving failed



## Problem 409

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 409: Given a point  $M_b$ , a point  $G$  and a point  $T_a$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 410

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 410: Given a point  $M_b$ , a point  $G$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $G$ , construct a point  $B$  (rule W01); ;
2. Using the point  $M_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $T_b$  are not the same;
3. Using the point  $T_b$  and the point  $B$ , construct a line  $s_b$  (rule W02); % DET: points  $T_b$  and  $B$  are not the same;
4. Using the point  $M_b$  and the line  $b$ , construct a line  $m_b$  (rule W10b); ;
5. Using the line  $m_b$  and the line  $s_b$ , construct a point  $N_b$  (rule W03); % NDG: lines  $m_b$  and  $s_b$  are not parallel % DET: lines  $m_b$  and  $s_b$  are not the same;
6. Using the point  $B$  and the point  $N_b$ , construct a line  $m(BN_b)$  (rule W14); % DET: points  $B$  and  $N_b$  are not the same;
7. Using the line  $m(BN_b)$  and the line  $m_b$ , construct a point  $O$  (rule W03); % NDG: lines  $m(BN_b)$  and  $m_b$  are not parallel % DET: lines  $m(BN_b)$  and  $m_b$  are not the same;
8. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
9. Using the circle  $k(O, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same; lines  $m(BN_b)$  and  $m_b$  are not parallel; lines  $m_b$  and  $s_b$  are not parallel.

Determination conditions: lines  $m(BN_b)$  and  $m_b$  are not the same; points  $B$  and  $N_b$  are not the same; lines  $m_b$  and  $s_b$  are not the same; points  $T_b$  and  $B$  are not the same; points  $M_b$  and  $T_b$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14]

Lemmas used: [D1,D12,D22,D24,D26,D48,GD01,GL01,GL03,GL04,GL09,L11,L12,L26,L56,L6]

Solving time: 1.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point G 70 58.33
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_t G
cmark_t T_{b}
color 0 0 0
fontsize 8

% Constructing a point B such that M_{b}B/M_{b}G=3
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

% DET: points M_{b} and T_{b} are not the same
% Constructing a line b which passes through point M_{b} and point T_{b}
line b M_{b} T_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points T_{b} and B are not the same
% Constructing a line s_{b} which passes through point T_{b} and point B
line s_{b} T_{b} B

color 200 200 200
drawline s_{b}
color 0 0 0
```

```

% Constructing a line  $m_{\{b\}}$  which is perpendicular to line  $b$  and which passes through point  $M_{\{b\}}$ 
perp  $m_{\{b\}}$   $M_{\{b\}}$   $b$ 

color 200 200 200
drawline  $m_{\{b\}}$ 
color 0 0 0

% NDG: lines  $m_{\{b\}}$  and  $s_{\{b\}}$  are not parallel% DET: lines  $m_{\{b\}}$  and  $s_{\{b\}}$  are not the same
% Constructing a point  $N_{\{b\}}$  which belongs to line  $m_{\{b\}}$  and line  $s_{\{b\}}$ 
intersec  $N_{\{b\}}$   $m_{\{b\}}$   $s_{\{b\}}$ 
cmark_rb  $N_{\{b\}}$ 

% DET: points  $B$  and  $N_{\{b\}}$  are not the same
% Constructing bisector  $m(BN_{\{b\}})$  of the segment  $BN_{\{b\}}$ 
med  $m(BN_{\{b\}})$   $B$   $N_{\{b\}}$ 

color 200 200 200
drawline  $m(BN_{\{b\}})$ 
color 0 0 0

color 200 200 200
drawsegment  $B$   $N_{\{b\}}$ 
color 0 0 0

% NDG: lines  $m(BN_{\{b\}})$  and  $m_{\{b\}}$  are not parallel% DET: lines  $m(BN_{\{b\}})$  and  $m_{\{b\}}$  are not the same
% Constructing a point  $O$  which belongs to line  $m(BN_{\{b\}})$  and line  $m_{\{b\}}$ 
intersec  $O$   $m(BN_{\{b\}})$   $m_{\{b\}}$ 
cmark_t  $O$ 

% NDG: points  $B$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $B$ 
circle  $k(O,C)$   $O$   $B$ 

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $b$  and circle  $k(O,C)$  intersect
% Constructing points  $C$  and  $A$  which are in intersection of  $k(O,C)$  and  $b$ 
intersec2  $C$   $A$   $k(O,C)$   $b$ 
cmark_l  $C$ 
cmark_t  $A$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

```

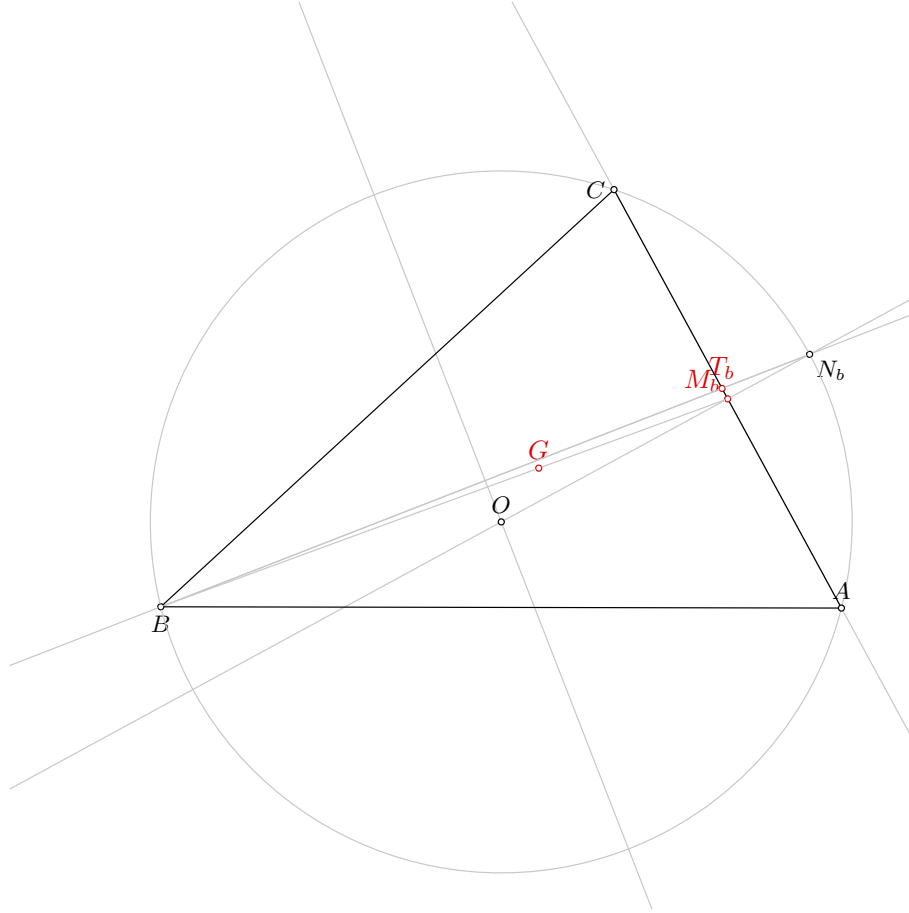


Figure 1: Illustration of the problem 0410

*% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;  
 lines m(BN\_{b}) and m\_{b} are not parallel; lines m\_{b} and s\_{b} are not parallel  
 % Determination conditions: lines m(BN\_{b}) and m\_{b} are not the same; points B and N\_{b} are not  
 the same; lines m\_{b} and s\_{b} are not the same; points T\_{b} and B are not the same; points M  
 \_{b} and T\_{b} are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = M_b$

Proving failed

#### 4.1.2 Proving $G = \neg G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2678 terms.

**Time Complexity:** Time spent by the prover is 22.767 seconds.

**NDG conditions** Line through points  $M_b$  and  $T_b$  is not perpendicular to line through points  $T_b$  and  $B$

Points  $T_b$  and  $G$  are not identical

Line through points  $M_b$  and  $T_b$  is not parallel with line through points  $B$  and  $N_b$

Line through points  $M_b$  and  $B$  is not perpendicular to line through points  $B$  and  $G$

Points  $T_b$  and  $G$  are not identical

Points  $T_b$  and  $G$  are not identical

Line through points  $A$  and  $\neg M_a$  is not parallel with line through points  $B$  and  $\neg M_b$

Line through points  $M_b$  and  $G$  is not parallel with line through points  $A$  and  $\neg M_a$

#### 4.1.3 Proving $T_b = \neg T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b = \neg M_b$

Proving failed

#### 4.2.2 Proving $G = \neg G$

Proving failed

#### 4.2.3 Proving $T_b = \neg T_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 1355 terms.

**Time Complexity:** Time spent by the prover is 1.380 seconds. There are no ndg conditions.

#### 4.3.2 Proving $G = \neg G$

Proving failed

#### 4.3.3 Proving $T_b = \neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 9188 terms.

**Time Complexity:** Time spent by the prover is 19.710 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_b = \neg M_b$

Proving failed

### 4.4.2 Proving $G = \neg G$

Proving failed

### 4.4.3 Proving $T_b = \neg T_b$

Proving failed

# Problem 411

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 411: Given a point  $M_b$ , a point  $G$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.



# Problem 412

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 412: Given a point  $M_b$ , a point  $G$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $G$ , construct a point  $B$  (rule W01); ;
2. Using the point  $M_b$  and the point  $I$ , construct a line  $IM_b$  (rule W02); % DET: points  $M_b$  and  $I$  are not the same;
3. Using the point  $I$  and the point  $B$ , construct a line  $s_b$  (rule W02); % DET: points  $I$  and  $B$  are not the same;
4. Using the point  $I$  and the point  $M_b$ , construct a circle  $k_{over}(I, M_b)$  (rule W09); % NDG: points  $I$  and  $M_b$  are not the same;
5. Using the point  $B$  and the line  $IM_b$ , construct a line  $BP'_b$  (rule W16); ;
6. Using the point  $M_b$ , the line  $BP'_b$  and the point  $B$ , construct a line  $h_{M_b, -1/1}(BP'_b)$  (rule W15); ;
7. Using the circle  $k_{over}(I, M_b)$  and the line  $h_{M_b, -1/1}(BP'_b)$ , construct a point  $B_{fo}$  and a point  $P_b$  (rule W04); % NDG: line  $h_{M_b, -1/1}(BP'_b)$  and circle  $k_{over}(I, M_b)$  intersect;
8. Using the point  $P_b$  and the point  $I$ , construct a circle  $k(I, P_a)$  (rule W06); % NDG: points  $P_b$  and  $I$  are not the same;
9. Using the circle  $k(I, P_a)$ , the point  $M_b$  and the point  $I$ , construct a line  $x_2$  and a line  $b$  (rule W12); % NDG: point  $M_b$  is outside the circle  $k(I, P_a)$ ;
10. Using the point  $M_b$  and the line  $b$ , construct a line  $m_b$  (rule W10b); ;
11. Using the line  $m_b$  and the line  $s_b$ , construct a point  $N_b$  (rule W03); % NDG: lines  $m_b$  and  $s_b$  are not parallel % DET: lines  $m_b$  and  $s_b$  are not the same;

12. Using the point  $I$  and the point  $N_b$ , construct a circle  $k(N_b, A)$  (rule W06); % NDG: points  $I$  and  $N_b$  are not the same;
13. Using the circle  $k(N_b, A)$  and the line  $b$ , construct a point  $A$  and a point  $C$  (rule W04); % NDG: line  $b$  and circle  $k(N_b, A)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(N_b, A)$  intersect; points  $I$  and  $N_b$  are not the same; lines  $m_b$  and  $s_b$  are not parallel; point  $M_b$  is outside the circle  $k(I, P_a)$ ; points  $P_b$  and  $I$  are not the same; line  $h_{M_b, -1/1}(BP'_b)$  and circle  $k_{over}(I, M_b)$  intersect; points  $I$  and  $M_b$  are not the same.

Determination conditions: lines  $m_b$  and  $s_b$  are not the same; points  $I$  and  $B$  are not the same; points  $M_b$  and  $I$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W09,W10b,W12,W15,W16]

Lemmas used: [D12,D2,D22,D27,D48,D60,D86,GD01,GD02,GL02,GL03,GL04,GL09,L13,L26,L33,L34,L56,L6,

Solving time: 2.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{b} 95 67.5
```

```
point G 70 58.33
```

```
point I 74.37 61.15
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_lt M_{b}
```

```
cmark_t G
```

```
cmark_b I
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point B such that M_{b}B/M_{b}G=3
```

```
towards B M_{b} G 3
```

```
cmark_b B
```

```
color 200 200 200
```

```
drawsegment M_{b} B
```

```
color 0 0 0
```

```
% DET: points M_{b} and I are not the same
```

```
% Constructing a line IM_{b} which passes through point M_{b} and point I
```

```
line IM_{b} M_{b} I
```

```
color 200 200 200
```

```
drawline IM_{b}
```

```
color 0 0 0
```

```
% DET: points I and B are not the same
```

```

% Constructing a line  $s_{\{b\}}$  which passes through point  $I$  and point  $B$ 
line  $s_{\{b\}}$  I B

color 200 200 200
drawline  $s_{\{b\}}$ 
color 0 0 0

% NDG: points  $I$  and  $M_{\{b\}}$  are not the same
% Constructing midpoint  $P_{\{\backslash\_G42630\}}$  of the segment  $IM_{\{b\}}$ 
midpoint  $P_{\{\backslash\_G42630\}}$  I  $M_{\{b\}}$ 
cmark_r  $P_{\{\backslash\_G42630\}}$ 

% Constructing a circle  $k_{\text{over}}(I, M_{\{b\}})$  whose center is at point  $P_{\{\backslash\_G42630\}}$  and which passes
    through point  $I$ 
circle  $k_{\text{over}}(I, M_{\{b\}})$   $P_{\{\backslash\_G42630\}}$  I

color 200 200 200
drawcircle  $k_{\text{over}}(I, M_{\{b\}})$ 
color 0 0 0

% Constructing a line  $BP'_{\{b\}}$  which contains the point  $B$  and is parallel to the line  $IM_{\{b\}}$ 
parallel  $BP'_{\{b\}}$  B  $IM_{\{b\}}$ 

color 200 200 200
drawline  $BP'_{\{b\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\backslash\_G42931\}}$  such that  $M_{\{b\}}P_{\{\backslash\_G42931\}}/M_{\{b\}}B=-1$ 
towards  $P_{\{\backslash\_G42931\}}$   $M_{\{b\}}$  B -1
cmark_r  $P_{\{\backslash\_G42931\}}$ 
color 200 200 200
drawsegment B  $P_{\{\backslash\_G42931\}}$ 
color 0 0 0

% Constructing a line  $h_{\{M_{\{b\}}, -1/1\}}(BP'_{\{b\}})$  which contains the point  $P_{\{\backslash\_G42931\}}$  and is parallel
    to the line  $BP'_{\{b\}}$ 
parallel  $h_{\{M_{\{b\}}, -1/1\}}(BP'_{\{b\}})$   $P_{\{\backslash\_G42931\}}$   $BP'_{\{b\}}$ 

color 200 200 200
drawline  $h_{\{M_{\{b\}}, -1/1\}}(BP'_{\{b\}})$ 
color 0 0 0

% NDG: line  $h_{\{M_{\{b\}}, -1/1\}}(BP'_{\{b\}})$  and circle  $k_{\text{over}}(I, M_{\{b\}})$  intersect
% Constructing points  $B_{\{fo\}}$  and  $P_{\{b\}}$  which are in intersection of  $k_{\text{over}}(I, M_{\{b\}})$  and  $h_{\{M_{\{b\}}, -1/1\}}(BP'_{\{b\}})$ 
intersec2  $B_{\{fo\}}$   $P_{\{b\}}$   $k_{\text{over}}(I, M_{\{b\}})$   $h_{\{M_{\{b\}}, -1/1\}}(BP'_{\{b\}})$ 

```

```

cmark_r B_{fo}
cmark_r P_{b}

% NDG: points  $P_{b}$  and  $I$  are not the same
% Constructing a circle  $k(I, P_{a})$  whose center is at point  $I$  and which passes through point  $P_{b}$ 
circle k(I, P_{a}) I P_{b}

color 200 200 200
drawcircle k(I, P_{a})
color 0 0 0

% NDG: point  $M_{b}$  is outside the circle  $k(I, P_{a})$ 
% Constructing midpoint  $P_{\backslash\_G43680}$  of the segment  $M_{b}I$ 
midpoint P_{\backslash\_G43680} M_{b} I
cmark_r P_{\backslash\_G43680}

% Constructing a circle  $C_{\backslash\_G43683}$  whose center is at point  $P_{\backslash\_G43680}$  and which passes through
point  $M_{b}$ 
circle C_{\backslash\_G43683} P_{\backslash\_G43680} M_{b}

color 200 200 200
drawcircle C_{\backslash\_G43683}
color 0 0 0

% Constructing points  $P_{\backslash\_G43686}$  and  $P_{\backslash\_G43689}$  which are in intersection of  $C_{\backslash\_G43683}$  and  $k$ 
( $I, P_{a}$ )
intersec2 P_{\backslash\_G43686} P_{\backslash\_G43689} C_{\backslash\_G43683} k(I, P_{a})
cmark_r P_{\backslash\_G43686}
cmark_r P_{\backslash\_G43689}

% Constructing a line  $x2$  which passes through point  $M_{b}$  and point  $P_{\backslash\_G43686}$ 
line x2 M_{b} P_{\backslash\_G43686}

color 200 200 200
drawline x2
color 0 0 0

% Constructing a line  $b$  which passes through point  $M_{b}$  and point  $P_{\backslash\_G43689}$ 
line b M_{b} P_{\backslash\_G43689}

color 200 200 200
drawline b
color 0 0 0

% Constructing a line  $m_{b}$  which is perpendicular to line  $b$  and which passes through point  $M_{b}$ 
perp m_{b} M_{b} b

color 200 200 200

```

```

drawline m_{b}
color 0 0 0

% NDG: lines m_{b} and s_{b} are not parallel% DET: lines m_{b} and s_{b} are not the same
% Constructing a point N_{b} which belongs to line m_{b} and line s_{b}
intersec N_{b} m_{b} s_{b}
cmark_rb N_{b}

% NDG: points I and N_{b} are not the same
% Constructing a circle k(N_{b},A) whose center is at point N_{b} and which passes through point I
circle k(N_{b},A) N_{b} I

color 200 200 200
drawcircle k(N_{b},A)
color 0 0 0

% NDG: line b and circle k(N_{b},A) intersect
% Constructing points A and C which are in intersection of k(N_{b},A) and b
intersec2 A C k(N_{b},A) b
cmark_t A
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k(N_{b},A) intersect; points I and N_{b} are not the
same; lines m_{b} and s_{b} are not parallel; point M_{b} is outside the circle k(I,P_{a});
points P_{b} and I are not the same; line h_{M_{b},-1/1}(BP'_{b}) and circle k_{over}(I,M_{b})
intersect; points I and M_{b} are not the same
% Determination conditions: lines m_{b} and s_{b} are not the same; points I and B are not the same
; points M_{b} and I are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = -M_b$

Proving failed

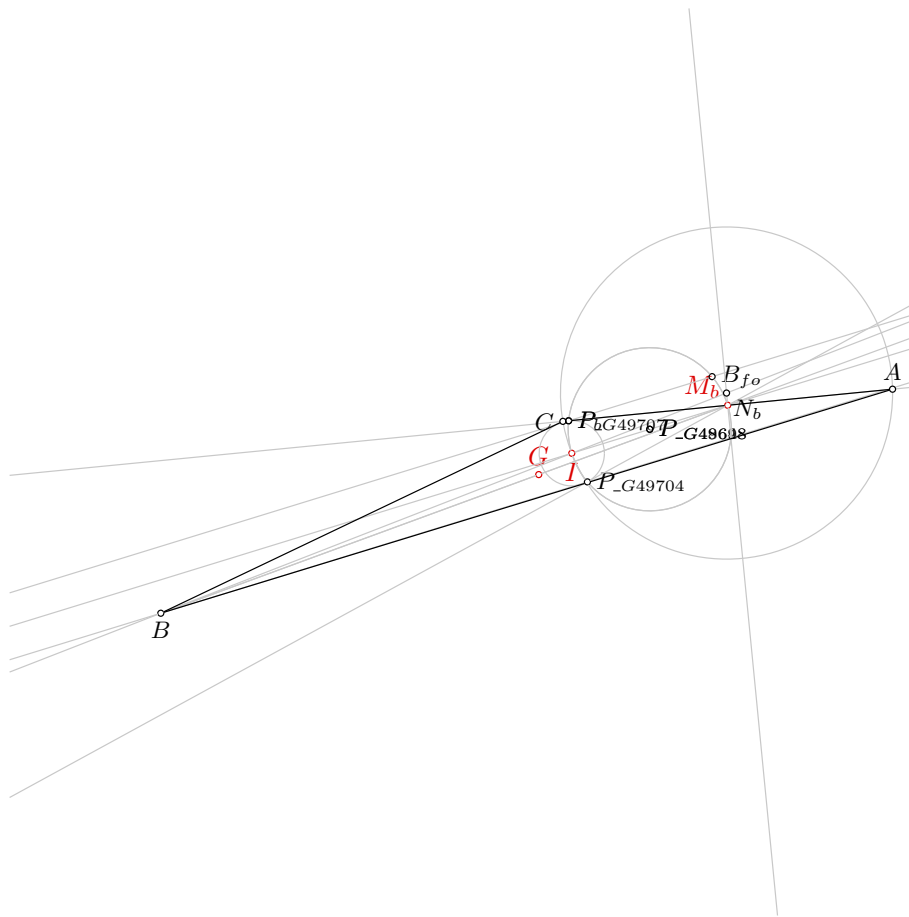


Figure 1: Illustration of the problem 0412

#### 4.1.2 Proving $G=_G$

Proving failed

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b=_M M_b$

Proving failed

#### 4.2.2 Proving $G=_G$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b=_M M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 1274 terms.

**Time Complexity:** Time spent by the prover is 2.890 seconds. There are no ndg conditions.

#### 4.3.2 Proving $G=_G$

Proving failed

#### 4.3.3 Proving $I=_I$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_b=_M M_b$

Proving failed

#### 4.4.2 Proving $G=_G$

Proving failed

#### 4.4.3 Proving $I=_I$

Proving failed

# Problem 413

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 413: Given a point  $M_b$ , a point  $H_a$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $H_b$  are not the same;
2. Using the point  $H_a$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_a$  and  $M_b$  are not the same;
3. Using the circle  $k(M_b, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(M_b, C)$  intersect;
4. Using the point  $H_a$  and the point  $C$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $C$  are not the same;
5. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
6. Using the line  $h_b$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $h_b$  and  $a$  are not parallel % DET: lines  $h_b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $h_b$  and  $a$  are not parallel; line  $b$  and circle  $k(M_b, C)$  intersect; points  $H_a$  and  $M_b$  are not the same.

Determination conditions: lines  $h_b$  and  $a$  are not the same; points  $H_a$  and  $C$  are not the same; points  $M_b$  and  $H_b$  are not the same.

Rules used: [W02,W03,W04,W06,W10b]

Lemmas used: [D22,D5,D6,D9,GD01,GD02,GL09,L43,L44]

Solving time: 1.1 seconds.



### 3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point H_{a} 80 40
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_r H_{a}
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points M_{b} and H_{b} are not the same
% Constructing a line b which passes through point M_{b} and point H_{b}
line b M_{b} H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points H_{a} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{a}
circle k(M_{b},C) M_{b} H_{a}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line b and circle k(M_{b},C) intersect
% Constructing points C and A which are in intersection of k(M_{b},C) and b
intersec2 C A k(M_{b},C) b
cmark_l C
cmark_t A

% DET: points H_{a} and C are not the same
% Constructing a line a which passes through point H_{a} and point C
line a H_{a} C

color 200 200 200
drawline a
color 0 0 0
```

```

% Constructing a line  $h_{\{b\}}$  which is perpendicular to line  $b$  and which passes through point  $H_{\{b\}}$ 
perp  $h_{\{b\}}$   $H_{\{b\}}$   $b$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: lines  $h_{\{b\}}$  and  $a$  are not parallel% DET: lines  $h_{\{b\}}$  and  $a$  are not the same
% Constructing a point  $B$  which belongs to line  $h_{\{b\}}$  and line  $a$ 
intersec  $B$   $h_{\{b\}}$   $a$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $h_{\{b\}}$  and  $a$  are not parallel; line  $b$  and circle  $k(M_{\{b\}}, C)$ 
% intersect; points  $H_{\{a\}}$  and  $M_{\{b\}}$  are not the same
% Determination conditions: lines  $h_{\{b\}}$  and  $a$  are not the same; points  $H_{\{a\}}$  and  $C$  are not the same
% ; points  $M_{\{b\}}$  and  $H_{\{b\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = \neg M_b$

Proving failed

#### 4.1.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.1.3 Proving $H_b = \neg H_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b = \neg M_b$

Proving failed

#### 4.2.2 Proving $H_a = \neg H_a$

Proving failed

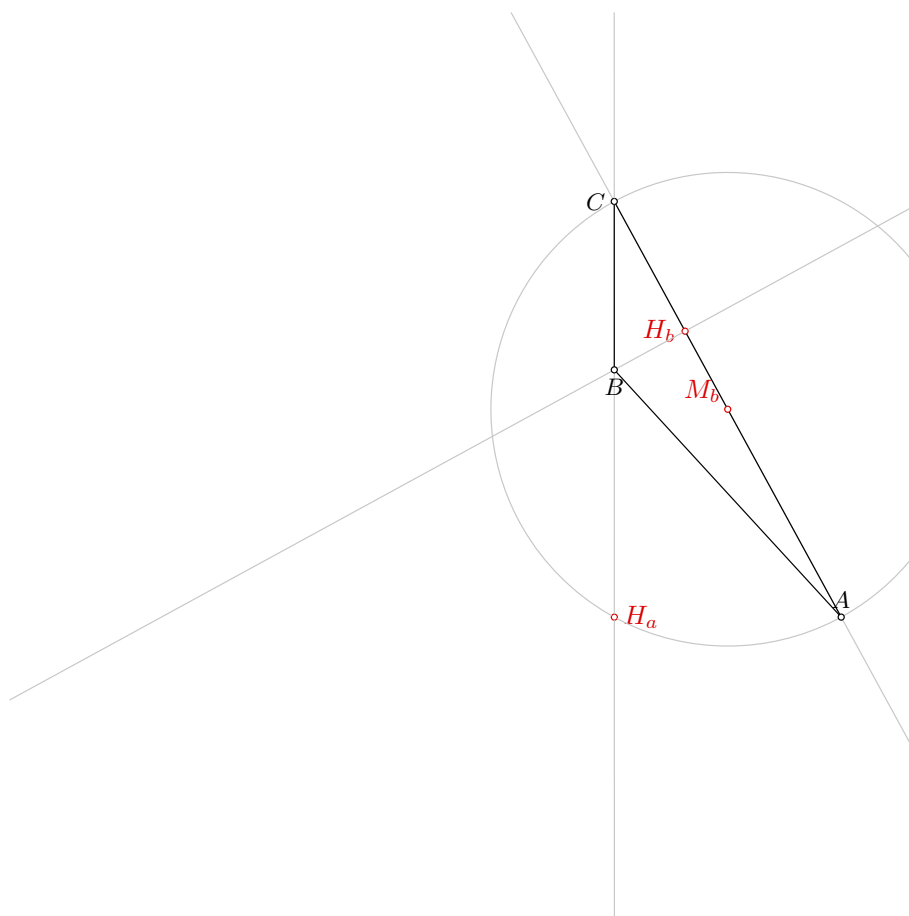


Figure 1: Illustration of the problem 0413

#### 4.2.3 Proving $H_b = \neg H_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 34 terms.

**Time Complexity:** Time spent by the prover is 0.040 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3080 terms.

**Time Complexity:** Time spent by the prover is 3.880 seconds. There are no ndg conditions.

#### 4.3.3 Proving $H_b = \neg H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 1247 terms.

**Time Complexity:** Time spent by the prover is 1.530 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_b = \neg M_b$

Proving failed

#### 4.4.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.4.3 Proving $H_b = \neg H_b$

Proving failed

# Problem 414

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 414: Given a point  $M_b$ , a point  $H_c$  and a point  $H_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_c$  and  $M_b$  are not the same;
2. Choose freely a point  $H_a$  on the circle  $k(M_b, C)$  (rule WOncircle);
3. Choose freely a point  $A$  on the circle  $k(M_b, C)$  (rule WOncircle);
4. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
5. Using the point  $A$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $H_c$  are not the same;
6. Using the point  $H_a$  and the point  $C$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $C$  are not the same;
7. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; points  $H_c$  and  $M_b$  are not the same.

Determination conditions: lines  $c$  and  $a$  are not the same; points  $H_a$  and  $C$  are not the same; points  $A$  and  $H_c$  are not the same.

Rules used: [W01,W02,W03,W06,WOncircle1]

Lemmas used: [D22,D5,D7,GD01,GD02,GL03,GL04,L45]

Solving time: 2.9 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point H_{c} 68.91 84.83
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_rt H_{c}
cmark_r H_{a}
color 0 0 0
fontsize 8

% NDG: points H_{c} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{c}
circle k(M_{b},C) M_{b} H_{c}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% Choosing randomly a point H_{a} on the circle with center M_{b} through point H_{c}
oncircle H_{a} M_{b} H_{c}
cmark_r H_{a}
color 200 200 200
drawcircle M_{b} H_{c}
color 0 0 0

% Choosing randomly a point A on the circle with center M_{b} through point H_{a}
oncircle A M_{b} H_{a}
cmark_t A
color 200 200 200
drawcircle M_{b} H_{a}
color 0 0 0

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points A and H_{c} are not the same
```

```

% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0

% DET: points H_{a} and C are not the same
% Constructing a line a which passes through point H_{a} and point C
line a H_{a} C

color 200 200 200
drawline a
color 0 0 0

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and a are not parallel; points H_{c} and M_{b} are not the
same
% Determination conditions: lines c and a are not the same; points H_{a} and C are not the same;
points A and H_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = \neg M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.135 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c = \neg H_c$

**Status:** Theorem has been proved.

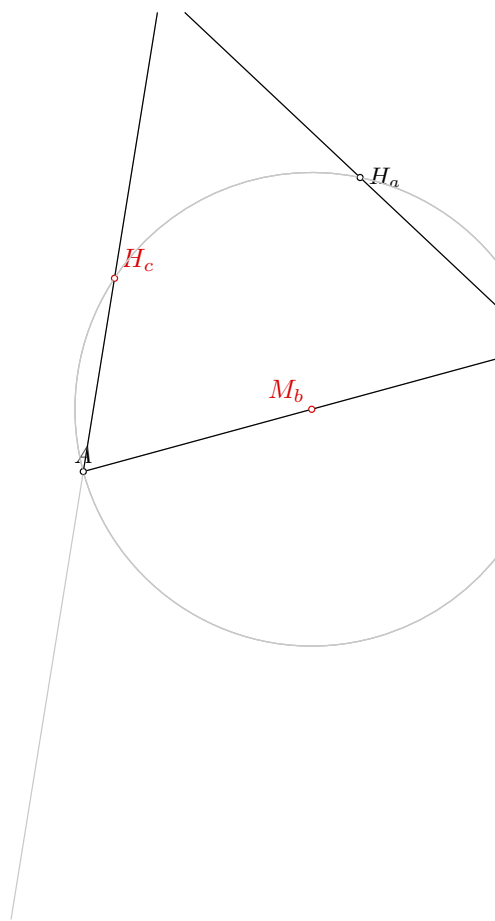


Figure 1: Illustration of the problem 0414



**Space Complexity:** The biggest polynomial obtained during prover execution contains 2162 terms.

**Time Complexity:** Time spent by the prover is 15.161 seconds.

**NDG conditions** Line through points  $A$  and  $H_c$  is not parallel with line through points  $C$  and  $H_a$

Points  $M_b$  and  $A$  are not identical

Points  $A$  and  $B$  are not identical

Line through points  $M_b$  and  $H_c$  is not parallel with line through points  $A$  and  $B$

#### 4.1.3 Proving $H_a = H_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b = M_b$

NDG conditions are:

$S_{AH_aC} \neq S_{H_cH_aC}$  i.e., lines  $AH_c$  and  $H_aC$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{H_c}^0} \neq S_{BCF_{H_c}^0}$  i.e., lines  $AB$  and  $CF_{H_c}^0$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{H_a}^1} \neq S_{CAF_{H_a}^1}$  i.e., lines  $BC$  and  $AF_{H_a}^1$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c = H_c$

Proving failed

#### 4.2.3 Proving $H_a = H_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 17 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_c = H_c$

Proving failed

#### 4.3.3 Proving $H_a = H_a$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_b = M_b$

Proving failed

#### 4.4.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.4.3 Proving $H_a = \neg H_a$

Proving failed

# Problem 415

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 415: Given a point  $M_b$ , a point  $H_a$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $H_a$  and  $H$  are not the same;
2. Using the point  $H_a$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_a$  and  $M_b$  are not the same;
3. Using the circle  $k(M_b, C)$ , the line  $h_a$ , the point  $M_b$  and the point  $H_a$ , construct a point  $A$  (rule W05); % NDG: line  $h_a$  and circle  $k(M_b, C)$  intersect % DET: points  $H_a$  and  $A$  must be different;
4. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
5. Using the point  $H_a$  and the point  $C$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $C$  are not the same;
6. Using the point  $H$  and the point  $C$ , construct a line  $h_c$  (rule W02); % DET: points  $H$  and  $C$  are not the same;
7. Using the circle  $k(M_b, C)$ , the line  $h_c$ , the point  $M_b$  and the point  $C$ , construct a point  $H_c$  (rule W05); % NDG: line  $h_c$  and circle  $k(M_b, C)$  intersect % DET: points  $C$  and  $H_c$  must be different;
8. Using the point  $A$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $H_c$  are not the same;
9. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; line  $h_c$  and circle  $k(M_b, C)$  intersect; line  $h_a$  and circle  $k(M_b, C)$  intersect; points  $H_a$  and  $M_b$  are not the same.

Determination conditions: lines  $a$  and  $c$  are not the same; points  $A$  and  $H_c$  are not the same; points  $C$  and  $H_c$  must be different; points  $H$  and  $C$  are not the same; points  $H_a$  and  $C$  are not the same; points  $H_a$  and  $A$  must be different; points  $H_a$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D22,D3,D5,D7,D8,GD01,GD02,GL03,GL04,L3,L43,L44,L45]

Solving time: 1.6 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{b} 95 67.5
```

```
point H_{a} 80 40
```

```
point H 80 72.73
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_lt M_{b}
```

```
cmark_r H_{a}
```

```
cmark_rt H
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{a} and H are not the same
```

```
% Constructing a line h_{a} which passes through point H_{a} and point H
```

```
line h_{a} H_{a} H
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```
% NDG: points H_{a} and M_{b} are not the same
```

```
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{a}
```

```
circle k(M_{b},C) M_{b} H_{a}
```

```
color 200 200 200
```

```
drawcircle k(M_{b},C)
```

```
color 0 0 0
```

```
% NDG: line h_{a} and circle k(M_{b},C) intersect% DET: points H_{a} and A must be different
```

```
% Constructing a point P_{\_G40476} which is a foot of the point M_{b} on the line h_{a}
```

```
foot P_{\_G40476} M_{b} h_{a}
```

```
cmark_r P_{\_G40476}
```

```
color 200 200 200
```

```
drawline M_{b} P_{\_G40476}
```

```
color 0 0 0
```

```

% Constructing a point A which is an image of the point H_{a} in the symmetry to point/line P_{\_G
40476}
sim A P_{\_G40476} H_{a}
cmark_t A

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points H_{a} and C are not the same
% Constructing a line a which passes through point H_{a} and point C
line a H_{a} C

color 200 200 200
drawline a
color 0 0 0

% DET: points H and C are not the same
% Constructing a line h_{c} which passes through point H and point C
line h_{c} H C

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: line h_{c} and circle k(M_{b},C) intersect% DET: points C and H_{c} must be different
% Constructing a point P_{\_G40839} which is a foot of the point M_{b} on the line h_{c}
foot P_{\_G40839} M_{b} h_{c}
cmark_r P_{\_G40839}
color 200 200 200
drawline M_{b} P_{\_G40839}
color 0 0 0

% Constructing a point H_{c} which is an image of the point C in the symmetry to point/line P_{\_G
40839}
sim H_{c} P_{\_G40839} C
cmark_rt H_{c}

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c

```

```
color 0 0 0
```

```
% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines a and c are not parallel; line h_{c} and circle k(M_{b},C)
    intersect; line h_{a} and circle k(M_{b},C) intersect; points H_{a} and M_{b} are not the same
% Determination conditions: lines a and c are not the same; points A and H_{c} are not the same;
    points C and H_{c} must be different; points H and C are not the same; points H_{a} and C are
    not the same; points H_{a} and A must be different; points H_{a} and H are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = \_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.227 seconds.

**NDG conditions** Points  $M_b$ ,  $H$  and  $H_a$  are not collinear

Points  $M_b$ ,  $H$  and  $H_a$  are not collinear

#### 4.1.2 Proving $H_a = \_H_a$

Proving failed

#### 4.1.3 Proving $H = \_H$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b = \_M_b$

NDG conditions are:

$S_{H_a A H_c} \neq S_{C A H_c}$  i.e., lines  $H_a C$  and  $A H_c$  are not parallel (construction based assumption)

$S_{A B C} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{B A F_{-H_a}^0} \neq S_{C A F_{-H_a}^0}$  i.e., lines  $BC$  and  $A F_{-H_a}^0$  are not parallel (construction based assumption)

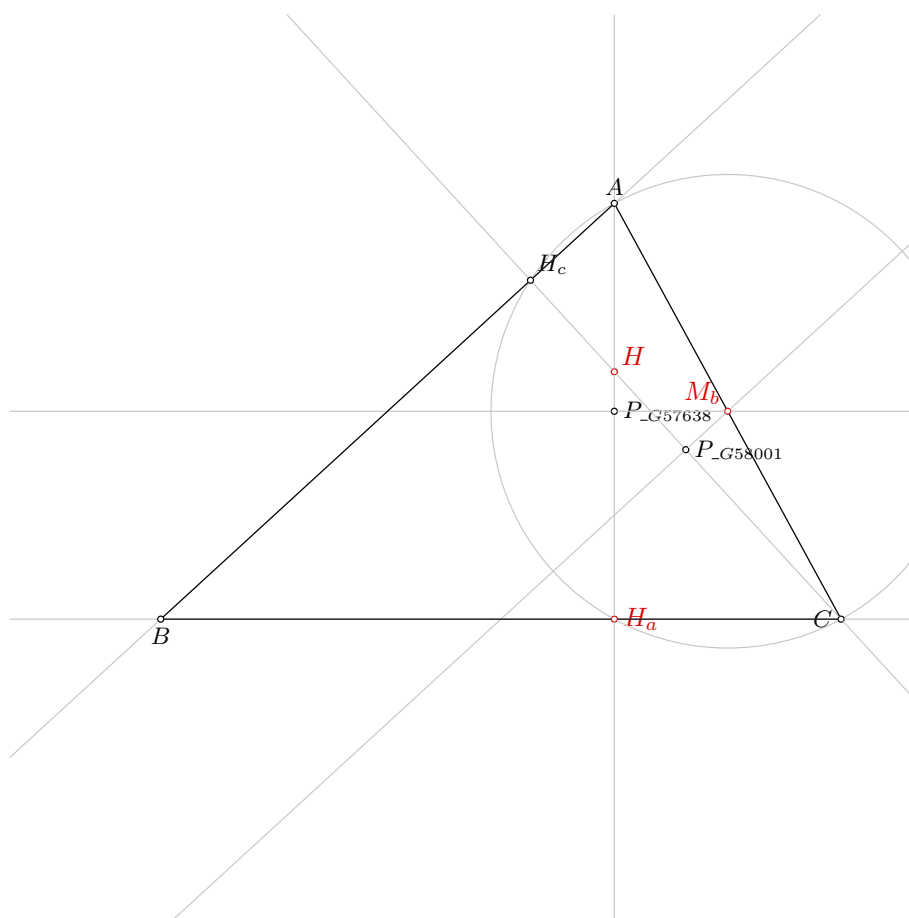


Figure 1: Illustration of the problem 0415

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{\perp h_b}} \neq S_{F^0_{\perp h_a}BF^1_{\perp h_b}}$  i.e., lines  $AF^0_{\perp h_a}$  and  $BF^1_{\perp h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a = \perp H_a$

Proving failed

#### 4.2.3 Proving $H = \perp H$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = \perp M_b$

Proving failed

#### 4.3.2 Proving $H_a = \perp H_a$

Proving failed

#### 4.3.3 Proving $H = \perp H$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_b = \perp M_b$

Proving failed

#### 4.4.2 Proving $H_a = \perp H_a$

Proving failed

#### 4.4.3 Proving $H = \perp H$

Proving failed



# Problem 416

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 416: Given a point  $M_b$ , a point  $H_a$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $T_a$  are not the same;
2. Using the point  $H_a$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_a$  and  $M_b$  are not the same;
3. Using the circle  $k(M_b, C)$ , the line  $a$ , the point  $M_b$  and the point  $H_a$ , construct a point  $C$  (rule W05); % NDG: line  $a$  and circle  $k(M_b, C)$  intersect % DET: points  $H_a$  and  $C$  must be different;
4. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); ;
5. Using the point  $M_b$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $C$  are not the same;
6. Using the point  $T_a$  and the line  $b$ , construct a circle  $k(T_a, foot[T_a, c])$  (rule W11); % NDG: point  $T_a$  is not incident to the line  $b$ ;
7. Using the circle  $k(T_a, foot[T_a, c])$ , the point  $A$ , the point  $T_a$  and the line  $b$ , construct a line  $c$  (rule W13); % NDG: point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ;
8. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ; point  $T_a$  is not incident to the line  $b$ ; line  $a$  and circle  $k(M_b, C)$  intersect; points  $H_a$  and  $M_b$  are not the same.

Determination conditions: lines  $c$  and  $a$  are not the same; points  $M_b$  and  $C$  are not the same; points  $H_a$  and  $C$  must be different; points  $H_a$  and  $T_a$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W11,W13]

Lemmas used: [D22,D23,D5,GD01,GD02,GL03,GL09,GL10,GL11,L44,L8]

Solving time: 1.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point H_{a} 80 40
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_r H_{a}
cmark_rb T_{a}
color 0 0 0
fontsize 8

% DET: points H_{a} and T_{a} are not the same
% Constructing a line a which passes through point H_{a} and point T_{a}
line a H_{a} T_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points H_{a} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{a}
circle k(M_{b},C) M_{b} H_{a}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line a and circle k(M_{b},C) intersect% DET: points H_{a} and C must be different
% Constructing a point P_{\G64295} which is a foot of the point M_{b} on the line a
foot P_{\G64295} M_{b} a
cmark_r P_{\G64295}
color 200 200 200
drawline M_{b} P_{\G64295}
color 0 0 0

% Constructing a point C which is an image of the point H_{a} in the symmetry to point/line P_{\G64295}
```

```

sim C P_{\_G64295} H_{a}
cmark_l C

% Constructing a point A such that  $CA/CM_{\{b\}}=2$ 
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

% DET: points  $M_{\{b\}}$  and  $C$  are not the same
% Constructing a line  $b$  which passes through point  $M_{\{b\}}$  and point  $C$ 
line b M_{b} C

color 200 200 200
drawline b
color 0 0 0

% NDG: point  $T_{\{a\}}$  is not incident to the line  $b$ 
% Constructing a point  $P_{\_G64579}$  which is a foot of the point  $T_{\{a\}}$  on the line  $b$ 
foot P_{\_G64579} T_{a} b
cmark_r P_{\_G64579}
color 200 200 200
drawline T_{a} P_{\_G64579}
color 0 0 0

% Constructing a circle  $k(T_{\{a\}}, foot[T_{\{a\}}, c])$  whose center is at point  $T_{\{a\}}$  and which passes
    through point  $P_{\_G64579}$ 
circle k(T_{a}, foot[T_{a}, c]) T_{a} P_{\_G64579}

color 200 200 200
drawcircle k(T_{a}, foot[T_{a}, c])
color 0 0 0

% NDG: point  $A$  is outside the circle  $k(T_{\{a\}}, foot[T_{\{a\}}, c])$ 
% Constructing a point  $P_{\_G64970}$  which is a foot of the point  $T_{\{a\}}$  on the line  $b$ 
foot P_{\_G37008} T_{a} b
cmark_r P_{\_G37008}
color 200 200 200
drawline T_{a} P_{\_G37008}
color 0 0 0

% Constructing a line  $L_{\_G64973}$  which passes through point  $T_{\{a\}}$  and point  $A$ 
line L_{\_G37011} T_{a} A

color 200 200 200
drawline L_{\_G37011}
color 0 0 0

```

```

% Constructing a point  $P_{\backslash\_G37014}$  which is an image of the point  $P_{\backslash\_G37008}$  in the symmetry to
    point/line  $L_{\backslash\_G37011}$ 
sim  $P_{\backslash\_G37014}$   $L_{\backslash\_G37011}$   $P_{\backslash\_G37008}$ 
cmark_r  $P_{\backslash\_G37014}$ 

% Constructing a line  $c$  which passes through point  $A$  and point  $P_{\backslash\_G37014}$ 
line c A  $P_{\backslash\_G37014}$ 

color 200 200 200
drawline c
color 0 0 0

% NDG: lines  $c$  and  $a$  are not parallel% DET: lines  $c$  and  $a$  are not the same
% Constructing a point  $B$  which belongs to line  $c$  and line  $a$ 
intersec B c a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(T_{\{a\}},$ 
     $foot[T_{\{a\}},c])$ ; point  $T_{\{a\}}$  is not incident to the line  $b$ ; line  $a$  and circle  $k(M_{\{b\}},C)$ 
    intersect; points  $H_{\{a\}}$  and  $M_{\{b\}}$  are not the same
% Determination conditions: lines  $c$  and  $a$  are not the same; points  $M_{\{b\}}$  and  $C$  are not the same;
    points  $H_{\{a\}}$  and  $C$  must be different; points  $H_{\{a\}}$  and  $T_{\{a\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.164 seconds.

**NDG conditions** Points  $T_a$  and  $H_a$  are not identical

Points  $T_a$  and  $H_a$  are not identical

#### 4.1.2 Proving $H_a = H_a$

Proving failed

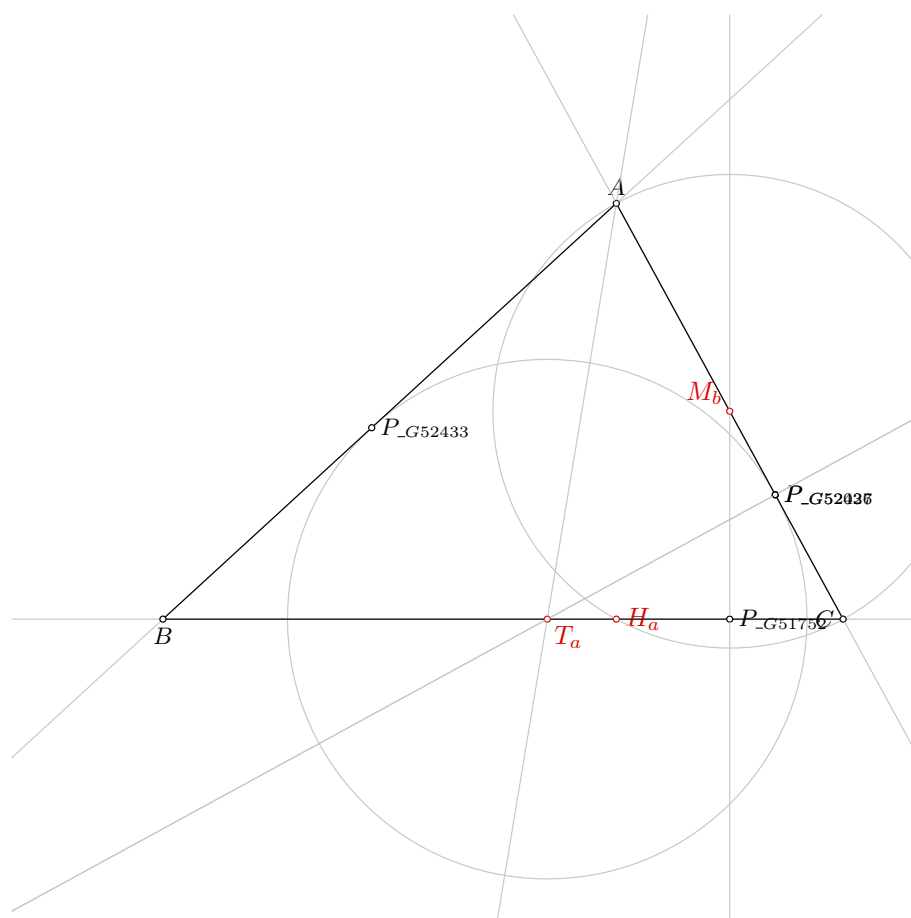


Figure 1: Illustration of the problem 0416

#### **4.1.3 Proving $T_a = \neg T_a$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $M_b = \neg M_b$**

Proving failed

#### **4.2.2 Proving $H_a = \neg H_a$**

Proving failed

#### **4.2.3 Proving $T_a = \neg T_a$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $M_b = \neg M_b$**

Proving failed

#### **4.3.2 Proving $H_a = \neg H_a$**

Proving failed

#### **4.3.3 Proving $T_a = \neg T_a$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $M_b = \neg M_b$**

Proving failed

#### **4.4.2 Proving $H_a = \neg H_a$**

Proving failed

#### **4.4.3 Proving $T_a = \neg T_a$**

Proving failed

# Problem 417

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 417: Given a point  $M_b$ , a point  $H_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $T_b$  are not the same;
2. Using the point  $H_a$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_a$  and  $M_b$  are not the same;
3. Using the circle  $k(M_b, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(M_b, C)$  intersect;
4. Using the point  $H_a$  and the point  $C$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $C$  are not the same;
5. Using the point  $A$ , the point  $C$ , the point  $T_b$  and the line  $b$ , construct a point  $T'_b$  (rule W19); % NDG: points  $A$  and  $C$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $\text{midpoint}([A, T_b])$  are not the same;
6. Using the point  $T_b$  and the point  $T'_b$ , construct a circle  $k_{over}(T_b, T'_b)$  (rule W09); % NDG: points  $T_b$  and  $T'_b$  are not the same;
7. Using the circle  $k_{over}(T_b, T'_b)$  and the line  $a$ , construct a point  $B_{wc}$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k_{over}(T_b, T'_b)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k_{over}(T_b, T'_b)$  intersect; points  $T_b$  and  $T'_b$  are not the same; points  $A$  and  $C$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $\text{midpoint}([A, T_b])$  are not the same; line  $b$  and circle  $k(M_b, C)$  intersect; points  $H_a$  and  $M_b$  are not the same.

Determination conditions: points  $H_a$  and  $C$  are not the same; points  $M_b$  and  $T_b$  are not the same.

Rules used: [W02,W04,W06,W09,W19]

Lemmas used: [D22,D24,D5,D56,GD01,GD02,GL09,L43,L44,L73,L78]

Solving time: 2589.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{b} 95 67.5
point H_{a} 80 40
point T_{b} 94.25 68.88
```

```
color 220 0 0
fontsize 9
```

```
cmark_lt M_{b}
cmark_r H_{a}
cmark_t T_{b}
color 0 0 0
fontsize 8
```

```
% DET: points M_{b} and T_{b} are not the same
% Constructing a line b which passes through point M_{b} and point T_{b}
line b M_{b} T_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% NDG: points H_{a} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{a}
circle k(M_{b},C) M_{b} H_{a}
```

```
color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0
```

```
% NDG: line b and circle k(M_{b},C) intersect
% Constructing points C and A which are in intersection of k(M_{b},C) and b
intersec2 C A k(M_{b},C) b
cmark_l C
cmark_t A
```

```
% DET: points H_{a} and C are not the same
% Constructing a line a which passes through point H_{a} and point C
line a H_{a} C
```

```
color 200 200 200
drawline a
```



```
color 0 0 0
```

```
% NDG: points A and C are not the same; points C and T_{b} are not the same; points C and midpoint
      ([A,T_{b}]) are not the same
% Constructing a point P_{\_G38641} which is an image of the point C in a rotation around the point
      T_{b} for the angle 90
rotate P_{\_G38641} T_{b} 90 C
cmark_r P_{\_G38641}
color 200 200 200
drawarc_p T_{b} C 90
color 0 0 0
```

```
% Constructing a line L_{\_G38644} which passes through point T_{b} and point P_{\_G38641}
line L_{\_G38644} T_{b} P_{\_G38641}
```

```
color 200 200 200
drawline L_{\_G38644}
color 0 0 0
```

```
% Constructing midpoint P_{\_G38647} of the segment CP_{\_G38641}
midpoint P_{\_G38647} C P_{\_G38641}
cmark_r P_{\_G38647}
```

```
% Constructing a line L_{\_G38650} which passes through point A and point P_{\_G38647}
line L_{\_G38650} A P_{\_G38647}
```

```
color 200 200 200
drawline L_{\_G38650}
color 0 0 0
```

```
% Constructing a line L_{\_G38653} which passes through point C and point P_{\_G38647}
line L_{\_G38653} C P_{\_G38647}
```

```
color 200 200 200
drawline L_{\_G38653}
color 0 0 0
```

```
% Constructing a point P_{\_G38656} which belongs to line L_{\_G38644} and line L_{\_G38650}
intersec P_{\_G38656} L_{\_G38644} L_{\_G38650}
cmark_r P_{\_G38656}
```

```
% Constructing a point P_{\_G38659} which belongs to line L_{\_G38644} and line L_{\_G38653}
intersec P_{\_G38659} L_{\_G38644} L_{\_G38653}
cmark_r P_{\_G38659}
```

```
% Constructing a line L_{\_G38662} which passes through point A and point P_{\_G38659}
line L_{\_G38662} A P_{\_G38659}
```

```
color 200 200 200
drawline L_{\_G38662}
color 0 0 0
```

```

% Constructing a line  $L_{G38665}$  which passes through point C and point  $P_{G38656}$ 
line  $L_{G38665}$  C  $P_{G38656}$ 

color 200 200 200
drawline  $L_{G38665}$ 
color 0 0 0

% Constructing a point  $P_{G38668}$  which belongs to line  $L_{G38662}$  and line  $L_{G38665}$ 
intersec  $P_{G38668}$   $L_{G38662}$   $L_{G38665}$ 
cmark_r  $P_{G38668}$ 

% Constructing a line  $L_{G38671}$  which passes through point  $P_{G38647}$  and point  $P_{G38668}$ 
line  $L_{G38671}$   $P_{G38647}$   $P_{G38668}$ 

color 200 200 200
drawline  $L_{G38671}$ 
color 0 0 0

% Constructing a point  $T'_b$  which belongs to line  $L_{G38671}$  and line b
intersec  $T'_b$   $L_{G38671}$  b
cmark_r  $T'_b$ 

% NDG: points  $T_b$  and  $T'_b$  are not the same
% Constructing midpoint  $P_{G39502}$  of the segment  $T_bT'_b$ 
midpoint  $P_{G39502}$   $T_b$   $T'_b$ 
cmark_r  $P_{G39502}$ 

% Constructing a circle  $k_{over}(T_b, T'_b)$  whose center is at point  $P_{G39502}$  and which passes
    through point  $T_b$ 
circle  $k_{over}(T_b, T'_b)$   $P_{G39502}$   $T_b$ 

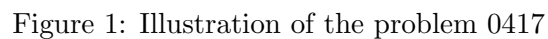
color 200 200 200
drawcircle  $k_{over}(T_b, T'_b)$ 
color 0 0 0

% NDG: line a and circle  $k_{over}(T_b, T'_b)$  intersect
% Constructing points  $B_{wc}$  and B which are in intersection of  $k_{over}(T_b, T'_b)$  and a
intersec2  $B_{wc}$  B  $k_{over}(T_b, T'_b)$  a
cmark_r  $B_{wc}$ 
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle  $k_{over}(T_b, T'_b)$  intersect; points  $T_b$  and  $T'_b$ 
    are not the same; points A and C are not the same; points C and  $T_b$  are not the same;

```



### 3.3 Illustration

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

#### 4.1 OGP - Wu method

### 4.1.1 Proving $M_b = -M_b$

Proving failed

#### **4.1.2 Proving $H_a = \neg H_a$**

Proving failed

#### **4.1.3 Proving $T_b = \neg T_b$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $M_b = \neg M_b$**

Proving failed

#### **4.2.2 Proving $H_a = \neg H_a$**

Proving failed

#### **4.2.3 Proving $T_b = \neg T_b$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $M_b = \neg M_b$**

Proving failed

#### **4.3.2 Proving $H_a = \neg H_a$**

Proving failed

#### **4.3.3 Proving $T_b = \neg T_b$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $M_b = \neg M_b$**

Proving failed

#### **4.4.2 Proving $H_a = \neg H_a$**

Proving failed

#### **4.4.3 Proving $T_b = \neg T_b$**

Proving failed

## Problem 418

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 418: Given a point  $M_b$ , a point  $H_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 419

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 419: Given a point  $M_b$ , a point  $H_a$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 420

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 420: Given a point  $M_b$ , a point  $H_b$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $H_b$  are not the same;
2. Using the point  $H_c$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_c$  and  $M_b$  are not the same;
3. Using the circle  $k(M_b, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(M_b, C)$  intersect;
4. Using the point  $H_c$  and the point  $A$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $A$  are not the same;
5. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
6. Using the line  $c$  and the line  $h_b$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $h_b$  are not parallel % DET: lines  $c$  and  $h_b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $h_b$  are not parallel; line  $b$  and circle  $k(M_b, C)$  intersect; points  $H_c$  and  $M_b$  are not the same.

Determination conditions: lines  $c$  and  $h_b$  are not the same; points  $H_c$  and  $A$  are not the same; points  $M_b$  and  $H_b$  are not the same.

Rules used: [W02,W03,W04,W06,W10b]

Lemmas used: [D22,D6,D7,D9,GD01,GD02,GL09,L43,L45]

Solving time: 1.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point H_{b} 89.36 77.83
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_l H_{b}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points M_{b} and H_{b} are not the same
% Constructing a line b which passes through point M_{b} and point H_{b}
line b M_{b} H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points H_{c} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{c}
circle k(M_{b},C) M_{b} H_{c}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line b and circle k(M_{b},C) intersect
% Constructing points C and A which are in intersection of k(M_{b},C) and b
intersec2 C A k(M_{b},C) b
cmark_l C
cmark_t A

% DET: points H_{c} and A are not the same
% Constructing a line c which passes through point H_{c} and point A
line c H_{c} A

color 200 200 200
drawline c
color 0 0 0
```



```

% Constructing a line  $h_{\{b\}}$  which is perpendicular to line  $b$  and which passes through point  $H_{\{b\}}$ 
perp  $h_{\{b\}}$   $H_{\{b\}}$   $b$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: lines  $c$  and  $h_{\{b\}}$  are not parallel% DET: lines  $c$  and  $h_{\{b\}}$  are not the same
% Constructing a point  $B$  which belongs to line  $c$  and line  $h_{\{b\}}$ 
intersec  $B$   $c$   $h_{\{b\}}$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $c$  and  $h_{\{b\}}$  are not parallel; line  $b$  and circle  $k(M_{\{b\}}, C)$ 
% intersect; points  $H_{\{c\}}$  and  $M_{\{b\}}$  are not the same
% Determination conditions: lines  $c$  and  $h_{\{b\}}$  are not the same; points  $H_{\{c\}}$  and  $A$  are not the same
% ; points  $M_{\{b\}}$  and  $H_{\{b\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = \_M_b$

Proving failed

#### 4.1.2 Proving $H_b = \_H_b$

Construction steps:

- Free point  $M_b$
- Free point  $H_b$
- Free point  $H_c$
- Line  $b$  through two points  $M_b$  and  $H_b$
- Circle  $k(M_b, C)$  with center  $M_b$  and one point  $H_c$
- Intersection point  $C$  of point sets  $k(M_b, C)$  and  $b$
- Intersection point  $A$  of point sets  $k(M_b, C)$  and  $b$
- Line  $c$  through two points  $H_c$  and  $A$
- Line  $h_b$  through point  $H_b$  perpendicular to line  $b$
- Intersection point  $B$  of point sets  $c$  and  $h_b$

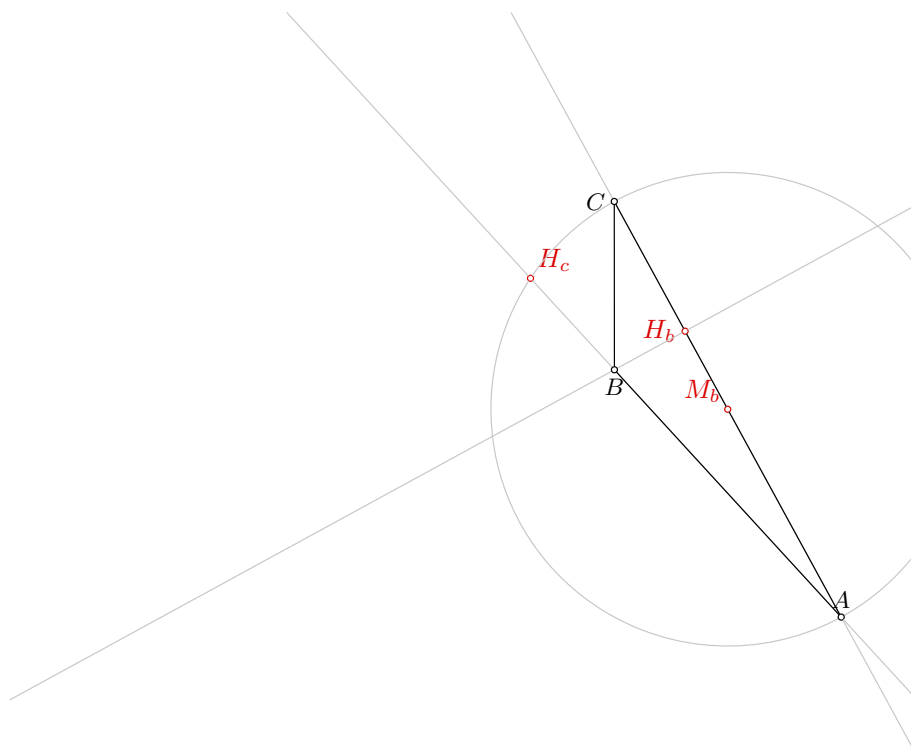


Figure 1: Illustration of the problem 0420

- Segment division point  $M_b$  of segment  $CA$  with division coefficient 1.0
- Line  $l_b$  through two points  $A$  and  $C$
- Line  $h_b$  through point  $B$  perpendicular to line  $l_b$
- Intersection point  $H_b$  of point sets  $l_b$  and  $h_b$
- Line  $l_c$  through two points  $A$  and  $B$
- Line  $h_c$  through point  $C$  perpendicular to line  $l_c$
- Intersection point  $H_c$  of point sets  $l_c$  and  $h_c$

### Theorem statement:

- Points  $H_b$  and  $H_c$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $M_b$  has been assigned following coordinates:  $(0, 0)$

#### 4.1.3 Proving $H_c = H_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b = M_b$

Proving failed

#### 4.2.2 Proving $H_b = H_b$

Proving failed

#### 4.2.3 Proving $H_c = H_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = M_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_b = H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $H_c = H_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 224 terms.

**Time Complexity:** Time spent by the prover is 0.350 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_b = \neg M_b$

Proving failed

### 4.4.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.3 Proving $H_c = \neg H_c$

Proving failed

# Problem 421

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 421: Given a point  $H_b$ , a point  $H$  and a point  $M_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $H_b$  and  $H$  are not the same;
2. Using the point  $H_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
3. Choose freely a point  $M_b$  on the line  $b$  (rule WOnline2);
4. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) ;
5. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
6. Using the point  $A$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H$  are not the same;
7. Using the point  $A$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $A$  and  $M_b$  are not the same;
8. Using the circle  $k(M_b, C)$ , the line  $h_a$ , the point  $M_b$  and the point  $A$ , construct a point  $H_a$  (rule W05); % NDG: line  $h_a$  and circle  $k(M_b, C)$  intersect % DET: points  $A$  and  $H_a$  must be different;
9. Using the point  $C$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $H_a$  are not the same;
10. Using the line  $h_b$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $h_b$  and  $a$  are not parallel % DET: lines  $h_b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $h_b$  and  $a$  are not parallel; line  $h_a$  and circle  $k(M_b, C)$  intersect; points  $A$  and  $M_b$  are not the same.

