

Solutions of Triangle Location Construction Problems

Part II - Connelly's corpus

Generated automatically by ArgoTriCS
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Problem 0637	382
Problem 0638	388
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Problem 0650	434
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Problem 0653	454
Problem 0654	455
Problem 0655	462
Problem 0656	469
Problem 0657	476
Problem 0658	483
Problem 0659	490
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Problem 0661	499
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Problem 0664	507
Problem 0665	513
Problem 0666	519
Problem 0667	526
Problem 0668	527
Problem 0669	534
Problem 0670	535
Problem 0671	542
Problem 0672	549
Problem 0673	550
Problem 0674	551
Problem 0675	552
Problem 0676	553
Problem 0677	559

Problem 0678	566
Problem 0679	574
Problem 0680	581
Problem 0681	588
Problem 0682	589
Problem 0683	597
Problem 0684	604
Problem 0685	612
Problem 0686	619
Problem 0687	620
Problem 0688	621
Problem 0689	622
Problem 0690	623
Problem 0691	629
Problem 0692	630
Problem 0693	636
Problem 0694	643
Problem 0695	649
Problem 0696	655
Problem 0697	656
Problem 0698	663
Problem 0699	670
Problem 0700	671
Problem 0701	678
Problem 0702	679

Problem 0703	680
Problem 0704	681
Problem 0705	682
Problem 0706	688
Problem 0707	689
Problem 0708	695
Problem 0709	701
Problem 0710	708
Problem 0711	714
Problem 0712	715
Problem 0713	722
Problem 0714	729
Problem 0715	730
Problem 0716	737
Problem 0717	738
Problem 0718	739
Problem 0719	740
Problem 0720	741
Problem 0721	747
Problem 0722	754
Problem 0723	760
Problem 0724	767
Problem 0725	774
Problem 0726	775
Problem 0727	782

Problem 0728	789
Problem 0729	796
Problem 0730	803
Problem 0731	810
Problem 0732	811
Problem 0733	812
Problem 0734	820
Problem 0735	826
Problem 0736	833
Problem 0737	840
Problem 0738	847
Problem 0739	854
Problem 0740	855
Problem 0741	862
Problem 0742	869
Problem 0743	876
Problem 0744	883
Problem 0745	884
Problem 0746	885
Problem 0747	886
Problem 0748	893
Problem 0749	894
Problem 0750	900
Problem 0751	906
Problem 0752	912

Problem 0753	918
Problem 0754	919
Problem 0755	925
Problem 0756	931
Problem 0757	939
Problem 0758	944
Problem 0759	945
Problem 0760	946
Problem 0761	947
Problem 0762	948
Problem 0763	949
Problem 0764	955
Problem 0765	961
Problem 0766	967
Problem 0767	973
Problem 0768	974
Problem 0769	980
Problem 0770	988
Problem 0771	994
Problem 0772	999
Problem 0773	1000
Problem 0774	1001
Problem 0775	1002
Problem 0776	1003
Problem 0777	1009

Problem 0778	1014
Problem 0779	1015
Problem 0780	1016
Problem 0781	1017
Problem 0782	1023
Problem 0783	1024
Problem 0784	1025
Problem 0785	1031
Problem 0786	1037
Problem 0787	1038
Problem 0788	1039
Problem 0789	1040
Problem 0790	1047
Problem 0791	1053
Problem 0792	1061
Problem 0793	1062
Problem 0794	1068
Problem 0795	1074
Problem 0796	1080
Problem 0797	1086
Problem 0798	1092
Problem 0799	1100
Problem 0800	1101
Problem 0801	1102
Problem 0802	1108

Problem 0803	1114
Problem 0804	1115
Problem 0805	1120
Problem 0806	1126
Problem 0807	1132
Problem 0808	1137
Problem 0809	1143
Problem 0810	1150
Problem 0811	1151
Problem 0812	1152
Problem 0813	1158
Problem 0814	1159
Problem 0815	1166
Problem 0816	1172
Problem 0817	1178
Problem 0818	1185
Problem 0819	1186
Problem 0820	1187
Problem 0821	1188
Problem 0822	1189
Problem 0823	1190
Problem 0824	1197
Problem 0825	1203
Problem 0826	1209
Problem 0827	1216

Problem 0828	1217
Problem 0829	1218
Problem 0830	1219
Problem 0831	1220
Problem 0832	1228
Problem 0833	1229
Problem 0834	1230
Problem 0835	1238
Problem 0836	1239
Problem 0837	1240
Problem 0838	1241
Problem 0839	1242
Problem 0840	1249
Problem 0841	1256
Problem 0842	1263
Problem 0843	1270
Problem 0844	1271
Problem 0845	1272
Problem 0846	1273
Problem 0847	1281
Problem 0848	1287
Problem 0849	1294
Problem 0850	1295
Problem 0851	1296
Problem 0852	1297

Problem 0853	1303
Problem 0854	1310
Problem 0855	1311
Problem 0856	1312
Problem 0857	1313
Problem 0858	1319
Problem 0859	1320
Problem 0860	1321
Problem 0861	1322
Problem 0862	1323
Problem 0863	1324
Problem 0864	1325
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Problem 0867	1328
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Problem 0869	1335
Problem 0870	1341
Problem 0871	1347
Problem 0872	1353
Problem 0873	1354
Problem 0874	1362
Problem 0875	1368
Problem 0876	1374
Problem 0877	1379

Problem 0878	1380
Problem 0879	1381
Problem 0880	1382
Problem 0881	1383
Problem 0882	1389
Problem 0883	1390
Problem 0884	1395
Problem 0885	1396
Problem 0886	1397
Problem 0887	1398
Problem 0888	1404
Problem 0889	1405
Problem 0890	1411
Problem 0891	1417
Problem 0892	1418
Problem 0893	1419
Problem 0894	1420
Problem 0895	1426
Problem 0896	1433
Problem 0897	1441
Problem 0898	1442
Problem 0899	1448
Problem 0900	1454
Problem 0901	1460
Problem 0902	1466

Problem 0903	1472
Problem 0904	1473
Problem 0905	1481
Problem 0906	1482
Problem 0907	1487
Problem 0908	1493
Problem 0909	1494
Problem 0910	1500
Problem 0911	1507
Problem 0912	1513
Problem 0913	1520
Problem 0914	1521
Problem 0915	1522
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Problem 0917	1524
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Problem 0923	1556
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Problem 0925	1563
Problem 0926	1569
Problem 0927	1570

Problem 0928	1571
Problem 0929	1577
Problem 0930	1584
Problem 0931	1590
Problem 0932	1596
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Problem 0934	1598
Problem 0935	1599
Problem 0936	1600
Problem 0937	1601
Problem 0938	1610
Problem 0939	1611
Problem 0940	1619
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Problem 0942	1621
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Problem 0945	1629
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Problem 0947	1643
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Problem 0951	1653
Problem 0952	1659

Problem 0953	1666
Problem 0954	1673
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Problem 0956	1675
Problem 0957	1676
Problem 0958	1681
Problem 0959	1688
Problem 0960	1689
Problem 0961	1690
Problem 0962	1691
Problem 0963	1697
Problem 0964	1698
Problem 0965	1699
Problem 0966	1700
Problem 0967	1701
Problem 0968	1702
Problem 0969	1703
Problem 0970	1704
Problem 0971	1705
Problem 0972	1706
Problem 0973	1712
Problem 0974	1713
Problem 0975	1714
Problem 0976	1719
Problem 0977	1720

Problem 0978	1721
Problem 0979	1722
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Problem 0981	1734
Problem 0982	1740
Problem 0983	1741
Problem 0984	1742
Problem 0985	1743
Problem 0986	1749
Problem 0987	1757
Problem 0988	1764
Problem 0989	1765
Problem 0990	1771
Problem 0991	1777
Problem 0992	1783
Problem 0993	1789
Problem 0994	1795
Problem 0995	1796
Problem 0996	1797
Problem 0997	1805
Problem 0998	1811
Problem 0999	1816
Problem 1000	1817
Problem 1001	1823
Problem 1002	1829

Problem 1003	1836
Problem 1004	1843
Problem 1005	1844
Problem 1006	1845
Problem 1007	1846
Problem 1008	1847
Problem 1009	1852
Problem 1010	1853
Problem 1011	1859
Problem 1012	1865
Problem 1013	1872
Problem 1014	1878
Problem 1015	1879
Problem 1016	1880
Problem 1017	1881
Problem 1018	1882
Problem 1019	1883
Problem 1020	1889
Problem 1021	1895
Problem 1022	1902
Problem 1023	1908
Problem 1024	1914
Problem 1025	1915
Problem 1026	1916
Problem 1027	1922

Problem 1028	1923
Problem 1029	1924
Problem 1030	1933
Problem 1031	1941
Problem 1032	1942
Problem 1033	1943
Problem 1034	1944
Problem 1035	1945
Problem 1036	1953
Problem 1037	1959
Problem 1038	1965
Problem 1039	1972
Problem 1040	1973
Problem 1041	1974
Problem 1042	1975
Problem 1043	1981
Problem 1044	1986
Problem 1045	1993
Problem 1046	1994
Problem 1047	1995
Problem 1048	1996
Problem 1049	2003
Problem 1050	2010
Problem 1051	2011
Problem 1052	2012

Problem 1053	2013
Problem 1054	2019
Problem 1055	2020
Problem 1056	2021
Problem 1057	2022
Problem 1058	2023
Problem 1059	2024
Problem 1060	2025
Problem 1061	2026
Problem 1062	2027
Problem 1063	2028
Problem 1064	2034
Problem 1065	2040
Problem 1066	2046
Problem 1067	2052
Problem 1068	2053
Problem 1069	2059
Problem 1070	2065
Problem 1071	2071
Problem 1072	2077
Problem 1073	2078
Problem 1074	2079
Problem 1075	2080
Problem 1076	2086
Problem 1077	2092

Problem 1078	2098
Problem 1079	2099
Problem 1080	2105
Problem 1081	2111
Problem 1082	2117
Problem 1083	2123
Problem 1084	2124
Problem 1085	2125
Problem 1086	2126
Problem 1087	2132
Problem 1088	2138
Problem 1089	2146
Problem 1090	2152
Problem 1091	2159
Problem 1092	2166
Problem 1093	2172
Problem 1094	2173
Problem 1095	2174
Problem 1096	2175
Problem 1097	2181
Problem 1098	2190
Problem 1099	2197
Problem 1100	2205
Problem 1101	2211
Problem 1102	2217

Problem 1103	2218
Problem 1104	2219
Problem 1105	2220
Problem 1106	2229
Problem 1107	2236
Problem 1108	2242
Problem 1109	2250
Problem 1110	2257
Problem 1111	2258
Problem 1112	2259
Problem 1113	2260
Problem 1114	2268
Problem 1115	2276
Problem 1116	2284
Problem 1117	2285
Problem 1118	2286
Problem 1119	2287
Problem 1120	2288
Problem 1121	2293
Problem 1122	2298
Problem 1123	2305
Problem 1124	2306
Problem 1125	2307
Problem 1126	2308
Problem 1127	2313

Problem 1128	2320
Problem 1129	2321
Problem 1130	2322
Problem 1131	2323
Problem 1132	2330
Problem 1133	2331
Problem 1134	2332
Problem 1135	2333
Problem 1136	2334
Problem 1137	2335
Problem 1138	2336
Problem 1139	2337
Problem 1140	2338

Problem 561

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1 Problem

Problem 561: Given a point A , a point B and a point E_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point E_a , construct a point H (rule W01); ;
2. Using the point A and the point B , construct a line c (rule W02); % DET: points A and B are not the same;
3. Using the point B and the point H , construct a line h_b (rule W02); % DET: points B and H are not the same;
4. Using the point A and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points A and E_a are not the same;
5. Using the circle $k(E_a, A)$, the line c , the point E_a and the point A , construct a point H_c (rule W05); % NDG: line c and circle $k(E_a, A)$ intersect % DET: points A and H_c must be different;
6. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
7. Using the circle $k(E_a, A)$, the line h_b , the point E_a and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_a, A)$ intersect % DET: points H and H_b must be different;
8. Using the point H_b and the point A , construct a line b (rule W02); % DET: points H_b and A are not the same;
9. Using the line h_c and the line b , construct a point C (rule W03); % NDG: lines h_c and b are not parallel % DET: lines h_c and b are not the same.

Non-degenerate conditions: lines h_c and b are not parallel; line h_b and circle $k(E_a, A)$ intersect; line c and circle $k(E_a, A)$ intersect; points A and E_a are not the same.

Determination conditions: lines h_c and b are not the same; points H_b and A are not the same; points H and H_b must be different; points H_c and H are not the same; points A and H_c must be different; points B and H are not the same; points A and B are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D28,D3,D6,D7,D9,GD01,GD02,GL03,L3,L46,L47,L48]

Solving time: 11.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
```

```
point B 20 40
```

```
point E_{a} 80 83.86
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_t A
```

```
cmark_b B
```

```
cmark_r E_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point H such that AH/AE_{a}=2
```

```
towards H A E_{a} 2
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment A H
```

```
color 0 0 0
```

```
% DET: points A and B are not the same
```

```
% Constructing a line c which passes through point A and point B
```

```
line c A B
```

```
color 200 200 200
```

```
drawline c
```

```
color 0 0 0
```

```
% DET: points B and H are not the same
```

```
% Constructing a line h_{b} which passes through point B and point H
```

```
line h_{b} B H
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```

% NDG: points A and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point A
circle k(E_{a},A) E_{a} A

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: line c and circle k(E_{a},A) intersect% DET: points A and H_{c} must be different
% Constructing a point P_{\G13260} which is a foot of the point E_{a} on the line c
foot P_{\G13260} E_{a} c
cmark_r P_{\G13260}
color 200 200 200
drawline E_{a} P_{\G13260}
color 0 0 0

% Constructing a point H_{c} which is an image of the point A in the symmetry to point/line P_{\G
13260}
sim H_{c} P_{\G13260} A
cmark_rt H_{c}

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: line h_{b} and circle k(E_{a},A) intersect% DET: points H and H_{b} must be different
% Constructing a point P_{\G13498} which is a foot of the point E_{a} on the line h_{b}
foot P_{\G13498} E_{a} h_{b}
cmark_r P_{\G13498}
color 200 200 200
drawline E_{a} P_{\G13498}
color 0 0 0

% Constructing a point H_{b} which is an image of the point H in the symmetry to point/line P_{\G
13498}
sim H_{b} P_{\G13498} H
cmark_l H_{b}

% DET: points H_{b} and A are not the same
% Constructing a line b which passes through point H_{b} and point A
line b H_{b} A

color 200 200 200
drawline b
color 0 0 0

```

```

% NDG: lines  $h_{\{c\}}$  and  $b$  are not parallel% DET: lines  $h_{\{c\}}$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $h_{\{c\}}$  and line  $b$ 
intersec C  $h_{\{c\}}$  b
cmark_1 C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{c\}}$  and  $b$  are not parallel; line  $h_{\{b\}}$  and circle  $k(E_{\{a\}}, A)$ 
% intersect; line  $c$  and circle  $k(E_{\{a\}}, A)$  intersect; points  $A$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $h_{\{c\}}$  and  $b$  are not the same; points  $H_{\{b\}}$  and  $A$  are not the same
% ; points  $H$  and  $H_{\{b\}}$  must be different; points  $H_{\{c\}}$  and  $H$  are not the same; points  $A$  and  $H_{\{c\}}$ 
% must be different; points  $B$  and  $H$  are not the same; points  $A$  and  $B$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $E_a = \neg E_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_c H_b A} \neq S_{H H_b A}$ i.e., lines $H_c H$ and $H_b A$ are not parallel (construction based assumption)

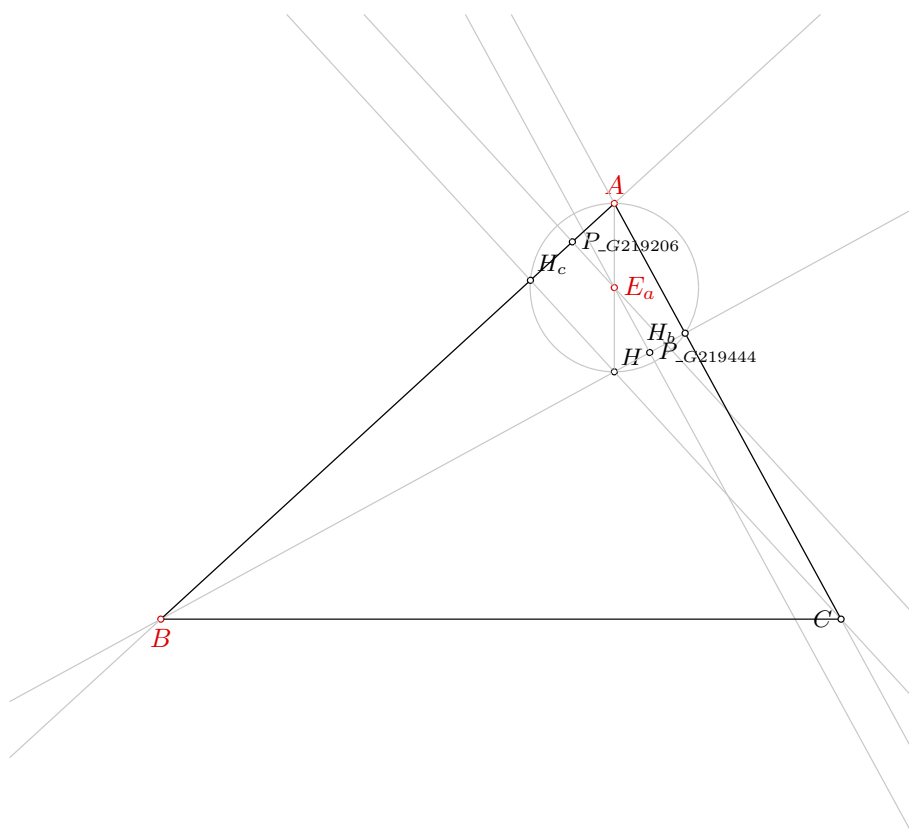


Figure 1: Illustration of the problem 0561

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a}BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $B=B$

NDG conditions are:

$S_{H_cH_bA} \neq S_{HH_bA}$ i.e., lines H_cH and H_bA are not parallel (construction based assumption)
 $S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a}BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.3 Proving $E_a=E_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.3 Proving $E_a=E_a$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 21 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.3 Proving $E_a = E_a$

Proving failed

Problem 562

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Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 562: Given a point A , a point B and a point E_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point B , construct a point M_c (rule W01); ;
2. Using the point B and the point E_b , construct a point H (rule W01); ;
3. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
4. Using the point B and the point E_b , construct a line h_b (rule W02); % DET: points B and E_b are not the same;
5. Using the point A and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points A and M_c are not the same;
6. Using the circle $k(M_c, A)$, the line h_a , the point M_c and the point A , construct a point H_a (rule W05); % NDG: line h_a and circle $k(M_c, A)$ intersect % DET: points A and H_a must be different;
7. Using the point H_a and the point B , construct a line a (rule W02); % DET: points H_a and B are not the same;
8. Using the circle $k(M_c, A)$, the line h_b , the point M_c and the point B , construct a point H_b (rule W05); % NDG: line h_b and circle $k(M_c, A)$ intersect % DET: points B and H_b must be different;
9. Using the point H_b and the point A , construct a line b (rule W02); % DET: points H_b and A are not the same;
10. Using the line a and the line b , construct a point C (rule W03); % NDG: lines a and b are not parallel % DET: lines a and b are not the same.

Non-degenerate conditions: lines a and b are not parallel; line h_b and circle $k(M_c, A)$ intersect; line h_a and circle $k(M_c, A)$ intersect; points A and M_c are not the same.

Determination conditions: lines a and b are not the same; points H_b and A are not the same; points B and H_b must be different; points H_a and B are not the same; points A and H_a must be different; points B and E_b are not the same; points A and H are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D20,D29,D3,D5,D6,D8,D9,GD01,GD02,GL03,GL09,L40,L41,L42]

Solving time: 10.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
```

```
point B 20 40
```

```
point E_{b} 50 56.36
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_t A
```

```
cmark_b B
```

```
cmark_r E_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point M_{c} such that AM_{c}/AB=0.5
```

```
towards M_{c} A B 0.5
```

```
cmark_lt M_{c}
```

```
color 200 200 200
```

```
drawsegment A B
```

```
color 0 0 0
```

```
% Constructing a point H such that BH/BE_{b}=2
```

```
towards H B E_{b} 2
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment B H
```

```
color 0 0 0
```

```
% DET: points A and H are not the same
```

```
% Constructing a line h_{a} which passes through point A and point H
```

```
line h_{a} A H
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```

% DET: points B and E_{b} are not the same
% Constructing a line h_{b} which passes through point B and point E_{b}
line h_{b} B E_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points A and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point A
circle k(M_{c},A) M_{c} A

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line h_{a} and circle k(M_{c},A) intersect% DET: points A and H_{a} must be different
% Constructing a point P_{\_G33470} which is a foot of the point M_{c} on the line h_{a}
foot P_{\_G33470} M_{c} h_{a}
cmark_r P_{\_G33470}
color 200 200 200
drawline M_{c} P_{\_G33470}
color 0 0 0

% Constructing a point H_{a} which is an image of the point A in the symmetry to point/line P_{\_G
33470}
sim H_{a} P_{\_G33470} A
cmark_r H_{a}

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% NDG: line h_{b} and circle k(M_{c},A) intersect% DET: points B and H_{b} must be different
% Constructing a point P_{\_G33708} which is a foot of the point M_{c} on the line h_{b}
foot P_{\_G33708} M_{c} h_{b}
cmark_r P_{\_G33708}
color 200 200 200
drawline M_{c} P_{\_G33708}
color 0 0 0

% Constructing a point H_{b} which is an image of the point B in the symmetry to point/line P_{\_G
33708}
sim H_{b} P_{\_G33708} B
cmark_l H_{b}

```

```

% DET: points  $H_{\{b\}}$  and  $A$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $A$ 
line b  $H_{\{b\}}$  A

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $a$  and  $b$  are not parallel% DET: lines  $a$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $a$  and line  $b$ 
intersec C a b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; line  $h_{\{b\}}$  and circle  $k(M_{\{c\}},A)$ 
% intersect; line  $h_{\{a\}}$  and circle  $k(M_{\{c\}},A)$  intersect; points  $A$  and  $M_{\{c\}}$  are not the same
% Determination conditions: lines  $a$  and  $b$  are not the same; points  $H_{\{b\}}$  and  $A$  are not the same;
% points  $B$  and  $H_{\{b\}}$  must be different; points  $H_{\{a\}}$  and  $B$  are not the same; points  $A$  and  $H_{\{a\}}$ 
% must be different; points  $B$  and  $E_{\{b\}}$  are not the same; points  $A$  and  $H$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.027 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

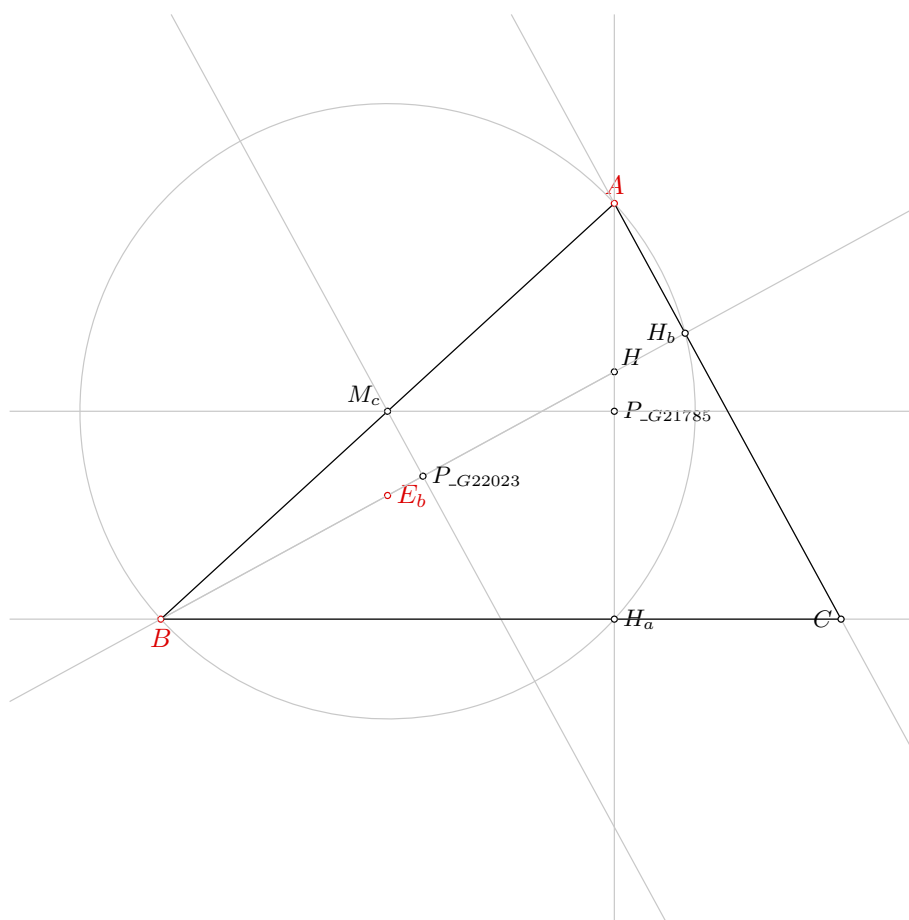


Figure 1: Illustration of the problem 0562

4.1.3 Proving $E_b = \neg E_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

- $S_{H_a H_b A} \neq S_{B H_b A}$ i.e., lines $H_a B$ and $H_b A$ are not parallel (construction based assumption)
- $S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)
- $S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)
- $S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $B=B$

NDG conditions are:

- $S_{H_a H_b A} \neq S_{B H_b A}$ i.e., lines $H_a B$ and $H_b A$ are not parallel (construction based assumption)
- $S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)
- $S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)
- $S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.3 Proving $E_b = \neg E_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.3 Proving $E_b = \neg E_b$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.3 Proving $E_b=\neg E_b$

Proving failed

Problem 563

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 563: Given a point A , a point B and a point E_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point B , construct a point M_c (rule W01); ;
2. Using the point A and the point B , construct a line c (rule W02); % DET: points A and B are not the same;
3. Using the point E_c and the point M_c , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and M_c are not the same;
4. Using the point E_c and the line c , construct a line h_c (rule W10b); ;
5. Using the line h_c and the line c , construct a point H_c (rule W03); % NDG: lines h_c and c are not parallel % DET: lines h_c and c are not the same;
6. Using the point A and the point H_c , construct a line $m(AH_c)$ (rule W14); % DET: points A and H_c are not the same;
7. Using the point E_c and the point M_c , construct a line $m(E_cM_c)$ (rule W14); % DET: points E_c and M_c are not the same;
8. Using the line $m(E_cM_c)$ and the line $m(H_bH_a)$, construct a point N (rule W03); % NDG: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel % DET: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same;
9. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
10. Using the circle $k(N, M_a)$ and the line $m(AH_c)$, construct a point M_b and a point E_a (rule W04); % NDG: line $m(AH_c)$ and circle $k(N, M_a)$ intersect;

11. Using the point M_b and the point A , construct a point C (rule W01); .

Non-degenerate conditions: line $m(AH_c)$ and circle $k(N, M_a)$ intersect; points E_c and N are not the same; lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel; lines h_c and c are not parallel.

Determination conditions: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same; points E_c and M_c are not the same; points A and H_c are not the same; lines h_c and c are not the same; points E_c and M_c are not the same; points A and B are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14]

Lemmas used: [D10,D20,D22,D30,D32,D7,GD01,GD02,GL01,GL03,GL04,GL09,L17,L18,L19,L20,L22,L24,L3,L10]

Solving time: 2.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point B 20 40
point E_{c} 95 56.36
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_b B
cmark_r E_{c}
color 0 0 0
fontsize 8
```

```
% Constructing a point M_{c} such that AM_{c}/AB=0.5
towards M_{c} A B 0.5
cmark_lt M_{c}
color 200 200 200
drawsegment A B
color 0 0 0
```

```
% DET: points A and B are not the same
% Constructing a line c which passes through point A and point B
line c A B
```

```
color 200 200 200
drawline c
color 0 0 0
```

```
% DET: points E_{c} and M_{c} are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point M_{c}
line m(H_{b}H_{a}) E_{c} M_{c}
```

```
color 200 200 200
drawline m(H_{b}H_{a})
```



```
color 0 0 0
```

```
% Constructing a line  $h_{\{c\}}$  which is perpendicular to line  $c$  and which passes through point  $E_{\{c\}}$   
perp  $h_{\{c\}}$   $E_{\{c\}}$   $c$ 
```

```
color 200 200 200  
drawline  $h_{\{c\}}$   
color 0 0 0
```

```
% NDG: lines  $h_{\{c\}}$  and  $c$  are not parallel% DET: lines  $h_{\{c\}}$  and  $c$  are not the same  
% Constructing a point  $H_{\{c\}}$  which belongs to line  $h_{\{c\}}$  and line  $c$   
intersec  $H_{\{c\}}$   $h_{\{c\}}$   $c$   
cmark_rt  $H_{\{c\}}$ 
```

```
% DET: points  $A$  and  $H_{\{c\}}$  are not the same  
% Constructing bisector  $m(AH_{\{c\}})$  of the segment  $AH_{\{c\}}$   
med  $m(AH_{\{c\}})$   $A$   $H_{\{c\}}$ 
```

```
color 200 200 200  
drawline  $m(AH_{\{c\}})$   
color 0 0 0
```

```
color 200 200 200  
drawsegment  $A$   $H_{\{c\}}$   
color 0 0 0
```

```
% DET: points  $E_{\{c\}}$  and  $M_{\{c\}}$  are not the same  
% Constructing bisector  $m(E_{\{c\}}M_{\{c\}})$  of the segment  $E_{\{c\}}M_{\{c\}}$   
med  $m(E_{\{c\}}M_{\{c\}})$   $E_{\{c\}}$   $M_{\{c\}}$ 
```

```
color 200 200 200  
drawline  $m(E_{\{c\}}M_{\{c\}})$   
color 0 0 0
```

```
color 200 200 200  
drawsegment  $E_{\{c\}}$   $M_{\{c\}}$   
color 0 0 0
```

```
% NDG: lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not parallel% DET: lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$   
 $H_{\{a\}}$  are not the same  
% Constructing a point  $N$  which belongs to line  $m(E_{\{c\}}M_{\{c\}})$  and line  $m(H_{\{b\}}H_{\{a\}})$   
intersec  $N$   $m(E_{\{c\}}M_{\{c\}})$   $m(H_{\{b\}}H_{\{a\}})$   
cmark_r  $N$ 
```

```
% NDG: points  $E_{\{c\}}$  and  $N$  are not the same  
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{c\}}$   
circle  $k(N, M_{\{a\}})$   $N$   $E_{\{c\}}$ 
```

```

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(AH_{c}) and circle k(N,M_{a}) intersect
% Constructing points M_{b} and E_{a} which are in intersection of k(N,M_{a}) and m(AH_{c})
intersec2 M_{b} E_{a} k(N,M_{a}) m(AH_{c})
cmark_lt M_{b}
cmark_r E_{a}

% Constructing a point C such that M_{b}C/M_{b}A=-1
towards C M_{b} A -1
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line m(AH_{c}) and circle k(N,M_{a}) intersect; points E_{c} and N are
% not the same; lines m(E_{c}M_{c}) and m(H_{b}H_{a}) are not parallel; lines h_{c} and c are
% not parallel
% Determination conditions: lines m(E_{c}M_{c}) and m(H_{b}H_{a}) are not the same; points E_{c}
% and M_{c} are not the same; points A and H_{c} are not the same; lines h_{c} and c are not the
% same; points E_{c} and M_{c} are not the same; points A and B are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.026 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $B=B$

Status: Theorem has been proved.

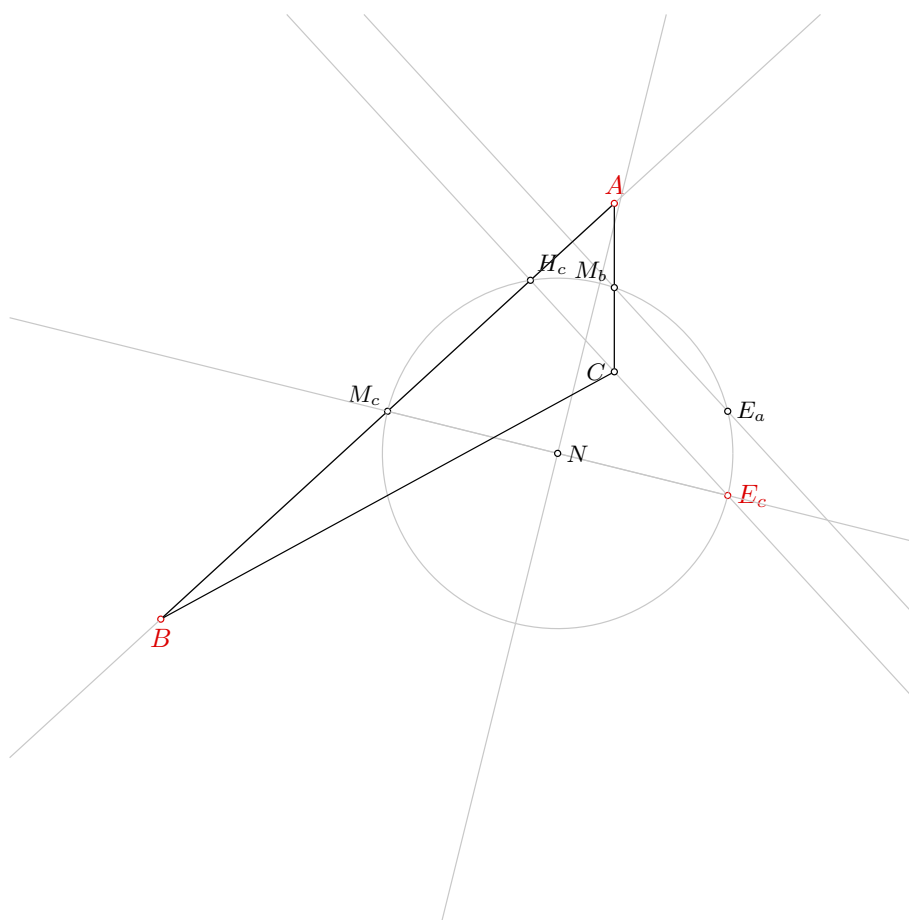


Figure 1: Illustration of the problem 0563

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.025 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $E_c = \neg E_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 36 terms.

Time Complexity: Time spent by the prover is 0.657 seconds.

NDG conditions Points E_c and M_c are not identical

Line through points A and H_c is not perpendicular to line through points H_c and B

Points A and B are not identical

Points B , C and M_c are not collinear

Line through points C and B is not perpendicular to line through points B and A

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{E_cAB} \neq 0$ i.e., points E_c , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{E_cAB} \neq S_{F_{h_c}^0 AB}$ i.e., lines $E_cF_{h_c}^0$ and AB are not parallel (construction based assumption)

$S_{M_{m(E_cM_c)}^3 E_cM_c} \neq S_{T_{m(E_cM_c)}^4 E_cM_c}$ i.e., lines $M_{m(E_cM_c)}^3 T_{m(E_cM_c)}^4$ and E_cM_c are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^6} \neq S_{F_{h_a}^5 BF_{h_b}^6}$ i.e., lines $AF_{h_a}^5$ and $BF_{h_b}^6$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $B=B$

NDG conditions are:

$S_{E_cAB} \neq 0$ i.e., points E_c , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{E_cAB} \neq S_{F_{h_c}^0 AB}$ i.e., lines $E_cF_{h_c}^0$ and AB are not parallel (construction based assumption)

$S_{M_{m(E_cM_c)}^3 E_cM_c} \neq S_{T_{m(E_cM_c)}^4 E_cM_c}$ i.e., lines $M_{m(E_cM_c)}^3 T_{m(E_cM_c)}^4$ and E_cM_c are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^6} \neq S_{F_{h_a}^5 BF_{h_b}^6}$ i.e., lines $AF_{h_a}^5$ and $BF_{h_b}^6$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.3 Proving $E_c = \neg E_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.3 Proving $E_c=_E_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 58 terms.

Time Complexity: Time spent by the prover is 0.050 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.3 Proving $E_c=_E_c$

Proving failed

Problem 564

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 564: Given a point A , a point B and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point B , construct a point M_c (rule W01); ;
2. Using the point A and the point B , construct a line c (rule W02); % DET: points A and B are not the same;
3. Using the point N and the point M_c , construct a line $m(H_aH_b)$ (rule W02); % DET: points N and M_c are not the same;
4. Using the point A and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points A and M_c are not the same;
5. Using the point M_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_c and N are not the same;
6. Using the circle $k(N, M_a)$, the line c , the point N and the point M_c , construct a point H_c (rule W05); % NDG: line c and circle $k(N, M_a)$ intersect % DET: points M_c and H_c must be different;
7. Using the circle $k(N, M_a)$, the line $m(H_aH_b)$, the point N and the point M_c , construct a point E_c (rule W05a); % NDG: line $m(H_aH_b)$ and circle $k(N, M_a)$ intersect % DET: points M_c and E_c must be different;
8. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;
9. Using the circle $k(M_c, A)$ and the circle $k(N, M_a)$, construct a point H_b and a point H_a (rule W07); % NDG: circles $k(M_c, A)$ and $k(N, M_a)$ intersect % DET: circles $k(M_c, A)$ and $k(N, M_a)$ are not the same;

10. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;
11. Using the line h_c and the line b , construct a point C (rule W03); % NDG: lines h_c and b are not parallel % DET: lines h_c and b are not the same.

Non-degenerate conditions: lines h_c and b are not parallel; circles $k(M_c, A)$ and $k(N, M_a)$ intersect; line $m(H_a H_b)$ and circle $k(N, M_a)$ intersect; line c and circle $k(N, M_a)$ intersect; points M_c and N are not the same; points A and M_c are not the same.

Determination conditions: lines h_c and b are not the same; points A and H_b are not the same; circles $k(M_c, A)$ and $k(N, M_a)$ are not the same; points E_c and H_c are not the same; points M_c and E_c must be different; points M_c and H_c must be different; points N and M_c are not the same; points A and B are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D10,D20,D30,D32,D6,D7,GD01,GD02,GL01,GL09,L18,L19,L20,L21,L24,L3,L41,L42,L53,L54]

Solving time: 4.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point B 20 40
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_b B
cmark_r N
color 0 0 0
fontsize 8
```

```
% Constructing a point M_{c} such that AM_{c}/AB=0.5
towards M_{c} A B 0.5
cmark_lt M_{c}
color 200 200 200
drawsegment A B
color 0 0 0
```

```
% DET: points A and B are not the same
% Constructing a line c which passes through point A and point B
line c A B
```

```
color 200 200 200
drawline c
color 0 0 0
```

```

% DET: points N and M_{c} are not the same
% Constructing a line m(H_{a}H_{b}) which passes through point N and point M_{c}
line m(H_{a}H_{b}) N M_{c}

color 200 200 200
drawline m(H_{a}H_{b})
color 0 0 0

% NDG: points A and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point A
circle k(M_{c},A) M_{c} A

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: points M_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{c}
circle k(N,M_{a}) N M_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line c and circle k(N,M_{a}) intersect% DET: points M_{c} and H_{c} must be different
% Constructing a point P_{\G84691} which is a foot of the point N on the line c
foot P_{\G84691} N c
cmark_r P_{\G84691}
color 200 200 200
drawline N P_{\G84691}
color 0 0 0

% Constructing a point H_{c} which is an image of the point M_{c} in the symmetry to point/line P
_{\G84691}
sim H_{c} P_{\G84691} M_{c}
cmark_rt H_{c}

% NDG: line m(H_{a}H_{b}) and circle k(N,M_{a}) intersect% DET: points M_{c} and E_{c} must be
different
% Constructing a point E_{c} which is an image of the point M_{c} in the symmetry to point/line N
sim E_{c} N M_{c}
cmark_r E_{c}

% DET: points E_{c} and H_{c} are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H_{c}
line h_{c} E_{c} H_{c}

```



```

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: circles  $k(M_{\{c\}}, A)$  and  $k(N, M_{\{a\}})$  intersect% DET: circles  $k(M_{\{c\}}, A)$  and  $k(N, M_{\{a\}})$  are not
the same
% Constructing points  $H_{\{b\}}$  and  $H_{\{a\}}$  which are in intersection of  $k(M_{\{c\}}, A)$  and  $k(N, M_{\{a\}})$ 
intersec2 H_{b} H_{a} k(M_{\{c\}}, A) k(N, M_{\{a\}})
cmark_l H_{b}
cmark_r H_{a}

% DET: points  $A$  and  $H_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $A$  and point  $H_{\{b\}}$ 
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $h_{\{c\}}$  and  $b$  are not parallel% DET: lines  $h_{\{c\}}$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $h_{\{c\}}$  and line  $b$ 
intersec C h_{c} b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{c\}}$  and  $b$  are not parallel; circles  $k(M_{\{c\}}, A)$  and  $k(N, M_{\{a\}})$ 
intersect; line  $m(H_{\{a\}}H_{\{b\}})$  and circle  $k(N, M_{\{a\}})$  intersect; line  $c$  and circle  $k(N, M_{\{a\}})$ 
intersect; points  $M_{\{c\}}$  and  $N$  are not the same; points  $A$  and  $M_{\{c\}}$  are not the same
% Determination conditions: lines  $h_{\{c\}}$  and  $b$  are not the same; points  $A$  and  $H_{\{b\}}$  are not the same
; circles  $k(M_{\{c\}}, A)$  and  $k(N, M_{\{a\}})$  are not the same; points  $E_{\{c\}}$  and  $H_{\{c\}}$  are not the same;
points  $M_{\{c\}}$  and  $E_{\{c\}}$  must be different; points  $M_{\{c\}}$  and  $H_{\{c\}}$  must be different; points  $N$ 
and  $M_{\{c\}}$  are not the same; points  $A$  and  $B$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

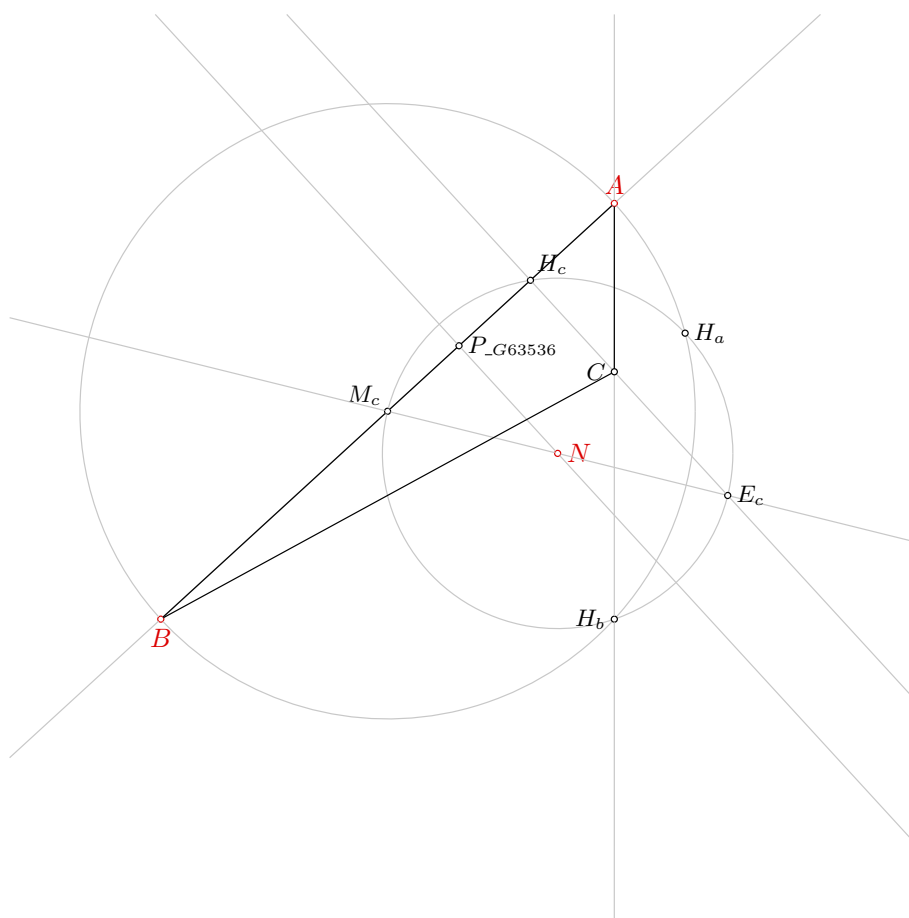


Figure 1: Illustration of the problem 0564

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $N=N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{E_cAH_b} \neq S_{H_cAH_b}$ i.e., lines E_cH_c and AH_b are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points M_a , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points M_b , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{F_{-h_a}^2BF_{-h_b}^3}$ i.e., lines $AF_{-h_a}^2$ and $BF_{-h_b}^3$ are not parallel (construction based assumption)

$S_{M_a-M_bF_{-m_b}^1} \neq S_{F_{-m_a}^0-M_bF_{-m_b}^1}$ i.e., lines $M_aF_{-m_a}^0$ and $M_bF_{-m_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $B=B$

NDG conditions are:

$S_{E_cAH_b} \neq S_{H_cAH_b}$ i.e., lines E_cH_c and AH_b are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points M_a , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points M_b , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{F_{-h_a}^2BF_{-h_b}^3}$ i.e., lines $AF_{-h_a}^2$ and $BF_{-h_b}^3$ are not parallel (construction based assumption)

$S_{M_a-M_bF_{-m_b}^1} \neq S_{F_{-m_a}^0-M_bF_{-m_b}^1}$ i.e., lines $M_aF_{-m_a}^0$ and $M_bF_{-m_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.3 Proving $N=N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.3 Proving $N=_N$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 778 terms.

Time Complexity: Time spent by the prover is 2.110 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.3 Proving $N=_N$

Proving failed

Problem 565

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 565: Given a point A , a point C and a point E_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point E_a , construct a point H (rule W01); ;
2. Using the point A and the point C , construct a line b (rule W02); % DET: points A and C are not the same;
3. Using the point C and the point H , construct a line h_c (rule W02); % DET: points C and H are not the same;
4. Using the point A and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points A and E_a are not the same;
5. Using the circle $k(E_a, A)$, the line b , the point E_a and the point A , construct a point H_b (rule W05); % NDG: line b and circle $k(E_a, A)$ intersect % DET: points A and H_b must be different;
6. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
7. Using the circle $k(E_a, A)$, the line h_c , the point E_a and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_a, A)$ intersect % DET: points H and H_c must be different;
8. Using the point H_c and the point A , construct a line c (rule W02); % DET: points H_c and A are not the same;
9. Using the line h_b and the line c , construct a point B (rule W03); % NDG: lines h_b and c are not parallel % DET: lines h_b and c are not the same.

Non-degenerate conditions: lines h_b and c are not parallel; line h_c and circle $k(E_a, A)$ intersect; line b and circle $k(E_a, A)$ intersect; points A and E_a are not the same.

Determination conditions: lines h_b and c are not the same; points H_c and A are not the same; points H and H_c must be different; points H_b and H are not the same; points A and H_b must be different; points C and H are not the same; points A and C are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D28,D3,D6,D7,D9,GD01,GD02,GL03,L3,L46,L47,L48]

Solving time: 11.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point C 110 40
point E_{a} 80 83.86
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_l C
cmark_r E_{a}
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0
```

```
% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% DET: points C and H are not the same
% Constructing a line h_{c} which passes through point C and point H
line h_{c} C H
```

```
color 200 200 200
drawline h_{c}
color 0 0 0
```

```

% NDG: points A and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point A
circle k(E_{a},A) E_{a} A

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: line b and circle k(E_{a},A) intersect% DET: points A and H_{b} must be different
% Constructing a point P_{\_G104880} which is a foot of the point E_{a} on the line b
foot P_{\_G104880} E_{a} b
cmark_r P_{\_G104880}
color 200 200 200
drawline E_{a} P_{\_G104880}
color 0 0 0

% Constructing a point H_{b} which is an image of the point A in the symmetry to point/line P_{\_G
104880}
sim H_{b} P_{\_G104880} A
cmark_l H_{b}

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{c} and circle k(E_{a},A) intersect% DET: points H and H_{c} must be different
% Constructing a point P_{\_G105118} which is a foot of the point E_{a} on the line h_{c}
foot P_{\_G105118} E_{a} h_{c}
cmark_r P_{\_G105118}
color 200 200 200
drawline E_{a} P_{\_G105118}
color 0 0 0

% Constructing a point H_{c} which is an image of the point H in the symmetry to point/line P_{\_G
105118}
sim H_{c} P_{\_G105118} H
cmark_rt H_{c}

% DET: points H_{c} and A are not the same
% Constructing a line c which passes through point H_{c} and point A
line c H_{c} A

color 200 200 200
drawline c
color 0 0 0

```

```

% NDG: lines  $h_{\{b\}}$  and  $c$  are not parallel% DET: lines  $h_{\{b\}}$  and  $c$  are not the same
% Constructing a point  $B$  which belongs to line  $h_{\{b\}}$  and line  $c$ 
intersec B  $h_{\{b\}}$  c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{b\}}$  and  $c$  are not parallel; line  $h_{\{c\}}$  and circle  $k(E_{\{a\}}, A)$ 
% intersect; line  $b$  and circle  $k(E_{\{a\}}, A)$  intersect; points  $A$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $h_{\{b\}}$  and  $c$  are not the same; points  $H_{\{c\}}$  and  $A$  are not the same
% ; points  $H$  and  $H_{\{c\}}$  must be different; points  $H_{\{b\}}$  and  $H$  are not the same; points  $A$  and  $H_{\{b\}}$ 
% must be different; points  $C$  and  $H$  are not the same; points  $A$  and  $C$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.023 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.027 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $E_a = \neg E_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_b H_c A} \neq S_{H H_c A}$ i.e., lines $H_b H$ and $H_c A$ are not parallel (construction based assumption)

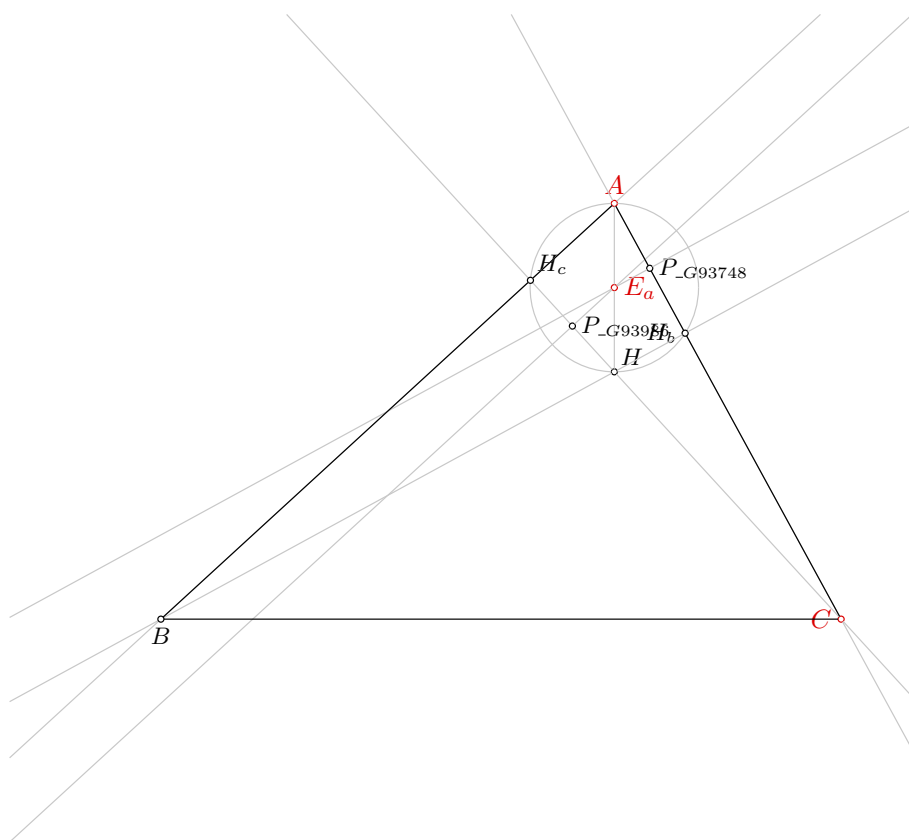


Figure 1: Illustration of the problem 0565

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a}BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $C=C$

NDG conditions are:

$S_{H_bH_cA} \neq S_{HH_cA}$ i.e., lines H_bH and H_cA are not parallel (construction based assumption)
 $S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a}BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.3 Proving $E_a=E_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.3 Proving $E_a=E_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 9 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.3 Proving $E_a = E_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 51 terms.

Time Complexity: Time spent by the prover is 0.260 seconds. There are no ndg conditions.

Problem 566

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 566: Given a point A , a point C and a point E_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point C , construct a point M_b (rule W01); ;
2. Using the point A and the point C , construct a line b (rule W02); % DET: points A and C are not the same;
3. Using the point E_b and the point M_b , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and M_b are not the same;
4. Using the point E_b and the line b , construct a line h_b (rule W10b); ;
5. Using the line h_b and the line b , construct a point H_b (rule W03); % NDG: lines h_b and b are not parallel % DET: lines h_b and b are not the same;
6. Using the point A and the point H_b , construct a line $m(AH_b)$ (rule W14); % DET: points A and H_b are not the same;
7. Using the point E_b and the point M_b , construct a line $m(E_bM_b)$ (rule W14); % DET: points E_b and M_b are not the same;
8. Using the line $m(E_bM_b)$ and the line $m(H_aH_c)$, construct a point N (rule W03); % NDG: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel % DET: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same;
9. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
10. Using the circle $k(N, M_a)$ and the line $m(AH_b)$, construct a point M_c and a point E_a (rule W04); % NDG: line $m(AH_b)$ and circle $k(N, M_a)$ intersect;

11. Using the point M_c and the point A , construct a point B (rule W01); .

Non-degenerate conditions: line $m(AH_b)$ and circle $k(N, M_a)$ intersect; points E_b and N are not the same; lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel; lines h_b and b are not parallel.

Determination conditions: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same; points E_b and M_b are not the same; points A and H_b are not the same; lines h_b and b are not the same; points E_b and M_b are not the same; points A and C are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14]

Lemmas used: [D20,D22,D29,D3,D32,D6,D9,GD01,GD02,GL01,GL03,GL04,GL09,L17,L18,L19,L21,L22,L23,L

Solving time: 2.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point C 110 40
point E_{b} 50 56.36
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_l C
cmark_r E_{b}
color 0 0 0
fontsize 8
```

```
% Constructing a point M_{b} such that AM_{b}/AC=0.5
towards M_{b} A C 0.5
cmark_lt M_{b}
color 200 200 200
drawsegment A C
color 0 0 0
```

```
% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% DET: points E_{b} and M_{b} are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point M_{b}
line m(H_{a}H_{c}) E_{b} M_{b}
```

```
color 200 200 200
drawline m(H_{a}H_{c})
```

```
color 0 0 0
```

```
% Constructing a line  $h_{\{b\}}$  which is perpendicular to line  $b$  and which passes through point  $E_{\{b\}}$ 
perp  $h_{\{b\}}$   $E_{\{b\}}$   $b$ 
```

```
color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0
```

```
% NDG: lines  $h_{\{b\}}$  and  $b$  are not parallel% DET: lines  $h_{\{b\}}$  and  $b$  are not the same
% Constructing a point  $H_{\{b\}}$  which belongs to line  $h_{\{b\}}$  and line  $b$ 
intersec  $H_{\{b\}}$   $h_{\{b\}}$   $b$ 
cmark_l  $H_{\{b\}}$ 
```

```
% DET: points  $A$  and  $H_{\{b\}}$  are not the same
% Constructing bisector  $m(AH_{\{b\}})$  of the segment  $AH_{\{b\}}$ 
med  $m(AH_{\{b\}})$   $A$   $H_{\{b\}}$ 
```

```
color 200 200 200
drawline  $m(AH_{\{b\}})$ 
color 0 0 0
```

```
color 200 200 200
drawsegment  $A$   $H_{\{b\}}$ 
color 0 0 0
```

```
% DET: points  $E_{\{b\}}$  and  $M_{\{b\}}$  are not the same
% Constructing bisector  $m(E_{\{b\}}M_{\{b\}})$  of the segment  $E_{\{b\}}M_{\{b\}}$ 
med  $m(E_{\{b\}}M_{\{b\}})$   $E_{\{b\}}$   $M_{\{b\}}$ 
```

```
color 200 200 200
drawline  $m(E_{\{b\}}M_{\{b\}})$ 
color 0 0 0
```

```
color 200 200 200
drawsegment  $E_{\{b\}}$   $M_{\{b\}}$ 
color 0 0 0
```

```
% NDG: lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not parallel% DET: lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}$ 
 $H_{\{c\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{b\}}M_{\{b\}})$  and line  $m(H_{\{a\}}H_{\{c\}})$ 
intersec  $N$   $m(E_{\{b\}}M_{\{b\}})$   $m(H_{\{a\}}H_{\{c\}})$ 
cmark_r  $N$ 
```

```
% NDG: points  $E_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{b\}}$ 
circle  $k(N, M_{\{a\}})$   $N$   $E_{\{b\}}$ 
```

```

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(AH_{b}) and circle k(N,M_{a}) intersect
% Constructing points M_{c} and E_{a} which are in intersection of k(N,M_{a}) and m(AH_{b})
intersec2 M_{c} E_{a} k(N,M_{a}) m(AH_{b})
cmark_lt M_{c}
cmark_r E_{a}

% Constructing a point B such that M_{c}B/M_{c}A=-1
towards B M_{c} A -1
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line m(AH_{b}) and circle k(N,M_{a}) intersect; points E_{b} and N are
% not the same; lines m(E_{b}M_{b}) and m(H_{a}H_{c}) are not parallel; lines h_{b} and b are
% not parallel
% Determination conditions: lines m(E_{b}M_{b}) and m(H_{a}H_{c}) are not the same; points E_{b}
% and M_{b} are not the same; points A and H_{b} are not the same; lines h_{b} and b are not the
% same; points E_{b} and M_{b} are not the same; points A and C are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

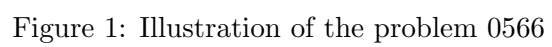
Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.026 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $C=C$

Status: Theorem has been proved.



Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.026 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $E_b = \neg E_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 36 terms.

Time Complexity: Time spent by the prover is 0.458 seconds.

NDG conditions Points M_b and E_b are not identical

Points M_b , H_b and E_b are not collinear

Points M_b , A and B are not collinear

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{E_bAC} \neq 0$ i.e., points E_b , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{E_bAC} \neq S_{F_{h_b}^0 AC}$ i.e., lines $E_bF_{h_b}^0$ and AC are not parallel (construction based assumption)

$S_{M_{m(E_bM_b)}^3 E_bM_b} \neq S_{T_{m(E_bM_b)}^4 E_bM_b}$ i.e., lines $M_{m(E_bM_b)}^3$ and $T_{m(E_bM_b)}^4$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^6} \neq S_{F_{h_a}^5 BF_{h_b}^6}$ i.e., lines $AF_{h_a}^5$ and $BF_{h_b}^6$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $C=C$

NDG conditions are:

$S_{E_bAC} \neq 0$ i.e., points E_b , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{E_bAC} \neq S_{F_{h_b}^0 AC}$ i.e., lines $E_bF_{h_b}^0$ and AC are not parallel (construction based assumption)

$S_{M_{m(E_bM_b)}^3 E_bM_b} \neq S_{T_{m(E_bM_b)}^4 E_bM_b}$ i.e., lines $M_{m(E_bM_b)}^3$ and $T_{m(E_bM_b)}^4$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^6} \neq S_{F_{h_a}^5 BF_{h_b}^6}$ i.e., lines $AF_{h_a}^5$ and $BF_{h_b}^6$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.3 Proving $E_b = \neg E_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.2 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.3 Proving $E_b = \neg E_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 42 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.3 Proving $E_b = \neg E_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 314 terms.

Time Complexity: Time spent by the prover is 0.590 seconds. There are no ndg conditions.

Problem 567

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 567: Given a point A , a point C and a point E_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point C , construct a point M_b (rule W01); ;
2. Using the point C and the point E_c , construct a point H (rule W01); ;
3. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
4. Using the point C and the point E_c , construct a line h_c (rule W02); % DET: points C and E_c are not the same;
5. Using the point A and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points A and M_b are not the same;
6. Using the circle $k(M_b, C)$, the line h_a , the point M_b and the point A , construct a point H_a (rule W05); % NDG: line h_a and circle $k(M_b, C)$ intersect % DET: points A and H_a must be different;
7. Using the point H_a and the point C , construct a line a (rule W02); % DET: points H_a and C are not the same;
8. Using the circle $k(M_b, C)$, the line h_c , the point M_b and the point C , construct a point H_c (rule W05); % NDG: line h_c and circle $k(M_b, C)$ intersect % DET: points C and H_c must be different;
9. Using the point H_c and the point A , construct a line c (rule W02); % DET: points H_c and A are not the same;
10. Using the line a and the line c , construct a point B (rule W03); % NDG: lines a and c are not parallel % DET: lines a and c are not the same.

Non-degenerate conditions: lines a and c are not parallel; line h_c and circle $k(M_b, C)$ intersect; line h_a and circle $k(M_b, C)$ intersect; points A and M_b are not the same.

Determination conditions: lines a and c are not the same; points H_c and A are not the same; points C and H_c must be different; points H_a and C are not the same; points A and H_a must be different; points C and E_c are not the same; points A and H are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D22,D3,D30,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L43,L44,L45]

Solving time: 10.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point C 110 40
point E_{c} 95 56.36
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_l C
cmark_r E_{c}
color 0 0 0
fontsize 8
```

```
% Constructing a point M_{b} such that AM_{b}/AC=0.5
towards M_{b} A C 0.5
cmark_lt M_{b}
color 200 200 200
drawsegment A C
color 0 0 0
```

```
% Constructing a point H such that CH/CE_{c}=2
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0
```

```
% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H
```

```
color 200 200 200
drawline h_{a}
color 0 0 0
```

```

% DET: points C and E_{c} are not the same
% Constructing a line h_{c} which passes through point C and point E_{c}
line h_{c} C E_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points A and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point A
circle k(M_{b},C) M_{b} A

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line h_{a} and circle k(M_{b},C) intersect% DET: points A and H_{a} must be different
% Constructing a point P_{\_G147834} which is a foot of the point M_{b} on the line h_{a}
foot P_{\_G147834} M_{b} h_{a}
cmark_r P_{\_G147834}
color 200 200 200
drawline M_{b} P_{\_G147834}
color 0 0 0

% Constructing a point H_{a} which is an image of the point A in the symmetry to point/line P_{\_G
147834}
sim H_{a} P_{\_G147834} A
cmark_r H_{a}

% DET: points H_{a} and C are not the same
% Constructing a line a which passes through point H_{a} and point C
line a H_{a} C

color 200 200 200
drawline a
color 0 0 0

% NDG: line h_{c} and circle k(M_{b},C) intersect% DET: points C and H_{c} must be different
% Constructing a point P_{\_G148072} which is a foot of the point M_{b} on the line h_{c}
foot P_{\_G148072} M_{b} h_{c}
cmark_r P_{\_G148072}
color 200 200 200
drawline M_{b} P_{\_G148072}
color 0 0 0

% Constructing a point H_{c} which is an image of the point C in the symmetry to point/line P_{\_G
148072}
sim H_{c} P_{\_G148072} C
cmark_rt H_{c}

```

```

% DET: points  $H_{\{c\}}$  and  $A$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $A$ 
line c  $H_{\{c\}}$  A

color 200 200 200
drawline c
color 0 0 0

% NDG: lines  $a$  and  $c$  are not parallel% DET: lines  $a$  and  $c$  are not the same
% Constructing a point  $B$  which belongs to line  $a$  and line  $c$ 
intersec B a c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; line  $h_{\{c\}}$  and circle  $k(M_{\{b\}}, C)$ 
% intersect; line  $h_{\{a\}}$  and circle  $k(M_{\{b\}}, C)$  intersect; points  $A$  and  $M_{\{b\}}$  are not the same
% Determination conditions: lines  $a$  and  $c$  are not the same; points  $H_{\{c\}}$  and  $A$  are not the same;
% points  $C$  and  $H_{\{c\}}$  must be different; points  $H_{\{a\}}$  and  $C$  are not the same; points  $A$  and  $H_{\{a\}}$ 
% must be different; points  $C$  and  $E_{\{c\}}$  are not the same; points  $A$  and  $H$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.019 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

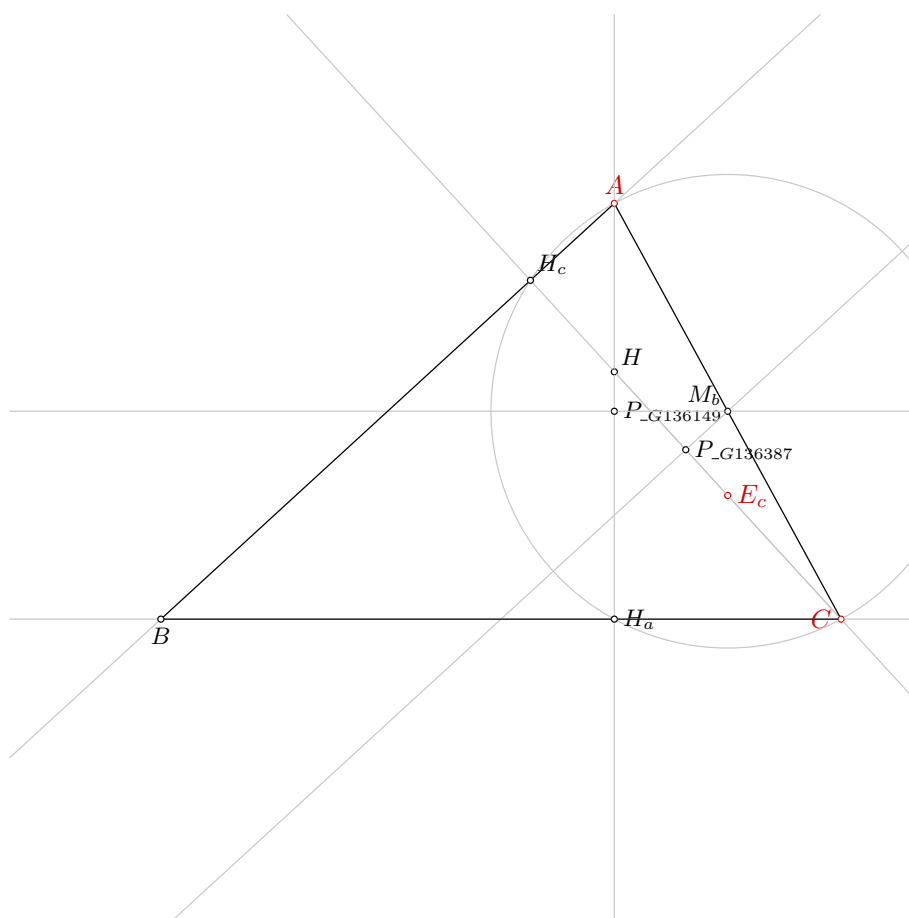


Figure 1: Illustration of the problem 0567

4.1.3 Proving $E_c = E_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 37 terms.

Time Complexity: Time spent by the prover is 4.501 seconds.

NDG conditions Points A and H are not identical

Points M_b and H are not identical

Points A , E_c and C are not collinear

Points A , E_c and C are not collinear

Line through points A and H_c is not parallel with line through points C and H_a

Points A and H_a are not identical

Points M_b , A and B are not collinear

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_a H_c A} \neq S_{CH_c A}$ i.e., lines $H_a C$ and $H_c A$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a} BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $C=C$

NDG conditions are:

$S_{H_a H_c A} \neq S_{CH_c A}$ i.e., lines $H_a C$ and $H_c A$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a} BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.3 Proving $E_c = E_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.3 Proving $E_c = \neg E_c$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A = A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.3 Proving $E_c = \neg E_c$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 97 terms.

Time Complexity: Time spent by the prover is 0.440 seconds. There are no ndg conditions.

Problem 568

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 568: Given a point A , a point C and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point C , construct a point M_b (rule W01); ;
2. Using the point A and the point C , construct a line b (rule W02); % DET: points A and C are not the same;
3. Using the point N and the point M_b , construct a line $m(H_aH_c)$ (rule W02); % DET: points N and M_b are not the same;
4. Using the point A and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points A and M_b are not the same;
5. Using the point M_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_b and N are not the same;
6. Using the circle $k(N, M_a)$, the line b , the point N and the point M_b , construct a point H_b (rule W05); % NDG: line b and circle $k(N, M_a)$ intersect % DET: points M_b and H_b must be different;
7. Using the circle $k(N, M_a)$, the line $m(H_aH_c)$, the point N and the point M_b , construct a point E_b (rule W05a); % NDG: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_b and E_b must be different;
8. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;
9. Using the circle $k(M_b, C)$ and the circle $k(N, M_a)$, construct a point H_a and a point H_c (rule W07); % NDG: circles $k(M_b, C)$ and $k(N, M_a)$ intersect % DET: circles $k(M_b, C)$ and $k(N, M_a)$ are not the same;

10. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
11. Using the line h_b and the line h_a , construct a point H (rule W03); % NDG: lines h_b and h_a are not parallel % DET: lines h_b and h_a are not the same;
12. Using the point N and the point H , construct a point G (rule W01); ;
13. Using the point C and the point G , construct a point M_c (rule W01); ;
14. Using the point A and the point M_c , construct a point B (rule W01); .

Non-degenerate conditions: lines h_b and h_a are not parallel; circles $k(M_b, C)$ and $k(N, M_a)$ intersect; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; line b and circle $k(N, M_a)$ intersect; points M_b and N are not the same; points A and M_b are not the same.

Determination conditions: lines h_b and h_a are not the same; points A and H_a are not the same; circles $k(M_b, C)$ and $k(N, M_a)$ are not the same; points E_b and H_b are not the same; points M_b and E_b must be different; points M_b and H_b must be different; points N and M_b are not the same; points A and C are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D20,D22,D29,D3,D32,D5,D6,D8,D9,GD01,GD02,GL01,GL03,GL04,GL09,L16,L17,L19,L20,L21]

Solving time: 5.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point C 110 40
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_l C
cmark_r N
color 0 0 0
fontsize 8
```

```
% Constructing a point M_{b} such that AM_{b}/AC=0.5
towards M_{b} A C 0.5
cmark_lt M_{b}
color 200 200 200
drawsegment A C
color 0 0 0
```

```
% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C
```

```

color 200 200 200
drawline b
color 0 0 0

```

```

% DET: points  $N$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $m(H_{\{a\}}H_{\{c\}})$  which passes through point  $N$  and point  $M_{\{b\}}$ 
line m( $H_{\{a\}}H_{\{c\}}$ ) N  $M_{\{b\}}$ 

```

```

color 200 200 200
drawline m( $H_{\{a\}}H_{\{c\}}$ )
color 0 0 0

```

```

% NDG: points  $A$  and  $M_{\{b\}}$  are not the same
% Constructing a circle  $k(M_{\{b\}}, C)$  whose center is at point  $M_{\{b\}}$  and which passes through point  $A$ 
circle k( $M_{\{b\}}, C$ )  $M_{\{b\}}$  A

```

```

color 200 200 200
drawcircle k( $M_{\{b\}}, C$ )
color 0 0 0

```

```

% NDG: points  $M_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $M_{\{b\}}$ 
circle k( $N, M_{\{a\}}$ ) N  $M_{\{b\}}$ 

```

```

color 200 200 200
drawcircle k( $N, M_{\{a\}}$ )
color 0 0 0

```

```

% NDG: line  $b$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{b\}}$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G186007\}}$  which is a foot of the point  $N$  on the line  $b$ 
foot  $P_{\{\backslash\_G186007\}}$  N b
cmark_r  $P_{\{\backslash\_G186007\}}$ 
color 200 200 200
drawline N  $P_{\{\backslash\_G186007\}}$ 
color 0 0 0

```

```

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $M_{\{b\}}$  in the symmetry to point/line  $P_{\{\backslash\_G186007\}}$ 
sim  $H_{\{b\}}$   $P_{\{\backslash\_G186007\}}$   $M_{\{b\}}$ 
cmark_l  $H_{\{b\}}$ 

```

```

% NDG: line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{b\}}$  and  $E_{\{b\}}$  must be different
% Constructing a point  $E_{\{b\}}$  which is an image of the point  $M_{\{b\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{b\}}$  N  $M_{\{b\}}$ 
cmark_r  $E_{\{b\}}$ 

```

```

% DET: points  $E_{\{b\}}$  and  $H_{\{b\}}$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $E_{\{b\}}$  and point  $H_{\{b\}}$ 
line  $h_{\{b\}}$   $E_{\{b\}}$   $H_{\{b\}}$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: circles  $k(M_{\{b\}}, C)$  and  $k(N, M_{\{a\}})$  intersect% DET: circles  $k(M_{\{b\}}, C)$  and  $k(N, M_{\{a\}})$  are not
the same
% Constructing points  $H_{\{a\}}$  and  $H_{\{c\}}$  which are in intersection of  $k(M_{\{b\}}, C)$  and  $k(N, M_{\{a\}})$ 
intersec2  $H_{\{a\}}$   $H_{\{c\}}$   $k(M_{\{b\}}, C)$   $k(N, M_{\{a\}})$ 
cmark_r  $H_{\{a\}}$ 
cmark_rt  $H_{\{c\}}$ 

% DET: points  $A$  and  $H_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $A$  and point  $H_{\{a\}}$ 
line  $h_{\{a\}}$   $A$   $H_{\{a\}}$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: lines  $h_{\{b\}}$  and  $h_{\{a\}}$  are not parallel% DET: lines  $h_{\{b\}}$  and  $h_{\{a\}}$  are not the same
% Constructing a point  $H$  which belongs to line  $h_{\{b\}}$  and line  $h_{\{a\}}$ 
intersec  $H$   $h_{\{b\}}$   $h_{\{a\}}$ 
cmark_rt  $H$ 

% Constructing a line  $L_{\{\_G186509\}}$  which passes through point  $N$  and point  $H$ 
line  $L_{\{\_G186509\}}$   $N$   $H$ 

color 200 200 200
drawline  $L_{\{\_G186509\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\_G186610\}}$  with coordinates  $(0,0)$ 
point  $P_{\{\_G186610\}}$  0 0
cmark_r  $P_{\{\_G186610\}}$ 

% Constructing a point  $P_{\{\_G186534\}}$  such that  $NP_{\{\_G186534\}}/NP_{\{\_G186610\}}=-1$ 
towards  $P_{\{\_G186534\}}$   $N$   $P_{\{\_G186610\}}$  -1
cmark_r  $P_{\{\_G186534\}}$ 
color 200 200 200
drawsegment  $P_{\{\_G186610\}}$   $P_{\{\_G186534\}}$ 
color 0 0 0

```

```

% Constructing a point  $P_{\backslash\_G186579}$  such that  $NP_{\backslash\_G186579}/NP_{\backslash\_G186610}=3$ 
towards P_{\backslash\_G186579} N P_{\backslash\_G186610} 3
cmark_r P_{\backslash\_G186579}
color 200 200 200
drawsegment N P_{\backslash\_G186579}
color 0 0 0

% Constructing a line  $L_{\backslash\_G186540}$  which passes through point H and point  $P_{\backslash\_G186579}$ 
line L_{\backslash\_G186540} H P_{\backslash\_G186579}

color 200 200 200
drawline L_{\backslash\_G186540}
color 0 0 0

% Constructing a line  $L_{\backslash\_G186503}$  which contains the point  $P_{\backslash\_G186534}$  and is parallel to the
line  $L_{\backslash\_G186540}$ 
parallel L_{\backslash\_G186503} P_{\backslash\_G186534} L_{\backslash\_G186540}

color 200 200 200
drawline L_{\backslash\_G186503}
color 0 0 0

% Constructing a point G which belongs to line  $L_{\backslash\_G186503}$  and line  $L_{\backslash\_G186509}$ 
intersec G L_{\backslash\_G186503} L_{\backslash\_G186509}
cmark_t G

% Constructing a point  $M_{\{c\}}$  such that  $CM_{\{c\}}/CG=1.5$ 
towards M_{\{c\}} C G 1.5
cmark_lt M_{\{c\}}
color 200 200 200
drawsegment C M_{\{c\}}
color 0 0 0

% Constructing a point B such that  $AB/AM_{\{c\}}=2$ 
towards B A M_{\{c\}} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{b\}}$  and  $h_{\{a\}}$  are not parallel; circles  $k(M_{\{b\}}, C)$  and  $k(N, M_{\{a\}})$  intersect; line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect; line b and circle  $k(N, M_{\{a\}})$  intersect; points  $M_{\{b\}}$  and N are not the same; points A and  $M_{\{b\}}$  are not the same

```

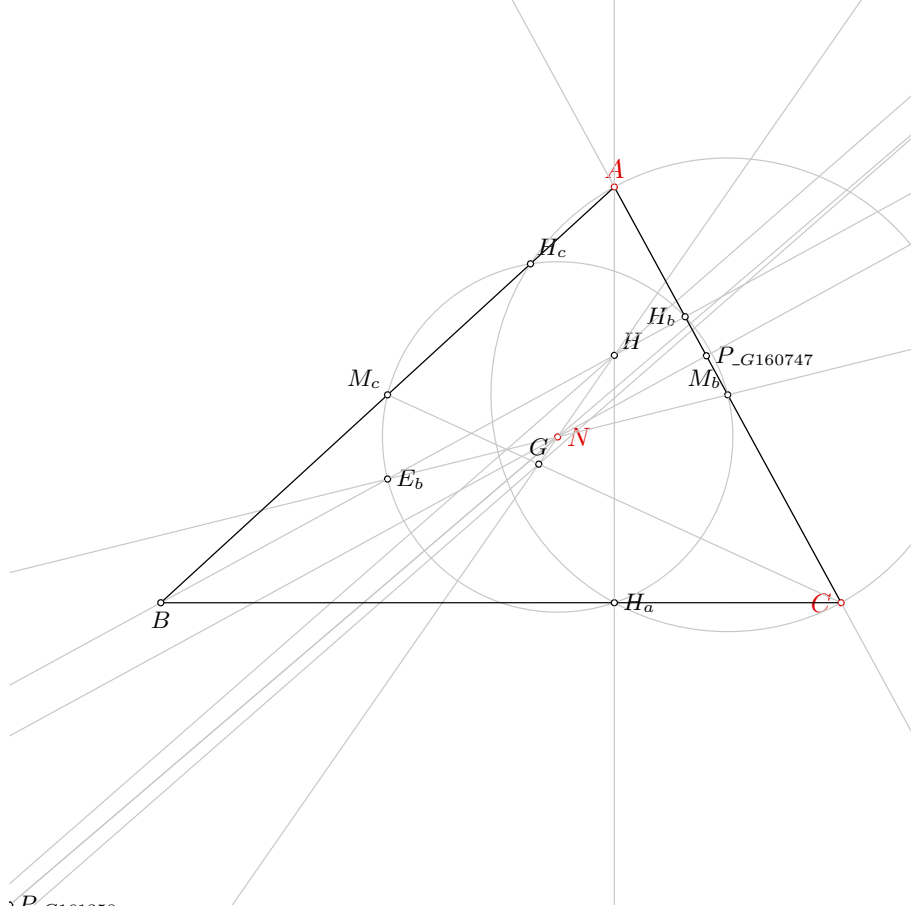


Figure 1: Illustration of the problem 0568

% Determination conditions: lines $h_{\{b\}}$ and $h_{\{a\}}$ are not the same; points A and $H_{\{a\}}$ are not the same; circles $k(M_{\{b\}}, C)$ and $k(N, M_{\{a\}})$ are not the same; points $E_{\{b\}}$ and $H_{\{b\}}$ are not the same; points $M_{\{b\}}$ and $E_{\{b\}}$ must be different; points $M_{\{b\}}$ and $H_{\{b\}}$ must be different; points N and $M_{\{b\}}$ are not the same; points A and C are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.03 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{E_bAH_a} \neq S_{H_bAH_a}$ i.e., lines E_bH_b and AH_a are not parallel (construction based assumption)

$S_{P_{-G163279}NH} \neq S_{P_{L_{-G163248}}^0NH}$ i.e., lines $P_{-G163279}$ and $P_{L_{-G163248}}^0$ are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points M_a, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points M_b, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^4} \neq S_{F_{-h_a}^3BF_{-h_b}^4}$ i.e., lines $AF_{-h_a}^3$ and $BF_{-h_b}^4$ are not parallel (construction based assumption)

$S_{M_a-M_bF_{-m_b}^2} \neq S_{F_{-m_a}^1-M_bF_{-m_b}^2}$ i.e., lines $M_aF_{-m_a}^1$ and $M_bF_{-m_b}^2$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $C=C$

NDG conditions are:

$S_{E_bAH_a} \neq S_{H_bAH_a}$ i.e., lines E_bH_b and AH_a are not parallel (construction based assumption)

$S_{P_{-G167616}NH} \neq S_{P_{L_{-G167585}}^0NH}$ i.e., lines $P_{-G167616}$ and $P_{L_{-G167585}}^0$ are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points M_a, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points M_b, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^4} \neq S_{F_{-h_a}^3BF_{-h_b}^4}$ i.e., lines $AF_{-h_a}^3$ and $BF_{-h_b}^4$ are not parallel (construction based assumption)

$S_{M_a-M_bF_{-m_b}^2} \neq S_{F_{-m_a}^1-M_bF_{-m_b}^2}$ i.e., lines $M_aF_{-m_a}^1$ and $M_bF_{-m_b}^2$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.2 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.3 Proving $N=_N$

Proving failed

Problem 569

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 569: Given a point A , a point E_a and a point E_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point E_a , construct a point H (rule W01); ;
2. Using the point E_b and the point H , construct a point B (rule W01); ;
3. Using the point A and the point B , construct a line c (rule W02); % DET: points A and B are not the same;
4. Using the point E_b and the point H , construct a line h_b (rule W02); % DET: points E_b and H are not the same;
5. Using the point A and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points A and E_a are not the same;
6. Using the circle $k(E_a, A)$, the line c , the point E_a and the point A , construct a point H_c (rule W05); % NDG: line c and circle $k(E_a, A)$ intersect % DET: points A and H_c must be different;
7. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
8. Using the circle $k(E_a, A)$, the line h_b , the point E_a and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_a, A)$ intersect % DET: points H and H_b must be different;
9. Using the point H_b and the point A , construct a line b (rule W02); % DET: points H_b and A are not the same;
10. Using the line h_c and the line b , construct a point C (rule W03); % NDG: lines h_c and b are not parallel % DET: lines h_c and b are not the same.

Non-degenerate conditions: lines h_c and b are not parallel; line h_b and circle $k(E_a, A)$ intersect; line c and circle $k(E_a, A)$ intersect; points A and E_a are not the same.

Determination conditions: lines h_c and b are not the same; points H_b and A are not the same; points H and H_b must be different; points H_c and H are not the same; points A and H_c must be different; points E_b and H are not the same; points A and B are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D28,D29,D3,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L46,L47,L48]

Solving time: 12.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point E_{a} 80 83.86
point E_{b} 50 56.36
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r E_{a}
cmark_r E_{b}
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0
```

```
% Constructing a point B such that E_{b}B/E_{b}H=-1
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0
```

```
% DET: points A and B are not the same
% Constructing a line c which passes through point A and point B
line c A B
```

```
color 200 200 200
drawline c
color 0 0 0
```

```

% DET: points  $E_{\{b\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $E_{\{b\}}$  and point  $H$ 
line  $h_{\{b\}}$   $E_{\{b\}}$   $H$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: points  $A$  and  $E_{\{a\}}$  are not the same
% Constructing a circle  $k(E_{\{a\}}, A)$  whose center is at point  $E_{\{a\}}$  and which passes through point  $A$ 
circle  $k(E_{\{a\}}, A)$   $E_{\{a\}}$   $A$ 

color 200 200 200
drawcircle  $k(E_{\{a\}}, A)$ 
color 0 0 0

% NDG: line  $c$  and circle  $k(E_{\{a\}}, A)$  intersect% DET: points  $A$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G213158\}}$  which is a foot of the point  $E_{\{a\}}$  on the line  $c$ 
foot  $P_{\{\backslash\_G213158\}}$   $E_{\{a\}}$   $c$ 
cmark_r  $P_{\{\backslash\_G213158\}}$ 
color 200 200 200
drawline  $E_{\{a\}}$   $P_{\{\backslash\_G213158\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $A$  in the symmetry to point/line  $P_{\{\backslash\_G213158\}}$ 
sim  $H_{\{c\}}$   $P_{\{\backslash\_G213158\}}$   $A$ 
cmark_rt  $H_{\{c\}}$ 

% DET: points  $H_{\{c\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $H_{\{c\}}$  and point  $H$ 
line  $h_{\{c\}}$   $H_{\{c\}}$   $H$ 

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(E_{\{a\}}, A)$  intersect% DET: points  $H$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G213396\}}$  which is a foot of the point  $E_{\{a\}}$  on the line  $h_{\{b\}}$ 
foot  $P_{\{\backslash\_G213396\}}$   $E_{\{a\}}$   $h_{\{b\}}$ 
cmark_r  $P_{\{\backslash\_G213396\}}$ 
color 200 200 200
drawline  $E_{\{a\}}$   $P_{\{\backslash\_G213396\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $H$  in the symmetry to point/line  $P_{\{\backslash\_G213396\}}$ 
sim  $H_{\{b\}}$   $P_{\{\backslash\_G213396\}}$   $H$ 
cmark_l  $H_{\{b\}}$ 

```

```

% DET: points  $H_{\{b\}}$  and  $A$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $A$ 
line b  $H_{\{b\}}$  A

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $h_{\{c\}}$  and  $b$  are not parallel% DET: lines  $h_{\{c\}}$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $h_{\{c\}}$  and line  $b$ 
intersec C  $h_{\{c\}}$  b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{c\}}$  and  $b$  are not parallel; line  $h_{\{b\}}$  and circle  $k(E_{\{a\}}, A)$ 
% intersect; line  $c$  and circle  $k(E_{\{a\}}, A)$  intersect; points  $A$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $h_{\{c\}}$  and  $b$  are not the same; points  $H_{\{b\}}$  and  $A$  are not the same
% ; points  $H$  and  $H_{\{b\}}$  must be different; points  $H_{\{c\}}$  and  $H$  are not the same; points  $A$  and  $H_{\{c\}}$ 
% must be different; points  $E_{\{b\}}$  and  $H$  are not the same; points  $A$  and  $B$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.035 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a = \neg E_a$

Proving failed

4.1.3 Proving $E_b = \neg E_b$

Proving failed

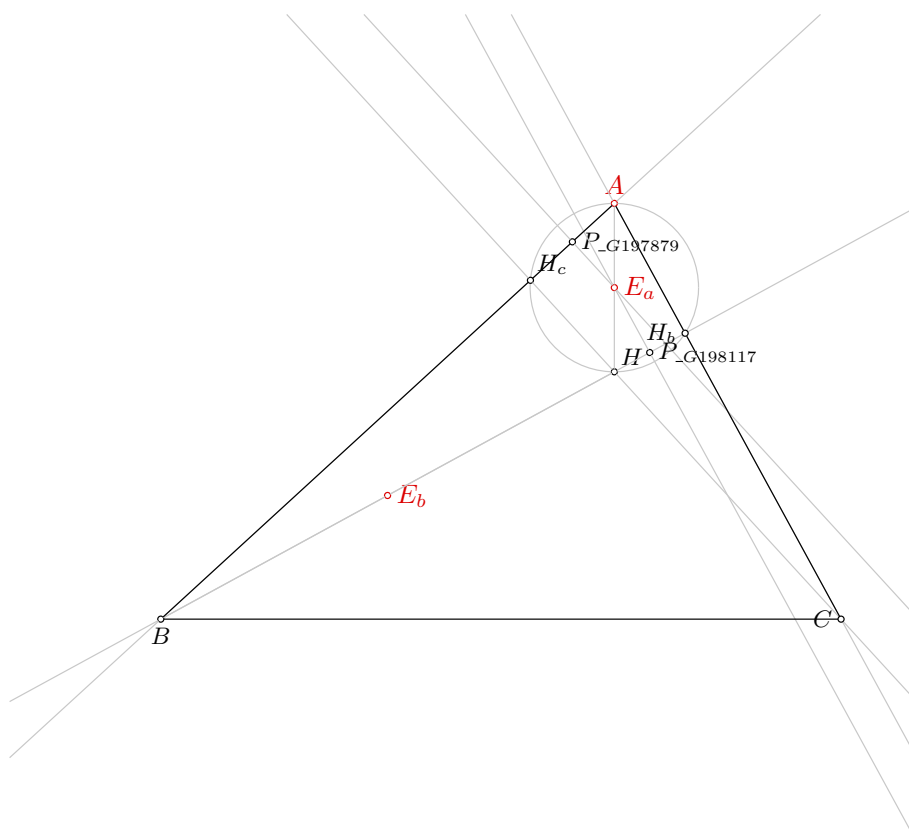


Figure 1: Illustration of the problem 0569

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_c H_b A} \neq S_{H H_b A}$ i.e., lines $H_c H$ and $H_b A$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a = \neg E_a$

Proving failed

4.2.3 Proving $E_b = \neg E_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_a = \neg E_a$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 1326 terms.

Time Complexity: Time spent by the prover is 2.250 seconds. There are no ndg conditions.

4.3.3 Proving $E_b = \neg E_b$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 804 terms.

Time Complexity: Time spent by the prover is 2.390 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_a = \neg E_a$

Proving failed

4.4.3 Proving $E_b = \neg E_b$

Proving failed

Problem 570

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 570: Given a point A , a point E_a and a point E_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point E_a , construct a point H (rule W01); ;
2. Using the point E_c and the point H , construct a point C (rule W01); ;
3. Using the point A and the point C , construct a line b (rule W02); % DET: points A and C are not the same;
4. Using the point E_c and the point H , construct a line h_c (rule W02); % DET: points E_c and H are not the same;
5. Using the point A and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points A and E_a are not the same;
6. Using the circle $k(E_a, A)$, the line b , the point E_a and the point A , construct a point H_b (rule W05); % NDG: line b and circle $k(E_a, A)$ intersect % DET: points A and H_b must be different;
7. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
8. Using the circle $k(E_a, A)$, the line h_c , the point E_a and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_a, A)$ intersect % DET: points H and H_c must be different;
9. Using the point H_c and the point A , construct a line c (rule W02); % DET: points H_c and A are not the same;
10. Using the line h_b and the line c , construct a point B (rule W03); % NDG: lines h_b and c are not parallel % DET: lines h_b and c are not the same.

Non-degenerate conditions: lines h_b and c are not parallel; line h_c and circle $k(E_a, A)$ intersect; line b and circle $k(E_a, A)$ intersect; points A and E_a are not the same.

Determination conditions: lines h_b and c are not the same; points H_c and A are not the same; points H and H_c must be different; points H_b and H are not the same; points A and H_b must be different; points E_c and H are not the same; points A and C are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D28,D3,D30,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L46,L47,L48]

Solving time: 12.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point E_{a} 80 83.86
point E_{c} 95 56.36
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r E_{a}
cmark_r E_{c}
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0
```

```
% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0
```

```
% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C
```

```
color 200 200 200
drawline b
color 0 0 0
```

```

% DET: points E_{c} and H are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H
line h_{c} E_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points A and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point A
circle k(E_{a},A) E_{a} A

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: line b and circle k(E_{a},A) intersect% DET: points A and H_{b} must be different
% Constructing a point P_{\_G238418} which is a foot of the point E_{a} on the line b
foot P_{\_G238418} E_{a} b
cmark_r P_{\_G238418}
color 200 200 200
drawline E_{a} P_{\_G238418}
color 0 0 0

% Constructing a point H_{b} which is an image of the point A in the symmetry to point/line P_{\_G
238418}
sim H_{b} P_{\_G238418} A
cmark_l H_{b}

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{c} and circle k(E_{a},A) intersect% DET: points H and H_{c} must be different
% Constructing a point P_{\_G238656} which is a foot of the point E_{a} on the line h_{c}
foot P_{\_G238656} E_{a} h_{c}
cmark_r P_{\_G238656}
color 200 200 200
drawline E_{a} P_{\_G238656}
color 0 0 0

% Constructing a point H_{c} which is an image of the point H in the symmetry to point/line P_{\_G
238656}
sim H_{c} P_{\_G238656} H
cmark_rt H_{c}

```

```

% DET: points  $H_{\{c\}}$  and  $A$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $A$ 
line c  $H_{\{c\}}$  A

color 200 200 200
drawline c
color 0 0 0

% NDG: lines  $h_{\{b\}}$  and  $c$  are not parallel% DET: lines  $h_{\{b\}}$  and  $c$  are not the same
% Constructing a point  $B$  which belongs to line  $h_{\{b\}}$  and line  $c$ 
intersec B  $h_{\{b\}}$  c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{b\}}$  and  $c$  are not parallel; line  $h_{\{c\}}$  and circle  $k(E_{\{a\}}, A)$ 
% intersect; line  $b$  and circle  $k(E_{\{a\}}, A)$  intersect; points  $A$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $h_{\{b\}}$  and  $c$  are not the same; points  $H_{\{c\}}$  and  $A$  are not the same
% ; points  $H$  and  $H_{\{c\}}$  must be different; points  $H_{\{b\}}$  and  $H$  are not the same; points  $A$  and  $H_{\{b\}}$ 
% must be different; points  $E_{\{c\}}$  and  $H$  are not the same; points  $A$  and  $C$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.034 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a = \neg E_a$

Proving failed

4.1.3 Proving $E_c = \neg E_c$

Proving failed

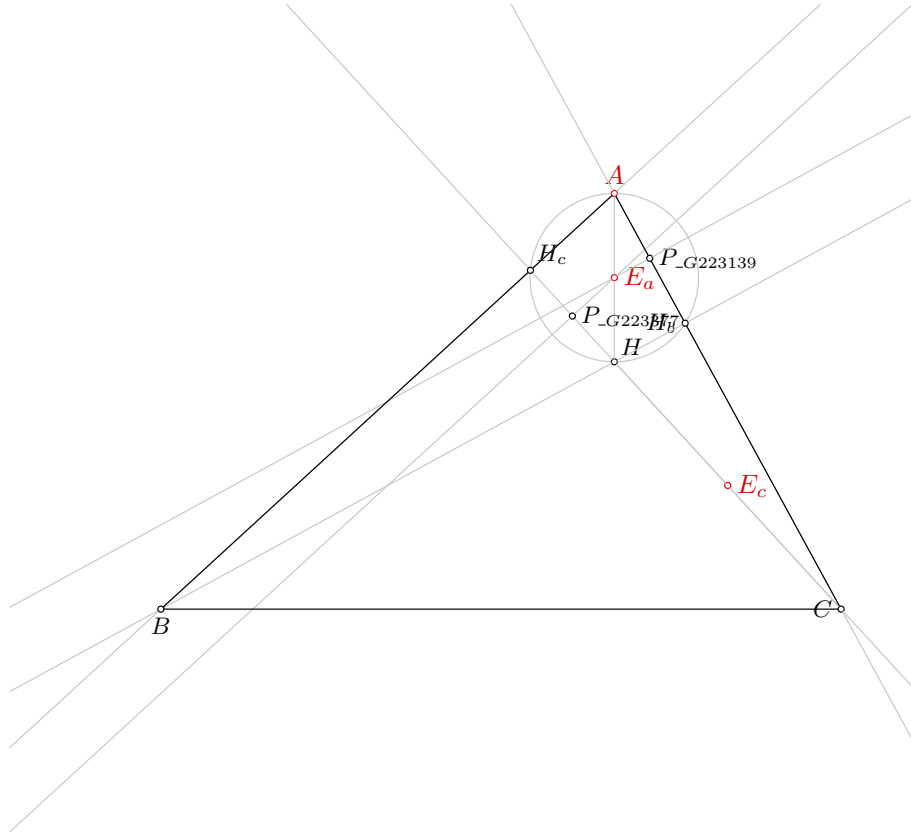


Figure 1: Illustration of the problem 0570

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_bH_cA} \neq S_{HH_cA}$ i.e., lines H_bH and H_cA are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a=_Ea$

Proving failed

4.2.3 Proving $E_c=_Ec$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_a=_Ea$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 1794 terms.

Time Complexity: Time spent by the prover is 4.540 seconds. There are no ndg conditions.

4.3.3 Proving $E_c=_Ec$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 1072 terms.

Time Complexity: Time spent by the prover is 5.730 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_a=_Ea$

Proving failed

4.4.3 Proving $E_c=_Ec$

Proving failed

Problem 571

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 571: Given a point A , a point E_a and a point G , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point E_a , construct a point H (rule W01); ;
2. Using the point A and the point G , construct a point M_a (rule W01); ;
3. Using the point G and the point H , construct a point N (rule W01); ;
4. Using the point G and the point H , construct a point O (rule W01); ;
5. Using the point A and the point E_a , construct a line h_a (rule W02); % DET: points A and E_a are not the same;
6. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
7. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
9. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; line h_a and circle $k(N, M_a)$ intersect; points E_a and N are not the same; points A and O are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points A and E_a are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L16,L19,L22,L55,L58]

Solving time: 9.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point E_{a} 80 83.86
point G 70 58.33

color 220 0 0
fontsize 9

cmark_t A
cmark_r E_{a}
cmark_t G
color 0 0 0
fontsize 8

% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% Constructing a point M_{a} such that AM_{a}/AG=1.5
towards M_{a} A G 1.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% Constructing a point N such that GN/GH=0.25
towards N G H 0.25
cmark_r N
color 200 200 200
drawsegment G H
color 0 0 0

% Constructing a point O such that GO/GH=-0.5
```

```

towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% DET: points A and E_{a} are not the same
% Constructing a line h_{a} which passes through point A and point E_{a}
line h_{a} A E_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points E_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{a}
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G6700} which is a foot of the point N on the line h_{a}
foot P_{\_G6700} N h_{a}
cmark_r P_{\_G6700}
color 200 200 200
drawline N P_{\_G6700}
color 0 0 0

% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
_{\_G6700}
sim H_{a} P_{\_G6700} E_{a}
cmark_r H_{a}

% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}

color 200 200 200

```



```
drawline a
color 0 0 0
```

```
% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; line h_{a} and circle k(N,M_{a})
% intersect; points E_{a} and N are not the same; points A and O are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
% different; points A and E_{a} are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.023 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a = E_a$

Proving failed

4.1.3 Proving $G = G$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a}BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

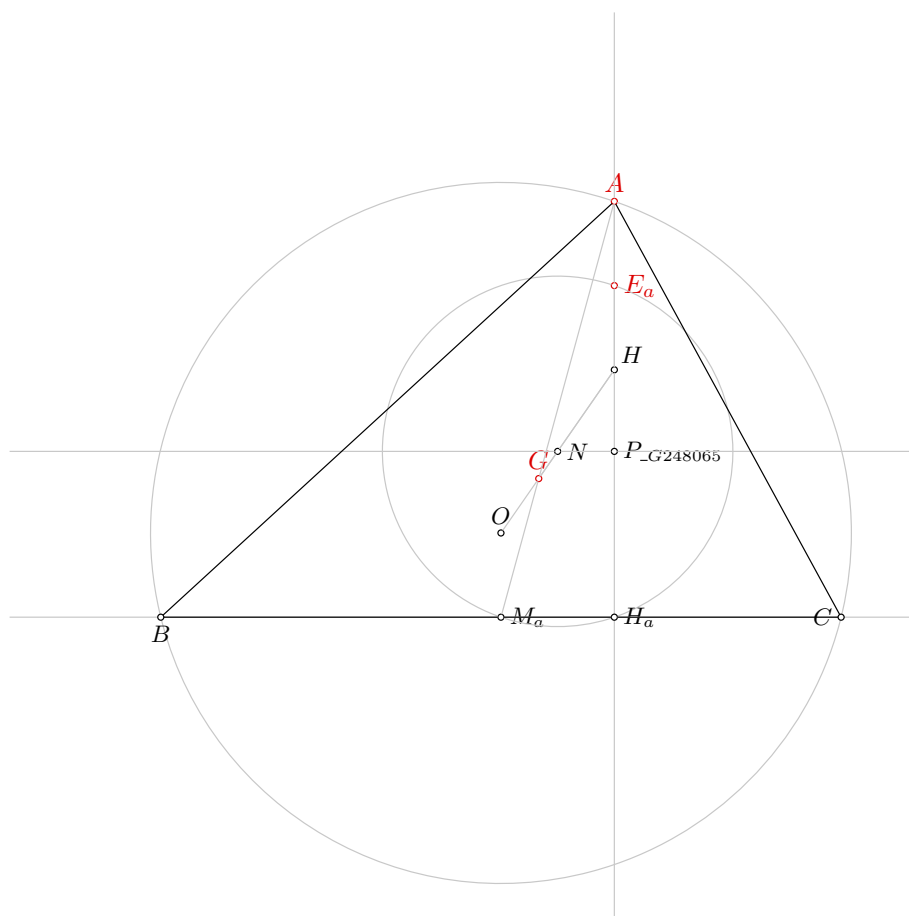


Figure 1: Illustration of the problem 0571

$S_{AB_M_b} \neq S_{_M_a B_M_b}$ i.e., lines A_M_a and B_M_b are not parallel (construction based assumption)
 Total number of proof steps: 1
 Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a =_E_a$

Proving failed

4.2.3 Proving $G =_G$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A = A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $E_a =_E_a$

Proving failed

4.3.3 Proving $G =_G$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A = A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $E_a =_E_a$

Proving failed

4.4.3 Proving $G =_G$

Proving failed

Problem 572

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 572: Given a point E_a , a point H and a point A , construct the triangle ABC .

2 Status

Problem is redundant.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H , construct a point A (rule W01); ;
2. Choose freely a point B (rule free);
3. Using the point B and the point H , construct a point E_b (rule W01); ;
4. Using the point B and the point A , construct a line c (rule W02); % DET: points B and A are not the same;
5. Using the point E_a and the point H , construct a line h_a (rule W02); % DET: points E_a and H are not the same;
6. Using the point B and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points B and E_b are not the same;
7. Using the circle $k(E_b, B)$, the line c , the point E_b and the point B , construct a point H_c (rule W05); % NDG: line c and circle $k(E_b, B)$ intersect % DET: points B and H_c must be different;
8. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
9. Using the circle $k(E_b, B)$, the line h_a , the point E_b and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_b, B)$ intersect % DET: points H and H_a must be different;
10. Using the point H_a and the point B , construct a line a (rule W02); % DET: points H_a and B are not the same;

11. Using the line h_c and the line a , construct a point C (rule W03); % NDG: lines h_c and a are not parallel % DET: lines h_c and a are not the same.

Non-degenerate conditions: lines h_c and a are not parallel; line h_a and circle $k(E_b, B)$ intersect; line c and circle $k(E_b, B)$ intersect; points B and E_b are not the same.

Determination conditions: lines h_c and a are not the same; points H_a and B are not the same; points H and H_a must be different; points H_c and H are not the same; points B and H_c must be different; points E_a and H are not the same; points B and A are not the same.

Rules used: [W01,W02,W03,W05,W06,free]

Lemmas used: [D10,D28,D29,D3,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L49,L50,L51]

Solving time: 10.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{a} 80 83.86
point H 80 72.73
point A 80 95

color 220 0 0
fontsize 9

cmark_r E_{a}
cmark_rt H
cmark_t A
color 0 0 0
fontsize 8

% Constructing a point A such that E_{a}A/E_{a}H=-1
towards A E_{a} H -1
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

% Constructing a free point B
point B 20 40

cmark_b B

% Constructing a point E_{b} such that BE_{b}/BH=0.5
towards E_{b} B H 0.5
cmark_r E_{b}
color 200 200 200
drawsegment B H
color 0 0 0
```

```

% DET: points B and A are not the same
% Constructing a line c which passes through point B and point A
line c B A

color 200 200 200
drawline c
color 0 0 0

% DET: points E_{a} and H are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H
line h_{a} E_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points B and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point B
circle k(E_{b},B) E_{b} B

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line c and circle k(E_{b},B) intersect% DET: points B and H_{c} must be different
% Constructing a point P_{\_G31089} which is a foot of the point E_{b} on the line c
foot P_{\_G31089} E_{b} c
cmark_r P_{\_G31089}
color 200 200 200
drawline E_{b} P_{\_G31089}
color 0 0 0

% Constructing a point H_{c} which is an image of the point B in the symmetry to point/line P_{\_G
31089}
sim H_{c} P_{\_G31089} B
cmark_rt H_{c}

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: line h_{a} and circle k(E_{b},B) intersect% DET: points H and H_{a} must be different

```

```

% Constructing a point  $P_{\{ \_G31327 \}}$  which is a foot of the point  $E_{\{b\}}$  on the line  $h_{\{a\}}$ 
foot  $P_{\{ \_G31327 \}}$   $E_{\{b\}}$   $h_{\{a\}}$ 
cmark_r  $P_{\{ \_G31327 \}}$ 
color 200 200 200
drawline  $E_{\{b\}}$   $P_{\{ \_G31327 \}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $H$  in the symmetry to point/line  $P_{\{ \_G31327 \}}$ 
sim  $H_{\{a\}}$   $P_{\{ \_G31327 \}}$   $H$ 
cmark_r  $H_{\{a\}}$ 

% DET: points  $H_{\{a\}}$  and  $B$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $B$ 
line  $a$   $H_{\{a\}}$   $B$ 

color 200 200 200
drawline  $a$ 
color 0 0 0

% NDG: lines  $h_{\{c\}}$  and  $a$  are not parallel% DET: lines  $h_{\{c\}}$  and  $a$  are not the same
% Constructing a point  $C$  which belongs to line  $h_{\{c\}}$  and line  $a$ 
intersec  $C$   $h_{\{c\}}$   $a$ 
cmark_l  $C$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $h_{\{c\}}$  and  $a$  are not parallel; line  $h_{\{a\}}$  and circle  $k(E_{\{b\}}, B)$ 
intersect; line  $c$  and circle  $k(E_{\{b\}}, B)$  intersect; points  $B$  and  $E_{\{b\}}$  are not the same
% Determination conditions: lines  $h_{\{c\}}$  and  $a$  are not the same; points  $H_{\{a\}}$  and  $B$  are not the same
; points  $H$  and  $H_{\{a\}}$  must be different; points  $H_{\{c\}}$  and  $H$  are not the same; points  $B$  and  $H_{\{c\}}$ 
must be different; points  $E_{\{a\}}$  and  $H$  are not the same; points  $B$  and  $A$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = _E_a$

Proving failed

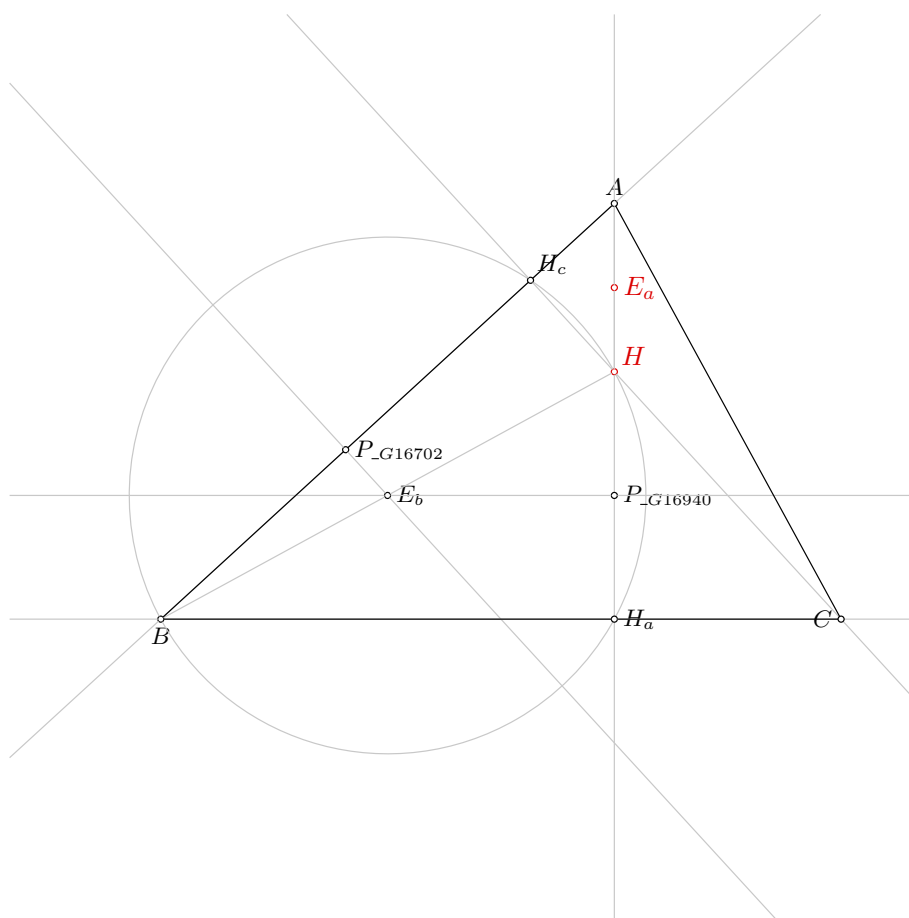


Figure 1: Illustration of the problem 0572

4.1.2 Proving $H=_H$

Proving failed

4.1.3 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2 terms.

Time Complexity: Time spent by the prover is 0.023 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $E_a=_Ea$

Proving failed

4.2.2 Proving $H=_H$

Proving failed

4.2.3 Proving $A=A$

NDG conditions are:

$S_{H_cH_aB} \neq S_{HH_aB}$ i.e., lines H_cH and H_aB are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{h_b}} \neq S_{F^0_{h_a}BF^1_{h_b}}$ i.e., lines $AF^0_{h_a}$ and $BF^1_{h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_a=_Ea$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 805 terms.

Time Complexity: Time spent by the prover is 1.410 seconds. There are no ndg conditions.

4.3.2 Proving $H=_H$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 152 terms.

Time Complexity: Time spent by the prover is 0.160 seconds. There are no ndg conditions.

4.3.3 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $H = \neg H$

Proving failed

4.4.3 Proving $A = A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

Problem 573

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 573: Given a point E_a , a point H_a and a point A , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;
2. Choose freely a point A on the line h_a (rule WOnline1) ;
3. Using the point A and the point E_a , construct a point H (rule W01); ;
4. Using the point A and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points A and E_a are not the same;
5. Using the point H_a and the line h_a , construct a line a (rule W10a); ;
6. Choose freely a point B on the line a (rule WOnline2);
7. Using the point B and the point A , construct a line c (rule W02); % DET: points B and A are not the same;
8. Using the circle $k(E_a, A)$, the line c , the point E_a and the point A , construct a point H_c (rule W05); % NDG: line c and circle $k(E_a, A)$ intersect % DET: points A and H_c must be different;
9. Using the point H and the point H_c , construct a line h_c (rule W02); % DET: points H and H_c are not the same;
10. Using the line a and the line h_c , construct a point C (rule W03); % NDG: lines a and h_c are not parallel % DET: lines a and h_c are not the same.

Non-degenerate conditions: lines a and h_c are not parallel; line c and circle $k(E_a, A)$ intersect; points A and E_a are not the same.

Determination conditions: lines a and h_c are not the same; points H and H_c are not the same; points A and H_c must be different; points B and A are not the same; points E_a and H_a are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D10,D28,D3,D5,D7,D8,GD01,GD02,GL03,GL09,L3,L48]

Solving time: 172.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point H_{a} 80 40
```

```
point A 80 95
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_r H_{a}
```

```
cmark_t A
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{a} and H_{a} are not the same
```

```
% Constructing a line h_{a} which passes through point E_{a} and point H_{a}
```

```
line h_{a} E_{a} H_{a}
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```
% Choosing randomly a point A on the line E_{a}H_{a}
```

```
online A E_{a} H_{a}
```

```
cmark_t A
```

```
color 200 200 200
```

```
drawline E_{a} H_{a}
```

```
color 0 0 0
```

```
% Constructing a point H such that AH/AE_{a}=2
```

```
towards H A E_{a} 2
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment A H
```

```
color 0 0 0
```

```

% NDG: points A and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point A
circle k(E_{a},A) E_{a} A

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
perp a H_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0

% Generating random value V[_G53006]
random V[_G53006]

% Calculating value V[_G53027] using formula V[_G53006]*20
expression V[_G53027] { V[_G53006]*20 }

% Constructing a point B which is a point for which holds  $H_{a}B = V[_G53027]$  and angle  $AH_{a}B = 90$ 
turtle B A H_{a} 90 V[_G53027]
cmark_b B

% DET: points B and A are not the same
% Constructing a line c which passes through point B and point A
line c B A

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(E_{a},A) intersect% DET: points A and H_{c} must be different
% Constructing a point P_{\_G53299} which is a foot of the point E_{a} on the line c
foot P_{\_G53299} E_{a} c
cmark_r P_{\_G53299}
color 200 200 200
drawline E_{a} P_{\_G53299}
color 0 0 0

% Constructing a point H_{c} which is an image of the point A in the symmetry to point/line P_{\_G53299}
sim H_{c} P_{\_G53299} A

```

```

cmark_rt H_{c}

% DET: points H and H_{c} are not the same
% Constructing a line h_{c} which passes through point H and point H_{c}
line h_{c} H H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines a and h_{c} are not parallel% DET: lines a and h_{c} are not the same
% Constructing a point C which belongs to line a and line h_{c}
intersec C a h_{c}
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and h_{c} are not parallel; line c and circle k(E_{a},A)
% intersect; points A and E_{a} are not the same
% Determination conditions: lines a and h_{c} are not the same; points H and H_{c} are not the same
% ; points A and H_{c} must be different; points B and A are not the same; points E_{a} and H_{a}
% are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 55 terms.

Time Complexity: Time spent by the prover is 0.997 seconds.

NDG conditions Points A and B are not identical

Points A and B are not identical

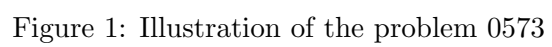
Line through points H_c and H_a is not perpendicular to line through points H_a and H

Line through points A and E_a is not parallel with line through points B and C

4.1.2 Proving $H_a = \neg H_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.



Time Complexity: Time spent by the prover is 0.155 seconds.

NDG conditions Points A and B are not identical

Points A and B are not identical

Line through points H_c and H_a is not perpendicular to line through points H_a and H

Line through points A and E_a is not parallel with line through points B and C

4.1.3 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.008 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $H_a = \neg H_a$

Proving failed

4.2.3 Proving $A=A$

NDG conditions are:

$S_{H_a H H_c} \neq S_{T_a^1 H H_c}$ i.e., lines $H_a T_a^1$ and $H H_c$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2 BF_{\neg h_b}^3}$ i.e., lines $AF_{\neg h_a}^2$ and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

$S_{BAF_{\neg h_a}^2} \neq S_{CAF_{\neg h_a}^2}$ i.e., lines BC and $AF_{\neg h_a}^2$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 9 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $H_a = \neg H_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 16 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.3 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 20 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $H_a = \neg H_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 20 terms.

Time Complexity: Time spent by the prover is 0.050 seconds. There are no ndg conditions.

4.4.3 Proving $A = A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

Problem 574

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 574: Given a point E_a , a point H_b and a point A , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_b and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_b and E_a are not the same;
2. Choose freely a point A on the circle $k(E_a, A)$ (rule WOncircle);
3. Using the point A and the point E_a , construct a point H (rule W01); ;
4. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;
5. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
6. Choose freely a point B on the line h_b (rule WOnline1) ;
7. Using the point B and the point A , construct a line c (rule W02); % DET: points B and A are not the same;
8. Using the circle $k(E_a, A)$, the line c , the point E_a and the point A , construct a point H_c (rule W05); % NDG: line c and circle $k(E_a, A)$ intersect % DET: points A and H_c must be different;
9. Using the point H and the point H_c , construct a line h_c (rule W02); % DET: points H and H_c are not the same;
10. Using the line b and the line h_c , construct a point C (rule W03); % NDG: lines b and h_c are not parallel % DET: lines b and h_c are not the same.

Non-degenerate conditions: lines b and h_c are not parallel; line c and circle $k(E_a, A)$ intersect; points H_b and E_a are not the same.

Determination conditions: lines b and h_c are not the same; points H and H_c are not the same; points A and H_c must be different; points B and A are not the same; points H_b and H are not the same; points A and H_b are not the same.

Rules used: [W01,W02,W03,W05,W06,WOncircle1,WOnline1]

Lemmas used: [D10,D28,D3,D6,D7,GD01,GD02,GL03,L3,L47,L48]

Solving time: 699.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{a} 80 83.86
point H_{b} 89.36 77.83
point A 80 95

color 220 0 0
fontsize 9

cmark_r E_{a}
cmark_l H_{b}
cmark_t A
color 0 0 0
fontsize 8

% NDG: points H_{b} and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H
%_{b}
circle k(E_{a},A) E_{a} H_{b}

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% Choosing randomly a point A on the circle with center E_{a} through point H_{b}
oncircle A E_{a} H_{b}
cmark_t A
color 200 200 200
drawcircle E_{a} H_{b}
color 0 0 0

% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0
```

```

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% Choosing randomly a point B on the line H_{b}H
online B H_{b} H
cmark_b B
color 200 200 200
drawline H_{b} H
color 0 0 0

% DET: points B and A are not the same
% Constructing a line c which passes through point B and point A
line c B A

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(E_{a},A) intersect% DET: points A and H_{c} must be different
% Constructing a point P_{\_G75738} which is a foot of the point E_{a} on the line c
foot P_{\_G75738} E_{a} c
cmark_r P_{\_G75738}
color 200 200 200
drawline E_{a} P_{\_G75738}
color 0 0 0

% Constructing a point H_{c} which is an image of the point A in the symmetry to point/line P_{\_G
75738}
sim H_{c} P_{\_G75738} A
cmark_rt H_{c}

% DET: points H and H_{c} are not the same
% Constructing a line h_{c} which passes through point H and point H_{c}

```

```

line h_{c} H H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines b and h_{c} are not parallel% DET: lines b and h_{c} are not the same
% Constructing a point C which belongs to line b and line h_{c}
intersec C b h_{c}
cmark_1 C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and h_{c} are not parallel; line c and circle k(E_{a},A)
% intersect; points H_{b} and E_{a} are not the same
% Determination conditions: lines b and h_{c} are not the same; points H and H_{c} are not the same
% ; points A and H_{c} must be different; points B and A are not the same; points H_{b} and H are
% not the same; points A and H_{b} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = _E_a$

Proving failed

4.1.2 Proving $H_b = _H_b$

Proving failed

4.1.3 Proving $A = A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2 terms.

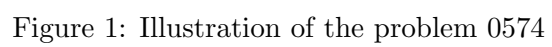
Time Complexity: Time spent by the prover is 0.025 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $E_a = _E_a$

Proving failed



4.2.2 Proving $H_b = \neg H_b$

Proving failed

4.2.3 Proving $A = A$

NDG conditions are:

$S_{AHH_c} \neq S_{H_bHH_c}$ i.e., lines AH_b and HH_c are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^2_{h_b}} \neq S_{F^1_{h_a}BF^2_{h_b}}$ i.e., lines $AF^1_{h_a}$ and $BF^2_{h_b}$ are not parallel (construction based assumption)

$S_{ABF^2_{h_b}} \neq S_{CBF^2_{h_b}}$ i.e., lines AC and $BF^2_{h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $H_b = \neg H_b$

Proving failed

4.3.3 Proving $A = A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $H_b = \neg H_b$

Proving failed

4.4.3 Proving $A = A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

Problem 575

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 575: Given a point E_a , a point H_c and a point A , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_c and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_c and E_a are not the same;
2. Choose freely a point A on the circle $k(E_a, A)$ (rule WOncircle);
3. Using the point A and the point E_a , construct a point H (rule W01); ;
4. Using the point A and the point H_c , construct a line c (rule W02); % DET: points A and H_c are not the same;
5. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
6. Choose freely a point B on the line c (rule WOnline1) ;
7. Using the point B and the point H , construct a line h_b (rule W02); % DET: points B and H are not the same;
8. Using the circle $k(E_a, A)$, the line h_b , the point E_a and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_a, A)$ intersect % DET: points H and H_b must be different;
9. Using the point H_b and the point A , construct a line b (rule W02); % DET: points H_b and A are not the same;
10. Using the line h_c and the line b , construct a point C (rule W03); % NDG: lines h_c and b are not parallel % DET: lines h_c and b are not the same.

Non-degenerate conditions: lines h_c and b are not parallel; line h_b and circle $k(E_a, A)$ intersect; points H_c and E_a are not the same.

Determination conditions: lines h_c and b are not the same; points H_b and A are not the same; points H and H_b must be different; points B and H are not the same; points H_c and H are not the same; points A and H_c are not the same.

Rules used: [W01,W02,W03,W05,W06,WOncircle1,WOnline1]

Lemmas used: [D10,D28,D3,D6,D7,D9,GD01,GD02,GL03,L3,L46,L47,L48]

Solving time: 694.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{a} 80 83.86
point H_{c} 68.91 84.83
point A 80 95

color 220 0 0
fontsize 9

cmark_r E_{a}
cmark_rt H_{c}
cmark_t A
color 0 0 0
fontsize 8

% NDG: points H_{c} and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H
%_{c}
circle k(E_{a},A) E_{a} H_{c}

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% Choosing randomly a point A on the circle with center E_{a} through point H_{c}
oncircle A E_{a} H_{c}
cmark_t A
color 200 200 200
drawcircle E_{a} H_{c}
color 0 0 0

% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0
```

```

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% Choosing randomly a point B on the line AH_{c}
online B A H_{c}
cmark_b B
color 200 200 200
drawline A H_{c}
color 0 0 0

% DET: points B and H are not the same
% Constructing a line h_{b} which passes through point B and point H
line h_{b} B H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{b} and circle k(E_{a},A) intersect% DET: points H and H_{b} must be different
% Constructing a point P_{\G98836} which is a foot of the point E_{a} on the line h_{b}
foot P_{\G98836} E_{a} h_{b}
cmark_r P_{\G98836}
color 200 200 200
drawline E_{a} P_{\G98836}
color 0 0 0

% Constructing a point H_{b} which is an image of the point H in the symmetry to point/line P_{\G98836}
sim H_{b} P_{\G98836} H
cmark_l H_{b}

% DET: points H_{b} and A are not the same
% Constructing a line b which passes through point H_{b} and point A

```

```

line b H_{b} A

color 200 200 200
drawline b
color 0 0 0

% NDG: lines h_{c} and b are not parallel% DET: lines h_{c} and b are not the same
% Constructing a point C which belongs to line h_{c} and line b
intersec C h_{c} b
cmark_1 C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and b are not parallel; line h_{b} and circle k(E_{a},A)
% intersect; points H_{c} and E_{a} are not the same
% Determination conditions: lines h_{c} and b are not the same; points H_{b} and A are not the same
% ; points H and H_{b} must be different; points B and H are not the same; points H_{c} and H are
% not the same; points A and H_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $H_c = \neg H_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2241 terms.

Time Complexity: Time spent by the prover is 14.996 seconds.

NDG conditions Points A , H_c and E_a are not collinear

Points B and H are not identical

Line through points H_c and E_a is not parallel with line through points B and H

Line through points H_b and A is not parallel with line through points H_c and H

Points H and E_a are not identical

Points A and B are not identical

Points A and B are not identical

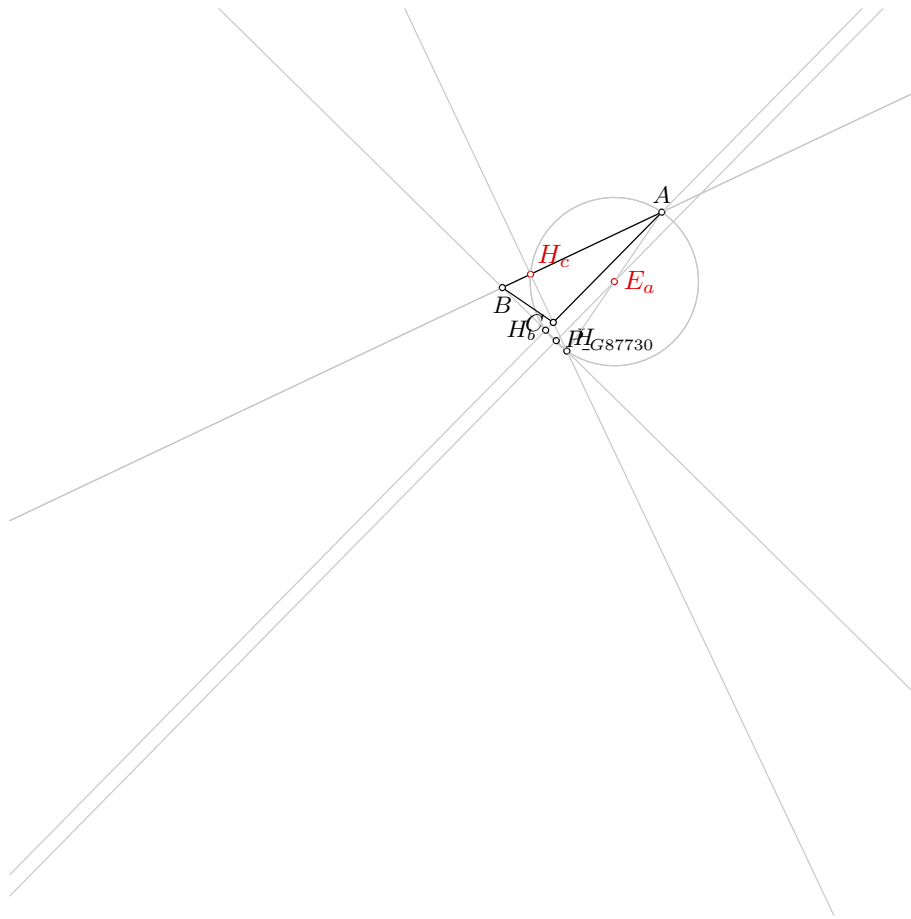


Figure 1: Illustration of the problem 0575

4.1.3 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2 terms.

Time Complexity: Time spent by the prover is 0.021 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $H_c = \neg H_c$

Proving failed

4.2.3 Proving $A=A$

NDG conditions are:

$S_{H_c H_b A} \neq S_{H H_b A}$ i.e., lines $H_c H$ and $H_b A$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^2} \neq S_{F_{h_a}^1 B F_{h_b}^2}$ i.e., lines $AF_{h_a}^1$ and $BF_{h_b}^2$ are not parallel (construction based assumption)

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{h_c}^3} \neq S_{BCF_{h_c}^3}$ i.e., lines AB and $CF_{h_c}^3$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $H_c = \neg H_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 280 terms.

Time Complexity: Time spent by the prover is 0.330 seconds. There are no ndg conditions.

4.3.3 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $H_c = \neg H_c$

Proving failed

4.4.3 Proving $A = A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

Problem 576

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 576: Given a point A , a point E_a and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 577

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 577: Given a point A , a point E_a and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point E_a , construct a point H (rule W01); ;
2. Using the point A and the point M_a , construct a point G (rule W01); ;
3. Using the point H and the point G , construct a point N (rule W01); ;
4. Using the point H and the point G , construct a point O (rule W01); ;
5. Using the point A and the point E_a , construct a line h_a (rule W02); % DET: points A and E_a are not the same;
6. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
7. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
9. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; line h_a and circle $k(N, M_a)$ intersect; points E_a and N are not the same; points A and O are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points A and E_a are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D32,D5,D8,GD01,GL03,GL09,L11,L12,L16,L19,L22,L55,L58]

Solving time: 9.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point E_{a} 80 83.86
point M_{a} 65 40

color 220 0 0
fontsize 9

cmark_t A
cmark_r E_{a}
cmark_r M_{a}
color 0 0 0
fontsize 8

% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% Constructing a line L_{\_G123557} which passes through point A and point M_{a}
line L_{\_G123557} A M_{a}

color 200 200 200
drawline L_{\_G123557}
color 0 0 0

% Constructing a point P_{\_G123658} with coordinates (0,0)
point P_{\_G123658} 0 0
cmark_r P_{\_G123658}

% Constructing a point P_{\_G123582} such that AP_{\_G123582}/AP_{\_G123658}=2
towards P_{\_G123582} A P_{\_G123658} 2
cmark_r P_{\_G123582}
color 200 200 200
drawsegment A P_{\_G123582}
color 0 0 0
```

```

% Constructing a point P_{\_G123627} such that AP_{\_G123627}/AP_{\_G123658}=3
towards P_{\_G123627} A P_{\_G123658} 3
cmark_r P_{\_G123627}
color 200 200 200
drawsegment A P_{\_G123627}
color 0 0 0

% Constructing a line L_{\_G123588} which passes through point M_{a} and point P_{\_G123627}
line L_{\_G123588} M_{a} P_{\_G123627}

color 200 200 200
drawline L_{\_G123588}
color 0 0 0

% Constructing a line L_{\_G123551} which contains the point P_{\_G123582} and is parallel to the
line L_{\_G123588}
parallel L_{\_G123551} P_{\_G123582} L_{\_G123588}

color 200 200 200
drawline L_{\_G123551}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G123551} and line L_{\_G123557}
intersec G L_{\_G123551} L_{\_G123557}
cmark_t G

% Constructing a point N such that HN/HG=0.75
towards N H G 0.75
cmark_r N
color 200 200 200
drawsegment H G
color 0 0 0

% Constructing a point O such that HO/HG=1.5
towards O H G 1.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% DET: points A and E_{a} are not the same
% Constructing a line h_{a} which passes through point A and point E_{a}
line h_{a} A E_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

```

```

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points E_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{a}
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G124628} which is a foot of the point N on the line h_{a}
foot P_{\_G124628} N h_{a}
cmark_r P_{\_G124628}
color 200 200 200
drawline N P_{\_G124628}
color 0 0 0

% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
\_G124628}
sim H_{a} P_{\_G124628} E_{a}
cmark_r H_{a}

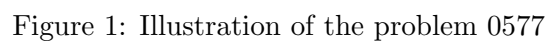
% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```



```
% Non-degenerate conditions: line a and circle k(O,C) intersect; line h_{a} and circle k(N,M_{a})
% intersect; points E_{a} and N are not the same; points A and O are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
% different; points A and E_{a} are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.026 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a = \neg E_a$

Proving failed

4.1.3 Proving $M_a = \neg M_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{P_{G111582}AM_a} \neq S_{P_{L_{G111551}}^0 AM_a}$ i.e., lines $P_{G111582}P_{L_{G111551}}^0$ and AM_a are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^2} \neq S_{F_{-h_a}^1 BF_{-h_b}^2}$ i.e., lines $AF_{-h_a}^1$ and $BF_{-h_b}^2$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a = \neg E_a$

Proving failed

4.2.3 Proving $M_a = \neg M_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.2 Proving $E_a = \neg E_a$

Proving failed

4.3.3 Proving $M_a = \neg M_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $E_a = \neg E_a$

Proving failed

4.4.3 Proving $M_a = \neg M_a$

Proving failed

Problem 578

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 578: Given a point A , a point E_a and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point E_a , construct a point H (rule W01); ;
2. Using the point A and the point M_b , construct a point C (rule W01); ;
3. Using the point A and the point M_b , construct a line b (rule W02); % DET: points A and M_b are not the same;
4. Using the point H and the point C , construct a line h_c (rule W02); % DET: points H and C are not the same;
5. Using the point A and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points A and E_a are not the same;
6. Using the circle $k(E_a, A)$, the line b , the point E_a and the point A , construct a point H_b (rule W05); % NDG: line b and circle $k(E_a, A)$ intersect % DET: points A and H_b must be different;
7. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
8. Using the circle $k(E_a, A)$, the line h_c , the point E_a and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_a, A)$ intersect % DET: points H and H_c must be different;
9. Using the point H_c and the point A , construct a line c (rule W02); % DET: points H_c and A are not the same;
10. Using the line h_b and the line c , construct a point B (rule W03); % NDG: lines h_b and c are not parallel % DET: lines h_b and c are not the same.