Determination conditions: lines  $h_b$  and  $a$  are not the same; points  $C$  and  $H_a$  are not the same; points  $A$  and  $H_a$  must be different; points  $A$  and  $H$  are not the same; points  $H_b$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D22,D3,D5,D6,D8,D9,GD01,GD02,GL03,GL04,L43,L44]

Solving time: 31.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{b} 89.36 77.83
```

```
point H 80 72.73
```

```
point M_{b} 95 67.5
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_l H_{b}
```

```
cmark_rt H
```

```
cmark_lt M_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{b} and H are not the same
```

```
% Constructing a line h_{b} which passes through point H_{b} and point H
```

```
line h_{b} H_{b} H
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```
% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
```

```
perp b H_{b} h_{b}
```

```
color 200 200 200
```

```
drawline b
```

```
color 0 0 0
```

```
% Generating random value V[_G62513]
```

```
random V[_G62513]
```

```
% Calculating value V[_G62534] using formula V[_G62513]*20
```

```
expression V[_G62534] { V[_G62513]*20 }
```

```

% Constructing a point  $M_{\{b\}}$  which is a point for which holds  $H_{\{b\}}M_{\{b\}} = V[_G62534]$  and angle  $HH_{\{b\}}M_{\{b\}} = 90$ 
turtle  $M_{\{b\}}$   $H_{\{b\}}$  90  $V[_G62534]$ 
cmark_lt  $M_{\{b\}}$ 

% Choosing randomly a point A on the line  $M_{\{b\}}H_{\{b\}}$ 
online A  $M_{\{b\}}$   $H_{\{b\}}$ 
cmark_t A
color 200 200 200
drawline  $M_{\{b\}}$   $H_{\{b\}}$ 
color 0 0 0

% Constructing a point C such that  $AC/AM_{\{b\}}=2$ 
towards C A  $M_{\{b\}}$  2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points A and H are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point A and point H
line  $h_{\{a\}}$  A H

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: points A and  $M_{\{b\}}$  are not the same
% Constructing a circle  $k(M_{\{b\}},C)$  whose center is at point  $M_{\{b\}}$  and which passes through point A
circle  $k(M_{\{b\}},C)$   $M_{\{b\}}$  A

color 200 200 200
drawcircle  $k(M_{\{b\}},C)$ 
color 0 0 0

% NDG: line  $h_{\{a\}}$  and circle  $k(M_{\{b\}},C)$  intersect% DET: points A and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G63054\}}$  which is a foot of the point  $M_{\{b\}}$  on the line  $h_{\{a\}}$ 
foot  $P_{\{\backslash\_G63054\}}$   $M_{\{b\}}$   $h_{\{a\}}$ 
cmark_r  $P_{\{\backslash\_G63054\}}$ 
color 200 200 200
drawline  $M_{\{b\}}$   $P_{\{\backslash\_G63054\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point A in the symmetry to point/line  $P_{\{\backslash\_G63054\}}$ 
sim  $H_{\{a\}}$   $P_{\{\backslash\_G63054\}}$  A

```

```

cmark_r H_{a}

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines h_{b} and a are not parallel% DET: lines h_{b} and a are not the same
% Constructing a point B which belongs to line h_{b} and line a
intersec B h_{b} a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{b} and a are not parallel; line h_{a} and circle k(M_{b},C)
% intersect; points A and M_{b} are not the same
% Determination conditions: lines h_{b} and a are not the same; points C and H_{a} are not the same
% ; points A and H_{a} must be different; points A and H are not the same; points H_{b} and H are
% not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_b = H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.145 seconds.

**NDG conditions** Points  $H_b$  and  $A$  are not identical

Points  $H_b$  and  $H$  are not identical

Line through points  $H_b$  and  $C$  is not perpendicular to line through points  $C$  and  $A$

Line through points  $H_b$  and  $H$  is not parallel with line through points  $A$  and  $C$

#### 4.1.2 Proving $H = H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 20 terms.



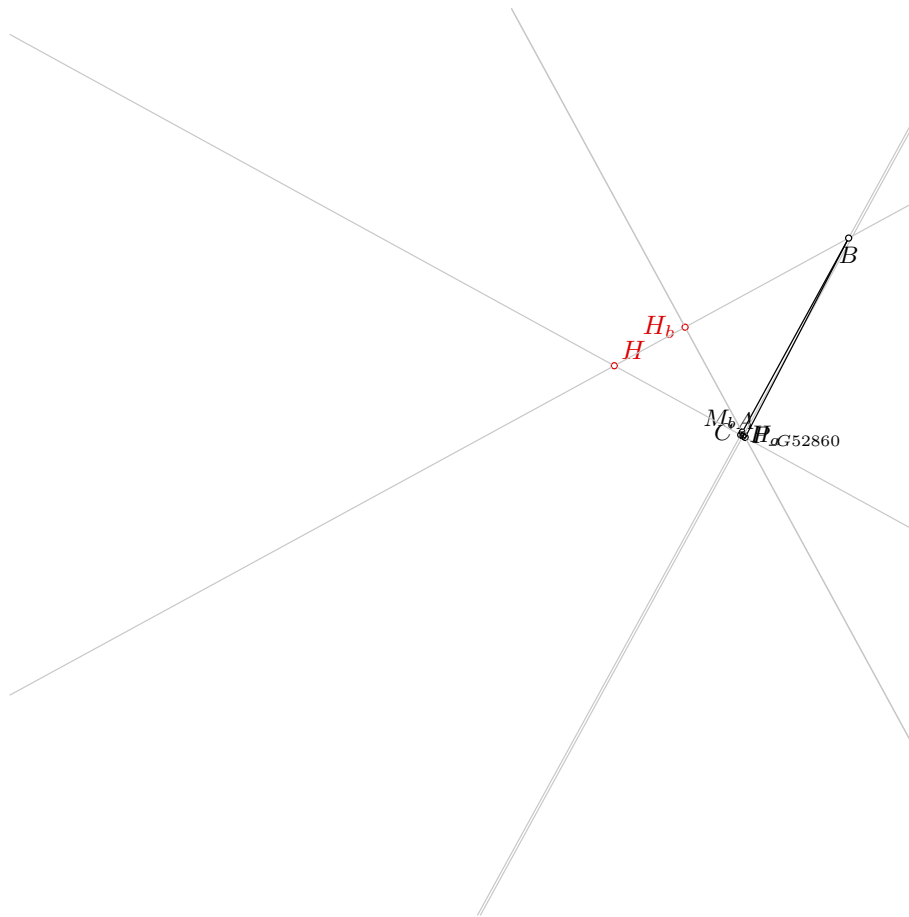


Figure 1: Illustration of the problem 0421

**Time Complexity:** Time spent by the prover is 0.313 seconds.

**NDG conditions** Points  $H_b$  and  $A$  are not identical

Points  $H_b$  and  $H$  are not identical

Line through points  $H_b$  and  $C$  is not perpendicular to line through points  $C$  and  $A$

Line through points  $H_b$  and  $H$  is not parallel with line through points  $A$  and  $C$

Line through points  $H$  and  $B$  is not perpendicular to line through points  $B$  and  $M_b$

#### 4.1.3 Proving $M_b = \neg M_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_b = \neg H_b$

Proving failed

#### 4.2.2 Proving $H = \neg H$

Proving failed

#### 4.2.3 Proving $M_b = \neg M_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_b = \neg H_b$

Proving failed

#### 4.3.2 Proving $H = \neg H$

Proving failed

#### 4.3.3 Proving $M_b = \neg M_b$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $H_b = \neg H_b$

Proving failed

#### 4.4.2 Proving $H = \neg H$

Proving failed

#### 4.4.3 Proving $M_b = \neg M_b$

Proving failed

# Problem 422

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 422: Given a point  $M_b$ , a point  $H_b$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 423

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 423: Given a point  $H_b$ , a point  $T_b$  and a point  $M_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $T_b$  are not the same;
2. Choose freely a point  $M_b$  on the line  $b$  (rule WOnline1) ;
3. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
4. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) ;
5. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
6. Choose freely a point  $B$  on the line  $h_b$  (rule WOnline2).

Non-degenerate conditions: .

Determination conditions: points  $H_b$  and  $T_b$  are not the same.

Rules used: [W01,W02,W10b,WOnline1,WOnline2]

Lemmas used: [D22,D24,D6,D9,GL03,GL04]

Solving time: 1897.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{b} 89.36 77.83
```

```
point T_{b} 94.25 68.88
```

```
point M_{b} 95 67.5
```

```
color 220 0 0
```

```

fontsize 9

cmark_l H_{b}
cmark_t T_{b}
cmark_lt M_{b}
color 0 0 0
fontsize 8

% DET: points H_{b} and T_{b} are not the same
% Constructing a line b which passes through point H_{b} and point T_{b}
line b H_{b} T_{b}

color 200 200 200
drawline b
color 0 0 0


% Choosing randomly a point M_{b} on the line H_{b}T_{b}
online M_{b} H_{b} T_{b}
cmark_lt M_{b}
color 200 200 200
drawline H_{b} T_{b}
color 0 0 0


% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0


% Choosing randomly a point A on the line M_{b}H_{b}
online A M_{b} H_{b}
cmark_t A
color 200 200 200
drawline M_{b} H_{b}
color 0 0 0


% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

```

```

% Generating random value V[_G57058]
random V[_G57058]

% Calculating value V[_G57079] using formula V[_G57058]*20
expression V[_G57079] { V[_G57058]*20 }

% Constructing a point B which is a point for which holds  $H_{\{b\}B} = V[_G57079]$  and angle  $AH_{\{b\}B} = 90$ 
turtle B A  $H_{\{b\}}$  90 V[_G57079]
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions: points  $H_{\{b\}}$  and  $T_{\{b\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_b = \_H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.065 seconds.

**NDG conditions** Line through points  $A$  and  $B$  is not perpendicular to line through points  $B$  and  $C$

#### 4.1.2 Proving $T_b = \_T_b$

Proving failed

#### 4.1.3 Proving $M_b = \_M_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_b = \_H_b$

Proving failed

#### 4.2.2 Proving $T_b = \_T_b$

Proving failed

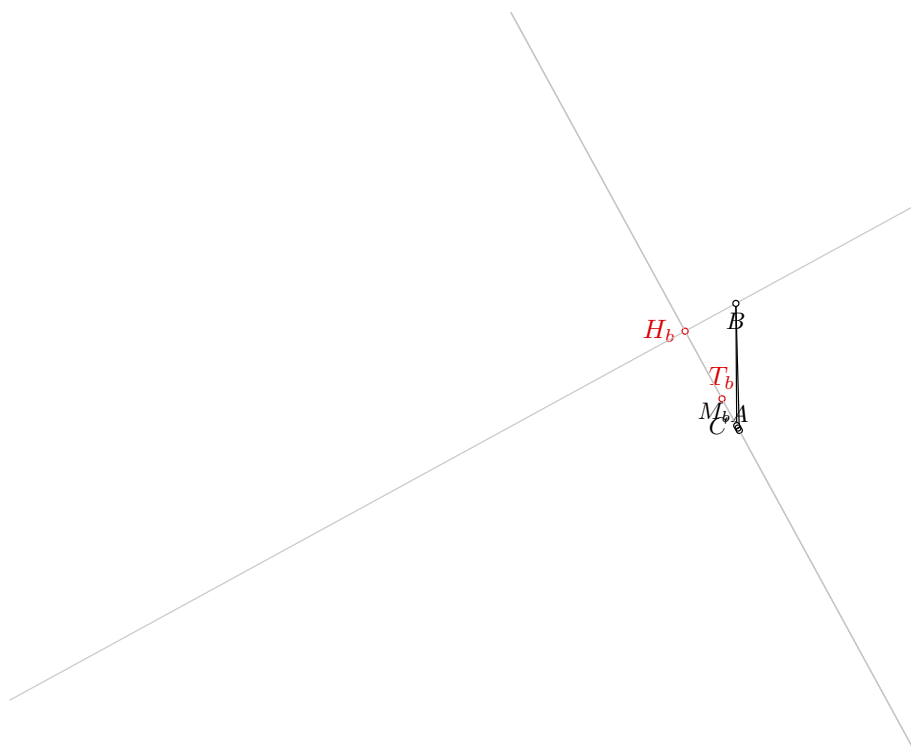


Figure 1: Illustration of the problem 0423

### 4.2.3 Proving $M_b = \neg M_b$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $H_b = \neg H_b$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 1 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $T_b = \neg T_b$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $H_b = \neg H_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $T_b = \neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.3 Proving $M_b = \neg M_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.



# Problem 424

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 424: Given a point  $M_b$ , a point  $H_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 425

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 425: Given a point  $M_b$ , a point  $H_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $H_b$  are not the same;
2. Using the point  $M_b$  and the point  $I$ , construct a line  $IM_b$  (rule W02); % DET: points  $M_b$  and  $I$  are not the same;
3. Using the point  $I$  and the point  $M_b$ , construct a circle  $k_{over}(I, M_b)$  (rule W09); % NDG: points  $I$  and  $M_b$  are not the same;
4. Using the point  $M_b$  and the line  $b$ , construct a line  $m_b$  (rule W10b); ;
5. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
6. Using the point  $I$  and the line  $b$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $b$ ;
7. Using the circle  $k(I, P_a)$  and the circle  $k_{over}(I, M_b)$ , construct a point  $B_{fi}$  and a point  $P_b$  (rule W07); % NDG: circles  $k(I, P_a)$  and  $k_{over}(I, M_b)$  intersect % DET: circles  $k(I, P_a)$  and  $k_{over}(I, M_b)$  are not the same;
8. Using the point  $P_b$  and the point  $M_b$ , construct a point  $P'_b$  (rule W01); ;
9. Using the point  $P'_b$  and the line  $IM_b$ , construct a line  $BP'_b$  (rule W16); ;
10. Using the line  $BP'_b$  and the line  $h_b$ , construct a point  $B$  (rule W03); % NDG: lines  $BP'_b$  and  $h_b$  are not parallel % DET: lines  $BP'_b$  and  $h_b$  are not the same;
11. Using the point  $I$  and the point  $B$ , construct a line  $s_b$  (rule W02); % DET: points  $I$  and  $B$  are not the same;

12. Using the line  $m_b$  and the line  $s_b$ , construct a point  $N_b$  (rule W03); % NDG: lines  $m_b$  and  $s_b$  are not parallel % DET: lines  $m_b$  and  $s_b$  are not the same;
13. Using the point  $I$  and the point  $N_b$ , construct a circle  $k(N_b, A)$  (rule W06); % NDG: points  $I$  and  $N_b$  are not the same;
14. Using the circle  $k(N_b, A)$  and the line  $b$ , construct a point  $A$  and a point  $C$  (rule W04); % NDG: line  $b$  and circle  $k(N_b, A)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(N_b, A)$  intersect; points  $I$  and  $N_b$  are not the same; lines  $m_b$  and  $s_b$  are not parallel; lines  $BP'_b$  and  $h_b$  are not parallel; circles  $k(I, P_a)$  and  $k_{over}(I, M_b)$  intersect; point  $I$  is not incident to the line  $b$ ; points  $I$  and  $M_b$  are not the same.

Determination conditions: lines  $m_b$  and  $s_b$  are not the same; points  $I$  and  $B$  are not the same; lines  $BP'_b$  and  $h_b$  are not the same; circles  $k(I, P_a)$  and  $k_{over}(I, M_b)$  are not the same; points  $M_b$  and  $I$  are not the same; points  $M_b$  and  $H_b$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W07,W09,W10b,W11,W16]

Lemmas used: [D12,D2,D22,D27,D48,D6,D66,D9,GD01,GD02,GL03,GL09,L13,L26,L33,L34,L6,L61,L63,L75]

Solving time: 5.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{b} 95 67.5
point H_{b} 89.36 77.83
point I 74.37 61.15
```

```
color 220 0 0
fontsize 9
```

```
cmark_lt M_{b}
cmark_l H_{b}
cmark_b I
color 0 0 0
fontsize 8
```

```
% DET: points M_{b} and H_{b} are not the same
% Constructing a line b which passes through point M_{b} and point H_{b}
line b M_{b} H_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% DET: points M_{b} and I are not the same
% Constructing a line IM_{b} which passes through point M_{b} and point I
line IM_{b} M_{b} I
```

```
color 200 200 200
drawline IM_{b}
```

```
color 0 0 0
```

```
% NDG: points I and M_{b} are not the same
% Constructing midpoint P_{\_G64085} of the segment IM_{b}
midpoint P_{\_G64085} I M_{b}
cmark_r P_{\_G64085}
```

```
% Constructing a circle k_{over(I,M_{b})} whose center is at point P_{\_G64085} and which passes
    through point I
circle k_{over(I,M_{b})} P_{\_G64085} I
```

```
color 200 200 200
drawcircle k_{over(I,M_{b})}
color 0 0 0
```

```
% Constructing a line m_{b} which is perpendicular to line b and which passes through point M_{b}
perp m_{b} M_{b} b
```

```
color 200 200 200
drawline m_{b}
color 0 0 0
```

```
% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b
```

```
color 200 200 200
drawline h_{b}
color 0 0 0
```

```
% NDG: point I is not incident to the line b
% Constructing a point P_{\_G64383} which is a foot of the point I on the line b
foot P_{\_G64383} I b
cmark_r P_{\_G64383}
color 200 200 200
drawline I P_{\_G64383}
color 0 0 0
```

```
% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
64383}
circle k(I,P_{a}) I P_{\_G64383}
```

```
color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0
```

```

% NDG: circles  $k(I, P_{\{a\}})$  and  $k_{\text{over}}(I, M_{\{b\}})$  intersect% DET: circles  $k(I, P_{\{a\}})$  and  $k_{\text{over}}(I, M_{\{b\}})$ 
%) are not the same
% Constructing points  $B_{\{fi\}}$  and  $P_{\{b\}}$  which are in intersection of  $k(I, P_{\{a\}})$  and  $k_{\text{over}}(I, M_{\{b\}})$ 
intersec2 B_{fi} P_{b} k(I, P_{a}) k_{\text{over}}(I, M_{b})
cmark_r B_{fi}
cmark_r P_{b}

% Constructing a point  $P'_{\{b\}}$  such that  $P_{\{b\}}P'_{\{b\}}/P_{\{b\}}M_{\{b\}}=2$ 
towards P'_{b} P_{b} M_{b} 2
cmark_r P'_{b}
color 200 200 200
drawsegment P_{b} P'_{b}
color 0 0 0

% Constructing a line  $BP'_{\{b\}}$  which contains the point  $P'_{\{b\}}$  and is parallel to the line  $IM_{\{b\}}$ 
parallel BP'_{b} P'_{b} IM_{b}

color 200 200 200
drawline BP'_{b}
color 0 0 0

% NDG: lines  $BP'_{\{b\}}$  and  $h_{\{b\}}$  are not parallel% DET: lines  $BP'_{\{b\}}$  and  $h_{\{b\}}$  are not the same
% Constructing a point  $B$  which belongs to line  $BP'_{\{b\}}$  and line  $h_{\{b\}}$ 
intersec B BP'_{b} h_{b}
cmark_b B

% DET: points  $I$  and  $B$  are not the same
% Constructing a line  $s_{\{b\}}$  which passes through point  $I$  and point  $B$ 
line s_{b} I B

color 200 200 200
drawline s_{b}
color 0 0 0

% NDG: lines  $m_{\{b\}}$  and  $s_{\{b\}}$  are not parallel% DET: lines  $m_{\{b\}}$  and  $s_{\{b\}}$  are not the same
% Constructing a point  $N_{\{b\}}$  which belongs to line  $m_{\{b\}}$  and line  $s_{\{b\}}$ 
intersec N_{b} m_{b} s_{b}
cmark_rb N_{b}

% NDG: points  $I$  and  $N_{\{b\}}$  are not the same
% Constructing a circle  $k(N_{\{b\}}, A)$  whose center is at point  $N_{\{b\}}$  and which passes through point  $I$ 
circle k(N_{b}, A) N_{b} I

color 200 200 200
drawcircle k(N_{b}, A)

```

```
color 0 0 0
```

```
% NDG: line b and circle k(N_{b},A) intersect
% Constructing points A and C which are in intersection of k(N_{b},A) and b
intersec2 A C k(N_{b},A) b
cmark_t A
cmark_l C
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line b and circle k(N_{b},A) intersect; points I and N_{b} are not the
same; lines m_{b} and s_{b} are not parallel; lines BP'_{b} and h_{b} are not parallel;
circles k(I,P_{a}) and k_{over}(I,M_{b}) intersect; point I is not incident to the line b; points
I and M_{b} are not the same
% Determination conditions: lines m_{b} and s_{b} are not the same; points I and B are not the same
; lines BP'_{b} and h_{b} are not the same; circles k(I,P_{a}) and k_{over}(I,M_{b}) are not the
same; points M_{b} and I are not the same; points M_{b} and H_{b} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

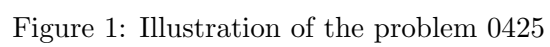
#### 4.1.1 Proving $M_b = \_M_b$

Proving failed

#### 4.1.2 Proving $H_b = \_H_b$

Construction steps:

- Free point  $M_b$
- Free point  $H_b$
- Free point  $I$
- Line  $b$  through two points  $M_b$  and  $H_b$
- Line  $IM_b$  through two points  $M_b$  and  $I$
- Midpoint  $P_{G59108}$  of segment  $IM_b$
- Circle  $k_{over}(I, M_b)$  with center  $P_{G59108}$  and one point  $I$
- Line  $m_b$  through point  $M_b$  perpendicular to line  $b$
- Line  $h_b$  through point  $H_b$  perpendicular to line  $b$
- Line footPointPerpLine345 through point  $I$  perpendicular to line  $b$
- Intersection point  $P_{G59234}$  of point sets footPointPerpLine345 and  $b$



- Circle  $k(I, P_a)$  with center  $I$  and one point  $P_{G59234}$
- Intersection point  $B_{fi}$  of point sets  $k(I, P_a)$  and  $k_{over}(I, M_b)$
- Intersection point  $P_b$  of point sets  $k(I, P_a)$  and  $k_{over}(I, M_b)$
- Segment division point  $P'_b$  of segment  $P_bM_b$  with division coefficient -2.0
- Line  $BP'_b$  through point  $P'_b$  parallel with line  $IM_b$
- Intersection point  $B$  of point sets  $BP'_b$  and  $h_b$
- Line  $s_b$  through two points  $I$  and  $B$
- Intersection point  $N_b$  of point sets  $m_b$  and  $s_b$
- Circle  $k(N_b, A)$  with center  $N_b$  and one point  $I$
- Intersection point  $A$  of point sets  $k(N_b, A)$  and  $b$
- Intersection point  $C$  of point sets  $k(N_b, A)$  and  $b$
- Segment division point  $_M_b$  of segment  $CA$  with division coefficient 1.0
- Line  $_b$  through two points  $A$  and  $C$
- Line  $_h_b$  through point  $B$  perpendicular to line  $_b$
- Intersection point  $_H_b$  of point sets  $_b$  and  $_h_b$
- Line  $_c$  through two points  $A$  and  $B$
- Line  $_a$  through two points  $B$  and  $C$
- Angle bisector  $_s_a$  of angle  $BAC$
- Angle bisector  $_s_b$  of angle  $CBA$
- Intersection point  $_I$  of point sets  $_s_a$  and  $_s_b$

### Theorem statement:

- Points  $H_b$  and  $_H_b$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $M_b$  has been assigned following coordinates:  $(0, 0)$

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b=_M_b$

Proving failed

#### 4.2.2 Proving $H_b=_H_b$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b=_M_b$

Proving failed



#### **4.3.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.3.3 Proving $I = \neg I$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $M_b = \neg M_b$**

Proving failed

#### **4.4.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.4.3 Proving $I = \neg I$**

Proving failed

# Problem 426

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 426: Given a point  $M_b$ , a point  $H_c$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $H_c$  and  $H$  are not the same;
2. Using the point  $H_c$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_c$  and  $M_b$  are not the same;
3. Using the circle  $k(M_b, C)$ , the line  $h_c$ , the point  $M_b$  and the point  $H_c$ , construct a point  $C$  (rule W05); % NDG: line  $h_c$  and circle  $k(M_b, C)$  intersect % DET: points  $H_c$  and  $C$  must be different;
4. Using the point  $C$  and the point  $M_b$ , construct a point  $A$  (rule W01); ;
5. Using the point  $H_c$  and the point  $A$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $A$  are not the same;
6. Using the point  $H$  and the point  $A$ , construct a line  $h_a$  (rule W02); % DET: points  $H$  and  $A$  are not the same;
7. Using the circle  $k(M_b, C)$ , the line  $h_a$ , the point  $M_b$  and the point  $A$ , construct a point  $H_a$  (rule W05); % NDG: line  $h_a$  and circle  $k(M_b, C)$  intersect % DET: points  $A$  and  $H_a$  must be different;
8. Using the point  $C$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $H_a$  are not the same;
9. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; line  $h_a$  and circle  $k(M_b, C)$  intersect; line  $h_c$  and circle  $k(M_b, C)$  intersect; points  $H_c$  and  $M_b$  are not the same.

Determination conditions: lines  $c$  and  $a$  are not the same; points  $C$  and  $H_a$  are not the same; points  $A$  and  $H_a$  must be different; points  $H$  and  $A$  are not the same; points  $H_c$  and  $A$  are not the same; points  $H_c$  and  $C$  must be different; points  $H_c$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D22,D3,D5,D7,D8,GD01,GD02,GL03,L3,L43,L44,L45]

Solving time: 1.6 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{b} 95 67.5
```

```
point H_{c} 68.91 84.83
```

```
point H 80 72.73
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_lt M_{b}
```

```
cmark_rt H_{c}
```

```
cmark_rt H
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{c} and H are not the same
```

```
% Constructing a line h_{c} which passes through point H_{c} and point H
```

```
line h_{c} H_{c} H
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```
% NDG: points H_{c} and M_{b} are not the same
```

```
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{c}
```

```
circle k(M_{b},C) M_{b} H_{c}
```

```
color 200 200 200
```

```
drawcircle k(M_{b},C)
```

```
color 0 0 0
```

```
% NDG: line h_{c} and circle k(M_{b},C) intersect% DET: points H_{c} and C must be different
```

```
% Constructing a point P_{\_G61683} which is a foot of the point M_{b} on the line h_{c}
```

```
foot P_{\_G61683} M_{b} h_{c}
```

```
cmark_r P_{\_G61683}
```

```
color 200 200 200
```

```
drawline M_{b} P_{\_G61683}
```

```
color 0 0 0
```

```

% Constructing a point C which is an image of the point H_{c} in the symmetry to point/line P_{\_G
61683}
sim C P_{\_G61683} H_{c}
cmark_l C

% Constructing a point A such that CA/CM_{b}=2
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

% DET: points H_{c} and A are not the same
% Constructing a line c which passes through point H_{c} and point A
line c H_{c} A

color 200 200 200
drawline c
color 0 0 0

% DET: points H and A are not the same
% Constructing a line h_{a} which passes through point H and point A
line h_{a} H A

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{a} and circle k(M_{b},C) intersect% DET: points A and H_{a} must be different
% Constructing a point P_{\_G62046} which is a foot of the point M_{b} on the line h_{a}
foot P_{\_G62046} M_{b} h_{a}
cmark_r P_{\_G62046}
color 200 200 200
drawline M_{b} P_{\_G62046}
color 0 0 0

% Constructing a point H_{a} which is an image of the point A in the symmetry to point/line P_{\_G
62046}
sim H_{a} P_{\_G62046} A
cmark_r H_{a}

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a

```

```
color 0 0 0
```

```
% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines c and a are not parallel; line h_{a} and circle k(M_{b},C)
    intersect; line h_{c} and circle k(M_{b},C) intersect; points H_{c} and M_{b} are not the same
% Determination conditions: lines c and a are not the same; points C and H_{a} are not the same;
    points A and H_{a} must be different; points H and A are not the same; points H_{c} and A are
    not the same; points H_{c} and C must be different; points H_{c} and H are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = \_M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.165 seconds.

**NDG conditions** Points  $H_c$  and  $H$  are not identical

Points  $H_c$  and  $H$  are not identical

#### 4.1.2 Proving $H_c = \_H_c$

Proving failed

#### 4.1.3 Proving $H = \_H$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b = \_M_b$

NDG conditions are:

$S_{H_cCH_a} \neq S_{ACH_a}$  i.e., lines  $H_cA$  and  $CH_a$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{-h_c}^0} \neq S_{BCF_{-h_c}^0}$  i.e., lines  $AB$  and  $CF_{-h_c}^0$  are not parallel (construction based assumption)

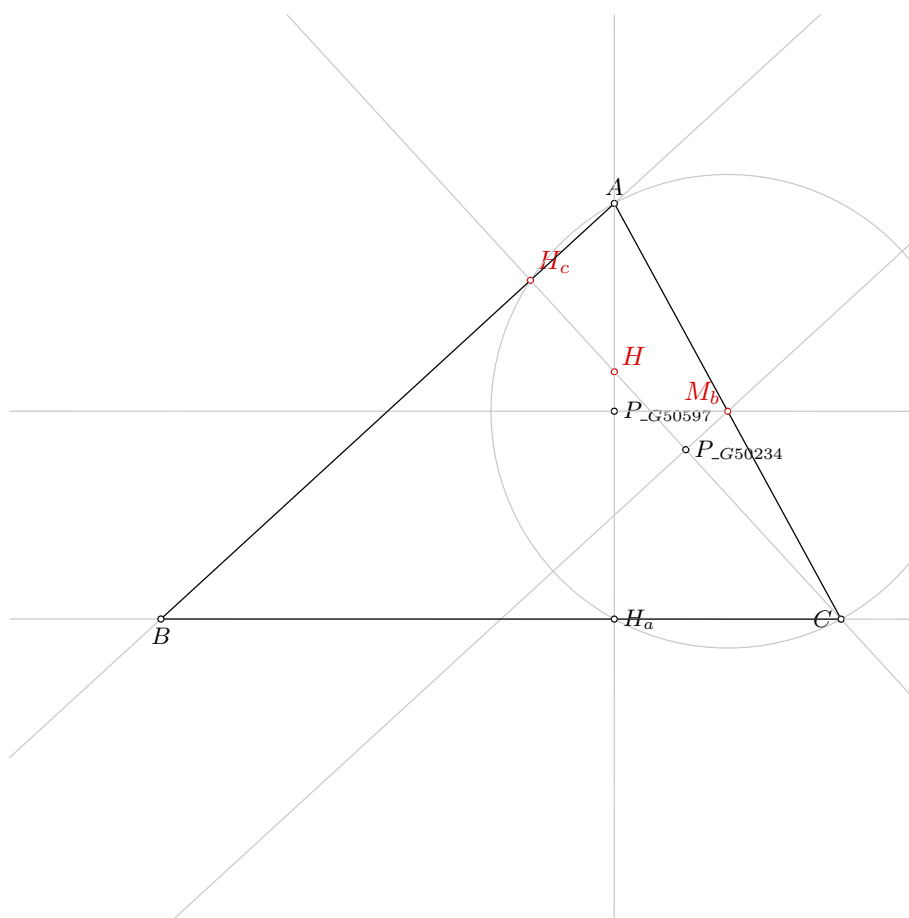


Figure 1: Illustration of the problem 0426

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^2_{-h_b}} \neq S_{F^1_{-h_a}BF^2_{-h_b}}$  i.e., lines  $AF^1_{-h_a}$  and  $BF^2_{-h_b}$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c = \_H_c$

Proving failed

#### 4.2.3 Proving $H = \_H$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = \_M_b$

Proving failed

#### 4.3.2 Proving $H_c = \_H_c$

Proving failed

#### 4.3.3 Proving $H = \_H$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_b = \_M_b$

Proving failed

#### 4.4.2 Proving $H_c = \_H_c$

Proving failed

#### 4.4.3 Proving $H = \_H$

Proving failed

## Problem 427

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 427: Given a point  $M_b$ , a point  $H_c$  and a point  $T_a$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.



# Problem 428

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 428: Given a point  $M_b$ , a point  $H_c$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $T_b$  are not the same;
2. Using the point  $H_c$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_c$  and  $M_b$  are not the same;
3. Using the circle  $k(M_b, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(M_b, C)$  intersect;
4. Using the point  $H_c$  and the point  $A$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $A$  are not the same;
5. Using the point  $A$ , the point  $C$ , the point  $T_b$  and the line  $b$ , construct a point  $T'_b$  (rule W19); % NDG: points  $A$  and  $C$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $\text{midpoint}([A, T_b])$  are not the same;
6. Using the point  $T_b$  and the point  $T'_b$ , construct a circle  $k_{over}(T_b, T'_b)$  (rule W09); % NDG: points  $T_b$  and  $T'_b$  are not the same;
7. Using the circle  $k_{over}(T_b, T'_b)$  and the line  $c$ , construct a point  $B_{wa}$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k_{over}(T_b, T'_b)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k_{over}(T_b, T'_b)$  intersect; points  $T_b$  and  $T'_b$  are not the same; points  $A$  and  $C$  are not the same; points  $C$  and  $T_b$  are not the same; points  $C$  and  $\text{midpoint}([A, T_b])$  are not the same; line  $b$  and circle  $k(M_b, C)$  intersect; points  $H_c$  and  $M_b$  are not the same.

Determination conditions: points  $H_c$  and  $A$  are not the same; points  $M_b$  and  $T_b$  are not the same.

Rules used: [W02,W04,W06,W09,W19]

Lemmas used: [D22,D24,D55,D7,GD01,GD02,GL09,L43,L45,L73,L78]

Solving time: 2709.5 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point H_{c} 68.91 84.83
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_rt H_{c}
cmark_t T_{b}
color 0 0 0
fontsize 8

% DET: points M_{b} and T_{b} are not the same
% Constructing a line b which passes through point M_{b} and point T_{b}
line b M_{b} T_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points H_{c} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{c}
circle k(M_{b},C) M_{b} H_{c}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line b and circle k(M_{b},C) intersect
% Constructing points C and A which are in intersection of k(M_{b},C) and b
intersec2 C A k(M_{b},C) b
cmark_l C
cmark_t A

% DET: points H_{c} and A are not the same
% Constructing a line c which passes through point H_{c} and point A
line c H_{c} A

color 200 200 200
drawline c
```

```
color 0 0 0
```

```
% NDG: points A and C are not the same; points C and T_{b} are not the same; points C and midpoint
      ([A,T_{b}]) are not the same
% Constructing a point P_{\_G43852} which is an image of the point C in a rotation around the point
      T_{b} for the angle 90
rotate P_{\_G43852} T_{b} 90 C
cmark_r P_{\_G43852}
color 200 200 200
drawarc_p T_{b} C 90
color 0 0 0
```

```
% Constructing a line L_{\_G43855} which passes through point T_{b} and point P_{\_G43852}
line L_{\_G43855} T_{b} P_{\_G43852}
```

```
color 200 200 200
drawline L_{\_G43855}
color 0 0 0
```

```
% Constructing midpoint P_{\_G43858} of the segment CP_{\_G43852}
midpoint P_{\_G43858} C P_{\_G43852}
cmark_r P_{\_G43858}
```

```
% Constructing a line L_{\_G43861} which passes through point A and point P_{\_G43858}
line L_{\_G43861} A P_{\_G43858}
```

```
color 200 200 200
drawline L_{\_G43861}
color 0 0 0
```

```
% Constructing a line L_{\_G43864} which passes through point C and point P_{\_G43858}
line L_{\_G43864} C P_{\_G43858}
```

```
color 200 200 200
drawline L_{\_G43864}
color 0 0 0
```

```
% Constructing a point P_{\_G43867} which belongs to line L_{\_G43855} and line L_{\_G43861}
intersec P_{\_G43867} L_{\_G43855} L_{\_G43861}
cmark_r P_{\_G43867}
```

```
% Constructing a point P_{\_G43870} which belongs to line L_{\_G43855} and line L_{\_G43864}
intersec P_{\_G43870} L_{\_G43855} L_{\_G43864}
cmark_r P_{\_G43870}
```

```
% Constructing a line L_{\_G43873} which passes through point A and point P_{\_G43870}
line L_{\_G43873} A P_{\_G43870}
```

```
color 200 200 200
drawline L_{\_G43873}
color 0 0 0
```

```

% Constructing a line  $L_{\{ \_G43876 \}}$  which passes through point C and point  $P_{\{ \_G43867 \}}$ 
line  $L_{\{ \_G43876 \}}$  C  $P_{\{ \_G43867 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G43876 \}}$ 
color 0 0 0

% Constructing a point  $P_{\{ \_G43879 \}}$  which belongs to line  $L_{\{ \_G43873 \}}$  and line  $L_{\{ \_G43876 \}}$ 
intersec  $P_{\{ \_G43879 \}}$   $L_{\{ \_G43873 \}}$   $L_{\{ \_G43876 \}}$ 
cmark_r  $P_{\{ \_G43879 \}}$ 

% Constructing a line  $L_{\{ \_G43882 \}}$  which passes through point  $P_{\{ \_G43858 \}}$  and point  $P_{\{ \_G43879 \}}$ 
line  $L_{\{ \_G43882 \}}$   $P_{\{ \_G43858 \}}$   $P_{\{ \_G43879 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G43882 \}}$ 
color 0 0 0

% Constructing a point  $T'_{\{b\}}$  which belongs to line  $L_{\{ \_G43882 \}}$  and line b
intersec  $T'_{\{b\}}$   $L_{\{ \_G43882 \}}$  b
cmark_r  $T'_{\{b\}}$ 

% NDG: points  $T_{\{b\}}$  and  $T'_{\{b\}}$  are not the same
% Constructing midpoint  $P_{\{ \_G44713 \}}$  of the segment  $T_{\{b\}}T'_{\{b\}}$ 
midpoint  $P_{\{ \_G44713 \}}$   $T_{\{b\}}$   $T'_{\{b\}}$ 
cmark_r  $P_{\{ \_G44713 \}}$ 

% Constructing a circle  $k_{\text{over}}(T_{\{b\}}, T'_{\{b\}})$  whose center is at point  $P_{\{ \_G44713 \}}$  and which passes
    through point  $T_{\{b\}}$ 
circle  $k_{\text{over}}(T_{\{b\}}, T'_{\{b\}})$   $P_{\{ \_G44713 \}}$   $T_{\{b\}}$ 

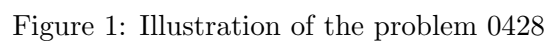
color 200 200 200
drawcircle  $k_{\text{over}}(T_{\{b\}}, T'_{\{b\}})$ 
color 0 0 0

% NDG: line c and circle  $k_{\text{over}}(T_{\{b\}}, T'_{\{b\}})$  intersect
% Constructing points  $B_{\{wa\}}$  and B which are in intersection of  $k_{\text{over}}(T_{\{b\}}, T'_{\{b\}})$  and c
intersec2  $B_{\{wa\}}$   $B$   $k_{\text{over}}(T_{\{b\}}, T'_{\{b\}})$  c
cmark_r  $B_{\{wa\}}$ 
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle  $k_{\text{over}}(T_{\{b\}}, T'_{\{b\}})$  intersect; points  $T_{\{b\}}$  and  $T'_{\{b\}}$  are not the same; points A and C are not the same; points C and  $T_{\{b\}}$  are not the same;

```



### 3.3 Illustration

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

## 4.1 OGP - Wu method

### 4.1.1 Proving $M_b = -M_b$

Proving failed

#### **4.1.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.1.3 Proving $T_b = \neg T_b$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $M_b = \neg M_b$**

Proving failed

#### **4.2.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.2.3 Proving $T_b = \neg T_b$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $M_b = \neg M_b$**

Proving failed

#### **4.3.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.3.3 Proving $T_b = \neg T_b$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $M_b = \neg M_b$**

Proving failed

#### **4.4.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.4.3 Proving $T_b = \neg T_b$**

Proving failed

# Problem 429

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 429: Given a point  $M_b$ , a point  $H_c$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $T_c$  are not the same;
2. Using the point  $H_c$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_c$  and  $M_b$  are not the same;
3. Using the circle  $k(M_b, C)$ , the line  $c$ , the point  $M_b$  and the point  $H_c$ , construct a point  $A$  (rule W05); % NDG: line  $c$  and circle  $k(M_b, C)$  intersect % DET: points  $H_c$  and  $A$  must be different;
4. Using the point  $A$  and the point  $M_b$ , construct a point  $C$  (rule W01); ;
5. Using the point  $M_b$  and the point  $A$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $A$  are not the same;
6. Using the point  $T_c$  and the line  $b$ , construct a circle  $k(T_c, foot[T_c, b])$  (rule W11); % NDG: point  $T_c$  is not incident to the line  $b$ ;
7. Using the circle  $k(T_c, foot[T_c, b])$ , the point  $C$ , the point  $T_c$  and the line  $b$ , construct a line  $a$  (rule W13); % NDG: point  $C$  is outside the circle  $k(T_c, foot[T_c, b])$ ;
8. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; point  $C$  is outside the circle  $k(T_c, foot[T_c, b])$ ; point  $T_c$  is not incident to the line  $b$ ; line  $c$  and circle  $k(M_b, C)$  intersect; points  $H_c$  and  $M_b$  are not the same.

Determination conditions: lines  $a$  and  $c$  are not the same; points  $M_b$  and  $A$  are not the same; points  $H_c$  and  $A$  must be different; points  $H_c$  and  $T_c$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W11,W13]

Lemmas used: [D22,D25,D7,GD01,GD02,GL03,GL04,GL09,GL10,GL11,L10,L43,L45]

Solving time: 1.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point H_{c} 68.91 84.83
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_rt H_{c}
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points H_{c} and T_{c} are not the same
% Constructing a line c which passes through point H_{c} and point T_{c}
line c H_{c} T_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points H_{c} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{c}
circle k(M_{b},C) M_{b} H_{c}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line c and circle k(M_{b},C) intersect% DET: points H_{c} and A must be different
% Constructing a point P_{\_G41800} which is a foot of the point M_{b} on the line c
foot P_{\_G41800} M_{b} c
cmark_r P_{\_G41800}
color 200 200 200
drawline M_{b} P_{\_G41800}
color 0 0 0

% Constructing a point A which is an image of the point H_{c} in the symmetry to point/line P_{\_G41800}
```



```

sim A P_{\_G41800} H_{c}
cmark_t A

% Constructing a point C such that  $AC/AM_{\{b\}}=2$ 
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points  $M_{\{b\}}$  and A are not the same
% Constructing a line b which passes through point  $M_{\{b\}}$  and point A
line b M_{b} A

color 200 200 200
drawline b
color 0 0 0

% NDG: point  $T_{\{c\}}$  is not incident to the line b
% Constructing a point  $P_{\{\_G42084\}}$  which is a foot of the point  $T_{\{c\}}$  on the line b
foot P_{\_G42084} T_{c} b
cmark_r P_{\_G42084}
color 200 200 200
drawline T_{c} P_{\_G42084}
color 0 0 0

% Constructing a circle  $k(T_{\{c\}}, foot[T_{\{c\}}, b])$  whose center is at point  $T_{\{c\}}$  and which passes
    through point  $P_{\{\_G42084\}}$ 
circle k(T_{c}, foot[T_{c}, b]) T_{c} P_{\_G42084}

color 200 200 200
drawcircle k(T_{c}, foot[T_{c}, b])
color 0 0 0

% NDG: point C is outside the circle  $k(T_{\{c\}}, foot[T_{\{c\}}, b])$ 
% Constructing a point  $P_{\{\_G42475\}}$  which is a foot of the point  $T_{\{c\}}$  on the line b
foot P_{\_G42475} T_{c} b
cmark_r P_{\_G42475}
color 200 200 200
drawline T_{c} P_{\_G42475}
color 0 0 0

% Constructing a line  $L_{\{\_G42478\}}$  which passes through point  $T_{\{c\}}$  and point C
line L_{\_G42478} T_{c} C

color 200 200 200
drawline L_{\_G42478}
color 0 0 0

```

```

% Constructing a point  $P_{\{G42481\}}$  which is an image of the point  $P_{\{G42475\}}$  in the symmetry to
    point/line  $L_{\{G42478\}}$ 
sim  $P_{\{G42481\}}$   $L_{\{G42478\}}$   $P_{\{G42475\}}$ 
cmark_r  $P_{\{G42481\}}$ 

% Constructing a line  $a$  which passes through point  $C$  and point  $P_{\{G42481\}}$ 
line a C  $P_{\{G42481\}}$ 

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $a$  and  $c$  are not parallel% DET: lines  $a$  and  $c$  are not the same
% Constructing a point  $B$  which belongs to line  $a$  and line  $c$ 
intersec B a c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; point  $C$  is outside the circle  $k(T_{\{c\}},$ 
     $foot[T_{\{c\}},b])$ ; point  $T_{\{c\}}$  is not incident to the line  $b$ ; line  $c$  and circle  $k(M_{\{b\}},C)$ 
    intersect; points  $H_{\{c\}}$  and  $M_{\{b\}}$  are not the same
% Determination conditions: lines  $a$  and  $c$  are not the same; points  $M_{\{b\}}$  and  $A$  are not the same;
    points  $H_{\{c\}}$  and  $A$  must be different; points  $H_{\{c\}}$  and  $T_{\{c\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = M_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.171 seconds.

**NDG conditions** Points  $H_c$  and  $T_c$  are not identical

Points  $H_c$  and  $T_c$  are not identical

#### 4.1.2 Proving $H_c = H_c$

Proving failed

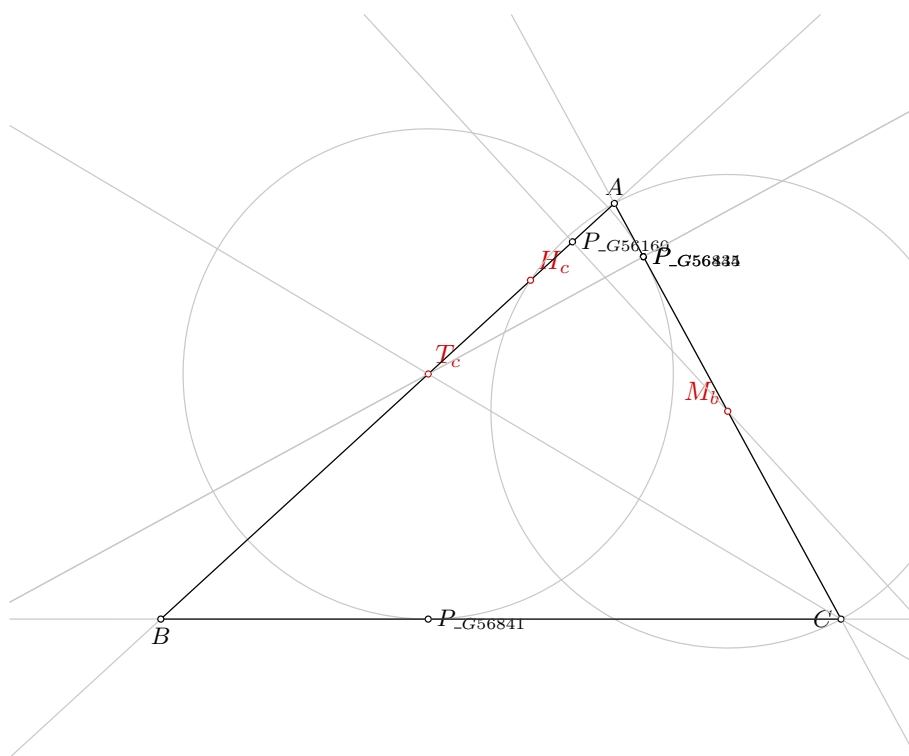


Figure 1: Illustration of the problem 0429

#### **4.1.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $M_b = \neg M_b$**

Proving failed

#### **4.2.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.2.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $M_b = \neg M_b$**

Proving failed

#### **4.3.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.3.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $M_b = \neg M_b$**

Proving failed

#### **4.4.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.4.3 Proving $T_c = \neg T_c$**

Proving failed

# Problem 430

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 430: Given a point  $M_b$ , a point  $H_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 431

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 431: Given a point  $M_b$ , a point  $H$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 432

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 432: Given a point  $M_b$ , a point  $H$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 433

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 433: Given a point  $M_b$ , a point  $H$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.



# Problem 434

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 434: Given a point  $M_b$ , a point  $H$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 435

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 435: Given a point  $M_b$ , a point  $T_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 436

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 436: Given a point  $M_b$ , a point  $T_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 437

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 437: Given a point  $M_b$ , a point  $T_a$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 438

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 438: Given a point  $M_b$ , a point  $T_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 439

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 439: Given a point  $M_b$ , a point  $T_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $M_b$  and  $T_b$  are not the same;
2. Using the point  $T_b$  and the point  $I$ , construct a line  $s_b$  (rule W02); % DET: points  $T_b$  and  $I$  are not the same;
3. Using the point  $M_b$  and the line  $b$ , construct a line  $m_b$  (rule W10b); ;
4. Using the line  $m_b$  and the line  $s_b$ , construct a point  $N_b$  (rule W03); % NDG: lines  $m_b$  and  $s_b$  are not parallel % DET: lines  $m_b$  and  $s_b$  are not the same;
5. Using the point  $I$  and the point  $N_b$ , construct a circle  $k(N_b, A)$  (rule W06); % NDG: points  $I$  and  $N_b$  are not the same;
6. Using the circle  $k(N_b, A)$  and the line  $b$ , construct a point  $A$  and a point  $C$  (rule W04); % NDG: line  $b$  and circle  $k(N_b, A)$  intersect;
7. Using the point  $I$  and the line  $b$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $b$ ;
8. Using the circle  $k(I, P_a)$ , the point  $A$ , the point  $I$  and the line  $b$ , construct a line  $c$  (rule W13); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
9. Using the line  $c$  and the line  $s_b$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $s_b$  are not parallel % DET: lines  $c$  and  $s_b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $s_b$  are not parallel; point  $A$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $b$ ; line  $b$  and circle  $k(N_b, A)$  intersect; points  $I$  and  $N_b$  are not the same; lines  $m_b$  and  $s_b$  are not parallel.

Determination conditions: lines  $c$  and  $s_b$  are not the same; lines  $m_b$  and  $s_b$  are not the same; points  $T_b$  and  $I$  are not the same; points  $M_b$  and  $T_b$  are not the same.

Rules used: [W02,W03,W04,W06,W10b,W11,W13]

Lemmas used: [D12,D2,D22,D24,D27,D48,GD01,GD02,GL09,L26,L33,L34,L59,L6,L61]

Solving time: 12.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{b} 95 67.5
```

```
point T_{b} 94.25 68.88
```

```
point I 74.37 61.15
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_lt M_{b}
```

```
cmark_t T_{b}
```

```
cmark_b I
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points M_{b} and T_{b} are not the same
```

```
% Constructing a line b which passes through point M_{b} and point T_{b}
```

```
line b M_{b} T_{b}
```

```
color 200 200 200
```

```
drawline b
```

```
color 0 0 0
```

```
% DET: points T_{b} and I are not the same
```

```
% Constructing a line s_{b} which passes through point T_{b} and point I
```

```
line s_{b} T_{b} I
```

```
color 200 200 200
```

```
drawline s_{b}
```

```
color 0 0 0
```

```
% Constructing a line m_{b} which is perpendicular to line b and which passes through point M_{b}
```

```
perp m_{b} M_{b} b
```

```
color 200 200 200
```

```
drawline m_{b}
```

```
color 0 0 0
```

```

% NDG: lines  $m_{\{b\}}$  and  $s_{\{b\}}$  are not parallel% DET: lines  $m_{\{b\}}$  and  $s_{\{b\}}$  are not the same
% Constructing a point  $N_{\{b\}}$  which belongs to line  $m_{\{b\}}$  and line  $s_{\{b\}}$ 
intersec  $N_{\{b\}}$   $m_{\{b\}}$   $s_{\{b\}}$ 
cmark_rb  $N_{\{b\}}$ 

% NDG: points  $I$  and  $N_{\{b\}}$  are not the same
% Constructing a circle  $k(N_{\{b\}},A)$  whose center is at point  $N_{\{b\}}$  and which passes through point  $I$ 
circle  $k(N_{\{b\}},A)$   $N_{\{b\}}$   $I$ 

color 200 200 200
drawcircle  $k(N_{\{b\}},A)$ 
color 0 0 0

% NDG: line  $b$  and circle  $k(N_{\{b\}},A)$  intersect
% Constructing points  $A$  and  $C$  which are in intersection of  $k(N_{\{b\}},A)$  and  $b$ 
intersec2  $A$   $C$   $k(N_{\{b\}},A)$   $b$ 
cmark_t  $A$ 
cmark_l  $C$ 

% NDG: point  $I$  is not incident to the line  $b$ 
% Constructing a point  $P_{\{\backslash\_G46606\}}$  which is a foot of the point  $I$  on the line  $b$ 
foot  $P_{\{\backslash\_G46606\}}$   $I$   $b$ 
cmark_r  $P_{\{\backslash\_G46606\}}$ 
color 200 200 200
drawline  $I$   $P_{\{\backslash\_G46606\}}$ 
color 0 0 0

% Constructing a circle  $k(I,P_{\{a\}})$  whose center is at point  $I$  and which passes through point  $P_{\{\backslash\_G46606\}}$ 
circle  $k(I,P_{\{a\}})$   $I$   $P_{\{\backslash\_G46606\}}$ 

color 200 200 200
drawcircle  $k(I,P_{\{a\}})$ 
color 0 0 0

% NDG: point  $A$  is outside the circle  $k(I,P_{\{a\}})$ 
% Constructing a point  $P_{\{\backslash\_G46813\}}$  which is a foot of the point  $I$  on the line  $b$ 
foot  $P_{\{\backslash\_G46813\}}$   $I$   $b$ 
cmark_r  $P_{\{\backslash\_G46813\}}$ 
color 200 200 200
drawline  $I$   $P_{\{\backslash\_G46813\}}$ 
color 0 0 0

% Constructing a line  $L_{\{\backslash\_G46816\}}$  which passes through point  $I$  and point  $A$ 
line  $L_{\{\backslash\_G46816\}}$   $I$   $A$ 

color 200 200 200
drawline  $L_{\{\backslash\_G46816\}}$ 

```



```

color 0 0 0

% Constructing a point  $P_{G46819}$  which is an image of the point  $P_{G46813}$  in the symmetry to
    point/line  $L_{G46816}$ 
sim  $P_{G46819}$   $L_{G46816}$   $P_{G46813}$ 
cmark_r  $P_{G46819}$ 

% Constructing a line  $c$  which passes through point  $A$  and point  $P_{G46819}$ 
line c A  $P_{G46819}$ 

color 200 200 200
drawline c
color 0 0 0

% NDG: lines  $c$  and  $s_b$  are not parallel% DET: lines  $c$  and  $s_b$  are not the same
% Constructing a point  $B$  which belongs to line  $c$  and line  $s_b$ 
intersec B c  $s_b$ 
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $c$  and  $s_b$  are not parallel; point  $A$  is outside the circle  $k(I, P_a)$ ;
    point  $I$  is not incident to the line  $b$ ; line  $b$  and circle  $k(N_b, A)$  intersect; points  $I$  and  $N_b$ 
    are not the same; lines  $m_b$  and  $s_b$  are not parallel
% Determination conditions: lines  $c$  and  $s_b$  are not the same; lines  $m_b$  and  $s_b$  are not the
    same; points  $T_b$  and  $I$  are not the same; points  $M_b$  and  $T_b$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_b = M_b$

Proving failed

#### 4.1.2 Proving $T_b = T_b$

Construction steps:

- Free point  $M_b$
- Free point  $T_b$
- Free point  $I$

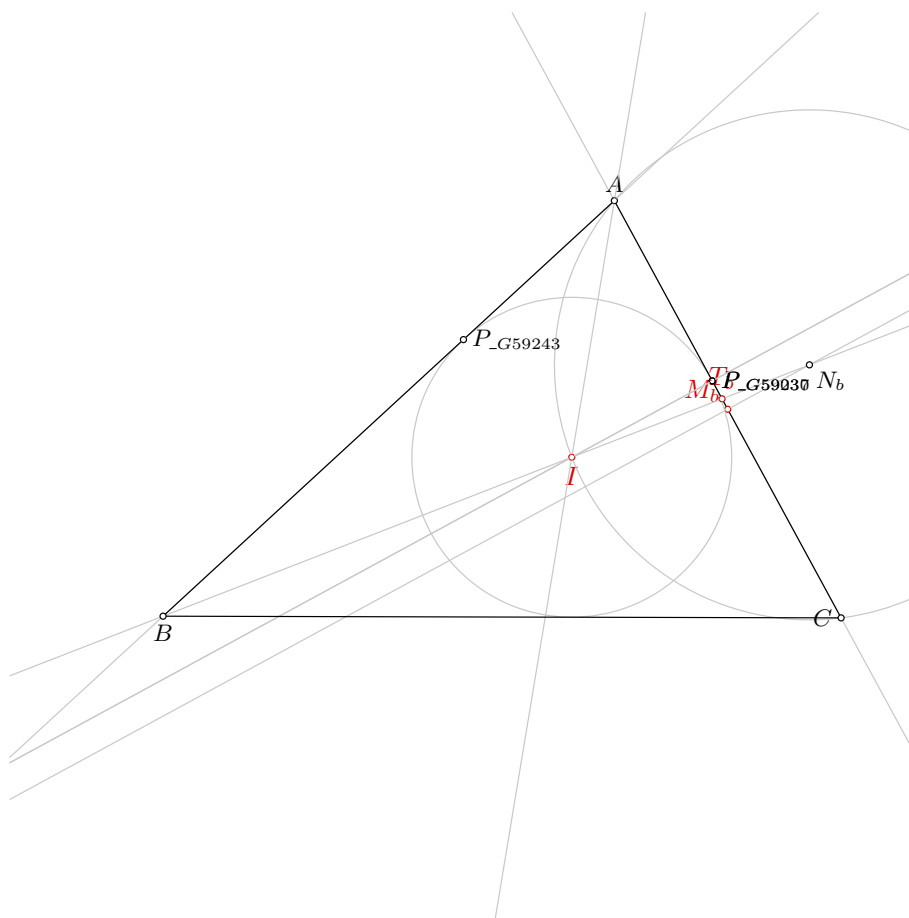


Figure 1: Illustration of the problem 0439

- Line  $b$  through two points  $M_b$  and  $T_b$
- Line  $s_b$  through two points  $T_b$  and  $I$
- Line  $m_b$  through point  $M_b$  perpendicular to line  $b$
- Intersection point  $N_b$  of point sets  $m_b$  and  $s_b$
- Circle  $k(N_b, A)$  with center  $N_b$  and one point  $I$
- Intersection point  $A$  of point sets  $k(N_b, A)$  and  $b$
- Intersection point  $C$  of point sets  $k(N_b, A)$  and  $b$
- Line footPointPerpLine899 through point  $I$  perpendicular to line  $b$
- Intersection point  $P_{G42149}$  of point sets footPointPerpLine899 and  $b$
- Circle  $k(I, P_a)$  with center  $I$  and one point  $P_{G42149}$
- Line footPointPerpLine692 through point  $I$  perpendicular to line  $b$
- Intersection point  $P_{G42252}$  of point sets footPointPerpLine692 and  $b$
- Line  $L_{G42255}$  through two points  $I$  and  $A$
- Line reflexivePointPerpLine265 through point  $P_{G42252}$  perpendicular to line  $L_{G42255}$
- Intersection point reflexivePointFootPoint787 of point sets reflexivePointPerpLine265 and  $L_{G42255}$
- Cental symmetric point  $P_{G42258}$  of point  $P_{G42252}$  with respect to center of symmetry reflexivePointFootPoint787
- Line  $c$  through two points  $A$  and  $P_{G42258}$
- Intersection point  $B$  of point sets  $c$  and  $s_b$
- Segment division point  $_M_b$  of segment  $CA$  with division coefficient 1.0
- Line  $_c$  through two points  $A$  and  $B$
- Line  $_a$  through two points  $B$  and  $C$
- Angle bisector  $_s_b$  of angle  $CBA$
- Line  $_b$  through two points  $A$  and  $C$
- Intersection point  $_T_b$  of point sets  $_s_b$  and  $_b$
- Angle bisector  $_s_a$  of angle  $BAC$
- Intersection point  $_I$  of point sets  $_s_a$  and  $_s_b$

### Theorem statement:

- Points  $T_b$  and  $_T_b$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $M_b$  has been assigned following coordinates: (0, 0)

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_b=_M_b$

Proving failed

#### 4.2.2 Proving $T_b=_T_b$

Proving failed

#### 4.2.3 Proving $I = \neg I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_b = \neg M_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 22 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

#### 4.3.2 Proving $T_b = \neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 28 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I = \neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 1738 terms.

**Time Complexity:** Time spent by the prover is 0.990 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_b = \neg M_b$

Proving failed

#### 4.4.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.4.3 Proving $I = \neg I$

Proving failed

# Problem 440

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 440: Given a point  $M_b$ , a point  $T_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 441

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 441: Given a point  $M_c$ , a point  $G$  and a point  $H_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $G$ , construct a point  $C$  (rule W01); ;
2. Using the point  $H_a$  and the point  $C$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $C$  are not the same;
3. Using the point  $H_a$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_a$  and  $M_c$  are not the same;
4. Using the circle  $k(M_c, A)$ , the line  $a$ , the point  $M_c$  and the point  $H_a$ , construct a point  $B$  (rule W05); % NDG: line  $a$  and circle  $k(M_c, A)$  intersect % DET: points  $H_a$  and  $B$  must be different;
5. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: line  $a$  and circle  $k(M_c, A)$  intersect; points  $H_a$  and  $M_c$  are not the same.

Determination conditions: points  $H_a$  and  $B$  must be different; points  $H_a$  and  $C$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D20,D5,GD01,GD02,GL03,GL04,L40,L42,L57]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```

dim 120 120

point M_{c} 50 67.5
point G 70 58.33
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_t G
cmark_r H_{a}
color 0 0 0
fontsize 8

% Constructing a point C such that  $M_{\{c\}}C/M_{\{c\}}G=3$ 
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

% DET: points  $H_{\{a\}}$  and  $C$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $C$ 
line a H_{a} C

color 200 200 200
drawline a
color 0 0 0

% NDG: points  $H_{\{a\}}$  and  $M_{\{c\}}$  are not the same
% Constructing a circle  $k(M_{\{c\}}, A)$  whose center is at point  $M_{\{c\}}$  and which passes through point  $H_{\{a\}}$ 
circle k(M_{c}, A) M_{c} H_{a}

color 200 200 200
drawcircle k(M_{c}, A)
color 0 0 0

% NDG: line  $a$  and circle  $k(M_{\{c\}}, A)$  intersect% DET: points  $H_{\{a\}}$  and  $B$  must be different
% Constructing a point  $P_{\{\backslash\_G44229\}}$  which is a foot of the point  $M_{\{c\}}$  on the line  $a$ 
foot P_{\_G44229} M_{c} a
cmark_r P_{\_G44229}
color 200 200 200
drawline M_{c} P_{\_G44229}
color 0 0 0

% Constructing a point  $B$  which is an image of the point  $H_{\{a\}}$  in the symmetry to point/line  $P_{\{\backslash\_G44229\}}$ 
sim B P_{\_G44229} H_{a}

```

```
cmark_b B
```

```
% Constructing a point A such that BA/BM_{c}=2
towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(M_{c},A) intersect; points H_{a} and M_{c} are not
the same
% Determination conditions: points H_{a} and B must be different; points H_{a} and C are not the
same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.123 seconds.

**NDG conditions** Points  $C$  and  $H_a$  are not identical

Line through points  $C$  and  $M_c$  is not perpendicular to line through points  $M_c$  and  $H_a$

#### 4.1.2 Proving $G = G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 599 terms.

**Time Complexity:** Time spent by the prover is 1.425 seconds.

**NDG conditions** Points  $C$  and  $H_a$  are not identical

Line through points  $C$  and  $M_c$  is not perpendicular to line through points  $M_c$  and  $H_a$

Line through points  $A$  and  $M_a$  is not parallel with line through points  $B$  and  $M_b$

Line through points  $A$  and  $M_a$  is not parallel with line through points  $M_c$  and  $G$



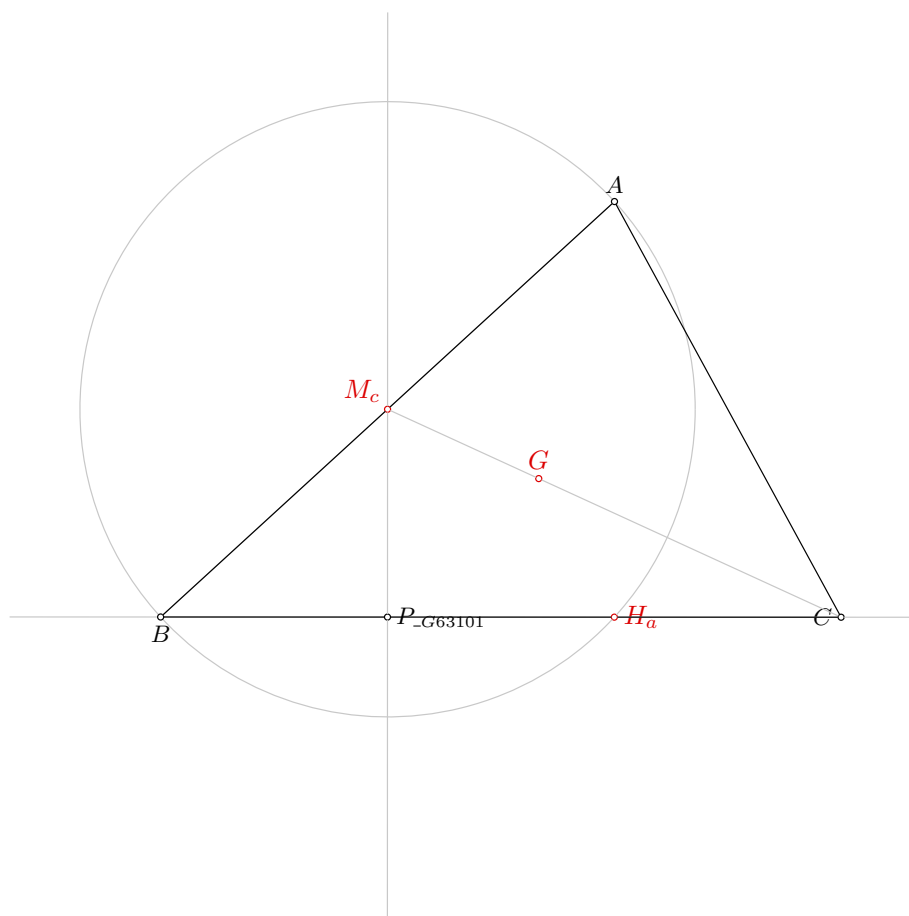


Figure 1: Illustration of the problem 0441

### 4.1.3 Proving $H_a = H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 174 terms.

**Time Complexity:** Time spent by the prover is 1.136 seconds.

**NDG conditions** Points  $C$  and  $H_a$  are not identical

Line through points  $C$  and  $M_c$  is not perpendicular to line through points  $M_c$  and  $H_a$

Points  $B$  and  $C$  are not identical

Points  $B$ ,  $C$  and  $M_c$  are not collinear

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_c = M_c$

NDG conditions are:

$S_{AB-M_b} \neq S_{-M_a B - M_b}$  i.e., lines  $A-M_a$  and  $B-M_b$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{-h_a}^0} \neq S_{CAF_{-h_a}^0}$  i.e., lines  $BC$  and  $AF_{-h_a}^0$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

### 4.2.2 Proving $G = G$

Proving failed

### 4.2.3 Proving $H_a = H_a$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_c = M_c$

Proving failed

### 4.3.2 Proving $G = G$

Proving failed

### 4.3.3 Proving $H_a = H_a$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_c = M_c$

Proving failed

### 4.4.2 Proving $G = G$

Proving failed

### 4.4.3 Proving $H_a = H_a$

Proving failed

# Problem 442

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 442: Given a point  $M_c$ , a point  $G$  and a point  $H_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $G$ , construct a point  $C$  (rule W01); ;
2. Using the point  $H_b$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $C$  are not the same;
3. Using the point  $H_b$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_b$  and  $M_c$  are not the same;
4. Using the circle  $k(M_c, A)$ , the line  $b$ , the point  $M_c$  and the point  $H_b$ , construct a point  $A$  (rule W05); % NDG: line  $b$  and circle  $k(M_c, A)$  intersect % DET: points  $H_b$  and  $A$  must be different;
5. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: line  $b$  and circle  $k(M_c, A)$  intersect; points  $H_b$  and  $M_c$  are not the same.

Determination conditions: points  $H_b$  and  $A$  must be different; points  $H_b$  and  $C$  are not the same.

Rules used: [W01,W02,W05,W06]

Lemmas used: [D20,D6,GD01,GD02,GL03,GL04,L41,L57]

Solving time: 0.1 seconds.

### 3.2 Construction in GCLC language

```

dim 120 120

point M_{c} 50 67.5
point G 70 58.33
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_t G
cmark_l H_{b}
color 0 0 0
fontsize 8

% Constructing a point C such that  $M_{\{c\}}C/M_{\{c\}}G=3$ 
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

% DET: points  $H_{\{b\}}$  and  $C$  are not the same
% Constructing a line b which passes through point  $H_{\{b\}}$  and point C
line b H_{b} C

color 200 200 200
drawline b
color 0 0 0

% NDG: points  $H_{\{b\}}$  and  $M_{\{c\}}$  are not the same
% Constructing a circle  $k(M_{\{c\}}, A)$  whose center is at point  $M_{\{c\}}$  and which passes through point  $H_{\{b\}}$ 
circle k(M_{c}, A) M_{c} H_{b}

color 200 200 200
drawcircle k(M_{c}, A)
color 0 0 0

% NDG: line b and circle  $k(M_{\{c\}}, A)$  intersect% DET: points  $H_{\{b\}}$  and  $A$  must be different
% Constructing a point  $P_{\{\backslash\_G63516\}}$  which is a foot of the point  $M_{\{c\}}$  on the line b
foot P_{\backslash\_G63516} M_{c} b
cmark_r P_{\backslash\_G63516}
color 200 200 200
drawline M_{c} P_{\backslash\_G63516}
color 0 0 0

% Constructing a point A which is an image of the point  $H_{\{b\}}$  in the symmetry to point/line  $P_{\{\backslash\_G63516\}}$ 
sim A P_{\backslash\_G63516} H_{b}

```

```
cmark_t A
```

```
% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line b and circle k(M_{c},A) intersect; points H_{b} and M_{c} are not
the same
% Determination conditions: points H_{b} and A must be different; points H_{b} and C are not the
same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.171 seconds.

**NDG conditions** Points  $H_b$  and  $C$  are not identical

Line through points  $H_b$  and  $M_c$  is not perpendicular to line through points  $M_c$  and  $C$

#### 4.1.2 Proving $G = G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 599 terms.

**Time Complexity:** Time spent by the prover is 1.3 seconds.

**NDG conditions** Points  $H_b$  and  $C$  are not identical

Line through points  $H_b$  and  $M_c$  is not perpendicular to line through points  $M_c$  and  $C$

Line through points  $A$  and  $M_a$  is not parallel with line through points  $B$  and  $M_b$

Line through points  $A$  and  $M_a$  is not parallel with line through points  $M_c$  and  $G$

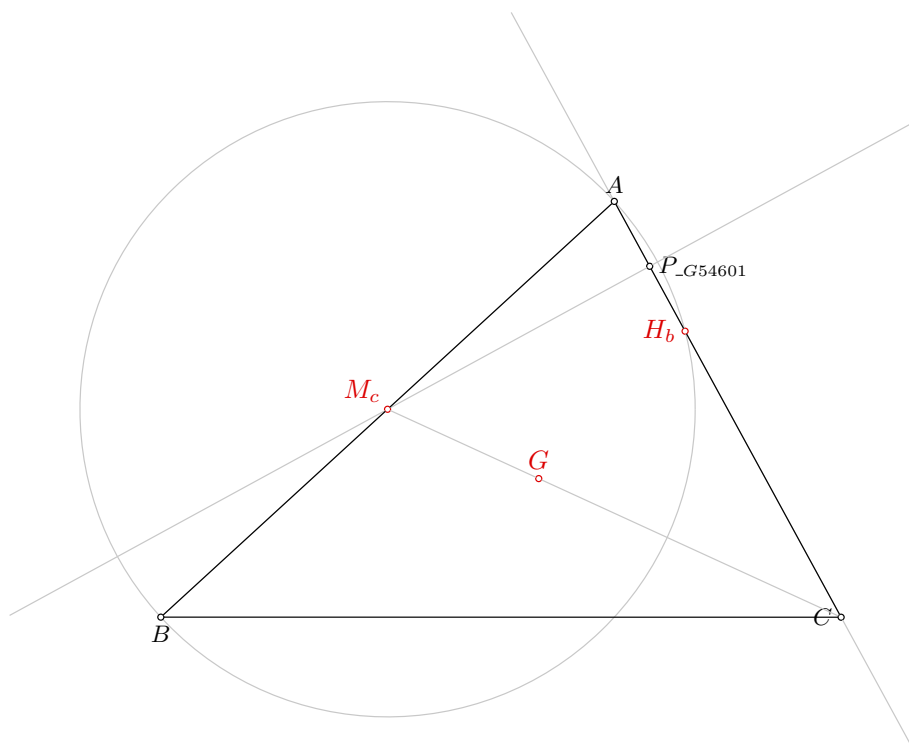


Figure 1: Illustration of the problem 0442

#### 4.1.3 Proving $H_b = \neg H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 174 terms.

**Time Complexity:** Time spent by the prover is 1.159 seconds.

**NDG conditions** Points  $H_b$  and  $C$  are not identical

Line through points  $H_b$  and  $M_c$  is not perpendicular to line through points  $M_c$  and  $C$

Points  $A$  and  $C$  are not identical

Points  $A$  and  $M_c$  are not identical

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{AB \neg M_b} \neq S_{\neg M_a B \neg M_b}$  i.e., lines  $A \neg M_a$  and  $B \neg M_b$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg H_b}^0} \neq S_{CBF_{\neg H_b}^0}$  i.e., lines  $AC$  and  $BF_{\neg H_b}^0$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $G = \neg G$

Proving failed

#### 4.2.3 Proving $H_b = \neg H_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c = \neg M_c$

Proving failed

#### 4.3.2 Proving $G = \neg G$

Proving failed

#### 4.3.3 Proving $H_b = \neg H_b$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_c = \neg M_c$

Proving failed

#### 4.4.2 Proving $G = \neg G$

Proving failed

#### 4.4.3 Proving $H_b = \neg H_b$

Proving failed

# Problem 443

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 443: Given a point  $M_c$ , a point  $H_c$  and a point  $G$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $H_c$  are not the same;
2. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
3. Using the point  $M_c$ , the line  $h_c$  and the point  $H_c$ , construct a line  $h_{M_c,1/3}(h_c)$  (rule W15); ;
4. Choose freely a point  $G$  on the line  $h_{M_c,1/3}(h_c)$  (rule WOnline4);
5. Using the point  $G$  and the point  $M_c$ , construct a point  $C$  (rule W01); ;
6. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) ;
7. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); .

Non-degenerate conditions: .

Determination conditions: points  $M_c$  and  $H_c$  are not the same.

Rules used: [W01,W02,W10b,W15,WOnline1,WOnline4]

Lemmas used: [D10,D20,D7,GD01,GL03,GL04,GL09,L57]

Solving time: 190.3 seconds.

### 3.2 Construction in GCLC language

dim 120 120

```
point M_{c} 50 67.5
point H_{c} 68.91 84.83
point G 70 58.33
```



```
color 220 0 0
fontsize 9
```

```
cmark_lt M_{c}
cmark_rt H_{c}
cmark_t G
color 0 0 0
fontsize 8
```

```
% DET: points M_{c} and H_{c} are not the same
% Constructing a line c which passes through point M_{c} and point H_{c}
line c M_{c} H_{c}
```

```
color 200 200 200
drawline c
color 0 0 0
```

```
% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c
```

```
color 200 200 200
drawline h_{c}
color 0 0 0
```

```
% Constructing a point P_{\_G57795} such that M_{c}P_{\_G57795}/M_{c}H_{c}=0.3333333333333333
towards P_{\_G57795} M_{c} H_{c} 0.3333333333333333
cmark_r P_{\_G57795}
color 200 200 200
drawsegment M_{c} H_{c}
color 0 0 0
```

```
% Constructing a line h_{M_{c},1/3}(h_{c}) which contains the point P_{\_G57795} and is parallel to
the line h_{c}
parallel h_{M_{c},1/3}(h_{c}) P_{\_G57795} h_{c}
```

```
color 200 200 200
drawline h_{M_{c},1/3}(h_{c})
color 0 0 0
```

```
% Constructing a point P_{\_G58117} such that M_{c}P_{\_G58117}/M_{c}H_{c}=0.3333333333333333
towards P_{\_G58117} M_{c} H_{c} 0.3333333333333333
cmark_r P_{\_G58117}
color 200 200 200
drawsegment M_{c} H_{c}
color 0 0 0
```

```

% Generating random value V[_G58056]
random V[_G58056]

% Calculating value V[_G58077] using formula V[_G58056]*20
expression V[_G58077] { V[_G58056]*20 }

% Constructing a point G which is a point for which holds  $P_{\backslash\_G58117}G = V[_G58077]$  and angle  $M_{\{c\}}$ 
 $P_{\backslash\_G58117}G = 90$ 
turtle G  $M_{\{c\}}$   $P_{\backslash\_G58117}$  90 V[_G58077]
cmark_t G

% Constructing a point C such that  $GC/GM_{\{c\}}=-2$ 
towards C G  $M_{\{c\}}$  -2
cmark_l C
color 200 200 200
drawsegment  $M_{\{c\}}$  C
color 0 0 0

% Choosing randomly a point A on the line  $M_{\{c\}}H_{\{c\}}$ 
online A  $M_{\{c\}}$   $H_{\{c\}}$ 
cmark_t A
color 200 200 200
drawline  $M_{\{c\}}$   $H_{\{c\}}$ 
color 0 0 0

% Constructing a point B such that  $AB/AM_{\{c\}}=2$ 
towards B A  $M_{\{c\}}$  2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions: points  $M_{\{c\}}$  and  $H_{\{c\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

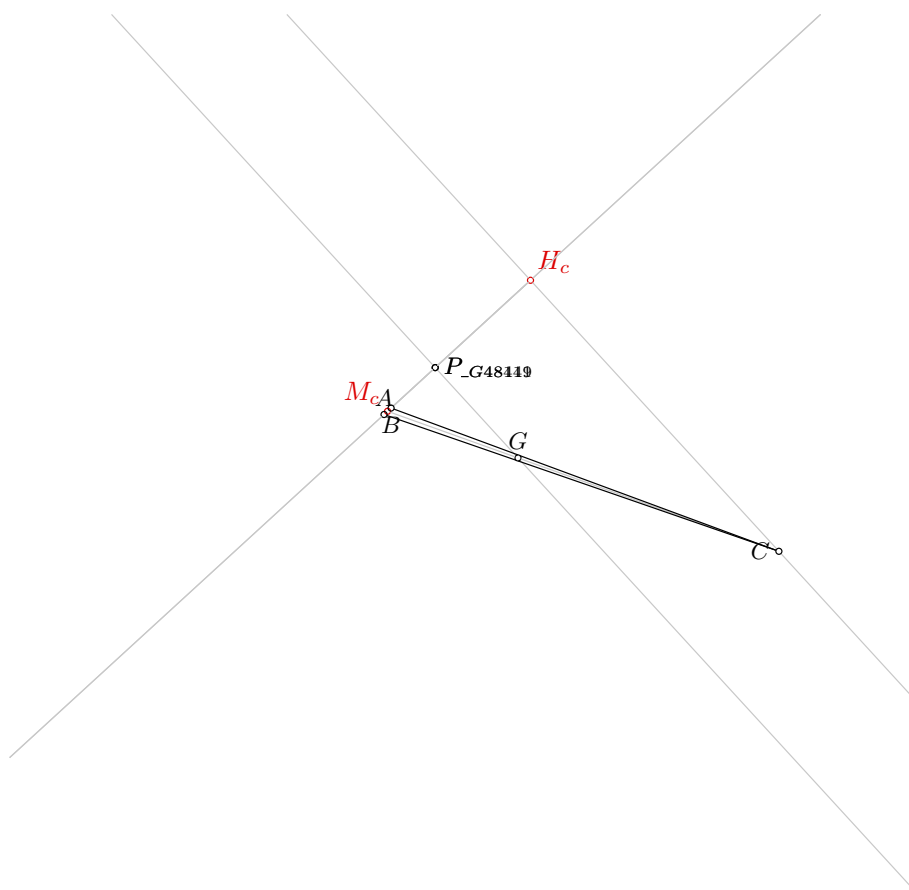


Figure 1: Illustration of the problem 0443

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = \neg M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.028 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_c = \neg H_c$

**Status:** Theorem has been disproved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 18 terms.

**Time Complexity:** Time spent by the prover is 0.295 seconds.

**NDG conditions** Points  $P_{G55820}$ ,  $M_c$  and  $G$  are not collinear

Points  $A$  and  $H_c$  are not identical

Line through points  $A$  and  $H_c$  is not perpendicular to line through points  $H_c$  and  $B$

#### 4.1.3 Proving $G = \neg G$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{H_c}^3} \neq S_{BCF_{H_c}^3}$  i.e., lines  $AB$  and  $CF_{H_c}^3$  are not parallel (construction based assumption)

$S_{AB_{M_b}} \neq S_{M_a B_{M_b}}$  i.e., lines  $A_{M_a}$  and  $B_{M_b}$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.2.3 Proving $G = \neg G$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c = \neg M_c$

Proving failed

#### 4.3.2 Proving $H_c = \neg H_c$

Proving failed

### 4.3.3 Proving $G=_G$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_c=_M M_c$

Proving failed

### 4.4.2 Proving $H_c=_H H_c$

Proving failed

### 4.4.3 Proving $G=_G$

Proving failed

# Problem 444

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 444: Given a point  $M_c$ , a point  $G$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $G$ , construct a point  $C$  (rule W01); ;
2. Using the point  $G$  and the point  $H$ , construct a point  $O$  (rule W01); ;
3. Using the point  $M_c$  and the point  $O$ , construct a line  $m_c$  (rule W02); % DET: points  $M_c$  and  $O$  are not the same;
4. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
5. Using the point  $M_c$  and the line  $m_c$ , construct a line  $c$  (rule W10a); ;
6. Using the circle  $k(O, C)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same.

Determination conditions: points  $M_c$  and  $O$  are not the same.

Rules used: [W01,W02,W04,W06,W10a]

Lemmas used: [D13,D20,D26,GD01,GL03,GL04,GL09,L1,L11,L12,L57,L58]

Solving time: 1.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{c} 50 67.5  
point G 70 58.33  
point H 80 72.73
```

```

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_t G
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a point C such that  $M_{\{c\}}C/M_{\{c\}}G=3$ 
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

% Constructing a point O such that  $GO/GH=-0.5$ 
towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% DET: points  $M_{\{c\}}$  and  $O$  are not the same
% Constructing a line  $m_{\{c\}}$  which passes through point  $M_{\{c\}}$  and point  $O$ 
line m_{c} M_{c} O

color 200 200 200
drawline m_{c}
color 0 0 0

% NDG: points  $C$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $C$ 
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% Constructing a line  $c$  which is perpendicular to line  $m_{\{c\}}$  and which passes through point  $M_{\{c\}}$ 
perp c M_{c} m_{c}

color 200 200 200
drawline c
color 0 0 0

```

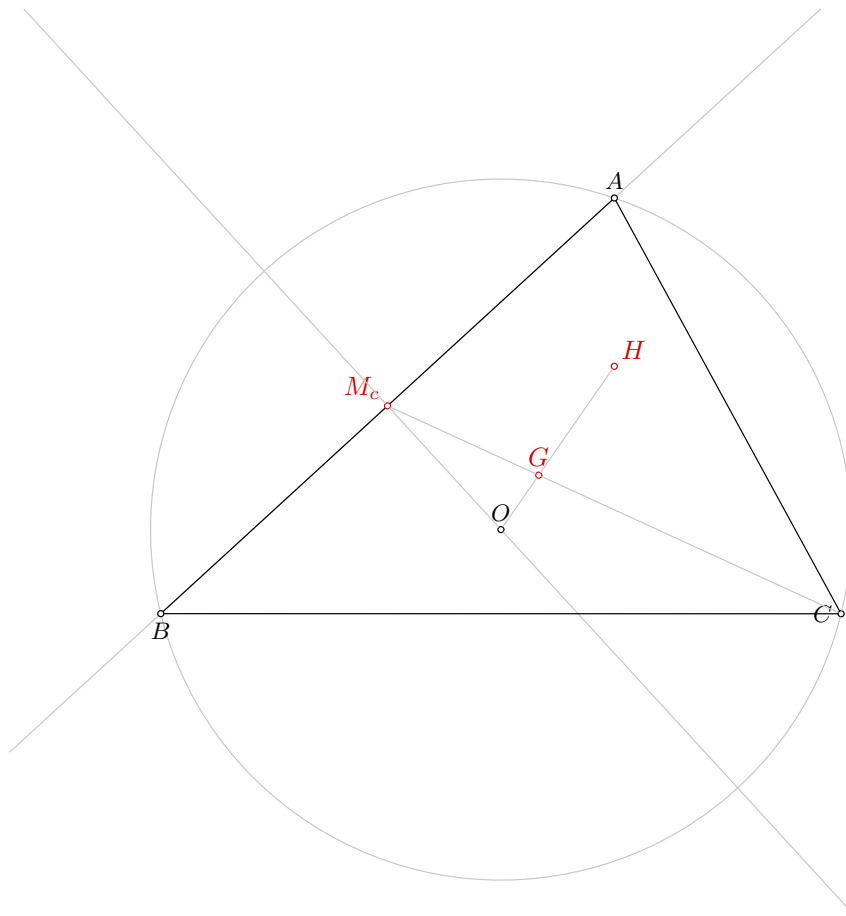


Figure 1: Illustration of the problem 0444

```
% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same
% Determination conditions: points M_{c} and O are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1



### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = \neg M_c$

Proving failed

#### 4.1.2 Proving $G = \neg G$

Proving failed

#### 4.1.3 Proving $H = \neg H$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_c = \neg M_c$

Proving failed

#### 4.2.2 Proving $G = \neg G$

Proving failed

#### 4.2.3 Proving $H = \neg H$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 60 terms.

**Time Complexity:** Time spent by the prover is 0.110 seconds. There are no ndg conditions.

#### 4.3.2 Proving $G = \neg G$

Proving failed

#### 4.3.3 Proving $H = \neg H$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_c = \neg M_c$

Proving failed

#### 4.4.2 Proving $G = \neg G$

Proving failed

### 4.4.3 Proving $H = \_H$

Proving failed

# Problem 445

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 445: Given a point  $M_c$ , a point  $G$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 446

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 446: Given a point  $M_c$ , a point  $G$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 447

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 447: Given a point  $M_c$ , a point  $G$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $G$ , construct a point  $C$  (rule W01); ;
2. Using the point  $M_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $T_c$  are not the same;
3. Using the point  $T_c$  and the point  $C$ , construct a line  $s_c$  (rule W02); % DET: points  $T_c$  and  $C$  are not the same;
4. Using the point  $M_c$  and the line  $c$ , construct a line  $m_c$  (rule W10b); ;
5. Using the line  $m_c$  and the line  $s_c$ , construct a point  $N_c$  (rule W03); % NDG: lines  $m_c$  and  $s_c$  are not parallel % DET: lines  $m_c$  and  $s_c$  are not the same;
6. Using the point  $C$  and the point  $N_c$ , construct a line  $m(CN_c)$  (rule W14); % DET: points  $C$  and  $N_c$  are not the same;
7. Using the line  $m(CN_c)$  and the line  $m_c$ , construct a point  $O$  (rule W03); % NDG: lines  $m(CN_c)$  and  $m_c$  are not parallel % DET: lines  $m(CN_c)$  and  $m_c$  are not the same;
8. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
9. Using the circle  $k(O, C)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same; lines  $m(CN_c)$  and  $m_c$  are not parallel; lines  $m_c$  and  $s_c$  are not parallel.

Determination conditions: lines  $m(CN_c)$  and  $m_c$  are not the same; points  $C$  and  $N_c$  are not the same; lines  $m_c$  and  $s_c$  are not the same; points  $T_c$  and  $C$  are not the same; points  $M_c$  and  $T_c$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14]

Lemmas used: [D13,D20,D25,D26,D49,GD01,GL01,GL03,GL04,GL09,L1,L11,L12,L27,L57,L7]

Solving time: 1.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point G 70 58.33
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_t G
cmark_rt T_{c}
color 0 0 0
fontsize 8

% Constructing a point C such that M_{c}C/M_{c}G=3
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

% DET: points M_{c} and T_{c} are not the same
% Constructing a line c which passes through point M_{c} and point T_{c}
line c M_{c} T_{c}

color 200 200 200
drawline c
color 0 0 0

% DET: points T_{c} and C are not the same
% Constructing a line s_{c} which passes through point T_{c} and point C
line s_{c} T_{c} C

color 200 200 200
drawline s_{c}
color 0 0 0
```

```

% Constructing a line  $m_{\{c\}}$  which is perpendicular to line  $c$  and which passes through point  $M_{\{c\}}$ 
perp  $m_{\{c\}}$   $M_{\{c\}}$   $c$ 

color 200 200 200
drawline  $m_{\{c\}}$ 
color 0 0 0

% NDG: lines  $m_{\{c\}}$  and  $s_{\{c\}}$  are not parallel% DET: lines  $m_{\{c\}}$  and  $s_{\{c\}}$  are not the same
% Constructing a point  $N_{\{c\}}$  which belongs to line  $m_{\{c\}}$  and line  $s_{\{c\}}$ 
intersec  $N_{\{c\}}$   $m_{\{c\}}$   $s_{\{c\}}$ 
cmark_b  $N_{\{c\}}$ 

% DET: points  $C$  and  $N_{\{c\}}$  are not the same
% Constructing bisector  $m(CN_{\{c\}})$  of the segment  $CN_{\{c\}}$ 
med  $m(CN_{\{c\}})$   $C$   $N_{\{c\}}$ 

color 200 200 200
drawline  $m(CN_{\{c\}})$ 
color 0 0 0

color 200 200 200
drawsegment  $C$   $N_{\{c\}}$ 
color 0 0 0

% NDG: lines  $m(CN_{\{c\}})$  and  $m_{\{c\}}$  are not parallel% DET: lines  $m(CN_{\{c\}})$  and  $m_{\{c\}}$  are not the same
% Constructing a point  $O$  which belongs to line  $m(CN_{\{c\}})$  and line  $m_{\{c\}}$ 
intersec  $O$   $m(CN_{\{c\}})$   $m_{\{c\}}$ 
cmark_t  $O$ 

% NDG: points  $C$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $C$ 
circle  $k(O,C)$   $O$   $C$ 

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $c$  and circle  $k(O,C)$  intersect
% Constructing points  $A$  and  $B$  which are in intersection of  $k(O,C)$  and  $c$ 
intersec2  $A$   $B$   $k(O,C)$   $c$ 
cmark_t  $A$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

```

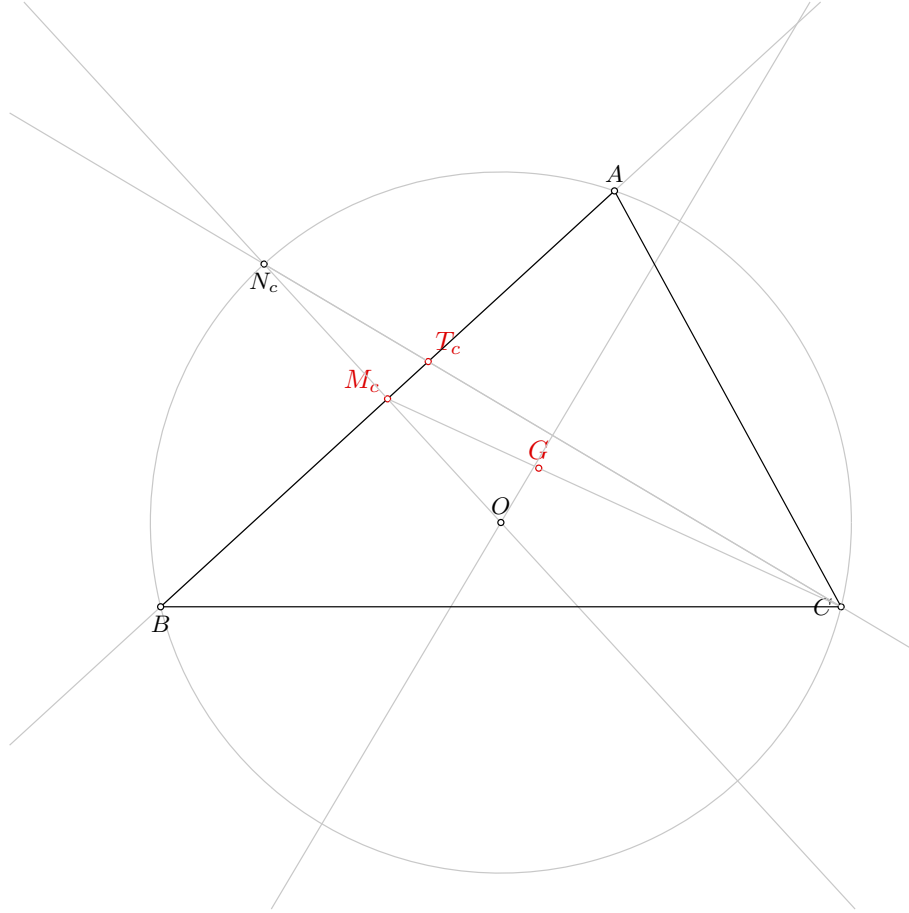


Figure 1: Illustration of the problem 0447

*% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;  
 lines m(CN\_{c}) and m\_{c} are not parallel; lines m\_{c} and s\_{c} are not parallel  
 % Determination conditions: lines m(CN\_{c}) and m\_{c} are not the same; points C and N\_{c} are not  
 the same; lines m\_{c} and s\_{c} are not the same; points T\_{c} and C are not the same; points M  
 \_{c} and T\_{c} are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = M_c$

Proving failed



#### 4.1.2 Proving $G = \neg G$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2678 terms.

**Time Complexity:** Time spent by the prover is 23.46 seconds.

**NDG conditions** Line through points  $C$  and  $T_c$  is not perpendicular to line through points  $T_c$  and  $M_c$

Points  $M_c$ ,  $G$  and  $T_c$  are not collinear

Line through points  $C$  and  $N_c$  is not parallel with line through points  $M_c$  and  $T_c$

Line through points  $C$  and  $M_c$  is not perpendicular to line through points  $M_c$  and  $N_c$

Points  $M_c$ ,  $G$  and  $T_c$  are not collinear

Points  $M_c$ ,  $G$  and  $T_c$  are not collinear

Line through points  $A$  and  $\neg M_a$  is not parallel with line through points  $B$  and  $\neg M_b$

Line through points  $A$  and  $\neg M_a$  is not parallel with line through points  $M_c$  and  $G$

#### 4.1.3 Proving $T_c = \neg T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_c = \neg M_c$

Proving failed

#### 4.2.2 Proving $G = \neg G$

Proving failed

#### 4.2.3 Proving $T_c = \neg T_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 1355 terms.

**Time Complexity:** Time spent by the prover is 1.370 seconds. There are no ndg conditions.

#### 4.3.2 Proving $G = \neg G$

Proving failed

#### 4.3.3 Proving $T_c = \neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 9188 terms.

**Time Complexity:** Time spent by the prover is 19.780 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_c = \neg M_c$

Proving failed

### 4.4.2 Proving $G = \neg G$

Proving failed

### 4.4.3 Proving $T_c = \neg T_c$

Proving failed

# Problem 448

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 448: Given a point  $M_c$ , a point  $G$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $G$ , construct a point  $C$  (rule W01); ;
2. Using the point  $M_c$  and the point  $I$ , construct a line  $IM_c$  (rule W02); % DET: points  $M_c$  and  $I$  are not the same;
3. Using the point  $I$  and the point  $C$ , construct a line  $s_c$  (rule W02); % DET: points  $I$  and  $C$  are not the same;
4. Using the point  $I$  and the point  $M_c$ , construct a circle  $k_{over}(I, M_c)$  (rule W09); % NDG: points  $I$  and  $M_c$  are not the same;
5. Using the point  $C$  and the line  $IM_c$ , construct a line  $CP'_c$  (rule W16); ;
6. Using the point  $M_c$ , the line  $CP'_c$  and the point  $C$ , construct a line  $h_{M_c, -1/1}(CP'_c)$  (rule W15); ;
7. Using the circle  $k_{over}(I, M_c)$  and the line  $h_{M_c, -1/1}(CP'_c)$ , construct a point  $C_{fo}$  and a point  $P_c$  (rule W04); % NDG: line  $h_{M_c, -1/1}(CP'_c)$  and circle  $k_{over}(I, M_c)$  intersect;
8. Using the point  $P_c$  and the point  $I$ , construct a circle  $k(I, P_a)$  (rule W06); % NDG: points  $P_c$  and  $I$  are not the same;
9. Using the circle  $k(I, P_a)$ , the point  $M_c$  and the point  $I$ , construct a line  $x3$  and a line  $c$  (rule W12); % NDG: point  $M_c$  is outside the circle  $k(I, P_a)$ ;
10. Using the point  $M_c$  and the line  $c$ , construct a line  $m_c$  (rule W10b); ;
11. Using the line  $m_c$  and the line  $s_c$ , construct a point  $N_c$  (rule W03); % NDG: lines  $m_c$  and  $s_c$  are not parallel % DET: lines  $m_c$  and  $s_c$  are not the same;

12. Using the point  $I$  and the point  $N_c$ , construct a circle  $k(N_c, B)$  (rule W06); % NDG: points  $I$  and  $N_c$  are not the same;
13. Using the circle  $k(N_c, B)$  and the line  $c$ , construct a point  $B$  and a point  $A$  (rule W04); % NDG: line  $c$  and circle  $k(N_c, B)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(N_c, B)$  intersect; points  $I$  and  $N_c$  are not the same; lines  $m_c$  and  $s_c$  are not parallel; point  $M_c$  is outside the circle  $k(I, P_a)$ ; points  $P_c$  and  $I$  are not the same; line  $h_{M_c, -1/1}(CP'_c)$  and circle  $k_{over}(I, M_c)$  intersect; points  $I$  and  $M_c$  are not the same.

Determination conditions: lines  $m_c$  and  $s_c$  are not the same; points  $I$  and  $C$  are not the same; points  $M_c$  and  $I$  are not the same.

Rules used: [W01, W02, W03, W04, W06, W09, W10b, W12, W15, W16]

Lemmas used: [D13, D20, D27, D49, D61, D87, GD01, GD02, GL02, GL03, GL04, GL09, L14, L2, L27, L35, L36, L57, L59]

Solving time: 2.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{c} 50 67.5
```

```
point G 70 58.33
```

```
point I 74.37 61.15
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_lt M_{c}
```

```
cmark_t G
```

```
cmark_b I
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point C such that M_{c}C/M_{c}G=3
```

```
towards C M_{c} G 3
```

```
cmark_l C
```

```
color 200 200 200
```

```
drawsegment M_{c} C
```

```
color 0 0 0
```

```
% DET: points M_{c} and I are not the same
```

```
% Constructing a line IM_{c} which passes through point M_{c} and point I
```

```
line IM_{c} M_{c} I
```

```
color 200 200 200
```

```
drawline IM_{c}
```

```
color 0 0 0
```

```
% DET: points I and C are not the same
```

```

% Constructing a line  $s_{\{c\}}$  which passes through point  $I$  and point  $C$ 
line  $s_{\{c\}}$  I C

color 200 200 200
drawline  $s_{\{c\}}$ 
color 0 0 0

% NDG: points  $I$  and  $M_{\{c\}}$  are not the same
% Constructing midpoint  $P_{\{\backslash\_G49959\}}$  of the segment  $IM_{\{c\}}$ 
midpoint  $P_{\{\backslash\_G49959\}}$  I  $M_{\{c\}}$ 
cmark_r  $P_{\{\backslash\_G49959\}}$ 

% Constructing a circle  $k_{\text{over}}(I, M_{\{c\}})$  whose center is at point  $P_{\{\backslash\_G49959\}}$  and which passes
    through point  $I$ 
circle  $k_{\text{over}}(I, M_{\{c\}})$   $P_{\{\backslash\_G49959\}}$  I

color 200 200 200
drawcircle  $k_{\text{over}}(I, M_{\{c\}})$ 
color 0 0 0

% Constructing a line  $CP'_{\{c\}}$  which contains the point  $C$  and is parallel to the line  $IM_{\{c\}}$ 
parallel  $CP'_{\{c\}}$  C  $IM_{\{c\}}$ 

color 200 200 200
drawline  $CP'_{\{c\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\backslash\_G50260\}}$  such that  $M_{\{c\}}P_{\{\backslash\_G50260\}}/M_{\{c\}}C=-1$ 
towards  $P_{\{\backslash\_G50260\}}$   $M_{\{c\}}$  C -1
cmark_r  $P_{\{\backslash\_G50260\}}$ 
color 200 200 200
drawsegment C  $P_{\{\backslash\_G50260\}}$ 
color 0 0 0

% Constructing a line  $h_{\{M_{\{c\}}, -1/1\}}(CP'_{\{c\}})$  which contains the point  $P_{\{\backslash\_G50260\}}$  and is parallel
    to the line  $CP'_{\{c\}}$ 
parallel  $h_{\{M_{\{c\}}, -1/1\}}(CP'_{\{c\}})$   $P_{\{\backslash\_G50260\}}$   $CP'_{\{c\}}$ 

color 200 200 200
drawline  $h_{\{M_{\{c\}}, -1/1\}}(CP'_{\{c\}})$ 
color 0 0 0

% NDG: line  $h_{\{M_{\{c\}}, -1/1\}}(CP'_{\{c\}})$  and circle  $k_{\text{over}}(I, M_{\{c\}})$  intersect
% Constructing points  $C_{\{fo\}}$  and  $P_{\{c\}}$  which are in intersection of  $k_{\text{over}}(I, M_{\{c\}})$  and  $h_{\{M_{\{c\}}, -1/1\}}(CP'_{\{c\}})$ 
intersec2  $C_{\{fo\}}$   $P_{\{c\}}$   $k_{\text{over}}(I, M_{\{c\}})$   $h_{\{M_{\{c\}}, -1/1\}}(CP'_{\{c\}})$ 

```

```

cmark_r C_{fo}
cmark_r P_{c}

% NDG: points  $P_{c}$  and  $I$  are not the same
% Constructing a circle  $k(I, P_{a})$  whose center is at point  $I$  and which passes through point  $P_{c}$ 
circle k(I, P_{a}) I P_{c}

color 200 200 200
drawcircle k(I, P_{a})
color 0 0 0

% NDG: point  $M_{c}$  is outside the circle  $k(I, P_{a})$ 
% Constructing midpoint  $P_{\backslash\_G51009}$  of the segment  $M_{c}I$ 
midpoint P_{\backslash\_G51009} M_{c} I
cmark_r P_{\backslash\_G51009}

% Constructing a circle  $C_{\backslash\_G51012}$  whose center is at point  $P_{\backslash\_G51009}$  and which passes through
point  $M_{c}$ 
circle C_{\backslash\_G51012} P_{\backslash\_G51009} M_{c}

color 200 200 200
drawcircle C_{\backslash\_G51012}
color 0 0 0

% Constructing points  $P_{\backslash\_G51015}$  and  $P_{\backslash\_G51018}$  which are in intersection of  $C_{\backslash\_G51012}$  and  $k$ 
( $I, P_{a}$ )
intersec2 P_{\backslash\_G51015} P_{\backslash\_G51018} C_{\backslash\_G51012} k(I, P_{a})
cmark_r P_{\backslash\_G51015}
cmark_r P_{\backslash\_G51018}

% Constructing a line  $x3$  which passes through point  $M_{c}$  and point  $P_{\backslash\_G51015}$ 
line x3 M_{c} P_{\backslash\_G51015}

color 200 200 200
drawline x3
color 0 0 0

% Constructing a line  $c$  which passes through point  $M_{c}$  and point  $P_{\backslash\_G51018}$ 
line c M_{c} P_{\backslash\_G51018}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line  $m_{c}$  which is perpendicular to line  $c$  and which passes through point  $M_{c}$ 
perp m_{c} M_{c} c

color 200 200 200

```

```

drawline m_{c}
color 0 0 0

% NDG: lines m_{c} and s_{c} are not parallel% DET: lines m_{c} and s_{c} are not the same
% Constructing a point N_{c} which belongs to line m_{c} and line s_{c}
intersec N_{c} m_{c} s_{c}
cmark_b N_{c}

% NDG: points I and N_{c} are not the same
% Constructing a circle k(N_{c},B) whose center is at point N_{c} and which passes through point I
circle k(N_{c},B) N_{c} I

color 200 200 200
drawcircle k(N_{c},B)
color 0 0 0

% NDG: line c and circle k(N_{c},B) intersect
% Constructing points B and A which are in intersection of k(N_{c},B) and c
intersec2 B A k(N_{c},B) c
cmark_b B
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k(N_{c},B) intersect; points I and N_{c} are not the
same; lines m_{c} and s_{c} are not parallel; point M_{c} is outside the circle k(I,P_{a});
points P_{c} and I are not the same; line h_{M_{c},-1/1}(CP'_{c}) and circle k_{over}(I,M_{c})
intersect; points I and M_{c} are not the same
% Determination conditions: lines m_{c} and s_{c} are not the same; points I and C are not the same
; points M_{c} and I are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

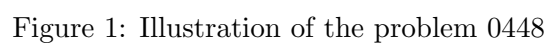
### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = M_c$

Proving failed





#### 4.1.2 Proving $G=_G$

Proving failed

#### 4.1.3 Proving $I=_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_c=_M_c$

Proving failed

#### 4.2.2 Proving $G=_G$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c=_M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 2361 terms.

**Time Complexity:** Time spent by the prover is 9.410 seconds. There are no ndg conditions.

#### 4.3.2 Proving $G=_G$

Proving failed

#### 4.3.3 Proving $I=_I$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_c=_M_c$

Proving failed

#### 4.4.2 Proving $G=_G$

Proving failed

#### 4.4.3 Proving $I=_I$

Proving failed

# Problem 449

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 449: Given a point  $M_c$ , a point  $H_b$  and a point  $H_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_b$  and  $M_c$  are not the same;
2. Choose freely a point  $H_a$  on the circle  $k(M_c, A)$  (rule WOncircle);
3. Choose freely a point  $A$  on the circle  $k(M_c, A)$  (rule WOncircle);
4. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); ;
5. Using the point  $A$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $H_b$  are not the same;
6. Using the point  $H_a$  and the point  $B$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $B$  are not the same;
7. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; points  $H_b$  and  $M_c$  are not the same.

Determination conditions: lines  $b$  and  $a$  are not the same; points  $H_a$  and  $B$  are not the same; points  $A$  and  $H_b$  are not the same.

Rules used: [W01,W02,W03,W06,WOncircle1]

Lemmas used: [D20,D5,D6,GD01,GD02,GL03,L41]

Solving time: 2.8 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point H_{b} 89.36 77.83
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_l H_{b}
cmark_r H_{a}
color 0 0 0
fontsize 8

% NDG: points H_{b} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{b}
circle k(M_{c},A) M_{c} H_{b}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% Choosing randomly a point H_{a} on the circle with center M_{c} through point H_{b}
oncircle H_{a} M_{c} H_{b}
cmark_r H_{a}
color 200 200 200
drawcircle M_{c} H_{b}
color 0 0 0

% Choosing randomly a point A on the circle with center M_{c} through point H_{a}
oncircle A M_{c} H_{a}
cmark_t A
color 200 200 200
drawcircle M_{c} H_{a}
color 0 0 0

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points A and H_{b} are not the same
```

```

% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_1 C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and a are not parallel; points H_{b} and M_{c} are not the
same
% Determination conditions: lines b and a are not the same; points H_{a} and B are not the same;
points A and H_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = \neg M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.13 seconds.

**NDG conditions** There are no NDG conditions for this theorem

#### 4.1.2 Proving $H_b = \neg H_b$

**Status:** Theorem has been proved.

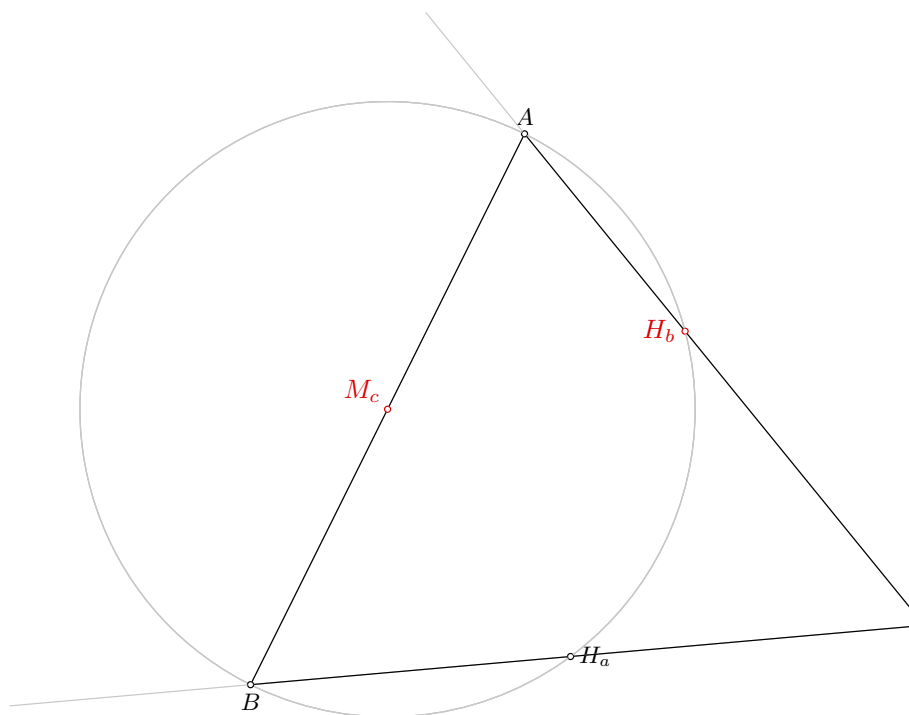


Figure 1: Illustration of the problem 0449

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2162 terms.

**Time Complexity:** Time spent by the prover is 15.017 seconds.

**NDG conditions** Line through points  $H_b$  and  $A$  is not parallel with line through points  $B$  and  $H_a$

Points  $A$  and  $M_c$  are not identical

Points  $A$  and  $C$  are not identical

Line through points  $H_b$  and  $M_c$  is not parallel with line through points  $A$  and  $C$

#### 4.1.3 Proving $H_a = H_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_c = M_c$

NDG conditions are:

$S_{AH_aB} \neq S_{H_bH_aB}$  i.e., lines  $AH_b$  and  $H_aB$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{H_b}^0} \neq S_{CBF_{H_b}^0}$  i.e., lines  $AC$  and  $BF_{H_b}^0$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{H_a}^1} \neq S_{CAF_{H_a}^1}$  i.e., lines  $BC$  and  $AF_{H_a}^1$  are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_b = H_b$

Proving failed

#### 4.2.3 Proving $H_a = H_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c = M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 17 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_b = H_b$

Proving failed

#### 4.3.3 Proving $H_a = H_a$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_c = M_c$

Proving failed

#### 4.4.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.4.3 Proving $H_a = \neg H_a$

Proving failed

# Problem 450

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 450: Given a point  $M_c$ , a point  $H_a$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $H_c$  are not the same;
2. Using the point  $H_a$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_a$  and  $M_c$  are not the same;
3. Using the circle  $k(M_c, A)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(M_c, A)$  intersect;
4. Using the point  $H_a$  and the point  $B$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $B$  are not the same;
5. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
6. Using the line  $a$  and the line  $h_c$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $h_c$  are not parallel % DET: lines  $a$  and  $h_c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $h_c$  are not parallel; line  $c$  and circle  $k(M_c, A)$  intersect; points  $H_a$  and  $M_c$  are not the same.

Determination conditions: lines  $a$  and  $h_c$  are not the same; points  $H_a$  and  $B$  are not the same; points  $M_c$  and  $H_c$  are not the same.

Rules used: [W02,W03,W04,W06,W10b]

Lemmas used: [D10,D20,D5,D7,GD01,GD02,GL09,L40,L42]

Solving time: 1.6 seconds.



### 3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point H_{a} 80 40
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_r H_{a}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points M_{c} and H_{c} are not the same
% Constructing a line c which passes through point M_{c} and point H_{c}
line c M_{c} H_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points H_{a} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{a}
circle k(M_{c},A) M_{c} H_{a}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line c and circle k(M_{c},A) intersect
% Constructing points A and B which are in intersection of k(M_{c},A) and c
intersec2 A B k(M_{c},A) c
cmark_t A
cmark_b B

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0
```

```

% Constructing a line  $h_{\{c\}}$  which is perpendicular to line  $c$  and which passes through point  $H_{\{c\}}$ 
perp  $h_{\{c\}}$   $H_{\{c\}}$   $c$ 

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: lines  $a$  and  $h_{\{c\}}$  are not parallel% DET: lines  $a$  and  $h_{\{c\}}$  are not the same
% Constructing a point  $C$  which belongs to line  $a$  and line  $h_{\{c\}}$ 
intersec  $C$   $a$   $h_{\{c\}}$ 
cmark_1  $C$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $a$  and  $h_{\{c\}}$  are not parallel; line  $c$  and circle  $k(M_{\{c\}}, A)$ 
% intersect; points  $H_{\{a\}}$  and  $M_{\{c\}}$  are not the same
% Determination conditions: lines  $a$  and  $h_{\{c\}}$  are not the same; points  $H_{\{a\}}$  and  $B$  are not the same
% ; points  $M_{\{c\}}$  and  $H_{\{c\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = M_c$

Proving failed

#### 4.1.2 Proving $H_a = H_a$

Proving failed

#### 4.1.3 Proving $H_c = H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 545 terms.

**Time Complexity:** Time spent by the prover is 2.084 seconds.

**NDG conditions** Points  $H_c$ ,  $M_c$  and  $H_a$  are not collinear

Points  $H_c$ ,  $M_c$  and  $H_a$  are not collinear

Line through points  $H_c$  and  $B$  is not perpendicular to line through points  $B$  and  $M_c$

Points  $B$ ,  $M_c$  and  $H_a$  are not collinear

Points  $A$  and  $B$  are not identical

Line through points  $A$  and  $B$  is not parallel with line through points  $M_c$  and  $H_a$

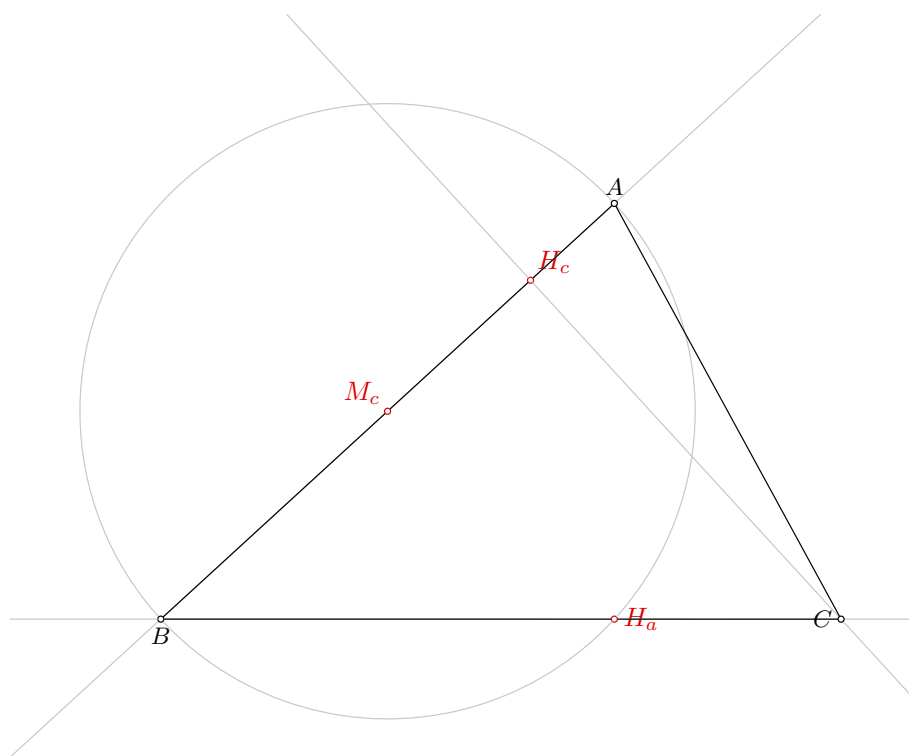


Figure 1: Illustration of the problem 0450

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_c = \neg M_c$

Proving failed

### 4.2.2 Proving $H_a = \neg H_a$

Proving failed

### 4.2.3 Proving $H_c = \neg H_c$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 34 terms.

**Time Complexity:** Time spent by the prover is 0.040 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_a = \neg H_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3080 terms.

**Time Complexity:** Time spent by the prover is 5.060 seconds. There are no ndg conditions.

### 4.3.3 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 1247 terms.

**Time Complexity:** Time spent by the prover is 1.820 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_c = \neg M_c$

Proving failed

### 4.4.2 Proving $H_a = \neg H_a$

Proving failed

### 4.4.3 Proving $H_c = \neg H_c$

Proving failed

# Problem 451

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 451: Given a point  $M_c$ , a point  $H_a$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $H_a$  and  $H$  are not the same;
2. Using the point  $H_a$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_a$  and  $M_c$  are not the same;
3. Using the circle  $k(M_c, A)$ , the line  $h_a$ , the point  $M_c$  and the point  $H_a$ , construct a point  $A$  (rule W05); % NDG: line  $h_a$  and circle  $k(M_c, A)$  intersect % DET: points  $H_a$  and  $A$  must be different;
4. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); ;
5. Using the point  $H_a$  and the point  $B$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $B$  are not the same;
6. Using the point  $H$  and the point  $B$ , construct a line  $h_b$  (rule W02); % DET: points  $H$  and  $B$  are not the same;
7. Using the circle  $k(M_c, A)$ , the line  $h_b$ , the point  $M_c$  and the point  $B$ , construct a point  $H_b$  (rule W05); % NDG: line  $h_b$  and circle  $k(M_c, A)$  intersect % DET: points  $B$  and  $H_b$  must be different;
8. Using the point  $A$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $H_b$  are not the same;
9. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; line  $h_b$  and circle  $k(M_c, A)$  intersect; line  $h_a$  and circle  $k(M_c, A)$  intersect; points  $H_a$  and  $M_c$  are not the same.

Determination conditions: lines  $a$  and  $b$  are not the same; points  $A$  and  $H_b$  are not the same; points  $B$  and  $H_b$  must be different; points  $H$  and  $B$  are not the same; points  $H_a$  and  $B$  are not the same; points  $H_a$  and  $A$  must be different; points  $H_a$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D20,D3,D5,D6,D8,D9,GD01,GD02,GL03,L40,L41,L42]

Solving time: 1.6 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point H_{a} 80 40
point H 80 72.73

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_r H_{a}
cmark_rt H
color 0 0 0
fontsize 8

% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points H_{a} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{a}
circle k(M_{c},A) M_{c} H_{a}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line h_{a} and circle k(M_{c},A) intersect% DET: points H_{a} and A must be different
% Constructing a point P_{\G54880} which is a foot of the point M_{c} on the line h_{a}
foot P_{\G54880} M_{c} h_{a}
cmark_r P_{\G54880}
color 200 200 200
drawline M_{c} P_{\G54880}
color 0 0 0
```

```

% Constructing a point A which is an image of the point H_{a} in the symmetry to point/line P_{\_G
54880}
sim A P_{\_G54880} H_{a}
cmark_t A

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% DET: points H and B are not the same
% Constructing a line h_{b} which passes through point H and point B
line h_{b} H B

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{b} and circle k(M_{c},A) intersect% DET: points B and H_{b} must be different
% Constructing a point P_{\_G55243} which is a foot of the point M_{c} on the line h_{b}
foot P_{\_G55243} M_{c} h_{b}
cmark_r P_{\_G55243}
color 200 200 200
drawline M_{c} P_{\_G55243}
color 0 0 0

% Constructing a point H_{b} which is an image of the point B in the symmetry to point/line P_{\_G
55243}
sim H_{b} P_{\_G55243} B
cmark_l H_{b}

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b

```

```
color 0 0 0
```

```
% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_l C
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines a and b are not parallel; line h_{b} and circle k(M_{c},A)
    intersect; line h_{a} and circle k(M_{c},A) intersect; points H_{a} and M_{c} are not the same
% Determination conditions: lines a and b are not the same; points A and H_{b} are not the same;
    points B and H_{b} must be different; points H and B are not the same; points H_{a} and B are
    not the same; points H_{a} and A must be different; points H_{a} and H are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.158 seconds.

**NDG conditions** Points  $M_c$ ,  $H$  and  $H_a$  are not collinear

Points  $M_c$ ,  $H$  and  $H_a$  are not collinear

#### 4.1.2 Proving $H_a = H_a$

Proving failed

#### 4.1.3 Proving $H = H$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_c = M_c$

NDG conditions are:

$S_{H_a A H_b} \neq S_{B A H_b}$  i.e., lines  $H_a B$  and  $A H_b$  are not parallel (construction based assumption)

$S_{A B C} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{B A F_{-H_a}^0} \neq S_{C A F_{-H_a}^0}$  i.e., lines  $BC$  and  $A F_{-H_a}^0$  are not parallel (construction based assumption)



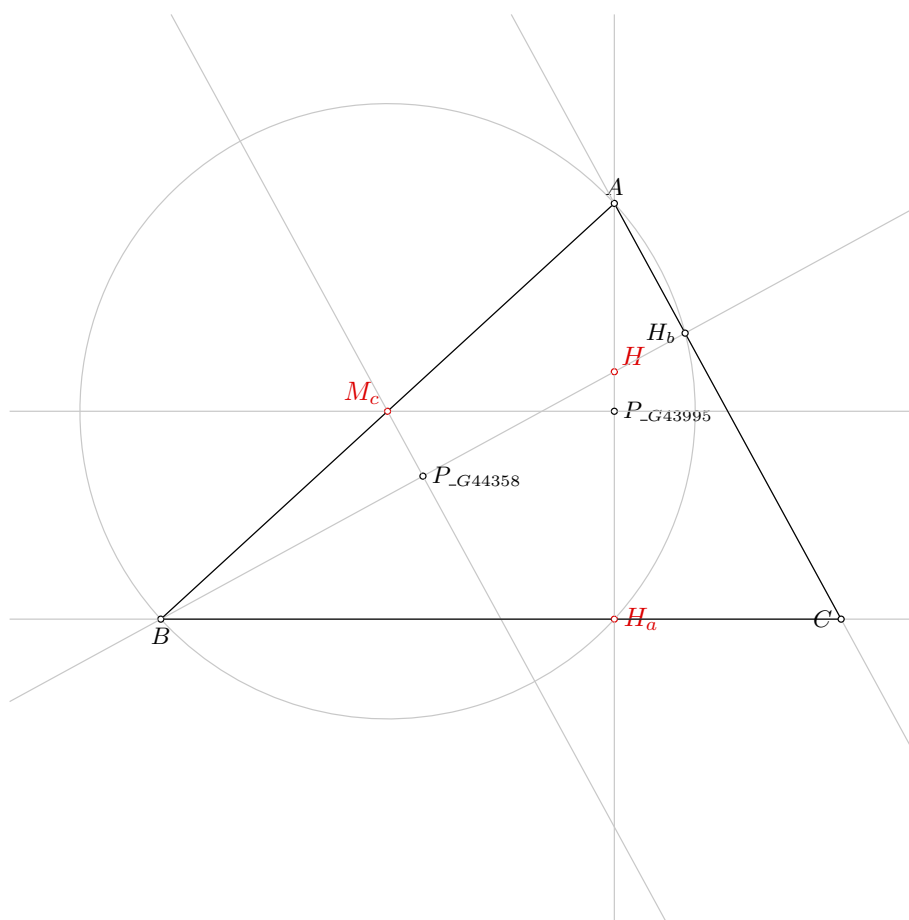


Figure 1: Illustration of the problem 0451

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)  
 $S_{ABF^1_{\perp h_b}} \neq S_{F^0_{\perp h_a}BF^1_{\perp h_b}}$  i.e., lines  $AF^0_{\perp h_a}$  and  $BF^1_{\perp h_b}$  are not parallel (construction based assumption)  
 Total number of proof steps: 41  
 Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.2.3 Proving $H = \neg H$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c = \neg M_c$

Proving failed

#### 4.3.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.3.3 Proving $H = \neg H$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_c = \neg M_c$

Proving failed

#### 4.4.2 Proving $H_a = \neg H_a$

Proving failed

#### 4.4.3 Proving $H = \neg H$

Proving failed

# Problem 452

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 452: Given a point  $M_c$ , a point  $H_a$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $T_a$  are not the same;
2. Using the point  $H_a$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_a$  and  $M_c$  are not the same;
3. Using the circle  $k(M_c, A)$ , the line  $a$ , the point  $M_c$  and the point  $H_a$ , construct a point  $B$  (rule W05); % NDG: line  $a$  and circle  $k(M_c, A)$  intersect % DET: points  $H_a$  and  $B$  must be different;
4. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01); ;
5. Using the point  $M_c$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $B$  are not the same;
6. Using the point  $T_a$  and the line  $c$ , construct a circle  $k(T_a, foot[T_a, c])$  (rule W11); % NDG: point  $T_a$  is not incident to the line  $c$ ;
7. Using the circle  $k(T_a, foot[T_a, c])$ , the point  $A$ , the point  $T_a$  and the line  $c$ , construct a line  $b$  (rule W13); % NDG: point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ;
8. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(T_a, foot[T_a, c])$ ; point  $T_a$  is not incident to the line  $c$ ; line  $a$  and circle  $k(M_c, A)$  intersect; points  $H_a$  and  $M_c$  are not the same.

Determination conditions: lines  $b$  and  $a$  are not the same; points  $M_c$  and  $B$  are not the same; points  $H_a$  and  $B$  must be different; points  $H_a$  and  $T_a$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W11,W13]

Lemmas used: [D20,D23,D5,GD01,GD02,GL03,GL04,GL09,GL10,GL11,L40,L42,L8]

Solving time: 1.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point H_{a} 80 40
point T_{a} 70.86 40

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_r H_{a}
cmark_rb T_{a}
color 0 0 0
fontsize 8

% DET: points H_{a} and T_{a} are not the same
% Constructing a line a which passes through point H_{a} and point T_{a}
line a H_{a} T_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points H_{a} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{a}
circle k(M_{c},A) M_{c} H_{a}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line a and circle k(M_{c},A) intersect% DET: points H_{a} and B must be different
% Constructing a point P_{\G54699} which is a foot of the point M_{c} on the line a
foot P_{\G54699} M_{c} a
cmark_r P_{\G54699}
color 200 200 200
drawline M_{c} P_{\G54699}
color 0 0 0

% Constructing a point B which is an image of the point H_{a} in the symmetry to point/line P_{\G54699}
```

```

sim B P_{\_G54699} H_{a}
cmark_b B

```

```

% Constructing a point A such that BA/BM_{c}=2
towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

```

```

% DET: points M_{c} and B are not the same
% Constructing a line c which passes through point M_{c} and point B
line c M_{c} B

```

```

color 200 200 200
drawline c
color 0 0 0

```

```

% NDG: point T_{a} is not incident to the line c
% Constructing a point P_{\_G54983} which is a foot of the point T_{a} on the line c
foot P_{\_G54983} T_{a} c
cmark_r P_{\_G54983}
color 200 200 200
drawline T_{a} P_{\_G54983}
color 0 0 0

```

```

% Constructing a circle k(T_{a},foot[T_{a},c]) whose center is at point T_{a} and which passes
  through point P_{\_G54983}
circle k(T_{a},foot[T_{a},c]) T_{a} P_{\_G54983}

color 200 200 200
drawcircle k(T_{a},foot[T_{a},c])
color 0 0 0

```

```

% NDG: point A is outside the circle k(T_{a},foot[T_{a},c])
% Constructing a point P_{\_G55374} which is a foot of the point T_{a} on the line c
foot P_{\_G55374} T_{a} c
cmark_r P_{\_G55374}
color 200 200 200
drawline T_{a} P_{\_G55374}
color 0 0 0

```

```

% Constructing a line L_{\_G55377} which passes through point T_{a} and point A
line L_{\_G55377} T_{a} A

color 200 200 200
drawline L_{\_G55377}
color 0 0 0

```

```

% Constructing a point  $P_{\setminus\_G55380}$  which is an image of the point  $P_{\setminus\_G55374}$  in the symmetry to
    point/line  $L_{\setminus\_G55377}$ 
sim  $P_{\setminus\_G55380}$   $L_{\setminus\_G55377}$   $P_{\setminus\_G55374}$ 
cmark_r  $P_{\setminus\_G55380}$ 

% Constructing a line  $b$  which passes through point  $A$  and point  $P_{\setminus\_G55380}$ 
line b A  $P_{\setminus\_G55380}$ 

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $b$  and  $a$  are not parallel% DET: lines  $b$  and  $a$  are not the same
% Constructing a point  $C$  which belongs to line  $b$  and line  $a$ 
intersec C b a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(T_{\{a\}},$ 
     $foot[T_{\{a\}},c]);$  point  $T_{\{a\}}$  is not incident to the line  $c$ ; line  $a$  and circle  $k(M_{\{c\}},A)$ 
    intersect; points  $H_{\{a\}}$  and  $M_{\{c\}}$  are not the same
% Determination conditions: lines  $b$  and  $a$  are not the same; points  $M_{\{c\}}$  and  $B$  are not the same;
    points  $H_{\{a\}}$  and  $B$  must be different; points  $H_{\{a\}}$  and  $T_{\{a\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.112 seconds.