Non-degenerate conditions: lines h_b and c are not parallel; line h_c and circle $k(E_a, A)$ intersect; line b and circle $k(E_a, A)$ intersect; points A and E_a are not the same.

Determination conditions: lines h_b and c are not the same; points H_c and A are not the same; points H and H_c must be different; points H_b and H are not the same; points A and H_b must be different; points H and C are not the same; points A and M_b are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D22,D28,D3,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L46,L47,L48]

Solving time: 12.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point E_{a} 80 83.86
point M_{b} 95 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r E_{a}
cmark_lt M_{b}
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0
```

```
% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0
```

```
% DET: points A and M_{b} are not the same
% Constructing a line b which passes through point A and point M_{b}
line b A M_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```



```

% DET: points H and C are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point H and point C
line  $h_{\{c\}}$  H C

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: points A and  $E_{\{a\}}$  are not the same
% Constructing a circle  $k(E_{\{a\}}, A)$  whose center is at point  $E_{\{a\}}$  and which passes through point A
circle  $k(E_{\{a\}}, A)$   $E_{\{a\}}$  A

color 200 200 200
drawcircle  $k(E_{\{a\}}, A)$ 
color 0 0 0

% NDG: line b and circle  $k(E_{\{a\}}, A)$  intersect% DET: points A and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\_G145962\}}$  which is a foot of the point  $E_{\{a\}}$  on the line b
foot  $P_{\{\_G145962\}}$   $E_{\{a\}}$  b
cmark_r  $P_{\{\_G145962\}}$ 
color 200 200 200
drawline  $E_{\{a\}}$   $P_{\{\_G145962\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point A in the symmetry to point/line  $P_{\{\_G145962\}}$ 
sim  $H_{\{b\}}$   $P_{\{\_G145962\}}$  A
cmark_l  $H_{\{b\}}$ 

% DET: points  $H_{\{b\}}$  and H are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $H_{\{b\}}$  and point H
line  $h_{\{b\}}$   $H_{\{b\}}$  H

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(E_{\{a\}}, A)$  intersect% DET: points H and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\_G146200\}}$  which is a foot of the point  $E_{\{a\}}$  on the line  $h_{\{c\}}$ 
foot  $P_{\{\_G146200\}}$   $E_{\{a\}}$   $h_{\{c\}}$ 
cmark_r  $P_{\{\_G146200\}}$ 
color 200 200 200
drawline  $E_{\{a\}}$   $P_{\{\_G146200\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point H in the symmetry to point/line  $P_{\{\_G146200\}}$ 
sim  $H_{\{c\}}$   $P_{\{\_G146200\}}$  H
cmark_rt  $H_{\{c\}}$ 

```

```

% DET: points  $H_{\{c\}}$  and  $A$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $A$ 
line c  $H_{\{c\}}$  A

color 200 200 200
drawline c
color 0 0 0

% NDG: lines  $h_{\{b\}}$  and  $c$  are not parallel% DET: lines  $h_{\{b\}}$  and  $c$  are not the same
% Constructing a point  $B$  which belongs to line  $h_{\{b\}}$  and line  $c$ 
intersec B  $h_{\{b\}}$  c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{b\}}$  and  $c$  are not parallel; line  $h_{\{c\}}$  and circle  $k(E_{\{a\}}, A)$ 
% intersect; line  $b$  and circle  $k(E_{\{a\}}, A)$  intersect; points  $A$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $h_{\{b\}}$  and  $c$  are not the same; points  $H_{\{c\}}$  and  $A$  are not the same
% ; points  $H$  and  $H_{\{c\}}$  must be different; points  $H_{\{b\}}$  and  $H$  are not the same; points  $A$  and  $H_{\{b\}}$ 
% must be different; points  $H$  and  $C$  are not the same; points  $A$  and  $M_{\{b\}}$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.032 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a = \neg E_a$

Proving failed

4.1.3 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

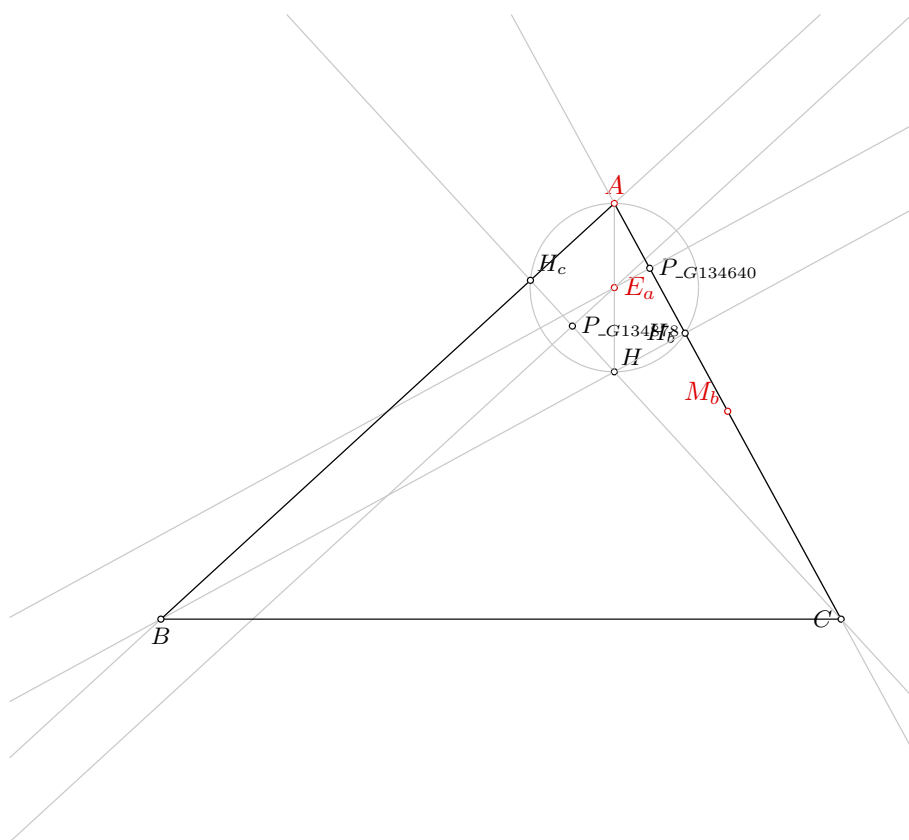


Figure 1: Illustration of the problem 0578

Time Complexity: Time spent by the prover is 0.027 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_bH_cA} \neq S_{HH_cA}$ i.e., lines H_bH and H_cA are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a=_Ea$

Proving failed

4.2.3 Proving $M_b=_Mb$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_a=_Ea$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 117 terms.

Time Complexity: Time spent by the prover is 0.410 seconds. There are no ndg conditions.

4.3.3 Proving $M_b=_Mb$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 8 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_a=_Ea$

Proving failed

4.4.3 Proving $M_b = -M_b$

Proving failed

Problem 579

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 579: Given a point A , a point E_a and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point E_a , construct a point H (rule W01); ;
2. Using the point A and the point M_c , construct a point B (rule W01); ;
3. Using the point A and the point M_c , construct a line c (rule W02); % DET: points A and M_c are not the same;
4. Using the point H and the point B , construct a line h_b (rule W02); % DET: points H and B are not the same;
5. Using the point A and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points A and E_a are not the same;
6. Using the circle $k(E_a, A)$, the line c , the point E_a and the point A , construct a point H_c (rule W05); % NDG: line c and circle $k(E_a, A)$ intersect % DET: points A and H_c must be different;
7. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
8. Using the circle $k(E_a, A)$, the line h_b , the point E_a and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_a, A)$ intersect % DET: points H and H_b must be different;
9. Using the point H_b and the point A , construct a line b (rule W02); % DET: points H_b and A are not the same;
10. Using the line h_c and the line b , construct a point C (rule W03); % NDG: lines h_c and b are not parallel % DET: lines h_c and b are not the same.

Non-degenerate conditions: lines h_c and b are not parallel; line h_b and circle $k(E_a, A)$ intersect; line c and circle $k(E_a, A)$ intersect; points A and E_a are not the same.

Determination conditions: lines h_c and b are not the same; points H_b and A are not the same; points H and H_b must be different; points H_c and H are not the same; points A and H_c must be different; points H and B are not the same; points A and M_c are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D20,D28,D3,D6,D7,D9,GD01,GD02,GL03,GL09,L3,L46,L47,L48]

Solving time: 12.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point E_{a} 80 83.86
point M_{c} 50 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r E_{a}
cmark_lt M_{c}
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0
```

```
% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0
```

```
% DET: points A and M_{c} are not the same
% Constructing a line c which passes through point A and point M_{c}
line c A M_{c}

color 200 200 200
drawline c
color 0 0 0
```

```

% DET: points H and B are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point H and point B
line  $h_{\{b\}}$  H B

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: points A and  $E_{\{a\}}$  are not the same
% Constructing a circle  $k(E_{\{a\}}, A)$  whose center is at point  $E_{\{a\}}$  and which passes through point A
circle  $k(E_{\{a\}}, A)$   $E_{\{a\}}$  A

color 200 200 200
drawcircle  $k(E_{\{a\}}, A)$ 
color 0 0 0

% NDG: line c and circle  $k(E_{\{a\}}, A)$  intersect% DET: points A and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G167563\}}$  which is a foot of the point  $E_{\{a\}}$  on the line c
foot  $P_{\{\backslash\_G167563\}}$   $E_{\{a\}}$  c
cmark_r  $P_{\{\backslash\_G167563\}}$ 
color 200 200 200
drawline  $E_{\{a\}}$   $P_{\{\backslash\_G167563\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point A in the symmetry to point/line  $P_{\{\backslash\_G167563\}}$ 
sim  $H_{\{c\}}$   $P_{\{\backslash\_G167563\}}$  A
cmark_rt  $H_{\{c\}}$ 

% DET: points  $H_{\{c\}}$  and H are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $H_{\{c\}}$  and point H
line  $h_{\{c\}}$   $H_{\{c\}}$  H

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(E_{\{a\}}, A)$  intersect% DET: points H and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G167801\}}$  which is a foot of the point  $E_{\{a\}}$  on the line  $h_{\{b\}}$ 
foot  $P_{\{\backslash\_G167801\}}$   $E_{\{a\}}$   $h_{\{b\}}$ 
cmark_r  $P_{\{\backslash\_G167801\}}$ 
color 200 200 200
drawline  $E_{\{a\}}$   $P_{\{\backslash\_G167801\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point H in the symmetry to point/line  $P_{\{\backslash\_G167801\}}$ 
sim  $H_{\{b\}}$   $P_{\{\backslash\_G167801\}}$  H
cmark_l  $H_{\{b\}}$ 

```



```

% DET: points  $H_{\{b\}}$  and  $A$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $A$ 
line b  $H_{\{b\}}$  A

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $h_{\{c\}}$  and  $b$  are not parallel% DET: lines  $h_{\{c\}}$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $h_{\{c\}}$  and line  $b$ 
intersec C  $h_{\{c\}}$  b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{c\}}$  and  $b$  are not parallel; line  $h_{\{b\}}$  and circle  $k(E_{\{a\}}, A)$ 
% intersect; line  $c$  and circle  $k(E_{\{a\}}, A)$  intersect; points  $A$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $h_{\{c\}}$  and  $b$  are not the same; points  $H_{\{b\}}$  and  $A$  are not the same
% ; points  $H$  and  $H_{\{b\}}$  must be different; points  $H_{\{c\}}$  and  $H$  are not the same; points  $A$  and  $H_{\{c\}}$ 
% must be different; points  $H$  and  $B$  are not the same; points  $A$  and  $M_{\{c\}}$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.034 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a=_E_a$

Proving failed

4.1.3 Proving $M_c=_M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

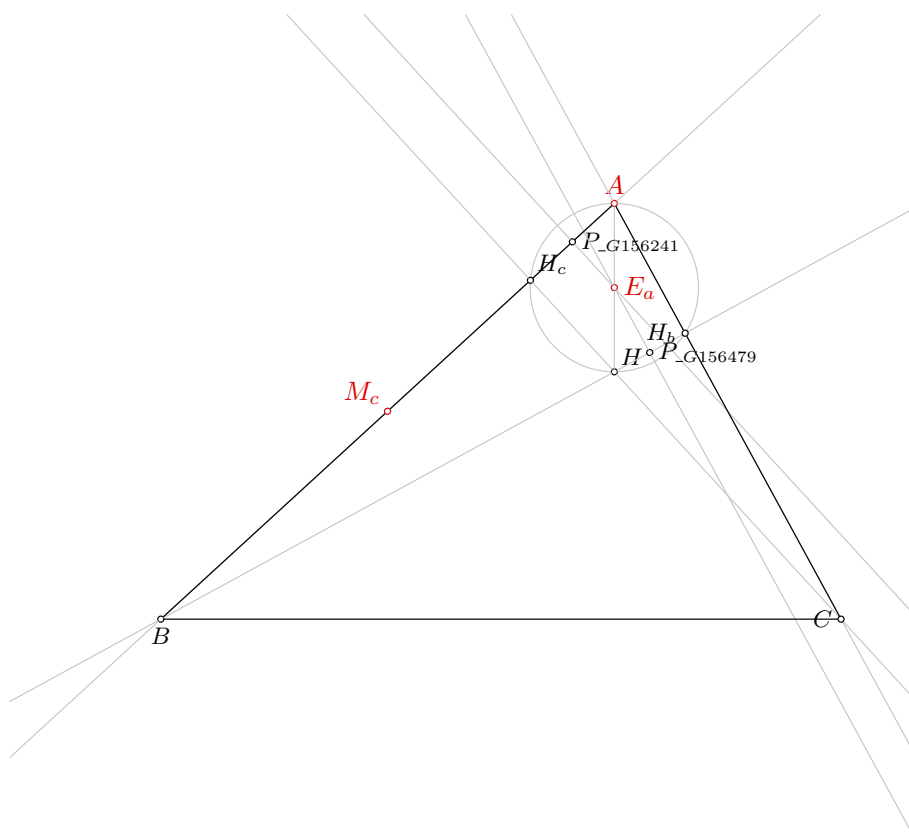


Figure 1: Illustration of the problem 0579

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_c H_b A} \neq S_{H H_b A}$ i.e., lines $H_c H$ and $H_b A$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a = E_a$

Proving failed

4.2.3 Proving $M_c = M_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_a = E_a$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 117 terms.

Time Complexity: Time spent by the prover is 0.350 seconds. There are no ndg conditions.

4.3.3 Proving $M_c = M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 8 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_a = E_a$

Proving failed

4.4.3 Proving $M_c = -M_c$

Proving failed

Problem 580

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 580: Given a point A , a point E_a and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point E_a , construct a point H (rule W01); ;
2. Using the point N and the point H , construct a point O (rule W01); ;
3. Using the point N and the point H , construct a point G (rule W01); ;
4. Using the point A and the point G , construct a point M_a (rule W01); ;
5. Using the point A and the point E_a , construct a line h_a (rule W02); % DET: points A and E_a are not the same;
6. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
7. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
9. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; line h_a and circle $k(N, M_a)$ intersect; points E_a and N are not the same; points A and O are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points A and E_a are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D31,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L16,L19,L22,L55]

Solving time: 9.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point E_{a} 80 83.86
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r E_{a}
cmark_r N
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0
```

```
% Constructing a point O such that NO/NH=-1
towards O N H -1
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0
```

```
% Constructing a line L_{\_G202589} which passes through point N and point H
line L_{\_G202589} N H

color 200 200 200
drawline L_{\_G202589}
color 0 0 0
```

```
% Constructing a point P_{\_G202690} with coordinates (0,0)
point P_{\_G202690} 0 0
```

```

cmark_r P_{\_G202690}

% Constructing a point P_{\_G202614} such that NP_{\_G202614}/NP_{\_G202690}=-1
towards P_{\_G202614} N P_{\_G202690} -1
cmark_r P_{\_G202614}
color 200 200 200
drawsegment P_{\_G202690} P_{\_G202614}
color 0 0 0

% Constructing a point P_{\_G202659} such that NP_{\_G202659}/NP_{\_G202690}=3
towards P_{\_G202659} N P_{\_G202690} 3
cmark_r P_{\_G202659}
color 200 200 200
drawsegment N P_{\_G202659}
color 0 0 0

% Constructing a line L_{\_G202620} which passes through point H and point P_{\_G202659}
line L_{\_G202620} H P_{\_G202659}

color 200 200 200
drawline L_{\_G202620}
color 0 0 0

% Constructing a line L_{\_G202583} which contains the point P_{\_G202614} and is parallel to the
line L_{\_G202620}
parallel L_{\_G202583} P_{\_G202614} L_{\_G202620}

color 200 200 200
drawline L_{\_G202583}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G202583} and line L_{\_G202589}
intersec G L_{\_G202583} L_{\_G202589}
cmark_t G

% Constructing a point M_{a} such that AM_{a}/AG=1.5
towards M_{a} A G 1.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% DET: points A and E_{a} are not the same
% Constructing a line h_{a} which passes through point A and point E_{a}
line h_{a} A E_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

```

```

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points E_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{a}
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G203585} which is a foot of the point N on the line h_{a}
foot P_{\_G203585} N h_{a}
cmark_r P_{\_G203585}
color 200 200 200
drawline N P_{\_G203585}
color 0 0 0

% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
\_G203585}
sim H_{a} P_{\_G203585} E_{a}
cmark_r H_{a}

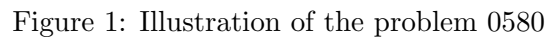
% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```

3.3 Illustration

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1.1 Proving $A=A$

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a = \neg E_a$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A = A$

NDG conditions are:

$S_{P_{G178624}NH} \neq S_{P_{L_{G178593}}^0}^{NH}$ i.e., lines $P_{G178624}P_{L_{G178593}}^0$ and NH are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^2} \neq S_{F_{h_a}^1 BF_{h_b}^2}$ i.e., lines $AF_{h_a}^1$ and $BF_{h_b}^2$ are not parallel (construction based assumption)

$S_{M_a BC} \neq 0$ i.e., points M_a, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_b AC} \neq 0$ i.e., points M_b, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_a M_b F_{m_b}^4} \neq S_{F_{m_a}^3 M_b F_{m_b}^4}$ i.e., lines $M_a F_{m_a}^3$ and $M_b F_{m_b}^4$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a = \neg E_a$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A = A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.3.2 Proving $E_a = \neg E_a$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.4.2 Proving $E_a=_Ea$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 581

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 581: Given a point A , a point E_a and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point E_a , construct a point H (rule W01); ;
2. Using the point O and the point H , construct a point N (rule W01); ;
3. Using the point O and the point H , construct a point G (rule W01); ;
4. Using the point A and the point G , construct a point M_a (rule W01); ;
5. Using the point A and the point E_a , construct a line h_a (rule W02); % DET: points A and E_a are not the same;
6. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
7. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
9. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; line h_a and circle $k(N, M_a)$ intersect; points E_a and N are not the same; points A and O are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points A and E_a are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D31,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L19,L22,L55,L58]

Solving time: 9.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point E_{a} 80 83.86
point O 65 51.14
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r E_{a}
cmark_t O
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0
```

```
% Constructing a point N such that ON/OH=0.5
towards N O H 0.5
cmark_r N
color 200 200 200
drawsegment O H
color 0 0 0
```

```
% Constructing a line L_{\_G231111} which passes through point O and point H
line L_{\_G231111} O H
```

```
color 200 200 200
drawline L_{\_G231111}
color 0 0 0
```

```
% Constructing a point P_{\_G231212} with coordinates (0,0)
point P_{\_G231212} 0 0
```

```

cmark_r P_{\_G231212}

% Constructing a point P_{\_G231136} such that OP_{\_G231136}/OP_{\_G231212}=1
towards P_{\_G231136} 0 P_{\_G231212} 1
cmark_r P_{\_G231136}
color 200 200 200
drawsegment 0 P_{\_G231136}
color 0 0 0

% Constructing a point P_{\_G231181} such that OP_{\_G231181}/OP_{\_G231212}=3
towards P_{\_G231181} 0 P_{\_G231212} 3
cmark_r P_{\_G231181}
color 200 200 200
drawsegment 0 P_{\_G231181}
color 0 0 0

% Constructing a line L_{\_G231142} which passes through point H and point P_{\_G231181}
line L_{\_G231142} H P_{\_G231181}

color 200 200 200
drawline L_{\_G231142}
color 0 0 0

% Constructing a line L_{\_G231105} which contains the point P_{\_G231136} and is parallel to the
line L_{\_G231142}
parallel L_{\_G231105} P_{\_G231136} L_{\_G231142}

color 200 200 200
drawline L_{\_G231105}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G231105} and line L_{\_G231111}
intersec G L_{\_G231105} L_{\_G231111}
cmark_t G

% Constructing a point M_{a} such that AM_{a}/AG=1.5
towards M_{a} A G 1.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% DET: points A and E_{a} are not the same
% Constructing a line h_{a} which passes through point A and point E_{a}
line h_{a} A E_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

```

```

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points E_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{a}
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G232099} which is a foot of the point N on the line h_{a}
foot P_{\_G232099} N h_{a}
cmark_r P_{\_G232099}
color 200 200 200
drawline N P_{\_G232099}
color 0 0 0

% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
\_G232099}
sim H_{a} P_{\_G232099} E_{a}
cmark_r H_{a}

% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```

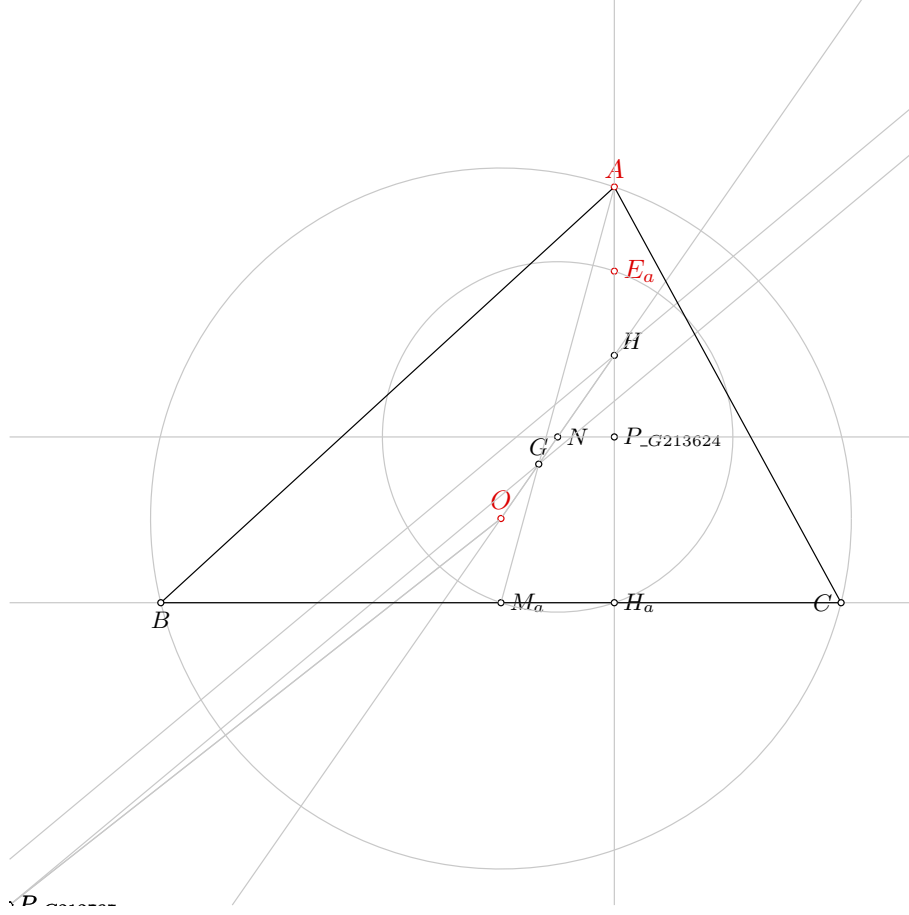


Figure 1: Illustration of the problem 0581

*% Non-degenerate conditions: line a and circle k(O,C) intersect; line h_{a} and circle k(N,M_{a}) intersect; points E_{a} and N are not the same; points A and O are not the same
 % Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be different; points A and E_{a} are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.031 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a = \neg E_a$

Proving failed

4.1.3 Proving $O = \neg O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A = A$

NDG conditions are:

$S_{P_{G214106}OH} \neq S_{P_{L_{G214075}}^0 OH}$ i.e., lines $P_{G214106}P_{L_{G214075}}^0$ and OH are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^2} \neq S_{F_{h_a}^1 BF_{h_b}^2}$ i.e., lines $AF_{h_a}^1$ and $BF_{h_b}^2$ are not parallel (construction based assumption)

$S_{M_a BC} \neq 0$ i.e., points M_a, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_b AC} \neq 0$ i.e., points M_b, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_a M_b F_{m_b}^4} \neq S_{F_{m_a}^3 M_b F_{m_b}^4}$ i.e., lines $M_a F_{m_a}^3$ and $M_b F_{m_b}^4$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a = \neg E_a$

Proving failed

4.2.3 Proving $O = \neg O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A = A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $E_a = \neg E_a$

Proving failed

4.3.3 Proving $O = \neg O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $E_a = -E_a$

Proving failed

4.4.3 Proving $O = -O$

Proving failed

Problem 582

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 582: Given a point A , a point E_a and a point T_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point E_a , construct a point H (rule W01); ;
2. Using the point A and the point E_a , construct a line h_a (rule W02); % DET: points A and E_a are not the same;
3. Using the point A and the point T_a , construct a line s_a (rule W02); % DET: points A and T_a are not the same;
4. Using the point T_a and the line h_a , construct a line a (rule W10a); ;
5. Using the point E_a , the point A , the point T_a , the line s_a and the line h_a , construct a line AO (rule W17); % NDG: points A and T_a are not the same; points E_a and A are not the same % DET: points A and T_a are not the same;
6. Using the point E_a and the line AO , construct a line $m(H_bH_c)$ (rule W16); ;
7. Using the line $m(H_bH_c)$ and the line a , construct a point M_a (rule W03); % NDG: lines $m(H_bH_c)$ and a are not parallel % DET: lines $m(H_bH_c)$ and a are not the same;
8. Using the point M_a and the point A , construct a point G (rule W01); ;
9. Using the point H and the point G , construct a point O (rule W01); ;
10. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
11. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; lines $m(H_bH_c)$ and a are not parallel; points A and T_a are not the same; points E_a and A are not the same.

Determination conditions: lines $m(H_bH_c)$ and a are not the same; points A and T_a are not the same; points A and E_a are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W16,W17]

Lemmas used: [D21,D23,D26,D28,D3,D8,GD01,GD02,GL01,GL03,GL04,GL09,GL17,L104,L11,L12,L38,L39,L4]

Solving time: 27.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point E_{a} 80 83.86
point T_{a} 70.86 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r E_{a}
cmark_rb T_{a}
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0
```

```
% DET: points A and E_{a} are not the same
% Constructing a line h_{a} which passes through point A and point E_{a}
line h_{a} A E_{a}
```

```
color 200 200 200
drawline h_{a}
color 0 0 0
```

```
% DET: points A and T_{a} are not the same
% Constructing a line s_{a} which passes through point A and point T_{a}
line s_{a} A T_{a}
```

```
color 200 200 200
drawline s_{a}
color 0 0 0
```

```

% Constructing a line a which is perpendicular to line h_{a} and which passes through point T_{a}
perp a T_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points A and T_{a} are not the same; points E_{a} and A are not the same% DET: points A and
T_{a} are not the same
% Constructing an angle V[_G6421] which is equal to the angle E_{a}AT_{a}
angle_o V[_G6421] E_{a} A T_{a}

% Calculating value angle[_G6500] using formula  $1/\text{pow}(2,0)*V[_G6421]+0/\text{pow}(2,0)*180$ 
expression angle[_G6500] {  $1/\text{pow}(2,0)*V[_G6421]+0/\text{pow}(2,0)*180$  }

% Constructing a point P_{\_G6497} which is an image of the point T_{a} in a rotation around the
point A for the angle  $1/\text{pow}(2,0)*V[_G6421]+0/\text{pow}(2,0)*180$ 
rotate P_{\_G6497} A angle[_G6500] T_{a}
cmark_r P_{\_G6497}
color 200 200 200
drawarc_p A T_{a} angle[_G6500]
color 0 0 0

% Constructing a line AO which passes through point A and point P_{\_G6497}
line AO A P_{\_G6497}

color 200 200 200
drawline AO
color 0 0 0

% Constructing a line m(H_{b}H_{c}) which contains the point E_{a} and is parallel to the line AO
parallel m(H_{b}H_{c}) E_{a} AO

color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0

% NDG: lines m(H_{b}H_{c}) and a are not parallel% DET: lines m(H_{b}H_{c}) and a are not the same
% Constructing a point M_{a} which belongs to line m(H_{b}H_{c}) and line a
intersec M_{a} m(H_{b}H_{c}) a
cmark_r M_{a}

% Constructing a line L_{\_G7014} which passes through point M_{a} and point A

```

```

line L_{\_G7014} M_{a} A

color 200 200 200
drawline L_{\_G7014}
color 0 0 0

% Constructing a point P_{\_G7115} with coordinates (0,0)
point P_{\_G7115} 0 0
cmark_r P_{\_G7115}

% Constructing a point P_{\_G7039} such that  $M_{a}P_{\_G7039}/M_{a}P_{\_G7115}=1$ 
towards P_{\_G7039} M_{a} P_{\_G7115} 1
cmark_r P_{\_G7039}
color 200 200 200
drawsegment M_{a} P_{\_G7039}
color 0 0 0

% Constructing a point P_{\_G7084} such that  $M_{a}P_{\_G7084}/M_{a}P_{\_G7115}=3$ 
towards P_{\_G7084} M_{a} P_{\_G7115} 3
cmark_r P_{\_G7084}
color 200 200 200
drawsegment M_{a} P_{\_G7084}
color 0 0 0

% Constructing a line L_{\_G7045} which passes through point A and point P_{\_G7084}
line L_{\_G7045} A P_{\_G7084}

color 200 200 200
drawline L_{\_G7045}
color 0 0 0

% Constructing a line L_{\_G7008} which contains the point P_{\_G7039} and is parallel to the line
L_{\_G7045}
parallel L_{\_G7008} P_{\_G7039} L_{\_G7045}

color 200 200 200
drawline L_{\_G7008}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G7008} and line L_{\_G7014}
intersec G L_{\_G7008} L_{\_G7014}
cmark_t G

% Constructing a point O such that  $HO/HG=1.5$ 
towards O H G 1.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

```

```

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% lines m(H_{b}H_{c}) and a are not parallel; points A and T_{a} are not the same; points E_{a}
% and A are not the same
% Determination conditions: lines m(H_{b}H_{c}) and a are not the same; points A and T_{a} are not
% the same; points A and T_{a} are not the same; points A and E_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a = E_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 103 terms.

Time Complexity: Time spent by the prover is 4.005 seconds.

NDG conditions Points A and B are not identical

Points T_a , A and H are not collinear

Line through points $P_{G256897}$ and A is not perpendicular to line through points A and E_a



Figure 1: Illustration of the problem 0582

Points A , $P_{G257295}$ and M_a are not collinear

Points A and $P_{G257295}$ are not identical

Line through points A and E_a is not parallel with line through points B and C

4.1.3 Proving $T_a = T_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A = A$

NDG conditions are:

$S_{T_a A E_a} \neq 0$ i.e., points T_a , A and E_a are not collinear (foot is not the point itself; construction based assumption)

$S_{E_a T_a F_a^0} \neq S_{P_{m(H_b H_c)}^1 T_a F_a^0}$ i.e., lines $E_a P_{m(H_b H_c)}^1$ and $T_a F_a^0$ are not parallel (construction based assumption)

$S_{P_{G246088} M_a A} \neq S_{P_{L_{G246057}}^2 M_a A}$ i.e., lines $P_{G246088} P_{L_{G246057}}^2$ and $M_a A$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^4} \neq S_{F_{-h_a}^3 BF_{-h_b}^4}$ i.e., lines $AF_{-h_a}^3$ and $BF_{-h_b}^4$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a = E_a$

Proving failed

4.2.3 Proving $T_a = T_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A = A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.2 Proving $E_a = E_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 3720 terms.

Time Complexity: Time spent by the prover is 8.090 seconds. There are no ndg conditions.

4.3.3 Proving $T_a = T_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 8 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $E_a = -E_a$

Proving failed

4.4.3 Proving $T_a = -T_a$

Proving failed

Problem 583

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 583: Given a point A , a point E_a and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 584

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 584: Given a point A , a point E_a and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 585

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 585: Given a point A , a point E_b and a point E_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 586

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 586: Given a point A , a point E_b and a point G , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 587

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 587: Given a point A , a point E_b and a point H , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point H , construct a point E_a (rule W01); ;
2. Using the point E_b and the point H , construct a point B (rule W01); ;
3. Using the point A and the point B , construct a line c (rule W02); % DET: points A and B are not the same;
4. Using the point E_b and the point H , construct a line h_b (rule W02); % DET: points E_b and H are not the same;
5. Using the point A and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points A and E_a are not the same;
6. Using the circle $k(E_a, A)$, the line c , the point E_a and the point A , construct a point H_c (rule W05); % NDG: line c and circle $k(E_a, A)$ intersect % DET: points A and H_c must be different;
7. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
8. Using the circle $k(E_a, A)$, the line h_b , the point E_a and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_a, A)$ intersect % DET: points H and H_b must be different;
9. Using the point H_b and the point A , construct a line b (rule W02); % DET: points H_b and A are not the same;
10. Using the line h_c and the line b , construct a point C (rule W03); % NDG: lines h_c and b are not parallel % DET: lines h_c and b are not the same.

Non-degenerate conditions: lines h_c and b are not parallel; line h_b and circle $k(E_a, A)$ intersect; line c and circle $k(E_a, A)$ intersect; points A and E_a are not the same.

Determination conditions: lines h_c and b are not the same; points H_b and A are not the same; points H and H_b must be different; points H_c and H are not the same; points A and H_c must be different; points E_b and H are not the same; points A and B are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D28,D29,D3,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L46,L47,L48]

Solving time: 11.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point E_{b} 50 56.36
point H 80 72.73
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r E_{b}
cmark_rt H
color 0 0 0
fontsize 8
```

```
% Constructing a point E_{a} such that AE_{a}/AH=0.5
towards E_{a} A H 0.5
cmark_r E_{a}
color 200 200 200
drawsegment A H
color 0 0 0
```

```
% Constructing a point B such that E_{b}B/E_{b}H=-1
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0
```

```
% DET: points A and B are not the same
% Constructing a line c which passes through point A and point B
line c A B
```

```
color 200 200 200
drawline c
color 0 0 0
```



```

% DET: points  $E_{\{b\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $E_{\{b\}}$  and point  $H$ 
line  $h_{\{b\}}$   $E_{\{b\}}$   $H$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: points  $A$  and  $E_{\{a\}}$  are not the same
% Constructing a circle  $k(E_{\{a\}}, A)$  whose center is at point  $E_{\{a\}}$  and which passes through point  $A$ 
circle  $k(E_{\{a\}}, A)$   $E_{\{a\}}$   $A$ 

color 200 200 200
drawcircle  $k(E_{\{a\}}, A)$ 
color 0 0 0

% NDG: line  $c$  and circle  $k(E_{\{a\}}, A)$  intersect% DET: points  $A$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\_G41621\}}$  which is a foot of the point  $E_{\{a\}}$  on the line  $c$ 
foot  $P_{\{\_G41621\}}$   $E_{\{a\}}$   $c$ 
cmark_r  $P_{\{\_G41621\}}$ 
color 200 200 200
drawline  $E_{\{a\}}$   $P_{\{\_G41621\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $A$  in the symmetry to point/line  $P_{\{\_G41621\}}$ 
sim  $H_{\{c\}}$   $P_{\{\_G41621\}}$   $A$ 
cmark_rt  $H_{\{c\}}$ 

% DET: points  $H_{\{c\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $H_{\{c\}}$  and point  $H$ 
line  $h_{\{c\}}$   $H_{\{c\}}$   $H$ 

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(E_{\{a\}}, A)$  intersect% DET: points  $H$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\_G41859\}}$  which is a foot of the point  $E_{\{a\}}$  on the line  $h_{\{b\}}$ 
foot  $P_{\{\_G41859\}}$   $E_{\{a\}}$   $h_{\{b\}}$ 
cmark_r  $P_{\{\_G41859\}}$ 
color 200 200 200
drawline  $E_{\{a\}}$   $P_{\{\_G41859\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $H$  in the symmetry to point/line  $P_{\{\_G41859\}}$ 
sim  $H_{\{b\}}$   $P_{\{\_G41859\}}$   $H$ 
cmark_l  $H_{\{b\}}$ 

```

```

% DET: points  $H_{\{b\}}$  and  $A$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $A$ 
line b  $H_{\{b\}}$  A

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $h_{\{c\}}$  and  $b$  are not parallel% DET: lines  $h_{\{c\}}$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $h_{\{c\}}$  and line  $b$ 
intersec C  $h_{\{c\}}$  b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{c\}}$  and  $b$  are not parallel; line  $h_{\{b\}}$  and circle  $k(E_{\{a\}}, A)$ 
  intersect; line  $c$  and circle  $k(E_{\{a\}}, A)$  intersect; points  $A$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $h_{\{c\}}$  and  $b$  are not the same; points  $H_{\{b\}}$  and  $A$  are not the same
  ; points  $H$  and  $H_{\{b\}}$  must be different; points  $H_{\{c\}}$  and  $H$  are not the same; points  $A$  and  $H_{\{c\}}$ 
  must be different; points  $E_{\{b\}}$  and  $H$  are not the same; points  $A$  and  $B$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b=_E_b$

Proving failed

4.1.3 Proving $H=_H$

Proving failed

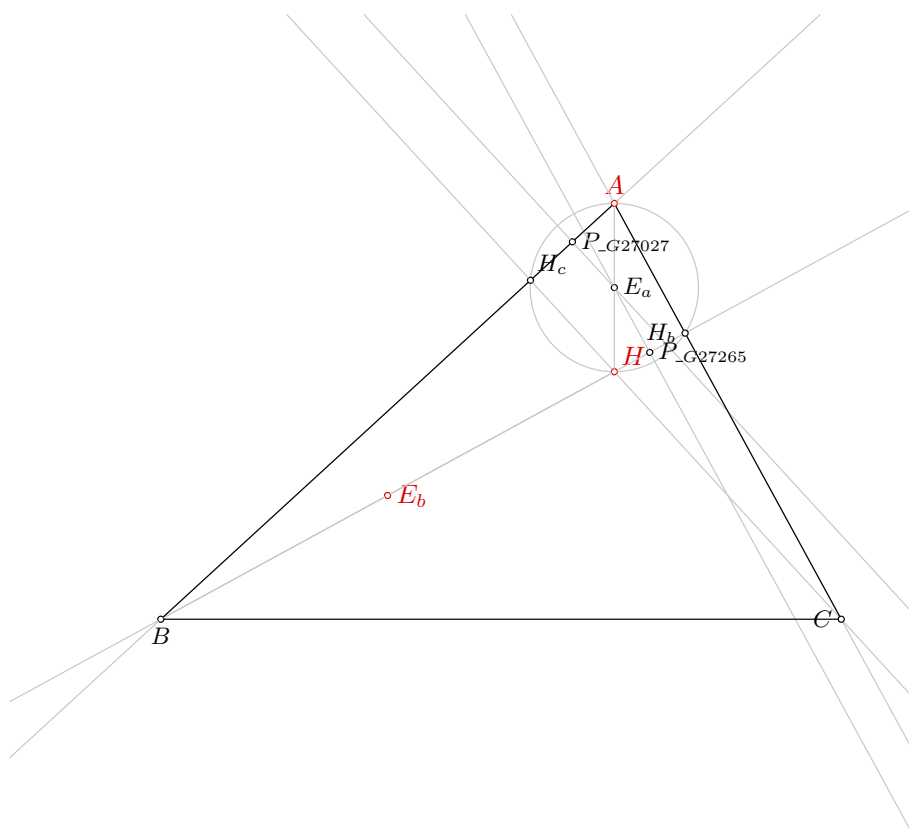


Figure 1: Illustration of the problem 0587

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_c H_b A} \neq S_{H H_b A}$ i.e., lines $H_c H$ and $H_b A$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b = \neg E_b$

Proving failed

4.2.3 Proving $H = \neg H$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_b = \neg E_b$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 767 terms.

Time Complexity: Time spent by the prover is 0.520 seconds. There are no ndg conditions.

4.3.3 Proving $H = \neg H$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 138 terms.

Time Complexity: Time spent by the prover is 0.090 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_b = \neg E_b$

Proving failed

4.4.3 Proving $H = \neg H$

Proving failed

Problem 588

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 588: Given a point A , a point E_b and a point H_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
2. Using the point H_a and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_a and E_b are not the same;
3. Using the circle $k(E_b, B)$, the line h_a , the point E_b and the point H_a , construct a point H (rule W05); % NDG: line h_a and circle $k(E_b, B)$ intersect % DET: points H_a and H must be different;
4. Using the point E_b and the point H , construct a point B (rule W01); ;
5. Using the point A and the point B , construct a line c (rule W02); % DET: points A and B are not the same;
6. Using the point H_a and the point B , construct a line a (rule W02); % DET: points H_a and B are not the same;
7. Using the circle $k(E_b, B)$, the line c , the point E_b and the point B , construct a point H_c (rule W05); % NDG: line c and circle $k(E_b, B)$ intersect % DET: points B and H_c must be different;
8. Using the point H and the point H_c , construct a line h_c (rule W02); % DET: points H and H_c are not the same;
9. Using the line a and the line h_c , construct a point C (rule W03); % NDG: lines a and h_c are not parallel % DET: lines a and h_c are not the same.

Non-degenerate conditions: lines a and h_c are not parallel; line c and circle $k(E_b, B)$ intersect; line h_a and circle $k(E_b, B)$ intersect; points H_a and E_b are not the same.

Determination conditions: lines a and h_c are not the same; points H and H_c are not the same; points B and H_c must be different; points H_a and B are not the same; points A and B are not the same; points H_a and H must be different; points A and H_a are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D29,D3,D5,D7,D8,GD01,GD02,GL03,GL04,L3,L49,L50,L51]

Solving time: 8.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point E_{b} 50 56.36
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_t A
cmark_r E_{b}
cmark_r H_{a}
color 0 0 0
fontsize 8

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points H_{a} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H_{a}
circle k(E_{b},B) E_{b} H_{a}

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line h_{a} and circle k(E_{b},B) intersect% DET: points H_{a} and H must be different
% Constructing a point P_{\_G63395} which is a foot of the point E_{b} on the line h_{a}
foot P_{\_G63395} E_{b} h_{a}
cmark_r P_{\_G63395}
color 200 200 200
drawline E_{b} P_{\_G63395}
color 0 0 0
```

```

% Constructing a point H which is an image of the point H_{a} in the symmetry to point/line P_{\_G
63395}
sim H P_{\_G63395} H_{a}
cmark_rt H

% Constructing a point B such that E_{b}B/E_{b}H=-1
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

% DET: points A and B are not the same
% Constructing a line c which passes through point A and point B
line c A B

color 200 200 200
drawline c
color 0 0 0

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% NDG: line c and circle k(E_{b},B) intersect% DET: points B and H_{c} must be different
% Constructing a point P_{\_G63758} which is a foot of the point E_{b} on the line c
foot P_{\_G63758} E_{b} c
cmark_r P_{\_G63758}
color 200 200 200
drawline E_{b} P_{\_G63758}
color 0 0 0

% Constructing a point H_{c} which is an image of the point B in the symmetry to point/line P_{\_G
63758}
sim H_{c} P_{\_G63758} B
cmark_rt H_{c}

% DET: points H and H_{c} are not the same
% Constructing a line h_{c} which passes through point H and point H_{c}
line h_{c} H H_{c}

color 200 200 200
drawline h_{c}

```

```
color 0 0 0
```

```
% NDG: lines a and h_{c} are not parallel% DET: lines a and h_{c} are not the same
% Constructing a point C which belongs to line a and line h_{c}
intersec C a h_{c}
cmark_l C
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines a and h_{c} are not parallel; line c and circle k(E_{b},B)
    intersect; line h_{a} and circle k(E_{b},B) intersect; points H_{a} and E_{b} are not the same
% Determination conditions: lines a and h_{c} are not the same; points H and H_{c} are not the same
    ; points B and H_{c} must be different; points H_{a} and B are not the same; points A and B are
    not the same; points H_{a} and H must be different; points A and H_{a} are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b = \neg E_b$

Proving failed

4.1.3 Proving $H_a = \neg H_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_a H H_c} \neq S_{B H H_c}$ i.e., lines $H_a B$ and $H H_c$ are not parallel (construction based assumption)
 $S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a} BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

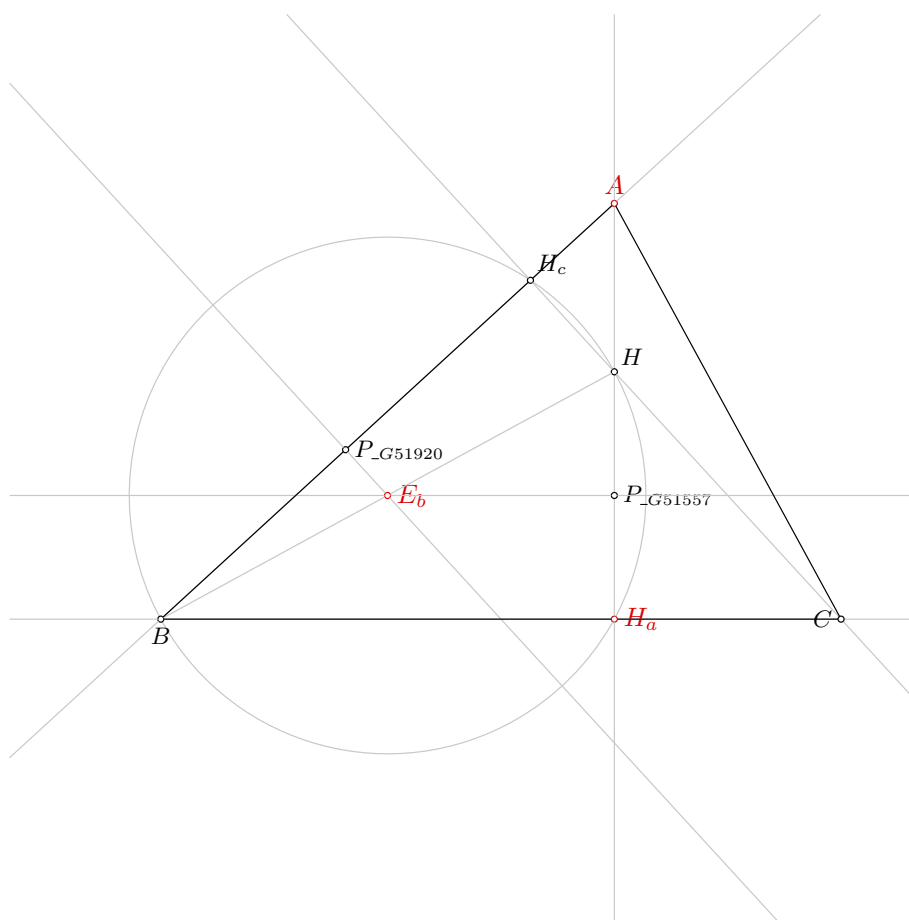


Figure 1: Illustration of the problem 0588

$S_{BAF^0_{\neg h_a}} \neq S_{CAF^0_{\neg h_a}}$ i.e., lines BC and $AF^0_{\neg h_a}$ are not parallel (construction based assumption)
 Total number of proof steps: 1
 Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b = \neg E_b$

Proving failed

4.2.3 Proving $H_a = \neg H_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A = A$

Proving failed

4.3.2 Proving $E_b = \neg E_b$

Proving failed

4.3.3 Proving $H_a = \neg H_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A = A$

Proving failed

4.4.2 Proving $E_b = \neg E_b$

Proving failed

4.4.3 Proving $H_a = \neg H_a$

Proving failed

Problem 589

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 589: Given a point E_b , a point H_b and a point A , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;
2. Using the point H_b and the line h_b , construct a line b (rule W10a); ;
3. Choose freely a point A on the line b (rule WOnline2);
4. Choose freely a point B on the line h_b (rule WOnline1) ;
5. Using the point B and the point E_b , construct a point H (rule W01); ;
6. Using the point B and the point A , construct a line c (rule W02); % DET: points B and A are not the same;
7. Using the point B and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points B and E_b are not the same;
8. Using the circle $k(E_b, B)$, the line c , the point E_b and the point B , construct a point H_c (rule W05); % NDG: line c and circle $k(E_b, B)$ intersect % DET: points B and H_c must be different;
9. Using the point H and the point H_c , construct a line h_c (rule W02); % DET: points H and H_c are not the same;
10. Using the line b and the line h_c , construct a point C (rule W03); % NDG: lines b and h_c are not parallel % DET: lines b and h_c are not the same.

Non-degenerate conditions: lines b and h_c are not parallel; line c and circle $k(E_b, B)$ intersect; points B and E_b are not the same.

Determination conditions: lines b and h_c are not the same; points H and H_c are not the same; points B and H_c must be different; points B and A are not the same; points E_b and H_b are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D10,D29,D3,D6,D7,D9,GD01,GD02,GL03,GL09,L3,L51]

Solving time: 63.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
```

```
point H_{b} 89.36 77.83
```

```
point A 80 95
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{b}
```

```
cmark_l H_{b}
```

```
cmark_t A
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{b} and H_{b} are not the same
```

```
% Constructing a line h_{b} which passes through point E_{b} and point H_{b}
```

```
line h_{b} E_{b} H_{b}
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```
% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
```

```
perp b H_{b} h_{b}
```

```
color 200 200 200
```

```
drawline b
```

```
color 0 0 0
```

```
% Generating random value V[_G85528]
```

```
random V[_G85528]
```

```
% Calculating value V[_G85549] using formula V[_G85528]*20
```

```
expression V[_G85549] { V[_G85528]*20 }
```

```

% Constructing a point A which is a point for which holds  $H_{\{b\}}A = V[_{G85549}]$  and angle  $E_{\{b\}}H_{\{b\}}A = 90$ 
turtle A  $E_{\{b\}}$   $H_{\{b\}}$  90  $V[_{G85549}]$ 
cmark_t A

% Choosing randomly a point B on the line  $E_{\{b\}}H_{\{b\}}$ 
online B  $E_{\{b\}}$   $H_{\{b\}}$ 
cmark_b B
color 200 200 200
drawline  $E_{\{b\}}$   $H_{\{b\}}$ 
color 0 0 0

% Constructing a point H such that  $BH/BE_{\{b\}}=2$ 
towards H B  $E_{\{b\}}$  2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0

% DET: points B and A are not the same
% Constructing a line c which passes through point B and point A
line c B A

color 200 200 200
drawline c
color 0 0 0

% NDG: points B and  $E_{\{b\}}$  are not the same
% Constructing a circle  $k(E_{\{b\}}, B)$  whose center is at point  $E_{\{b\}}$  and which passes through point B
circle k( $E_{\{b\}}$ , B)  $E_{\{b\}}$  B

color 200 200 200
drawcircle k( $E_{\{b\}}$ , B)
color 0 0 0

% NDG: line c and circle  $k(E_{\{b\}}, B)$  intersect% DET: points B and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G86069\}}$  which is a foot of the point  $E_{\{b\}}$  on the line c
foot  $P_{\{\backslash\_G86069\}}$   $E_{\{b\}}$  c
cmark_r  $P_{\{\backslash\_G86069\}}$ 
color 200 200 200
drawline  $E_{\{b\}}$   $P_{\{\backslash\_G86069\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point B in the symmetry to point/line  $P_{\{\backslash\_G86069\}}$ 
sim  $H_{\{c\}}$   $P_{\{\backslash\_G86069\}}$  B

```

```

cmark_rt H_{c}

% DET: points H and H_{c} are not the same
% Constructing a line h_{c} which passes through point H and point H_{c}
line h_{c} H H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines b and h_{c} are not parallel% DET: lines b and h_{c} are not the same
% Constructing a point C which belongs to line b and line h_{c}
intersec C b h_{c}
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and h_{c} are not parallel; line c and circle k(E_{b},B)
% intersect; points B and E_{b} are not the same
% Determination conditions: lines b and h_{c} are not the same; points H and H_{c} are not the same
% ; points B and H_{c} must be different; points B and A are not the same; points E_{b} and H_{b}
% are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 55 terms.

Time Complexity: Time spent by the prover is 0.955 seconds.

NDG conditions Points A and B are not identical

Point E_b is not on circle with center H_b and point from it B

Line through points B and H_c is not perpendicular to line through points H_c and E_b

Points A , C and E_b are not collinear

4.1.2 Proving $H_b = \neg H_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

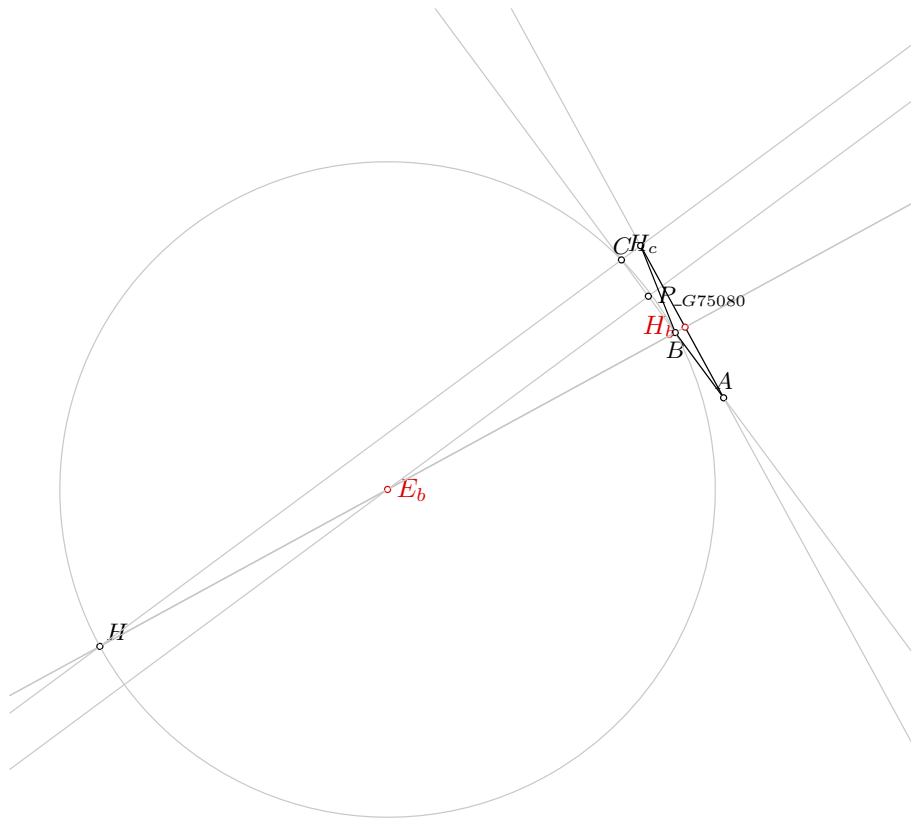


Figure 1: Illustration of the problem 0589

Time Complexity: Time spent by the prover is 0.152 seconds.

NDG conditions Points A and B are not identical

Point E_b is not on circle with center H_b and point from it B

Line through points B and H_c is not perpendicular to line through points H_c and E_b

Points A , C and E_b are not collinear

4.1.3 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.013 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $H_b = \neg H_b$

Proving failed

4.2.3 Proving $A=A$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 9 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $H_b = \neg H_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 16 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.3 Proving $A=A$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 20 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $H_b = \neg H_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 20 terms.

Time Complexity: Time spent by the prover is 0.050 seconds. There are no ndg conditions.

4.4.3 Proving $A = A$

Proving failed

Problem 590

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 590: Given a point A , a point E_b and a point H_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point H_c , construct a line c (rule W02); % DET: points A and H_c are not the same;
2. Using the point H_c and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_c and E_b are not the same;
3. Using the circle $k(E_b, B)$, the line c , the point E_b and the point H_c , construct a point B (rule W05); % NDG: line c and circle $k(E_b, B)$ intersect % DET: points H_c and B must be different;
4. Using the point E_b and the point B , construct a point H (rule W01); ;
5. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
6. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
7. Using the circle $k(E_b, B)$, the line h_a , the point E_b and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_b, B)$ intersect % DET: points H and H_a must be different;
8. Using the point B and the point H_a , construct a line a (rule W02); % DET: points B and H_a are not the same;
9. Using the line h_c and the line a , construct a point C (rule W03); % NDG: lines h_c and a are not parallel % DET: lines h_c and a are not the same.

Non-degenerate conditions: lines h_c and a are not parallel; line h_a and circle $k(E_b, B)$ intersect; line c and circle $k(E_b, B)$ intersect; points H_c and E_b are not the same.

Determination conditions: lines h_c and a are not the same; points B and H_a are not the same; points H and H_a must be different; points H_c and H are not the same; points A and H are not the same; points H_c and B must be different; points A and H_c are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D29,D3,D5,D7,D8,GD01,GD02,GL03,GL04,L3,L49,L50,L51]

Solving time: 8.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point E_{b} 50 56.36
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_t A
cmark_r E_{b}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points H_{c} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H
_{c}
circle k(E_{b},B) E_{b} H_{c}

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line c and circle k(E_{b},B) intersect% DET: points H_{c} and B must be different
% Constructing a point P_{\_G108276} which is a foot of the point E_{b} on the line c
foot P_{\_G108276} E_{b} c
cmark_r P_{\_G108276}
color 200 200 200
drawline E_{b} P_{\_G108276}
color 0 0 0
```

```

% Constructing a point B which is an image of the point H_{c} in the symmetry to point/line P_{\_G
108276}
sim B P_{\_G108276} H_{c}
cmark_b B

% Constructing a point H such that E_{b}H/E_{b}B=-1
towards H E_{b} B -1
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: line h_{a} and circle k(E_{b},B) intersect% DET: points H and H_{a} must be different
% Constructing a point P_{\_G108639} which is a foot of the point E_{b} on the line h_{a}
foot P_{\_G108639} E_{b} h_{a}
cmark_r P_{\_G108639}
color 200 200 200
drawline E_{b} P_{\_G108639}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H in the symmetry to point/line P_{\_G
108639}
sim H_{a} P_{\_G108639} H
cmark_r H_{a}

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a

```

```
color 0 0 0
```

```
% NDG: lines h_{c} and a are not parallel% DET: lines h_{c} and a are not the same
% Constructing a point C which belongs to line h_{c} and line a
intersec C h_{c} a
cmark_l C
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines h_{c} and a are not parallel; line h_{a} and circle k(E_{b},B)
    intersect; line c and circle k(E_{b},B) intersect; points H_{c} and E_{b} are not the same
% Determination conditions: lines h_{c} and a are not the same; points B and H_{a} are not the same
    ; points H and H_{a} must be different; points H_{c} and H are not the same; points A and H are
    not the same; points H_{c} and B must be different; points A and H_{c} are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.033 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b = \neg E_b$

Proving failed

4.1.3 Proving $H_c = \neg H_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 1176 terms.

Time Complexity: Time spent by the prover is 7.687 seconds.

NDG conditions Points A , H_c and E_b are not collinear

Points A , H_c and E_b are not collinear

Points A and H are not identical

Points A and H are not identical

Line through points B and H_a is not parallel with line through points H_c and H

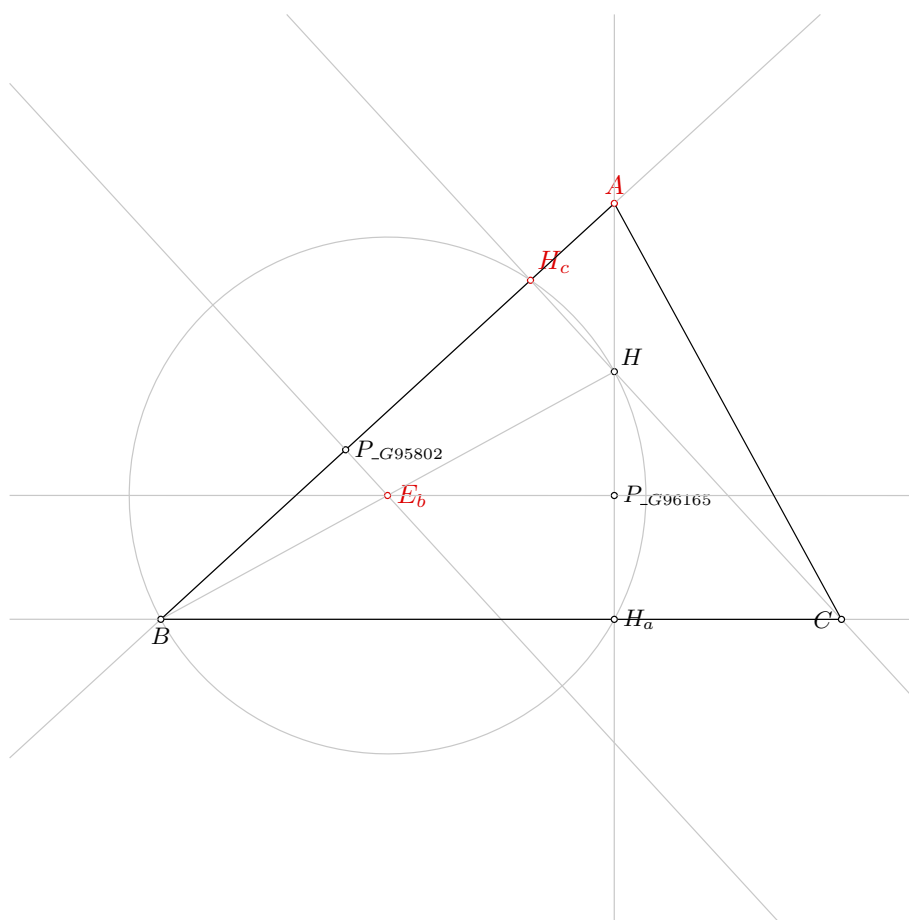


Figure 1: Illustration of the problem 0590

Line through points A and E_b is not parallel with line through points H_c and H

Points A and B are not identical

Points A , B and E_b are not collinear

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_c B H_a} \neq S_{H B H_a}$ i.e., lines $H_c H$ and $B H_a$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a} B F^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF^2_{-h_c}} \neq S_{BCF^2_{-h_c}}$ i.e., lines AB and $CF^2_{-h_c}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b=_E E_b$

Proving failed

4.2.3 Proving $H_c=_H H_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Proving failed

4.3.2 Proving $E_b=_E E_b$

Proving failed

4.3.3 Proving $H_c=_H H_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Proving failed

4.4.2 Proving $E_b=_E E_b$

Proving failed

4.4.3 Proving $H_c=_H H_c$

Proving failed

Problem 591

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 591: Given a point A , a point E_b and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 592

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 592: Given a point A , a point E_b and a point M_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 593

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 593: Given a point A , a point E_b and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point M_b , construct a point C (rule W01); ;
2. Using the point A and the point M_b , construct a line b (rule W02); % DET: points A and M_b are not the same;
3. Using the point E_b and the point M_b , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and M_b are not the same;
4. Using the point E_b and the line b , construct a line h_b (rule W10b); ;
5. Using the line h_b and the line b , construct a point H_b (rule W03); % NDG: lines h_b and b are not parallel % DET: lines h_b and b are not the same;
6. Using the point A and the point H_b , construct a line $m(AH_b)$ (rule W14); % DET: points A and H_b are not the same;
7. Using the point E_b and the point M_b , construct a line $m(E_bM_b)$ (rule W14); % DET: points E_b and M_b are not the same;
8. Using the line $m(E_bM_b)$ and the line $m(H_aH_c)$, construct a point N (rule W03); % NDG: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel % DET: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same;
9. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
10. Using the circle $k(N, M_a)$ and the line $m(AH_b)$, construct a point M_c and a point E_a (rule W04); % NDG: line $m(AH_b)$ and circle $k(N, M_a)$ intersect;

11. Using the point M_c and the point A , construct a point B (rule W01); .

Non-degenerate conditions: line $m(AH_b)$ and circle $k(N, M_a)$ intersect; points E_b and N are not the same; lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel; lines h_b and b are not parallel.

Determination conditions: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same; points E_b and M_b are not the same; points A and H_b are not the same; lines h_b and b are not the same; points E_b and M_b are not the same; points A and M_b are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14]

Lemmas used: [D20,D22,D29,D3,D32,D6,D9,GD01,GD02,GL01,GL03,GL04,GL09,L17,L18,L19,L21,L22,L23,L

Solving time: 2.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point E_{b} 50 56.36
point M_{b} 95 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r E_{b}
cmark_lt M_{b}
color 0 0 0
fontsize 8
```

```
% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0
```

```
% DET: points A and M_{b} are not the same
% Constructing a line b which passes through point A and point M_{b}
line b A M_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% DET: points E_{b} and M_{b} are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point M_{b}
line m(H_{a}H_{c}) E_{b} M_{b}
```

```
color 200 200 200
drawline m(H_{a}H_{c})
```

```
color 0 0 0
```

```
% Constructing a line  $h_{\{b\}}$  which is perpendicular to line  $b$  and which passes through point  $E_{\{b\}}$ 
perp  $h_{\{b\}}$   $E_{\{b\}}$   $b$ 
```

```
color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0
```

```
% NDG: lines  $h_{\{b\}}$  and  $b$  are not parallel% DET: lines  $h_{\{b\}}$  and  $b$  are not the same
% Constructing a point  $H_{\{b\}}$  which belongs to line  $h_{\{b\}}$  and line  $b$ 
intersec  $H_{\{b\}}$   $h_{\{b\}}$   $b$ 
cmark_l  $H_{\{b\}}$ 
```

```
% DET: points  $A$  and  $H_{\{b\}}$  are not the same
% Constructing bisector  $m(AH_{\{b\}})$  of the segment  $AH_{\{b\}}$ 
med  $m(AH_{\{b\}})$   $A$   $H_{\{b\}}$ 
```

```
color 200 200 200
drawline  $m(AH_{\{b\}})$ 
color 0 0 0
```

```
color 200 200 200
drawsegment  $A$   $H_{\{b\}}$ 
color 0 0 0
```

```
% DET: points  $E_{\{b\}}$  and  $M_{\{b\}}$  are not the same
% Constructing bisector  $m(E_{\{b\}}M_{\{b\}})$  of the segment  $E_{\{b\}}M_{\{b\}}$ 
med  $m(E_{\{b\}}M_{\{b\}})$   $E_{\{b\}}$   $M_{\{b\}}$ 
```

```
color 200 200 200
drawline  $m(E_{\{b\}}M_{\{b\}})$ 
color 0 0 0
```

```
color 200 200 200
drawsegment  $E_{\{b\}}$   $M_{\{b\}}$ 
color 0 0 0
```

```
% NDG: lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not parallel% DET: lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}$ 
 $H_{\{c\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{b\}}M_{\{b\}})$  and line  $m(H_{\{a\}}H_{\{c\}})$ 
intersec  $N$   $m(E_{\{b\}}M_{\{b\}})$   $m(H_{\{a\}}H_{\{c\}})$ 
cmark_r  $N$ 
```

```
% NDG: points  $E_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{b\}}$ 
circle  $k(N, M_{\{a\}})$   $N$   $E_{\{b\}}$ 
```

```

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(AH_{b}) and circle k(N,M_{a}) intersect
% Constructing points M_{c} and E_{a} which are in intersection of k(N,M_{a}) and m(AH_{b})
intersec2 M_{c} E_{a} k(N,M_{a}) m(AH_{b})
cmark_lt M_{c}
cmark_r E_{a}

% Constructing a point B such that M_{c}B/M_{c}A=-1
towards B M_{c} A -1
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line m(AH_{b}) and circle k(N,M_{a}) intersect; points E_{b} and N are
% not the same; lines m(E_{b}M_{b}) and m(H_{a}H_{c}) are not parallel; lines h_{b} and b are
% not parallel
% Determination conditions: lines m(E_{b}M_{b}) and m(H_{a}H_{c}) are not the same; points E_{b}
% and M_{b} are not the same; points A and H_{b} are not the same; lines h_{b} and b are not the
% same; points E_{b} and M_{b} are not the same; points A and M_{b} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b = E_b$

Proving failed

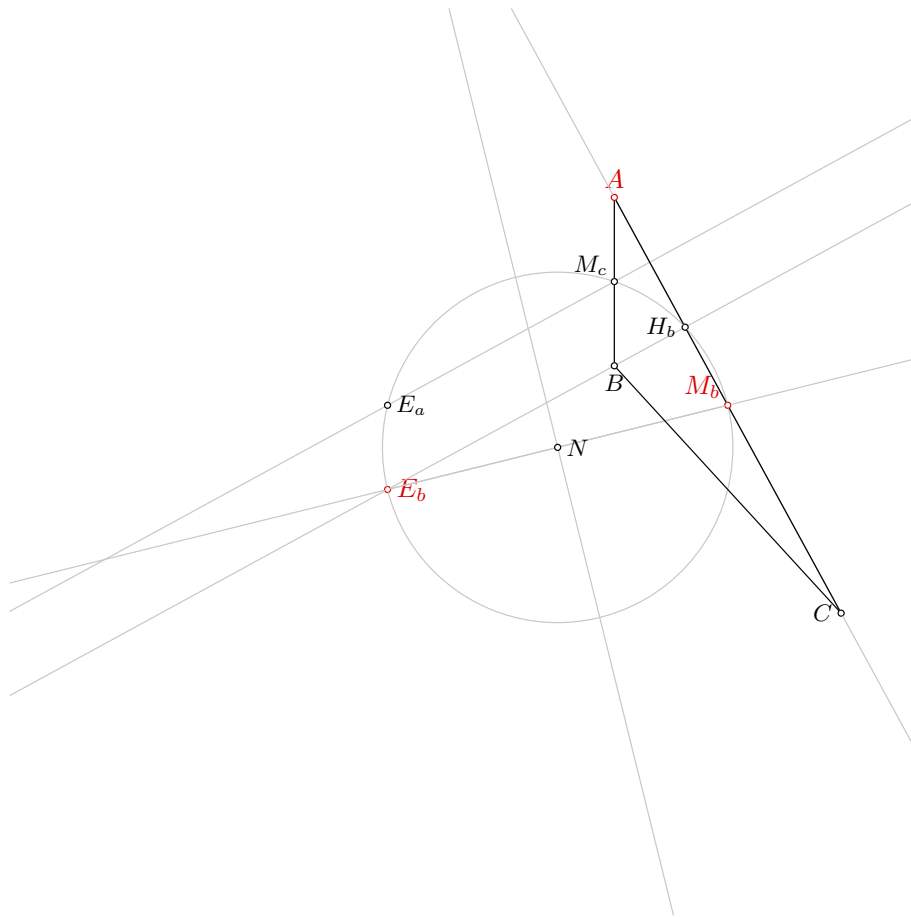


Figure 1: Illustration of the problem 0593

4.1.3 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.032 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{E_b A M_b} \neq 0$ i.e., points E_b , A and M_b are not collinear (foot is not the point itself; construction based assumption)

$S_{E_b A M_b} \neq S_{F_{h_b}^0 A M_b}$ i.e., lines $E_b F_{h_b}^0$ and $A M_b$ are not parallel (construction based assumption)

$S_{M_{m(E_b M_b)}^3 E_b M_b} \neq S_{T_{m(E_b M_b)}^4 E_b M_b}$ i.e., lines $M_{m(E_b M_b)}^3$ and $T_{m(E_b M_b)}^4$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{AB F_{h_b}^6} \neq S_{F_{h_a}^5 B F_{h_b}^6}$ i.e., lines $A F_{h_a}^5$ and $B F_{h_b}^6$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b = \neg E_b$

Proving failed

4.2.3 Proving $M_b = \neg M_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.2 Proving $E_b = \neg E_b$

Proving failed

4.3.3 Proving $M_b = \neg M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 13 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $E_b = \neg E_b$

Proving failed

4.4.3 Proving $M_b = \neg M_b$

Proving failed

Problem 594

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 594: Given a point A , a point E_b and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point M_c , construct a point B (rule W01); ;
2. Using the point E_b and the point B , construct a point H (rule W01); ;
3. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
4. Using the point E_b and the point B , construct a line h_b (rule W02); % DET: points E_b and B are not the same;
5. Using the point A and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points A and M_c are not the same;
6. Using the circle $k(M_c, A)$, the line h_a , the point M_c and the point A , construct a point H_a (rule W05); % NDG: line h_a and circle $k(M_c, A)$ intersect % DET: points A and H_a must be different;
7. Using the point H_a and the point B , construct a line a (rule W02); % DET: points H_a and B are not the same;
8. Using the circle $k(M_c, A)$, the line h_b , the point M_c and the point B , construct a point H_b (rule W05); % NDG: line h_b and circle $k(M_c, A)$ intersect % DET: points B and H_b must be different;
9. Using the point H_b and the point A , construct a line b (rule W02); % DET: points H_b and A are not the same;
10. Using the line a and the line b , construct a point C (rule W03); % NDG: lines a and b are not parallel % DET: lines a and b are not the same.

Non-degenerate conditions: lines a and b are not parallel; line h_b and circle $k(M_c, A)$ intersect; line h_a and circle $k(M_c, A)$ intersect; points A and M_c are not the same.

Determination conditions: lines a and b are not the same; points H_b and A are not the same; points B and H_b must be different; points H_a and B are not the same; points A and H_a must be different; points E_b and B are not the same; points A and H are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D20,D29,D3,D5,D6,D8,D9,GD01,GD02,GL03,GL04,GL09,L40,L41,L42]

Solving time: 10.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point E_{b} 50 56.36
point M_{c} 50 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r E_{b}
cmark_lt M_{c}
color 0 0 0
fontsize 8
```

```
% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0
```

```
% Constructing a point H such that E_{b}H/E_{b}B=-1
towards H E_{b} B -1
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0
```

```
% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H
```

```
color 200 200 200
drawline h_{a}
color 0 0 0
```

```

% DET: points E_{b} and B are not the same
% Constructing a line h_{b} which passes through point E_{b} and point B
line h_{b} E_{b} B

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points A and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point A
circle k(M_{c},A) M_{c} A

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line h_{a} and circle k(M_{c},A) intersect% DET: points A and H_{a} must be different
% Constructing a point P_{\_G156024} which is a foot of the point M_{c} on the line h_{a}
foot P_{\_G156024} M_{c} h_{a}
cmark_r P_{\_G156024}
color 200 200 200
drawline M_{c} P_{\_G156024}
color 0 0 0

% Constructing a point H_{a} which is an image of the point A in the symmetry to point/line P_{\_G
156024}
sim H_{a} P_{\_G156024} A
cmark_r H_{a}

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% NDG: line h_{b} and circle k(M_{c},A) intersect% DET: points B and H_{b} must be different
% Constructing a point P_{\_G156262} which is a foot of the point M_{c} on the line h_{b}
foot P_{\_G156262} M_{c} h_{b}
cmark_r P_{\_G156262}
color 200 200 200
drawline M_{c} P_{\_G156262}
color 0 0 0

% Constructing a point H_{b} which is an image of the point B in the symmetry to point/line P_{\_G
156262}
sim H_{b} P_{\_G156262} B
cmark_l H_{b}

```

```

% DET: points  $H_{\{b\}}$  and  $A$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $A$ 
line b  $H_{\{b\}}$  A

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $a$  and  $b$  are not parallel% DET: lines  $a$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $a$  and line  $b$ 
intersec C a b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $a$  and  $b$  are not parallel; line  $h_{\{b\}}$  and circle  $k(M_{\{c\}},A)$ 
% intersect; line  $h_{\{a\}}$  and circle  $k(M_{\{c\}},A)$  intersect; points  $A$  and  $M_{\{c\}}$  are not the same
% Determination conditions: lines  $a$  and  $b$  are not the same; points  $H_{\{b\}}$  and  $A$  are not the same;
% points  $B$  and  $H_{\{b\}}$  must be different; points  $H_{\{a\}}$  and  $B$  are not the same; points  $A$  and  $H_{\{a\}}$ 
% must be different; points  $E_{\{b\}}$  and  $B$  are not the same; points  $A$  and  $H$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.034 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b = \neg E_b$

Proving failed

4.1.3 Proving $M_c = \neg M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

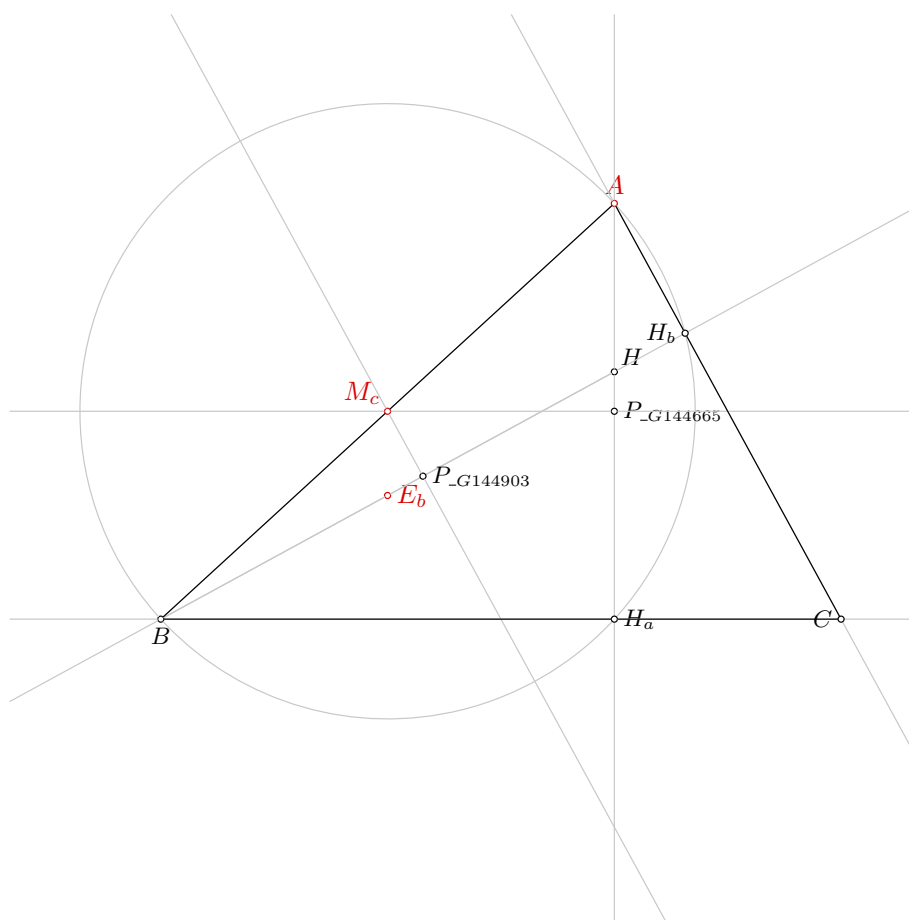


Figure 1: Illustration of the problem 0594

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_a H_b A} \neq S_{B H_b A}$ i.e., lines $H_a B$ and $H_b A$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b = \neg E_b$

Proving failed

4.2.3 Proving $M_c = \neg M_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_b = \neg E_b$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 124 terms.

Time Complexity: Time spent by the prover is 0.470 seconds. There are no ndg conditions.

4.3.3 Proving $M_c = \neg M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 9 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_b = \neg E_b$

Proving failed

4.4.3 Proving $M_c = -M_c$

Proving failed

Problem 595

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 595: Given a point A , a point E_b and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point N , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and N are not the same;
2. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
3. Using the circle $k(N, M_a)$, the line $m(H_aH_c)$, the point N and the point E_b , construct a point M_b (rule W05a); % NDG: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect % DET: points E_b and M_b must be different;
4. Using the point M_b and the point A , construct a point C (rule W01); ;
5. Using the point A and the point M_b , construct a line b (rule W02); % DET: points A and M_b are not the same;
6. Using the circle $k(N, M_a)$, the line b , the point N and the point M_b , construct a point H_b (rule W05); % NDG: line b and circle $k(N, M_a)$ intersect % DET: points M_b and H_b must be different;
7. Using the point H_b and the point E_b , construct a line h_b (rule W02); % DET: points H_b and E_b are not the same;
8. Using the point A and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points A and M_b are not the same;
9. Using the circle $k(N, M_a)$ and the circle $k(M_b, C)$, construct a point H_a and a point H_c (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_b, C)$ intersect % DET: circles $k(N, M_a)$ and $k(M_b, C)$ are not the same;

10. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
11. Using the line h_b and the line h_a , construct a point H (rule W03); % NDG: lines h_b and h_a are not parallel % DET: lines h_b and h_a are not the same;
12. Using the point E_b and the point H , construct a point B (rule W01); .

Non-degenerate conditions: lines h_b and h_a are not parallel; circles $k(N, M_a)$ and $k(M_b, C)$ intersect; points A and M_b are not the same; line b and circle $k(N, M_a)$ intersect; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; points E_b and N are not the same.

Determination conditions: lines h_b and h_a are not the same; points A and H_a are not the same; circles $k(N, M_a)$ and $k(M_b, C)$ are not the same; points H_b and E_b are not the same; points M_b and H_b must be different; points A and M_b are not the same; points E_b and M_b must be different; points E_b and N are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D22,D29,D3,D32,D5,D6,D8,D9,GD01,GD02,GL01,GL03,GL04,GL09,L17,L19,L20,L21,L23,L43]

Solving time: 4.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point E_{b} 50 56.36
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r E_{b}
cmark_r N
color 0 0 0
fontsize 8
```

```
% DET: points E_{b} and N are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point N
line m(H_{a}H_{c}) E_{b} N
```

```
color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0
```

```
% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}
```

```
color 200 200 200
drawcircle k(N,M_{a})
```

```

color 0 0 0

% NDG: line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $E_{\{b\}}$  and  $M_{\{b\}}$  must be
different
% Constructing a point  $M_{\{b\}}$  which is an image of the point  $E_{\{b\}}$  in the symmetry to point/line  $N$ 
sim  $M_{\{b\}}$   $N$   $E_{\{b\}}$ 
cmark_lt  $M_{\{b\}}$ 

% Constructing a point  $C$  such that  $M_{\{b\}}C/M_{\{b\}}A=-1$ 
towards  $C$   $M_{\{b\}}$   $A$  -1
cmark_l  $C$ 
color 200 200 200
drawsegment  $A$   $C$ 
color 0 0 0

% DET: points  $A$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $A$  and point  $M_{\{b\}}$ 
line  $b$   $A$   $M_{\{b\}}$ 

color 200 200 200
drawline  $b$ 
color 0 0 0

% NDG: line  $b$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $M_{\{b\}}$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G192447\}}$  which is a foot of the point  $N$  on the line  $b$ 
foot  $P_{\{\backslash\_G192447\}}$   $N$   $b$ 
cmark_r  $P_{\{\backslash\_G192447\}}$ 
color 200 200 200
drawline  $N$   $P_{\{\backslash\_G192447\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $M_{\{b\}}$  in the symmetry to point/line  $P_{\{\backslash\_G192447\}}$ 
sim  $H_{\{b\}}$   $P_{\{\backslash\_G192447\}}$   $M_{\{b\}}$ 
cmark_l  $H_{\{b\}}$ 

% DET: points  $H_{\{b\}}$  and  $E_{\{b\}}$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $H_{\{b\}}$  and point  $E_{\{b\}}$ 
line  $h_{\{b\}}$   $H_{\{b\}}$   $E_{\{b\}}$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: points  $A$  and  $M_{\{b\}}$  are not the same
% Constructing a circle  $k(M_{\{b\}},C)$  whose center is at point  $M_{\{b\}}$  and which passes through point  $A$ 

```

```

circle k(M_{b},C) M_{b} A

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(M_{b},C) intersect% DET: circles k(N,M_{a}) and k(M_{b},C) are not
the same
% Constructing points H_{a} and H_{c} which are in intersection of k(N,M_{a}) and k(M_{b},C)
intersec2 H_{a} H_{c} k(N,M_{a}) k(M_{b},C)
cmark_r H_{a}
cmark_rt H_{c}

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: lines h_{b} and h_{a} are not parallel% DET: lines h_{b} and h_{a} are not the same
% Constructing a point H which belongs to line h_{b} and line h_{a}
intersec H h_{b} h_{a}
cmark_rt H

% Constructing a point B such that E_{b}B/E_{b}H=-1
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{b} and h_{a} are not parallel; circles k(N,M_{a}) and k(M_{b}
,C) intersect; points A and M_{b} are not the same; line b and circle k(N,M_{a}) intersect;
line m(H_{a}H_{c}) and circle k(N,M_{a}) intersect; points E_{b} and N are not the same
% Determination conditions: lines h_{b} and h_{a} are not the same; points A and H_{a} are not the
same; circles k(N,M_{a}) and k(M_{b},C) are not the same; points H_{b} and E_{b} are not the
same; points M_{b} and H_{b} must be different; points A and M_{b} are not the same; points E_{
b} and M_{b} must be different; points E_{b} and N are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

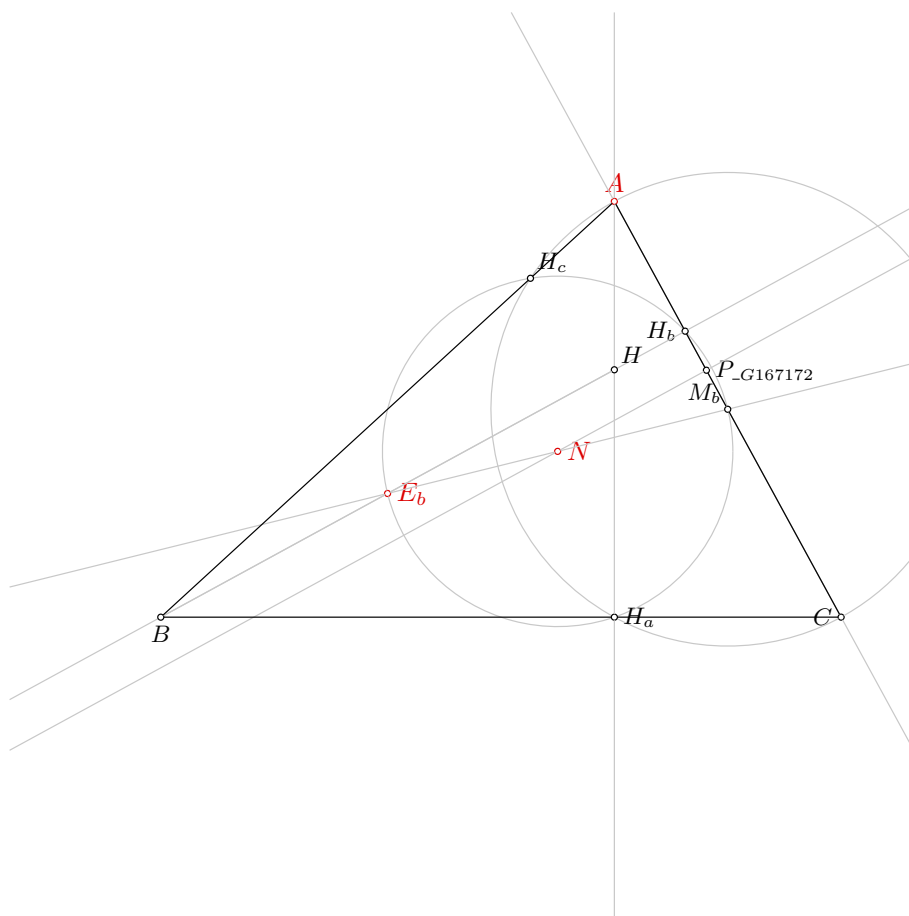


Figure 1: Illustration of the problem 0595

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.03 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b=_E_b$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_bAH_a} \neq S_{E_bAH_a}$ i.e., lines H_bE_b and AH_a are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

$S_{_M_aBC} \neq 0$ i.e., points $_M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$ i.e., points $_M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a_M_bF_{-m_b}^3} \neq S_{F_{-m_a}^2_M_bF_{-m_b}^3}$ i.e., lines $_M_aF_{-m_a}^2$ and $_M_bF_{-m_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b=_E_b$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Proving failed

4.3.2 Proving $E_b=_E_b$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Proving failed

4.4.2 Proving $E_b=_E E_b$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 596

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 596: Given a point A , a point E_b and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 597

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 597: Given a point A , a point E_b and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 598

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 598: Given a point A , a point E_b and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 599

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 599: Given a point A , a point E_b and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 600

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 600: Given a point A , a point E_c and a point G , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 601

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 601: Given a point A , a point E_c and a point H , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point H , construct a point E_a (rule W01); ;
2. Using the point E_c and the point H , construct a point C (rule W01); ;
3. Using the point A and the point C , construct a line b (rule W02); % DET: points A and C are not the same;
4. Using the point E_c and the point H , construct a line h_c (rule W02); % DET: points E_c and H are not the same;
5. Using the point A and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points A and E_a are not the same;
6. Using the circle $k(E_a, A)$, the line b , the point E_a and the point A , construct a point H_b (rule W05); % NDG: line b and circle $k(E_a, A)$ intersect % DET: points A and H_b must be different;
7. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
8. Using the circle $k(E_a, A)$, the line h_c , the point E_a and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_a, A)$ intersect % DET: points H and H_c must be different;
9. Using the point H_c and the point A , construct a line c (rule W02); % DET: points H_c and A are not the same;
10. Using the line h_b and the line c , construct a point B (rule W03); % NDG: lines h_b and c are not parallel % DET: lines h_b and c are not the same.

Non-degenerate conditions: lines h_b and c are not parallel; line h_c and circle $k(E_a, A)$ intersect; line b and circle $k(E_a, A)$ intersect; points A and E_a are not the same.

Determination conditions: lines h_b and c are not the same; points H_c and A are not the same; points H and H_c must be different; points H_b and H are not the same; points A and H_b must be different; points E_c and H are not the same; points A and C are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D28,D3,D30,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L46,L47,L48]

Solving time: 10.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point E_{c} 95 56.36
point H 80 72.73
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r E_{c}
cmark_rt H
color 0 0 0
fontsize 8
```

```
% Constructing a point E_{a} such that AE_{a}/AH=0.5
towards E_{a} A H 0.5
cmark_r E_{a}
color 200 200 200
drawsegment A H
color 0 0 0
```

```
% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0
```

```
% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C
```

```
color 200 200 200
drawline b
color 0 0 0
```

```

% DET: points E_{c} and H are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H
line h_{c} E_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points A and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point A
circle k(E_{a},A) E_{a} A

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: line b and circle k(E_{a},A) intersect% DET: points A and H_{b} must be different
% Constructing a point P_{\_G229866} which is a foot of the point E_{a} on the line b
foot P_{\_G229866} E_{a} b
cmark_r P_{\_G229866}
color 200 200 200
drawline E_{a} P_{\_G229866}
color 0 0 0

% Constructing a point H_{b} which is an image of the point A in the symmetry to point/line P_{\_G
229866}
sim H_{b} P_{\_G229866} A
cmark_l H_{b}

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{c} and circle k(E_{a},A) intersect% DET: points H and H_{c} must be different
% Constructing a point P_{\_G230104} which is a foot of the point E_{a} on the line h_{c}
foot P_{\_G230104} E_{a} h_{c}
cmark_r P_{\_G230104}
color 200 200 200
drawline E_{a} P_{\_G230104}
color 0 0 0

% Constructing a point H_{c} which is an image of the point H in the symmetry to point/line P_{\_G
230104}
sim H_{c} P_{\_G230104} H
cmark_rt H_{c}

```

```

% DET: points  $H_{\{c\}}$  and  $A$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $A$ 
line c  $H_{\{c\}}$  A

color 200 200 200
drawline c
color 0 0 0

% NDG: lines  $h_{\{b\}}$  and  $c$  are not parallel% DET: lines  $h_{\{b\}}$  and  $c$  are not the same
% Constructing a point  $B$  which belongs to line  $h_{\{b\}}$  and line  $c$ 
intersec B  $h_{\{b\}}$  c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{b\}}$  and  $c$  are not parallel; line  $h_{\{c\}}$  and circle  $k(E_{\{a\}}, A)$ 
% intersect; line  $b$  and circle  $k(E_{\{a\}}, A)$  intersect; points  $A$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $h_{\{b\}}$  and  $c$  are not the same; points  $H_{\{c\}}$  and  $A$  are not the same
% ; points  $H$  and  $H_{\{c\}}$  must be different; points  $H_{\{b\}}$  and  $H$  are not the same; points  $A$  and  $H_{\{b\}}$ 
% must be different; points  $E_{\{c\}}$  and  $H$  are not the same; points  $A$  and  $C$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.027 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c=_E_c$

Proving failed

4.1.3 Proving $H=_H$

Proving failed

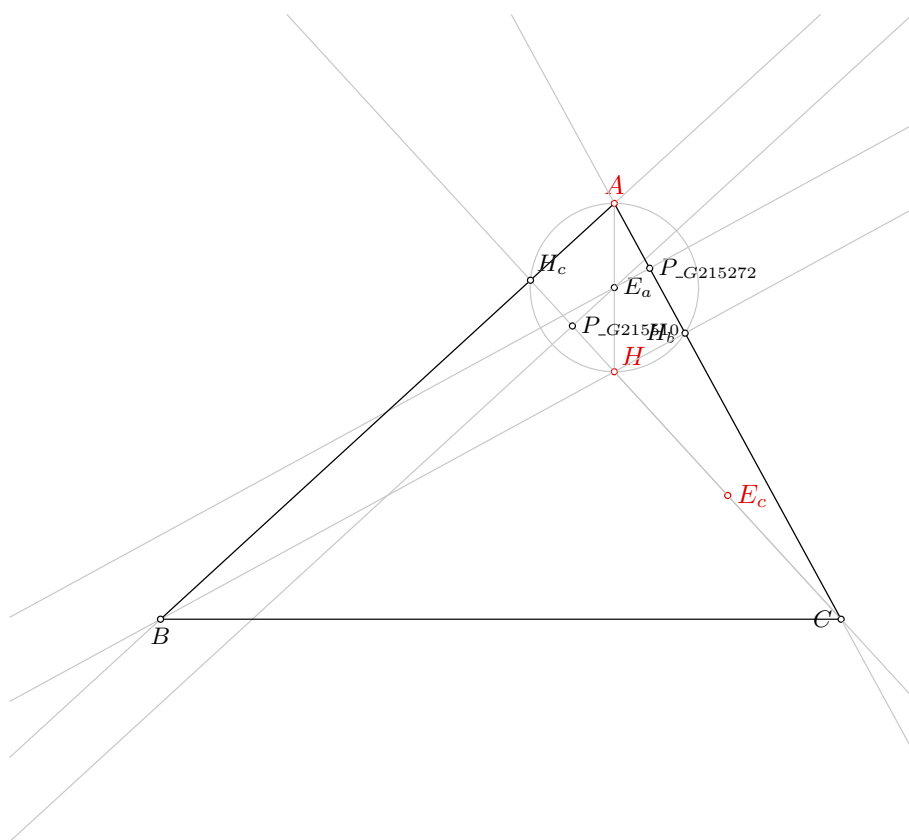


Figure 1: Illustration of the problem 0601

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_bH_cA} \neq S_{HH_cA}$ i.e., lines H_bH and H_cA are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c=_E E_c$

Proving failed

4.2.3 Proving $H=_H H$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_c=_E E_c$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 1023 terms.

Time Complexity: Time spent by the prover is 0.760 seconds. There are no ndg conditions.

4.3.3 Proving $H=_H H$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 182 terms.

Time Complexity: Time spent by the prover is 0.150 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_c=_E E_c$

Proving failed

4.4.3 Proving $H=_H H$

Proving failed

Problem 602

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 602: Given a point A , a point E_c and a point H_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
2. Using the point H_a and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_a and E_c are not the same;
3. Using the circle $k(E_c, C)$, the line h_a , the point E_c and the point H_a , construct a point H (rule W05); % NDG: line h_a and circle $k(E_c, C)$ intersect % DET: points H_a and H must be different;
4. Using the point E_c and the point H , construct a point C (rule W01); ;
5. Using the point A and the point C , construct a line b (rule W02); % DET: points A and C are not the same;
6. Using the point H_a and the point C , construct a line a (rule W02); % DET: points H_a and C are not the same;
7. Using the circle $k(E_c, C)$, the line b , the point E_c and the point C , construct a point H_b (rule W05); % NDG: line b and circle $k(E_c, C)$ intersect % DET: points C and H_b must be different;
8. Using the point H and the point H_b , construct a line h_b (rule W02); % DET: points H and H_b are not the same;
9. Using the line a and the line h_b , construct a point B (rule W03); % NDG: lines a and h_b are not parallel % DET: lines a and h_b are not the same.

Non-degenerate conditions: lines a and h_b are not parallel; line b and circle $k(E_c, C)$ intersect; line h_a and circle $k(E_c, C)$ intersect; points H_a and E_c are not the same.

Determination conditions: lines a and h_b are not the same; points H and H_b are not the same; points C and H_b must be different; points H_a and C are not the same; points A and C are not the same; points H_a and H must be different; points A and H_a are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,GL04,L52,L53,L54]

Solving time: 8.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point E_{c} 95 56.36
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_t A
cmark_r E_{c}
cmark_r H_{a}
color 0 0 0
fontsize 8

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points H_{a} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H_{a}
circle k(E_{c},C) E_{c} H_{a}

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% NDG: line h_{a} and circle k(E_{c},C) intersect% DET: points H_{a} and H must be different
% Constructing a point P_{\G252719} which is a foot of the point E_{c} on the line h_{a}
foot P_{\G252719} E_{c} h_{a}
cmark_r P_{\G252719}
color 200 200 200
drawline E_{c} P_{\G252719}
color 0 0 0
```

```

% Constructing a point H which is an image of the point H_{a} in the symmetry to point/line P_{\_G
252719}
sim H P_{\_G252719} H_{a}
cmark_rt H

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C

color 200 200 200
drawline b
color 0 0 0

% DET: points H_{a} and C are not the same
% Constructing a line a which passes through point H_{a} and point C
line a H_{a} C

color 200 200 200
drawline a
color 0 0 0

% NDG: line b and circle k(E_{c},C) intersect% DET: points C and H_{b} must be different
% Constructing a point P_{\_G253082} which is a foot of the point E_{c} on the line b
foot P_{\_G253082} E_{c} b
cmark_r P_{\_G253082}
color 200 200 200
drawline E_{c} P_{\_G253082}
color 0 0 0

% Constructing a point H_{b} which is an image of the point C in the symmetry to point/line P_{\_G
253082}
sim H_{b} P_{\_G253082} C
cmark_l H_{b}

% DET: points H and H_{b} are not the same
% Constructing a line h_{b} which passes through point H and point H_{b}
line h_{b} H H_{b}

color 200 200 200
drawline h_{b}

```

```
color 0 0 0
```

```
% NDG: lines a and h_{b} are not parallel% DET: lines a and h_{b} are not the same
% Constructing a point B which belongs to line a and line h_{b}
intersec B a h_{b}
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines a and h_{b} are not parallel; line b and circle k(E_{c},C)
    intersect; line h_{a} and circle k(E_{c},C) intersect; points H_{a} and E_{c} are not the same
% Determination conditions: lines a and h_{b} are not the same; points H and H_{b} are not the same
    ; points C and H_{b} must be different; points H_{a} and C are not the same; points A and C are
    not the same; points H_{a} and H must be different; points A and H_{a} are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c = E_c$

Proving failed

4.1.3 Proving $H_a = H_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_a H H_b} \neq S_{C H H_b}$ i.e., lines $H_a C$ and $H H_b$ are not parallel (construction based assumption)
 $S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a} BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

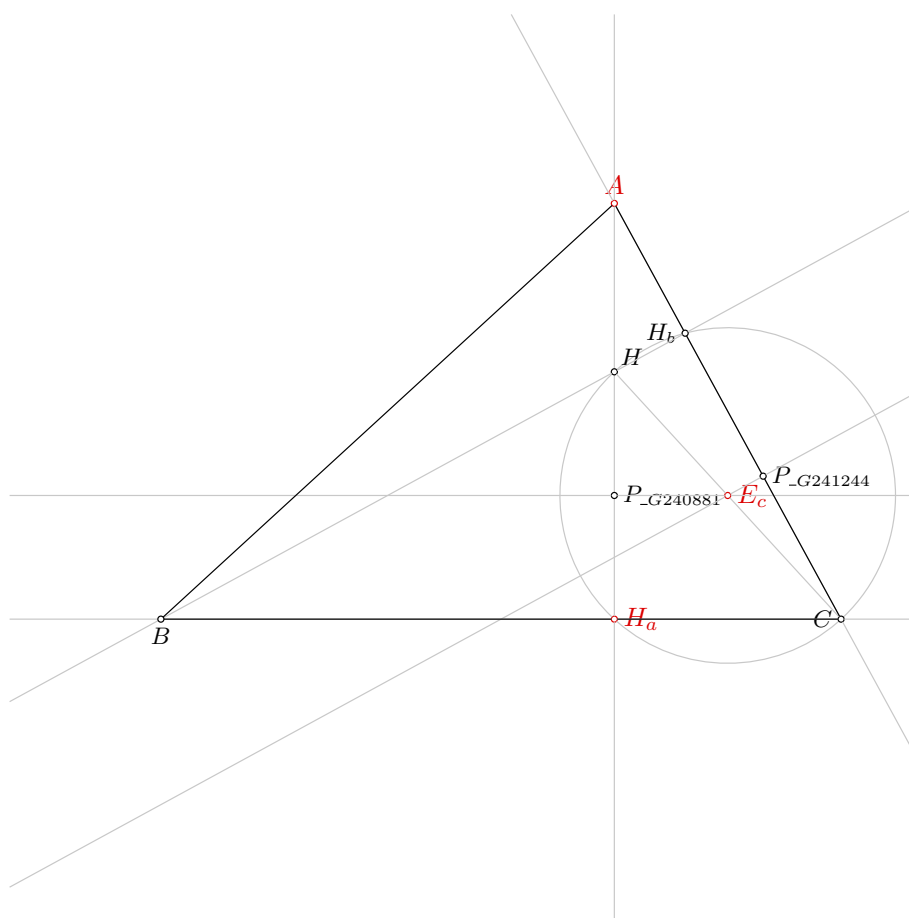


Figure 1: Illustration of the problem 0602

$S_{BAF^0_{\neg h_a}} \neq S_{CAF^0_{\neg h_a}}$ i.e., lines BC and $AF^0_{\neg h_a}$ are not parallel (construction based assumption)
Total number of proof steps: 1
Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c = \neg E_c$

Proving failed

4.2.3 Proving $H_a = \neg H_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A = A$

Proving failed

4.3.2 Proving $E_c = \neg E_c$

Proving failed

4.3.3 Proving $H_a = \neg H_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A = A$

Proving failed

4.4.2 Proving $E_c = \neg E_c$

Proving failed

4.4.3 Proving $H_a = \neg H_a$

Proving failed

Problem 603

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 603: Given a point A , a point E_c and a point H_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;
2. Using the point H_b and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_b and E_c are not the same;
3. Using the circle $k(E_c, C)$, the line b , the point E_c and the point H_b , construct a point C (rule W05); % NDG: line b and circle $k(E_c, C)$ intersect % DET: points H_b and C must be different;
4. Using the point E_c and the point C , construct a point H (rule W01); ;
5. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
6. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
7. Using the circle $k(E_c, C)$, the line h_a , the point E_c and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_c, C)$ intersect % DET: points H and H_a must be different;
8. Using the point C and the point H_a , construct a line a (rule W02); % DET: points C and H_a are not the same;
9. Using the line h_b and the line a , construct a point B (rule W03); % NDG: lines h_b and a are not parallel % DET: lines h_b and a are not the same.

Non-degenerate conditions: lines h_b and a are not parallel; line h_a and circle $k(E_c, C)$ intersect; line b and circle $k(E_c, C)$ intersect; points H_b and E_c are not the same.

Determination conditions: lines h_b and a are not the same; points C and H_a are not the same; points H and H_a must be different; points H_b and H are not the same; points A and H are not the same; points H_b and C must be different; points A and H_b are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,GL04,L52,L53,L54]

Solving time: 8.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point E_{c} 95 56.36
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_t A
cmark_r E_{c}
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points H_{b} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H_{b}
circle k(E_{c},C) E_{c} H_{b}

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% NDG: line b and circle k(E_{c},C) intersect% DET: points H_{b} and C must be different
% Constructing a point P_{\_G19665} which is a foot of the point E_{c} on the line b
foot P_{\_G19665} E_{c} b
cmark_r P_{\_G19665}
color 200 200 200
drawline E_{c} P_{\_G19665}
color 0 0 0
```

```

% Constructing a point C which is an image of the point H_{b} in the symmetry to point/line P_{\_G
19665}
sim C P_{\_G19665} H_{b}
cmark_l C

% Constructing a point H such that E_{c}H/E_{c}C=-1
towards H E_{c} C -1
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{a} and circle k(E_{c},C) intersect% DET: points H and H_{a} must be different
% Constructing a point P_{\_G20028} which is a foot of the point E_{c} on the line h_{a}
foot P_{\_G20028} E_{c} h_{a}
cmark_r P_{\_G20028}
color 200 200 200
drawline E_{c} P_{\_G20028}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H in the symmetry to point/line P_{\_G
20028}
sim H_{a} P_{\_G20028} H
cmark_r H_{a}

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a

```

```
color 0 0 0
```

```
% NDG: lines h_{b} and a are not parallel% DET: lines h_{b} and a are not the same
% Constructing a point B which belongs to line h_{b} and line a
intersec B h_{b} a
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines h_{b} and a are not parallel; line h_{a} and circle k(E_{c},C)
    intersect; line b and circle k(E_{c},C) intersect; points H_{b} and E_{c} are not the same
% Determination conditions: lines h_{b} and a are not the same; points C and H_{a} are not the same
    ; points H and H_{a} must be different; points H_{b} and H are not the same; points A and H are
    not the same; points H_{b} and C must be different; points A and H_{b} are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c = E_c$

Proving failed

4.1.3 Proving $H_b = H_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 1176 terms.

Time Complexity: Time spent by the prover is 7.679 seconds.

NDG conditions Points H_b and E_c are not identical

Points H_b and E_c are not identical

Points A and H are not identical

Points A and H are not identical

Line through points H_b and H is not parallel with line through points C and H_a

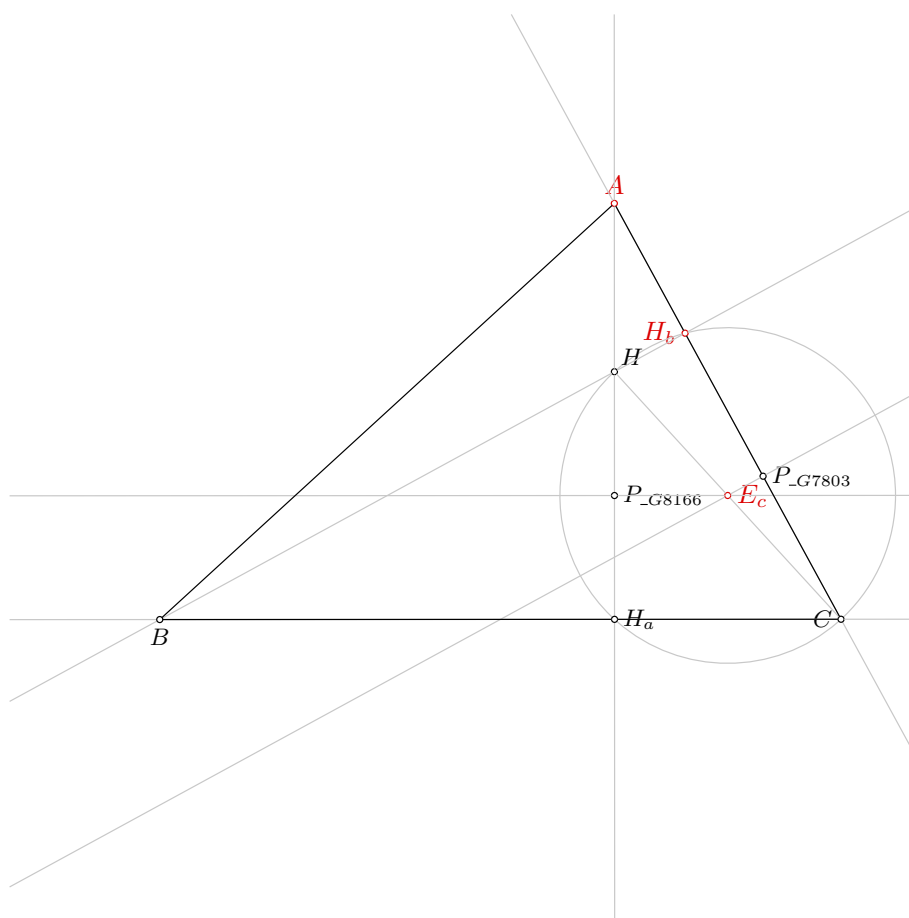


Figure 1: Illustration of the problem 0603

Line through points H_b and H is not parallel with line through points E_c and A

Points A and C are not identical

Points A and C are not identical

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_bCH_a} \neq S_{HCH_a}$ i.e., lines H_bH and CH_a are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a}BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

$S_{ABF^1_{-h_b}} \neq S_{CBF^1_{-h_b}}$ i.e., lines AC and $BF^1_{-h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c = \neg E_c$

Proving failed

4.2.3 Proving $H_b = \neg H_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Proving failed

4.3.2 Proving $E_c = \neg E_c$

Proving failed

4.3.3 Proving $H_b = \neg H_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Proving failed

4.4.2 Proving $E_c = \neg E_c$

Proving failed

4.4.3 Proving $H_b = \neg H_b$

Proving failed

Problem 604

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 604: Given a point E_c , a point H_c and a point A , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;
2. Using the point H_c and the line h_c , construct a line c (rule W10a); ;
3. Choose freely a point A on the line c (rule WOnline2);
4. Using the point A and the point H_c , construct a line $m(AH_c)$ (rule W14); % DET: points A and H_c are not the same;
5. Using the point E_c and the point H_c , construct a line $m(E_cH_c)$ (rule W14); % DET: points E_c and H_c are not the same;
6. Choose freely a point B on the line c (rule WOnline1) ;
7. Using the point B and the point A , construct a point M_c (rule W01); ;
8. Using the point E_c and the point M_c , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and M_c are not the same;
9. Using the line $m(E_cH_c)$ and the line $m(H_bH_a)$, construct a point N (rule W03); % NDG: lines $m(E_cH_c)$ and $m(H_bH_a)$ are not parallel % DET: lines $m(E_cH_c)$ and $m(H_bH_a)$ are not the same;
10. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
11. Using the circle $k(N, M_a)$ and the line $m(AH_c)$, construct a point M_b and a point E_a (rule W04); % NDG: line $m(AH_c)$ and circle $k(N, M_a)$ intersect;

12. Using the point A and the point M_b , construct a point C (rule W01); .

Non-degenerate conditions: line $m(AH_c)$ and circle $k(N, M_a)$ intersect; points E_c and N are not the same; lines $m(E_cH_c)$ and $m(H_bH_a)$ are not parallel.

Determination conditions: lines $m(E_cH_c)$ and $m(H_bH_a)$ are not the same; points E_c and M_c are not the same; points E_c and H_c are not the same; points A and H_c are not the same; points E_c and H_c are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W14,WOnline1,WOnline2]

Lemmas used: [D10,D20,D22,D30,D32,D7,GD02,GL01,GL03,GL04,GL09,L17,L19,L20,L21,L22,L24,L3,L41,L42]

Solving time: 54.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
point H_{c} 68.91 84.83
point A 80 95
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{c}
cmark_rt H_{c}
cmark_t A
color 0 0 0
fontsize 8
```

```
% DET: points E_{c} and H_{c} are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H_{c}
line h_{c} E_{c} H_{c}
```

```
color 200 200 200
drawline h_{c}
color 0 0 0
```

```
% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}
```

```
color 200 200 200
drawline c
color 0 0 0
```

```
% Generating random value V[_G43770]
random V[_G43770]
```

```

% Calculating value  $V[_{G43791}]$  using formula  $V[_{G43770}]*20$ 
expression  $V[_{G43791}] \{ V[_{G43770}]*20 \}$ 

% Constructing a point A which is a point for which holds  $H_{\{c\}}A = V[_{G43791}]$  and angle  $E_{\{c\}}H_{\{c\}}A = 90$ 
turtle A  $E_{\{c\}} H_{\{c\}} 90 V[_{G43791}]$ 
cmark_t A

% DET: points A and  $H_{\{c\}}$  are not the same
% Constructing bisector  $m(AH_{\{c\}})$  of the segment  $AH_{\{c\}}$ 
med m( $AH_{\{c\}}$ ) A  $H_{\{c\}}$ 

color 200 200 200
drawline m( $AH_{\{c\}}$ )
color 0 0 0

color 200 200 200
drawsegment A  $H_{\{c\}}$ 
color 0 0 0

% DET: points  $E_{\{c\}}$  and  $H_{\{c\}}$  are not the same
% Constructing bisector  $m(E_{\{c\}}H_{\{c\}})$  of the segment  $E_{\{c\}}H_{\{c\}}$ 
med m( $E_{\{c\}}H_{\{c\}}$ )  $E_{\{c\}}$   $H_{\{c\}}$ 

color 200 200 200
drawline m( $E_{\{c\}}H_{\{c\}}$ )
color 0 0 0

color 200 200 200
drawsegment  $E_{\{c\}}$   $H_{\{c\}}$ 
color 0 0 0

% Choosing randomly a point B on the line  $AH_{\{c\}}$ 
online B A  $H_{\{c\}}$ 
cmark_b B
color 200 200 200
drawline A  $H_{\{c\}}$ 
color 0 0 0

% Constructing a point  $M_{\{c\}}$  such that  $BM_{\{c\}}/BA=0.5$ 
towards  $M_{\{c\}}$  B A 0.5
cmark_lt  $M_{\{c\}}$ 
color 200 200 200
drawsegment B A
color 0 0 0

% DET: points  $E_{\{c\}}$  and  $M_{\{c\}}$  are not the same
% Constructing a line  $m(H_{\{b\}}H_{\{a\}})$  which passes through point  $E_{\{c\}}$  and point  $M_{\{c\}}$ 

```



```

line m(H_{b}H_{a}) E_{c} M_{c}

color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0

% NDG: lines m(E_{c}H_{c}) and m(H_{b}H_{a}) are not parallel% DET: lines m(E_{c}H_{c}) and m(H_{b}
H_{a}) are not the same
% Constructing a point N which belongs to line m(E_{c}H_{c}) and line m(H_{b}H_{a})
intersec N m(E_{c}H_{c}) m(H_{b}H_{a})
cmark_r N

% NDG: points E_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{c}
circle k(N,M_{a}) N E_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(AH_{c}) and circle k(N,M_{a}) intersect
% Constructing points M_{b} and E_{a} which are in intersection of k(N,M_{a}) and m(AH_{c})
intersec2 M_{b} E_{a} k(N,M_{a}) m(AH_{c})
cmark_lt M_{b}
cmark_r E_{a}

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line m(AH_{c}) and circle k(N,M_{a}) intersect; points E_{c} and N are
not the same; lines m(E_{c}H_{c}) and m(H_{b}H_{a}) are not parallel
% Determination conditions: lines m(E_{c}H_{c}) and m(H_{b}H_{a}) are not the same; points E_{c}
and M_{c} are not the same; points E_{c} and H_{c} are not the same; points A and H_{c} are not
the same; points E_{c} and H_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

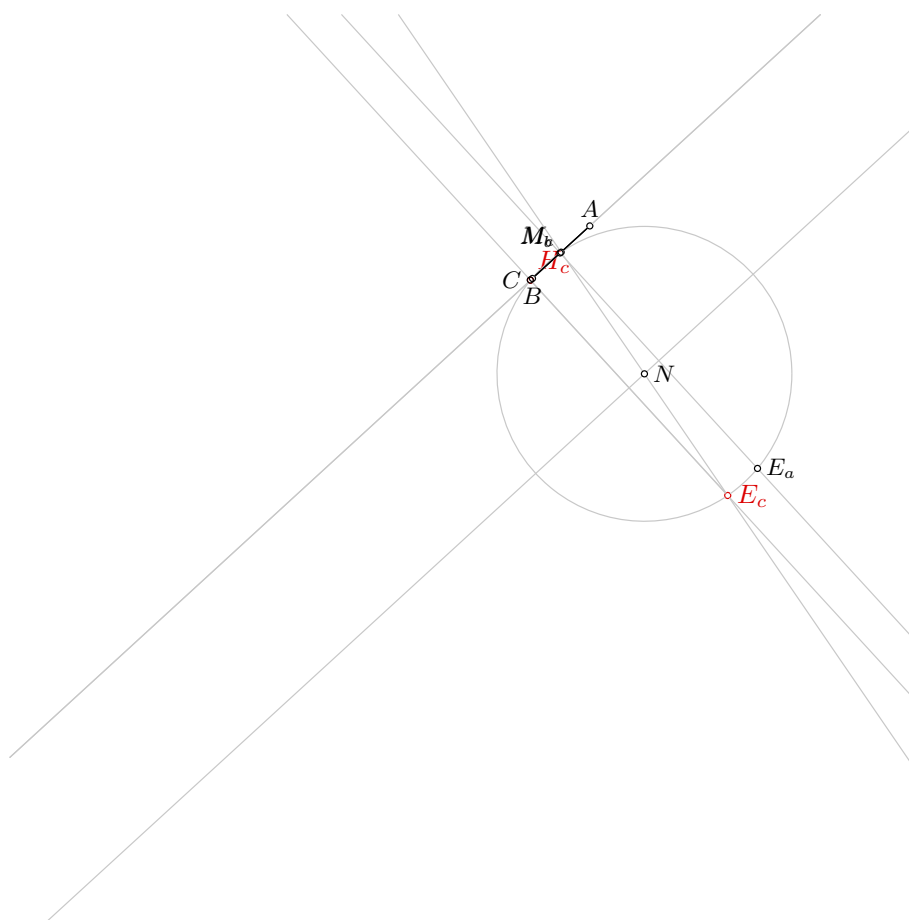


Figure 1: Illustration of the problem 0604

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 59 terms.

Time Complexity: Time spent by the prover is 0.717 seconds.

NDG conditions Points A and B are not identical

Points A , B and C are not collinear

Point M_c is not the midpoint of segment with endpoints B and C

4.1.2 Proving $H_c = \neg H_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 4 terms.

Time Complexity: Time spent by the prover is 0.141 seconds.

NDG conditions Points A and B are not identical

4.1.3 Proving $A = A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.007 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $H_c = \neg H_c$

Proving failed

4.2.3 Proving $A = A$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $H_c = \neg H_c$

Proving failed

4.3.3 Proving $A=A$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = E_c$

Proving failed

4.4.2 Proving $H_c = H_c$

Proving failed

4.4.3 Proving $A=A$

Proving failed

Problem 605

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 605: Given a point A , a point E_c and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 606

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 606: Given a point A , a point E_c and a point M_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 607

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 607: Given a point A , a point E_c and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point M_b , construct a point C (rule W01); ;
2. Using the point E_c and the point C , construct a point H (rule W01); ;
3. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
4. Using the point E_c and the point C , construct a line h_c (rule W02); % DET: points E_c and C are not the same;
5. Using the point A and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points A and M_b are not the same;
6. Using the circle $k(M_b, C)$, the line h_a , the point M_b and the point A , construct a point H_a (rule W05); % NDG: line h_a and circle $k(M_b, C)$ intersect % DET: points A and H_a must be different;
7. Using the point H_a and the point C , construct a line a (rule W02); % DET: points H_a and C are not the same;
8. Using the circle $k(M_b, C)$, the line h_c , the point M_b and the point C , construct a point H_c (rule W05); % NDG: line h_c and circle $k(M_b, C)$ intersect % DET: points C and H_c must be different;
9. Using the point H_c and the point A , construct a line c (rule W02); % DET: points H_c and A are not the same;
10. Using the line a and the line c , construct a point B (rule W03); % NDG: lines a and c are not parallel % DET: lines a and c are not the same.

Non-degenerate conditions: lines a and c are not parallel; line h_c and circle $k(M_b, C)$ intersect; line h_a and circle $k(M_b, C)$ intersect; points A and M_b are not the same.

Determination conditions: lines a and c are not the same; points H_c and A are not the same; points C and H_c must be different; points H_a and C are not the same; points A and H_a must be different; points E_c and C are not the same; points A and H are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D22,D3,D30,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L43,L44,L45]

Solving time: 10.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point E_{c} 95 56.36
point M_{b} 95 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r E_{c}
cmark_lt M_{b}
color 0 0 0
fontsize 8
```

```
% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0
```

```
% Constructing a point H such that E_{c}H/E_{c}C=-1
towards H E_{c} C -1
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0
```

```
% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H
```

```
color 200 200 200
drawline h_{a}
color 0 0 0
```



```

% DET: points E_{c} and C are not the same
% Constructing a line h_{c} which passes through point E_{c} and point C
line h_{c} E_{c} C

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points A and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point A
circle k(M_{b},C) M_{b} A

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line h_{a} and circle k(M_{b},C) intersect% DET: points A and H_{a} must be different
% Constructing a point P_{\_G72966} which is a foot of the point M_{b} on the line h_{a}
foot P_{\_G72966} M_{b} h_{a}
cmark_r P_{\_G72966}
color 200 200 200
drawline M_{b} P_{\_G72966}
color 0 0 0

% Constructing a point H_{a} which is an image of the point A in the symmetry to point/line P_{\_G
72966}
sim H_{a} P_{\_G72966} A
cmark_r H_{a}

% DET: points H_{a} and C are not the same
% Constructing a line a which passes through point H_{a} and point C
line a H_{a} C

color 200 200 200
drawline a
color 0 0 0

% NDG: line h_{c} and circle k(M_{b},C) intersect% DET: points C and H_{c} must be different
% Constructing a point P_{\_G73204} which is a foot of the point M_{b} on the line h_{c}
foot P_{\_G73204} M_{b} h_{c}
cmark_r P_{\_G73204}
color 200 200 200
drawline M_{b} P_{\_G73204}
color 0 0 0

% Constructing a point H_{c} which is an image of the point C in the symmetry to point/line P_{\_G
73204}
sim H_{c} P_{\_G73204} C
cmark_rt H_{c}

```

```

% DET: points  $H_{\{c\}}$  and  $A$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $A$ 
line c  $H_{\{c\}}$  A

color 200 200 200
drawline c
color 0 0 0

% NDG: lines  $a$  and  $c$  are not parallel% DET: lines  $a$  and  $c$  are not the same
% Constructing a point  $B$  which belongs to line  $a$  and line  $c$ 
intersec B a c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $a$  and  $c$  are not parallel; line  $h_{\{c\}}$  and circle  $k(M_{\{b\}}, C)$ 
% intersect; line  $h_{\{a\}}$  and circle  $k(M_{\{b\}}, C)$  intersect; points  $A$  and  $M_{\{b\}}$  are not the same
% Determination conditions: lines  $a$  and  $c$  are not the same; points  $H_{\{c\}}$  and  $A$  are not the same;
% points  $C$  and  $H_{\{c\}}$  must be different; points  $H_{\{a\}}$  and  $C$  are not the same; points  $A$  and  $H_{\{a\}}$ 
% must be different; points  $E_{\{c\}}$  and  $C$  are not the same; points  $A$  and  $H$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.032 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c = \neg E_c$

Proving failed

4.1.3 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

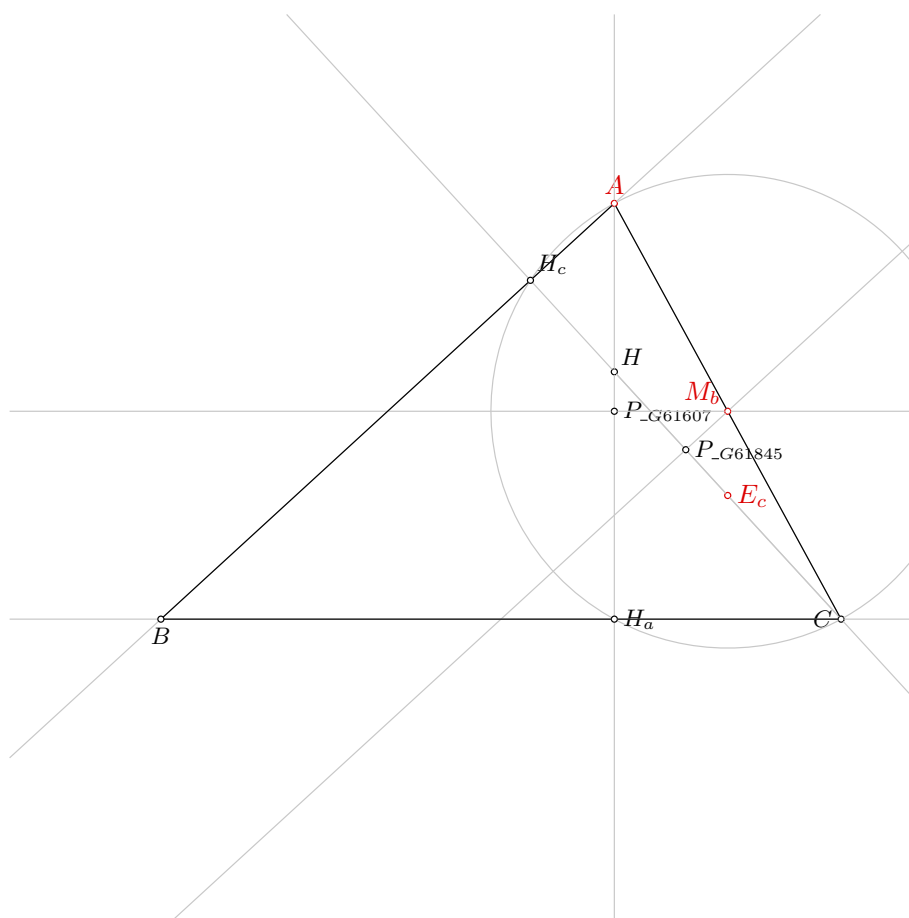


Figure 1: Illustration of the problem 0607

Time Complexity: Time spent by the prover is 0.034 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_a H_c A} \neq S_{CH_c A}$ i.e., lines $H_a C$ and $H_c A$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c = \neg E_c$

Proving failed

4.2.3 Proving $M_b = \neg M_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_c = \neg E_c$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 124 terms.

Time Complexity: Time spent by the prover is 0.420 seconds. There are no ndg conditions.

4.3.3 Proving $M_b = \neg M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 9 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $E_c = \neg E_c$

Proving failed

4.4.3 Proving $M_b = -M_b$

Proving failed

Problem 608

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 608: Given a point A , a point E_c and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point M_c , construct a point B (rule W01); ;
2. Using the point A and the point M_c , construct a line c (rule W02); % DET: points A and M_c are not the same;
3. Using the point E_c and the point M_c , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and M_c are not the same;
4. Using the point E_c and the line c , construct a line h_c (rule W10b); ;
5. Using the line h_c and the line c , construct a point H_c (rule W03); % NDG: lines h_c and c are not parallel % DET: lines h_c and c are not the same;
6. Using the point A and the point H_c , construct a line $m(AH_c)$ (rule W14); % DET: points A and H_c are not the same;
7. Using the point E_c and the point M_c , construct a line $m(E_cM_c)$ (rule W14); % DET: points E_c and M_c are not the same;
8. Using the line $m(E_cM_c)$ and the line $m(H_bH_a)$, construct a point N (rule W03); % NDG: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel % DET: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same;
9. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
10. Using the circle $k(N, M_a)$ and the line $m(AH_c)$, construct a point M_b and a point E_a (rule W04); % NDG: line $m(AH_c)$ and circle $k(N, M_a)$ intersect;

11. Using the point M_b and the point A , construct a point C (rule W01); .

Non-degenerate conditions: line $m(AH_c)$ and circle $k(N, M_a)$ intersect; points E_c and N are not the same; lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel; lines h_c and c are not parallel.

Determination conditions: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same; points E_c and M_c are not the same; points A and H_c are not the same; lines h_c and c are not the same; points E_c and M_c are not the same; points A and M_c are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14]

Lemmas used: [D10,D20,D22,D30,D32,D7,GD01,GD02,GL01,GL03,GL04,GL09,L17,L18,L19,L20,L22,L24,L3,I

Solving time: 2.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point E_{c} 95 56.36
point M_{c} 50 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r E_{c}
cmark_lt M_{c}
color 0 0 0
fontsize 8
```

```
% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0
```

```
% DET: points A and M_{c} are not the same
% Constructing a line c which passes through point A and point M_{c}
line c A M_{c}
```

```
color 200 200 200
drawline c
color 0 0 0
```

```
% DET: points E_{c} and M_{c} are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point M_{c}
line m(H_{b}H_{a}) E_{c} M_{c}
```

```
color 200 200 200
drawline m(H_{b}H_{a})
```

```
color 0 0 0
```

```
% Constructing a line  $h_{\{c\}}$  which is perpendicular to line  $c$  and which passes through point  $E_{\{c\}}$ 
perp  $h_{\{c\}}$   $E_{\{c\}}$   $c$ 
```

```
color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0
```

```
% NDG: lines  $h_{\{c\}}$  and  $c$  are not parallel% DET: lines  $h_{\{c\}}$  and  $c$  are not the same
% Constructing a point  $H_{\{c\}}$  which belongs to line  $h_{\{c\}}$  and line  $c$ 
intersec  $H_{\{c\}}$   $h_{\{c\}}$   $c$ 
cmark_rt  $H_{\{c\}}$ 
```

```
% DET: points  $A$  and  $H_{\{c\}}$  are not the same
% Constructing bisector  $m(AH_{\{c\}})$  of the segment  $AH_{\{c\}}$ 
med  $m(AH_{\{c\}})$   $A$   $H_{\{c\}}$ 
```

```
color 200 200 200
drawline  $m(AH_{\{c\}})$ 
color 0 0 0
```

```
color 200 200 200
drawsegment  $A$   $H_{\{c\}}$ 
color 0 0 0
```

```
% DET: points  $E_{\{c\}}$  and  $M_{\{c\}}$  are not the same
% Constructing bisector  $m(E_{\{c\}}M_{\{c\}})$  of the segment  $E_{\{c\}}M_{\{c\}}$ 
med  $m(E_{\{c\}}M_{\{c\}})$   $E_{\{c\}}$   $M_{\{c\}}$ 
```

```
color 200 200 200
drawline  $m(E_{\{c\}}M_{\{c\}})$ 
color 0 0 0
```

```
color 200 200 200
drawsegment  $E_{\{c\}}$   $M_{\{c\}}$ 
color 0 0 0
```

```
% NDG: lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not parallel% DET: lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}$ 
 $H_{\{a\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{c\}}M_{\{c\}})$  and line  $m(H_{\{b\}}H_{\{a\}})$ 
intersec  $N$   $m(E_{\{c\}}M_{\{c\}})$   $m(H_{\{b\}}H_{\{a\}})$ 
cmark_r  $N$ 
```

```
% NDG: points  $E_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{c\}}$ 
circle  $k(N, M_{\{a\}})$   $N$   $E_{\{c\}}$ 
```



```

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(AH_{c}) and circle k(N,M_{a}) intersect
% Constructing points M_{b} and E_{a} which are in intersection of k(N,M_{a}) and m(AH_{c})
intersec2 M_{b} E_{a} k(N,M_{a}) m(AH_{c})
cmark_lt M_{b}
cmark_r E_{a}

% Constructing a point C such that M_{b}C/M_{b}A=-1
towards C M_{b} A -1
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line m(AH_{c}) and circle k(N,M_{a}) intersect; points E_{c} and N are
% not the same; lines m(E_{c}M_{c}) and m(H_{b}H_{a}) are not parallel; lines h_{c} and c are
% not parallel
% Determination conditions: lines m(E_{c}M_{c}) and m(H_{b}H_{a}) are not the same; points E_{c}
% and M_{c} are not the same; points A and H_{c} are not the same; lines h_{c} and c are not the
% same; points E_{c} and M_{c} are not the same; points A and M_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.03 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c = E_c$

Proving failed

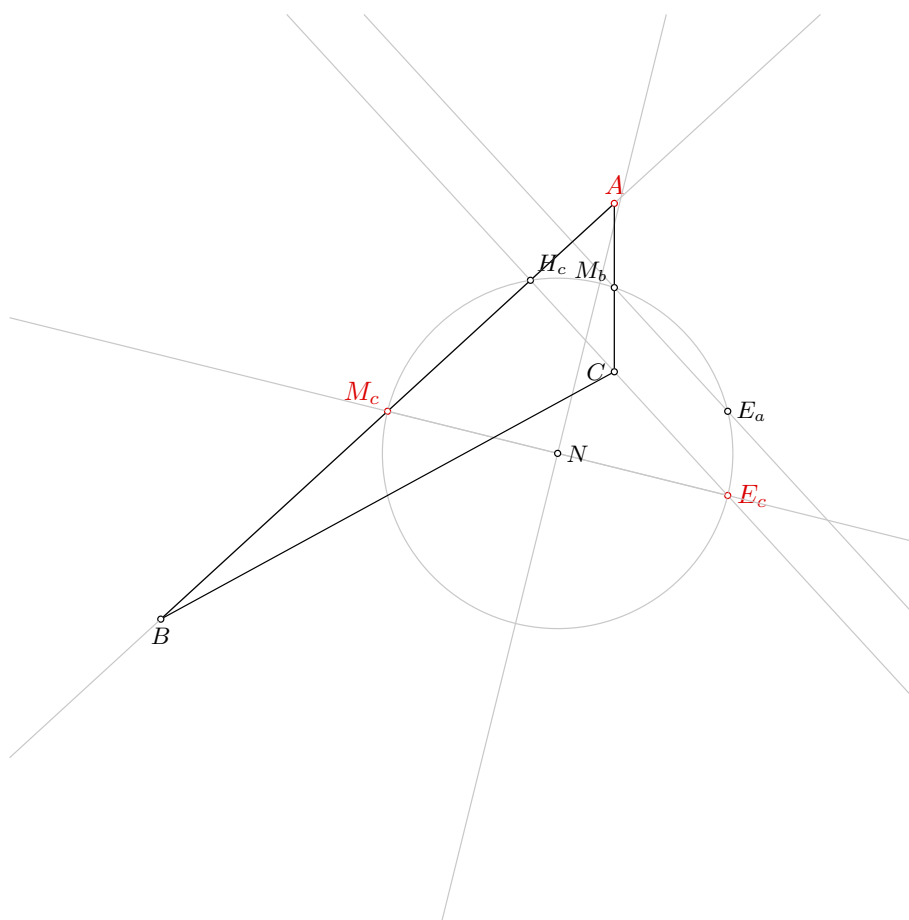


Figure 1: Illustration of the problem 0608

4.1.3 Proving $M_c = \neg M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $A = A$

NDG conditions are:

$S_{E_c A M_c} \neq 0$ i.e., points E_c , A and M_c are not collinear (foot is not the point itself; construction based assumption)

$S_{E_c A M_c} \neq S_{F_{h_c}^0 A M_c}$ i.e., lines $E_c F_{h_c}^0$ and $A M_c$ are not parallel (construction based assumption)

$S_{M_{m(E_c M_c)}^3 E_c M_c} \neq S_{T_{m(E_c M_c)}^4 E_c M_c}$ i.e., lines $M_{m(E_c M_c)}^3 T_{m(E_c M_c)}^4$ and $E_c M_c$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^6} \neq S_{F_{h_a}^5 B F_{h_b}^6}$ i.e., lines $AF_{h_a}^5$ and $BF_{h_b}^6$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c = \neg E_c$

Proving failed

4.2.3 Proving $M_c = \neg M_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A = A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.2 Proving $E_c = \neg E_c$

Proving failed

4.3.3 Proving $M_c = \neg M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 13 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A = A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_c = E_c$

Proving failed

4.4.3 Proving $M_c = M_c$

Proving failed

Problem 609

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 609: Given a point A , a point E_c and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point N , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and N are not the same;
2. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
3. Using the circle $k(N, M_a)$, the line $m(H_bH_a)$, the point N and the point E_c , construct a point M_c (rule W05a); % NDG: line $m(H_bH_a)$ and circle $k(N, M_a)$ intersect % DET: points E_c and M_c must be different;
4. Using the point M_c and the point A , construct a point B (rule W01); ;
5. Using the point A and the point M_c , construct a line c (rule W02); % DET: points A and M_c are not the same;
6. Using the circle $k(N, M_a)$, the line c , the point N and the point M_c , construct a point H_c (rule W05); % NDG: line c and circle $k(N, M_a)$ intersect % DET: points M_c and H_c must be different;
7. Using the point H_c and the point E_c , construct a line h_c (rule W02); % DET: points H_c and E_c are not the same;
8. Using the point A and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points A and M_c are not the same;
9. Using the circle $k(N, M_a)$ and the circle $k(M_c, A)$, construct a point H_a and a point H_b (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_c, A)$ intersect % DET: circles $k(N, M_a)$ and $k(M_c, A)$ are not the same;

10. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
11. Using the line h_c and the line h_a , construct a point H (rule W03); % NDG: lines h_c and h_a are not parallel % DET: lines h_c and h_a are not the same;
12. Using the point E_c and the point H , construct a point C (rule W01); .

Non-degenerate conditions: lines h_c and h_a are not parallel; circles $k(N, M_a)$ and $k(M_c, A)$ intersect; points A and M_c are not the same; line c and circle $k(N, M_a)$ intersect; line $m(H_b H_a)$ and circle $k(N, M_a)$ intersect; points E_c and N are not the same.

Determination conditions: lines h_c and h_a are not the same; points A and H_a are not the same; circles $k(N, M_a)$ and $k(M_c, A)$ are not the same; points H_c and E_c are not the same; points M_c and H_c must be different; points A and M_c are not the same; points E_c and M_c must be different; points E_c and N are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D10,D20,D3,D30,D32,D5,D7,D8,GD01,GD02,GL01,GL03,GL04,GL09,L18,L19,L20,L21,L24,L3]

Solving time: 4.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point E_{c} 95 56.36
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r E_{c}
cmark_r N
color 0 0 0
fontsize 8
```

```
% DET: points E_{c} and N are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point N
line m(H_{b}H_{a}) E_{c} N
```

```
color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0
```

```
% NDG: points E_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{c}
circle k(N,M_{a}) N E_{c}
```

```
color 200 200 200
drawcircle k(N,M_{a})
```

```

color 0 0 0

% NDG: line  $m(H_{\{b\}}H_{\{a\}})$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $E_{\{c\}}$  and  $M_{\{c\}}$  must be
different
% Constructing a point  $M_{\{c\}}$  which is an image of the point  $E_{\{c\}}$  in the symmetry to point/line  $N$ 
sim  $M_{\{c\}}$   $N$   $E_{\{c\}}$ 
cmark_lt  $M_{\{c\}}$ 

% Constructing a point  $B$  such that  $M_{\{c\}}B/M_{\{c\}}A=-1$ 
towards  $B$   $M_{\{c\}}$   $A$  -1
cmark_b  $B$ 
color 200 200 200
drawsegment  $A$   $B$ 
color 0 0 0

% DET: points  $A$  and  $M_{\{c\}}$  are not the same
% Constructing a line  $c$  which passes through point  $A$  and point  $M_{\{c\}}$ 
line  $c$   $A$   $M_{\{c\}}$ 

color 200 200 200
drawline  $c$ 
color 0 0 0

% NDG: line  $c$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $M_{\{c\}}$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G131808\}}$  which is a foot of the point  $N$  on the line  $c$ 
foot  $P_{\{\backslash\_G131808\}}$   $N$   $c$ 
cmark_r  $P_{\{\backslash\_G131808\}}$ 
color 200 200 200
drawline  $N$   $P_{\{\backslash\_G131808\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $M_{\{c\}}$  in the symmetry to point/line  $P_{\{\backslash\_G131808\}}$ 
sim  $H_{\{c\}}$   $P_{\{\backslash\_G131808\}}$   $M_{\{c\}}$ 
cmark_rt  $H_{\{c\}}$ 

% DET: points  $H_{\{c\}}$  and  $E_{\{c\}}$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $H_{\{c\}}$  and point  $E_{\{c\}}$ 
line  $h_{\{c\}}$   $H_{\{c\}}$   $E_{\{c\}}$ 

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: points  $A$  and  $M_{\{c\}}$  are not the same
% Constructing a circle  $k(M_{\{c\}},A)$  whose center is at point  $M_{\{c\}}$  and which passes through point  $A$ 

```

```

circle k(M_{c},A) M_{c} A

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(M_{c},A) intersect% DET: circles k(N,M_{a}) and k(M_{c},A) are not
the same
% Constructing points H_{a} and H_{b} which are in intersection of k(N,M_{a}) and k(M_{c},A)
intersec2 H_{a} H_{b} k(N,M_{a}) k(M_{c},A)
cmark_r H_{a}
cmark_l H_{b}

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: lines h_{c} and h_{a} are not parallel% DET: lines h_{c} and h_{a} are not the same
% Constructing a point H which belongs to line h_{c} and line h_{a}
intersec H h_{c} h_{a}
cmark_rt H

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and h_{a} are not parallel; circles k(N,M_{a}) and k(M_{c}
),A) intersect; points A and M_{c} are not the same; line c and circle k(N,M_{a}) intersect;
line m(H_{b}H_{a}) and circle k(N,M_{a}) intersect; points E_{c} and N are not the same
% Determination conditions: lines h_{c} and h_{a} are not the same; points A and H_{a} are not the
same; circles k(N,M_{a}) and k(M_{c},A) are not the same; points H_{c} and E_{c} are not the
same; points M_{c} and H_{c} must be different; points A and M_{c} are not the same; points E_{
c} and M_{c} must be different; points E_{c} and N are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

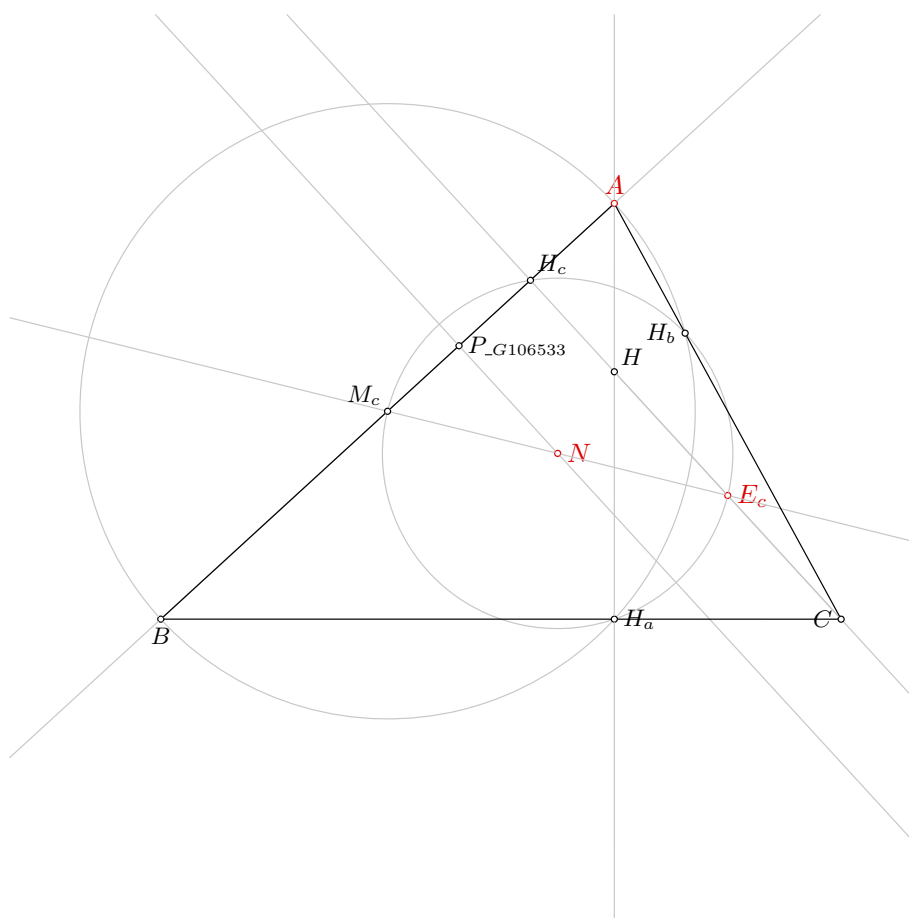


Figure 1: Illustration of the problem 0609

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.026 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c=_E_c$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{H_cAH_a} \neq S_{E_cAH_a}$ i.e., lines H_cE_c and AH_a are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

$S_{_M_aBC} \neq 0$ i.e., points $_M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$ i.e., points $_M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a_M_bF_{-m_b}^3} \neq S_{F_{-m_a}^2_M_bF_{-m_b}^3}$ i.e., lines $_M_aF_{-m_a}^2$ and $_M_bF_{-m_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c=_E_c$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Proving failed

4.3.2 Proving $E_c=_E_c$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Proving failed

4.4.2 Proving $E_c=_E E_c$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 610

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 610: Given a point A , a point E_c and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 611

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 611: Given a point A , a point E_c and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 612

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 612: Given a point A , a point E_c and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 613

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 613: Given a point A , a point E_c and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 614

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 614: Given a point A , a point G and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point G , construct a point M_a (rule W01); ;
2. Using the point G and the point N , construct a point H (rule W01); ;
3. Using the point A and the point H , construct a point E_a (rule W01); ;
4. Using the point G and the point N , construct a point O (rule W01); ;
5. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
6. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
7. Using the point M_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_a and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
9. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; line h_a and circle $k(N, M_a)$ intersect; points M_a and N are not the same; points A and O are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points A and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L15,L16,L19,L22,L55]

Solving time: 9.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
```

```
point G 70 58.33
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_t A
```

```
cmark_t G
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point M_{a} such that AM_{a}/AG=1.5
```

```
towards M_{a} A G 1.5
```

```
cmark_r M_{a}
```

```
color 200 200 200
```

```
drawsegment A M_{a}
```

```
color 0 0 0
```

```
% Constructing a point H such that GH/GN=4
```

```
towards H G N 4
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment G H
```

```
color 0 0 0
```

```
% Constructing a point E_{a} such that AE_{a}/AH=0.5
```

```
towards E_{a} A H 0.5
```

```
cmark_r E_{a}
```

```
color 200 200 200
```

```
drawsegment A H
```

```
color 0 0 0
```

```
% Constructing a point O such that GO/GN=-2
```

```

towards O G N -2
cmark_t O
color 200 200 200
drawsegment N O
color 0 0 0

% DET: points A and H are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point A and point H
line  $h_{\{a\}}$  A H

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point A
circle  $k(O,C)$  O A

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: points  $M_{\{a\}}$  and N are not the same
% Constructing a circle  $k(N,M_{\{a\}})$  whose center is at point N and which passes through point  $M_{\{a\}}$ 
circle  $k(N,M_{\{a\}})$  N  $M_{\{a\}}$ 

color 200 200 200
drawcircle  $k(N,M_{\{a\}})$ 
color 0 0 0

% NDG: line  $h_{\{a\}}$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $E_{\{a\}}$  and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G180067\}}$  which is a foot of the point N on the line  $h_{\{a\}}$ 
foot  $P_{\{\backslash\_G180067\}}$  N  $h_{\{a\}}$ 
cmark_r  $P_{\{\backslash\_G180067\}}$ 
color 200 200 200
drawline N  $P_{\{\backslash\_G180067\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $E_{\{a\}}$  in the symmetry to point/line  $P_{\{\backslash\_G180067\}}$ 
sim  $H_{\{a\}}$   $P_{\{\backslash\_G180067\}}$   $E_{\{a\}}$ 
cmark_r  $H_{\{a\}}$ 

% DET: points  $H_{\{a\}}$  and  $M_{\{a\}}$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $M_{\{a\}}$ 
line  $a$   $H_{\{a\}}$   $M_{\{a\}}$ 

color 200 200 200

```

```
drawline a
color 0 0 0
```

```
% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; line h_{a} and circle k(N,M_{a})
    intersect; points M_{a} and N are not the same; points A and O are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
    different; points A and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $G=_G$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{AB_M_b} \neq S_{_M_a B_M_b}$ i.e., lines A_M_a and B_M_b are not parallel (construction based assumption)
 $S_{_M_a BC} \neq 0$ i.e., points $_M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{_M_b AC} \neq 0$ i.e., points $_M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

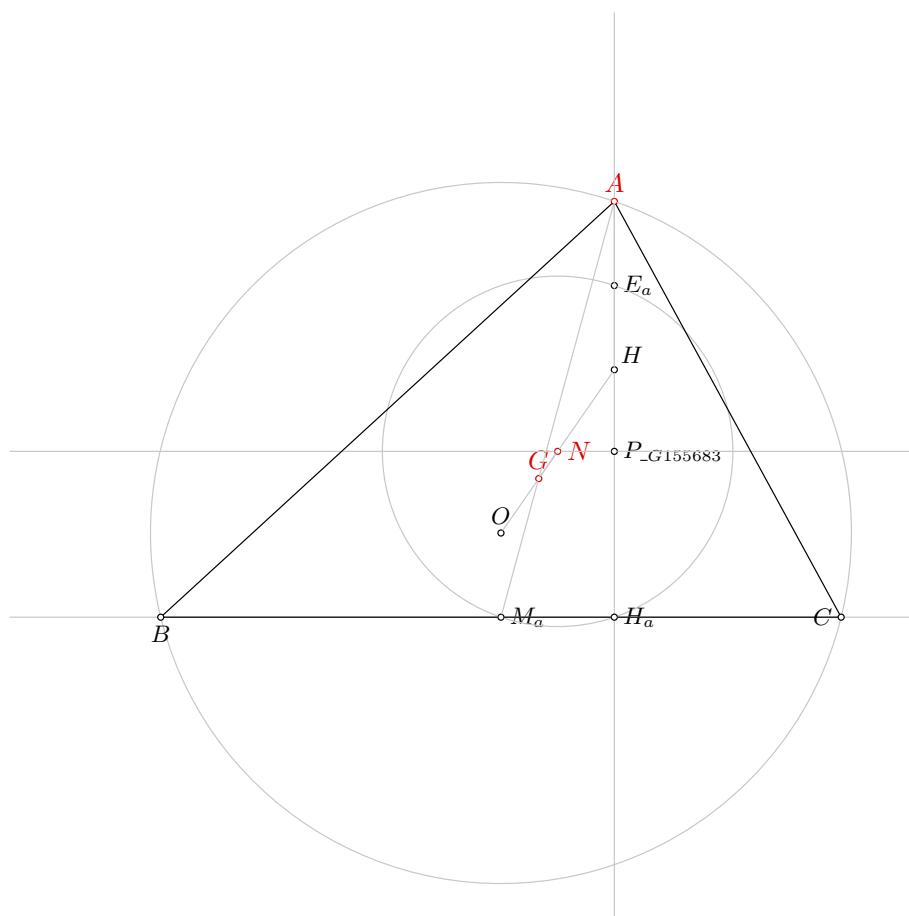


Figure 1: Illustration of the problem 0614

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF^3_{-h_b}} \neq S_{F^2_{-h_a}BF^3_{-h_b}}$ i.e., lines $AF^2_{-h_a}$ and $BF^3_{-h_b}$ are not parallel (construction based assumption)
 $S_{-M_a-M_bF^1_{-m_b}} \neq S_{F^0_{-m_a}-M_bF^1_{-m_b}}$ i.e., lines $-M_aF^0_{-m_a}$ and $-M_bF^1_{-m_b}$ are not parallel (construction based assumption)
 Total number of proof steps: 1
 Time spent by the prover: 0.000 seconds

4.2.2 Proving $G=_G$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $G=_G$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $G=_G$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 615

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 615: Given a point A , a point H and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point H , construct a point E_a (rule W01); ;
2. Using the point H and the point N , construct a point O (rule W01); ;
3. Using the point H and the point N , construct a point G (rule W01); ;
4. Using the point A and the point G , construct a point M_a (rule W01); ;
5. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
6. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
7. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
9. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; line h_a and circle $k(N, M_a)$ intersect; points E_a and N are not the same; points A and O are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points A and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D31,D32,D5,D8,GD01,GL03,GL09,L11,L12,L16,L19,L22,L55]

Solving time: 9.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
```

```
point H 80 72.73
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_t A
```

```
cmark_rt H
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point E_{a} such that AE_{a}/AH=0.5
```

```
towards E_{a} A H 0.5
```

```
cmark_r E_{a}
```

```
color 200 200 200
```

```
drawsegment A H
```

```
color 0 0 0
```

```
% Constructing a point O such that HO/HN=2
```

```
towards O H N 2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment H O
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G214237} which passes through point H and point N
```

```
line L_{\_G214237} H N
```

```
color 200 200 200
```

```
drawline L_{\_G214237}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G214338} with coordinates (0,0)
```

```
point P_{\_G214338} 0 0
```

```

cmark_r P_{\_G214338}

% Constructing a point P_{\_G214262} such that HP_{\_G214262}/HP_{\_G214338}=4
towards P_{\_G214262} H P_{\_G214338} 4
cmark_r P_{\_G214262}
color 200 200 200
drawsegment H P_{\_G214262}
color 0 0 0

% Constructing a point P_{\_G214307} such that HP_{\_G214307}/HP_{\_G214338}=3
towards P_{\_G214307} H P_{\_G214338} 3
cmark_r P_{\_G214307}
color 200 200 200
drawsegment H P_{\_G214307}
color 0 0 0

% Constructing a line L_{\_G214268} which passes through point N and point P_{\_G214307}
line L_{\_G214268} N P_{\_G214307}

color 200 200 200
drawline L_{\_G214268}
color 0 0 0

% Constructing a line L_{\_G214231} which contains the point P_{\_G214262} and is parallel to the
line L_{\_G214268}
parallel L_{\_G214231} P_{\_G214262} L_{\_G214268}

color 200 200 200
drawline L_{\_G214231}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G214231} and line L_{\_G214237}
intersec G L_{\_G214231} L_{\_G214237}
cmark_t G

% Constructing a point M_{a} such that AM_{a}/AG=1.5
towards M_{a} A G 1.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

```



```

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points E_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{a}
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G215225} which is a foot of the point N on the line h_{a}
foot P_{\_G215225} N h_{a}
cmark_r P_{\_G215225}
color 200 200 200
drawline N P_{\_G215225}
color 0 0 0

% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
\_G215225}
sim H_{a} P_{\_G215225} E_{a}
cmark_r H_{a}

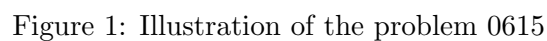
% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```



```
% Non-degenerate conditions: line a and circle k(O,C) intersect; line h_{a} and circle k(N,M_{a})
% intersect; points E_{a} and N are not the same; points A and O are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
% different; points A and H are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.032 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $H=_H$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{P_{G190900}HN} \neq S_{P_{L_{G190869}}^0}^{HN}$ i.e., lines $P_{G190900}P_{L_{G190869}}^0$ and HN are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^2} \neq S_{F_{h_a}^1BF_{h_b}^2}$ i.e., lines $AF_{h_a}^1$ and $BF_{h_b}^2$ are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points M_a, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points M_b, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aM_bF_{m_b}^4} \neq S_{F_{m_a}^3M_bF_{m_b}^4}$ i.e., lines $M_aF_{m_a}^3$ and $M_bF_{m_b}^4$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $H=_H$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $H=_H$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $H=_H$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 616

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 616: Given a point A , a point H_a and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
2. Using the point H_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_a and N are not the same;
3. Using the circle $k(N, M_a)$, the line h_a , the point N and the point H_a , construct a point E_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points H_a and E_a must be different;
4. Using the point E_a and the point A , construct a point H (rule W01); ;
5. Using the point N and the point H , construct a point O (rule W01); ;
6. Using the point N and the point H , construct a point G (rule W01); ;
7. Using the point A and the point G , construct a point M_a (rule W01); ;
8. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
9. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; line h_a and circle $k(N, M_a)$ intersect; points H_a and N are not the same.

Determination conditions: points H_a and M_a are not the same; points H_a and E_a must be different; points A and H_a are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D31,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L16,L19,L22,L55]

Solving time: 3.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point H_{a} 80 40
point N 72.5 61.93

color 220 0 0
fontsize 9

cmark_t A
cmark_r H_{a}
cmark_r N
color 0 0 0
fontsize 8

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points H_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{a}
circle k(N,M_{a}) N H_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points H_{a} and E_{a} must be different
% Constructing a point P_{\_G246728} which is a foot of the point N on the line h_{a}
foot P_{\_G246728} N h_{a}
cmark_r P_{\_G246728}
color 200 200 200
drawline N P_{\_G246728}
color 0 0 0
```

```

% Constructing a point E_{a} which is an image of the point H_{a} in the symmetry to point/line P
_{\_G246728}
sim E_{a} P_{\_G246728} H_{a}
cmark_r E_{a}

% Constructing a point H such that E_{a}H/E_{a}A=-1
towards H E_{a} A -1
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% Constructing a point O such that NO/NH=-1
towards O N H -1
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% Constructing a line L_{\_G246990} which passes through point N and point H
line L_{\_G246990} N H

color 200 200 200
drawline L_{\_G246990}
color 0 0 0

% Constructing a point P_{\_G247091} with coordinates (0,0)
point P_{\_G247091} 0 0
cmark_r P_{\_G247091}

% Constructing a point P_{\_G247015} such that NP_{\_G247015}/NP_{\_G247091}=-1
towards P_{\_G247015} N P_{\_G247091} -1
cmark_r P_{\_G247015}
color 200 200 200
drawsegment P_{\_G247091} P_{\_G247015}
color 0 0 0

% Constructing a point P_{\_G247060} such that NP_{\_G247060}/NP_{\_G247091}=3
towards P_{\_G247060} N P_{\_G247091} 3
cmark_r P_{\_G247060}
color 200 200 200
drawsegment N P_{\_G247060}
color 0 0 0

% Constructing a line L_{\_G247021} which passes through point H and point P_{\_G247060}
line L_{\_G247021} H P_{\_G247060}

color 200 200 200
drawline L_{\_G247021}

```

```

color 0 0 0

% Constructing a line  $L_{\backslash\_G246984}$  which contains the point  $P_{\backslash\_G247015}$  and is parallel to the
    line  $L_{\backslash\_G247021}$ 
parallel  $L_{\backslash\_G246984}$   $P_{\backslash\_G247015}$   $L_{\backslash\_G247021}$ 

color 200 200 200
drawline  $L_{\backslash\_G246984}$ 
color 0 0 0

% Constructing a point  $G$  which belongs to line  $L_{\backslash\_G246984}$  and line  $L_{\backslash\_G246990}$ 
intersec G  $L_{\backslash\_G246984}$   $L_{\backslash\_G246990}$ 
cmark_t G

% Constructing a point  $M_{\{a\}}$  such that  $AM_{\{a\}}/AG=1.5$ 
towards  $M_{\{a\}}$  A G 1.5
cmark_r  $M_{\{a\}}$ 
color 200 200 200
drawsegment A  $M_{\{a\}}$ 
color 0 0 0

% DET: points  $H_{\{a\}}$  and  $M_{\{a\}}$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $M_{\{a\}}$ 
line a  $H_{\{a\}}$   $M_{\{a\}}$ 

color 200 200 200
drawline a
color 0 0 0

% NDG: points  $A$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $A$ 
circle  $k(O,C)$  O A

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $a$  and circle  $k(O,C)$  intersect
% Constructing points  $C$  and  $B$  which are in intersection of  $k(O,C)$  and  $a$ 
intersec2 C B  $k(O,C)$  a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C

```

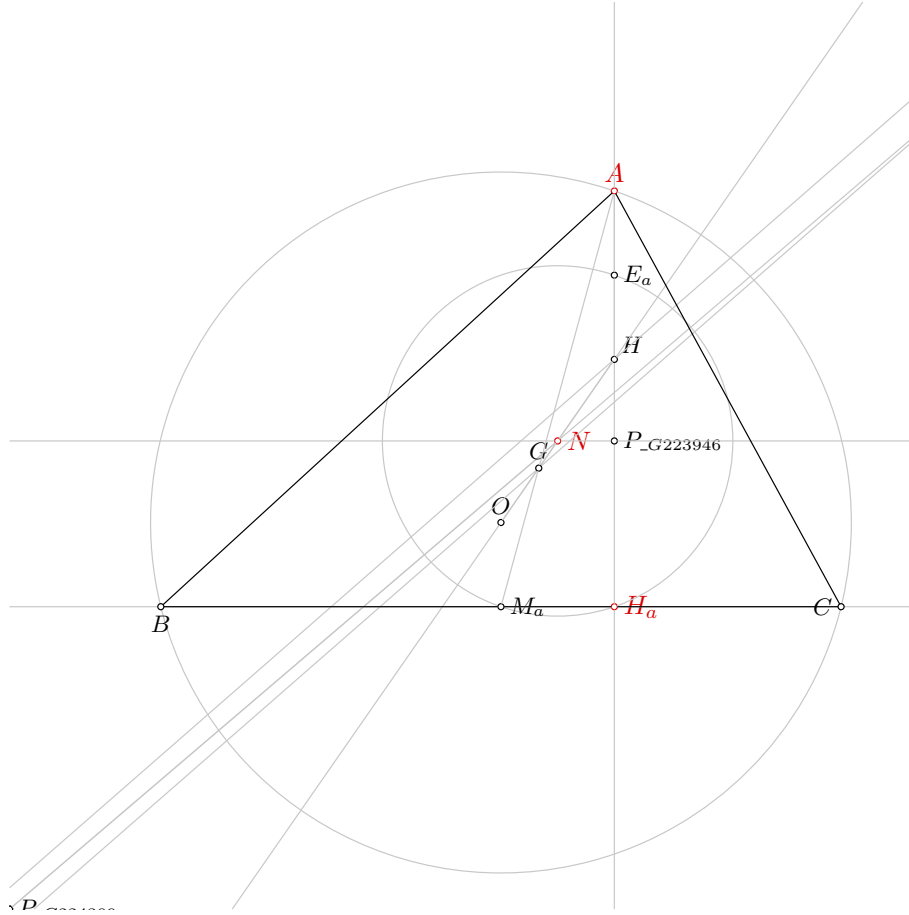



Figure 1: Illustration of the problem 0616

drawsegment B C

*% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
line h_{a} and circle k(N,M_{a}) intersect; points H_{a} and N are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points H_{a} and E_{a} must be
different; points A and H_{a} are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.031 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $H_a = \neg H_a$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A = A$

NDG conditions are:

$S_{P_{-G225677}NH} \neq S_{P_{L_{-G225646}}^0}^{NH}$ i.e., lines $P_{-G225677}P_{L_{-G225646}}^0$ and NH are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{-ha}^1} \neq S_{CAF_{-ha}^1}$ i.e., lines BC and AF_{-ha}^1 are not parallel (construction based assumption)

$S_{MaBC} \neq 0$ i.e., points M_a, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{MbAC} \neq 0$ i.e., points M_b, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-hb}^4} \neq S_{F_{-ha}^1BF_{-hb}^4}$ i.e., lines AF_{-ha}^1 and BF_{-hb}^4 are not parallel (construction based assumption)

$S_{MaMbF_{-mb}^3} \neq S_{F_{-ma}^2MbF_{-mb}^3}$ i.e., lines $M_aF_{-ma}^2$ and $M_bF_{-mb}^3$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $H_a = \neg H_a$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A = A$

Proving failed

4.3.2 Proving $H_a = \neg H_a$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A = A$

Proving failed

4.4.2 Proving $H_a = H_a$

Proving failed

4.4.3 Proving $N = N$

Proving failed

Problem 617

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 617: Given a point A , a point H_b and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;
2. Using the point H_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_b and N are not the same;
3. Using the circle $k(N, M_a)$, the line b , the point N and the point H_b , construct a point M_b (rule W05); % NDG: line b and circle $k(N, M_a)$ intersect % DET: points H_b and M_b must be different;
4. Using the point M_b and the point A , construct a point C (rule W01); ;
5. Using the point N and the point M_b , construct a line $m(H_a H_c)$ (rule W02); % DET: points N and M_b are not the same;
6. Using the circle $k(N, M_a)$, the line $m(H_a H_c)$, the point N and the point M_b , construct a point E_b (rule W05a); % NDG: line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect % DET: points M_b and E_b must be different;
7. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;
8. Using the point A and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points A and M_b are not the same;
9. Using the circle $k(N, M_a)$ and the circle $k(M_b, C)$, construct a point H_a and a point H_c (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_b, C)$ intersect % DET: circles $k(N, M_a)$ and $k(M_b, C)$ are not the same;

10. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
11. Using the line h_b and the line h_a , construct a point H (rule W03); % NDG: lines h_b and h_a are not parallel % DET: lines h_b and h_a are not the same;
12. Using the point N and the point H , construct a point G (rule W01); ;
13. Using the point M_b and the point G , construct a point B (rule W01); .

Non-degenerate conditions: lines h_b and h_a are not parallel; circles $k(N, M_a)$ and $k(M_b, C)$ intersect; points A and M_b are not the same; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; line b and circle $k(N, M_a)$ intersect; points H_b and N are not the same.

Determination conditions: lines h_b and h_a are not the same; points A and H_a are not the same; circles $k(N, M_a)$ and $k(M_b, C)$ are not the same; points E_b and H_b are not the same; points M_b and E_b must be different; points N and M_b are not the same; points H_b and M_b must be different; points A and H_b are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D22,D29,D3,D32,D5,D6,D8,D9,GD01,GD02,GL01,GL03,GL04,GL09,L16,L17,L19,L20,L21,L23]

Solving time: 4.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point H_{b} 89.36 77.83
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_l H_{b}
cmark_r N
color 0 0 0
fontsize 8
```

```
% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% NDG: points H_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{b}
circle k(N,M_{a}) N H_{b}
```

```

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line b and circle k(N,M_{a}) intersect% DET: points H_{b} and M_{b} must be different
% Constructing a point P_{\G31606} which is a foot of the point N on the line b
foot P_{\G31606} N b
cmark_r P_{\G31606}
color 200 200 200
drawline N P_{\G31606}
color 0 0 0

% Constructing a point M_{b} which is an image of the point H_{b} in the symmetry to point/line P
_{\G31606}
sim M_{b} P_{\G31606} H_{b}
cmark_lt M_{b}

% Constructing a point C such that M_{b}C/M_{b}A=-1
towards C M_{b} A -1
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points N and M_{b} are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point N and point M_{b}
line m(H_{a}H_{c}) N M_{b}

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

% NDG: line m(H_{a}H_{c}) and circle k(N,M_{a}) intersect% DET: points M_{b} and E_{b} must be
different
% Constructing a point E_{b} which is an image of the point M_{b} in the symmetry to point/line N
sim E_{b} N M_{b}
cmark_r E_{b}

% DET: points E_{b} and H_{b} are not the same
% Constructing a line h_{b} which passes through point E_{b} and point H_{b}
line h_{b} E_{b} H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

```

```

% NDG: points A and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point A
circle k(M_{b},C) M_{b} A

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(M_{b},C) intersect% DET: circles k(N,M_{a}) and k(M_{b},C) are not
the same
% Constructing points H_{a} and H_{c} which are in intersection of k(N,M_{a}) and k(M_{b},C)
intersec2 H_{a} H_{c} k(N,M_{a}) k(M_{b},C)
cmark_r H_{a}
cmark_rt H_{c}

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: lines h_{b} and h_{a} are not parallel% DET: lines h_{b} and h_{a} are not the same
% Constructing a point H which belongs to line h_{b} and line h_{a}
intersec H h_{b} h_{a}
cmark_rt H

% Constructing a line L_{\_G32404} which passes through point N and point H
line L_{\_G32404} N H

color 200 200 200
drawline L_{\_G32404}
color 0 0 0

% Constructing a point P_{\_G32505} with coordinates (0,0)
point P_{\_G32505} 0 0
cmark_r P_{\_G32505}

% Constructing a point P_{\_G32429} such that NP_{\_G32429}/NP_{\_G32505}=-1
towards P_{\_G32429} N P_{\_G32505} -1
cmark_r P_{\_G32429}
color 200 200 200
drawsegment P_{\_G32505} P_{\_G32429}
color 0 0 0

% Constructing a point P_{\_G32474} such that NP_{\_G32474}/NP_{\_G32505}=3
towards P_{\_G32474} N P_{\_G32505} 3

```

```

cmark_r P_{\_G32474}
color 200 200 200
drawsegment N P_{\_G32474}
color 0 0 0

% Constructing a line L_{\_G32435} which passes through point H and point P_{\_G32474}
line L_{\_G32435} H P_{\_G32474}

color 200 200 200
drawline L_{\_G32435}
color 0 0 0

% Constructing a line L_{\_G32398} which contains the point P_{\_G32429} and is parallel to the
line L_{\_G32435}
parallel L_{\_G32398} P_{\_G32429} L_{\_G32435}

color 200 200 200
drawline L_{\_G32398}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G32398} and line L_{\_G32404}
intersec G L_{\_G32398} L_{\_G32404}
cmark_t G

% Constructing a point B such that  $M_{\{b\}}B/M_{\{b\}}G=3$ 
towards B M_{\{b\}} G 3
cmark_b B
color 200 200 200
drawsegment M_{\{b\}} B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{b\}}$  and  $h_{\{a\}}$  are not parallel; circles  $k(N, M_{\{a\}})$  and  $k(M_{\{b\}}, C)$  intersect; points A and  $M_{\{b\}}$  are not the same; line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect; line b and circle  $k(N, M_{\{a\}})$  intersect; points  $H_{\{b\}}$  and N are not the same
% Determination conditions: lines  $h_{\{b\}}$  and  $h_{\{a\}}$  are not the same; points A and  $H_{\{a\}}$  are not the same; circles  $k(N, M_{\{a\}})$  and  $k(M_{\{b\}}, C)$  are not the same; points  $E_{\{b\}}$  and  $H_{\{b\}}$  are not the same; points  $M_{\{b\}}$  and  $E_{\{b\}}$  must be different; points N and  $M_{\{b\}}$  are not the same; points  $H_{\{b\}}$  and  $M_{\{b\}}$  must be different; points A and  $H_{\{b\}}$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

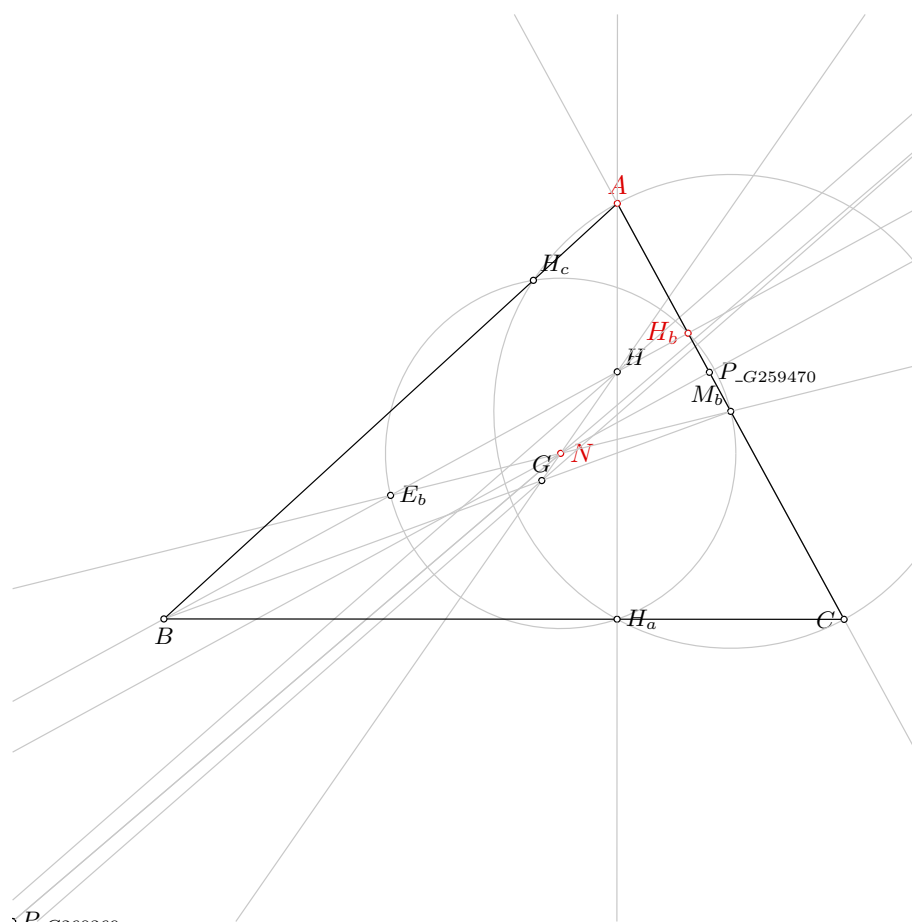


Figure 1: Illustration of the problem 0617

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.026 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $H_b=_H H_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 76 terms.

Time Complexity: Time spent by the prover is 0.756 seconds.

NDG conditions Points M_b and N are not identical

Line through points M_b and H_b is not perpendicular to line through points H_b and N

Line through points H_b and E_b is not parallel with line through points A and H_a

Points A , E_b and P_{G23985} are not collinear

Points A and C are not identical

4.1.3 Proving $N=_N N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{E_b A H_a} \neq S_{H_b A H_a}$ i.e., lines $E_b H_b$ and $A H_a$ are not parallel (construction based assumption)

$S_{P_{G8676} N H} \neq S_{P_{L_{G8645}}^0 N H}$ i.e., lines $P_{G8676} P_{L_{G8645}}^0$ and $N H$ are not parallel (construction based assumption)

$S_{B A C} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{A B F_{-h_b}^1} \neq S_{C B F_{-h_b}^1}$ i.e., lines $A C$ and $B F_{-h_b}^1$ are not parallel (construction based assumption)

$S_{-M_a B C} \neq 0$ i.e., points $-M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{-M_b A C} \neq 0$ i.e., points $-M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{A B C} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{A B F_{-h_b}^1} \neq S_{F_{-h_a}^4 B F_{-h_b}^1}$ i.e., lines $A F_{-h_a}^4$ and $B F_{-h_b}^1$ are not parallel (construction based assumption)

$S_{-M_a -M_b F_{-m_b}^3} \neq S_{F_{-m_a}^2 -M_b F_{-m_b}^3}$ i.e., lines $-M_a F_{-m_a}^2$ and $-M_b F_{-m_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $H_b=_H H_b$

Proving failed

4.2.3 Proving $N=_N N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Proving failed

4.3.2 Proving $H_b=_H H_b$

Proving failed

4.3.3 Proving $N=_N N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Proving failed

4.4.2 Proving $H_b=_H H_b$

Proving failed

4.4.3 Proving $N=_N N$

Proving failed

Problem 618

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 618: Given a point A , a point H_c and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point H_c , construct a line c (rule W02); % DET: points A and H_c are not the same;
2. Using the point H_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_c and N are not the same;
3. Using the circle $k(N, M_a)$, the line c , the point N and the point H_c , construct a point M_c (rule W05); % NDG: line c and circle $k(N, M_a)$ intersect % DET: points H_c and M_c must be different;
4. Using the point M_c and the point A , construct a point B (rule W01); ;
5. Using the point N and the point M_c , construct a line $m(H_a H_b)$ (rule W02); % DET: points N and M_c are not the same;
6. Using the circle $k(N, M_a)$, the line $m(H_a H_b)$, the point N and the point M_c , construct a point E_c (rule W05a); % NDG: line $m(H_a H_b)$ and circle $k(N, M_a)$ intersect % DET: points M_c and E_c must be different;
7. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;
8. Using the point A and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points A and M_c are not the same;
9. Using the circle $k(N, M_a)$ and the circle $k(M_c, A)$, construct a point H_a and a point H_b (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_c, A)$ intersect % DET: circles $k(N, M_a)$ and $k(M_c, A)$ are not the same;

10. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
11. Using the line h_c and the line h_a , construct a point H (rule W03); % NDG: lines h_c and h_a are not parallel % DET: lines h_c and h_a are not the same;
12. Using the point N and the point H , construct a point G (rule W01); ;
13. Using the point M_c and the point G , construct a point C (rule W01); .

Non-degenerate conditions: lines h_c and h_a are not parallel; circles $k(N, M_a)$ and $k(M_c, A)$ intersect; points A and M_c are not the same; line $m(H_a H_b)$ and circle $k(N, M_a)$ intersect; line c and circle $k(N, M_a)$ intersect; points H_c and N are not the same.

Determination conditions: lines h_c and h_a are not the same; points A and H_a are not the same; circles $k(N, M_a)$ and $k(M_c, A)$ are not the same; points E_c and H_c are not the same; points M_c and E_c must be different; points N and M_c are not the same; points H_c and M_c must be different; points A and H_c are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D10,D20,D3,D30,D32,D5,D7,D8,GD01,GD02,GL01,GL03,GL04,GL09,L16,L18,L19,L20,L21,L22]

Solving time: 4.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point H_{c} 68.91 84.83
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_rt H_{c}
cmark_r N
color 0 0 0
fontsize 8
```

```
% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}
```

```
color 200 200 200
drawline c
color 0 0 0
```

```
% NDG: points H_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{c}
circle k(N,M_{a}) N H_{c}
```

```

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line c and circle k(N,M_{a}) intersect% DET: points H_{c} and M_{c} must be different
% Constructing a point P_{\G71818} which is a foot of the point N on the line c
foot P_{\G71818} N c
cmark_r P_{\G71818}
color 200 200 200
drawline N P_{\G71818}
color 0 0 0

% Constructing a point M_{c} which is an image of the point H_{c} in the symmetry to point/line P
_{\G71818}
sim M_{c} P_{\G71818} H_{c}
cmark_lt M_{c}

% Constructing a point B such that M_{c}B/M_{c}A=-1
towards B M_{c} A -1
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points N and M_{c} are not the same
% Constructing a line m(H_{a}H_{b}) which passes through point N and point M_{c}
line m(H_{a}H_{b}) N M_{c}

color 200 200 200
drawline m(H_{a}H_{b})
color 0 0 0

% NDG: line m(H_{a}H_{b}) and circle k(N,M_{a}) intersect% DET: points M_{c} and E_{c} must be
different
% Constructing a point E_{c} which is an image of the point M_{c} in the symmetry to point/line N
sim E_{c} N M_{c}
cmark_r E_{c}

% DET: points E_{c} and H_{c} are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H_{c}
line h_{c} E_{c} H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

```

```

% NDG: points A and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point A
circle k(M_{c},A) M_{c} A

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(M_{c},A) intersect% DET: circles k(N,M_{a}) and k(M_{c},A) are not
the same
% Constructing points H_{a} and H_{b} which are in intersection of k(N,M_{a}) and k(M_{c},A)
intersec2 H_{a} H_{b} k(N,M_{a}) k(M_{c},A)
cmark_r H_{a}
cmark_l H_{b}

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: lines h_{c} and h_{a} are not parallel% DET: lines h_{c} and h_{a} are not the same
% Constructing a point H which belongs to line h_{c} and line h_{a}
intersec H h_{c} h_{a}
cmark_rt H

% Constructing a line L_{\_G72616} which passes through point N and point H
line L_{\_G72616} N H

color 200 200 200
drawline L_{\_G72616}
color 0 0 0

% Constructing a point P_{\_G72717} with coordinates (0,0)
point P_{\_G72717} 0 0
cmark_r P_{\_G72717}

% Constructing a point P_{\_G72641} such that NP_{\_G72641}/NP_{\_G72717}=-1
towards P_{\_G72641} N P_{\_G72717} -1
cmark_r P_{\_G72641}
color 200 200 200
drawsegment P_{\_G72717} P_{\_G72641}
color 0 0 0

% Constructing a point P_{\_G72686} such that NP_{\_G72686}/NP_{\_G72717}=3
towards P_{\_G72686} N P_{\_G72717} 3

```

```

cmark_r P_{\_G72686}
color 200 200 200
drawsegment N P_{\_G72686}
color 0 0 0

% Constructing a line L_{\_G72647} which passes through point H and point P_{\_G72686}
line L_{\_G72647} H P_{\_G72686}

color 200 200 200
drawline L_{\_G72647}
color 0 0 0

% Constructing a line L_{\_G72610} which contains the point P_{\_G72641} and is parallel to the
line L_{\_G72647}
parallel L_{\_G72610} P_{\_G72641} L_{\_G72647}

color 200 200 200
drawline L_{\_G72610}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G72610} and line L_{\_G72616}
intersec G L_{\_G72610} L_{\_G72616}
cmark_t G

% Constructing a point C such that M_{c}C/M_{c}G=3
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and h_{a} are not parallel; circles k(N,M_{a}) and k(M_{c},A) intersect; points A and M_{c} are not the same; line m(H_{a}H_{b}) and circle k(N,M_{a}) intersect; line c and circle k(N,M_{a}) intersect; points H_{c} and N are not the same
% Determination conditions: lines h_{c} and h_{a} are not the same; points A and H_{a} are not the same; circles k(N,M_{a}) and k(M_{c},A) are not the same; points E_{c} and H_{c} are not the same; points M_{c} and E_{c} must be different; points N and M_{c} are not the same; points H_{c} and M_{c} must be different; points A and H_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

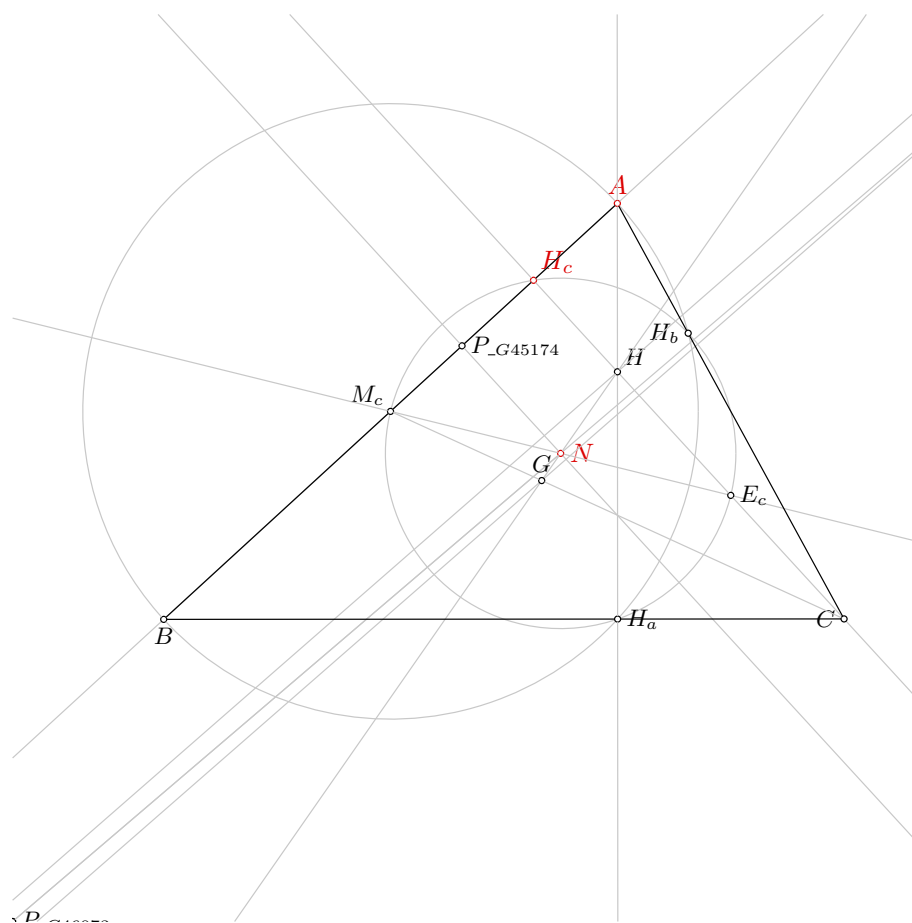


Figure 1: Illustration of the problem 0618

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.032 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $H_c=_Hc$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 76 terms.

Time Complexity: Time spent by the prover is 0.855 seconds.

NDG conditions Points M_c and N are not identical

Line through points A and P_{G63873} is not perpendicular to line through points P_{G63873} and M_c

Line through points A and H_a is not parallel with line through points E_c and H_c

Points E_c , H_c and P_{G63873} are not collinear

Line through points A and H_c is not perpendicular to line through points H_c and B

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{E_cAH_a} \neq S_{H_cAH_a}$ i.e., lines E_cH_c and AH_a are not parallel (construction based assumption)

$S_{P_{G47916}NH} \neq S_{P_{L_{G47885}}^0NH}$ i.e., lines P_{G47916} and NH are not parallel (construction based assumption)

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{-h_c}^1} \neq S_{BCF_{-h_c}^1}$ i.e., lines AB and $CF_{-h_c}^1$ are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points M_a , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points M_b , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^5} \neq S_{F_{-h_a}^4BF_{-h_b}^5}$ i.e., lines $AF_{-h_a}^4$ and $BF_{-h_b}^5$ are not parallel (construction based assumption)

$S_{M_aM_bF_{-m_b}^3} \neq S_{F_{-m_a}^2M_bF_{-m_b}^3}$ i.e., lines $M_aF_{-m_a}^2$ and $M_bF_{-m_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $H_c=_Hc$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Proving failed

4.3.2 Proving $H_c=_Hc$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Proving failed

4.4.2 Proving $H_c=_Hc$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 619

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 619: Given a point A , a point I and a point N , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 620

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 620: Given a point A , a point M_a and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point M_a , construct a point G (rule W01); ;
2. Using the point N and the point G , construct a point O (rule W01); ;
3. Using the point N and the point G , construct a point H (rule W01); ;
4. Using the point A and the point H , construct a point E_a (rule W01); ;
5. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
6. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
7. Using the point M_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_a and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
9. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; line h_a and circle $k(N, M_a)$ intersect; points M_a and N are not the same; points A and O are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points A and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L15,L16,L19,L22,L55]

Solving time: 8.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point A 80 95
point M_{a} 65 40
point N 72.5 61.93

color 220 0 0
fontsize 9

cmark_t A
cmark_r M_{a}
cmark_r N
color 0 0 0
fontsize 8

% Constructing a line L_{\_G109567} which passes through point A and point M_{a}
line L_{\_G109567} A M_{a}

color 200 200 200
drawline L_{\_G109567}
color 0 0 0

% Constructing a point P_{\_G109668} with coordinates (0,0)
point P_{\_G109668} 0 0
cmark_r P_{\_G109668}

% Constructing a point P_{\_G109592} such that AP_{\_G109592}/AP_{\_G109668}=2
towards P_{\_G109592} A P_{\_G109668} 2
cmark_r P_{\_G109592}
color 200 200 200
drawsegment A P_{\_G109592}
color 0 0 0

% Constructing a point P_{\_G109637} such that AP_{\_G109637}/AP_{\_G109668}=3
towards P_{\_G109637} A P_{\_G109668} 3
cmark_r P_{\_G109637}
color 200 200 200
drawsegment A P_{\_G109637}
color 0 0 0

% Constructing a line L_{\_G109598} which passes through point M_{a} and point P_{\_G109637}
line L_{\_G109598} M_{a} P_{\_G109637}
```

```

color 200 200 200
drawline L_{\_G109598}
color 0 0 0

% Constructing a line L_{\_G109561} which contains the point P_{\_G109592} and is parallel to the
  line L_{\_G109598}
parallel L_{\_G109561} P_{\_G109592} L_{\_G109598}

color 200 200 200
drawline L_{\_G109561}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G109561} and line L_{\_G109567}
intersec G L_{\_G109561} L_{\_G109567}
cmark_t G

% Constructing a point O such that NO/NG=3
towards O N G 3
cmark_t O
color 200 200 200
drawsegment N O
color 0 0 0

% Constructing a point H such that NH/NG=-3
towards H N G -3
cmark_rt H
color 200 200 200
drawsegment G H
color 0 0 0

% Constructing a point E_{a} such that AE_{a}/AH=0.5
towards E_{a} A H 0.5
cmark_r E_{a}
color 200 200 200
drawsegment A H
color 0 0 0

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

```

```

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points M_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{a}
circle k(N,M_{a}) N M_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G110703} which is a foot of the point N on the line h_{a}
foot P_{\_G110703} N h_{a}
cmark_r P_{\_G110703}
color 200 200 200
drawline N P_{\_G110703}
color 0 0 0

% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P_{\_G110703}
sim H_{a} P_{\_G110703} E_{a}
cmark_r H_{a}

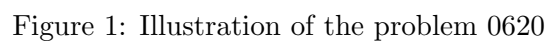
% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; line h_{a} and circle k(N,M_{a})
% intersect; points M_{a} and N are not the same; points A and O are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
% different; points A and H are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.031 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $M_a = \neg M_a$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A = A$

NDG conditions are:

$S_{P_{G88926}AM_a} \neq S_{P_{L_{G88895}}^0AM_a}$ i.e., lines $P_{G88926}P_{L_{G88895}}^0$ and AM_a are not parallel (construction based assumption)

$S_{\neg M_aBC} \neq 0$ i.e., points $\neg M_a, B$ and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_bAC} \neq 0$ i.e., points $\neg M_b, A$ and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^4} \neq S_{F_{\neg h_a}^3BF_{\neg h_b}^4}$ i.e., lines $AF_{\neg h_a}^3$ and $BF_{\neg h_b}^4$ are not parallel (construction based assumption)

$S_{\neg M_a\neg M_bF_{\neg m_b}^2} \neq S_{F_{\neg m_a}^1\neg M_bF_{\neg m_b}^2}$ i.e., lines $\neg M_aF_{\neg m_a}^1$ and $\neg M_bF_{\neg m_b}^2$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $M_a = \neg M_a$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A = A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.3.2 Proving $M_a = \neg M_a$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $M_a=_M_a$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 621

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 621: Given a point A , a point M_b and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point M_b , construct a point C (rule W01); ;
2. Using the point A and the point M_b , construct a line b (rule W02); % DET: points A and M_b are not the same;
3. Using the point M_b and the point N , construct a line $m(H_aH_c)$ (rule W02); % DET: points M_b and N are not the same;
4. Using the point A and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points A and M_b are not the same;
5. Using the point M_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_b and N are not the same;
6. Using the circle $k(N, M_a)$, the line b , the point N and the point M_b , construct a point H_b (rule W05); % NDG: line b and circle $k(N, M_a)$ intersect % DET: points M_b and H_b must be different;
7. Using the circle $k(N, M_a)$, the line $m(H_aH_c)$, the point N and the point M_b , construct a point E_b (rule W05a); % NDG: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_b and E_b must be different;
8. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;
9. Using the circle $k(M_b, C)$ and the circle $k(N, M_a)$, construct a point H_a and a point H_c (rule W07); % NDG: circles $k(M_b, C)$ and $k(N, M_a)$ intersect % DET: circles $k(M_b, C)$ and $k(N, M_a)$ are not the same;

10. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
11. Using the line h_b and the line h_a , construct a point H (rule W03); % NDG: lines h_b and h_a are not parallel % DET: lines h_b and h_a are not the same;
12. Using the point N and the point H , construct a point G (rule W01); ;
13. Using the point M_b and the point G , construct a point B (rule W01); .

Non-degenerate conditions: lines h_b and h_a are not parallel; circles $k(M_b, C)$ and $k(N, M_a)$ intersect; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; line b and circle $k(N, M_a)$ intersect; points M_b and N are not the same; points A and M_b are not the same.

Determination conditions: lines h_b and h_a are not the same; points A and H_a are not the same; circles $k(M_b, C)$ and $k(N, M_a)$ are not the same; points E_b and H_b are not the same; points M_b and E_b must be different; points M_b and H_b must be different; points M_b and N are not the same; points A and M_b are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D22,D29,D3,D32,D5,D6,D8,D9,GD01,GD02,GL01,GL03,GL04,GL09,L16,L17,L19,L20,L21,L23]

Solving time: 5.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point M_{b} 95 67.5
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_lt M_{b}
cmark_r N
color 0 0 0
fontsize 8
```

```
% Constructing a point C such that AC/AM_{b}=2
```

```
towards C A M_{b} 2
```

```
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0
```

```
% DET: points A and M_{b} are not the same
```

```
% Constructing a line b which passes through point A and point M_{b}
```

```
line b A M_{b}
```

```
color 200 200 200
```

```

drawline b
color 0 0 0

% DET: points  $M_{\{b\}}$  and  $N$  are not the same
% Constructing a line  $m(H_{\{a\}}H_{\{c\}})$  which passes through point  $M_{\{b\}}$  and point  $N$ 
line m( $H_{\{a\}}H_{\{c\}}$ )  $M_{\{b\}}$  N

color 200 200 200
drawline m( $H_{\{a\}}H_{\{c\}}$ )
color 0 0 0

% NDG: points  $A$  and  $M_{\{b\}}$  are not the same
% Constructing a circle  $k(M_{\{b\}}, C)$  whose center is at point  $M_{\{b\}}$  and which passes through point  $A$ 
circle k( $M_{\{b\}}, C$ )  $M_{\{b\}}$  A

color 200 200 200
drawcircle k( $M_{\{b\}}, C$ )
color 0 0 0

% NDG: points  $M_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $M_{\{b\}}$ 
circle k( $N, M_{\{a\}}$ )  $N$   $M_{\{b\}}$ 

color 200 200 200
drawcircle k( $N, M_{\{a\}}$ )
color 0 0 0

% NDG: line  $b$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{b\}}$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G147508\}}$  which is a foot of the point  $N$  on the line  $b$ 
foot  $P_{\{\backslash\_G147508\}}$   $N$   $b$ 
cmark_r  $P_{\{\backslash\_G147508\}}$ 
color 200 200 200
drawline  $N$   $P_{\{\backslash\_G147508\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $M_{\{b\}}$  in the symmetry to point/line  $P_{\{\backslash\_G147508\}}$ 
sim  $H_{\{b\}}$   $P_{\{\backslash\_G147508\}}$   $M_{\{b\}}$ 
cmark_l  $H_{\{b\}}$ 

% NDG: line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{b\}}$  and  $E_{\{b\}}$  must be
different
% Constructing a point  $E_{\{b\}}$  which is an image of the point  $M_{\{b\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{b\}}$   $N$   $M_{\{b\}}$ 
cmark_r  $E_{\{b\}}$ 

```

```

% DET: points  $E_{\{b\}}$  and  $H_{\{b\}}$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $E_{\{b\}}$  and point  $H_{\{b\}}$ 
line  $h_{\{b\}}$   $E_{\{b\}}$   $H_{\{b\}}$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: circles  $k(M_{\{b\}}, C)$  and  $k(N, M_{\{a\}})$  intersect% DET: circles  $k(M_{\{b\}}, C)$  and  $k(N, M_{\{a\}})$  are not
the same
% Constructing points  $H_{\{a\}}$  and  $H_{\{c\}}$  which are in intersection of  $k(M_{\{b\}}, C)$  and  $k(N, M_{\{a\}})$ 
intersec2  $H_{\{a\}}$   $H_{\{c\}}$   $k(M_{\{b\}}, C)$   $k(N, M_{\{a\}})$ 
cmark_r  $H_{\{a\}}$ 
cmark_rt  $H_{\{c\}}$ 

% DET: points  $A$  and  $H_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $A$  and point  $H_{\{a\}}$ 
line  $h_{\{a\}}$   $A$   $H_{\{a\}}$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: lines  $h_{\{b\}}$  and  $h_{\{a\}}$  are not parallel% DET: lines  $h_{\{b\}}$  and  $h_{\{a\}}$  are not the same
% Constructing a point  $H$  which belongs to line  $h_{\{b\}}$  and line  $h_{\{a\}}$ 
intersec  $H$   $h_{\{b\}}$   $h_{\{a\}}$ 
cmark_rt  $H$ 

% Constructing a line  $L_{\{\backslash\_G148010\}}$  which passes through point  $N$  and point  $H$ 
line  $L_{\{\backslash\_G148010\}}$   $N$   $H$ 

color 200 200 200
drawline  $L_{\{\backslash\_G148010\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\backslash\_G148111\}}$  with coordinates  $(0,0)$ 
point  $P_{\{\backslash\_G148111\}}$  0 0
cmark_r  $P_{\{\backslash\_G148111\}}$ 

% Constructing a point  $P_{\{\backslash\_G148035\}}$  such that  $NP_{\{\backslash\_G148035\}}/NP_{\{\backslash\_G148111\}}=-1$ 
towards  $P_{\{\backslash\_G148035\}}$   $N$   $P_{\{\backslash\_G148111\}}$  -1
cmark_r  $P_{\{\backslash\_G148035\}}$ 
color 200 200 200
drawsegment  $P_{\{\backslash\_G148111\}}$   $P_{\{\backslash\_G148035\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\backslash\_G148080\}}$  such that  $NP_{\{\backslash\_G148080\}}/NP_{\{\backslash\_G148111\}}=3$ 
towards  $P_{\{\backslash\_G148080\}}$   $N$   $P_{\{\backslash\_G148111\}}$  3

```

```

cmark_r P_{\_G148080}
color 200 200 200
drawsegment N P_{\_G148080}
color 0 0 0

% Constructing a line L_{\_G148041} which passes through point H and point P_{\_G148080}
line L_{\_G148041} H P_{\_G148080}

color 200 200 200
drawline L_{\_G148041}
color 0 0 0

% Constructing a line L_{\_G148004} which contains the point P_{\_G148035} and is parallel to the
line L_{\_G148041}
parallel L_{\_G148004} P_{\_G148035} L_{\_G148041}

color 200 200 200
drawline L_{\_G148004}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G148004} and line L_{\_G148010}
intersec G L_{\_G148004} L_{\_G148010}
cmark_t G

% Constructing a point B such that M_{b}B/M_{b}G=3
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{b} and h_{a} are not parallel; circles k(M_{b},C) and k(N,M_{a}) intersect; line m(H_{a}H_{c}) and circle k(N,M_{a}) intersect; line b and circle k(N,M_{a}) intersect; points M_{b} and N are not the same; points A and M_{b} are not the same
% Determination conditions: lines h_{b} and h_{a} are not the same; points A and H_{a} are not the same; circles k(M_{b},C) and k(N,M_{a}) are not the same; points E_{b} and H_{b} are not the same; points M_{b} and E_{b} must be different; points M_{b} and H_{b} must be different; points M_{b} and N are not the same; points A and M_{b} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

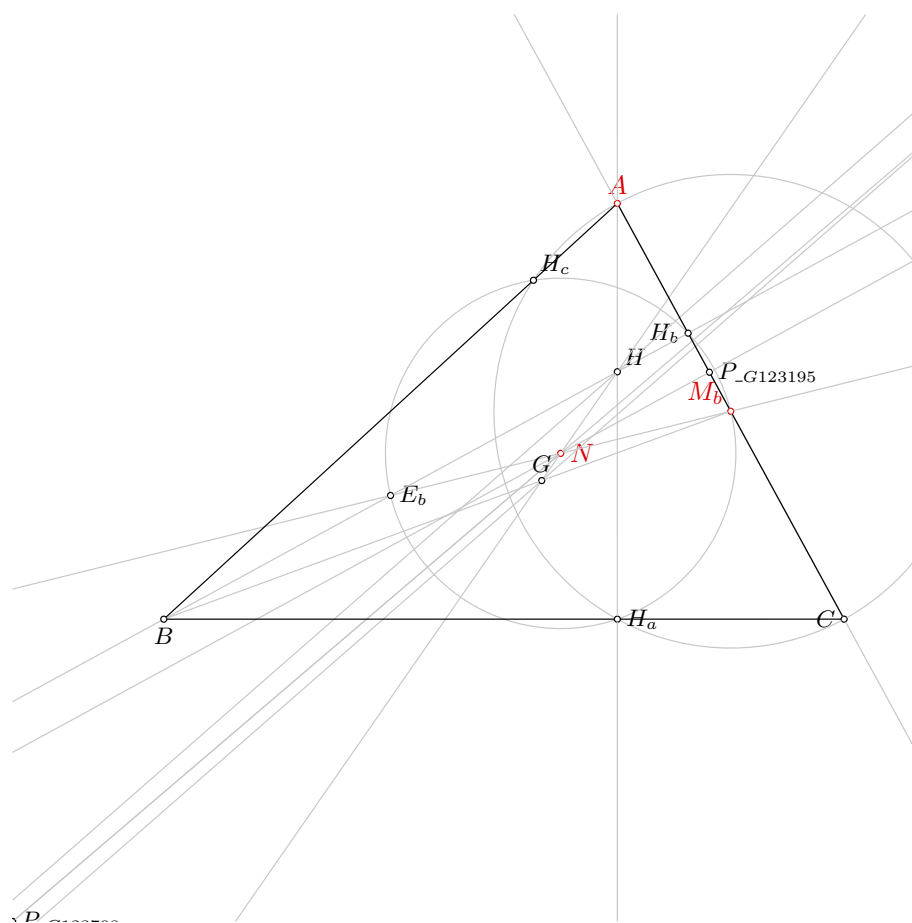


Figure 1: Illustration of the problem 0621

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.026 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $M_b=_M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.036 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{E_bAH_a} \neq S_{H_bAH_a}$ i.e., lines E_bH_b and AH_a are not parallel (construction based assumption)

$S_{P_{-G125638}NH} \neq S_{P_{L_{-G125607}}^0NH}$ i.e., lines $P_{-G125638}$ and NH are not parallel (construction based assumption)

$S_{_M_aBC} \neq 0$ i.e., points $_M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$ i.e., points $_M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^4} \neq S_{F_{-h_a}^3BF_{-h_b}^4}$ i.e., lines $AF_{-h_a}^3$ and $BF_{-h_b}^4$ are not parallel (construction based assumption)

$S_{_M_a-_M_bF_{-m_b}^2} \neq S_{F_{-m_a}^1-_M_bF_{-m_b}^2}$ i.e., lines $_M_aF_{-m_a}^1$ and $_M_bF_{-m_b}^2$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $M_b=_M_b$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $M_b = \neg M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 18 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A = A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $M_b = \neg M_b$

Proving failed

4.4.3 Proving $N = \neg N$

Proving failed

Problem 622

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 622: Given a point A , a point M_c and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point A and the point M_c , construct a point B (rule W01); ;
2. Using the point A and the point M_c , construct a line c (rule W02); % DET: points A and M_c are not the same;
3. Using the point M_c and the point N , construct a line $m(H_bH_a)$ (rule W02); % DET: points M_c and N are not the same;
4. Using the point A and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points A and M_c are not the same;
5. Using the point M_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_c and N are not the same;
6. Using the circle $k(N, M_a)$, the line c , the point N and the point M_c , construct a point H_c (rule W05); % NDG: line c and circle $k(N, M_a)$ intersect % DET: points M_c and H_c must be different;
7. Using the circle $k(N, M_a)$, the line $m(H_bH_a)$, the point N and the point M_c , construct a point E_c (rule W05a); % NDG: line $m(H_bH_a)$ and circle $k(N, M_a)$ intersect % DET: points M_c and E_c must be different;
8. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;
9. Using the circle $k(M_c, A)$ and the circle $k(N, M_a)$, construct a point H_b and a point H_a (rule W07); % NDG: circles $k(M_c, A)$ and $k(N, M_a)$ intersect % DET: circles $k(M_c, A)$ and $k(N, M_a)$ are not the same;

10. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;
11. Using the line h_c and the line b , construct a point C (rule W03); % NDG: lines h_c and b are not parallel % DET: lines h_c and b are not the same.

Non-degenerate conditions: lines h_c and b are not parallel; circles $k(M_c, A)$ and $k(N, M_a)$ intersect; line $m(H_b H_a)$ and circle $k(N, M_a)$ intersect; line c and circle $k(N, M_a)$ intersect; points M_c and N are not the same; points A and M_c are not the same.

Determination conditions: lines h_c and b are not the same; points A and H_b are not the same; circles $k(M_c, A)$ and $k(N, M_a)$ are not the same; points E_c and H_c are not the same; points M_c and E_c must be different; points M_c and H_c must be different; points M_c and N are not the same; points A and M_c are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D10,D20,D30,D32,D6,D7,GD01,GD02,GL01,GL03,GL09,L18,L19,L20,L21,L24,L3,L41,L42,L53]

Solving time: 4.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point M_{c} 50 67.5
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_lt M_{c}
cmark_r N
color 0 0 0
fontsize 8
```

```
% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0
```

```
% DET: points A and M_{c} are not the same
% Constructing a line c which passes through point A and point M_{c}
line c A M_{c}
```

```
color 200 200 200
drawline c
color 0 0 0
```

```

% DET: points  $M_{\{c\}}$  and  $N$  are not the same
% Constructing a line  $m(H_{\{b\}}H_{\{a\}})$  which passes through point  $M_{\{c\}}$  and point  $N$ 
line  $m(H_{\{b\}}H_{\{a\}})$   $M_{\{c\}}$   $N$ 

color 200 200 200
drawline  $m(H_{\{b\}}H_{\{a\}})$ 
color 0 0 0

% NDG: points  $A$  and  $M_{\{c\}}$  are not the same
% Constructing a circle  $k(M_{\{c\}},A)$  whose center is at point  $M_{\{c\}}$  and which passes through point  $A$ 
circle  $k(M_{\{c\}},A)$   $M_{\{c\}}$   $A$ 

color 200 200 200
drawcircle  $k(M_{\{c\}},A)$ 
color 0 0 0

% NDG: points  $M_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N,M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $M_{\{c\}}$ 
circle  $k(N,M_{\{a\}})$   $N$   $M_{\{c\}}$ 

color 200 200 200
drawcircle  $k(N,M_{\{a\}})$ 
color 0 0 0

% NDG: line  $c$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $M_{\{c\}}$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G180208\}}$  which is a foot of the point  $N$  on the line  $c$ 
foot  $P_{\{\backslash\_G180208\}}$   $N$   $c$ 
cmark_r  $P_{\{\backslash\_G180208\}}$ 
color 200 200 200
drawline  $N$   $P_{\{\backslash\_G180208\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $M_{\{c\}}$  in the symmetry to point/line  $P_{\{\backslash\_G180208\}}$ 
sim  $H_{\{c\}}$   $P_{\{\backslash\_G180208\}}$   $M_{\{c\}}$ 
cmark_rt  $H_{\{c\}}$ 

% NDG: line  $m(H_{\{b\}}H_{\{a\}})$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $M_{\{c\}}$  and  $E_{\{c\}}$  must be
different
% Constructing a point  $E_{\{c\}}$  which is an image of the point  $M_{\{c\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{c\}}$   $N$   $M_{\{c\}}$ 
cmark_r  $E_{\{c\}}$ 

% DET: points  $E_{\{c\}}$  and  $H_{\{c\}}$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $E_{\{c\}}$  and point  $H_{\{c\}}$ 
line  $h_{\{c\}}$   $E_{\{c\}}$   $H_{\{c\}}$ 

```

```

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: circles  $k(M_{\{c\}}, A)$  and  $k(N, M_{\{a\}})$  intersect% DET: circles  $k(M_{\{c\}}, A)$  and  $k(N, M_{\{a\}})$  are not
the same
% Constructing points  $H_{\{b\}}$  and  $H_{\{a\}}$  which are in intersection of  $k(M_{\{c\}}, A)$  and  $k(N, M_{\{a\}})$ 
intersec2 H_{b} H_{a} k(M_{\{c\}}, A) k(N, M_{\{a\}})
cmark_l H_{b}
cmark_r H_{a}

% DET: points  $A$  and  $H_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $A$  and point  $H_{\{b\}}$ 
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $h_{\{c\}}$  and  $b$  are not parallel% DET: lines  $h_{\{c\}}$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $h_{\{c\}}$  and line  $b$ 
intersec C h_{c} b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{c\}}$  and  $b$  are not parallel; circles  $k(M_{\{c\}}, A)$  and  $k(N, M_{\{a\}})$ 
intersect; line  $m(H_{\{b\}}H_{\{a\}})$  and circle  $k(N, M_{\{a\}})$  intersect; line  $c$  and circle  $k(N, M_{\{a\}})$ 
intersect; points  $M_{\{c\}}$  and  $N$  are not the same; points  $A$  and  $M_{\{c\}}$  are not the same
% Determination conditions: lines  $h_{\{c\}}$  and  $b$  are not the same; points  $A$  and  $H_{\{b\}}$  are not the same
; circles  $k(M_{\{c\}}, A)$  and  $k(N, M_{\{a\}})$  are not the same; points  $E_{\{c\}}$  and  $H_{\{c\}}$  are not the same;
points  $M_{\{c\}}$  and  $E_{\{c\}}$  must be different; points  $M_{\{c\}}$  and  $H_{\{c\}}$  must be different; points  $M_{\{c\}}$ 
and  $N$  are not the same; points  $A$  and  $M_{\{c\}}$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

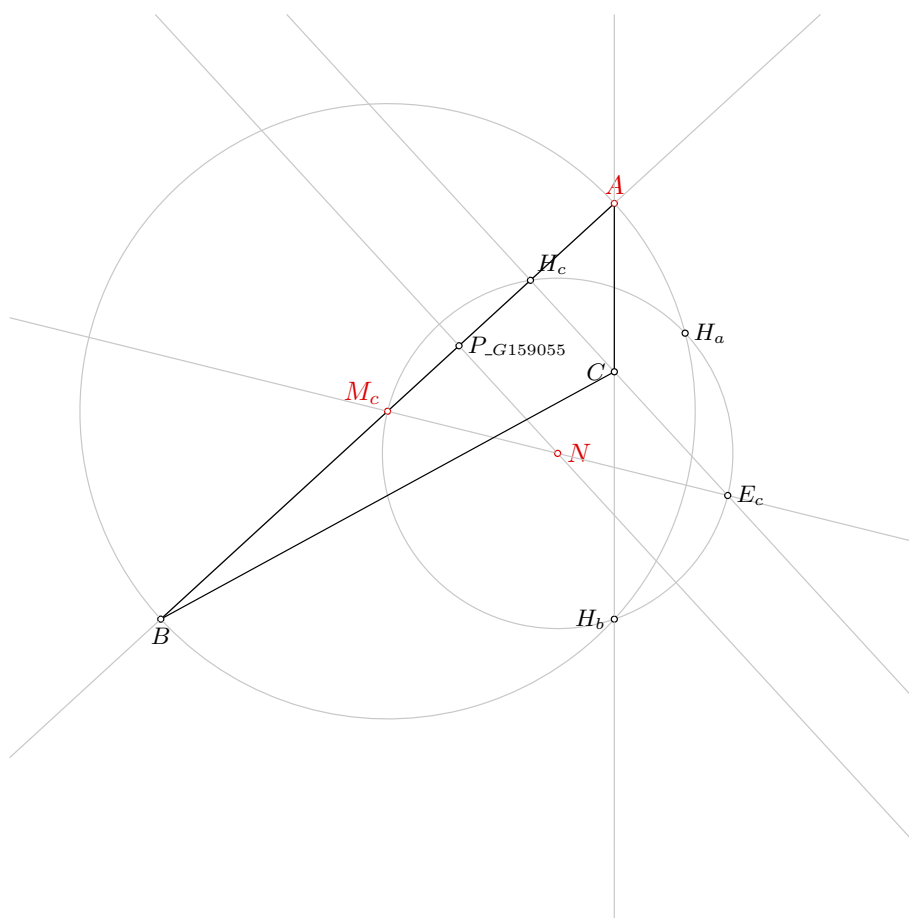


Figure 1: Illustration of the problem 0622

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.03 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $M_c = \neg M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.033 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A = A$

NDG conditions are:

$S_{E_c A H_b} \neq S_{H_c A H_b}$ i.e., lines $E_c H_c$ and $A H_b$ are not parallel (construction based assumption)

$S_{\neg M_a B C} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_b A C} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{A B C} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{B A C} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{A B F_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2 B F_{\neg h_b}^3}$ i.e., lines $A F_{\neg h_a}^2$ and $B F_{\neg h_b}^3$ are not parallel (construction based assumption)

$S_{\neg M_a \neg M_b F_{\neg m_b}^1} \neq S_{F_{\neg m_a}^0 \neg M_b F_{\neg m_b}^1}$ i.e., lines $\neg M_a F_{\neg m_a}^0$ and $\neg M_b F_{\neg m_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $M_c = \neg M_c$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A = A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $M_c = \neg M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 8 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.3 Proving $N=_N$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 778 terms.

Time Complexity: Time spent by the prover is 2.710 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $M_c=_M_c$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 623

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 623: Given a point A , a point N and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point N and the point O , construct a point G (rule W01); ;
2. Using the point A and the point G , construct a point M_a (rule W01); ;
3. Using the point N and the point O , construct a point H (rule W01); ;
4. Using the point A and the point H , construct a point E_a (rule W01); ;
5. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
6. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
7. Using the point M_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_a and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
9. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; line h_a and circle $k(N, M_a)$ intersect; points M_a and N are not the same; points A and O are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points A and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D31,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L15,L19,L22,L55]

Solving time: 8.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point A 80 95
point N 72.5 61.93
point O 65 51.14
```

```
color 220 0 0
fontsize 9
```

```
cmark_t A
cmark_r N
cmark_t O
color 0 0 0
fontsize 8
```

```
% Constructing a line L_{\_G217840} which passes through point N and point O
line L_{\_G217840} N O
```

```
color 200 200 200
drawline L_{\_G217840}
color 0 0 0
```

```
% Constructing a point P_{\_G217941} with coordinates (0,0)
point P_{\_G217941} 0 0
cmark_r P_{\_G217941}
```

```
% Constructing a point P_{\_G217865} such that NP_{\_G217865}/NP_{\_G217941}=1
towards P_{\_G217865} N P_{\_G217941} 1
cmark_r P_{\_G217865}
color 200 200 200
drawsegment N P_{\_G217865}
color 0 0 0
```

```
% Constructing a point P_{\_G217910} such that NP_{\_G217910}/NP_{\_G217941}=3
towards P_{\_G217910} N P_{\_G217941} 3
cmark_r P_{\_G217910}
color 200 200 200
drawsegment N P_{\_G217910}
color 0 0 0
```

```
% Constructing a line L_{\_G217871} which passes through point O and point P_{\_G217910}
line L_{\_G217871} O P_{\_G217910}
```

```

color 200 200 200
drawline L_{\_G217871}
color 0 0 0

% Constructing a line L_{\_G217834} which contains the point P_{\_G217865} and is parallel to the
  line L_{\_G217871}
parallel L_{\_G217834} P_{\_G217865} L_{\_G217871}

color 200 200 200
drawline L_{\_G217834}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G217834} and line L_{\_G217840}
intersec G L_{\_G217834} L_{\_G217840}
cmark_t G

% Constructing a point M_{a} such that  $AM_{a}/AG=1.5$ 
towards M_{a} A G 1.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% Constructing a point H such that  $NH/NO=-1$ 
towards H N O -1
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0

% Constructing a point E_{a} such that  $AE_{a}/AH=0.5$ 
towards E_{a} A H 0.5
cmark_r E_{a}
color 200 200 200
drawsegment A H
color 0 0 0

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

```

```

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points M_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{a}
circle k(N,M_{a}) N M_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G218985} which is a foot of the point N on the line h_{a}
foot P_{\_G218985} N h_{a}
cmark_r P_{\_G218985}
color 200 200 200
drawline N P_{\_G218985}
color 0 0 0

% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
\_G218985}
sim H_{a} P_{\_G218985} E_{a}
cmark_r H_{a}

% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```

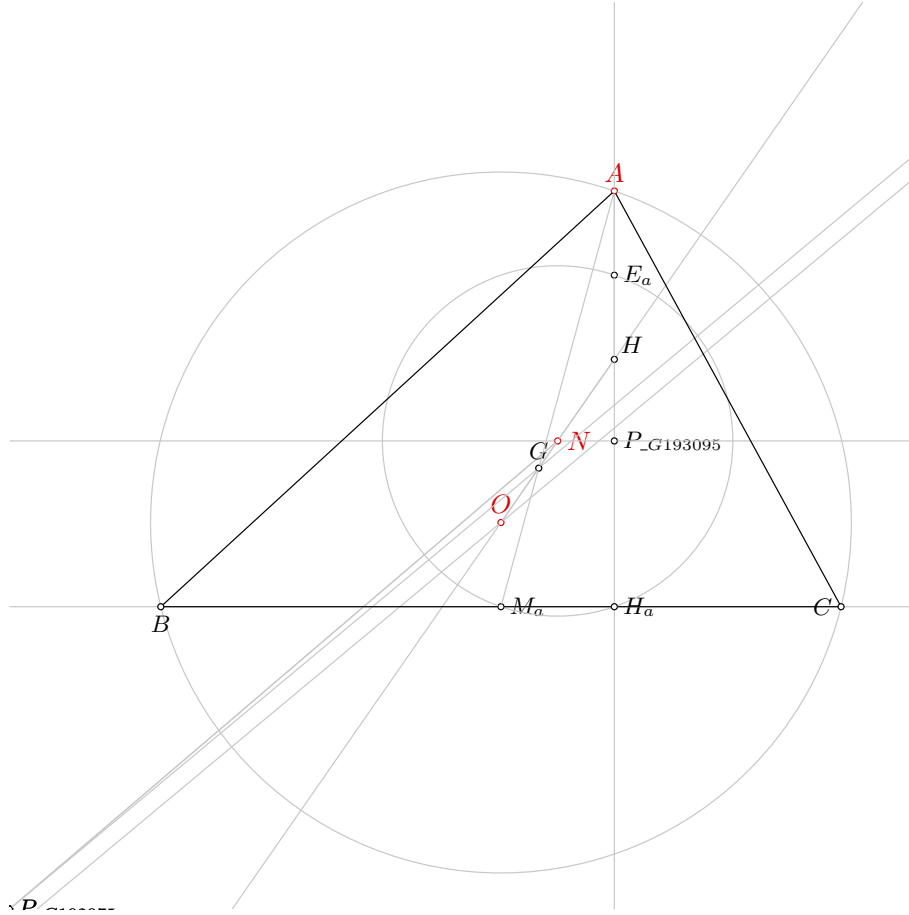


Figure 1: Illustration of the problem 0623

*% Non-degenerate conditions: line a and circle $k(O, C)$ intersect; line $h_{\{a\}}$ and circle $k(N, M_{\{a\}})$ intersect; points $M_{\{a\}}$ and N are not the same; points A and O are not the same
 % Determination conditions: points $H_{\{a\}}$ and $M_{\{a\}}$ are not the same; points $E_{\{a\}}$ and $H_{\{a\}}$ must be different; points A and H are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $A=A$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.031 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $O=_O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $A=A$

NDG conditions are:

$S_{P_{G193423}NO} \neq S_{P_{L_{G193392}}^0 NO}$ i.e., lines $P_{G193423}P_{L_{G193392}}^0$ and NO are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points M_a, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points M_b, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aM_bF_{M_b}^2} \neq S_{F_{M_a}^1M_bF_{M_b}^2}$ i.e., lines $M_aF_{M_a}^1$ and $M_bF_{M_b}^2$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^4} \neq S_{F_{h_a}^3BF_{h_b}^4}$ i.e., lines $AF_{h_a}^3$ and $BF_{h_b}^4$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $O=_O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $O=_O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $A=A$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $O=_O$

Proving failed

Problem 624

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 624: Given a point A , a point N and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 625

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 625: Given a point A , a point N and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 626

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 626: Given a point A , a point N and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 627

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 627: Given a point B , a point C and a point E_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point C , construct a point M_a (rule W01); ;
2. Using the point B and the point C , construct a line a (rule W02); % DET: points B and C are not the same;
3. Using the point E_a and the point M_a , construct a line $m(H_bH_c)$ (rule W02); % DET: points E_a and M_a are not the same;
4. Using the point E_a and the line a , construct a line h_a (rule W10b); ;
5. Using the line h_a and the line a , construct a point H_a (rule W03); % NDG: lines h_a and a are not parallel % DET: lines h_a and a are not the same;
6. Using the point B and the point H_a , construct a line $m(BH_a)$ (rule W14); % DET: points B and H_a are not the same;
7. Using the point E_a and the point M_a , construct a line $m(E_aM_a)$ (rule W14); % DET: points E_a and M_a are not the same;
8. Using the line $m(E_aM_a)$ and the line $m(H_bH_c)$, construct a point N (rule W03); % NDG: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not parallel % DET: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not the same;
9. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
10. Using the circle $k(N, M_a)$ and the line $m(BH_a)$, construct a point M_c and a point E_b (rule W04); % NDG: line $m(BH_a)$ and circle $k(N, M_a)$ intersect;

11. Using the point M_c and the point B , construct a point A (rule W01); .

Non-degenerate conditions: line $m(BH_a)$ and circle $k(N, M_a)$ intersect; points E_a and N are not the same; lines $m(E_aM_a)$ and $m(H_bH_c)$ are not parallel; lines h_a and a are not parallel.

Determination conditions: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not the same; points E_a and M_a are not the same; points B and H_a are not the same; lines h_a and a are not the same; points E_a and M_a are not the same; points B and C are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14]

Lemmas used: [D20,D21,D28,D3,D32,D5,D8,GD01,GD02,GL01,GL03,GL04,GL09,L18,L20,L21,L22,L23,L38,L

Solving time: 2.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point C 110 40
point E_{a} 80 83.86
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_l C
cmark_r E_{a}
color 0 0 0
fontsize 8
```

```
% Constructing a point M_{a} such that BM_{a}/BC=0.5
towards M_{a} B C 0.5
cmark_r M_{a}
color 200 200 200
drawsegment B C
color 0 0 0
```

```
% DET: points B and C are not the same
% Constructing a line a which passes through point B and point C
line a B C
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% DET: points E_{a} and M_{a} are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point E_{a} and point M_{a}
line m(H_{b}H_{c}) E_{a} M_{a}
```

```
color 200 200 200
drawline m(H_{b}H_{c})
```

```
color 0 0 0
```

```
% Constructing a line  $h_{\{a\}}$  which is perpendicular to line  $a$  and which passes through point  $E_{\{a\}}$ 
perp  $h_{\{a\}}$   $E_{\{a\}}$   $a$ 
```

```
color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0
```

```
% NDG: lines  $h_{\{a\}}$  and  $a$  are not parallel% DET: lines  $h_{\{a\}}$  and  $a$  are not the same
% Constructing a point  $H_{\{a\}}$  which belongs to line  $h_{\{a\}}$  and line  $a$ 
intersec  $H_{\{a\}}$   $h_{\{a\}}$   $a$ 
cmark_r  $H_{\{a\}}$ 
```

```
% DET: points  $B$  and  $H_{\{a\}}$  are not the same
% Constructing bisector  $m(BH_{\{a\}})$  of the segment  $BH_{\{a\}}$ 
med  $m(BH_{\{a\}})$   $B$   $H_{\{a\}}$ 
```

```
color 200 200 200
drawline  $m(BH_{\{a\}})$ 
color 0 0 0
```

```
color 200 200 200
drawsegment  $B$   $H_{\{a\}}$ 
color 0 0 0
```

```
% DET: points  $E_{\{a\}}$  and  $M_{\{a\}}$  are not the same
% Constructing bisector  $m(E_{\{a\}}M_{\{a\}})$  of the segment  $E_{\{a\}}M_{\{a\}}$ 
med  $m(E_{\{a\}}M_{\{a\}})$   $E_{\{a\}}$   $M_{\{a\}}$ 
```

```
color 200 200 200
drawline  $m(E_{\{a\}}M_{\{a\}})$ 
color 0 0 0
```

```
color 200 200 200
drawsegment  $E_{\{a\}}$   $M_{\{a\}}$ 
color 0 0 0
```

```
% NDG: lines  $m(E_{\{a\}}M_{\{a\}})$  and  $m(H_{\{b\}}H_{\{c\}})$  are not parallel% DET: lines  $m(E_{\{a\}}M_{\{a\}})$  and  $m(H_{\{b\}}H_{\{c\}})$ 
 $H_{\{c\}}$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{a\}}M_{\{a\}})$  and line  $m(H_{\{b\}}H_{\{c\}})$ 
intersec  $N$   $m(E_{\{a\}}M_{\{a\}})$   $m(H_{\{b\}}H_{\{c\}})$ 
cmark_r  $N$ 
```

```
% NDG: points  $E_{\{a\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{a\}}$ 
circle  $k(N, M_{\{a\}})$   $N$   $E_{\{a\}}$ 
```

```

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(BH_{a}) and circle k(N,M_{a}) intersect
% Constructing points M_{c} and E_{b} which are in intersection of k(N,M_{a}) and m(BH_{a})
intersec2 M_{c} E_{b} k(N,M_{a}) m(BH_{a})
cmark_lt M_{c}
cmark_r E_{b}

% Constructing a point A such that M_{c}A/M_{c}B=-1
towards A M_{c} B -1
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line m(BH_{a}) and circle k(N,M_{a}) intersect; points E_{a} and N are
% not the same; lines m(E_{a}M_{a}) and m(H_{b}H_{c}) are not parallel; lines h_{a} and a are
% not parallel
% Determination conditions: lines m(E_{a}M_{a}) and m(H_{b}H_{c}) are not the same; points E_{a}
% and M_{a} are not the same; points B and H_{a} are not the same; lines h_{a} and a are not the
% same; points E_{a} and M_{a} are not the same; points B and C are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.026 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $C=C$

Status: Theorem has been proved.

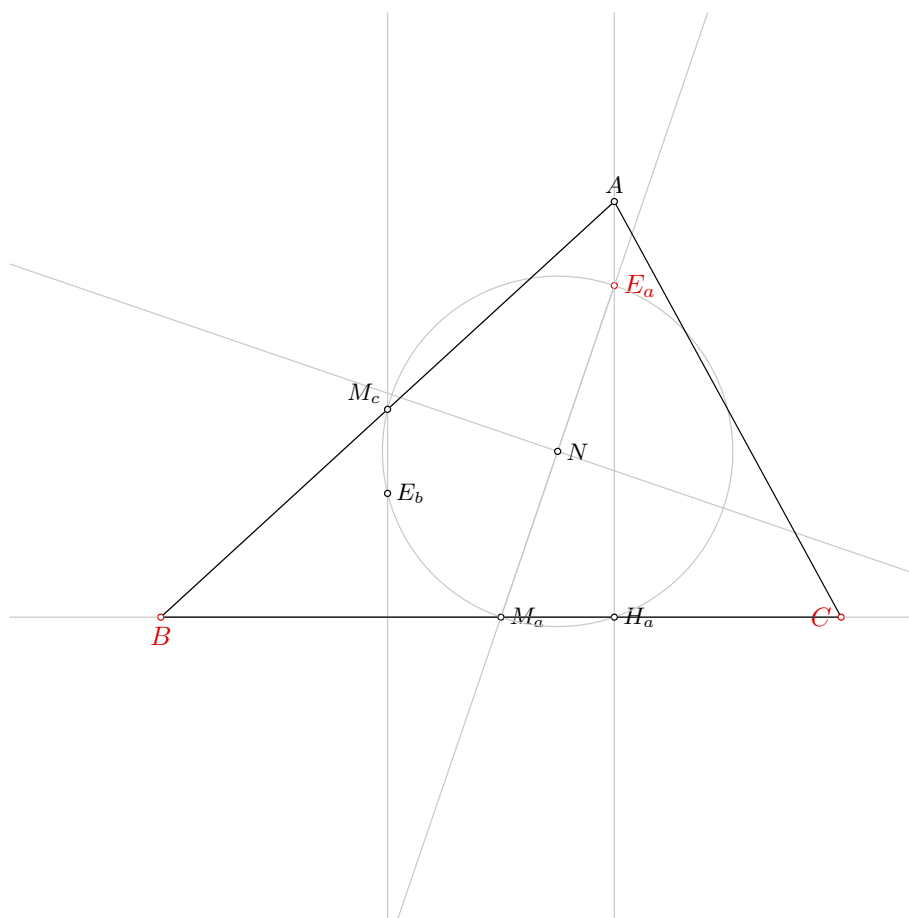


Figure 1: Illustration of the problem 0627

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.026 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $E_a = \neg E_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 36 terms.

Time Complexity: Time spent by the prover is 0.433 seconds.

NDG conditions Points M_a and E_a are not identical

Line through points B and M_a is not perpendicular to line through points M_a and C

Points A , B and C are not collinear

4.2 GCLC - Area method

4.2.1 Proving $B = B$

NDG conditions are:

$S_{E_a BC} \neq 0$ i.e., points E_a , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{E_a BC} \neq S_{F_{ha}^0 BC}$ i.e., lines $E_a F_{ha}^0$ and BC are not parallel (construction based assumption)

$S_{M_{m(E_a M_a)}^3 E_a M_a} \neq S_{T_{m(E_a M_a)}^4 E_a M_a}$ i.e., lines $M_{m(E_a M_a)}^3$ and $T_{m(E_a M_a)}^4$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{ha}^6} \neq S_{F_{ha}^5 BF_{hb}^6}$ i.e., lines AF_{ha}^5 and BF_{hb}^6 are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $C = C$

NDG conditions are:

$S_{E_a BC} \neq 0$ i.e., points E_a , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{E_a BC} \neq S_{F_{ha}^0 BC}$ i.e., lines $E_a F_{ha}^0$ and BC are not parallel (construction based assumption)

$S_{M_{m(E_a M_a)}^3 E_a M_a} \neq S_{T_{m(E_a M_a)}^4 E_a M_a}$ i.e., lines $M_{m(E_a M_a)}^3$ and $T_{m(E_a M_a)}^4$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{ha}^6} \neq S_{F_{ha}^5 BF_{hb}^6}$ i.e., lines AF_{ha}^5 and BF_{hb}^6 are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.3 Proving $E_a = \neg E_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.2 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.3 Proving $E_a = \neg E_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 42 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.3 Proving $E_a = \neg E_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 314 terms.

Time Complexity: Time spent by the prover is 0.580 seconds. There are no ndg conditions.

Problem 628

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 628: Given a point B , a point C and a point E_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point E_b , construct a point H (rule W01); ;
2. Using the point B and the point C , construct a line a (rule W02); % DET: points B and C are not the same;
3. Using the point C and the point H , construct a line h_c (rule W02); % DET: points C and H are not the same;
4. Using the point B and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points B and E_b are not the same;
5. Using the circle $k(E_b, B)$, the line a , the point E_b and the point B , construct a point H_a (rule W05); % NDG: line a and circle $k(E_b, B)$ intersect % DET: points B and H_a must be different;
6. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
7. Using the circle $k(E_b, B)$, the line h_c , the point E_b and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_b, B)$ intersect % DET: points H and H_c must be different;
8. Using the point H_c and the point B , construct a line c (rule W02); % DET: points H_c and B are not the same;
9. Using the line h_a and the line c , construct a point A (rule W03); % NDG: lines h_a and c are not parallel % DET: lines h_a and c are not the same.

Non-degenerate conditions: lines h_a and c are not parallel; line h_c and circle $k(E_b, B)$ intersect; line a and circle $k(E_b, B)$ intersect; points B and E_b are not the same.

Determination conditions: lines h_a and c are not the same; points H_c and B are not the same; points H and H_c must be different; points H_a and H are not the same; points B and H_a must be different; points C and H are not the same; points B and C are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D29,D3,D5,D7,D8,GD01,GD02,GL03,L3,L49,L50,L51]

Solving time: 11.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point C 110 40
point E_{b} 50 56.36
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_l C
cmark_r E_{b}
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that BH/BE_{b}=2
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0
```

```
% DET: points B and C are not the same
% Constructing a line a which passes through point B and point C
line a B C
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% DET: points C and H are not the same
% Constructing a line h_{c} which passes through point C and point H
line h_{c} C H
```

```
color 200 200 200
drawline h_{c}
color 0 0 0
```

```

% NDG: points B and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point B
circle k(E_{b},B) E_{b} B

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line a and circle k(E_{b},B) intersect% DET: points B and H_{a} must be different
% Constructing a point P_{\G18897} which is a foot of the point E_{b} on the line a
foot P_{\G18897} E_{b} a
cmark_r P_{\G18897}
color 200 200 200
drawline E_{b} P_{\G18897}
color 0 0 0

% Constructing a point H_{a} which is an image of the point B in the symmetry to point/line P_{\G
18897}
sim H_{a} P_{\G18897} B
cmark_r H_{a}

% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{c} and circle k(E_{b},B) intersect% DET: points H and H_{c} must be different
% Constructing a point P_{\G19135} which is a foot of the point E_{b} on the line h_{c}
foot P_{\G19135} E_{b} h_{c}
cmark_r P_{\G19135}
color 200 200 200
drawline E_{b} P_{\G19135}
color 0 0 0

% Constructing a point H_{c} which is an image of the point H in the symmetry to point/line P_{\G
19135}
sim H_{c} P_{\G19135} H
cmark_rt H_{c}

% DET: points H_{c} and B are not the same
% Constructing a line c which passes through point H_{c} and point B
line c H_{c} B

color 200 200 200
drawline c
color 0 0 0

```

```

% NDG: lines  $h_{\{a\}}$  and  $c$  are not parallel% DET: lines  $h_{\{a\}}$  and  $c$  are not the same
% Constructing a point  $A$  which belongs to line  $h_{\{a\}}$  and line  $c$ 
intersec A  $h_{\{a\}}$  c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{a\}}$  and  $c$  are not parallel; line  $h_{\{c\}}$  and circle  $k(E_{\{b\}}, B)$ 
% intersect; line  $a$  and circle  $k(E_{\{b\}}, B)$  intersect; points  $B$  and  $E_{\{b\}}$  are not the same
% Determination conditions: lines  $h_{\{a\}}$  and  $c$  are not the same; points  $H_{\{c\}}$  and  $B$  are not the same
% ; points  $H$  and  $H_{\{c\}}$  must be different; points  $H_{\{a\}}$  and  $H$  are not the same; points  $B$  and  $H_{\{a\}}$ 
% must be different; points  $C$  and  $H$  are not the same; points  $B$  and  $C$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.03 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $E_b = \neg E_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_a H_c B} \neq S_{H H_c B}$ i.e., lines $H_a H$ and $H_c B$ are not parallel (construction based assumption)

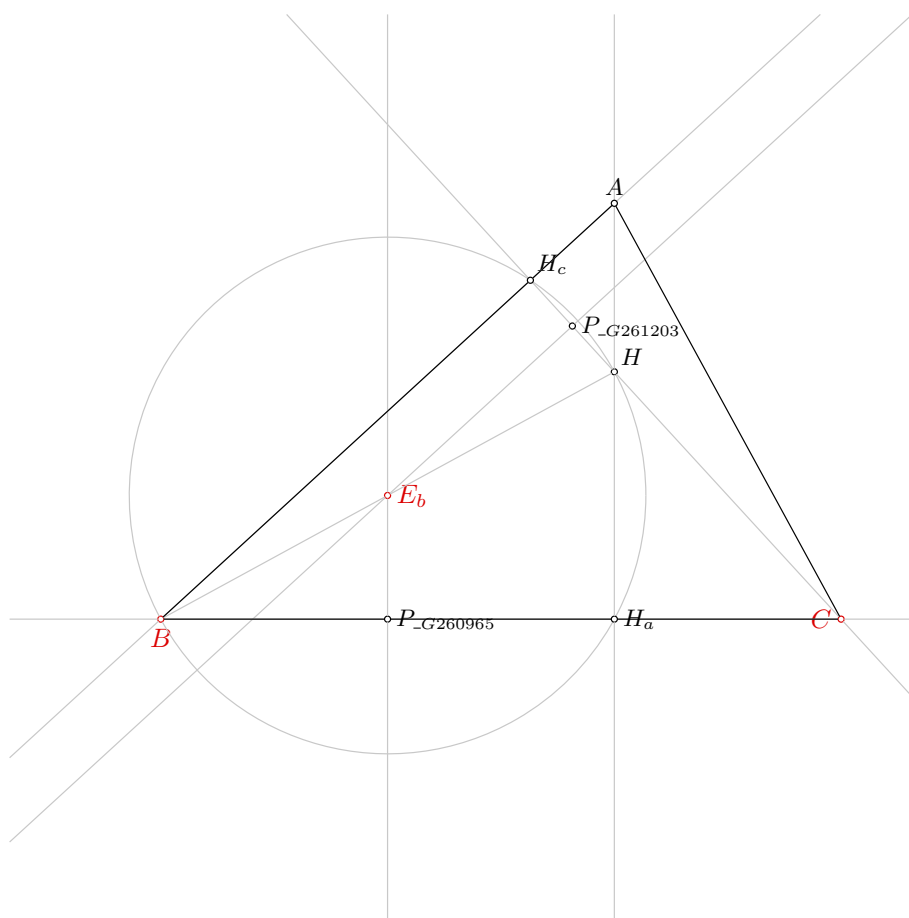


Figure 1: Illustration of the problem 0628

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a}BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $C=C$

NDG conditions are:

$S_{H_aH_cB} \neq S_{HH_cB}$ i.e., lines H_aH and H_cB are not parallel (construction based assumption)
 $S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a}BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.3 Proving $E_b=_E b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.3 Proving $E_b=_E b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 9 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.3 Proving $E_b = \neg E_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 51 terms.

Time Complexity: Time spent by the prover is 0.250 seconds. There are no ndg conditions.

Problem 629

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 629: Given a point B , a point C and a point E_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point C , construct a point M_a (rule W01); ;
2. Using the point C and the point E_c , construct a point H (rule W01); ;
3. Using the point B and the point H , construct a line h_b (rule W02); % DET: points B and H are not the same;
4. Using the point C and the point E_c , construct a line h_c (rule W02); % DET: points C and E_c are not the same;
5. Using the point B and the point M_a , construct a circle $k(M_a, B)$ (rule W06); % NDG: points B and M_a are not the same;
6. Using the circle $k(M_a, B)$, the line h_b , the point M_a and the point B , construct a point H_b (rule W05); % NDG: line h_b and circle $k(M_a, B)$ intersect % DET: points B and H_b must be different;
7. Using the point H_b and the point C , construct a line b (rule W02); % DET: points H_b and C are not the same;
8. Using the circle $k(M_a, B)$, the line h_c , the point M_a and the point C , construct a point H_c (rule W05); % NDG: line h_c and circle $k(M_a, B)$ intersect % DET: points C and H_c must be different;
9. Using the point H_c and the point B , construct a line c (rule W02); % DET: points H_c and B are not the same;
10. Using the line b and the line c , construct a point A (rule W03); % NDG: lines b and c are not parallel % DET: lines b and c are not the same.

Non-degenerate conditions: lines b and c are not parallel; line h_c and circle $k(M_a, B)$ intersect; line h_b and circle $k(M_a, B)$ intersect; points B and M_a are not the same.

Determination conditions: lines b and c are not the same; points H_c and B are not the same; points C and H_c must be different; points H_b and C are not the same; points B and H_b must be different; points C and E_c are not the same; points B and H are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D21,D3,D30,D6,D7,D9,GD01,GD02,GL03,GL09,L3,L37,L38,L39]

Solving time: 10.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
```

```
point C 110 40
```

```
point E_{c} 95 56.36
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_b B
```

```
cmark_l C
```

```
cmark_r E_{c}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point M_{a} such that BM_{a}/BC=0.5
```

```
towards M_{a} B C 0.5
```

```
cmark_r M_{a}
```

```
color 200 200 200
```

```
drawsegment B C
```

```
color 0 0 0
```

```
% Constructing a point H such that CH/CE_{c}=2
```

```
towards H C E_{c} 2
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment C H
```

```
color 0 0 0
```

```
% DET: points B and H are not the same
```

```
% Constructing a line h_{b} which passes through point B and point H
```

```
line h_{b} B H
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```

% DET: points C and E_{c} are not the same
% Constructing a line h_{c} which passes through point C and point E_{c}
line h_{c} C E_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points B and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point B
circle k(M_{a},B) M_{a} B

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line h_{b} and circle k(M_{a},B) intersect% DET: points B and H_{b} must be different
% Constructing a point P_{\_G41482} which is a foot of the point M_{a} on the line h_{b}
foot P_{\_G41482} M_{a} h_{b}
cmark_r P_{\_G41482}
color 200 200 200
drawline M_{a} P_{\_G41482}
color 0 0 0

% Constructing a point H_{b} which is an image of the point B in the symmetry to point/line P_{\_G
41482}
sim H_{b} P_{\_G41482} B
cmark_l H_{b}

% DET: points H_{b} and C are not the same
% Constructing a line b which passes through point H_{b} and point C
line b H_{b} C

color 200 200 200
drawline b
color 0 0 0

% NDG: line h_{c} and circle k(M_{a},B) intersect% DET: points C and H_{c} must be different
% Constructing a point P_{\_G41720} which is a foot of the point M_{a} on the line h_{c}
foot P_{\_G41720} M_{a} h_{c}
cmark_r P_{\_G41720}
color 200 200 200
drawline M_{a} P_{\_G41720}
color 0 0 0

% Constructing a point H_{c} which is an image of the point C in the symmetry to point/line P_{\_G
41720}
sim H_{c} P_{\_G41720} C
cmark_rt H_{c}