**NDG conditions** Points  $T_a$  and  $H_a$  are not identical

Points  $T_a$  and  $H_a$  are not identical

#### 4.1.2 Proving $H_a = H_a$

Proving failed

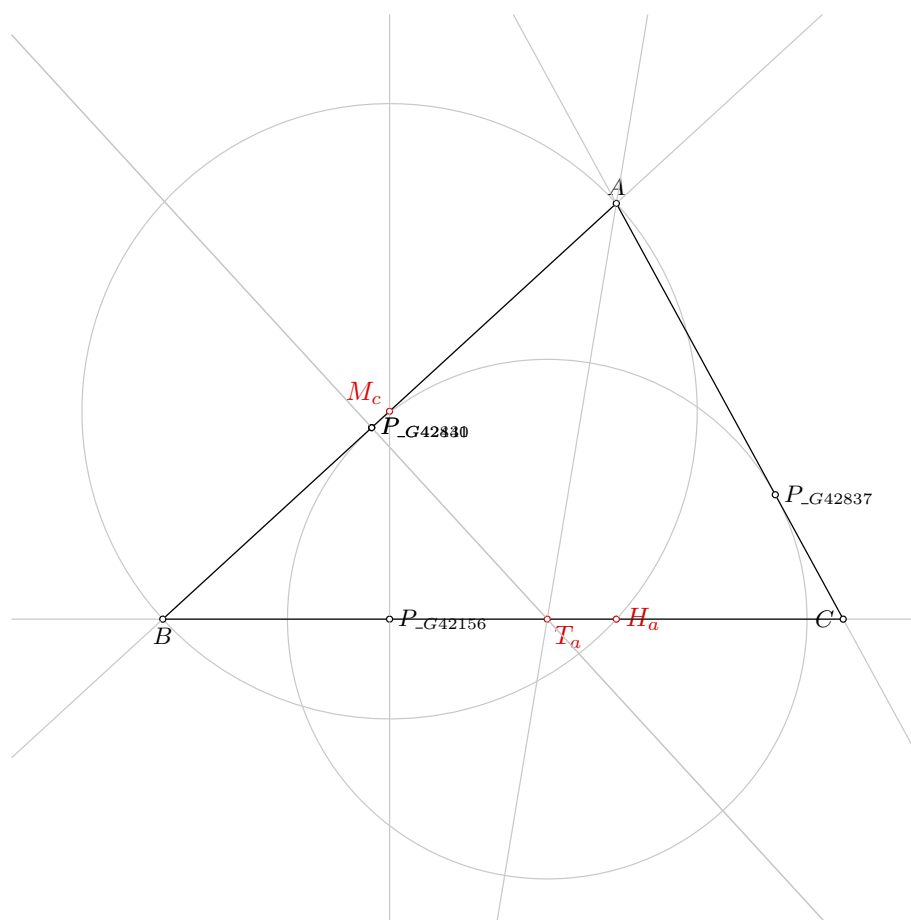


Figure 1: Illustration of the problem 0452

#### **4.1.3 Proving $T_a = \neg T_a$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $M_c = \neg M_c$**

Proving failed

#### **4.2.2 Proving $H_a = \neg H_a$**

Proving failed

#### **4.2.3 Proving $T_a = \neg T_a$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $M_c = \neg M_c$**

Proving failed

#### **4.3.2 Proving $H_a = \neg H_a$**

Proving failed

#### **4.3.3 Proving $T_a = \neg T_a$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $M_c = \neg M_c$**

Proving failed

#### **4.4.2 Proving $H_a = \neg H_a$**

Proving failed

#### **4.4.3 Proving $T_a = \neg T_a$**

Proving failed



# Problem 453

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 453: Given a point  $M_c$ , a point  $H_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 454

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 454: Given a point  $M_c$ , a point  $H_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $T_c$  are not the same;
2. Using the point  $H_a$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_a$  and  $M_c$  are not the same;
3. Using the circle  $k(M_c, A)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(M_c, A)$  intersect;
4. Using the point  $H_a$  and the point  $B$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $B$  are not the same;
5. Using the point  $A$ , the point  $B$ , the point  $T_c$  and the line  $c$ , construct a point  $T'_c$  (rule W19); % NDG: points  $A$  and  $B$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $\text{midpoint}([A, T_c])$  are not the same;
6. Using the point  $T_c$  and the point  $T'_c$ , construct a circle  $k_{over}(T_c, T'_c)$  (rule W09); % NDG: points  $T_c$  and  $T'_c$  are not the same;
7. Using the circle  $k_{over}(T_c, T'_c)$  and the line  $a$ , construct a point  $C_{wb}$  and a point  $C$  (rule W04); % NDG: line  $a$  and circle  $k_{over}(T_c, T'_c)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k_{over}(T_c, T'_c)$  intersect; points  $T_c$  and  $T'_c$  are not the same; points  $A$  and  $B$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $\text{midpoint}([A, T_c])$  are not the same; line  $c$  and circle  $k(M_c, A)$  intersect; points  $H_a$  and  $M_c$  are not the same.

Determination conditions: points  $H_a$  and  $B$  are not the same; points  $M_c$  and  $T_c$  are not the same.

Rules used: [W02,W04,W06,W09,W19]

Lemmas used: [D20,D25,D5,D58,GD01,GD02,GL09,L40,L42,L72,L79]

Solving time: 2722.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point H_{a} 80 40
point T_{c} 55.38 72.43

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_r H_{a}
cmark_rt T_{c}
color 0 0 0
fontsize 8

% DET: points M_{c} and T_{c} are not the same
% Constructing a line c which passes through point M_{c} and point T_{c}
line c M_{c} T_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points H_{a} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{a}
circle k(M_{c},A) M_{c} H_{a}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line c and circle k(M_{c},A) intersect
% Constructing points A and B which are in intersection of k(M_{c},A) and c
intersec2 A B k(M_{c},A) c
cmark_t A
cmark_b B

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
```

```
color 0 0 0
```

```
% NDG: points A and B are not the same; points B and T_{c} are not the same; points B and midpoint
      ([A,T_{c}]) are not the same
% Constructing a point P_{\_G63684} which is an image of the point B in a rotation around the point
      T_{c} for the angle 90
rotate P_{\_G63684} T_{c} 90 B
cmark_r P_{\_G63684}
color 200 200 200
drawarc_p T_{c} B 90
color 0 0 0
```

```
% Constructing a line L_{\_G63687} which passes through point T_{c} and point P_{\_G63684}
line L_{\_G63687} T_{c} P_{\_G63684}

color 200 200 200
drawline L_{\_G63687}
color 0 0 0
```

```
% Constructing midpoint P_{\_G63690} of the segment BP_{\_G63684}
midpoint P_{\_G63690} B P_{\_G63684}
cmark_r P_{\_G63690}
```

```
% Constructing a line L_{\_G63693} which passes through point A and point P_{\_G63690}
line L_{\_G63693} A P_{\_G63690}

color 200 200 200
drawline L_{\_G63693}
color 0 0 0
```

```
% Constructing a line L_{\_G63696} which passes through point B and point P_{\_G63690}
line L_{\_G63696} B P_{\_G63690}

color 200 200 200
drawline L_{\_G63696}
color 0 0 0
```

```
% Constructing a point P_{\_G63699} which belongs to line L_{\_G63687} and line L_{\_G63693}
intersec P_{\_G63699} L_{\_G63687} L_{\_G63693}
cmark_r P_{\_G63699}
```

```
% Constructing a point P_{\_G63702} which belongs to line L_{\_G63687} and line L_{\_G63696}
intersec P_{\_G63702} L_{\_G63687} L_{\_G63696}
cmark_r P_{\_G63702}
```

```
% Constructing a line L_{\_G63705} which passes through point A and point P_{\_G63702}
line L_{\_G63705} A P_{\_G63702}

color 200 200 200
drawline L_{\_G63705}
color 0 0 0
```

```

% Constructing a line  $L_{\backslash\_G63708}$  which passes through point B and point  $P_{\backslash\_G63699}$ 
line  $L_{\backslash\_G63708}$  B  $P_{\backslash\_G63699}$ 

color 200 200 200
drawline  $L_{\backslash\_G63708}$ 
color 0 0 0

% Constructing a point  $P_{\backslash\_G63711}$  which belongs to line  $L_{\backslash\_G63705}$  and line  $L_{\backslash\_G63708}$ 
intersec  $P_{\backslash\_G63711}$   $L_{\backslash\_G63705}$   $L_{\backslash\_G63708}$ 
cmark_r  $P_{\backslash\_G63711}$ 

% Constructing a line  $L_{\backslash\_G63714}$  which passes through point  $P_{\backslash\_G63690}$  and point  $P_{\backslash\_G63711}$ 
line  $L_{\backslash\_G63714}$   $P_{\backslash\_G63690}$   $P_{\backslash\_G63711}$ 

color 200 200 200
drawline  $L_{\backslash\_G63714}$ 
color 0 0 0

% Constructing a point  $T'_{\{c\}}$  which belongs to line  $L_{\backslash\_G63714}$  and line c
intersec  $T'_{\{c\}}$   $L_{\backslash\_G63714}$  c
cmark_r  $T'_{\{c\}}$ 

% NDG: points  $T_{\{c\}}$  and  $T'_{\{c\}}$  are not the same
% Constructing midpoint  $P_{\backslash\_G64545}$  of the segment  $T_{\{c\}}T'_{\{c\}}$ 
midpoint  $P_{\backslash\_G64545}$   $T_{\{c\}}$   $T'_{\{c\}}$ 
cmark_r  $P_{\backslash\_G64545}$ 

% Constructing a circle  $k_{\text{over}}(T_{\{c\}}, T'_{\{c\}})$  whose center is at point  $P_{\backslash\_G64545}$  and which passes
    through point  $T_{\{c\}}$ 
circle  $k_{\text{over}}(T_{\{c\}}, T'_{\{c\}})$   $P_{\backslash\_G64545}$   $T_{\{c\}}$ 

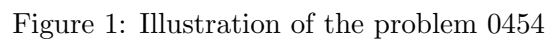
color 200 200 200
drawcircle  $k_{\text{over}}(T_{\{c\}}, T'_{\{c\}})$ 
color 0 0 0

% NDG: line a and circle  $k_{\text{over}}(T_{\{c\}}, T'_{\{c\}})$  intersect
% Constructing points  $C_{\{wb\}}$  and C which are in intersection of  $k_{\text{over}}(T_{\{c\}}, T'_{\{c\}})$  and a
intersec2  $C_{\{wb\}}$  C  $k_{\text{over}}(T_{\{c\}}, T'_{\{c\}})$  a
cmark_r  $C_{\{wb\}}$ 
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle  $k_{\text{over}}(T_{\{c\}}, T'_{\{c\}})$  intersect; points  $T_{\{c\}}$  and  $T'_{\{c\}}$ 
    are not the same; points A and B are not the same; points B and  $T_{\{c\}}$  are not the same;

```



### 3.3 Illustration

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

#### 4.1 OGP - Wu method

### 4.1.1 Proving $M_c = -M_c$

2078

#### **4.1.2 Proving $H_a = \neg H_a$**

Proving failed

#### **4.1.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $M_c = \neg M_c$**

Proving failed

#### **4.2.2 Proving $H_a = \neg H_a$**

Proving failed

#### **4.2.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $M_c = \neg M_c$**

Proving failed

#### **4.3.2 Proving $H_a = \neg H_a$**

Proving failed

#### **4.3.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $M_c = \neg M_c$**

Proving failed

#### **4.4.2 Proving $H_a = \neg H_a$**

Proving failed

#### **4.4.3 Proving $T_c = \neg T_c$**

Proving failed

# Problem 455

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 455: Given a point  $M_c$ , a point  $H_a$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.



# Problem 456

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 456: Given a point  $M_c$ , a point  $H_b$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $H_c$  are not the same;
2. Using the point  $H_b$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_b$  and  $M_c$  are not the same;
3. Using the circle  $k(M_c, A)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(M_c, A)$  intersect;
4. Using the point  $H_b$  and the point  $A$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $A$  are not the same;
5. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
6. Using the line  $h_c$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $h_c$  and  $b$  are not parallel % DET: lines  $h_c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $h_c$  and  $b$  are not parallel; line  $c$  and circle  $k(M_c, A)$  intersect; points  $H_b$  and  $M_c$  are not the same.

Determination conditions: lines  $h_c$  and  $b$  are not the same; points  $H_b$  and  $A$  are not the same; points  $M_c$  and  $H_c$  are not the same.

Rules used: [W02,W03,W04,W06,W10b]

Lemmas used: [D10,D20,D6,D7,GD01,GD02,GL09,L40,L41]

Solving time: 1.1 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point H_{b} 89.36 77.83
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_l H_{b}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points M_{c} and H_{c} are not the same
% Constructing a line c which passes through point M_{c} and point H_{c}
line c M_{c} H_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points H_{b} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{b}
circle k(M_{c},A) M_{c} H_{b}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line c and circle k(M_{c},A) intersect
% Constructing points A and B which are in intersection of k(M_{c},A) and c
intersec2 A B k(M_{c},A) c
cmark_t A
cmark_b B

% DET: points H_{b} and A are not the same
% Constructing a line b which passes through point H_{b} and point A
line b H_{b} A

color 200 200 200
drawline b
color 0 0 0
```

```

% Constructing a line  $h_{\{c\}}$  which is perpendicular to line  $c$  and which passes through point  $H_{\{c\}}$ 
perp  $h_{\{c\}}$   $H_{\{c\}}$   $c$ 

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: lines  $h_{\{c\}}$  and  $b$  are not parallel% DET: lines  $h_{\{c\}}$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $h_{\{c\}}$  and line  $b$ 
intersec  $C$   $h_{\{c\}}$   $b$ 
cmark_1  $C$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $h_{\{c\}}$  and  $b$  are not parallel; line  $c$  and circle  $k(M_{\{c\}}, A)$ 
% intersect; points  $H_{\{b\}}$  and  $M_{\{c\}}$  are not the same
% Determination conditions: lines  $h_{\{c\}}$  and  $b$  are not the same; points  $H_{\{b\}}$  and  $A$  are not the same
% ; points  $M_{\{c\}}$  and  $H_{\{c\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = M_c$

Proving failed

#### 4.1.2 Proving $H_b = H_b$

Proving failed

#### 4.1.3 Proving $H_c = H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 472 terms.

**Time Complexity:** Time spent by the prover is 1.717 seconds.

**NDG conditions** Points  $H_c$  and  $M_c$  are not identical

Points  $H_c$  and  $M_c$  are not identical

Line through points  $H_b$  and  $H_c$  is not perpendicular to line through points  $H_c$  and  $A$

Points  $H_c$  and  $M_c$  are not identical

Points  $A$  and  $B$  are not identical

Line through points  $H_b$  and  $M_c$  is not parallel with line through points  $A$  and  $B$

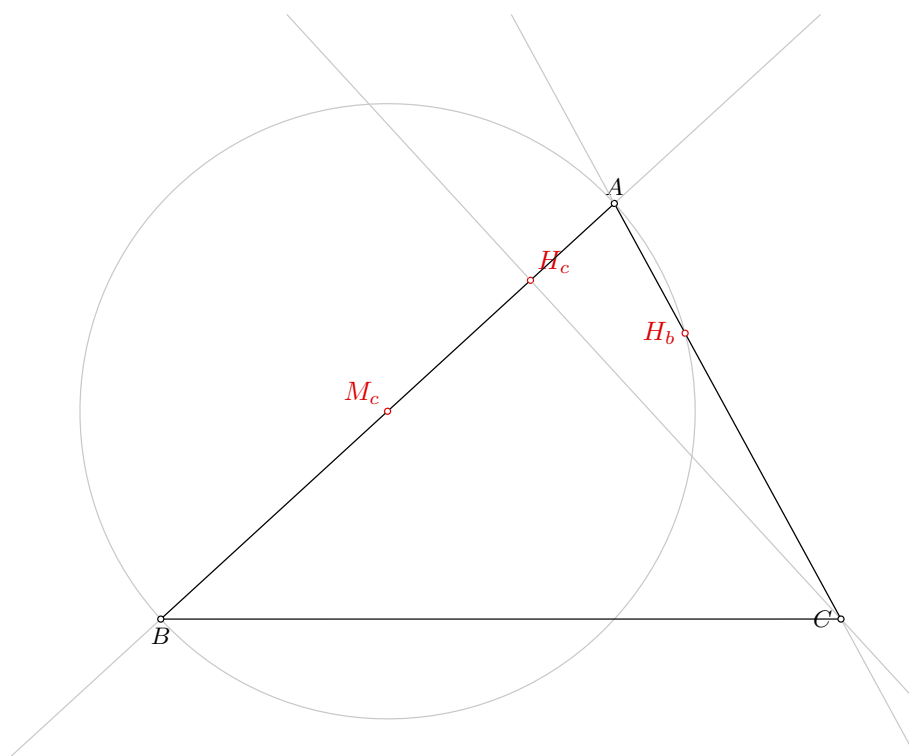


Figure 1: Illustration of the problem 0456

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_c = \neg M_c$

Proving failed

### 4.2.2 Proving $H_b = \neg H_b$

Proving failed

### 4.2.3 Proving $H_c = \neg H_c$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 34 terms.

**Time Complexity:** Time spent by the prover is 0.040 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3080 terms.

**Time Complexity:** Time spent by the prover is 3.910 seconds. There are no ndg conditions.

### 4.3.3 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 1247 terms.

**Time Complexity:** Time spent by the prover is 1.550 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_c = \neg M_c$

Proving failed

### 4.4.2 Proving $H_b = \neg H_b$

Proving failed

### 4.4.3 Proving $H_c = \neg H_c$

Proving failed

# Problem 457

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 457: Given a point  $M_c$ , a point  $H_b$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $H_b$  and  $H$  are not the same;
2. Using the point  $H_b$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_b$  and  $M_c$  are not the same;
3. Using the circle  $k(M_c, A)$ , the line  $h_b$ , the point  $M_c$  and the point  $H_b$ , construct a point  $B$  (rule W05); % NDG: line  $h_b$  and circle  $k(M_c, A)$  intersect % DET: points  $H_b$  and  $B$  must be different;
4. Using the point  $B$  and the point  $M_c$ , construct a point  $A$  (rule W01); ;
5. Using the point  $H_b$  and the point  $A$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $A$  are not the same;
6. Using the point  $H$  and the point  $A$ , construct a line  $h_a$  (rule W02); % DET: points  $H$  and  $A$  are not the same;
7. Using the circle  $k(M_c, A)$ , the line  $h_a$ , the point  $M_c$  and the point  $A$ , construct a point  $H_a$  (rule W05); % NDG: line  $h_a$  and circle  $k(M_c, A)$  intersect % DET: points  $A$  and  $H_a$  must be different;
8. Using the point  $B$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $H_a$  are not the same;
9. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; line  $h_a$  and circle  $k(M_c, A)$  intersect; line  $h_b$  and circle  $k(M_c, A)$  intersect; points  $H_b$  and  $M_c$  are not the same.

Determination conditions: lines  $b$  and  $a$  are not the same; points  $B$  and  $H_a$  are not the same; points  $A$  and  $H_a$  must be different; points  $H$  and  $A$  are not the same; points  $H_b$  and  $A$  are not the same; points  $H_b$  and  $B$  must be different; points  $H_b$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D20,D3,D5,D6,D8,D9,GD01,GD02,GL03,GL04,L40,L41,L42]

Solving time: 1.6 seconds.

## 3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point H_{b} 89.36 77.83
point H 80 72.73

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_l H_{b}
cmark_rt H
color 0 0 0
fontsize 8

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points H_{b} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{b}
circle k(M_{c},A) M_{c} H_{b}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line h_{b} and circle k(M_{c},A) intersect% DET: points H_{b} and B must be different
% Constructing a point P_{\_G57556} which is a foot of the point M_{c} on the line h_{b}
foot P_{\_G57556} M_{c} h_{b}
cmark_r P_{\_G57556}
color 200 200 200
drawline M_{c} P_{\_G57556}
color 0 0 0
```

```

% Constructing a point B which is an image of the point  $H_{\{b\}}$  in the symmetry to point/line  $P_{\{\backslash\_G57556\}}$ 
sim B  $P_{\{\backslash\_G57556\}}$   $H_{\{b\}}$ 
cmark_b B

```

```

% Constructing a point A such that  $BA/BM_{\{c\}}=2$ 
towards A B  $M_{\{c\}}$  2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

```

```

% DET: points  $H_{\{b\}}$  and A are not the same
% Constructing a line b which passes through point  $H_{\{b\}}$  and point A
line b  $H_{\{b\}}$  A

```

```

color 200 200 200
drawline b
color 0 0 0

```

```

% DET: points H and A are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point H and point A
line  $h_{\{a\}}$  H A

```

```

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

```

```

% NDG: line  $h_{\{a\}}$  and circle  $k(M_{\{c\}}, A)$  intersect% DET: points A and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G57919\}}$  which is a foot of the point  $M_{\{c\}}$  on the line  $h_{\{a\}}$ 
foot  $P_{\{\backslash\_G57919\}}$   $M_{\{c\}}$   $h_{\{a\}}$ 
cmark_r  $P_{\{\backslash\_G57919\}}$ 
color 200 200 200
drawline  $M_{\{c\}}$   $P_{\{\backslash\_G57919\}}$ 
color 0 0 0

```

```

% Constructing a point  $H_{\{a\}}$  which is an image of the point A in the symmetry to point/line  $P_{\{\backslash\_G57919\}}$ 
sim  $H_{\{a\}}$   $P_{\{\backslash\_G57919\}}$  A
cmark_r  $H_{\{a\}}$ 

```

```

% DET: points B and  $H_{\{a\}}$  are not the same
% Constructing a line a which passes through point B and point  $H_{\{a\}}$ 
line a B  $H_{\{a\}}$ 

```

```

color 200 200 200
drawline a

```



```
color 0 0 0
```

```
% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines b and a are not parallel; line h_{a} and circle k(M_{c},A)
    intersect; line h_{b} and circle k(M_{c},A) intersect; points H_{b} and M_{c} are not the same
% Determination conditions: lines b and a are not the same; points B and H_{a} are not the same;
    points A and H_{a} must be different; points H and A are not the same; points H_{b} and A are
    not the same; points H_{b} and B must be different; points H_{b} and H are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = \_M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.16 seconds.

**NDG conditions** Points  $H_b$ ,  $M_c$  and  $H$  are not collinear

Points  $H_b$ ,  $M_c$  and  $H$  are not collinear

#### 4.1.2 Proving $H_b = \_H_b$

Proving failed

#### 4.1.3 Proving $H = \_H$

Proving failed

### 4.2 GCLC - Area method

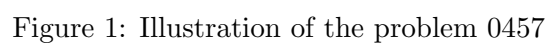
#### 4.2.1 Proving $M_c = \_M_c$

NDG conditions are:

$S_{H_b B H_a} \neq S_{A B H_a}$  i.e., lines  $H_b A$  and  $B H_a$  are not parallel (construction based assumption)

$S_{B A C} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{A B F_{-h_b}^0} \neq S_{C B F_{-h_b}^0}$  i.e., lines  $AC$  and  $B F_{-h_b}^0$  are not parallel (construction based assumption)



$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^0} \neq S_{F_{-h_a}^1 BF_{-h_b}^0}$  i.e., lines  $AF_{-h_a}^1$  and  $BF_{-h_b}^0$  are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

#### 4.2.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.2.3 Proving $H = \neg H$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c = \neg M_c$

Proving failed

#### 4.3.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.3.3 Proving $H = \neg H$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_c = \neg M_c$

Proving failed

#### 4.4.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.4.3 Proving $H = \neg H$

Proving failed

# Problem 458

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 458: Given a point  $M_c$ , a point  $H_b$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 459

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 459: Given a point  $M_c$ , a point  $H_b$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $T_b$  are not the same;
2. Using the point  $H_b$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_b$  and  $M_c$  are not the same;
3. Using the circle  $k(M_c, A)$ , the line  $b$ , the point  $M_c$  and the point  $H_b$ , construct a point  $A$  (rule W05); % NDG: line  $b$  and circle  $k(M_c, A)$  intersect % DET: points  $H_b$  and  $A$  must be different;
4. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); ;
5. Using the point  $M_c$  and the point  $A$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $A$  are not the same;
6. Using the point  $T_b$  and the line  $c$ , construct a circle  $k(T_b, foot[T_b, c])$  (rule W11); % NDG: point  $T_b$  is not incident to the line  $c$ ;
7. Using the circle  $k(T_b, foot[T_b, c])$ , the point  $B$ , the point  $T_b$  and the line  $c$ , construct a line  $a$  (rule W13); % NDG: point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ;
8. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; point  $B$  is outside the circle  $k(T_b, foot[T_b, c])$ ; point  $T_b$  is not incident to the line  $c$ ; line  $b$  and circle  $k(M_c, A)$  intersect; points  $H_b$  and  $M_c$  are not the same.

Determination conditions: lines  $a$  and  $b$  are not the same; points  $M_c$  and  $A$  are not the same; points  $H_b$  and  $A$  must be different; points  $H_b$  and  $T_b$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W11,W13]

Lemmas used: [D20,D24,D6,GD01,GD02,GL03,GL09,GL10,GL11,L41,L9]

Solving time: 1.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point H_{b} 89.36 77.83
point T_{b} 94.25 68.88

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_l H_{b}
cmark_t T_{b}
color 0 0 0
fontsize 8

% DET: points H_{b} and T_{b} are not the same
% Constructing a line b which passes through point H_{b} and point T_{b}
line b H_{b} T_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points H_{b} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{b}
circle k(M_{c},A) M_{c} H_{b}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line b and circle k(M_{c},A) intersect% DET: points H_{b} and A must be different
% Constructing a point P_{\_G63381} which is a foot of the point M_{c} on the line b
foot P_{\_G63381} M_{c} b
cmark_r P_{\_G63381}
color 200 200 200
drawline M_{c} P_{\_G63381}
color 0 0 0

% Constructing a point A which is an image of the point H_{b} in the symmetry to point/line P_{\_G63381}
```

```

sim A P_{\_G63381} H_{b}
cmark_t A

% Constructing a point B such that  $AB/AM_{\{c\}}=2$ 
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points  $M_{\{c\}}$  and A are not the same
% Constructing a line c which passes through point  $M_{\{c\}}$  and point A
line c M_{c} A

color 200 200 200
drawline c
color 0 0 0

% NDG: point  $T_{\{b\}}$  is not incident to the line c
% Constructing a point  $P_{\_G63665}$  which is a foot of the point  $T_{\{b\}}$  on the line c
foot P_{\_G63665} T_{b} c
cmark_r P_{\_G63665}
color 200 200 200
drawline T_{b} P_{\_G63665}
color 0 0 0

% Constructing a circle  $k(T_{\{b\}}, foot[T_{\{b\}}, c])$  whose center is at point  $T_{\{b\}}$  and which passes
    through point  $P_{\_G63665}$ 
circle k(T_{b}, foot[T_{b}, c]) T_{b} P_{\_G63665}

color 200 200 200
drawcircle k(T_{b}, foot[T_{b}, c])
color 0 0 0

% NDG: point B is outside the circle  $k(T_{\{b\}}, foot[T_{\{b\}}, c])$ 
% Constructing a point  $P_{\_G64056}$  which is a foot of the point  $T_{\{b\}}$  on the line c
foot P_{\_G64056} T_{b} c
cmark_r P_{\_G64056}
color 200 200 200
drawline T_{b} P_{\_G64056}
color 0 0 0

% Constructing a line  $L_{\_G64059}$  which passes through point  $T_{\{b\}}$  and point B
line L_{\_G64059} T_{b} B

color 200 200 200
drawline L_{\_G64059}
color 0 0 0

```

```

% Constructing a point  $P_{\{G64062\}}$  which is an image of the point  $P_{\{G64056\}}$  in the symmetry to
    point/line  $L_{\{G64059\}}$ 
sim  $P_{\{G64062\}}$   $L_{\{G64059\}}$   $P_{\{G64056\}}$ 
cmark_r  $P_{\{G64062\}}$ 

% Constructing a line  $a$  which passes through point  $B$  and point  $P_{\{G64062\}}$ 
line a B  $P_{\{G64062\}}$ 

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $a$  and  $b$  are not parallel% DET: lines  $a$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $a$  and line  $b$ 
intersec C a b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; point  $B$  is outside the circle  $k(T_{\{b\}},$ 
     $foot[T_{\{b\}},c])$ ; point  $T_{\{b\}}$  is not incident to the line  $c$ ; line  $b$  and circle  $k(M_{\{c\}},A)$ 
    intersect; points  $H_{\{b\}}$  and  $M_{\{c\}}$  are not the same
% Determination conditions: lines  $a$  and  $b$  are not the same; points  $M_{\{c\}}$  and  $A$  are not the same;
    points  $H_{\{b\}}$  and  $A$  must be different; points  $H_{\{b\}}$  and  $T_{\{b\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = M_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 6 terms.

**Time Complexity:** Time spent by the prover is 0.113 seconds.

**NDG conditions** Points  $H_b$ ,  $M_c$  and  $T_b$  are not collinear

Points  $H_b$ ,  $M_c$  and  $T_b$  are not collinear

#### 4.1.2 Proving $H_b = H_b$

Proving failed



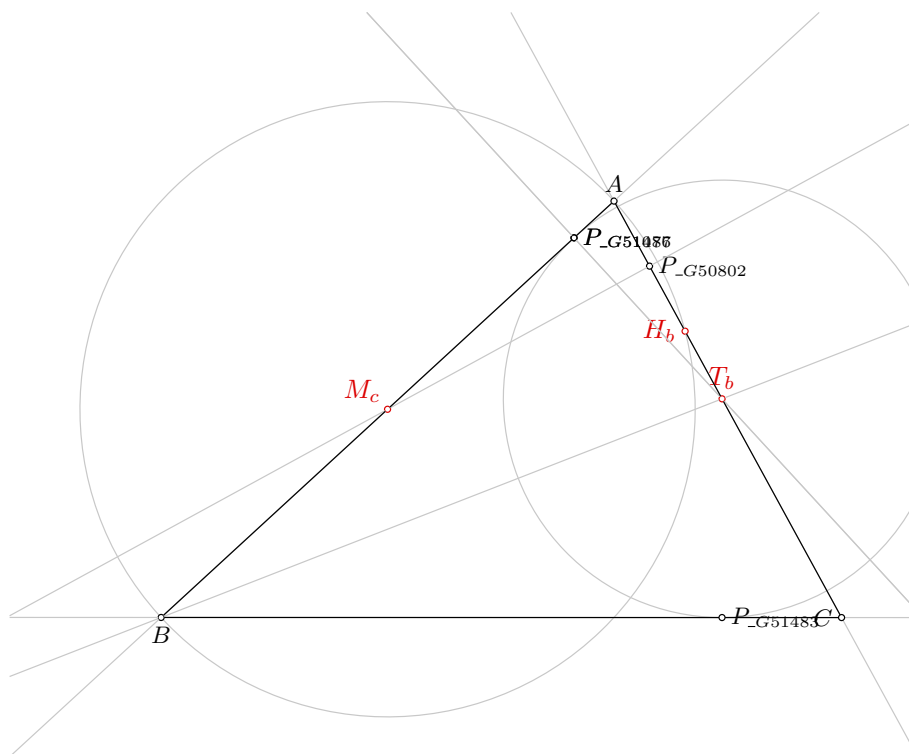


Figure 1: Illustration of the problem 0459

#### **4.1.3 Proving $T_b = \neg T_b$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $M_c = \neg M_c$**

Proving failed

#### **4.2.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.2.3 Proving $T_b = \neg T_b$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $M_c = \neg M_c$**

Proving failed

#### **4.3.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.3.3 Proving $T_b = \neg T_b$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $M_c = \neg M_c$**

Proving failed

#### **4.4.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.4.3 Proving $T_b = \neg T_b$**

Proving failed

# Problem 460

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 460: Given a point  $M_c$ , a point  $H_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $T_c$  are not the same;
2. Using the point  $H_b$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_b$  and  $M_c$  are not the same;
3. Using the circle  $k(M_c, A)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(M_c, A)$  intersect;
4. Using the point  $H_b$  and the point  $A$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $A$  are not the same;
5. Using the point  $A$ , the point  $B$ , the point  $T_c$  and the line  $c$ , construct a point  $T'_c$  (rule W19); % NDG: points  $A$  and  $B$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $\text{midpoint}([A, T_c])$  are not the same;
6. Using the point  $T_c$  and the point  $T'_c$ , construct a circle  $k_{over}(T_c, T'_c)$  (rule W09); % NDG: points  $T_c$  and  $T'_c$  are not the same;
7. Using the circle  $k_{over}(T_c, T'_c)$  and the line  $b$ , construct a point  $C_{wa}$  and a point  $C$  (rule W04); % NDG: line  $b$  and circle  $k_{over}(T_c, T'_c)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k_{over}(T_c, T'_c)$  intersect; points  $T_c$  and  $T'_c$  are not the same; points  $A$  and  $B$  are not the same; points  $B$  and  $T_c$  are not the same; points  $B$  and  $\text{midpoint}([A, T_c])$  are not the same; line  $c$  and circle  $k(M_c, A)$  intersect; points  $H_b$  and  $M_c$  are not the same.

Determination conditions: points  $H_b$  and  $A$  are not the same; points  $M_c$  and  $T_c$  are not the same.

Rules used: [W02,W04,W06,W09,W19]

Lemmas used: [D20,D25,D57,D6,GD01,GD02,GL09,L40,L41,L72,L79]

Solving time: 2619.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{c} 50 67.5
point H_{b} 89.36 77.83
point T_{c} 55.38 72.43
```

```
color 220 0 0
fontsize 9
```

```
cmark_lt M_{c}
cmark_l H_{b}
cmark_rt T_{c}
color 0 0 0
fontsize 8
```

```
% DET: points M_{c} and T_{c} are not the same
% Constructing a line c which passes through point M_{c} and point T_{c}
line c M_{c} T_{c}
```

```
color 200 200 200
drawline c
color 0 0 0
```

```
% NDG: points H_{b} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{b}
circle k(M_{c},A) M_{c} H_{b}
```

```
color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0
```

```
% NDG: line c and circle k(M_{c},A) intersect
% Constructing points A and B which are in intersection of k(M_{c},A) and c
intersec2 A B k(M_{c},A) c
cmark_t A
cmark_b B
```

```
% DET: points H_{b} and A are not the same
% Constructing a line b which passes through point H_{b} and point A
line b H_{b} A
```

```
color 200 200 200
drawline b
```

```
color 0 0 0
```

```
% NDG: points A and B are not the same; points B and T_{c} are not the same; points B and midpoint
      ([A,T_{c}]) are not the same
% Constructing a point P_{\_G42785} which is an image of the point B in a rotation around the point
      T_{c} for the angle 90
rotate P_{\_G42785} T_{c} 90 B
cmark_r P_{\_G42785}
color 200 200 200
drawarc_p T_{c} B 90
color 0 0 0
```

```
% Constructing a line L_{\_G42788} which passes through point T_{c} and point P_{\_G42785}
line L_{\_G42788} T_{c} P_{\_G42785}

color 200 200 200
drawline L_{\_G42788}
color 0 0 0
```

```
% Constructing midpoint P_{\_G42791} of the segment BP_{\_G42785}
midpoint P_{\_G42791} B P_{\_G42785}
cmark_r P_{\_G42791}
```

```
% Constructing a line L_{\_G42794} which passes through point A and point P_{\_G42791}
line L_{\_G42794} A P_{\_G42791}

color 200 200 200
drawline L_{\_G42794}
color 0 0 0
```

```
% Constructing a line L_{\_G42797} which passes through point B and point P_{\_G42791}
line L_{\_G42797} B P_{\_G42791}

color 200 200 200
drawline L_{\_G42797}
color 0 0 0
```

```
% Constructing a point P_{\_G42800} which belongs to line L_{\_G42788} and line L_{\_G42794}
intersec P_{\_G42800} L_{\_G42788} L_{\_G42794}
cmark_r P_{\_G42800}
```

```
% Constructing a point P_{\_G42803} which belongs to line L_{\_G42788} and line L_{\_G42797}
intersec P_{\_G42803} L_{\_G42788} L_{\_G42797}
cmark_r P_{\_G42803}
```

```
% Constructing a line L_{\_G42806} which passes through point A and point P_{\_G42803}
line L_{\_G42806} A P_{\_G42803}

color 200 200 200
drawline L_{\_G42806}
color 0 0 0
```

```

% Constructing a line  $L_{G42809}$  which passes through point B and point  $P_{G42800}$ 
line  $L_{G42809}$  B  $P_{G42800}$ 

color 200 200 200
drawline  $L_{G42809}$ 
color 0 0 0

% Constructing a point  $P_{G42812}$  which belongs to line  $L_{G42806}$  and line  $L_{G42809}$ 
intersec  $P_{G42812}$   $L_{G42806}$   $L_{G42809}$ 
cmark_r  $P_{G42812}$ 

% Constructing a line  $L_{G42815}$  which passes through point  $P_{G42791}$  and point  $P_{G42812}$ 
line  $L_{G42815}$   $P_{G42791}$   $P_{G42812}$ 

color 200 200 200
drawline  $L_{G42815}$ 
color 0 0 0

% Constructing a point  $T'_c$  which belongs to line  $L_{G42815}$  and line c
intersec  $T'_c$   $L_{G42815}$  c
cmark_r  $T'_c$ 

% NDG: points  $T_c$  and  $T'_c$  are not the same
% Constructing midpoint  $P_{G43646}$  of the segment  $T_cT'_c$ 
midpoint  $P_{G43646}$   $T_c$   $T'_c$ 
cmark_r  $P_{G43646}$ 

% Constructing a circle  $k_{over}(T_c, T'_c)$  whose center is at point  $P_{G43646}$  and which passes
    through point  $T_c$ 
circle  $k_{over}(T_c, T'_c)$   $P_{G43646}$   $T_c$ 

color 200 200 200
drawcircle  $k_{over}(T_c, T'_c)$ 
color 0 0 0

% NDG: line b and circle  $k_{over}(T_c, T'_c)$  intersect
% Constructing points  $C_{wa}$  and C which are in intersection of  $k_{over}(T_c, T'_c)$  and b
intersec2  $C_{wa}$  C  $k_{over}(T_c, T'_c)$  b
cmark_r  $C_{wa}$ 
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle  $k_{over}(T_c, T'_c)$  intersect; points  $T_c$  and  $T'_c$ 
    are not the same; points A and B are not the same; points B and  $T_c$  are not the same;

```

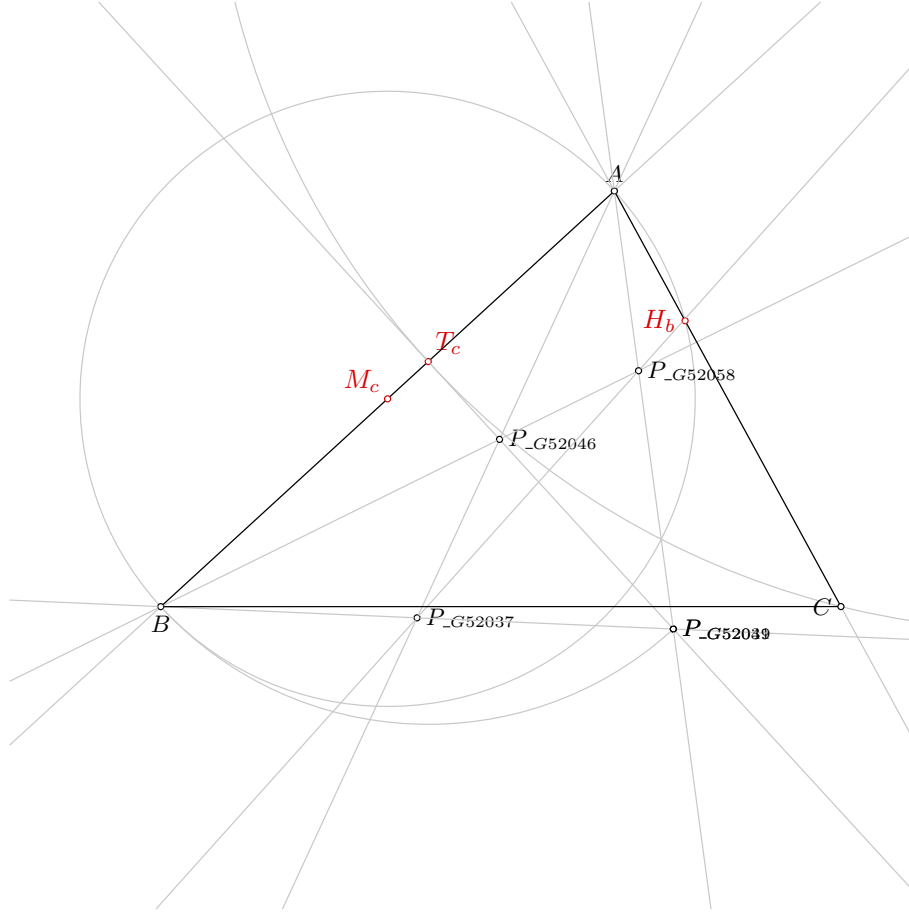


Figure 1: Illustration of the problem 0460

*points B and midpoint([A,T\_{c}]) are not the same; line c and circle k(M\_{c},A) intersect;  
points H\_{b} and M\_{c} are not the same  
% Determination conditions: points H\_{b} and A are not the same; points M\_{c} and T\_{c} are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = M_c$

Proving failed

#### **4.1.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.1.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $M_c = \neg M_c$**

Proving failed

#### **4.2.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.2.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $M_c = \neg M_c$**

Proving failed

#### **4.3.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.3.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $M_c = \neg M_c$**

Proving failed

#### **4.4.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.4.3 Proving $T_c = \neg T_c$**

Proving failed



# Problem 461

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 461: Given a point  $M_c$ , a point  $H_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 462

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 462: Given a point  $H_c$ , a point  $H$  and a point  $M_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $H_c$  and  $H$  are not the same;
2. Using the point  $H_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
3. Choose freely a point  $M_c$  on the line  $c$  (rule WOnline2);
4. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) ;
5. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); ;
6. Using the point  $A$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H$  are not the same;
7. Using the point  $A$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $A$  and  $M_c$  are not the same;
8. Using the circle  $k(M_c, A)$ , the line  $h_a$ , the point  $M_c$  and the point  $A$ , construct a point  $H_a$  (rule W05); % NDG: line  $h_a$  and circle  $k(M_c, A)$  intersect % DET: points  $A$  and  $H_a$  must be different;
9. Using the point  $B$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $H_a$  are not the same;
10. Using the line  $h_c$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $h_c$  and  $a$  are not parallel % DET: lines  $h_c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $h_c$  and  $a$  are not parallel; line  $h_a$  and circle  $k(M_c, A)$  intersect; points  $A$  and  $M_c$  are not the same.

Determination conditions: lines  $h_c$  and  $a$  are not the same; points  $B$  and  $H_a$  are not the same; points  $A$  and  $H_a$  must be different; points  $A$  and  $H$  are not the same; points  $H_c$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D10,D20,D3,D5,D7,D8,GD01,GD02,GL03,L3,L42]

Solving time: 31.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{c} 68.91 84.83
```

```
point H 80 72.73
```

```
point M_{c} 50 67.5
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H_{c}
```

```
cmark_rt H
```

```
cmark_lt M_{c}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{c} and H are not the same
```

```
% Constructing a line h_{c} which passes through point H_{c} and point H
```

```
line h_{c} H_{c} H
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```
% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
```

```
perp c H_{c} h_{c}
```

```
color 200 200 200
```

```
drawline c
```

```
color 0 0 0
```

```
% Generating random value V[_G50506]
```

```
random V[_G50506]
```

```
% Calculating value V[_G50527] using formula V[_G50506]*20
```

```
expression V[_G50527] { V[_G50506]*20 }
```

```

% Constructing a point  $M_{\{c\}}$  which is a point for which holds  $H_{\{c\}}M_{\{c\}} = V[_G50527]$  and angle  $HH_{\{c\}}M_{\{c\}} = 90$ 
turtle M_{c} H H_{c} 90 V[_G50527]
cmark_lt M_{c}

% Choosing randomly a point A on the line  $M_{\{c\}}H_{\{c\}}$ 
online A M_{c} H_{c}
cmark_t A
color 200 200 200
drawline M_{c} H_{c}
color 0 0 0

% Constructing a point B such that  $AB/AM_{\{c\}}=2$ 
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points A and H are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points A and  $M_{\{c\}}$  are not the same
% Constructing a circle  $k(M_{\{c\}},A)$  whose center is at point  $M_{\{c\}}$  and which passes through point A
circle k(M_{c},A) M_{c} A

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line  $h_{\{a\}}$  and circle  $k(M_{\{c\}},A)$  intersect% DET: points A and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G51047\}}$  which is a foot of the point  $M_{\{c\}}$  on the line  $h_{\{a\}}$ 
foot P_{\backslash\_G51047} M_{c} h_{a}
cmark_r P_{\backslash\_G51047}
color 200 200 200
drawline M_{c} P_{\backslash\_G51047}
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point A in the symmetry to point/line  $P_{\{\backslash\_G51047\}}$ 
sim H_{a} P_{\backslash\_G51047} A

```

```

cmark_r H_{a}

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines h_{c} and a are not parallel% DET: lines h_{c} and a are not the same
% Constructing a point C which belongs to line h_{c} and line a
intersec C h_{c} a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and a are not parallel; line h_{a} and circle k(M_{c},A)
% intersect; points A and M_{c} are not the same
% Determination conditions: lines h_{c} and a are not the same; points B and H_{a} are not the same
% ; points A and H_{a} must be different; points A and H are not the same; points H_{c} and H are
% not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_c = \_H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.147 seconds.

**NDG conditions** Points  $A$  and  $H_c$  are not identical

Points  $H_c$  and  $H$  are not identical

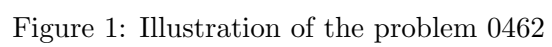
Line through points  $A$  and  $B$  is not perpendicular to line through points  $B$  and  $H_c$

Points  $A$ ,  $B$  and  $H$  are not collinear

#### 4.1.2 Proving $H = \_H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 20 terms.



**Time Complexity:** Time spent by the prover is 0.379 seconds.

**NDG conditions** Points  $A$  and  $H_c$  are not identical

Points  $H_c$  and  $H$  are not identical

Line through points  $A$  and  $B$  is not perpendicular to line through points  $B$  and  $H_c$

Points  $A$ ,  $B$  and  $C$  are not collinear

Points  $B$ ,  $C$  and  $M_c$  are not collinear

#### 4.1.3 Proving $M_c = M_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_c = H_c$

Proving failed

#### 4.2.2 Proving $H = H$

Proving failed

#### 4.2.3 Proving $M_c = M_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_c = H_c$

Proving failed

#### 4.3.2 Proving $H = H$

Proving failed

#### 4.3.3 Proving $M_c = M_c$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $H_c = H_c$

Proving failed

#### 4.4.2 Proving $H = H$

Proving failed

#### 4.4.3 Proving $M_c = M_c$

Proving failed

# Problem 463

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 463: Given a point  $M_c$ , a point  $H_c$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.



# Problem 464

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 464: Given a point  $M_c$ , a point  $H_c$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 465

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 465: Given a point  $H_c$ , a point  $T_c$  and a point  $M_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $T_c$  are not the same;
2. Choose freely a point  $M_c$  on the line  $c$  (rule WOnline1) ;
3. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
4. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) ;
5. Using the point  $A$  and the point  $M_c$ , construct a point  $B$  (rule W01); ;
6. Choose freely a point  $C$  on the line  $h_c$  (rule WOnline2).

Non-degenerate conditions: .

Determination conditions: points  $H_c$  and  $T_c$  are not the same.

Rules used: [W01,W02,W10b,WOnline1,WOnline2]

Lemmas used: [D10,D20,D25,D7,GL03]

Solving time: 1903.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{c} 68.91 84.83
```

```
point T_{c} 55.38 72.43
```

```
point M_{c} 50 67.5
```

```
color 220 0 0
```

```

fontsize 9

cmark_rt H_{c}
cmark_rt T_{c}
cmark_lt M_{c}
color 0 0 0
fontsize 8

% DET: points H_{c} and T_{c} are not the same
% Constructing a line c which passes through point H_{c} and point T_{c}
line c H_{c} T_{c}

color 200 200 200
drawline c
color 0 0 0


% Choosing randomly a point M_{c} on the line H_{c}T_{c}
online M_{c} H_{c} T_{c}
cmark_lt M_{c}
color 200 200 200
drawline H_{c} T_{c}
color 0 0 0


% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0


% Choosing randomly a point A on the line M_{c}H_{c}
online A M_{c} H_{c}
cmark_t A
color 200 200 200
drawline M_{c} H_{c}
color 0 0 0


% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

```

```

% Generating random value V[_G56635]
random V[_G56635]

% Calculating value V[_G56656] using formula V[_G56635]*20
expression V[_G56656] { V[_G56635]*20 }

% Constructing a point C which is a point for which holds  $H_{\{c\}C} = V[_G56656]$  and angle  $AH_{\{c\}C} = 90$ 
turtle C A H_{c} 90 V[_G56656]
cmark_1 C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions:
% Determination conditions: points  $H_{\{c\}}$  and  $T_{\{c\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_c = \_H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 2 terms.

**Time Complexity:** Time spent by the prover is 0.062 seconds.

**NDG conditions** Line through points  $A$  and  $H_c$  is not perpendicular to line through points  $H_c$  and  $B$

#### 4.1.2 Proving $T_c = \_T_c$

Proving failed

#### 4.1.3 Proving $M_c = \_M_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_c = \_H_c$

Proving failed

#### 4.2.2 Proving $T_c = \_T_c$

Proving failed

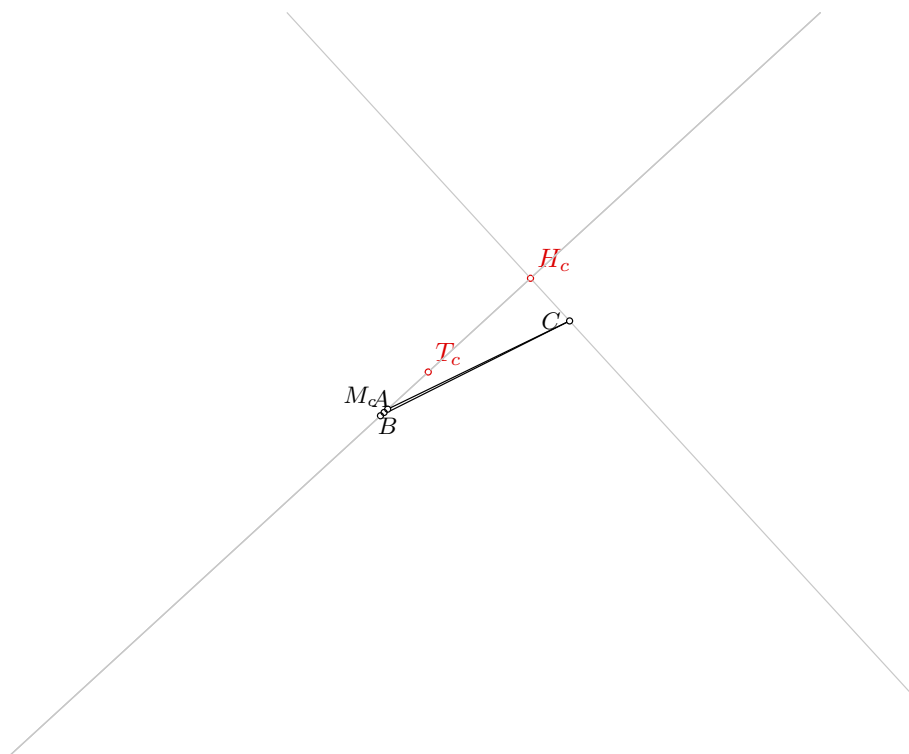


Figure 1: Illustration of the problem 0465

### 4.2.3 Proving $M_c = \neg M_c$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $H_c = \neg H_c$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 1 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.2 Proving $T_c = \neg T_c$

**Status:** The conjecture has been disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 3 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.3.3 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 6 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $H_c = \neg H_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.2 Proving $T_c = \neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.3 Proving $M_c = \neg M_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 4 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

# Problem 466

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 466: Given a point  $M_c$ , a point  $H_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $H_c$  are not the same;
2. Using the point  $M_c$  and the point  $I$ , construct a line  $IM_c$  (rule W02); % DET: points  $M_c$  and  $I$  are not the same;
3. Using the point  $I$  and the point  $M_c$ , construct a circle  $k_{over}(I, M_c)$  (rule W09); % NDG: points  $I$  and  $M_c$  are not the same;
4. Using the point  $M_c$  and the line  $c$ , construct a line  $m_c$  (rule W10b); ;
5. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
6. Using the point  $I$  and the line  $c$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $c$ ;
7. Using the circle  $k(I, P_a)$  and the circle  $k_{over}(I, M_c)$ , construct a point  $C_{fi}$  and a point  $P_c$  (rule W07); % NDG: circles  $k(I, P_a)$  and  $k_{over}(I, M_c)$  intersect % DET: circles  $k(I, P_a)$  and  $k_{over}(I, M_c)$  are not the same;
8. Using the point  $P_c$  and the point  $M_c$ , construct a point  $P'_c$  (rule W01); ;
9. Using the point  $P'_c$  and the line  $IM_c$ , construct a line  $CP'_c$  (rule W16); ;
10. Using the line  $CP'_c$  and the line  $h_c$ , construct a point  $C$  (rule W03); % NDG: lines  $CP'_c$  and  $h_c$  are not parallel % DET: lines  $CP'_c$  and  $h_c$  are not the same;
11. Using the point  $I$  and the point  $C$ , construct a line  $s_c$  (rule W02); % DET: points  $I$  and  $C$  are not the same;

12. Using the line  $m_c$  and the line  $s_c$ , construct a point  $N_c$  (rule W03); % NDG: lines  $m_c$  and  $s_c$  are not parallel % DET: lines  $m_c$  and  $s_c$  are not the same;
13. Using the point  $I$  and the point  $N_c$ , construct a circle  $k(N_c, B)$  (rule W06); % NDG: points  $I$  and  $N_c$  are not the same;
14. Using the circle  $k(N_c, B)$  and the line  $c$ , construct a point  $B$  and a point  $A$  (rule W04); % NDG: line  $c$  and circle  $k(N_c, B)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(N_c, B)$  intersect; points  $I$  and  $N_c$  are not the same; lines  $m_c$  and  $s_c$  are not parallel; lines  $CP'_c$  and  $h_c$  are not parallel; circles  $k(I, P_a)$  and  $k_{over}(I, M_c)$  intersect; point  $I$  is not incident to the line  $c$ ; points  $I$  and  $M_c$  are not the same.

Determination conditions: lines  $m_c$  and  $s_c$  are not the same; points  $I$  and  $C$  are not the same; lines  $CP'_c$  and  $h_c$  are not the same; circles  $k(I, P_a)$  and  $k_{over}(I, M_c)$  are not the same; points  $M_c$  and  $I$  are not the same; points  $M_c$  and  $H_c$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W07,W09,W10b,W11,W16]

Lemmas used: [D10,D13,D20,D27,D49,D67,D7,GD01,GD02,GL03,GL09,L14,L2,L27,L35,L36,L59,L64,L7,L76]

Solving time: 5.4 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{c} 50 67.5
point H_{c} 68.91 84.83
point I 74.37 61.15
```

```
color 220 0 0
fontsize 9
```

```
cmark_lt M_{c}
cmark_rt H_{c}
cmark_b I
color 0 0 0
fontsize 8
```

```
% DET: points M_{c} and H_{c} are not the same
% Constructing a line c which passes through point M_{c} and point H_{c}
line c M_{c} H_{c}
```

```
color 200 200 200
drawline c
color 0 0 0
```

```
% DET: points M_{c} and I are not the same
% Constructing a line IM_{c} which passes through point M_{c} and point I
line IM_{c} M_{c} I
```

```
color 200 200 200
drawline IM_{c}
```



```
color 0 0 0
```

```
% NDG: points I and M_{c} are not the same
% Constructing midpoint P_{\_G62205} of the segment IM_{c}
midpoint P_{\_G62205} I M_{c}
cmark_r P_{\_G62205}
```

```
% Constructing a circle k_{over(I,M_{c})} whose center is at point P_{\_G62205} and which passes
through point I
circle k_{over(I,M_{c})} P_{\_G62205} I
```

```
color 200 200 200
drawcircle k_{over(I,M_{c})}
color 0 0 0
```

```
% Constructing a line m_{c} which is perpendicular to line c and which passes through point M_{c}
perp m_{c} M_{c} c
```

```
color 200 200 200
drawline m_{c}
color 0 0 0
```

```
% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c
```

```
color 200 200 200
drawline h_{c}
color 0 0 0
```

```
% NDG: point I is not incident to the line c
% Constructing a point P_{\_G62503} which is a foot of the point I on the line c
foot P_{\_G62503} I c
cmark_r P_{\_G62503}
color 200 200 200
drawline I P_{\_G62503}
color 0 0 0
```

```
% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
62503}
circle k(I,P_{a}) I P_{\_G62503}
```

```
color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0
```

```

% NDG: circles  $k(I, P_{\{a\}})$  and  $k_{\text{over}}(I, M_{\{c\}})$  intersect% DET: circles  $k(I, P_{\{a\}})$  and  $k_{\text{over}}(I, M_{\{c\}})$ 
%) are not the same
% Constructing points  $C_{\{fi\}}$  and  $P_{\{c\}}$  which are in intersection of  $k(I, P_{\{a\}})$  and  $k_{\text{over}}(I, M_{\{c\}})$ 
intersec2 C_{fi} P_{c} k(I, P_{a}) k_{\text{over}}(I, M_{c})
cmark_r C_{fi}
cmark_r P_{c}

% Constructing a point  $P'_{\{c\}}$  such that  $P_{\{c\}}P'_{\{c\}}/P_{\{c\}}M_{\{c\}}=2$ 
towards P'_{c} P_{c} M_{c} 2
cmark_r P'_{c}
color 200 200 200
drawsegment P_{c} P'_{c}
color 0 0 0

% Constructing a line  $CP'_{\{c\}}$  which contains the point  $P'_{\{c\}}$  and is parallel to the line  $IM_{\{c\}}$ 
parallel CP'_{c} P'_{c} IM_{c}

color 200 200 200
drawline CP'_{c}
color 0 0 0

% NDG: lines  $CP'_{\{c\}}$  and  $h_{\{c\}}$  are not parallel% DET: lines  $CP'_{\{c\}}$  and  $h_{\{c\}}$  are not the same
% Constructing a point  $C$  which belongs to line  $CP'_{\{c\}}$  and line  $h_{\{c\}}$ 
intersec C CP'_{c} h_{c}
cmark_l C

% DET: points  $I$  and  $C$  are not the same
% Constructing a line  $s_{\{c\}}$  which passes through point  $I$  and point  $C$ 
line s_{c} I C

color 200 200 200
drawline s_{c}
color 0 0 0

% NDG: lines  $m_{\{c\}}$  and  $s_{\{c\}}$  are not parallel% DET: lines  $m_{\{c\}}$  and  $s_{\{c\}}$  are not the same
% Constructing a point  $N_{\{c\}}$  which belongs to line  $m_{\{c\}}$  and line  $s_{\{c\}}$ 
intersec N_{c} m_{c} s_{c}
cmark_b N_{c}

% NDG: points  $I$  and  $N_{\{c\}}$  are not the same
% Constructing a circle  $k(N_{\{c\}}, B)$  whose center is at point  $N_{\{c\}}$  and which passes through point  $I$ 
circle k(N_{c}, B) N_{c} I

color 200 200 200
drawcircle k(N_{c}, B)

```

```
color 0 0 0
```

```
% NDG: line c and circle k(N_{c},B) intersect
% Constructing points B and A which are in intersection of k(N_{c},B) and c
intersec2 B A k(N_{c},B) c
cmark_b B
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line c and circle k(N_{c},B) intersect; points I and N_{c} are not the
same; lines m_{c} and s_{c} are not parallel; lines CP'_{c} and h_{c} are not parallel;
circles k(I,P_{a}) and k_{over}(I,M_{c}) intersect; point I is not incident to the line c; points
I and M_{c} are not the same
% Determination conditions: lines m_{c} and s_{c} are not the same; points I and C are not the same
; lines CP'_{c} and h_{c} are not the same; circles k(I,P_{a}) and k_{over}(I,M_{c}) are not the
same; points M_{c} and I are not the same; points M_{c} and H_{c} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

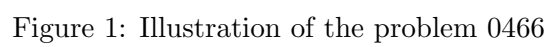
#### 4.1.1 Proving $M_c = M_c$

Proving failed

#### 4.1.2 Proving $H_c = H_c$

**Construction steps:**

- Free point  $M_c$
- Free point  $H_c$
- Free point  $I$
- Line  $c$  through two points  $M_c$  and  $H_c$
- Line  $IM_c$  through two points  $M_c$  and  $I$
- Midpoint  $P_{G57228}$  of segment  $IM_c$
- Circle  $k_{over}(I, M_c)$  with center  $P_{G57228}$  and one point  $I$
- Line  $m_c$  through point  $M_c$  perpendicular to line  $c$
- Line  $h_c$  through point  $H_c$  perpendicular to line  $c$
- Line footPointPerpLine358 through point  $I$  perpendicular to line  $c$
- Intersection point  $P_{G57354}$  of point sets footPointPerpLine358 and  $c$



- Circle  $k(I, P_a)$  with center  $I$  and one point  $P_{G57354}$
- Intersection point  $C_{fi}$  of point sets  $k(I, P_a)$  and  $k_{over}(I, M_c)$
- Intersection point  $P_c$  of point sets  $k(I, P_a)$  and  $k_{over}(I, M_c)$
- Segment division point  $P'_c$  of segment  $P_c M_c$  with division coefficient -2.0
- Line  $CP'_c$  through point  $P'_c$  parallel with line  $IM_c$
- Intersection point  $C$  of point sets  $CP'_c$  and  $h_c$
- Line  $s_c$  through two points  $I$  and  $C$
- Intersection point  $N_c$  of point sets  $m_c$  and  $s_c$
- Circle  $k(N_c, B)$  with center  $N_c$  and one point  $I$
- Intersection point  $B$  of point sets  $k(N_c, B)$  and  $c$
- Intersection point  $A$  of point sets  $k(N_c, B)$  and  $c$
- Segment division point  $\_M_c$  of segment  $AB$  with division coefficient 1.0
- Line  $\_c$  through two points  $A$  and  $B$
- Line  $\_h_c$  through point  $C$  perpendicular to line  $\_c$
- Intersection point  $\_H_c$  of point sets  $\_c$  and  $\_h_c$
- Line  $\_a$  through two points  $B$  and  $C$
- Line  $\_b$  through two points  $A$  and  $C$
- Angle bisector  $\_s_a$  of angle  $BAC$
- Angle bisector  $\_s_b$  of angle  $CBA$
- Intersection point  $\_I$  of point sets  $\_s_a$  and  $\_s_b$

### Theorem statement:

- Points  $H_c$  and  $\_H_c$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $M_c$  has been assigned following coordinates: (0, 0)

### 4.1.3 Proving $I = \_I$

#### Construction steps:

- Free point  $M_c$
- Free point  $H_c$
- Free point  $I$
- Line  $c$  through two points  $M_c$  and  $H_c$
- Line  $IM_c$  through two points  $M_c$  and  $I$
- Midpoint  $P_{G58887}$  of segment  $IM_c$
- Circle  $k_{over}(I, M_c)$  with center  $P_{G58887}$  and one point  $I$
- Line  $m_c$  through point  $M_c$  perpendicular to line  $c$
- Line  $h_c$  through point  $H_c$  perpendicular to line  $c$
- Line footPointPerpLine82 through point  $I$  perpendicular to line  $c$
- Intersection point  $P_{G59013}$  of point sets footPointPerpLine82 and  $c$
- Circle  $k(I, P_a)$  with center  $I$  and one point  $P_{G59013}$
- Intersection point  $C_{fi}$  of point sets  $k(I, P_a)$  and  $k_{over}(I, M_c)$
- Intersection point  $P_c$  of point sets  $k(I, P_a)$  and  $k_{over}(I, M_c)$

- Segment division point  $P'_c$  of segment  $P_cM_c$  with division coefficient -2.0
- Line  $CP'_c$  through point  $P'_c$  parallel with line  $IM_c$
- Intersection point  $C$  of point sets  $CP'_c$  and  $h_c$
- Line  $s_c$  through two points  $I$  and  $C$
- Intersection point  $N_c$  of point sets  $m_c$  and  $s_c$
- Circle  $k(N_c, B)$  with center  $N_c$  and one point  $I$
- Intersection point  $B$  of point sets  $k(N_c, B)$  and  $c$
- Intersection point  $A$  of point sets  $k(N_c, B)$  and  $c$
- Segment division point  $_M_c$  of segment  $AB$  with division coefficient 1.0
- Line  $_c$  through two points  $A$  and  $B$
- Line  $_h_c$  through point  $C$  perpendicular to line  $_c$
- Intersection point  $_H_c$  of point sets  $_c$  and  $_h_c$
- Line  $_a$  through two points  $B$  and  $C$
- Line  $_b$  through two points  $A$  and  $C$
- Angle bisector  $_s_a$  of angle  $BAC$
- Angle bisector  $_s_b$  of angle  $CBA$
- Intersection point  $_I$  of point sets  $_s_a$  and  $_s_b$

#### Theorem statement:

- Points  $I$  and  $_I$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $M_c$  has been assigned following coordinates:  $(0, 0)$

### 4.2 GCLC - Area method

#### 4.2.1 Proving $M_c=_M_c$

Proving failed

#### 4.2.2 Proving $H_c=_H_c$

Proving failed

#### 4.2.3 Proving $I=_I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c=_M_c$

Proving failed

#### 4.3.2 Proving $H_c=_H_c$

Proving failed

#### 4.3.3 Proving $I=_I$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $M_c = M_c$

Proving failed

### 4.4.2 Proving $H_c = H_c$

Proving failed

### 4.4.3 Proving $I = I$

Proving failed

## Problem 467

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 467: Given a point  $M_c$ , a point  $H$  and a point  $T_a$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.