```

```

% DET: points  $H_{\{c\}}$  and  $B$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $B$ 
line c  $H_{\{c\}}$  B

color 200 200 200
drawline c
color 0 0 0

% NDG: lines  $b$  and  $c$  are not parallel% DET: lines  $b$  and  $c$  are not the same
% Constructing a point  $A$  which belongs to line  $b$  and line  $c$ 
intersec A b c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $b$  and  $c$  are not parallel; line  $h_{\{c\}}$  and circle  $k(M_{\{a\}},B)$ 
% intersect; line  $h_{\{b\}}$  and circle  $k(M_{\{a\}},B)$  intersect; points  $B$  and  $M_{\{a\}}$  are not the same
% Determination conditions: lines  $b$  and  $c$  are not the same; points  $H_{\{c\}}$  and  $B$  are not the same;
% points  $C$  and  $H_{\{c\}}$  must be different; points  $H_{\{b\}}$  and  $C$  are not the same; points  $B$  and  $H_{\{b\}}$ 
% must be different; points  $C$  and  $E_{\{c\}}$  are not the same; points  $B$  and  $H$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.027 seconds.

NDG conditions There are no NDG conditions for this theorem

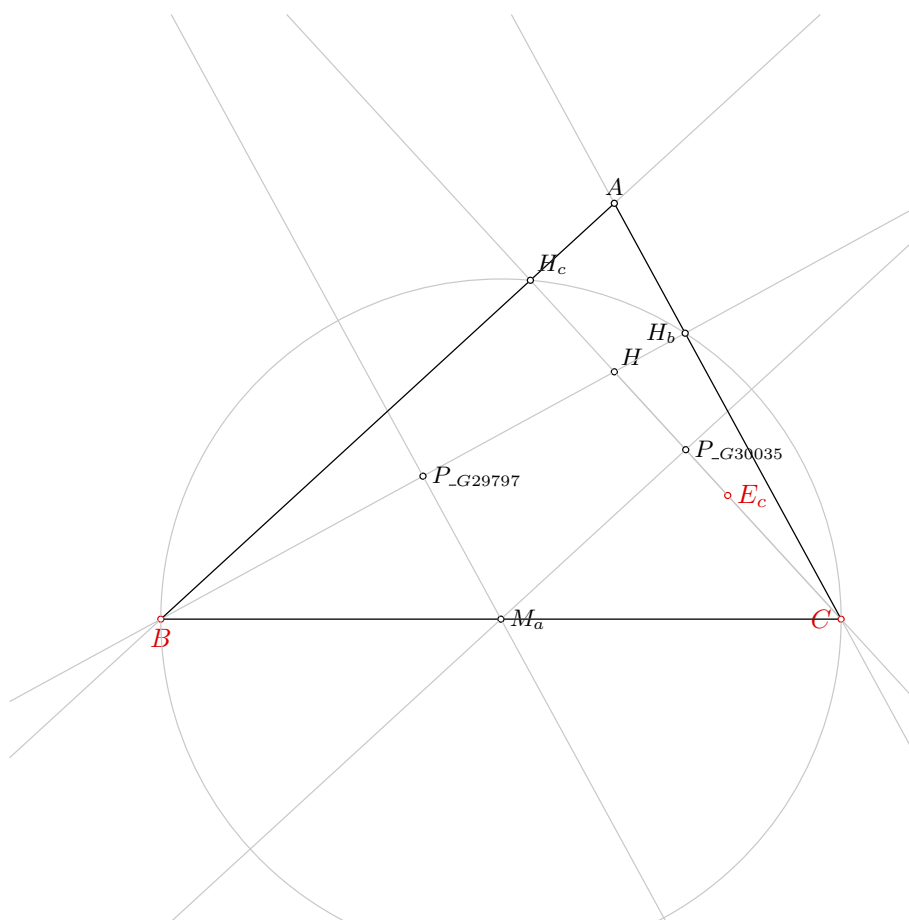


Figure 1: Illustration of the problem 0629

4.1.3 Proving $E_c = \neg E_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 37 terms.

Time Complexity: Time spent by the prover is 4.716 seconds.

NDG conditions Points B and H are not identical

Points B , C and H are not collinear

Points E_c , B and C are not collinear

Points E_c , B and C are not collinear

Line through points H_b and C is not parallel with line through points B and H_c

Points H_b , B and C are not collinear

Points A and B are not identical

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_b H_c B} \neq S_{CH_c B}$ i.e., lines $H_b C$ and $H_c B$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a} BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $C=C$

NDG conditions are:

$S_{H_b H_c B} \neq S_{CH_c B}$ i.e., lines $H_b C$ and $H_c B$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a} BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.3 Proving $E_c = \neg E_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.3 Proving $E_c = \neg E_c$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.3 Proving $E_c = \neg E_c$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 97 terms.

Time Complexity: Time spent by the prover is 0.480 seconds. There are no ndg conditions.

Problem 630

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 630: Given a point B , a point C and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point C , construct a point M_a (rule W01); ;
2. Using the point B and the point C , construct a line a (rule W02); % DET: points B and C are not the same;
3. Using the point N and the point M_a , construct a line $m(H_bH_c)$ (rule W02); % DET: points N and M_a are not the same;
4. Using the point B and the point M_a , construct a circle $k(M_a, B)$ (rule W06); % NDG: points B and M_a are not the same;
5. Using the point M_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_a and N are not the same;
6. Using the circle $k(N, M_a)$, the line a , the point N and the point M_a , construct a point H_a (rule W05); % NDG: line a and circle $k(N, M_a)$ intersect % DET: points M_a and H_a must be different;
7. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point M_a , construct a point E_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_a and E_a must be different;
8. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;
9. Using the circle $k(M_a, B)$ and the circle $k(N, M_a)$, construct a point H_b and a point H_c (rule W07); % NDG: circles $k(M_a, B)$ and $k(N, M_a)$ intersect % DET: circles $k(M_a, B)$ and $k(N, M_a)$ are not the same;

10. Using the point B and the point H_b , construct a line h_b (rule W02); % DET: points B and H_b are not the same;
11. Using the line h_a and the line h_b , construct a point H (rule W03); % NDG: lines h_a and h_b are not parallel % DET: lines h_a and h_b are not the same;
12. Using the point N and the point H , construct a point G (rule W01); ;
13. Using the point C and the point G , construct a point M_c (rule W01); ;
14. Using the point B and the point M_c , construct a point A (rule W01); .

Non-degenerate conditions: lines h_a and h_b are not parallel; circles $k(M_a, B)$ and $k(N, M_a)$ intersect; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; line a and circle $k(N, M_a)$ intersect; points M_a and N are not the same; points B and M_a are not the same.

Determination conditions: lines h_a and h_b are not the same; points B and H_b are not the same; circles $k(M_a, B)$ and $k(N, M_a)$ are not the same; points E_a and H_a are not the same; points M_a and E_a must be different; points M_a and H_a must be different; points N and M_a are not the same; points B and C are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D20,D21,D28,D3,D32,D5,D6,D8,D9,GD01,GD02,GL01,GL03,GL04,GL09,L16,L19,L20,L21,L22]

Solving time: 5.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point C 110 40
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_l C
cmark_r N
color 0 0 0
fontsize 8
```

```
% Constructing a point M_{a} such that BM_{a}/BC=0.5
towards M_{a} B C 0.5
cmark_r M_{a}
color 200 200 200
drawsegment B C
color 0 0 0
```

```
% DET: points B and C are not the same
% Constructing a line a which passes through point B and point C
line a B C
```

```

color 200 200 200
drawline a
color 0 0 0

```

```

% DET: points  $N$  and  $M_{\{a\}}$  are not the same
% Constructing a line  $m(H_{\{b\}}H_{\{c\}})$  which passes through point  $N$  and point  $M_{\{a\}}$ 
line m( $H_{\{b\}}H_{\{c\}}$ ) N  $M_{\{a\}}$ 

```

```

color 200 200 200
drawline m( $H_{\{b\}}H_{\{c\}}$ )
color 0 0 0

```

```

% NDG: points  $B$  and  $M_{\{a\}}$  are not the same
% Constructing a circle  $k(M_{\{a\}}, B)$  whose center is at point  $M_{\{a\}}$  and which passes through point  $B$ 
circle k( $M_{\{a\}}, B$ )  $M_{\{a\}}$  B

```

```

color 200 200 200
drawcircle k( $M_{\{a\}}, B$ )
color 0 0 0

```

```

% NDG: points  $M_{\{a\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $M_{\{a\}}$ 
circle k( $N, M_{\{a\}}$ ) N  $M_{\{a\}}$ 

```

```

color 200 200 200
drawcircle k( $N, M_{\{a\}}$ )
color 0 0 0

```

```

% NDG: line  $a$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{a\}}$  and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G80192\}}$  which is a foot of the point  $N$  on the line  $a$ 
foot  $P_{\{\backslash\_G80192\}}$  N a
cmark_r  $P_{\{\backslash\_G80192\}}$ 
color 200 200 200
drawline N  $P_{\{\backslash\_G80192\}}$ 
color 0 0 0

```

```

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $M_{\{a\}}$  in the symmetry to point/line  $P_{\{\backslash\_G80192\}}$ 
sim  $H_{\{a\}}$   $P_{\{\backslash\_G80192\}}$   $M_{\{a\}}$ 
cmark_r  $H_{\{a\}}$ 

```

```

% NDG: line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{a\}}$  and  $E_{\{a\}}$  must be different
% Constructing a point  $E_{\{a\}}$  which is an image of the point  $M_{\{a\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{a\}}$  N  $M_{\{a\}}$ 
cmark_r  $E_{\{a\}}$ 

```

```

% DET: points  $E_{\{a\}}$  and  $H_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $E_{\{a\}}$  and point  $H_{\{a\}}$ 
line h_{a} E_{a} H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: circles  $k(M_{\{a\}}, B)$  and  $k(N, M_{\{a\}})$  intersect% DET: circles  $k(M_{\{a\}}, B)$  and  $k(N, M_{\{a\}})$  are not
the same
% Constructing points  $H_{\{b\}}$  and  $H_{\{c\}}$  which are in intersection of  $k(M_{\{a\}}, B)$  and  $k(N, M_{\{a\}})$ 
intersec2 H_{b} H_{c} k(M_{a}, B) k(N, M_{a})
cmark_l H_{b}
cmark_rt H_{c}

% DET: points  $B$  and  $H_{\{b\}}$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $B$  and point  $H_{\{b\}}$ 
line h_{b} B H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines  $h_{\{a\}}$  and  $h_{\{b\}}$  are not parallel% DET: lines  $h_{\{a\}}$  and  $h_{\{b\}}$  are not the same
% Constructing a point  $H$  which belongs to line  $h_{\{a\}}$  and line  $h_{\{b\}}$ 
intersec H h_{a} h_{b}
cmark_rt H

% Constructing a line  $L_{\{\_G80694\}}$  which passes through point  $N$  and point  $H$ 
line L_{\_G80694} N H

color 200 200 200
drawline L_{\_G80694}
color 0 0 0

% Constructing a point  $P_{\{\_G80795\}}$  with coordinates  $(0,0)$ 
point P_{\_G80795} 0 0
cmark_r P_{\_G80795}

% Constructing a point  $P_{\{\_G80719\}}$  such that  $NP_{\{\_G80719\}}/NP_{\{\_G80795\}}=-1$ 
towards P_{\_G80719} N P_{\_G80795} -1
cmark_r P_{\_G80719}
color 200 200 200
drawsegment P_{\_G80795} P_{\_G80719}
color 0 0 0

```

```

% Constructing a point  $P_{\{G80764\}}$  such that  $NP_{\{G80764\}}/NP_{\{G80795\}}=3$ 
towards P_{\_G80764} N P_{\_G80795} 3
cmark_r P_{\_G80764}
color 200 200 200
drawsegment N P_{\_G80764}
color 0 0 0

% Constructing a line  $L_{\{G80725\}}$  which passes through point H and point  $P_{\{G80764\}}$ 
line L_{\_G80725} H P_{\_G80764}

color 200 200 200
drawline L_{\_G80725}
color 0 0 0

% Constructing a line  $L_{\{G80688\}}$  which contains the point  $P_{\{G80719\}}$  and is parallel to the
line  $L_{\{G80725\}}$ 
parallel L_{\_G80688} P_{\_G80719} L_{\_G80725}

color 200 200 200
drawline L_{\_G80688}
color 0 0 0

% Constructing a point G which belongs to line  $L_{\{G80688\}}$  and line  $L_{\{G80694\}}$ 
intersec G L_{\_G80688} L_{\_G80694}
cmark_t G

% Constructing a point  $M_{\{c\}}$  such that  $CM_{\{c\}}/CG=1.5$ 
towards M_{\_c} C G 1.5
cmark_lt M_{\_c}
color 200 200 200
drawsegment C M_{\_c}
color 0 0 0

% Constructing a point A such that  $BA/BM_{\{c\}}=2$ 
towards A B M_{\_c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{a\}}$  and  $h_{\{b\}}$  are not parallel; circles  $k(M_{\{a\}}, B)$  and  $k(N, M_{\{a\}})$  intersect; line  $m(H_{\{b\}}, H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect; line a and circle  $k(N, M_{\{a\}})$  intersect; points  $M_{\{a\}}$  and N are not the same; points B and  $M_{\{a\}}$  are not the same

```

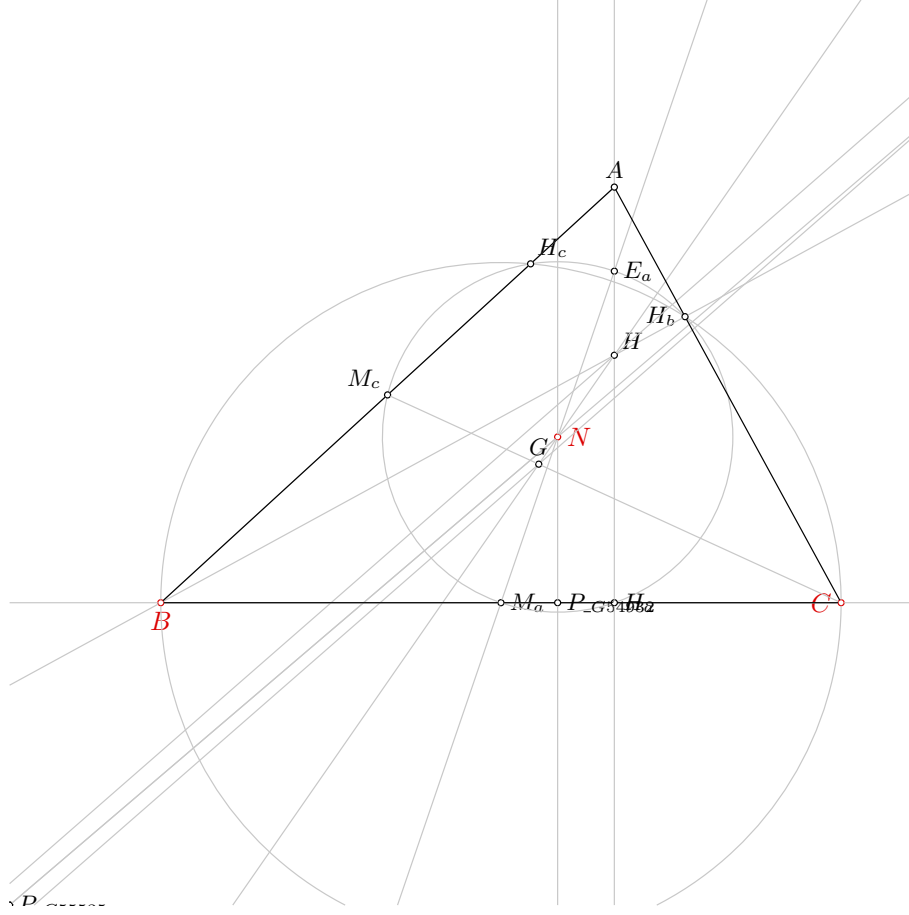


Figure 1: Illustration of the problem 0630

% Determination conditions: lines $h_{\{a\}}$ and $h_{\{b\}}$ are not the same; points B and $H_{\{b\}}$ are not the same; circles $k(M_{\{a\}}, B)$ and $k(N, M_{\{a\}})$ are not the same; points $E_{\{a\}}$ and $H_{\{a\}}$ are not the same; points $M_{\{a\}}$ and $E_{\{a\}}$ must be different; points $M_{\{a\}}$ and $H_{\{a\}}$ must be different; points N and $M_{\{a\}}$ are not the same; points B and C are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{E_aBH_b} \neq S_{H_aBH_b}$ i.e., lines E_aH_a and BH_b are not parallel (construction based assumption)

$S_{P_{-G57464}NH} \neq S_{P_{L_{-G57433}}^0NH}$ i.e., lines $P_{-G57464}P_{L_{-G57433}}^0$ and NH are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points M_a, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points M_b, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^4} \neq S_{F_{-h_a}^3BF_{-h_b}^4}$ i.e., lines $AF_{-h_a}^3$ and $BF_{-h_b}^4$ are not parallel (construction based assumption)

$S_{M_a-M_bF_{-m_b}^2} \neq S_{F_{-m_a}^1-M_bF_{-m_b}^2}$ i.e., lines $M_aF_{-m_a}^1$ and $M_bF_{-m_b}^2$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $C=C$

NDG conditions are:

$S_{E_aBH_b} \neq S_{H_aBH_b}$ i.e., lines E_aH_a and BH_b are not parallel (construction based assumption)

$S_{P_{-G61801}NH} \neq S_{P_{L_{-G61770}}^0NH}$ i.e., lines $P_{-G61801}P_{L_{-G61770}}^0$ and NH are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points M_a, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points M_b, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^4} \neq S_{F_{-h_a}^3BF_{-h_b}^4}$ i.e., lines $AF_{-h_a}^3$ and $BF_{-h_b}^4$ are not parallel (construction based assumption)

$S_{M_a-M_bF_{-m_b}^2} \neq S_{F_{-m_a}^1-M_bF_{-m_b}^2}$ i.e., lines $M_aF_{-m_a}^1$ and $M_bF_{-m_b}^2$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.3 Proving $N=_N$

Proving failed

Problem 631

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 631: Given a point B , a point E_a and a point E_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point E_b , construct a point H (rule W01); ;
2. Using the point E_a and the point H , construct a point A (rule W01); ;
3. Using the point B and the point A , construct a line c (rule W02); % DET: points B and A are not the same;
4. Using the point E_a and the point H , construct a line h_a (rule W02); % DET: points E_a and H are not the same;
5. Using the point B and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points B and E_b are not the same;
6. Using the circle $k(E_b, B)$, the line c , the point E_b and the point B , construct a point H_c (rule W05); % NDG: line c and circle $k(E_b, B)$ intersect % DET: points B and H_c must be different;
7. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
8. Using the circle $k(E_b, B)$, the line h_a , the point E_b and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_b, B)$ intersect % DET: points H and H_a must be different;
9. Using the point H_a and the point B , construct a line a (rule W02); % DET: points H_a and B are not the same;
10. Using the line h_c and the line a , construct a point C (rule W03); % NDG: lines h_c and a are not parallel % DET: lines h_c and a are not the same.

Non-degenerate conditions: lines h_c and a are not parallel; line h_a and circle $k(E_b, B)$ intersect; line c and circle $k(E_b, B)$ intersect; points B and E_b are not the same.

Determination conditions: lines h_c and a are not the same; points H_a and B are not the same; points H and H_a must be different; points H_c and H are not the same; points B and H_c must be different; points E_a and H are not the same; points B and A are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D28,D29,D3,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L49,L50,L51]

Solving time: 11.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point E_{a} 80 83.86
point E_{b} 50 56.36
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_r E_{a}
cmark_r E_{b}
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that BH/BE_{b}=2
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0
```

```
% Constructing a point A such that E_{a}A/E_{a}H=-1
towards A E_{a} H -1
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0
```

```
% DET: points B and A are not the same
% Constructing a line c which passes through point B and point A
line c B A
```

```
color 200 200 200
drawline c
color 0 0 0
```

```

% DET: points E_{a} and H are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H
line h_{a} E_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points B and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point B
circle k(E_{b},B) E_{b} B

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line c and circle k(E_{b},B) intersect% DET: points B and H_{c} must be different
% Constructing a point P_{\_G108061} which is a foot of the point E_{b} on the line c
foot P_{\_G108061} E_{b} c
cmark_r P_{\_G108061}
color 200 200 200
drawline E_{b} P_{\_G108061}
color 0 0 0

% Constructing a point H_{c} which is an image of the point B in the symmetry to point/line P_{\_G
108061}
sim H_{c} P_{\_G108061} B
cmark_rt H_{c}

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: line h_{a} and circle k(E_{b},B) intersect% DET: points H and H_{a} must be different
% Constructing a point P_{\_G108299} which is a foot of the point E_{b} on the line h_{a}
foot P_{\_G108299} E_{b} h_{a}
cmark_r P_{\_G108299}
color 200 200 200
drawline E_{b} P_{\_G108299}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H in the symmetry to point/line P_{\_G
108299}
sim H_{a} P_{\_G108299} H
cmark_r H_{a}

```

```

% DET: points  $H_{\{a\}}$  and  $B$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $B$ 
line a  $H_{\{a\}}$  B

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $h_{\{c\}}$  and  $a$  are not parallel% DET: lines  $h_{\{c\}}$  and  $a$  are not the same
% Constructing a point  $C$  which belongs to line  $h_{\{c\}}$  and line  $a$ 
intersec C  $h_{\{c\}}$  a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{c\}}$  and  $a$  are not parallel; line  $h_{\{a\}}$  and circle  $k(E_{\{b\}}, B)$ 
% intersect; line  $c$  and circle  $k(E_{\{b\}}, B)$  intersect; points  $B$  and  $E_{\{b\}}$  are not the same
% Determination conditions: lines  $h_{\{c\}}$  and  $a$  are not the same; points  $H_{\{a\}}$  and  $B$  are not the same
% ; points  $H$  and  $H_{\{a\}}$  must be different; points  $H_{\{c\}}$  and  $H$  are not the same; points  $B$  and  $H_{\{c\}}$ 
% must be different; points  $E_{\{a\}}$  and  $H$  are not the same; points  $B$  and  $A$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.032 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a = \neg E_a$

Proving failed

4.1.3 Proving $E_b = \neg E_b$

Proving failed

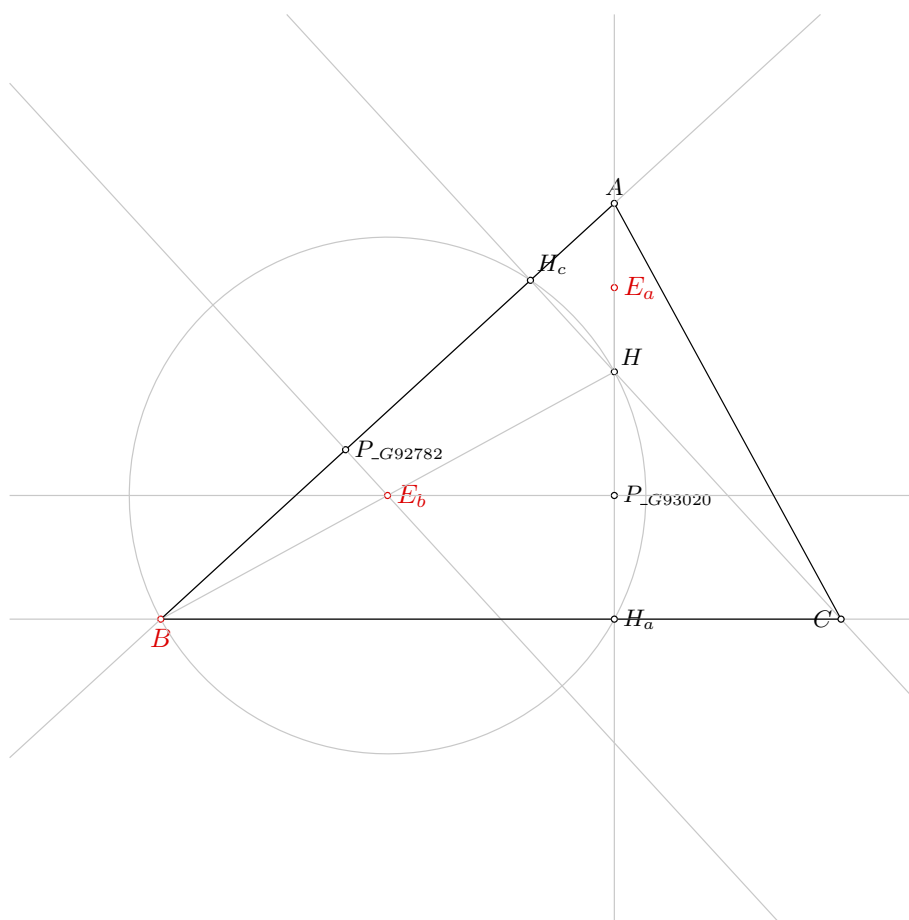


Figure 1: Illustration of the problem 0631

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_cH_aB} \neq S_{HH_aB}$ i.e., lines H_cH and H_aB are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a=\neg E_a$

Proving failed

4.2.3 Proving $E_b=\neg E_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_a=\neg E_a$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 2174 terms.

Time Complexity: Time spent by the prover is 25.070 seconds. There are no ndg conditions.

4.3.3 Proving $E_b=\neg E_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_a=\neg E_a$

Proving failed

4.4.3 Proving $E_b=\neg E_b$

Proving failed

Problem 632

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 632: Given a point B , a point E_a and a point E_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 633

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 633: Given a point B , a point E_a and a point G , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 634

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 634: Given a point B , a point E_a and a point H , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point H , construct a point E_b (rule W01); ;
2. Using the point E_a and the point H , construct a point A (rule W01); ;
3. Using the point B and the point A , construct a line c (rule W02); % DET: points B and A are not the same;
4. Using the point E_a and the point H , construct a line h_a (rule W02); % DET: points E_a and H are not the same;
5. Using the point B and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points B and E_b are not the same;
6. Using the circle $k(E_b, B)$, the line c , the point E_b and the point B , construct a point H_c (rule W05); % NDG: line c and circle $k(E_b, B)$ intersect % DET: points B and H_c must be different;
7. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
8. Using the circle $k(E_b, B)$, the line h_a , the point E_b and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_b, B)$ intersect % DET: points H and H_a must be different;
9. Using the point H_a and the point B , construct a line a (rule W02); % DET: points H_a and B are not the same;
10. Using the line h_c and the line a , construct a point C (rule W03); % NDG: lines h_c and a are not parallel % DET: lines h_c and a are not the same.

Non-degenerate conditions: lines h_c and a are not parallel; line h_a and circle $k(E_b, B)$ intersect; line c and circle $k(E_b, B)$ intersect; points B and E_b are not the same.

Determination conditions: lines h_c and a are not the same; points H_a and B are not the same; points H and H_a must be different; points H_c and H are not the same; points B and H_c must be different; points E_a and H are not the same; points B and A are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D28,D29,D3,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L49,L50,L51]

Solving time: 10.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point E_{a} 80 83.86
point H 80 72.73

color 220 0 0
fontsize 9

cmark_b B
cmark_r E_{a}
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a point E_{b} such that BE_{b}/BH=0.5
towards E_{b} B H 0.5
cmark_r E_{b}
color 200 200 200
drawsegment B H
color 0 0 0

% Constructing a point A such that E_{a}A/E_{a}H=-1
towards A E_{a} H -1
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

% DET: points B and A are not the same
% Constructing a line c which passes through point B and point A
line c B A

color 200 200 200
drawline c
color 0 0 0
```

```

% DET: points E_{a} and H are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H
line h_{a} E_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points B and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point B
circle k(E_{b},B) E_{b} B

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line c and circle k(E_{b},B) intersect% DET: points B and H_{c} must be different
% Constructing a point P_{\_G139107} which is a foot of the point E_{b} on the line c
foot P_{\_G139107} E_{b} c
cmark_r P_{\_G139107}
color 200 200 200
drawline E_{b} P_{\_G139107}
color 0 0 0

% Constructing a point H_{c} which is an image of the point B in the symmetry to point/line P_{\_G
139107}
sim H_{c} P_{\_G139107} B
cmark_rt H_{c}

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: line h_{a} and circle k(E_{b},B) intersect% DET: points H and H_{a} must be different
% Constructing a point P_{\_G139345} which is a foot of the point E_{b} on the line h_{a}
foot P_{\_G139345} E_{b} h_{a}
cmark_r P_{\_G139345}
color 200 200 200
drawline E_{b} P_{\_G139345}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H in the symmetry to point/line P_{\_G
139345}
sim H_{a} P_{\_G139345} H
cmark_r H_{a}

```

```

% DET: points  $H_{\{a\}}$  and  $B$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $B$ 
line a  $H_{\{a\}}$  B

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $h_{\{c\}}$  and  $a$  are not parallel% DET: lines  $h_{\{c\}}$  and  $a$  are not the same
% Constructing a point  $C$  which belongs to line  $h_{\{c\}}$  and line  $a$ 
intersec C  $h_{\{c\}}$  a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{c\}}$  and  $a$  are not parallel; line  $h_{\{a\}}$  and circle  $k(E_{\{b\}}, B)$ 
% intersect; line  $c$  and circle  $k(E_{\{b\}}, B)$  intersect; points  $B$  and  $E_{\{b\}}$  are not the same
% Determination conditions: lines  $h_{\{c\}}$  and  $a$  are not the same; points  $H_{\{a\}}$  and  $B$  are not the same
% ; points  $H$  and  $H_{\{a\}}$  must be different; points  $H_{\{c\}}$  and  $H$  are not the same; points  $B$  and  $H_{\{c\}}$ 
% must be different; points  $E_{\{a\}}$  and  $H$  are not the same; points  $B$  and  $A$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a=_E_a$

Proving failed

4.1.3 Proving $H=_H$

Proving failed

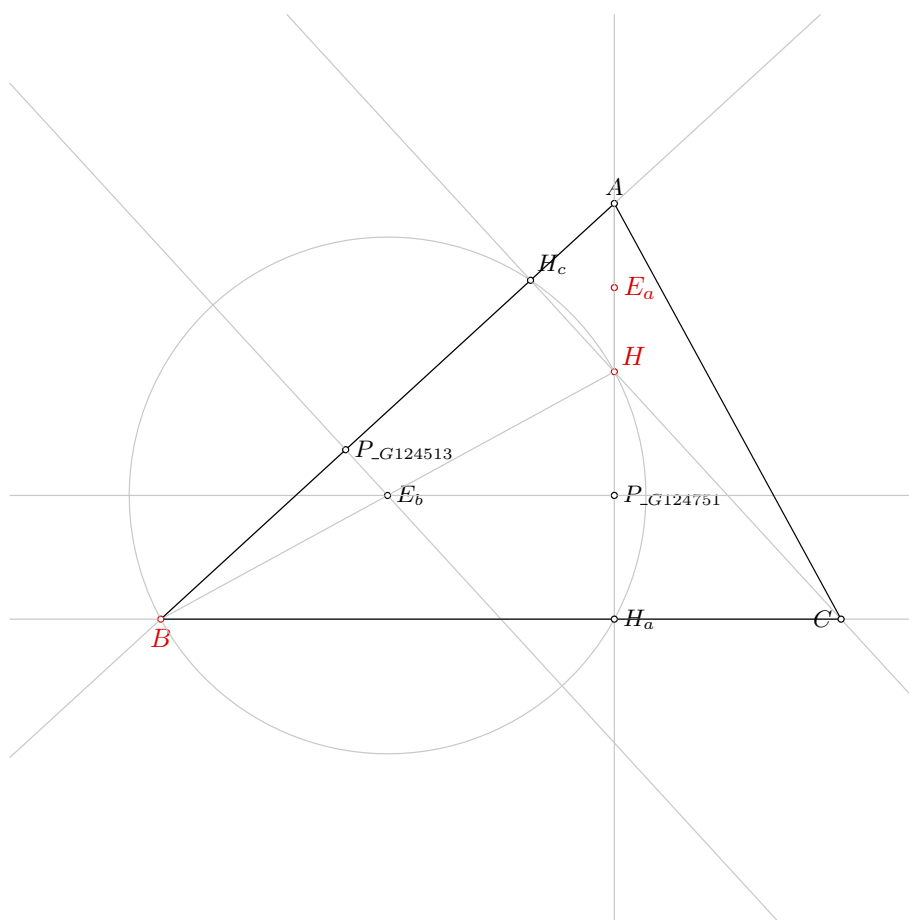


Figure 1: Illustration of the problem 0634

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_c H_a B} \neq S_{H H_a B}$ i.e., lines $H_c H$ and $H_a B$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a = \neg E_a$

Proving failed

4.2.3 Proving $H = \neg H$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_a = \neg E_a$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 2174 terms.

Time Complexity: Time spent by the prover is 1.770 seconds. There are no ndg conditions.

4.3.3 Proving $H = \neg H$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 315 terms.

Time Complexity: Time spent by the prover is 0.270 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_a = \neg E_a$

Proving failed

4.4.3 Proving $H = \neg H$

Proving failed

Problem 635

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 635: Given a point E_a , a point H_a and a point B , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;
2. Using the point H_a and the line h_a , construct a line a (rule W10a); ;
3. Choose freely a point B on the line a (rule WOnline2);
4. Choose freely a point A on the line h_a (rule WOnline1) ;
5. Using the point A and the point E_a , construct a point H (rule W01); ;
6. Using the point A and the point B , construct a line c (rule W02); % DET: points A and B are not the same;
7. Using the point A and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points A and E_a are not the same;
8. Using the circle $k(E_a, A)$, the line c , the point E_a and the point A , construct a point H_c (rule W05); % NDG: line c and circle $k(E_a, A)$ intersect % DET: points A and H_c must be different;
9. Using the point H and the point H_c , construct a line h_c (rule W02); % DET: points H and H_c are not the same;
10. Using the line a and the line h_c , construct a point C (rule W03); % NDG: lines a and h_c are not parallel % DET: lines a and h_c are not the same.

Non-degenerate conditions: lines a and h_c are not parallel; line c and circle $k(E_a, A)$ intersect; points A and E_a are not the same.

Determination conditions: lines a and h_c are not the same; points H and H_c are not the same; points A and H_c must be different; points A and B are not the same; points E_a and H_a are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D10,D28,D3,D5,D7,D8,GD01,GD02,GL03,GL09,L3,L48]

Solving time: 62.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point H_{a} 80 40
```

```
point B 20 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_r H_{a}
```

```
cmark_b B
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{a} and H_{a} are not the same
```

```
% Constructing a line h_{a} which passes through point E_{a} and point H_{a}
```

```
line h_{a} E_{a} H_{a}
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```
% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
```

```
perp a H_{a} h_{a}
```

```
color 200 200 200
```

```
drawline a
```

```
color 0 0 0
```

```
% Generating random value V[_G161621]
```

```
random V[_G161621]
```

```
% Calculating value V[_G161642] using formula V[_G161621]*20
```

```
expression V[_G161642] { V[_G161621]*20 }
```

```

% Constructing a point B which is a point for which holds  $H_{\{a\}}B = V[_G161642]$  and angle  $E_{\{a\}}H_{\{a\}}$ 
    B = 90
turtle B E_{a} H_{a} 90 V[_G161642]
cmark_b B

% Choosing randomly a point A on the line  $E_{\{a\}}H_{\{a\}}$ 
online A E_{a} H_{a}
cmark_t A
color 200 200 200
drawline E_{a} H_{a}
color 0 0 0

% Constructing a point H such that  $AH/AE_{\{a\}}=2$ 
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% DET: points A and B are not the same
% Constructing a line c which passes through point A and point B
line c A B

color 200 200 200
drawline c
color 0 0 0

% NDG: points A and  $E_{\{a\}}$  are not the same
% Constructing a circle  $k(E_{\{a\}},A)$  whose center is at point  $E_{\{a\}}$  and which passes through point A
circle k(E_{a},A) E_{a} A

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: line c and circle  $k(E_{\{a\}},A)$  intersect% DET: points A and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G162162\}}$  which is a foot of the point  $E_{\{a\}}$  on the line c
foot P_{\_G162162} E_{a} c
cmark_r P_{\_G162162}
color 200 200 200
drawline E_{a} P_{\_G162162}
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point A in the symmetry to point/line  $P_{\{\backslash\_G162162\}}$ 
sim H_{c} P_{\_G162162} A

```

```

cmark_rt H_{c}

% DET: points H and H_{c} are not the same
% Constructing a line h_{c} which passes through point H and point H_{c}
line h_{c} H H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines a and h_{c} are not parallel% DET: lines a and h_{c} are not the same
% Constructing a point C which belongs to line a and line h_{c}
intersec C a h_{c}
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and h_{c} are not parallel; line c and circle k(E_{a},A)
% intersect; points A and E_{a} are not the same
% Determination conditions: lines a and h_{c} are not the same; points H and H_{c} are not the same
% ; points A and H_{c} must be different; points A and B are not the same; points E_{a} and H_{a}
% are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 55 terms.

Time Complexity: Time spent by the prover is 0.935 seconds.

NDG conditions Points A and B are not identical

Points A and B are not identical

Line through points H_c and H_a is not perpendicular to line through points H_a and H

Line through points A and E_a is not parallel with line through points B and C

4.1.2 Proving $H_a = \neg H_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

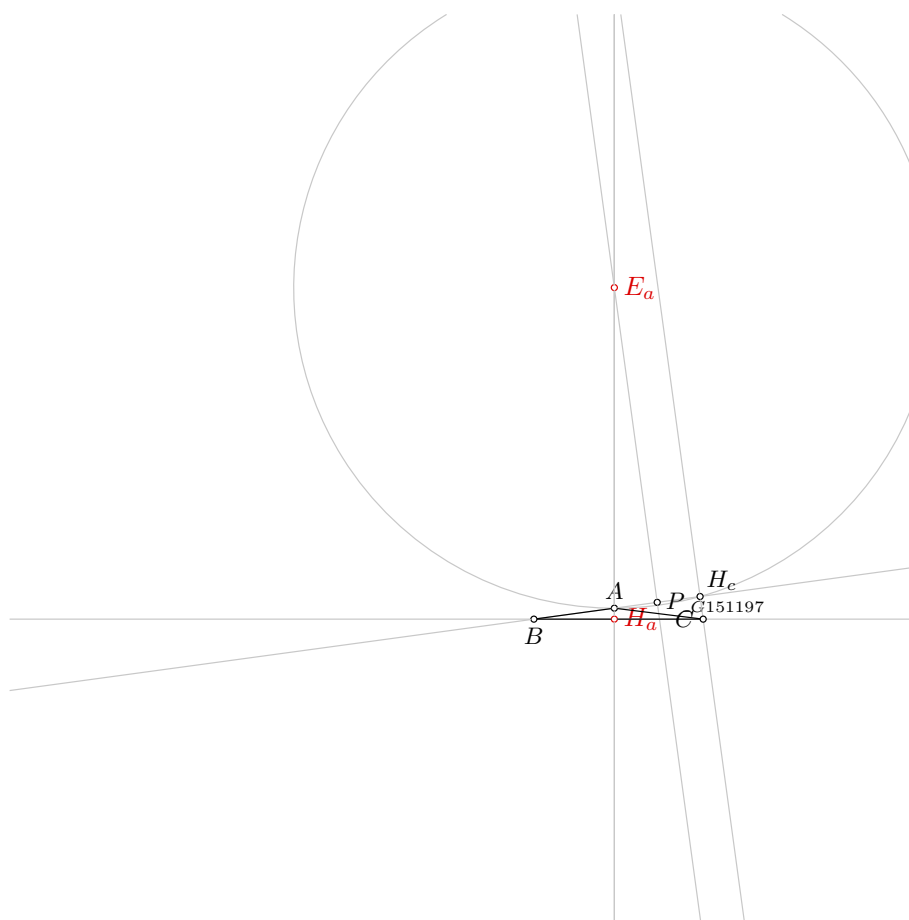


Figure 1: Illustration of the problem 0635

Time Complexity: Time spent by the prover is 0.169 seconds.

NDG conditions Points A and B are not identical

Points A and B are not identical

Line through points H_c and H_a is not perpendicular to line through points H_a and H

Line through points A and E_a is not parallel with line through points B and C

4.1.3 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.007 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $H_a = \neg H_a$

Proving failed

4.2.3 Proving $B=B$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 9 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $H_a = \neg H_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 16 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.3 Proving $B=B$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 20 terms.

Time Complexity: Time spent by the prover is 0.050 seconds. There are no ndg conditions.

4.4.2 Proving $H_a = \neg H_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 20 terms.

Time Complexity: Time spent by the prover is 0.060 seconds. There are no ndg conditions.

4.4.3 Proving $B = B$

Proving failed

Problem 636

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 636: Given a point B , a point E_a and a point H_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point H_b , construct a line h_b (rule W02); % DET: points B and H_b are not the same;
2. Using the point H_b and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_b and E_a are not the same;
3. Using the circle $k(E_a, A)$, the line h_b , the point E_a and the point H_b , construct a point H (rule W05); % NDG: line h_b and circle $k(E_a, A)$ intersect % DET: points H_b and H must be different;
4. Using the point E_a and the point H , construct a point A (rule W01); ;
5. Using the point B and the point A , construct a line c (rule W02); % DET: points B and A are not the same;
6. Using the point H_b and the point A , construct a line b (rule W02); % DET: points H_b and A are not the same;
7. Using the circle $k(E_a, A)$, the line c , the point E_a and the point A , construct a point H_c (rule W05); % NDG: line c and circle $k(E_a, A)$ intersect % DET: points A and H_c must be different;
8. Using the point H and the point H_c , construct a line h_c (rule W02); % DET: points H and H_c are not the same;
9. Using the line b and the line h_c , construct a point C (rule W03); % NDG: lines b and h_c are not parallel % DET: lines b and h_c are not the same.

Non-degenerate conditions: lines b and h_c are not parallel; line c and circle $k(E_a, A)$ intersect; line h_b and circle $k(E_a, A)$ intersect; points H_b and E_a are not the same.

Determination conditions: lines b and h_c are not the same; points H and H_c are not the same; points A and H_c must be different; points H_b and A are not the same; points B and A are not the same; points H_b and H must be different; points B and H_b are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D28,D3,D6,D7,D9,GD01,GD02,GL03,GL04,L3,L46,L47,L48]

Solving time: 8.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point E_{a} 80 83.86
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_b B
cmark_r E_{a}
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points B and H_{b} are not the same
% Constructing a line h_{b} which passes through point B and point H_{b}
line h_{b} B H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points H_{b} and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H_{b}
circle k(E_{a},A) E_{a} H_{b}

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: line h_{b} and circle k(E_{a},A) intersect% DET: points H_{b} and H must be different
% Constructing a point P_{\_G184218} which is a foot of the point E_{a} on the line h_{b}
foot P_{\_G184218} E_{a} h_{b}
cmark_r P_{\_G184218}
color 200 200 200
drawline E_{a} P_{\_G184218}
color 0 0 0
```



```

% Constructing a point H which is an image of the point H_{b} in the symmetry to point/line P_{\_G
184218}
sim H P_{\_G184218} H_{b}
cmark_rt H

% Constructing a point A such that E_{a}A/E_{a}H=-1
towards A E_{a} H -1
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

% DET: points B and A are not the same
% Constructing a line c which passes through point B and point A
line c B A

color 200 200 200
drawline c
color 0 0 0

% DET: points H_{b} and A are not the same
% Constructing a line b which passes through point H_{b} and point A
line b H_{b} A

color 200 200 200
drawline b
color 0 0 0

% NDG: line c and circle k(E_{a},A) intersect% DET: points A and H_{c} must be different
% Constructing a point P_{\_G184581} which is a foot of the point E_{a} on the line c
foot P_{\_G184581} E_{a} c
cmark_r P_{\_G184581}
color 200 200 200
drawline E_{a} P_{\_G184581}
color 0 0 0

% Constructing a point H_{c} which is an image of the point A in the symmetry to point/line P_{\_G
184581}
sim H_{c} P_{\_G184581} A
cmark_rt H_{c}

% DET: points H and H_{c} are not the same
% Constructing a line h_{c} which passes through point H and point H_{c}
line h_{c} H H_{c}

color 200 200 200
drawline h_{c}

```

```
color 0 0 0
```

```
% NDG: lines b and h_{c} are not parallel% DET: lines b and h_{c} are not the same
% Constructing a point C which belongs to line b and line h_{c}
intersec C b h_{c}
cmark_1 C
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines b and h_{c} are not parallel; line c and circle k(E_{a},A)
    intersect; line h_{b} and circle k(E_{a},A) intersect; points H_{b} and E_{a} are not the same
% Determination conditions: lines b and h_{c} are not the same; points H and H_{c} are not the same
    ; points A and H_{c} must be different; points H_{b} and A are not the same; points B and A are
    not the same; points H_{b} and H must be different; points B and H_{b} are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.031 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a = E_a$

Proving failed

4.1.3 Proving $H_b = H_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_bHH_c} \neq S_{AHH_c}$ i.e., lines H_bA and HH_c are not parallel (construction based assumption)
 $S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

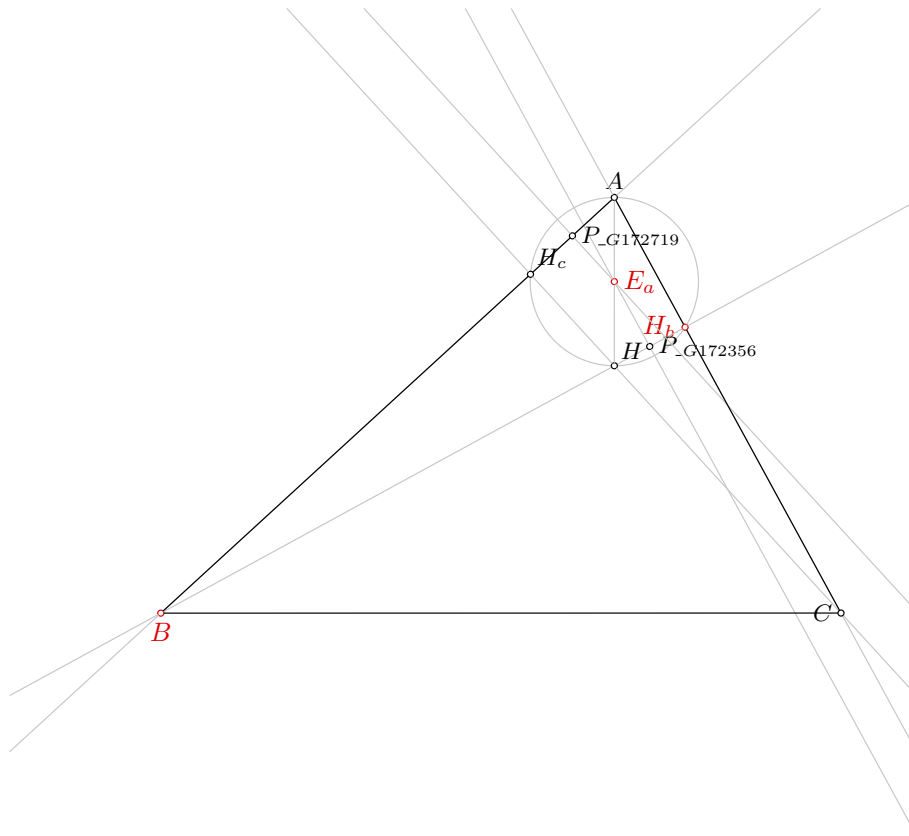


Figure 1: Illustration of the problem 0636

$S_{ABF^1_{\neg h_b}} \neq S_{CBF^1_{\neg h_b}}$ i.e., lines AC and $BF^1_{\neg h_b}$ are not parallel (construction based assumption)
Total number of proof steps: 1
Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a = \neg E_a$

Proving failed

4.2.3 Proving $H_b = \neg H_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B = B$

Proving failed

4.3.2 Proving $E_a = \neg E_a$

Proving failed

4.3.3 Proving $H_b = \neg H_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B = B$

Proving failed

4.4.2 Proving $E_a = \neg E_a$

Proving failed

4.4.3 Proving $H_b = \neg H_b$

Proving failed

Problem 637

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 637: Given a point B , a point E_a and a point H_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point H_c , construct a line c (rule W02); % DET: points B and H_c are not the same;
2. Using the point H_c and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_c and E_a are not the same;
3. Using the circle $k(E_a, A)$, the line c , the point E_a and the point H_c , construct a point A (rule W05); % NDG: line c and circle $k(E_a, A)$ intersect % DET: points H_c and A must be different;
4. Using the point E_a and the point A , construct a point H (rule W01); ;
5. Using the point B and the point H , construct a line h_b (rule W02); % DET: points B and H are not the same;
6. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
7. Using the circle $k(E_a, A)$, the line h_b , the point E_a and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_a, A)$ intersect % DET: points H and H_b must be different;
8. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;
9. Using the line h_c and the line b , construct a point C (rule W03); % NDG: lines h_c and b are not parallel % DET: lines h_c and b are not the same.

Non-degenerate conditions: lines h_c and b are not parallel; line h_b and circle $k(E_a, A)$ intersect; line c and circle $k(E_a, A)$ intersect; points H_c and E_a are not the same.

Determination conditions: lines h_c and b are not the same; points A and H_b are not the same; points H and H_b must be different; points H_c and H are not the same; points B and H are not the same; points H_c and A must be different; points B and H_c are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D28,D3,D6,D7,D9,GD01,GD02,GL03,GL04,L3,L46,L47,L48]

Solving time: 8.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point E_{a} 80 83.86
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_b B
cmark_r E_{a}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points H_{c} and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H_{c}
circle k(E_{a},A) E_{a} H_{c}

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: line c and circle k(E_{a},A) intersect% DET: points H_{c} and A must be different
% Constructing a point P_{\G207349} which is a foot of the point E_{a} on the line c
foot P_{\G207349} E_{a} c
cmark_r P_{\G207349}
color 200 200 200
drawline E_{a} P_{\G207349}
color 0 0 0
```

```

% Constructing a point A which is an image of the point H_{c} in the symmetry to point/line P_{\_G
207349}
sim A P_{\_G207349} H_{c}
cmark_t A

% Constructing a point H such that E_{a}H/E_{a}A=-1
towards H E_{a} A -1
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% DET: points B and H are not the same
% Constructing a line h_{b} which passes through point B and point H
line h_{b} B H

color 200 200 200
drawline h_{b}
color 0 0 0

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: line h_{b} and circle k(E_{a},A) intersect% DET: points H and H_{b} must be different
% Constructing a point P_{\_G207712} which is a foot of the point E_{a} on the line h_{b}
foot P_{\_G207712} E_{a} h_{b}
cmark_r P_{\_G207712}
color 200 200 200
drawline E_{a} P_{\_G207712}
color 0 0 0

% Constructing a point H_{b} which is an image of the point H in the symmetry to point/line P_{\_G
207712}
sim H_{b} P_{\_G207712} H
cmark_l H_{b}

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b

```

```
color 0 0 0
```

```
% NDG: lines  $h_{\{c\}}$  and  $b$  are not parallel% DET: lines  $h_{\{c\}}$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $h_{\{c\}}$  and line  $b$ 
intersec C  $h_{\{c\}}$  b
cmark_1 C
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines  $h_{\{c\}}$  and  $b$  are not parallel; line  $h_{\{b\}}$  and circle  $k(E_{\{a\}}, A)$ 
    intersect; line  $c$  and circle  $k(E_{\{a\}}, A)$  intersect; points  $H_{\{c\}}$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $h_{\{c\}}$  and  $b$  are not the same; points  $A$  and  $H_{\{b\}}$  are not the same
    ; points  $H$  and  $H_{\{b\}}$  must be different; points  $H_{\{c\}}$  and  $H$  are not the same; points  $B$  and  $H$  are
    not the same; points  $H_{\{c\}}$  and  $A$  must be different; points  $B$  and  $H_{\{c\}}$  are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.031 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a = \neg E_a$

Proving failed

4.1.3 Proving $H_c = \neg H_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 1176 terms.

Time Complexity: Time spent by the prover is 7.805 seconds.

NDG conditions Points B , H_c and E_a are not collinear

Points B , H_c and E_a are not collinear

Points B and H are not identical

Points B and H are not identical

Line through points H_b and A is not parallel with line through points H_c and H

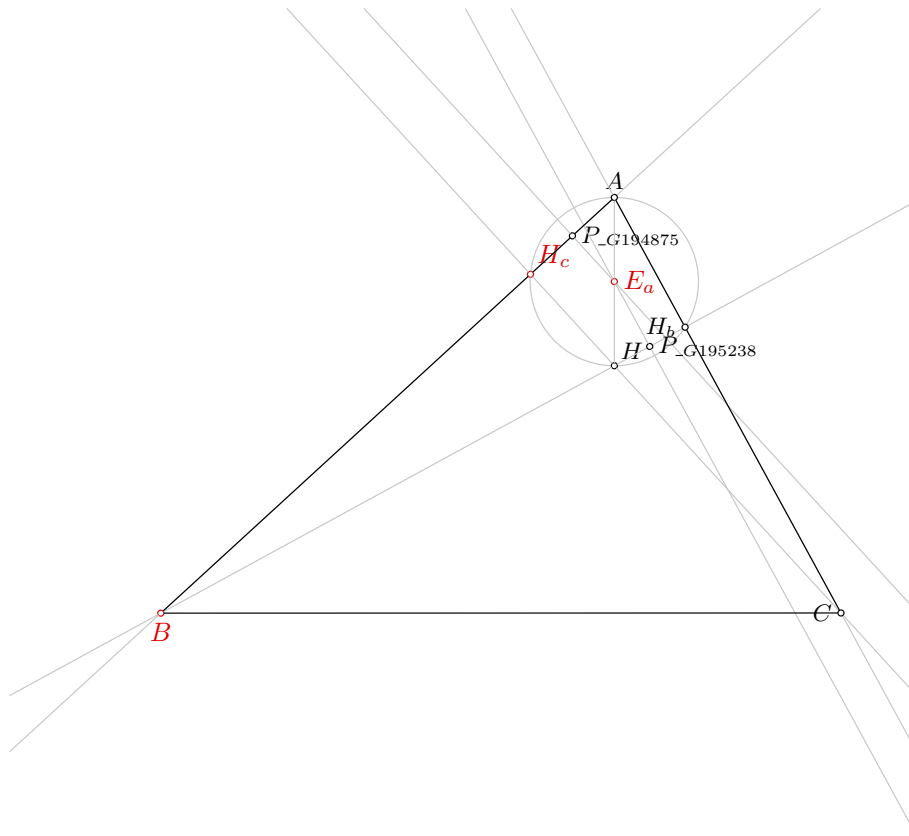


Figure 1: Illustration of the problem 0637

Line through points B and E_a is not parallel with line through points H_c and H

Points A and B are not identical

Points A and B are not identical

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_cAH_b} \neq S_{HAH_b}$ i.e., lines H_cH and AH_b are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a}BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF^2_{-h_c}} \neq S_{BCF^2_{-h_c}}$ i.e., lines AB and $CF^2_{-h_c}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a=-E_a$

Proving failed

4.2.3 Proving $H_c=-H_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Proving failed

4.3.2 Proving $E_a=-E_a$

Proving failed

4.3.3 Proving $H_c=-H_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Proving failed

4.4.2 Proving $E_a=-E_a$

Proving failed

4.4.3 Proving $H_c=-H_c$

Proving failed

Problem 638

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 638: Given a point B , a point E_a and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 639

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 639: Given a point B , a point E_a and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point M_a , construct a point C (rule W01); ;
2. Using the point B and the point M_a , construct a line a (rule W02); % DET: points B and M_a are not the same;
3. Using the point E_a and the point M_a , construct a line $m(H_bH_c)$ (rule W02); % DET: points E_a and M_a are not the same;
4. Using the point E_a and the line a , construct a line h_a (rule W10b); ;
5. Using the line h_a and the line a , construct a point H_a (rule W03); % NDG: lines h_a and a are not parallel % DET: lines h_a and a are not the same;
6. Using the point B and the point H_a , construct a line $m(BH_a)$ (rule W14); % DET: points B and H_a are not the same;
7. Using the point E_a and the point M_a , construct a line $m(E_aM_a)$ (rule W14); % DET: points E_a and M_a are not the same;
8. Using the line $m(E_aM_a)$ and the line $m(H_bH_c)$, construct a point N (rule W03); % NDG: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not parallel % DET: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not the same;
9. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
10. Using the circle $k(N, M_a)$ and the line $m(BH_a)$, construct a point M_c and a point E_b (rule W04); % NDG: line $m(BH_a)$ and circle $k(N, M_a)$ intersect;

11. Using the point M_c and the point B , construct a point A (rule W01); .

Non-degenerate conditions: line $m(BH_a)$ and circle $k(N, M_a)$ intersect; points E_a and N are not the same; lines $m(E_aM_a)$ and $m(H_bH_c)$ are not parallel; lines h_a and a are not parallel.

Determination conditions: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not the same; points E_a and M_a are not the same; points B and H_a are not the same; lines h_a and a are not the same; points E_a and M_a are not the same; points B and M_a are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14]

Lemmas used: [D20,D21,D28,D3,D32,D5,D8,GD01,GD02,GL01,GL03,GL04,GL09,L18,L20,L21,L22,L23,L38,L

Solving time: 2.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point E_{a} 80 83.86
point M_{a} 65 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_r E_{a}
cmark_r M_{a}
color 0 0 0
fontsize 8
```

```
% Constructing a point C such that BC/BM_{a}=2
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0
```

```
% DET: points B and M_{a} are not the same
% Constructing a line a which passes through point B and point M_{a}
line a B M_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% DET: points E_{a} and M_{a} are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point E_{a} and point M_{a}
line m(H_{b}H_{c}) E_{a} M_{a}
```

```
color 200 200 200
drawline m(H_{b}H_{c})
```

```
color 0 0 0
```

```
% Constructing a line  $h_{\{a\}}$  which is perpendicular to line  $a$  and which passes through point  $E_{\{a\}}$ 
perp  $h_{\{a\}}$   $E_{\{a\}}$   $a$ 
```

```
color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0
```

```
% NDG: lines  $h_{\{a\}}$  and  $a$  are not parallel% DET: lines  $h_{\{a\}}$  and  $a$  are not the same
% Constructing a point  $H_{\{a\}}$  which belongs to line  $h_{\{a\}}$  and line  $a$ 
intersec  $H_{\{a\}}$   $h_{\{a\}}$   $a$ 
cmark_r  $H_{\{a\}}$ 
```

```
% DET: points  $B$  and  $H_{\{a\}}$  are not the same
% Constructing bisector  $m(BH_{\{a\}})$  of the segment  $BH_{\{a\}}$ 
med  $m(BH_{\{a\}})$   $B$   $H_{\{a\}}$ 
```

```
color 200 200 200
drawline  $m(BH_{\{a\}})$ 
color 0 0 0
```

```
color 200 200 200
drawsegment  $B$   $H_{\{a\}}$ 
color 0 0 0
```

```
% DET: points  $E_{\{a\}}$  and  $M_{\{a\}}$  are not the same
% Constructing bisector  $m(E_{\{a\}}M_{\{a\}})$  of the segment  $E_{\{a\}}M_{\{a\}}$ 
med  $m(E_{\{a\}}M_{\{a\}})$   $E_{\{a\}}$   $M_{\{a\}}$ 
```

```
color 200 200 200
drawline  $m(E_{\{a\}}M_{\{a\}})$ 
color 0 0 0
```

```
color 200 200 200
drawsegment  $E_{\{a\}}$   $M_{\{a\}}$ 
color 0 0 0
```

```
% NDG: lines  $m(E_{\{a\}}M_{\{a\}})$  and  $m(H_{\{b\}}H_{\{c\}})$  are not parallel% DET: lines  $m(E_{\{a\}}M_{\{a\}})$  and  $m(H_{\{b\}}H_{\{c\}})$ 
 $H_{\{c\}}$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{a\}}M_{\{a\}})$  and line  $m(H_{\{b\}}H_{\{c\}})$ 
intersec  $N$   $m(E_{\{a\}}M_{\{a\}})$   $m(H_{\{b\}}H_{\{c\}})$ 
cmark_r  $N$ 
```

```
% NDG: points  $E_{\{a\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{a\}}$ 
circle  $k(N, M_{\{a\}})$   $N$   $E_{\{a\}}$ 
```

```

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(BH_{a}) and circle k(N,M_{a}) intersect
% Constructing points M_{c} and E_{b} which are in intersection of k(N,M_{a}) and m(BH_{a})
intersec2 M_{c} E_{b} k(N,M_{a}) m(BH_{a})
cmark_lt M_{c}
cmark_r E_{b}

% Constructing a point A such that M_{c}A/M_{c}B=-1
towards A M_{c} B -1
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line m(BH_{a}) and circle k(N,M_{a}) intersect; points E_{a} and N are
% not the same; lines m(E_{a}M_{a}) and m(H_{b}H_{c}) are not parallel; lines h_{a} and a are
% not parallel
% Determination conditions: lines m(E_{a}M_{a}) and m(H_{b}H_{c}) are not the same; points E_{a}
% and M_{a} are not the same; points B and H_{a} are not the same; lines h_{a} and a are not the
% same; points E_{a} and M_{a} are not the same; points B and M_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a = E_a$

Proving failed

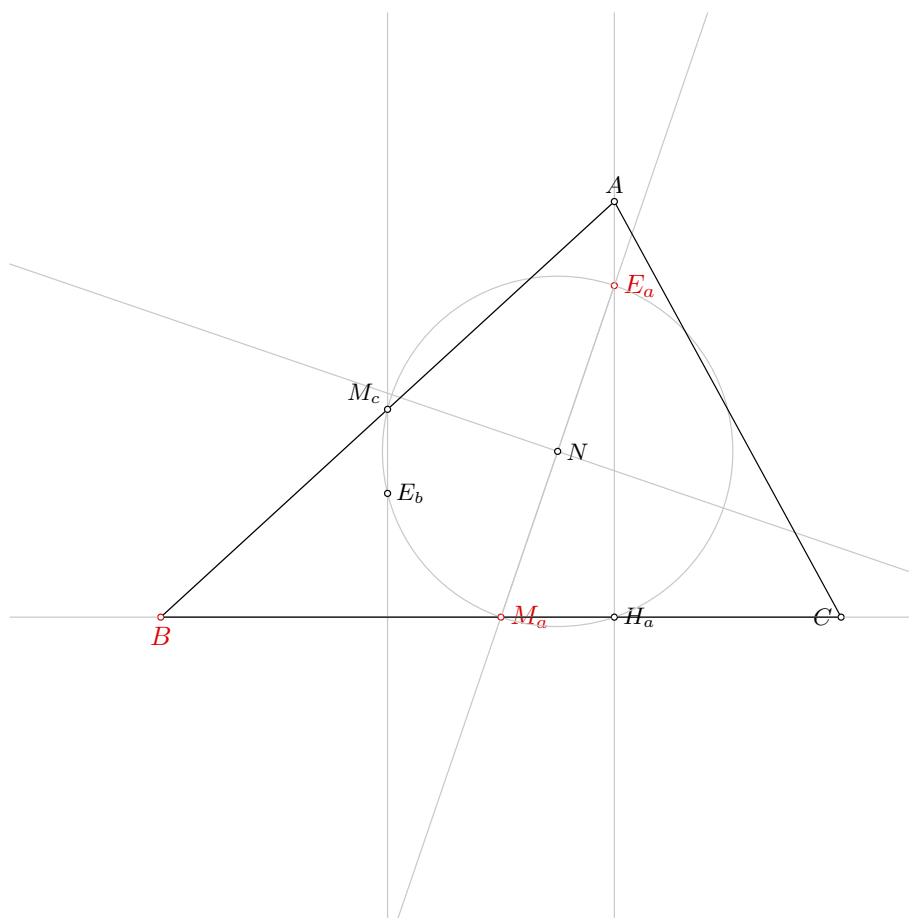


Figure 1: Illustration of the problem 0639

4.1.3 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.032 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $B = B$

NDG conditions are:

$S_{E_a B M_a} \neq 0$ i.e., points E_a , B and M_a are not collinear (foot is not the point itself; construction based assumption)

$S_{E_a B M_a} \neq S_{F_{h_a}^0 B M_a}$ i.e., lines $E_a F_{h_a}^0$ and $B M_a$ are not parallel (construction based assumption)

$S_{M_{m(E_a M_a)}^3 E_a M_a} \neq S_{T_{m(E_a M_a)}^4 E_a M_a}$ i.e., lines $M_{m(E_a M_a)}^3$ and $T_{m(E_a M_a)}^4$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{AB F_{h_b}^6} \neq S_{F_{h_a}^5 B F_{h_b}^6}$ i.e., lines AB and $F_{h_a}^5$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a = \neg E_a$

Proving failed

4.2.3 Proving $M_a = \neg M_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.2 Proving $E_a = \neg E_a$

Proving failed

4.3.3 Proving $M_a = \neg M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 13 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $E_a = -E_a$

Proving failed

4.4.3 Proving $M_a = -M_a$

Proving failed

Problem 640

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 640: Given a point B , a point E_a and a point M_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 641

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 641: Given a point B , a point E_a and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point M_c , construct a point A (rule W01); ;
2. Using the point E_a and the point A , construct a point H (rule W01); ;
3. Using the point B and the point H , construct a line h_b (rule W02); % DET: points B and H are not the same;
4. Using the point E_a and the point A , construct a line h_a (rule W02); % DET: points E_a and A are not the same;
5. Using the point B and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points B and M_c are not the same;
6. Using the circle $k(M_c, A)$, the line h_b , the point M_c and the point B , construct a point H_b (rule W05); % NDG: line h_b and circle $k(M_c, A)$ intersect % DET: points B and H_b must be different;
7. Using the point H_b and the point A , construct a line b (rule W02); % DET: points H_b and A are not the same;
8. Using the circle $k(M_c, A)$, the line h_a , the point M_c and the point A , construct a point H_a (rule W05); % NDG: line h_a and circle $k(M_c, A)$ intersect % DET: points A and H_a must be different;
9. Using the point H_a and the point B , construct a line a (rule W02); % DET: points H_a and B are not the same;
10. Using the line b and the line a , construct a point C (rule W03); % NDG: lines b and a are not parallel % DET: lines b and a are not the same.

Non-degenerate conditions: lines b and a are not parallel; line h_a and circle $k(M_c, A)$ intersect; line h_b and circle $k(M_c, A)$ intersect; points B and M_c are not the same.

Determination conditions: lines b and a are not the same; points H_a and B are not the same; points A and H_a must be different; points H_b and A are not the same; points B and H_b must be different; points E_a and A are not the same; points B and H are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D20,D28,D3,D5,D6,D8,D9,GD01,GD02,GL03,GL04,GL09,L40,L41,L42]

Solving time: 10.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point E_{a} 80 83.86
point M_{c} 50 67.5

color 220 0 0
fontsize 9

cmark_b B
cmark_r E_{a}
cmark_lt M_{c}
color 0 0 0
fontsize 8

% Constructing a point A such that BA/BM_{c}=2
towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

% Constructing a point H such that E_{a}H/E_{a}A=-1
towards H E_{a} A -1
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% DET: points B and H are not the same
% Constructing a line h_{b} which passes through point B and point H
line h_{b} B H

color 200 200 200
drawline h_{b}
color 0 0 0
```

```

% DET: points E_{a} and A are not the same
% Constructing a line h_{a} which passes through point E_{a} and point A
line h_{a} E_{a} A

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points B and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point B
circle k(M_{c},A) M_{c} B

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: line h_{b} and circle k(M_{c},A) intersect% DET: points B and H_{b} must be different
% Constructing a point P_{\_G257178} which is a foot of the point M_{c} on the line h_{b}
foot P_{\_G257178} M_{c} h_{b}
cmark_r P_{\_G257178}
color 200 200 200
drawline M_{c} P_{\_G257178}
color 0 0 0

% Constructing a point H_{b} which is an image of the point B in the symmetry to point/line P_{\_G
257178}
sim H_{b} P_{\_G257178} B
cmark_l H_{b}

% DET: points H_{b} and A are not the same
% Constructing a line b which passes through point H_{b} and point A
line b H_{b} A

color 200 200 200
drawline b
color 0 0 0

% NDG: line h_{a} and circle k(M_{c},A) intersect% DET: points A and H_{a} must be different
% Constructing a point P_{\_G257416} which is a foot of the point M_{c} on the line h_{a}
foot P_{\_G257416} M_{c} h_{a}
cmark_r P_{\_G257416}
color 200 200 200
drawline M_{c} P_{\_G257416}
color 0 0 0

% Constructing a point H_{a} which is an image of the point A in the symmetry to point/line P_{\_G
257416}
sim H_{a} P_{\_G257416} A
cmark_r H_{a}

```

```

% DET: points  $H_{\{a\}}$  and  $B$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $B$ 
line a  $H_{\{a\}}$  B

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $b$  and  $a$  are not parallel% DET: lines  $b$  and  $a$  are not the same
% Constructing a point  $C$  which belongs to line  $b$  and line  $a$ 
intersec C b a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $b$  and  $a$  are not parallel; line  $h_{\{a\}}$  and circle  $k(M_{\{c\}}, A)$ 
% intersect; line  $h_{\{b\}}$  and circle  $k(M_{\{c\}}, A)$  intersect; points  $B$  and  $M_{\{c\}}$  are not the same
% Determination conditions: lines  $b$  and  $a$  are not the same; points  $H_{\{a\}}$  and  $B$  are not the same;
% points  $A$  and  $H_{\{a\}}$  must be different; points  $H_{\{b\}}$  and  $A$  are not the same; points  $B$  and  $H_{\{b\}}$ 
% must be different; points  $E_{\{a\}}$  and  $A$  are not the same; points  $B$  and  $H$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.038 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a = \neg E_a$

Proving failed

4.1.3 Proving $M_c = \neg M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

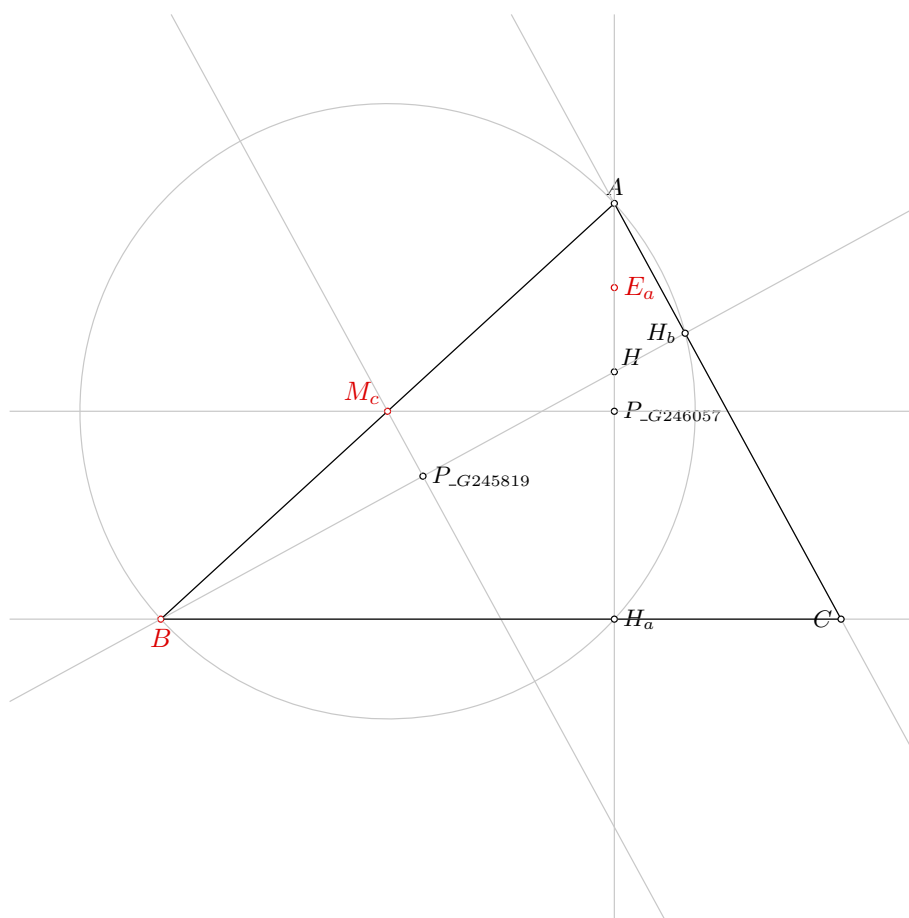


Figure 1: Illustration of the problem 0641

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_bH_aB} \neq S_{AH_aB}$ i.e., lines H_bA and H_aB are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a=_Ea$

Proving failed

4.2.3 Proving $M_c=_M c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_a=_Ea$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 331 terms.

Time Complexity: Time spent by the prover is 1.040 seconds. There are no ndg conditions.

4.3.3 Proving $M_c=_M c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 9 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_a=_Ea$

Proving failed

4.4.3 Proving $M_c = -M_c$

Proving failed

Problem 642

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 642: Given a point B , a point E_a and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point N , construct a line $m(H_bH_c)$ (rule W02); % DET: points E_a and N are not the same;
2. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
3. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point E_a , construct a point M_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points E_a and M_a must be different;
4. Using the point M_a and the point B , construct a point C (rule W01); ;
5. Using the point B and the point M_a , construct a line a (rule W02); % DET: points B and M_a are not the same;
6. Using the circle $k(N, M_a)$, the line a , the point N and the point M_a , construct a point H_a (rule W05); % NDG: line a and circle $k(N, M_a)$ intersect % DET: points M_a and H_a must be different;
7. Using the point H_a and the point E_a , construct a line h_a (rule W02); % DET: points H_a and E_a are not the same;
8. Using the point B and the point M_a , construct a circle $k(M_a, B)$ (rule W06); % NDG: points B and M_a are not the same;
9. Using the circle $k(N, M_a)$ and the circle $k(M_a, B)$, construct a point H_b and a point H_c (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_a, B)$ intersect % DET: circles $k(N, M_a)$ and $k(M_a, B)$ are not the same;

10. Using the point B and the point H_b , construct a line h_b (rule W02); % DET: points B and H_b are not the same;
11. Using the line h_a and the line h_b , construct a point H (rule W03); % NDG: lines h_a and h_b are not parallel % DET: lines h_a and h_b are not the same;
12. Using the point E_a and the point H , construct a point A (rule W01); .

Non-degenerate conditions: lines h_a and h_b are not parallel; circles $k(N, M_a)$ and $k(M_a, B)$ intersect; points B and M_a are not the same; line a and circle $k(N, M_a)$ intersect; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; points E_a and N are not the same.

Determination conditions: lines h_a and h_b are not the same; points B and H_b are not the same; circles $k(N, M_a)$ and $k(M_a, B)$ are not the same; points H_a and E_a are not the same; points M_a and H_a must be different; points B and M_a are not the same; points E_a and M_a must be different; points E_a and N are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D21,D28,D3,D32,D5,D6,D8,D9,GD01,GD02,GL01,GL03,GL04,GL09,L19,L20,L21,L22,L38,L39]

Solving time: 4.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point E_{a} 80 83.86
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_r E_{a}
cmark_r N
color 0 0 0
fontsize 8
```

```
% DET: points E_{a} and N are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point E_{a} and point N
line m(H_{b}H_{c}) E_{a} N
```

```
color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0
```

```
% NDG: points E_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{a}
circle k(N,M_{a}) N E_{a}
```

```
color 200 200 200
drawcircle k(N,M_{a})
```

```

color 0 0 0

% NDG: line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $E_{\{a\}}$  and  $M_{\{a\}}$  must be
different
% Constructing a point  $M_{\{a\}}$  which is an image of the point  $E_{\{a\}}$  in the symmetry to point/line  $N$ 
sim  $M_{\{a\}}$   $N$   $E_{\{a\}}$ 
cmark_r  $M_{\{a\}}$ 

% Constructing a point  $C$  such that  $M_{\{a\}}C/M_{\{a\}}B=-1$ 
towards  $C$   $M_{\{a\}}$   $B$  -1
cmark_l  $C$ 
color 200 200 200
drawsegment  $B$   $C$ 
color 0 0 0

% DET: points  $B$  and  $M_{\{a\}}$  are not the same
% Constructing a line  $a$  which passes through point  $B$  and point  $M_{\{a\}}$ 
line  $a$   $B$   $M_{\{a\}}$ 

color 200 200 200
drawline  $a$ 
color 0 0 0

% NDG: line  $a$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $M_{\{a\}}$  and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G40062\}}$  which is a foot of the point  $N$  on the line  $a$ 
foot  $P_{\{\backslash\_G40062\}}$   $N$   $a$ 
cmark_r  $P_{\{\backslash\_G40062\}}$ 
color 200 200 200
drawline  $N$   $P_{\{\backslash\_G40062\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $M_{\{a\}}$  in the symmetry to point/line  $P_{\{\backslash\_G40062\}}$ 
sim  $H_{\{a\}}$   $P_{\{\backslash\_G40062\}}$   $M_{\{a\}}$ 
cmark_r  $H_{\{a\}}$ 

% DET: points  $H_{\{a\}}$  and  $E_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $H_{\{a\}}$  and point  $E_{\{a\}}$ 
line  $h_{\{a\}}$   $H_{\{a\}}$   $E_{\{a\}}$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: points  $B$  and  $M_{\{a\}}$  are not the same
% Constructing a circle  $k(M_{\{a\}},B)$  whose center is at point  $M_{\{a\}}$  and which passes through point  $B$ 

```

```

circle k(M_{a},B) M_{a} B

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(M_{a},B) intersect% DET: circles k(N,M_{a}) and k(M_{a},B) are not
the same
% Constructing points H_{b} and H_{c} which are in intersection of k(N,M_{a}) and k(M_{a},B)
intersec2 H_{b} H_{c} k(N,M_{a}) k(M_{a},B)
cmark_l H_{b}
cmark_rt H_{c}

% DET: points B and H_{b} are not the same
% Constructing a line h_{b} which passes through point B and point H_{b}
line h_{b} B H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines h_{a} and h_{b} are not parallel% DET: lines h_{a} and h_{b} are not the same
% Constructing a point H which belongs to line h_{a} and line h_{b}
intersec H h_{a} h_{b}
cmark_rt H

% Constructing a point A such that E_{a}A/E_{a}H=-1
towards A E_{a} H -1
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{a} and h_{b} are not parallel; circles k(N,M_{a}) and k(M_{a}
,B) intersect; points B and M_{a} are not the same; line a and circle k(N,M_{a}) intersect;
line m(H_{b}H_{c}) and circle k(N,M_{a}) intersect; points E_{a} and N are not the same
% Determination conditions: lines h_{a} and h_{b} are not the same; points B and H_{b} are not the
same; circles k(N,M_{a}) and k(M_{a},B) are not the same; points H_{a} and E_{a} are not the
same; points M_{a} and H_{a} must be different; points B and M_{a} are not the same; points E_{
a} and M_{a} must be different; points E_{a} and N are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

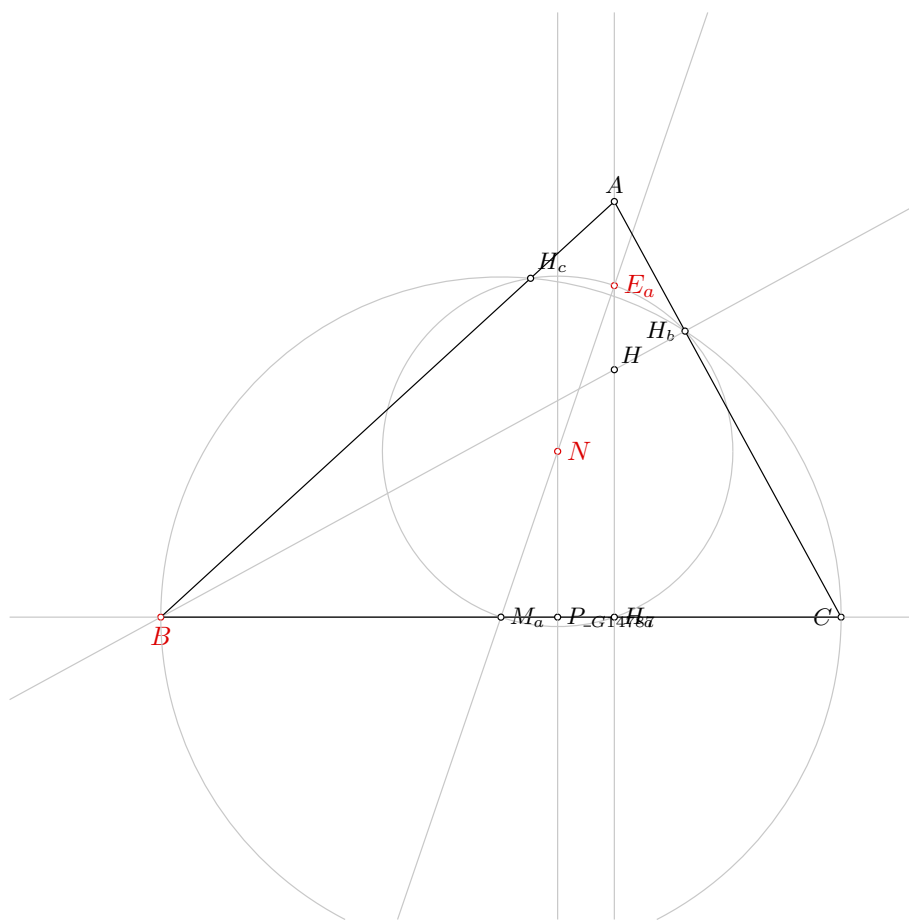


Figure 1: Illustration of the problem 0642

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.026 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a=_Ea$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_aBH_b} \neq S_{E_aBH_b}$ i.e., lines H_aE_a and BH_b are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

$S_{_M_aBC} \neq 0$ i.e., points $_M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$ i.e., points $_M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a_M_bF_{-m_b}^3} \neq S_{F_{-m_a}^2_M_bF_{-m_b}^3}$ i.e., lines $_M_aF_{-m_a}^2$ and $_M_bF_{-m_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a=_Ea$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Proving failed

4.3.2 Proving $E_a=_Ea$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Proving failed

4.4.2 Proving $E_a=_Ea$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 643

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 643: Given a point B , a point E_a and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 644

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 644: Given a point B , a point E_a and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 645

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 645: Given a point B , a point E_a and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 646

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 646: Given a point B , a point E_a and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 647

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 647: Given a point B , a point E_b and a point E_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point E_b , construct a point H (rule W01); ;
2. Using the point E_c and the point H , construct a point C (rule W01); ;
3. Using the point B and the point C , construct a line a (rule W02); % DET: points B and C are not the same;
4. Using the point E_c and the point H , construct a line h_c (rule W02); % DET: points E_c and H are not the same;
5. Using the point B and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points B and E_b are not the same;
6. Using the circle $k(E_b, B)$, the line a , the point E_b and the point B , construct a point H_a (rule W05); % NDG: line a and circle $k(E_b, B)$ intersect % DET: points B and H_a must be different;
7. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
8. Using the circle $k(E_b, B)$, the line h_c , the point E_b and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_b, B)$ intersect % DET: points H and H_c must be different;
9. Using the point H_c and the point B , construct a line c (rule W02); % DET: points H_c and B are not the same;
10. Using the line h_a and the line c , construct a point A (rule W03); % NDG: lines h_a and c are not parallel % DET: lines h_a and c are not the same.

Non-degenerate conditions: lines h_a and c are not parallel; line h_c and circle $k(E_b, B)$ intersect; line a and circle $k(E_b, B)$ intersect; points B and E_b are not the same.

Determination conditions: lines h_a and c are not the same; points H_c and B are not the same; points H and H_c must be different; points H_a and H are not the same; points B and H_a must be different; points E_c and H are not the same; points B and C are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D29,D3,D30,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L49,L50,L51]

Solving time: 12.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point E_{b} 50 56.36
point E_{c} 95 56.36
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_r E_{b}
cmark_r E_{c}
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that BH/BE_{b}=2
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0
```

```
% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0
```

```
% DET: points B and C are not the same
% Constructing a line a which passes through point B and point C
line a B C
```

```
color 200 200 200
drawline a
color 0 0 0
```

```

% DET: points E_{c} and H are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H
line h_{c} E_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points B and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point B
circle k(E_{b},B) E_{b} B

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line a and circle k(E_{b},B) intersect% DET: points B and H_{a} must be different
% Constructing a point P_{\_G77886} which is a foot of the point E_{b} on the line a
foot P_{\_G77886} E_{b} a
cmark_r P_{\_G77886}
color 200 200 200
drawline E_{b} P_{\_G77886}
color 0 0 0

% Constructing a point H_{a} which is an image of the point B in the symmetry to point/line P_{\_G
77886}
sim H_{a} P_{\_G77886} B
cmark_r H_{a}

% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{c} and circle k(E_{b},B) intersect% DET: points H and H_{c} must be different
% Constructing a point P_{\_G78124} which is a foot of the point E_{b} on the line h_{c}
foot P_{\_G78124} E_{b} h_{c}
cmark_r P_{\_G78124}
color 200 200 200
drawline E_{b} P_{\_G78124}
color 0 0 0

% Constructing a point H_{c} which is an image of the point H in the symmetry to point/line P_{\_G
78124}
sim H_{c} P_{\_G78124} H
cmark_rt H_{c}

```



```

% DET: points  $H_{\{c\}}$  and  $B$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $B$ 
line c  $H_{\{c\}}$  B

color 200 200 200
drawline c
color 0 0 0

% NDG: lines  $h_{\{a\}}$  and  $c$  are not parallel% DET: lines  $h_{\{a\}}$  and  $c$  are not the same
% Constructing a point  $A$  which belongs to line  $h_{\{a\}}$  and line  $c$ 
intersec A  $h_{\{a\}}$  c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{a\}}$  and  $c$  are not parallel; line  $h_{\{c\}}$  and circle  $k(E_{\{b\}}, B)$ 
% intersect; line  $a$  and circle  $k(E_{\{b\}}, B)$  intersect; points  $B$  and  $E_{\{b\}}$  are not the same
% Determination conditions: lines  $h_{\{a\}}$  and  $c$  are not the same; points  $H_{\{c\}}$  and  $B$  are not the same
% ; points  $H$  and  $H_{\{c\}}$  must be different; points  $H_{\{a\}}$  and  $H$  are not the same; points  $B$  and  $H_{\{a\}}$ 
% must be different; points  $E_{\{c\}}$  and  $H$  are not the same; points  $B$  and  $C$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.033 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b = \neg E_b$

Proving failed

4.1.3 Proving $E_c = \neg E_c$

Proving failed

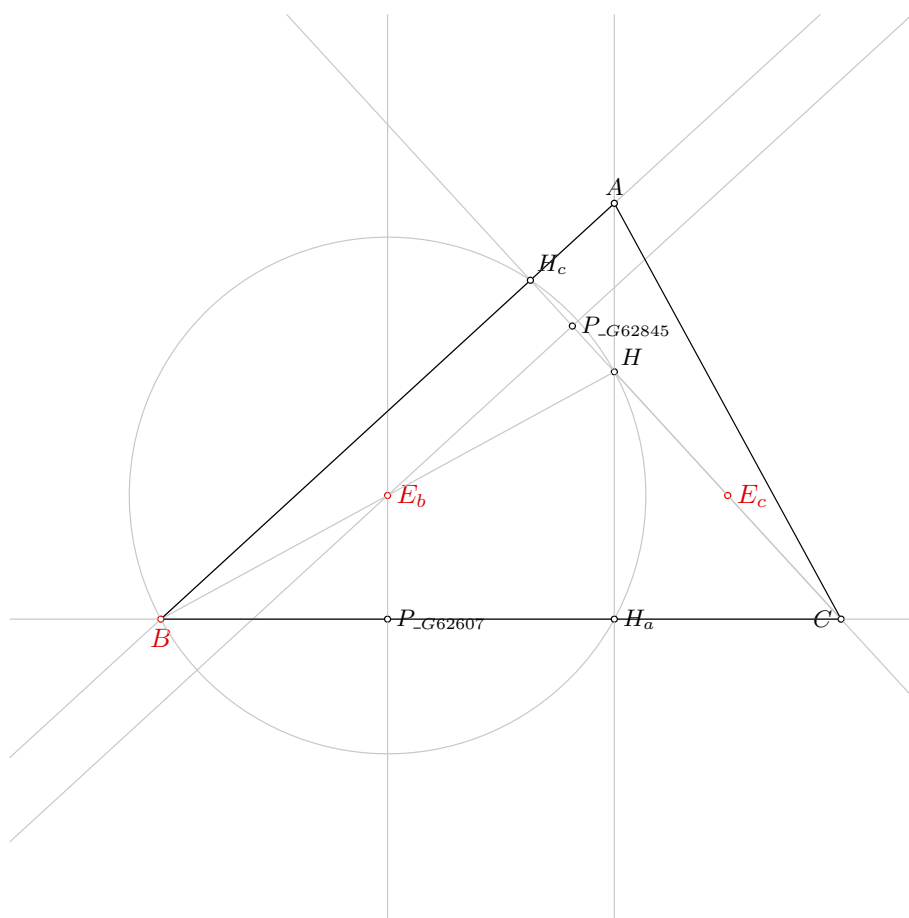


Figure 1: Illustration of the problem 0647

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_a H_c B} \neq S_{H H_c B}$ i.e., lines $H_a H$ and $H_c B$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b = \neg E_b$

Proving failed

4.2.3 Proving $E_c = \neg E_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_b = \neg E_b$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 4794 terms.

Time Complexity: Time spent by the prover is 13.460 seconds. There are no ndg conditions.

4.3.3 Proving $E_c = \neg E_c$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 2170 terms.

Time Complexity: Time spent by the prover is 14.490 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_b = \neg E_b$

Proving failed

4.4.3 Proving $E_c = \neg E_c$

Proving failed

Problem 648

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 648: Given a point B , a point E_b and a point G , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point E_b , construct a point H (rule W01); ;
2. Using the point B and the point G , construct a point M_b (rule W01); ;
3. Using the point G and the point H , construct a point N (rule W01); ;
4. Using the point G and the point H , construct a point O (rule W01); ;
5. Using the point B and the point E_b , construct a line h_b (rule W02); % DET: points B and E_b are not the same;
6. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
7. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
9. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; line h_b and circle $k(N, M_a)$ intersect; points E_b and N are not the same; points B and O are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points B and E_b are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L16,L20,L23,L56,L58]

Solving time: 9.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point E_{b} 50 56.36
point G 70 58.33

color 220 0 0
fontsize 9

cmark_b B
cmark_r E_{b}
cmark_t G
color 0 0 0
fontsize 8

% Constructing a point H such that BH/BE_{b}=2
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0

% Constructing a point M_{b} such that BM_{b}/BG=1.5
towards M_{b} B G 1.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% Constructing a point N such that GN/GH=0.25
towards N G H 0.25
cmark_r N
color 200 200 200
drawsegment G H
color 0 0 0

% Constructing a point O such that GO/GH=-0.5
```

```

towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% DET: points B and E_{b} are not the same
% Constructing a line h_{b} which passes through point B and point E_{b}
line h_{b} B E_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{b} and circle k(N,M_{a}) intersect% DET: points E_{b} and H_{b} must be different
% Constructing a point P_{\_G104487} which is a foot of the point N on the line h_{b}
foot P_{\_G104487} N h_{b}
cmark_r P_{\_G104487}
color 200 200 200
drawline N P_{\_G104487}
color 0 0 0

% Constructing a point H_{b} which is an image of the point E_{b} in the symmetry to point/line P
\_G104487}
sim H_{b} P_{\_G104487} E_{b}
cmark_l H_{b}

% DET: points H_{b} and M_{b} are not the same
% Constructing a line b which passes through point H_{b} and point M_{b}
line b H_{b} M_{b}

color 200 200 200

```

```
drawline b
color 0 0 0
```

```
% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line b and circle k(O,C) intersect; line h_{b} and circle k(N,M_{a})
% intersect; points E_{b} and N are not the same; points B and O are not the same
% Determination conditions: points H_{b} and M_{b} are not the same; points E_{b} and H_{b} must be
% different; points B and E_{b} are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.027 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b=_E_b$

Proving failed

4.1.3 Proving $G=_G$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a}BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

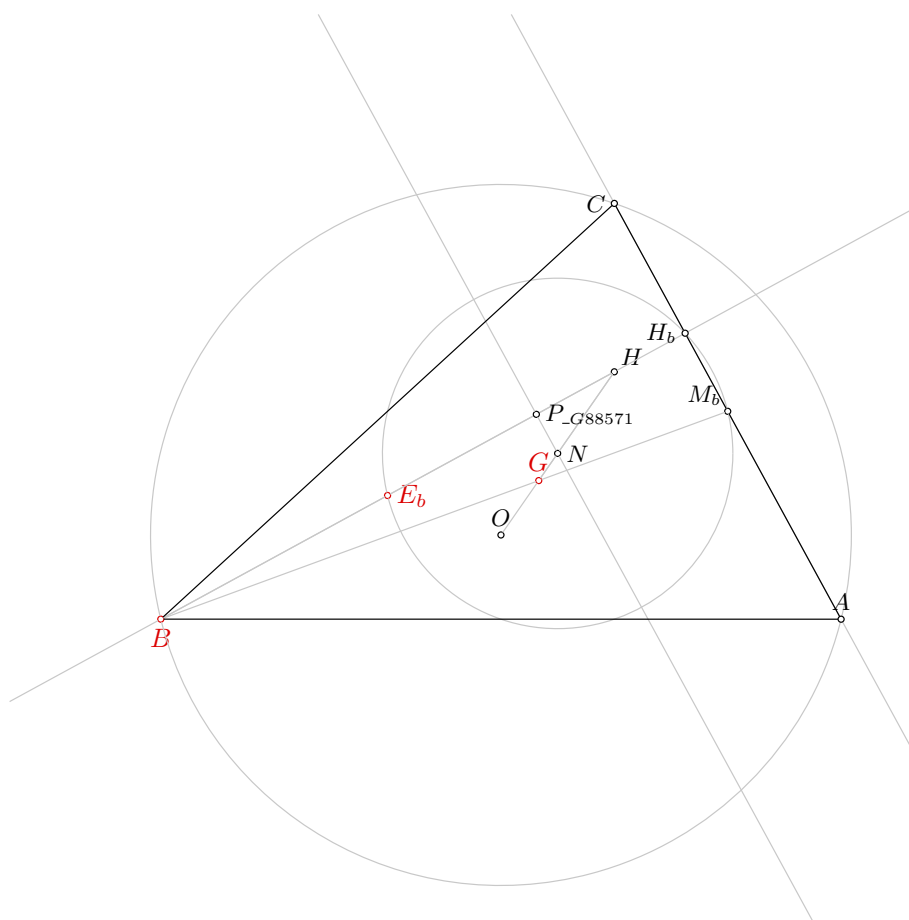


Figure 1: Illustration of the problem 0648

$S_{AB_M_b} \neq S_{_M_a B_M_b}$ i.e., lines A_M_a and B_M_b are not parallel (construction based assumption)
Total number of proof steps: 1
Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b = _E_b$

Proving failed

4.2.3 Proving $G = _G$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.2 Proving $E_b = _E_b$

Proving failed

4.3.3 Proving $G = _G$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $E_b = _E_b$

Proving failed

4.4.3 Proving $G = _G$

Proving failed

Problem 649

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 649: Given a point E_b , a point H and a point B , construct the triangle ABC .

2 Status

Problem is redundant.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H , construct a point B (rule W01); ;
2. Choose freely a point A (rule free);
3. Using the point A and the point H , construct a point E_a (rule W01); ;
4. Using the point A and the point B , construct a line c (rule W02); % DET: points A and B are not the same;
5. Using the point E_b and the point H , construct a line h_b (rule W02); % DET: points E_b and H are not the same;
6. Using the point A and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points A and E_a are not the same;
7. Using the circle $k(E_a, A)$, the line c , the point E_a and the point A , construct a point H_c (rule W05); % NDG: line c and circle $k(E_a, A)$ intersect % DET: points A and H_c must be different;
8. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
9. Using the circle $k(E_a, A)$, the line h_b , the point E_a and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_a, A)$ intersect % DET: points H and H_b must be different;
10. Using the point H_b and the point A , construct a line b (rule W02); % DET: points H_b and A are not the same;

11. Using the line h_c and the line b , construct a point C (rule W03); % NDG: lines h_c and b are not parallel % DET: lines h_c and b are not the same.

Non-degenerate conditions: lines h_c and b are not parallel; line h_b and circle $k(E_a, A)$ intersect; line c and circle $k(E_a, A)$ intersect; points A and E_a are not the same.

Determination conditions: lines h_c and b are not the same; points H_b and A are not the same; points H and H_b must be different; points H_c and H are not the same; points A and H_c must be different; points E_b and H are not the same; points A and B are not the same.

Rules used: [W01,W02,W03,W05,W06,free]

Lemmas used: [D10,D28,D29,D3,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L46,L47,L48]

Solving time: 10.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{b} 50 56.36
point H 80 72.73
point B 20 40

color 220 0 0
fontsize 9

cmark_r E_{b}
cmark_rt H
cmark_b B
color 0 0 0
fontsize 8

% Constructing a point B such that E_{b}B/E_{b}H=-1
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

% Constructing a free point A
point A 80 95

cmark_t A

% Constructing a point E_{a} such that AE_{a}/AH=0.5
towards E_{a} A H 0.5
cmark_r E_{a}
color 200 200 200
drawsegment A H
color 0 0 0
```

```

% DET: points A and B are not the same
% Constructing a line c which passes through point A and point B
line c A B

color 200 200 200
drawline c
color 0 0 0

% DET: points E_{b} and H are not the same
% Constructing a line h_{b} which passes through point E_{b} and point H
line h_{b} E_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points A and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point A
circle k(E_{a},A) E_{a} A

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: line c and circle k(E_{a},A) intersect% DET: points A and H_{c} must be different
% Constructing a point P_{\_G129688} which is a foot of the point E_{a} on the line c
foot P_{\_G129688} E_{a} c
cmark_r P_{\_G129688}
color 200 200 200
drawline E_{a} P_{\_G129688}
color 0 0 0

% Constructing a point H_{c} which is an image of the point A in the symmetry to point/line P_{\_G
129688}
sim H_{c} P_{\_G129688} A
cmark_rt H_{c}

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: line h_{b} and circle k(E_{a},A) intersect% DET: points H and H_{b} must be different

```

```

% Constructing a point  $P_{\{ \_G129926 \}}$  which is a foot of the point  $E_{\{a\}}$  on the line  $h_{\{b\}}$ 
foot  $P_{\{ \_G129926 \}}$   $E_{\{a\}}$   $h_{\{b\}}$ 
cmark_r  $P_{\{ \_G129926 \}}$ 
color 200 200 200
drawline  $E_{\{a\}}$   $P_{\{ \_G129926 \}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $H$  in the symmetry to point/line  $P_{\{ \_G129926 \}}$ 
sim  $H_{\{b\}}$   $P_{\{ \_G129926 \}}$   $H$ 
cmark_l  $H_{\{b\}}$ 

% DET: points  $H_{\{b\}}$  and  $A$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $A$ 
line  $b$   $H_{\{b\}}$   $A$ 

color 200 200 200
drawline  $b$ 
color 0 0 0

% NDG: lines  $h_{\{c\}}$  and  $b$  are not parallel% DET: lines  $h_{\{c\}}$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $h_{\{c\}}$  and line  $b$ 
intersec  $C$   $h_{\{c\}}$   $b$ 
cmark_l  $C$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $h_{\{c\}}$  and  $b$  are not parallel; line  $h_{\{b\}}$  and circle  $k(E_{\{a\}}, A)$ 
intersect; line  $c$  and circle  $k(E_{\{a\}}, A)$  intersect; points  $A$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $h_{\{c\}}$  and  $b$  are not the same; points  $H_{\{b\}}$  and  $A$  are not the same
; points  $H$  and  $H_{\{b\}}$  must be different; points  $H_{\{c\}}$  and  $H$  are not the same; points  $A$  and  $H_{\{c\}}$ 
must be different; points  $E_{\{b\}}$  and  $H$  are not the same; points  $A$  and  $B$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = _E_b$

Proving failed

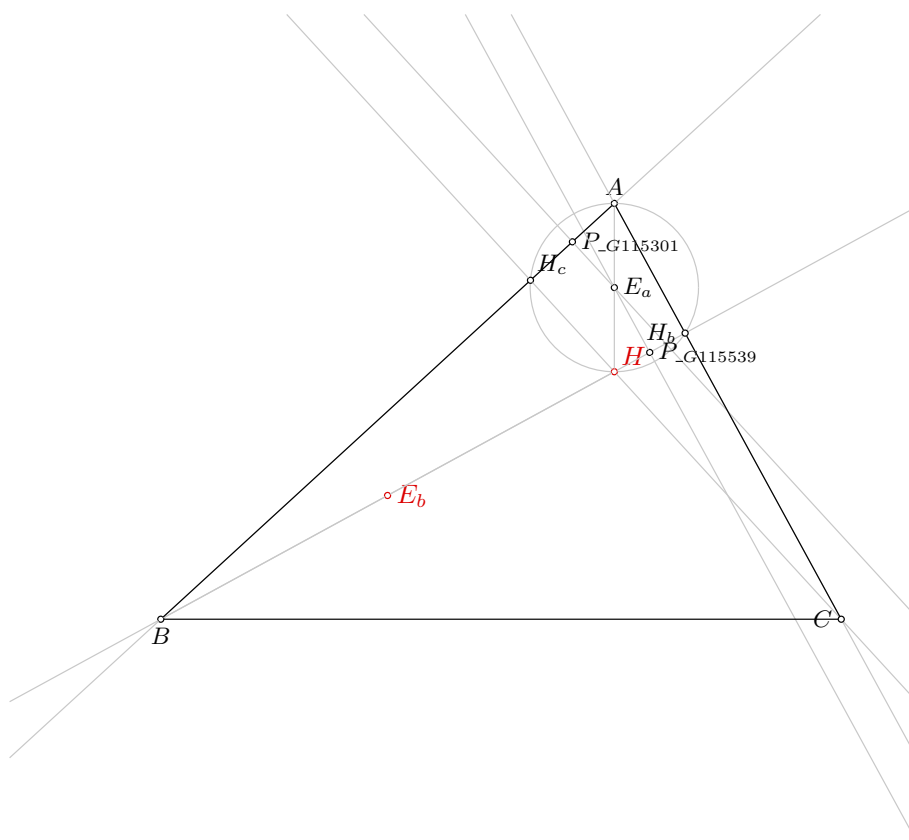


Figure 1: Illustration of the problem 0649

4.1.2 Proving $H=_H$

Proving failed

4.1.3 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2 terms.

Time Complexity: Time spent by the prover is 0.022 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $E_b=_E_b$

Proving failed

4.2.2 Proving $H=_H$

Proving failed

4.2.3 Proving $B=B$

NDG conditions are:

$S_{H_cH_bA} \neq S_{HH_bA}$ i.e., lines H_cH and H_bA are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a}BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_b=_E_b$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 974 terms.

Time Complexity: Time spent by the prover is 1.210 seconds. There are no ndg conditions.

4.3.2 Proving $H=_H$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 167 terms.

Time Complexity: Time spent by the prover is 0.160 seconds. There are no ndg conditions.

4.3.3 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $H = \neg H$

Proving failed

4.4.3 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

Problem 650

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 650: Given a point E_b , a point H_a and a point B , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_a and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_a and E_b are not the same;
2. Choose freely a point B on the circle $k(E_b, B)$ (rule WOncircle);
3. Using the point B and the point E_b , construct a point H (rule W01); ;
4. Using the point B and the point H_a , construct a line a (rule W02); % DET: points B and H_a are not the same;
5. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
6. Choose freely a point A on the line h_a (rule WOnline1) ;
7. Using the point A and the point B , construct a line c (rule W02); % DET: points A and B are not the same;
8. Using the circle $k(E_b, B)$, the line c , the point E_b and the point B , construct a point H_c (rule W05); % NDG: line c and circle $k(E_b, B)$ intersect % DET: points B and H_c must be different;
9. Using the point H and the point H_c , construct a line h_c (rule W02); % DET: points H and H_c are not the same;
10. Using the line a and the line h_c , construct a point C (rule W03); % NDG: lines a and h_c are not parallel % DET: lines a and h_c are not the same.

Non-degenerate conditions: lines a and h_c are not parallel; line c and circle $k(E_b, B)$ intersect; points H_a and E_b are not the same.

Determination conditions: lines a and h_c are not the same; points H and H_c are not the same; points B and H_c must be different; points A and B are not the same; points H_a and H are not the same; points B and H_a are not the same.

Rules used: [W01,W02,W03,W05,W06,WOncircle1,WOnline1]

Lemmas used: [D10,D29,D3,D5,D7,GD01,GD02,GL03,L3,L50,L51]

Solving time: 683.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{b} 50 56.36
point H_{a} 80 40
point B 20 40

color 220 0 0
fontsize 9

cmark_r E_{b}
cmark_r H_{a}
cmark_b B
color 0 0 0
fontsize 8

% NDG: points H_{a} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H
%_{a}
circle k(E_{b},B) E_{b} H_{a}

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% Choosing randomly a point B on the circle with center E_{b} through point H_{a}
oncircle B E_{b} H_{a}
cmark_b B
color 200 200 200
drawcircle E_{b} H_{a}
color 0 0 0

% Constructing a point H such that BH/BE_{b}=2
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0
```

```

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% Choosing randomly a point A on the line H_{a}H
online A H_{a} H
cmark_t A
color 200 200 200
drawline H_{a} H
color 0 0 0

% DET: points A and B are not the same
% Constructing a line c which passes through point A and point B
line c A B

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(E_{b},B) intersect% DET: points B and H_{c} must be different
% Constructing a point P_{\G153175} which is a foot of the point E_{b} on the line c
foot P_{\G153175} E_{b} c
cmark_r P_{\G153175}
color 200 200 200
drawline E_{b} P_{\G153175}
color 0 0 0

% Constructing a point H_{c} which is an image of the point B in the symmetry to point/line P_{\G
153175}
sim H_{c} P_{\G153175} B
cmark_rt H_{c}

% DET: points H and H_{c} are not the same
% Constructing a line h_{c} which passes through point H and point H_{c}

```

```

line h_{c} H H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines a and h_{c} are not parallel% DET: lines a and h_{c} are not the same
% Constructing a point C which belongs to line a and line h_{c}
intersec C a h_{c}
cmark_1 C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and h_{c} are not parallel; line c and circle k(E_{b},B)
% intersect; points H_{a} and E_{b} are not the same
% Determination conditions: lines a and h_{c} are not the same; points H and H_{c} are not the same
% ; points B and H_{c} must be different; points A and B are not the same; points H_{a} and H are
% not the same; points B and H_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $H_a = \neg H_a$

Proving failed

4.1.3 Proving $B = B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2 terms.

Time Complexity: Time spent by the prover is 0.023 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

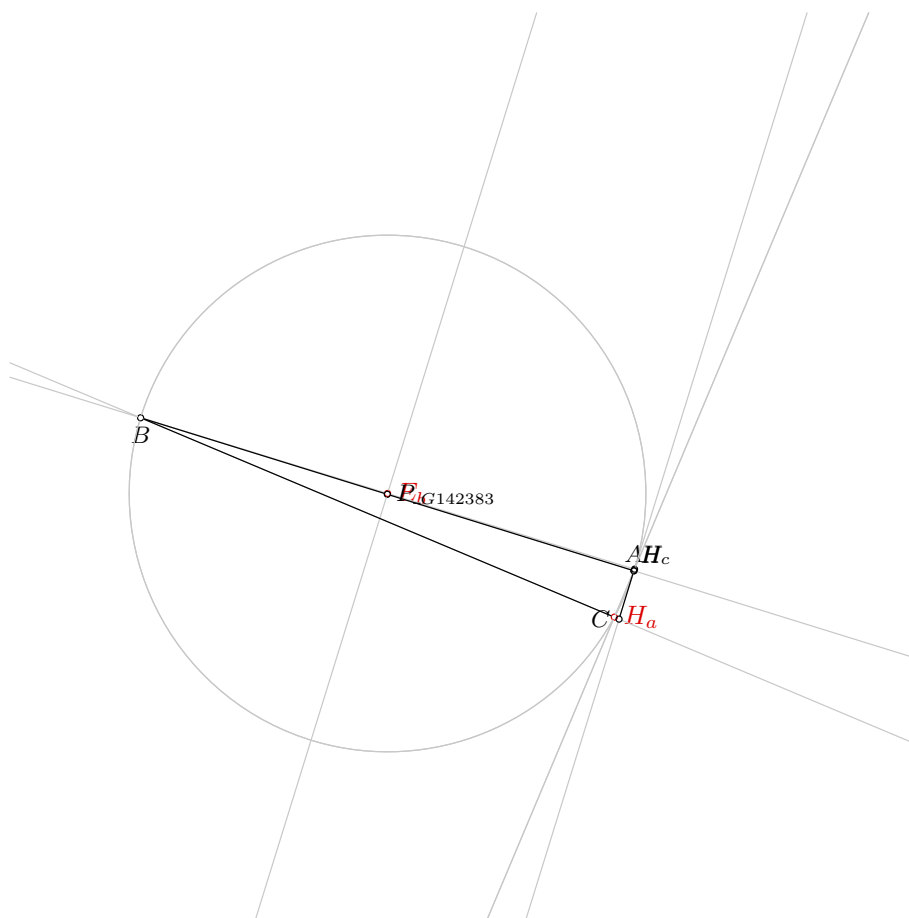


Figure 1: Illustration of the problem 0650

4.2.2 Proving $H_a = \neg H_a$

Proving failed

4.2.3 Proving $B = B$

NDG conditions are:

$S_{BHH_c} \neq S_{H_aHH_c}$ i.e., lines BH_a and HH_c are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^2} \neq S_{F_{\neg h_a}^1 BF_{\neg h_b}^2}$ i.e., lines $AF_{\neg h_a}^1$ and $BF_{\neg h_b}^2$ are not parallel (construction based assumption)

$S_{BAF_{\neg h_a}^1} \neq S_{CAF_{\neg h_a}^1}$ i.e., lines BC and $AF_{\neg h_a}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $H_a = \neg H_a$

Proving failed

4.3.3 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $H_a = \neg H_a$

Proving failed

4.4.3 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

Problem 651

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 651: Given a point E_b , a point H_b and a point B , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;
2. Choose freely a point B on the line h_b (rule WOnline1) ;
3. Using the point B and the point E_b , construct a point H (rule W01); ;
4. Using the point B and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points B and E_b are not the same;
5. Using the point H_b and the line h_b , construct a line b (rule W10a); ;
6. Choose freely a point A on the line b (rule WOnline2);
7. Using the point A and the point B , construct a line c (rule W02); % DET: points A and B are not the same;
8. Using the circle $k(E_b, B)$, the line c , the point E_b and the point B , construct a point H_c (rule W05); % NDG: line c and circle $k(E_b, B)$ intersect % DET: points B and H_c must be different;
9. Using the point H and the point H_c , construct a line h_c (rule W02); % DET: points H and H_c are not the same;
10. Using the line b and the line h_c , construct a point C (rule W03); % NDG: lines b and h_c are not parallel % DET: lines b and h_c are not the same.

Non-degenerate conditions: lines b and h_c are not parallel; line c and circle $k(E_b, B)$ intersect; points B and E_b are not the same.

Determination conditions: lines b and h_c are not the same; points H and H_c are not the same; points B and H_c must be different; points A and B are not the same; points E_b and H_b are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D10,D29,D3,D6,D7,D9,GD01,GD02,GL03,GL09,L3,L51]

Solving time: 174.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
```

```
point H_{b} 89.36 77.83
```

```
point B 20 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{b}
```

```
cmark_l H_{b}
```

```
cmark_b B
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{b} and H_{b} are not the same
```

```
% Constructing a line h_{b} which passes through point E_{b} and point H_{b}
```

```
line h_{b} E_{b} H_{b}
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```
% Choosing randomly a point B on the line E_{b}H_{b}
```

```
online B E_{b} H_{b}
```

```
cmark_b B
```

```
color 200 200 200
```

```
drawline E_{b} H_{b}
```

```
color 0 0 0
```

```
% Constructing a point H such that BH/BE_{b}=2
```

```
towards H B E_{b} 2
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment B H
```

```
color 0 0 0
```



```

% NDG: points B and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point B
circle k(E_{b},B) E_{b} B

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0


% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
perp b H_{b} h_{b}

color 200 200 200
drawline b
color 0 0 0


% Generating random value V[_G175698]
random V[_G175698]


% Calculating value V[_G175719] using formula V[_G175698]*20
expression V[_G175719] { V[_G175698]*20 }


% Constructing a point A which is a point for which holds  $H_{b}A = V[_G175719]$  and angle  $BH_{b}A = 90$ 
turtle A B H_{b} 90 V[_G175719]
cmark_t A


% DET: points A and B are not the same
% Constructing a line c which passes through point A and point B
line c A B

color 200 200 200
drawline c
color 0 0 0


% NDG: line c and circle k(E_{b},B) intersect% DET: points B and H_{c} must be different
% Constructing a point P_{\_G175991} which is a foot of the point E_{b} on the line c
foot P_{\_G175991} E_{b} c
cmark_r P_{\_G175991}
color 200 200 200
drawline E_{b} P_{\_G175991}
color 0 0 0


% Constructing a point H_{c} which is an image of the point B in the symmetry to point/line P_{\_G175991}
sim H_{c} P_{\_G175991} B

```

```

cmark_rt H_{c}

% DET: points H and H_{c} are not the same
% Constructing a line h_{c} which passes through point H and point H_{c}
line h_{c} H H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines b and h_{c} are not parallel% DET: lines b and h_{c} are not the same
% Constructing a point C which belongs to line b and line h_{c}
intersec C b h_{c}
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and h_{c} are not parallel; line c and circle k(E_{b},B)
% intersect; points B and E_{b} are not the same
% Determination conditions: lines b and h_{c} are not the same; points H and H_{c} are not the same
% ; points B and H_{c} must be different; points A and B are not the same; points E_{b} and H_{b}
% are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 55 terms.

Time Complexity: Time spent by the prover is 0.986 seconds.

NDG conditions Points A and B are not identical

Point E_b is not on circle with center H_b and point from it B

Line through points B and H_c is not perpendicular to line through points H_c and E_b

Points A , C and E_b are not collinear

4.1.2 Proving $H_b = \neg H_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

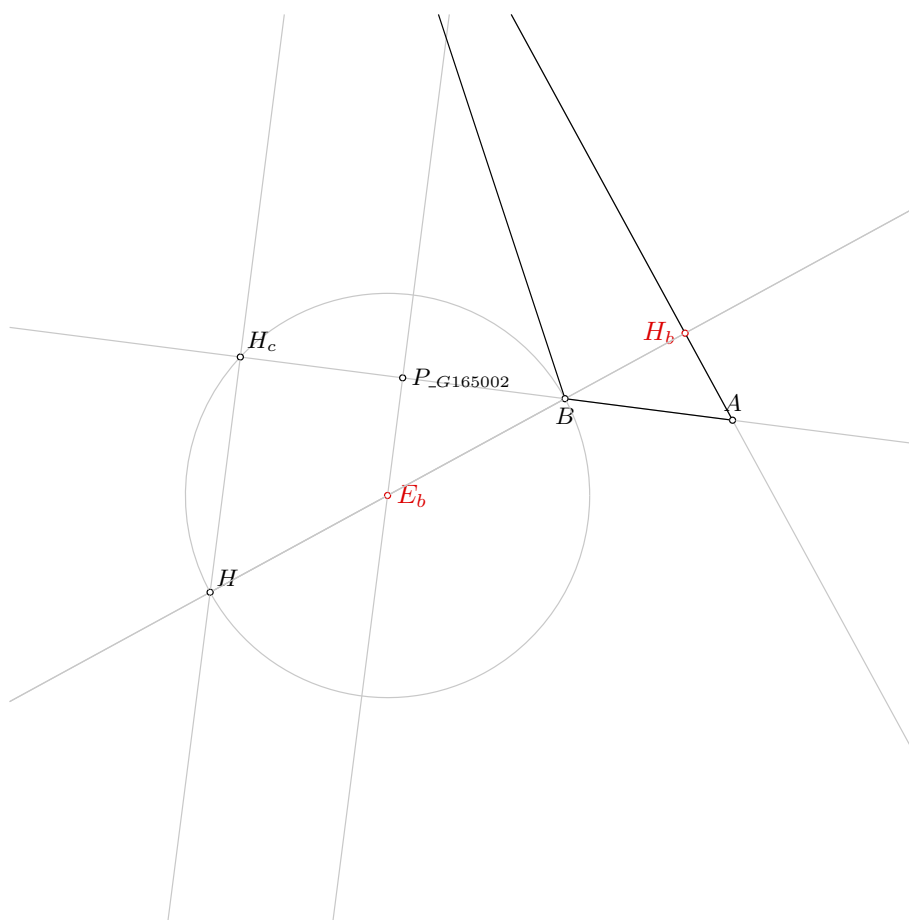


Figure 1: Illustration of the problem 0651

Time Complexity: Time spent by the prover is 0.174 seconds.

NDG conditions Points A and B are not identical

Point E_b is not on circle with center H_b and point from it B

Line through points B and H_c is not perpendicular to line through points H_c and E_b

Points A , C and E_b are not collinear

4.1.3 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.015 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $E_b=_E_b$

Proving failed

4.2.2 Proving $H_b=_H_b$

Proving failed

4.2.3 Proving $B=B$

NDG conditions are:

$S_{H_bHH_c} \neq S_{T_b^1HH_c}$ i.e., lines $H_bT_b^1$ and HH_c are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{F_{-h_a}^2BF_{-h_b}^3}$ i.e., lines $AF_{-h_a}^2$ and $BF_{-h_b}^3$ are not parallel (construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{CBF_{-h_b}^3}$ i.e., lines AC and $BF_{-h_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_b=_E_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 9 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $H_b=_H_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 16 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.3.3 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 20 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $H_b = \neg H_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 20 terms.

Time Complexity: Time spent by the prover is 0.060 seconds. There are no ndg conditions.

4.4.3 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

Problem 652

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 652: Given a point E_b , a point H_c and a point B , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_c and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_c and E_b are not the same;
2. Choose freely a point B on the circle $k(E_b, B)$ (rule WOncircle);
3. Using the point B and the point E_b , construct a point H (rule W01); ;
4. Using the point B and the point H_c , construct a line c (rule W02); % DET: points B and H_c are not the same;
5. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
6. Choose freely a point A on the line c (rule WOnline1) ;
7. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
8. Using the circle $k(E_b, B)$, the line h_a , the point E_b and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_b, B)$ intersect % DET: points H and H_a must be different;
9. Using the point H_a and the point B , construct a line a (rule W02); % DET: points H_a and B are not the same;
10. Using the line h_c and the line a , construct a point C (rule W03); % NDG: lines h_c and a are not parallel % DET: lines h_c and a are not the same.

Non-degenerate conditions: lines h_c and a are not parallel; line h_a and circle $k(E_b, B)$ intersect; points H_c and E_b are not the same.

Determination conditions: lines h_c and a are not the same; points H_a and B are not the same; points H and H_a must be different; points A and H are not the same; points H_c and H are not the same; points B and H_c are not the same.

Rules used: [W01,W02,W03,W05,W06,WOncircle1,WOnline1]

Lemmas used: [D10,D29,D3,D5,D7,D8,GD01,GD02,GL03,L3,L49,L50,L51]

Solving time: 687.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{b} 50 56.36
point H_{c} 68.91 84.83
point B 20 40

color 220 0 0
fontsize 9

cmark_r E_{b}
cmark_rt H_{c}
cmark_b B
color 0 0 0
fontsize 8

% NDG: points H_{c} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H
%_{c}
circle k(E_{b},B) E_{b} H_{c}

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% Choosing randomly a point B on the circle with center E_{b} through point H_{c}
oncircle B E_{b} H_{c}
cmark_b B
color 200 200 200
drawcircle E_{b} H_{c}
color 0 0 0

% Constructing a point H such that BH/BE_{b}=2
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0
```

```

% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

color 200 200 200
drawline c
color 0 0 0

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% Choosing randomly a point A on the line BH_{c}
online A B H_{c}
cmark_t A
color 200 200 200
drawline B H_{c}
color 0 0 0

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{a} and circle k(E_{b},B) intersect% DET: points H and H_{a} must be different
% Constructing a point P_{\G199847} which is a foot of the point E_{b} on the line h_{a}
foot P_{\G199847} E_{b} h_{a}
cmark_r P_{\G199847}
color 200 200 200
drawline E_{b} P_{\G199847}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H in the symmetry to point/line P_{\G199847}
sim H_{a} P_{\G199847} H
cmark_r H_{a}

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B

```



```

line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% NDG: lines h_{c} and a are not parallel% DET: lines h_{c} and a are not the same
% Constructing a point C which belongs to line h_{c} and line a
intersec C h_{c} a
cmark_1 C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and a are not parallel; line h_{a} and circle k(E_{b},B)
% intersect; points H_{c} and E_{b} are not the same
% Determination conditions: lines h_{c} and a are not the same; points H_{a} and B are not the same
% ; points H and H_{a} must be different; points A and H are not the same; points H_{c} and H are
% not the same; points B and H_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = E_b$

Proving failed

4.1.2 Proving $H_c = H_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2241 terms.

Time Complexity: Time spent by the prover is 15.129 seconds.

NDG conditions Points H_c , B and E_b are not collinear

Points A and H are not identical

Line through points A and H is not parallel with line through points H_c and E_b

Line through points B and H_a is not parallel with line through points H_c and H

Points E_b and H are not identical

Points A and B are not identical

Points A , B and E_b are not collinear

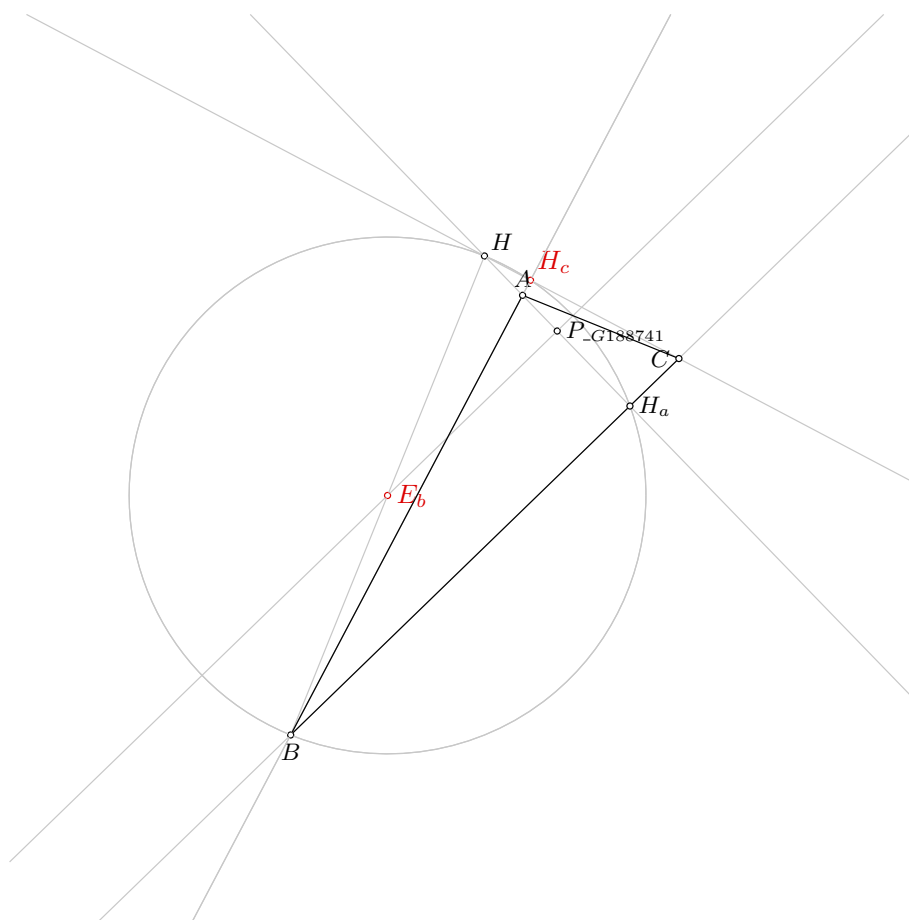


Figure 1: Illustration of the problem 0652

4.1.3 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2 terms.

Time Complexity: Time spent by the prover is 0.026 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $E_b=_E_b$

Proving failed

4.2.2 Proving $H_c=_H_c$

Proving failed

4.2.3 Proving $B=B$

NDG conditions are:

$S_{H_cH_aB} \neq S_{HH_aB}$ i.e., lines H_cH and H_aB are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^2_{h_b}} \neq S_{F^1_{h_a}BF^2_{h_b}}$ i.e., lines $AF^1_{h_a}$ and $BF^2_{h_b}$ are not parallel (construction based assumption)

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF^3_{h_c}} \neq S_{BCF^3_{h_c}}$ i.e., lines AB and $CF^3_{h_c}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_b=_E_b$

Proving failed

4.3.2 Proving $H_c=_H_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 280 terms.

Time Complexity: Time spent by the prover is 0.140 seconds. There are no ndg conditions.

4.3.3 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b=_E_b$

Proving failed

4.4.2 Proving $H_c = H_c$

Proving failed

4.4.3 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

Problem 653

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 653: Given a point B , a point E_b and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 654

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 654: Given a point B , a point E_b and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point E_b , construct a point H (rule W01); ;
2. Using the point B and the point M_a , construct a point C (rule W01); ;
3. Using the point B and the point M_a , construct a line a (rule W02); % DET: points B and M_a are not the same;
4. Using the point H and the point C , construct a line h_c (rule W02); % DET: points H and C are not the same;
5. Using the point B and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points B and E_b are not the same;
6. Using the circle $k(E_b, B)$, the line a , the point E_b and the point B , construct a point H_a (rule W05); % NDG: line a and circle $k(E_b, B)$ intersect % DET: points B and H_a must be different;
7. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
8. Using the circle $k(E_b, B)$, the line h_c , the point E_b and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_b, B)$ intersect % DET: points H and H_c must be different;
9. Using the point H_c and the point B , construct a line c (rule W02); % DET: points H_c and B are not the same;
10. Using the line h_a and the line c , construct a point A (rule W03); % NDG: lines h_a and c are not parallel % DET: lines h_a and c are not the same.

Non-degenerate conditions: lines h_a and c are not parallel; line h_c and circle $k(E_b, B)$ intersect; line a and circle $k(E_b, B)$ intersect; points B and E_b are not the same.

Determination conditions: lines h_a and c are not the same; points H_c and B are not the same; points H and H_c must be different; points H_a and H are not the same; points B and H_a must be different; points H and C are not the same; points B and M_a are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D21,D29,D3,D5,D7,D8,GD01,GD02,GL03,GL09,L3,L49,L50,L51]

Solving time: 12.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
```

```
point E_{b} 50 56.36
```

```
point M_{a} 65 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_b B
```

```
cmark_r E_{b}
```

```
cmark_r M_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point H such that BH/BE_{b}=2
```

```
towards H B E_{b} 2
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment B H
```

```
color 0 0 0
```

```
% Constructing a point C such that BC/BM_{a}=2
```

```
towards C B M_{a} 2
```

```
cmark_l C
```

```
color 200 200 200
```

```
drawsegment B C
```

```
color 0 0 0
```

```
% DET: points B and M_{a} are not the same
```

```
% Constructing a line a which passes through point B and point M_{a}
```

```
line a B M_{a}
```

```
color 200 200 200
```

```
drawline a
```

```
color 0 0 0
```

```

% DET: points H and C are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point H and point C
line  $h_{\{c\}}$  H C

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: points B and  $E_{\{b\}}$  are not the same
% Constructing a circle  $k(E_{\{b\}}, B)$  whose center is at point  $E_{\{b\}}$  and which passes through point B
circle  $k(E_{\{b\}}, B)$   $E_{\{b\}}$  B

color 200 200 200
drawcircle  $k(E_{\{b\}}, B)$ 
color 0 0 0

% NDG: line a and circle  $k(E_{\{b\}}, B)$  intersect% DET: points B and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G224792\}}$  which is a foot of the point  $E_{\{b\}}$  on the line a
foot  $P_{\{\backslash\_G224792\}}$   $E_{\{b\}}$  a
cmark_r  $P_{\{\backslash\_G224792\}}$ 
color 200 200 200
drawline  $E_{\{b\}}$   $P_{\{\backslash\_G224792\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point B in the symmetry to point/line  $P_{\{\backslash\_G224792\}}$ 
sim  $H_{\{a\}}$   $P_{\{\backslash\_G224792\}}$  B
cmark_r  $H_{\{a\}}$ 

% DET: points  $H_{\{a\}}$  and H are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $H_{\{a\}}$  and point H
line  $h_{\{a\}}$   $H_{\{a\}}$  H

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(E_{\{b\}}, B)$  intersect% DET: points H and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G225030\}}$  which is a foot of the point  $E_{\{b\}}$  on the line  $h_{\{c\}}$ 
foot  $P_{\{\backslash\_G225030\}}$   $E_{\{b\}}$   $h_{\{c\}}$ 
cmark_r  $P_{\{\backslash\_G225030\}}$ 
color 200 200 200
drawline  $E_{\{b\}}$   $P_{\{\backslash\_G225030\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point H in the symmetry to point/line  $P_{\{\backslash\_G225030\}}$ 
sim  $H_{\{c\}}$   $P_{\{\backslash\_G225030\}}$  H
cmark_rt  $H_{\{c\}}$ 

```



```

% DET: points  $H_{\{c\}}$  and  $B$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $B$ 
line c  $H_{\{c\}}$  B

color 200 200 200
drawline c
color 0 0 0

% NDG: lines  $h_{\{a\}}$  and  $c$  are not parallel% DET: lines  $h_{\{a\}}$  and  $c$  are not the same
% Constructing a point  $A$  which belongs to line  $h_{\{a\}}$  and line  $c$ 
intersec A  $h_{\{a\}}$  c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{a\}}$  and  $c$  are not parallel; line  $h_{\{c\}}$  and circle  $k(E_{\{b\}}, B)$ 
% intersect; line  $a$  and circle  $k(E_{\{b\}}, B)$  intersect; points  $B$  and  $E_{\{b\}}$  are not the same
% Determination conditions: lines  $h_{\{a\}}$  and  $c$  are not the same; points  $H_{\{c\}}$  and  $B$  are not the same
% ; points  $H$  and  $H_{\{c\}}$  must be different; points  $H_{\{a\}}$  and  $H$  are not the same; points  $B$  and  $H_{\{a\}}$ 
% must be different; points  $H$  and  $C$  are not the same; points  $B$  and  $M_{\{a\}}$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.034 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b = E_b$

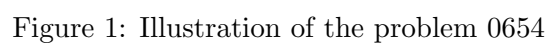
Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 1741 terms.

Time Complexity: Time spent by the prover is 14.964 seconds.

NDG conditions Points E_b and M_a are not identical

Points E_b and M_a are not identical



Points C and H are not identical

Line through points B and C is not perpendicular to line through points C and E_b

Line through points B and H_c is not parallel with line through points H and H_a

Points B , E_b and H are not collinear

Points A , B and C are not collinear

Points B , C and H are not collinear

4.1.3 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $B = B$

NDG conditions are:

$S_{H_a H_c B} \neq S_{H H_c B}$ i.e., lines $H_a H$ and $H_c B$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b = \neg E_b$

Proving failed

4.2.3 Proving $M_a = \neg M_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_b = \neg E_b$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 363 terms.

Time Complexity: Time spent by the prover is 0.960 seconds. There are no ndg conditions.

4.3.3 Proving $M_a = \neg M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 8 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_b=_E_b$

Proving failed

4.4.3 Proving $M_a=_M_a$

Proving failed

Problem 655

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 655: Given a point B , a point E_b and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point E_b , construct a point H (rule W01); ;
2. Using the point B and the point M_b , construct a point G (rule W01); ;
3. Using the point H and the point G , construct a point N (rule W01); ;
4. Using the point H and the point G , construct a point O (rule W01); ;
5. Using the point B and the point E_b , construct a line h_b (rule W02); % DET: points B and E_b are not the same;
6. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
7. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
9. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; line h_b and circle $k(N, M_a)$ intersect; points E_b and N are not the same; points B and O are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points B and E_b are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D32,D6,D9,GD01,GL03,GL09,L11,L12,L16,L20,L23,L56,L58]

Solving time: 9.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point E_{b} 50 56.36
point M_{b} 95 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_r E_{b}
cmark_lt M_{b}
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that BH/BE_{b}=2
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0
```

```
% Constructing a line L_{\_G248357} which passes through point B and point M_{b}
line L_{\_G248357} B M_{b}
```

```
color 200 200 200
drawline L_{\_G248357}
color 0 0 0
```

```
% Constructing a point P_{\_G248458} with coordinates (0,0)
point P_{\_G248458} 0 0
cmark_r P_{\_G248458}
```

```
% Constructing a point P_{\_G248382} such that BP_{\_G248382}/BP_{\_G248458}=2
towards P_{\_G248382} B P_{\_G248458} 2
cmark_r P_{\_G248382}
color 200 200 200
drawsegment B P_{\_G248382}
color 0 0 0
```

```

% Constructing a point  $P_{\{G248427\}}$  such that  $BP_{\{G248427\}}/BP_{\{G248458\}}=3$ 
towards P_{\_G248427} B P_{\_G248458} 3
cmark_r P_{\_G248427}
color 200 200 200
drawsegment B P_{\_G248427}
color 0 0 0

% Constructing a line  $L_{\{G248388\}}$  which passes through point  $M_{\{b\}}$  and point  $P_{\{G248427\}}$ 
line L_{\_G248388} M_{\{b\}} P_{\_G248427}

color 200 200 200
drawline L_{\_G248388}
color 0 0 0

% Constructing a line  $L_{\{G248351\}}$  which contains the point  $P_{\{G248382\}}$  and is parallel to the
line  $L_{\{G248388\}}$ 
parallel L_{\_G248351} P_{\_G248382} L_{\_G248388}

color 200 200 200
drawline L_{\_G248351}
color 0 0 0

% Constructing a point  $G$  which belongs to line  $L_{\{G248351\}}$  and line  $L_{\{G248357\}}$ 
intersec G L_{\_G248351} L_{\_G248357}
cmark_t G

% Constructing a point  $N$  such that  $HN/HG=0.75$ 
towards N H G 0.75
cmark_r N
color 200 200 200
drawsegment H G
color 0 0 0

% Constructing a point  $O$  such that  $HO/HG=1.5$ 
towards O H G 1.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% DET: points  $B$  and  $E_{\{b\}}$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $B$  and point  $E_{\{b\}}$ 
line h_{\{b\}} B E_{\{b\}}

color 200 200 200
drawline h_{\{b\}}
color 0 0 0

```

```

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{b} and circle k(N,M_{a}) intersect% DET: points E_{b} and H_{b} must be different
% Constructing a point P_{\_G249428} which is a foot of the point N on the line h_{b}
foot P_{\_G249428} N h_{b}
cmark_r P_{\_G249428}
color 200 200 200
drawline N P_{\_G249428}
color 0 0 0

% Constructing a point H_{b} which is an image of the point E_{b} in the symmetry to point/line P
\_G249428}
sim H_{b} P_{\_G249428} E_{b}
cmark_l H_{b}

% DET: points H_{b} and M_{b} are not the same
% Constructing a line b which passes through point H_{b} and point M_{b}
line b H_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

```

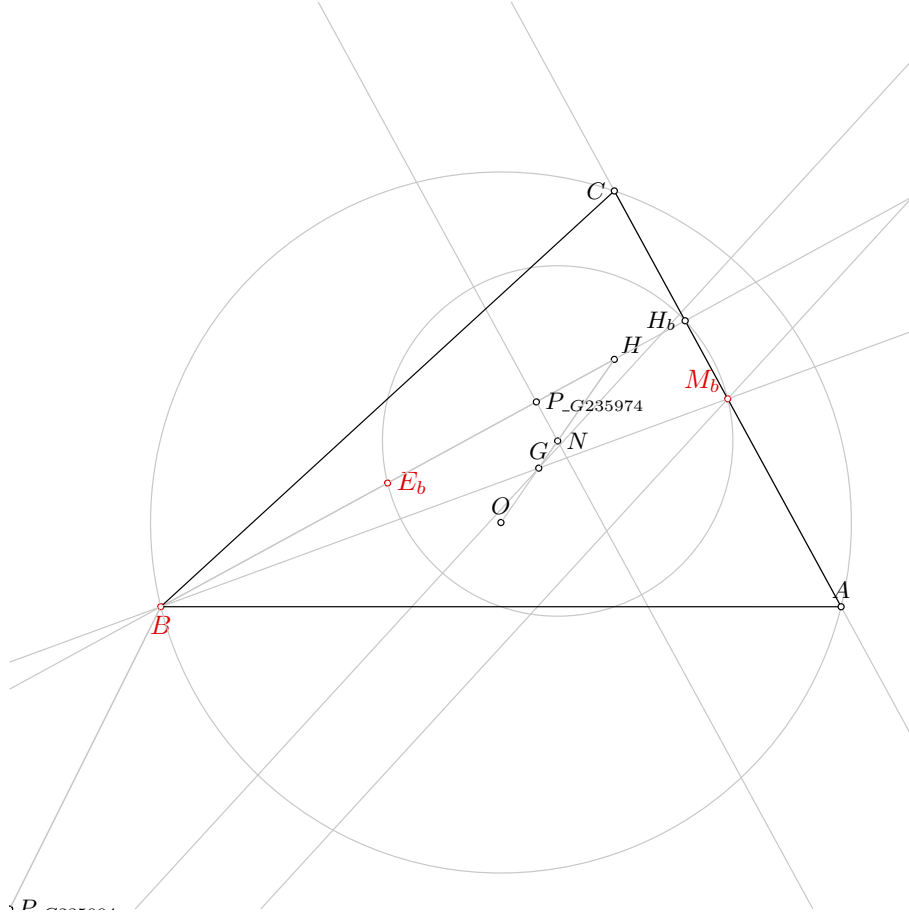



Figure 1: Illustration of the problem 0655

*% Non-degenerate conditions: line b and circle $k(O,C)$ intersect; line $h_{\{b\}}$ and circle $k(N,M_{\{a\}})$ intersect; points $E_{\{b\}}$ and N are not the same; points B and O are not the same
 % Determination conditions: points $H_{\{b\}}$ and $M_{\{b\}}$ are not the same; points $E_{\{b\}}$ and $H_{\{b\}}$ must be different; points B and $E_{\{b\}}$ are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.027 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b = \neg E_b$

Proving failed

4.1.3 Proving $M_b = \neg M_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B = B$

NDG conditions are:

$S_{P_{G236373}BM_b} \neq S_{P_{L_{G236342}}^0 BM_b}$ i.e., lines $P_{G236373}P_{L_{G236342}}^0$ and BM_b are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^2} \neq S_{F_{-h_a}^1 BF_{-h_b}^2}$ i.e., lines $AF_{-h_a}^1$ and $BF_{-h_b}^2$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b = \neg E_b$

Proving failed

4.2.3 Proving $M_b = \neg M_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $E_b = \neg E_b$

Proving failed

4.3.3 Proving $M_b = \neg M_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $E_b = \neg E_b$

Proving failed

4.4.3 Proving $M_b = \neg M_b$

Proving failed

Problem 656

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 656: Given a point B , a point E_b and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point E_b , construct a point H (rule W01); ;
2. Using the point B and the point M_c , construct a point A (rule W01); ;
3. Using the point B and the point M_c , construct a line c (rule W02); % DET: points B and M_c are not the same;
4. Using the point H and the point A , construct a line h_a (rule W02); % DET: points H and A are not the same;
5. Using the point B and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points B and E_b are not the same;
6. Using the circle $k(E_b, B)$, the line c , the point E_b and the point B , construct a point H_c (rule W05); % NDG: line c and circle $k(E_b, B)$ intersect % DET: points B and H_c must be different;
7. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
8. Using the circle $k(E_b, B)$, the line h_a , the point E_b and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_b, B)$ intersect % DET: points H and H_a must be different;
9. Using the point H_a and the point B , construct a line a (rule W02); % DET: points H_a and B are not the same;
10. Using the line h_c and the line a , construct a point C (rule W03); % NDG: lines h_c and a are not parallel % DET: lines h_c and a are not the same.

Non-degenerate conditions: lines h_c and a are not parallel; line h_a and circle $k(E_b, B)$ intersect; line c and circle $k(E_b, B)$ intersect; points B and E_b are not the same.

Determination conditions: lines h_c and a are not the same; points H_a and B are not the same; points H and H_a must be different; points H_c and H are not the same; points B and H_c must be different; points H and A are not the same; points B and M_c are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D20,D29,D3,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L49,L50,L51]

Solving time: 12.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point E_{b} 50 56.36
point M_{c} 50 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_r E_{b}
cmark_lt M_{c}
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that BH/BE_{b}=2
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0
```

```
% Constructing a point A such that BA/BM_{c}=2
towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0
```

```
% DET: points B and M_{c} are not the same
% Constructing a line c which passes through point B and point M_{c}
line c B M_{c}
```

```
color 200 200 200
drawline c
color 0 0 0
```

```

% DET: points H and A are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point H and point A
line  $h_{\{a\}}$  H A

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: points B and  $E_{\{b\}}$  are not the same
% Constructing a circle  $k(E_{\{b\}}, B)$  whose center is at point  $E_{\{b\}}$  and which passes through point B
circle  $k(E_{\{b\}}, B)$   $E_{\{b\}}$  B

color 200 200 200
drawcircle  $k(E_{\{b\}}, B)$ 
color 0 0 0

% NDG: line c and circle  $k(E_{\{b\}}, B)$  intersect% DET: points B and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G20797\}}$  which is a foot of the point  $E_{\{b\}}$  on the line c
foot  $P_{\{\backslash\_G20797\}}$   $E_{\{b\}}$  c
cmark_r  $P_{\{\backslash\_G20797\}}$ 
color 200 200 200
drawline  $E_{\{b\}}$   $P_{\{\backslash\_G20797\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point B in the symmetry to point/line  $P_{\{\backslash\_G20797\}}$ 
sim  $H_{\{c\}}$   $P_{\{\backslash\_G20797\}}$  B
cmark_rt  $H_{\{c\}}$ 

% DET: points  $H_{\{c\}}$  and H are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $H_{\{c\}}$  and point H
line  $h_{\{c\}}$   $H_{\{c\}}$  H

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: line  $h_{\{a\}}$  and circle  $k(E_{\{b\}}, B)$  intersect% DET: points H and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G21035\}}$  which is a foot of the point  $E_{\{b\}}$  on the line  $h_{\{a\}}$ 
foot  $P_{\{\backslash\_G21035\}}$   $E_{\{b\}}$   $h_{\{a\}}$ 
cmark_r  $P_{\{\backslash\_G21035\}}$ 
color 200 200 200
drawline  $E_{\{b\}}$   $P_{\{\backslash\_G21035\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point H in the symmetry to point/line  $P_{\{\backslash\_G21035\}}$ 
sim  $H_{\{a\}}$   $P_{\{\backslash\_G21035\}}$  H
cmark_r  $H_{\{a\}}$ 

```

```

% DET: points  $H_{\{a\}}$  and  $B$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $B$ 
line a  $H_{\{a\}}$  B

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $h_{\{c\}}$  and  $a$  are not parallel% DET: lines  $h_{\{c\}}$  and  $a$  are not the same
% Constructing a point  $C$  which belongs to line  $h_{\{c\}}$  and line  $a$ 
intersec C  $h_{\{c\}}$  a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{c\}}$  and  $a$  are not parallel; line  $h_{\{a\}}$  and circle  $k(E_{\{b\}}, B)$ 
% intersect; line  $c$  and circle  $k(E_{\{b\}}, B)$  intersect; points  $B$  and  $E_{\{b\}}$  are not the same
% Determination conditions: lines  $h_{\{c\}}$  and  $a$  are not the same; points  $H_{\{a\}}$  and  $B$  are not the same
% ; points  $H$  and  $H_{\{a\}}$  must be different; points  $H_{\{c\}}$  and  $H$  are not the same; points  $B$  and  $H_{\{c\}}$ 
% must be different; points  $H$  and  $A$  are not the same; points  $B$  and  $M_{\{c\}}$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.035 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b = \neg E_b$

Proving failed

4.1.3 Proving $M_c = \neg M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

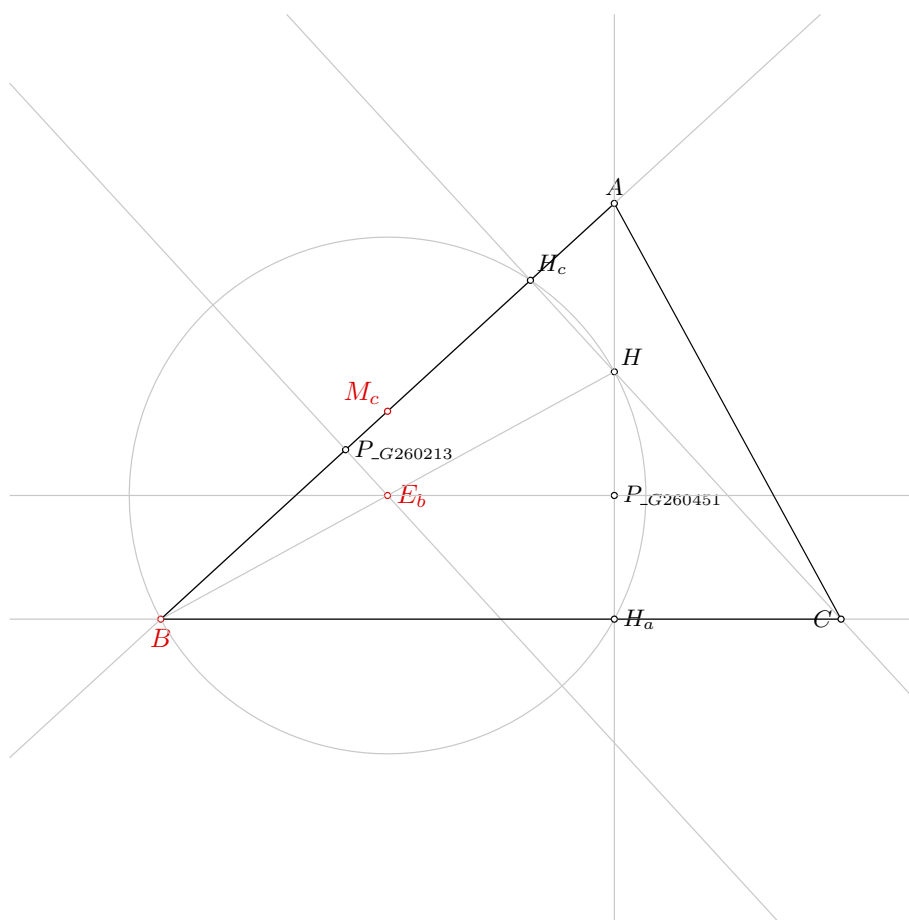


Figure 1: Illustration of the problem 0656

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_c H_a B} \neq S_{H H_a B}$ i.e., lines $H_c H$ and $H_a B$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b=_E b$

Proving failed

4.2.3 Proving $M_c=_M c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_b=_E b$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 363 terms.

Time Complexity: Time spent by the prover is 1.000 seconds. There are no ndg conditions.

4.3.3 Proving $M_c=_M c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 8 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_b=_E b$

Proving failed

4.4.3 Proving $M_c = -M_c$

Proving failed

Problem 657

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 657: Given a point B , a point E_b and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point E_b , construct a point H (rule W01); ;
2. Using the point N and the point H , construct a point O (rule W01); ;
3. Using the point N and the point H , construct a point G (rule W01); ;
4. Using the point B and the point G , construct a point M_b (rule W01); ;
5. Using the point B and the point E_b , construct a line h_b (rule W02); % DET: points B and E_b are not the same;
6. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
7. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
9. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; line h_b and circle $k(N, M_a)$ intersect; points E_b and N are not the same; points B and O are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points B and E_b are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D31,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L16,L20,L23,L56]

Solving time: 9.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point E_{b} 50 56.36
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_r E_{b}
cmark_r N
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that BH/BE_{b}=2
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0
```

```
% Constructing a point O such that NO/NH=-1
towards O N H -1
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0
```

```
% Constructing a line L_{\_G56414} which passes through point N and point H
line L_{\_G56414} N H

color 200 200 200
drawline L_{\_G56414}
color 0 0 0
```

```
% Constructing a point P_{\_G56515} with coordinates (0,0)
point P_{\_G56515} 0 0
```

```

cmark_r P_{\_G56515}

% Constructing a point P_{\_G56439} such that NP_{\_G56439}/NP_{\_G56515}=-1
towards P_{\_G56439} N P_{\_G56515} -1
cmark_r P_{\_G56439}
color 200 200 200
drawsegment P_{\_G56515} P_{\_G56439}
color 0 0 0

% Constructing a point P_{\_G56484} such that NP_{\_G56484}/NP_{\_G56515}=3
towards P_{\_G56484} N P_{\_G56515} 3
cmark_r P_{\_G56484}
color 200 200 200
drawsegment N P_{\_G56484}
color 0 0 0

% Constructing a line L_{\_G56445} which passes through point H and point P_{\_G56484}
line L_{\_G56445} H P_{\_G56484}

color 200 200 200
drawline L_{\_G56445}
color 0 0 0

% Constructing a line L_{\_G56408} which contains the point P_{\_G56439} and is parallel to the
line L_{\_G56445}
parallel L_{\_G56408} P_{\_G56439} L_{\_G56445}

color 200 200 200
drawline L_{\_G56408}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G56408} and line L_{\_G56414}
intersec G L_{\_G56408} L_{\_G56414}
cmark_t G

% Constructing a point M_{b} such that BM_{b}/BG=1.5
towards M_{b} B G 1.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% DET: points B and E_{b} are not the same
% Constructing a line h_{b} which passes through point B and point E_{b}
line h_{b} B E_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

```

```

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{b} and circle k(N,M_{a}) intersect% DET: points E_{b} and H_{b} must be different
% Constructing a point P_{\_G57410} which is a foot of the point N on the line h_{b}
foot P_{\_G57410} N h_{b}
cmark_r P_{\_G57410}
color 200 200 200
drawline N P_{\_G57410}
color 0 0 0

% Constructing a point H_{b} which is an image of the point E_{b} in the symmetry to point/line P
\_G57410}
sim H_{b} P_{\_G57410} E_{b}
cmark_l H_{b}

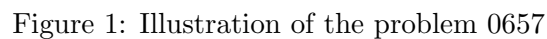
% DET: points H_{b} and M_{b} are not the same
% Constructing a line b which passes through point H_{b} and point M_{b}
line b H_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

```



```
% Non-degenerate conditions: line b and circle k(O,C) intersect; line h_{b} and circle k(N,M_{a})
% intersect; points E_{b} and N are not the same; points B and O are not the same
% Determination conditions: points H_{b} and M_{b} are not the same; points E_{b} and H_{b} must be
% different; points B and E_{b} are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b = \neg E_b$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B = B$

NDG conditions are:

$S_{P_{G32449}NH} \neq S_{P_{L_{G32418}}^0}^{NH}$ i.e., lines $P_{G32449}P_{L_{G32418}}^0$ and NH are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^2} \neq S_{F_{h_a}^1 BF_{h_b}^2}$ i.e., lines $AF_{h_a}^1$ and $BF_{h_b}^2$ are not parallel (construction based assumption)

$S_{M_a BC} \neq 0$ i.e., points M_a, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_b AC} \neq 0$ i.e., points M_b, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_a M_b F_{m_b}^4} \neq S_{F_{m_a}^3 M_b F_{m_b}^4}$ i.e., lines $M_a F_{m_a}^3$ and $M_b F_{m_b}^4$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b = \neg E_b$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $E_b = \neg E_b$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.4.2 Proving $E_b=_E_b$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 658

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 658: Given a point B , a point E_b and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point E_b , construct a point H (rule W01); ;
2. Using the point O and the point H , construct a point N (rule W01); ;
3. Using the point O and the point H , construct a point G (rule W01); ;
4. Using the point B and the point G , construct a point M_b (rule W01); ;
5. Using the point B and the point E_b , construct a line h_b (rule W02); % DET: points B and E_b are not the same;
6. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
7. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
9. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; line h_b and circle $k(N, M_a)$ intersect; points E_b and N are not the same; points B and O are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points B and E_b are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D31,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L20,L23,L56,L58]

Solving time: 9.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point E_{b} 50 56.36
point O 65 51.14
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_r E_{b}
cmark_t O
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that BH/BE_{b}=2
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0
```

```
% Constructing a point N such that ON/OH=0.5
towards N O H 0.5
cmark_r N
color 200 200 200
drawsegment O H
color 0 0 0
```

```
% Constructing a line L_{\_G85769} which passes through point O and point H
line L_{\_G85769} O H

color 200 200 200
drawline L_{\_G85769}
color 0 0 0
```

```
% Constructing a point P_{\_G85870} with coordinates (0,0)
point P_{\_G85870} 0 0
```

```

cmark_r P_{\_G85870}

% Constructing a point P_{\_G85794} such that  $OP_{\_G85794}/OP_{\_G85870}=1$ 
towards P_{\_G85794} 0 P_{\_G85870} 1
cmark_r P_{\_G85794}
color 200 200 200
drawsegment 0 P_{\_G85794}
color 0 0 0

% Constructing a point P_{\_G85839} such that  $OP_{\_G85839}/OP_{\_G85870}=3$ 
towards P_{\_G85839} 0 P_{\_G85870} 3
cmark_r P_{\_G85839}
color 200 200 200
drawsegment 0 P_{\_G85839}
color 0 0 0

% Constructing a line L_{\_G85800} which passes through point H and point P_{\_G85839}
line L_{\_G85800} H P_{\_G85839}

color 200 200 200
drawline L_{\_G85800}
color 0 0 0

% Constructing a line L_{\_G85763} which contains the point P_{\_G85794} and is parallel to the
line L_{\_G85800}
parallel L_{\_G85763} P_{\_G85794} L_{\_G85800}

color 200 200 200
drawline L_{\_G85763}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G85763} and line L_{\_G85769}
intersec G L_{\_G85763} L_{\_G85769}
cmark_t G

% Constructing a point M_{b} such that  $BM_b/BG=1.5$ 
towards M_{b} B G 1.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% DET: points B and E_{b} are not the same
% Constructing a line h_{b} which passes through point B and point E_{b}
line h_{b} B E_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

```

```

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{b} and circle k(N,M_{a}) intersect% DET: points E_{b} and H_{b} must be different
% Constructing a point P_{\_G86757} which is a foot of the point N on the line h_{b}
foot P_{\_G86757} N h_{b}
cmark_r P_{\_G86757}
color 200 200 200
drawline N P_{\_G86757}
color 0 0 0

% Constructing a point H_{b} which is an image of the point E_{b} in the symmetry to point/line P
\_G86757}
sim H_{b} P_{\_G86757} E_{b}
cmark_l H_{b}

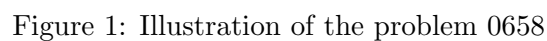
% DET: points H_{b} and M_{b} are not the same
% Constructing a line b which passes through point H_{b} and point M_{b}
line b H_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

```



```
% Non-degenerate conditions: line b and circle k(O,C) intersect; line h_{b} and circle k(N,M_{a})
% intersect; points E_{b} and N are not the same; points B and O are not the same
% Determination conditions: points H_{b} and M_{b} are not the same; points E_{b} and H_{b} must be
% different; points B and E_{b} are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.032 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b = \neg E_b$

Proving failed

4.1.3 Proving $O = \neg O$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2435 terms.

Time Complexity: Time spent by the prover is 8.008 seconds.

NDG conditions Points H , O and P_{G82740} are not collinear

Points H_b , E_b and P_{G83034} are not collinear

Points M_b and H_b are not identical

Points M_b and B are not identical

Points A , B and C are not collinear

Points C , E_b and H are not collinear

4.2 GCLC - Area method

4.2.1 Proving $B = B$

NDG conditions are:

$S_{P_{G68764}OH} \neq S_{P_{L_{G68733}}^0 OH}$ i.e., lines $P_{G68764}P_{L_{G68733}}^0$ and OH are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^2} \neq S_{F_{\neg h_a}^1 BF_{\neg h_b}^2}$ i.e., lines $AF_{\neg h_a}^1$ and $BF_{\neg h_b}^2$ are not parallel (construction based assumption)

$S_{\neg M_a BC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_b AC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_a \neg M_b F_{\neg m_b}^4} \neq S_{F_{\neg m_a}^3 \neg M_b F_{\neg m_b}^4}$ i.e., lines $\neg M_a F_{\neg m_a}^3$ and $\neg M_b F_{\neg m_b}^4$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b = \neg E_b$

Proving failed

4.2.3 Proving $O = \neg O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $E_b = \neg E_b$

Proving failed

4.3.3 Proving $O = \neg O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $E_b = \neg E_b$

Proving failed

4.4.3 Proving $O = \neg O$

Proving failed

Problem 659

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 659: Given a point B , a point E_b and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 660

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 660: Given a point B , a point E_b and a point T_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point E_b , construct a point H (rule W01); ;
2. Using the point B and the point E_b , construct a line h_b (rule W02); % DET: points B and E_b are not the same;
3. Using the point B and the point T_b , construct a line s_b (rule W02); % DET: points B and T_b are not the same;
4. Using the point T_b and the line h_b , construct a line b (rule W10a); ;
5. Using the point E_b , the point B , the point T_b , the line s_b and the line h_b , construct a line BO (rule W17); % NDG: points B and T_b are not the same; points E_b and B are not the same % DET: points B and T_b are not the same;
6. Using the point E_b and the line BO , construct a line $m(H_aH_c)$ (rule W16); ;
7. Using the line $m(H_aH_c)$ and the line b , construct a point M_b (rule W03); % NDG: lines $m(H_aH_c)$ and b are not parallel % DET: lines $m(H_aH_c)$ and b are not the same;
8. Using the point M_b and the point B , construct a point G (rule W01); ;
9. Using the point H and the point G , construct a point O (rule W01); ;
10. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
11. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; lines $m(H_a H_c)$ and b are not parallel; points B and T_b are not the same; points E_b and B are not the same.

Determination conditions: lines $m(H_a H_c)$ and b are not the same; points B and T_b are not the same; points B and E_b are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W16,W17]

Lemmas used: [D22,D24,D26,D29,D3,D9,GD01,GD02,GL01,GL03,GL04,GL09,GL17,L105,L11,L12,L44,L45,L5]

Solving time: 27.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
```

```
point E_{b} 50 56.36
```

```
point T_{b} 94.25 68.88
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_b B
```

```
cmark_r E_{b}
```

```
cmark_t T_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point H such that BH/BE_{b}=2
```

```
towards H B E_{b} 2
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment B H
```

```
color 0 0 0
```

```
% DET: points B and E_{b} are not the same
```

```
% Constructing a line h_{b} which passes through point B and point E_{b}
```

```
line h_{b} B E_{b}
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```
% DET: points B and T_{b} are not the same
```

```
% Constructing a line s_{b} which passes through point B and point T_{b}
```

```
line s_{b} B T_{b}
```

```
color 200 200 200
```

```
drawline s_{b}
```

```
color 0 0 0
```

```

% Constructing a line b which is perpendicular to line h_{b} and which passes through point T_{b}
perp b T_{b} h_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points B and T_{b} are not the same; points E_{b} and B are not the same% DET: points B and
T_{b} are not the same
% Constructing an angle V[_G120609] which is equal to the angle E_{b}BT_{b}
angle_o V[_G120609] E_{b} B T_{b}

% Calculating value angle[_G120688] using formula  $1/\text{pow}(2,0)*V[_G120609]+0/\text{pow}(2,0)*180$ 
expression angle[_G120688] { 1/pow(2,0)*V[_G120609]+0/pow(2,0)*180 }

% Constructing a point P_{\_G120685} which is an image of the point T_{b} in a rotation around the
point B for the angle  $1/\text{pow}(2,0)*V[_G120609]+0/\text{pow}(2,0)*180$ 
rotate P_{\_G120685} B angle[_G120688] T_{b}
cmark_r P_{\_G120685}
color 200 200 200
drawarc_p B T_{b} angle[_G120688]
color 0 0 0

% Constructing a line BO which passes through point B and point P_{\_G120685}
line BO B P_{\_G120685}

color 200 200 200
drawline BO
color 0 0 0

% Constructing a line m(H_{a}H_{c}) which contains the point E_{b} and is parallel to the line BO
parallel m(H_{a}H_{c}) E_{b} BO

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

% NDG: lines m(H_{a}H_{c}) and b are not parallel% DET: lines m(H_{a}H_{c}) and b are not the same
% Constructing a point M_{b} which belongs to line m(H_{a}H_{c}) and line b
intersec M_{b} m(H_{a}H_{c}) b
cmark_lt M_{b}

% Constructing a line L_{\_G121210} which passes through point M_{b} and point B

```

```

line L_{\_G121210} M_{b} B

color 200 200 200
drawline L_{\_G121210}
color 0 0 0

% Constructing a point P_{\_G121311} with coordinates (0,0)
point P_{\_G121311} 0 0
cmark_r P_{\_G121311}

% Constructing a point P_{\_G121235} such that  $M_{b}P_{\_G121235}/M_{b}P_{\_G121311}=1$ 
towards P_{\_G121235} M_{b} P_{\_G121311} 1
cmark_r P_{\_G121235}
color 200 200 200
drawsegment M_{b} P_{\_G121235}
color 0 0 0

% Constructing a point P_{\_G121280} such that  $M_{b}P_{\_G121280}/M_{b}P_{\_G121311}=3$ 
towards P_{\_G121280} M_{b} P_{\_G121311} 3
cmark_r P_{\_G121280}
color 200 200 200
drawsegment M_{b} P_{\_G121280}
color 0 0 0

% Constructing a line L_{\_G121241} which passes through point B and point P_{\_G121280}
line L_{\_G121241} B P_{\_G121280}

color 200 200 200
drawline L_{\_G121241}
color 0 0 0

% Constructing a line L_{\_G121204} which contains the point P_{\_G121235} and is parallel to the
line L_{\_G121241}
parallel L_{\_G121204} P_{\_G121235} L_{\_G121241}

color 200 200 200
drawline L_{\_G121204}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G121204} and line L_{\_G121210}
intersec G L_{\_G121204} L_{\_G121210}
cmark_t G

% Constructing a point O such that  $HO/HG=1.5$ 
towards O H G 1.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

```

```

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;
% lines m(H_{a}H_{c}) and b are not parallel; points B and T_{b} are not the same; points E_{b}
% and B are not the same
% Determination conditions: lines m(H_{a}H_{c}) and b are not the same; points B and T_{b} are not
% the same; points B and T_{b} are not the same; points B and E_{b} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b = E_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 103 terms.

Time Complexity: Time spent by the prover is 4.026 seconds.

NDG conditions Points A and B are not identical

Points T_b and E_b are not identical

Points B, $P_{G115198}$ and H are not collinear

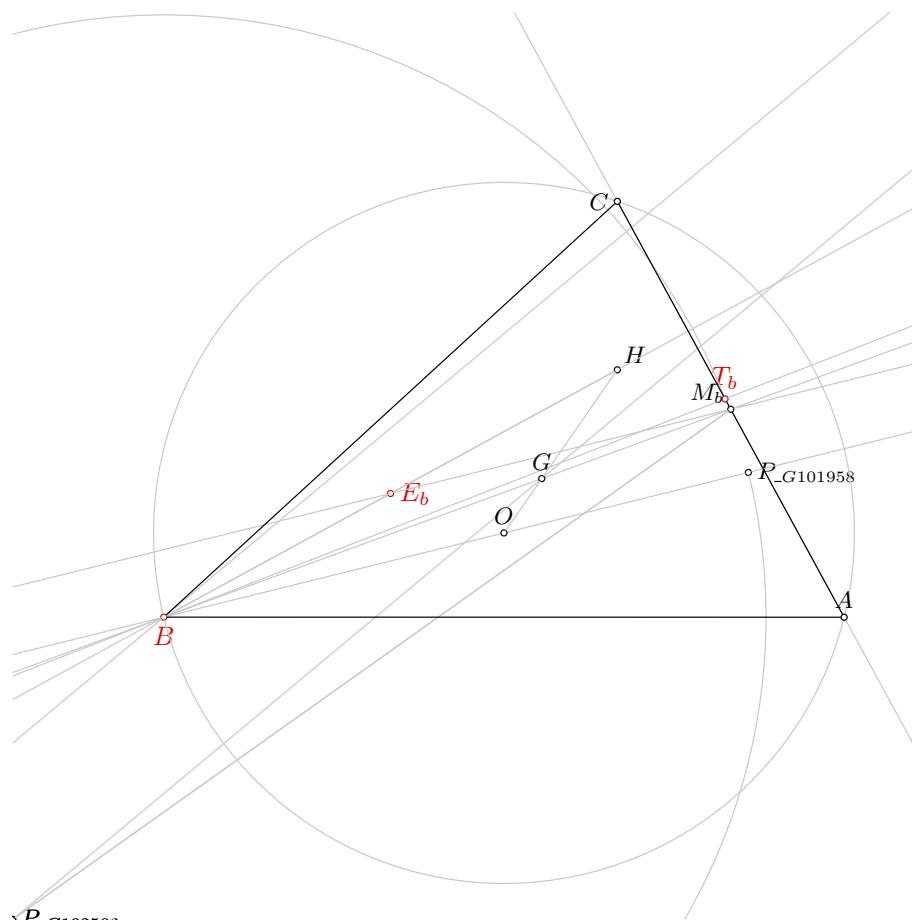


Figure 1: Illustration of the problem 0660

Points M_b , $P_{G115596}$ and B are not collinear

Points $P_{G115596}$ and B are not identical

Points A , B and C are not collinear

4.1.3 Proving $T_b = \neg T_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B = B$

NDG conditions are:

$S_{T_b B E_b} \neq 0$ i.e., points T_b , B and E_b are not collinear (foot is not the point itself; construction based assumption)

$S_{E_b T_b F_b^0} \neq S_{P_{m(H_a H_c)}^1 T_b F_b^0}$ i.e., lines $E_b P_{m(H_a H_c)}^1$ and $T_b F_b^0$ are not parallel (construction based assumption)

$S_{P_{G104383} M_b B} \neq S_{P_{L_{G104352}}^2 M_b B}$ i.e., lines $P_{G104383} P_{L_{G104352}}^2$ and $M_b B$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{AB F_{-h_b}^4} \neq S_{F_{-h_a}^3 B F_{-h_b}^4}$ i.e., lines $A F_{-h_a}^3$ and $B F_{-h_b}^4$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b = \neg E_b$

Proving failed

4.2.3 Proving $T_b = \neg T_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.2 Proving $E_b = \neg E_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 3720 terms.

Time Complexity: Time spent by the prover is 8.080 seconds. There are no ndg conditions.

4.3.3 Proving $T_b = \neg T_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 8 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $E_b=\neg E_b$

Proving failed

4.4.3 Proving $T_b=\neg T_b$

Proving failed

Problem 661

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 661: Given a point B , a point E_b and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 662

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 662: Given a point B , a point E_c and a point G , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 663

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 663: Given a point B , a point E_c and a point H , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point H , construct a point E_b (rule W01); ;
2. Using the point E_c and the point H , construct a point C (rule W01); ;
3. Using the point B and the point C , construct a line a (rule W02); % DET: points B and C are not the same;
4. Using the point E_c and the point H , construct a line h_c (rule W02); % DET: points E_c and H are not the same;
5. Using the point B and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points B and E_b are not the same;
6. Using the circle $k(E_b, B)$, the line a , the point E_b and the point B , construct a point H_a (rule W05); % NDG: line a and circle $k(E_b, B)$ intersect % DET: points B and H_a must be different;
7. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
8. Using the circle $k(E_b, B)$, the line h_c , the point E_b and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_b, B)$ intersect % DET: points H and H_c must be different;
9. Using the point H_c and the point B , construct a line c (rule W02); % DET: points H_c and B are not the same;
10. Using the line h_a and the line c , construct a point A (rule W03); % NDG: lines h_a and c are not parallel % DET: lines h_a and c are not the same.

Non-degenerate conditions: lines h_a and c are not parallel; line h_c and circle $k(E_b, B)$ intersect; line a and circle $k(E_b, B)$ intersect; points B and E_b are not the same.

Determination conditions: lines h_a and c are not the same; points H_c and B are not the same; points H and H_c must be different; points H_a and H are not the same; points B and H_a must be different; points E_c and H are not the same; points B and C are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D29,D3,D30,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L49,L50,L51]

Solving time: 10.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point E_{c} 95 56.36
point H 80 72.73
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_r E_{c}
cmark_rt H
color 0 0 0
fontsize 8
```

```
% Constructing a point E_{b} such that BE_{b}/BH=0.5
towards E_{b} B H 0.5
cmark_r E_{b}
color 200 200 200
drawsegment B H
color 0 0 0
```

```
% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0
```

```
% DET: points B and C are not the same
% Constructing a line a which passes through point B and point C
line a B C
```

```
color 200 200 200
drawline a
color 0 0 0
```

```

% DET: points E_{c} and H are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H
line h_{c} E_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points B and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point B
circle k(E_{b},B) E_{b} B

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line a and circle k(E_{b},B) intersect% DET: points B and H_{a} must be different
% Constructing a point P_{\_G154117} which is a foot of the point E_{b} on the line a
foot P_{\_G154117} E_{b} a
cmark_r P_{\_G154117}
color 200 200 200
drawline E_{b} P_{\_G154117}
color 0 0 0

% Constructing a point H_{a} which is an image of the point B in the symmetry to point/line P_{\_G
154117}
sim H_{a} P_{\_G154117} B
cmark_r H_{a}

% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{c} and circle k(E_{b},B) intersect% DET: points H and H_{c} must be different
% Constructing a point P_{\_G154355} which is a foot of the point E_{b} on the line h_{c}
foot P_{\_G154355} E_{b} h_{c}
cmark_r P_{\_G154355}
color 200 200 200
drawline E_{b} P_{\_G154355}
color 0 0 0

% Constructing a point H_{c} which is an image of the point H in the symmetry to point/line P_{\_G
154355}
sim H_{c} P_{\_G154355} H
cmark_rt H_{c}

```

```

% DET: points  $H_{\{c\}}$  and  $B$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $B$ 
line c  $H_{\{c\}}$  B

color 200 200 200
drawline c
color 0 0 0

% NDG: lines  $h_{\{a\}}$  and  $c$  are not parallel% DET: lines  $h_{\{a\}}$  and  $c$  are not the same
% Constructing a point  $A$  which belongs to line  $h_{\{a\}}$  and line  $c$ 
intersec A  $h_{\{a\}}$  c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{a\}}$  and  $c$  are not parallel; line  $h_{\{c\}}$  and circle  $k(E_{\{b\}}, B)$ 
% intersect; line  $a$  and circle  $k(E_{\{b\}}, B)$  intersect; points  $B$  and  $E_{\{b\}}$  are not the same
% Determination conditions: lines  $h_{\{a\}}$  and  $c$  are not the same; points  $H_{\{c\}}$  and  $B$  are not the same
% ; points  $H$  and  $H_{\{c\}}$  must be different; points  $H_{\{a\}}$  and  $H$  are not the same; points  $B$  and  $H_{\{a\}}$ 
% must be different; points  $E_{\{c\}}$  and  $H$  are not the same; points  $B$  and  $C$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c=_E_c$

Proving failed

4.1.3 Proving $H=_H$

Proving failed

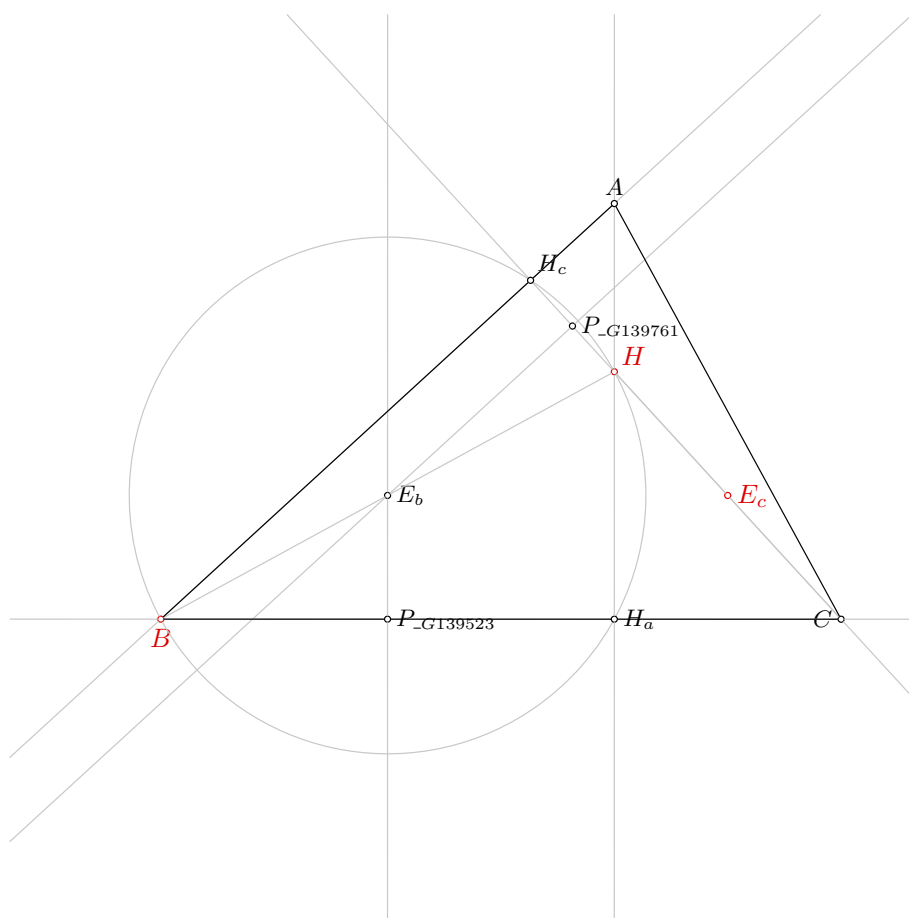


Figure 1: Illustration of the problem 0663

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_aH_cB} \neq S_{HH_cB}$ i.e., lines H_aH and H_cB are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c=_E_c$

Proving failed

4.2.3 Proving $H=_H$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_c=_E_c$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 2174 terms.

Time Complexity: Time spent by the prover is 1.680 seconds. There are no ndg conditions.

4.3.3 Proving $H=_H$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 315 terms.

Time Complexity: Time spent by the prover is 0.270 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_c=_E_c$

Proving failed

4.4.3 Proving $H=_H$

Proving failed

Problem 664

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 664: Given a point B , a point E_c and a point H_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point H_a , construct a line a (rule W02); % DET: points B and H_a are not the same;
2. Using the point H_a and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_a and E_c are not the same;
3. Using the circle $k(E_c, C)$, the line a , the point E_c and the point H_a , construct a point C (rule W05); % NDG: line a and circle $k(E_c, C)$ intersect % DET: points H_a and C must be different;
4. Using the point E_c and the point C , construct a point H (rule W01); ;
5. Using the point B and the point H , construct a line h_b (rule W02); % DET: points B and H are not the same;
6. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
7. Using the circle $k(E_c, C)$, the line h_b , the point E_c and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_c, C)$ intersect % DET: points H and H_b must be different;
8. Using the point C and the point H_b , construct a line b (rule W02); % DET: points C and H_b are not the same;
9. Using the line h_a and the line b , construct a point A (rule W03); % NDG: lines h_a and b are not parallel % DET: lines h_a and b are not the same.

Non-degenerate conditions: lines h_a and b are not parallel; line h_b and circle $k(E_c, C)$ intersect; line a and circle $k(E_c, C)$ intersect; points H_a and E_c are not the same.

Determination conditions: lines h_a and b are not the same; points C and H_b are not the same; points H and H_b must be different; points H_a and H are not the same; points B and H are not the same; points H_a and C must be different; points B and H_a are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,GL04,L52,L53,L54]

Solving time: 8.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point E_{c} 95 56.36
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_b B
cmark_r E_{c}
cmark_r H_{a}
color 0 0 0
fontsize 8

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points H_{a} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H_{a}
circle k(E_{c},C) E_{c} H_{a}

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% NDG: line a and circle k(E_{c},C) intersect% DET: points H_{a} and C must be different
% Constructing a point P_{\_G177504} which is a foot of the point E_{c} on the line a
foot P_{\_G177504} E_{c} a
cmark_r P_{\_G177504}
color 200 200 200
drawline E_{c} P_{\_G177504}
color 0 0 0
```

```

% Constructing a point C which is an image of the point H_{a} in the symmetry to point/line P_{\_G
177504}
sim C P_{\_G177504} H_{a}
cmark_l C

% Constructing a point H such that E_{c}H/E_{c}C=-1
towards H E_{c} C -1
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0

% DET: points B and H are not the same
% Constructing a line h_{b} which passes through point B and point H
line h_{b} B H

color 200 200 200
drawline h_{b}
color 0 0 0

% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{b} and circle k(E_{c},C) intersect% DET: points H and H_{b} must be different
% Constructing a point P_{\_G177867} which is a foot of the point E_{c} on the line h_{b}
foot P_{\_G177867} E_{c} h_{b}
cmark_r P_{\_G177867}
color 200 200 200
drawline E_{c} P_{\_G177867}
color 0 0 0

% Constructing a point H_{b} which is an image of the point H in the symmetry to point/line P_{\_G
177867}
sim H_{b} P_{\_G177867} H
cmark_l H_{b}

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b

```

```
color 0 0 0
```

```
% NDG: lines h_{a} and b are not parallel% DET: lines h_{a} and b are not the same
% Constructing a point A which belongs to line h_{a} and line b
intersec A h_{a} b
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines h_{a} and b are not parallel; line h_{b} and circle k(E_{c},C)
    intersect; line a and circle k(E_{c},C) intersect; points H_{a} and E_{c} are not the same
% Determination conditions: lines h_{a} and b are not the same; points C and H_{b} are not the same
    ; points H and H_{b} must be different; points H_{a} and H are not the same; points B and H are
    not the same; points H_{a} and C must be different; points B and H_{a} are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c=_E_c$

Proving failed

4.1.3 Proving $H_a=_H_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 1176 terms.

Time Complexity: Time spent by the prover is 6.569 seconds.

NDG conditions Points B and H_a are not identical

Points B and H_a are not identical

Points B and H are not identical

Points B and H are not identical

Line through points H_b and C is not parallel with line through points H and H_a

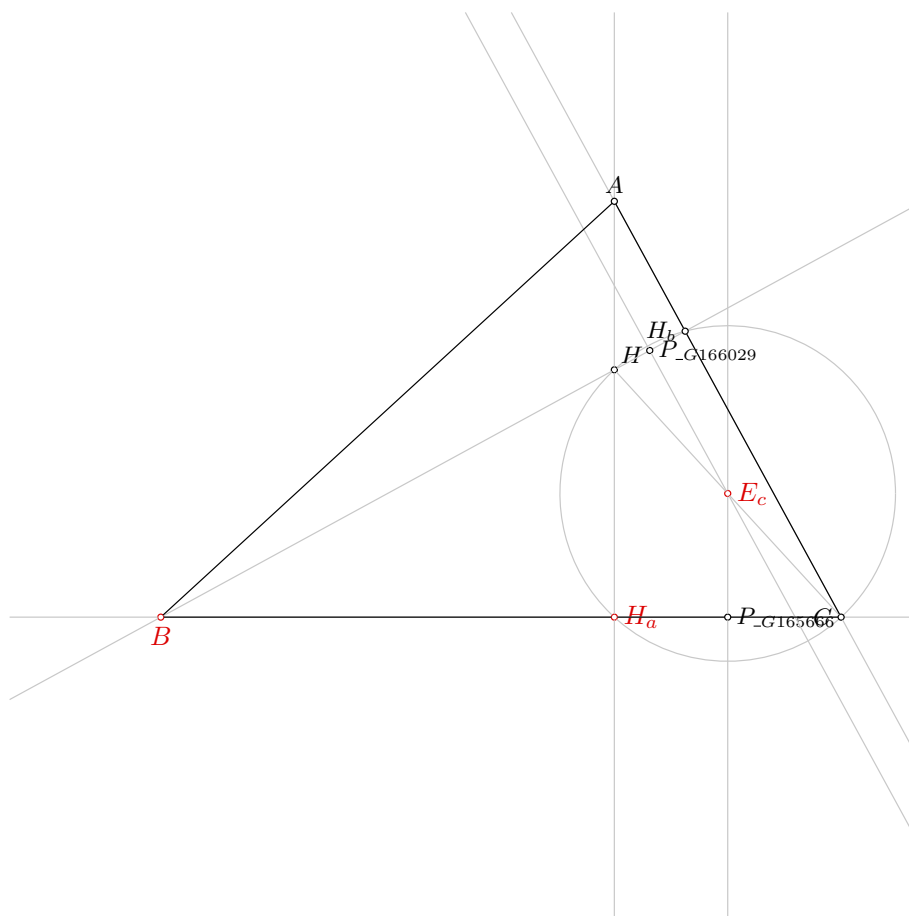


Figure 1: Illustration of the problem 0664

Line through points E_c and B is not parallel with line through points H and H_a

Points B and C are not identical

Points E_c , B and C are not collinear

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_aCH_b} \neq S_{HCH_b}$ i.e., lines H_aH and CH_b are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a}BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

$S_{BAF^0_{-h_a}} \neq S_{CAF^0_{-h_a}}$ i.e., lines BC and $AF^0_{-h_a}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c=_E E_c$

Proving failed

4.2.3 Proving $H_a=_H H_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Proving failed

4.3.2 Proving $E_c=_E E_c$

Proving failed

4.3.3 Proving $H_a=_H H_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Proving failed

4.4.2 Proving $E_c=_E E_c$

Proving failed

4.4.3 Proving $H_a=_H H_a$

Proving failed

Problem 665

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 665: Given a point B , a point E_c and a point H_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point H_b , construct a line h_b (rule W02); % DET: points B and H_b are not the same;
2. Using the point H_b and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_b and E_c are not the same;
3. Using the circle $k(E_c, C)$, the line h_b , the point E_c and the point H_b , construct a point H (rule W05); % NDG: line h_b and circle $k(E_c, C)$ intersect % DET: points H_b and H must be different;
4. Using the point E_c and the point H , construct a point C (rule W01); ;
5. Using the point B and the point C , construct a line a (rule W02); % DET: points B and C are not the same;
6. Using the point H_b and the point C , construct a line b (rule W02); % DET: points H_b and C are not the same;
7. Using the circle $k(E_c, C)$, the line a , the point E_c and the point C , construct a point H_a (rule W05); % NDG: line a and circle $k(E_c, C)$ intersect % DET: points C and H_a must be different;
8. Using the point H and the point H_a , construct a line h_a (rule W02); % DET: points H and H_a are not the same;
9. Using the line b and the line h_a , construct a point A (rule W03); % NDG: lines b and h_a are not parallel % DET: lines b and h_a are not the same.

Non-degenerate conditions: lines b and h_a are not parallel; line a and circle $k(E_c, C)$ intersect; line h_b and circle $k(E_c, C)$ intersect; points H_b and E_c are not the same.

Determination conditions: lines b and h_a are not the same; points H and H_a are not the same; points C and H_a must be different; points H_b and C are not the same; points B and C are not the same; points H_b and H must be different; points B and H_b are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,GL04,L52,L53,L54]

Solving time: 8.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point E_{c} 95 56.36
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_b B
cmark_r E_{c}
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points B and H_{b} are not the same
% Constructing a line h_{b} which passes through point B and point H_{b}
line h_{b} B H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points H_{b} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H_{b}
circle k(E_{c},C) E_{c} H_{b}

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% NDG: line h_{b} and circle k(E_{c},C) intersect% DET: points H_{b} and H must be different
% Constructing a point P_{\_G200167} which is a foot of the point E_{c} on the line h_{b}
foot P_{\_G200167} E_{c} h_{b}
cmark_r P_{\_G200167}
color 200 200 200
drawline E_{c} P_{\_G200167}
color 0 0 0
```

```

% Constructing a point H which is an image of the point H_{b} in the symmetry to point/line P_{\_G
200167}
sim H P_{\_G200167} H_{b}
cmark_rt H

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

% DET: points B and C are not the same
% Constructing a line a which passes through point B and point C
line a B C

color 200 200 200
drawline a
color 0 0 0

% DET: points H_{b} and C are not the same
% Constructing a line b which passes through point H_{b} and point C
line b H_{b} C

color 200 200 200
drawline b
color 0 0 0

% NDG: line a and circle k(E_{c},C) intersect% DET: points C and H_{a} must be different
% Constructing a point P_{\_G200530} which is a foot of the point E_{c} on the line a
foot P_{\_G200530} E_{c} a
cmark_r P_{\_G200530}
color 200 200 200
drawline E_{c} P_{\_G200530}
color 0 0 0

% Constructing a point H_{a} which is an image of the point C in the symmetry to point/line P_{\_G
200530}
sim H_{a} P_{\_G200530} C
cmark_r H_{a}

% DET: points H and H_{a} are not the same
% Constructing a line h_{a} which passes through point H and point H_{a}
line h_{a} H H_{a}

color 200 200 200
drawline h_{a}

```

```
color 0 0 0
```

```
% NDG: lines b and h_{a} are not parallel% DET: lines b and h_{a} are not the same
% Constructing a point A which belongs to line b and line h_{a}
intersec A b h_{a}
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines b and h_{a} are not parallel; line a and circle k(E_{c},C)
    intersect; line h_{b} and circle k(E_{c},C) intersect; points H_{b} and E_{c} are not the same
% Determination conditions: lines b and h_{a} are not the same; points H and H_{a} are not the same
    ; points C and H_{a} must be different; points H_{b} and C are not the same; points B and C are
    not the same; points H_{b} and H must be different; points B and H_{b} are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c = E_c$

Proving failed

4.1.3 Proving $H_b = H_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_b H H_a} \neq S_{C H H_a}$ i.e., lines $H_b C$ and $H H_a$ are not parallel (construction based assumption)
 $S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{AB F_{-h_b}^1} \neq S_{F_{-h_a}^0 F_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

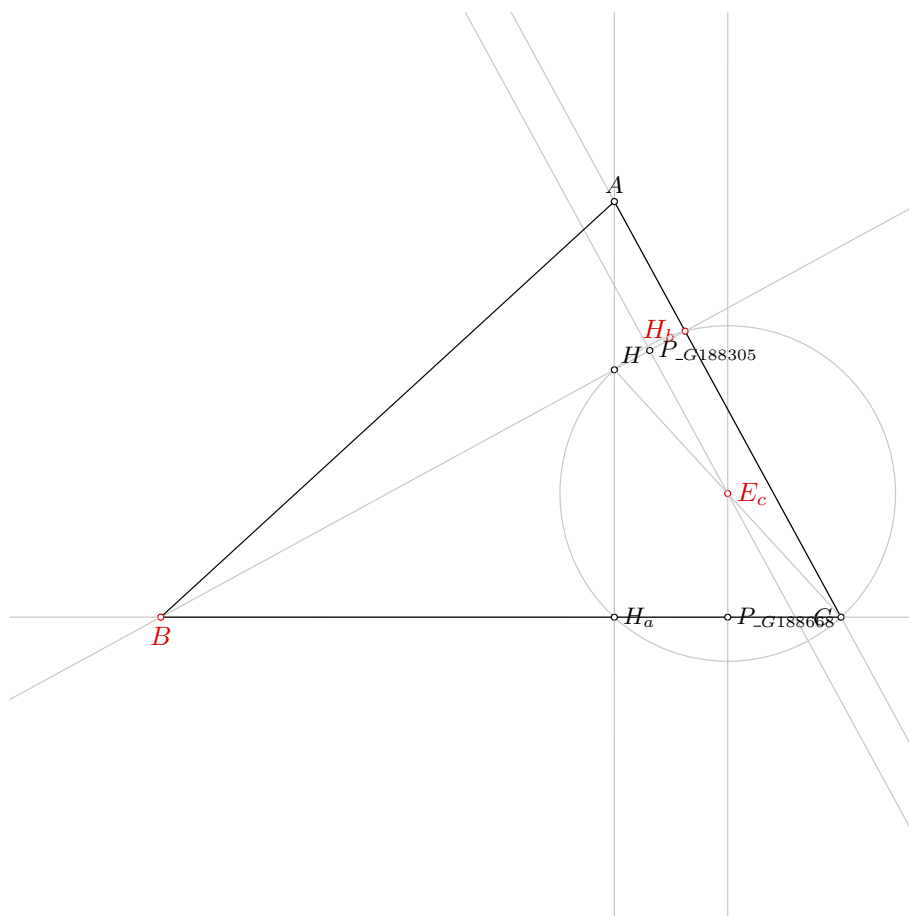


Figure 1: Illustration of the problem 0665

$S_{ABF^1_{\neg h_b}} \neq S_{CBF^1_{\neg h_b}}$ i.e., lines AC and $BF^1_{\neg h_b}$ are not parallel (construction based assumption)
Total number of proof steps: 1
Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c = \neg E_c$

Proving failed

4.2.3 Proving $H_b = \neg H_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B = B$

Proving failed

4.3.2 Proving $E_c = \neg E_c$

Proving failed

4.3.3 Proving $H_b = \neg H_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B = B$

Proving failed

4.4.2 Proving $E_c = \neg E_c$

Proving failed

4.4.3 Proving $H_b = \neg H_b$

Proving failed

Problem 666

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 666: Given a point E_c , a point H_c and a point B , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;
2. Using the point H_c and the line h_c , construct a line c (rule W10a); ;
3. Choose freely a point B on the line c (rule WOnline2);
4. Using the point B and the point H_c , construct a line $m(BH_c)$ (rule W14); % DET: points B and H_c are not the same;
5. Using the point E_c and the point H_c , construct a line $m(E_cH_c)$ (rule W14); % DET: points E_c and H_c are not the same;
6. Choose freely a point A on the line c (rule WOnline1) ;
7. Using the point A and the point B , construct a point M_c (rule W01); ;
8. Using the point E_c and the point M_c , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and M_c are not the same;
9. Using the line $m(E_cH_c)$ and the line $m(H_bH_a)$, construct a point N (rule W03); % NDG: lines $m(E_cH_c)$ and $m(H_bH_a)$ are not parallel % DET: lines $m(E_cH_c)$ and $m(H_bH_a)$ are not the same;
10. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
11. Using the circle $k(N, M_a)$ and the line $m(BH_c)$, construct a point M_a and a point E_b (rule W04); % NDG: line $m(BH_c)$ and circle $k(N, M_a)$ intersect;

12. Using the point B and the point M_a , construct a point C (rule W01); .

Non-degenerate conditions: line $m(BH_c)$ and circle $k(N, M_a)$ intersect; points E_c and N are not the same; lines $m(E_cH_c)$ and $m(H_bH_a)$ are not parallel.

Determination conditions: lines $m(E_cH_c)$ and $m(H_bH_a)$ are not the same; points E_c and M_c are not the same; points E_c and H_c are not the same; points B and H_c are not the same; points E_c and H_c are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W14,WOnline1,WOnline2]

Lemmas used: [D10,D20,D21,D30,D32,D7,GD02,GL01,GL03,GL09,L19,L20,L21,L23,L24,L3,L39,L41,L42,L51,

Solving time: 54.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
point H_{c} 68.91 84.83
point B 20 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{c}
cmark_rt H_{c}
cmark_b B
color 0 0 0
fontsize 8
```

```
% DET: points E_{c} and H_{c} are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H_{c}
line h_{c} E_{c} H_{c}
```

```
color 200 200 200
drawline h_{c}
color 0 0 0
```

```
% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}
```

```
color 200 200 200
drawline c
color 0 0 0
```

```
% Generating random value V[_G224437]
random V[_G224437]
```

```

% Calculating value  $V[_{G224458}]$  using formula  $V[_{G224437}]*20$ 
expression  $V[_{G224458}]$  {  $V[_{G224437}]*20$  }

% Constructing a point B which is a point for which holds  $H_{\{c\}}B = V[_{G224458}]$  and angle  $E_{\{c\}}H_{\{c\}}$ 
 $B = 90$ 
turtle B  $E_{\{c\}}$   $H_{\{c\}}$  90  $V[_{G224458}]$ 
cmark_b B

% DET: points B and  $H_{\{c\}}$  are not the same
% Constructing bisector  $m(BH_{\{c\}})$  of the segment  $BH_{\{c\}}$ 
med m( $BH_{\{c\}}$ ) B  $H_{\{c\}}$ 

color 200 200 200
drawline m( $BH_{\{c\}}$ )
color 0 0 0

color 200 200 200
drawsegment B  $H_{\{c\}}$ 
color 0 0 0

% DET: points  $E_{\{c\}}$  and  $H_{\{c\}}$  are not the same
% Constructing bisector  $m(E_{\{c\}}H_{\{c\}})$  of the segment  $E_{\{c\}}H_{\{c\}}$ 
med m( $E_{\{c\}}H_{\{c\}}$ )  $E_{\{c\}}$   $H_{\{c\}}$ 

color 200 200 200
drawline m( $E_{\{c\}}H_{\{c\}}$ )
color 0 0 0

color 200 200 200
drawsegment  $E_{\{c\}}$   $H_{\{c\}}$ 
color 0 0 0

% Choosing randomly a point A on the line  $BH_{\{c\}}$ 
online A B  $H_{\{c\}}$ 
cmark_t A
color 200 200 200
drawline B  $H_{\{c\}}$ 
color 0 0 0

% Constructing a point  $M_{\{c\}}$  such that  $AM_{\{c\}}/AB=0.5$ 
towards  $M_{\{c\}}$  A B 0.5
cmark_lt  $M_{\{c\}}$ 
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points  $E_{\{c\}}$  and  $M_{\{c\}}$  are not the same
% Constructing a line  $m(H_{\{b\}}H_{\{a\}})$  which passes through point  $E_{\{c\}}$  and point  $M_{\{c\}}$ 

```



```

line m(H_{b}H_{a}) E_{c} M_{c}

color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0

% NDG: lines m(E_{c}H_{c}) and m(H_{b}H_{a}) are not parallel% DET: lines m(E_{c}H_{c}) and m(H_{b}
H_{a}) are not the same
% Constructing a point N which belongs to line m(E_{c}H_{c}) and line m(H_{b}H_{a})
intersec N m(E_{c}H_{c}) m(H_{b}H_{a})
cmark_r N

% NDG: points E_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{c}
circle k(N,M_{a}) N E_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(BH_{c}) and circle k(N,M_{a}) intersect
% Constructing points M_{a} and E_{b} which are in intersection of k(N,M_{a}) and m(BH_{c})
intersec2 M_{a} E_{b} k(N,M_{a}) m(BH_{c})
cmark_r M_{a}
cmark_r E_{b}

% Constructing a point C such that BC/BM_{a}=2
towards C B M_{a} 2
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line m(BH_{c}) and circle k(N,M_{a}) intersect; points E_{c} and N are
not the same; lines m(E_{c}H_{c}) and m(H_{b}H_{a}) are not parallel
% Determination conditions: lines m(E_{c}H_{c}) and m(H_{b}H_{a}) are not the same; points E_{c}
and M_{c} are not the same; points E_{c} and H_{c} are not the same; points B and H_{c} are not
the same; points E_{c} and H_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

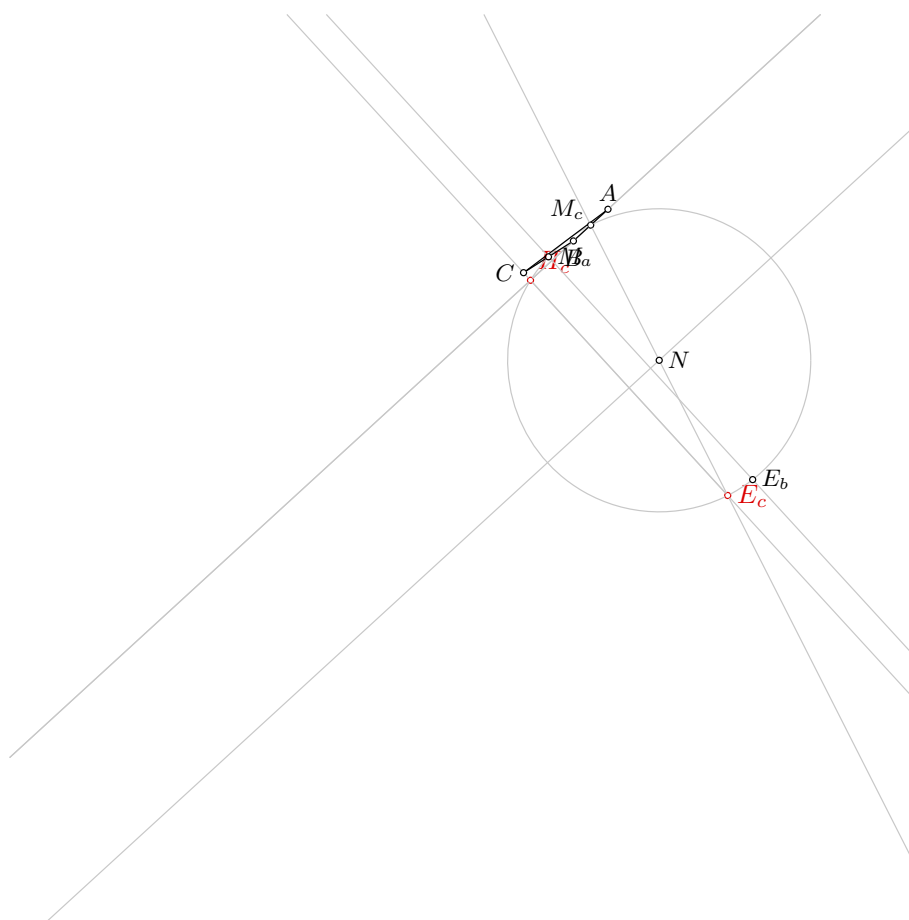


Figure 1: Illustration of the problem 0666

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 59 terms.

Time Complexity: Time spent by the prover is 0.669 seconds.

NDG conditions Points A and B are not identical

Points A , B and C are not collinear

Point M_c is not the midpoint of segment with endpoints B and C

4.1.2 Proving $H_c = \neg H_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 4 terms.

Time Complexity: Time spent by the prover is 0.119 seconds.

NDG conditions Points A and B are not identical

4.1.3 Proving $B = B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.007 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $H_c = \neg H_c$

Proving failed

4.2.3 Proving $B = B$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $H_c = \neg H_c$

Proving failed

4.3.3 Proving $B=B$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c=E_c$

Proving failed

4.4.2 Proving $H_c=H_c$

Proving failed

4.4.3 Proving $B=B$

Proving failed

Problem 667

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 667: Given a point B , a point E_c and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 668

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 668: Given a point B , a point E_c and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point M_a , construct a point C (rule W01); ;
2. Using the point E_c and the point C , construct a point H (rule W01); ;
3. Using the point B and the point H , construct a line h_b (rule W02); % DET: points B and H are not the same;
4. Using the point E_c and the point C , construct a line h_c (rule W02); % DET: points E_c and C are not the same;
5. Using the point B and the point M_a , construct a circle $k(M_a, B)$ (rule W06); % NDG: points B and M_a are not the same;
6. Using the circle $k(M_a, B)$, the line h_b , the point M_a and the point B , construct a point H_b (rule W05); % NDG: line h_b and circle $k(M_a, B)$ intersect % DET: points B and H_b must be different;
7. Using the point H_b and the point C , construct a line b (rule W02); % DET: points H_b and C are not the same;
8. Using the circle $k(M_a, B)$, the line h_c , the point M_a and the point C , construct a point H_c (rule W05); % NDG: line h_c and circle $k(M_a, B)$ intersect % DET: points C and H_c must be different;
9. Using the point H_c and the point B , construct a line c (rule W02); % DET: points H_c and B are not the same;
10. Using the line b and the line c , construct a point A (rule W03); % NDG: lines b and c are not parallel % DET: lines b and c are not the same.

Non-degenerate conditions: lines b and c are not parallel; line h_c and circle $k(M_a, B)$ intersect; line h_b and circle $k(M_a, B)$ intersect; points B and M_a are not the same.

Determination conditions: lines b and c are not the same; points H_c and B are not the same; points C and H_c must be different; points H_b and C are not the same; points B and H_b must be different; points E_c and C are not the same; points B and H are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D21,D3,D30,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L37,L38,L39]

Solving time: 10.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
```

```
point E_{c} 95 56.36
```

```
point M_{a} 65 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_b B
```

```
cmark_r E_{c}
```

```
cmark_r M_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point C such that BC/BM_{a}=2
```

```
towards C B M_{a} 2
```

```
cmark_l C
```

```
color 200 200 200
```

```
drawsegment B C
```

```
color 0 0 0
```

```
% Constructing a point H such that E_{c}H/E_{c}C=-1
```

```
towards H E_{c} C -1
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment C H
```

```
color 0 0 0
```

```
% DET: points B and H are not the same
```

```
% Constructing a line h_{b} which passes through point B and point H
```

```
line h_{b} B H
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```

% DET: points E_{c} and C are not the same
% Constructing a line h_{c} which passes through point E_{c} and point C
line h_{c} E_{c} C

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points B and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point B
circle k(M_{a},B) M_{a} B

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: line h_{b} and circle k(M_{a},B) intersect% DET: points B and H_{b} must be different
% Constructing a point P_{\_G251377} which is a foot of the point M_{a} on the line h_{b}
foot P_{\_G251377} M_{a} h_{b}
cmark_r P_{\_G251377}
color 200 200 200
drawline M_{a} P_{\_G251377}
color 0 0 0

% Constructing a point H_{b} which is an image of the point B in the symmetry to point/line P_{\_G
251377}
sim H_{b} P_{\_G251377} B
cmark_l H_{b}

% DET: points H_{b} and C are not the same
% Constructing a line b which passes through point H_{b} and point C
line b H_{b} C

color 200 200 200
drawline b
color 0 0 0

% NDG: line h_{c} and circle k(M_{a},B) intersect% DET: points C and H_{c} must be different
% Constructing a point P_{\_G251615} which is a foot of the point M_{a} on the line h_{c}
foot P_{\_G251615} M_{a} h_{c}
cmark_r P_{\_G251615}
color 200 200 200
drawline M_{a} P_{\_G251615}
color 0 0 0

% Constructing a point H_{c} which is an image of the point C in the symmetry to point/line P_{\_G
251615}
sim H_{c} P_{\_G251615} C
cmark_rt H_{c}

```



```

% DET: points  $H_{\{c\}}$  and  $B$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $B$ 
line c  $H_{\{c\}}$  B

color 200 200 200
drawline c
color 0 0 0

% NDG: lines  $b$  and  $c$  are not parallel% DET: lines  $b$  and  $c$  are not the same
% Constructing a point  $A$  which belongs to line  $b$  and line  $c$ 
intersec A b c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $b$  and  $c$  are not parallel; line  $h_{\{c\}}$  and circle  $k(M_{\{a\}}, B)$ 
% intersect; line  $h_{\{b\}}$  and circle  $k(M_{\{a\}}, B)$  intersect; points  $B$  and  $M_{\{a\}}$  are not the same
% Determination conditions: lines  $b$  and  $c$  are not the same; points  $H_{\{c\}}$  and  $B$  are not the same;
% points  $C$  and  $H_{\{c\}}$  must be different; points  $H_{\{b\}}$  and  $C$  are not the same; points  $B$  and  $H_{\{b\}}$ 
% must be different; points  $E_{\{c\}}$  and  $C$  are not the same; points  $B$  and  $H$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.033 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c = \neg E_c$

Proving failed

4.1.3 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

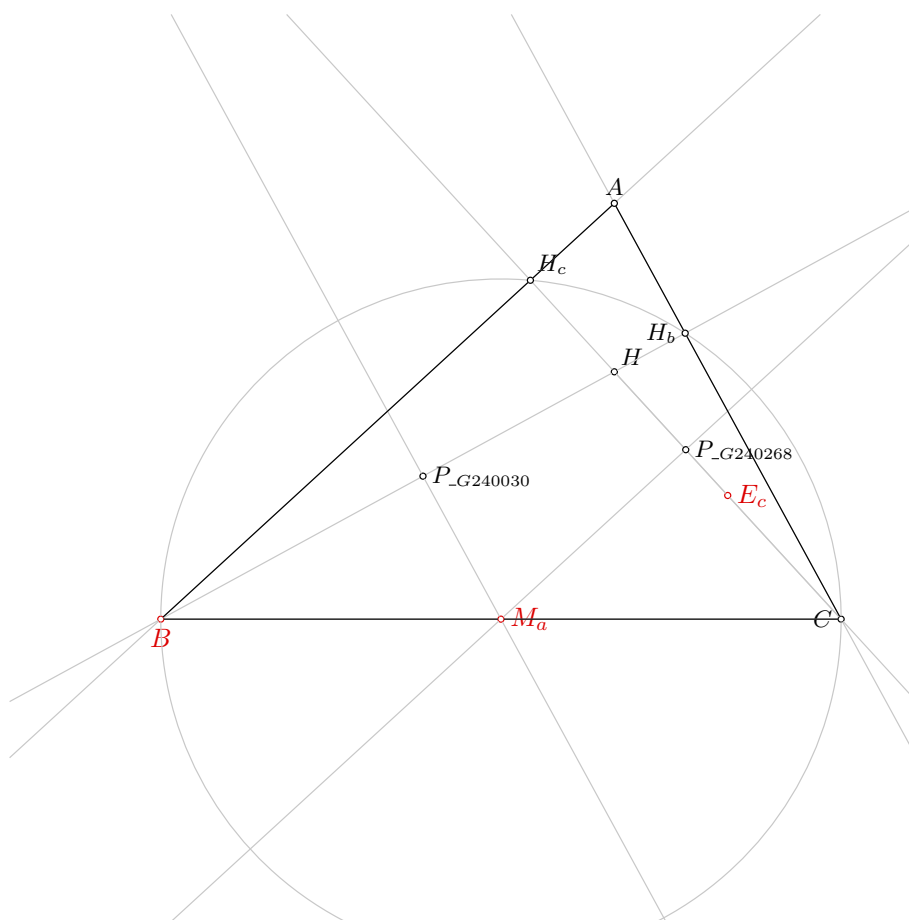


Figure 1: Illustration of the problem 0668

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{H_bH_cB} \neq S_{CH_cB}$ i.e., lines H_bC and H_cB are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c=_E_c$

Proving failed

4.2.3 Proving $M_a=_M_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_c=_E_c$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 331 terms.

Time Complexity: Time spent by the prover is 1.070 seconds. There are no ndg conditions.

4.3.3 Proving $M_a=_M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 9 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_c=_E_c$

Proving failed

4.4.3 Proving $M_a = -M_a$

Proving failed

Problem 669

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 669: Given a point B , a point E_c and a point M_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 670

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 670: Given a point B , a point E_c and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point M_c , construct a point A (rule W01); ;
2. Using the point B and the point M_c , construct a line c (rule W02); % DET: points B and M_c are not the same;
3. Using the point E_c and the point M_c , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and M_c are not the same;
4. Using the point E_c and the line c , construct a line h_c (rule W10b); ;
5. Using the line h_c and the line c , construct a point H_c (rule W03); % NDG: lines h_c and c are not parallel % DET: lines h_c and c are not the same;
6. Using the point B and the point H_c , construct a line $m(BH_c)$ (rule W14); % DET: points B and H_c are not the same;
7. Using the point E_c and the point M_c , construct a line $m(E_cM_c)$ (rule W14); % DET: points E_c and M_c are not the same;
8. Using the line $m(E_cM_c)$ and the line $m(H_bH_a)$, construct a point N (rule W03); % NDG: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel % DET: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same;
9. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
10. Using the circle $k(N, M_a)$ and the line $m(BH_c)$, construct a point M_a and a point E_b (rule W04); % NDG: line $m(BH_c)$ and circle $k(N, M_a)$ intersect;

11. Using the point M_a and the point B , construct a point C (rule W01); .

Non-degenerate conditions: line $m(BH_c)$ and circle $k(N, M_a)$ intersect; points E_c and N are not the same; lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel; lines h_c and c are not parallel.

Determination conditions: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same; points E_c and M_c are not the same; points B and H_c are not the same; lines h_c and c are not the same; points E_c and M_c are not the same; points B and M_c are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14]

Lemmas used: [D10,D20,D21,D30,D32,D7,GD01,GD02,GL01,GL03,GL04,GL09,L18,L19,L20,L23,L24,L3,L39,I

Solving time: 2.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point E_{c} 95 56.36
point M_{c} 50 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_r E_{c}
cmark_lt M_{c}
color 0 0 0
fontsize 8
```

```
% Constructing a point A such that BA/BM_{c}=2
towards A B M_{c} 2
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0
```

```
% DET: points B and M_{c} are not the same
% Constructing a line c which passes through point B and point M_{c}
line c B M_{c}
```

```
color 200 200 200
drawline c
color 0 0 0
```

```
% DET: points E_{c} and M_{c} are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point M_{c}
line m(H_{b}H_{a}) E_{c} M_{c}
```

```
color 200 200 200
drawline m(H_{b}H_{a})
```

```
color 0 0 0
```

```
% Constructing a line  $h_{\{c\}}$  which is perpendicular to line  $c$  and which passes through point  $E_{\{c\}}$ 
perp  $h_{\{c\}}$   $E_{\{c\}}$   $c$ 
```

```
color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0
```

```
% NDG: lines  $h_{\{c\}}$  and  $c$  are not parallel% DET: lines  $h_{\{c\}}$  and  $c$  are not the same
% Constructing a point  $H_{\{c\}}$  which belongs to line  $h_{\{c\}}$  and line  $c$ 
intersec  $H_{\{c\}}$   $h_{\{c\}}$   $c$ 
cmark_rt  $H_{\{c\}}$ 
```

```
% DET: points  $B$  and  $H_{\{c\}}$  are not the same
% Constructing bisector  $m(BH_{\{c\}})$  of the segment  $BH_{\{c\}}$ 
med  $m(BH_{\{c\}})$   $B$   $H_{\{c\}}$ 
```

```
color 200 200 200
drawline  $m(BH_{\{c\}})$ 
color 0 0 0
```

```
color 200 200 200
drawsegment  $B$   $H_{\{c\}}$ 
color 0 0 0
```

```
% DET: points  $E_{\{c\}}$  and  $M_{\{c\}}$  are not the same
% Constructing bisector  $m(E_{\{c\}}M_{\{c\}})$  of the segment  $E_{\{c\}}M_{\{c\}}$ 
med  $m(E_{\{c\}}M_{\{c\}})$   $E_{\{c\}}$   $M_{\{c\}}$ 
```

```
color 200 200 200
drawline  $m(E_{\{c\}}M_{\{c\}})$ 
color 0 0 0
```

```
color 200 200 200
drawsegment  $E_{\{c\}}$   $M_{\{c\}}$ 
color 0 0 0
```

```
% NDG: lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not parallel% DET: lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$ 
 $H_{\{a\}}$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{c\}}M_{\{c\}})$  and line  $m(H_{\{b\}}H_{\{a\}})$ 
intersec  $N$   $m(E_{\{c\}}M_{\{c\}})$   $m(H_{\{b\}}H_{\{a\}})$ 
cmark_r  $N$ 
```

```
% NDG: points  $E_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{c\}}$ 
circle  $k(N, M_{\{a\}})$   $N$   $E_{\{c\}}$ 
```



```

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(BH_{c}) and circle k(N,M_{a}) intersect
% Constructing points M_{a} and E_{b} which are in intersection of k(N,M_{a}) and m(BH_{c})
intersec2 M_{a} E_{b} k(N,M_{a}) m(BH_{c})
cmark_r M_{a}
cmark_r E_{b}

% Constructing a point C such that M_{a}C/M_{a}B=-1
towards C M_{a} B -1
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line m(BH_{c}) and circle k(N,M_{a}) intersect; points E_{c} and N are
% not the same; lines m(E_{c}M_{c}) and m(H_{b}H_{a}) are not parallel; lines h_{c} and c are
% not parallel
% Determination conditions: lines m(E_{c}M_{c}) and m(H_{b}H_{a}) are not the same; points E_{c}
% and M_{c} are not the same; points B and H_{c} are not the same; lines h_{c} and c are not the
% same; points E_{c} and M_{c} are not the same; points B and M_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c = E_c$

Proving failed

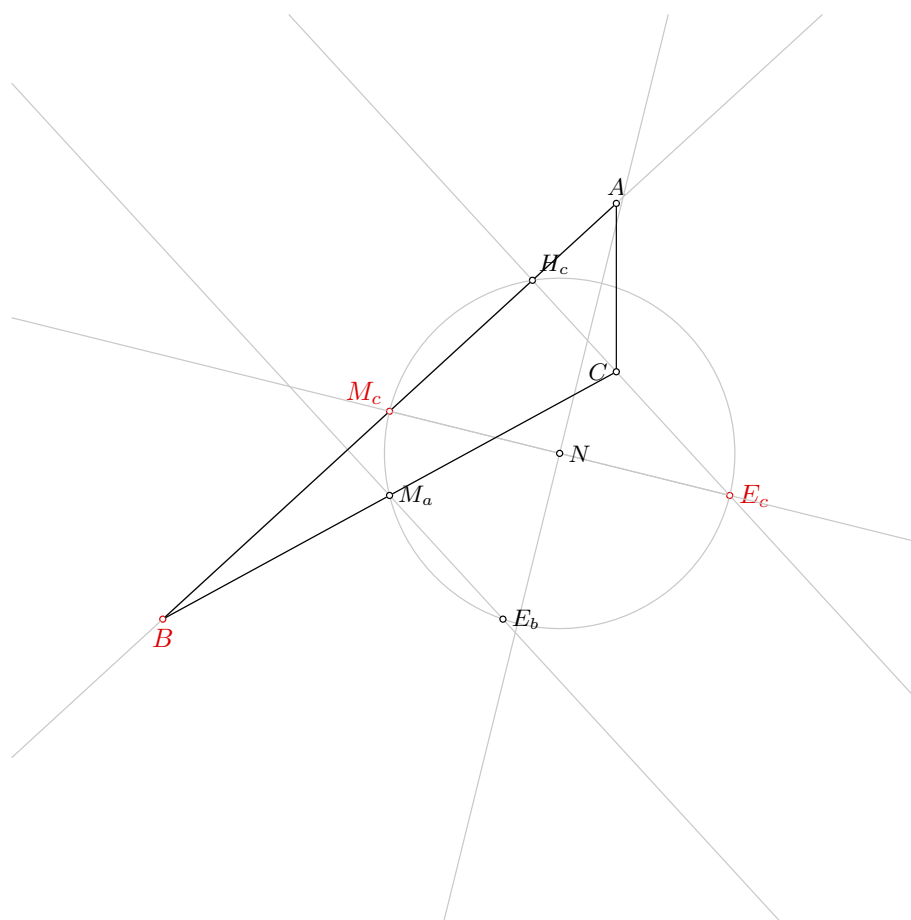


Figure 1: Illustration of the problem 0670

4.1.3 Proving $M_c = \neg M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $B = B$

NDG conditions are:

$S_{E_c B M_c} \neq 0$ i.e., points E_c , B and M_c are not collinear (foot is not the point itself; construction based assumption)

$S_{E_c B M_c} \neq S_{F_{h_c}^0 B M_c}$ i.e., lines $E_c F_{h_c}^0$ and $B M_c$ are not parallel (construction based assumption)

$S_{M_{m(E_c M_c)}^3 E_c M_c} \neq S_{T_{m(E_c M_c)}^4 E_c M_c}$ i.e., lines $M_{m(E_c M_c)}^3 T_{m(E_c M_c)}^4$ and $E_c M_c$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^6} \neq S_{F_{h_a}^5 B F_{h_b}^6}$ i.e., lines $AF_{h_a}^5$ and $BF_{h_b}^6$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c = \neg E_c$

Proving failed

4.2.3 Proving $M_c = \neg M_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.2 Proving $E_c = \neg E_c$

Proving failed

4.3.3 Proving $M_c = \neg M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 13 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_c = E_c$

Proving failed

4.4.3 Proving $M_c = M_c$

Proving failed

Problem 671

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 671: Given a point B , a point E_c and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point N , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and N are not the same;
2. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
3. Using the circle $k(N, M_a)$, the line $m(H_bH_a)$, the point N and the point E_c , construct a point M_c (rule W05a); % NDG: line $m(H_bH_a)$ and circle $k(N, M_a)$ intersect % DET: points E_c and M_c must be different;
4. Using the point M_c and the point B , construct a point A (rule W01); ;
5. Using the point B and the point M_c , construct a line c (rule W02); % DET: points B and M_c are not the same;
6. Using the circle $k(N, M_a)$, the line c , the point N and the point M_c , construct a point H_c (rule W05); % NDG: line c and circle $k(N, M_a)$ intersect % DET: points M_c and H_c must be different;
7. Using the point H_c and the point E_c , construct a line h_c (rule W02); % DET: points H_c and E_c are not the same;
8. Using the point B and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points B and M_c are not the same;
9. Using the circle $k(N, M_a)$ and the circle $k(M_c, A)$, construct a point H_a and a point H_b (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_c, A)$ intersect % DET: circles $k(N, M_a)$ and $k(M_c, A)$ are not the same;

10. Using the point B and the point H_a , construct a line a (rule W02); % DET: points B and H_a are not the same;
11. Using the line h_c and the line a , construct a point C (rule W03); % NDG: lines h_c and a are not parallel % DET: lines h_c and a are not the same.

Non-degenerate conditions: lines h_c and a are not parallel; circles $k(N, M_a)$ and $k(M_c, A)$ intersect; points B and M_c are not the same; line c and circle $k(N, M_a)$ intersect; line $m(H_b H_a)$ and circle $k(N, M_a)$ intersect; points E_c and N are not the same.

Determination conditions: lines h_c and a are not the same; points B and H_a are not the same; circles $k(N, M_a)$ and $k(M_c, A)$ are not the same; points H_c and E_c are not the same; points M_c and H_c must be different; points B and M_c are not the same; points E_c and M_c must be different; points E_c and N are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D10,D20,D30,D32,D5,D7,GD01,GD02,GL01,GL03,GL04,GL09,L18,L19,L20,L21,L24,L3,L40,L41]

Solving time: 4.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point E_{c} 95 56.36
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_r E_{c}
cmark_r N
color 0 0 0
fontsize 8
```

```
% DET: points E_{c} and N are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point N
line m(H_{b}H_{a}) E_{c} N
```

```
color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0
```

```
% NDG: points E_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{c}
circle k(N,M_{a}) N E_{c}
```

```
color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```

% NDG: line  $m(H_b H_a)$  and circle  $k(N, M_a)$  intersect% DET: points  $E_c$  and  $M_c$  must be
different
% Constructing a point  $M_c$  which is an image of the point  $E_c$  in the symmetry to point/line  $N$ 
sim  $M_c$   $N$   $E_c$ 
cmark_lt  $M_c$ 

% Constructing a point  $A$  such that  $M_c A / M_c B = -1$ 
towards  $A$   $M_c$   $B$  -1
cmark_t  $A$ 
color 200 200 200
drawsegment  $B$   $A$ 
color 0 0 0

% DET: points  $B$  and  $M_c$  are not the same
% Constructing a line  $c$  which passes through point  $B$  and point  $M_c$ 
line  $c$   $B$   $M_c$ 

color 200 200 200
drawline  $c$ 
color 0 0 0

% NDG: line  $c$  and circle  $k(N, M_a)$  intersect% DET: points  $M_c$  and  $H_c$  must be different
% Constructing a point  $P_{\setminus G61352}$  which is a foot of the point  $N$  on the line  $c$ 
foot  $P_{\setminus G61352}$   $N$   $c$ 
cmark_r  $P_{\setminus G61352}$ 
color 200 200 200
drawline  $N$   $P_{\setminus G61352}$ 
color 0 0 0

% Constructing a point  $H_c$  which is an image of the point  $M_c$  in the symmetry to point/line  $P_{\setminus G61352}$ 
sim  $H_c$   $P_{\setminus G61352}$   $M_c$ 
cmark_rt  $H_c$ 

% DET: points  $H_c$  and  $E_c$  are not the same
% Constructing a line  $h_c$  which passes through point  $H_c$  and point  $E_c$ 
line  $h_c$   $H_c$   $E_c$ 

color 200 200 200
drawline  $h_c$ 
color 0 0 0

% NDG: points  $B$  and  $M_c$  are not the same
% Constructing a circle  $k(M_c, A)$  whose center is at point  $M_c$  and which passes through point  $B$ 
circle  $k(M_c, A)$   $M_c$   $B$ 

```

```

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(M_{c},A) intersect% DET: circles k(N,M_{a}) and k(M_{c},A) are not
the same
% Constructing points H_{a} and H_{b} which are in intersection of k(N,M_{a}) and k(M_{c},A)
intersec2 H_{a} H_{b} k(N,M_{a}) k(M_{c},A)
cmark_r H_{a}
cmark_l H_{b}

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines h_{c} and a are not parallel% DET: lines h_{c} and a are not the same
% Constructing a point C which belongs to line h_{c} and line a
intersec C h_{c} a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and a are not parallel; circles k(N,M_{a}) and k(M_{c},A)
intersect; points B and M_{c} are not the same; line c and circle k(N,M_{a}) intersect; line m(
H_{b}H_{a}) and circle k(N,M_{a}) intersect; points E_{c} and N are not the same
% Determination conditions: lines h_{c} and a are not the same; points B and H_{a} are not the same
; circles k(N,M_{a}) and k(M_{c},A) are not the same; points H_{c} and E_{c} are not the same;
points M_{c} and H_{c} must be different; points B and M_{c} are not the same; points E_{c} and
M_{c} must be different; points E_{c} and N are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

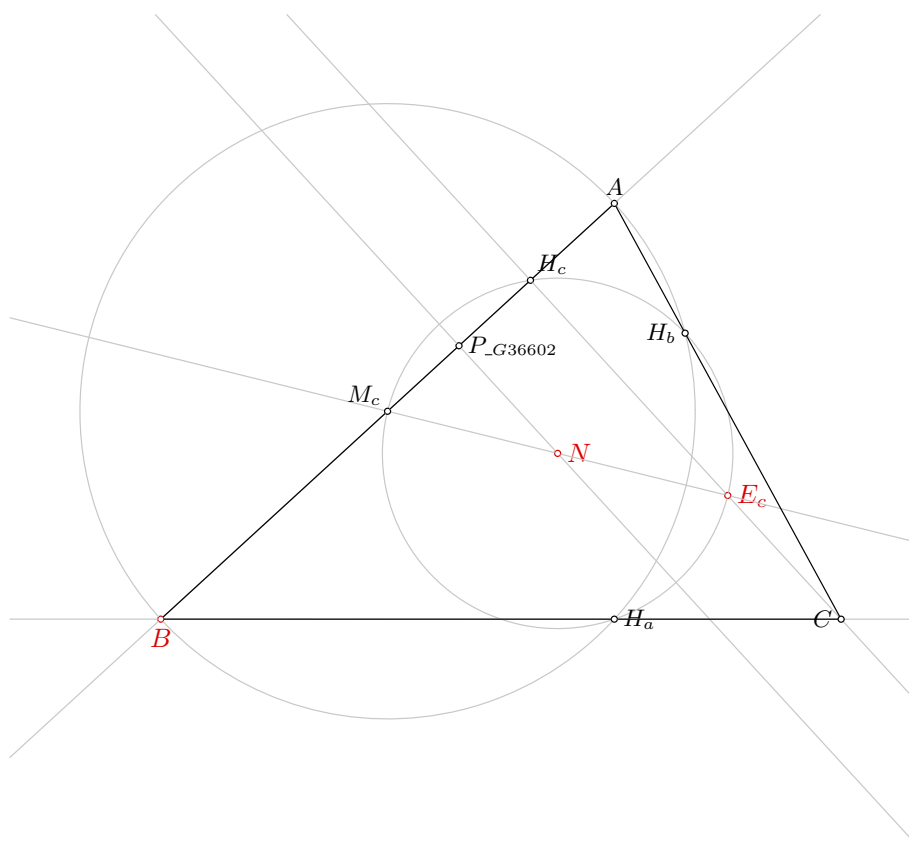


Figure 1: Illustration of the problem 0671

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c = \neg E_c$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B = B$

NDG conditions are:

$S_{H_c B H_a} \neq S_{E_c B H_a}$ i.e., lines $H_c E_c$ and $B H_a$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 B F_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

$S_{\neg M_a B C} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_b A C} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_a \neg M_b F_{-m_b}^3} \neq S_{F_{-m_a}^2 \neg M_b F_{-m_b}^3}$ i.e., lines $\neg M_a F_{-m_a}^2$ and $\neg M_b F_{-m_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c = \neg E_c$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B = B$

Proving failed

4.3.2 Proving $E_c = \neg E_c$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B = B$

Proving failed

4.4.2 Proving $E_c = E_c$

Proving failed

4.4.3 Proving $N = N$

Proving failed

Problem 672

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 672: Given a point B , a point E_c and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 673

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 673: Given a point B , a point E_c and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 674

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 674: Given a point B , a point E_c and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 675

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 675: Given a point B , a point E_c and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 676

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 676: Given a point B , a point G and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point G , construct a point M_b (rule W01); ;
2. Using the point G and the point N , construct a point H (rule W01); ;
3. Using the point B and the point H , construct a point E_b (rule W01); ;
4. Using the point G and the point N , construct a point O (rule W01); ;
5. Using the point B and the point H , construct a line h_b (rule W02); % DET: points B and H are not the same;
6. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
7. Using the point M_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_b and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
9. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; line h_b and circle $k(N, M_a)$ intersect; points M_b and N are not the same; points B and O are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points B and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L15,L16,L17,L20,L23,L56]

Solving time: 9.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
```

```
point G 70 58.33
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_b B
```

```
cmark_t G
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point M_{b} such that BM_{b}/BG=1.5
```

```
towards M_{b} B G 1.5
```

```
cmark_lt M_{b}
```

```
color 200 200 200
```

```
drawsegment B M_{b}
```

```
color 0 0 0
```

```
% Constructing a point H such that GH/GN=4
```

```
towards H G N 4
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment G H
```

```
color 0 0 0
```

```
% Constructing a point E_{b} such that BE_{b}/BH=0.5
```

```
towards E_{b} B H 0.5
```

```
cmark_r E_{b}
```

```
color 200 200 200
```

```
drawsegment B H
```

```
color 0 0 0
```

```
% Constructing a point O such that GO/GN=-2
```

```

towards O G N -2
cmark_t O
color 200 200 200
drawsegment N O
color 0 0 0

% DET: points B and H are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point B and point H
line  $h_{\{b\}}$  B H

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point B
circle  $k(O,C)$  O B

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: points  $M_{\{b\}}$  and N are not the same
% Constructing a circle  $k(N,M_{\{a\}})$  whose center is at point N and which passes through point  $M_{\{b\}}$ 
circle  $k(N,M_{\{a\}})$  N  $M_{\{b\}}$ 

color 200 200 200
drawcircle  $k(N,M_{\{a\}})$ 
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $E_{\{b\}}$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G110875\}}$  which is a foot of the point N on the line  $h_{\{b\}}$ 
foot  $P_{\{\backslash\_G110875\}}$  N  $h_{\{b\}}$ 
cmark_r  $P_{\{\backslash\_G110875\}}$ 
color 200 200 200
drawline N  $P_{\{\backslash\_G110875\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $E_{\{b\}}$  in the symmetry to point/line  $P_{\{\backslash\_G110875\}}$ 
sim  $H_{\{b\}}$   $P_{\{\backslash\_G110875\}}$   $E_{\{b\}}$ 
cmark_l  $H_{\{b\}}$ 

% DET: points  $H_{\{b\}}$  and  $M_{\{b\}}$  are not the same
% Constructing a line b which passes through point  $H_{\{b\}}$  and point  $M_{\{b\}}$ 
line b  $H_{\{b\}}$   $M_{\{b\}}$ 

color 200 200 200

```

```
drawline b
color 0 0 0
```

```
% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line b and circle k(O,C) intersect; line h_{b} and circle k(N,M_{a})
    intersect; points M_{b} and N are not the same; points B and O are not the same
% Determination conditions: points H_{b} and M_{b} are not the same; points E_{b} and H_{b} must be
    different; points B and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $G=_G$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{AB_M_b} \neq S_{_M_a B_M_b}$ i.e., lines A_M_a and B_M_b are not parallel (construction based assumption)
 $S_{_M_a BC} \neq 0$ i.e., points $_M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{_M_b AC} \neq 0$ i.e., points $_M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

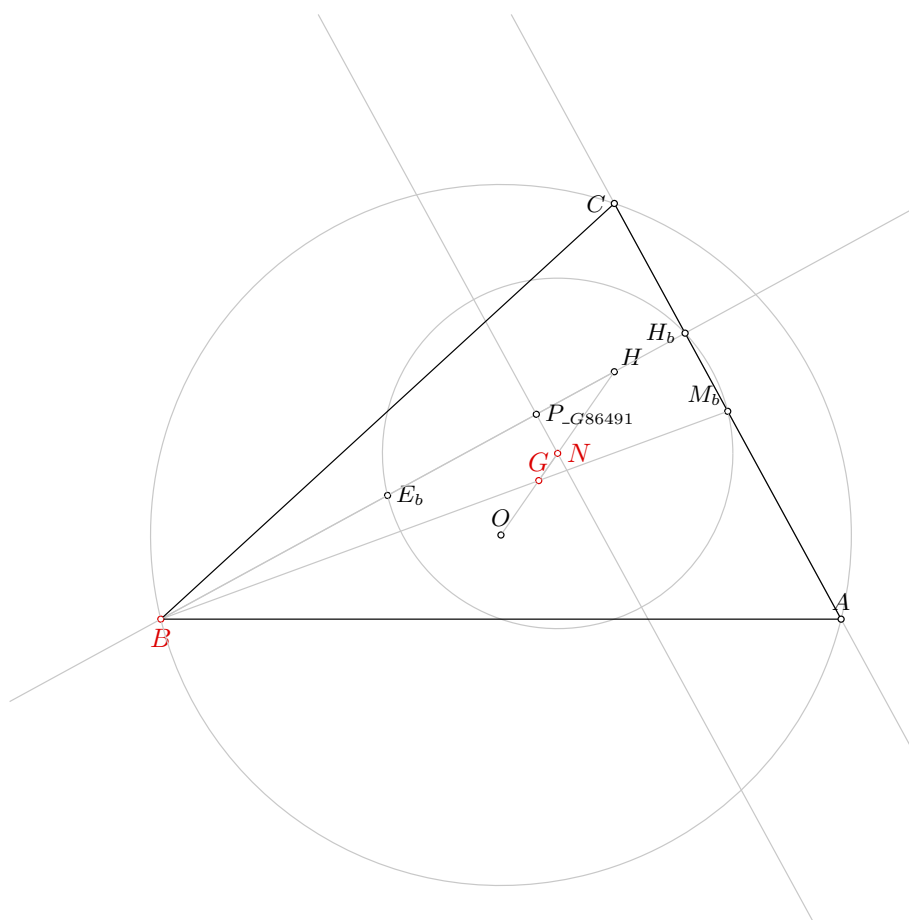


Figure 1: Illustration of the problem 0676

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF^3_{-h_b}} \neq S_{F^2_{-h_a}BF^3_{-h_b}}$ i.e., lines $AF^2_{-h_a}$ and $BF^3_{-h_b}$ are not parallel (construction based assumption)
 $S_{-M_a-M_bF^1_{-m_b}} \neq S_{F^0_{-m_a}-M_bF^1_{-m_b}}$ i.e., lines $-M_aF^0_{-m_a}$ and $-M_bF^1_{-m_b}$ are not parallel (construction based assumption)
 Total number of proof steps: 1
 Time spent by the prover: 0.000 seconds

4.2.2 Proving $G=_G$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $G=_G$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $G=_G$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 677

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 677: Given a point B , a point H and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point H , construct a point E_b (rule W01); ;
2. Using the point H and the point N , construct a point O (rule W01); ;
3. Using the point H and the point N , construct a point G (rule W01); ;
4. Using the point B and the point G , construct a point M_b (rule W01); ;
5. Using the point B and the point H , construct a line h_b (rule W02); % DET: points B and H are not the same;
6. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
7. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
9. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; line h_b and circle $k(N, M_a)$ intersect; points E_b and N are not the same; points B and O are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points B and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D31,D32,D6,D9,GD01,GL03,GL09,L11,L12,L16,L20,L23,L56]

Solving time: 9.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
```

```
point H 80 72.73
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_b B
```

```
cmark_rt H
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point E_{b} such that BE_{b}/BH=0.5
```

```
towards E_{b} B H 0.5
```

```
cmark_r E_{b}
```

```
color 200 200 200
```

```
drawsegment B H
```

```
color 0 0 0
```

```
% Constructing a point O such that HO/HN=2
```

```
towards O H N 2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment H O
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G145689} which passes through point H and point N
```

```
line L_{\_G145689} H N
```

```
color 200 200 200
```

```
drawline L_{\_G145689}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G145790} with coordinates (0,0)
```

```
point P_{\_G145790} 0 0
```

```

cmark_r P_{\_G145790}

% Constructing a point P_{\_G145714} such that HP_{\_G145714}/HP_{\_G145790}=4
towards P_{\_G145714} H P_{\_G145790} 4
cmark_r P_{\_G145714}
color 200 200 200
drawsegment H P_{\_G145714}
color 0 0 0

% Constructing a point P_{\_G145759} such that HP_{\_G145759}/HP_{\_G145790}=3
towards P_{\_G145759} H P_{\_G145790} 3
cmark_r P_{\_G145759}
color 200 200 200
drawsegment H P_{\_G145759}
color 0 0 0

% Constructing a line L_{\_G145720} which passes through point N and point P_{\_G145759}
line L_{\_G145720} N P_{\_G145759}

color 200 200 200
drawline L_{\_G145720}
color 0 0 0

% Constructing a line L_{\_G145683} which contains the point P_{\_G145714} and is parallel to the
line L_{\_G145720}
parallel L_{\_G145683} P_{\_G145714} L_{\_G145720}

color 200 200 200
drawline L_{\_G145683}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G145683} and line L_{\_G145689}
intersec G L_{\_G145683} L_{\_G145689}
cmark_t G

% Constructing a point M_{b} such that BM_{b}/BG=1.5
towards M_{b} B G 1.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% DET: points B and H are not the same
% Constructing a line h_{b} which passes through point B and point H
line h_{b} B H

color 200 200 200
drawline h_{b}
color 0 0 0

```



```

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{b} and circle k(N,M_{a}) intersect% DET: points E_{b} and H_{b} must be different
% Constructing a point P_{\_G146677} which is a foot of the point N on the line h_{b}
foot P_{\_G146677} N h_{b}
cmark_r P_{\_G146677}
color 200 200 200
drawline N P_{\_G146677}
color 0 0 0

% Constructing a point H_{b} which is an image of the point E_{b} in the symmetry to point/line P
\_G146677}
sim H_{b} P_{\_G146677} E_{b}
cmark_l H_{b}

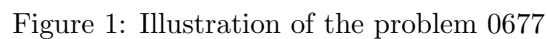
% DET: points H_{b} and M_{b} are not the same
% Constructing a line b which passes through point H_{b} and point M_{b}
line b H_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

```



```
% Non-degenerate conditions: line b and circle k(O,C) intersect; line h_{b} and circle k(N,M_{a})
    intersect; points E_{b} and N are not the same; points B and O are not the same
% Determination conditions: points H_{b} and M_{b} are not the same; points E_{b} and H_{b} must be
    different; points B and H are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.032 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $H=_H$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{P_{G122352}HN} \neq S_{P_{L_{G122321}}^0}^{HN}$ i.e., lines $P_{G122352}P_{L_{G122321}}^0$ and HN are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^2} \neq S_{F_{h_a}^1 BF_{h_b}^2}$ i.e., lines $AF_{h_a}^1$ and $BF_{h_b}^2$ are not parallel (construction based assumption)

$S_{M_a BC} \neq 0$ i.e., points M_a, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_b AC} \neq 0$ i.e., points M_b, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_a M_b F_{m_b}^4} \neq S_{F_{m_a}^3 M_b F_{m_b}^4}$ i.e., lines $M_a F_{m_a}^3$ and $M_b F_{m_b}^4$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $H=_H$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $H=_H$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $H=_H$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 678

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 678: Given a point B , a point H_a and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point H_a , construct a line a (rule W02); % DET: points B and H_a are not the same;
2. Using the point H_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_a and N are not the same;
3. Using the circle $k(N, M_a)$, the line a , the point N and the point H_a , construct a point M_a (rule W05); % NDG: line a and circle $k(N, M_a)$ intersect % DET: points H_a and M_a must be different;
4. Using the point M_a and the point B , construct a point C (rule W01); ;
5. Using the point N and the point M_a , construct a line $m(H_b H_c)$ (rule W02); % DET: points N and M_a are not the same;
6. Using the circle $k(N, M_a)$, the line $m(H_b H_c)$, the point N and the point M_a , construct a point E_a (rule W05a); % NDG: line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect % DET: points M_a and E_a must be different;
7. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;
8. Using the point B and the point M_a , construct a circle $k(M_a, B)$ (rule W06); % NDG: points B and M_a are not the same;
9. Using the circle $k(N, M_a)$ and the circle $k(M_a, B)$, construct a point H_b and a point H_c (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_a, B)$ intersect % DET: circles $k(N, M_a)$ and $k(M_a, B)$ are not the same;

10. Using the point B and the point H_b , construct a line h_b (rule W02); % DET: points B and H_b are not the same;
11. Using the line h_a and the line h_b , construct a point H (rule W03); % NDG: lines h_a and h_b are not parallel % DET: lines h_a and h_b are not the same;
12. Using the point N and the point H , construct a point G (rule W01); ;
13. Using the point M_a and the point G , construct a point A (rule W01); .

Non-degenerate conditions: lines h_a and h_b are not parallel; circles $k(N, M_a)$ and $k(M_a, B)$ intersect; points B and M_a are not the same; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; line a and circle $k(N, M_a)$ intersect; points H_a and N are not the same.

Determination conditions: lines h_a and h_b are not the same; points B and H_b are not the same; circles $k(N, M_a)$ and $k(M_a, B)$ are not the same; points E_a and H_a are not the same; points M_a and E_a must be different; points N and M_a are not the same; points H_a and M_a must be different; points B and H_a are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D21,D28,D3,D32,D5,D6,D8,D9,GD01,GD02,GL01,GL03,GL04,GL09,L16,L19,L20,L21,L22,L38]

Solving time: 4.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point H_{a} 80 40
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_r H_{a}
cmark_r N
color 0 0 0
fontsize 8
```

```
% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% NDG: points H_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{a}
circle k(N,M_{a}) N H_{a}
```

```

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line a and circle k(N,M_{a}) intersect% DET: points H_{a} and M_{a} must be different
% Constructing a point P_{\_G184600} which is a foot of the point N on the line a
foot P_{\_G184600} N a
cmark_r P_{\_G184600}
color 200 200 200
drawline N P_{\_G184600}
color 0 0 0

% Constructing a point M_{a} which is an image of the point H_{a} in the symmetry to point/line P
\_G184600}
sim M_{a} P_{\_G184600} H_{a}
cmark_r M_{a}

% Constructing a point C such that M_{a}C/M_{a}B=-1
towards C M_{a} B -1
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% DET: points N and M_{a} are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point N and point M_{a}
line m(H_{b}H_{c}) N M_{a}

color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0

% NDG: line m(H_{b}H_{c}) and circle k(N,M_{a}) intersect% DET: points M_{a} and E_{a} must be
different
% Constructing a point E_{a} which is an image of the point M_{a} in the symmetry to point/line N
sim E_{a} N M_{a}
cmark_r E_{a}

% DET: points E_{a} and H_{a} are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H_{a}
line h_{a} E_{a} H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

```

```

% NDG: points B and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point B
circle k(M_{a},B) M_{a} B

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(M_{a},B) intersect% DET: circles k(N,M_{a}) and k(M_{a},B) are not
the same
% Constructing points H_{b} and H_{c} which are in intersection of k(N,M_{a}) and k(M_{a},B)
intersec2 H_{b} H_{c} k(N,M_{a}) k(M_{a},B)
cmark_l H_{b}
cmark_rt H_{c}

% DET: points B and H_{b} are not the same
% Constructing a line h_{b} which passes through point B and point H_{b}
line h_{b} B H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines h_{a} and h_{b} are not parallel% DET: lines h_{a} and h_{b} are not the same
% Constructing a point H which belongs to line h_{a} and line h_{b}
intersec H h_{a} h_{b}
cmark_rt H

% Constructing a line L_{\_G185398} which passes through point N and point H
line L_{\_G185398} N H

color 200 200 200
drawline L_{\_G185398}
color 0 0 0

% Constructing a point P_{\_G185499} with coordinates (0,0)
point P_{\_G185499} 0 0
cmark_r P_{\_G185499}

% Constructing a point P_{\_G185423} such that NP_{\_G185423}/NP_{\_G185499}=-1
towards P_{\_G185423} N P_{\_G185499} -1
cmark_r P_{\_G185423}
color 200 200 200
drawsegment P_{\_G185499} P_{\_G185423}
color 0 0 0

% Constructing a point P_{\_G185468} such that NP_{\_G185468}/NP_{\_G185499}=3
towards P_{\_G185468} N P_{\_G185499} 3

```



```

cmark_r P_{\_G185468}
color 200 200 200
drawsegment N P_{\_G185468}
color 0 0 0

% Constructing a line L_{\_G185429} which passes through point H and point P_{\_G185468}
line L_{\_G185429} H P_{\_G185468}

color 200 200 200
drawline L_{\_G185429}
color 0 0 0

% Constructing a line L_{\_G185392} which contains the point P_{\_G185423} and is parallel to the
line L_{\_G185429}
parallel L_{\_G185392} P_{\_G185423} L_{\_G185429}

color 200 200 200
drawline L_{\_G185392}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G185392} and line L_{\_G185398}
intersec G L_{\_G185392} L_{\_G185398}
cmark_t G

% Constructing a point A such that M_{a}A/M_{a}G=3
towards A M_{a} G 3
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{a} and h_{b} are not parallel; circles k(N,M_{a}) and k(M_{a}
,B) intersect; points B and M_{a} are not the same; line m(H_{b}H_{c}) and circle k(N,M_{a})
intersect; line a and circle k(N,M_{a}) intersect; points H_{a} and N are not the same
% Determination conditions: lines h_{a} and h_{b} are not the same; points B and H_{b} are not the
same; circles k(N,M_{a}) and k(M_{a},B) are not the same; points E_{a} and H_{a} are not the
same; points M_{a} and E_{a} must be different; points N and M_{a} are not the same; points H_{
a} and M_{a} must be different; points B and H_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

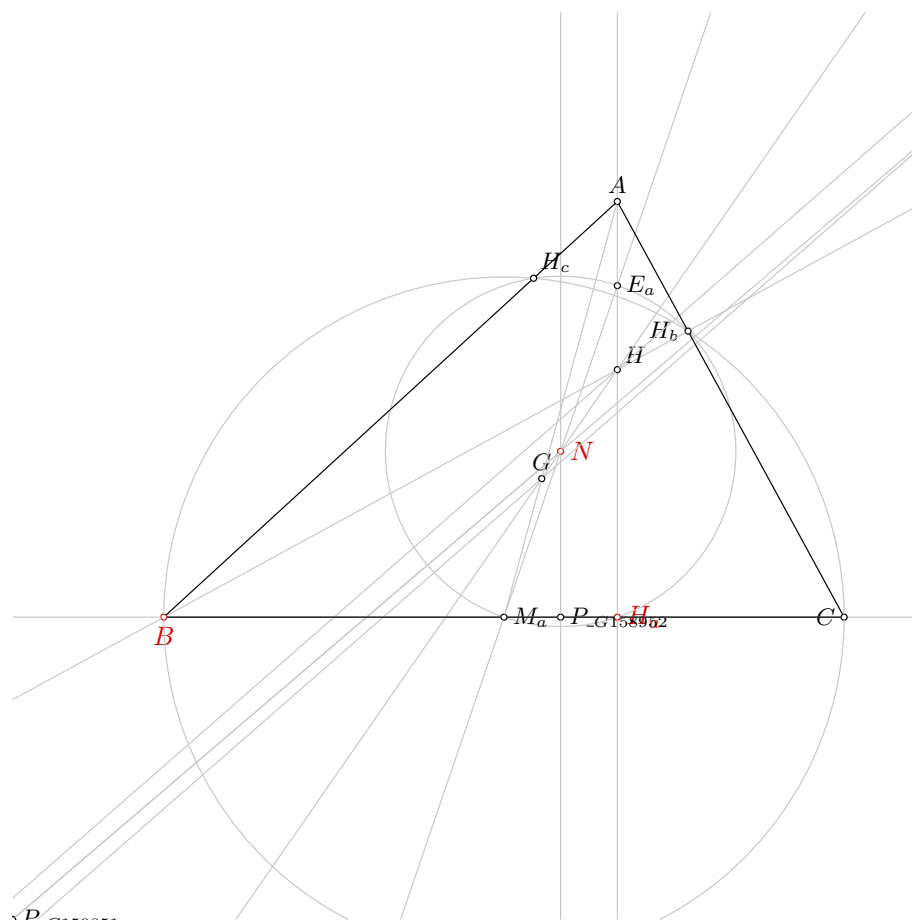


Figure 1: Illustration of the problem 0678

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.03 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $H_a=_H_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 76 terms.

Time Complexity: Time spent by the prover is 0.741 seconds.

NDG conditions Points N and M_a are not identical

Line through points B and N is not perpendicular to line through points N and $P_{G176985}$

Line through points H_b and B is not parallel with line through points E_a and H_a

Points B , H_a and E_a are not collinear

Point B is not on circle with center H_a and point from it C

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{E_a B H_b} \neq S_{H_a B H_b}$ i.e., lines $E_a H_a$ and $B H_b$ are not parallel (construction based assumption)

$S_{P_{G161688} N H} \neq S_{P_{L_{G161657}}^0 N H}$ i.e., lines $P_{G161688} P_{L_{G161657}}^0$ and $N H$ are not parallel (construction based assumption)

$S_{A B C} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{B A F_{H_a}^1} \neq S_{C A F_{H_a}^1}$ i.e., lines $B C$ and $A F_{H_a}^1$ are not parallel (construction based assumption)

$S_{M_a B C} \neq 0$ i.e., points M_a , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_b A C} \neq 0$ i.e., points M_b , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{B A C} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{A B F_{H_b}^4} \neq S_{F_{H_a}^1 B F_{H_b}^4}$ i.e., lines $A F_{H_a}^1$ and $B F_{H_b}^4$ are not parallel (construction based assumption)

$S_{M_a M_b F_{M_b}^3} \neq S_{F_{M_a}^2 M_b F_{M_b}^3}$ i.e., lines $M_a F_{M_a}^2$ and $M_b F_{M_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $H_a=_H_a$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Proving failed

4.3.2 Proving $H_a=_H_a$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Proving failed

4.4.2 Proving $H_a=_H_a$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 679

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 679: Given a point B , a point H_b and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point H_b , construct a line h_b (rule W02); % DET: points B and H_b are not the same;
2. Using the point H_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_b and N are not the same;
3. Using the circle $k(N, M_a)$, the line h_b , the point N and the point H_b , construct a point E_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points H_b and E_b must be different;
4. Using the point E_b and the point B , construct a point H (rule W01); ;
5. Using the point N and the point H , construct a point O (rule W01); ;
6. Using the point N and the point H , construct a point G (rule W01); ;
7. Using the point B and the point G , construct a point M_b (rule W01); ;
8. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
9. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; line h_b and circle $k(N, M_a)$ intersect; points H_b and N are not the same.

Determination conditions: points H_b and M_b are not the same; points H_b and E_b must be different; points B and H_b are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D31,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L16,L20,L23,L56]

Solving time: 3.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point H_{b} 89.36 77.83
point N 72.5 61.93

color 220 0 0
fontsize 9

cmark_b B
cmark_l H_{b}
cmark_r N
color 0 0 0
fontsize 8

% DET: points B and H_{b} are not the same
% Constructing a line h_{b} which passes through point B and point H_{b}
line h_{b} B H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points H_{b} and N are not the same
% Constructing a circle k(N, M_{a}) whose center is at point N and which passes through point H_{b}
circle k(N, M_{a}) N H_{b}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line h_{b} and circle k(N, M_{a}) intersect% DET: points H_{b} and E_{b} must be different
% Constructing a point P_{\G218216} which is a foot of the point N on the line h_{b}
foot P_{\G218216} N h_{b}
cmark_r P_{\G218216}
color 200 200 200
drawline N P_{\G218216}
color 0 0 0
```

```

% Constructing a point E_{b} which is an image of the point H_{b} in the symmetry to point/line P
_{\_G218216}
sim E_{b} P_{\_G218216} H_{b}
cmark_r E_{b}

% Constructing a point H such that E_{b}H/E_{b}B=-1
towards H E_{b} B -1
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0

% Constructing a point O such that NO/NH=-1
towards O N H -1
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% Constructing a line L_{\_G218478} which passes through point N and point H
line L_{\_G218478} N H

color 200 200 200
drawline L_{\_G218478}
color 0 0 0

% Constructing a point P_{\_G218579} with coordinates (0,0)
point P_{\_G218579} 0 0
cmark_r P_{\_G218579}

% Constructing a point P_{\_G218503} such that NP_{\_G218503}/NP_{\_G218579}=-1
towards P_{\_G218503} N P_{\_G218579} -1
cmark_r P_{\_G218503}
color 200 200 200
drawsegment P_{\_G218579} P_{\_G218503}
color 0 0 0

% Constructing a point P_{\_G218548} such that NP_{\_G218548}/NP_{\_G218579}=3
towards P_{\_G218548} N P_{\_G218579} 3
cmark_r P_{\_G218548}
color 200 200 200
drawsegment N P_{\_G218548}
color 0 0 0

% Constructing a line L_{\_G218509} which passes through point H and point P_{\_G218548}
line L_{\_G218509} H P_{\_G218548}

color 200 200 200
drawline L_{\_G218509}

```

```

color 0 0 0

% Constructing a line  $L_{\backslash\_G218472}$  which contains the point  $P_{\backslash\_G218503}$  and is parallel to the
    line  $L_{\backslash\_G218509}$ 
parallel  $L_{\backslash\_G218472}$   $P_{\backslash\_G218503}$   $L_{\backslash\_G218509}$ 

color 200 200 200
drawline  $L_{\backslash\_G218472}$ 
color 0 0 0

% Constructing a point  $G$  which belongs to line  $L_{\backslash\_G218472}$  and line  $L_{\backslash\_G218478}$ 
intersec G  $L_{\backslash\_G218472}$   $L_{\backslash\_G218478}$ 
cmark_t G

% Constructing a point  $M_{\{b\}}$  such that  $BM_{\{b\}}/BG=1.5$ 
towards  $M_{\{b\}}$  B G 1.5
cmark_lt  $M_{\{b\}}$ 
color 200 200 200
drawsegment B  $M_{\{b\}}$ 
color 0 0 0

% DET: points  $H_{\{b\}}$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $M_{\{b\}}$ 
line b  $H_{\{b\}}$   $M_{\{b\}}$ 

color 200 200 200
drawline b
color 0 0 0

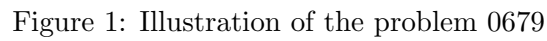
% NDG: points  $B$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $B$ 
circle  $k(O,C)$  O B

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $b$  and circle  $k(O,C)$  intersect
% Constructing points  $C$  and  $A$  which are in intersection of  $k(O,C)$  and  $b$ 
intersec2 C A  $k(O,C)$  b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C

```

```
% Non-degenerate conditions: line b and circle k(D,C) intersect; points B and O are not the same;
    line h_{b} and circle k(N,M_{a}) intersect; points H_{b} and N are not the same
% Determination conditions: points H_{b} and M_{b} are not the same; points H_{b} and E_{b} must be
    different; points B and H_{b} are not the same
```

Illustration of the constructed figure is given in Figure 1

4 Correctness proof

4.1.1 Proving $B=B$

601

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $H_b = \neg H_b$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B = B$

NDG conditions are:

$S_{P_{G197147}NH} \neq S_{P_{L_{G197116}}^0}^{NH}$ i.e., lines $P_{G197147}P_{L_{G197116}}^0$ and NH are not parallel (construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{CBF_{-h_b}^1}$ i.e., lines AC and $BF_{-h_b}^1$ are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points M_a , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points M_b , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^4BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^4$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

$S_{M_aM_bF_{-m_b}^3} \neq S_{F_{-m_a}^2M_bF_{-m_b}^3}$ i.e., lines $M_aF_{-m_a}^2$ and $M_bF_{-m_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $H_b = \neg H_b$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B = B$

Proving failed

4.3.2 Proving $H_b = \neg H_b$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B = B$

Proving failed

4.4.2 Proving $H_b = \neg H_b$

Proving failed

4.4.3 Proving $N = \neg N$

Proving failed

Problem 680

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 680: Given a point B , a point H_c and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point H_c , construct a line c (rule W02); % DET: points B and H_c are not the same;
2. Using the point H_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_c and N are not the same;
3. Using the circle $k(N, M_a)$, the line c , the point N and the point H_c , construct a point M_c (rule W05); % NDG: line c and circle $k(N, M_a)$ intersect % DET: points H_c and M_c must be different;
4. Using the point M_c and the point B , construct a point A (rule W01); ;
5. Using the point N and the point M_c , construct a line $m(H_a H_b)$ (rule W02); % DET: points N and M_c are not the same;
6. Using the circle $k(N, M_a)$, the line $m(H_a H_b)$, the point N and the point M_c , construct a point E_c (rule W05a); % NDG: line $m(H_a H_b)$ and circle $k(N, M_a)$ intersect % DET: points M_c and E_c must be different;
7. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;
8. Using the point B and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points B and M_c are not the same;
9. Using the circle $k(N, M_a)$ and the circle $k(M_c, A)$, construct a point H_a and a point H_b (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_c, A)$ intersect % DET: circles $k(N, M_a)$ and $k(M_c, A)$ are not the same;

10. Using the point B and the point H_a , construct a line a (rule W02); % DET: points B and H_a are not the same;
11. Using the line h_c and the line a , construct a point C (rule W03); % NDG: lines h_c and a are not parallel % DET: lines h_c and a are not the same.