# Problem 468

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 468: Given a point  $M_c$ , a point  $H$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 469

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 469: Given a point  $M_c$ , a point  $H$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 470

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 470: Given a point  $M_c$ , a point  $H$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 471

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 471: Given a point  $M_c$ , a point  $T_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 472

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 472: Given a point  $M_c$ , a point  $T_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 473

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 473: Given a point  $M_c$ , a point  $T_a$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 474

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 474: Given a point  $M_c$ , a point  $T_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 475

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 475: Given a point  $M_c$ , a point  $T_b$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.



# Problem 476

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 476: Given a point  $M_c$ , a point  $T_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $M_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $M_c$  and  $T_c$  are not the same;
2. Using the point  $T_c$  and the point  $I$ , construct a line  $s_c$  (rule W02); % DET: points  $T_c$  and  $I$  are not the same;
3. Using the point  $M_c$  and the line  $c$ , construct a line  $m_c$  (rule W10b); ;
4. Using the line  $m_c$  and the line  $s_c$ , construct a point  $N_c$  (rule W03); % NDG: lines  $m_c$  and  $s_c$  are not parallel % DET: lines  $m_c$  and  $s_c$  are not the same;
5. Using the point  $I$  and the point  $N_c$ , construct a circle  $k(N_c, B)$  (rule W06); % NDG: points  $I$  and  $N_c$  are not the same;
6. Using the circle  $k(N_c, B)$  and the line  $c$ , construct a point  $B$  and a point  $A$  (rule W04); % NDG: line  $c$  and circle  $k(N_c, B)$  intersect;
7. Using the point  $I$  and the line  $c$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $c$ ;
8. Using the circle  $k(I, P_a)$ , the point  $B$ , the point  $I$  and the line  $c$ , construct a line  $a$  (rule W13); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
9. Using the line  $a$  and the line  $s_c$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $s_c$  are not parallel % DET: lines  $a$  and  $s_c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $s_c$  are not parallel; point  $B$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $c$ ; line  $c$  and circle  $k(N_c, B)$  intersect; points  $I$  and  $N_c$  are not the same; lines  $m_c$  and  $s_c$  are not parallel.

Determination conditions: lines  $a$  and  $s_c$  are not the same; lines  $m_c$  and  $s_c$  are not the same; points  $T_c$  and  $I$  are not the same; points  $M_c$  and  $T_c$  are not the same.

Rules used: [W02,W03,W04,W06,W10b,W11,W13]

Lemmas used: [D13,D20,D25,D27,D49,GD01,GD02,GL09,L2,L27,L35,L36,L59,L60,L7]

Solving time: 13.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{c} 50 67.5
```

```
point T_{c} 55.38 72.43
```

```
point I 74.37 61.15
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_lt M_{c}
```

```
cmark_rt T_{c}
```

```
cmark_b I
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points M_{c} and T_{c} are not the same
```

```
% Constructing a line c which passes through point M_{c} and point T_{c}
```

```
line c M_{c} T_{c}
```

```
color 200 200 200
```

```
drawline c
```

```
color 0 0 0
```

```
% DET: points T_{c} and I are not the same
```

```
% Constructing a line s_{c} which passes through point T_{c} and point I
```

```
line s_{c} T_{c} I
```

```
color 200 200 200
```

```
drawline s_{c}
```

```
color 0 0 0
```

```
% Constructing a line m_{c} which is perpendicular to line c and which passes through point M_{c}
```

```
perp m_{c} M_{c} c
```

```
color 200 200 200
```

```
drawline m_{c}
```

```
color 0 0 0
```

```

% NDG: lines  $m_{\{c\}}$  and  $s_{\{c\}}$  are not parallel% DET: lines  $m_{\{c\}}$  and  $s_{\{c\}}$  are not the same
% Constructing a point  $N_{\{c\}}$  which belongs to line  $m_{\{c\}}$  and line  $s_{\{c\}}$ 
intersec  $N_{\{c\}}$   $m_{\{c\}}$   $s_{\{c\}}$ 
cmark_b  $N_{\{c\}}$ 

% NDG: points  $I$  and  $N_{\{c\}}$  are not the same
% Constructing a circle  $k(N_{\{c\}},B)$  whose center is at point  $N_{\{c\}}$  and which passes through point  $I$ 
circle  $k(N_{\{c\}},B)$   $N_{\{c\}}$   $I$ 

color 200 200 200
drawcircle  $k(N_{\{c\}},B)$ 
color 0 0 0

% NDG: line  $c$  and circle  $k(N_{\{c\}},B)$  intersect
% Constructing points  $B$  and  $A$  which are in intersection of  $k(N_{\{c\}},B)$  and  $c$ 
intersec2  $B$   $A$   $k(N_{\{c\}},B)$   $c$ 
cmark_b  $B$ 
cmark_t  $A$ 

% NDG: point  $I$  is not incident to the line  $c$ 
% Constructing a point  $P_{\{\backslash\_G56914\}}$  which is a foot of the point  $I$  on the line  $c$ 
foot  $P_{\{\backslash\_G56914\}}$   $I$   $c$ 
cmark_r  $P_{\{\backslash\_G56914\}}$ 
color 200 200 200
drawline  $I$   $P_{\{\backslash\_G56914\}}$ 
color 0 0 0

% Constructing a circle  $k(I,P_{\{a\}})$  whose center is at point  $I$  and which passes through point  $P_{\{\backslash\_G56914\}}$ 
circle  $k(I,P_{\{a\}})$   $I$   $P_{\{\backslash\_G56914\}}$ 

color 200 200 200
drawcircle  $k(I,P_{\{a\}})$ 
color 0 0 0

% NDG: point  $B$  is outside the circle  $k(I,P_{\{a\}})$ 
% Constructing a point  $P_{\{\backslash\_G57121\}}$  which is a foot of the point  $I$  on the line  $c$ 
foot  $P_{\{\backslash\_G57121\}}$   $I$   $c$ 
cmark_r  $P_{\{\backslash\_G57121\}}$ 
color 200 200 200
drawline  $I$   $P_{\{\backslash\_G57121\}}$ 
color 0 0 0

% Constructing a line  $L_{\{\backslash\_G57124\}}$  which passes through point  $I$  and point  $B$ 
line  $L_{\{\backslash\_G57124\}}$   $I$   $B$ 

color 200 200 200
drawline  $L_{\{\backslash\_G57124\}}$ 

```

```

color 0 0 0

% Constructing a point  $P_{\backslash\_G57127}$  which is an image of the point  $P_{\backslash\_G57121}$  in the symmetry to
  point/line  $L_{\backslash\_G57124}$ 
sim  $P_{\backslash\_G57127}$   $L_{\backslash\_G57124}$   $P_{\backslash\_G57121}$ 
cmark_r  $P_{\backslash\_G57127}$ 

% Constructing a line  $a$  which passes through point  $B$  and point  $P_{\backslash\_G57127}$ 
line a B  $P_{\backslash\_G57127}$ 

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $a$  and  $s_{\{c\}}$  are not parallel% DET: lines  $a$  and  $s_{\{c\}}$  are not the same
% Constructing a point  $C$  which belongs to line  $a$  and line  $s_{\{c\}}$ 
intersec C a  $s_{\{c\}}$ 
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $a$  and  $s_{\{c\}}$  are not parallel; point  $B$  is outside the circle  $k(I, P_{\{a\}})$ ; point  $I$  is not incident to the line  $c$ ; line  $c$  and circle  $k(N_{\{c\}}, B)$  intersect; points  $I$  and  $N_{\{c\}}$  are not the same; lines  $m_{\{c\}}$  and  $s_{\{c\}}$  are not parallel
% Determination conditions: lines  $a$  and  $s_{\{c\}}$  are not the same; lines  $m_{\{c\}}$  and  $s_{\{c\}}$  are not the same; points  $T_{\{c\}}$  and  $I$  are not the same; points  $M_{\{c\}}$  and  $T_{\{c\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $M_c = M_c$

Proving failed

#### 4.1.2 Proving $T_c = T_c$

Construction steps:

- Free point  $M_c$
- Free point  $T_c$
- Free point  $I$

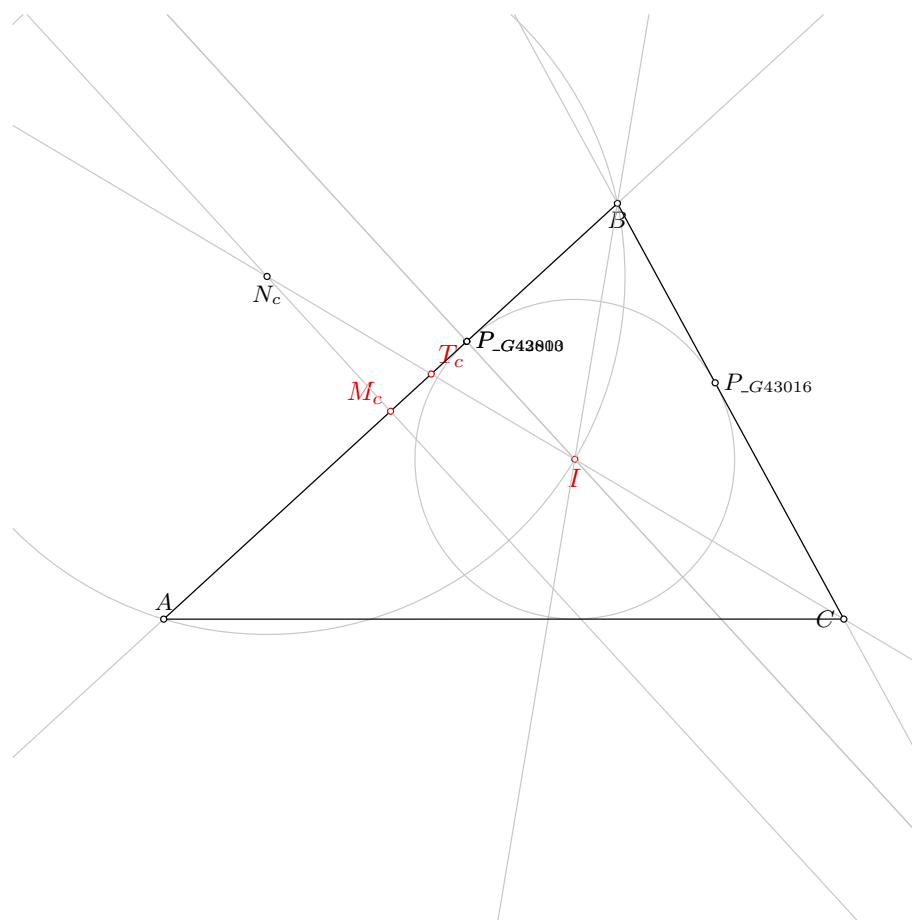


Figure 1: Illustration of the problem 0476

- Line  $c$  through two points  $M_c$  and  $T_c$
- Line  $s_c$  through two points  $T_c$  and  $I$
- Line  $m_c$  through point  $M_c$  perpendicular to line  $c$
- Intersection point  $N_c$  of point sets  $m_c$  and  $s_c$
- Circle  $k(N_c, B)$  with center  $N_c$  and one point  $I$
- Intersection point  $B$  of point sets  $k(N_c, B)$  and  $c$
- Intersection point  $A$  of point sets  $k(N_c, B)$  and  $c$
- Line footPointPerpLine354 through point  $I$  perpendicular to line  $c$
- Intersection point  $P_{G52361}$  of point sets footPointPerpLine354 and  $c$
- Circle  $k(I, P_a)$  with center  $I$  and one point  $P_{G52361}$
- Line footPointPerpLine296 through point  $I$  perpendicular to line  $c$
- Intersection point  $P_{G52464}$  of point sets footPointPerpLine296 and  $c$
- Line  $L_{G52467}$  through two points  $I$  and  $B$
- Line reflexivePointPerpLine101 through point  $P_{G52464}$  perpendicular to line  $L_{G52467}$
- Intersection point reflexivePointFootPoint743 of point sets reflexivePointPerpLine101 and  $L_{G52467}$
- Cental symmetric point  $P_{G52470}$  of point  $P_{G52464}$  with respect to center of symmetry reflexivePointFootPoint743
- Line  $a$  through two points  $B$  and  $P_{G52470}$
- Intersection point  $C$  of point sets  $a$  and  $s_c$
- Segment division point  $_M_c$  of segment  $AB$  with division coefficient 1.0
- Line  $_a$  through two points  $B$  and  $C$
- Line  $_b$  through two points  $A$  and  $C$
- Angle bisector  $_s_c$  of angle  $ACB$
- Line  $_c$  through two points  $A$  and  $B$
- Intersection point  $_T_c$  of point sets  $_s_c$  and  $_c$
- Angle bisector  $_s_a$  of angle  $BAC$
- Angle bisector  $_s_b$  of angle  $CBA$
- Intersection point  $_I$  of point sets  $_s_a$  and  $_s_b$

### Theorem statement:

- Points  $T_c$  and  $_T_c$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $M_c$  has been assigned following coordinates: (0, 0)

### 4.1.3 Proving $I=_I$

#### Construction steps:

- Free point  $M_c$
- Free point  $T_c$
- Free point  $I$
- Line  $c$  through two points  $M_c$  and  $T_c$
- Line  $s_c$  through two points  $T_c$  and  $I$
- Line  $m_c$  through point  $M_c$  perpendicular to line  $c$

- Intersection point  $N_c$  of point sets  $m_c$  and  $s_c$
- Circle  $k(N_c, B)$  with center  $N_c$  and one point  $I$
- Intersection point  $B$  of point sets  $k(N_c, B)$  and  $c$
- Intersection point  $A$  of point sets  $k(N_c, B)$  and  $c$
- Line footPointPerpLine57 through point  $I$  perpendicular to line  $c$
- Intersection point  $P_{G54032}$  of point sets footPointPerpLine57 and  $c$
- Circle  $k(I, P_a)$  with center  $I$  and one point  $P_{G54032}$
- Line footPointPerpLine19 through point  $I$  perpendicular to line  $c$
- Intersection point  $P_{G54135}$  of point sets footPointPerpLine19 and  $c$
- Line  $L_{G54138}$  through two points  $I$  and  $B$
- Line reflexivePointPerpLine533 through point  $P_{G54135}$  perpendicular to line  $L_{G54138}$
- Intersection point reflexivePointFootPoint664 of point sets reflexivePointPerpLine533 and  $L_{G54138}$
- Cental symmetric point  $P_{G54141}$  of point  $P_{G54135}$  with respect to center of symmetry reflexivePointFootPoint664
- Line  $a$  through two points  $B$  and  $P_{G54141}$
- Intersection point  $C$  of point sets  $a$  and  $s_c$
- Segment division point  $_M_c$  of segment  $AB$  with division coefficient 1.0
- Line  $_a$  through two points  $B$  and  $C$
- Line  $_b$  through two points  $A$  and  $C$
- Angle bisector  $_s_c$  of angle  $ACB$
- Line  $_c$  through two points  $A$  and  $B$
- Intersection point  $_T_c$  of point sets  $_s_c$  and  $_c$
- Angle bisector  $_s_a$  of angle  $BAC$
- Angle bisector  $_s_b$  of angle  $CBA$
- Intersection point  $_I$  of point sets  $_s_a$  and  $_s_b$

### Theorem statement:

- Points  $I$  and  $_I$  are identical

**Validation result:** Theorem protocol is valid.

**NDG conditions** Point  $M_c$  has been assigned following coordinates:  $(0, 0)$

## 4.2 GCLC - Area method

### 4.2.1 Proving $M_c = _M_c$

Proving failed

### 4.2.2 Proving $T_c = _T_c$

Proving failed

### 4.2.3 Proving $I = _I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $M_c = \neg M_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 94 terms.

**Time Complexity:** Time spent by the prover is 0.120 seconds. There are no ndg conditions.

#### 4.3.2 Proving $T_c = \neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 94 terms.

**Time Complexity:** Time spent by the prover is 0.120 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I = \neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 5547 terms.

**Time Complexity:** Time spent by the prover is 5.560 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $M_c = \neg M_c$

Proving failed

#### 4.4.2 Proving $T_c = \neg T_c$

Proving failed

#### 4.4.3 Proving $I = \neg I$

Proving failed



## Problem 477

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 477: Given a point  $G$ , a point  $H_a$  and a point  $H_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 478

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 478: Given a point  $G$ , a point  $H_a$  and a point  $H_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 479

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 479: Given a point  $G$ , a point  $H_a$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $G$  and the point  $H$ , construct a point  $O$  (rule W01); ;
2. Using the point  $H_a$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $H_a$  and  $H$  are not the same;
3. Using the point  $H_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
4. Using the point  $O$  and the line  $a$ , construct a line  $m_a$  (rule W10b); ;
5. Using the line  $m_a$  and the line  $a$ , construct a point  $M_a$  (rule W03); % NDG: lines  $m_a$  and  $a$  are not parallel % DET: lines  $m_a$  and  $a$  are not the same;
6. Using the point  $M_a$  and the point  $G$ , construct a point  $A$  (rule W01); ;
7. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;
8. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); % NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same; lines  $m_a$  and  $a$  are not parallel.

Determination conditions: lines  $m_a$  and  $a$  are not the same; points  $H_a$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W10b]

Lemmas used: [D1,D11,D21,D26,D3,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L55,L58]

Solving time: 0.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point G 70 58.33
point H_{a} 80 40
point H 80 72.73
```

```
color 220 0 0
fontsize 9
```

```
cmark_t G
cmark_r H_{a}
cmark_rt H
color 0 0 0
fontsize 8
```

```
% Constructing a point O such that GO/GH=-0.5
towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0
```

```
% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H
```

```
color 200 200 200
drawline h_{a}
color 0 0 0
```

```
% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
perp a H_{a} h_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% Constructing a line m_{a} which is perpendicular to line a and which passes through point O
perp m_{a} O a
```

```
color 200 200 200
drawline m_{a}
color 0 0 0
```

```

% NDG: lines  $m_{\{a\}}$  and  $a$  are not parallel% DET: lines  $m_{\{a\}}$  and  $a$  are not the same
% Constructing a point  $M_{\{a\}}$  which belongs to line  $m_{\{a\}}$  and line  $a$ 
intersec  $M_{\{a\}}$   $m_{\{a\}}$   $a$ 
cmark_r  $M_{\{a\}}$ 

% Constructing a point  $A$  such that  $M_{\{a\}}A/M_{\{a\}}G=3$ 
towards  $A$   $M_{\{a\}}$   $G$  3
cmark_t  $A$ 
color 200 200 200
drawsegment  $M_{\{a\}}$   $A$ 
color 0 0 0

% NDG: points  $A$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $A$ 
circle  $k(O,C)$   $O$   $A$ 

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $a$  and circle  $k(O,C)$  intersect
% Constructing points  $C$  and  $B$  which are in intersection of  $k(O,C)$  and  $a$ 
intersec2  $C$   $B$   $k(O,C)$   $a$ 
cmark_l  $C$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: line  $a$  and circle  $k(O,C)$  intersect; points  $A$  and  $O$  are not the same;
    lines  $m_{\{a\}}$  and  $a$  are not parallel
% Determination conditions: lines  $m_{\{a\}}$  and  $a$  are not the same; points  $H_{\{a\}}$  and  $H$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $G=_G$

Proving failed

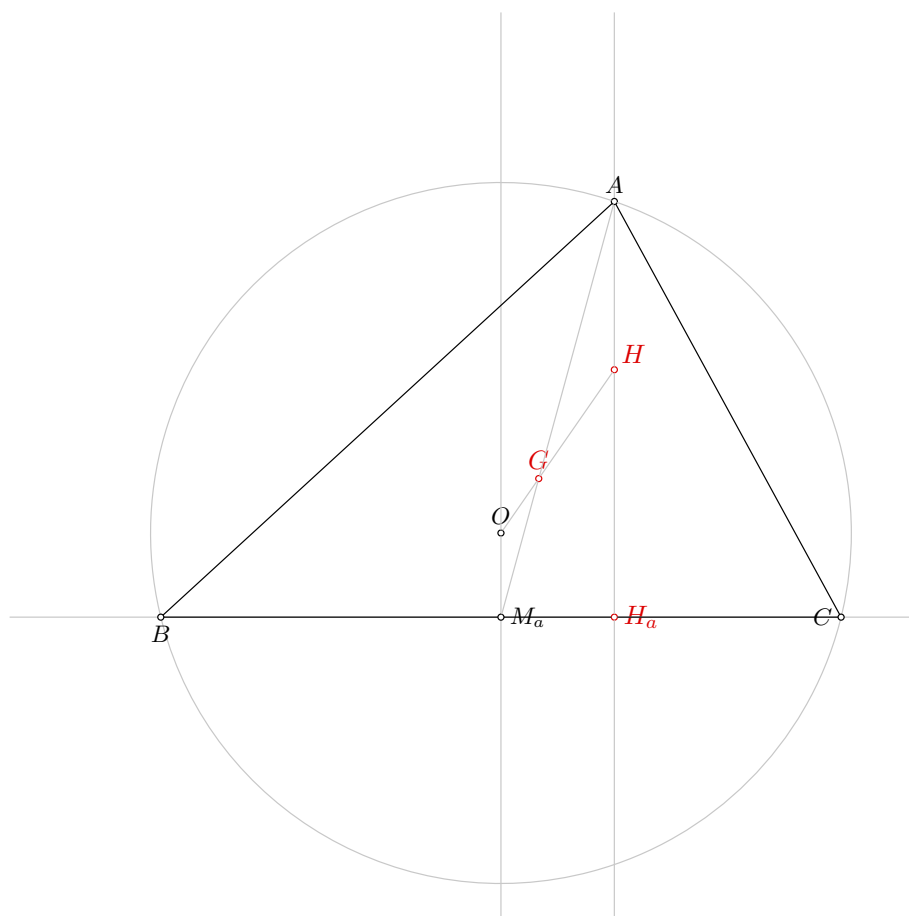


Figure 1: Illustration of the problem 0479

#### **4.1.2 Proving $H_a =_H H_a$**

Proving failed

#### **4.1.3 Proving $H =_H H$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $G =_G G$**

Proving failed

#### **4.2.2 Proving $H_a =_H H_a$**

Proving failed

#### **4.2.3 Proving $H =_H H$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $G =_G G$**

Proving failed

#### **4.3.2 Proving $H_a =_H H_a$**

Proving failed

#### **4.3.3 Proving $H =_H H$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $G =_G G$**

Proving failed

#### **4.4.2 Proving $H_a =_H H_a$**

Proving failed

#### **4.4.3 Proving $H =_H H$**

Proving failed

# Problem 480

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 480: Given a point  $G$ , a point  $H_a$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $T_a$  are not the same;
2. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
3. Using the point  $G$ , the line  $a$  and the point  $H_a$ , construct a line  $h_{G,-2/1}(a)$  (rule W15); ;
4. Using the line  $h_{G,-2/1}(a)$  and the line  $h_a$ , construct a point  $A$  (rule W03); % NDG: lines  $h_{G,-2/1}(a)$  and  $h_a$  are not parallel % DET: lines  $h_{G,-2/1}(a)$  and  $h_a$  are not the same;
5. Using the point  $A$  and the point  $G$ , construct a point  $M_a$  (rule W01); ;
6. Using the point  $T_a$  and the point  $A$ , construct a line  $s_a$  (rule W02); % DET: points  $T_a$  and  $A$  are not the same;
7. Using the point  $M_a$  and the line  $a$ , construct a line  $m_a$  (rule W10b); ;
8. Using the line  $m_a$  and the line  $s_a$ , construct a point  $N_a$  (rule W03); % NDG: lines  $m_a$  and  $s_a$  are not parallel % DET: lines  $m_a$  and  $s_a$  are not the same;
9. Using the point  $A$  and the point  $N_a$ , construct a line  $m(AN_a)$  (rule W14); % DET: points  $A$  and  $N_a$  are not the same;
10. Using the line  $m(AN_a)$  and the line  $m_a$ , construct a point  $O$  (rule W03); % NDG: lines  $m(AN_a)$  and  $m_a$  are not parallel % DET: lines  $m(AN_a)$  and  $m_a$  are not the same;
11. Using the point  $A$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $A$  and  $O$  are not the same;



12. Using the circle  $k(O, C)$  and the line  $a$ , construct a point  $C$  and a point  $B$  (rule W04); %  
NDG: line  $a$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $a$  and circle  $k(O, C)$  intersect; points  $A$  and  $O$  are not the same; lines  $m(AN_a)$  and  $m_a$  are not parallel; lines  $m_a$  and  $s_a$  are not parallel; lines  $h_{G,-2/1}(a)$  and  $h_a$  are not parallel.

Determination conditions: lines  $m(AN_a)$  and  $m_a$  are not the same; points  $A$  and  $N_a$  are not the same; lines  $m_a$  and  $s_a$  are not the same; points  $T_a$  and  $A$  are not the same; lines  $h_{G,-2/1}(a)$  and  $h_a$  are not the same; points  $H_a$  and  $T_a$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14,W15]

Lemmas used: [D1,D11,D21,D23,D26,D47,D5,D8,GD01,GL01,GL02,GL03,GL09,L11,L12,L25,L5,L55]

Solving time: 1.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point G 70 58.33
point H_{a} 80 40
point T_{a} 70.86 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_t G
cmark_r H_{a}
cmark_rb T_{a}
color 0 0 0
fontsize 8
```

```
% DET: points H_{a} and T_{a} are not the same
% Constructing a line a which passes through point H_{a} and point T_{a}
line a H_{a} T_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a
```

```
color 200 200 200
drawline h_{a}
color 0 0 0
```

```
% Constructing a point P_{\_G49553} such that GP_{\_G49553}/GH_{a}=-2
towards P_{\_G49553} G H_{a} -2
```

```

cmark_r P_{\_G49553}
color 200 200 200
drawsegment H_{a} P_{\_G49553}
color 0 0 0

% Constructing a line  $h_{G,-2/1}(a)$  which contains the point  $P_{\_G49553}$  and is parallel to the
line a
parallel h_{G,-2/1}(a) P_{\_G49553} a

color 200 200 200
drawline h_{G,-2/1}(a)
color 0 0 0

% NDG: lines  $h_{G,-2/1}(a)$  and  $h_{a}$  are not parallel% DET: lines  $h_{G,-2/1}(a)$  and  $h_{a}$  are not
the same
% Constructing a point A which belongs to line  $h_{G,-2/1}(a)$  and line  $h_{a}$ 
intersec A h_{G,-2/1}(a) h_{a}
cmark_t A

% Constructing a point  $M_{a}$  such that  $AM_{a}/AG=1.5$ 
towards M_{a} A G 1.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% DET: points  $T_{a}$  and A are not the same
% Constructing a line  $s_{a}$  which passes through point  $T_{a}$  and point A
line s_{a} T_{a} A

color 200 200 200
drawline s_{a}
color 0 0 0

% Constructing a line  $m_{a}$  which is perpendicular to line a and which passes through point  $M_{a}$ 
perp m_{a} M_{a} a

color 200 200 200
drawline m_{a}
color 0 0 0

% NDG: lines  $m_{a}$  and  $s_{a}$  are not parallel% DET: lines  $m_{a}$  and  $s_{a}$  are not the same
% Constructing a point  $N_{a}$  which belongs to line  $m_{a}$  and line  $s_{a}$ 
intersec N_{a} m_{a} s_{a}
cmark_b N_{a}

```

```

% DET: points A and N_{a} are not the same
% Constructing bisector m(AN_{a}) of the segment AN_{a}
med m(AN_{a}) A N_{a}

color 200 200 200
drawline m(AN_{a})
color 0 0 0

color 200 200 200
drawsegment A N_{a}
color 0 0 0

% NDG: lines m(AN_{a}) and m_{a} are not parallel% DET: lines m(AN_{a}) and m_{a} are not the same
% Constructing a point O which belongs to line m(AN_{a}) and line m_{a}
intersec O m(AN_{a}) m_{a}
cmark_t O

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

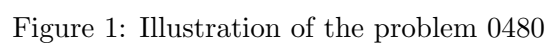
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% lines m(AN_{a}) and m_{a} are not parallel; lines m_{a} and s_{a} are not parallel; lines h_{G, -2/1}(a) and h_{a} are not parallel
% Determination conditions: lines m(AN_{a}) and m_{a} are not the same; points A and N_{a} are not the same; lines m_{a} and s_{a} are not the same; points T_{a} and A are not the same; lines h_{G, -2/1}(a) and h_{a} are not the same; points H_{a} and T_{a} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format



## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $G=_G$

Proving failed

#### 4.1.2 Proving $H_a=_H_a$

Proving failed

#### 4.1.3 Proving $T_a=_T_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $G=_G$

Proving failed

#### 4.2.2 Proving $H_a=_H_a$

Proving failed

#### 4.2.3 Proving $T_a=_T_a$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $G=_G$

Proving failed

#### 4.3.2 Proving $H_a=_H_a$

Proving failed

#### 4.3.3 Proving $T_a=_T_a$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $G=_G$

Proving failed

#### 4.4.2 Proving $H_a=_H_a$

Proving failed

#### 4.4.3 Proving $T_a=_T_a$

Proving failed

# Problem 481

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 481: Given a point  $G$ , a point  $H_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 482

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 482: Given a point  $G$ , a point  $H_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 483

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 483: Given a point  $G$ , a point  $H_a$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.



## Problem 484

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 484: Given a point  $G$ , a point  $H_b$  and a point  $H_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 485

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 485: Given a point  $G$ , a point  $H_b$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $G$  and the point  $H$ , construct a point  $O$  (rule W01); ;
2. Using the point  $H_b$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $H_b$  and  $H$  are not the same;
3. Using the point  $H_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
4. Using the point  $O$  and the line  $b$ , construct a line  $m_b$  (rule W10b); ;
5. Using the line  $m_b$  and the line  $b$ , construct a point  $M_b$  (rule W03); % NDG: lines  $m_b$  and  $b$  are not parallel % DET: lines  $m_b$  and  $b$  are not the same;
6. Using the point  $M_b$  and the point  $G$ , construct a point  $B$  (rule W01); ;
7. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;
8. Using the circle  $k(O, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); % NDG: line  $b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same; lines  $m_b$  and  $b$  are not parallel.

Determination conditions: lines  $m_b$  and  $b$  are not the same; points  $H_b$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W10b]

Lemmas used: [D1,D12,D22,D26,D3,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L56,L58]

Solving time: 0.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point G 70 58.33
point H_{b} 89.36 77.83
point H 80 72.73
```

```
color 220 0 0
fontsize 9
```

```
cmark_t G
cmark_l H_{b}
cmark_rt H
color 0 0 0
fontsize 8
```

```
% Constructing a point O such that GO/GH=-0.5
towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0
```

```
% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H
```

```
color 200 200 200
drawline h_{b}
color 0 0 0
```

```
% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
perp b H_{b} h_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% Constructing a line m_{b} which is perpendicular to line b and which passes through point O
perp m_{b} O b
```

```
color 200 200 200
drawline m_{b}
color 0 0 0
```

```

% NDG: lines  $m_{\{b\}}$  and  $b$  are not parallel% DET: lines  $m_{\{b\}}$  and  $b$  are not the same
% Constructing a point  $M_{\{b\}}$  which belongs to line  $m_{\{b\}}$  and line  $b$ 
intersec  $M_{\{b\}}$   $m_{\{b\}}$   $b$ 
cmark_lt  $M_{\{b\}}$ 

% Constructing a point  $B$  such that  $M_{\{b\}}B/M_{\{b\}}G=3$ 
towards B  $M_{\{b\}}$  G 3
cmark_b B
color 200 200 200
drawsegment  $M_{\{b\}}$  B
color 0 0 0

% NDG: points  $B$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $B$ 
circle  $k(O,C)$  O B

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $b$  and circle  $k(O,C)$  intersect
% Constructing points  $C$  and  $A$  which are in intersection of  $k(O,C)$  and  $b$ 
intersec2 C A  $k(O,C)$   $b$ 
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line  $b$  and circle  $k(O,C)$  intersect; points  $B$  and  $O$  are not the same;
    lines  $m_{\{b\}}$  and  $b$  are not parallel
% Determination conditions: lines  $m_{\{b\}}$  and  $b$  are not the same; points  $H_{\{b\}}$  and  $H$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $G=_G$

Proving failed

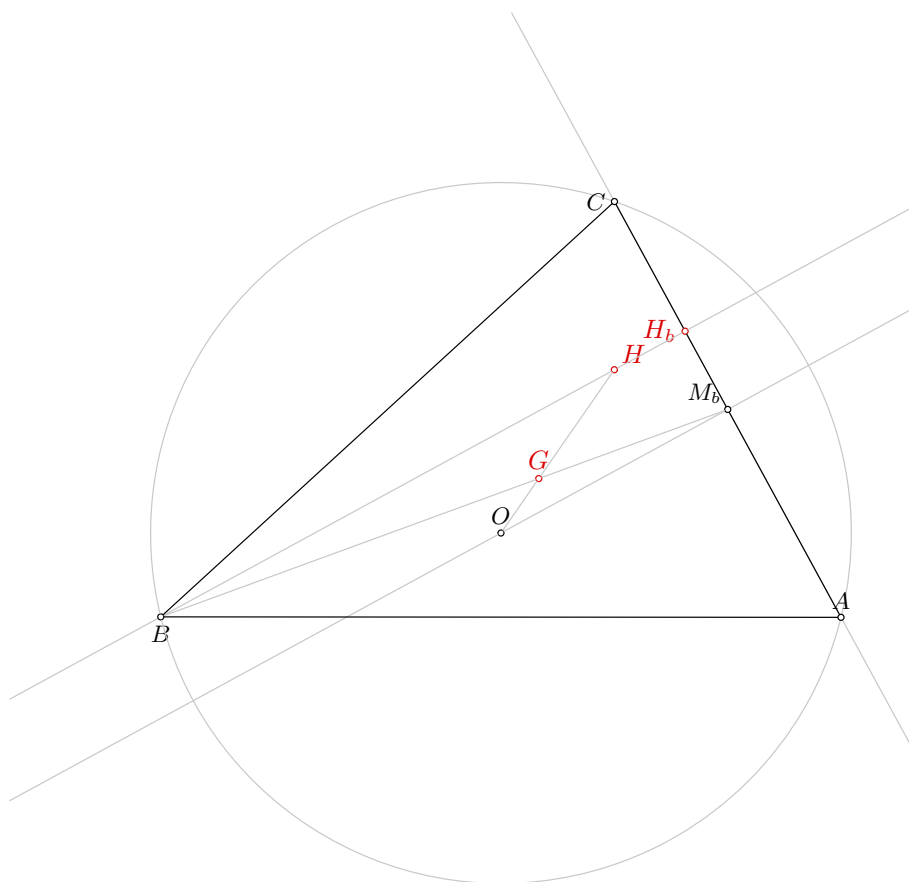


Figure 1: Illustration of the problem 0485

#### **4.1.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.1.3 Proving $H = \neg H$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $G = \neg G$**

Proving failed

#### **4.2.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.2.3 Proving $H = \neg H$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $G = \neg G$**

Proving failed

#### **4.3.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.3.3 Proving $H = \neg H$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $G = \neg G$**

Proving failed

#### **4.4.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.4.3 Proving $H = \neg H$**

Proving failed

# Problem 486

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 486: Given a point  $G$ , a point  $H_b$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 487

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 487: Given a point  $G$ , a point  $H_b$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $T_b$  are not the same;
2. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
3. Using the point  $G$ , the line  $b$  and the point  $H_b$ , construct a line  $h_{G,-2/1}(b)$  (rule W15); ;
4. Using the line  $h_{G,-2/1}(b)$  and the line  $h_b$ , construct a point  $B$  (rule W03); % NDG: lines  $h_{G,-2/1}(b)$  and  $h_b$  are not parallel % DET: lines  $h_{G,-2/1}(b)$  and  $h_b$  are not the same;
5. Using the point  $B$  and the point  $G$ , construct a point  $M_b$  (rule W01); ;
6. Using the point  $T_b$  and the point  $B$ , construct a line  $s_b$  (rule W02); % DET: points  $T_b$  and  $B$  are not the same;
7. Using the point  $M_b$  and the line  $b$ , construct a line  $m_b$  (rule W10b); ;
8. Using the line  $m_b$  and the line  $s_b$ , construct a point  $N_b$  (rule W03); % NDG: lines  $m_b$  and  $s_b$  are not parallel % DET: lines  $m_b$  and  $s_b$  are not the same;
9. Using the point  $B$  and the point  $N_b$ , construct a line  $m(BN_b)$  (rule W14); % DET: points  $B$  and  $N_b$  are not the same;
10. Using the line  $m(BN_b)$  and the line  $m_b$ , construct a point  $O$  (rule W03); % NDG: lines  $m(BN_b)$  and  $m_b$  are not parallel % DET: lines  $m(BN_b)$  and  $m_b$  are not the same;
11. Using the point  $B$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $B$  and  $O$  are not the same;



12. Using the circle  $k(O, C)$  and the line  $b$ , construct a point  $C$  and a point  $A$  (rule W04); %  
NDG: line  $b$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $b$  and circle  $k(O, C)$  intersect; points  $B$  and  $O$  are not the same; lines  $m(BN_b)$  and  $m_b$  are not parallel; lines  $m_b$  and  $s_b$  are not parallel; lines  $h_{G,-2/1}(b)$  and  $h_b$  are not parallel.

Determination conditions: lines  $m(BN_b)$  and  $m_b$  are not the same; points  $B$  and  $N_b$  are not the same; lines  $m_b$  and  $s_b$  are not the same; points  $T_b$  and  $B$  are not the same; lines  $h_{G,-2/1}(b)$  and  $h_b$  are not the same; points  $H_b$  and  $T_b$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14,W15]

Lemmas used: [D1,D12,D22,D24,D26,D48,D6,D9,GD01,GL01,GL02,GL03,GL09,L11,L12,L26,L56,L6]

Solving time: 1.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point G 70 58.33
point H_{b} 89.36 77.83
point T_{b} 94.25 68.88
```

```
color 220 0 0
fontsize 9
```

```
cmark_t G
cmark_l H_{b}
cmark_t T_{b}
color 0 0 0
fontsize 8
```

```
% DET: points H_{b} and T_{b} are not the same
% Constructing a line b which passes through point H_{b} and point T_{b}
line b H_{b} T_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b
```

```
color 200 200 200
drawline h_{b}
color 0 0 0
```

```
% Constructing a point P_{\_G61329} such that GP_{\_G61329}/GH_{b}=-2
towards P_{\_G61329} G H_{b} -2
```

```

cmark_r P_{\_G61329}
color 200 200 200
drawsegment H_{b} P_{\_G61329}
color 0 0 0

% Constructing a line h_{G,-2/1}(b) which contains the point P_{\_G61329} and is parallel to the
line b
parallel h_{G,-2/1}(b) P_{\_G61329} b

color 200 200 200
drawline h_{G,-2/1}(b)
color 0 0 0

% NDG: lines h_{G,-2/1}(b) and h_{b} are not parallel% DET: lines h_{G,-2/1}(b) and h_{b} are not
the same
% Constructing a point B which belongs to line h_{G,-2/1}(b) and line h_{b}
intersec B h_{G,-2/1}(b) h_{b}
cmark_b B

% Constructing a point M_{b} such that BM_{b}/BG=1.5
towards M_{b} B G 1.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% DET: points T_{b} and B are not the same
% Constructing a line s_{b} which passes through point T_{b} and point B
line s_{b} T_{b} B

color 200 200 200
drawline s_{b}
color 0 0 0

% Constructing a line m_{b} which is perpendicular to line b and which passes through point M_{b}
perp m_{b} M_{b} b

color 200 200 200
drawline m_{b}
color 0 0 0

% NDG: lines m_{b} and s_{b} are not parallel% DET: lines m_{b} and s_{b} are not the same
% Constructing a point N_{b} which belongs to line m_{b} and line s_{b}
intersec N_{b} m_{b} s_{b}
cmark_rb N_{b}

```

```

% DET: points B and N_{b} are not the same
% Constructing bisector m(BN_{b}) of the segment BN_{b}
med m(BN_{b}) B N_{b}

color 200 200 200
drawline m(BN_{b})
color 0 0 0

color 200 200 200
drawsegment B N_{b}
color 0 0 0

% NDG: lines m(BN_{b}) and m_{b} are not parallel% DET: lines m(BN_{b}) and m_{b} are not the same
% Constructing a point O which belongs to line m(BN_{b}) and line m_{b}
intersec O m(BN_{b}) m_{b}
cmark_t O

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;
% lines m(BN_{b}) and m_{b} are not parallel; lines m_{b} and s_{b} are not parallel; lines h_{G}
% , -2/1}(b) and h_{b} are not parallel
% Determination conditions: lines m(BN_{b}) and m_{b} are not the same; points B and N_{b} are not
% the same; lines m_{b} and s_{b} are not the same; points T_{b} and B are not the same; lines h
% _{G, -2/1}(b) and h_{b} are not the same; points H_{b} and T_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

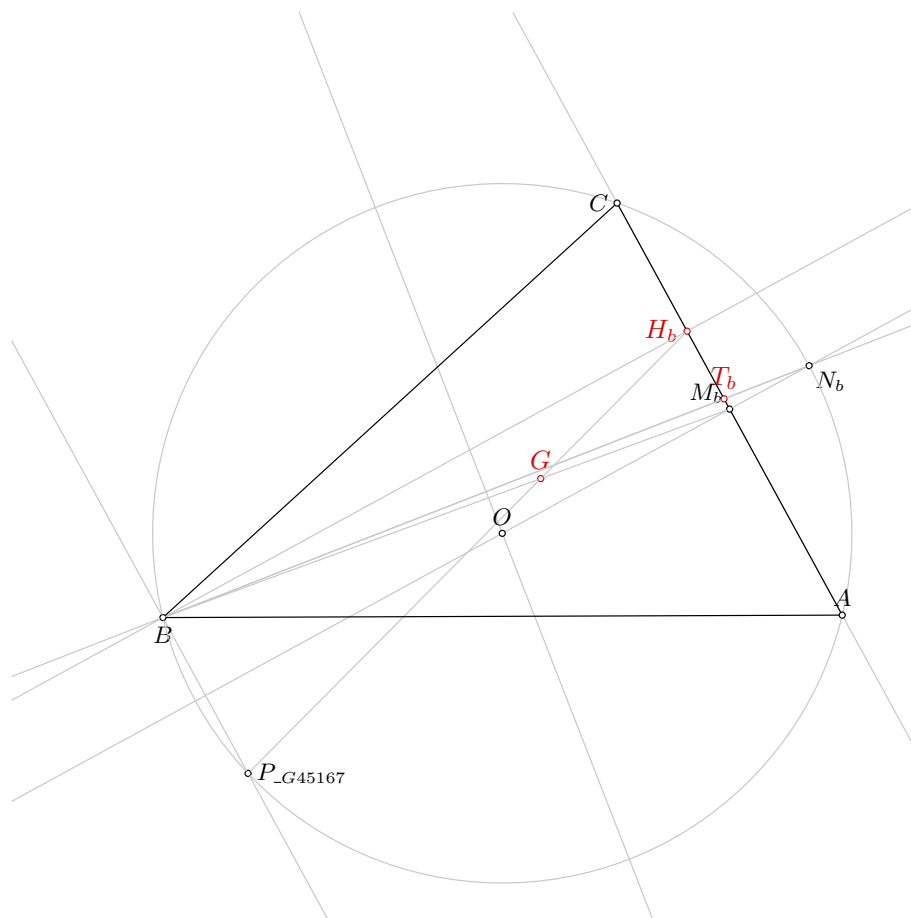


Figure 1: Illustration of the problem 0487

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $G=_G$

Proving failed

#### 4.1.2 Proving $H_b=_H H_b$

Proving failed

#### 4.1.3 Proving $T_b=_T T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $G=_G$

Proving failed

#### 4.2.2 Proving $H_b=_H H_b$

Proving failed

#### 4.2.3 Proving $T_b=_T T_b$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $G=_G$

Proving failed

#### 4.3.2 Proving $H_b=_H H_b$

Proving failed

#### 4.3.3 Proving $T_b=_T T_b$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $G=_G$

Proving failed

#### 4.4.2 Proving $H_b=_H H_b$

Proving failed

#### 4.4.3 Proving $T_b=_T T_b$

Proving failed

## Problem 488

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 488: Given a point  $G$ , a point  $H_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 489

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 489: Given a point  $G$ , a point  $H_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 490

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 490: Given a point  $G$ , a point  $H_c$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $G$  and the point  $H$ , construct a point  $O$  (rule W01); ;
2. Using the point  $H_c$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $H_c$  and  $H$  are not the same;
3. Using the point  $H_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
4. Using the point  $O$  and the line  $c$ , construct a line  $m_c$  (rule W10b); ;
5. Using the line  $m_c$  and the line  $c$ , construct a point  $M_c$  (rule W03); % NDG: lines  $m_c$  and  $c$  are not parallel % DET: lines  $m_c$  and  $c$  are not the same;
6. Using the point  $M_c$  and the point  $G$ , construct a point  $C$  (rule W01); ;
7. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;
8. Using the circle  $k(O, C)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); % NDG: line  $c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same; lines  $m_c$  and  $c$  are not parallel.

Determination conditions: lines  $m_c$  and  $c$  are not the same; points  $H_c$  and  $H$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W10b]

Lemmas used: [D10,D13,D20,D26,D7,GD01,GL03,GL04,GL09,L1,L11,L12,L3,L57,L58]

Solving time: 0.6 seconds.



### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point G 70 58.33
point H_{c} 68.91 84.83
point H 80 72.73
```

```
color 220 0 0
fontsize 9
```

```
cmark_t G
cmark_rt H_{c}
cmark_rt H
color 0 0 0
fontsize 8
```

```
% Constructing a point O such that GO/GH=-0.5
towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0
```

```
% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H
```

```
color 200 200 200
drawline h_{c}
color 0 0 0
```

```
% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}
```

```
color 200 200 200
drawline c
color 0 0 0
```

```
% Constructing a line m_{c} which is perpendicular to line c and which passes through point O
perp m_{c} O c
```

```
color 200 200 200
drawline m_{c}
color 0 0 0
```

```

% NDG: lines  $m_{\{c\}}$  and  $c$  are not parallel% DET: lines  $m_{\{c\}}$  and  $c$  are not the same
% Constructing a point  $M_{\{c\}}$  which belongs to line  $m_{\{c\}}$  and line  $c$ 
intersec  $M_{\{c\}}$   $m_{\{c\}}$   $c$ 
cmark_lt  $M_{\{c\}}$ 

% Constructing a point  $C$  such that  $M_{\{c\}}C/M_{\{c\}}G=3$ 
towards C  $M_{\{c\}}$  G 3
cmark_l C
color 200 200 200
drawsegment  $M_{\{c\}}$  C
color 0 0 0

% NDG: points  $C$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $C$ 
circle k( $O,C$ ) O C

color 200 200 200
drawcircle k( $O,C$ )
color 0 0 0

% NDG: line  $c$  and circle  $k(O,C)$  intersect
% Constructing points  $A$  and  $B$  which are in intersection of  $k(O,C)$  and  $c$ 
intersec2 A B k( $O,C$ )  $c$ 
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line  $c$  and circle  $k(O,C)$  intersect; points  $C$  and  $O$  are not the same;
% lines  $m_{\{c\}}$  and  $c$  are not parallel
% Determination conditions: lines  $m_{\{c\}}$  and  $c$  are not the same; points  $H_{\{c\}}$  and  $H$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $G=_G$

Proving failed

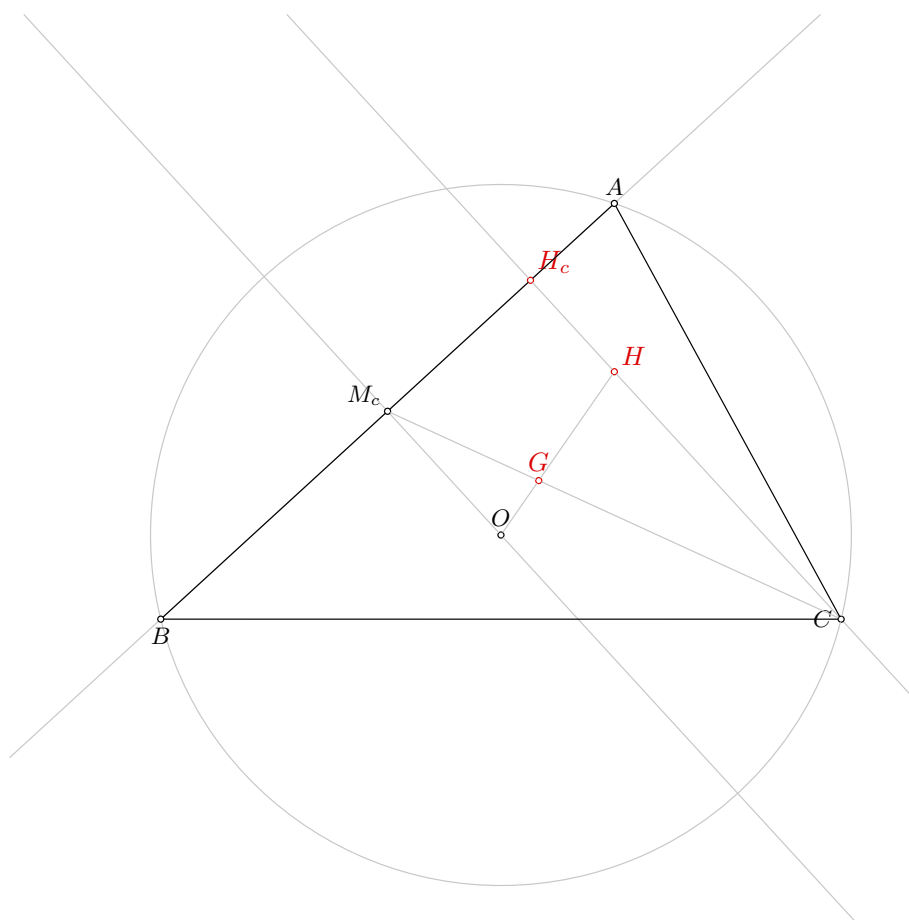


Figure 1: Illustration of the problem 0490

#### **4.1.2 Proving $H_c = H_c$**

Proving failed

#### **4.1.3 Proving $H = H$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $G = G$**

Proving failed

#### **4.2.2 Proving $H_c = H_c$**

Proving failed

#### **4.2.3 Proving $H = H$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $G = G$**

Proving failed

#### **4.3.2 Proving $H_c = H_c$**

Proving failed

#### **4.3.3 Proving $H = H$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $G = G$**

Proving failed

#### **4.4.2 Proving $H_c = H_c$**

Proving failed

#### **4.4.3 Proving $H = H$**

Proving failed

# Problem 491

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 491: Given a point  $G$ , a point  $H_c$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 492

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 492: Given a point  $G$ , a point  $H_c$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 493

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 493: Given a point  $G$ , a point  $H_c$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $T_c$  are not the same;
2. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
3. Using the point  $G$ , the line  $c$  and the point  $H_c$ , construct a line  $h_{G,-2/1}(c)$  (rule W15); ;
4. Using the line  $h_{G,-2/1}(c)$  and the line  $h_c$ , construct a point  $C$  (rule W03); % NDG: lines  $h_{G,-2/1}(c)$  and  $h_c$  are not parallel % DET: lines  $h_{G,-2/1}(c)$  and  $h_c$  are not the same;
5. Using the point  $C$  and the point  $G$ , construct a point  $M_c$  (rule W01); ;
6. Using the point  $T_c$  and the point  $C$ , construct a line  $s_c$  (rule W02); % DET: points  $T_c$  and  $C$  are not the same;
7. Using the point  $M_c$  and the line  $c$ , construct a line  $m_c$  (rule W10b); ;
8. Using the line  $m_c$  and the line  $s_c$ , construct a point  $N_c$  (rule W03); % NDG: lines  $m_c$  and  $s_c$  are not parallel % DET: lines  $m_c$  and  $s_c$  are not the same;
9. Using the point  $C$  and the point  $N_c$ , construct a line  $m(CN_c)$  (rule W14); % DET: points  $C$  and  $N_c$  are not the same;
10. Using the line  $m(CN_c)$  and the line  $m_c$ , construct a point  $O$  (rule W03); % NDG: lines  $m(CN_c)$  and  $m_c$  are not parallel % DET: lines  $m(CN_c)$  and  $m_c$  are not the same;
11. Using the point  $C$  and the point  $O$ , construct a circle  $k(O, C)$  (rule W06); % NDG: points  $C$  and  $O$  are not the same;

12. Using the circle  $k(O, C)$  and the line  $c$ , construct a point  $A$  and a point  $B$  (rule W04); %  
NDG: line  $c$  and circle  $k(O, C)$  intersect.

Non-degenerate conditions: line  $c$  and circle  $k(O, C)$  intersect; points  $C$  and  $O$  are not the same; lines  $m(CN_c)$  and  $m_c$  are not parallel; lines  $m_c$  and  $s_c$  are not parallel; lines  $h_{G,-2/1}(c)$  and  $h_c$  are not parallel.

Determination conditions: lines  $m(CN_c)$  and  $m_c$  are not the same; points  $C$  and  $N_c$  are not the same; lines  $m_c$  and  $s_c$  are not the same; points  $T_c$  and  $C$  are not the same; lines  $h_{G,-2/1}(c)$  and  $h_c$  are not the same; points  $H_c$  and  $T_c$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14,W15]

Lemmas used: [D10,D13,D20,D25,D26,D49,D7,GD01,GL01,GL02,GL03,GL09,L1,L11,L12,L27,L57,L7]

Solving time: 1.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point G 70 58.33
point H_{c} 68.91 84.83
point T_{c} 55.38 72.43
```

```
color 220 0 0
fontsize 9
```

```
cmark_t G
cmark_rt H_{c}
cmark_rt T_{c}
color 0 0 0
fontsize 8
```

```
% DET: points H_{c} and T_{c} are not the same
% Constructing a line c which passes through point H_{c} and point T_{c}
line c H_{c} T_{c}
```

```
color 200 200 200
drawline c
color 0 0 0
```

```
% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c
```

```
color 200 200 200
drawline h_{c}
color 0 0 0
```

```
% Constructing a point P_{\_G48547} such that GP_{\_G48547}/GH_{c}=-2
towards P_{\_G48547} G H_{c} -2
```



```

cmark_r P_{\_G48547}
color 200 200 200
drawsegment H_{c} P_{\_G48547}
color 0 0 0

% Constructing a line  $h_{G,-2/1}(c)$  which contains the point  $P_{\_G48547}$  and is parallel to the
line c
parallel h_{G,-2/1}(c) P_{\_G48547} c

color 200 200 200
drawline h_{G,-2/1}(c)
color 0 0 0

% NDG: lines  $h_{G,-2/1}(c)$  and  $h_{c}$  are not parallel% DET: lines  $h_{G,-2/1}(c)$  and  $h_{c}$  are not
the same
% Constructing a point C which belongs to line  $h_{G,-2/1}(c)$  and line  $h_{c}$ 
intersec C h_{G,-2/1}(c) h_{c}
cmark_l C

% Constructing a point  $M_{c}$  such that  $CM_{c}/CG=1.5$ 
towards M_{c} C G 1.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% DET: points  $T_{c}$  and C are not the same
% Constructing a line  $s_{c}$  which passes through point  $T_{c}$  and point C
line s_{c} T_{c} C

color 200 200 200
drawline s_{c}
color 0 0 0

% Constructing a line  $m_{c}$  which is perpendicular to line c and which passes through point  $M_{c}$ 
perp m_{c} M_{c} c

color 200 200 200
drawline m_{c}
color 0 0 0

% NDG: lines  $m_{c}$  and  $s_{c}$  are not parallel% DET: lines  $m_{c}$  and  $s_{c}$  are not the same
% Constructing a point  $N_{c}$  which belongs to line  $m_{c}$  and line  $s_{c}$ 
intersec N_{c} m_{c} s_{c}
cmark_b N_{c}

```

```

% DET: points C and N_{c} are not the same
% Constructing bisector m(CN_{c}) of the segment CN_{c}
med m(CN_{c}) C N_{c}

color 200 200 200
drawline m(CN_{c})
color 0 0 0

color 200 200 200
drawsegment C N_{c}
color 0 0 0

% NDG: lines m(CN_{c}) and m_{c} are not parallel% DET: lines m(CN_{c}) and m_{c} are not the same
% Constructing a point O which belongs to line m(CN_{c}) and line m_{c}
intersec O m(CN_{c}) m_{c}
cmark_t O

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
% lines m(CN_{c}) and m_{c} are not parallel; lines m_{c} and s_{c} are not parallel; lines h_{G}
% , -2/1}(c) and h_{c} are not parallel
% Determination conditions: lines m(CN_{c}) and m_{c} are not the same; points C and N_{c} are not
% the same; lines m_{c} and s_{c} are not the same; points T_{c} and C are not the same; lines h
% _{G, -2/1}(c) and h_{c} are not the same; points H_{c} and T_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

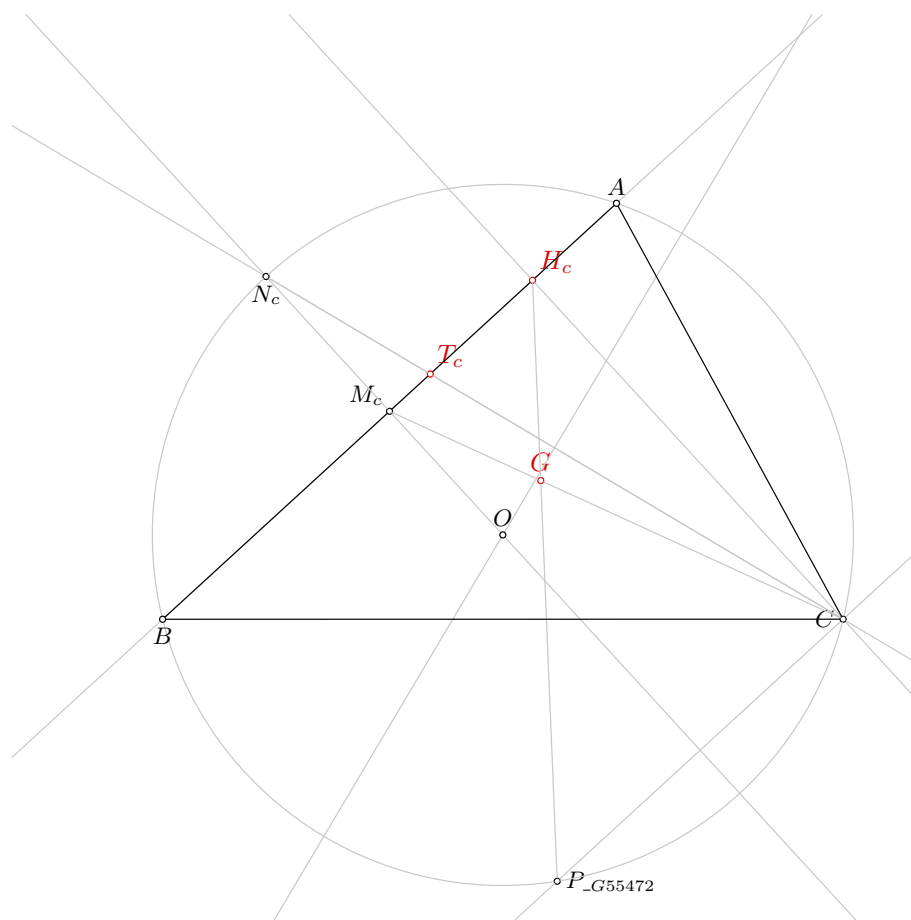


Figure 1: Illustration of the problem 0493

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $G=_G$

Proving failed

#### 4.1.2 Proving $H_c=_Hc$

Proving failed

#### 4.1.3 Proving $T_c=_Tc$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $G=_G$

Proving failed

#### 4.2.2 Proving $H_c=_Hc$

Proving failed

#### 4.2.3 Proving $T_c=_Tc$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $G=_G$

Proving failed

#### 4.3.2 Proving $H_c=_Hc$

Proving failed

#### 4.3.3 Proving $T_c=_Tc$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $G=_G$

Proving failed

#### 4.4.2 Proving $H_c=_Hc$

Proving failed

#### 4.4.3 Proving $T_c=_Tc$

Proving failed

## Problem 494

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 494: Given a point  $G$ , a point  $H_c$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 495

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 495: Given a point  $G$ , a point  $H$  and a point  $T_a$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 496

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 496: Given a point  $G$ , a point  $H$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 497

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 497: Given a point  $G$ , a point  $H$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.



# Problem 498

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 498: Given a point  $G$ , a point  $H$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 499

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 499: Given a point  $G$ , a point  $T_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 500

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 500: Given a point  $G$ , a point  $T_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 501

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 501: Given a point  $G$ , a point  $T_a$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 502

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 502: Given a point  $G$ , a point  $T_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 503

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 503: Given a point  $G$ , a point  $T_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 504

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 504: Given a point  $G$ , a point  $T_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 505

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 505: Given a point  $H_a$ , a point  $H_b$  and a point  $H_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $H_b$ , construct a line  $H_aH_b$  (rule W02); % DET: points  $H_a$  and  $H_b$  are not the same;
2. Using the point  $H_a$  and the point  $H_c$ , construct a line  $H_cH_a$  (rule W02); % DET: points  $H_a$  and  $H_c$  are not the same;
3. Using the point  $H_a$  and the point  $H_b$ , construct a line  $m(H_aH_b)$  (rule W14); % DET: points  $H_a$  and  $H_b$  are not the same;
4. Using the point  $H_b$  and the point  $H_c$ , construct a line  $m(H_bH_c)$  (rule W14); % DET: points  $H_b$  and  $H_c$  are not the same;
5. Using the line  $H_cH_a$ , the point  $H_b$ , the point  $H_a$ , the point  $H_c$  and the line  $H_aH_b$ , construct a line  $h_a$  (rule W17); % NDG: points  $H_a$  and  $H_c$  are not the same; points  $H_b$  and  $H_a$  are not the same % DET: points  $H_a$  and  $H_b$  are not the same;
6. Using the point  $H_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
7. Using the line  $a$  and the line  $m(H_bH_c)$ , construct a point  $M_a$  (rule W03); % NDG: lines  $a$  and  $m(H_bH_c)$  are not parallel % DET: lines  $a$  and  $m(H_bH_c)$  are not the same;
8. Using the point  $H_b$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_b$  and  $M_a$  are not the same;
9. Using the circle  $k(M_a, B)$  and the line  $a$ , construct a point  $B$  and a point  $C$  (rule W04); % NDG: line  $a$  and circle  $k(M_a, B)$  intersect;
10. Using the point  $H_c$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $B$  are not the same;



11. Using the line  $m(H_aH_b)$  and the line  $c$ , construct a point  $M_c$  (rule W03); % NDG: lines  $m(H_aH_b)$  and  $c$  are not parallel % DET: lines  $m(H_aH_b)$  and  $c$  are not the same;
12. Using the point  $M_c$  and the point  $B$ , construct a point  $A$  (rule W01); .

Non-degenerate conditions: lines  $m(H_aH_b)$  and  $c$  are not parallel; line  $a$  and circle  $k(M_a, B)$  intersect; points  $H_b$  and  $M_a$  are not the same; lines  $a$  and  $m(H_bH_c)$  are not parallel; points  $H_a$  and  $H_c$  are not the same; points  $H_b$  and  $H_a$  are not the same.

Determination conditions: lines  $m(H_aH_b)$  and  $c$  are not the same; points  $H_c$  and  $B$  are not the same; lines  $a$  and  $m(H_bH_c)$  are not the same; points  $H_a$  and  $H_b$  are not the same; points  $H_b$  and  $H_c$  are not the same; points  $H_a$  and  $H_b$  are not the same; points  $H_a$  and  $H_c$  are not the same; points  $H_a$  and  $H_b$  are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W14,W17]

Lemmas used: [D20,D21,D5,D7,D8,GD01,GD02,GL01,GL03,GL04,GL09,L113,L37,L38,L39,L41,L42]

Solving time: 1.7 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{a} 80 40
point H_{b} 89.36 77.83
point H_{c} 68.91 84.83
```

```
color 220 0 0
fontsize 9
```

```
cmark_r H_{a}
cmark_l H_{b}
cmark_rt H_{c}
color 0 0 0
fontsize 8
```

```
% DET: points H_{a} and H_{b} are not the same
% Constructing a line H_{a}H_{b} which passes through point H_{a} and point H_{b}
line H_{a}H_{b} H_{a} H_{b}
```

```
color 200 200 200
drawline H_{a}H_{b}
color 0 0 0
```

```
% DET: points H_{a} and H_{c} are not the same
% Constructing a line H_{c}H_{a} which passes through point H_{a} and point H_{c}
line H_{c}H_{a} H_{a} H_{c}
```

```
color 200 200 200
drawline H_{c}H_{a}
color 0 0 0
```

```

% DET: points  $H_{\{a\}}$  and  $H_{\{b\}}$  are not the same
% Constructing bisector  $m(H_{\{a\}}H_{\{b\}})$  of the segment  $H_{\{a\}}H_{\{b\}}$ 
med m( $H_{\{a\}}H_{\{b\}}$ )  $H_{\{a\}}$   $H_{\{b\}}$ 

color 200 200 200
drawline m( $H_{\{a\}}H_{\{b\}}$ )
color 0 0 0

color 200 200 200
drawsegment  $H_{\{a\}}$   $H_{\{b\}}$ 
color 0 0 0

% DET: points  $H_{\{b\}}$  and  $H_{\{c\}}$  are not the same
% Constructing bisector  $m(H_{\{b\}}H_{\{c\}})$  of the segment  $H_{\{b\}}H_{\{c\}}$ 
med m( $H_{\{b\}}H_{\{c\}}$ )  $H_{\{b\}}$   $H_{\{c\}}$ 

color 200 200 200
drawline m( $H_{\{b\}}H_{\{c\}}$ )
color 0 0 0

color 200 200 200
drawsegment  $H_{\{b\}}$   $H_{\{c\}}$ 
color 0 0 0

% NDG: points  $H_{\{a\}}$  and  $H_{\{c\}}$  are not the same; points  $H_{\{b\}}$  and  $H_{\{a\}}$  are not the same% DET:
      points  $H_{\{a\}}$  and  $H_{\{b\}}$  are not the same
% Constructing an angle  $V[_G57867]$  which is equal to the angle  $H_{\{b\}}H_{\{a\}}H_{\{c\}}$ 
angle_o  $V[_G57867]$   $H_{\{b\}}$   $H_{\{a\}}$   $H_{\{c\}}$ 

% Calculating value  $angle[_G57946]$  using formula  $1/\text{pow}(2,1)*V[_G57867]+0/\text{pow}(2,0)*180$ 
expression angle[_G57946] {  $1/\text{pow}(2,1)*V[_G57867]+0/\text{pow}(2,0)*180$  }

% Constructing a point  $P_{\{\_G57943\}}$  which is an image of the point  $H_{\{b\}}$  in a rotation around the
      point  $H_{\{a\}}$  for the angle  $1/\text{pow}(2,1)*V[_G57867]+0/\text{pow}(2,0)*180$ 
rotate  $P_{\{\_G57943\}}$   $H_{\{a\}}$  angle[_G57946]  $H_{\{b\}}$ 
cmark_r  $P_{\{\_G57943\}}$ 
color 200 200 200
drawarc_p  $H_{\{a\}}$   $H_{\{b\}}$  angle[_G57946]
color 0 0 0

% Constructing a line  $h_{\{a\}}$  which passes through point  $H_{\{a\}}$  and point  $P_{\{\_G57943\}}$ 
line  $h_{\{a\}}$   $H_{\{a\}}$   $P_{\{\_G57943\}}$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% Constructing a line  $a$  which is perpendicular to line  $h_{\{a\}}$  and which passes through point  $H_{\{a\}}$ 
perp a  $H_{\{a\}}$   $h_{\{a\}}$ 

```

```

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and m(H_{b}H_{c}) are not parallel% DET: lines a and m(H_{b}H_{c}) are not the same
% Constructing a point M_{a} which belongs to line a and line m(H_{b}H_{c})
intersec M_{a} a m(H_{b}H_{c})
cmark_r M_{a}

% NDG: points H_{b} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{b}
circle k(M_{a},B) M_{a} H_{b}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line a and circle k(M_{a},B) intersect
% Constructing points B and C which are in intersection of k(M_{a},B) and a
intersec2 B C k(M_{a},B) a
cmark_b B
cmark_l C

% DET: points H_{c} and B are not the same
% Constructing a line c which passes through point H_{c} and point B
line c H_{c} B

color 200 200 200
drawline c
color 0 0 0

% NDG: lines m(H_{a}H_{b}) and c are not parallel% DET: lines m(H_{a}H_{b}) and c are not the same
% Constructing a point M_{c} which belongs to line m(H_{a}H_{b}) and line c
intersec M_{c} m(H_{a}H_{b}) c
cmark_lt M_{c}

% Constructing a point A such that M_{c}A/M_{c}B=-1
towards A M_{c} B -1
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

```

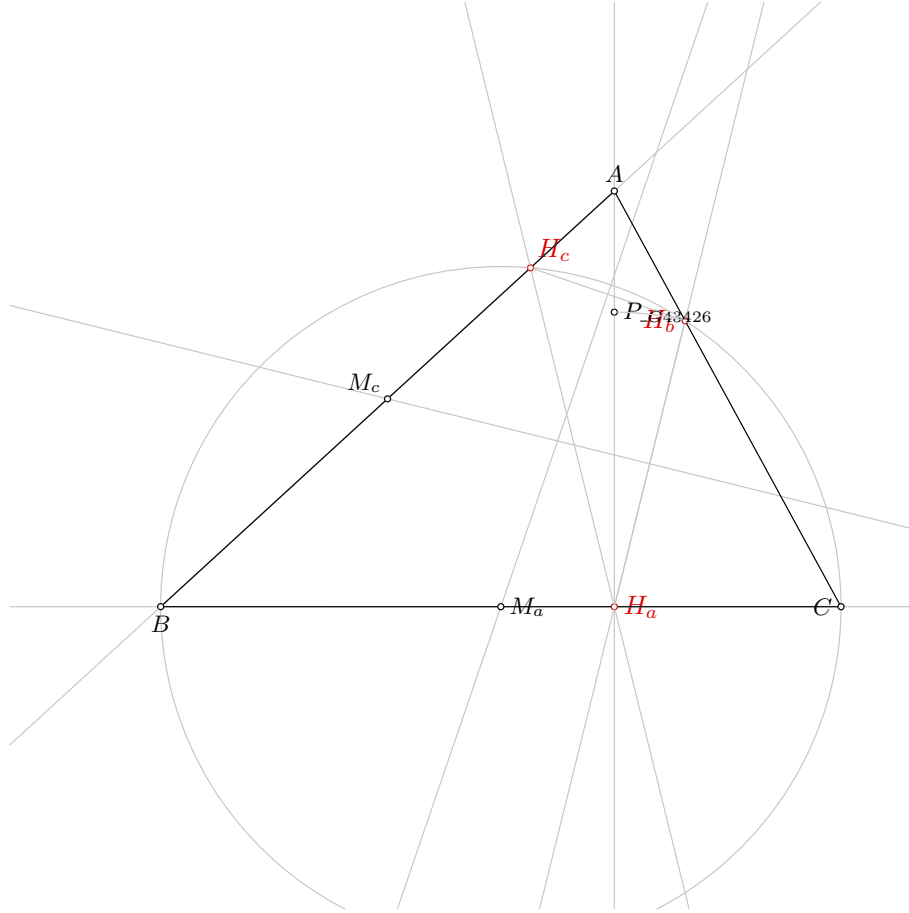


Figure 1: Illustration of the problem 0505

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines m(H_{a}H_{b}) and c are not parallel; line a and circle k(M_{a},
B) intersect; points H_{b} and M_{a} are not the same; lines a and m(H_{b}H_{c}) are not
parallel; points H_{a} and H_{c} are not the same; points H_{b} and H_{a} are not the same
% Determination conditions: lines m(H_{a}H_{b}) and c are not the same; points H_{c} and B are not
the same; lines a and m(H_{b}H_{c}) are not the same; points H_{a} and H_{b} are not the same;
points H_{b} and H_{c} are not the same; points H_{a} and H_{b} are not the same; points H_{a}
and H_{c} are not the same; points H_{a} and H_{b} are not the same
```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_a = \neg H_a$

Proving failed

#### 4.1.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.1.3 Proving $H_c = \neg H_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_a = \neg H_a$

Proving failed

#### 4.2.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.2.3 Proving $H_c = \neg H_c$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_a = \neg H_a$

Proving failed

#### 4.3.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.3.3 Proving $H_c = \neg H_c$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $H_a = \neg H_a$

Proving failed

#### 4.4.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.4.3 Proving $H_c = \neg H_c$

Proving failed

# Problem 506

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 506: Given a point  $H_a$ , a point  $H_b$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $H_a$  and  $H$  are not the same;
2. Using the point  $H_b$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $H_b$  and  $H$  are not the same;
3. Using the point  $H_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
4. Using the line  $a$  and the line  $h_b$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $h_b$  are not parallel % DET: lines  $a$  and  $h_b$  are not the same;
5. Using the point  $H_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
6. Using the line  $b$  and the line  $h_a$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $h_a$  are not parallel % DET: lines  $b$  and  $h_a$  are not the same;
7. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; lines  $b$  and  $h_a$  are not parallel; lines  $a$  and  $h_b$  are not parallel.

Determination conditions: lines  $a$  and  $b$  are not the same; lines  $b$  and  $h_a$  are not the same; lines  $a$  and  $h_b$  are not the same; points  $H_b$  and  $H$  are not the same; points  $H_a$  and  $H$  are not the same.

Rules used: [W02,W03,W10a]

Lemmas used: [D3,D5,D6,D8,D9,GD01]

Solving time: 0.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point H_{a} 80 40
point H_{b} 89.36 77.83
point H 80 72.73

color 220 0 0
fontsize 9

cmark_r H_{a}
cmark_l H_{b}
cmark_rt H
color 0 0 0
fontsize 8

% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
perp a H_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and h_{b} are not parallel% DET: lines a and h_{b} are not the same
% Constructing a point B which belongs to line a and line h_{b}
intersec B a h_{b}
cmark_b B

% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
perp b H_{b} h_{b}
```

```

color 200 200 200
drawline b
color 0 0 0

% NDG: lines b and h_{a} are not parallel% DET: lines b and h_{a} are not the same
% Constructing a point A which belongs to line b and line h_{a}
intersec A b h_{a}
cmark_t A

% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and b are not parallel; lines b and h_{a} are not parallel;
%                               lines a and h_{b} are not parallel
% Determination conditions: lines a and b are not the same; lines b and h_{a} are not the same;
%                               lines a and h_{b} are not the same; points H_{b} and H are not the same; points H_{a} and H are
%                               not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_a = H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 114 terms.

**Time Complexity:** Time spent by the prover is 1.282 seconds.

**NDG conditions** Points  $H_b$  and  $H$  are not identical

Points  $H_b$  and  $H$  are not identical

Points  $H_b$  and  $H$  are not identical

Points  $H_b$  and  $H$  are not identical

Points  $H_b$  and  $H_a$  are not identical

Points  $B$  and  $C$  are not identical

Line through points  $H_b$  and  $H_a$  is not parallel with line through points  $B$  and  $C$



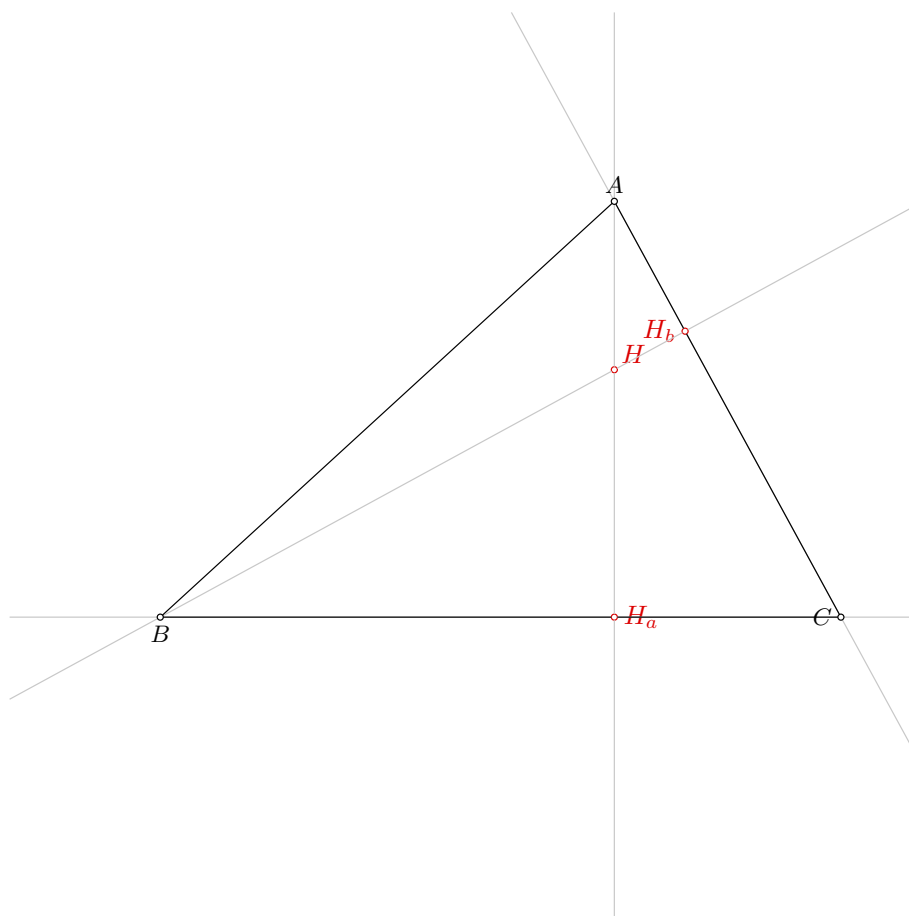


Figure 1: Illustration of the problem 0506

### 4.1.2 Proving $H_b = \_H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 239 terms.

**Time Complexity:** Time spent by the prover is 1.872 seconds.

**NDG conditions** Points  $H_b$  and  $H$  are not identical

Points  $H_b$  and  $H$  are not identical

Points  $H_b$  and  $H$  are not identical

Points  $H_b$  and  $H$  are not identical

Points  $H_b$  and  $H_a$  are not identical

Points  $A$  and  $C$  are not identical

Line through points  $H_b$  and  $H_a$  is not parallel with line through points  $A$  and  $C$

### 4.1.3 Proving $H = \_H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 541 terms.

**Time Complexity:** Time spent by the prover is 3.575 seconds.

**NDG conditions** Points  $H_b$  and  $H$  are not identical

Points  $H_b$  and  $H$  are not identical

Points  $H_b$  and  $H$  are not identical

Points  $H_b$  and  $H$  are not identical

Points  $H_b$  and  $H_a$  are not identical

Points  $A$ ,  $B$  and  $C$  are not collinear

Line through points  $H_b$  and  $B$  is not perpendicular to line through points  $B$  and  $H_a$

## 4.2 GCLC - Area method

### 4.2.1 Proving $H_a = \_H_a$

NDG conditions are:

$S_{H_a H_b H} \neq S_{T_a^0 H_b H}$  i.e., lines  $H_a T_a^0$  and  $H_b H$  are not parallel (construction based assumption)

$S_{H_b H_a H} \neq S_{T_b^1 H_a H}$  i.e., lines  $H_b T_b^1$  and  $H_a H$  are not parallel (construction based assumption)

$S_{H_a H_b T_b^1} \neq S_{T_a^0 H_b T_b^1}$  i.e., lines  $H_a T_a^0$  and  $H_b T_b^1$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{-h_a}^2} \neq S_{CAF_{-h_a}^2}$  i.e., lines  $BC$  and  $AF_{-h_a}^2$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{CBF_{-h_b}^3}$  i.e., lines  $AC$  and  $BF_{-h_b}^3$  are not parallel (construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{F_{-h_a}^2 BF_{-h_b}^3}$  i.e., lines  $AF_{-h_a}^2$  and  $BF_{-h_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 2976

Time spent by the prover: 5.840 seconds

### 4.2.2 Proving $H_b = \neg H_b$

NDG conditions are:

$S_{H_a H_b H} \neq S_{T_a^0 H_b H}$  i.e., lines  $H_a T_a^0$  and  $H_b H$  are not parallel (construction based assumption)

$S_{H_b H_a H} \neq S_{T_b^1 H_a H}$  i.e., lines  $H_b T_b^1$  and  $H_a H$  are not parallel (construction based assumption)

$S_{H_a H_b T_b^1} \neq S_{T_a^0 H_b T_b^1}$  i.e., lines  $H_a T_a^0$  and  $H_b T_b^1$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{\neg h_a}^2} \neq S_{CAF_{\neg h_a}^2}$  i.e., lines  $BC$  and  $AF_{\neg h_a}^2$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{CBF_{\neg h_b}^3}$  i.e., lines  $AC$  and  $BF_{\neg h_b}^3$  are not parallel (construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2 BF_{\neg h_b}^3}$  i.e., lines  $AF_{\neg h_a}^2$  and  $BF_{\neg h_b}^3$  are not parallel (construction based assumption)

Total number of proof steps: 3372

Time spent by the prover: 9.400 seconds

### 4.2.3 Proving $H = \neg H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 980 terms.

**Time Complexity:** Time spent by the prover is 4.220 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_b = \neg H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 716 terms.

**Time Complexity:** Time spent by the prover is 2.540 seconds. There are no ndg conditions.

### 4.3.3 Proving $H = \neg H$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $H_a = \neg H_a$

Proving failed

### 4.4.2 Proving $H_b = \neg H_b$

Proving failed

### 4.4.3 Proving $H = \neg H$

Proving failed

# Problem 507

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 507: Given a point  $H_a$ , a point  $H_b$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $H_b$ , construct a line  $H_aH_b$  (rule W02); % DET: points  $H_a$  and  $H_b$  are not the same;
2. Using the point  $H_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $T_a$  are not the same;
3. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
4. Using the line  $a$ , the line  $H_aH_b$ , the point  $T_a$ , the point  $H_a$  and the point  $H_b$ , construct a line  $H_cH_a$  (rule W18); % NDG: points  $T_a$  and  $H_a$  are not the same; points  $H_a$  and  $H_b$  are not the same % DET: points  $H_a$  and  $T_a$  are not the same;
5. Using the point  $T_a$ , the point  $H_a$  and the point  $H_b$ , construct a circle  $circle[T_a, H_b, angle[s[a]][b]]$  (rule W20); % NDG: points  $T_a$  and  $H_a$  are not the same; points  $H_a$  and  $H_b$  are not the same; points  $T_a$  and  $H_b$  are not the same;
6. Using the circle  $circle[T_a, H_b, angle[s[a]][b]]$  and the line  $h_a$ , construct a point  $A_{ca1}$  and a point  $A$  (rule W04); % NDG: line  $h_a$  and circle  $circle[T_a, H_b, angle[s[a]][b]]$  intersect;
7. Using the point  $A$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $H_b$  are not the same;
8. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same;
9. Using the point  $C$  and the point  $A$ , construct a point  $M_b$  (rule W01); ;
10. Using the point  $H_a$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_a$  and  $M_b$  are not the same;

11. Using the circle  $k(M_b, C)$ , the line  $H_c H_a$ , the point  $M_b$  and the point  $H_a$ , construct a point  $H_c$  (rule W05); % NDG: line  $H_c H_a$  and circle  $k(M_b, C)$  intersect % DET: points  $H_a$  and  $H_c$  must be different;
12. Using the point  $A$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $H_c$  are not the same;
13. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; line  $H_c H_a$  and circle  $k(M_b, C)$  intersect; points  $H_a$  and  $M_b$  are not the same; lines  $a$  and  $b$  are not parallel; line  $h_a$  and circle  $circle[T_a, H_b, angle[s[a]][b]]$  intersect; points  $T_a$  and  $H_a$  are not the same; points  $H_a$  and  $H_b$  are not the same; points  $T_a$  and  $H_b$  are not the same; points  $T_a$  and  $H_a$  are not the same; points  $H_a$  and  $H_b$  are not the same.

Determination conditions: lines  $a$  and  $c$  are not the same; points  $A$  and  $H_c$  are not the same; points  $H_a$  and  $H_c$  must be different; lines  $a$  and  $b$  are not the same; points  $A$  and  $H_b$  are not the same; points  $H_a$  and  $T_a$  are not the same; points  $H_a$  and  $T_a$  are not the same; points  $H_a$  and  $H_b$  are not the same.

Rules used: [W01,W02,W03,W04,W05,W06,W10b,W18,W20]

Lemmas used: [D22,D23,D41,D5,D6,D7,D8,GD01,GD02,GL12,L107,L122,L44,L45,L80,L99]

Solving time: 9.5 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{a} 80 40
point H_{b} 89.36 77.83
point T_{a} 70.86 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_r H_{a}
cmark_l H_{b}
cmark_rb T_{a}
color 0 0 0
fontsize 8
```

```
% DET: points H_{a} and H_{b} are not the same
% Constructing a line H_{a}H_{b} which passes through point H_{a} and point H_{b}
line H_{a}H_{b} H_{a} H_{b}
```

```
color 200 200 200
drawline H_{a}H_{b}
color 0 0 0
```

```
% DET: points H_{a} and T_{a} are not the same
```

```

% Constructing a line a which passes through point H_{a} and point T_{a}
line a H_{a} T_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points T_{a} and H_{a} are not the same; points H_{a} and H_{b} are not the same% DET:
      points H_{a} and T_{a} are not the same
% Constructing an angle V[_G47691] which is equal to the angle T_{a}H_{a}H_{b}
angle_o V[_G47691] T_{a} H_{a} H_{b}

% Calculating value angle[_G47770] using formula  $-1/(\text{pow}(2,0))*V[_G47691]-0/(\text{pow}(2,0))*180$ 
expression angle[_G47770] {  $-1/(\text{pow}(2,0))*V[_G47691]-0/(\text{pow}(2,0))*180$  }

% Constructing a point P_{\_G47767} which is an image of the point T_{a} in a rotation around the
      point H_{a} for the angle  $-1/(\text{pow}(2,0))*V[_G47691]-0/(\text{pow}(2,0))*180$ 
rotate P_{\_G47767} H_{a} angle[_G47770] T_{a}
cmark_r P_{\_G47767}
color 200 200 200
drawarc_p H_{a} T_{a} angle[_G47770]
color 0 0 0

% Constructing a line H_{c}H_{a} which passes through point H_{a} and point P_{\_G47767}
line H_{c}H_{a} H_{a} P_{\_G47767}

color 200 200 200
drawline H_{c}H_{a}
color 0 0 0

% NDG: points T_{a} and H_{a} are not the same; points H_{a} and H_{b} are not the same; points T_{a}
      and H_{b} are not the same
% Constructing an angle angle[_G48254] which is equal to the angle T_{a}H_{a}H_{b}
angle_o angle[_G48254] T_{a} H_{a} H_{b}

% Calculating value angle[_G48265] using formula  $90-(1/\text{pow}(2,1))*angle[_G48254]+0/\text{pow}(2,0)*180$ 
expression angle[_G48265] {  $90-(1/\text{pow}(2,1))*angle[_G48254]+0/\text{pow}(2,0)*180$  }

```

```

% Constructing a line L_{\_G48336} which passes through point T_{a} and point H_{b}
line L_{\_G48336} T_{a} H_{b}

color 200 200 200
drawline L_{\_G48336}
color 0 0 0

% Constructing bisector L_{\_G48339} of the segment T_{a}H_{b}
med L_{\_G48339} T_{a} H_{b}

color 200 200 200
drawline L_{\_G48339}
color 0 0 0

color 200 200 200
drawsegment T_{a} H_{b}
color 0 0 0

% Constructing a point P_{\_G48342} which is an image of the point H_{b} in a rotation around the
    point T_{a} for the angle angle[_G48265]
rotate P_{\_G48342} T_{a} angle[_G48265] H_{b}
cmark_r P_{\_G48342}
color 200 200 200
drawarc_p T_{a} H_{b} angle[_G48265]
color 0 0 0

% Constructing a line L_{\_G48345} which passes through point T_{a} and point P_{\_G48342}
line L_{\_G48345} T_{a} P_{\_G48342}

color 200 200 200
drawline L_{\_G48345}
color 0 0 0

% Constructing a point P_{\_G48348} which belongs to line L_{\_G48339} and line L_{\_G48345}
intersec P_{\_G48348} L_{\_G48339} L_{\_G48345}
cmark_r P_{\_G48348}

% Constructing a circle circle[T_{a},H_{b},angle[s[a]][b]] whose center is at point P_{\_G48348}
    and which passes through point T_{a}
circle circle[T_{a},H_{b},angle[s[a]][b]] P_{\_G48348} T_{a}

color 200 200 200
drawcircle circle[T_{a},H_{b},angle[s[a]][b]]
color 0 0 0

% NDG: line h_{a} and circle circle[T_{a},H_{b},angle[s[a]][b]] intersect
% Constructing points A_{ca1} and A which are in intersection of circle[T_{a},H_{b},angle[s[a]][b]]
    and h_{a}
intersec2 A_{ca1} A circle[T_{a},H_{b},angle[s[a]][b]] h_{a}
cmark_r A_{ca1}
cmark_t A

```

```

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_l C

% Constructing a point M_{b} such that CM_{b}/CA=0.5
towards M_{b} C A 0.5
cmark_lt M_{b}
color 200 200 200
drawsegment C A
color 0 0 0

% NDG: points H_{a} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{a}
circle k(M_{b},C) M_{b} H_{a}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line H_{c}H_{a} and circle k(M_{b},C) intersect% DET: points H_{a} and H_{c} must be
different
% Constructing a point P_{\_G49846} which is a foot of the point M_{b} on the line H_{c}H_{a}
foot P_{\_G49846} M_{b} H_{c}H_{a}
cmark_r P_{\_G49846}
color 200 200 200
drawline M_{b} P_{\_G49846}
color 0 0 0

% Constructing a point H_{c} which is an image of the point H_{a} in the symmetry to point/line P_{\_G49846}
sim H_{c} P_{\_G49846} H_{a}
cmark_rt H_{c}

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

```



```

color 200 200 200
drawline c
color 0 0 0

% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and c are not parallel; line H_{c}H_{a} and circle k(M_{b},C)
% intersect; points H_{a} and M_{b} are not the same; lines a and b are not parallel; line h_{a}
% and circle circle[T_{a},H_{b},angle[s[a]][b]] intersect; points T_{a} and H_{a} are not the
% same; points H_{a} and H_{b} are not the same; points T_{a} and H_{b} are not the same; points
% T_{a} and H_{a} are not the same; points H_{a} and H_{b} are not the same
% Determination conditions: lines a and c are not the same; points A and H_{c} are not the same;
% points H_{a} and H_{c} must be different; lines a and b are not the same; points A and H_{b}
% are not the same; points H_{a} and T_{a} are not the same; points H_{a} and T_{a} are not the
% same; points H_{a} and H_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_a = H_a$

Proving failed

#### 4.1.2 Proving $H_b = H_b$

Proving failed

#### 4.1.3 Proving $T_a = T_a$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_a = H_a$

Proving failed

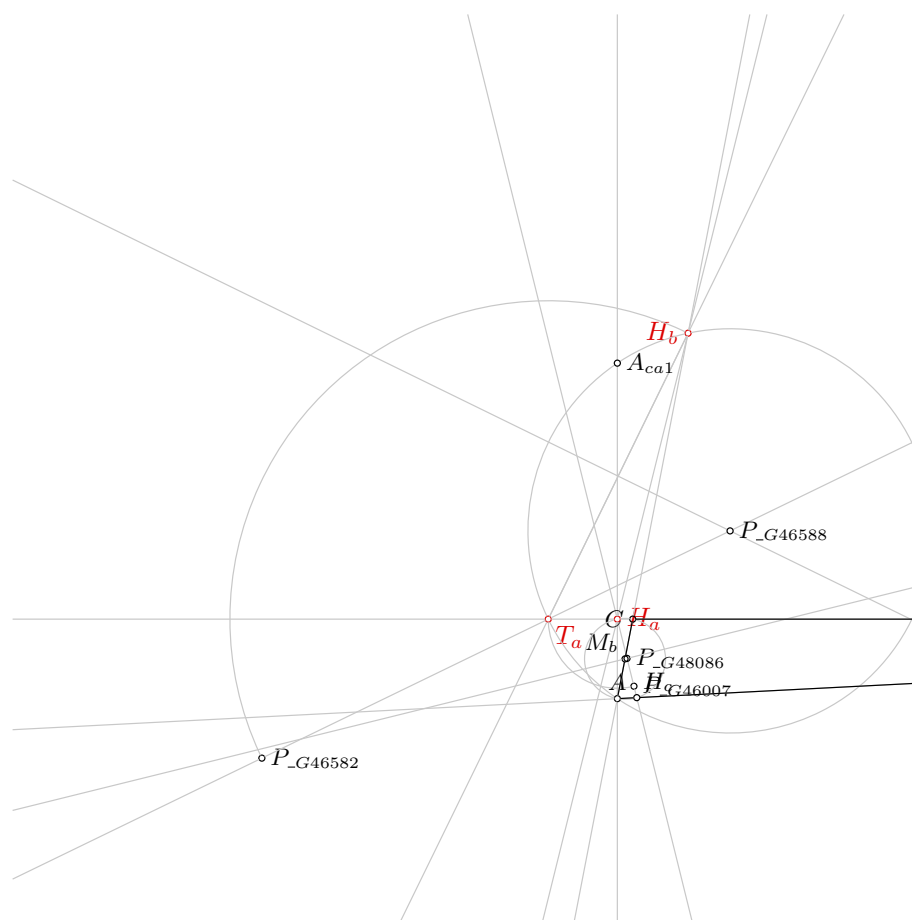


Figure 1: Illustration of the problem 0507

#### **4.2.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.2.3 Proving $T_a = \neg T_a$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $H_a = \neg H_a$**

Proving failed

#### **4.3.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.3.3 Proving $T_a = \neg T_a$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $H_a = \neg H_a$**

Proving failed

#### **4.4.2 Proving $H_b = \neg H_b$**

Proving failed

#### **4.4.3 Proving $T_a = \neg T_a$**

Proving failed

# Problem 508

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 508: Given a point  $H_a$ , a point  $H_b$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $H_b$ , construct a line  $H_aH_b$  (rule W02); % DET: points  $H_a$  and  $H_b$  are not the same;
2. Using the point  $H_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $T_b$  are not the same;
3. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
4. Using the point  $H_a$ , the point  $H_b$ , the point  $T_b$ , the line  $b$  and the line  $H_aH_b$ , construct a line  $H_bH_c$  (rule W17); % NDG: points  $H_b$  and  $T_b$  are not the same; points  $H_a$  and  $H_b$  are not the same % DET: points  $H_b$  and  $T_b$  are not the same;
5. Using the point  $H_a$ , the point  $H_b$  and the point  $T_b$ , construct a circle  $circle[T_b, H_a, angle[a][s[b]]]$  (rule W20); % NDG: points  $H_a$  and  $H_b$  are not the same; points  $H_b$  and  $T_b$  are not the same; points  $T_b$  and  $H_a$  are not the same;
6. Using the circle  $circle[T_b, H_a, angle[a][s[b]]]$  and the line  $h_b$ , construct a point  $B_{ca2}$  and a point  $B$  (rule W04); % NDG: line  $h_b$  and circle  $circle[T_b, H_a, angle[a][s[b]]]$  intersect;
7. Using the point  $B$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $H_a$  are not the same;
8. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same;
9. Using the point  $C$  and the point  $B$ , construct a point  $M_a$  (rule W01); ;
10. Using the point  $H_b$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_b$  and  $M_a$  are not the same;

11. Using the circle  $k(M_a, B)$ , the line  $H_b H_c$ , the point  $M_a$  and the point  $H_b$ , construct a point  $H_c$  (rule W05); % NDG: line  $H_b H_c$  and circle  $k(M_a, B)$  intersect % DET: points  $H_b$  and  $H_c$  must be different;
12. Using the point  $B$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $H_c$  are not the same;
13. Using the line  $b$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $c$  are not parallel % DET: lines  $b$  and  $c$  are not the same.

Non-degenerate conditions: lines  $b$  and  $c$  are not parallel; line  $H_b H_c$  and circle  $k(M_a, B)$  intersect; points  $H_b$  and  $M_a$  are not the same; lines  $b$  and  $a$  are not parallel; line  $h_b$  and circle  $circle[T_b, H_a, angle[a][s[b]]]$  intersect; points  $H_a$  and  $H_b$  are not the same; points  $H_b$  and  $T_b$  are not the same; points  $T_b$  and  $H_a$  are not the same; points  $H_b$  and  $T_b$  are not the same; points  $H_a$  and  $H_b$  are not the same.

Determination conditions: lines  $b$  and  $c$  are not the same; points  $B$  and  $H_c$  are not the same; points  $H_b$  and  $H_c$  must be different; lines  $b$  and  $a$  are not the same; points  $B$  and  $H_a$  are not the same; points  $H_b$  and  $T_b$  are not the same; points  $H_b$  and  $T_b$  are not the same; points  $H_a$  and  $H_b$  are not the same.

Rules used: [W01,W02,W03,W04,W05,W06,W10b,W17,W20]

Lemmas used: [D21,D24,D44,D5,D6,D7,D9,GD01,GD02,GL04,GL12,L100,L108,L120,L38,L39,L83]

Solving time: 8.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{a} 80 40
point H_{b} 89.36 77.83
point T_{b} 94.25 68.88
```

```
color 220 0 0
fontsize 9
```

```
cmark_r H_{a}
cmark_l H_{b}
cmark_t T_{b}
color 0 0 0
fontsize 8
```

```
% DET: points H_{a} and H_{b} are not the same
% Constructing a line H_{a}H_{b} which passes through point H_{a} and point H_{b}
line H_{a}H_{b} H_{a} H_{b}
```

```
color 200 200 200
drawline H_{a}H_{b}
color 0 0 0
```

```
% DET: points H_{b} and T_{b} are not the same
```

```
% Constructing a line b which passes through point H_{b} and point T_{b}
line b H_{b} T_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b
```

```
color 200 200 200
drawline h_{b}
color 0 0 0
```

```
% NDG: points H_{b} and T_{b} are not the same; points H_{a} and H_{b} are not the same% DET:
points H_{b} and T_{b} are not the same
% Constructing an angle V[_G89032] which is equal to the angle H_{a}H_{b}T_{b}
angle_o V[_G89032] H_{a} H_{b} T_{b}
```

```
% Calculating value angle[_G89111] using formula  $1/\text{pow}(2,0)*V[_G89032]+0/\text{pow}(2,0)*180$ 
expression angle[_G89111] {  $1/\text{pow}(2,0)*V[_G89032]+0/\text{pow}(2,0)*180$  }
```

```
% Constructing a point P_{\_G89108} which is an image of the point T_{b} in a rotation around the
point H_{b} for the angle  $1/\text{pow}(2,0)*V[_G89032]+0/\text{pow}(2,0)*180$ 
rotate P_{\_G89108} H_{b} angle[_G89111] T_{b}
cmark_r P_{\_G89108}
color 200 200 200
drawarc_p H_{b} T_{b} angle[_G89111]
color 0 0 0
```

```
% Constructing a line H_{b}H_{c} which passes through point H_{b} and point P_{\_G89108}
line H_{b}H_{c} H_{b} P_{\_G89108}
```

```
color 200 200 200
drawline H_{b}H_{c}
color 0 0 0
```

```
% NDG: points H_{a} and H_{b} are not the same; points H_{b} and T_{b} are not the same; points T_{b}
b} and H_{a} are not the same
% Constructing an angle angle[_G89588] which is equal to the angle H_{a}H_{b}T_{b}
angle_o angle[_G89588] H_{a} H_{b} T_{b}
```

```
% Calculating value angle[_G89599] using formula  $90-(1/\text{pow}(2,1)*\text{angle}[_G89588]+0/\text{pow}(2,0)*180)$ 
expression angle[_G89599] {  $90-(1/\text{pow}(2,1)*\text{angle}[_G89588]+0/\text{pow}(2,0)*180)$  }
```

```

% Constructing a line L_{\_G89670} which passes through point T_{b} and point H_{a}
line L_{\_G89670} T_{b} H_{a}

color 200 200 200
drawline L_{\_G89670}
color 0 0 0

% Constructing bisector L_{\_G89673} of the segment T_{b}H_{a}
med L_{\_G89673} T_{b} H_{a}

color 200 200 200
drawline L_{\_G89673}
color 0 0 0

color 200 200 200
drawsegment T_{b} H_{a}
color 0 0 0

% Constructing a point P_{\_G89676} which is an image of the point H_{a} in a rotation around the
    point T_{b} for the angle angle[_G89599]
rotate P_{\_G89676} T_{b} angle[_G89599] H_{a}
cmark_r P_{\_G89676}
color 200 200 200
drawarc_p T_{b} H_{a} angle[_G89599]
color 0 0 0

% Constructing a line L_{\_G89679} which passes through point T_{b} and point P_{\_G89676}
line L_{\_G89679} T_{b} P_{\_G89676}

color 200 200 200
drawline L_{\_G89679}
color 0 0 0

% Constructing a point P_{\_G89682} which belongs to line L_{\_G89673} and line L_{\_G89679}
intersec P_{\_G89682} L_{\_G89673} L_{\_G89679}
cmark_r P_{\_G89682}

% Constructing a circle circle[T_{b},H_{a},angle[a][s[b]]] whose center is at point P_{\_G89682}
    and which passes through point T_{b}
circle circle[T_{b},H_{a},angle[a][s[b]]] P_{\_G89682} T_{b}

color 200 200 200
drawcircle circle[T_{b},H_{a},angle[a][s[b]]]
color 0 0 0

% NDG: line h_{b} and circle circle[T_{b},H_{a},angle[a][s[b]]] intersect
% Constructing points B_{ca2} and B which are in intersection of circle[T_{b},H_{a},angle[a][s[b]]]
    and h_{b}
intersec2 B_{ca2} B circle[T_{b},H_{a},angle[a][s[b]]] h_{b}
cmark_r B_{ca2}
cmark_b B

```

```

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

% Constructing a point M_{a} such that CM_{a}/CB=0.5
towards M_{a} C B 0.5
cmark_r M_{a}
color 200 200 200
drawsegment C B
color 0 0 0

% NDG: points H_{b} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{b}
circle k(M_{a},B) M_{a} H_{b}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line H_{b}H_{c} and circle k(M_{a},B) intersect% DET: points H_{b} and H_{c} must be
different
% Constructing a point P_{\G91180} which is a foot of the point M_{a} on the line H_{b}H_{c}
foot P_{\G91180} M_{a} H_{b}H_{c}
cmark_r P_{\G91180}
color 200 200 200
drawline M_{a} P_{\G91180}
color 0 0 0

% Constructing a point H_{c} which is an image of the point H_{b} in the symmetry to point/line P_{\G91180}
sim H_{c} P_{\G91180} H_{b}
cmark_rt H_{c}

% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

```



Figure 1: Illustration of the problem 0508

```

color 200 200 200
drawline c
color 0 0 0

% NDG: lines b and c are not parallel% DET: lines b and c are not the same
% Constructing a point A which belongs to line b and line c
intersec A b c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and c are not parallel; line H_{b}H_{c} and circle k(M_{a},B)
% intersect; points H_{b} and M_{a} are not the same; lines b and a are not parallel; line h_{b}
% and circle circle[T_{b},H_{a},angle[a][s[b]]] intersect; points H_{a} and H_{b} are not the
% same; points H_{b} and T_{b} are not the same; points T_{b} and H_{a} are not the same; points
% H_{b} and T_{b} are not the same; points H_{a} and H_{b} are not the same
% Determination conditions: lines b and c are not the same; points B and H_{c} are not the same;
% points H_{b} and H_{c} must be different; lines b and a are not the same; points B and H_{a}
% are not the same; points H_{b} and T_{b} are not the same; points H_{b} and T_{b} are not the
% same; points H_{a} and H_{b} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_a = \neg H_a$

Proving failed

#### 4.1.2 Proving $H_b = \neg H_b$

Proving failed

#### 4.1.3 Proving $T_b = \neg T_b$

Proving failed

## 4.2 GCLC - Area method

### 4.2.1 Proving $H_a = \neg H_a$

Proving failed

### 4.2.2 Proving $H_b = \neg H_b$

Proving failed

### 4.2.3 Proving $T_b = \neg T_b$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $H_a = \neg H_a$

Proving failed

### 4.3.2 Proving $H_b = \neg H_b$

Proving failed

### 4.3.3 Proving $T_b = \neg T_b$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $H_a = \neg H_a$

Proving failed

### 4.4.2 Proving $H_b = \neg H_b$

Proving failed

### 4.4.3 Proving $T_b = \neg T_b$

Proving failed

# Problem 509

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 509: Given a point  $H_a$ , a point  $H_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 510

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 510: Given a point  $H_a$ , a point  $H_b$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 511

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 511: Given a point  $H_a$ , a point  $H_c$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $H_a$  and  $H$  are not the same;
2. Using the point  $H_c$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $H_c$  and  $H$  are not the same;
3. Using the point  $H_a$  and the line  $h_a$ , construct a line  $a$  (rule W10a); ;
4. Using the line  $a$  and the line  $h_c$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $h_c$  are not parallel % DET: lines  $a$  and  $h_c$  are not the same;
5. Using the point  $H_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
6. Using the line  $c$  and the line  $h_a$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $h_a$  are not parallel % DET: lines  $c$  and  $h_a$  are not the same;
7. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same.

Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; lines  $c$  and  $h_a$  are not parallel; lines  $a$  and  $h_c$  are not parallel.

Determination conditions: lines  $a$  and  $c$  are not the same; lines  $c$  and  $h_a$  are not the same; lines  $a$  and  $h_c$  are not the same; points  $H_c$  and  $H$  are not the same; points  $H_a$  and  $H$  are not the same.

Rules used: [W02,W03,W10a]

Lemmas used: [D10,D3,D5,D7,D8,GD01,L3]

Solving time: 0.9 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point H_{a} 80 40
point H_{c} 68.91 84.83
point H 80 72.73

color 220 0 0
fontsize 9

cmark_r H_{a}
cmark_rt H_{c}
cmark_rt H
color 0 0 0
fontsize 8

% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
perp a H_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and h_{c} are not parallel% DET: lines a and h_{c} are not the same
% Constructing a point C which belongs to line a and line h_{c}
intersec C a h_{c}
cmark_l C

% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}
```

```

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and h_{a} are not parallel% DET: lines c and h_{a} are not the same
% Constructing a point A which belongs to line c and line h_{a}
intersec A c h_{a}
cmark_t A

% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and c are not parallel; lines c and h_{a} are not parallel;
% lines a and h_{c} are not parallel
% Determination conditions: lines a and c are not the same; lines c and h_{a} are not the same;
% lines a and h_{c} are not the same; points H_{c} and H are not the same; points H_{a} and H are
% not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_a = H_a$

Proving failed

#### 4.1.2 Proving $H_c = H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 239 terms.

**Time Complexity:** Time spent by the prover is 1.884 seconds.

**NDG conditions** Points  $H_c$  and  $H$  are not identical

Points  $H_c$  and  $H$  are not identical

Points  $H_c$  and  $H$  are not identical

Points  $H_c$  and  $H$  are not identical

Points  $H_c$  and  $H_a$  are not identical

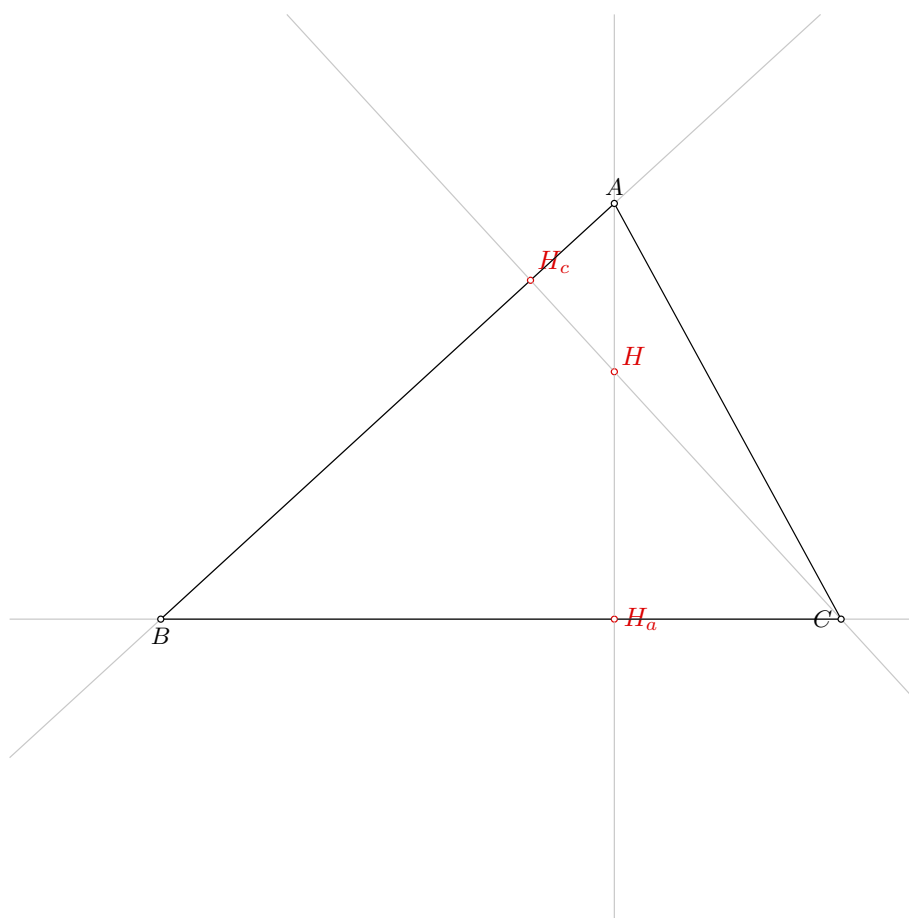


Figure 1: Illustration of the problem 0511



Points  $A$  and  $B$  are not identical

Line through points  $A$  and  $B$  is not parallel with line through points  $H_c$  and  $H_a$

### 4.1.3 Proving $H=_H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 541 terms.

**Time Complexity:** Time spent by the prover is 3.491 seconds.

**NDG conditions** Points  $H_c$  and  $H$  are not identical

Points  $H_c$  and  $H$  are not identical

Points  $H_c$  and  $H$  are not identical

Points  $H_c$  and  $H$  are not identical

Points  $H_c$  and  $H_a$  are not identical

Points  $A$ ,  $B$  and  $C$  are not collinear

Line through points  $H_c$  and  $B$  is not perpendicular to line through points  $B$  and  $H_a$

## 4.2 GCLC - Area method

### 4.2.1 Proving $H_a=_H$

Proving failed

### 4.2.2 Proving $H_c=_H$

NDG conditions are:

$S_{H_a H_c H} \neq S_{T_a^0 H_c H}$  i.e., lines  $H_a T_a^0$  and  $H_c H$  are not parallel (construction based assumption)

$S_{H_c H_a H} \neq S_{T_c^1 H_a H}$  i.e., lines  $H_c T_c^1$  and  $H_a H$  are not parallel (construction based assumption)

$S_{H_a H_c T_c^1} \neq S_{T_a^0 H_c T_c^1}$  i.e., lines  $H_a T_a^0$  and  $H_c T_c^1$  are not parallel (construction based assumption)

$S_{ABC} \neq 0$  i.e., points  $A$ ,  $B$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{H_a}^2} \neq S_{CAF_{H_a}^2}$  i.e., lines  $BC$  and  $AF_{H_a}^2$  are not parallel (construction based assumption)

$S_{CAB} \neq 0$  i.e., points  $C$ ,  $A$  and  $B$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{H_c}^3} \neq S_{BCF_{H_c}^3}$  i.e., lines  $AB$  and  $CF_{H_c}^3$  are not parallel (construction based assumption)

$S_{BAC} \neq 0$  i.e., points  $B$ ,  $A$  and  $C$  are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{H_b}^4} \neq S_{F_{H_a}^2 BF_{H_b}^4}$  i.e., lines  $AF_{H_a}^2$  and  $BF_{H_b}^4$  are not parallel (construction based assumption)

Total number of proof steps: 3372

Time spent by the prover: 9.340 seconds

### 4.2.3 Proving $H=_H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $H_a=_H$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 980 terms.

**Time Complexity:** Time spent by the prover is 3.940 seconds. There are no ndg conditions.

#### 4.3.2 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 716 terms.

**Time Complexity:** Time spent by the prover is 2.380 seconds. There are no ndg conditions.

#### 4.3.3 Proving $H = \neg H$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $H_a = \neg H_a$

Proving failed

#### 4.4.2 Proving $H_c = \neg H_c$

Proving failed

#### 4.4.3 Proving $H = \neg H$

Proving failed

# Problem 512

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 512: Given a point  $H_a$ , a point  $H_c$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $H_c$ , construct a line  $H_cH_a$  (rule W02); % DET: points  $H_a$  and  $H_c$  are not the same;
2. Using the point  $H_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $T_a$  are not the same;
3. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
4. Using the point  $H_c$ , the point  $H_a$ , the point  $T_a$ , the line  $a$  and the line  $H_cH_a$ , construct a line  $H_aH_b$  (rule W17); % NDG: points  $H_a$  and  $T_a$  are not the same; points  $H_c$  and  $H_a$  are not the same % DET: points  $H_a$  and  $T_a$  are not the same;
5. Using the point  $H_c$ , the point  $H_a$  and the point  $T_a$ , construct a circle  $circle[T_a, H_c, angle[c][s[a]]]$  (rule W20); % NDG: points  $H_c$  and  $H_a$  are not the same; points  $H_a$  and  $T_a$  are not the same; points  $T_a$  and  $H_c$  are not the same;
6. Using the circle  $circle[T_a, H_c, angle[c][s[a]]]$  and the line  $h_a$ , construct a point  $A_{ca2}$  and a point  $A$  (rule W04); % NDG: line  $h_a$  and circle  $circle[T_a, H_c, angle[c][s[a]]]$  intersect;
7. Using the point  $A$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $A$  and  $H_c$  are not the same;
8. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same;
9. Using the point  $B$  and the point  $A$ , construct a point  $M_c$  (rule W01); ;
10. Using the point  $H_a$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_a$  and  $M_c$  are not the same;

11. Using the circle  $k(M_c, A)$ , the line  $H_a H_b$ , the point  $M_c$  and the point  $H_a$ , construct a point  $H_b$  (rule W05); % NDG: line  $H_a H_b$  and circle  $k(M_c, A)$  intersect % DET: points  $H_a$  and  $H_b$  must be different;
12. Using the point  $A$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $H_b$  are not the same;
13. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; line  $H_a H_b$  and circle  $k(M_c, A)$  intersect; points  $H_a$  and  $M_c$  are not the same; lines  $a$  and  $c$  are not parallel; line  $h_a$  and circle  $circle[T_a, H_c, angle[c][s[a]]]$  intersect; points  $H_c$  and  $H_a$  are not the same; points  $H_a$  and  $T_a$  are not the same; points  $T_a$  and  $H_c$  are not the same; points  $H_a$  and  $T_a$  are not the same; points  $H_c$  and  $H_a$  are not the same.

Determination conditions: lines  $a$  and  $b$  are not the same; points  $A$  and  $H_b$  are not the same; points  $H_a$  and  $H_b$  must be different; lines  $a$  and  $c$  are not the same; points  $A$  and  $H_c$  are not the same; points  $H_a$  and  $T_a$  are not the same; points  $H_a$  and  $T_a$  are not the same; points  $H_a$  and  $H_c$  are not the same.

Rules used: [W01,W02,W03,W04,W05,W06,W10b,W17,W20]

Lemmas used: [D20,D23,D42,D5,D6,D7,D8,GD01,GD02,GL04,GL12,L107,L122,L41,L42,L81,L98]

Solving time: 8.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{a} 80 40
point H_{c} 68.91 84.83
point T_{a} 70.86 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_r H_{a}
cmark_rt H_{c}
cmark_rb T_{a}
color 0 0 0
fontsize 8
```

```
% DET: points H_{a} and H_{c} are not the same
% Constructing a line H_{c}H_{a} which passes through point H_{a} and point H_{c}
line H_{c}H_{a} H_{a} H_{c}
```

```
color 200 200 200
drawline H_{c}H_{a}
color 0 0 0
```

```
% DET: points H_{a} and T_{a} are not the same
```

```

% Constructing a line a which passes through point H_{a} and point T_{a}
line a H_{a} T_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points H_{a} and T_{a} are not the same; points H_{c} and H_{a} are not the same% DET:
      points H_{a} and T_{a} are not the same
% Constructing an angle V[_G79222] which is equal to the angle H_{c}H_{a}T_{a}
angle_o V[_G79222] H_{c} H_{a} T_{a}

% Calculating value angle[_G79301] using formula  $1/\text{pow}(2,0)*V[_G79222]+0/\text{pow}(2,0)*180$ 
expression angle[_G79301] {  $1/\text{pow}(2,0)*V[_G79222]+0/\text{pow}(2,0)*180$  }

% Constructing a point P_{\_G79298} which is an image of the point T_{a} in a rotation around the
      point H_{a} for the angle  $1/\text{pow}(2,0)*V[_G79222]+0/\text{pow}(2,0)*180$ 
rotate P_{\_G79298} H_{a} angle[_G79301] T_{a}
cmark_r P_{\_G79298}
color 200 200 200
drawarc_p H_{a} T_{a} angle[_G79301]
color 0 0 0

% Constructing a line H_{a}H_{b} which passes through point H_{a} and point P_{\_G79298}
line H_{a}H_{b} H_{a} P_{\_G79298}

color 200 200 200
drawline H_{a}H_{b}
color 0 0 0

% NDG: points H_{c} and H_{a} are not the same; points H_{a} and T_{a} are not the same; points T_{a}
      a} and H_{c} are not the same
% Constructing an angle angle[_G79778] which is equal to the angle H_{c}H_{a}T_{a}
angle_o angle[_G79778] H_{c} H_{a} T_{a}

% Calculating value angle[_G79789] using formula  $90-(1/\text{pow}(2,1)*\text{angle}[_G79778]+0/\text{pow}(2,0)*180)$ 
expression angle[_G79789] {  $90-(1/\text{pow}(2,1)*\text{angle}[_G79778]+0/\text{pow}(2,0)*180)$  }

```

```

% Constructing a line L_{\_G79860} which passes through point T_{a} and point H_{c}
line L_{\_G79860} T_{a} H_{c}

color 200 200 200
drawline L_{\_G79860}
color 0 0 0

% Constructing bisector L_{\_G79863} of the segment T_{a}H_{c}
med L_{\_G79863} T_{a} H_{c}

color 200 200 200
drawline L_{\_G79863}
color 0 0 0

color 200 200 200
drawsegment T_{a} H_{c}
color 0 0 0

% Constructing a point P_{\_G79866} which is an image of the point H_{c} in a rotation around the
    point T_{a} for the angle angle[_G79789]
rotate P_{\_G79866} T_{a} angle[_G79789] H_{c}
cmark_r P_{\_G79866}
color 200 200 200
drawarc_p T_{a} H_{c} angle[_G79789]
color 0 0 0

% Constructing a line L_{\_G79869} which passes through point T_{a} and point P_{\_G79866}
line L_{\_G79869} T_{a} P_{\_G79866}

color 200 200 200
drawline L_{\_G79869}
color 0 0 0

% Constructing a point P_{\_G79872} which belongs to line L_{\_G79863} and line L_{\_G79869}
intersec P_{\_G79872} L_{\_G79863} L_{\_G79869}
cmark_r P_{\_G79872}

% Constructing a circle circle[T_{a},H_{c},angle[c][s[a]]] whose center is at point P_{\_G79872}
    and which passes through point T_{a}
circle circle[T_{a},H_{c},angle[c][s[a]]] P_{\_G79872} T_{a}

color 200 200 200
drawcircle circle[T_{a},H_{c},angle[c][s[a]]]
color 0 0 0

% NDG: line h_{a} and circle circle[T_{a},H_{c},angle[c][s[a]]] intersect
% Constructing points A_{ca2} and A which are in intersection of circle[T_{a},H_{c},angle[c][s[a]]]
    and h_{a}
intersec2 A_{ca2} A circle[T_{a},H_{c},angle[c][s[a]]] h_{a}
cmark_r A_{ca2}
cmark_t A

```

```

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B

% Constructing a point M_{c} such that BM_{c}/BA=0.5
towards M_{c} B A 0.5
cmark_lt M_{c}
color 200 200 200
drawsegment B A
color 0 0 0

% NDG: points H_{a} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{a}
circle k(M_{c},A) M_{c} H_{a}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line H_{a}H_{b} and circle k(M_{c},A) intersect% DET: points H_{a} and H_{b} must be
different
% Constructing a point P_{\G81370} which is a foot of the point M_{c} on the line H_{a}H_{b}
foot P_{\G81370} M_{c} H_{a}H_{b}
cmark_r P_{\G81370}
color 200 200 200
drawline M_{c} P_{\G81370}
color 0 0 0

% Constructing a point H_{b} which is an image of the point H_{a} in the symmetry to point/line P_{\G81370}
sim H_{b} P_{\G81370} H_{a}
cmark_l H_{b}

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

```

Figure 1: Illustration of the problem 0512

```

color 200 200 200
drawline b
color 0 0 0

% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_1 C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and b are not parallel; line H_{a}H_{b} and circle k(M_{c},A)
% intersect; points H_{a} and M_{c} are not the same; lines a and c are not parallel; line h_{a}
% and circle circle[T_{a},H_{c},angle[c][s[a]]] intersect; points H_{c} and H_{a} are not the
% same; points H_{a} and T_{a} are not the same; points T_{a} and H_{c} are not the same; points
% H_{a} and T_{a} are not the same; points H_{c} and H_{a} are not the same
% Determination conditions: lines a and b are not the same; points A and H_{b} are not the same;
% points H_{a} and H_{b} must be different; lines a and c are not the same; points A and H_{c}
% are not the same; points H_{a} and T_{a} are not the same; points H_{a} and T_{a} are not the
% same; points H_{a} and H_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_a = H_a$

Proving failed

#### 4.1.2 Proving $H_c = H_c$

Proving failed

#### 4.1.3 Proving $T_a = T_a$

Proving failed



## 4.2 GCLC - Area method

### 4.2.1 Proving $H_a = \neg H_a$

Proving failed

### 4.2.2 Proving $H_c = \neg H_c$

Proving failed

### 4.2.3 Proving $T_a = \neg T_a$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $H_a = \neg H_a$

Proving failed

### 4.3.2 Proving $H_c = \neg H_c$

Proving failed

### 4.3.3 Proving $T_a = \neg T_a$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $H_a = \neg H_a$

Proving failed

### 4.4.2 Proving $H_c = \neg H_c$

Proving failed

### 4.4.3 Proving $T_a = \neg T_a$

Proving failed

## Problem 513

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 513: Given a point  $H_a$ , a point  $H_c$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 514

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 514: Given a point  $H_a$ , a point  $H_c$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $H_c$ , construct a line  $H_cH_a$  (rule W02); % DET: points  $H_a$  and  $H_c$  are not the same;
2. Using the point  $H_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $T_c$  are not the same;
3. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
4. Using the line  $c$ , the line  $H_cH_a$ , the point  $T_c$ , the point  $H_c$  and the point  $H_a$ , construct a line  $H_bH_c$  (rule W18); % NDG: points  $T_c$  and  $H_c$  are not the same; points  $H_c$  and  $H_a$  are not the same % DET: points  $H_c$  and  $T_c$  are not the same;
5. Using the point  $T_c$ , the point  $H_c$  and the point  $H_a$ , construct a circle  $circle[T_c, H_a, angle[s[c]][a]]$  (rule W20); % NDG: points  $T_c$  and  $H_c$  are not the same; points  $H_c$  and  $H_a$  are not the same; points  $T_c$  and  $H_a$  are not the same;
6. Using the circle  $circle[T_c, H_a, angle[s[c]][a]]$  and the line  $h_c$ , construct a point  $C_{ca1}$  and a point  $C$  (rule W04); % NDG: line  $h_c$  and circle  $circle[T_c, H_a, angle[s[c]][a]]$  intersect;
7. Using the point  $C$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $H_a$  are not the same;
8. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same;
9. Using the point  $B$  and the point  $C$ , construct a point  $M_a$  (rule W01); ;
10. Using the point  $H_c$  and the point  $M_a$ , construct a circle  $k(M_a, B)$  (rule W06); % NDG: points  $H_c$  and  $M_a$  are not the same;

11. Using the circle  $k(M_a, B)$ , the line  $H_bH_c$ , the point  $M_a$  and the point  $H_c$ , construct a point  $H_b$  (rule W05); % NDG: line  $H_bH_c$  and circle  $k(M_a, B)$  intersect % DET: points  $H_c$  and  $H_b$  must be different;
12. Using the point  $C$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $H_b$  are not the same;
13. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; line  $H_bH_c$  and circle  $k(M_a, B)$  intersect; points  $H_c$  and  $M_a$  are not the same; lines  $c$  and  $a$  are not parallel; line  $h_c$  and circle  $circle[T_c, H_a, angle[s[c]][a]]$  intersect; points  $T_c$  and  $H_c$  are not the same; points  $H_c$  and  $H_a$  are not the same; points  $T_c$  and  $H_a$  are not the same; points  $T_c$  and  $H_c$  are not the same; points  $H_c$  and  $H_a$  are not the same.

Determination conditions: lines  $c$  and  $b$  are not the same; points  $C$  and  $H_b$  are not the same; points  $H_c$  and  $H_b$  must be different; lines  $c$  and  $a$  are not the same; points  $C$  and  $H_a$  are not the same; points  $H_c$  and  $T_c$  are not the same; points  $H_c$  and  $T_c$  are not the same; points  $H_a$  and  $H_c$  are not the same.