Non-degenerate conditions: lines h_c and a are not parallel; circles $k(N, M_a)$ and $k(M_c, A)$ intersect; points B and M_c are not the same; line $m(H_a H_b)$ and circle $k(N, M_a)$ intersect; line c and circle $k(N, M_a)$ intersect; points H_c and N are not the same.

Determination conditions: lines h_c and a are not the same; points B and H_a are not the same; circles $k(N, M_a)$ and $k(M_c, A)$ are not the same; points E_c and H_c are not the same; points M_c and E_c must be different; points N and M_c are not the same; points H_c and M_c must be different; points B and H_c are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D10,D20,D30,D32,D5,D7,GD01,GD02,GL01,GL03,GL04,GL09,L18,L19,L20,L21,L24,L3,L40,L41]

Solving time: 4.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point H_{c} 68.91 84.83
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_rt H_{c}
cmark_r N
color 0 0 0
fontsize 8
```

```
% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}
```

```
color 200 200 200
drawline c
color 0 0 0
```

```
% NDG: points H_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{c}
circle k(N,M_{a}) N H_{c}
```

```
color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```

% NDG: line c and circle k(N,M_{a}) intersect% DET: points H_{c} and M_{c} must be different
% Constructing a point P_{\_G252842} which is a foot of the point N on the line c
foot P_{\_G252842} N c
cmark_r P_{\_G252842}
color 200 200 200
drawline N P_{\_G252842}
color 0 0 0

% Constructing a point M_{c} which is an image of the point H_{c} in the symmetry to point/line P
\_G252842}
sim M_{c} P_{\_G252842} H_{c}
cmark_lt M_{c}

% Constructing a point A such that M_{c}A/M_{c}B=-1
towards A M_{c} B -1
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

% DET: points N and M_{c} are not the same
% Constructing a line m(H_{a}H_{b}) which passes through point N and point M_{c}
line m(H_{a}H_{b}) N M_{c}

color 200 200 200
drawline m(H_{a}H_{b})
color 0 0 0

% NDG: line m(H_{a}H_{b}) and circle k(N,M_{a}) intersect% DET: points M_{c} and E_{c} must be
different
% Constructing a point E_{c} which is an image of the point M_{c} in the symmetry to point/line N
sim E_{c} N M_{c}
cmark_r E_{c}

% DET: points E_{c} and H_{c} are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H_{c}
line h_{c} E_{c} H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points B and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point B
circle k(M_{c},A) M_{c} B

```

```

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(M_{c},A) intersect% DET: circles k(N,M_{a}) and k(M_{c},A) are not
the same
% Constructing points H_{a} and H_{b} which are in intersection of k(N,M_{a}) and k(M_{c},A)
intersec2 H_{a} H_{b} k(N,M_{a}) k(M_{c},A)
cmark_r H_{a}
cmark_l H_{b}

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines h_{c} and a are not parallel% DET: lines h_{c} and a are not the same
% Constructing a point C which belongs to line h_{c} and line a
intersec C h_{c} a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and a are not parallel; circles k(N,M_{a}) and k(M_{c},A)
intersect; points B and M_{c} are not the same; line m(H_{a}H_{b}) and circle k(N,M_{a})
intersect; line c and circle k(N,M_{a}) intersect; points H_{c} and N are not the same
% Determination conditions: lines h_{c} and a are not the same; points B and H_{a} are not the same
; circles k(N,M_{a}) and k(M_{c},A) are not the same; points E_{c} and H_{c} are not the same;
points M_{c} and E_{c} must be different; points N and M_{c} are not the same; points H_{c} and
M_{c} must be different; points B and H_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

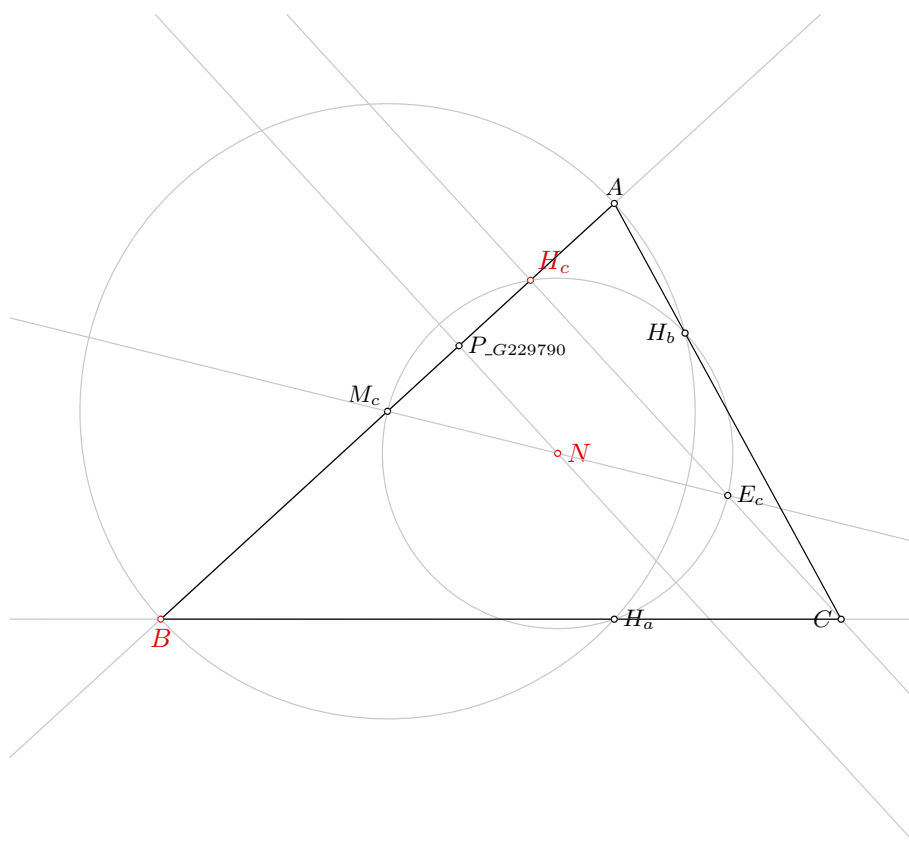


Figure 1: Illustration of the problem 0680

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.027 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $H_c = \neg H_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 18 terms.

Time Complexity: Time spent by the prover is 0.345 seconds.

NDG conditions Points M_c and N are not identical

Line through points B and $P_{G245242}$ is not perpendicular to line through points $P_{G245242}$ and M_c

Line through points E_c and H_c is not parallel with line through points B and H_a

Points A , E_c and M_c are not collinear

Points A and B are not identical

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B = B$

NDG conditions are:

$S_{E_c B H_a} \neq S_{H_c B H_a}$ i.e., lines $E_c H_c$ and $B H_a$ are not parallel (construction based assumption)

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF^0_{H_c}} \neq S_{BCF^0_{H_c}}$ i.e., lines AB and $CF^0_{H_c}$ are not parallel (construction based assumption)

$S_{M_a B C} \neq 0$ i.e., points M_a , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_b A C} \neq 0$ i.e., points M_b , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^4_{H_b}} \neq S_{F^3_{H_a} B F^4_{H_b}}$ i.e., lines $AF^3_{H_a}$ and $BF^4_{H_b}$ are not parallel (construction based assumption)

$S_{M_a M_b F^2_{m_b}} \neq S_{F^1_{m_a} M_b F^2_{m_b}}$ i.e., lines $M_a F^1_{m_a}$ and $M_b F^2_{m_b}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $H_c = \neg H_c$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B = B$

Proving failed

4.3.2 Proving $H_c = \neg H_c$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Proving failed

4.4.2 Proving $H_c=_H H_c$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 681

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 681: Given a point B , a point I and a point N , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 682

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 682: Given a point B , a point M_a and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point M_a , construct a point C (rule W01); ;
2. Using the point B and the point M_a , construct a line a (rule W02); % DET: points B and M_a are not the same;
3. Using the point M_a and the point N , construct a line $m(H_bH_c)$ (rule W02); % DET: points M_a and N are not the same;
4. Using the point B and the point M_a , construct a circle $k(M_a, B)$ (rule W06); % NDG: points B and M_a are not the same;
5. Using the point M_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_a and N are not the same;
6. Using the circle $k(N, M_a)$, the line a , the point N and the point M_a , construct a point H_a (rule W05); % NDG: line a and circle $k(N, M_a)$ intersect % DET: points M_a and H_a must be different;
7. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point M_a , construct a point E_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_a and E_a must be different;
8. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;
9. Using the circle $k(M_a, B)$ and the circle $k(N, M_a)$, construct a point H_b and a point H_c (rule W07); % NDG: circles $k(M_a, B)$ and $k(N, M_a)$ intersect % DET: circles $k(M_a, B)$ and $k(N, M_a)$ are not the same;

10. Using the point B and the point H_b , construct a line h_b (rule W02); % DET: points B and H_b are not the same;
11. Using the line h_a and the line h_b , construct a point H (rule W03); % NDG: lines h_a and h_b are not parallel % DET: lines h_a and h_b are not the same;
12. Using the point N and the point H , construct a point G (rule W01); ;
13. Using the point M_a and the point G , construct a point A (rule W01); .

Non-degenerate conditions: lines h_a and h_b are not parallel; circles $k(M_a, B)$ and $k(N, M_a)$ intersect; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; line a and circle $k(N, M_a)$ intersect; points M_a and N are not the same; points B and M_a are not the same.

Determination conditions: lines h_a and h_b are not the same; points B and H_b are not the same; circles $k(M_a, B)$ and $k(N, M_a)$ are not the same; points E_a and H_a are not the same; points M_a and E_a must be different; points M_a and H_a must be different; points M_a and N are not the same; points B and M_a are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D21,D28,D3,D32,D5,D6,D8,D9,GD01,GD02,GL01,GL03,GL04,GL09,L16,L19,L20,L21,L22,L38]

Solving time: 5.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point M_{a} 65 40
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_r M_{a}
cmark_r N
color 0 0 0
fontsize 8
```

```
% Constructing a point C such that BC/BM_{a}=2
```

```
towards C B M_{a} 2
```

```
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0
```

```
% DET: points B and M_{a} are not the same
```

```
% Constructing a line a which passes through point B and point M_{a}
```

```
line a B M_{a}
```

```
color 200 200 200
```

```

drawline a
color 0 0 0

% DET: points  $M_{\{a\}}$  and  $N$  are not the same
% Constructing a line  $m(H_{\{b\}}H_{\{c\}})$  which passes through point  $M_{\{a\}}$  and point  $N$ 
line m( $H_{\{b\}}H_{\{c\}}$ )  $M_{\{a\}}$   $N$ 

color 200 200 200
drawline m( $H_{\{b\}}H_{\{c\}}$ )
color 0 0 0

% NDG: points  $B$  and  $M_{\{a\}}$  are not the same
% Constructing a circle  $k(M_{\{a\}}, B)$  whose center is at point  $M_{\{a\}}$  and which passes through point  $B$ 
circle k( $M_{\{a\}}, B$ )  $M_{\{a\}}$   $B$ 

color 200 200 200
drawcircle k( $M_{\{a\}}, B$ )
color 0 0 0

% NDG: points  $M_{\{a\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $M_{\{a\}}$ 
circle k( $N, M_{\{a\}}$ )  $N$   $M_{\{a\}}$ 

color 200 200 200
drawcircle k( $N, M_{\{a\}}$ )
color 0 0 0

% NDG: line  $a$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{a\}}$  and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G43394\}}$  which is a foot of the point  $N$  on the line  $a$ 
foot  $P_{\{\backslash\_G43394\}}$   $N$   $a$ 
cmark_r  $P_{\{\backslash\_G43394\}}$ 
color 200 200 200
drawline  $N$   $P_{\{\backslash\_G43394\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $M_{\{a\}}$  in the symmetry to point/line  $P_{\{\backslash\_G43394\}}$ 
sim  $H_{\{a\}}$   $P_{\{\backslash\_G43394\}}$   $M_{\{a\}}$ 
cmark_r  $H_{\{a\}}$ 

% NDG: line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{a\}}$  and  $E_{\{a\}}$  must be
different
% Constructing a point  $E_{\{a\}}$  which is an image of the point  $M_{\{a\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{a\}}$   $N$   $M_{\{a\}}$ 
cmark_r  $E_{\{a\}}$ 

```

```

% DET: points  $E_{\{a\}}$  and  $H_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $E_{\{a\}}$  and point  $H_{\{a\}}$ 
line  $h_{\{a\}}$   $E_{\{a\}}$   $H_{\{a\}}$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: circles  $k(M_{\{a\}}, B)$  and  $k(N, M_{\{a\}})$  intersect% DET: circles  $k(M_{\{a\}}, B)$  and  $k(N, M_{\{a\}})$  are not
the same
% Constructing points  $H_{\{b\}}$  and  $H_{\{c\}}$  which are in intersection of  $k(M_{\{a\}}, B)$  and  $k(N, M_{\{a\}})$ 
intersec2  $H_{\{b\}}$   $H_{\{c\}}$   $k(M_{\{a\}}, B)$   $k(N, M_{\{a\}})$ 
cmark_l  $H_{\{b\}}$ 
cmark_rt  $H_{\{c\}}$ 

% DET: points  $B$  and  $H_{\{b\}}$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $B$  and point  $H_{\{b\}}$ 
line  $h_{\{b\}}$   $B$   $H_{\{b\}}$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: lines  $h_{\{a\}}$  and  $h_{\{b\}}$  are not parallel% DET: lines  $h_{\{a\}}$  and  $h_{\{b\}}$  are not the same
% Constructing a point  $H$  which belongs to line  $h_{\{a\}}$  and line  $h_{\{b\}}$ 
intersec  $H$   $h_{\{a\}}$   $h_{\{b\}}$ 
cmark_rt  $H$ 

% Constructing a line  $L_{\{G43896\}}$  which passes through point  $N$  and point  $H$ 
line  $L_{\{G43896\}}$   $N$   $H$ 

color 200 200 200
drawline  $L_{\{G43896\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G43997\}}$  with coordinates  $(0,0)$ 
point  $P_{\{G43997\}}$  0 0
cmark_r  $P_{\{G43997\}}$ 

% Constructing a point  $P_{\{G43921\}}$  such that  $NP_{\{G43921\}}/NP_{\{G43997\}}=-1$ 
towards  $P_{\{G43921\}}$   $N$   $P_{\{G43997\}}$  -1
cmark_r  $P_{\{G43921\}}$ 
color 200 200 200
drawsegment  $P_{\{G43997\}}$   $P_{\{G43921\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G43966\}}$  such that  $NP_{\{G43966\}}/NP_{\{G43997\}}=3$ 
towards  $P_{\{G43966\}}$   $N$   $P_{\{G43997\}}$  3

```

```

cmark_r P_{\_G43966}
color 200 200 200
drawsegment N P_{\_G43966}
color 0 0 0

% Constructing a line L_{\_G43927} which passes through point H and point P_{\_G43966}
line L_{\_G43927} H P_{\_G43966}

color 200 200 200
drawline L_{\_G43927}
color 0 0 0

% Constructing a line L_{\_G43890} which contains the point P_{\_G43921} and is parallel to the
line L_{\_G43927}
parallel L_{\_G43890} P_{\_G43921} L_{\_G43927}

color 200 200 200
drawline L_{\_G43890}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G43890} and line L_{\_G43896}
intersec G L_{\_G43890} L_{\_G43896}
cmark_t G

% Constructing a point A such that M_{a}A/M_{a}G=3
towards A M_{a} G 3
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{a} and h_{b} are not parallel; circles k(M_{a},B) and k(N,M_{a}) intersect; line m(H_{b}H_{c}) and circle k(N,M_{a}) intersect; line a and circle k(N,M_{a}) intersect; points M_{a} and N are not the same; points B and M_{a} are not the same
% Determination conditions: lines h_{a} and h_{b} are not the same; points B and H_{b} are not the same; circles k(M_{a},B) and k(N,M_{a}) are not the same; points E_{a} and H_{a} are not the same; points M_{a} and E_{a} must be different; points M_{a} and H_{a} must be different; points M_{a} and N are not the same; points B and M_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

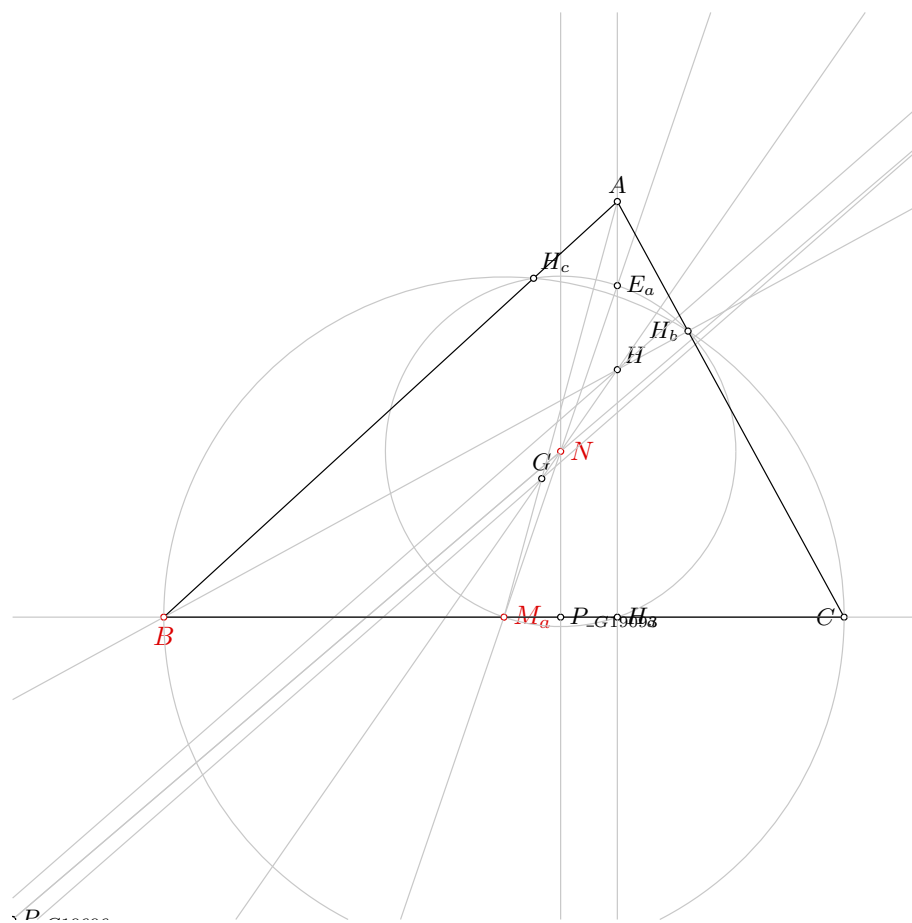


Figure 1: Illustration of the problem 0682

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.027 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $M_a=_M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.037 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{E_aBH_b} \neq S_{H_aBH_b}$ i.e., lines E_aH_a and BH_b are not parallel (construction based assumption)

$S_{P_{-G21533}NH} \neq S_{P_{L_{-G21502}}^0NH}$ i.e., lines $P_{-G21533}P_{L_{-G21502}}^0$ and NH are not parallel (construction based assumption)

$S_{_M_aBC} \neq 0$ i.e., points $_M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$ i.e., points $_M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{_h_b}^4} \neq S_{F_{_h_a}^3BF_{_h_b}^4}$ i.e., lines $AF_{_h_a}^3$ and $BF_{_h_b}^4$ are not parallel (construction based assumption)

$S_{_M_a-_M_bF_{_m_b}^2} \neq S_{F_{_m_a}^1-_M_bF_{_m_b}^2}$ i.e., lines $_M_aF_{_m_a}^1$ and $_M_bF_{_m_b}^2$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $M_a=_M_a$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.2 Proving $M_a = \neg M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 18 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $M_a = \neg M_a$

Proving failed

4.4.3 Proving $N = \neg N$

Proving failed

Problem 683

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 683: Given a point B , a point M_b and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point M_b , construct a point G (rule W01); ;
2. Using the point N and the point G , construct a point O (rule W01); ;
3. Using the point N and the point G , construct a point H (rule W01); ;
4. Using the point B and the point H , construct a point E_b (rule W01); ;
5. Using the point B and the point H , construct a line h_b (rule W02); % DET: points B and H are not the same;
6. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
7. Using the point M_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_b and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
9. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; line h_b and circle $k(N, M_a)$ intersect; points M_b and N are not the same; points B and O are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points B and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L15,L16,L17,L20,L23,L56]

Solving time: 8.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point B 20 40
point M_{b} 95 67.5
point N 72.5 61.93

color 220 0 0
fontsize 9

cmark_b B
cmark_lt M_{b}
cmark_r N
color 0 0 0
fontsize 8

% Constructing a line L_{\_G76710} which passes through point B and point M_{b}
line L_{\_G76710} B M_{b}

color 200 200 200
drawline L_{\_G76710}
color 0 0 0

% Constructing a point P_{\_G76811} with coordinates (0,0)
point P_{\_G76811} 0 0
cmark_r P_{\_G76811}

% Constructing a point P_{\_G76735} such that BP_{\_G76735}/BP_{\_G76811}=2
towards P_{\_G76735} B P_{\_G76811} 2
cmark_r P_{\_G76735}
color 200 200 200
drawsegment B P_{\_G76735}
color 0 0 0

% Constructing a point P_{\_G76780} such that BP_{\_G76780}/BP_{\_G76811}=3
towards P_{\_G76780} B P_{\_G76811} 3
cmark_r P_{\_G76780}
color 200 200 200
drawsegment B P_{\_G76780}
color 0 0 0

% Constructing a line L_{\_G76741} which passes through point M_{b} and point P_{\_G76780}
line L_{\_G76741} M_{b} P_{\_G76780}
```

```

color 200 200 200
drawline L_{\_G76741}
color 0 0 0

% Constructing a line L_{\_G76704} which contains the point P_{\_G76735} and is parallel to the
line L_{\_G76741}
parallel L_{\_G76704} P_{\_G76735} L_{\_G76741}

color 200 200 200
drawline L_{\_G76704}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G76704} and line L_{\_G76710}
intersec G L_{\_G76704} L_{\_G76710}
cmark_t G

% Constructing a point O such that NO/NG=3
towards O N G 3
cmark_t O
color 200 200 200
drawsegment N O
color 0 0 0

% Constructing a point H such that NH/NG=-3
towards H N G -3
cmark_rt H
color 200 200 200
drawsegment G H
color 0 0 0

% Constructing a point E_{b} such that BE_{b}/BH=0.5
towards E_{b} B H 0.5
cmark_r E_{b}
color 200 200 200
drawsegment B H
color 0 0 0

% DET: points B and H are not the same
% Constructing a line h_{b} which passes through point B and point H
line h_{b} B H

color 200 200 200
drawline h_{b}
color 0 0 0

```

```

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points M_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{b}
circle k(N,M_{a}) N M_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{b} and circle k(N,M_{a}) intersect% DET: points E_{b} and H_{b} must be different
% Constructing a point P_{\_G77846} which is a foot of the point N on the line h_{b}
foot P_{\_G77846} N h_{b}
cmark_r P_{\_G77846}
color 200 200 200
drawline N P_{\_G77846}
color 0 0 0

% Constructing a point H_{b} which is an image of the point E_{b} in the symmetry to point/line P_{\_G77846}
sim H_{b} P_{\_G77846} E_{b}
cmark_l H_{b}

% DET: points H_{b} and M_{b} are not the same
% Constructing a line b which passes through point H_{b} and point M_{b}
line b H_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

```

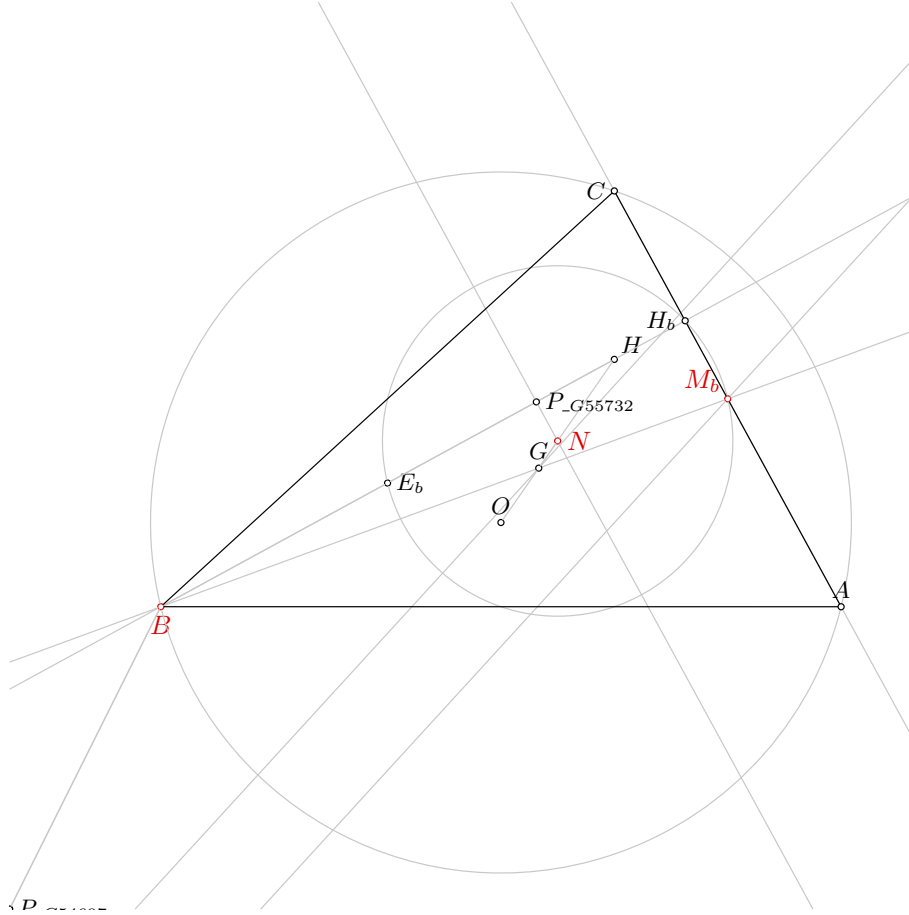


Figure 1: Illustration of the problem 0683

*% Non-degenerate conditions: line b and circle $k(O, C)$ intersect; line $h_{\{b\}}$ and circle $k(N, M_{\{a\}})$ intersect; points $M_{\{b\}}$ and N are not the same; points B and O are not the same
 % Determination conditions: points $H_{\{b\}}$ and $M_{\{b\}}$ are not the same; points $E_{\{b\}}$ and $H_{\{b\}}$ must be different; points B and H are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.031 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $M_b = \neg M_b$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B = B$

NDG conditions are:

$S_{P_{G56060}BM_b} \neq S_{P_{L_{G56029}}^0 BM_b}$ i.e., lines $P_{G56060}P_{L_{G56029}}^0$ and BM_b are not parallel (construction based assumption)

$S_{\neg M_a BC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_b AC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^4} \neq S_{F_{\neg h_a}^3 BF_{\neg h_b}^4}$ i.e., lines $AF_{\neg h_a}^3$ and $BF_{\neg h_b}^4$ are not parallel (construction based assumption)

$S_{\neg M_a \neg M_b F_{\neg m_b}^2} \neq S_{F_{\neg m_a}^1 \neg M_b F_{\neg m_b}^2}$ i.e., lines $\neg M_a F_{\neg m_a}^1$ and $\neg M_b F_{\neg m_b}^2$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $M_b = \neg M_b$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $M_b = \neg M_b$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $M_b=_M M_b$

Proving failed

4.4.3 Proving $N=_N N$

Proving failed

Problem 684

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 684: Given a point B , a point M_c and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point B and the point M_c , construct a point A (rule W01); ;
2. Using the point B and the point M_c , construct a line c (rule W02); % DET: points B and M_c are not the same;
3. Using the point M_c and the point N , construct a line $m(H_bH_a)$ (rule W02); % DET: points M_c and N are not the same;
4. Using the point B and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points B and M_c are not the same;
5. Using the point M_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_c and N are not the same;
6. Using the circle $k(N, M_a)$, the line c , the point N and the point M_c , construct a point H_c (rule W05); % NDG: line c and circle $k(N, M_a)$ intersect % DET: points M_c and H_c must be different;
7. Using the circle $k(N, M_a)$, the line $m(H_bH_a)$, the point N and the point M_c , construct a point E_c (rule W05a); % NDG: line $m(H_bH_a)$ and circle $k(N, M_a)$ intersect % DET: points M_c and E_c must be different;
8. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;
9. Using the circle $k(M_c, A)$ and the circle $k(N, M_a)$, construct a point H_b and a point H_a (rule W07); % NDG: circles $k(M_c, A)$ and $k(N, M_a)$ intersect % DET: circles $k(M_c, A)$ and $k(N, M_a)$ are not the same;

10. Using the point B and the point H_b , construct a line h_b (rule W02); % DET: points B and H_b are not the same;
11. Using the line h_c and the line h_b , construct a point H (rule W03); % NDG: lines h_c and h_b are not parallel % DET: lines h_c and h_b are not the same;
12. Using the point N and the point H , construct a point G (rule W01); ;
13. Using the point M_c and the point G , construct a point C (rule W01); .

Non-degenerate conditions: lines h_c and h_b are not parallel; circles $k(M_c, A)$ and $k(N, M_a)$ intersect; line $m(H_b H_a)$ and circle $k(N, M_a)$ intersect; line c and circle $k(N, M_a)$ intersect; points M_c and N are not the same; points B and M_c are not the same.

Determination conditions: lines h_c and h_b are not the same; points B and H_b are not the same; circles $k(M_c, A)$ and $k(N, M_a)$ are not the same; points E_c and H_c are not the same; points M_c and E_c must be different; points M_c and H_c must be different; points M_c and N are not the same; points B and M_c are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D10,D20,D3,D30,D32,D6,D7,D9,GD01,GD02,GL01,GL03,GL04,GL09,L16,L18,L19,L20,L21,L22]

Solving time: 5.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point M_{c} 50 67.5
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_lt M_{c}
cmark_r N
color 0 0 0
fontsize 8
```

```
% Constructing a point A such that BA/BM_{c}=2
```

```
towards A B M_{c} 2
```

```
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0
```

```
% DET: points B and M_{c} are not the same
```

```
% Constructing a line c which passes through point B and point M_{c}
```

```
line c B M_{c}
```

```
color 200 200 200
```

```

drawline c
color 0 0 0

% DET: points  $M_{\{c\}}$  and  $N$  are not the same
% Constructing a line  $m(H_{\{b\}}H_{\{a\}})$  which passes through point  $M_{\{c\}}$  and point  $N$ 
line m( $H_{\{b\}}H_{\{a\}}$ )  $M_{\{c\}}$  N

color 200 200 200
drawline m( $H_{\{b\}}H_{\{a\}}$ )
color 0 0 0

% NDG: points  $B$  and  $M_{\{c\}}$  are not the same
% Constructing a circle  $k(M_{\{c\}}, A)$  whose center is at point  $M_{\{c\}}$  and which passes through point  $B$ 
circle k( $M_{\{c\}}, A$ )  $M_{\{c\}}$  B

color 200 200 200
drawcircle k( $M_{\{c\}}, A$ )
color 0 0 0

% NDG: points  $M_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $M_{\{c\}}$ 
circle k( $N, M_{\{a\}}$ )  $N$   $M_{\{c\}}$ 

color 200 200 200
drawcircle k( $N, M_{\{a\}}$ )
color 0 0 0

% NDG: line  $c$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{c\}}$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G115370\}}$  which is a foot of the point  $N$  on the line  $c$ 
foot  $P_{\{\backslash\_G115370\}}$   $N$   $c$ 
cmark_r  $P_{\{\backslash\_G115370\}}$ 
color 200 200 200
drawline  $N$   $P_{\{\backslash\_G115370\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $M_{\{c\}}$  in the symmetry to point/line  $P_{\{\backslash\_G115370\}}$ 
sim  $H_{\{c\}}$   $P_{\{\backslash\_G115370\}}$   $M_{\{c\}}$ 
cmark_rt  $H_{\{c\}}$ 

% NDG: line  $m(H_{\{b\}}H_{\{a\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{c\}}$  and  $E_{\{c\}}$  must be
different
% Constructing a point  $E_{\{c\}}$  which is an image of the point  $M_{\{c\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{c\}}$   $N$   $M_{\{c\}}$ 
cmark_r  $E_{\{c\}}$ 

```

```

% DET: points  $E_{\{c\}}$  and  $H_{\{c\}}$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $E_{\{c\}}$  and point  $H_{\{c\}}$ 
line  $h_{\{c\}}$   $E_{\{c\}}$   $H_{\{c\}}$ 

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: circles  $k(M_{\{c\}}, A)$  and  $k(N, M_{\{a\}})$  intersect% DET: circles  $k(M_{\{c\}}, A)$  and  $k(N, M_{\{a\}})$  are not
the same
% Constructing points  $H_{\{b\}}$  and  $H_{\{a\}}$  which are in intersection of  $k(M_{\{c\}}, A)$  and  $k(N, M_{\{a\}})$ 
intersec2  $H_{\{b\}}$   $H_{\{a\}}$   $k(M_{\{c\}}, A)$   $k(N, M_{\{a\}})$ 
cmark_l  $H_{\{b\}}$ 
cmark_r  $H_{\{a\}}$ 

% DET: points  $B$  and  $H_{\{b\}}$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $B$  and point  $H_{\{b\}}$ 
line  $h_{\{b\}}$   $B$   $H_{\{b\}}$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: lines  $h_{\{c\}}$  and  $h_{\{b\}}$  are not parallel% DET: lines  $h_{\{c\}}$  and  $h_{\{b\}}$  are not the same
% Constructing a point  $H$  which belongs to line  $h_{\{c\}}$  and line  $h_{\{b\}}$ 
intersec  $H$   $h_{\{c\}}$   $h_{\{b\}}$ 
cmark_rt  $H$ 

% Constructing a line  $L_{\{\backslash\_G115872\}}$  which passes through point  $N$  and point  $H$ 
line  $L_{\{\backslash\_G115872\}}$   $N$   $H$ 

color 200 200 200
drawline  $L_{\{\backslash\_G115872\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\backslash\_G115973\}}$  with coordinates  $(0,0)$ 
point  $P_{\{\backslash\_G115973\}}$  0 0
cmark_r  $P_{\{\backslash\_G115973\}}$ 

% Constructing a point  $P_{\{\backslash\_G115897\}}$  such that  $NP_{\{\backslash\_G115897\}}/NP_{\{\backslash\_G115973\}}=-1$ 
towards  $P_{\{\backslash\_G115897\}}$   $N$   $P_{\{\backslash\_G115973\}}$  -1
cmark_r  $P_{\{\backslash\_G115897\}}$ 
color 200 200 200
drawsegment  $P_{\{\backslash\_G115973\}}$   $P_{\{\backslash\_G115897\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\backslash\_G115942\}}$  such that  $NP_{\{\backslash\_G115942\}}/NP_{\{\backslash\_G115973\}}=3$ 
towards  $P_{\{\backslash\_G115942\}}$   $N$   $P_{\{\backslash\_G115973\}}$  3

```

```

cmark_r P_{\_G115942}
color 200 200 200
drawsegment N P_{\_G115942}
color 0 0 0

% Constructing a line L_{\_G115903} which passes through point H and point P_{\_G115942}
line L_{\_G115903} H P_{\_G115942}

color 200 200 200
drawline L_{\_G115903}
color 0 0 0

% Constructing a line L_{\_G115866} which contains the point P_{\_G115897} and is parallel to the
line L_{\_G115903}
parallel L_{\_G115866} P_{\_G115897} L_{\_G115903}

color 200 200 200
drawline L_{\_G115866}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G115866} and line L_{\_G115872}
intersec G L_{\_G115866} L_{\_G115872}
cmark_t G

% Constructing a point C such that M_{c}C/M_{c}G=3
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and h_{b} are not parallel; circles k(M_{c},A) and k(N,M_{a}) intersect; line m(H_{b}H_{a}) and circle k(N,M_{a}) intersect; line c and circle k(N,M_{a}) intersect; points M_{c} and N are not the same; points B and M_{c} are not the same
% Determination conditions: lines h_{c} and h_{b} are not the same; points B and H_{b} are not the same; circles k(M_{c},A) and k(N,M_{a}) are not the same; points E_{c} and H_{c} are not the same; points M_{c} and E_{c} must be different; points M_{c} and H_{c} must be different; points M_{c} and N are not the same; points B and M_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format



Figure 1: Illustration of the problem 0684

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.033 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $M_c=_M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.037 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{E_cBH_b} \neq S_{H_cBH_b}$ i.e., lines E_cH_c and BH_b are not parallel (construction based assumption)

$S_{P_{-G93068}NH} \neq S_{P_{L_{-G93037}}^0NH}$ i.e., lines $P_{-G93068}$ and NH are not parallel (construction based assumption)

$S_{_M_aBC} \neq 0$ i.e., points $_M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$ i.e., points $_M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^4} \neq S_{F_{-h_a}^3BF_{-h_b}^4}$ i.e., lines $AF_{-h_a}^3$ and $BF_{-h_b}^4$ are not parallel (construction based assumption)

$S_{_M_a-_M_bF_{-m_b}^2} \neq S_{F_{-m_a}^1-_M_bF_{-m_b}^2}$ i.e., lines $_M_aF_{-m_a}^1$ and $_M_bF_{-m_b}^2$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $M_c=_M_c$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.2 Proving $M_c = M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 18 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.3.3 Proving $N = N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B = B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $M_c = M_c$

Proving failed

4.4.3 Proving $N = N$

Proving failed

Problem 685

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 685: Given a point B , a point N and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point N and the point O , construct a point G (rule W01); ;
2. Using the point B and the point G , construct a point M_b (rule W01); ;
3. Using the point N and the point O , construct a point H (rule W01); ;
4. Using the point B and the point H , construct a point E_b (rule W01); ;
5. Using the point B and the point H , construct a line h_b (rule W02); % DET: points B and H are not the same;
6. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
7. Using the point M_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_b and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
9. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; line h_b and circle $k(N, M_a)$ intersect; points M_b and N are not the same; points B and O are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points B and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D31,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L15,L17,L20,L23,L56]

Solving time: 8.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point B 20 40
point N 72.5 61.93
point O 65 51.14
```

```
color 220 0 0
fontsize 9
```

```
cmark_b B
cmark_r N
cmark_t O
color 0 0 0
fontsize 8
```

```
% Constructing a line L_{\_G153998} which passes through point N and point O
line L_{\_G153998} N O
```

```
color 200 200 200
drawline L_{\_G153998}
color 0 0 0
```

```
% Constructing a point P_{\_G154099} with coordinates (0,0)
point P_{\_G154099} 0 0
cmark_r P_{\_G154099}
```

```
% Constructing a point P_{\_G154023} such that NP_{\_G154023}/NP_{\_G154099}=1
towards P_{\_G154023} N P_{\_G154099} 1
cmark_r P_{\_G154023}
color 200 200 200
drawsegment N P_{\_G154023}
color 0 0 0
```

```
% Constructing a point P_{\_G154068} such that NP_{\_G154068}/NP_{\_G154099}=3
towards P_{\_G154068} N P_{\_G154099} 3
cmark_r P_{\_G154068}
color 200 200 200
drawsegment N P_{\_G154068}
color 0 0 0
```

```
% Constructing a line L_{\_G154029} which passes through point O and point P_{\_G154068}
line L_{\_G154029} O P_{\_G154068}
```

```

color 200 200 200
drawline L_{\_G154029}
color 0 0 0

% Constructing a line L_{\_G153992} which contains the point P_{\_G154023} and is parallel to the
line L_{\_G154029}
parallel L_{\_G153992} P_{\_G154023} L_{\_G154029}

color 200 200 200
drawline L_{\_G153992}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G153992} and line L_{\_G153998}
intersec G L_{\_G153992} L_{\_G153998}
cmark_t G

% Constructing a point M_{b} such that BM_{b}/BG=1.5
towards M_{b} B G 1.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% Constructing a point H such that NH/NO=-1
towards H N O -1
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0

% Constructing a point E_{b} such that BE_{b}/BH=0.5
towards E_{b} B H 0.5
cmark_r E_{b}
color 200 200 200
drawsegment B H
color 0 0 0

% DET: points B and H are not the same
% Constructing a line h_{b} which passes through point B and point H
line h_{b} B H

color 200 200 200
drawline h_{b}
color 0 0 0

```

```

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points M_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{b}
circle k(N,M_{a}) N M_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{b} and circle k(N,M_{a}) intersect% DET: points E_{b} and H_{b} must be different
% Constructing a point P_{\_G155143} which is a foot of the point N on the line h_{b}
foot P_{\_G155143} N h_{b}
cmark_r P_{\_G155143}
color 200 200 200
drawline N P_{\_G155143}
color 0 0 0

% Constructing a point H_{b} which is an image of the point E_{b} in the symmetry to point/line P
_{\_G155143}
sim H_{b} P_{\_G155143} E_{b}
cmark_l H_{b}

% DET: points H_{b} and M_{b} are not the same
% Constructing a line b which passes through point H_{b} and point M_{b}
line b H_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

```

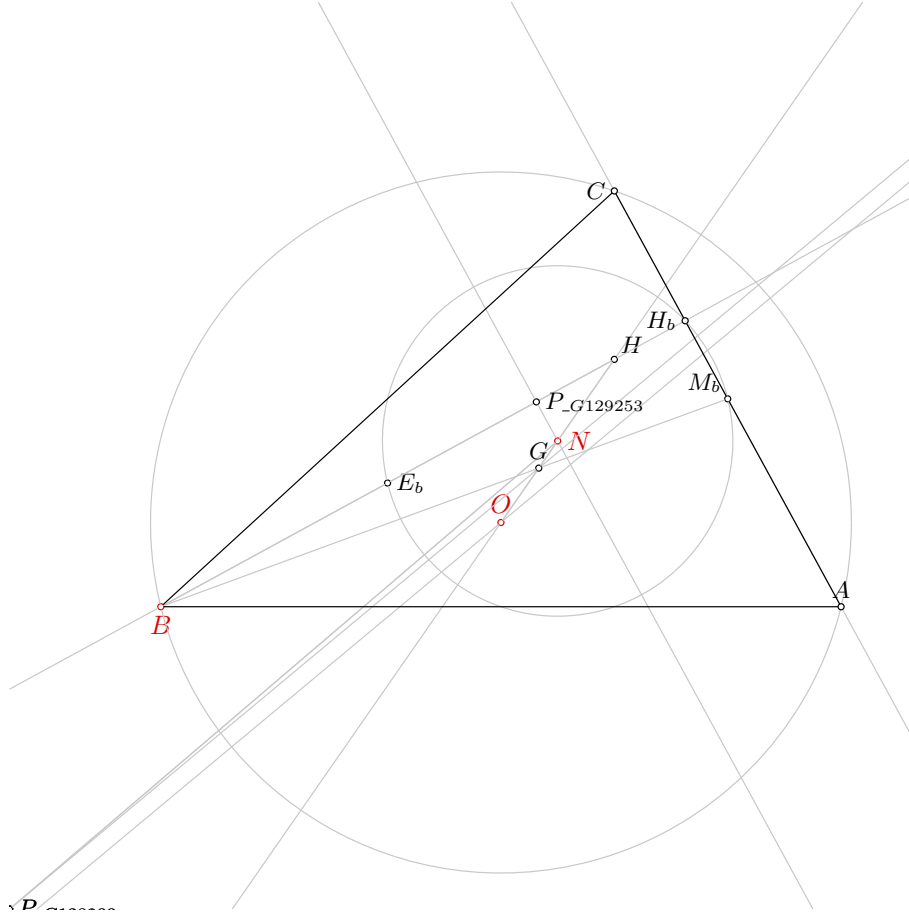


Figure 1: Illustration of the problem 0685

*% Non-degenerate conditions: line b and circle k(O,C) intersect; line h_{b} and circle k(N,M_{a}) intersect; points M_{b} and N are not the same; points B and O are not the same
 % Determination conditions: points H_{b} and M_{b} are not the same; points E_{b} and H_{b} must be different; points B and H are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $B=B$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.031 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $O=_O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $B=B$

NDG conditions are:

$S_{P_{G129581}NO} \neq S_{P_{L_{G129550}}^0NO}$ i.e., lines $P_{G129581}P_{L_{G129550}}^0$ and NO are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points M_a, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points M_b, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aM_bF_{m_b}^2} \neq S_{F_{m_a}^1M_bF_{m_b}^2}$ i.e., lines $M_aF_{m_a}^1$ and $M_bF_{m_b}^2$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^4} \neq S_{F_{h_a}^3BF_{h_b}^4}$ i.e., lines $AF_{h_a}^3$ and $BF_{h_b}^4$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $O=_O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $O=_O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $B=B$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $O=_O$

Proving failed

Problem 686

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 686: Given a point B , a point N and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 687

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 687: Given a point B , a point N and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 688

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 688: Given a point B , a point N and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 689

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 689: Given a point C , a point E_a and a point E_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 690

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 690: Given a point C , a point E_a and a point E_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point E_c , construct a point H (rule W01); ;
2. Using the point E_a and the point H , construct a point A (rule W01); ;
3. Using the point C and the point A , construct a line b (rule W02); % DET: points C and A are not the same;
4. Using the point E_a and the point H , construct a line h_a (rule W02); % DET: points E_a and H are not the same;
5. Using the point C and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points C and E_c are not the same;
6. Using the circle $k(E_c, C)$, the line b , the point E_c and the point C , construct a point H_b (rule W05); % NDG: line b and circle $k(E_c, C)$ intersect % DET: points C and H_b must be different;
7. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
8. Using the circle $k(E_c, C)$, the line h_a , the point E_c and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_c, C)$ intersect % DET: points H and H_a must be different;
9. Using the point H_a and the point C , construct a line a (rule W02); % DET: points H_a and C are not the same;
10. Using the line h_b and the line a , construct a point B (rule W03); % NDG: lines h_b and a are not parallel % DET: lines h_b and a are not the same.

Non-degenerate conditions: lines h_b and a are not parallel; line h_a and circle $k(E_c, C)$ intersect; line b and circle $k(E_c, C)$ intersect; points C and E_c are not the same.

Determination conditions: lines h_b and a are not the same; points H_a and C are not the same; points H and H_a must be different; points H_b and H are not the same; points C and H_b must be different; points E_a and H are not the same; points C and A are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D28,D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,GL04,GL09,L52,L53,L54]

Solving time: 11.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point E_{a} 80 83.86
point E_{c} 95 56.36
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_r E_{a}
cmark_r E_{c}
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that CH/CE_{c}=2
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0
```

```
% Constructing a point A such that E_{a}A/E_{a}H=-1
towards A E_{a} H -1
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0
```

```
% DET: points C and A are not the same
% Constructing a line b which passes through point C and point A
line b C A
```

```
color 200 200 200
drawline b
color 0 0 0
```

```

% DET: points E_{a} and H are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H
line h_{a} E_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points C and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point C
circle k(E_{c},C) E_{c} C

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% NDG: line b and circle k(E_{c},C) intersect% DET: points C and H_{b} must be different
% Constructing a point P_{\_G194274} which is a foot of the point E_{c} on the line b
foot P_{\_G194274} E_{c} b
cmark_r P_{\_G194274}
color 200 200 200
drawline E_{c} P_{\_G194274}
color 0 0 0

% Constructing a point H_{b} which is an image of the point C in the symmetry to point/line P_{\_G
194274}
sim H_{b} P_{\_G194274} C
cmark_l H_{b}

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{a} and circle k(E_{c},C) intersect% DET: points H and H_{a} must be different
% Constructing a point P_{\_G194512} which is a foot of the point E_{c} on the line h_{a}
foot P_{\_G194512} E_{c} h_{a}
cmark_r P_{\_G194512}
color 200 200 200
drawline E_{c} P_{\_G194512}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H in the symmetry to point/line P_{\_G
194512}
sim H_{a} P_{\_G194512} H
cmark_r H_{a}

```



```

% DET: points  $H_{\{a\}}$  and  $C$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $C$ 
line a  $H_{\{a\}}$  C

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $h_{\{b\}}$  and  $a$  are not parallel% DET: lines  $h_{\{b\}}$  and  $a$  are not the same
% Constructing a point  $B$  which belongs to line  $h_{\{b\}}$  and line  $a$ 
intersec B  $h_{\{b\}}$  a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{b\}}$  and  $a$  are not parallel; line  $h_{\{a\}}$  and circle  $k(E_{\{c\}}, C)$ 
% intersect; line  $b$  and circle  $k(E_{\{c\}}, C)$  intersect; points  $C$  and  $E_{\{c\}}$  are not the same
% Determination conditions: lines  $h_{\{b\}}$  and  $a$  are not the same; points  $H_{\{a\}}$  and  $C$  are not the same
% ; points  $H$  and  $H_{\{a\}}$  must be different; points  $H_{\{b\}}$  and  $H$  are not the same; points  $C$  and  $H_{\{b\}}$ 
% must be different; points  $E_{\{a\}}$  and  $H$  are not the same; points  $C$  and  $A$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.033 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a=_E_a$

Proving failed

4.1.3 Proving $E_c=_E_c$

Proving failed

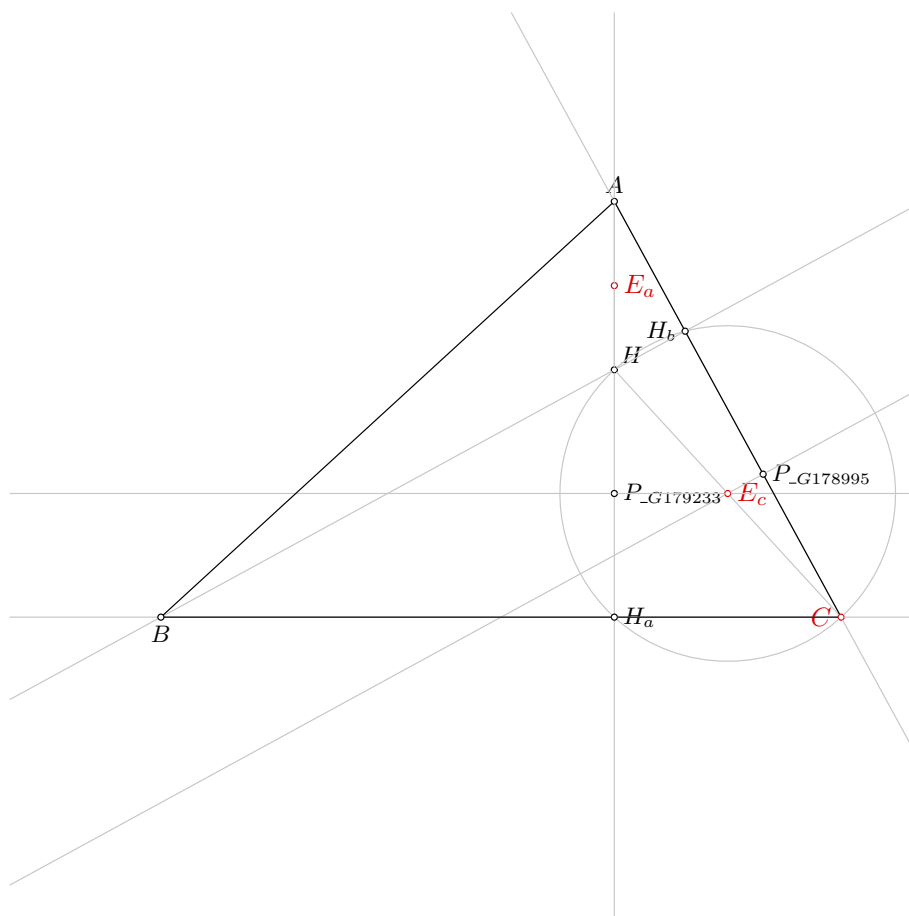


Figure 1: Illustration of the problem 0690

4.2 GCLC - Area method

4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_bH_aC} \neq S_{HH_aC}$ i.e., lines H_bH and H_aC are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a=_Ea$

Proving failed

4.2.3 Proving $E_c=_Ec$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_a=_Ea$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 1023 terms.

Time Complexity: Time spent by the prover is 6.570 seconds. There are no ndg conditions.

4.3.3 Proving $E_c=_Ec$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 2819 terms.

Time Complexity: Time spent by the prover is 16.930 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_a=_Ea$

Proving failed

4.4.3 Proving $E_c=_Ec$

Proving failed

Problem 691

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 691: Given a point C , a point E_a and a point G , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 692

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 692: Given a point C , a point E_a and a point H , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point H , construct a point E_c (rule W01); ;
2. Using the point E_a and the point H , construct a point A (rule W01); ;
3. Using the point C and the point A , construct a line b (rule W02); % DET: points C and A are not the same;
4. Using the point E_a and the point H , construct a line h_a (rule W02); % DET: points E_a and H are not the same;
5. Using the point C and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points C and E_c are not the same;
6. Using the circle $k(E_c, C)$, the line b , the point E_c and the point C , construct a point H_b (rule W05); % NDG: line b and circle $k(E_c, C)$ intersect % DET: points C and H_b must be different;
7. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
8. Using the circle $k(E_c, C)$, the line h_a , the point E_c and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_c, C)$ intersect % DET: points H and H_a must be different;
9. Using the point H_a and the point C , construct a line a (rule W02); % DET: points H_a and C are not the same;
10. Using the line h_b and the line a , construct a point B (rule W03); % NDG: lines h_b and a are not parallel % DET: lines h_b and a are not the same.

Non-degenerate conditions: lines h_b and a are not parallel; line h_a and circle $k(E_c, C)$ intersect; line b and circle $k(E_c, C)$ intersect; points C and E_c are not the same.

Determination conditions: lines h_b and a are not the same; points H_a and C are not the same; points H and H_a must be different; points H_b and H are not the same; points C and H_b must be different; points E_a and H are not the same; points C and A are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D28,D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,GL04,GL09,L52,L53,L54]

Solving time: 10.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point E_{a} 80 83.86
point H 80 72.73

color 220 0 0
fontsize 9

cmark_l C
cmark_r E_{a}
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a point E_{c} such that CE_{c}/CH=0.5
towards E_{c} C H 0.5
cmark_r E_{c}
color 200 200 200
drawsegment C H
color 0 0 0

% Constructing a point A such that E_{a}A/E_{a}H=-1
towards A E_{a} H -1
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

% DET: points C and A are not the same
% Constructing a line b which passes through point C and point A
line b C A

color 200 200 200
drawline b
color 0 0 0
```

```

% DET: points E_{a} and H are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H
line h_{a} E_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points C and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point C
circle k(E_{c},C) E_{c} C

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% NDG: line b and circle k(E_{c},C) intersect% DET: points C and H_{b} must be different
% Constructing a point P_{\_G223752} which is a foot of the point E_{c} on the line b
foot P_{\_G223752} E_{c} b
cmark_r P_{\_G223752}
color 200 200 200
drawline E_{c} P_{\_G223752}
color 0 0 0

% Constructing a point H_{b} which is an image of the point C in the symmetry to point/line P_{\_G
223752}
sim H_{b} P_{\_G223752} C
cmark_l H_{b}

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{a} and circle k(E_{c},C) intersect% DET: points H and H_{a} must be different
% Constructing a point P_{\_G223990} which is a foot of the point E_{c} on the line h_{a}
foot P_{\_G223990} E_{c} h_{a}
cmark_r P_{\_G223990}
color 200 200 200
drawline E_{c} P_{\_G223990}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H in the symmetry to point/line P_{\_G
223990}
sim H_{a} P_{\_G223990} H
cmark_r H_{a}

```

```

% DET: points  $H_{\{a\}}$  and  $C$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $C$ 
line a  $H_{\{a\}}$  C

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $h_{\{b\}}$  and  $a$  are not parallel% DET: lines  $h_{\{b\}}$  and  $a$  are not the same
% Constructing a point  $B$  which belongs to line  $h_{\{b\}}$  and line  $a$ 
intersec B  $h_{\{b\}}$  a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{b\}}$  and  $a$  are not parallel; line  $h_{\{a\}}$  and circle  $k(E_{\{c\}}, C)$ 
% intersect; line  $b$  and circle  $k(E_{\{c\}}, C)$  intersect; points  $C$  and  $E_{\{c\}}$  are not the same
% Determination conditions: lines  $h_{\{b\}}$  and  $a$  are not the same; points  $H_{\{a\}}$  and  $C$  are not the same
% ; points  $H$  and  $H_{\{a\}}$  must be different; points  $H_{\{b\}}$  and  $H$  are not the same; points  $C$  and  $H_{\{b\}}$ 
% must be different; points  $E_{\{a\}}$  and  $H$  are not the same; points  $C$  and  $A$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a=_E_a$

Proving failed

4.1.3 Proving $H=_H$

Proving failed

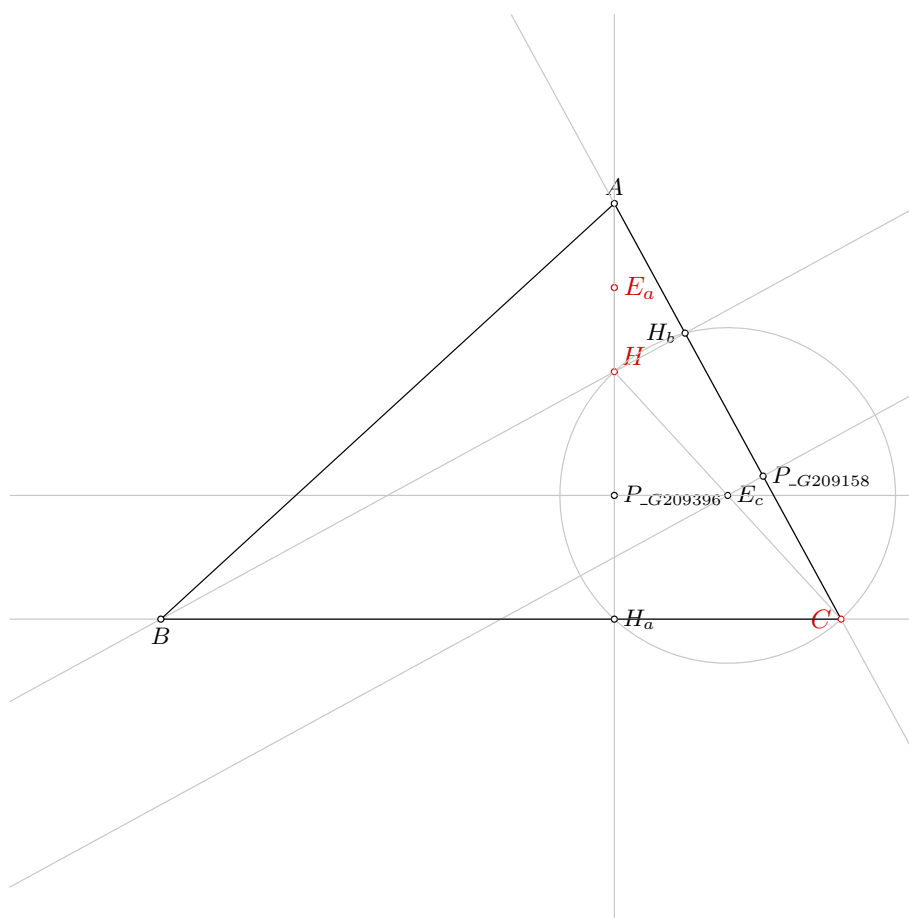


Figure 1: Illustration of the problem 0692

4.2 GCLC - Area method

4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_bH_aC} \neq S_{HH_aC}$ i.e., lines H_bH and H_aC are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a=_Ea$

Proving failed

4.2.3 Proving $H=_H$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.2 Proving $E_a=_Ea$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 1023 terms.

Time Complexity: Time spent by the prover is 0.880 seconds. There are no ndg conditions.

4.3.3 Proving $H=_H$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 182 terms.

Time Complexity: Time spent by the prover is 0.150 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_a=_Ea$

Proving failed

4.4.3 Proving $H=_H$

Proving failed

Problem 693

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 693: Given a point E_a , a point H_a and a point C , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;
2. Using the point H_a and the line h_a , construct a line a (rule W10a); ;
3. Choose freely a point C on the line a (rule WOnline2);
4. Choose freely a point A on the line h_a (rule WOnline1) ;
5. Using the point A and the point E_a , construct a point H (rule W01); ;
6. Using the point A and the point C , construct a line b (rule W02); % DET: points A and C are not the same;
7. Using the point A and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points A and E_a are not the same;
8. Using the circle $k(E_a, A)$, the line b , the point E_a and the point A , construct a point H_b (rule W05); % NDG: line b and circle $k(E_a, A)$ intersect % DET: points A and H_b must be different;
9. Using the point H and the point H_b , construct a line h_b (rule W02); % DET: points H and H_b are not the same;
10. Using the line a and the line h_b , construct a point B (rule W03); % NDG: lines a and h_b are not parallel % DET: lines a and h_b are not the same.

Non-degenerate conditions: lines a and h_b are not parallel; line b and circle $k(E_a, A)$ intersect; points A and E_a are not the same.

Determination conditions: lines a and h_b are not the same; points H and H_b are not the same; points A and H_b must be different; points A and C are not the same; points E_a and H_a are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D28,D3,D5,D6,D8,D9,GD01,GD02,GL03,GL09,L47]

Solving time: 63.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point H_{a} 80 40
```

```
point C 110 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_r H_{a}
```

```
cmark_l C
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{a} and H_{a} are not the same
```

```
% Constructing a line h_{a} which passes through point E_{a} and point H_{a}
```

```
line h_{a} E_{a} H_{a}
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```
% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
```

```
perp a H_{a} h_{a}
```

```
color 200 200 200
```

```
drawline a
```

```
color 0 0 0
```

```
% Generating random value V[_G247307]
```

```
random V[_G247307]
```

```
% Calculating value V[_G247328] using formula V[_G247307]*20
```

```
expression V[_G247328] { V[_G247307]*20 }
```

```

% Constructing a point C which is a point for which holds  $H_{\{a\}}C = V[_G247328]$  and angle  $E_{\{a\}}H_{\{a\}}$ 
C = 90
turtle C E_{a} H_{a} 90 V[_G247328]
cmark_l C

% Choosing randomly a point A on the line  $E_{\{a\}}H_{\{a\}}$ 
online A E_{a} H_{a}
cmark_t A
color 200 200 200
drawline E_{a} H_{a}
color 0 0 0

% Constructing a point H such that  $AH/AE_{\{a\}}=2$ 
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C

color 200 200 200
drawline b
color 0 0 0

% NDG: points A and  $E_{\{a\}}$  are not the same
% Constructing a circle  $k(E_{\{a\}}, A)$  whose center is at point  $E_{\{a\}}$  and which passes through point A
circle k(E_{a}, A) E_{a} A

color 200 200 200
drawcircle k(E_{a}, A)
color 0 0 0

% NDG: line b and circle  $k(E_{\{a\}}, A)$  intersect% DET: points A and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\_G247848\}}$  which is a foot of the point  $E_{\{a\}}$  on the line b
foot P_{\_G247848} E_{a} b
cmark_r P_{\_G247848}
color 200 200 200
drawline E_{a} P_{\_G247848}
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point A in the symmetry to point/line  $P_{\{\_G247848\}}$ 
sim H_{b} P_{\_G247848} A

```

```

cmark_l H_{b}

% DET: points H and H_{b} are not the same
% Constructing a line h_{b} which passes through point H and point H_{b}
line h_{b} H H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines a and h_{b} are not parallel% DET: lines a and h_{b} are not the same
% Constructing a point B which belongs to line a and line h_{b}
intersec B a h_{b}
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and h_{b} are not parallel; line b and circle k(E_{a},A)
% intersect; points A and E_{a} are not the same
% Determination conditions: lines a and h_{b} are not the same; points H and H_{b} are not the same
% ; points A and H_{b} must be different; points A and C are not the same; points E_{a} and H_{a}
% are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = E_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 55 terms.

Time Complexity: Time spent by the prover is 0.976 seconds.

NDG conditions Points A and C are not identical

Points A and B are not identical

Line through points H_b and H_a is not perpendicular to line through points H_a and H

Line through points A and E_a is not parallel with line through points B and C

4.1.2 Proving $H_a = H_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

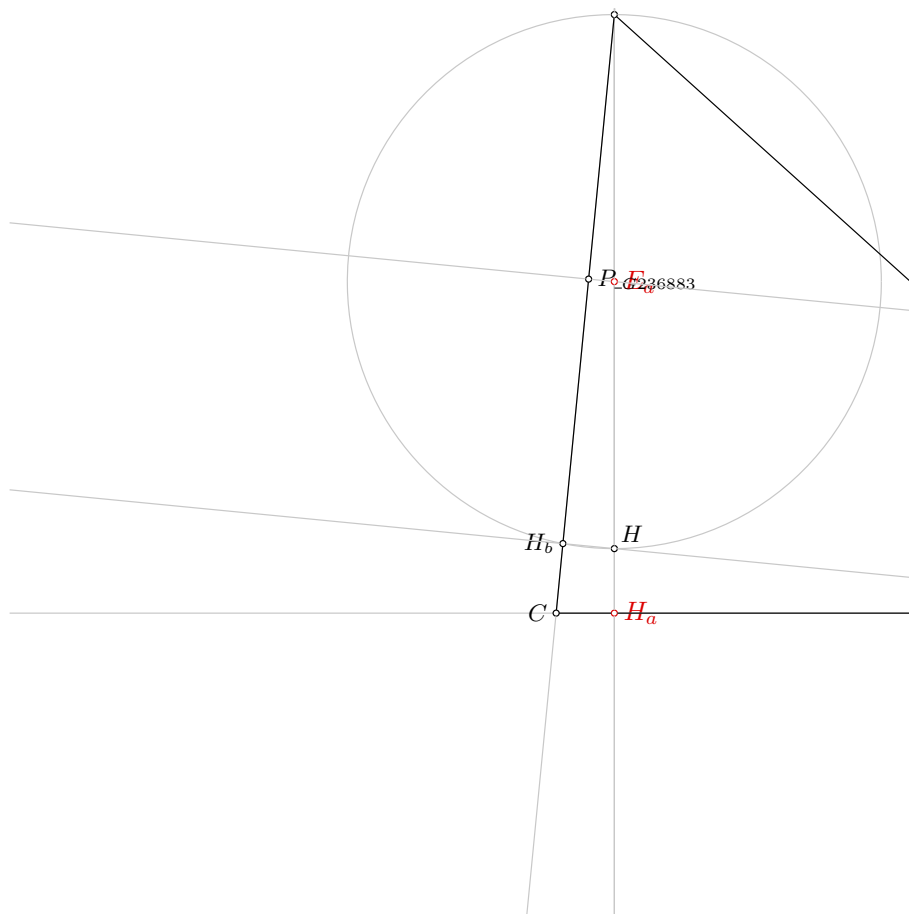


Figure 1: Illustration of the problem 0693

Time Complexity: Time spent by the prover is 0.136 seconds.

NDG conditions Points A and C are not identical

Points A and B are not identical

Line through points H_b and H_a is not perpendicular to line through points H_a and H

Line through points A and E_a is not parallel with line through points B and C

4.1.3 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.006 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $E_a=_Ea$

Proving failed

4.2.2 Proving $H_a=_Ha$

Proving failed

4.2.3 Proving $C=C$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a=_Ea$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 9 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $H_a=_Ha$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 16 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.3 Proving $C=C$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a=_Ea$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 20 terms.

Time Complexity: Time spent by the prover is 0.060 seconds. There are no ndg conditions.

4.4.2 Proving $H_a = \neg H_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 20 terms.

Time Complexity: Time spent by the prover is 0.050 seconds. There are no ndg conditions.

4.4.3 Proving $C = C$

Proving failed

Problem 694

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 694: Given a point C , a point E_a and a point H_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point H_b , construct a line b (rule W02); % DET: points C and H_b are not the same;
2. Using the point H_b and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_b and E_a are not the same;
3. Using the circle $k(E_a, A)$, the line b , the point E_a and the point H_b , construct a point A (rule W05); % NDG: line b and circle $k(E_a, A)$ intersect % DET: points H_b and A must be different;
4. Using the point E_a and the point A , construct a point H (rule W01); ;
5. Using the point C and the point H , construct a line h_c (rule W02); % DET: points C and H are not the same;
6. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
7. Using the circle $k(E_a, A)$, the line h_c , the point E_a and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_a, A)$ intersect % DET: points H and H_c must be different;
8. Using the point A and the point H_c , construct a line c (rule W02); % DET: points A and H_c are not the same;
9. Using the line h_b and the line c , construct a point B (rule W03); % NDG: lines h_b and c are not parallel % DET: lines h_b and c are not the same.

Non-degenerate conditions: lines h_b and c are not parallel; line h_c and circle $k(E_a, A)$ intersect; line b and circle $k(E_a, A)$ intersect; points H_b and E_a are not the same.

Determination conditions: lines h_b and c are not the same; points A and H_c are not the same; points H and H_c must be different; points H_b and H are not the same; points C and H are not the same; points H_b and A must be different; points C and H_b are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D28,D3,D6,D7,D9,GD01,GD02,GL03,GL04,L3,L46,L47,L48]

Solving time: 8.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point E_{a} 80 83.86
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_l C
cmark_r E_{a}
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points H_{b} and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H_{b}
circle k(E_{a},A) E_{a} H_{b}

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: line b and circle k(E_{a},A) intersect% DET: points H_{b} and A must be different
% Constructing a point P_{\_G22130} which is a foot of the point E_{a} on the line b
foot P_{\_G22130} E_{a} b
cmark_r P_{\_G22130}
color 200 200 200
drawline E_{a} P_{\_G22130}
color 0 0 0
```

```

% Constructing a point A which is an image of the point H_{b} in the symmetry to point/line P_{\_G
22130}
sim A P_{\_G22130} H_{b}
cmark_t A

% Constructing a point H such that E_{a}H/E_{a}A=-1
towards H E_{a} A -1
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% DET: points C and H are not the same
% Constructing a line h_{c} which passes through point C and point H
line h_{c} C H

color 200 200 200
drawline h_{c}
color 0 0 0

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{c} and circle k(E_{a},A) intersect% DET: points H and H_{c} must be different
% Constructing a point P_{\_G22493} which is a foot of the point E_{a} on the line h_{c}
foot P_{\_G22493} E_{a} h_{c}
cmark_r P_{\_G22493}
color 200 200 200
drawline E_{a} P_{\_G22493}
color 0 0 0

% Constructing a point H_{c} which is an image of the point H in the symmetry to point/line P_{\_G
22493}
sim H_{c} P_{\_G22493} H
cmark_rt H_{c}

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c

```

```
color 0 0 0
```

```
% NDG: lines h_{b} and c are not parallel% DET: lines h_{b} and c are not the same
% Constructing a point B which belongs to line h_{b} and line c
intersec B h_{b} c
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines h_{b} and c are not parallel; line h_{c} and circle k(E_{a},A)
    intersect; line b and circle k(E_{a},A) intersect; points H_{b} and E_{a} are not the same
% Determination conditions: lines h_{b} and c are not the same; points A and H_{c} are not the same
    ; points H and H_{c} must be different; points H_{b} and H are not the same; points C and H are
    not the same; points H_{b} and A must be different; points C and H_{b} are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.027 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a=_E_a$

Proving failed

4.1.3 Proving $H_b=_H_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 1176 terms.

Time Complexity: Time spent by the prover is 7.638 seconds.

NDG conditions Points H_b and C are not identical

Points H_b and C are not identical

Points C and H are not identical

Points C and H are not identical

Line through points H_b and H is not parallel with line through points A and H_c

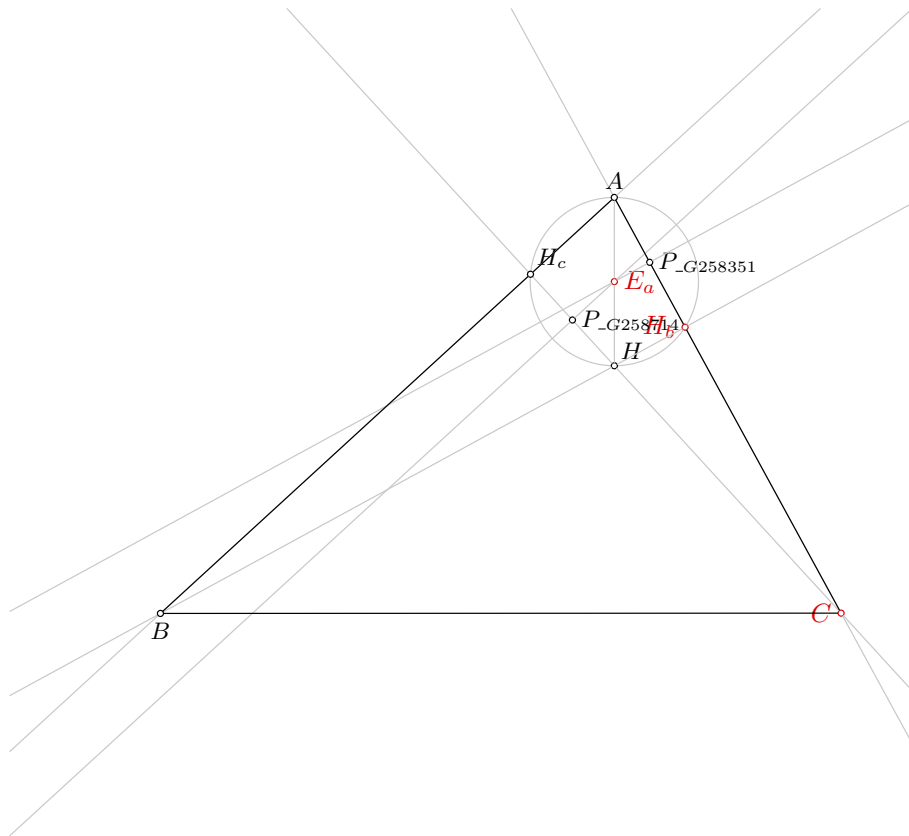


Figure 1: Illustration of the problem 0694

Line through points H_b and H is not parallel with line through points C and E_a

Points A and C are not identical

Points A and C are not identical

4.2 GCLC - Area method

4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_bAH_c} \neq S_{HAH_c}$ i.e., lines H_bH and AH_c are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{CBF_{-h_b}^1}$ i.e., lines AC and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a=_Ea$

Proving failed

4.2.3 Proving $H_b=_Hb$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C=C$

Proving failed

4.3.2 Proving $E_a=_Ea$

Proving failed

4.3.3 Proving $H_b=_Hb$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Proving failed

4.4.2 Proving $E_a=_Ea$

Proving failed

4.4.3 Proving $H_b=_Hb$

Proving failed

Problem 695

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 695: Given a point C , a point E_a and a point H_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point H_c , construct a line h_c (rule W02); % DET: points C and H_c are not the same;
2. Using the point H_c and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_c and E_a are not the same;
3. Using the circle $k(E_a, A)$, the line h_c , the point E_a and the point H_c , construct a point H (rule W05); % NDG: line h_c and circle $k(E_a, A)$ intersect % DET: points H_c and H must be different;
4. Using the point E_a and the point H , construct a point A (rule W01); ;
5. Using the point C and the point A , construct a line b (rule W02); % DET: points C and A are not the same;
6. Using the point H_c and the point A , construct a line c (rule W02); % DET: points H_c and A are not the same;
7. Using the circle $k(E_a, A)$, the line b , the point E_a and the point A , construct a point H_b (rule W05); % NDG: line b and circle $k(E_a, A)$ intersect % DET: points A and H_b must be different;
8. Using the point H and the point H_b , construct a line h_b (rule W02); % DET: points H and H_b are not the same;
9. Using the line c and the line h_b , construct a point B (rule W03); % NDG: lines c and h_b are not parallel % DET: lines c and h_b are not the same.

Non-degenerate conditions: lines c and h_b are not parallel; line b and circle $k(E_a, A)$ intersect; line h_c and circle $k(E_a, A)$ intersect; points H_c and E_a are not the same.

Determination conditions: lines c and h_b are not the same; points H and H_b are not the same; points A and H_b must be different; points H_c and A are not the same; points C and A are not the same; points H_c and H must be different; points C and H_c are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D28,D3,D6,D7,D9,GD01,GD02,GL03,GL04,L3,L46,L47,L48]

Solving time: 8.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point E_{a} 80 83.86
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_l C
cmark_r E_{a}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points C and H_{c} are not the same
% Constructing a line h_{c} which passes through point C and point H_{c}
line h_{c} C H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points H_{c} and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H_{c}
circle k(E_{a},A) E_{a} H_{c}

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: line h_{c} and circle k(E_{a},A) intersect% DET: points H_{c} and H must be different
% Constructing a point P_{\_G45633} which is a foot of the point E_{a} on the line h_{c}
foot P_{\_G45633} E_{a} h_{c}
cmark_r P_{\_G45633}
color 200 200 200
drawline E_{a} P_{\_G45633}
color 0 0 0
```

```

% Constructing a point H which is an image of the point H_{c} in the symmetry to point/line P_{\_G
45633}
sim H P_{\_G45633} H_{c}
cmark_rt H

% Constructing a point A such that E_{a}A/E_{a}H=-1
towards A E_{a} H -1
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

% DET: points C and A are not the same
% Constructing a line b which passes through point C and point A
line b C A

color 200 200 200
drawline b
color 0 0 0

% DET: points H_{c} and A are not the same
% Constructing a line c which passes through point H_{c} and point A
line c H_{c} A

color 200 200 200
drawline c
color 0 0 0

% NDG: line b and circle k(E_{a},A) intersect% DET: points A and H_{b} must be different
% Constructing a point P_{\_G45996} which is a foot of the point E_{a} on the line b
foot P_{\_G45996} E_{a} b
cmark_r P_{\_G45996}
color 200 200 200
drawline E_{a} P_{\_G45996}
color 0 0 0

% Constructing a point H_{b} which is an image of the point A in the symmetry to point/line P_{\_G
45996}
sim H_{b} P_{\_G45996} A
cmark_l H_{b}

% DET: points H and H_{b} are not the same
% Constructing a line h_{b} which passes through point H and point H_{b}
line h_{b} H H_{b}

color 200 200 200
drawline h_{b}

```

```
color 0 0 0
```

```
% NDG: lines c and h_{b} are not parallel% DET: lines c and h_{b} are not the same
% Constructing a point B which belongs to line c and line h_{b}
intersec B c h_{b}
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines c and h_{b} are not parallel; line b and circle k(E_{a},A)
    intersect; line h_{c} and circle k(E_{a},A) intersect; points H_{c} and E_{a} are not the same
% Determination conditions: lines c and h_{b} are not the same; points H and H_{b} are not the same
    ; points A and H_{b} must be different; points H_{c} and A are not the same; points C and A are
    not the same; points H_{c} and H must be different; points C and H_{c} are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.032 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a = E_a$

Proving failed

4.1.3 Proving $H_c = H_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_c H H_b} \neq S_{A H H_b}$ i.e., lines $H_c A$ and $H H_b$ are not parallel (construction based assumption)
 $S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

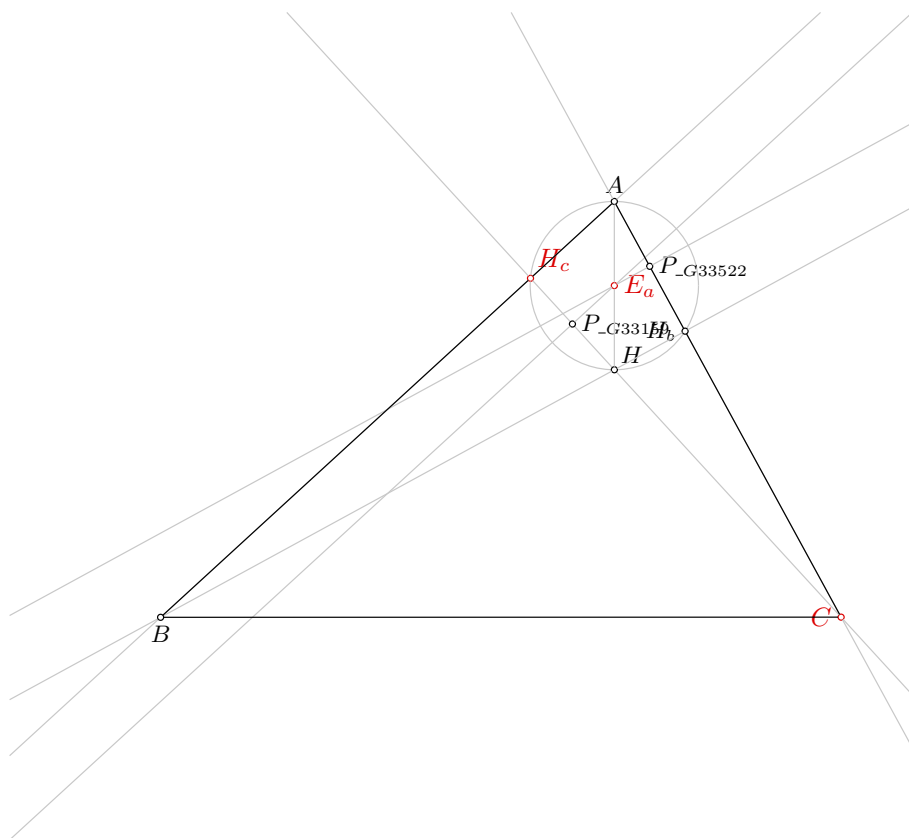


Figure 1: Illustration of the problem 0695

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)
 $S_{ACF_{h_c}^2} \neq S_{BCF_{h_c}^2}$ i.e., lines AB and $CF_{h_c}^2$ are not parallel (construction based assumption)
 Total number of proof steps: 1
 Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a = \neg E_a$

Proving failed

4.2.3 Proving $H_c = \neg H_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C = C$

Proving failed

4.3.2 Proving $E_a = \neg E_a$

Proving failed

4.3.3 Proving $H_c = \neg H_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $C = C$

Proving failed

4.4.2 Proving $E_a = \neg E_a$

Proving failed

4.4.3 Proving $H_c = \neg H_c$

Proving failed

Problem 696

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 696: Given a point C , a point E_a and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 697

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 697: Given a point C , a point E_a and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point M_a , construct a point B (rule W01); ;
2. Using the point C and the point M_a , construct a line a (rule W02); % DET: points C and M_a are not the same;
3. Using the point E_a and the point M_a , construct a line $m(H_bH_c)$ (rule W02); % DET: points E_a and M_a are not the same;
4. Using the point E_a and the line a , construct a line h_a (rule W10b); ;
5. Using the line h_a and the line a , construct a point H_a (rule W03); % NDG: lines h_a and a are not parallel % DET: lines h_a and a are not the same;
6. Using the point C and the point H_a , construct a line $m(CH_a)$ (rule W14); % DET: points C and H_a are not the same;
7. Using the point E_a and the point M_a , construct a line $m(E_aM_a)$ (rule W14); % DET: points E_a and M_a are not the same;
8. Using the line $m(E_aM_a)$ and the line $m(H_bH_c)$, construct a point N (rule W03); % NDG: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not parallel % DET: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not the same;
9. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
10. Using the circle $k(N, M_a)$ and the line $m(CH_a)$, construct a point M_b and a point E_c (rule W04); % NDG: line $m(CH_a)$ and circle $k(N, M_a)$ intersect;

11. Using the point M_b and the point C , construct a point A (rule W01); .

Non-degenerate conditions: line $m(CH_a)$ and circle $k(N, M_a)$ intersect; points E_a and N are not the same; lines $m(E_a M_a)$ and $m(H_b H_c)$ are not parallel; lines h_a and a are not parallel.

Determination conditions: lines $m(E_a M_a)$ and $m(H_b H_c)$ are not the same; points E_a and M_a are not the same; points C and H_a are not the same; lines h_a and a are not the same; points E_a and M_a are not the same; points C and M_a are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14]

Lemmas used: [D21,D22,D28,D3,D32,D5,D8,GD01,GD02,GL01,GL03,GL04,GL09,L17,L20,L21,L22,L24,L38,L

Solving time: 2.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point E_{a} 80 83.86
point M_{a} 65 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_r E_{a}
cmark_r M_{a}
color 0 0 0
fontsize 8
```

```
% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0
```

```
% DET: points C and M_{a} are not the same
% Constructing a line a which passes through point C and point M_{a}
line a C M_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% DET: points E_{a} and M_{a} are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point E_{a} and point M_{a}
line m(H_{b}H_{c}) E_{a} M_{a}
```

```
color 200 200 200
drawline m(H_{b}H_{c})
```



```
color 0 0 0
```

```
% Constructing a line  $h_{\{a\}}$  which is perpendicular to line  $a$  and which passes through point  $E_{\{a\}}$ 
perp  $h_{\{a\}}$   $E_{\{a\}}$   $a$ 
```

```
color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0
```

```
% NDG: lines  $h_{\{a\}}$  and  $a$  are not parallel% DET: lines  $h_{\{a\}}$  and  $a$  are not the same
% Constructing a point  $H_{\{a\}}$  which belongs to line  $h_{\{a\}}$  and line  $a$ 
intersec  $H_{\{a\}}$   $h_{\{a\}}$   $a$ 
cmark_r  $H_{\{a\}}$ 
```

```
% DET: points  $C$  and  $H_{\{a\}}$  are not the same
% Constructing bisector  $m(CH_{\{a\}})$  of the segment  $CH_{\{a\}}$ 
med  $m(CH_{\{a\}})$   $C$   $H_{\{a\}}$ 
```

```
color 200 200 200
drawline  $m(CH_{\{a\}})$ 
color 0 0 0
```

```
color 200 200 200
drawsegment  $C$   $H_{\{a\}}$ 
color 0 0 0
```

```
% DET: points  $E_{\{a\}}$  and  $M_{\{a\}}$  are not the same
% Constructing bisector  $m(E_{\{a\}}M_{\{a\}})$  of the segment  $E_{\{a\}}M_{\{a\}}$ 
med  $m(E_{\{a\}}M_{\{a\}})$   $E_{\{a\}}$   $M_{\{a\}}$ 
```

```
color 200 200 200
drawline  $m(E_{\{a\}}M_{\{a\}})$ 
color 0 0 0
```

```
color 200 200 200
drawsegment  $E_{\{a\}}$   $M_{\{a\}}$ 
color 0 0 0
```

```
% NDG: lines  $m(E_{\{a\}}M_{\{a\}})$  and  $m(H_{\{b\}}H_{\{c\}})$  are not parallel% DET: lines  $m(E_{\{a\}}M_{\{a\}})$  and  $m(H_{\{b\}}H_{\{c\}})$ 
 $H_{\{c\}}$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{a\}}M_{\{a\}})$  and line  $m(H_{\{b\}}H_{\{c\}})$ 
intersec  $N$   $m(E_{\{a\}}M_{\{a\}})$   $m(H_{\{b\}}H_{\{c\}})$ 
cmark_r  $N$ 
```

```
% NDG: points  $E_{\{a\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{a\}}$ 
circle  $k(N, M_{\{a\}})$   $N$   $E_{\{a\}}$ 
```

```

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(CH_{a}) and circle k(N,M_{a}) intersect
% Constructing points M_{b} and E_{c} which are in intersection of k(N,M_{a}) and m(CH_{a})
intersec2 M_{b} E_{c} k(N,M_{a}) m(CH_{a})
cmark_lt M_{b}
cmark_r E_{c}

% Constructing a point A such that M_{b}A/M_{b}C=-1
towards A M_{b} C -1
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line m(CH_{a}) and circle k(N,M_{a}) intersect; points E_{a} and N are
% not the same; lines m(E_{a}M_{a}) and m(H_{b}H_{c}) are not parallel; lines h_{a} and a are
% not parallel
% Determination conditions: lines m(E_{a}M_{a}) and m(H_{b}H_{c}) are not the same; points E_{a}
% and M_{a} are not the same; points C and H_{a} are not the same; lines h_{a} and a are not the
% same; points E_{a} and M_{a} are not the same; points C and M_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a = E_a$

Proving failed

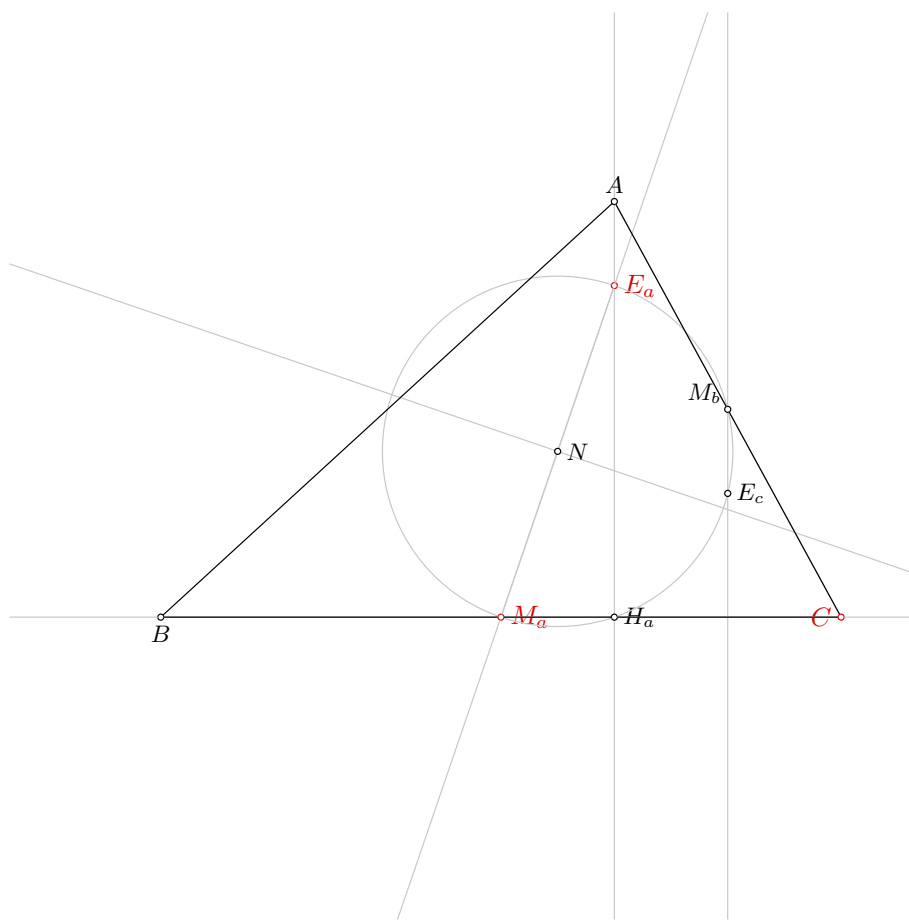


Figure 1: Illustration of the problem 0697

4.1.3 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $C = C$

NDG conditions are:

$S_{E_a C M_a} \neq 0$ i.e., points E_a , C and M_a are not collinear (foot is not the point itself; construction based assumption)

$S_{E_a C M_a} \neq S_{F_{h_a}^0 C M_a}$ i.e., lines $E_a F_{h_a}^0$ and $C M_a$ are not parallel (construction based assumption)

$S_{M_{m(E_a M_a)}^3 E_a M_a} \neq S_{T_{m(E_a M_a)}^4 E_a M_a}$ i.e., lines $M_{m(E_a M_a)}^3$ and $T_{m(E_a M_a)}^4$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{AB F_{h_b}^6} \neq S_{F_{h_a}^5 B F_{h_b}^6}$ i.e., lines $A F_{h_a}^5$ and $B F_{h_b}^6$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a = \neg E_a$

Proving failed

4.2.3 Proving $M_a = \neg M_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.2 Proving $E_a = \neg E_a$

Proving failed

4.3.3 Proving $M_a = \neg M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 13 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $E_a = \neg E_a$

Proving failed

4.4.3 Proving $M_a = \neg M_a$

Proving failed

Problem 698

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 698: Given a point C , a point E_a and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point M_b , construct a point A (rule W01); ;
2. Using the point E_a and the point A , construct a point H (rule W01); ;
3. Using the point C and the point H , construct a line h_c (rule W02); % DET: points C and H are not the same;
4. Using the point E_a and the point A , construct a line h_a (rule W02); % DET: points E_a and A are not the same;
5. Using the point C and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points C and M_b are not the same;
6. Using the circle $k(M_b, C)$, the line h_c , the point M_b and the point C , construct a point H_c (rule W05); % NDG: line h_c and circle $k(M_b, C)$ intersect % DET: points C and H_c must be different;
7. Using the point H_c and the point A , construct a line c (rule W02); % DET: points H_c and A are not the same;
8. Using the circle $k(M_b, C)$, the line h_a , the point M_b and the point A , construct a point H_a (rule W05); % NDG: line h_a and circle $k(M_b, C)$ intersect % DET: points A and H_a must be different;
9. Using the point H_a and the point C , construct a line a (rule W02); % DET: points H_a and C are not the same;
10. Using the line c and the line a , construct a point B (rule W03); % NDG: lines c and a are not parallel % DET: lines c and a are not the same.

Non-degenerate conditions: lines c and a are not parallel; line h_a and circle $k(M_b, C)$ intersect; line h_c and circle $k(M_b, C)$ intersect; points C and M_b are not the same.

Determination conditions: lines c and a are not the same; points H_a and C are not the same; points A and H_a must be different; points H_c and A are not the same; points C and H_c must be different; points E_a and A are not the same; points C and H are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D22,D28,D3,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L43,L44,L45]

Solving time: 10.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
```

```
point E_{a} 80 83.86
```

```
point M_{b} 95 67.5
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_l C
```

```
cmark_r E_{a}
```

```
cmark_lt M_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point A such that CA/CM_{b}=2
```

```
towards A C M_{b} 2
```

```
cmark_t A
```

```
color 200 200 200
```

```
drawsegment C A
```

```
color 0 0 0
```

```
% Constructing a point H such that E_{a}H/E_{a}A=-1
```

```
towards H E_{a} A -1
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment A H
```

```
color 0 0 0
```

```
% DET: points C and H are not the same
```

```
% Constructing a line h_{c} which passes through point C and point H
```

```
line h_{c} C H
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```

% DET: points E_{a} and A are not the same
% Constructing a line h_{a} which passes through point E_{a} and point A
line h_{a} E_{a} A

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points C and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point C
circle k(M_{b},C) M_{b} C

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: line h_{c} and circle k(M_{b},C) intersect% DET: points C and H_{c} must be different
% Constructing a point P_{\_G94160} which is a foot of the point M_{b} on the line h_{c}
foot P_{\_G94160} M_{b} h_{c}
cmark_r P_{\_G94160}
color 200 200 200
drawline M_{b} P_{\_G94160}
color 0 0 0

% Constructing a point H_{c} which is an image of the point C in the symmetry to point/line P_{\_G
94160}
sim H_{c} P_{\_G94160} C
cmark_rt H_{c}

% DET: points H_{c} and A are not the same
% Constructing a line c which passes through point H_{c} and point A
line c H_{c} A

color 200 200 200
drawline c
color 0 0 0

% NDG: line h_{a} and circle k(M_{b},C) intersect% DET: points A and H_{a} must be different
% Constructing a point P_{\_G94398} which is a foot of the point M_{b} on the line h_{a}
foot P_{\_G94398} M_{b} h_{a}
cmark_r P_{\_G94398}
color 200 200 200
drawline M_{b} P_{\_G94398}
color 0 0 0

% Constructing a point H_{a} which is an image of the point A in the symmetry to point/line P_{\_G
94398}
sim H_{a} P_{\_G94398} A
cmark_r H_{a}

```



```

% DET: points  $H_{\{a\}}$  and  $C$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $C$ 
line a  $H_{\{a\}}$  C

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $c$  and  $a$  are not parallel% DET: lines  $c$  and  $a$  are not the same
% Constructing a point  $B$  which belongs to line  $c$  and line  $a$ 
intersec B c a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $c$  and  $a$  are not parallel; line  $h_{\{a\}}$  and circle  $k(M_{\{b\}}, C)$ 
% intersect; line  $h_{\{c\}}$  and circle  $k(M_{\{b\}}, C)$  intersect; points  $C$  and  $M_{\{b\}}$  are not the same
% Determination conditions: lines  $c$  and  $a$  are not the same; points  $H_{\{a\}}$  and  $C$  are not the same;
% points  $A$  and  $H_{\{a\}}$  must be different; points  $H_{\{c\}}$  and  $A$  are not the same; points  $C$  and  $H_{\{c\}}$ 
% must be different; points  $E_{\{a\}}$  and  $A$  are not the same; points  $C$  and  $H$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.033 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a=_E_a$

Proving failed

4.1.3 Proving $M_b=_M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

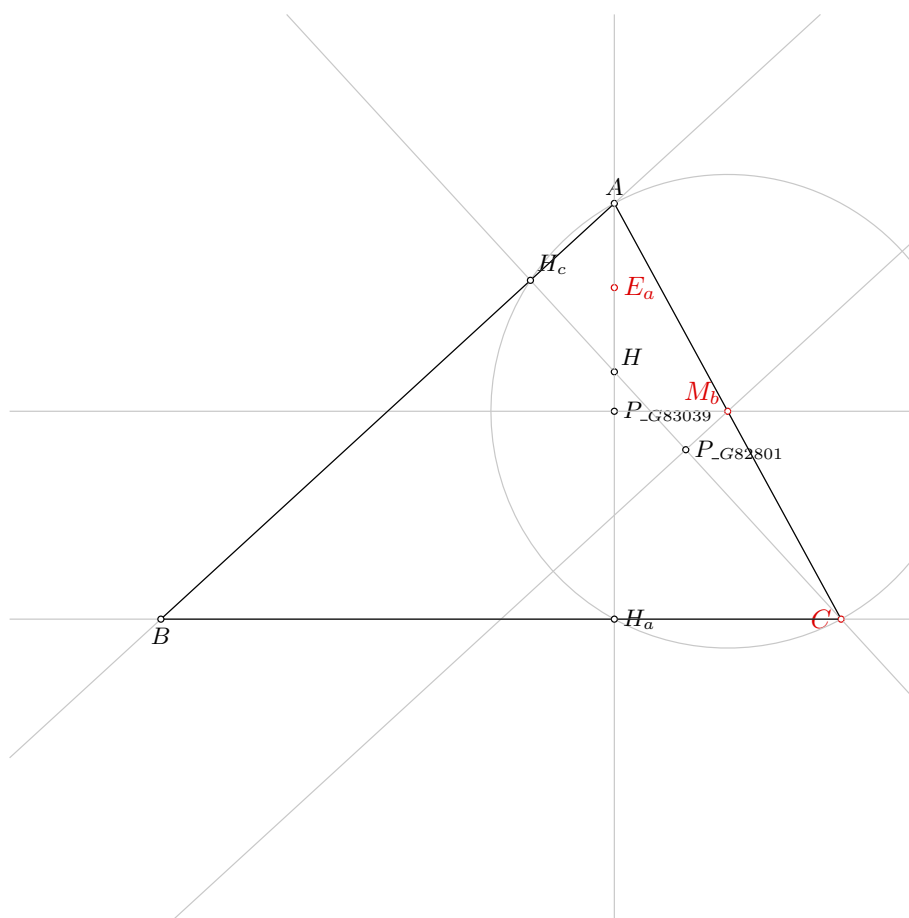


Figure 1: Illustration of the problem 0698

Time Complexity: Time spent by the prover is 0.033 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_c H_a C} \neq S_{A H_a C}$ i.e., lines $H_c A$ and $H_a C$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a = E_a$

Proving failed

4.2.3 Proving $M_b = M_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_a = E_a$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 124 terms.

Time Complexity: Time spent by the prover is 0.470 seconds. There are no ndg conditions.

4.3.3 Proving $M_b = M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 9 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $E_a = E_a$

Proving failed

4.4.3 Proving $M_b = -M_b$

Proving failed

Problem 699

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 699: Given a point C , a point E_a and a point M_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 700

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 700: Given a point C , a point E_a and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point N , construct a line $m(H_bH_c)$ (rule W02); % DET: points E_a and N are not the same;
2. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
3. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point E_a , construct a point M_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points E_a and M_a must be different;
4. Using the point M_a and the point C , construct a point B (rule W01); ;
5. Using the point C and the point M_a , construct a line a (rule W02); % DET: points C and M_a are not the same;
6. Using the circle $k(N, M_a)$, the line a , the point N and the point M_a , construct a point H_a (rule W05); % NDG: line a and circle $k(N, M_a)$ intersect % DET: points M_a and H_a must be different;
7. Using the point H_a and the point E_a , construct a line h_a (rule W02); % DET: points H_a and E_a are not the same;
8. Using the point C and the point M_a , construct a circle $k(M_a, B)$ (rule W06); % NDG: points C and M_a are not the same;
9. Using the circle $k(N, M_a)$ and the circle $k(M_a, B)$, construct a point H_b and a point H_c (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_a, B)$ intersect % DET: circles $k(N, M_a)$ and $k(M_a, B)$ are not the same;

10. Using the point C and the point H_b , construct a line b (rule W02); % DET: points C and H_b are not the same;
11. Using the line h_a and the line b , construct a point A (rule W03); % NDG: lines h_a and b are not parallel % DET: lines h_a and b are not the same.

Non-degenerate conditions: lines h_a and b are not parallel; circles $k(N, M_a)$ and $k(M_a, B)$ intersect; points C and M_a are not the same; line a and circle $k(N, M_a)$ intersect; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; points E_a and N are not the same.

Determination conditions: lines h_a and b are not the same; points C and H_b are not the same; circles $k(N, M_a)$ and $k(M_a, B)$ are not the same; points H_a and E_a are not the same; points M_a and H_a must be different; points C and M_a are not the same; points E_a and M_a must be different; points E_a and N are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D21,D28,D3,D32,D5,D6,D8,GD01,GD02,GL01,GL03,GL04,GL09,L19,L20,L21,L22,L37,L38,L39]

Solving time: 4.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point E_{a} 80 83.86
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_r E_{a}
cmark_r N
color 0 0 0
fontsize 8
```

```
% DET: points E_{a} and N are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point E_{a} and point N
line m(H_{b}H_{c}) E_{a} N
```

```
color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0
```

```
% NDG: points E_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{a}
circle k(N,M_{a}) N E_{a}
```

```
color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```

% NDG: line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{a\}}$  and  $M_{\{a\}}$  must be
different
% Constructing a point  $M_{\{a\}}$  which is an image of the point  $E_{\{a\}}$  in the symmetry to point/line  $N$ 
sim  $M_{\{a\}}$   $N$   $E_{\{a\}}$ 
cmark_r  $M_{\{a\}}$ 

% Constructing a point  $B$  such that  $M_{\{a\}}B/M_{\{a\}}C=-1$ 
towards  $B$   $M_{\{a\}}$   $C$  -1
cmark_b  $B$ 
color 200 200 200
drawsegment  $C$   $B$ 
color 0 0 0

% DET: points  $C$  and  $M_{\{a\}}$  are not the same
% Constructing a line  $a$  which passes through point  $C$  and point  $M_{\{a\}}$ 
line  $a$   $C$   $M_{\{a\}}$ 

color 200 200 200
drawline  $a$ 
color 0 0 0

% NDG: line  $a$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{a\}}$  and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G132935\}}$  which is a foot of the point  $N$  on the line  $a$ 
foot  $P_{\{\backslash\_G132935\}}$   $N$   $a$ 
cmark_r  $P_{\{\backslash\_G132935\}}$ 
color 200 200 200
drawline  $N$   $P_{\{\backslash\_G132935\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $M_{\{a\}}$  in the symmetry to point/line  $P_{\{\backslash\_G132935\}}$ 
sim  $H_{\{a\}}$   $P_{\{\backslash\_G132935\}}$   $M_{\{a\}}$ 
cmark_r  $H_{\{a\}}$ 

% DET: points  $H_{\{a\}}$  and  $E_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $H_{\{a\}}$  and point  $E_{\{a\}}$ 
line  $h_{\{a\}}$   $H_{\{a\}}$   $E_{\{a\}}$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: points  $C$  and  $M_{\{a\}}$  are not the same
% Constructing a circle  $k(M_{\{a\}}, B)$  whose center is at point  $M_{\{a\}}$  and which passes through point  $C$ 
circle  $k(M_{\{a\}}, B)$   $M_{\{a\}}$   $C$ 

```



```

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(M_{a},B) intersect% DET: circles k(N,M_{a}) and k(M_{a},B) are not
the same
% Constructing points H_{b} and H_{c} which are in intersection of k(N,M_{a}) and k(M_{a},B)
intersec2 H_{b} H_{c} k(N,M_{a}) k(M_{a},B)
cmark_l H_{b}
cmark_rt H_{c}

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines h_{a} and b are not parallel% DET: lines h_{a} and b are not the same
% Constructing a point A which belongs to line h_{a} and line b
intersec A h_{a} b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{a} and b are not parallel; circles k(N,M_{a}) and k(M_{a},B)
intersect; points C and M_{a} are not the same; line a and circle k(N,M_{a}) intersect; line m(
H_{b}H_{c}) and circle k(N,M_{a}) intersect; points E_{a} and N are not the same
% Determination conditions: lines h_{a} and b are not the same; points C and H_{b} are not the same
; circles k(N,M_{a}) and k(M_{a},B) are not the same; points H_{a} and E_{a} are not the same;
points M_{a} and H_{a} must be different; points C and M_{a} are not the same; points E_{a} and
M_{a} must be different; points E_{a} and N are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

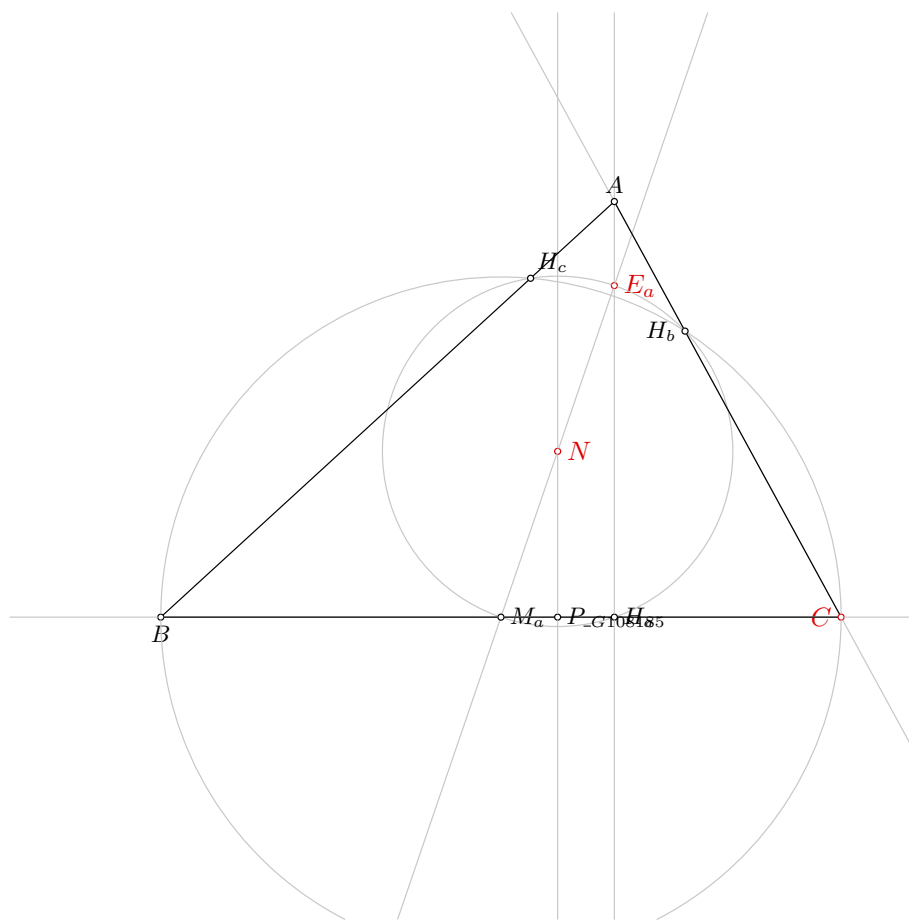


Figure 1: Illustration of the problem 0700

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.028 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_a = \neg E_a$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $C = C$

NDG conditions are:

$S_{H_aCH_b} \neq S_{E_aCH_b}$ i.e., lines H_aE_a and CH_b are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

$S_{\neg M_aBC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_bAC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_a\neg M_bF_{-m_b}^3} \neq S_{F_{-m_a}^2\neg M_bF_{-m_b}^3}$ i.e., lines $\neg M_aF_{-m_a}^2$ and $\neg M_bF_{-m_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_a = \neg E_a$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C = C$

Proving failed

4.3.2 Proving $E_a = \neg E_a$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $C = C$

Proving failed

4.4.2 Proving $E_a = \neg E_a$

Proving failed

4.4.3 Proving $N = \neg N$

Proving failed

Problem 701

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 701: Given a point C , a point E_a and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 702

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 702: Given a point C , a point E_a and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 703

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 703: Given a point C , a point E_a and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 704

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 704: Given a point C , a point E_a and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 705

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 705: Given a point C , a point E_b and a point E_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point E_c , construct a point H (rule W01); ;
2. Using the point E_b and the point H , construct a point B (rule W01); ;
3. Using the point C and the point B , construct a line a (rule W02); % DET: points C and B are not the same;
4. Using the point E_b and the point H , construct a line h_b (rule W02); % DET: points E_b and H are not the same;
5. Using the point C and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points C and E_c are not the same;
6. Using the circle $k(E_c, C)$, the line a , the point E_c and the point C , construct a point H_a (rule W05); % NDG: line a and circle $k(E_c, C)$ intersect % DET: points C and H_a must be different;
7. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
8. Using the circle $k(E_c, C)$, the line h_b , the point E_c and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_c, C)$ intersect % DET: points H and H_b must be different;
9. Using the point H_b and the point C , construct a line b (rule W02); % DET: points H_b and C are not the same;
10. Using the line h_a and the line b , construct a point A (rule W03); % NDG: lines h_a and b are not parallel % DET: lines h_a and b are not the same.

Non-degenerate conditions: lines h_a and b are not parallel; line h_b and circle $k(E_c, C)$ intersect; line a and circle $k(E_c, C)$ intersect; points C and E_c are not the same.

Determination conditions: lines h_a and b are not the same; points H_b and C are not the same; points H and H_b must be different; points H_a and H are not the same; points C and H_a must be different; points E_b and H are not the same; points C and B are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D29,D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,GL04,GL09,L52,L53,L54]

Solving time: 11.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point E_{b} 50 56.36
point E_{c} 95 56.36

color 220 0 0
fontsize 9

cmark_l C
cmark_r E_{b}
cmark_r E_{c}
color 0 0 0
fontsize 8

% Constructing a point H such that CH/CE_{c}=2
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0

% Constructing a point B such that E_{b}B/E_{b}H=-1
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

% DET: points C and B are not the same
% Constructing a line a which passes through point C and point B
line a C B

color 200 200 200
drawline a
color 0 0 0
```

```

% DET: points E_{b} and H are not the same
% Constructing a line h_{b} which passes through point E_{b} and point H
line h_{b} E_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points C and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point C
circle k(E_{c},C) E_{c} C

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% NDG: line a and circle k(E_{c},C) intersect% DET: points C and H_{a} must be different
% Constructing a point P_{\_G173602} which is a foot of the point E_{c} on the line a
foot P_{\_G173602} E_{c} a
cmark_r P_{\_G173602}
color 200 200 200
drawline E_{c} P_{\_G173602}
color 0 0 0

% Constructing a point H_{a} which is an image of the point C in the symmetry to point/line P_{\_G
173602}
sim H_{a} P_{\_G173602} C
cmark_r H_{a}

% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{b} and circle k(E_{c},C) intersect% DET: points H and H_{b} must be different
% Constructing a point P_{\_G173840} which is a foot of the point E_{c} on the line h_{b}
foot P_{\_G173840} E_{c} h_{b}
cmark_r P_{\_G173840}
color 200 200 200
drawline E_{c} P_{\_G173840}
color 0 0 0

% Constructing a point H_{b} which is an image of the point H in the symmetry to point/line P_{\_G
173840}
sim H_{b} P_{\_G173840} H
cmark_l H_{b}

```

```

% DET: points  $H_{\{b\}}$  and  $C$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $C$ 
line b  $H_{\{b\}}$  C

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $h_{\{a\}}$  and  $b$  are not parallel% DET: lines  $h_{\{a\}}$  and  $b$  are not the same
% Constructing a point  $A$  which belongs to line  $h_{\{a\}}$  and line  $b$ 
intersec A  $h_{\{a\}}$  b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{a\}}$  and  $b$  are not parallel; line  $h_{\{b\}}$  and circle  $k(E_{\{c\}}, C)$ 
% intersect; line  $a$  and circle  $k(E_{\{c\}}, C)$  intersect; points  $C$  and  $E_{\{c\}}$  are not the same
% Determination conditions: lines  $h_{\{a\}}$  and  $b$  are not the same; points  $H_{\{b\}}$  and  $C$  are not the same
% ; points  $H$  and  $H_{\{b\}}$  must be different; points  $H_{\{a\}}$  and  $H$  are not the same; points  $C$  and  $H_{\{a\}}$ 
% must be different; points  $E_{\{b\}}$  and  $H$  are not the same; points  $C$  and  $B$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.034 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b = \neg E_b$

Proving failed

4.1.3 Proving $E_c = \neg E_c$

Proving failed

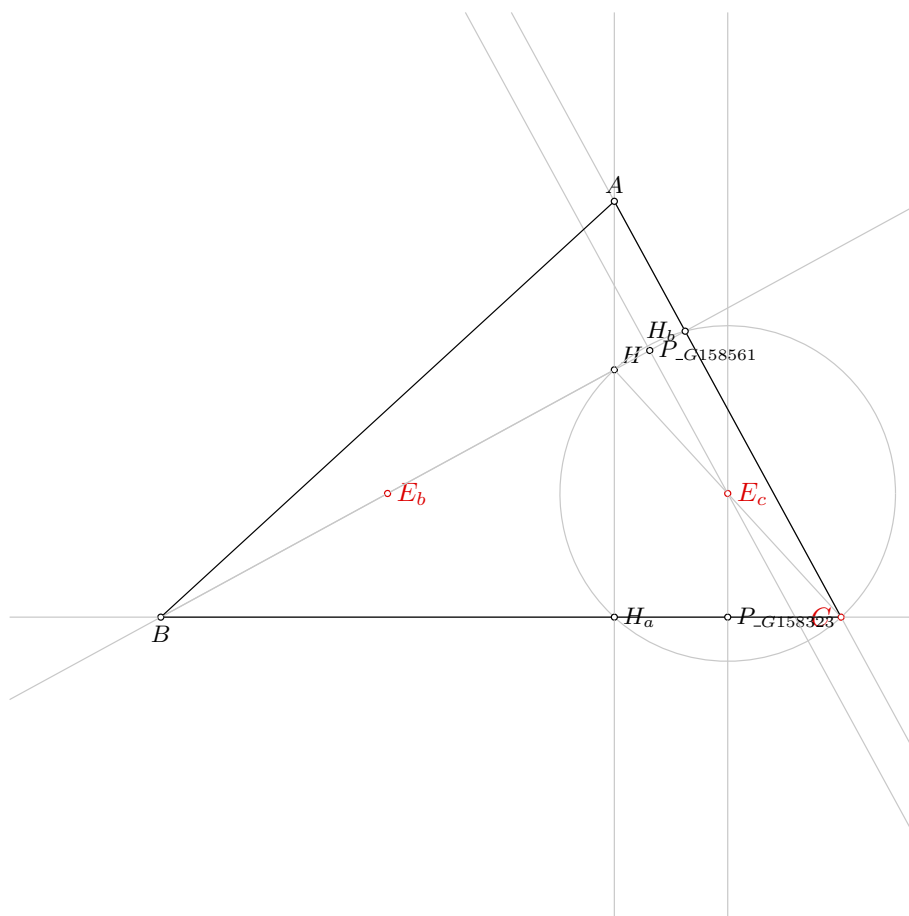


Figure 1: Illustration of the problem 0705

4.2 GCLC - Area method

4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_a H_b C} \neq S_{H H_b C}$ i.e., lines $H_a H$ and $H_b C$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b=\neg E_b$

Proving failed

4.2.3 Proving $E_c=\neg E_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_b=\neg E_b$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 767 terms.

Time Complexity: Time spent by the prover is 3.220 seconds. There are no ndg conditions.

4.3.3 Proving $E_c=\neg E_c$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 2083 terms.

Time Complexity: Time spent by the prover is 7.660 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_b=\neg E_b$

Proving failed

4.4.3 Proving $E_c=\neg E_c$

Proving failed

Problem 706

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 706: Given a point C , a point E_b and a point G , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 707

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 707: Given a point C , a point E_b and a point H , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point H , construct a point E_c (rule W01); ;
2. Using the point E_b and the point H , construct a point B (rule W01); ;
3. Using the point C and the point B , construct a line a (rule W02); % DET: points C and B are not the same;
4. Using the point E_b and the point H , construct a line h_b (rule W02); % DET: points E_b and H are not the same;
5. Using the point C and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points C and E_c are not the same;
6. Using the circle $k(E_c, C)$, the line a , the point E_c and the point C , construct a point H_a (rule W05); % NDG: line a and circle $k(E_c, C)$ intersect % DET: points C and H_a must be different;
7. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
8. Using the circle $k(E_c, C)$, the line h_b , the point E_c and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_c, C)$ intersect % DET: points H and H_b must be different;
9. Using the point H_b and the point C , construct a line b (rule W02); % DET: points H_b and C are not the same;
10. Using the line h_a and the line b , construct a point A (rule W03); % NDG: lines h_a and b are not parallel % DET: lines h_a and b are not the same.

Non-degenerate conditions: lines h_a and b are not parallel; line h_b and circle $k(E_c, C)$ intersect; line a and circle $k(E_c, C)$ intersect; points C and E_c are not the same.

Determination conditions: lines h_a and b are not the same; points H_b and C are not the same; points H and H_b must be different; points H_a and H are not the same; points C and H_a must be different; points E_b and H are not the same; points C and B are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D29,D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,GL04,GL09,L52,L53,L54]

Solving time: 10.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point E_{b} 50 56.36
point H 80 72.73
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_r E_{b}
cmark_rt H
color 0 0 0
fontsize 8
```

```
% Constructing a point E_{c} such that CE_{c}/CH=0.5
towards E_{c} C H 0.5
cmark_r E_{c}
color 200 200 200
drawsegment C H
color 0 0 0
```

```
% Constructing a point B such that E_{b}B/E_{b}H=-1
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0
```

```
% DET: points C and B are not the same
% Constructing a line a which passes through point C and point B
line a C B
```

```
color 200 200 200
drawline a
color 0 0 0
```

```

% DET: points E_{b} and H are not the same
% Constructing a line h_{b} which passes through point E_{b} and point H
line h_{b} E_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points C and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point C
circle k(E_{c},C) E_{c} C

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% NDG: line a and circle k(E_{c},C) intersect% DET: points C and H_{a} must be different
% Constructing a point P_{\_G203158} which is a foot of the point E_{c} on the line a
foot P_{\_G203158} E_{c} a
cmark_r P_{\_G203158}
color 200 200 200
drawline E_{c} P_{\_G203158}
color 0 0 0

% Constructing a point H_{a} which is an image of the point C in the symmetry to point/line P_{\_G
203158}
sim H_{a} P_{\_G203158} C
cmark_r H_{a}

% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{b} and circle k(E_{c},C) intersect% DET: points H and H_{b} must be different
% Constructing a point P_{\_G203396} which is a foot of the point E_{c} on the line h_{b}
foot P_{\_G203396} E_{c} h_{b}
cmark_r P_{\_G203396}
color 200 200 200
drawline E_{c} P_{\_G203396}
color 0 0 0

% Constructing a point H_{b} which is an image of the point H in the symmetry to point/line P_{\_G
203396}
sim H_{b} P_{\_G203396} H
cmark_l H_{b}

```

```

% DET: points  $H_{\{b\}}$  and  $C$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $C$ 
line b  $H_{\{b\}}$  C

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $h_{\{a\}}$  and  $b$  are not parallel% DET: lines  $h_{\{a\}}$  and  $b$  are not the same
% Constructing a point  $A$  which belongs to line  $h_{\{a\}}$  and line  $b$ 
intersec A  $h_{\{a\}}$  b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{a\}}$  and  $b$  are not parallel; line  $h_{\{b\}}$  and circle  $k(E_{\{c\}}, C)$ 
% intersect; line  $a$  and circle  $k(E_{\{c\}}, C)$  intersect; points  $C$  and  $E_{\{c\}}$  are not the same
% Determination conditions: lines  $h_{\{a\}}$  and  $b$  are not the same; points  $H_{\{b\}}$  and  $C$  are not the same
% ; points  $H$  and  $H_{\{b\}}$  must be different; points  $H_{\{a\}}$  and  $H$  are not the same; points  $C$  and  $H_{\{a\}}$ 
% must be different; points  $E_{\{b\}}$  and  $H$  are not the same; points  $C$  and  $B$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.027 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b=_E_b$

Proving failed

4.1.3 Proving $H=_H$

Proving failed

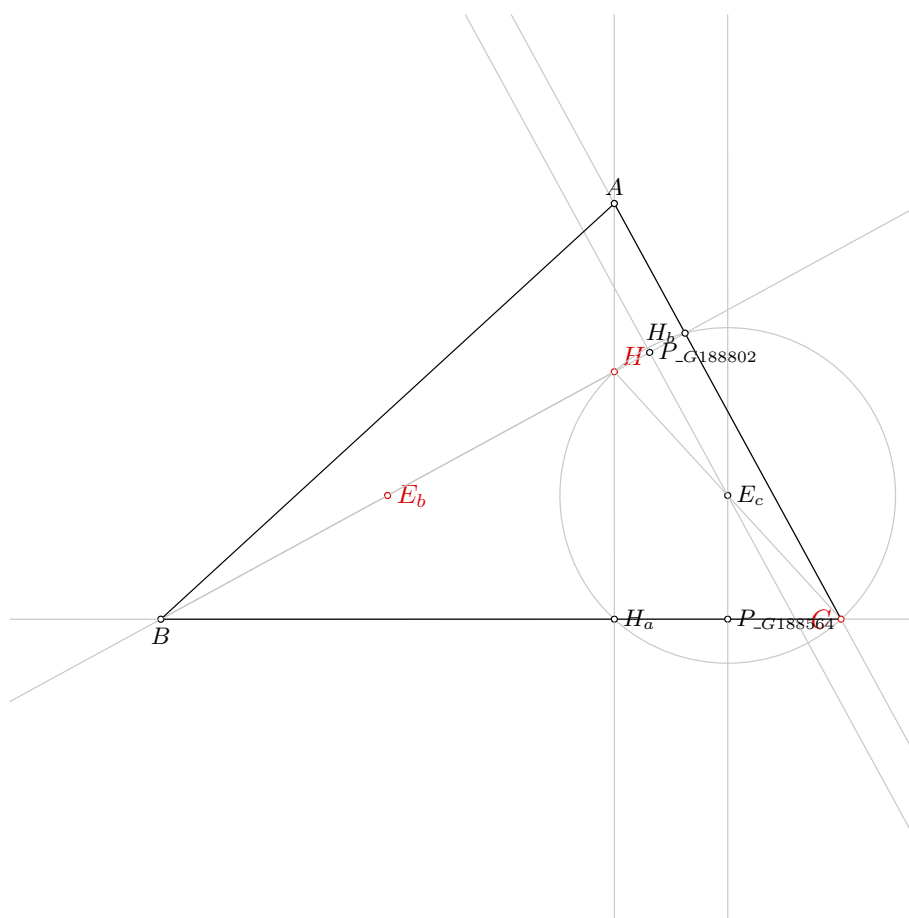


Figure 1: Illustration of the problem 0707

4.2 GCLC - Area method

4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_a H_b C} \neq S_{H H_b C}$ i.e., lines $H_a H$ and $H_b C$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b=_E E_b$

Proving failed

4.2.3 Proving $H=_H H$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_b=_E E_b$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 767 terms.

Time Complexity: Time spent by the prover is 0.520 seconds. There are no ndg conditions.

4.3.3 Proving $H=_H H$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 138 terms.

Time Complexity: Time spent by the prover is 0.090 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_b=_E E_b$

Proving failed

4.4.3 Proving $H=_H H$

Proving failed

Problem 708

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 708: Given a point C , a point E_b and a point H_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point H_a , construct a line a (rule W02); % DET: points C and H_a are not the same;
2. Using the point H_a and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_a and E_b are not the same;
3. Using the circle $k(E_b, B)$, the line a , the point E_b and the point H_a , construct a point B (rule W05); % NDG: line a and circle $k(E_b, B)$ intersect % DET: points H_a and B must be different;
4. Using the point E_b and the point B , construct a point H (rule W01); ;
5. Using the point C and the point H , construct a line h_c (rule W02); % DET: points C and H are not the same;
6. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
7. Using the circle $k(E_b, B)$, the line h_c , the point E_b and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_b, B)$ intersect % DET: points H and H_c must be different;
8. Using the point B and the point H_c , construct a line c (rule W02); % DET: points B and H_c are not the same;
9. Using the line h_a and the line c , construct a point A (rule W03); % NDG: lines h_a and c are not parallel % DET: lines h_a and c are not the same.

Non-degenerate conditions: lines h_a and c are not parallel; line h_c and circle $k(E_b, B)$ intersect; line a and circle $k(E_b, B)$ intersect; points H_a and E_b are not the same.

Determination conditions: lines h_a and c are not the same; points B and H_c are not the same; points H and H_c must be different; points H_a and H are not the same; points C and H are not the same; points H_a and B must be different; points C and H_a are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D29,D3,D5,D7,D8,GD01,GD02,GL03,GL04,L3,L49,L50,L51]

Solving time: 8.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point E_{b} 50 56.36
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_l C
cmark_r E_{b}
cmark_r H_{a}
color 0 0 0
fontsize 8

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points H_{a} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H
_{a}
circle k(E_{b},B) E_{b} H_{a}

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line a and circle k(E_{b},B) intersect% DET: points H_{a} and B must be different
% Constructing a point P_{\_G226629} which is a foot of the point E_{b} on the line a
foot P_{\_G226629} E_{b} a
cmark_r P_{\_G226629}
color 200 200 200
drawline E_{b} P_{\_G226629}
color 0 0 0
```

```

% Constructing a point B which is an image of the point H_{a} in the symmetry to point/line P_{\_G
226629}
sim B P_{\_G226629} H_{a}
cmark_b B

% Constructing a point H such that E_{b}H/E_{b}B=-1
towards H E_{b} B -1
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0

% DET: points C and H are not the same
% Constructing a line h_{c} which passes through point C and point H
line h_{c} C H

color 200 200 200
drawline h_{c}
color 0 0 0

% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{c} and circle k(E_{b},B) intersect% DET: points H and H_{c} must be different
% Constructing a point P_{\_G226992} which is a foot of the point E_{b} on the line h_{c}
foot P_{\_G226992} E_{b} h_{c}
cmark_r P_{\_G226992}
color 200 200 200
drawline E_{b} P_{\_G226992}
color 0 0 0

% Constructing a point H_{c} which is an image of the point H in the symmetry to point/line P_{\_G
226992}
sim H_{c} P_{\_G226992} H
cmark_rt H_{c}

% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

color 200 200 200
drawline c

```



```
color 0 0 0
```

```
% NDG: lines h_{a} and c are not parallel% DET: lines h_{a} and c are not the same
% Constructing a point A which belongs to line h_{a} and line c
intersec A h_{a} c
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines h_{a} and c are not parallel; line h_{c} and circle k(E_{b},B)
    intersect; line a and circle k(E_{b},B) intersect; points H_{a} and E_{b} are not the same
% Determination conditions: lines h_{a} and c are not the same; points B and H_{c} are not the same
    ; points H and H_{c} must be different; points H_{a} and H are not the same; points C and H are
    not the same; points H_{a} and B must be different; points C and H_{a} are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b = \neg E_b$

Proving failed

4.1.3 Proving $H_a = \neg H_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 1176 terms.

Time Complexity: Time spent by the prover is 7.762 seconds.

NDG conditions Points C , E_b and H_a are not collinear

Points C , E_b and H_a are not collinear

Points C and H are not identical

Points C , E_b and H are not collinear

Line through points B and H_c is not parallel with line through points H and H_a

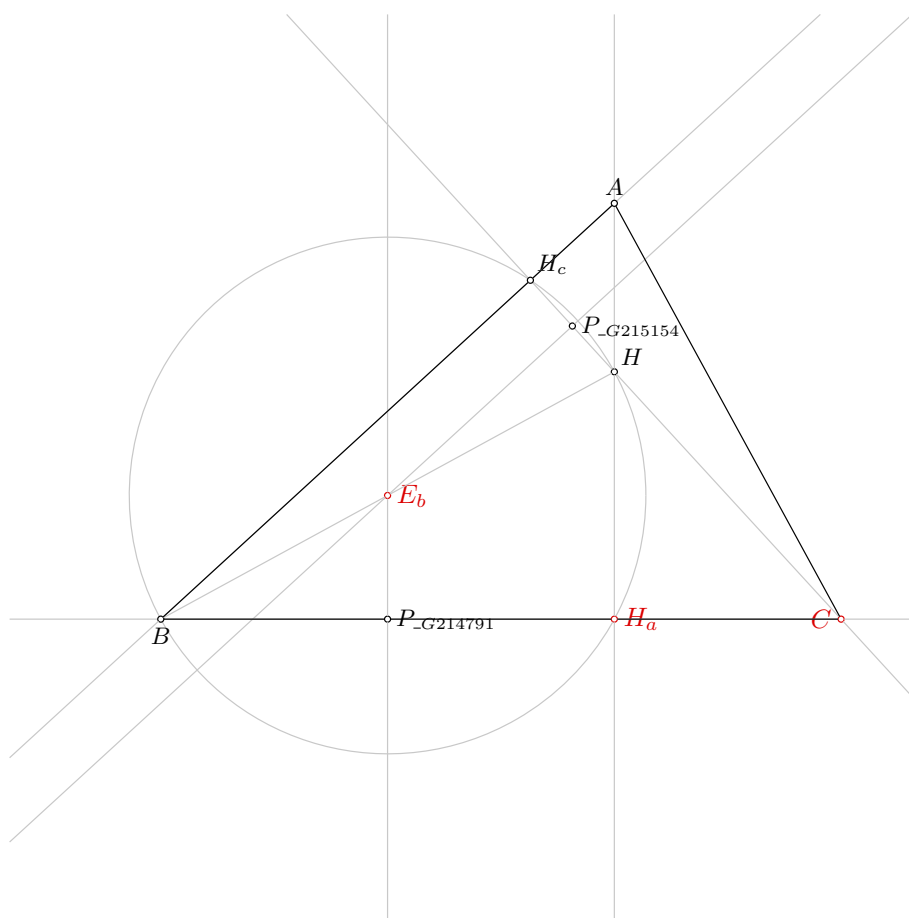


Figure 1: Illustration of the problem 0708

Line through points C and E_b is not parallel with line through points H and H_a

Points B and C are not identical

Points B and C are not identical

4.2 GCLC - Area method

4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_aBH_c} \neq S_{HBH_c}$ i.e., lines H_aH and BH_c are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a}BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

$S_{BAF^0_{-h_a}} \neq S_{CAF^0_{-h_a}}$ i.e., lines BC and $AF^0_{-h_a}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b=\neg E_b$

Proving failed

4.2.3 Proving $H_a=\neg H_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C=C$

Proving failed

4.3.2 Proving $E_b=\neg E_b$

Proving failed

4.3.3 Proving $H_a=\neg H_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Proving failed

4.4.2 Proving $E_b=\neg E_b$

Proving failed

4.4.3 Proving $H_a=\neg H_a$

Proving failed

Problem 709

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 709: Given a point E_b , a point H_b and a point C , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;
2. Using the point H_b and the line h_b , construct a line b (rule W10a); ;
3. Choose freely a point C on the line b (rule WOnline2);
4. Using the point C and the point H_b , construct a line $m(CH_b)$ (rule W14); % DET: points C and H_b are not the same;
5. Using the point E_b and the point H_b , construct a line $m(E_bH_b)$ (rule W14); % DET: points E_b and H_b are not the same;
6. Choose freely a point A on the line b (rule WOnline1) ;
7. Using the point A and the point C , construct a point M_b (rule W01); ;
8. Using the point E_b and the point M_b , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and M_b are not the same;
9. Using the line $m(E_bH_b)$ and the line $m(H_aH_c)$, construct a point N (rule W03); % NDG: lines $m(E_bH_b)$ and $m(H_aH_c)$ are not parallel % DET: lines $m(E_bH_b)$ and $m(H_aH_c)$ are not the same;
10. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
11. Using the circle $k(N, M_a)$ and the line $m(CH_b)$, construct a point M_a and a point E_c (rule W04); % NDG: line $m(CH_b)$ and circle $k(N, M_a)$ intersect;

12. Using the point C and the point M_a , construct a point B (rule W01); .

Non-degenerate conditions: line $m(CH_b)$ and circle $k(N, M_a)$ intersect; points E_b and N are not the same; lines $m(E_bH_b)$ and $m(H_aH_c)$ are not parallel.

Determination conditions: lines $m(E_bH_b)$ and $m(H_aH_c)$ are not the same; points E_b and M_b are not the same; points E_b and H_b are not the same; points C and H_b are not the same; points E_b and H_b are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W14,WOnline1,WOnline2]

Lemmas used: [D21,D22,D29,D3,D32,D6,D9,GD02,GL01,GL03,GL04,GL09,L19,L20,L21,L23,L24,L37,L38,L44]

Solving time: 55.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
point H_{b} 89.36 77.83
point C 110 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{b}
cmark_l H_{b}
cmark_l C
color 0 0 0
fontsize 8
```

```
% DET: points E_{b} and H_{b} are not the same
% Constructing a line h_{b} which passes through point E_{b} and point H_{b}
line h_{b} E_{b} H_{b}
```

```
color 200 200 200
drawline h_{b}
color 0 0 0
```

```
% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
perp b H_{b} h_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% Generating random value V[_G250743]
random V[_G250743]
```

```

% Calculating value  $V[_G250764]$  using formula  $V[_G250743]*20$ 
expression  $V[_G250764]$  {  $V[_G250743]*20$  }

% Constructing a point  $C$  which is a point for which holds  $H_{\{b\}}C = V[_G250764]$  and angle  $E_{\{b\}}H_{\{b\}}$ 
 $C = 90$ 
turtle C  $E_{\{b\}}$   $H_{\{b\}}$  90  $V[_G250764]$ 
cmark_l C

% DET: points  $C$  and  $H_{\{b\}}$  are not the same
% Constructing bisector  $m(CH_{\{b\}})$  of the segment  $CH_{\{b\}}$ 
med m( $CH_{\{b\}}$ ) C  $H_{\{b\}}$ 

color 200 200 200
drawline m( $CH_{\{b\}}$ )
color 0 0 0

color 200 200 200
drawsegment C  $H_{\{b\}}$ 
color 0 0 0

% DET: points  $E_{\{b\}}$  and  $H_{\{b\}}$  are not the same
% Constructing bisector  $m(E_{\{b\}}H_{\{b\}})$  of the segment  $E_{\{b\}}H_{\{b\}}$ 
med m( $E_{\{b\}}H_{\{b\}}$ )  $E_{\{b\}}$   $H_{\{b\}}$ 

color 200 200 200
drawline m( $E_{\{b\}}H_{\{b\}}$ )
color 0 0 0

color 200 200 200
drawsegment  $E_{\{b\}}$   $H_{\{b\}}$ 
color 0 0 0

% Choosing randomly a point  $A$  on the line  $CH_{\{b\}}$ 
online A C  $H_{\{b\}}$ 
cmark_t A
color 200 200 200
drawline C  $H_{\{b\}}$ 
color 0 0 0

% Constructing a point  $M_{\{b\}}$  such that  $AM_{\{b\}}/AC=0.5$ 
towards  $M_{\{b\}}$  A C 0.5
cmark_lt  $M_{\{b\}}$ 
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points  $E_{\{b\}}$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $m(H_{\{a\}}H_{\{c\}})$  which passes through point  $E_{\{b\}}$  and point  $M_{\{b\}}$ 

```

```

line m(H_{a}H_{c}) E_{b} M_{b}

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

% NDG: lines m(E_{b}H_{b}) and m(H_{a}H_{c}) are not parallel% DET: lines m(E_{b}H_{b}) and m(H_{a}
H_{c}) are not the same
% Constructing a point N which belongs to line m(E_{b}H_{b}) and line m(H_{a}H_{c})
intersec N m(E_{b}H_{b}) m(H_{a}H_{c})
cmark_r N

% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(CH_{b}) and circle k(N,M_{a}) intersect
% Constructing points M_{a} and E_{c} which are in intersection of k(N,M_{a}) and m(CH_{b})
intersec2 M_{a} E_{c} k(N,M_{a}) m(CH_{b})
cmark_r M_{a}
cmark_r E_{c}

% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line m(CH_{b}) and circle k(N,M_{a}) intersect; points E_{b} and N are
not the same; lines m(E_{b}H_{b}) and m(H_{a}H_{c}) are not parallel
% Determination conditions: lines m(E_{b}H_{b}) and m(H_{a}H_{c}) are not the same; points E_{b}
and M_{b} are not the same; points E_{b} and H_{b} are not the same; points C and H_{b} are not
the same; points E_{b} and H_{b} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

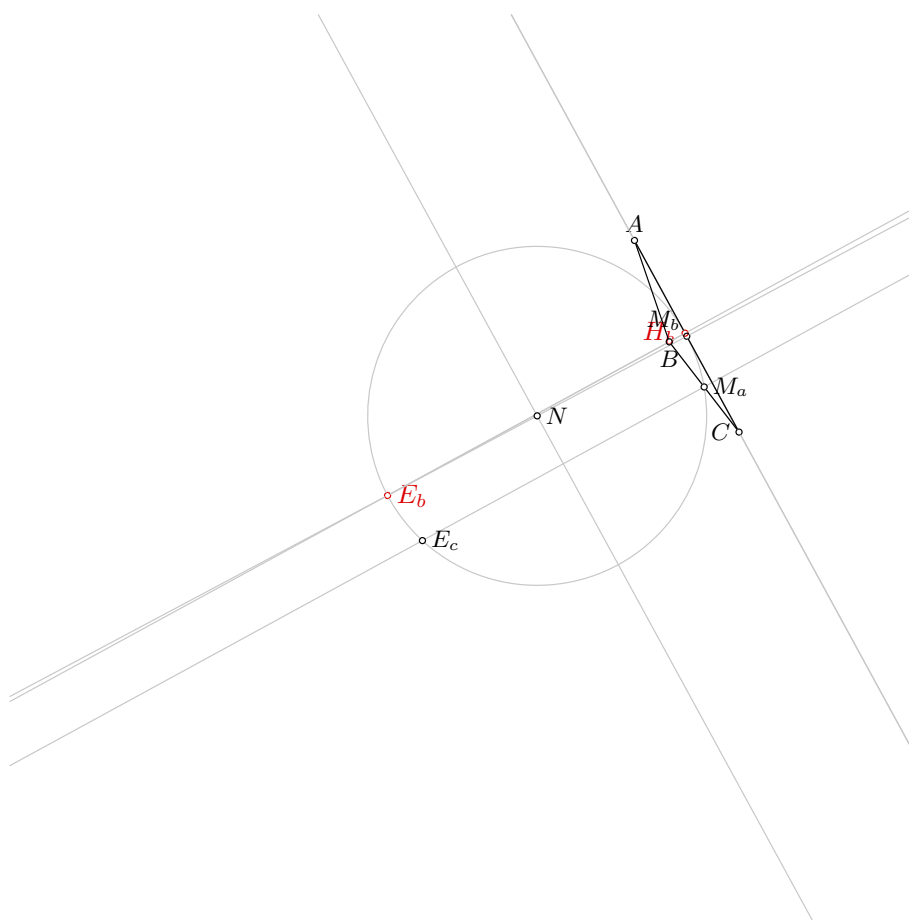


Figure 1: Illustration of the problem 0709

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 20 terms.

Time Complexity: Time spent by the prover is 0.305 seconds.

NDG conditions Points H_b and C are not identical

Line through points B and H_b is not perpendicular to line through points H_b and E_b

4.1.2 Proving $H_b = \neg H_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 4 terms.

Time Complexity: Time spent by the prover is 0.139 seconds.

NDG conditions Point $\neg H_b$ is not on circle with center H_b and point from it C

4.1.3 Proving $C = C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.007 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $H_b = \neg H_b$

Proving failed

4.2.3 Proving $C = C$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $H_b = \neg H_b$

Proving failed

4.3.3 Proving $C=C$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b=_E E_b$

Proving failed

4.4.2 Proving $H_b=_H H_b$

Proving failed

4.4.3 Proving $C=C$

Proving failed

Problem 710

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 710: Given a point C , a point E_b and a point H_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point H_c , construct a line h_c (rule W02); % DET: points C and H_c are not the same;
2. Using the point H_c and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_c and E_b are not the same;
3. Using the circle $k(E_b, B)$, the line h_c , the point E_b and the point H_c , construct a point H (rule W05); % NDG: line h_c and circle $k(E_b, B)$ intersect % DET: points H_c and H must be different;
4. Using the point E_b and the point H , construct a point B (rule W01); ;
5. Using the point C and the point B , construct a line a (rule W02); % DET: points C and B are not the same;
6. Using the point H_c and the point B , construct a line c (rule W02); % DET: points H_c and B are not the same;
7. Using the circle $k(E_b, B)$, the line a , the point E_b and the point B , construct a point H_a (rule W05); % NDG: line a and circle $k(E_b, B)$ intersect % DET: points B and H_a must be different;
8. Using the point H and the point H_a , construct a line h_a (rule W02); % DET: points H and H_a are not the same;
9. Using the line c and the line h_a , construct a point A (rule W03); % NDG: lines c and h_a are not parallel % DET: lines c and h_a are not the same.

Non-degenerate conditions: lines c and h_a are not parallel; line a and circle $k(E_b, B)$ intersect; line h_c and circle $k(E_b, B)$ intersect; points H_c and E_b are not the same.

Determination conditions: lines c and h_a are not the same; points H and H_a are not the same; points B and H_a must be different; points H_c and B are not the same; points C and B are not the same; points H_c and H must be different; points C and H_c are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D29,D3,D5,D7,D8,GD01,GD02,GL03,GL04,L3,L49,L50,L51]

Solving time: 8.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point E_{b} 50 56.36
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_l C
cmark_r E_{b}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points C and H_{c} are not the same
% Constructing a line h_{c} which passes through point C and point H_{c}
line h_{c} C H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points H_{c} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H_{c}
circle k(E_{b},B) E_{b} H_{c}

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line h_{c} and circle k(E_{b},B) intersect% DET: points H_{c} and H must be different
% Constructing a point P_{\_G27748} which is a foot of the point E_{b} on the line h_{c}
foot P_{\_G27748} E_{b} h_{c}
cmark_r P_{\_G27748}
color 200 200 200
drawline E_{b} P_{\_G27748}
color 0 0 0
```

```

% Constructing a point H which is an image of the point H_{c} in the symmetry to point/line P_{\_G
27748}
sim H P_{\_G27748} H_{c}
cmark_rt H

% Constructing a point B such that E_{b}B/E_{b}H=-1
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

% DET: points C and B are not the same
% Constructing a line a which passes through point C and point B
line a C B

color 200 200 200
drawline a
color 0 0 0

% DET: points H_{c} and B are not the same
% Constructing a line c which passes through point H_{c} and point B
line c H_{c} B

color 200 200 200
drawline c
color 0 0 0

% NDG: line a and circle k(E_{b},B) intersect% DET: points B and H_{a} must be different
% Constructing a point P_{\_G28111} which is a foot of the point E_{b} on the line a
foot P_{\_G28111} E_{b} a
cmark_r P_{\_G28111}
color 200 200 200
drawline E_{b} P_{\_G28111}
color 0 0 0

% Constructing a point H_{a} which is an image of the point B in the symmetry to point/line P_{\_G
28111}
sim H_{a} P_{\_G28111} B
cmark_r H_{a}

% DET: points H and H_{a} are not the same
% Constructing a line h_{a} which passes through point H and point H_{a}
line h_{a} H H_{a}

color 200 200 200
drawline h_{a}

```

```
color 0 0 0
```

```
% NDG: lines c and h_{a} are not parallel% DET: lines c and h_{a} are not the same
% Constructing a point A which belongs to line c and line h_{a}
intersec A c h_{a}
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines c and h_{a} are not parallel; line a and circle k(E_{b},B)
    intersect; line h_{c} and circle k(E_{b},B) intersect; points H_{c} and E_{b} are not the same
% Determination conditions: lines c and h_{a} are not the same; points H and H_{a} are not the same
    ; points B and H_{a} must be different; points H_{c} and B are not the same; points C and B are
    not the same; points H_{c} and H must be different; points C and H_{c} are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.032 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b = \neg E_b$

Proving failed

4.1.3 Proving $H_c = \neg H_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_c H H_a} \neq S_{B H H_a}$ i.e., lines $H_c B$ and $H H_a$ are not parallel (construction based assumption)
 $S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a} BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

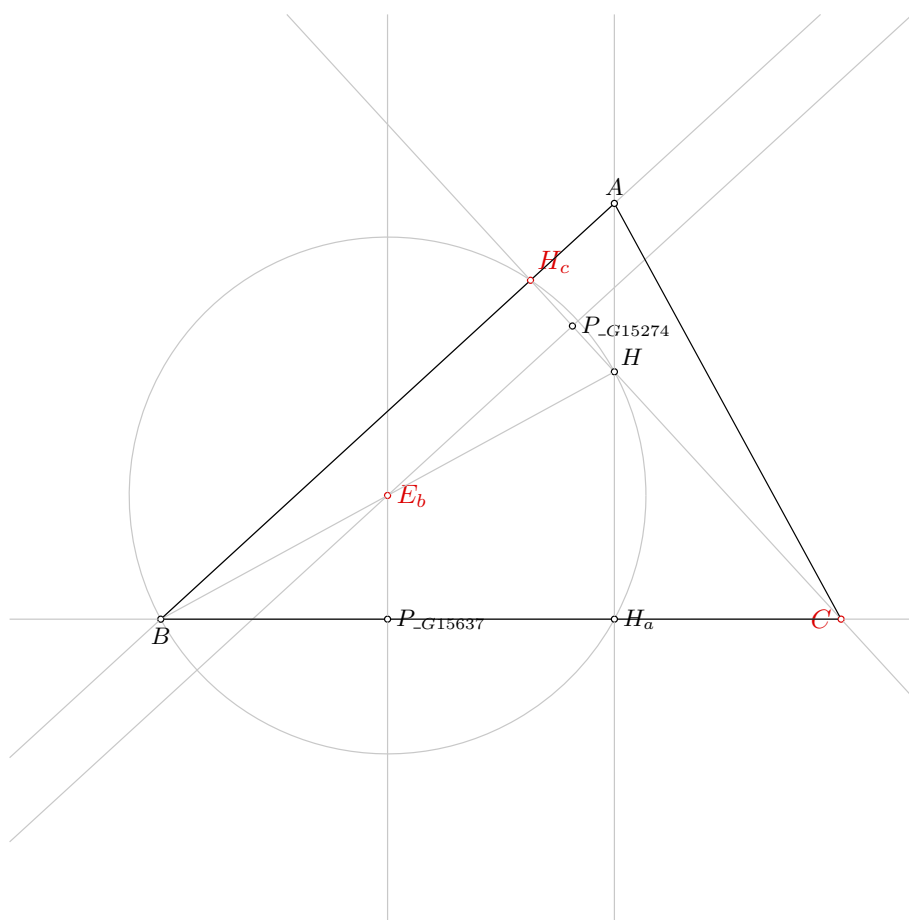


Figure 1: Illustration of the problem 0710

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)
 $S_{ACF_{h_c}^2} \neq S_{BCF_{h_c}^2}$ i.e., lines AB and $CF_{h_c}^2$ are not parallel (construction based assumption)
 Total number of proof steps: 1
 Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b = \neg E_b$

Proving failed

4.2.3 Proving $H_c = \neg H_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C = C$

Proving failed

4.3.2 Proving $E_b = \neg E_b$

Proving failed

4.3.3 Proving $H_c = \neg H_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $C = C$

Proving failed

4.4.2 Proving $E_b = \neg E_b$

Proving failed

4.4.3 Proving $H_c = \neg H_c$

Proving failed

Problem 711

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 711: Given a point C , a point E_b and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 712

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 712: Given a point C , a point E_b and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point M_a , construct a point B (rule W01); ;
2. Using the point E_b and the point B , construct a point H (rule W01); ;
3. Using the point C and the point H , construct a line h_c (rule W02); % DET: points C and H are not the same;
4. Using the point E_b and the point B , construct a line h_b (rule W02); % DET: points E_b and B are not the same;
5. Using the point C and the point M_a , construct a circle $k(M_a, B)$ (rule W06); % NDG: points C and M_a are not the same;
6. Using the circle $k(M_a, B)$, the line h_c , the point M_a and the point C , construct a point H_c (rule W05); % NDG: line h_c and circle $k(M_a, B)$ intersect % DET: points C and H_c must be different;
7. Using the point H_c and the point B , construct a line c (rule W02); % DET: points H_c and B are not the same;
8. Using the circle $k(M_a, B)$, the line h_b , the point M_a and the point B , construct a point H_b (rule W05); % NDG: line h_b and circle $k(M_a, B)$ intersect % DET: points B and H_b must be different;
9. Using the point H_b and the point C , construct a line b (rule W02); % DET: points H_b and C are not the same;
10. Using the line c and the line b , construct a point A (rule W03); % NDG: lines c and b are not parallel % DET: lines c and b are not the same.

Non-degenerate conditions: lines c and b are not parallel; line h_b and circle $k(M_a, B)$ intersect; line h_c and circle $k(M_a, B)$ intersect; points C and M_a are not the same.

Determination conditions: lines c and b are not the same; points H_b and C are not the same; points B and H_b must be different; points H_c and B are not the same; points C and H_c must be different; points E_b and B are not the same; points C and H are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D21,D29,D3,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L37,L38,L39]

Solving time: 10.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point E_{b} 50 56.36
point M_{a} 65 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_r E_{b}
cmark_r M_{a}
color 0 0 0
fontsize 8
```

```
% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0
```

```
% Constructing a point H such that E_{b}H/E_{b}B=-1
towards H E_{b} B -1
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0
```

```
% DET: points C and H are not the same
% Constructing a line h_{c} which passes through point C and point H
line h_{c} C H

color 200 200 200
drawline h_{c}
color 0 0 0
```

```

% DET: points  $E_{\{b\}}$  and  $B$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $E_{\{b\}}$  and point  $B$ 
line  $h_{\{b\}}$   $E_{\{b\}}$   $B$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: points  $C$  and  $M_{\{a\}}$  are not the same
% Constructing a circle  $k(M_{\{a\}}, B)$  whose center is at point  $M_{\{a\}}$  and which passes through point  $C$ 
circle  $k(M_{\{a\}}, B)$   $M_{\{a\}}$   $C$ 

color 200 200 200
drawcircle  $k(M_{\{a\}}, B)$ 
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(M_{\{a\}}, B)$  intersect% DET: points  $C$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\_G54239\}}$  which is a foot of the point  $M_{\{a\}}$  on the line  $h_{\{c\}}$ 
foot  $P_{\{\_G54239\}}$   $M_{\{a\}}$   $h_{\{c\}}$ 
cmark_r  $P_{\{\_G54239\}}$ 
color 200 200 200
drawline  $M_{\{a\}}$   $P_{\{\_G54239\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $C$  in the symmetry to point/line  $P_{\{\_G54239\}}$ 
sim  $H_{\{c\}}$   $P_{\{\_G54239\}}$   $C$ 
cmark_rt  $H_{\{c\}}$ 

% DET: points  $H_{\{c\}}$  and  $B$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $B$ 
line  $c$   $H_{\{c\}}$   $B$ 

color 200 200 200
drawline  $c$ 
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(M_{\{a\}}, B)$  intersect% DET: points  $B$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\_G54477\}}$  which is a foot of the point  $M_{\{a\}}$  on the line  $h_{\{b\}}$ 
foot  $P_{\{\_G54477\}}$   $M_{\{a\}}$   $h_{\{b\}}$ 
cmark_r  $P_{\{\_G54477\}}$ 
color 200 200 200
drawline  $M_{\{a\}}$   $P_{\{\_G54477\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $B$  in the symmetry to point/line  $P_{\{\_G54477\}}$ 
sim  $H_{\{b\}}$   $P_{\{\_G54477\}}$   $B$ 
cmark_l  $H_{\{b\}}$ 

```

```

% DET: points  $H_{\{b\}}$  and  $C$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $C$ 
line b  $H_{\{b\}}$  C

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $c$  and  $b$  are not parallel% DET: lines  $c$  and  $b$  are not the same
% Constructing a point  $A$  which belongs to line  $c$  and line  $b$ 
intersec A c b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $c$  and  $b$  are not parallel; line  $h_{\{b\}}$  and circle  $k(M_{\{a\}}, B)$ 
% intersect; line  $h_{\{c\}}$  and circle  $k(M_{\{a\}}, B)$  intersect; points  $C$  and  $M_{\{a\}}$  are not the same
% Determination conditions: lines  $c$  and  $b$  are not the same; points  $H_{\{b\}}$  and  $C$  are not the same;
% points  $B$  and  $H_{\{b\}}$  must be different; points  $H_{\{c\}}$  and  $B$  are not the same; points  $C$  and  $H_{\{c\}}$ 
% must be different; points  $E_{\{b\}}$  and  $B$  are not the same; points  $C$  and  $H$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.046 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b = \neg E_b$

Proving failed

4.1.3 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

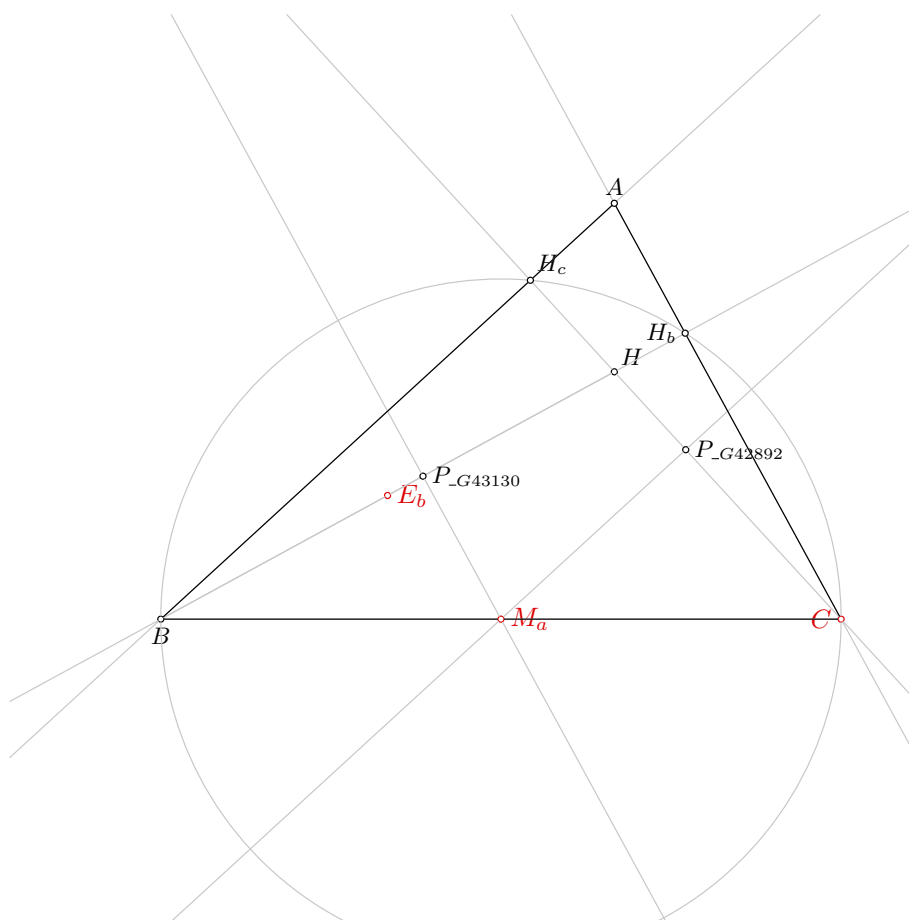


Figure 1: Illustration of the problem 0712

Time Complexity: Time spent by the prover is 0.022 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_cH_bC} \neq S_{BH_bC}$ i.e., lines H_cB and H_bC are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b=_E_b$

Proving failed

4.2.3 Proving $M_a=_M_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_b=_E_b$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 124 terms.

Time Complexity: Time spent by the prover is 0.370 seconds. There are no ndg conditions.

4.3.3 Proving $M_a=_M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 9 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_b=_E_b$

Proving failed

4.4.3 Proving $M_a = -M_a$

Proving failed

Problem 713

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 713: Given a point C , a point E_b and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point M_b , construct a point A (rule W01); ;
2. Using the point C and the point M_b , construct a line b (rule W02); % DET: points C and M_b are not the same;
3. Using the point E_b and the point M_b , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and M_b are not the same;
4. Using the point E_b and the line b , construct a line h_b (rule W10b); ;
5. Using the line h_b and the line b , construct a point H_b (rule W03); % NDG: lines h_b and b are not parallel % DET: lines h_b and b are not the same;
6. Using the point C and the point H_b , construct a line $m(CH_b)$ (rule W14); % DET: points C and H_b are not the same;
7. Using the point E_b and the point M_b , construct a line $m(E_bM_b)$ (rule W14); % DET: points E_b and M_b are not the same;
8. Using the line $m(E_bM_b)$ and the line $m(H_aH_c)$, construct a point N (rule W03); % NDG: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel % DET: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same;
9. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
10. Using the circle $k(N, M_a)$ and the line $m(CH_b)$, construct a point M_a and a point E_c (rule W04); % NDG: line $m(CH_b)$ and circle $k(N, M_a)$ intersect;

11. Using the point M_a and the point C , construct a point B (rule W01); .

Non-degenerate conditions: line $m(CH_b)$ and circle $k(N, M_a)$ intersect; points E_b and N are not the same; lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel; lines h_b and b are not parallel.

Determination conditions: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same; points E_b and M_b are not the same; points C and H_b are not the same; lines h_b and b are not the same; points E_b and M_b are not the same; points C and M_b are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14]

Lemmas used: [D21,D22,D29,D3,D32,D6,D9,GD01,GD02,GL01,GL03,GL04,GL09,L17,L19,L21,L23,L24,L37,L

Solving time: 2.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point E_{b} 50 56.36
point M_{b} 95 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_r E_{b}
cmark_lt M_{b}
color 0 0 0
fontsize 8
```

```
% Constructing a point A such that CA/CM_{b}=2
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0
```

```
% DET: points C and M_{b} are not the same
% Constructing a line b which passes through point C and point M_{b}
line b C M_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% DET: points E_{b} and M_{b} are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point M_{b}
line m(H_{a}H_{c}) E_{b} M_{b}
```

```
color 200 200 200
drawline m(H_{a}H_{c})
```

```
color 0 0 0
```

```
% Constructing a line  $h_{\{b\}}$  which is perpendicular to line  $b$  and which passes through point  $E_{\{b\}}$ 
perp  $h_{\{b\}}$   $E_{\{b\}}$   $b$ 
```

```
color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0
```

```
% NDG: lines  $h_{\{b\}}$  and  $b$  are not parallel% DET: lines  $h_{\{b\}}$  and  $b$  are not the same
% Constructing a point  $H_{\{b\}}$  which belongs to line  $h_{\{b\}}$  and line  $b$ 
intersec  $H_{\{b\}}$   $h_{\{b\}}$   $b$ 
cmark_l  $H_{\{b\}}$ 
```

```
% DET: points  $C$  and  $H_{\{b\}}$  are not the same
% Constructing bisector  $m(CH_{\{b\}})$  of the segment  $CH_{\{b\}}$ 
med  $m(CH_{\{b\}})$   $C$   $H_{\{b\}}$ 
```

```
color 200 200 200
drawline  $m(CH_{\{b\}})$ 
color 0 0 0
```

```
color 200 200 200
drawsegment  $C$   $H_{\{b\}}$ 
color 0 0 0
```

```
% DET: points  $E_{\{b\}}$  and  $M_{\{b\}}$  are not the same
% Constructing bisector  $m(E_{\{b\}}M_{\{b\}})$  of the segment  $E_{\{b\}}M_{\{b\}}$ 
med  $m(E_{\{b\}}M_{\{b\}})$   $E_{\{b\}}$   $M_{\{b\}}$ 
```

```
color 200 200 200
drawline  $m(E_{\{b\}}M_{\{b\}})$ 
color 0 0 0
```

```
color 200 200 200
drawsegment  $E_{\{b\}}$   $M_{\{b\}}$ 
color 0 0 0
```

```
% NDG: lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not parallel% DET: lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}$ 
 $H_{\{c\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{b\}}M_{\{b\}})$  and line  $m(H_{\{a\}}H_{\{c\}})$ 
intersec  $N$   $m(E_{\{b\}}M_{\{b\}})$   $m(H_{\{a\}}H_{\{c\}})$ 
cmark_r  $N$ 
```

```
% NDG: points  $E_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{b\}}$ 
circle  $k(N, M_{\{a\}})$   $N$   $E_{\{b\}}$ 
```

```

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(CH_{b}) and circle k(N,M_{a}) intersect
% Constructing points M_{a} and E_{c} which are in intersection of k(N,M_{a}) and m(CH_{b})
intersec2 M_{a} E_{c} k(N,M_{a}) m(CH_{b})
cmark_r M_{a}
cmark_r E_{c}

% Constructing a point B such that M_{a}B/M_{a}C=-1
towards B M_{a} C -1
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line m(CH_{b}) and circle k(N,M_{a}) intersect; points E_{b} and N are
% not the same; lines m(E_{b}M_{b}) and m(H_{a}H_{c}) are not parallel; lines h_{b} and b are
% not parallel
% Determination conditions: lines m(E_{b}M_{b}) and m(H_{a}H_{c}) are not the same; points E_{b}
% and M_{b} are not the same; points C and H_{b} are not the same; lines h_{b} and b are not the
% same; points E_{b} and M_{b} are not the same; points C and M_{b} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.022 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b = E_b$

Proving failed

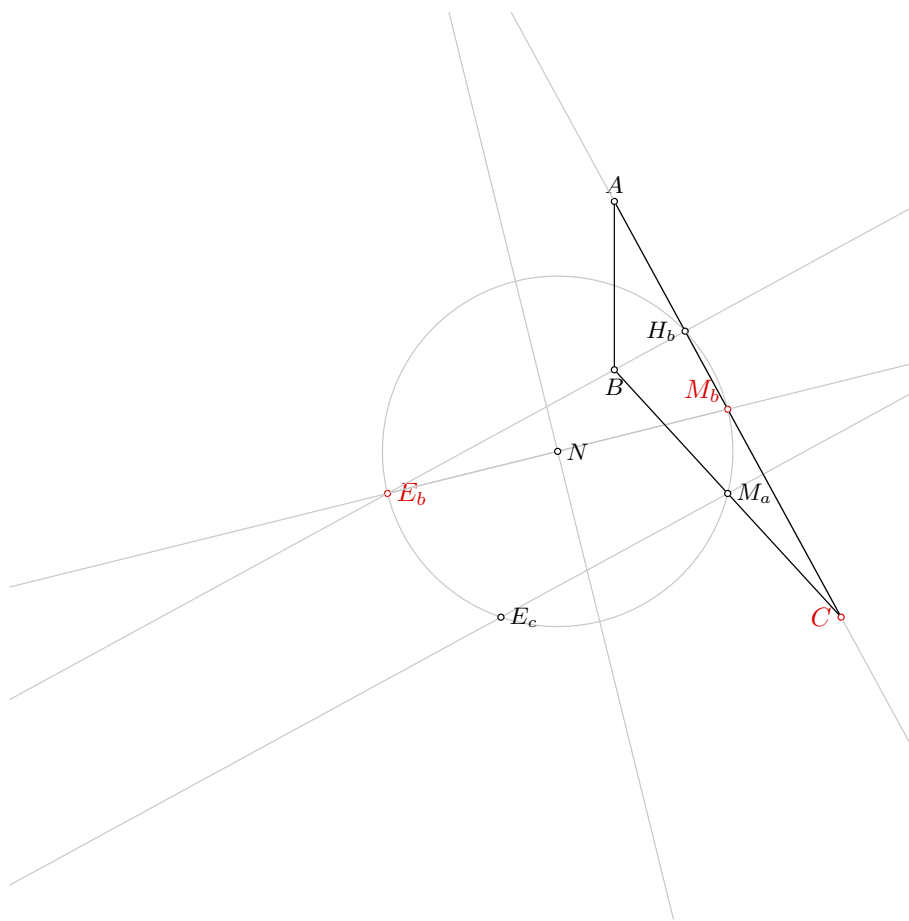


Figure 1: Illustration of the problem 0713

4.1.3 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.033 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $C = C$

NDG conditions are:

$S_{E_b C M_b} \neq 0$ i.e., points E_b , C and M_b are not collinear (foot is not the point itself; construction based assumption)

$S_{E_b C M_b} \neq S_{F_{h_b}^0 C M_b}$ i.e., lines $E_b F_{h_b}^0$ and $C M_b$ are not parallel (construction based assumption)

$S_{M_{m(E_b M_b)}^3 E_b M_b} \neq S_{T_{m(E_b M_b)}^4 E_b M_b}$ i.e., lines $M_{m(E_b M_b)}^3$ and $T_{m(E_b M_b)}^4$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^6} \neq S_{F_{-h_a}^5 B F_{-h_b}^6}$ i.e., lines $AF_{-h_a}^5$ and $BF_{-h_b}^6$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b = \neg E_b$

Proving failed

4.2.3 Proving $M_b = \neg M_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.2 Proving $E_b = \neg E_b$

Proving failed

4.3.3 Proving $M_b = \neg M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 13 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $E_b = \neg E_b$

Proving failed

4.4.3 Proving $M_b = \neg M_b$

Proving failed

Problem 714

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 714: Given a point C , a point E_b and a point M_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 715

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 715: Given a point C , a point E_b and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point N , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and N are not the same;
2. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
3. Using the circle $k(N, M_a)$, the line $m(H_aH_c)$, the point N and the point E_b , construct a point M_b (rule W05a); % NDG: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect % DET: points E_b and M_b must be different;
4. Using the point M_b and the point C , construct a point A (rule W01); ;
5. Using the point C and the point M_b , construct a line b (rule W02); % DET: points C and M_b are not the same;
6. Using the circle $k(N, M_a)$, the line b , the point N and the point M_b , construct a point H_b (rule W05); % NDG: line b and circle $k(N, M_a)$ intersect % DET: points M_b and H_b must be different;
7. Using the point H_b and the point E_b , construct a line h_b (rule W02); % DET: points H_b and E_b are not the same;
8. Using the point C and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points C and M_b are not the same;
9. Using the circle $k(N, M_a)$ and the circle $k(M_b, C)$, construct a point H_a and a point H_c (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_b, C)$ intersect % DET: circles $k(N, M_a)$ and $k(M_b, C)$ are not the same;

10. Using the point C and the point H_a , construct a line a (rule W02); % DET: points C and H_a are not the same;
11. Using the line h_b and the line a , construct a point B (rule W03); % NDG: lines h_b and a are not parallel % DET: lines h_b and a are not the same.

Non-degenerate conditions: lines h_b and a are not parallel; circles $k(N, M_a)$ and $k(M_b, C)$ intersect; points C and M_b are not the same; line b and circle $k(N, M_a)$ intersect; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; points E_b and N are not the same.

Determination conditions: lines h_b and a are not the same; points C and H_a are not the same; circles $k(N, M_a)$ and $k(M_b, C)$ are not the same; points H_b and E_b are not the same; points M_b and H_b must be different; points C and M_b are not the same; points E_b and M_b must be different; points E_b and N are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D22,D29,D3,D32,D5,D6,D9,GD01,GD02,GL01,GL03,GL04,GL09,L17,L19,L20,L21,L23,L44,L45]

Solving time: 4.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point E_{b} 50 56.36
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_r E_{b}
cmark_r N
color 0 0 0
fontsize 8
```

```
% DET: points E_{b} and N are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point N
line m(H_{a}H_{c}) E_{b} N
```

```
color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0
```

```
% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}
```

```
color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```

% NDG: line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $E_{\{b\}}$  and  $M_{\{b\}}$  must be
different
% Constructing a point  $M_{\{b\}}$  which is an image of the point  $E_{\{b\}}$  in the symmetry to point/line  $N$ 
sim  $M_{\{b\}}$   $N$   $E_{\{b\}}$ 
cmark_lt  $M_{\{b\}}$ 

% Constructing a point  $A$  such that  $M_{\{b\}}A/M_{\{b\}}C=-1$ 
towards  $A$   $M_{\{b\}}$   $C$  -1
cmark_t  $A$ 
color 200 200 200
drawsegment  $C$   $A$ 
color 0 0 0

% DET: points  $C$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $C$  and point  $M_{\{b\}}$ 
line  $b$   $C$   $M_{\{b\}}$ 

color 200 200 200
drawline  $b$ 
color 0 0 0

% NDG: line  $b$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $M_{\{b\}}$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G115236\}}$  which is a foot of the point  $N$  on the line  $b$ 
foot  $P_{\{\backslash\_G115236\}}$   $N$   $b$ 
cmark_r  $P_{\{\backslash\_G115236\}}$ 
color 200 200 200
drawline  $N$   $P_{\{\backslash\_G115236\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $M_{\{b\}}$  in the symmetry to point/line  $P_{\{\backslash\_G115236\}}$ 
sim  $H_{\{b\}}$   $P_{\{\backslash\_G115236\}}$   $M_{\{b\}}$ 
cmark_l  $H_{\{b\}}$ 

% DET: points  $H_{\{b\}}$  and  $E_{\{b\}}$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $H_{\{b\}}$  and point  $E_{\{b\}}$ 
line  $h_{\{b\}}$   $H_{\{b\}}$   $E_{\{b\}}$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: points  $C$  and  $M_{\{b\}}$  are not the same
% Constructing a circle  $k(M_{\{b\}},C)$  whose center is at point  $M_{\{b\}}$  and which passes through point  $C$ 
circle  $k(M_{\{b\}},C)$   $M_{\{b\}}$   $C$ 

```

```

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(M_{b},C) intersect% DET: circles k(N,M_{a}) and k(M_{b},C) are not
the same
% Constructing points H_{a} and H_{c} which are in intersection of k(N,M_{a}) and k(M_{b},C)
intersec2 H_{a} H_{c} k(N,M_{a}) k(M_{b},C)
cmark_r H_{a}
cmark_rt H_{c}

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines h_{b} and a are not parallel% DET: lines h_{b} and a are not the same
% Constructing a point B which belongs to line h_{b} and line a
intersec B h_{b} a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{b} and a are not parallel; circles k(N,M_{a}) and k(M_{b},C)
intersect; points C and M_{b} are not the same; line b and circle k(N,M_{a}) intersect; line m(
H_{a}H_{c}) and circle k(N,M_{a}) intersect; points E_{b} and N are not the same
% Determination conditions: lines h_{b} and a are not the same; points C and H_{a} are not the same
; circles k(N,M_{a}) and k(M_{b},C) are not the same; points H_{b} and E_{b} are not the same;
points M_{b} and H_{b} must be different; points C and M_{b} are not the same; points E_{b} and
M_{b} must be different; points E_{b} and N are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

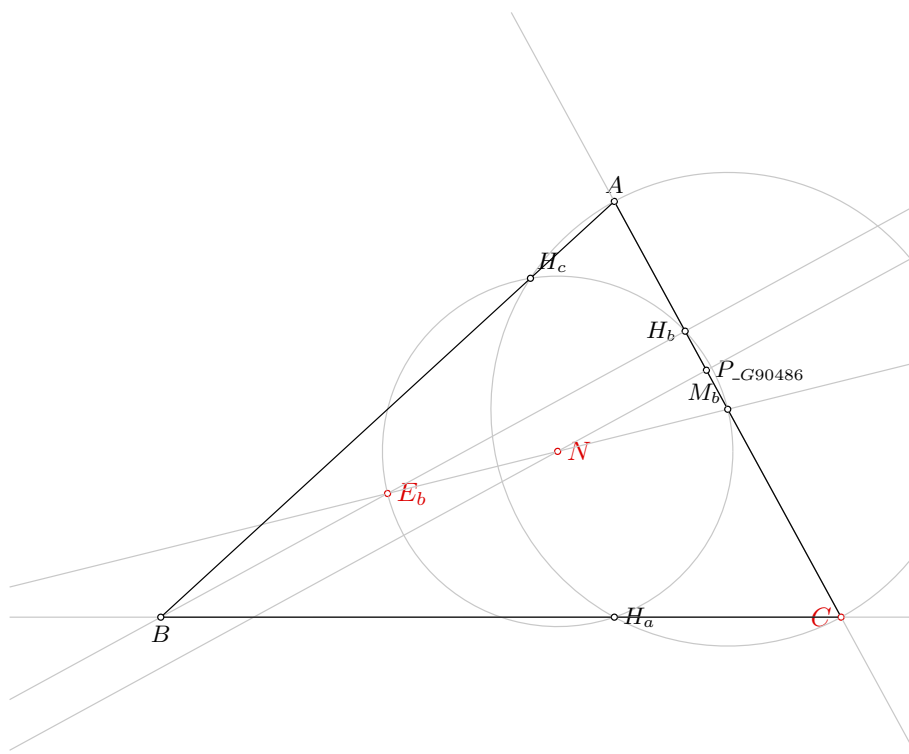


Figure 1: Illustration of the problem 0715

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.03 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_b = \neg E_b$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $C = C$

NDG conditions are:

$S_{H_bCH_a} \neq S_{E_bCH_a}$ i.e., lines H_bE_b and CH_a are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

$S_{\neg M_aBC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_bAC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_a\neg M_bF_{-m_b}^3} \neq S_{F_{-m_a}^2\neg M_bF_{-m_b}^3}$ i.e., lines $\neg M_aF_{-m_a}^2$ and $\neg M_bF_{-m_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_b = \neg E_b$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C = C$

Proving failed

4.3.2 Proving $E_b = \neg E_b$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $C = C$

Proving failed

4.4.2 Proving $E_b = E_b$

Proving failed

4.4.3 Proving $N = N$

Proving failed

Problem 716

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 716: Given a point C , a point E_b and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 717

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 717: Given a point C , a point E_b and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 718

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 718: Given a point C , a point E_b and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 719

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 719: Given a point C , a point E_b and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 720

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 720: Given a point C , a point E_c and a point G , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point E_c , construct a point H (rule W01); ;
2. Using the point C and the point G , construct a point M_c (rule W01); ;
3. Using the point G and the point H , construct a point N (rule W01); ;
4. Using the point G and the point H , construct a point O (rule W01); ;
5. Using the point C and the point E_c , construct a line h_c (rule W02); % DET: points C and E_c are not the same;
6. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
7. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
9. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; line h_c and circle $k(N, M_a)$ intersect; points E_c and N are not the same; points C and O are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points C and E_c are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L16,L21,L24,L3,L57,L58]

Solving time: 9.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point E_{c} 95 56.36
point G 70 58.33
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_r E_{c}
cmark_t G
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that CH/CE_{c}=2
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0
```

```
% Constructing a point M_{c} such that CM_{c}/CG=1.5
towards M_{c} C G 1.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0
```

```
% Constructing a point N such that GN/GH=0.25
towards N G H 0.25
cmark_r N
color 200 200 200
drawsegment G H
color 0 0 0
```

```
% Constructing a point O such that GO/GH=-0.5
```

```

towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% DET: points C and E_{c} are not the same
% Constructing a line h_{c} which passes through point C and point E_{c}
line h_{c} C E_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points E_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{c}
circle k(N,M_{a}) N E_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{c} and circle k(N,M_{a}) intersect% DET: points E_{c} and H_{c} must be different
% Constructing a point P_{\_G156485} which is a foot of the point N on the line h_{c}
foot P_{\_G156485} N h_{c}
cmark_r P_{\_G156485}
color 200 200 200
drawline N P_{\_G156485}
color 0 0 0

% Constructing a point H_{c} which is an image of the point E_{c} in the symmetry to point/line P
\_G156485}
sim H_{c} P_{\_G156485} E_{c}
cmark_rt H_{c}

% DET: points H_{c} and M_{c} are not the same
% Constructing a line c which passes through point H_{c} and point M_{c}
line c H_{c} M_{c}

color 200 200 200

```

```
drawline c
color 0 0 0
```

```
% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line c and circle k(O,C) intersect; line h_{c} and circle k(N,M_{a})
% intersect; points E_{c} and N are not the same; points C and O are not the same
% Determination conditions: points H_{c} and M_{c} are not the same; points E_{c} and H_{c} must be
% different; points C and E_{c} are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.023 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c=_E_c$

Proving failed

4.1.3 Proving $G=_G$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $C=C$

NDG conditions are:

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a}BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

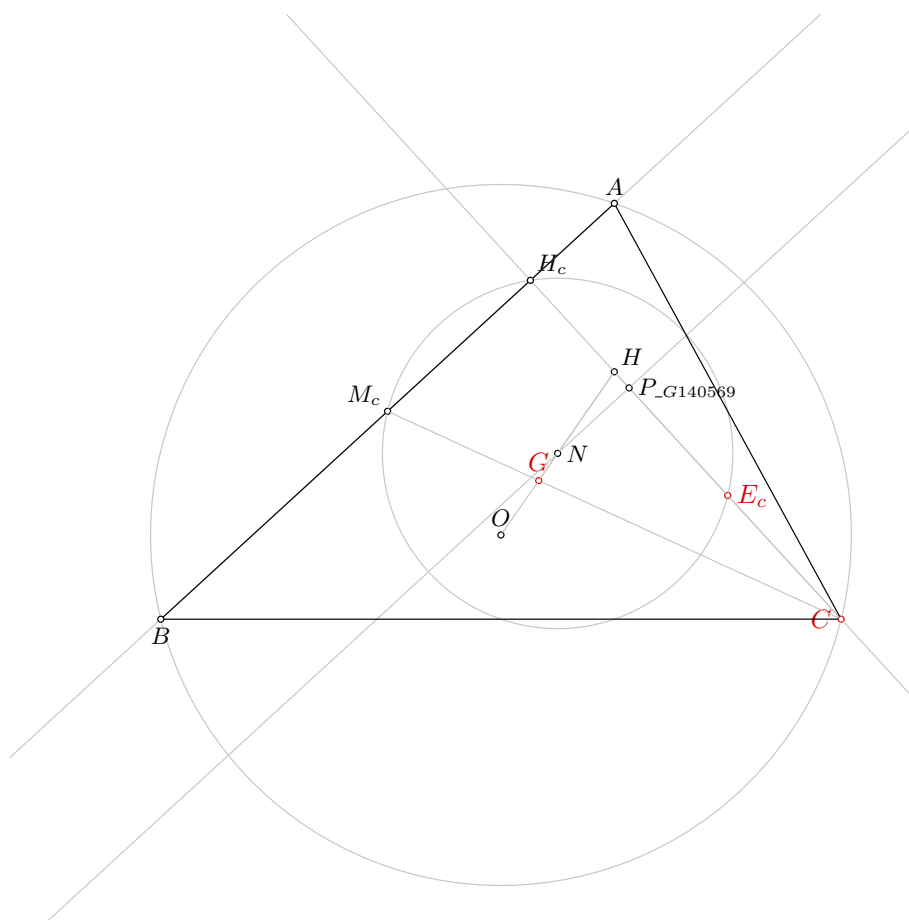


Figure 1: Illustration of the problem 0720

$S_{AB_M_b} \neq S_{_M_a B_M_b}$ i.e., lines A_M_a and B_M_b are not parallel (construction based assumption)
Total number of proof steps: 1
Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c =_ E_c$

Proving failed

4.2.3 Proving $G =_ G$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.2 Proving $E_c =_ E_c$

Proving failed

4.3.3 Proving $G =_ G$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $E_c =_ E_c$

Proving failed

4.4.3 Proving $G =_ G$

Proving failed

Problem 721

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 721: Given a point E_c , a point H and a point C , construct the triangle ABC .

2 Status

Problem is redundant.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H , construct a point C (rule W01); ;
2. Choose freely a point A (rule free);
3. Using the point A and the point H , construct a point E_a (rule W01); ;
4. Using the point A and the point C , construct a line b (rule W02); % DET: points A and C are not the same;
5. Using the point E_c and the point H , construct a line h_c (rule W02); % DET: points E_c and H are not the same;
6. Using the point A and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points A and E_a are not the same;
7. Using the circle $k(E_a, A)$, the line b , the point E_a and the point A , construct a point H_b (rule W05); % NDG: line b and circle $k(E_a, A)$ intersect % DET: points A and H_b must be different;
8. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
9. Using the circle $k(E_a, A)$, the line h_c , the point E_a and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_a, A)$ intersect % DET: points H and H_c must be different;
10. Using the point H_c and the point A , construct a line c (rule W02); % DET: points H_c and A are not the same;

11. Using the line h_b and the line c , construct a point B (rule W03); % NDG: lines h_b and c are not parallel % DET: lines h_b and c are not the same.

Non-degenerate conditions: lines h_b and c are not parallel; line h_c and circle $k(E_a, A)$ intersect; line b and circle $k(E_a, A)$ intersect; points A and E_a are not the same.

Determination conditions: lines h_b and c are not the same; points H_c and A are not the same; points H and H_c must be different; points H_b and H are not the same; points A and H_b must be different; points E_c and H are not the same; points A and C are not the same.

Rules used: [W01,W02,W03,W05,W06,free]

Lemmas used: [D10,D28,D3,D30,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L46,L47,L48]

Solving time: 10.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{c} 95 56.36
point H 80 72.73
point C 110 40

color 220 0 0
fontsize 9

cmark_r E_{c}
cmark_rt H
cmark_l C
color 0 0 0
fontsize 8

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

% Constructing a free point A
point A 80 95

cmark_t A

% Constructing a point E_{a} such that AE_{a}/AH=0.5
towards E_{a} A H 0.5
cmark_r E_{a}
color 200 200 200
drawsegment A H
color 0 0 0
```

```

% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C

color 200 200 200
drawline b
color 0 0 0

% DET: points E_{c} and H are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H
line h_{c} E_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points A and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point A
circle k(E_{a},A) E_{a} A

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: line b and circle k(E_{a},A) intersect% DET: points A and H_{b} must be different
% Constructing a point P_{\_G182772} which is a foot of the point E_{a} on the line b
foot P_{\_G182772} E_{a} b
cmark_r P_{\_G182772}
color 200 200 200
drawline E_{a} P_{\_G182772}
color 0 0 0

% Constructing a point H_{b} which is an image of the point A in the symmetry to point/line P_{\_G
182772}
sim H_{b} P_{\_G182772} A
cmark_l H_{b}

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{c} and circle k(E_{a},A) intersect% DET: points H and H_{c} must be different

```

```

% Constructing a point  $P_{\{ \_G183010 \}}$  which is a foot of the point  $E_{\{a\}}$  on the line  $h_{\{c\}}$ 
foot  $P_{\{ \_G183010 \}}$   $E_{\{a\}}$   $h_{\{c\}}$ 
cmark_r  $P_{\{ \_G183010 \}}$ 
color 200 200 200
drawline  $E_{\{a\}}$   $P_{\{ \_G183010 \}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $H$  in the symmetry to point/line  $P_{\{ \_G183010 \}}$ 
sim  $H_{\{c\}}$   $P_{\{ \_G183010 \}}$   $H$ 
cmark_rt  $H_{\{c\}}$ 

% DET: points  $H_{\{c\}}$  and  $A$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $A$ 
line  $c$   $H_{\{c\}}$   $A$ 

color 200 200 200
drawline  $c$ 
color 0 0 0

% NDG: lines  $h_{\{b\}}$  and  $c$  are not parallel% DET: lines  $h_{\{b\}}$  and  $c$  are not the same
% Constructing a point  $B$  which belongs to line  $h_{\{b\}}$  and line  $c$ 
intersec  $B$   $h_{\{b\}}$   $c$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $h_{\{b\}}$  and  $c$  are not parallel; line  $h_{\{c\}}$  and circle  $k(E_{\{a\}}, A)$ 
intersec; line  $b$  and circle  $k(E_{\{a\}}, A)$  intersect; points  $A$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $h_{\{b\}}$  and  $c$  are not the same; points  $H_{\{c\}}$  and  $A$  are not the same
; points  $H$  and  $H_{\{c\}}$  must be different; points  $H_{\{b\}}$  and  $H$  are not the same; points  $A$  and  $H_{\{b\}}$ 
must be different; points  $E_{\{c\}}$  and  $H$  are not the same; points  $A$  and  $C$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = E_c$

Proving failed

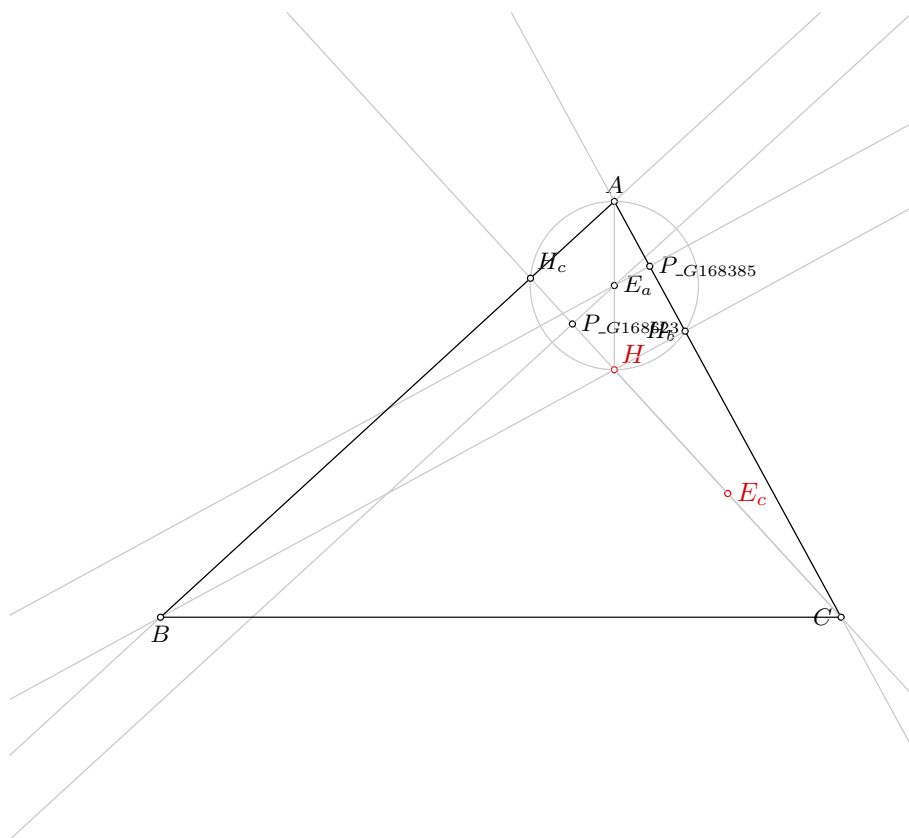


Figure 1: Illustration of the problem 0721

4.1.2 Proving $H=_H$

Proving failed

4.1.3 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2 terms.

Time Complexity: Time spent by the prover is 0.022 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $E_c=_E_c$

Proving failed

4.2.2 Proving $H=_H$

Proving failed

4.2.3 Proving $C=C$

NDG conditions are:

$S_{H_bH_cA} \neq S_{HH_cA}$ i.e., lines H_bH and H_cA are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a}BF^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_c=_E_c$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 805 terms.

Time Complexity: Time spent by the prover is 1.220 seconds. There are no ndg conditions.

4.3.2 Proving $H=_H$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 152 terms.

Time Complexity: Time spent by the prover is 0.150 seconds. There are no ndg conditions.

4.3.3 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $H = \neg H$

Proving failed

4.4.3 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

Problem 722

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 722: Given a point E_c , a point H_a and a point C , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_a and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_a and E_c are not the same;
2. Choose freely a point C on the circle $k(E_c, C)$ (rule WOncircle);
3. Using the point C and the point E_c , construct a point H (rule W01); ;
4. Using the point C and the point H_a , construct a line a (rule W02); % DET: points C and H_a are not the same;
5. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
6. Choose freely a point A on the line h_a (rule WOnline1) ;
7. Using the point A and the point C , construct a line b (rule W02); % DET: points A and C are not the same;
8. Using the circle $k(E_c, C)$, the line b , the point E_c and the point C , construct a point H_b (rule W05); % NDG: line b and circle $k(E_c, C)$ intersect % DET: points C and H_b must be different;
9. Using the point H and the point H_b , construct a line h_b (rule W02); % DET: points H and H_b are not the same;
10. Using the line a and the line h_b , construct a point B (rule W03); % NDG: lines a and h_b are not parallel % DET: lines a and h_b are not the same.

Non-degenerate conditions: lines a and h_b are not parallel; line b and circle $k(E_c, C)$ intersect; points H_a and E_c are not the same.

Determination conditions: lines a and h_b are not the same; points H and H_b are not the same; points C and H_b must be different; points A and C are not the same; points H_a and H are not the same; points C and H_a are not the same.

Rules used: [W01,W02,W03,W05,W06,WOncircle1,WOnline1]

Lemmas used: [D3,D30,D5,D6,D9,GD01,GD02,GL03,L53,L54]

Solving time: 687.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{c} 95 56.36
point H_{a} 80 40
point C 110 40

color 220 0 0
fontsize 9

cmark_r E_{c}
cmark_r H_{a}
cmark_l C
color 0 0 0
fontsize 8

% NDG: points H_{a} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H
%_{a}
circle k(E_{c},C) E_{c} H_{a}

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% Choosing randomly a point C on the circle with center E_{c} through point H_{a}
oncircle C E_{c} H_{a}
cmark_l C
color 200 200 200
drawcircle E_{c} H_{a}
color 0 0 0

% Constructing a point H such that CH/CE_{c}=2
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0
```

```

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% Choosing randomly a point A on the line H_{a}H
online A H_{a} H
cmark_t A
color 200 200 200
drawline H_{a} H
color 0 0 0

% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(E_{c},C) intersect% DET: points C and H_{b} must be different
% Constructing a point P_{\G207351} which is a foot of the point E_{c} on the line b
foot P_{\G207351} E_{c} b
cmark_r P_{\G207351}
color 200 200 200
drawline E_{c} P_{\G207351}
color 0 0 0

% Constructing a point H_{b} which is an image of the point C in the symmetry to point/line P_{\G
207351}
sim H_{b} P_{\G207351} C
cmark_l H_{b}

% DET: points H and H_{b} are not the same
% Constructing a line h_{b} which passes through point H and point H_{b}

```

```

line h_{b} H H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines a and h_{b} are not parallel% DET: lines a and h_{b} are not the same
% Constructing a point B which belongs to line a and line h_{b}
intersec B a h_{b}
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and h_{b} are not parallel; line b and circle k(E_{c},C)
% intersect; points H_{a} and E_{c} are not the same
% Determination conditions: lines a and h_{b} are not the same; points H and H_{b} are not the same
% ; points C and H_{b} must be different; points A and C are not the same; points H_{a} and H are
% not the same; points C and H_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $H_a = \neg H_a$

Proving failed

4.1.3 Proving $C = C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2 terms.

Time Complexity: Time spent by the prover is 0.02 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

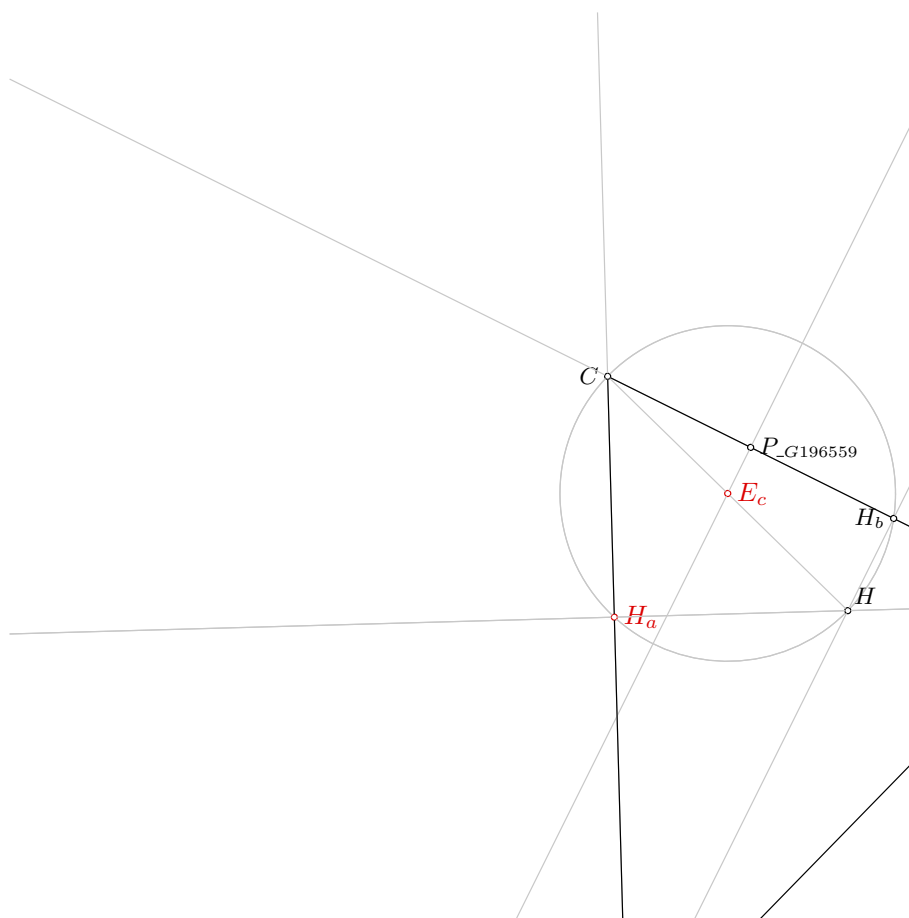


Figure 1: Illustration of the problem 0722

4.2.2 Proving $H_a = \neg H_a$

Proving failed

4.2.3 Proving $C = C$

NDG conditions are:

$S_{CHH_b} \neq S_{H_aHH_b}$ i.e., lines CH_a and HH_b are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^2_{h_b}} \neq S_{F^1_{h_a}BF^2_{h_b}}$ i.e., lines $AF^1_{h_a}$ and $BF^2_{h_b}$ are not parallel (construction based assumption)

$S_{BAF^1_{h_a}} \neq S_{CAF^1_{h_a}}$ i.e., lines BC and $AF^1_{h_a}$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $H_a = \neg H_a$

Proving failed

4.3.3 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $H_a = \neg H_a$

Proving failed

4.4.3 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

Problem 723

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 723: Given a point E_c , a point H_b and a point C , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_b and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_b and E_c are not the same;
2. Choose freely a point C on the circle $k(E_c, C)$ (rule W0ncircle);
3. Using the point C and the point E_c , construct a point H (rule W01); ;
4. Using the point C and the point H_b , construct a line b (rule W02); % DET: points C and H_b are not the same;
5. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
6. Choose freely a point A on the line b (rule W0online1) ;
7. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
8. Using the circle $k(E_c, C)$, the line h_a , the point E_c and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_c, C)$ intersect % DET: points H and H_a must be different;
9. Using the point H_a and the point C , construct a line a (rule W02); % DET: points H_a and C are not the same;
10. Using the line h_b and the line a , construct a point B (rule W03); % NDG: lines h_b and a are not parallel % DET: lines h_b and a are not the same.

Non-degenerate conditions: lines h_b and a are not parallel; line h_a and circle $k(E_c, C)$ intersect; points H_b and E_c are not the same.

Determination conditions: lines h_b and a are not the same; points H_a and C are not the same; points H and H_a must be different; points A and H are not the same; points H_b and H are not the same; points C and H_b are not the same.

Rules used: [W01,W02,W03,W05,W06,WOncircle1,WOnline1]

Lemmas used: [D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,L52,L53,L54]

Solving time: 691.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{c} 95 56.36
point H_{b} 89.36 77.83
point C 110 40

color 220 0 0
fontsize 9

cmark_r E_{c}
cmark_l H_{b}
cmark_l C
color 0 0 0
fontsize 8

% NDG: points H_{b} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H
%_{b}
circle k(E_{c},C) E_{c} H_{b}

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% Choosing randomly a point C on the circle with center E_{c} through point H_{b}
oncircle C E_{c} H_{b}
cmark_l C
color 200 200 200
drawcircle E_{c} H_{b}
color 0 0 0

% Constructing a point H such that CH/CE_{c}=2
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0
```



```

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% Choosing randomly a point A on the line CH_{b}
online A C H_{b}
cmark_t A
color 200 200 200
drawline C H_{b}
color 0 0 0

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{a} and circle k(E_{c},C) intersect% DET: points H and H_{a} must be different
% Constructing a point P_{\G231135} which is a foot of the point E_{c} on the line h_{a}
foot P_{\G231135} E_{c} h_{a}
cmark_r P_{\G231135}
color 200 200 200
drawline E_{c} P_{\G231135}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H in the symmetry to point/line P_{\G
231135}
sim H_{a} P_{\G231135} H
cmark_r H_{a}

% DET: points H_{a} and C are not the same
% Constructing a line a which passes through point H_{a} and point C

```

```

line a H_{a} C

color 200 200 200
drawline a
color 0 0 0

% NDG: lines h_{b} and a are not parallel% DET: lines h_{b} and a are not the same
% Constructing a point B which belongs to line h_{b} and line a
intersec B h_{b} a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{b} and a are not parallel; line h_{a} and circle k(E_{c},C)
% intersect; points H_{b} and E_{c} are not the same
% Determination conditions: lines h_{b} and a are not the same; points H_{a} and C are not the same
% ; points H and H_{a} must be different; points A and H are not the same; points H_{b} and H are
% not the same; points C and H_{b} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = E_c$

Proving failed

4.1.2 Proving $H_b = H_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2241 terms.

Time Complexity: Time spent by the prover is 15.109 seconds.

NDG conditions Points H_b and C are not identical

Points A and H are not identical

Line through points H_b and E_c is not parallel with line through points A and H

Line through points H_b and H is not parallel with line through points C and H_a

Points H_b , E_c and H are not collinear

Points A and C are not identical

Points H_b and E_c are not identical

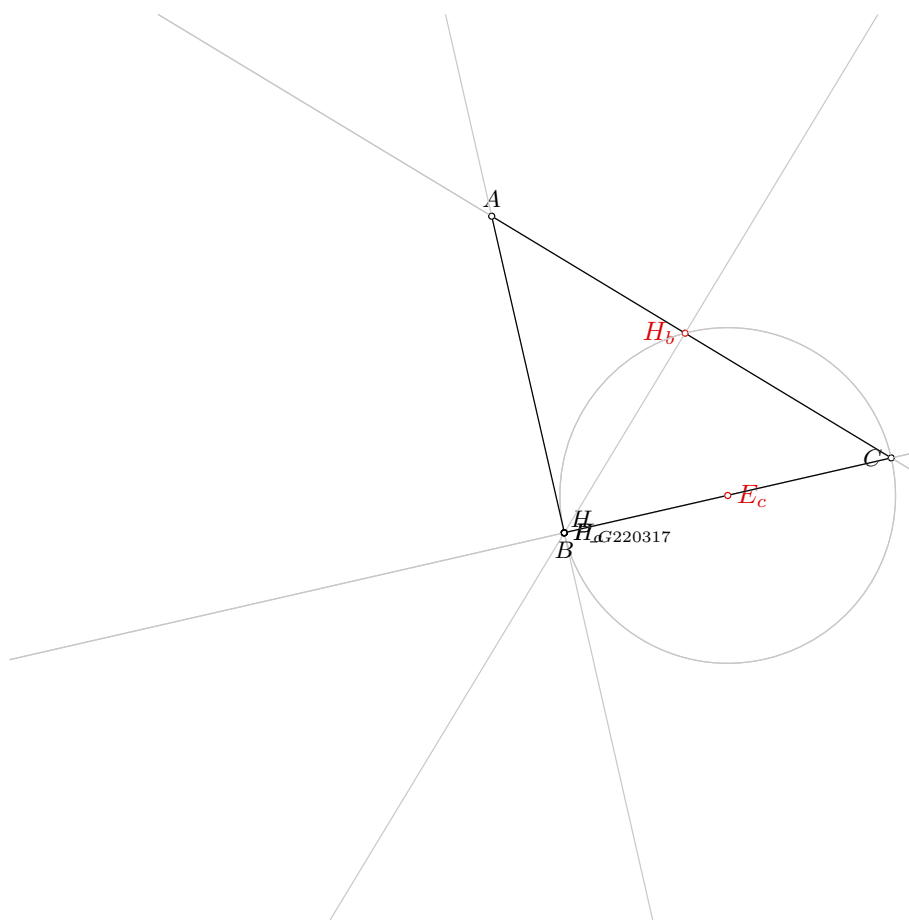


Figure 1: Illustration of the problem 0723

4.1.3 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2 terms.

Time Complexity: Time spent by the prover is 0.031 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $E_c=_E_c$

Proving failed

4.2.2 Proving $H_b=_H_b$

Proving failed

4.2.3 Proving $C=C$

NDG conditions are:

$S_{H_bH_aC} \neq S_{HH_aC}$ i.e., lines H_bH and H_aC are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^2} \neq S_{F_{-h_a}^1BF_{-h_b}^2}$ i.e., lines $AF_{-h_a}^1$ and $BF_{-h_b}^2$ are not parallel (construction based assumption)

$S_{ABF_{-h_b}^2} \neq S_{CBF_{-h_b}^2}$ i.e., lines AC and $BF_{-h_b}^2$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_c=_E_c$

Proving failed

4.3.2 Proving $H_b=_H_b$

Proving failed

4.3.3 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c=_E_c$

Proving failed

4.4.2 Proving $H_b=_H_b$

Proving failed

4.4.3 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

Problem 724

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 724: Given a point E_c , a point H_c and a point C , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;
2. Choose freely a point C on the line h_c (rule WOnline1) ;
3. Using the point C and the point E_c , construct a point H (rule W01); ;
4. Using the point C and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points C and E_c are not the same;
5. Using the point H_c and the line h_c , construct a line c (rule W10a); ;
6. Choose freely a point A on the line c (rule WOnline2);
7. Using the point A and the point C , construct a line b (rule W02); % DET: points A and C are not the same;
8. Using the circle $k(E_c, C)$, the line b , the point E_c and the point C , construct a point H_b (rule W05); % NDG: line b and circle $k(E_c, C)$ intersect % DET: points C and H_b must be different;
9. Using the point H and the point H_b , construct a line h_b (rule W02); % DET: points H and H_b are not the same;
10. Using the line c and the line h_b , construct a point B (rule W03); % NDG: lines c and h_b are not parallel % DET: lines c and h_b are not the same.

Non-degenerate conditions: lines c and h_b are not parallel; line b and circle $k(E_c, C)$ intersect; points C and E_c are not the same.

Determination conditions: lines c and h_b are not the same; points H and H_b are not the same; points C and H_b must be different; points A and C are not the same; points E_c and H_c are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D10,D3,D30,D6,D7,D9,GD01,GD02,GL03,GL09,L3,L53]

Solving time: 172.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
```

```
point H_{c} 68.91 84.83
```

```
point C 110 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{c}
```

```
cmark_rt H_{c}
```

```
cmark_l C
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{c} and H_{c} are not the same
```

```
% Constructing a line h_{c} which passes through point E_{c} and point H_{c}
```

```
line h_{c} E_{c} H_{c}
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```
% Choosing randomly a point C on the line E_{c}H_{c}
```

```
online C E_{c} H_{c}
```

```
cmark_l C
```

```
color 200 200 200
```

```
drawline E_{c} H_{c}
```

```
color 0 0 0
```

```
% Constructing a point H such that CH/CE_{c}=2
```

```
towards H C E_{c} 2
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment C H
```

```
color 0 0 0
```

```

% NDG: points C and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point C
circle k(E_{c},C) E_{c} C

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0


% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0


% Generating random value V[_G254177]
random V[_G254177]


% Calculating value V[_G254198] using formula V[_G254177]*20
expression V[_G254198] { V[_G254177]*20 }


% Constructing a point A which is a point for which holds  $H_{c}A = V[_G254198]$  and angle  $CH_{c}A = 90$ 
turtle A C H_{c} 90 V[_G254198]
cmark_t A


% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C

color 200 200 200
drawline b
color 0 0 0


% NDG: line b and circle k(E_{c},C) intersect% DET: points C and H_{b} must be different
% Constructing a point P_{\_G254470} which is a foot of the point E_{c} on the line b
foot P_{\_G254470} E_{c} b
cmark_r P_{\_G254470}
color 200 200 200
drawline E_{c} P_{\_G254470}
color 0 0 0


% Constructing a point H_{b} which is an image of the point C in the symmetry to point/line P_{\_G
254470}
sim H_{b} P_{\_G254470} C

```



```

cmark_l H_{b}

% DET: points H and H_{b} are not the same
% Constructing a line h_{b} which passes through point H and point H_{b}
line h_{b} H H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines c and h_{b} are not parallel% DET: lines c and h_{b} are not the same
% Constructing a point B which belongs to line c and line h_{b}
intersec B c h_{b}
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and h_{b} are not parallel; line b and circle k(E_{c},C)
% intersect; points C and E_{c} are not the same
% Determination conditions: lines c and h_{b} are not the same; points H and H_{b} are not the same
% ; points C and H_{b} must be different; points A and C are not the same; points E_{c} and H_{c}
% are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = E_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 55 terms.

Time Complexity: Time spent by the prover is 1.301 seconds.

NDG conditions Points E_c and A are not identical

Points E_c , B and C are not collinear

Line through points H_b and E_c is not perpendicular to line through points E_c and H

Points A , B and C are not collinear

4.1.2 Proving $H_c = H_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

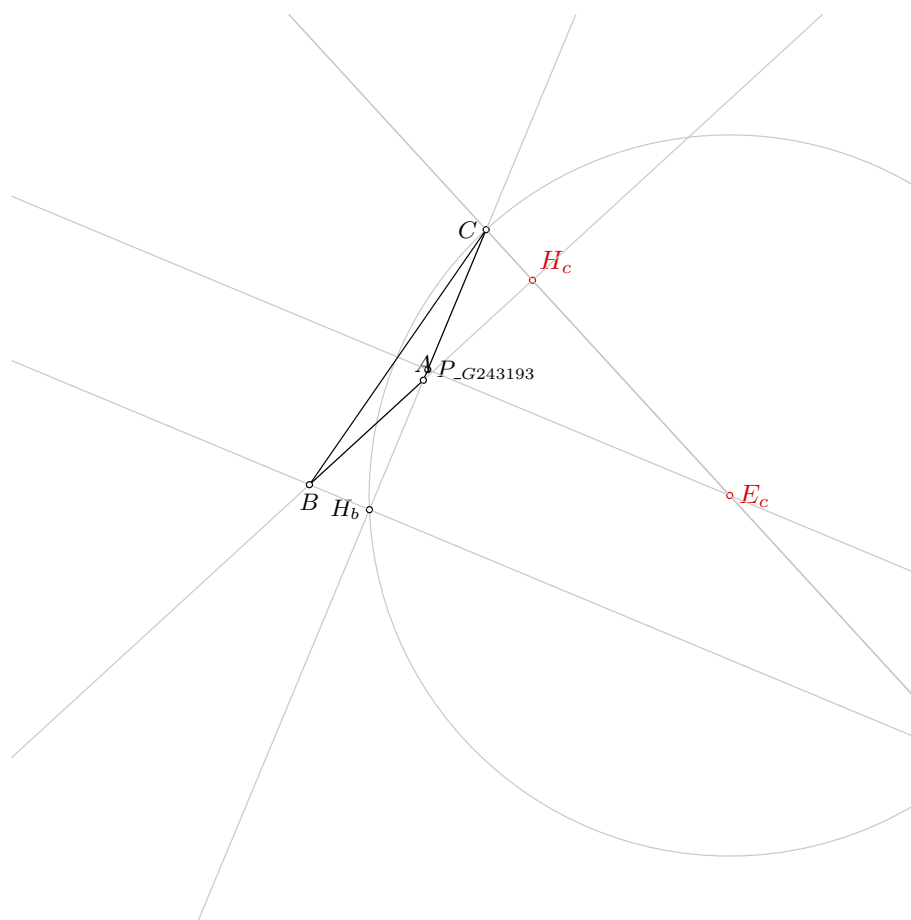


Figure 1: Illustration of the problem 0724

Time Complexity: Time spent by the prover is 0.151 seconds.

NDG conditions Points E_c and A are not identical

Points E_c , B and C are not collinear

Line through points H_b and E_c is not perpendicular to line through points E_c and H

Points E_c , A and B are not collinear

4.1.3 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.019 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $E_c=_E E_c$

Proving failed

4.2.2 Proving $H_c=_H H_c$

Proving failed

4.2.3 Proving $C=C$

NDG conditions are:

$S_{H_c H H_b} \neq S_{T_c^1 H H_b}$ i.e., lines $H_c T_c^1$ and $H H_b$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-H_b}^3} \neq S_{F_{-H_a}^2 BF_{-H_b}^3}$ i.e., lines $AF_{-H_a}^2$ and $BF_{-H_b}^3$ are not parallel (construction based assumption)

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{-H_c}^4} \neq S_{BCF_{-H_c}^4}$ i.e., lines AB and $CF_{-H_c}^4$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_c=_E E_c$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 6 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $H_c=_H H_c$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 6 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.3 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c=_E_c$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 4 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $H_c=_H_c$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 5 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.3 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

Problem 725

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 725: Given a point C , a point E_c and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 726

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 726: Given a point C , a point E_c and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point E_c , construct a point H (rule W01); ;
2. Using the point C and the point M_a , construct a point B (rule W01); ;
3. Using the point C and the point M_a , construct a line a (rule W02); % DET: points C and M_a are not the same;
4. Using the point H and the point B , construct a line h_b (rule W02); % DET: points H and B are not the same;
5. Using the point C and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points C and E_c are not the same;
6. Using the circle $k(E_c, C)$, the line a , the point E_c and the point C , construct a point H_a (rule W05); % NDG: line a and circle $k(E_c, C)$ intersect % DET: points C and H_a must be different;
7. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
8. Using the circle $k(E_c, C)$, the line h_b , the point E_c and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_c, C)$ intersect % DET: points H and H_b must be different;
9. Using the point H_b and the point C , construct a line b (rule W02); % DET: points H_b and C are not the same;
10. Using the line h_a and the line b , construct a point A (rule W03); % NDG: lines h_a and b are not parallel % DET: lines h_a and b are not the same.

Non-degenerate conditions: lines h_a and b are not parallel; line h_b and circle $k(E_c, C)$ intersect; line a and circle $k(E_c, C)$ intersect; points C and E_c are not the same.

Determination conditions: lines h_a and b are not the same; points H_b and C are not the same; points H and H_b must be different; points H_a and H are not the same; points C and H_a must be different; points H and B are not the same; points C and M_a are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D21,D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,GL04,GL09,L52,L53,L54]

Solving time: 12.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point E_{c} 95 56.36
point M_{a} 65 40

color 220 0 0
fontsize 9

cmark_l C
cmark_r E_{c}
cmark_r M_{a}
color 0 0 0
fontsize 8

% Constructing a point H such that CH/CE_{c}=2
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0

% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% DET: points C and M_{a} are not the same
% Constructing a line a which passes through point C and point M_{a}
line a C M_{a}

color 200 200 200
drawline a
color 0 0 0
```

```

% DET: points H and B are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point H and point B
line  $h_{\{b\}}$  H B

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: points C and  $E_{\{c\}}$  are not the same
% Constructing a circle  $k(E_{\{c\}}, C)$  whose center is at point  $E_{\{c\}}$  and which passes through point C
circle  $k(E_{\{c\}}, C)$   $E_{\{c\}}$  C

color 200 200 200
drawcircle  $k(E_{\{c\}}, C)$ 
color 0 0 0

% NDG: line a and circle  $k(E_{\{c\}}, C)$  intersect% DET: points C and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\_G31845\}}$  which is a foot of the point  $E_{\{c\}}$  on the line a
foot  $P_{\{\_G31845\}}$   $E_{\{c\}}$  a
cmark_r  $P_{\{\_G31845\}}$ 
color 200 200 200
drawline  $E_{\{c\}}$   $P_{\{\_G31845\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point C in the symmetry to point/line  $P_{\{\_G31845\}}$ 
sim  $H_{\{a\}}$   $P_{\{\_G31845\}}$  C
cmark_r  $H_{\{a\}}$ 

% DET: points  $H_{\{a\}}$  and H are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $H_{\{a\}}$  and point H
line  $h_{\{a\}}$   $H_{\{a\}}$  H

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(E_{\{c\}}, C)$  intersect% DET: points H and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\_G32083\}}$  which is a foot of the point  $E_{\{c\}}$  on the line  $h_{\{b\}}$ 
foot  $P_{\{\_G32083\}}$   $E_{\{c\}}$   $h_{\{b\}}$ 
cmark_r  $P_{\{\_G32083\}}$ 
color 200 200 200
drawline  $E_{\{c\}}$   $P_{\{\_G32083\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point H in the symmetry to point/line  $P_{\{\_G32083\}}$ 
sim  $H_{\{b\}}$   $P_{\{\_G32083\}}$  H
cmark_l  $H_{\{b\}}$ 

```



```

% DET: points  $H_{\{b\}}$  and  $C$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $C$ 
line b  $H_{\{b\}}$  C

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $h_{\{a\}}$  and  $b$  are not parallel% DET: lines  $h_{\{a\}}$  and  $b$  are not the same
% Constructing a point  $A$  which belongs to line  $h_{\{a\}}$  and line  $b$ 
intersec A  $h_{\{a\}}$  b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{a\}}$  and  $b$  are not parallel; line  $h_{\{b\}}$  and circle  $k(E_{\{c\}}, C)$ 
% intersect; line  $a$  and circle  $k(E_{\{c\}}, C)$  intersect; points  $C$  and  $E_{\{c\}}$  are not the same
% Determination conditions: lines  $h_{\{a\}}$  and  $b$  are not the same; points  $H_{\{b\}}$  and  $C$  are not the same
% ; points  $H$  and  $H_{\{b\}}$  must be different; points  $H_{\{a\}}$  and  $H$  are not the same; points  $C$  and  $H_{\{a\}}$ 
% must be different; points  $H$  and  $B$  are not the same; points  $C$  and  $M_{\{a\}}$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.033 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c = E_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 1741 terms.

Time Complexity: Time spent by the prover is 16.322 seconds.

NDG conditions Points C and M_a are not identical

Points C and M_a are not identical

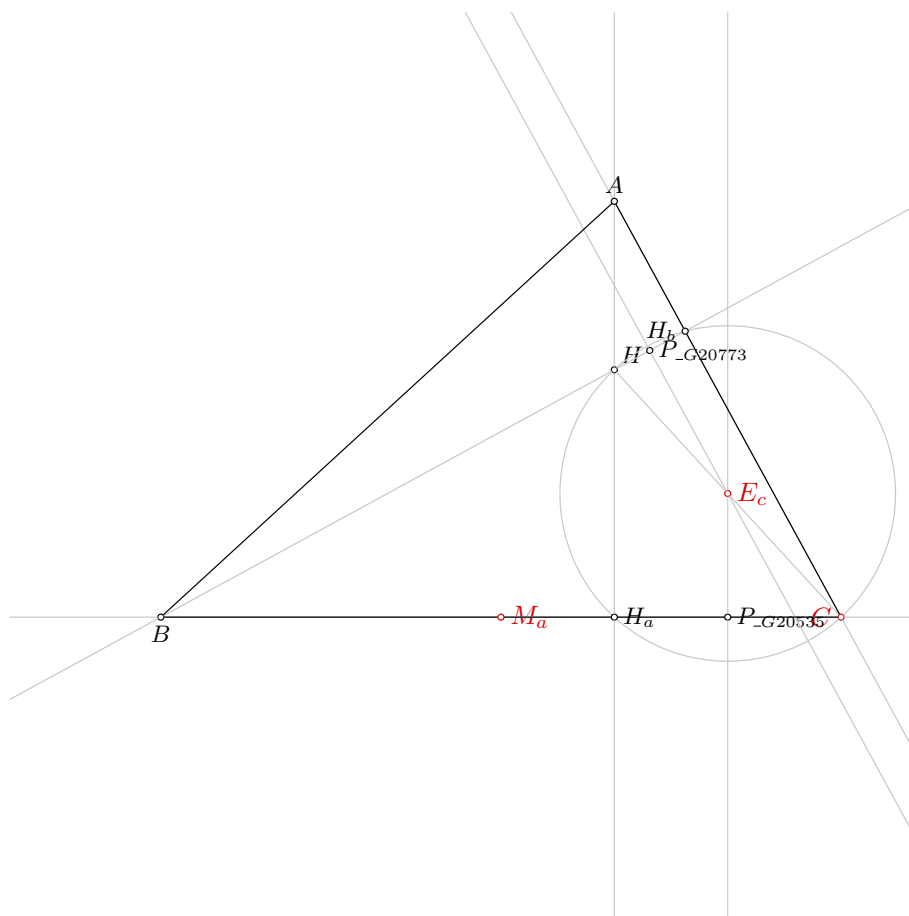


Figure 1: Illustration of the problem 0726

Points B and H are not identical

Line through points E_c and B is not perpendicular to line through points B and C

Line through points H_b and C is not parallel with line through points H and H_a

Points C and H_a are not identical

Points A , B and C are not collinear

Points B , C and H are not collinear

4.1.3 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.032 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $C = C$

NDG conditions are:

$S_{H_a H_b C} \neq S_{HH_b C}$ i.e., lines $H_a H$ and $H_b C$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c = \neg E_c$

Proving failed

4.2.3 Proving $M_a = \neg M_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_c = \neg E_c$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 117 terms.

Time Complexity: Time spent by the prover is 0.360 seconds. There are no ndg conditions.

4.3.3 Proving $M_a = \neg M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 8 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_c=E_c$

Proving failed

4.4.3 Proving $M_a=M_a$

Proving failed

Problem 727

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 727: Given a point C , a point E_c and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point E_c , construct a point H (rule W01); ;
2. Using the point C and the point M_b , construct a point A (rule W01); ;
3. Using the point C and the point M_b , construct a line b (rule W02); % DET: points C and M_b are not the same;
4. Using the point H and the point A , construct a line h_a (rule W02); % DET: points H and A are not the same;
5. Using the point C and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points C and E_c are not the same;
6. Using the circle $k(E_c, C)$, the line b , the point E_c and the point C , construct a point H_b (rule W05); % NDG: line b and circle $k(E_c, C)$ intersect % DET: points C and H_b must be different;
7. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
8. Using the circle $k(E_c, C)$, the line h_a , the point E_c and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_c, C)$ intersect % DET: points H and H_a must be different;
9. Using the point H_a and the point C , construct a line a (rule W02); % DET: points H_a and C are not the same;
10. Using the line h_b and the line a , construct a point B (rule W03); % NDG: lines h_b and a are not parallel % DET: lines h_b and a are not the same.

Non-degenerate conditions: lines h_b and a are not parallel; line h_a and circle $k(E_c, C)$ intersect; line b and circle $k(E_c, C)$ intersect; points C and E_c are not the same.

Determination conditions: lines h_b and a are not the same; points H_a and C are not the same; points H and H_a must be different; points H_b and H are not the same; points C and H_b must be different; points H and A are not the same; points C and M_b are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D22,D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,GL09,L52,L53,L54]

Solving time: 12.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point E_{c} 95 56.36
point M_{b} 95 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_r E_{c}
cmark_lt M_{b}
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that CH/CE_{c}=2
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0
```

```
% Constructing a point A such that CA/CM_{b}=2
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0
```

```
% DET: points C and M_{b} are not the same
% Constructing a line b which passes through point C and point M_{b}
line b C M_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```

% DET: points H and A are not the same
% Constructing a line h_{a} which passes through point H and point A
line h_{a} H A

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points C and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point C
circle k(E_{c},C) E_{c} C

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% NDG: line b and circle k(E_{c},C) intersect% DET: points C and H_{b} must be different
% Constructing a point P_{\_G54417} which is a foot of the point E_{c} on the line b
foot P_{\_G54417} E_{c} b
cmark_r P_{\_G54417}
color 200 200 200
drawline E_{c} P_{\_G54417}
color 0 0 0

% Constructing a point H_{b} which is an image of the point C in the symmetry to point/line P_{\_G
54417}
sim H_{b} P_{\_G54417} C
cmark_l H_{b}

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{a} and circle k(E_{c},C) intersect% DET: points H and H_{a} must be different
% Constructing a point P_{\_G54655} which is a foot of the point E_{c} on the line h_{a}
foot P_{\_G54655} E_{c} h_{a}
cmark_r P_{\_G54655}
color 200 200 200
drawline E_{c} P_{\_G54655}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H in the symmetry to point/line P_{\_G
54655}
sim H_{a} P_{\_G54655} H
cmark_r H_{a}

```

```

% DET: points  $H_{\{a\}}$  and  $C$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $C$ 
line a  $H_{\{a\}}$  C

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $h_{\{b\}}$  and  $a$  are not parallel% DET: lines  $h_{\{b\}}$  and  $a$  are not the same
% Constructing a point  $B$  which belongs to line  $h_{\{b\}}$  and line  $a$ 
intersec B  $h_{\{b\}}$  a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{b\}}$  and  $a$  are not parallel; line  $h_{\{a\}}$  and circle  $k(E_{\{c\}}, C)$ 
% intersect; line  $b$  and circle  $k(E_{\{c\}}, C)$  intersect; points  $C$  and  $E_{\{c\}}$  are not the same
% Determination conditions: lines  $h_{\{b\}}$  and  $a$  are not the same; points  $H_{\{a\}}$  and  $C$  are not the same
% ; points  $H$  and  $H_{\{a\}}$  must be different; points  $H_{\{b\}}$  and  $H$  are not the same; points  $C$  and  $H_{\{b\}}$ 
% must be different; points  $H$  and  $A$  are not the same; points  $C$  and  $M_{\{b\}}$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.034 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c=_E_c$

Proving failed

4.1.3 Proving $M_b=_M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

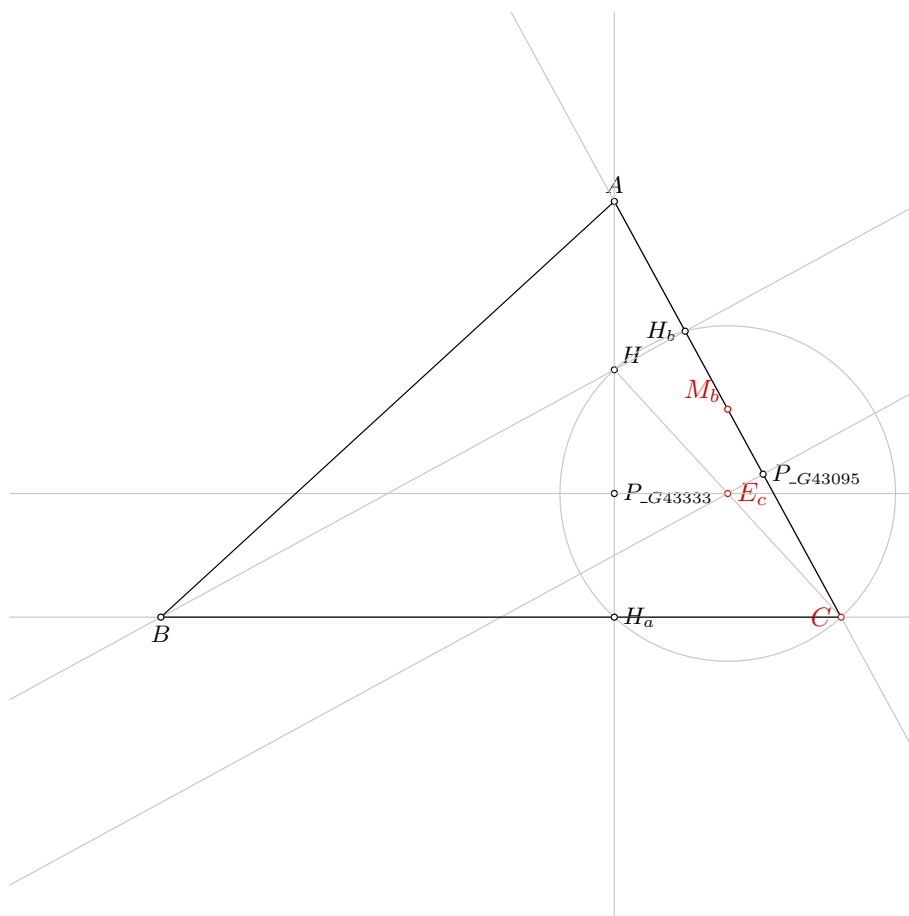


Figure 1: Illustration of the problem 0727

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $C=C$

NDG conditions are:

$S_{H_bH_aC} \neq S_{HH_aC}$ i.e., lines H_bH and H_aC are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c=_E_c$

Proving failed

4.2.3 Proving $M_b=_M_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $E_c=_E_c$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 117 terms.

Time Complexity: Time spent by the prover is 0.410 seconds. There are no ndg conditions.