Rules used: [W01,W02,W03,W04,W05,W06,W10b,W18,W20]

Lemmas used: [D10,D21,D25,D45,D5,D6,D7,GD01,GD02,GL12,L103,L109,L121,L38,L39,L84]

Solving time: 9.5 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{a} 80 40
point H_{c} 68.91 84.83
point T_{c} 55.38 72.43
```

```
color 220 0 0
fontsize 9
```

```
cmark_r H_{a}
cmark_rt H_{c}
cmark_rt T_{c}
color 0 0 0
fontsize 8
```

```
% DET: points H_{a} and H_{c} are not the same
% Constructing a line H_{c}H_{a} which passes through point H_{a} and point H_{c}
line H_{c}H_{a} H_{a} H_{c}
```

```
color 200 200 200
drawline H_{c}H_{a}
color 0 0 0
```

```
% DET: points H_{c} and T_{c} are not the same
```

```

% Constructing a line c which passes through point H_{c} and point T_{c}
line c H_{c} T_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points T_{c} and H_{c} are not the same; points H_{c} and H_{a} are not the same% DET:
% points H_{c} and T_{c} are not the same
% Constructing an angle V[_G44013] which is equal to the angle T_{c}H_{c}H_{a}
angle_o V[_G44013] T_{c} H_{c} H_{a}

% Calculating value angle[_G130447] using formula  $-1/(\text{pow}(2,0))*V[_G130368]-0/(\text{pow}(2,0))*180$ 
expression angle[_G44092] {  $-1/(\text{pow}(2,0))*V[_G130368]-0/(\text{pow}(2,0))*180$  }

% Constructing a point P_{\_G130444} which is an image of the point T_{c} in a rotation around the
% point H_{c} for the angle  $-1/(\text{pow}(2,0))*V[_G130368]-0/(\text{pow}(2,0))*180$ 
rotate P_{\_G44089} H_{c} angle[_G44092] T_{c}
cmark_r P_{\_G44089}
color 200 200 200
drawarc_p H_{c} T_{c} angle[_G44092]
color 0 0 0

% Constructing a line H_{b}H_{c} which passes through point H_{c} and point P_{\_G44089}
line H_{b}H_{c} H_{c} P_{\_G44089}

color 200 200 200
drawline H_{b}H_{c}
color 0 0 0

% NDG: points T_{c} and H_{c} are not the same; points H_{c} and H_{a} are not the same; points T_{c}
% and H_{a} are not the same
% Constructing an angle angle[_G44487] which is equal to the angle T_{c}H_{c}H_{a}
angle_o angle[_G44487] T_{c} H_{c} H_{a}

% Calculating value angle[_G44498] using formula  $90-(1/\text{pow}(2,1))*\text{angle}[_G44487]+0/\text{pow}(2,0)*180$ 
expression angle[_G44498] {  $90-(1/\text{pow}(2,1))*\text{angle}[_G44487]+0/\text{pow}(2,0)*180$  }

```

```

% Constructing a line L_{\_G44569} which passes through point T_{c} and point H_{a}
line L_{\_G44569} T_{c} H_{a}

color 200 200 200
drawline L_{\_G44569}
color 0 0 0

% Constructing bisector L_{\_G44572} of the segment T_{c}H_{a}
med L_{\_G44572} T_{c} H_{a}

color 200 200 200
drawline L_{\_G44572}
color 0 0 0

color 200 200 200
drawsegment T_{c} H_{a}
color 0 0 0

% Constructing a point P_{\_G44575} which is an image of the point H_{a} in a rotation around the
    point T_{c} for the angle angle[_G44498]
rotate P_{\_G44575} T_{c} angle[_G44498] H_{a}
cmark_r P_{\_G44575}
color 200 200 200
drawarc_p T_{c} H_{a} angle[_G44498]
color 0 0 0

% Constructing a line L_{\_G44578} which passes through point T_{c} and point P_{\_G44575}
line L_{\_G44578} T_{c} P_{\_G44575}

color 200 200 200
drawline L_{\_G44578}
color 0 0 0

% Constructing a point P_{\_G44581} which belongs to line L_{\_G44572} and line L_{\_G44578}
intersec P_{\_G44581} L_{\_G44572} L_{\_G44578}
cmark_r P_{\_G44581}

% Constructing a circle circle[T_{c},H_{a},angle[s[c]] [a]] whose center is at point P_{\_G44581}
    and which passes through point T_{c}
circle circle[T_{c},H_{a},angle[s[c]] [a]] P_{\_G44581} T_{c}

color 200 200 200
drawcircle circle[T_{c},H_{a},angle[s[c]] [a]]
color 0 0 0

% NDG: line h_{c} and circle circle[T_{c},H_{a},angle[s[c]] [a]] intersect
% Constructing points C_{ca1} and C which are in intersection of circle[T_{c},H_{a},angle[s[c]] [a]]
    and h_{c}
intersec2 C_{ca1} C circle[T_{c},H_{a},angle[s[c]] [a]] h_{c}
cmark_r C_{ca1}
cmark_l C

```

```

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

% Constructing a point M_{a} such that BM_{a}/BC=0.5
towards M_{a} B C 0.5
cmark_r M_{a}
color 200 200 200
drawsegment B C
color 0 0 0

% NDG: points H_{c} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{c}
circle k(M_{a},B) M_{a} H_{c}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line H_{b}H_{c} and circle k(M_{a},B) intersect% DET: points H_{c} and H_{b} must be
different
% Constructing a point P_{\G46079} which is a foot of the point M_{a} on the line H_{b}H_{c}
foot P_{\G46079} M_{a} H_{b}H_{c}
cmark_r P_{\G46079}
color 200 200 200
drawline M_{a} P_{\G46079}
color 0 0 0

% Constructing a point H_{b} which is an image of the point H_{c} in the symmetry to point/line P_{\G46079}
sim H_{b} P_{\G46079} H_{c}
cmark_l H_{b}

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

```

```

color 200 200 200
drawline b
color 0 0 0

```

```

% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions: lines c and b are not parallel; line H_{b}H_{c} and circle k(M_{a},B)
% intersect; points H_{c} and M_{a} are not the same; lines c and a are not parallel; line h_{c}
% and circle circle[T_{c},H_{a},angle[s[c]][a]] intersect; points T_{c} and H_{c} are not the
% same; points H_{c} and H_{a} are not the same; points T_{c} and H_{a} are not the same; points
% T_{c} and H_{c} are not the same; points H_{c} and H_{a} are not the same
% Determination conditions: lines c and b are not the same; points C and H_{b} are not the same;
% points H_{c} and H_{b} must be different; lines c and a are not the same; points C and H_{a}
% are not the same; points H_{c} and T_{c} are not the same; points H_{c} and T_{c} are not the
% same; points H_{a} and H_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_a = H_a$

Proving failed

#### 4.1.2 Proving $H_c = H_c$

Proving failed

#### 4.1.3 Proving $T_c = T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_a = H_a$

Proving failed



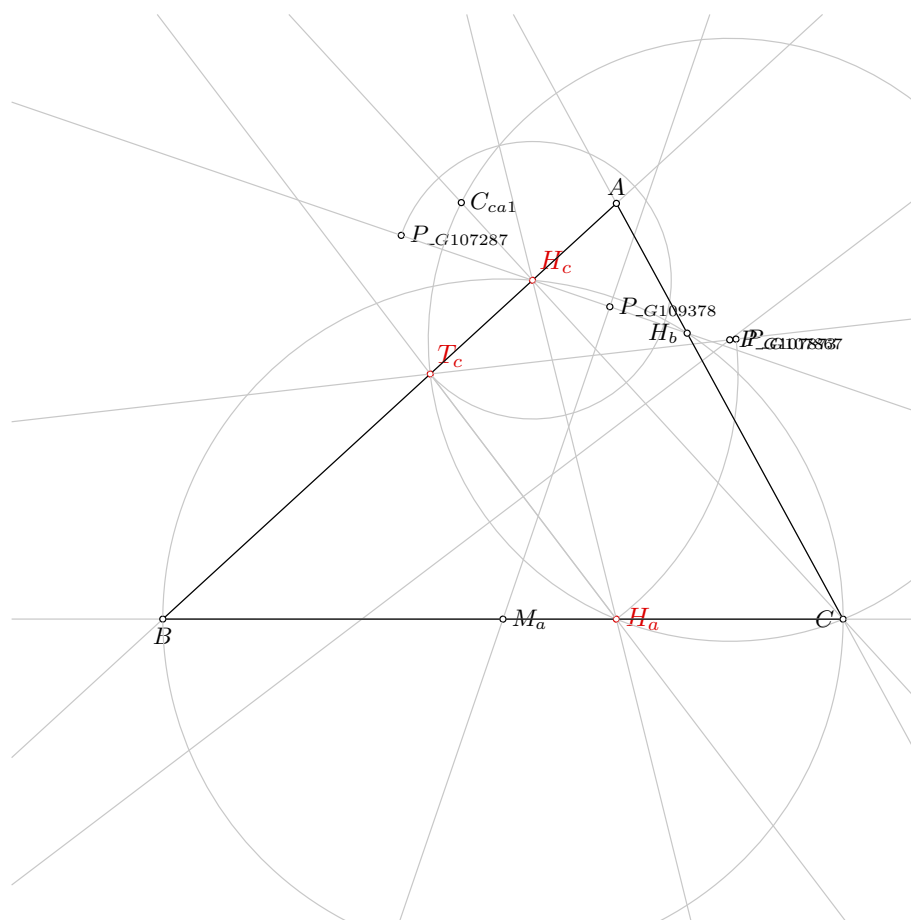


Figure 1: Illustration of the problem 0514

#### **4.2.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.2.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $H_a = \neg H_a$**

Proving failed

#### **4.3.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.3.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $H_a = \neg H_a$**

Proving failed

#### **4.4.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.4.3 Proving $T_c = \neg T_c$**

Proving failed

## Problem 515

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 515: Given a point  $H_a$ , a point  $H_c$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 516

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 516: Given a point  $H_a$ , a point  $T_a$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $T_a$  are not the same;
2. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
3. Choose freely a point  $H$  on the line  $h_a$  (rule WOnline2);
4. Choose freely a point  $A$  on the line  $h_a$  (rule WOnline1) ;
5. Choose freely a point  $B$  on the line  $a$  (rule WOnline1) ;
6. Using the point  $B$  and the point  $A$ , construct a point  $M_c$  (rule W01); ;
7. Using the point  $B$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $B$  and  $H$  are not the same;
8. Using the point  $B$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $B$  and  $M_c$  are not the same;
9. Using the circle  $k(M_c, A)$ , the line  $h_b$ , the point  $M_c$  and the point  $B$ , construct a point  $H_b$  (rule W05); % NDG: line  $h_b$  and circle  $k(M_c, A)$  intersect % DET: points  $B$  and  $H_b$  must be different;
10. Using the point  $H_b$  and the point  $A$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $A$  are not the same;
11. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; line  $h_b$  and circle  $k(M_c, A)$  intersect; points  $B$  and  $M_c$  are not the same.

Determination conditions: lines  $a$  and  $b$  are not the same; points  $H_b$  and  $A$  are not the same; points  $B$  and  $H_b$  must be different; points  $B$  and  $H$  are not the same; points  $H_a$  and  $T_a$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W10b,WOnline1,WOnline2]

Lemmas used: [D20,D23,D3,D5,D6,D8,D9,GD01,GD02,GL04,L40,L41]

Solving time: 69.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{a} 80 40
point T_{a} 70.86 40
point H 80 72.73
```

```
color 220 0 0
fontsize 9
```

```
cmark_r H_{a}
cmark_rb T_{a}
cmark_rt H
color 0 0 0
fontsize 8
```

```
% DET: points H_{a} and T_{a} are not the same
% Constructing a line a which passes through point H_{a} and point T_{a}
line a H_{a} T_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a
```

```
color 200 200 200
drawline h_{a}
color 0 0 0
```

```
% Generating random value V[_G84665]
random V[_G84665]
```

```
% Calculating value V[_G84686] using formula V[_G84665]*20
expression V[_G84686] { V[_G84665]*20 }
```

```

% Constructing a point H which is a point for which holds  $H_{\{a\}}H = V[_{G84686}]$  and angle  $T_{\{a\}}H_{\{a\}}H = 90$ 
turtle H  $T_{\{a\}}$   $H_{\{a\}}$  90  $V[_{G84686}]$ 
cmark_rt H

% Choosing randomly a point A on the line  $HH_{\{a\}}$ 
online A H  $H_{\{a\}}$ 
cmark_t A
color 200 200 200
drawline H  $H_{\{a\}}$ 
color 0 0 0

% Choosing randomly a point B on the line  $H_{\{a\}}T_{\{a\}}$ 
online B  $H_{\{a\}}$   $T_{\{a\}}$ 
cmark_b B
color 200 200 200
drawline  $H_{\{a\}}$   $T_{\{a\}}$ 
color 0 0 0

% Constructing a point  $M_{\{c\}}$  such that  $BM_{\{c\}}/BA=0.5$ 
towards  $M_{\{c\}}$  B A 0.5
cmark_lt  $M_{\{c\}}$ 
color 200 200 200
drawsegment B A
color 0 0 0

% DET: points B and H are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point B and point H
line  $h_{\{b\}}$  B H

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: points B and  $M_{\{c\}}$  are not the same
% Constructing a circle  $k(M_{\{c\}},A)$  whose center is at point  $M_{\{c\}}$  and which passes through point B
circle  $k(M_{\{c\}},A)$   $M_{\{c\}}$  B

color 200 200 200
drawcircle  $k(M_{\{c\}},A)$ 
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(M_{\{c\}},A)$  intersect% DET: points B and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{G85272\}}$  which is a foot of the point  $M_{\{c\}}$  on the line  $h_{\{b\}}$ 

```

```

foot P_{\_G85272} M_{c} h_{b}
cmark_r P_{\_G85272}
color 200 200 200
drawline M_{c} P_{\_G85272}
color 0 0 0

% Constructing a point H_{b} which is an image of the point B in the symmetry to point/line P_{\_G
85272}
sim H_{b} P_{\_G85272} B
cmark_l H_{b}

% DET: points H_{b} and A are not the same
% Constructing a line b which passes through point H_{b} and point A
line b H_{b} A

color 200 200 200
drawline b
color 0 0 0

% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and b are not parallel; line h_{b} and circle k(M_{c},A)
intersect; points B and M_{c} are not the same
% Determination conditions: lines a and b are not the same; points H_{b} and A are not the same;
points B and H_{b} must be different; points B and H are not the same; points H_{a} and T_{a}
are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_a = H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.16 seconds.

**NDG conditions** Points  $T_a$  and  $A$  are not identical

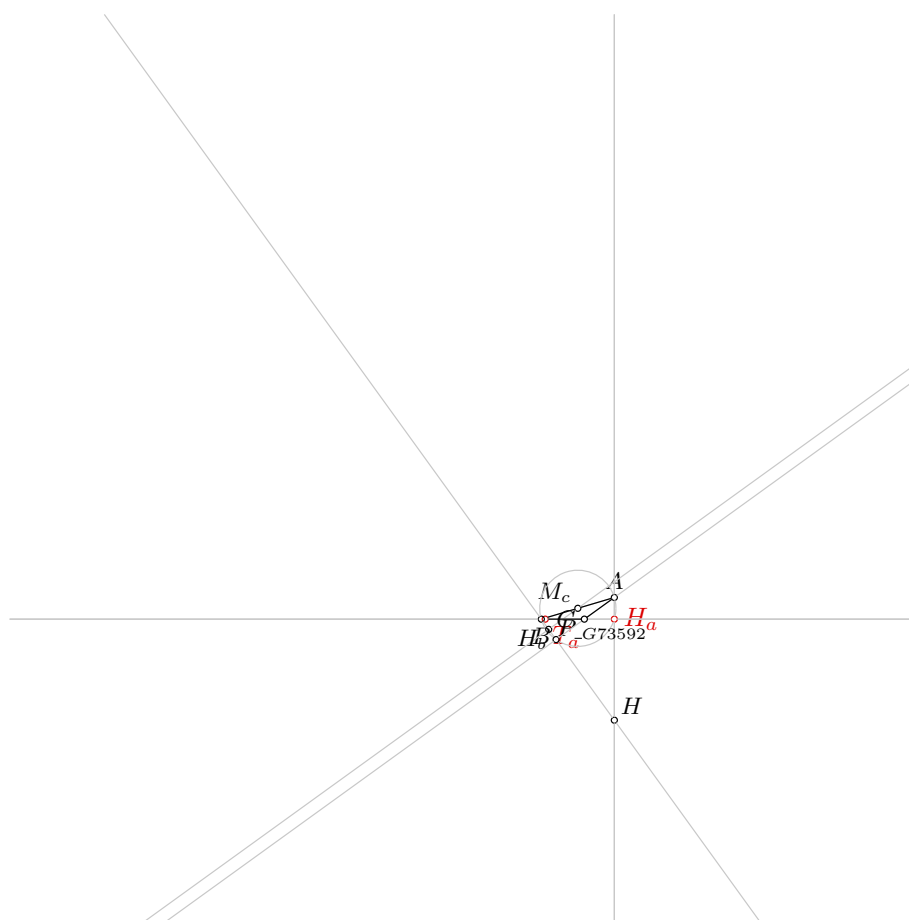


Figure 1: Illustration of the problem 0516



Points  $A$  and  $B$  are not identical

Line through points  $H_b$  and  $H$  is not perpendicular to line through points  $H$  and  $A$

Line through points  $A$  and  $H_a$  is not parallel with line through points  $B$  and  $C$

#### **4.1.2 Proving $T_a = \neg T_a$**

Proving failed

#### **4.1.3 Proving $H = \neg H$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $H_a = \neg H_a$**

Proving failed

#### **4.2.2 Proving $T_a = \neg T_a$**

Proving failed

#### **4.2.3 Proving $H = \neg H$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $H_a = \neg H_a$**

Proving failed

#### **4.3.2 Proving $T_a = \neg T_a$**

Proving failed

#### **4.3.3 Proving $H = \neg H$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $H_a = \neg H_a$**

Proving failed

#### **4.4.2 Proving $T_a = \neg T_a$**

Proving failed

#### **4.4.3 Proving $H = \neg H$**

Proving failed

## Problem 517

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 517: Given a point  $H_a$ , a point  $H$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 518

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 518: Given a point  $H_a$ , a point  $H$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 519

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 519: Given a point  $H_a$ , a point  $H$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 520

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 520: Given a point  $H_a$ , a point  $T_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 521

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 521: Given a point  $H_a$ , a point  $T_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 522

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 522: Given a point  $H_a$ , a point  $T_a$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_a$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $T_a$  are not the same;
2. Using the point  $T_a$  and the point  $I$ , construct a line  $s_a$  (rule W02); % DET: points  $T_a$  and  $I$  are not the same;
3. Using the point  $H_a$  and the line  $a$ , construct a line  $h_a$  (rule W10b); ;
4. Using the line  $h_a$  and the line  $s_a$ , construct a point  $A$  (rule W03); % NDG: lines  $h_a$  and  $s_a$  are not parallel % DET: lines  $h_a$  and  $s_a$  are not the same;
5. Using the point  $I$  and the line  $a$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $a$ ;
6. Using the circle  $k(I, P_a)$ , the point  $A$  and the point  $I$ , construct a line  $c$  and a line  $b$  (rule W12); % NDG: point  $A$  is outside the circle  $k(I, P_a)$ ;
7. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same;
8. Using the line  $a$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $a$  and  $b$  are not parallel % DET: lines  $a$  and  $b$  are not the same.

Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; lines  $c$  and  $a$  are not parallel; point  $A$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $a$ ; lines  $h_a$  and  $s_a$  are not parallel.

Determination conditions: lines  $a$  and  $b$  are not the same; lines  $c$  and  $a$  are not the same; lines  $h_a$  and  $s_a$  are not the same; points  $T_a$  and  $I$  are not the same; points  $H_a$  and  $T_a$  are not the same.

Rules used: [W02,W03,W10b,W11,W12]

Lemmas used: [D2,D23,D27,D5,D8,GD01,L5,L59,L60,L61]  
Solving time: 1.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{a} 80 40  
point T_{a} 70.86 40  
point I 74.37 61.15
```

```
color 220 0 0  
fontsize 9
```

```
cmark_r H_{a}  
cmark_rb T_{a}  
cmark_b I  
color 0 0 0  
fontsize 8
```

```
% DET: points H_{a} and T_{a} are not the same  
% Constructing a line a which passes through point H_{a} and point T_{a}  
line a H_{a} T_{a}
```

```
color 200 200 200  
drawline a  
color 0 0 0
```

```
% DET: points T_{a} and I are not the same  
% Constructing a line s_{a} which passes through point T_{a} and point I  
line s_{a} T_{a} I
```

```
color 200 200 200  
drawline s_{a}  
color 0 0 0
```

```
% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}  
perp h_{a} H_{a} a
```

```
color 200 200 200  
drawline h_{a}  
color 0 0 0
```

```
% NDG: lines h_{a} and s_{a} are not parallel% DET: lines h_{a} and s_{a} are not the same  
% Constructing a point A which belongs to line h_{a} and line s_{a}  
intersec A h_{a} s_{a}  
cmark_t A
```



```

% NDG: point I is not incident to the line a
% Constructing a point  $P_{\{G68186\}}$  which is a foot of the point I on the line a
foot  $P_{\{G68186\}}$  I a
cmark_r  $P_{\{G68186\}}$ 
color 200 200 200
drawline I  $P_{\{G68186\}}$ 
color 0 0 0

% Constructing a circle  $k(I, P_{\{a\}})$  whose center is at point I and which passes through point  $P_{\{G68186\}}$ 
circle k(I,  $P_{\{a\}}$ ) I  $P_{\{G68186\}}$ 

color 200 200 200
drawcircle k(I,  $P_{\{a\}}$ )
color 0 0 0

% NDG: point A is outside the circle  $k(I, P_{\{a\}})$ 
% Constructing midpoint  $P_{\{G68420\}}$  of the segment AI
midpoint  $P_{\{G68420\}}$  A I
cmark_r  $P_{\{G68420\}}$ 

% Constructing a circle  $C_{\{G68423\}}$  whose center is at point  $P_{\{G68420\}}$  and which passes through
point A
circle  $C_{\{G68423\}}$   $P_{\{G68420\}}$  A

color 200 200 200
drawcircle  $C_{\{G68423\}}$ 
color 0 0 0

% Constructing points  $P_{\{G68426\}}$  and  $P_{\{G68429\}}$  which are in intersection of  $C_{\{G68423\}}$  and k
( $I, P_{\{a\}}$ )
intersec2  $P_{\{G68426\}}$   $P_{\{G68429\}}$   $C_{\{G68423\}}$  k(I,  $P_{\{a\}}$ )
cmark_r  $P_{\{G68426\}}$ 
cmark_r  $P_{\{G68429\}}$ 

% Constructing a line c which passes through point A and point  $P_{\{G68426\}}$ 
line c A  $P_{\{G68426\}}$ 

color 200 200 200
drawline c
color 0 0 0

% Constructing a line b which passes through point A and point  $P_{\{G68429\}}$ 
line b A  $P_{\{G68429\}}$ 

color 200 200 200
drawline b
color 0 0 0

```

```

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and b are not parallel; lines c and a are not parallel; point
  A is outside the circle k(I,P_{a}); point I is not incident to the line a; lines h_{a} and s_{a}
  are not parallel
% Determination conditions: lines a and b are not the same; lines c and a are not the same; lines h
  _{a} and s_{a} are not the same; points T_{a} and I are not the same; points H_{a} and T_{a}
  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_a = \_H_a$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.096 seconds.

**NDG conditions** Points  $T_a$  and  $A$  are not identical

Line through points  $T_a$  and  $B$  is not perpendicular to line through points  $B$  and  $A$

#### 4.1.2 Proving $T_a = \_T_a$

Proving failed

#### 4.1.3 Proving $I = \_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_a = \_H_a$

Proving failed

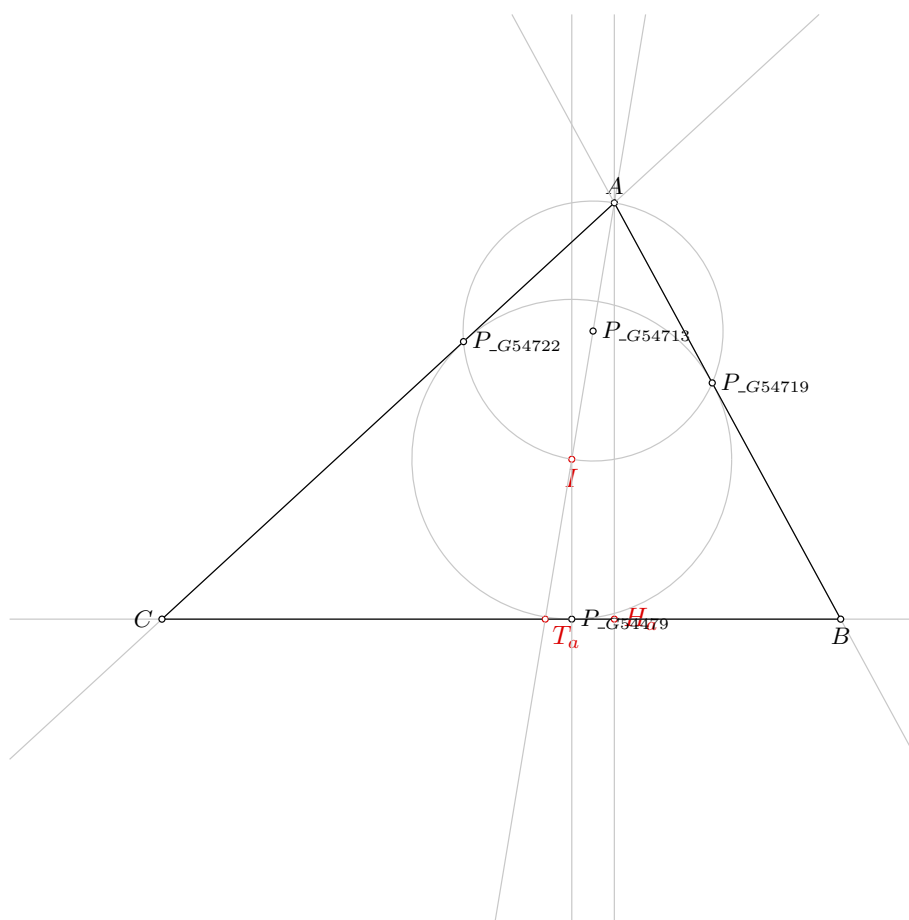


Figure 1: Illustration of the problem 0522

#### 4.2.2 Proving $T_a = \neg T_a$

Proving failed

#### 4.2.3 Proving $I = \neg I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $T_a = \neg T_a$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 18 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I = \neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 187 terms.

**Time Complexity:** Time spent by the prover is 0.290 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $H_a = \neg H_a$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $T_a = \neg T_a$

Proving failed

#### 4.4.3 Proving $I = \neg I$

Proving failed

## Problem 523

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 523: Given a point  $H_a$ , a point  $T_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 524

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 524: Given a point  $H_a$ , a point  $T_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 525

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 525: Given a point  $H_a$ , a point  $T_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 526

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 526: Given a point  $H_b$ , a point  $H_c$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $H$ , construct a line  $h_b$  (rule W02); % DET: points  $H_b$  and  $H$  are not the same;
2. Using the point  $H_c$  and the point  $H$ , construct a line  $h_c$  (rule W02); % DET: points  $H_c$  and  $H$  are not the same;
3. Using the point  $H_b$  and the line  $h_b$ , construct a line  $b$  (rule W10a); ;
4. Using the line  $b$  and the line  $h_c$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $h_c$  are not parallel % DET: lines  $b$  and  $h_c$  are not the same;
5. Using the point  $H_c$  and the line  $h_c$ , construct a line  $c$  (rule W10a); ;
6. Using the line  $c$  and the line  $h_b$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $h_b$  are not parallel % DET: lines  $c$  and  $h_b$  are not the same;
7. Using the line  $b$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $c$  are not parallel % DET: lines  $b$  and  $c$  are not the same.

Non-degenerate conditions: lines  $b$  and  $c$  are not parallel; lines  $c$  and  $h_b$  are not parallel; lines  $b$  and  $h_c$  are not parallel.

Determination conditions: lines  $b$  and  $c$  are not the same; lines  $c$  and  $h_b$  are not the same; lines  $b$  and  $h_c$  are not the same; points  $H_c$  and  $H$  are not the same; points  $H_b$  and  $H$  are not the same.

Rules used: [W02,W03,W10a]

Lemmas used: [D10,D3,D6,D7,D9,GD01,L3]

Solving time: 0.9 seconds.



### 3.2 Construction in GCLC language

```
dim 120 120

point H_{b} 89.36 77.83
point H_{c} 68.91 84.83
point H 80 72.73

color 220 0 0
fontsize 9

cmark_l H_{b}
cmark_rt H_{c}
cmark_rt H
color 0 0 0
fontsize 8

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
perp b H_{b} h_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines b and h_{c} are not parallel% DET: lines b and h_{c} are not the same
% Constructing a point C which belongs to line b and line h_{c}
intersec C b h_{c}
cmark_l C

% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}
```

```

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and h_{b} are not parallel% DET: lines c and h_{b} are not the same
% Constructing a point B which belongs to line c and line h_{b}
intersec B c h_{b}
cmark_b B

% NDG: lines b and c are not parallel% DET: lines b and c are not the same
% Constructing a point A which belongs to line b and line c
intersec A b c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and c are not parallel; lines c and h_{b} are not parallel;
% lines b and h_{c} are not parallel
% Determination conditions: lines b and c are not the same; lines c and h_{b} are not the same;
% lines b and h_{c} are not the same; points H_{c} and H are not the same; points H_{b} and H are
% not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_b = H_b$

Proving failed

#### 4.1.2 Proving $H_c = H_c$

Proving failed

#### 4.1.3 Proving $H = H$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 541 terms.

**Time Complexity:** Time spent by the prover is 4.903 seconds.

**NDG conditions** Points  $H_c$  and  $H$  are not identical

Points  $H_c$  and  $H$  are not identical

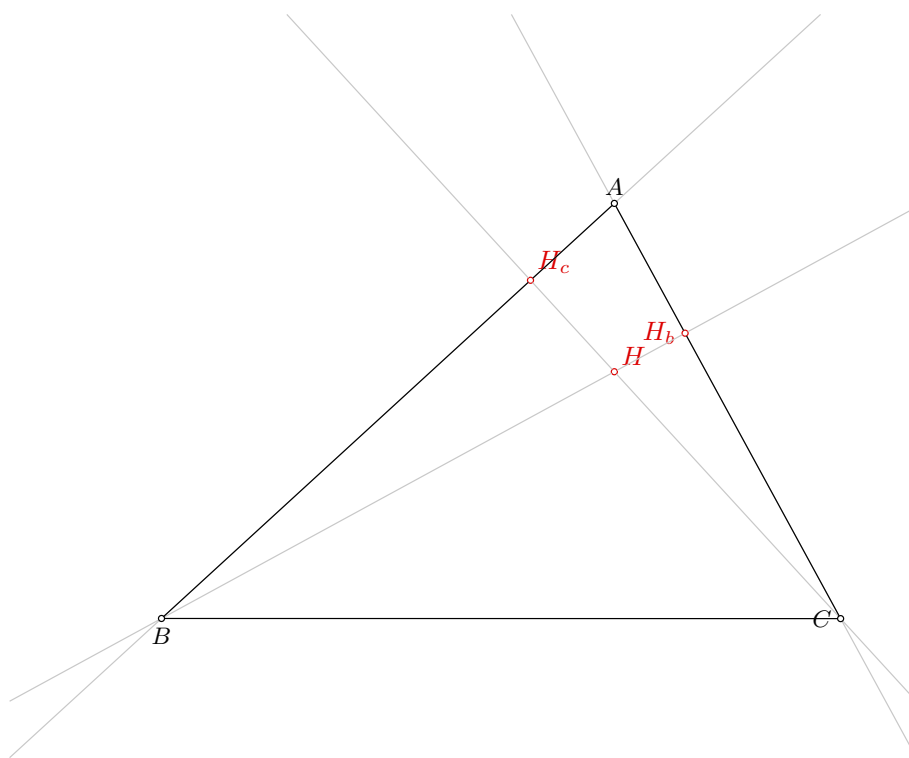


Figure 1: Illustration of the problem 0526

Points  $H_c$  and  $H$  are not identical

Points  $H_c$  and  $H$  are not identical

Points  $H_b$  and  $H_c$  are not identical

Points  $A$ ,  $B$  and  $C$  are not collinear

Line through points  $H_b$  and  $B$  is not perpendicular to line through points  $B$  and  $H_c$

## 4.2 GCLC - Area method

### 4.2.1 Proving $H_b = H_b$

Proving failed

### 4.2.2 Proving $H_c = H_c$

Proving failed

### 4.2.3 Proving $H = H$

Proving failed

## 4.3 GCLC - Wu method

### 4.3.1 Proving $H_b = H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 980 terms.

**Time Complexity:** Time spent by the prover is 4.150 seconds. There are no ndg conditions.

### 4.3.2 Proving $H_c = H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 716 terms.

**Time Complexity:** Time spent by the prover is 2.550 seconds. There are no ndg conditions.

### 4.3.3 Proving $H = H$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $H_b = H_b$

Proving failed

### 4.4.2 Proving $H_c = H_c$

Proving failed

### 4.4.3 Proving $H = H$

Proving failed

# Problem 527

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 527: Given a point  $H_b$ , a point  $H_c$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 528

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 528: Given a point  $H_b$ , a point  $H_c$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $H_c$ , construct a line  $H_bH_c$  (rule W02); % DET: points  $H_b$  and  $H_c$  are not the same;
2. Using the point  $H_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $T_b$  are not the same;
3. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
4. Using the line  $b$ , the line  $H_bH_c$ , the point  $T_b$ , the point  $H_b$  and the point  $H_c$ , construct a line  $H_aH_b$  (rule W18); % NDG: points  $T_b$  and  $H_b$  are not the same; points  $H_b$  and  $H_c$  are not the same % DET: points  $H_b$  and  $T_b$  are not the same;
5. Using the point  $T_b$ , the point  $H_b$  and the point  $H_c$ , construct a circle  $circle[T_b, H_c, angle[s[b]][c]]$  (rule W20); % NDG: points  $T_b$  and  $H_b$  are not the same; points  $H_b$  and  $H_c$  are not the same; points  $T_b$  and  $H_c$  are not the same;
6. Using the circle  $circle[T_b, H_c, angle[s[b]][c]]$  and the line  $h_b$ , construct a point  $B_{ca1}$  and a point  $B$  (rule W04); % NDG: line  $h_b$  and circle  $circle[T_b, H_c, angle[s[b]][c]]$  intersect;
7. Using the point  $B$  and the point  $H_c$ , construct a line  $c$  (rule W02); % DET: points  $B$  and  $H_c$  are not the same;
8. Using the line  $b$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $b$  and  $c$  are not parallel % DET: lines  $b$  and  $c$  are not the same;
9. Using the point  $A$  and the point  $B$ , construct a point  $M_c$  (rule W01); ;
10. Using the point  $H_b$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $H_b$  and  $M_c$  are not the same;

11. Using the circle  $k(M_c, A)$ , the line  $H_a H_b$ , the point  $M_c$  and the point  $H_b$ , construct a point  $H_a$  (rule W05); % NDG: line  $H_a H_b$  and circle  $k(M_c, A)$  intersect % DET: points  $H_b$  and  $H_a$  must be different;
12. Using the point  $B$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $H_a$  are not the same;
13. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; line  $H_a H_b$  and circle  $k(M_c, A)$  intersect; points  $H_b$  and  $M_c$  are not the same; lines  $b$  and  $c$  are not parallel; line  $h_b$  and circle  $circle[T_b, H_c, angle[s[b]][c]]$  intersect; points  $T_b$  and  $H_b$  are not the same; points  $H_b$  and  $H_c$  are not the same; points  $T_b$  and  $H_c$  are not the same; points  $T_b$  and  $H_b$  are not the same; points  $H_b$  and  $H_c$  are not the same.

Determination conditions: lines  $b$  and  $a$  are not the same; points  $B$  and  $H_a$  are not the same; points  $H_b$  and  $H_a$  must be different; lines  $b$  and  $c$  are not the same; points  $B$  and  $H_c$  are not the same; points  $H_b$  and  $T_b$  are not the same; points  $H_b$  and  $T_b$  are not the same; points  $H_b$  and  $H_c$  are not the same.

Rules used: [W01,W02,W03,W04,W05,W06,W10b,W18,W20]

Lemmas used: [D20,D24,D43,D5,D6,D7,D9,GD01,GD02,GL12,L101,L108,L120,L41,L42,L82]

Solving time: 9.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{b} 89.36 77.83
point H_{c} 68.91 84.83
point T_{b} 94.25 68.88
```

```
color 220 0 0
fontsize 9
```

```
cmark_l H_{b}
cmark_rt H_{c}
cmark_t T_{b}
color 0 0 0
fontsize 8
```

```
% DET: points H_{b} and H_{c} are not the same
% Constructing a line H_{b}H_{c} which passes through point H_{b} and point H_{c}
line H_{b}H_{c} H_{b} H_{c}
```

```
color 200 200 200
drawline H_{b}H_{c}
color 0 0 0
```

```
% DET: points H_{b} and T_{b} are not the same
```

```
% Constructing a line b which passes through point H_{b} and point T_{b}
line b H_{b} T_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b
```

```
color 200 200 200
drawline h_{b}
color 0 0 0
```

```
% NDG: points T_{b} and H_{b} are not the same; points H_{b} and H_{c} are not the same% DET:
points H_{b} and T_{b} are not the same
% Constructing an angle V[_G82072] which is equal to the angle T_{b}H_{b}H_{c}
angle_o V[_G82072] T_{b} H_{b} H_{c}
```

```
% Calculating value angle[_G82151] using formula  $-1/(\text{pow}(2,0))*V[_G82072]-0/(\text{pow}(2,0))*180$ 
expression angle[_G82151] {  $-1/(\text{pow}(2,0))*V[_G82072]-0/(\text{pow}(2,0))*180$  }
```

```
% Constructing a point P_{\_G82148} which is an image of the point T_{b} in a rotation around the
point H_{b} for the angle  $-1/(\text{pow}(2,0))*V[_G82072]-0/(\text{pow}(2,0))*180$ 
rotate P_{\_G82148} H_{b} angle[_G82151] T_{b}
cmark_r P_{\_G82148}
color 200 200 200
drawarc_p H_{b} T_{b} angle[_G82151]
color 0 0 0
```

```
% Constructing a line H_{a}H_{b} which passes through point H_{b} and point P_{\_G82148}
line H_{a}H_{b} H_{b} P_{\_G82148}
```

```
color 200 200 200
drawline H_{a}H_{b}
color 0 0 0
```

```
% NDG: points T_{b} and H_{b} are not the same; points H_{b} and H_{c} are not the same; points T_{b}
b} and H_{c} are not the same
% Constructing an angle angle[_G82635] which is equal to the angle T_{b}H_{b}H_{c}
angle_o angle[_G82635] T_{b} H_{b} H_{c}
```

```
% Calculating value angle[_G82646] using formula  $90-(1/\text{pow}(2,1))*\text{angle}[_G82635]+0/\text{pow}(2,0)*180$ 
expression angle[_G82646] {  $90-(1/\text{pow}(2,1))*\text{angle}[_G82635]+0/\text{pow}(2,0)*180$  }
```



```

% Constructing a line L_{\_G82717} which passes through point T_{b} and point H_{c}
line L_{\_G82717} T_{b} H_{c}

color 200 200 200
drawline L_{\_G82717}
color 0 0 0

% Constructing bisector L_{\_G82720} of the segment T_{b}H_{c}
med L_{\_G82720} T_{b} H_{c}

color 200 200 200
drawline L_{\_G82720}
color 0 0 0

color 200 200 200
drawsegment T_{b} H_{c}
color 0 0 0

% Constructing a point P_{\_G82723} which is an image of the point H_{c} in a rotation around the
    point T_{b} for the angle angle[_G82646]
rotate P_{\_G82723} T_{b} angle[_G82646] H_{c}
cmark_r P_{\_G82723}
color 200 200 200
drawarc_p T_{b} H_{c} angle[_G82646]
color 0 0 0

% Constructing a line L_{\_G82726} which passes through point T_{b} and point P_{\_G82723}
line L_{\_G82726} T_{b} P_{\_G82723}

color 200 200 200
drawline L_{\_G82726}
color 0 0 0

% Constructing a point P_{\_G82729} which belongs to line L_{\_G82720} and line L_{\_G82726}
intersec P_{\_G82729} L_{\_G82720} L_{\_G82726}
cmark_r P_{\_G82729}

% Constructing a circle circle[T_{b},H_{c},angle[s[b]] [c]] whose center is at point P_{\_G82729}
    and which passes through point T_{b}
circle circle[T_{b},H_{c},angle[s[b]] [c]] P_{\_G82729} T_{b}

color 200 200 200
drawcircle circle[T_{b},H_{c},angle[s[b]] [c]]
color 0 0 0

% NDG: line h_{b} and circle circle[T_{b},H_{c},angle[s[b]] [c]] intersect
% Constructing points B_{ca1} and B which are in intersection of circle[T_{b},H_{c},angle[s[b]] [c]]
    and h_{b}
intersec2 B_{ca1} B circle[T_{b},H_{c},angle[s[b]] [c]] h_{b}
cmark_r B_{ca1}
cmark_b B

```

```

% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines b and c are not parallel% DET: lines b and c are not the same
% Constructing a point A which belongs to line b and line c
intersec A b c
cmark_t A

% Constructing a point M_{c} such that AM_{c}/AB=0.5
towards M_{c} A B 0.5
cmark_lt M_{c}
color 200 200 200
drawsegment A B
color 0 0 0

% NDG: points H_{b} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{b}
circle k(M_{c},A) M_{c} H_{b}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line H_{a}H_{b} and circle k(M_{c},A) intersect% DET: points H_{b} and H_{a} must be
different
% Constructing a point P_{\G84227} which is a foot of the point M_{c} on the line H_{a}H_{b}
foot P_{\G84227} M_{c} H_{a}H_{b}
cmark_r P_{\G84227}
color 200 200 200
drawline M_{c} P_{\G84227}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H_{b} in the symmetry to point/line P_{\G84227}
sim H_{a} P_{\G84227} H_{b}
cmark_r H_{a}

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

```

```

color 200 200 200
drawline a
color 0 0 0

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and a are not parallel; line  $H_{\{a\}}H_{\{b\}}$  and circle  $k(M_{\{c\}},A)$ 
% intersect; points  $H_{\{b\}}$  and  $M_{\{c\}}$  are not the same; lines b and c are not parallel; line  $h_{\{b\}}$ 
% and circle  $\text{circle}[T_{\{b\}},H_{\{c\}},\text{angle}[s[b]][c]]$  intersect; points  $T_{\{b\}}$  and  $H_{\{b\}}$  are not the
% same; points  $H_{\{b\}}$  and  $H_{\{c\}}$  are not the same; points  $T_{\{b\}}$  and  $H_{\{c\}}$  are not the same; points
%  $T_{\{b\}}$  and  $H_{\{b\}}$  are not the same; points  $H_{\{b\}}$  and  $H_{\{c\}}$  are not the same
% Determination conditions: lines b and a are not the same; points B and  $H_{\{a\}}$  are not the same;
% points  $H_{\{b\}}$  and  $H_{\{a\}}$  must be different; lines b and c are not the same; points B and  $H_{\{c\}}$ 
% are not the same; points  $H_{\{b\}}$  and  $T_{\{b\}}$  are not the same; points  $H_{\{b\}}$  and  $T_{\{b\}}$  are not the
% same; points  $H_{\{b\}}$  and  $H_{\{c\}}$  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_b = \_H_b$

Proving failed

#### 4.1.2 Proving $H_c = \_H_c$

Proving failed

#### 4.1.3 Proving $T_b = \_T_b$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_b = \_H_b$

Proving failed

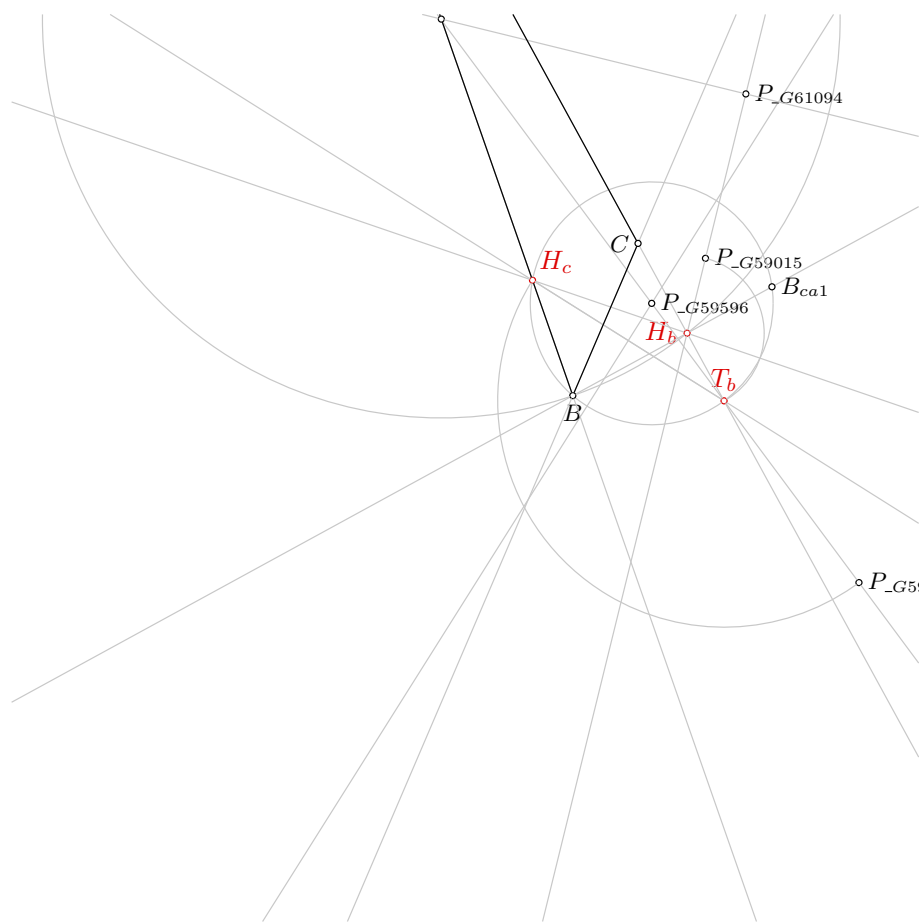


Figure 1: Illustration of the problem 0528

#### **4.2.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.2.3 Proving $T_b = \neg T_b$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $H_b = \neg H_b$**

Proving failed

#### **4.3.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.3.3 Proving $T_b = \neg T_b$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $H_b = \neg H_b$**

Proving failed

#### **4.4.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.4.3 Proving $T_b = \neg T_b$**

Proving failed

# Problem 529

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 529: Given a point  $H_b$ , a point  $H_c$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $H_c$ , construct a line  $H_bH_c$  (rule W02); % DET: points  $H_b$  and  $H_c$  are not the same;
2. Using the point  $H_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $T_c$  are not the same;
3. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
4. Using the point  $H_b$ , the point  $H_c$ , the point  $T_c$ , the line  $c$  and the line  $H_bH_c$ , construct a line  $H_cH_a$  (rule W17); % NDG: points  $H_c$  and  $T_c$  are not the same; points  $H_b$  and  $H_c$  are not the same % DET: points  $H_c$  and  $T_c$  are not the same;
5. Using the point  $H_b$ , the point  $H_c$  and the point  $T_c$ , construct a circle  $circle[T_c, H_b, angle[b][s[c]]]$  (rule W20); % NDG: points  $H_b$  and  $H_c$  are not the same; points  $H_c$  and  $T_c$  are not the same; points  $T_c$  and  $H_b$  are not the same;
6. Using the circle  $circle[T_c, H_b, angle[b][s[c]]]$  and the line  $h_c$ , construct a point  $C_{ca2}$  and a point  $C$  (rule W04); % NDG: line  $h_c$  and circle  $circle[T_c, H_b, angle[b][s[c]]]$  intersect;
7. Using the point  $C$  and the point  $H_b$ , construct a line  $b$  (rule W02); % DET: points  $C$  and  $H_b$  are not the same;
8. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same;
9. Using the point  $A$  and the point  $C$ , construct a point  $M_b$  (rule W01); ;
10. Using the point  $H_c$  and the point  $M_b$ , construct a circle  $k(M_b, C)$  (rule W06); % NDG: points  $H_c$  and  $M_b$  are not the same;

11. Using the circle  $k(M_b, C)$ , the line  $H_c H_a$ , the point  $M_b$  and the point  $H_c$ , construct a point  $H_a$  (rule W05); % NDG: line  $H_c H_a$  and circle  $k(M_b, C)$  intersect % DET: points  $H_c$  and  $H_a$  must be different;
12. Using the point  $C$  and the point  $H_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $H_a$  are not the same;
13. Using the line  $c$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $c$  and  $a$  are not parallel % DET: lines  $c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; line  $H_c H_a$  and circle  $k(M_b, C)$  intersect; points  $H_c$  and  $M_b$  are not the same; lines  $c$  and  $b$  are not parallel; line  $h_c$  and circle  $circle[T_c, H_b, angle[b][s[c]]]$  intersect; points  $H_b$  and  $H_c$  are not the same; points  $H_c$  and  $T_c$  are not the same; points  $T_c$  and  $H_b$  are not the same; points  $H_c$  and  $T_c$  are not the same; points  $H_b$  and  $H_c$  are not the same.

Determination conditions: lines  $c$  and  $a$  are not the same; points  $C$  and  $H_a$  are not the same; points  $H_c$  and  $H_a$  must be different; lines  $c$  and  $b$  are not the same; points  $C$  and  $H_b$  are not the same; points  $H_c$  and  $T_c$  are not the same; points  $H_c$  and  $T_c$  are not the same; points  $H_b$  and  $H_c$  are not the same.

Rules used: [W01,W02,W03,W04,W05,W06,W10b,W17,W20]

Lemmas used: [D10,D22,D25,D46,D5,D6,D7,GD01,GD02,GL04,GL12,L102,L109,L121,L44,L45,L85]

Solving time: 8.3 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{b} 89.36 77.83
point H_{c} 68.91 84.83
point T_{c} 55.38 72.43
```

```
color 220 0 0
fontsize 9
```

```
cmark_l H_{b}
cmark_rt H_{c}
cmark_rt T_{c}
color 0 0 0
fontsize 8
```

```
% DET: points H_{b} and H_{c} are not the same
% Constructing a line H_{b}H_{c} which passes through point H_{b} and point H_{c}
line H_{b}H_{c} H_{b} H_{c}
```

```
color 200 200 200
drawline H_{b}H_{c}
color 0 0 0
```

```
% DET: points H_{c} and T_{c} are not the same
```

```

% Constructing a line c which passes through point H_{c} and point T_{c}
line c H_{c} T_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points H_{c} and T_{c} are not the same; points H_{b} and H_{c} are not the same% DET:
% points H_{c} and T_{c} are not the same
% Constructing an angle V[_G125283] which is equal to the angle H_{b}H_{c}T_{c}
angle_o V[_G125283] H_{b} H_{c} T_{c}

% Calculating value angle[_G125362] using formula  $1/\text{pow}(2,0)*V[_G125283]+0/\text{pow}(2,0)*180$ 
expression angle[_G125362] { 1/pow(2,0)*V[_G125283]+0/pow(2,0)*180 }

% Constructing a point P_{\_G125359} which is an image of the point T_{c} in a rotation around the
% point H_{c} for the angle  $1/\text{pow}(2,0)*V[_G125283]+0/\text{pow}(2,0)*180$ 
rotate P_{\_G125359} H_{c} angle[_G125362] T_{c}
cmark_r P_{\_G125359}
color 200 200 200
drawarc_p H_{c} T_{c} angle[_G125362]
color 0 0 0

% Constructing a line H_{c}H_{a} which passes through point H_{c} and point P_{\_G125359}
line H_{c}H_{a} H_{c} P_{\_G125359}

color 200 200 200
drawline H_{c}H_{a}
color 0 0 0

% NDG: points H_{b} and H_{c} are not the same; points H_{c} and T_{c} are not the same; points T_{c}
% and H_{b} are not the same
% Constructing an angle angle[_G125845] which is equal to the angle H_{b}H_{c}T_{c}
angle_o angle[_G125845] H_{b} H_{c} T_{c}

% Calculating value angle[_G125857] using formula  $90-(1/\text{pow}(2,1)*\text{angle}[_G125845]+0/\text{pow}(2,0)*180)$ 
expression angle[_G125857] { 90-(1/pow(2,1)*angle[_G125845]+0/pow(2,0)*180) }

```



```

% Constructing a line  $L_{\{c\}}$  which passes through point  $T_{\{c\}}$  and point  $H_{\{b\}}$ 
line L_{\_G125928} T_{\{c\}} H_{\{b\}}

color 200 200 200
drawline L_{\_G125928}
color 0 0 0

% Constructing bisector  $L_{\{c\}}$  of the segment  $T_{\{c\}}H_{\{b\}}$ 
med L_{\_G125931} T_{\{c\}} H_{\{b\}}

color 200 200 200
drawline L_{\_G125931}
color 0 0 0

color 200 200 200
drawsegment T_{\{c\}} H_{\{b\}}
color 0 0 0

% Constructing a point  $P_{\{c\}}$  which is an image of the point  $H_{\{b\}}$  in a rotation around the
    point  $T_{\{c\}}$  for the angle  $\angle_{G125857}$ 
rotate P_{\_G125934} T_{\{c\}} angle[_G125857] H_{\{b\}}
cmark_r P_{\_G125934}
color 200 200 200
drawarc_p T_{\{c\}} H_{\{b\}} angle[_G125857]
color 0 0 0

% Constructing a line  $L_{\{c\}}$  which passes through point  $T_{\{c\}}$  and point  $P_{\{c\}}$ 
line L_{\_G125937} T_{\{c\}} P_{\_G125934}

color 200 200 200
drawline L_{\_G125937}
color 0 0 0

% Constructing a point  $P_{\{c\}}$  which belongs to line  $L_{\{c\}}$  and line  $L_{\{c\}}$ 
intersec P_{\_G125940} L_{\_G125931} L_{\_G125937}
cmark_r P_{\_G125940}

% Constructing a circle  $circle[T_{\{c\}}, H_{\{b\}}, \angle[b][s[c]]]$  whose center is at point  $P_{\{c\}}$ 
    and which passes through point  $T_{\{c\}}$ 
circle circle[T_{\{c\}}, H_{\{b\}}, angle[b][s[c]]] P_{\_G125940} T_{\{c\}}

color 200 200 200
drawcircle circle[T_{\{c\}}, H_{\{b\}}, angle[b][s[c]]]
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $circle[T_{\{c\}}, H_{\{b\}}, \angle[b][s[c]]]$  intersect
% Constructing points  $C_{\{ca2\}}$  and  $C$  which are in intersection of  $circle[T_{\{c\}}, H_{\{b\}}, \angle[b][s[c]]]$ 
    and  $h_{\{c\}}$ 
intersec2 C_{\{ca2\}} C circle[T_{\{c\}}, H_{\{b\}}, angle[b][s[c]]] h_{\{c\}}
cmark_r C_{\{ca2\}}
cmark_l C

```

```

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A

% Constructing a point M_{b} such that AM_{b}/AC=0.5
towards M_{b} A C 0.5
cmark_lt M_{b}
color 200 200 200
drawsegment A C
color 0 0 0

% NDG: points H_{c} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{c}
circle k(M_{b},C) M_{b} H_{c}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line H_{c}H_{a} and circle k(M_{b},C) intersect% DET: points H_{c} and H_{a} must be
different
% Constructing a point P_{\_G127445} which is a foot of the point M_{b} on the line H_{c}H_{a}
foot P_{\_G127445} M_{b} H_{c}H_{a}
cmark_r P_{\_G127445}
color 200 200 200
drawline M_{b} P_{\_G127445}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H_{c} in the symmetry to point/line P_{\_G127445}
sim H_{a} P_{\_G127445} H_{c}
cmark_r H_{a}

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

```

```

color 200 200 200
drawline a
color 0 0 0

```

```

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions: lines c and a are not parallel; line H_{c}H_{a} and circle k(M_{b},C)
% intersect; points H_{c} and M_{b} are not the same; lines c and b are not parallel; line h_{c}
% and circle circle[T_{c},H_{b},angle[b][s[c]]] intersect; points H_{b} and H_{c} are not the
% same; points H_{c} and T_{c} are not the same; points T_{c} and H_{b} are not the same; points
% H_{c} and T_{c} are not the same; points H_{b} and H_{c} are not the same
% Determination conditions: lines c and a are not the same; points C and H_{a} are not the same;
% points H_{c} and H_{a} must be different; lines c and b are not the same; points C and H_{b}
% are not the same; points H_{c} and T_{c} are not the same; points H_{c} and T_{c} are not the
% same; points H_{b} and H_{c} are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_b = H_b$

Proving failed

#### 4.1.2 Proving $H_c = H_c$

Proving failed

#### 4.1.3 Proving $T_c = T_c$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_b = H_b$

Proving failed

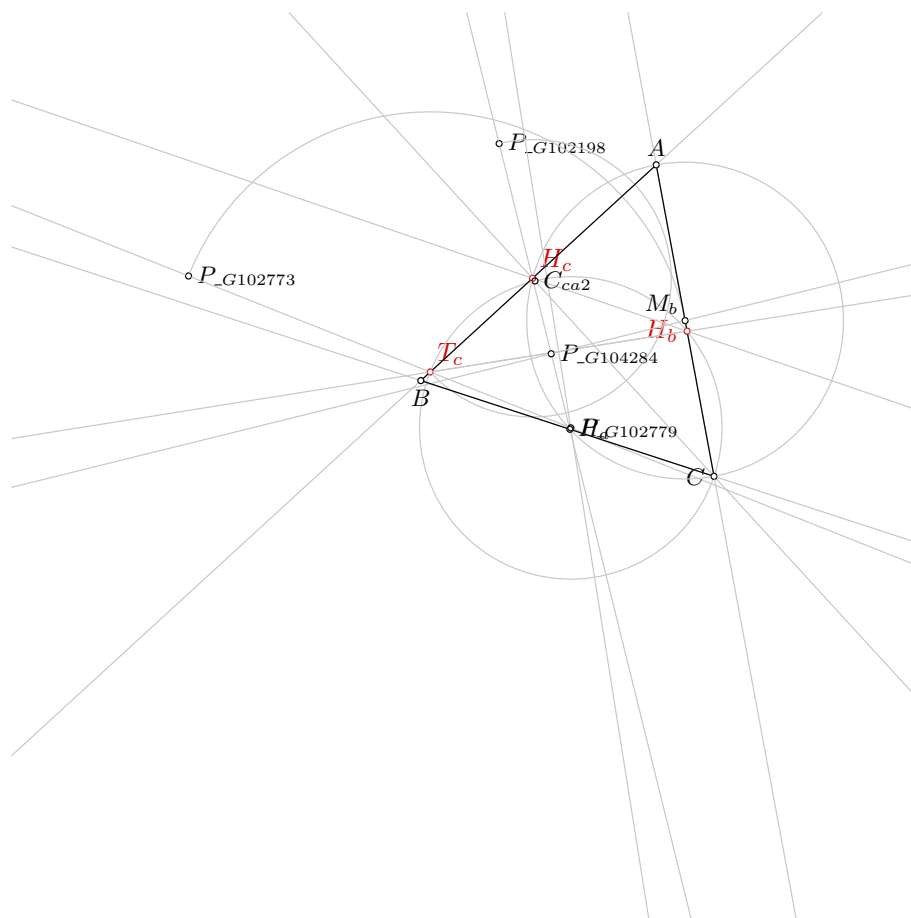


Figure 1: Illustration of the problem 0529

#### **4.2.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.2.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $H_b = \neg H_b$**

Proving failed

#### **4.3.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.3.3 Proving $T_c = \neg T_c$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $H_b = \neg H_b$**

Proving failed

#### **4.4.2 Proving $H_c = \neg H_c$**

Proving failed

#### **4.4.3 Proving $T_c = \neg T_c$**

Proving failed

## Problem 530

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 530: Given a point  $H_b$ , a point  $H_c$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 531

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 531: Given a point  $H_b$ , a point  $H$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 532

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 532: Given a point  $H_b$ , a point  $T_b$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $T_b$  are not the same;
2. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
3. Choose freely a point  $H$  on the line  $h_b$  (rule WOnline2);
4. Choose freely a point  $A$  on the line  $b$  (rule WOnline1) ;
5. Using the point  $A$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H$  are not the same;
6. Choose freely a point  $B$  on the line  $h_b$  (rule WOnline1) ;
7. Using the point  $B$  and the point  $A$ , construct a point  $M_c$  (rule W01); ;
8. Using the point  $B$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $B$  and  $M_c$  are not the same;
9. Using the circle  $k(M_c, A)$ , the line  $h_a$ , the point  $M_c$  and the point  $A$ , construct a point  $H_a$  (rule W05); % NDG: line  $h_a$  and circle  $k(M_c, A)$  intersect % DET: points  $A$  and  $H_a$  must be different;
10. Using the point  $H_a$  and the point  $B$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $B$  are not the same;
11. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.



Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; line  $h_a$  and circle  $k(M_c, A)$  intersect; points  $B$  and  $M_c$  are not the same.

Determination conditions: lines  $b$  and  $a$  are not the same; points  $H_a$  and  $B$  are not the same; points  $A$  and  $H_a$  must be different; points  $A$  and  $H$  are not the same; points  $H_b$  and  $T_b$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W10b,WOnline1,WOnline2]

Lemmas used: [D20,D24,D3,D5,D6,D8,D9,GD01,GD02,GL04,L40,L42]

Solving time: 88.2 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{b} 89.36 77.83
```

```
point T_{b} 94.25 68.88
```

```
point H 80 72.73
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_l H_{b}
```

```
cmark_t T_{b}
```

```
cmark_rt H
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{b} and T_{b} are not the same
```

```
% Constructing a line b which passes through point H_{b} and point T_{b}
```

```
line b H_{b} T_{b}
```

```
color 200 200 200
```

```
drawline b
```

```
color 0 0 0
```

```
% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
```

```
perp h_{b} H_{b} b
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```
% Generating random value V[_G87448]
```

```
random V[_G87448]
```

```
% Calculating value V[_G87469] using formula V[_G87448]*20
```

```
expression V[_G87469] { V[_G87448]*20 }
```

```

% Constructing a point H which is a point for which holds  $H_{\{b\}}H = V[_{G87469}]$  and angle  $T_{\{b\}}H_{\{b\}}H = 90$ 
turtle H  $T_{\{b\}}$   $H_{\{b\}}$  90  $V[_{G87469}]$ 
cmark_rt H

```

```

% Choosing randomly a point A on the line  $H_{\{b\}}T_{\{b\}}$ 
online A  $H_{\{b\}}$   $T_{\{b\}}$ 
cmark_t A
color 200 200 200
drawline  $H_{\{b\}}$   $T_{\{b\}}$ 
color 0 0 0

```

```

% DET: points A and H are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point A and point H
line  $h_{\{a\}}$  A H

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

```

```

% Choosing randomly a point B on the line  $HH_{\{b\}}$ 
online B H  $H_{\{b\}}$ 
cmark_b B
color 200 200 200
drawline H  $H_{\{b\}}$ 
color 0 0 0

```

```

% Constructing a point  $M_{\{c\}}$  such that  $BM_{\{c\}}/BA=0.5$ 
towards  $M_{\{c\}}$  B A 0.5
cmark_lt  $M_{\{c\}}$ 
color 200 200 200
drawsegment B A
color 0 0 0

```

```

% NDG: points B and  $M_{\{c\}}$  are not the same
% Constructing a circle  $k(M_{\{c\}},A)$  whose center is at point  $M_{\{c\}}$  and which passes through point B
circle  $k(M_{\{c\}},A)$   $M_{\{c\}}$  B

color 200 200 200
drawcircle  $k(M_{\{c\}},A)$ 
color 0 0 0

```

```

% NDG: line  $h_{\{a\}}$  and circle  $k(M_{\{c\}},A)$  intersect% DET: points A and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{G88055\}}$  which is a foot of the point  $M_{\{c\}}$  on the line  $h_{\{a\}}$ 

```

```

foot P_{\_G88055} M_{c} h_{a}
cmark_r P_{\_G88055}
color 200 200 200
drawline M_{c} P_{\_G88055}
color 0 0 0

% Constructing a point H_{a} which is an image of the point A in the symmetry to point/line P_{\_G
88055}
sim H_{a} P_{\_G88055} A
cmark_r H_{a}

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and a are not parallel; line h_{a} and circle k(M_{c},A)
intersect; points B and M_{c} are not the same
% Determination conditions: lines b and a are not the same; points H_{a} and B are not the same;
points A and H_{a} must be different; points A and H are not the same; points H_{b} and T_{b}
are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_b = H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 4 terms.

**Time Complexity:** Time spent by the prover is 0.156 seconds.

**NDG conditions** Points  $H_b$  and  $A$  are not identical

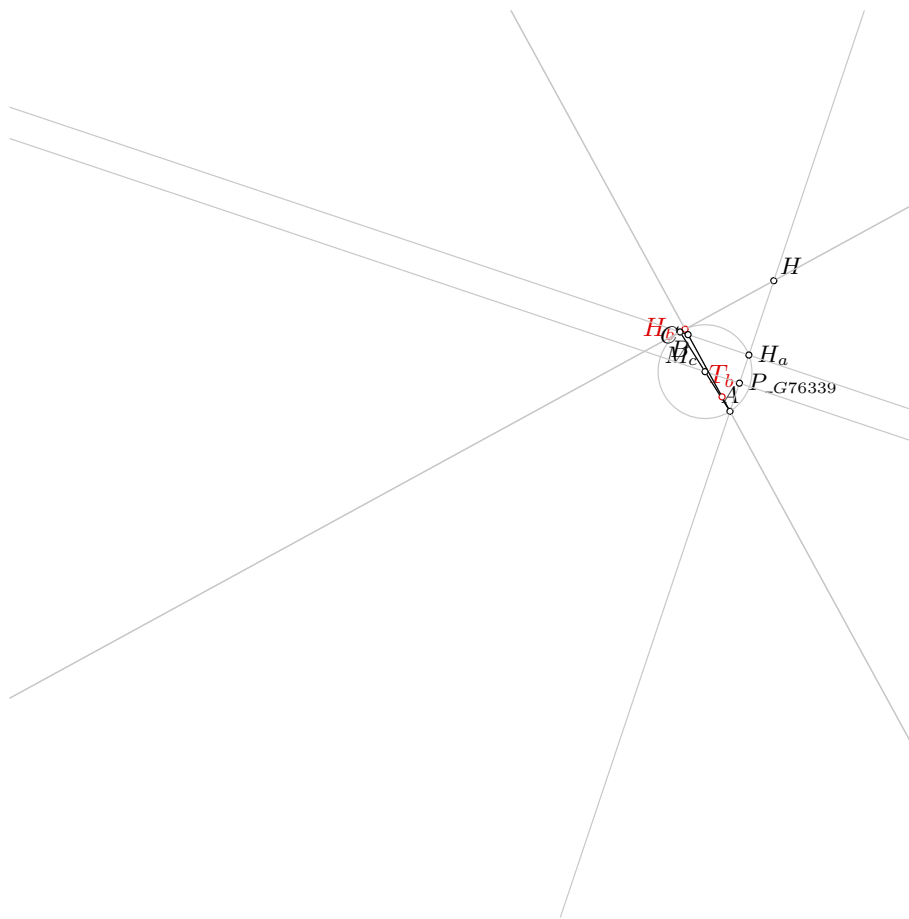


Figure 1: Illustration of the problem 0532

Points  $H_b$  and  $A$  are not identical

Line through points  $H_a$  and  $B$  is not perpendicular to line through points  $B$  and  $H_b$

Line through points  $A$  and  $B$  is not perpendicular to line through points  $B$  and  $C$

#### **4.1.2 Proving $T_b = \neg T_b$**

Proving failed

#### **4.1.3 Proving $H = \neg H$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $H_b = \neg H_b$**

Proving failed

#### **4.2.2 Proving $T_b = \neg T_b$**

Proving failed

#### **4.2.3 Proving $H = \neg H$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $H_b = \neg H_b$**

Proving failed

#### **4.3.2 Proving $T_b = \neg T_b$**

Proving failed

#### **4.3.3 Proving $H = \neg H$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $H_b = \neg H_b$**

Proving failed

#### **4.4.2 Proving $T_b = \neg T_b$**

Proving failed

#### **4.4.3 Proving $H = \neg H$**

Proving failed

# Problem 533

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 533: Given a point  $H_b$ , a point  $H$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 534

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 534: Given a point  $H_b$ , a point  $H$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 535

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 535: Given a point  $H_b$ , a point  $T_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.



## Problem 536

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 536: Given a point  $H_b$ , a point  $T_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 537

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 537: Given a point  $H_b$ , a point  $T_a$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 538

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 538: Given a point  $H_b$ , a point  $T_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 539

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 539: Given a point  $H_b$ , a point  $T_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_b$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $H_b$  and  $T_b$  are not the same;
2. Using the point  $T_b$  and the point  $I$ , construct a line  $s_b$  (rule W02); % DET: points  $T_b$  and  $I$  are not the same;
3. Using the point  $H_b$  and the line  $b$ , construct a line  $h_b$  (rule W10b); ;
4. Using the line  $h_b$  and the line  $s_b$ , construct a point  $B$  (rule W03); % NDG: lines  $h_b$  and  $s_b$  are not parallel % DET: lines  $h_b$  and  $s_b$  are not the same;
5. Using the point  $I$  and the line  $b$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $b$ ;
6. Using the circle  $k(I, P_a)$ , the point  $B$  and the point  $I$ , construct a line  $c$  and a line  $a$  (rule W12); % NDG: point  $B$  is outside the circle  $k(I, P_a)$ ;
7. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same;
8. Using the line  $b$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $b$  and  $a$  are not parallel % DET: lines  $b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; lines  $c$  and  $b$  are not parallel; point  $B$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $b$ ; lines  $h_b$  and  $s_b$  are not parallel.

Determination conditions: lines  $b$  and  $a$  are not the same; lines  $c$  and  $b$  are not the same; lines  $h_b$  and  $s_b$  are not the same; points  $T_b$  and  $I$  are not the same; points  $H_b$  and  $T_b$  are not the same.

Rules used: [W02,W03,W10b,W11,W12]

Lemmas used: [D2,D24,D27,D6,D9,GD01,L59,L6,L60,L61]  
Solving time: 1.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point H_{b} 89.36 77.83
point T_{b} 94.25 68.88
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_l H_{b}
cmark_t T_{b}
cmark_b I
color 0 0 0
fontsize 8

% DET: points H_{b} and T_{b} are not the same
% Constructing a line b which passes through point H_{b} and point T_{b}
line b H_{b} T_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points T_{b} and I are not the same
% Constructing a line s_{b} which passes through point T_{b} and point I
line s_{b} T_{b} I

color 200 200 200
drawline s_{b}
color 0 0 0

% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines h_{b} and s_{b} are not parallel% DET: lines h_{b} and s_{b} are not the same
% Constructing a point B which belongs to line h_{b} and line s_{b}
intersec B h_{b} s_{b}
cmark_b B
```

```

% NDG: point I is not incident to the line b
% Constructing a point  $P_{\{G84613\}}$  which is a foot of the point I on the line b
foot  $P_{\{G84613\}}$  I b
cmark_r  $P_{\{G84613\}}$ 
color 200 200 200
drawline I  $P_{\{G84613\}}$ 
color 0 0 0

% Constructing a circle  $k(I, P_{\{a\}})$  whose center is at point I and which passes through point  $P_{\{G84613\}}$ 
circle k(I,  $P_{\{a\}}$ ) I  $P_{\{G84613\}}$ 

color 200 200 200
drawcircle k(I,  $P_{\{a\}}$ )
color 0 0 0

% NDG: point B is outside the circle  $k(I, P_{\{a\}})$ 
% Constructing midpoint  $P_{\{G84847\}}$  of the segment BI
midpoint  $P_{\{G84847\}}$  B I
cmark_r  $P_{\{G84847\}}$ 

% Constructing a circle  $C_{\{G84850\}}$  whose center is at point  $P_{\{G84847\}}$  and which passes through
point B
circle  $C_{\{G84850\}}$   $P_{\{G84847\}}$  B

color 200 200 200
drawcircle  $C_{\{G84850\}}$ 
color 0 0 0

% Constructing points  $P_{\{G84853\}}$  and  $P_{\{G84856\}}$  which are in intersection of  $C_{\{G84850\}}$  and  $k(I, P_{\{a\}})$ 
intersec2  $P_{\{G84853\}}$   $P_{\{G84856\}}$   $C_{\{G84850\}}$  k(I,  $P_{\{a\}}$ )
cmark_r  $P_{\{G84853\}}$ 
cmark_r  $P_{\{G84856\}}$ 

% Constructing a line c which passes through point B and point  $P_{\{G84853\}}$ 
line c B  $P_{\{G84853\}}$ 

color 200 200 200
drawline c
color 0 0 0

% Constructing a line a which passes through point B and point  $P_{\{G84856\}}$ 
line a B  $P_{\{G84856\}}$ 

color 200 200 200
drawline a
color 0 0 0

```

```

% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A

```

```

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions: lines b and a are not parallel; lines c and b are not parallel; point
  B is outside the circle  $k(I, P_{\{a\}})$ ; point I is not incident to the line b; lines  $h_{\{b\}}$  and  $s_{\{b\}}$ 
  are not parallel
% Determination conditions: lines b and a are not the same; lines c and b are not the same; lines  $h_{\{b\}}$ 
  and  $s_{\{b\}}$  are not the same; points  $T_{\{b\}}$  and I are not the same; points  $H_{\{b\}}$  and  $T_{\{b\}}$ 
  are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_b = \_H_b$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.09 seconds.

**NDG conditions** Points  $B$ ,  $T_b$  and  $P_{G78722}$  are not collinear

Line through points  $A$  and  $B$  is not perpendicular to line through points  $B$  and  $C$

#### 4.1.2 Proving $T_b = \_T_b$

Proving failed

#### 4.1.3 Proving $I = \_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_b = \_H_b$

Proving failed

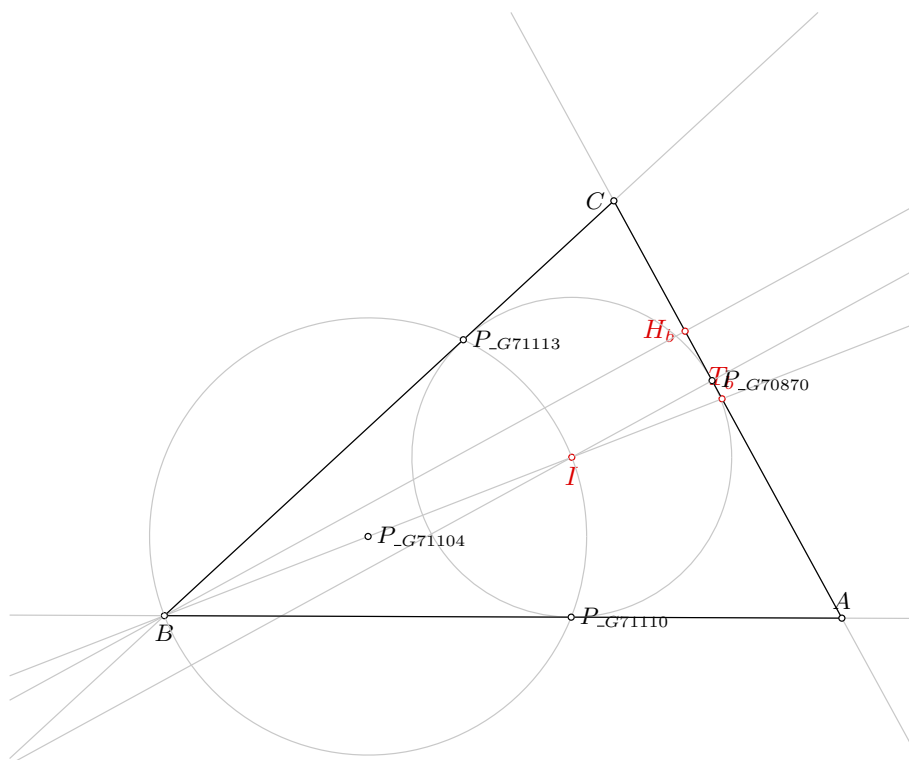


Figure 1: Illustration of the problem 0539



#### 4.2.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.2.3 Proving $I = \neg I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_b = \neg H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $T_b = \neg T_b$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 18 terms.

**Time Complexity:** Time spent by the prover is 0.020 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I = \neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 187 terms.

**Time Complexity:** Time spent by the prover is 0.330 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $H_b = \neg H_b$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.4.3 Proving $I = \neg I$

Proving failed

## Problem 540

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 540: Given a point  $H_b$ , a point  $T_c$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 541

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 541: Given a point  $H_c$ , a point  $H$  and a point  $T_a$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 542

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 542: Given a point  $H_c$ , a point  $H$  and a point  $T_b$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 543

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 543: Given a point  $H_c$ , a point  $T_c$  and a point  $H$ , construct the triangle  $ABC$ .

## 2 Status

Problem is locus dependent.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $T_c$  are not the same;
2. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
3. Choose freely a point  $H$  on the line  $h_c$  (rule WOnline2);
4. Choose freely a point  $A$  on the line  $c$  (rule WOnline1) ;
5. Using the point  $A$  and the point  $H$ , construct a line  $h_a$  (rule W02); % DET: points  $A$  and  $H$  are not the same;
6. Choose freely a point  $B$  on the line  $c$  (rule WOnline1) ;
7. Using the point  $B$  and the point  $A$ , construct a point  $M_c$  (rule W01); ;
8. Using the point  $B$  and the point  $M_c$ , construct a circle  $k(M_c, A)$  (rule W06); % NDG: points  $B$  and  $M_c$  are not the same;
9. Using the circle  $k(M_c, A)$ , the line  $h_a$ , the point  $M_c$  and the point  $A$ , construct a point  $H_a$  (rule W05); % NDG: line  $h_a$  and circle  $k(M_c, A)$  intersect % DET: points  $A$  and  $H_a$  must be different;
10. Using the point  $H_a$  and the point  $B$ , construct a line  $a$  (rule W02); % DET: points  $H_a$  and  $B$  are not the same;
11. Using the line  $h_c$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $h_c$  and  $a$  are not parallel % DET: lines  $h_c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $h_c$  and  $a$  are not parallel; line  $h_a$  and circle  $k(M_c, A)$  intersect; points  $B$  and  $M_c$  are not the same.

Determination conditions: lines  $h_c$  and  $a$  are not the same; points  $H_a$  and  $B$  are not the same; points  $A$  and  $H_a$  must be different; points  $A$  and  $H$  are not the same; points  $H_c$  and  $T_c$  are not the same.

Rules used: [W01,W02,W03,W05,W06,W10b,WOnline1,WOnline2]

Lemmas used: [D10,D20,D25,D3,D5,D7,D8,GD01,GD02,GL04,L40,L42]

Solving time: 87.0 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{c} 68.91 84.83
```

```
point T_{c} 55.38 72.43
```

```
point H 80 72.73
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H_{c}
```

```
cmark_rt T_{c}
```

```
cmark_rt H
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{c} and T_{c} are not the same
```

```
% Constructing a line c which passes through point H_{c} and point T_{c}
```

```
line c H_{c} T_{c}
```

```
color 200 200 200
```

```
drawline c
```

```
color 0 0 0
```

```
% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
```

```
perp h_{c} H_{c} c
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```
% Generating random value V[_G57353]
```

```
random V[_G57353]
```

```
% Calculating value V[_G57374] using formula V[_G57353]*20
```

```
expression V[_G57374] { V[_G57353]*20 }
```

```

% Constructing a point H which is a point for which holds  $H_{\{c\}}H = V[_{G57374}]$  and angle  $T_{\{c\}}H_{\{c\}}H = 90$ 
turtle H  $T_{\{c\}}$   $H_{\{c\}}$  90  $V[_{G57374}]$ 
cmark_rt H

```

```

% Choosing randomly a point A on the line  $H_{\{c\}}T_{\{c\}}$ 
online A  $H_{\{c\}}$   $T_{\{c\}}$ 
cmark_t A
color 200 200 200
drawline  $H_{\{c\}}$   $T_{\{c\}}$ 
color 0 0 0

```

```

% DET: points A and H are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point A and point H
line  $h_{\{a\}}$  A H

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

```

```

% Choosing randomly a point B on the line  $AH_{\{c\}}$ 
online B A  $H_{\{c\}}$ 
cmark_b B
color 200 200 200
drawline A  $H_{\{c\}}$ 
color 0 0 0

```

```

% Constructing a point  $M_{\{c\}}$  such that  $BM_{\{c\}}/BA=0.5$ 
towards  $M_{\{c\}}$  B A 0.5
cmark_lt  $M_{\{c\}}$ 
color 200 200 200
drawsegment B A
color 0 0 0

```

```

% NDG: points B and  $M_{\{c\}}$  are not the same
% Constructing a circle  $k(M_{\{c\}},A)$  whose center is at point  $M_{\{c\}}$  and which passes through point B
circle  $k(M_{\{c\}},A)$   $M_{\{c\}}$  B

color 200 200 200
drawcircle  $k(M_{\{c\}},A)$ 
color 0 0 0

```

```

% NDG: line  $h_{\{a\}}$  and circle  $k(M_{\{c\}},A)$  intersect% DET: points A and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{G57960\}}$  which is a foot of the point  $M_{\{c\}}$  on the line  $h_{\{a\}}$ 

```

```

foot P_{\_G57960} M_{c} h_{a}
cmark_r P_{\_G57960}
color 200 200 200
drawline M_{c} P_{\_G57960}
color 0 0 0

% Constructing a point H_{a} which is an image of the point A in the symmetry to point/line P_{\_G
57960}
sim H_{a} P_{\_G57960} A
cmark_r H_{a}

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% NDG: lines h_{c} and a are not parallel% DET: lines h_{c} and a are not the same
% Constructing a point C which belongs to line h_{c} and line a
intersec C h_{c} a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and a are not parallel; line h_{a} and circle k(M_{c},A)
intersect; points B and M_{c} are not the same
% Determination conditions: lines h_{c} and a are not the same; points H_{a} and B are not the same
; points A and H_{a} must be different; points A and H are not the same; points H_{c} and T_{c}
are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_c = \neg H_c$

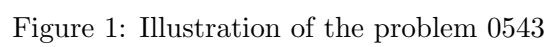
**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.129 seconds.

**NDG conditions** Points  $A$  and  $\neg H_c$  are not identical





Points  $A$  and  $B$  are not identical

Line through points  $H_a$  and  $B$  is not perpendicular to line through points  $B$  and  $H_c$

#### 4.1.2 Proving $T_c = \neg T_c$

Proving failed

#### 4.1.3 Proving $H = \neg H$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_c = \neg H_c$

Proving failed

#### 4.2.2 Proving $T_c = \neg T_c$

Proving failed

#### 4.2.3 Proving $H = \neg H$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $T_c = \neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 8 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.3 Proving $H = \neg H$

Proving failed

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.4.2 Proving $T_c = \neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 28 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

### 4.4.3 Proving $H = \_H$

Proving failed

# Problem 544

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 544: Given a point  $H_c$ , a point  $H$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 545

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 545: Given a point  $H_c$ , a point  $T_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 546

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 546: Given a point  $H_c$ , a point  $T_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 547

*Generated automatically by ArgoTriCS  
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### 1 Problem

Problem 547: Given a point  $H_c$ , a point  $T_a$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

## Problem 548

*Generated automatically by ArgoTriCS  
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### 1 Problem

Problem 548: Given a point  $H_c$ , a point  $T_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.



## Problem 549

*Generated automatically by ArgoTriCS  
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### 1 Problem

Problem 549: Given a point  $H_c$ , a point  $T_b$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 550

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 550: Given a point  $H_c$ , a point  $T_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $H_c$  and the point  $T_c$ , construct a line  $c$  (rule W02); % DET: points  $H_c$  and  $T_c$  are not the same;
2. Using the point  $T_c$  and the point  $I$ , construct a line  $s_c$  (rule W02); % DET: points  $T_c$  and  $I$  are not the same;
3. Using the point  $H_c$  and the line  $c$ , construct a line  $h_c$  (rule W10b); ;
4. Using the line  $h_c$  and the line  $s_c$ , construct a point  $C$  (rule W03); % NDG: lines  $h_c$  and  $s_c$  are not parallel % DET: lines  $h_c$  and  $s_c$  are not the same;
5. Using the point  $I$  and the line  $c$ , construct a circle  $k(I, P_a)$  (rule W11); % NDG: point  $I$  is not incident to the line  $c$ ;
6. Using the circle  $k(I, P_a)$ , the point  $C$  and the point  $I$ , construct a line  $a$  and a line  $b$  (rule W12); % NDG: point  $C$  is outside the circle  $k(I, P_a)$ ;
7. Using the line  $a$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $a$  and  $c$  are not parallel % DET: lines  $a$  and  $c$  are not the same;
8. Using the line  $c$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $c$  and  $b$  are not parallel % DET: lines  $c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; lines  $a$  and  $c$  are not parallel; point  $C$  is outside the circle  $k(I, P_a)$ ; point  $I$  is not incident to the line  $c$ ; lines  $h_c$  and  $s_c$  are not parallel.

Determination conditions: lines  $c$  and  $b$  are not the same; lines  $a$  and  $c$  are not the same; lines  $h_c$  and  $s_c$  are not the same; points  $T_c$  and  $I$  are not the same; points  $H_c$  and  $T_c$  are not the same.

Rules used: [W02,W03,W10b,W11,W12]

Lemmas used: [D10,D25,D27,D7,GD01,L2,L59,L60,L61,L7]  
Solving time: 1.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point H_{c} 68.91 84.83
point T_{c} 55.38 72.43
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_rt H_{c}
cmark_rt T_{c}
cmark_b I
color 0 0 0
fontsize 8

% DET: points H_{c} and T_{c} are not the same
% Constructing a line c which passes through point H_{c} and point T_{c}
line c H_{c} T_{c}

color 200 200 200
drawline c
color 0 0 0

% DET: points T_{c} and I are not the same
% Constructing a line s_{c} which passes through point T_{c} and point I
line s_{c} T_{c} I

color 200 200 200
drawline s_{c}
color 0 0 0

% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines h_{c} and s_{c} are not parallel% DET: lines h_{c} and s_{c} are not the same
% Constructing a point C which belongs to line h_{c} and line s_{c}
intersec C h_{c} s_{c}
cmark_l C
```

```

% NDG: point I is not incident to the line c
% Constructing a point P_{\_G60109} which is a foot of the point I on the line c
foot P_{\_G60109} I c
cmark_r P_{\_G60109}
color 200 200 200
drawline I P_{\_G60109}
color 0 0 0

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
60109}
circle k(I,P_{a}) I P_{\_G60109}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point C is outside the circle k(I,P_{a})
% Constructing midpoint P_{\_G60343} of the segment CI
midpoint P_{\_G60343} C I
cmark_r P_{\_G60343}

% Constructing a circle C_{\_G60346} whose center is at point P_{\_G60343} and which passes through
point C
circle C_{\_G60346} P_{\_G60343} C

color 200 200 200
drawcircle C_{\_G60346}
color 0 0 0

% Constructing points P_{\_G60349} and P_{\_G60352} which are in intersection of C_{\_G60346} and k
(I,P_{a})
intersec2 P_{\_G60349} P_{\_G60352} C_{\_G60346} k(I,P_{a})
cmark_r P_{\_G60349}
cmark_r P_{\_G60352}

% Constructing a line a which passes through point C and point P_{\_G60349}
line a C P_{\_G60349}

color 200 200 200
drawline a
color 0 0 0

% Constructing a line b which passes through point C and point P_{\_G60352}
line b C P_{\_G60352}

color 200 200 200
drawline b
color 0 0 0

```

```

% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B

% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and b are not parallel; lines a and c are not parallel; point
% C is outside the circle k(I,P_{a}); point I is not incident to the line c; lines h_{c} and s_{c}
% are not parallel
% Determination conditions: lines c and b are not the same; lines a and c are not the same; lines h
% _{c} and s_{c} are not the same; points T_{c} and I are not the same; points H_{c} and T_{c}
% are not the same

```

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $H_c = \_H_c$

**Status:** Theorem has been proved.

**Space Complexity:** The biggest polynomial obtained during prover execution contains 3 terms.

**Time Complexity:** Time spent by the prover is 0.092 seconds.

**NDG conditions** Points  $H_c$ ,  $I$  and  $T_c$  are not collinear

Line through points  $A$  and  $C$  is not perpendicular to line through points  $C$  and  $B$

#### 4.1.2 Proving $T_c = \_T_c$

Proving failed

#### 4.1.3 Proving $I = \_I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $H_c = \_H_c$

Proving failed

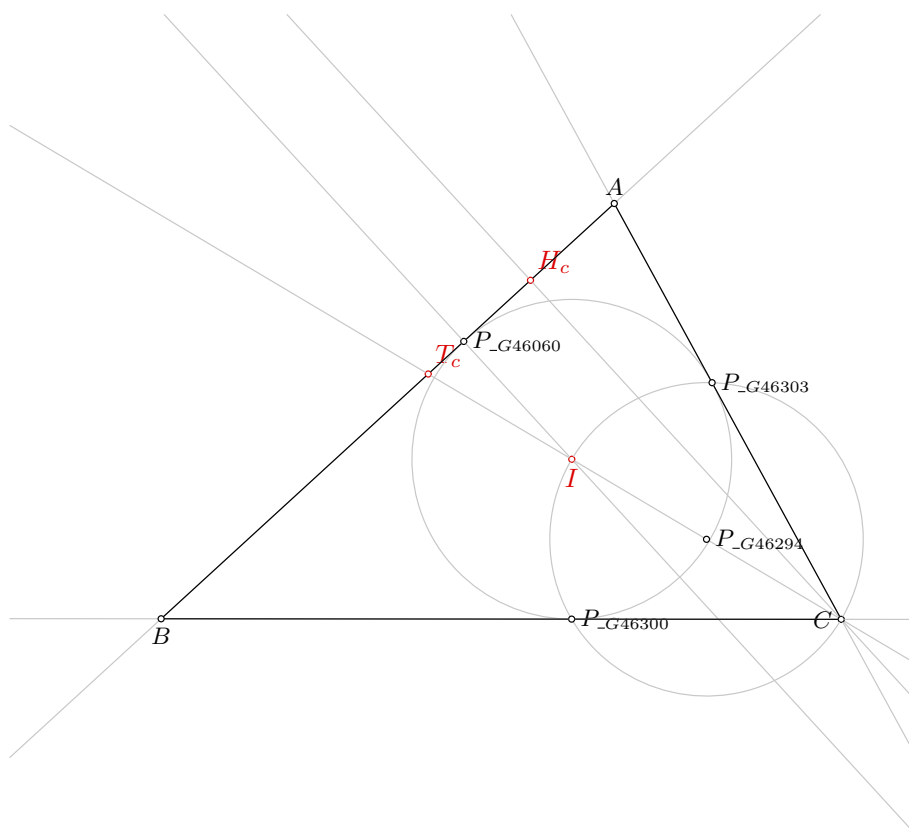


Figure 1: Illustration of the problem 0550

#### 4.2.2 Proving $T_c = \neg T_c$

Proving failed

#### 4.2.3 Proving $I = \neg I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.000 seconds. There are no ndg conditions.

#### 4.3.2 Proving $T_c = \neg T_c$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 62 terms.

**Time Complexity:** Time spent by the prover is 0.040 seconds. There are no ndg conditions.

#### 4.3.3 Proving $I = \neg I$

**Status:** The conjecture has not been proved nor disproved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 446 terms.

**Time Complexity:** Time spent by the prover is 0.680 seconds. There are no ndg conditions.

### 4.4 GCLC - Grobner basis method

#### 4.4.1 Proving $H_c = \neg H_c$

**Status:** The conjecture has been proved.

**Space Complexity:** The biggest polynomial obtained during proof process contained 0 terms.

**Time Complexity:** Time spent by the prover is 0.010 seconds. There are no ndg conditions.

#### 4.4.2 Proving $T_c = \neg T_c$

Proving failed

#### 4.4.3 Proving $I = \neg I$

Proving failed

# Problem 551

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 551: Given a point  $H$ , a point  $T_a$  and a point  $T_b$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.



# Problem 552

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 552: Given a point  $H$ , a point  $T_a$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

## Problem 553

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

### 1 Problem

Problem 553: Given a point  $H$ , a point  $T_a$  and a point  $I$ , construct the triangle  $ABC$ .

### 2 Status

Status of the problem is unknown.

# Problem 554

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 554: Given a point  $H$ , a point  $T_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 555

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 555: Given a point  $H$ , a point  $T_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 556

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 556: Given a point  $H$ , a point  $T_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 557

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 557: Given a point  $T_a$ , a point  $T_b$  and a point  $T_c$ , construct the triangle  $ABC$ .

## 2 Status

Status of the problem is unknown.

# Problem 558

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 558: Given a point  $T_a$ , a point  $T_b$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $T_a$  and the point  $I$ , construct a line  $s_a$  (rule W02); % DET: points  $T_a$  and  $I$  are not the same;
2. Using the point  $T_b$  and the point  $I$ , construct a line  $s_b$  (rule W02); % DET: points  $T_b$  and  $I$  are not the same;
3. Using the point  $T_a$ , the point  $I$  and the point  $T_b$ , construct a circle  $circle[T_a, T_b, angle[b][a]]$  (rule W20); % NDG: points  $T_a$  and  $I$  are not the same; points  $I$  and  $T_b$  are not the same; points  $T_a$  and  $T_b$  are not the same;
4. Using the point  $T_a$ , the point  $I$  and the point  $T_b$ , construct a circle  $circle[T_b, I, angle[b][s[c]]]$  (rule W20); % NDG: points  $T_a$  and  $I$  are not the same; points  $I$  and  $T_b$  are not the same; points  $T_b$  and  $I$  are not the same;
5. Using the circle  $circle[T_b, I, angle[b][s[c]]]$  and the circle  $circle[T_a, T_b, angle[b][a]]$ , construct a point  $C_{c1}$  and a point  $C$  (rule W07); % NDG: circles  $circle[T_b, I, angle[b][s[c]]]$  and  $circle[T_a, T_b, angle[b][a]]$  intersect % DET: circles  $circle[T_b, I, angle[b][s[c]]]$  and  $circle[T_a, T_b, angle[b][a]]$  are not the same;
6. Using the point  $C$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $C$  and  $T_a$  are not the same;
7. Using the point  $T_b$  and the point  $C$ , construct a line  $b$  (rule W02); % DET: points  $T_b$  and  $C$  are not the same;
8. Using the line  $s_a$  and the line  $b$ , construct a point  $A$  (rule W03); % NDG: lines  $s_a$  and  $b$  are not parallel % DET: lines  $s_a$  and  $b$  are not the same;

9. Using the line  $s_b$  and the line  $a$ , construct a point  $B$  (rule W03); % NDG: lines  $s_b$  and  $a$  are not parallel % DET: lines  $s_b$  and  $a$  are not the same.

Non-degenerate conditions: lines  $s_b$  and  $a$  are not parallel; lines  $s_a$  and  $b$  are not parallel; circles  $circle[T_b, I, angle[b][s[c]]]$  and  $circle[T_a, T_b, angle[b][a]]$  intersect; points  $T_a$  and  $I$  are not the same; points  $I$  and  $T_b$  are not the same; points  $T_b$  and  $I$  are not the same; points  $T_a$  and  $I$  are not the same; points  $I$  and  $T_b$  are not the same; points  $T_a$  and  $T_b$  are not the same.

Determination conditions: lines  $s_b$  and  $a$  are not the same; lines  $s_a$  and  $b$  are not the same; points  $T_b$  and  $C$  are not the same; points  $C$  and  $T_a$  are not the same; circles  $circle[T_b, I, angle[b][s[c]]]$  and  $circle[T_a, T_b, angle[b][a]]$  are not the same; points  $T_b$  and  $I$  are not the same; points  $T_a$  and  $I$  are not the same.

Rules used: [W02,W03,W07,W20]

Lemmas used: [D2,D23,D24,D72,GD01,GL13,GL20,L102,L112,L5,L6,L91,L96]

Solving time: 2.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point T_{a} 70.86 40
point T_{b} 94.25 68.88
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_rb T_{a}
cmark_t T_{b}
cmark_b I
color 0 0 0
fontsize 8

% DET: points T_{a} and I are not the same
% Constructing a line s_{a} which passes through point T_{a} and point I
line s_{a} T_{a} I

color 200 200 200
drawline s_{a}
color 0 0 0

% DET: points T_{b} and I are not the same
% Constructing a line s_{b} which passes through point T_{b} and point I
line s_{b} T_{b} I

color 200 200 200
drawline s_{b}
color 0 0 0
```



```

% NDG: points T_{a} and I are not the same; points I and T_{b} are not the same; points T_{a} and T
_{b} are not the same
% Constructing an angle angle[_G89385] which is equal to the angle T_{a}IT_{b}
angle_o angle[_G89385] T_{a} I T_{b}

% Calculating value angle[_G89396] using formula  $90 - (2/\text{pow}(2,0) * \text{angle}[_G89385] + -1/\text{pow}(2,0) * 180)$ 
expression angle[_G89396] {  $90 - (2/\text{pow}(2,0) * \text{angle}[_G89385] + -1/\text{pow}(2,0) * 180)$  }

% Constructing a line L_{\_G89467} which passes through point T_{a} and point T_{b}
line L_{\_G89467} T_{a} T_{b}

color 200 200 200
drawline L_{\_G89467}
color 0 0 0

% Constructing bisector L_{\_G89470} of the segment T_{a}T_{b}
med L_{\_G89470} T_{a} T_{b}

color 200 200 200
drawline L_{\_G89470}
color 0 0 0

color 200 200 200
drawsegment T_{a} T_{b}
color 0 0 0

% Constructing a point P_{\_G89473} which is an image of the point T_{b} in a rotation around the
point T_{a} for the angle angle[_G89396]
rotate P_{\_G89473} T_{a} angle[_G89396] T_{b}
cmark_r P_{\_G89473}
color 200 200 200
drawarc_p T_{a} T_{b} angle[_G89396]
color 0 0 0

% Constructing a line L_{\_G89476} which passes through point T_{a} and point P_{\_G89473}
line L_{\_G89476} T_{a} P_{\_G89473}

color 200 200 200
drawline L_{\_G89476}
color 0 0 0

% Constructing a point P_{\_G89479} which belongs to line L_{\_G89470} and line L_{\_G89476}
intersec P_{\_G89479} L_{\_G89470} L_{\_G89476}
cmark_r P_{\_G89479}

% Constructing a circle circle[T_{a},T_{b},angle[b][a]] whose center is at point P_{\_G89479} and
which passes through point T_{a}
circle circle[T_{a},T_{b},angle[b][a]] P_{\_G89479} T_{a}

color 200 200 200
drawcircle circle[T_{a},T_{b},angle[b][a]]
color 0 0 0

```

```

% NDG: points T_{a} and I are not the same; points I and T_{b} are not the same; points T_{b} and I
      are not the same
% Constructing an angle angle[_G90358] which is equal to the angle T_{a}IT_{b}
angle_o angle[_G90358] T_{a} I T_{b}

% Calculating value angle[_G90369] using formula  $90 - (2/\sqrt{2}) * \text{angle}[_G90358] + 1/\sqrt{2} * 180$ 
expression angle[_G90369] {  $90 - (2/\sqrt{2}) * \text{angle}[_G90358] + 1/\sqrt{2} * 180$  }

% Constructing a line L_{\_G90440} which passes through point T_{b} and point I
line L_{\_G90440} T_{b} I

color 200 200 200
drawline L_{\_G90440}
color 0 0 0

% Constructing bisector L_{\_G90443} of the segment T_{b}I
med L_{\_G90443} T_{b} I

color 200 200 200
drawline L_{\_G90443}
color 0 0 0

color 200 200 200
drawsegment T_{b} I
color 0 0 0

% Constructing a point P_{\_G90446} which is an image of the point I in a rotation around the point
      T_{b} for the angle angle[_G90369]
rotate P_{\_G90446} T_{b} angle[_G90369] I
cmark_r P_{\_G90446}
color 200 200 200
drawarc_p T_{b} I angle[_G90369]
color 0 0 0

% Constructing a line L_{\_G90449} which passes through point T_{b} and point P_{\_G90446}
line L_{\_G90449} T_{b} P_{\_G90446}

color 200 200 200
drawline L_{\_G90449}
color 0 0 0

% Constructing a point P_{\_G90452} which belongs to line L_{\_G90443} and line L_{\_G90449}
intersec P_{\_G90452} L_{\_G90443} L_{\_G90449}
cmark_r P_{\_G90452}

% Constructing a circle circle[T_{b},I,angle[b][s[c]]] whose center is at point P_{\_G90452} and
      which passes through point T_{b}
circle circle[T_{b},I,angle[b][s[c]]] P_{\_G90452} T_{b}

```

```

color 200 200 200
drawcircle circle[T_{b}],I,angle[b][s[c]]]
color 0 0 0

% NDG: circles circle[T_{b}],I,angle[b][s[c]]] and circle[T_{a}],T_{b},angle[b][a]] intersect% DET:
% circles circle[T_{b}],I,angle[b][s[c]]] and circle[T_{a}],T_{b},angle[b][a]] are not the same
% Constructing points C_{c1} and C which are in intersection of circle[T_{b}],I,angle[b][s[c]]] and
% circle[T_{a}],T_{b},angle[b][a]]
intersec2 C_{c1} C circle[T_{b}],I,angle[b][s[c]]] circle[T_{a}],T_{b},angle[b][a]]
cmark_r C_{c1}
cmark_l C

% DET: points C and T_{a} are not the same
% Constructing a line a which passes through point C and point T_{a}
line a C T_{a}

color 200 200 200
drawline a
color 0 0 0

% DET: points T_{b} and C are not the same
% Constructing a line b which passes through point T_{b} and point C
line b T_{b} C

color 200 200 200
drawline b
color 0 0 0

% NDG: lines s_{a} and b are not parallel% DET: lines s_{a} and b are not the same
% Constructing a point A which belongs to line s_{a} and line b
intersec A s_{a} b
cmark_t A

% NDG: lines s_{b} and a are not parallel% DET: lines s_{b} and a are not the same
% Constructing a point B which belongs to line s_{b} and line a
intersec B s_{b} a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines s_{b} and a are not parallel; lines s_{a} and b are not parallel
% ; circles circle[T_{b}],I,angle[b][s[c]]] and circle[T_{a}],T_{b},angle[b][a]] intersect; points
% T_{a} and I are not the same; points I and T_{b} are not the same; points T_{b} and I are not
% the same; points T_{a} and I are not the same; points I and T_{b} are not the same; points T_{a}

```

Figure 1: Illustration of the problem 0558

*} and  $T_{\{b\}}$  are not the same*  
% Determination conditions: lines  $s_{\{b\}}$  and  $a$  are not the same; lines  $s_{\{a\}}$  and  $b$  are not the same;  
points  $T_{\{b\}}$  and  $C$  are not the same; points  $C$  and  $T_{\{a\}}$  are not the same; circles  $\text{circle}[T_{\{b\}}$   
 $\{, I, \text{angle}[b][s[c]]]$  and  $\text{circle}[T_{\{a\}}, T_{\{b\}}, \text{angle}[b][a]]$  are not the same; points  $T_{\{b\}}$  and  $I$   
are not the same; points  $T_{\{a\}}$  and  $I$  are not the same

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $T_a = \neg T_a$

Proving failed

#### 4.1.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.1.3 Proving $I = \neg I$

Proving failed

### 4.2 GCLC - Area method

#### 4.2.1 Proving $T_a = \neg T_a$

Proving failed

#### 4.2.2 Proving $T_b = \neg T_b$

Proving failed

#### 4.2.3 Proving $I = \neg I$

Proving failed

### 4.3 GCLC - Wu method

#### 4.3.1 Proving $T_a = \neg T_a$

Proving failed

#### 4.3.2 Proving $T_b = \neg T_b$

Proving failed

### 4.3.3 Proving $I = \perp$

Proving failed

## 4.4 GCLC - Grobner basis method

### 4.4.1 Proving $T_a = \perp T_a$

Proving failed

### 4.4.2 Proving $T_b = \perp T_b$

Proving failed

### 4.4.3 Proving $I = \perp$

Proving failed

# Problem 559

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 559: Given a point  $T_a$ , a point  $T_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $T_a$  and the point  $I$ , construct a line  $s_a$  (rule W02); % DET: points  $T_a$  and  $I$  are not the same;
2. Using the point  $T_c$  and the point  $I$ , construct a line  $s_c$  (rule W02); % DET: points  $T_c$  and  $I$  are not the same;
3. Using the point  $T_c$ , the point  $I$  and the point  $T_a$ , construct a circle  $circle[T_a, I, angle[a][s[b]]]$  (rule W20); % NDG: points  $T_c$  and  $I$  are not the same; points  $I$  and  $T_a$  are not the same; points  $T_a$  and  $I$  are not the same;
4. Using the point  $T_c$ , the point  $I$  and the point  $T_a$ , construct a circle  $circle[T_c, T_a, angle[a][c]]$  (rule W20); % NDG: points  $T_c$  and  $I$  are not the same; points  $I$  and  $T_a$  are not the same; points  $T_c$  and  $T_a$  are not the same;
5. Using the circle  $circle[T_c, T_a, angle[a][c]]$  and the circle  $circle[T_a, I, angle[a][s[b]]]$ , construct a point  $B_{c1}$  and a point  $B$  (rule W07); % NDG: circles  $circle[T_c, T_a, angle[a][c]]$  and  $circle[T_a, I, angle[a][s[b]]]$  intersect % DET: circles  $circle[T_c, T_a, angle[a][c]]$  and  $circle[T_a, I, angle[a][s[b]]]$  are not the same;
6. Using the point  $B$  and the point  $T_a$ , construct a line  $a$  (rule W02); % DET: points  $B$  and  $T_a$  are not the same;
7. Using the point  $T_c$  and the point  $B$ , construct a line  $c$  (rule W02); % DET: points  $T_c$  and  $B$  are not the same;
8. Using the line  $s_a$  and the line  $c$ , construct a point  $A$  (rule W03); % NDG: lines  $s_a$  and  $c$  are not parallel % DET: lines  $s_a$  and  $c$  are not the same;

9. Using the line  $s_c$  and the line  $a$ , construct a point  $C$  (rule W03); % NDG: lines  $s_c$  and  $a$  are not parallel % DET: lines  $s_c$  and  $a$  are not the same.

Non-degenerate conditions: lines  $s_c$  and  $a$  are not parallel; lines  $s_a$  and  $c$  are not parallel; circles  $circle[T_c, T_a, angle[a][c]]$  and  $circle[T_a, I, angle[a][s[b]]]$  intersect; points  $T_c$  and  $I$  are not the same; points  $I$  and  $T_a$  are not the same; points  $T_c$  and  $T_a$  are not the same; points  $T_c$  and  $I$  are not the same; points  $I$  and  $T_a$  are not the same; points  $T_a$  and  $I$  are not the same.

Determination conditions: lines  $s_c$  and  $a$  are not the same; lines  $s_a$  and  $c$  are not the same; points  $T_c$  and  $B$  are not the same; points  $B$  and  $T_a$  are not the same; circles  $circle[T_c, T_a, angle[a][c]]$  and  $circle[T_a, I, angle[a][s[b]]]$  are not the same; points  $T_c$  and  $I$  are not the same; points  $T_a$  and  $I$  are not the same.

Rules used: [W02,W03,W07,W20]

Lemmas used: [D2,D23,D25,D75,GD01,GL13,GL20,L100,L111,L2,L5,L7,L90,L94]

Solving time: 2.5 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point T_{a} 70.86 40
point T_{c} 55.38 72.43
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_rb T_{a}
cmark_rt T_{c}
cmark_b I
color 0 0 0
fontsize 8

% DET: points T_{a} and I are not the same
% Constructing a line s_{a} which passes through point T_{a} and point I
line s_{a} T_{a} I

color 200 200 200
drawline s_{a}
color 0 0 0

% DET: points T_{c} and I are not the same
% Constructing a line s_{c} which passes through point T_{c} and point I
line s_{c} T_{c} I

color 200 200 200
drawline s_{c}
color 0 0 0
```

```

% NDG: points T_{c} and I are not the same; points I and T_{a} are not the same; points T_{a} and I
      are not the same
% Constructing an angle angle[_G56266] which is equal to the angle T_{c}IT_{a}
angle_o angle[_G56266] T_{c} I T_{a}

% Calculating value angle[_G56277] using formula  $90 - (2/\text{pow}(2,1) * \text{angle}[_G56266] + -1/\text{pow}(2,1) * 180)$ 
expression angle[_G56277] {  $90 - (2/\text{pow}(2,1) * \text{angle}[_G56266] + -1/\text{pow}(2,1) * 180)$  }

% Constructing a line L_{\_G56348} which passes through point T_{a} and point I
line L_{\_G56348} T_{a} I

color 200 200 200
drawline L_{\_G56348}
color 0 0 0

% Constructing bisector L_{\_G56351} of the segment T_{a}I
med L_{\_G56351} T_{a} I

color 200 200 200
drawline L_{\_G56351}
color 0 0 0

color 200 200 200
drawsegment T_{a} I
color 0 0 0

% Constructing a point P_{\_G56354} which is an image of the point I in a rotation around the point
      T_{a} for the angle angle[_G56277]
rotate P_{\_G56354} T_{a} angle[_G56277] I
cmark_r P_{\_G56354}
color 200 200 200
drawarc_p T_{a} I angle[_G56277]
color 0 0 0

% Constructing a line L_{\_G56357} which passes through point T_{a} and point P_{\_G56354}
line L_{\_G56357} T_{a} P_{\_G56354}

color 200 200 200
drawline L_{\_G56357}
color 0 0 0

% Constructing a point P_{\_G56360} which belongs to line L_{\_G56351} and line L_{\_G56357}
intersec P_{\_G56360} L_{\_G56351} L_{\_G56357}
cmark_r P_{\_G56360}

% Constructing a circle circle[T_{a},I,angle[a][s[b]]] whose center is at point P_{\_G56360} and
      which passes through point T_{a}
circle circle[T_{a},I,angle[a][s[b]]] P_{\_G56360} T_{a}

color 200 200 200
drawcircle circle[T_{a},I,angle[a][s[b]]]
color 0 0 0

```



```

% NDG: points T_{c} and I are not the same; points I and T_{a} are not the same; points T_{c} and T
_{a} are not the same
% Constructing an angle angle[_G57296] which is equal to the angle T_{c}IT_{a}
angle_o angle[_G57296] T_{c} I T_{a}

% Calculating value angle[_G57307] using formula  $90 - (2/\sqrt{2}) * \text{angle}[_G57296] + 1/\sqrt{2} * 180$ 
expression angle[_G57307] {  $90 - (2/\sqrt{2}) * \text{angle}[_G57296] + 1/\sqrt{2} * 180$  }

% Constructing a line L_{\_G57378} which passes through point T_{c} and point T_{a}
line L_{\_G57378} T_{c} T_{a}

color 200 200 200
drawline L_{\_G57378}
color 0 0 0

% Constructing bisector L_{\_G57381} of the segment T_{c}T_{a}
med L_{\_G57381} T_{c} T_{a}

color 200 200 200
drawline L_{\_G57381}
color 0 0 0

color 200 200 200
drawsegment T_{c} T_{a}
color 0 0 0

% Constructing a point P_{\_G57384} which is an image of the point T_{a} in a rotation around the
point T_{c} for the angle angle[_G57307]
rotate P_{\_G57384} T_{c} angle[_G57307] T_{a}
cmark_r P_{\_G57384}
color 200 200 200
drawarc_p T_{c} T_{a} angle[_G57307]
color 0 0 0

% Constructing a line L_{\_G57387} which passes through point T_{c} and point P_{\_G57384}
line L_{\_G57387} T_{c} P_{\_G57384}

color 200 200 200
drawline L_{\_G57387}
color 0 0 0

% Constructing a point P_{\_G57390} which belongs to line L_{\_G57381} and line L_{\_G57387}
intersec P_{\_G57390} L_{\_G57381} L_{\_G57387}
cmark_r P_{\_G57390}

% Constructing a circle circle[T_{c},T_{a},angle[a][c]] whose center is at point P_{\_G57390} and
which passes through point T_{c}
circle circle[T_{c},T_{a},angle[a][c]] P_{\_G57390} T_{c}

```

```

color 200 200 200
drawcircle circle[T_{c},T_{a},angle[a][c]]
color 0 0 0

% NDG: circles circle[T_{c},T_{a},angle[a][c]] and circle[T_{a},I,angle[a][s[b]]] intersect% DET:
% circles circle[T_{c},T_{a},angle[a][c]] and circle[T_{a},I,angle[a][s[b]]] are not the same
% Constructing points B_{c1} and B which are in intersection of circle[T_{c},T_{a},angle[a][c]] and
% circle[T_{a},I,angle[a][s[b]]]
intersec2 B_{c1} B circle[T_{c},T_{a},angle[a][c]] circle[T_{a},I,angle[a][s[b]]]
cmark_r B_{c1}
cmark_b B

% DET: points B and T_{a} are not the same
% Constructing a line a which passes through point B and point T_{a}
line a B T_{a}

color 200 200 200
drawline a
color 0 0 0

% DET: points T_{c} and B are not the same
% Constructing a line c which passes through point T_{c} and point B
line c T_{c} B

color 200 200 200
drawline c
color 0 0 0

% NDG: lines s_{a} and c are not parallel% DET: lines s_{a} and c are not the same
% Constructing a point A which belongs to line s_{a} and line c
intersec A s_{a} c
cmark_t A

% NDG: lines s_{c} and a are not parallel% DET: lines s_{c} and a are not the same
% Constructing a point C which belongs to line s_{c} and line a
intersec C s_{c} a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines s_{c} and a are not parallel; lines s_{a} and c are not parallel
% ; circles circle[T_{c},T_{a},angle[a][c]] and circle[T_{a},I,angle[a][s[b]]] intersect; points
% T_{c} and I are not the same; points I and T_{a} are not the same; points T_{c} and T_{a} are
% not the same; points T_{c} and I are not the same; points I and T_{a} are not the same; points

```

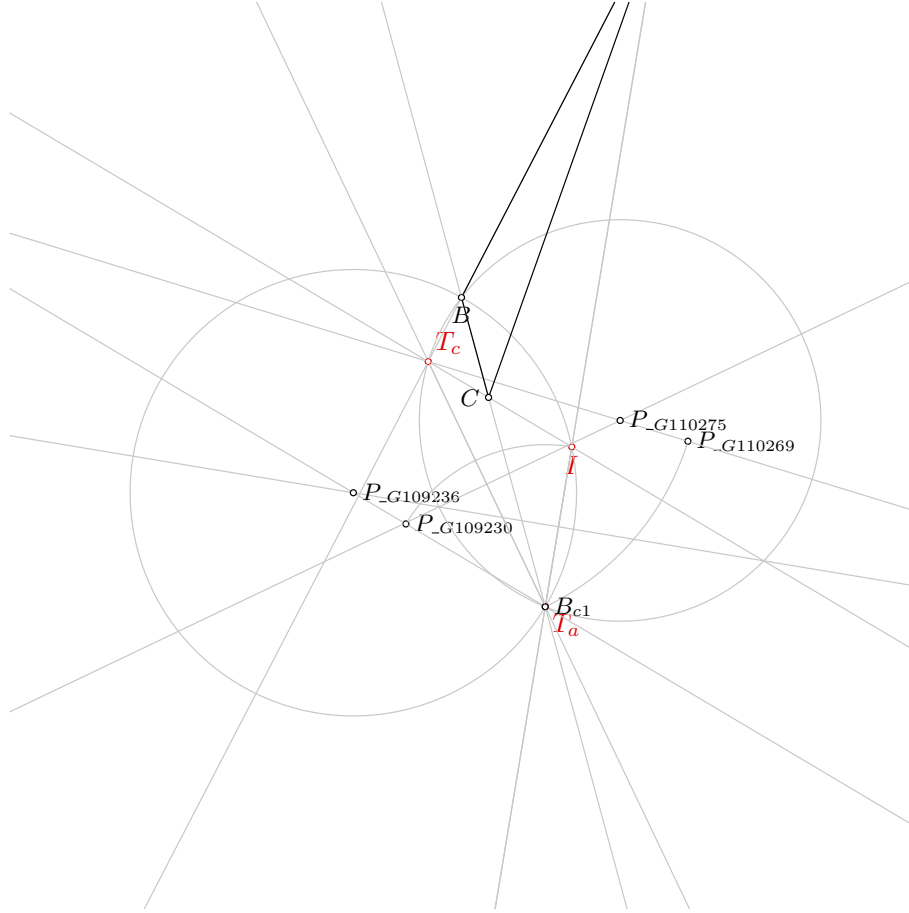


Figure 1: Illustration of the problem 0559

$T_{\{a\}}$  and  $I$  are not the same  
 % Determination conditions: lines  $s_{\{c\}}$  and  $a$  are not the same; lines  $s_{\{a\}}$  and  $c$  are not the same;  
 points  $T_{\{c\}}$  and  $B$  are not the same; points  $B$  and  $T_{\{a\}}$  are not the same; circles  $\text{circle}[T_{\{c\}}$   
 $\{, T_{\{a\}}, \text{angle}[a][c]]$  and  $\text{circle}[T_{\{a\}}, I, \text{angle}[a][s[b]]]$  are not the same; points  $T_{\{c\}}$  and  $I$   
 are not the same; points  $T_{\{a\}}$  and  $I$  are not the same

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $T_a = T_a$

Proving failed

#### **4.1.2 Proving $T_c = \neg T_c$**

Proving failed

#### **4.1.3 Proving $I = \neg I$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $T_a = \neg T_a$**

Proving failed

#### **4.2.2 Proving $T_c = \neg T_c$**

Proving failed

#### **4.2.3 Proving $I = \neg I$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $T_a = \neg T_a$**

Proving failed

#### **4.3.2 Proving $T_c = \neg T_c$**

Proving failed

#### **4.3.3 Proving $I = \neg I$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $T_a = \neg T_a$**

Proving failed

#### **4.4.2 Proving $T_c = \neg T_c$**

Proving failed

#### **4.4.3 Proving $I = \neg I$**

Proving failed

# Problem 560

*Generated automatically by ArgoTriCS  
Developed by Vesna Marinković, University of Belgrade*

## 1 Problem

Problem 560: Given a point  $T_b$ , a point  $T_c$  and a point  $I$ , construct the triangle  $ABC$ .

## 2 Status

Problem is solvable.

## 3 Solution

### 3.1 Construction in natural language form

1. Using the point  $T_b$  and the point  $I$ , construct a line  $s_b$  (rule W02); % DET: points  $T_b$  and  $I$  are not the same;
2. Using the point  $T_c$  and the point  $I$ , construct a line  $s_c$  (rule W02); % DET: points  $T_c$  and  $I$  are not the same;
3. Using the point  $T_b$ , the point  $I$  and the point  $T_c$ , construct a circle  $circle[T_b, T_c, angle[c][b]]$  (rule W20); % NDG: points  $T_b$  and  $I$  are not the same; points  $I$  and  $T_c$  are not the same; points  $T_b$  and  $T_c$  are not the same;
4. Using the point  $T_b$ , the point  $I$  and the point  $T_c$ , construct a circle  $circle[T_c, I, angle[c][s[a]]]$  (rule W20); % NDG: points  $T_b$  and  $I$  are not the same; points  $I$  and  $T_c$  are not the same; points  $T_c$  and  $I$  are not the same;
5. Using the circle  $circle[T_c, I, angle[c][s[a]]]$  and the circle  $circle[T_b, T_c, angle[c][b]]$ , construct a point  $A_{c2}$  and a point  $A$  (rule W07); % NDG: circles  $circle[T_c, I, angle[c][s[a]]]$  and  $circle[T_b, T_c, angle[c][b]]$  intersect % DET: circles  $circle[T_c, I, angle[c][s[a]]]$  and  $circle[T_b, T_c, angle[c][b]]$  are not the same;
6. Using the point  $A$  and the point  $T_b$ , construct a line  $b$  (rule W02); % DET: points  $A$  and  $T_b$  are not the same;
7. Using the point  $T_c$  and the point  $A$ , construct a line  $c$  (rule W02); % DET: points  $T_c$  and  $A$  are not the same;
8. Using the line  $s_b$  and the line  $c$ , construct a point  $B$  (rule W03); % NDG: lines  $s_b$  and  $c$  are not parallel % DET: lines  $s_b$  and  $c$  are not the same;

9. Using the line  $s_c$  and the line  $b$ , construct a point  $C$  (rule W03); % NDG: lines  $s_c$  and  $b$  are not parallel % DET: lines  $s_c$  and  $b$  are not the same.

Non-degenerate conditions: lines  $s_c$  and  $b$  are not parallel; lines  $s_b$  and  $c$  are not parallel; circles  $circle[T_c, I, angle[c][s[a]]]$  and  $circle[T_b, T_c, angle[c][b]]$  intersect; points  $T_b$  and  $I$  are not the same; points  $I$  and  $T_c$  are not the same; points  $T_c$  and  $I$  are not the same; points  $T_b$  and  $I$  are not the same; points  $I$  and  $T_c$  are not the same; points  $T_b$  and  $T_c$  are not the same.

Determination conditions: lines  $s_c$  and  $b$  are not the same; lines  $s_b$  and  $c$  are not the same; points  $T_c$  and  $A$  are not the same; points  $A$  and  $T_b$  are not the same; circles  $circle[T_c, I, angle[c][s[a]]]$  and  $circle[T_b, T_c, angle[c][b]]$  are not the same; points  $T_c$  and  $I$  are not the same; points  $T_b$  and  $I$  are not the same.

Rules used: [W02,W03,W07,W20]

Lemmas used: [D2,D24,D25,D69,GD01,GL13,GL20,L110,L2,L6,L7,L89,L93,L98]

Solving time: 2.6 seconds.

### 3.2 Construction in GCLC language

```
dim 120 120

point T_{b} 94.25 68.88
point T_{c} 55.38 72.43
point I 74.37 61.15

color 220 0 0
fontsize 9

cmark_t T_{b}
cmark_rt T_{c}
cmark_b I
color 0 0 0
fontsize 8

% DET: points T_{b} and I are not the same
% Constructing a line s_{b} which passes through point T_{b} and point I
line s_{b} T_{b} I

color 200 200 200
drawline s_{b}
color 0 0 0

% DET: points T_{c} and I are not the same
% Constructing a line s_{c} which passes through point T_{c} and point I
line s_{c} T_{c} I

color 200 200 200
drawline s_{c}
color 0 0 0
```

```

% NDG: points  $T_b$  and  $I$  are not the same; points  $I$  and  $T_c$  are not the same; points  $T_b$  and  $T_c$  are not the same
% Constructing an angle  $\angle_{G109740}$  which is equal to the angle  $\angle T_b I T_c$ 
angle_o angle[_G109740] T_b I T_c

% Calculating value  $\angle_{G109752}$  using formula  $90 - (2/\sqrt{2}) * \angle_{G109740} + (-1/\sqrt{2}) * 180$ 
expression angle[_G109752] { 90 - (2/pow(2,0)*angle[_G109740] + -1/pow(2,0)*180) }

% Constructing a line  $L_{G109824}$  which passes through point  $T_b$  and point  $T_c$ 
line L[_G109824] T_b T_c

color 200 200 200
drawline L[_G109824]
color 0 0 0

% Constructing bisector  $L_{G109827}$  of the segment  $T_b T_c$ 
med L[_G109827] T_b T_c

color 200 200 200
drawline L[_G109827]
color 0 0 0

color 200 200 200
drawsegment T_b T_c
color 0 0 0

% Constructing a point  $P_{G109830}$  which is an image of the point  $T_c$  in a rotation around the
point  $T_b$  for the angle  $\angle_{G109752}$ 
rotate P[_G109830] T_b angle[_G109752] T_c
cmark_r P[_G109830]
color 200 200 200
drawarc_p T_b T_c angle[_G109752]
color 0 0 0

% Constructing a line  $L_{G109833}$  which passes through point  $T_b$  and point  $P_{G109830}$ 
line L[_G109833] T_b P[_G109830]

color 200 200 200
drawline L[_G109833]
color 0 0 0

% Constructing a point  $P_{G109836}$  which belongs to line  $L_{G109827}$  and line  $L_{G109833}$ 
intersec P[_G109836] L[_G109827] L[_G109833]
cmark_r P[_G109836]

% Constructing a circle  $\text{circle}[T_b, T_c, \angle_c[b]]$  whose center is at point  $P_{G109836}$  and
which passes through point  $T_b$ 
circle circle[T_b, T_c, angle[c][b]] P[_G109836] T_b

color 200 200 200
drawcircle circle[T_b, T_c, angle[c][b]]
color 0 0 0

```

```

% NDG: points T_{b} and I are not the same; points I and T_{c} are not the same; points T_{c} and I
    are not the same
% Constructing an angle angle[_G110722] which is equal to the angle T_{b}IT_{c}
angle_o angle[_G110722] T_{b} I T_{c}

% Calculating value angle[_G110734] using formula  $90 - (2/\sqrt{2}) * \text{angle}[_G110722] + (-1/\sqrt{2}) * 180$ 
expression angle[_G110734] { 90 - (2/pow(2,1)*angle[_G110722] + -1/pow(2,1)*180) }

% Constructing a line L_{\_G110806} which passes through point T_{c} and point I
line L_{\_G110806} T_{c} I

color 200 200 200
drawline L_{\_G110806}
color 0 0 0

% Constructing bisector L_{\_G110809} of the segment T_{c}I
med L_{\_G110809} T_{c} I

color 200 200 200
drawline L_{\_G110809}
color 0 0 0

color 200 200 200
drawsegment T_{c} I
color 0 0 0

% Constructing a point P_{\_G110812} which is an image of the point I in a rotation around the
    point T_{c} for the angle angle[_G110734]
rotate P_{\_G110812} T_{c} angle[_G110734] I
cmark_r P_{\_G110812}
color 200 200 200
drawarc_p T_{c} I angle[_G110734]
color 0 0 0

% Constructing a line L_{\_G110815} which passes through point T_{c} and point P_{\_G110812}
line L_{\_G110815} T_{c} P_{\_G110812}

color 200 200 200
drawline L_{\_G110815}
color 0 0 0

% Constructing a point P_{\_G110818} which belongs to line L_{\_G110809} and line L_{\_G110815}
intersec P_{\_G110818} L_{\_G110809} L_{\_G110815}
cmark_r P_{\_G110818}

% Constructing a circle circle[T_{c},I,angle[c][s[a]]] whose center is at point P_{\_G110818} and
    which passes through point T_{c}
circle circle[T_{c},I,angle[c][s[a]]] P_{\_G110818} T_{c}

```



```

color 200 200 200
drawcircle circle[T_{c},I,angle[c][s[a]]]
color 0 0 0

% NDG: circles circle[T_{c},I,angle[c][s[a]]] and circle[T_{b},T_{c},angle[c][b]] intersect% DET:
% circles circle[T_{c},I,angle[c][s[a]]] and circle[T_{b},T_{c},angle[c][b]] are not the same
% Constructing points A_{c2} and A which are in intersection of circle[T_{c},I,angle[c][s[a]]] and
% circle[T_{b},T_{c},angle[c][b]]
intersec2 A_{c2} A circle[T_{c},I,angle[c][s[a]]] circle[T_{b},T_{c},angle[c][b]]
cmark_r A_{c2}
cmark_t A

% DET: points A and T_{b} are not the same
% Constructing a line b which passes through point A and point T_{b}
line b A T_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points T_{c} and A are not the same
% Constructing a line c which passes through point T_{c} and point A
line c T_{c} A

color 200 200 200
drawline c
color 0 0 0

% NDG: lines s_{b} and c are not parallel% DET: lines s_{b} and c are not the same
% Constructing a point B which belongs to line s_{b} and line c
intersec B s_{b} c
cmark_b B

% NDG: lines s_{c} and b are not parallel% DET: lines s_{c} and b are not the same
% Constructing a point C which belongs to line s_{c} and line b
intersec C s_{c} b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines s_{c} and b are not parallel; lines s_{b} and c are not parallel
; circles circle[T_{c},I,angle[c][s[a]]] and circle[T_{b},T_{c},angle[c][b]] intersect; points
T_{b} and I are not the same; points I and T_{c} are not the same; points T_{c} and I are not
the same; points T_{b} and I are not the same; points I and T_{c} are not the same; points T_{b}

```

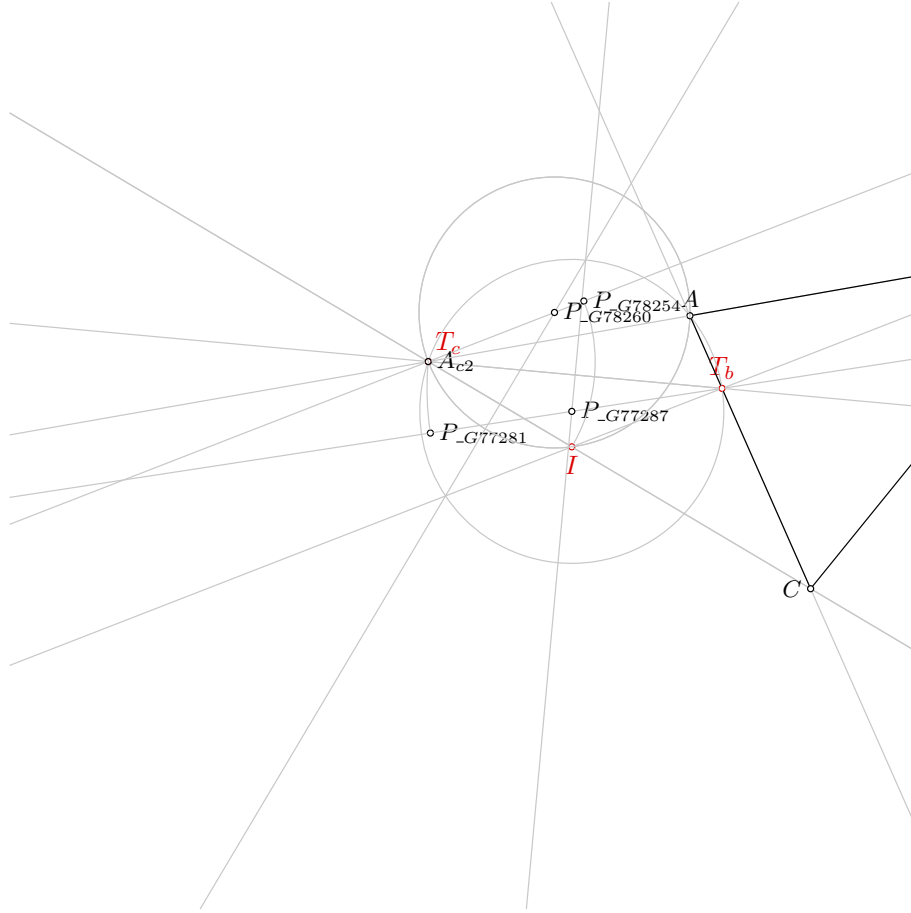


Figure 1: Illustration of the problem 0560

*} and  $T_{\{c\}}$  are not the same*  
*% Determination conditions: lines  $s_{\{c\}}$  and  $b$  are not the same; lines  $s_{\{b\}}$  and  $c$  are not the same;*  
*points  $T_{\{c\}}$  and  $A$  are not the same; points  $A$  and  $T_{\{b\}}$  are not the same; circles  $\text{circle}[T_{\{c\}}$*   
 *$\}, I, \text{angle}[c][s[a]]]$  and  $\text{circle}[T_{\{b\}}, T_{\{c\}}, \text{angle}[c][b]]$  are not the same; points  $T_{\{c\}}$  and  $I$*   
*are not the same; points  $T_{\{b\}}$  and  $I$  are not the same*

### 3.3 Illustration

Illustration of the constructed figure is given in Figure 1

### 3.4 Construction in OpenGeoProver format

## 4 Correctness proof

### 4.1 OGP - Wu method

#### 4.1.1 Proving $T_b = T_b$

Proving failed

#### **4.1.2 Proving $T_c = \neg T_c$**

Proving failed

#### **4.1.3 Proving $I = \neg I$**

Proving failed

### **4.2 GCLC - Area method**

#### **4.2.1 Proving $T_b = \neg T_b$**

Proving failed

#### **4.2.2 Proving $T_c = \neg T_c$**

Proving failed

#### **4.2.3 Proving $I = \neg I$**

Proving failed

### **4.3 GCLC - Wu method**

#### **4.3.1 Proving $T_b = \neg T_b$**

Proving failed

#### **4.3.2 Proving $T_c = \neg T_c$**

Proving failed

#### **4.3.3 Proving $I = \neg I$**

Proving failed

### **4.4 GCLC - Grobner basis method**

#### **4.4.1 Proving $T_b = \neg T_b$**

Proving failed

#### **4.4.2 Proving $T_c = \neg T_c$**

Proving failed

#### **4.4.3 Proving $I = \neg I$**

Proving failed