4.3.3 Proving $M_b=_M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 8 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $E_c=_E_c$

Proving failed

4.4.3 Proving $M_b = -M_b$

Proving failed

Problem 728

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 728: Given a point C , a point E_c and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point E_c , construct a point H (rule W01); ;
2. Using the point C and the point M_c , construct a point G (rule W01); ;
3. Using the point H and the point G , construct a point N (rule W01); ;
4. Using the point H and the point G , construct a point O (rule W01); ;
5. Using the point C and the point E_c , construct a line h_c (rule W02); % DET: points C and E_c are not the same;
6. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
7. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
9. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; line h_c and circle $k(N, M_a)$ intersect; points E_c and N are not the same; points C and O are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points C and E_c are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D32,D7,GD01,GL03,GL09,L11,L12,L16,L21,L24,L3,L57,L58]

Solving time: 9.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point E_{c} 95 56.36
point M_{c} 50 67.5

color 220 0 0
fontsize 9

cmark_l C
cmark_r E_{c}
cmark_lt M_{c}
color 0 0 0
fontsize 8

% Constructing a point H such that CH/CE_{c}=2
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0

% Constructing a line L_{\_G78192} which passes through point C and point M_{c}
line L_{\_G78192} C M_{c}

color 200 200 200
drawline L_{\_G78192}
color 0 0 0

% Constructing a point P_{\_G78293} with coordinates (0,0)
point P_{\_G78293} 0 0
cmark_r P_{\_G78293}

% Constructing a point P_{\_G78217} such that CP_{\_G78217}/CP_{\_G78293}=2
towards P_{\_G78217} C P_{\_G78293} 2
cmark_r P_{\_G78217}
color 200 200 200
drawsegment C P_{\_G78217}
color 0 0 0
```

```

% Constructing a point P_{\_G78262} such that CP_{\_G78262}/CP_{\_G78293}=3
towards P_{\_G78262} C P_{\_G78293} 3
cmark_r P_{\_G78262}
color 200 200 200
drawsegment C P_{\_G78262}
color 0 0 0

% Constructing a line L_{\_G78223} which passes through point M_{c} and point P_{\_G78262}
line L_{\_G78223} M_{c} P_{\_G78262}

color 200 200 200
drawline L_{\_G78223}
color 0 0 0

% Constructing a line L_{\_G78186} which contains the point P_{\_G78217} and is parallel to the
line L_{\_G78223}
parallel L_{\_G78186} P_{\_G78217} L_{\_G78223}

color 200 200 200
drawline L_{\_G78186}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G78186} and line L_{\_G78192}
intersec G L_{\_G78186} L_{\_G78192}
cmark_t G

% Constructing a point N such that HN/HG=0.75
towards N H G 0.75
cmark_r N
color 200 200 200
drawsegment H G
color 0 0 0

% Constructing a point O such that HO/HG=1.5
towards O H G 1.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% DET: points C and E_{c} are not the same
% Constructing a line h_{c} which passes through point C and point E_{c}
line h_{c} C E_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

```

```

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points E_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{c}
circle k(N,M_{a}) N E_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{c} and circle k(N,M_{a}) intersect% DET: points E_{c} and H_{c} must be different
% Constructing a point P_{\_G79263} which is a foot of the point N on the line h_{c}
foot P_{\_G79263} N h_{c}
cmark_r P_{\_G79263}
color 200 200 200
drawline N P_{\_G79263}
color 0 0 0

% Constructing a point H_{c} which is an image of the point E_{c} in the symmetry to point/line P
\_G79263}
sim H_{c} P_{\_G79263} E_{c}
cmark_rt H_{c}

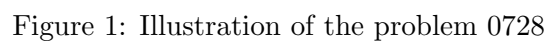
% DET: points H_{c} and M_{c} are not the same
% Constructing a line c which passes through point H_{c} and point M_{c}
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```



```
% Non-degenerate conditions: line c and circle k(O,C) intersect; line h_{c} and circle k(N,M_{a})
% intersect; points E_{c} and N are not the same; points C and O are not the same
% Determination conditions: points H_{c} and M_{c} are not the same; points E_{c} and H_{c} must be
% different; points C and E_{c} are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.033 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c = \neg E_c$

Proving failed

4.1.3 Proving $M_c = \neg M_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $C = C$

NDG conditions are:

$S_{P_{G66208}CM_c} \neq S_{P_{L_{G66177}}^0 CM_c}$ i.e., lines $P_{G66208}P_{L_{G66177}}^0$ and CM_c are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^2} \neq S_{F_{-h_a}^1 BF_{-h_b}^2}$ i.e., lines $AF_{-h_a}^1$ and $BF_{-h_b}^2$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c = \neg E_c$

Proving failed

4.2.3 Proving $M_c = \neg M_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.2 Proving $E_c = \neg E_c$

Proving failed

4.3.3 Proving $M_c = \neg M_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $E_c = E_c$

Proving failed

4.4.3 Proving $M_c = M_c$

Proving failed

Problem 729

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 729: Given a point C , a point E_c and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point E_c , construct a point H (rule W01); ;
2. Using the point N and the point H , construct a point O (rule W01); ;
3. Using the point N and the point H , construct a point G (rule W01); ;
4. Using the point C and the point G , construct a point M_c (rule W01); ;
5. Using the point C and the point E_c , construct a line h_c (rule W02); % DET: points C and E_c are not the same;
6. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
7. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
9. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; line h_c and circle $k(N, M_a)$ intersect; points E_c and N are not the same; points C and O are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points C and E_c are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D31,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L16,L21,L24,L3,L57]

Solving time: 9.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point E_{c} 95 56.36
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_r E_{c}
cmark_r N
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that CH/CE_{c}=2
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0
```

```
% Constructing a point O such that NO/NH=-1
towards O N H -1
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0
```

```
% Constructing a line L_{\_G114709} which passes through point N and point H
line L_{\_G114709} N H

color 200 200 200
drawline L_{\_G114709}
color 0 0 0
```

```
% Constructing a point P_{\_G114810} with coordinates (0,0)
point P_{\_G114810} 0 0
```

```

cmark_r P_{\_G114810}

% Constructing a point P_{\_G114734} such that NP_{\_G114734}/NP_{\_G114810}=-1
towards P_{\_G114734} N P_{\_G114810} -1
cmark_r P_{\_G114734}
color 200 200 200
drawsegment P_{\_G114810} P_{\_G114734}
color 0 0 0

% Constructing a point P_{\_G114779} such that NP_{\_G114779}/NP_{\_G114810}=3
towards P_{\_G114779} N P_{\_G114810} 3
cmark_r P_{\_G114779}
color 200 200 200
drawsegment N P_{\_G114779}
color 0 0 0

% Constructing a line L_{\_G114740} which passes through point H and point P_{\_G114779}
line L_{\_G114740} H P_{\_G114779}

color 200 200 200
drawline L_{\_G114740}
color 0 0 0

% Constructing a line L_{\_G114703} which contains the point P_{\_G114734} and is parallel to the
line L_{\_G114740}
parallel L_{\_G114703} P_{\_G114734} L_{\_G114740}

color 200 200 200
drawline L_{\_G114703}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G114703} and line L_{\_G114709}
intersec G L_{\_G114703} L_{\_G114709}
cmark_t G

% Constructing a point M_{c} such that CM_{c}/CG=1.5
towards M_{c} C G 1.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% DET: points C and E_{c} are not the same
% Constructing a line h_{c} which passes through point C and point E_{c}
line h_{c} C E_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

```

```

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points E_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{c}
circle k(N,M_{a}) N E_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{c} and circle k(N,M_{a}) intersect% DET: points E_{c} and H_{c} must be different
% Constructing a point P_{\_G115705} which is a foot of the point N on the line h_{c}
foot P_{\_G115705} N h_{c}
cmark_r P_{\_G115705}
color 200 200 200
drawline N P_{\_G115705}
color 0 0 0

% Constructing a point H_{c} which is an image of the point E_{c} in the symmetry to point/line P
\_G115705}
sim H_{c} P_{\_G115705} E_{c}
cmark_rt H_{c}

% DET: points H_{c} and M_{c} are not the same
% Constructing a line c which passes through point H_{c} and point M_{c}
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```

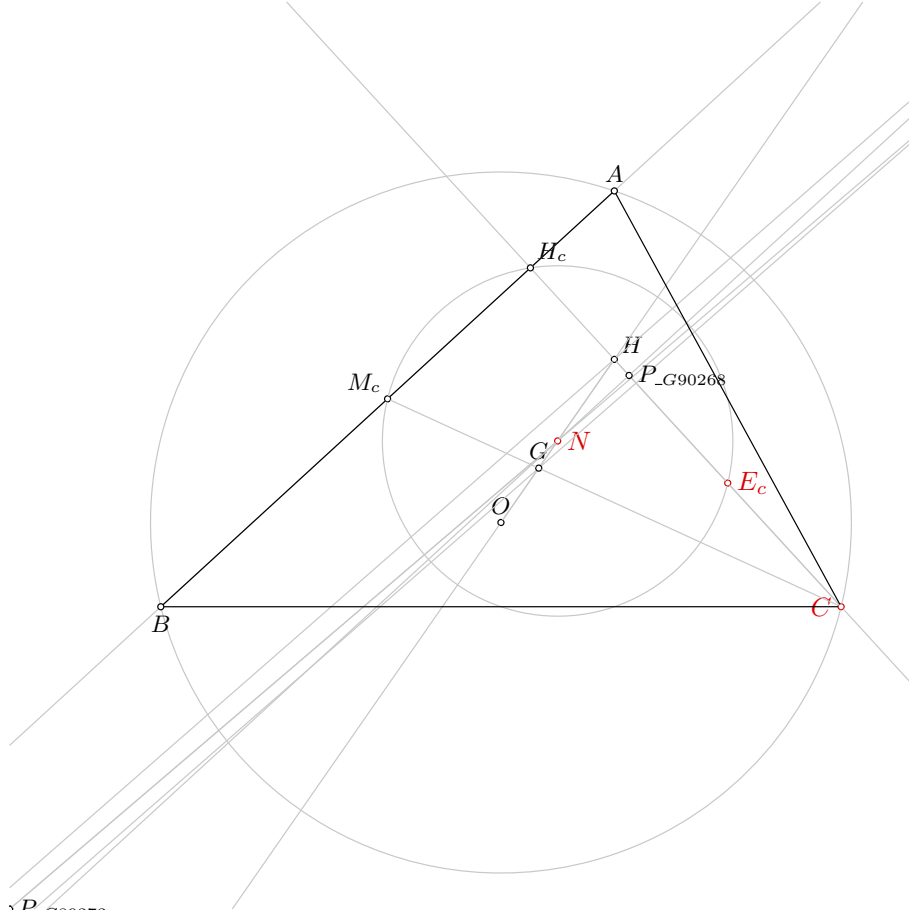


Figure 1: Illustration of the problem 0729

% Non-degenerate conditions: line c and circle $k(O,C)$ intersect; line $h_{\{c\}}$ and circle $k(N,M_{\{a\}})$ intersect; points $E_{\{c\}}$ and N are not the same; points C and O are not the same
% Determination conditions: points $H_{\{c\}}$ and $M_{\{c\}}$ are not the same; points $E_{\{c\}}$ and $H_{\{c\}}$ must be different; points C and $E_{\{c\}}$ are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.03 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c = \neg E_c$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $C = C$

NDG conditions are:

$S_{P_{G90744}NH} \neq S_{P_{L_{G90713}}^0}^{NH}$ i.e., lines $P_{G90744}P_{L_{G90713}}^0$ and NH are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{ha}^2} \neq S_{F_{ha}^1BF_{hb}^2}$ i.e., lines AF_{ha}^1 and BF_{hb}^2 are not parallel (construction based assumption)

$S_{MaBC} \neq 0$ i.e., points M_a, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{MbAC} \neq 0$ i.e., points M_b, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{MaMbF_{ma}^4} \neq S_{F_{ma}^3MbF_{mb}^4}$ i.e., lines $M_aF_{ma}^3$ and $M_bF_{mb}^4$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c = \neg E_c$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $E_c = \neg E_c$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $E_c=_E_c$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 730

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 730: Given a point C , a point E_c and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point E_c , construct a point H (rule W01); ;
2. Using the point O and the point H , construct a point N (rule W01); ;
3. Using the point O and the point H , construct a point G (rule W01); ;
4. Using the point C and the point G , construct a point M_c (rule W01); ;
5. Using the point C and the point E_c , construct a line h_c (rule W02); % DET: points C and E_c are not the same;
6. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
7. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
9. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; line h_c and circle $k(N, M_a)$ intersect; points E_c and N are not the same; points C and O are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points C and E_c are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D31,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L21,L24,L3,L57,L58]

Solving time: 9.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point E_{c} 95 56.36
point O 65 51.14
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_r E_{c}
cmark_t O
color 0 0 0
fontsize 8
```

```
% Constructing a point H such that CH/CE_{c}=2
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0
```

```
% Constructing a point N such that ON/OH=0.5
towards N O H 0.5
cmark_r N
color 200 200 200
drawsegment O H
color 0 0 0
```

```
% Constructing a line L_{\_G145711} which passes through point O and point H
line L_{\_G145711} O H
```

```
color 200 200 200
drawline L_{\_G145711}
color 0 0 0
```

```
% Constructing a point P_{\_G145812} with coordinates (0,0)
point P_{\_G145812} 0 0
```

```

cmark_r P_{\_G145812}

% Constructing a point P_{\_G145736} such that  $OP_{\_G145736}/OP_{\_G145812}=1$ 
towards P_{\_G145736} O P_{\_G145812} 1
cmark_r P_{\_G145736}
color 200 200 200
drawsegment O P_{\_G145736}
color 0 0 0

% Constructing a point P_{\_G145781} such that  $OP_{\_G145781}/OP_{\_G145812}=3$ 
towards P_{\_G145781} O P_{\_G145812} 3
cmark_r P_{\_G145781}
color 200 200 200
drawsegment O P_{\_G145781}
color 0 0 0

% Constructing a line L_{\_G145742} which passes through point H and point P_{\_G145781}
line L_{\_G145742} H P_{\_G145781}

color 200 200 200
drawline L_{\_G145742}
color 0 0 0

% Constructing a line L_{\_G145705} which contains the point P_{\_G145736} and is parallel to the
line L_{\_G145742}
parallel L_{\_G145705} P_{\_G145736} L_{\_G145742}

color 200 200 200
drawline L_{\_G145705}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G145705} and line L_{\_G145711}
intersec G L_{\_G145705} L_{\_G145711}
cmark_t G

% Constructing a point M_{c} such that  $CM_{c}/CG=1.5$ 
towards M_{c} C G 1.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% DET: points C and E_{c} are not the same
% Constructing a line h_{c} which passes through point C and point E_{c}
line h_{c} C E_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

```

```

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points E_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{c}
circle k(N,M_{a}) N E_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{c} and circle k(N,M_{a}) intersect% DET: points E_{c} and H_{c} must be different
% Constructing a point P_{\_G146699} which is a foot of the point N on the line h_{c}
foot P_{\_G146699} N h_{c}
cmark_r P_{\_G146699}
color 200 200 200
drawline N P_{\_G146699}
color 0 0 0

% Constructing a point H_{c} which is an image of the point E_{c} in the symmetry to point/line P
\_G146699}
sim H_{c} P_{\_G146699} E_{c}
cmark_rt H_{c}

% DET: points H_{c} and M_{c} are not the same
% Constructing a line c which passes through point H_{c} and point M_{c}
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```

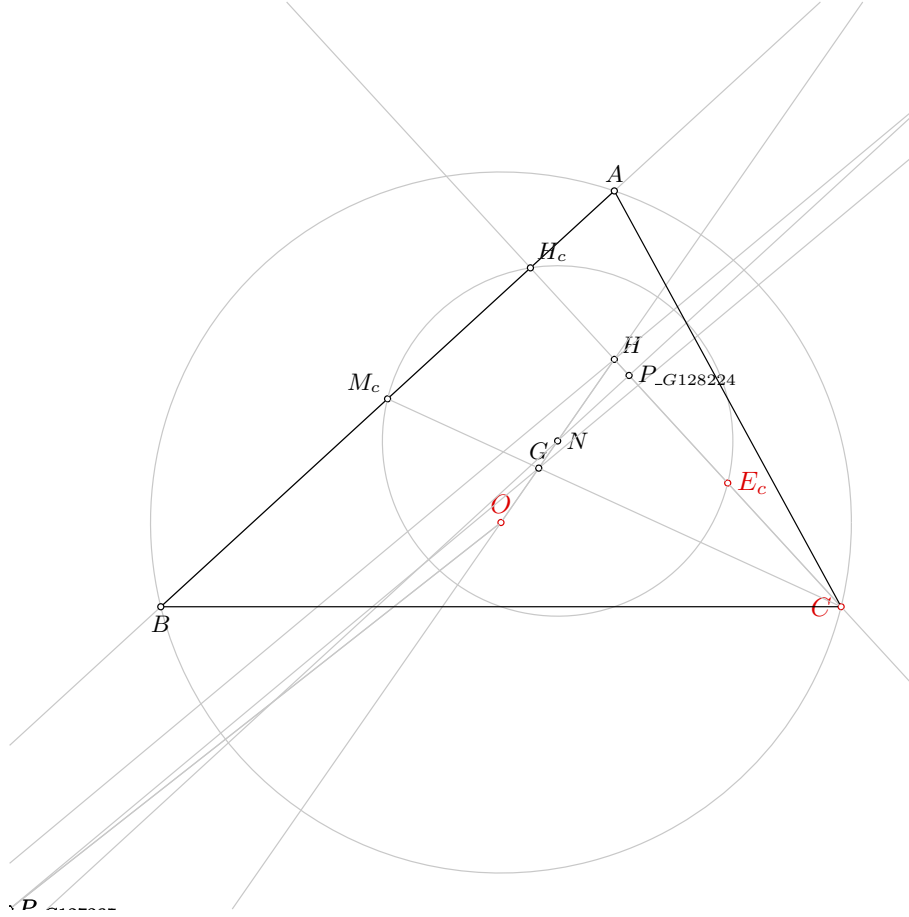


Figure 1: Illustration of the problem 0730

*% Non-degenerate conditions: line c and circle k(O,C) intersect; line h_{c} and circle k(N,M_{a}) intersect; points E_{c} and N are not the same; points C and O are not the same
 % Determination conditions: points H_{c} and M_{c} are not the same; points E_{c} and H_{c} must be different; points C and E_{c} are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.036 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c = \neg E_c$

Proving failed

4.1.3 Proving $O = \neg O$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 4143 terms.

Time Complexity: Time spent by the prover is 17.205 seconds.

NDG conditions Points H , O and $P_{G142682}$ are not collinear

Points H_c , C and $P_{G142682}$ are not collinear

Points H_c and M_c are not identical

Points M_c and H are not identical

Points A , B and C are not collinear

Points E_c , B and C are not collinear

4.2 GCLC - Area method

4.2.1 Proving $C = C$

NDG conditions are:

$S_{P_{G128706}OH} \neq S_{P_{L_{G128675}}^0 OH}$ i.e., lines $P_{G128706}P_{L_{G128675}}^0$ and OH are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^2} \neq S_{F_{\neg h_a}^1 BF_{\neg h_b}^2}$ i.e., lines $AF_{\neg h_a}^1$ and $BF_{\neg h_b}^2$ are not parallel (construction based assumption)

$S_{\neg M_a BC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_b AC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_a \neg M_b F_{\neg m_b}^4} \neq S_{F_{\neg m_a}^3 \neg M_b F_{\neg m_b}^4}$ i.e., lines $\neg M_a F_{\neg m_a}^3$ and $\neg M_b F_{\neg m_b}^4$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c = \neg E_c$

Proving failed

4.2.3 Proving $O = \neg O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $E_c = E_c$

Proving failed

4.3.3 Proving $O = O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $E_c = E_c$

Proving failed

4.4.3 Proving $O = O$

Proving failed

Problem 731

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 731: Given a point C , a point E_c and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 732

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 732: Given a point C , a point E_c and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 733

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 733: Given a point C , a point E_c and a point T_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point E_c , construct a point H (rule W01); ;
2. Using the point C and the point E_c , construct a line h_c (rule W02); % DET: points C and E_c are not the same;
3. Using the point C and the point T_c , construct a line s_c (rule W02); % DET: points C and T_c are not the same;
4. Using the point T_c and the line h_c , construct a line c (rule W10a); ;
5. Using the point E_c , the point C , the point T_c , the line s_c and the line h_c , construct a line CO (rule W17); % NDG: points C and T_c are not the same; points E_c and C are not the same % DET: points C and T_c are not the same;
6. Using the point E_c and the line CO , construct a line $m(H_bH_a)$ (rule W16); ;
7. Using the line $m(H_bH_a)$ and the line c , construct a point M_c (rule W03); % NDG: lines $m(H_bH_a)$ and c are not parallel % DET: lines $m(H_bH_a)$ and c are not the same;
8. Using the point M_c and the point C , construct a point G (rule W01); ;
9. Using the point H and the point G , construct a point O (rule W01); ;
10. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
11. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; lines $m(H_b H_a)$ and c are not parallel; points C and T_c are not the same; points E_c and C are not the same.

Determination conditions: lines $m(H_b H_a)$ and c are not the same; points C and T_c are not the same; points C and E_c are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W16,W17]

Lemmas used: [D10,D20,D25,D26,D30,GD01,GD02,GL01,GL03,GL04,GL09,GL17,L106,L11,L12,L3,L41,L42,L

Solving time: 27.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
```

```
point E_{c} 95 56.36
```

```
point T_{c} 55.38 72.43
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_l C
```

```
cmark_r E_{c}
```

```
cmark_rt T_{c}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point H such that CH/CE_{c}=2
```

```
towards H C E_{c} 2
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment C H
```

```
color 0 0 0
```

```
% DET: points C and E_{c} are not the same
```

```
% Constructing a line h_{c} which passes through point C and point E_{c}
```

```
line h_{c} C E_{c}
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```
% DET: points C and T_{c} are not the same
```

```
% Constructing a line s_{c} which passes through point C and point T_{c}
```

```
line s_{c} C T_{c}
```

```
color 200 200 200
```

```
drawline s_{c}
```

```
color 0 0 0
```

```

% Constructing a line c which is perpendicular to line h_{c} and which passes through point T_{c}
perp c T_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points C and T_{c} are not the same; points E_{c} and C are not the same% DET: points C and
T_{c} are not the same
% Constructing an angle V[_G184813] which is equal to the angle E_{c}CT_{c}
angle_o V[_G184813] E_{c} C T_{c}

% Calculating value angle[_G184892] using formula  $1/\text{pow}(2,0)*V[_G184813]+0/\text{pow}(2,0)*180$ 
expression angle[_G184892] {  $1/\text{pow}(2,0)*V[_G184813]+0/\text{pow}(2,0)*180$  }

% Constructing a point P_{\_G184889} which is an image of the point T_{c} in a rotation around the
point C for the angle  $1/\text{pow}(2,0)*V[_G184813]+0/\text{pow}(2,0)*180$ 
rotate P_{\_G184889} C angle[_G184892] T_{c}
cmark_r P_{\_G184889}
color 200 200 200
drawarc_p C T_{c} angle[_G184892]
color 0 0 0

% Constructing a line CO which passes through point C and point P_{\_G184889}
line CO C P_{\_G184889}

color 200 200 200
drawline CO
color 0 0 0

% Constructing a line m(H_{b}H_{a}) which contains the point E_{c} and is parallel to the line CO
parallel m(H_{b}H_{a}) E_{c} CO

color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0

% NDG: lines m(H_{b}H_{a}) and c are not parallel% DET: lines m(H_{b}H_{a}) and c are not the same
% Constructing a point M_{c} which belongs to line m(H_{b}H_{a}) and line c
intersec M_{c} m(H_{b}H_{a}) c
cmark_lt M_{c}

% Constructing a line L_{\_G185414} which passes through point M_{c} and point C

```

```

line L_{\_G185414} M_{c} C

color 200 200 200
drawline L_{\_G185414}
color 0 0 0

% Constructing a point P_{\_G185515} with coordinates (0,0)
point P_{\_G185515} 0 0
cmark_r P_{\_G185515}

% Constructing a point P_{\_G185439} such that  $M_{\{c\}}P_{\{\_G185439\}}/M_{\{c\}}P_{\{\_G185515\}}=1$ 
towards P_{\_G185439} M_{c} P_{\_G185515} 1
cmark_r P_{\_G185439}
color 200 200 200
drawsegment M_{c} P_{\_G185439}
color 0 0 0

% Constructing a point P_{\_G185484} such that  $M_{\{c\}}P_{\{\_G185484\}}/M_{\{c\}}P_{\{\_G185515\}}=3$ 
towards P_{\_G185484} M_{c} P_{\_G185515} 3
cmark_r P_{\_G185484}
color 200 200 200
drawsegment M_{c} P_{\_G185484}
color 0 0 0

% Constructing a line L_{\_G185445} which passes through point C and point P_{\_G185484}
line L_{\_G185445} C P_{\_G185484}

color 200 200 200
drawline L_{\_G185445}
color 0 0 0

% Constructing a line L_{\_G185408} which contains the point P_{\_G185439} and is parallel to the
line L_{\_G185445}
parallel L_{\_G185408} P_{\_G185439} L_{\_G185445}

color 200 200 200
drawline L_{\_G185408}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G185408} and line L_{\_G185414}
intersec G L_{\_G185408} L_{\_G185414}
cmark_t G

% Constructing a point O such that  $HO/HG=1.5$ 
towards O H G 1.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

```

```

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
% lines m(H_{b}H_{a}) and c are not parallel; points C and T_{c} are not the same; points E_{c}
% and C are not the same
% Determination conditions: lines m(H_{b}H_{a}) and c are not the same; points C and T_{c} are not
% the same; points C and T_{c} are not the same; points C and E_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $E_c = E_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 103 terms.

Time Complexity: Time spent by the prover is 3.815 seconds.

NDG conditions Points A and B are not identical

Points H and T_c are not identical

Line through points E_c and H is not perpendicular to line through points H and C

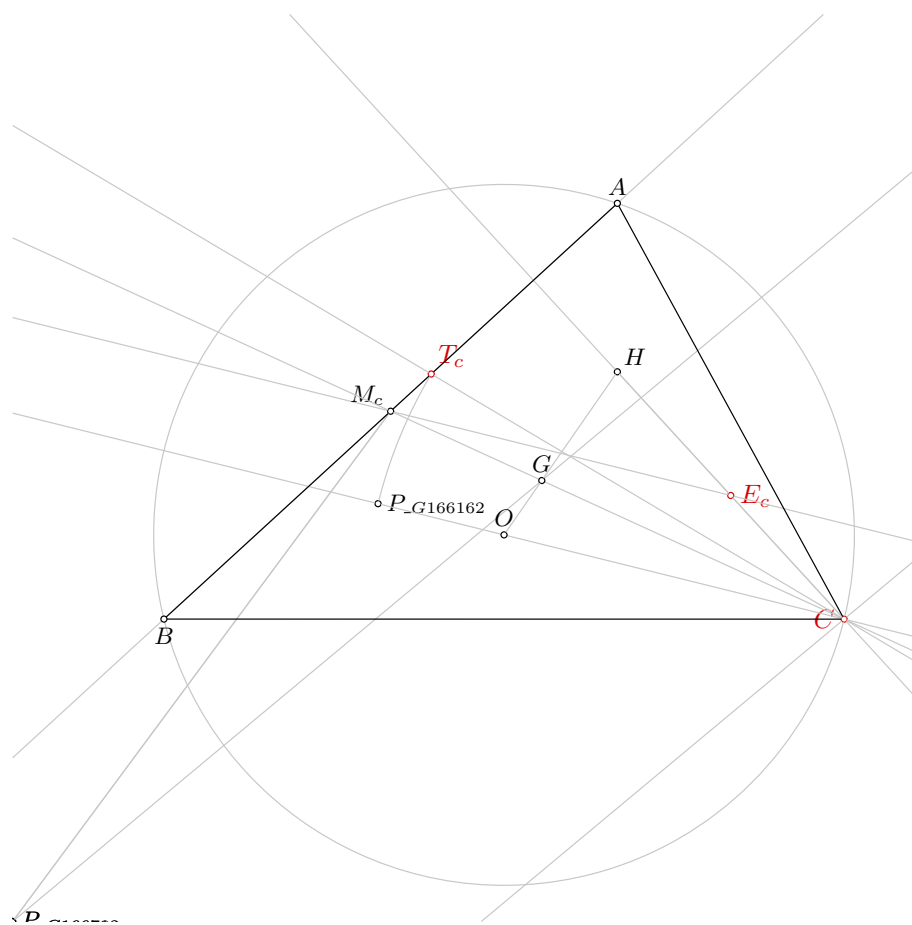


Figure 1: Illustration of the problem 0733

Points $P_{G179800}$, C and M_c are not collinear

Points $P_{G179800}$ and H are not identical

Points A , B and C are not collinear

4.1.3 Proving $T_c = \neg T_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $C = C$

NDG conditions are:

$S_{T_c C E_c} \neq 0$ i.e., points T_c , C and E_c are not collinear (foot is not the point itself; construction based assumption)

$S_{E_c T_c F_c^0} \neq S_{P_{m(H_b H_a)}^1 T_c F_c^0}$ i.e., lines $E_c P_{m(H_b H_a)}^1$ and $T_c F_c^0$ are not parallel (construction based assumption)

$S_{P_{G168587} M_c C} \neq S_{P_{L_{G168556}}^2 M_c C}$ i.e., lines $P_{G168587} P_{L_{G168556}}^2$ and $M_c C$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^4} \neq S_{F_{-h_a}^3 BF_{-h_b}^4}$ i.e., lines $AF_{-h_a}^3$ and $BF_{-h_b}^4$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $E_c = \neg E_c$

Proving failed

4.2.3 Proving $T_c = \neg T_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $E_c = \neg E_c$

Proving failed

4.3.3 Proving $T_c = \neg T_c$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 8 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $E_c=\neg E_c$

Proving failed

4.4.3 Proving $T_c=\neg T_c$

Proving failed

Problem 734

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 734: Given a point C , a point G and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point G , construct a point M_c (rule W01); ;
2. Using the point G and the point N , construct a point H (rule W01); ;
3. Using the point C and the point H , construct a point E_c (rule W01); ;
4. Using the point G and the point N , construct a point O (rule W01); ;
5. Using the point C and the point H , construct a line h_c (rule W02); % DET: points C and H are not the same;
6. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
7. Using the point M_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_c and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
9. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; line h_c and circle $k(N, M_a)$ intersect; points M_c and N are not the same; points C and O are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points C and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L15,L16,L18,L21,L24,L3,L57]

Solving time: 9.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
```

```
point G 70 58.33
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_l C
```

```
cmark_t G
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point M_{c} such that CM_{c}/CG=1.5
```

```
towards M_{c} C G 1.5
```

```
cmark_lt M_{c}
```

```
color 200 200 200
```

```
drawsegment C M_{c}
```

```
color 0 0 0
```

```
% Constructing a point H such that GH/GN=4
```

```
towards H G N 4
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment G H
```

```
color 0 0 0
```

```
% Constructing a point E_{c} such that CE_{c}/CH=0.5
```

```
towards E_{c} C H 0.5
```

```
cmark_r E_{c}
```

```
color 200 200 200
```

```
drawsegment C H
```

```
color 0 0 0
```

```
% Constructing a point O such that GO/GN=-2
```

```

towards O G N -2
cmark_t O
color 200 200 200
drawsegment N O
color 0 0 0

% DET: points C and H are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point C and point H
line  $h_{\{c\}}$  C H

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point C
circle  $k(O,C)$  O C

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: points  $M_{\{c\}}$  and N are not the same
% Constructing a circle  $k(N,M_{\{a\}})$  whose center is at point N and which passes through point  $M_{\{c\}}$ 
circle  $k(N,M_{\{a\}})$  N  $M_{\{c\}}$ 

color 200 200 200
drawcircle  $k(N,M_{\{a\}})$ 
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $E_{\{c\}}$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G221439\}}$  which is a foot of the point N on the line  $h_{\{c\}}$ 
foot  $P_{\{\backslash\_G221439\}}$  N  $h_{\{c\}}$ 
cmark_r  $P_{\{\backslash\_G221439\}}$ 
color 200 200 200
drawline N  $P_{\{\backslash\_G221439\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $E_{\{c\}}$  in the symmetry to point/line  $P_{\{\backslash\_G221439\}}$ 
sim  $H_{\{c\}}$   $P_{\{\backslash\_G221439\}}$   $E_{\{c\}}$ 
cmark_rt  $H_{\{c\}}$ 

% DET: points  $H_{\{c\}}$  and  $M_{\{c\}}$  are not the same
% Constructing a line c which passes through point  $H_{\{c\}}$  and point  $M_{\{c\}}$ 
line c  $H_{\{c\}}$   $M_{\{c\}}$ 

color 200 200 200

```

```
drawline c
color 0 0 0
```

```
% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line c and circle k(O,C) intersect; line h_{c} and circle k(N,M_{a})
    intersect; points M_{c} and N are not the same; points C and O are not the same
% Determination conditions: points H_{c} and M_{c} are not the same; points E_{c} and H_{c} must be
    different; points C and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.027 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $G=_G$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $C=C$

NDG conditions are:

$S_{AB_M_b} \neq S_{M_aB_M_b}$ i.e., lines A_M_a and B_M_b are not parallel (construction based assumption)
 $S_{M_aBC} \neq 0$ i.e., points M_a , B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{M_bAC} \neq 0$ i.e., points M_b , A and C are not collinear (foot is not the point itself; construction based assumption)

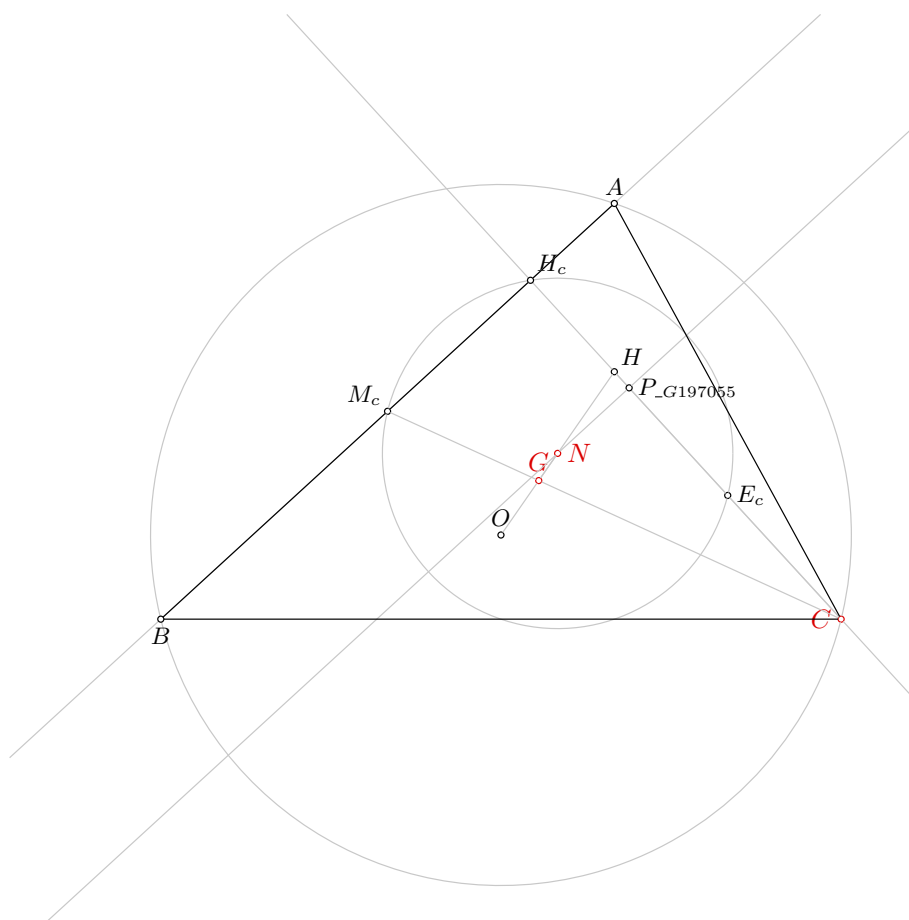


Figure 1: Illustration of the problem 0734

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF^3_{-h_b}} \neq S_{F^2_{-h_a}BF^3_{-h_b}}$ i.e., lines $AF^2_{-h_a}$ and $BF^3_{-h_b}$ are not parallel (construction based assumption)
 $S_{-M_a-M_bF^1_{-m_b}} \neq S_{F^0_{-m_a}-M_bF^1_{-m_b}}$ i.e., lines $-M_aF^0_{-m_a}$ and $-M_bF^1_{-m_b}$ are not parallel (construction based assumption)
 Total number of proof steps: 1
 Time spent by the prover: 0.000 seconds

4.2.2 Proving $G=_G$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $G=_G$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $G=_G$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 735

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 735: Given a point C , a point H and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point H , construct a point E_c (rule W01); ;
2. Using the point H and the point N , construct a point O (rule W01); ;
3. Using the point H and the point N , construct a point G (rule W01); ;
4. Using the point C and the point G , construct a point M_c (rule W01); ;
5. Using the point C and the point H , construct a line h_c (rule W02); % DET: points C and H are not the same;
6. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
7. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
9. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; line h_c and circle $k(N, M_a)$ intersect; points E_c and N are not the same; points C and O are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points C and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D31,D32,D7,GD01,GL03,GL09,L11,L12,L16,L21,L24,L3,L57]

Solving time: 9.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
```

```
point H 80 72.73
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_l C
```

```
cmark_rt H
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point E_{c} such that CE_{c}/CH=0.5
```

```
towards E_{c} C H 0.5
```

```
cmark_r E_{c}
```

```
color 200 200 200
```

```
drawsegment C H
```

```
color 0 0 0
```

```
% Constructing a point O such that HO/HN=2
```

```
towards O H N 2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment H O
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G257771} which passes through point H and point N
```

```
line L_{\_G257771} H N
```

```
color 200 200 200
```

```
drawline L_{\_G257771}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G257872} with coordinates (0,0)
```

```
point P_{\_G257872} 0 0
```

```

cmark_r P_{\_G257872}

% Constructing a point P_{\_G257796} such that HP_{\_G257796}/HP_{\_G257872}=4
towards P_{\_G257796} H P_{\_G257872} 4
cmark_r P_{\_G257796}
color 200 200 200
drawsegment H P_{\_G257796}
color 0 0 0

% Constructing a point P_{\_G257841} such that HP_{\_G257841}/HP_{\_G257872}=3
towards P_{\_G257841} H P_{\_G257872} 3
cmark_r P_{\_G257841}
color 200 200 200
drawsegment H P_{\_G257841}
color 0 0 0

% Constructing a line L_{\_G257802} which passes through point N and point P_{\_G257841}
line L_{\_G257802} N P_{\_G257841}

color 200 200 200
drawline L_{\_G257802}
color 0 0 0

% Constructing a line L_{\_G257765} which contains the point P_{\_G257796} and is parallel to the
line L_{\_G257802}
parallel L_{\_G257765} P_{\_G257796} L_{\_G257802}

color 200 200 200
drawline L_{\_G257765}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G257765} and line L_{\_G257771}
intersec G L_{\_G257765} L_{\_G257771}
cmark_t G

% Constructing a point M_{c} such that CM_{c}/CG=1.5
towards M_{c} C G 1.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% DET: points C and H are not the same
% Constructing a line h_{c} which passes through point C and point H
line h_{c} C H

color 200 200 200
drawline h_{c}
color 0 0 0

```

```

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points E_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{c}
circle k(N,M_{a}) N E_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{c} and circle k(N,M_{a}) intersect% DET: points E_{c} and H_{c} must be different
% Constructing a point P_{\_G258759} which is a foot of the point N on the line h_{c}
foot P_{\_G258759} N h_{c}
cmark_r P_{\_G258759}
color 200 200 200
drawline N P_{\_G258759}
color 0 0 0

% Constructing a point H_{c} which is an image of the point E_{c} in the symmetry to point/line P
\_G258759}
sim H_{c} P_{\_G258759} E_{c}
cmark_rt H_{c}

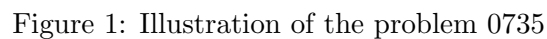
% DET: points H_{c} and M_{c} are not the same
% Constructing a line c which passes through point H_{c} and point M_{c}
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```



```
% Non-degenerate conditions: line c and circle k(O,C) intersect; line h_{c} and circle k(N,M_{a})
% intersect; points E_{c} and N are not the same; points C and O are not the same
% Determination conditions: points H_{c} and M_{c} are not the same; points E_{c} and H_{c} must be
% different; points C and H are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.034 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $H=_H$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $C=C$

NDG conditions are:

$S_{P_{G234434}HN} \neq S_{P_{L_{G234403}}^0 HN}$ i.e., lines $P_{G234434}P_{L_{G234403}}^0$ and HN are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^2} \neq S_{F_{-h_a}^1 BF_{-h_b}^2}$ i.e., lines $AF_{-h_a}^1$ and $BF_{-h_b}^2$ are not parallel (construction based assumption)

$S_{_M_a BC} \neq 0$ i.e., points $_M_a, B$ and C are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_b AC} \neq 0$ i.e., points $_M_b, A$ and C are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_a _M_b F_{-m_b}^4} \neq S_{F_{-m_a}^3 _M_b F_{-m_b}^4}$ i.e., lines $_M_a F_{-m_a}^3$ and $_M_b F_{-m_b}^4$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $H=_H$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $H=_H$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $H=_H$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 736

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 736: Given a point C , a point H_a and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point H_a , construct a line a (rule W02); % DET: points C and H_a are not the same;
2. Using the point H_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_a and N are not the same;
3. Using the circle $k(N, M_a)$, the line a , the point N and the point H_a , construct a point M_a (rule W05); % NDG: line a and circle $k(N, M_a)$ intersect % DET: points H_a and M_a must be different;
4. Using the point M_a and the point C , construct a point B (rule W01); ;
5. Using the point N and the point M_a , construct a line $m(H_b H_c)$ (rule W02); % DET: points N and M_a are not the same;
6. Using the circle $k(N, M_a)$, the line $m(H_b H_c)$, the point N and the point M_a , construct a point E_a (rule W05a); % NDG: line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect % DET: points M_a and E_a must be different;
7. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;
8. Using the point C and the point M_a , construct a circle $k(M_a, B)$ (rule W06); % NDG: points C and M_a are not the same;
9. Using the circle $k(N, M_a)$ and the circle $k(M_a, B)$, construct a point H_b and a point H_c (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_a, B)$ intersect % DET: circles $k(N, M_a)$ and $k(M_a, B)$ are not the same;

10. Using the point C and the point H_b , construct a line b (rule W02); % DET: points C and H_b are not the same;
11. Using the line h_a and the line b , construct a point A (rule W03); % NDG: lines h_a and b are not parallel % DET: lines h_a and b are not the same.

Non-degenerate conditions: lines h_a and b are not parallel; circles $k(N, M_a)$ and $k(M_a, B)$ intersect; points C and M_a are not the same; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; line a and circle $k(N, M_a)$ intersect; points H_a and N are not the same.

Determination conditions: lines h_a and b are not the same; points C and H_b are not the same; circles $k(N, M_a)$ and $k(M_a, B)$ are not the same; points E_a and H_a are not the same; points M_a and E_a must be different; points N and M_a are not the same; points H_a and M_a must be different; points C and H_a are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D21,D28,D3,D32,D5,D6,D8,GD01,GD02,GL01,GL03,GL04,GL09,L19,L20,L21,L22,L37,L38,L39]

Solving time: 4.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point H_{a} 80 40
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_r H_{a}
cmark_r N
color 0 0 0
fontsize 8
```

```
% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% NDG: points H_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{a}
circle k(N,M_{a}) N H_{a}
```

```
color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```

% NDG: line a and circle k(N,M_{a}) intersect% DET: points H_{a} and M_{a} must be different
% Constructing a point P_{\_G45212} which is a foot of the point N on the line a
foot P_{\_G45212} N a
cmark_r P_{\_G45212}
color 200 200 200
drawline N P_{\_G45212}
color 0 0 0

% Constructing a point M_{a} which is an image of the point H_{a} in the symmetry to point/line P
\_G45212}
sim M_{a} P_{\_G45212} H_{a}
cmark_r M_{a}

% Constructing a point B such that M_{a}B/M_{a}C=-1
towards B M_{a} C -1
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% DET: points N and M_{a} are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point N and point M_{a}
line m(H_{b}H_{c}) N M_{a}

color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0

% NDG: line m(H_{b}H_{c}) and circle k(N,M_{a}) intersect% DET: points M_{a} and E_{a} must be
different
% Constructing a point E_{a} which is an image of the point M_{a} in the symmetry to point/line N
sim E_{a} N M_{a}
cmark_r E_{a}

% DET: points E_{a} and H_{a} are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H_{a}
line h_{a} E_{a} H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points C and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point C
circle k(M_{a},B) M_{a} C

```

```

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(M_{a},B) intersect% DET: circles k(N,M_{a}) and k(M_{a},B) are not
the same
% Constructing points H_{b} and H_{c} which are in intersection of k(N,M_{a}) and k(M_{a},B)
intersec2 H_{b} H_{c} k(N,M_{a}) k(M_{a},B)
cmark_l H_{b}
cmark_rt H_{c}

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines h_{a} and b are not parallel% DET: lines h_{a} and b are not the same
% Constructing a point A which belongs to line h_{a} and line b
intersec A h_{a} b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{a} and b are not parallel; circles k(N,M_{a}) and k(M_{a},B)
intersect; points C and M_{a} are not the same; line m(H_{b}H_{c}) and circle k(N,M_{a})
intersect; line a and circle k(N,M_{a}) intersect; points H_{a} and N are not the same
% Determination conditions: lines h_{a} and b are not the same; points C and H_{b} are not the same
; circles k(N,M_{a}) and k(M_{a},B) are not the same; points E_{a} and H_{a} are not the same;
points M_{a} and E_{a} must be different; points N and M_{a} are not the same; points H_{a} and
M_{a} must be different; points C and H_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

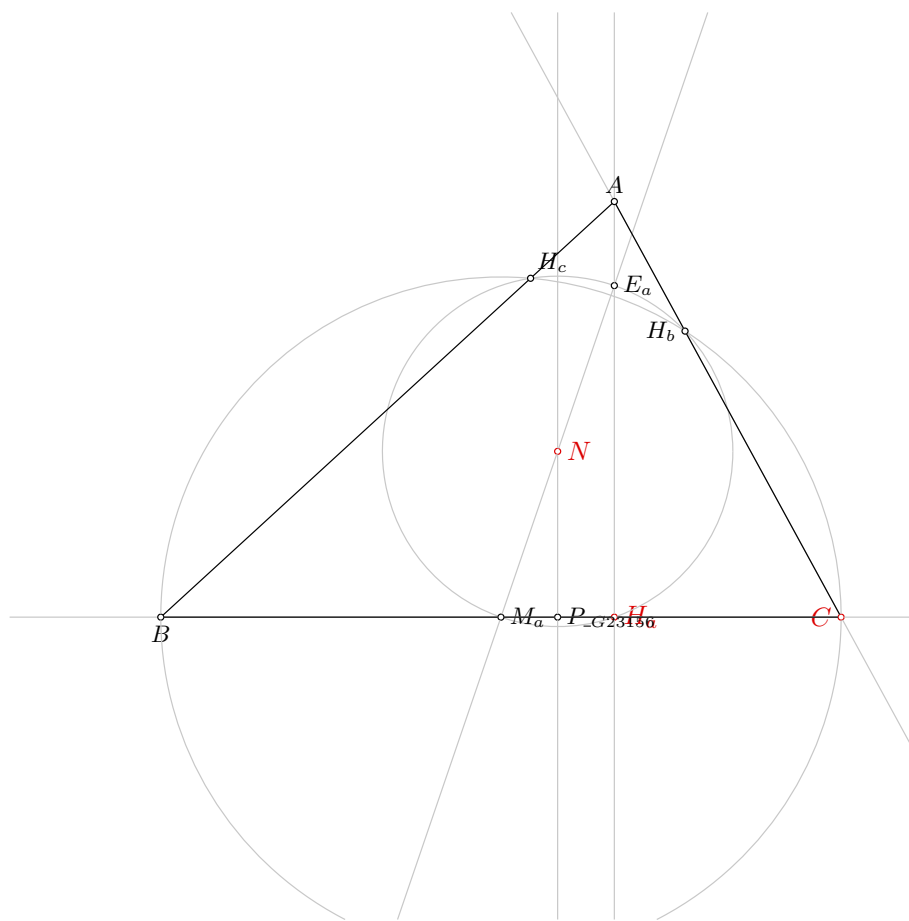


Figure 1: Illustration of the problem 0736

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $H_a = \neg H_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 18 terms.

Time Complexity: Time spent by the prover is 0.407 seconds.

NDG conditions Points N and M_a are not identical

Line through points C and N is not perpendicular to line through points N and H_a

Line through points H_b and C is not parallel with line through points E_a and H_a

Points B , H_a and E_a are not collinear

Point B is not on circle with center H_a and point from it C

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $C = C$

NDG conditions are:

$S_{E_aCH_b} \neq S_{H_aCH_b}$ i.e., lines E_aH_a and CH_b are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAF_{\neg h_a}^0} \neq S_{CAF_{\neg h_a}^0}$ i.e., lines BC and $AF_{\neg h_a}^0$ are not parallel (construction based assumption)

$S_{\neg M_aBC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_bAC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^0BF_{\neg h_b}^3}$ i.e., lines $AF_{\neg h_a}^0$ and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

$S_{\neg M_a\neg M_bF_{\neg m_b}^2} \neq S_{F_{\neg m_a}^1\neg M_bF_{\neg m_b}^2}$ i.e., lines $\neg M_aF_{\neg m_a}^1$ and $\neg M_bF_{\neg m_b}^2$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $H_a = \neg H_a$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C = C$

Proving failed

4.3.2 Proving $H_a = \neg H_a$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Proving failed

4.4.2 Proving $H_a=_H H_a$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 737

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 737: Given a point C , a point H_b and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point H_b , construct a line b (rule W02); % DET: points C and H_b are not the same;
2. Using the point H_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_b and N are not the same;
3. Using the circle $k(N, M_a)$, the line b , the point N and the point H_b , construct a point M_b (rule W05); % NDG: line b and circle $k(N, M_a)$ intersect % DET: points H_b and M_b must be different;
4. Using the point M_b and the point C , construct a point A (rule W01); ;
5. Using the point N and the point M_b , construct a line $m(H_a H_c)$ (rule W02); % DET: points N and M_b are not the same;
6. Using the circle $k(N, M_a)$, the line $m(H_a H_c)$, the point N and the point M_b , construct a point E_b (rule W05a); % NDG: line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect % DET: points M_b and E_b must be different;
7. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;
8. Using the point C and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points C and M_b are not the same;
9. Using the circle $k(N, M_a)$ and the circle $k(M_b, C)$, construct a point H_a and a point H_c (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_b, C)$ intersect % DET: circles $k(N, M_a)$ and $k(M_b, C)$ are not the same;

10. Using the point C and the point H_a , construct a line a (rule W02); % DET: points C and H_a are not the same;
11. Using the line h_b and the line a , construct a point B (rule W03); % NDG: lines h_b and a are not parallel % DET: lines h_b and a are not the same.

Non-degenerate conditions: lines h_b and a are not parallel; circles $k(N, M_a)$ and $k(M_b, C)$ intersect; points C and M_b are not the same; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; line b and circle $k(N, M_a)$ intersect; points H_b and N are not the same.

Determination conditions: lines h_b and a are not the same; points C and H_a are not the same; circles $k(N, M_a)$ and $k(M_b, C)$ are not the same; points E_b and H_b are not the same; points M_b and E_b must be different; points N and M_b are not the same; points H_b and M_b must be different; points C and H_b are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D22,D29,D3,D32,D5,D6,D9,GD01,GD02,GL01,GL03,GL04,GL09,L17,L19,L20,L21,L23,L44,L45]

Solving time: 4.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point H_{b} 89.36 77.83
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_l H_{b}
cmark_r N
color 0 0 0
fontsize 8
```

```
% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% NDG: points H_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{b}
circle k(N,M_{a}) N H_{b}
```

```
color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```



```

% NDG: line b and circle k(N,M_{a}) intersect% DET: points H_{b} and M_{b} must be different
% Constructing a point P_{\_G80580} which is a foot of the point N on the line b
foot P_{\_G80580} N b
cmark_r P_{\_G80580}
color 200 200 200
drawline N P_{\_G80580}
color 0 0 0

% Constructing a point M_{b} which is an image of the point H_{b} in the symmetry to point/line P
\_G80580}
sim M_{b} P_{\_G80580} H_{b}
cmark_lt M_{b}

% Constructing a point A such that M_{b}A/M_{b}C=-1
towards A M_{b} C -1
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

% DET: points N and M_{b} are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point N and point M_{b}
line m(H_{a}H_{c}) N M_{b}

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

% NDG: line m(H_{a}H_{c}) and circle k(N,M_{a}) intersect% DET: points M_{b} and E_{b} must be
different
% Constructing a point E_{b} which is an image of the point M_{b} in the symmetry to point/line N
sim E_{b} N M_{b}
cmark_r E_{b}

% DET: points E_{b} and H_{b} are not the same
% Constructing a line h_{b} which passes through point E_{b} and point H_{b}
line h_{b} E_{b} H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points C and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point C
circle k(M_{b},C) M_{b} C

```

```

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(M_{b},C) intersect% DET: circles k(N,M_{a}) and k(M_{b},C) are not
the same
% Constructing points H_{a} and H_{c} which are in intersection of k(N,M_{a}) and k(M_{b},C)
intersec2 H_{a} H_{c} k(N,M_{a}) k(M_{b},C)
cmark_r H_{a}
cmark_rt H_{c}

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines h_{b} and a are not parallel% DET: lines h_{b} and a are not the same
% Constructing a point B which belongs to line h_{b} and line a
intersec B h_{b} a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{b} and a are not parallel; circles k(N,M_{a}) and k(M_{b},C)
intersect; points C and M_{b} are not the same; line m(H_{a}H_{c}) and circle k(N,M_{a})
intersect; line b and circle k(N,M_{a}) intersect; points H_{b} and N are not the same
% Determination conditions: lines h_{b} and a are not the same; points C and H_{a} are not the same
; circles k(N,M_{a}) and k(M_{b},C) are not the same; points E_{b} and H_{b} are not the same;
points M_{b} and E_{b} must be different; points N and M_{b} are not the same; points H_{b} and
M_{b} must be different; points C and H_{b} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

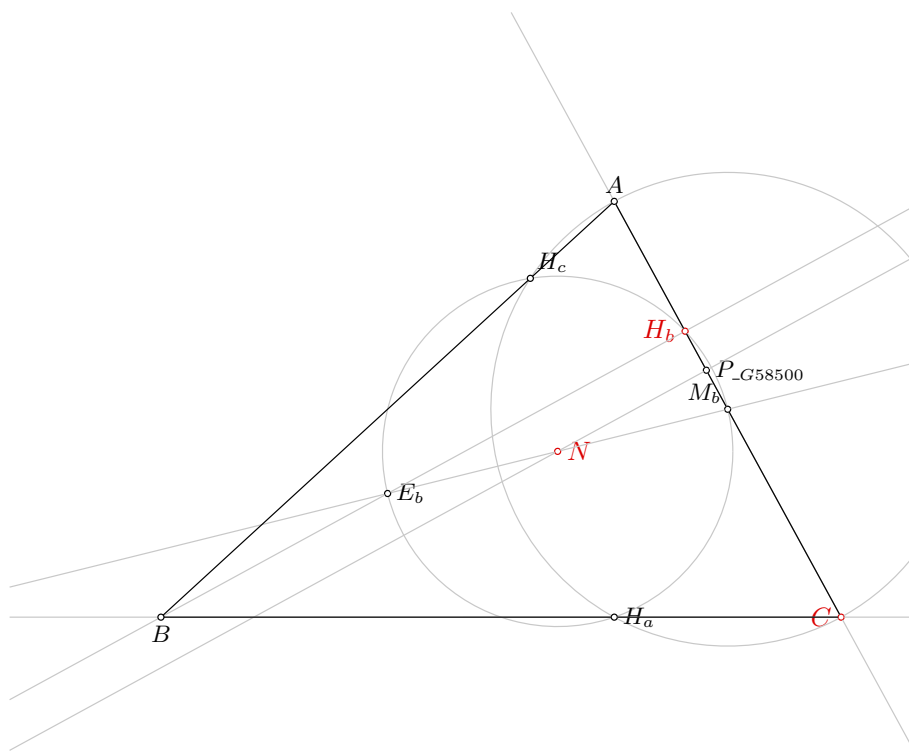


Figure 1: Illustration of the problem 0737

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.031 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $H_b = \neg H_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 18 terms.

Time Complexity: Time spent by the prover is 0.387 seconds.

NDG conditions Points M_b and N are not identical

Points M_b , P_{G73304} and N are not collinear

Line through points H_b and E_b is not parallel with line through points C and H_a

Points P_{G73304} , C and E_b are not collinear

Points A and C are not identical

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $C = C$

NDG conditions are:

$S_{E_bCH_a} \neq S_{H_bCH_a}$ i.e., lines E_bH_b and CH_a are not parallel (construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^0} \neq S_{CBF_{\neg h_b}^0}$ i.e., lines AC and $BF_{\neg h_b}^0$ are not parallel (construction based assumption)

$S_{\neg M_aBC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_bAC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^0} \neq S_{F_{\neg h_a}^3BF_{\neg h_b}^0}$ i.e., lines $AF_{\neg h_a}^3$ and $BF_{\neg h_b}^0$ are not parallel (construction based assumption)

$S_{\neg M_a\neg M_bF_{\neg m_b}^2} \neq S_{F_{\neg m_a}^1\neg M_bF_{\neg m_b}^2}$ i.e., lines $\neg M_aF_{\neg m_a}^1$ and $\neg M_bF_{\neg m_b}^2$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $H_b = \neg H_b$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C = C$

Proving failed

4.3.2 Proving $H_b = \neg H_b$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Proving failed

4.4.2 Proving $H_b=_H H_b$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 738

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 738: Given a point C , a point H_c and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point H_c , construct a line h_c (rule W02); % DET: points C and H_c are not the same;
2. Using the point H_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_c and N are not the same;
3. Using the circle $k(N, M_a)$, the line h_c , the point N and the point H_c , construct a point E_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points H_c and E_c must be different;
4. Using the point E_c and the point C , construct a point H (rule W01); ;
5. Using the point N and the point H , construct a point O (rule W01); ;
6. Using the point N and the point H , construct a point G (rule W01); ;
7. Using the point C and the point G , construct a point M_c (rule W01); ;
8. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
9. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; line h_c and circle $k(N, M_a)$ intersect; points H_c and N are not the same.

Determination conditions: points H_c and M_c are not the same; points H_c and E_c must be different; points C and H_c are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D31,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L16,L21,L24,L3,L57]

Solving time: 3.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point H_{c} 68.91 84.83
point N 72.5 61.93

color 220 0 0
fontsize 9

cmark_l C
cmark_rt H_{c}
cmark_r N
color 0 0 0
fontsize 8

% DET: points C and H_{c} are not the same
% Constructing a line h_{c} which passes through point C and point H_{c}
line h_{c} C H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points H_{c} and N are not the same
% Constructing a circle k(N, M_{a}) whose center is at point N and which passes through point H_{c}
circle k(N, M_{a}) N H_{c}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line h_{c} and circle k(N, M_{a}) intersect% DET: points H_{c} and E_{c} must be different
% Constructing a point P_{\G116575} which is a foot of the point N on the line h_{c}
foot P_{\G116575} N h_{c}
cmark_r P_{\G116575}
color 200 200 200
drawline N P_{\G116575}
color 0 0 0
```

```

% Constructing a point  $E_{\{c\}}$  which is an image of the point  $H_{\{c\}}$  in the symmetry to point/line  $P_{\{\_G116575\}}$ 
sim  $E_{\{c\}}$   $P_{\{\_G116575\}}$   $H_{\{c\}}$ 
cmark_r  $E_{\{c\}}$ 

% Constructing a point  $H$  such that  $E_{\{c\}}H/E_{\{c\}}C=-1$ 
towards  $H$   $E_{\{c\}}$   $C$  -1
cmark_rt  $H$ 
color 200 200 200
drawsegment  $C$   $H$ 
color 0 0 0

% Constructing a point  $O$  such that  $NO/NH=-1$ 
towards  $O$   $N$   $H$  -1
cmark_t  $O$ 
color 200 200 200
drawsegment  $H$   $O$ 
color 0 0 0

% Constructing a line  $L_{\{\_G116837\}}$  which passes through point  $N$  and point  $H$ 
line  $L_{\{\_G116837\}}$   $N$   $H$ 

color 200 200 200
drawline  $L_{\{\_G116837\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\_G116938\}}$  with coordinates  $(0,0)$ 
point  $P_{\{\_G116938\}}$  0 0
cmark_r  $P_{\{\_G116938\}}$ 

% Constructing a point  $P_{\{\_G116862\}}$  such that  $NP_{\{\_G116862\}}/NP_{\{\_G116938\}}=-1$ 
towards  $P_{\{\_G116862\}}$   $N$   $P_{\{\_G116938\}}$  -1
cmark_r  $P_{\{\_G116862\}}$ 
color 200 200 200
drawsegment  $P_{\{\_G116938\}}$   $P_{\{\_G116862\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\_G116907\}}$  such that  $NP_{\{\_G116907\}}/NP_{\{\_G116938\}}=3$ 
towards  $P_{\{\_G116907\}}$   $N$   $P_{\{\_G116938\}}$  3
cmark_r  $P_{\{\_G116907\}}$ 
color 200 200 200
drawsegment  $N$   $P_{\{\_G116907\}}$ 
color 0 0 0

% Constructing a line  $L_{\{\_G116868\}}$  which passes through point  $H$  and point  $P_{\{\_G116907\}}$ 
line  $L_{\{\_G116868\}}$   $H$   $P_{\{\_G116907\}}$ 

color 200 200 200
drawline  $L_{\{\_G116868\}}$ 

```



```

color 0 0 0

% Constructing a line  $L_{\backslash\_G116831}$  which contains the point  $P_{\backslash\_G116862}$  and is parallel to the
    line  $L_{\backslash\_G116868}$ 
parallel  $L_{\backslash\_G116831}$   $P_{\backslash\_G116862}$   $L_{\backslash\_G116868}$ 

color 200 200 200
drawline  $L_{\backslash\_G116831}$ 
color 0 0 0

% Constructing a point  $G$  which belongs to line  $L_{\backslash\_G116831}$  and line  $L_{\backslash\_G116837}$ 
intersec G  $L_{\backslash\_G116831}$   $L_{\backslash\_G116837}$ 
cmark_t G

% Constructing a point  $M_{\{c\}}$  such that  $CM_{\{c\}}/CG=1.5$ 
towards  $M_{\{c\}}$  C G 1.5
cmark_lt  $M_{\{c\}}$ 
color 200 200 200
drawsegment C  $M_{\{c\}}$ 
color 0 0 0

% DET: points  $H_{\{c\}}$  and  $M_{\{c\}}$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $M_{\{c\}}$ 
line c  $H_{\{c\}}$   $M_{\{c\}}$ 

color 200 200 200
drawline c
color 0 0 0

% NDG: points  $C$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $C$ 
circle  $k(O,C)$  O C

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $c$  and circle  $k(O,C)$  intersect
% Constructing points  $A$  and  $B$  which are in intersection of  $k(O,C)$  and  $c$ 
intersec2 A B  $k(O,C)$  c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C

```

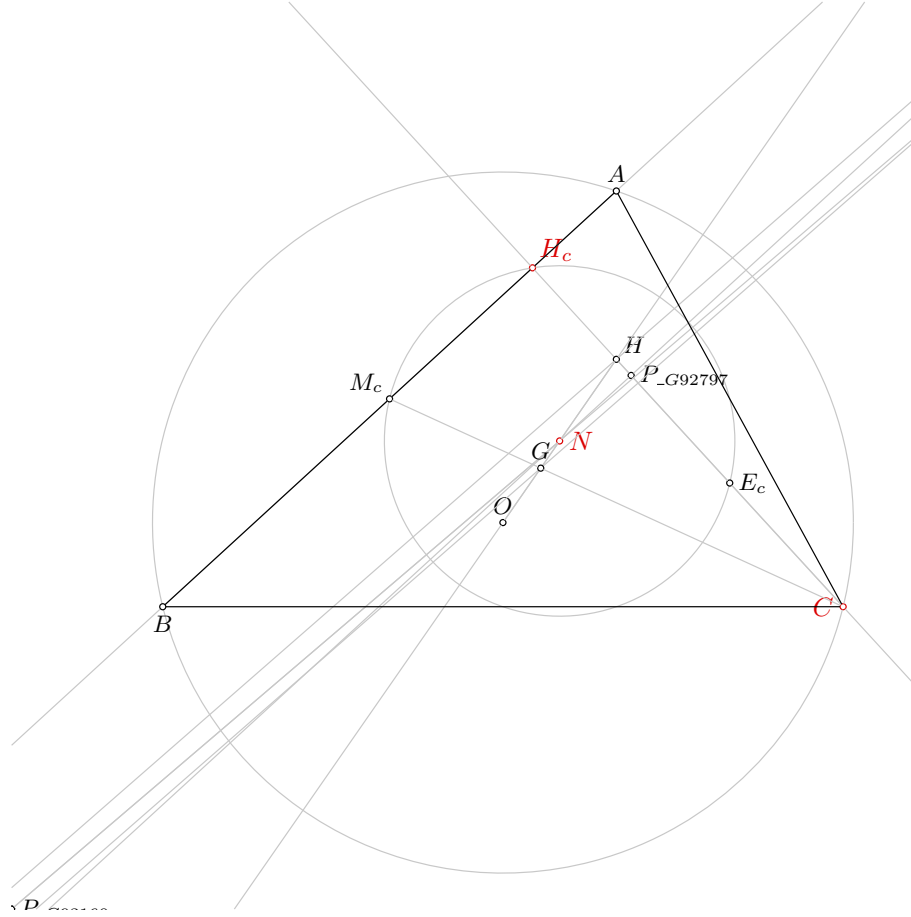


Figure 1: Illustration of the problem 0738

drawsegment B C

*% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
 line h_{c} and circle k(N,M_{a}) intersect; points H_{c} and N are not the same
 % Determination conditions: points H_{c} and M_{c} are not the same; points H_{c} and E_{c} must be
 different; points C and H_{c} are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $H_c = \neg H_c$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $C = C$

NDG conditions are:

$S_{P_{-G94534}NH} \neq S_{P_{L_{-G94503}}^0}^{NH}$ i.e., lines $P_{-G94534}P_{L_{-G94503}}^0$ and NH are not parallel (construction based assumption)

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{-h_c}^1} \neq S_{BCF_{-h_c}^1}$ i.e., lines AB and $CF_{-h_c}^1$ are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points M_a , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points M_b , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^5} \neq S_{F_{-h_a}^4BF_{-h_b}^5}$ i.e., lines $AF_{-h_a}^4$ and $BF_{-h_b}^5$ are not parallel (construction based assumption)

$S_{M_aM_bF_{-m_b}^3} \neq S_{F_{-m_a}^2M_bF_{-m_b}^3}$ i.e., lines $M_aF_{-m_a}^2$ and $M_bF_{-m_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $H_c = \neg H_c$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C = C$

Proving failed

4.3.2 Proving $H_c = \neg H_c$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Proving failed

4.4.2 Proving $H_c=_Hc$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 739

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 739: Given a point C , a point I and a point N , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 740

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 740: Given a point C , a point M_a and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point M_a , construct a point B (rule W01); ;
2. Using the point C and the point M_a , construct a line a (rule W02); % DET: points C and M_a are not the same;
3. Using the point M_a and the point N , construct a line $m(H_bH_c)$ (rule W02); % DET: points M_a and N are not the same;
4. Using the point C and the point M_a , construct a circle $k(M_a, B)$ (rule W06); % NDG: points C and M_a are not the same;
5. Using the point M_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_a and N are not the same;
6. Using the circle $k(N, M_a)$, the line a , the point N and the point M_a , construct a point H_a (rule W05); % NDG: line a and circle $k(N, M_a)$ intersect % DET: points M_a and H_a must be different;
7. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point M_a , construct a point E_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_a and E_a must be different;
8. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;
9. Using the circle $k(M_a, B)$ and the circle $k(N, M_a)$, construct a point H_b and a point H_c (rule W07); % NDG: circles $k(M_a, B)$ and $k(N, M_a)$ intersect % DET: circles $k(M_a, B)$ and $k(N, M_a)$ are not the same;

10. Using the point C and the point H_b , construct a line b (rule W02); % DET: points C and H_b are not the same;
11. Using the line h_a and the line b , construct a point A (rule W03); % NDG: lines h_a and b are not parallel % DET: lines h_a and b are not the same.

Non-degenerate conditions: lines h_a and b are not parallel; circles $k(M_a, B)$ and $k(N, M_a)$ intersect; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; line a and circle $k(N, M_a)$ intersect; points M_a and N are not the same; points C and M_a are not the same.

Determination conditions: lines h_a and b are not the same; points C and H_b are not the same; circles $k(M_a, B)$ and $k(N, M_a)$ are not the same; points E_a and H_a are not the same; points M_a and E_a must be different; points M_a and H_a must be different; points M_a and N are not the same; points C and M_a are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D21,D28,D3,D32,D5,D6,D8,GD01,GD02,GL01,GL03,GL04,GL09,L19,L20,L21,L22,L37,L38,L39]

Solving time: 4.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point M_{a} 65 40
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_r M_{a}
cmark_r N
color 0 0 0
fontsize 8
```

```
% Constructing a point B such that CB/CM_{a}=2
towards B C M_{a} 2
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0
```

```
% DET: points C and M_{a} are not the same
% Constructing a line a which passes through point C and point M_{a}
line a C M_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```

% DET: points  $M_{\{a\}}$  and  $N$  are not the same
% Constructing a line  $m(H_{\{b\}}H_{\{c\}})$  which passes through point  $M_{\{a\}}$  and point  $N$ 
line m( $H_{\{b\}}H_{\{c\}}$ )  $M_{\{a\}}$  N

color 200 200 200
drawline m( $H_{\{b\}}H_{\{c\}}$ )
color 0 0 0

% NDG: points  $C$  and  $M_{\{a\}}$  are not the same
% Constructing a circle  $k(M_{\{a\}}, B)$  whose center is at point  $M_{\{a\}}$  and which passes through point  $C$ 
circle k( $M_{\{a\}}, B$ )  $M_{\{a\}}$  C

color 200 200 200
drawcircle k( $M_{\{a\}}, B$ )
color 0 0 0

% NDG: points  $M_{\{a\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $M_{\{a\}}$ 
circle k( $N, M_{\{a\}}$ )  $N$   $M_{\{a\}}$ 

color 200 200 200
drawcircle k( $N, M_{\{a\}}$ )
color 0 0 0

% NDG: line  $a$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{a\}}$  and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G155520\}}$  which is a foot of the point  $N$  on the line  $a$ 
foot  $P_{\{\backslash\_G155520\}}$   $N$   $a$ 
cmark_r  $P_{\{\backslash\_G155520\}}$ 
color 200 200 200
drawline  $N$   $P_{\{\backslash\_G155520\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $M_{\{a\}}$  in the symmetry to point/line  $P_{\{\backslash\_G155520\}}$ 
sim  $H_{\{a\}}$   $P_{\{\backslash\_G155520\}}$   $M_{\{a\}}$ 
cmark_r  $H_{\{a\}}$ 

% NDG: line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{a\}}$  and  $E_{\{a\}}$  must be
different
% Constructing a point  $E_{\{a\}}$  which is an image of the point  $M_{\{a\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{a\}}$   $N$   $M_{\{a\}}$ 
cmark_r  $E_{\{a\}}$ 

% DET: points  $E_{\{a\}}$  and  $H_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $E_{\{a\}}$  and point  $H_{\{a\}}$ 
line  $h_{\{a\}}$   $E_{\{a\}}$   $H_{\{a\}}$ 

```



```

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: circles  $k(M_{\{a\}}, B)$  and  $k(N, M_{\{a\}})$  intersect% DET: circles  $k(M_{\{a\}}, B)$  and  $k(N, M_{\{a\}})$  are not
the same
% Constructing points  $H_{\{b\}}$  and  $H_{\{c\}}$  which are in intersection of  $k(M_{\{a\}}, B)$  and  $k(N, M_{\{a\}})$ 
intersec2 H_{b} H_{c} k(M_{\{a\}}, B) k(N, M_{\{a\}})
cmark_l H_{b}
cmark_rt H_{c}

% DET: points  $C$  and  $H_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $C$  and point  $H_{\{b\}}$ 
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $h_{\{a\}}$  and  $b$  are not parallel% DET: lines  $h_{\{a\}}$  and  $b$  are not the same
% Constructing a point  $A$  which belongs to line  $h_{\{a\}}$  and line  $b$ 
intersec A h_{a} b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{a\}}$  and  $b$  are not parallel; circles  $k(M_{\{a\}}, B)$  and  $k(N, M_{\{a\}})$ 
intersect; line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect; line  $a$  and circle  $k(N, M_{\{a\}})$ 
intersect; points  $M_{\{a\}}$  and  $N$  are not the same; points  $C$  and  $M_{\{a\}}$  are not the same
% Determination conditions: lines  $h_{\{a\}}$  and  $b$  are not the same; points  $C$  and  $H_{\{b\}}$  are not the same
; circles  $k(M_{\{a\}}, B)$  and  $k(N, M_{\{a\}})$  are not the same; points  $E_{\{a\}}$  and  $H_{\{a\}}$  are not the same;
points  $M_{\{a\}}$  and  $E_{\{a\}}$  must be different; points  $M_{\{a\}}$  and  $H_{\{a\}}$  must be different; points  $M_{\{a\}}$ 
and  $N$  are not the same; points  $C$  and  $M_{\{a\}}$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

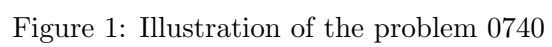
3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.



Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.033 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $N = \neg N$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 140 terms.

Time Complexity: Time spent by the prover is 4.114 seconds.

NDG conditions Points B , N and M_a are not collinear

Points B , N and M_a are not collinear

Line through points H_b and C is not parallel with line through points E_a and H_a

Points $\neg M_a$ and E_a are not identical

Points A , B and C are not collinear

Points B and C are not identical

4.2 GCLC - Area method

4.2.1 Proving $C = C$

NDG conditions are:

$S_{E_aCH_b} \neq S_{H_aCH_b}$ i.e., lines E_aH_a and CH_b are not parallel (construction based assumption)

$S_{\neg M_aBC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_bAC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2BF_{\neg h_b}^3}$ i.e., lines $AF_{\neg h_a}^2$ and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

$S_{\neg M_a\neg M_bF_{\neg m_b}^1} \neq S_{F_{\neg m_a}^0\neg M_bF_{\neg m_b}^1}$ i.e., lines $\neg M_aF_{\neg m_a}^0$ and $\neg M_bF_{\neg m_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $M_a = \neg M_a$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $M_a = \neg M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 6 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.3 Proving $N = \neg N$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 158 terms.

Time Complexity: Time spent by the prover is 0.310 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $M_a = \neg M_a$

Proving failed

4.4.3 Proving $N = \neg N$

Proving failed

Problem 741

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 741: Given a point C , a point M_b and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point M_b , construct a point A (rule W01); ;
2. Using the point C and the point M_b , construct a line b (rule W02); % DET: points C and M_b are not the same;
3. Using the point M_b and the point N , construct a line $m(H_aH_c)$ (rule W02); % DET: points M_b and N are not the same;
4. Using the point C and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points C and M_b are not the same;
5. Using the point M_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_b and N are not the same;
6. Using the circle $k(N, M_a)$, the line b , the point N and the point M_b , construct a point H_b (rule W05); % NDG: line b and circle $k(N, M_a)$ intersect % DET: points M_b and H_b must be different;
7. Using the circle $k(N, M_a)$, the line $m(H_aH_c)$, the point N and the point M_b , construct a point E_b (rule W05a); % NDG: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_b and E_b must be different;
8. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;
9. Using the circle $k(M_b, C)$ and the circle $k(N, M_a)$, construct a point H_a and a point H_c (rule W07); % NDG: circles $k(M_b, C)$ and $k(N, M_a)$ intersect % DET: circles $k(M_b, C)$ and $k(N, M_a)$ are not the same;

10. Using the point C and the point H_a , construct a line a (rule W02); % DET: points C and H_a are not the same;
11. Using the line h_b and the line a , construct a point B (rule W03); % NDG: lines h_b and a are not parallel % DET: lines h_b and a are not the same.

Non-degenerate conditions: lines h_b and a are not parallel; circles $k(M_b, C)$ and $k(N, M_a)$ intersect; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; line b and circle $k(N, M_a)$ intersect; points M_b and N are not the same; points C and M_b are not the same.

Determination conditions: lines h_b and a are not the same; points C and H_a are not the same; circles $k(M_b, C)$ and $k(N, M_a)$ are not the same; points E_b and H_b are not the same; points M_b and E_b must be different; points M_b and H_b must be different; points M_b and N are not the same; points C and M_b are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07]

Lemmas used: [D22,D29,D3,D32,D5,D6,D9,GD01,GD02,GL01,GL03,GL09,L17,L19,L20,L21,L23,L44,L45,L50,

Solving time: 4.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point C 110 40
point M_{b} 95 67.5
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_l C
cmark_lt M_{b}
cmark_r N
color 0 0 0
fontsize 8
```

```
% Constructing a point A such that CA/CM_{b}=2
towards A C M_{b} 2
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0
```

```
% DET: points C and M_{b} are not the same
% Constructing a line b which passes through point C and point M_{b}
line b C M_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```

% DET: points  $M_{\{b\}}$  and  $N$  are not the same
% Constructing a line  $m(H_{\{a\}}H_{\{c\}})$  which passes through point  $M_{\{b\}}$  and point  $N$ 
line m( $H_{\{a\}}H_{\{c\}}$ )  $M_{\{b\}}$  N

color 200 200 200
drawline m( $H_{\{a\}}H_{\{c\}}$ )
color 0 0 0

% NDG: points  $C$  and  $M_{\{b\}}$  are not the same
% Constructing a circle  $k(M_{\{b\}}, C)$  whose center is at point  $M_{\{b\}}$  and which passes through point  $C$ 
circle k( $M_{\{b\}}$ ,  $C$ )  $M_{\{b\}}$  C

color 200 200 200
drawcircle k( $M_{\{b\}}$ ,  $C$ )
color 0 0 0

% NDG: points  $M_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $M_{\{b\}}$ 
circle k( $N, M_{\{a\}}$ )  $N$   $M_{\{b\}}$ 

color 200 200 200
drawcircle k( $N, M_{\{a\}}$ )
color 0 0 0

% NDG: line  $b$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{b\}}$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G189339\}}$  which is a foot of the point  $N$  on the line  $b$ 
foot  $P_{\{\backslash\_G189339\}}$   $N$   $b$ 
cmark_r  $P_{\{\backslash\_G189339\}}$ 
color 200 200 200
drawline  $N$   $P_{\{\backslash\_G189339\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $M_{\{b\}}$  in the symmetry to point/line  $P_{\{\backslash\_G189339\}}$ 
sim  $H_{\{b\}}$   $P_{\{\backslash\_G189339\}}$   $M_{\{b\}}$ 
cmark_l  $H_{\{b\}}$ 

% NDG: line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{b\}}$  and  $E_{\{b\}}$  must be
different
% Constructing a point  $E_{\{b\}}$  which is an image of the point  $M_{\{b\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{b\}}$   $N$   $M_{\{b\}}$ 
cmark_r  $E_{\{b\}}$ 

% DET: points  $E_{\{b\}}$  and  $H_{\{b\}}$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $E_{\{b\}}$  and point  $H_{\{b\}}$ 
line  $h_{\{b\}}$   $E_{\{b\}}$   $H_{\{b\}}$ 

```

```

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: circles  $k(M_{\{b\}}, C)$  and  $k(N, M_{\{a\}})$  intersect% DET: circles  $k(M_{\{b\}}, C)$  and  $k(N, M_{\{a\}})$  are not
the same
% Constructing points  $H_{\{a\}}$  and  $H_{\{c\}}$  which are in intersection of  $k(M_{\{b\}}, C)$  and  $k(N, M_{\{a\}})$ 
intersec2 H_{a} H_{c} k(M_{\{b\}}, C) k(N, M_{\{a\}})
cmark_r H_{a}
cmark_rt H_{c}

% DET: points  $C$  and  $H_{\{a\}}$  are not the same
% Constructing a line  $a$  which passes through point  $C$  and point  $H_{\{a\}}$ 
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $h_{\{b\}}$  and  $a$  are not parallel% DET: lines  $h_{\{b\}}$  and  $a$  are not the same
% Constructing a point  $B$  which belongs to line  $h_{\{b\}}$  and line  $a$ 
intersec B h_{b} a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{b\}}$  and  $a$  are not parallel; circles  $k(M_{\{b\}}, C)$  and  $k(N, M_{\{a\}})$ 
intersect; line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect; line  $b$  and circle  $k(N, M_{\{a\}})$ 
intersect; points  $M_{\{b\}}$  and  $N$  are not the same; points  $C$  and  $M_{\{b\}}$  are not the same
% Determination conditions: lines  $h_{\{b\}}$  and  $a$  are not the same; points  $C$  and  $H_{\{a\}}$  are not the same
; circles  $k(M_{\{b\}}, C)$  and  $k(N, M_{\{a\}})$  are not the same; points  $E_{\{b\}}$  and  $H_{\{b\}}$  are not the same;
points  $M_{\{b\}}$  and  $E_{\{b\}}$  must be different; points  $M_{\{b\}}$  and  $H_{\{b\}}$  must be different; points  $M_{\{b\}}$ 
and  $N$  are not the same; points  $C$  and  $M_{\{b\}}$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

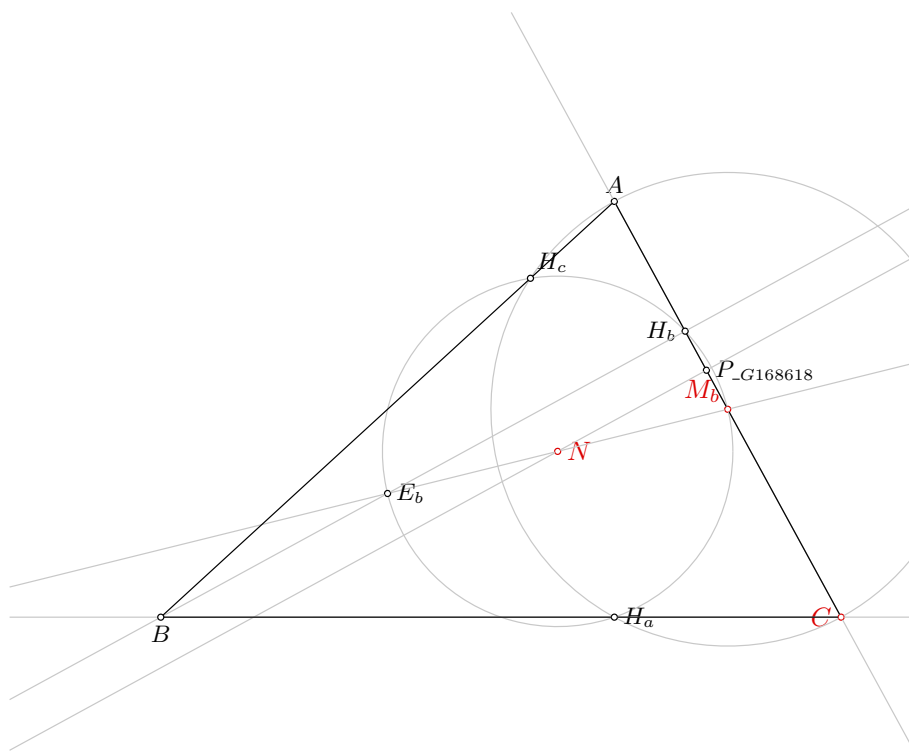


Figure 1: Illustration of the problem 0741

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.027 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.032 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $C = C$

NDG conditions are:

$S_{E_bCH_a} \neq S_{H_bCH_a}$ i.e., lines E_bH_b and CH_a are not parallel (construction based assumption)

$S_{\neg M_aBC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_bAC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2BF_{\neg h_b}^3}$ i.e., lines $AF_{\neg h_a}^2$ and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

$S_{\neg M_a\neg M_bF_{\neg m_b}^1} \neq S_{F_{\neg m_a}^0\neg M_bF_{\neg m_b}^1}$ i.e., lines $\neg M_aF_{\neg m_a}^0$ and $\neg M_bF_{\neg m_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $M_b = \neg M_b$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $M_b = \neg M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 6 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.3.3 Proving $N=_N$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 59 terms.

Time Complexity: Time spent by the prover is 0.080 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.2 Proving $M_b=_M M_b$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 742

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 742: Given a point C , a point M_c and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point C and the point M_c , construct a point G (rule W01); ;
2. Using the point N and the point G , construct a point O (rule W01); ;
3. Using the point N and the point G , construct a point H (rule W01); ;
4. Using the point C and the point H , construct a point E_c (rule W01); ;
5. Using the point C and the point H , construct a line h_c (rule W02); % DET: points C and H are not the same;
6. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
7. Using the point M_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_c and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
9. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; line h_c and circle $k(N, M_a)$ intersect; points M_c and N are not the same; points C and O are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points C and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L15,L16,L18,L21,L24,L3,L57]

Solving time: 8.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point M_{c} 50 67.5
point N 72.5 61.93

color 220 0 0
fontsize 9

cmark_l C
cmark_lt M_{c}
cmark_r N
color 0 0 0
fontsize 8

% Constructing a line L_{\_G224459} which passes through point C and point M_{c}
line L_{\_G224459} C M_{c}

color 200 200 200
drawline L_{\_G224459}
color 0 0 0

% Constructing a point P_{\_G224560} with coordinates (0,0)
point P_{\_G224560} 0 0
cmark_r P_{\_G224560}

% Constructing a point P_{\_G224484} such that CP_{\_G224484}/CP_{\_G224560}=2
towards P_{\_G224484} C P_{\_G224560} 2
cmark_r P_{\_G224484}
color 200 200 200
drawsegment C P_{\_G224484}
color 0 0 0

% Constructing a point P_{\_G224529} such that CP_{\_G224529}/CP_{\_G224560}=3
towards P_{\_G224529} C P_{\_G224560} 3
cmark_r P_{\_G224529}
color 200 200 200
drawsegment C P_{\_G224529}
color 0 0 0

% Constructing a line L_{\_G224490} which passes through point M_{c} and point P_{\_G224529}
line L_{\_G224490} M_{c} P_{\_G224529}
```

```

color 200 200 200
drawline L_{\_G224490}
color 0 0 0

% Constructing a line L_{\_G224453} which contains the point P_{\_G224484} and is parallel to the
line L_{\_G224490}
parallel L_{\_G224453} P_{\_G224484} L_{\_G224490}

color 200 200 200
drawline L_{\_G224453}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G224453} and line L_{\_G224459}
intersec G L_{\_G224453} L_{\_G224459}
cmark_t G

% Constructing a point O such that NO/NG=3
towards O N G 3
cmark_t O
color 200 200 200
drawsegment N O
color 0 0 0

% Constructing a point H such that NH/NG=-3
towards H N G -3
cmark_rt H
color 200 200 200
drawsegment G H
color 0 0 0

% Constructing a point E_{c} such that CE_{c}/CH=0.5
towards E_{c} C H 0.5
cmark_r E_{c}
color 200 200 200
drawsegment C H
color 0 0 0

% DET: points C and H are not the same
% Constructing a line h_{c} which passes through point C and point H
line h_{c} C H

color 200 200 200
drawline h_{c}
color 0 0 0

```

```

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points M_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{c}
circle k(N,M_{a}) N M_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{c} and circle k(N,M_{a}) intersect% DET: points E_{c} and H_{c} must be different
% Constructing a point P_{\_G225595} which is a foot of the point N on the line h_{c}
foot P_{\_G225595} N h_{c}
cmark_r P_{\_G225595}
color 200 200 200
drawline N P_{\_G225595}
color 0 0 0

% Constructing a point H_{c} which is an image of the point E_{c} in the symmetry to point/line P
\_G225595}
sim H_{c} P_{\_G225595} E_{c}
cmark_rt H_{c}

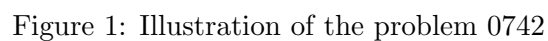
% DET: points H_{c} and M_{c} are not the same
% Constructing a line c which passes through point H_{c} and point M_{c}
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```



```
% Non-degenerate conditions: line c and circle k(O,C) intersect; line h_{c} and circle k(N,M_{a})
% intersect; points M_{c} and N are not the same; points C and O are not the same
% Determination conditions: points H_{c} and M_{c} are not the same; points E_{c} and H_{c} must be
% different; points C and H are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.037 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $M_c = \neg M_c$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $C = C$

NDG conditions are:

$S_{P_{G203377}CM_c} \neq S_{P_{L_{G203346}}^0 CM_c}$ i.e., lines $P_{G203377}P_{L_{G203346}}^0$ and CM_c are not parallel (construction based assumption)

$S_{\neg M_a BC} \neq 0$ i.e., points $\neg M_a, B$ and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_b AC} \neq 0$ i.e., points $\neg M_b, A$ and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^4} \neq S_{F_{\neg h_a}^3 BF_{\neg h_b}^4}$ i.e., lines $AF_{\neg h_a}^3$ and $BF_{\neg h_b}^4$ are not parallel (construction based assumption)

$S_{\neg M_a \neg M_b F_{\neg m_b}^2} \neq S_{F_{\neg m_a}^1 \neg M_b F_{\neg m_b}^2}$ i.e., lines $\neg M_a F_{\neg m_a}^1$ and $\neg M_b F_{\neg m_b}^2$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $M_c = \neg M_c$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C = C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.3.2 Proving $M_c = \neg M_c$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $M_c=_M_c$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 743

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 743: Given a point C , a point N and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point N and the point O , construct a point G (rule W01); ;
2. Using the point C and the point G , construct a point M_c (rule W01); ;
3. Using the point N and the point O , construct a point H (rule W01); ;
4. Using the point C and the point H , construct a point E_c (rule W01); ;
5. Using the point C and the point H , construct a line h_c (rule W02); % DET: points C and H are not the same;
6. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
7. Using the point M_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_c and N are not the same;
8. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
9. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; line h_c and circle $k(N, M_a)$ intersect; points M_c and N are not the same; points C and O are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points C and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D31,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L15,L18,L21,L24,L3,L57]

Solving time: 8.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point C 110 40
point N 72.5 61.93
point O 65 51.14

color 220 0 0
fontsize 9

cmark_l C
cmark_r N
cmark_t O
color 0 0 0
fontsize 8

% Constructing a line L_{\_G20203} which passes through point N and point O
line L_{\_G20203} N O

color 200 200 200
drawline L_{\_G20203}
color 0 0 0

% Constructing a point P_{\_G20304} with coordinates (0,0)
point P_{\_G20304} 0 0
cmark_r P_{\_G20304}

% Constructing a point P_{\_G20228} such that NP_{\_G20228}/NP_{\_G20304}=1
towards P_{\_G20228} N P_{\_G20304} 1
cmark_r P_{\_G20228}
color 200 200 200
drawsegment N P_{\_G20228}
color 0 0 0

% Constructing a point P_{\_G20273} such that NP_{\_G20273}/NP_{\_G20304}=3
towards P_{\_G20273} N P_{\_G20304} 3
cmark_r P_{\_G20273}
color 200 200 200
drawsegment N P_{\_G20273}
color 0 0 0

% Constructing a line L_{\_G20234} which passes through point O and point P_{\_G20273}
line L_{\_G20234} O P_{\_G20273}
```

```

color 200 200 200
drawline L_{\_G20234}
color 0 0 0

% Constructing a line L_{\_G20197} which contains the point P_{\_G20228} and is parallel to the
  line L_{\_G20234}
parallel L_{\_G20197} P_{\_G20228} L_{\_G20234}

color 200 200 200
drawline L_{\_G20197}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G20197} and line L_{\_G20203}
intersec G L_{\_G20197} L_{\_G20203}
cmark_t G

% Constructing a point M_{c} such that CM_{c}/CG=1.5
towards M_{c} C G 1.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% Constructing a point H such that NH/NO=-1
towards H N O -1
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0

% Constructing a point E_{c} such that CE_{c}/CH=0.5
towards E_{c} C H 0.5
cmark_r E_{c}
color 200 200 200
drawsegment C H
color 0 0 0

% DET: points C and H are not the same
% Constructing a line h_{c} which passes through point C and point H
line h_{c} C H

color 200 200 200
drawline h_{c}
color 0 0 0

```

```

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points M_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{c}
circle k(N,M_{a}) N M_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{c} and circle k(N,M_{a}) intersect% DET: points E_{c} and H_{c} must be different
% Constructing a point P_{\_G21348} which is a foot of the point N on the line h_{c}
foot P_{\_G21348} N h_{c}
cmark_r P_{\_G21348}
color 200 200 200
drawline N P_{\_G21348}
color 0 0 0

% Constructing a point H_{c} which is an image of the point E_{c} in the symmetry to point/line P_{\_G21348}
sim H_{c} P_{\_G21348} E_{c}
cmark_rt H_{c}

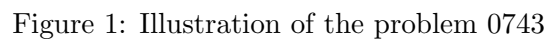
% DET: points H_{c} and M_{c} are not the same
% Constructing a line c which passes through point H_{c} and point M_{c}
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```



```
% Non-degenerate conditions: line c and circle k(O,C) intersect; line h_{c} and circle k(N,M_{a})
% intersect; points M_{c} and N are not the same; points C and O are not the same
% Determination conditions: points H_{c} and M_{c} are not the same; points E_{c} and H_{c} must be
% different; points C and H are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $C=C$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 0 terms.

Time Complexity: Time spent by the prover is 0.031 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $O=_O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $C=C$

NDG conditions are:

$S_{P_{G239032}NO} \neq S_{P_{L_{G239001}}^0 NO}$ i.e., lines $P_{G239032}P_{L_{G239001}}^0$ and NO are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points M_a, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points M_b, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aM_bF_{M_b}^2} \neq S_{F_{M_a}^1M_bF_{M_b}^2}$ i.e., lines $M_aF_{M_a}^1$ and $M_bF_{M_b}^2$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{M_b}^4} \neq S_{F_{M_a}^3BF_{M_b}^4}$ i.e., lines $AF_{M_a}^3$ and $BF_{M_b}^4$ are not parallel (construction based assumption)

Total number of proof steps: 1

Time spent by the prover: 0.000 seconds

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $O=_O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $O=_O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $C=C$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $O=_O$

Proving failed

Problem 744

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 744: Given a point C , a point N and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 745

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 745: Given a point C , a point N and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 746

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 746: Given a point C , a point N and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 747

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 747: Given a point E_a , a point E_b and a point E_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point E_b , construct a line $m(E_aE_b)$ (rule W14); % DET: points E_a and E_b are not the same;
2. Using the point E_a and the point E_c , construct a line $m(E_aE_c)$ (rule W14); % DET: points E_a and E_c are not the same;
3. Using the line $m(E_aE_c)$ and the line $m(E_aE_b)$, construct a point N (rule W03); % NDG: lines $m(E_aE_c)$ and $m(E_aE_b)$ are not parallel % DET: lines $m(E_aE_c)$ and $m(E_aE_b)$ are not the same;
4. Using the point E_b and the point N , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and N are not the same;
5. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
6. Using the circle $k(N, M_a)$, the line $m(H_aH_c)$, the point N and the point E_b , construct a point M_b (rule W05a); % NDG: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect % DET: points E_b and M_b must be different;
7. Using the point E_c and the point M_b , construct a line $m(CH_a)$ (rule W02); % DET: points E_c and M_b are not the same;
8. Using the point E_a and the line $m(CH_a)$, construct a line h_a (rule W16); ;
9. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;

10. Using the point H_a and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_a and E_b are not the same;
11. Using the circle $k(E_b, B)$, the line h_a , the point E_b and the point H_a , construct a point H (rule W05); % NDG: line h_a and circle $k(E_b, B)$ intersect % DET: points H_a and H must be different;
12. Using the point H and the point E_a , construct a point A (rule W01); ;
13. Using the point E_b and the point H , construct a point B (rule W01); ;
14. Using the point E_c and the point H , construct a point C (rule W01); .

Non-degenerate conditions: line h_a and circle $k(E_b, B)$ intersect; points H_a and E_b are not the same; line h_a and circle $k(N, M_a)$ intersect; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; points E_a and N are not the same; lines $m(E_a E_c)$ and $m(E_a E_b)$ are not parallel.

Determination conditions: points H_a and H must be different; points E_a and H_a must be different; points E_c and M_b are not the same; points E_b and M_b must be different; points E_b and N are not the same; lines $m(E_a E_c)$ and $m(E_a E_b)$ are not the same; points E_a and E_c are not the same; points E_a and E_b are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W14,W16]

Lemmas used: [D28,D29,D3,D30,D32,D5,D8,GD02,GL01,GL03,GL04,GL09,L17,L19,L21,L22,L23,L24,L44,L45]

Solving time: 24.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
point E_{b} 50 56.36
point E_{c} 95 56.36
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{a}
cmark_r E_{b}
cmark_r E_{c}
color 0 0 0
fontsize 8
```

```
% DET: points E_{a} and E_{b} are not the same
% Constructing bisector m(E_{a}E_{b}) of the segment E_{a}E_{b}
med m(E_{a}E_{b}) E_{a} E_{b}
```

```
color 200 200 200
drawline m(E_{a}E_{b})
color 0 0 0
```

```
color 200 200 200
drawsegment E_{a} E_{b}
color 0 0 0
```

```

% DET: points  $E_{\{a\}}$  and  $E_{\{c\}}$  are not the same
% Constructing bisector  $m(E_{\{a\}}E_{\{c\}})$  of the segment  $E_{\{a\}}E_{\{c\}}$ 
med m( $E_{\{a\}}E_{\{c\}}$ )  $E_{\{a\}}$   $E_{\{c\}}$ 

color 200 200 200
drawline m( $E_{\{a\}}E_{\{c\}}$ )
color 0 0 0

color 200 200 200
drawsegment  $E_{\{a\}}$   $E_{\{c\}}$ 
color 0 0 0

% NDG: lines  $m(E_{\{a\}}E_{\{c\}})$  and  $m(E_{\{a\}}E_{\{b\}})$  are not parallel% DET: lines  $m(E_{\{a\}}E_{\{c\}})$  and  $m(E_{\{a\}}$ 
 $E_{\{b\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{a\}}E_{\{c\}})$  and line  $m(E_{\{a\}}E_{\{b\}})$ 
intersec N m( $E_{\{a\}}E_{\{c\}}$ ) m( $E_{\{a\}}E_{\{b\}}$ )
cmark_r N

% DET: points  $E_{\{b\}}$  and  $N$  are not the same
% Constructing a line  $m(H_{\{a\}}H_{\{c\}})$  which passes through point  $E_{\{b\}}$  and point  $N$ 
line m( $H_{\{a\}}H_{\{c\}}$ )  $E_{\{b\}}$  N

color 200 200 200
drawline m( $H_{\{a\}}H_{\{c\}}$ )
color 0 0 0

% NDG: points  $E_{\{a\}}$  and  $N$  are not the same
% Constructing a circle  $k(N,M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{a\}}$ 
circle k( $N,M_{\{a\}}$ ) N  $E_{\{a\}}$ 

color 200 200 200
drawcircle k( $N,M_{\{a\}}$ )
color 0 0 0

% NDG: line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $E_{\{b\}}$  and  $M_{\{b\}}$  must be
different
% Constructing a point  $M_{\{b\}}$  which is an image of the point  $E_{\{b\}}$  in the symmetry to point/line  $N$ 
sim  $M_{\{b\}}$  N  $E_{\{b\}}$ 
cmark_lt  $M_{\{b\}}$ 

% DET: points  $E_{\{c\}}$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $m(CH_{\{a\}})$  which passes through point  $E_{\{c\}}$  and point  $M_{\{b\}}$ 
line m( $CH_{\{a\}}$ )  $E_{\{c\}}$   $M_{\{b\}}$ 

color 200 200 200
drawline m( $CH_{\{a\}}$ )
color 0 0 0

```

```

% Constructing a line h_{a} which contains the point E_{a} and is parallel to the line m(CH_{a})
parallel h_{a} E_{a} m(CH_{a})

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G77888} which is a foot of the point N on the line h_{a}
foot P_{\_G77888} N h_{a}
cmark_r P_{\_G77888}
color 200 200 200
drawline N P_{\_G77888}
color 0 0 0

% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
\_G77888}
sim H_{a} P_{\_G77888} E_{a}
cmark_r H_{a}

% NDG: points H_{a} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H
_{a}
circle k(E_{b},B) E_{b} H_{a}

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line h_{a} and circle k(E_{b},B) intersect% DET: points H_{a} and H must be different
% Constructing a point P_{\_G78192} which is a foot of the point E_{b} on the line h_{a}
foot P_{\_G78192} E_{b} h_{a}
cmark_r P_{\_G78192}
color 200 200 200
drawline E_{b} P_{\_G78192}
color 0 0 0

% Constructing a point H which is an image of the point H_{a} in the symmetry to point/line P_{\_G
78192}
sim H P_{\_G78192} H_{a}
cmark_rt H

% Constructing a point A such that HA/HE_{a}=2
towards A H E_{a} 2
cmark_t A

```



```

color 200 200 200
drawsegment H A
color 0 0 0

```

```

% Constructing a point B such that  $E_{\{b\}}B/E_{\{b\}}H=-1$ 
towards B  $E_{\{b\}}$  H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

```

```

% Constructing a point C such that  $E_{\{c\}}C/E_{\{c\}}H=-1$ 
towards C  $E_{\{c\}}$  H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

% Non-degenerate conditions: line $h_{\{a\}}$ and circle $k(E_{\{b\}}, B)$ intersect; points $H_{\{a\}}$ and $E_{\{b\}}$ are not the same; line $h_{\{a\}}$ and circle $k(N, M_{\{a\}})$ intersect; line $m(H_{\{a\}}H_{\{c\}})$ and circle $k(N, M_{\{a\}})$ intersect; points $E_{\{a\}}$ and N are not the same; lines $m(E_{\{a\}}E_{\{c\}})$ and $m(E_{\{a\}}E_{\{b\}})$ are not parallel

% Determination conditions: points $H_{\{a\}}$ and H must be different; points $E_{\{a\}}$ and $H_{\{a\}}$ must be different; points $E_{\{c\}}$ and $M_{\{b\}}$ are not the same; points $E_{\{b\}}$ and $M_{\{b\}}$ must be different; points $E_{\{b\}}$ and N are not the same; lines $m(E_{\{a\}}E_{\{c\}})$ and $m(E_{\{a\}}E_{\{b\}})$ are not the same; points $E_{\{a\}}$ and $E_{\{c\}}$ are not the same; points $E_{\{a\}}$ and $E_{\{b\}}$ are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $E_b = \neg E_b$

Proving failed

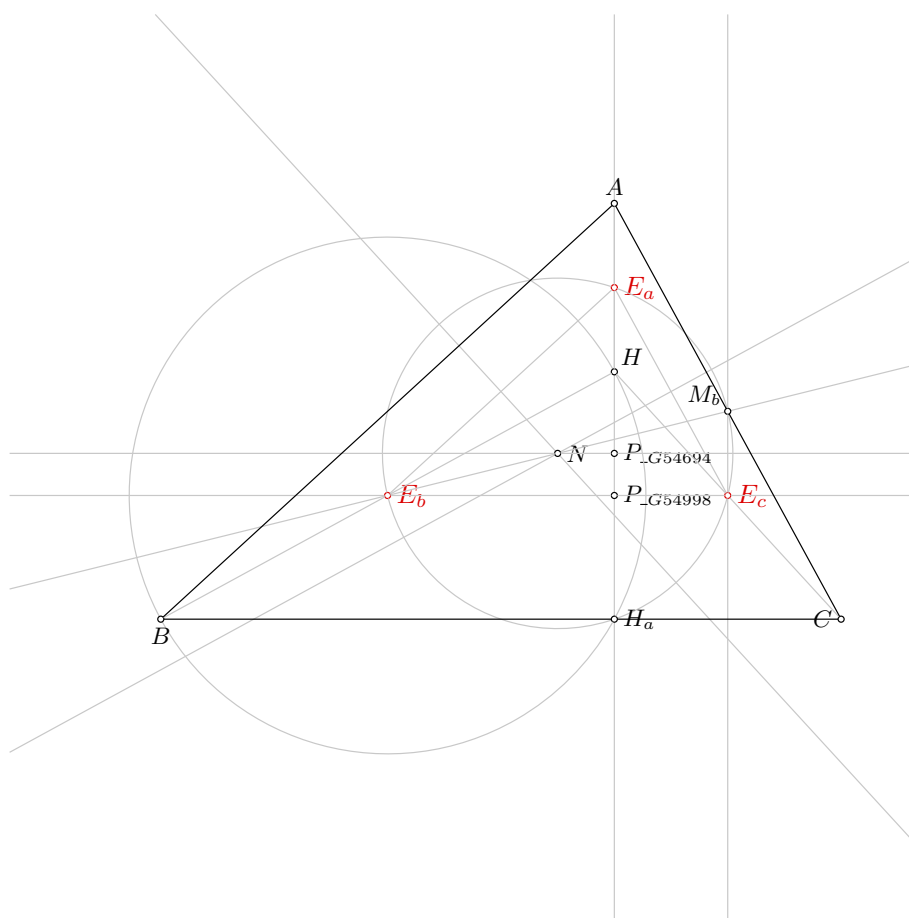


Figure 1: Illustration of the problem 0747

4.1.3 Proving $E_c = \neg E_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $E_b = \neg E_b$

Proving failed

4.2.3 Proving $E_c = \neg E_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $E_b = \neg E_b$

Proving failed

4.3.3 Proving $E_c = \neg E_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $E_b = \neg E_b$

Proving failed

4.4.3 Proving $E_c = \neg E_c$

Proving failed

Problem 748

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 748: Given a point E_a , a point E_b and a point G , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 749

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 749: Given a point E_a , a point E_b and a point H , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H , construct a point A (rule W01); ;
2. Using the point E_b and the point H , construct a point B (rule W01); ;
3. Using the point E_b and the point H , construct a line h_b (rule W02); % DET: points E_b and H are not the same;
4. Using the point A and the point B , construct a line c (rule W02); % DET: points A and B are not the same;
5. Using the point H and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H and E_a are not the same;
6. Using the circle $k(E_a, A)$, the line h_b , the point E_a and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_a, A)$ intersect % DET: points H and H_b must be different;
7. Using the point H_b and the point A , construct a line b (rule W02); % DET: points H_b and A are not the same;
8. Using the circle $k(E_a, A)$, the line c , the point E_a and the point A , construct a point H_c (rule W05); % NDG: line c and circle $k(E_a, A)$ intersect % DET: points A and H_c must be different;
9. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
10. Using the line b and the line h_c , construct a point C (rule W03); % NDG: lines b and h_c are not parallel % DET: lines b and h_c are not the same.

Non-degenerate conditions: lines b and h_c are not parallel; line c and circle $k(E_a, A)$ intersect; line h_b and circle $k(E_a, A)$ intersect; points H and E_a are not the same.

Determination conditions: lines b and h_c are not the same; points H_c and H are not the same; points A and H_c must be different; points H_b and A are not the same; points H and H_b must be different; points A and B are not the same; points E_b and H are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D28,D29,D3,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L46,L47,L48]

Solving time: 12.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point E_{b} 50 56.36
```

```
point H 80 72.73
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_r E_{b}
```

```
cmark_rt H
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point A such that E_{a}A/E_{a}H=-1
```

```
towards A E_{a} H -1
```

```
cmark_t A
```

```
color 200 200 200
```

```
drawsegment H A
```

```
color 0 0 0
```

```
% Constructing a point B such that E_{b}B/E_{b}H=-1
```

```
towards B E_{b} H -1
```

```
cmark_b B
```

```
color 200 200 200
```

```
drawsegment H B
```

```
color 0 0 0
```

```
% DET: points E_{b} and H are not the same
```

```
% Constructing a line h_{b} which passes through point E_{b} and point H
```

```
line h_{b} E_{b} H
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```

% DET: points A and B are not the same
% Constructing a line c which passes through point A and point B
line c A B

color 200 200 200
drawline c
color 0 0 0

% NDG: points H and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H
circle k(E_{a},A) E_{a} H

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: line h_{b} and circle k(E_{a},A) intersect% DET: points H and H_{b} must be different
% Constructing a point P_{\_G112031} which is a foot of the point E_{a} on the line h_{b}
foot P_{\_G112031} E_{a} h_{b}
cmark_r P_{\_G112031}
color 200 200 200
drawline E_{a} P_{\_G112031}
color 0 0 0

% Constructing a point H_{b} which is an image of the point H in the symmetry to point/line P_{\_G
112031}
sim H_{b} P_{\_G112031} H
cmark_l H_{b}

% DET: points H_{b} and A are not the same
% Constructing a line b which passes through point H_{b} and point A
line b H_{b} A

color 200 200 200
drawline b
color 0 0 0

% NDG: line c and circle k(E_{a},A) intersect% DET: points A and H_{c} must be different
% Constructing a point P_{\_G112269} which is a foot of the point E_{a} on the line c
foot P_{\_G112269} E_{a} c
cmark_r P_{\_G112269}
color 200 200 200
drawline E_{a} P_{\_G112269}
color 0 0 0

% Constructing a point H_{c} which is an image of the point A in the symmetry to point/line P_{\_G
112269}
sim H_{c} P_{\_G112269} A
cmark_rt H_{c}

```

```

% DET: points  $H_{\{c\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $H_{\{c\}}$  and point  $H$ 
line  $h_{\{c\}}$   $H_{\{c\}}$   $H$ 

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: lines  $b$  and  $h_{\{c\}}$  are not parallel% DET: lines  $b$  and  $h_{\{c\}}$  are not the same
% Constructing a point  $C$  which belongs to line  $b$  and line  $h_{\{c\}}$ 
intersec C b  $h_{\{c\}}$ 
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $b$  and  $h_{\{c\}}$  are not parallel; line  $c$  and circle  $k(E_{\{a\}}, A)$ 
% intersect; line  $h_{\{b\}}$  and circle  $k(E_{\{a\}}, A)$  intersect; points  $H$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $b$  and  $h_{\{c\}}$  are not the same; points  $H_{\{c\}}$  and  $H$  are not the same
% ; points  $A$  and  $H_{\{c\}}$  must be different; points  $H_{\{b\}}$  and  $A$  are not the same; points  $H$  and  $H_{\{b\}}$ 
% must be different; points  $A$  and  $B$  are not the same; points  $E_{\{b\}}$  and  $H$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $E_b = \neg E_b$

Proving failed

4.1.3 Proving $H = \neg H$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

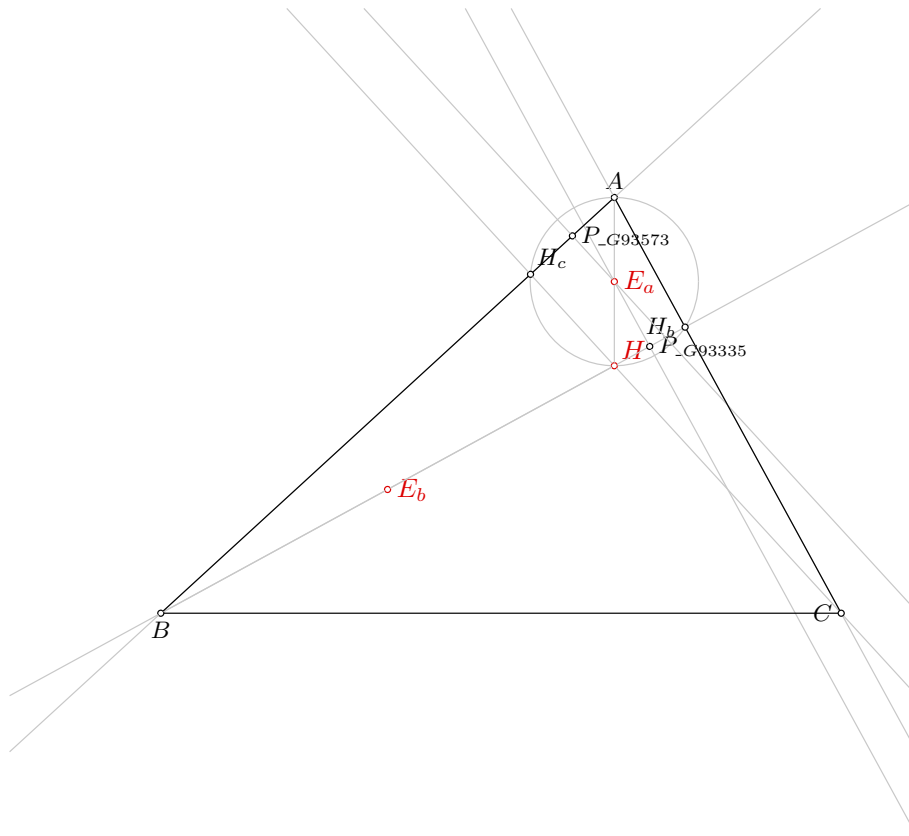


Figure 1: Illustration of the problem 0749

4.2.2 Proving $E_b = \neg E_b$

Proving failed

4.2.3 Proving $H = \neg H$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $E_b = \neg E_b$

Proving failed

4.3.3 Proving $H = \neg H$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 1916 terms.

Time Complexity: Time spent by the prover is 1.640 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $E_b = \neg E_b$

Proving failed

4.4.3 Proving $H = \neg H$

Proving failed

Problem 750

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 750: Given a point E_a , a point E_b and a point H_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;
2. Using the point H_a and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_a and E_b are not the same;
3. Using the circle $k(E_b, B)$, the line h_a , the point E_b and the point H_a , construct a point H (rule W05); % NDG: line h_a and circle $k(E_b, B)$ intersect % DET: points H_a and H must be different;
4. Using the point H and the point E_a , construct a point A (rule W01); ;
5. Using the point E_b and the point H , construct a point B (rule W01); ;
6. Using the point H_a and the point B , construct a line a (rule W02); % DET: points H_a and B are not the same;
7. Using the point A and the point B , construct a line c (rule W02); % DET: points A and B are not the same;
8. Using the circle $k(E_b, B)$, the line c , the point E_b and the point B , construct a point H_c (rule W05); % NDG: line c and circle $k(E_b, B)$ intersect % DET: points B and H_c must be different;
9. Using the point H and the point H_c , construct a line h_c (rule W02); % DET: points H and H_c are not the same;
10. Using the line a and the line h_c , construct a point C (rule W03); % NDG: lines a and h_c are not parallel % DET: lines a and h_c are not the same.

Non-degenerate conditions: lines a and h_c are not parallel; line c and circle $k(E_b, B)$ intersect; line h_a and circle $k(E_b, B)$ intersect; points H_a and E_b are not the same.

Determination conditions: lines a and h_c are not the same; points H and H_c are not the same; points B and H_c must be different; points A and B are not the same; points H_a and B are not the same; points H_a and H must be different; points E_a and H_a are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D28,D29,D3,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L49,L50,L51]

Solving time: 9.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{a} 80 83.86
point E_{b} 50 56.36
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_r E_{a}
cmark_r E_{b}
cmark_r H_{a}
color 0 0 0
fontsize 8

% DET: points E_{a} and H_{a} are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H_{a}
line h_{a} E_{a} H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points H_{a} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H_{a}
circle k(E_{b},B) E_{b} H_{a}

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line h_{a} and circle k(E_{b},B) intersect% DET: points H_{a} and H must be different
% Constructing a point P_{\G139857} which is a foot of the point E_{b} on the line h_{a}
foot P_{\G139857} E_{b} h_{a}
cmark_r P_{\G139857}
color 200 200 200
drawline E_{b} P_{\G139857}
color 0 0 0
```

```

% Constructing a point H which is an image of the point H_{a} in the symmetry to point/line P_{\_G
139857}
sim H P_{\_G139857} H_{a}
cmark_rt H

% Constructing a point A such that HA/HE_{a}=2
towards A H E_{a} 2
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

% Constructing a point B such that E_{b}B/E_{b}H=-1
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% DET: points A and B are not the same
% Constructing a line c which passes through point A and point B
line c A B

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(E_{b},B) intersect% DET: points B and H_{c} must be different
% Constructing a point P_{\_G140294} which is a foot of the point E_{b} on the line c
foot P_{\_G140294} E_{b} c
cmark_r P_{\_G140294}
color 200 200 200
drawline E_{b} P_{\_G140294}
color 0 0 0

% Constructing a point H_{c} which is an image of the point B in the symmetry to point/line P_{\_G
140294}
sim H_{c} P_{\_G140294} B

```

```

cmark_rt H_{c}

% DET: points H and H_{c} are not the same
% Constructing a line h_{c} which passes through point H and point H_{c}
line h_{c} H H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines a and h_{c} are not parallel% DET: lines a and h_{c} are not the same
% Constructing a point C which belongs to line a and line h_{c}
intersec C a h_{c}
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and h_{c} are not parallel; line c and circle k(E_{b},B)
% intersect; line h_{a} and circle k(E_{b},B) intersect; points H_{a} and E_{b} are not the same
% Determination conditions: lines a and h_{c} are not the same; points H and H_{c} are not the same
% ; points B and H_{c} must be different; points A and B are not the same; points H_{a} and B are
% not the same; points H_{a} and H must be different; points E_{a} and H_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $E_b = \neg E_b$

Proving failed

4.1.3 Proving $H_a = \neg H_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

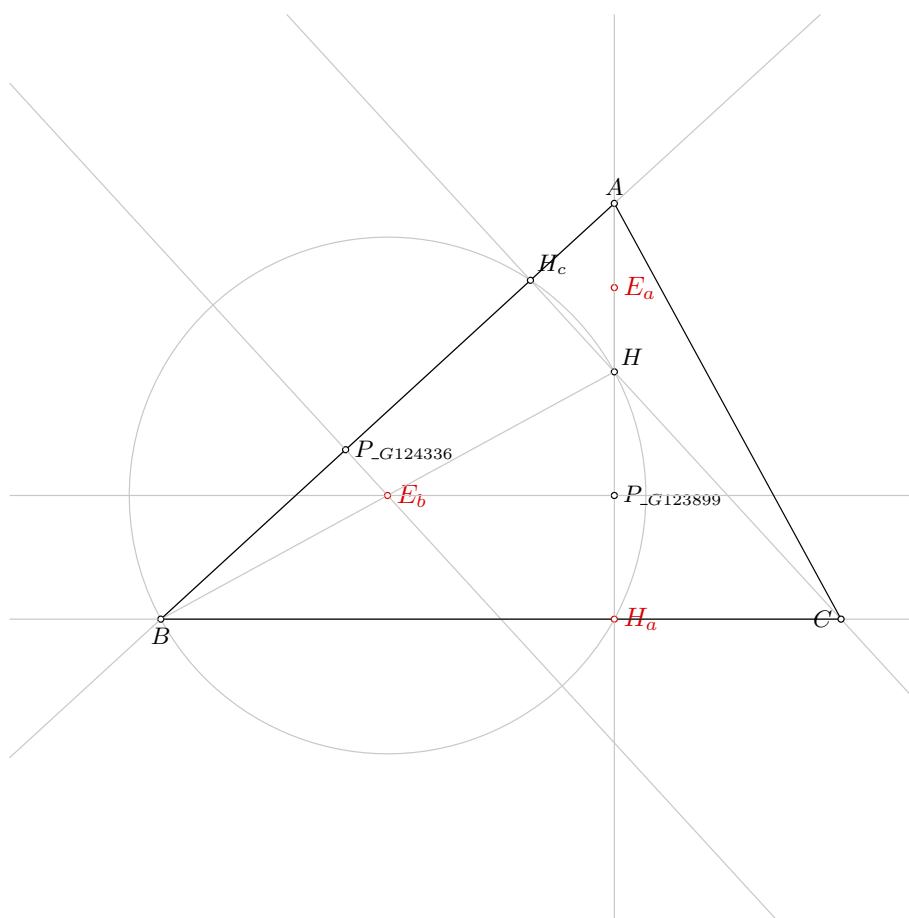


Figure 1: Illustration of the problem 0750

4.2.2 Proving $E_b = \neg E_b$

Proving failed

4.2.3 Proving $H_a = \neg H_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $E_b = \neg E_b$

Proving failed

4.3.3 Proving $H_a = \neg H_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $E_b = \neg E_b$

Proving failed

4.4.3 Proving $H_a = \neg H_a$

Proving failed

Problem 751

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 751: Given a point E_a , a point E_b and a point H_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;
2. Using the point H_b and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_b and E_a are not the same;
3. Using the circle $k(E_a, A)$, the line h_b , the point E_a and the point H_b , construct a point H (rule W05); % NDG: line h_b and circle $k(E_a, A)$ intersect % DET: points H_b and H must be different;
4. Using the point H and the point E_a , construct a point A (rule W01); ;
5. Using the point E_b and the point H , construct a point B (rule W01); ;
6. Using the point H_b and the point A , construct a line b (rule W02); % DET: points H_b and A are not the same;
7. Using the point A and the point B , construct a line c (rule W02); % DET: points A and B are not the same;
8. Using the circle $k(E_a, A)$, the line c , the point E_a and the point A , construct a point H_c (rule W05); % NDG: line c and circle $k(E_a, A)$ intersect % DET: points A and H_c must be different;
9. Using the point H and the point H_c , construct a line h_c (rule W02); % DET: points H and H_c are not the same;
10. Using the line b and the line h_c , construct a point C (rule W03); % NDG: lines b and h_c are not parallel % DET: lines b and h_c are not the same.

Non-degenerate conditions: lines b and h_c are not parallel; line c and circle $k(E_a, A)$ intersect; line h_b and circle $k(E_a, A)$ intersect; points H_b and E_a are not the same.

Determination conditions: lines b and h_c are not the same; points H and H_c are not the same; points A and H_c must be different; points A and B are not the same; points H_b and A are not the same; points H_b and H must be different; points E_b and H_b are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D28,D29,D3,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L46,L47,L48]

Solving time: 9.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
point E_{b} 50 56.36
point H_{b} 89.36 77.83
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{a}
cmark_r E_{b}
cmark_l H_{b}
color 0 0 0
fontsize 8
```

```
% DET: points E_{b} and H_{b} are not the same
% Constructing a line h_{b} which passes through point E_{b} and point H_{b}
line h_{b} E_{b} H_{b}
```

```
color 200 200 200
drawline h_{b}
color 0 0 0
```

```
% NDG: points H_{b} and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H_{b}
circle k(E_{a},A) E_{a} H_{b}
```

```
color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0
```

```
% NDG: line h_{b} and circle k(E_{a},A) intersect% DET: points H_{b} and H must be different
% Constructing a point P_{\_G167915} which is a foot of the point E_{a} on the line h_{b}
foot P_{\_G167915} E_{a} h_{b}
cmark_r P_{\_G167915}
color 200 200 200
drawline E_{a} P_{\_G167915}
color 0 0 0
```

```

% Constructing a point H which is an image of the point H_{b} in the symmetry to point/line P_{\_G
167915}
sim H P_{\_G167915} H_{b}
cmark_rt H

% Constructing a point A such that HA/HE_{a}=2
towards A H E_{a} 2
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

% Constructing a point B such that E_{b}B/E_{b}H=-1
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

% DET: points H_{b} and A are not the same
% Constructing a line b which passes through point H_{b} and point A
line b H_{b} A

color 200 200 200
drawline b
color 0 0 0

% DET: points A and B are not the same
% Constructing a line c which passes through point A and point B
line c A B

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(E_{a},A) intersect% DET: points A and H_{c} must be different
% Constructing a point P_{\_G168352} which is a foot of the point E_{a} on the line c
foot P_{\_G168352} E_{a} c
cmark_r P_{\_G168352}
color 200 200 200
drawline E_{a} P_{\_G168352}
color 0 0 0

% Constructing a point H_{c} which is an image of the point A in the symmetry to point/line P_{\_G
168352}
sim H_{c} P_{\_G168352} A

```

```

cmark_rt H_{c}

% DET: points H and H_{c} are not the same
% Constructing a line h_{c} which passes through point H and point H_{c}
line h_{c} H H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines b and h_{c} are not parallel% DET: lines b and h_{c} are not the same
% Constructing a point C which belongs to line b and line h_{c}
intersec C b h_{c}
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and h_{c} are not parallel; line c and circle k(E_{a},A)
% intersect; line h_{b} and circle k(E_{a},A) intersect; points H_{b} and E_{a} are not the same
% Determination conditions: lines b and h_{c} are not the same; points H and H_{c} are not the same
% ; points A and H_{c} must be different; points A and B are not the same; points H_{b} and A are
% not the same; points H_{b} and H must be different; points E_{b} and H_{b} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $E_b = \neg E_b$

Proving failed

4.1.3 Proving $H_b = \neg H_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

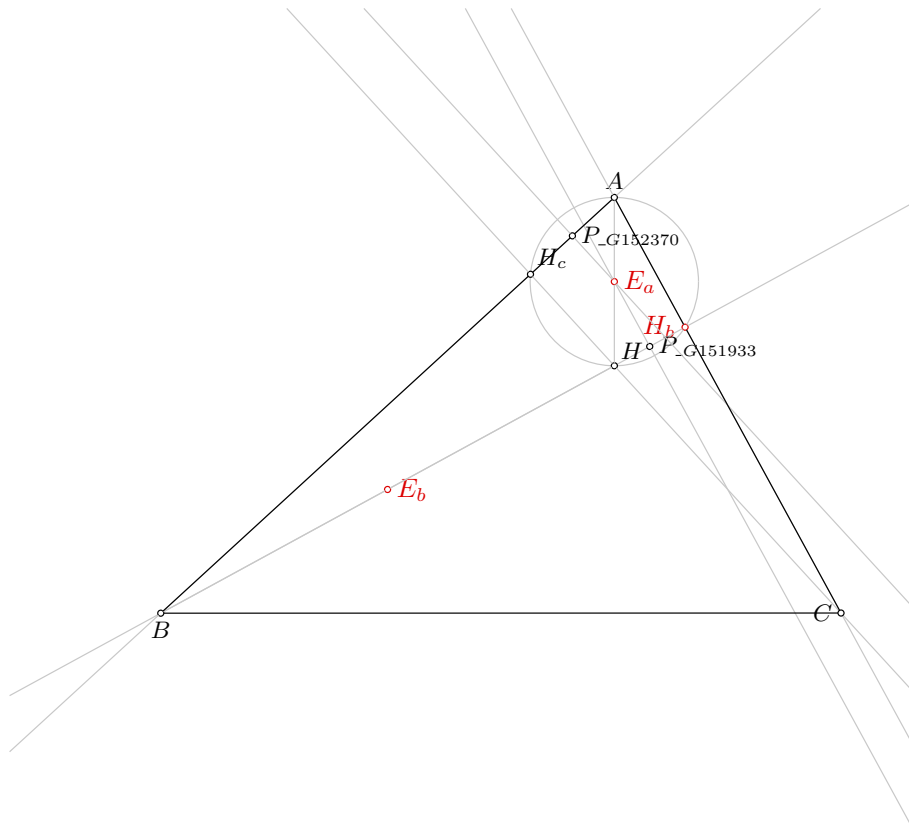


Figure 1: Illustration of the problem 0751

4.2.2 Proving $E_b = \neg E_b$

Proving failed

4.2.3 Proving $H_b = \neg H_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $E_b = \neg E_b$

Proving failed

4.3.3 Proving $H_b = \neg H_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $E_b = \neg E_b$

Proving failed

4.4.3 Proving $H_b = \neg H_b$

Proving failed

Problem 752

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 752: Given a point E_a , a point E_b and a point H_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H_c and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_c and E_a are not the same;
2. Using the point H_c and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_c and E_b are not the same;
3. Using the circle $k(E_a, A)$, the circle $k(E_b, B)$, the point H_c , the point E_a and the point E_b , construct a point H (rule W08); % NDG: circles $k(E_a, A)$ and $k(E_b, B)$ intersect % DET: circles $k(E_a, A)$ and $k(E_b, B)$ are not the same; points H_c and H must be different;
4. Using the point H and the point E_a , construct a point A (rule W01); ;
5. Using the point E_b and the point H , construct a point B (rule W01); ;
6. Using the point E_b and the point H , construct a line h_b (rule W02); % DET: points E_b and H are not the same;
7. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
8. Using the circle $k(E_a, A)$, the line h_b , the point E_a and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_a, A)$ intersect % DET: points H and H_b must be different;
9. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;
10. Using the line h_c and the line b , construct a point C (rule W03); % NDG: lines h_c and b are not parallel % DET: lines h_c and b are not the same.

Non-degenerate conditions: lines h_c and b are not parallel; line h_b and circle $k(E_a, A)$ intersect; circles $k(E_a, A)$ and $k(E_b, B)$ intersect; points H_c and E_b are not the same; points H_c and E_a are not the same.

Determination conditions: lines h_c and b are not the same; points A and H_b are not the same; points H and H_b must be different; points H_c and H are not the same; points E_b and H are not the same; circles $k(E_a, A)$ and $k(E_b, B)$ are not the same; points H_c and H must be different.

Rules used: [W01,W02,W03,W05,W06,W08]

Lemmas used: [D10,D28,D29,D3,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L46,L47,L48,L49,L51]

Solving time: 10.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
point E_{b} 50 56.36
point H_{c} 68.91 84.83
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{a}
cmark_r E_{b}
cmark_rt H_{c}
color 0 0 0
fontsize 8
```

```
% NDG: points H_{c} and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H
_{c}
circle k(E_{a},A) E_{a} H_{c}
```

```
color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0
```

```
% NDG: points H_{c} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H
_{c}
circle k(E_{b},B) E_{b} H_{c}
```

```
color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0
```

```
% NDG: circles k(E_{a},A) and k(E_{b},B) intersect% DET: circles k(E_{a},A) and k(E_{b},B) are not
the same; points H_{c} and H must be different
% Constructing a line L_{\_G198894} which passes through point E_{a} and point E_{b}
line L_{\_G198894} E_{a} E_{b}
```



```

color 200 200 200
drawline L_{\_G198894}
color 0 0 0

% Constructing a point H which is an image of the point H_{c} in the symmetry to point/line L_{\_G
198894}
sim H L_{\_G198894} H_{c}
cmark_rt H

% Constructing a point A such that HA/HE_{a}=2
towards A H E_{a} 2
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

% Constructing a point B such that E_{b}B/E_{b}H=-1
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

% DET: points E_{b} and H are not the same
% Constructing a line h_{b} which passes through point E_{b} and point H
line h_{b} E_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: line h_{b} and circle k(E_{a},A) intersect% DET: points H and H_{b} must be different
% Constructing a point P_{\_G199311} which is a foot of the point E_{a} on the line h_{b}
foot P_{\_G199311} E_{a} h_{b}
cmark_r P_{\_G199311}
color 200 200 200
drawline E_{a} P_{\_G199311}

```

```

color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $H$  in the symmetry to point/line  $P_{\{\backslash\_G$ 
199311}
sim  $H_{\{b\}}$   $P_{\{\backslash\_G199311\}}$   $H$ 
cmark_1  $H_{\{b\}}$ 

% DET: points  $A$  and  $H_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $A$  and point  $H_{\{b\}}$ 
line b A  $H_{\{b\}}$ 

color 200 200 200
drawline b
color 0 0 0

% NDG: lines  $h_{\{c\}}$  and  $b$  are not parallel% DET: lines  $h_{\{c\}}$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $h_{\{c\}}$  and line  $b$ 
intersec C  $h_{\{c\}}$  b
cmark_1 C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{c\}}$  and  $b$  are not parallel; line  $h_{\{b\}}$  and circle  $k(E_{\{a\}},A)$ 
intersect; circles  $k(E_{\{a\}},A)$  and  $k(E_{\{b\}},B)$  intersect; points  $H_{\{c\}}$  and  $E_{\{b\}}$  are not the same
; points  $H_{\{c\}}$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $h_{\{c\}}$  and  $b$  are not the same; points  $A$  and  $H_{\{b\}}$  are not the same
; points  $H$  and  $H_{\{b\}}$  must be different; points  $H_{\{c\}}$  and  $H$  are not the same; points  $E_{\{b\}}$  and  $H$ 
are not the same; circles  $k(E_{\{a\}},A)$  and  $k(E_{\{b\}},B)$  are not the same; points  $H_{\{c\}}$  and  $H$  must
be different

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $E_b = \neg E_b$

Proving failed

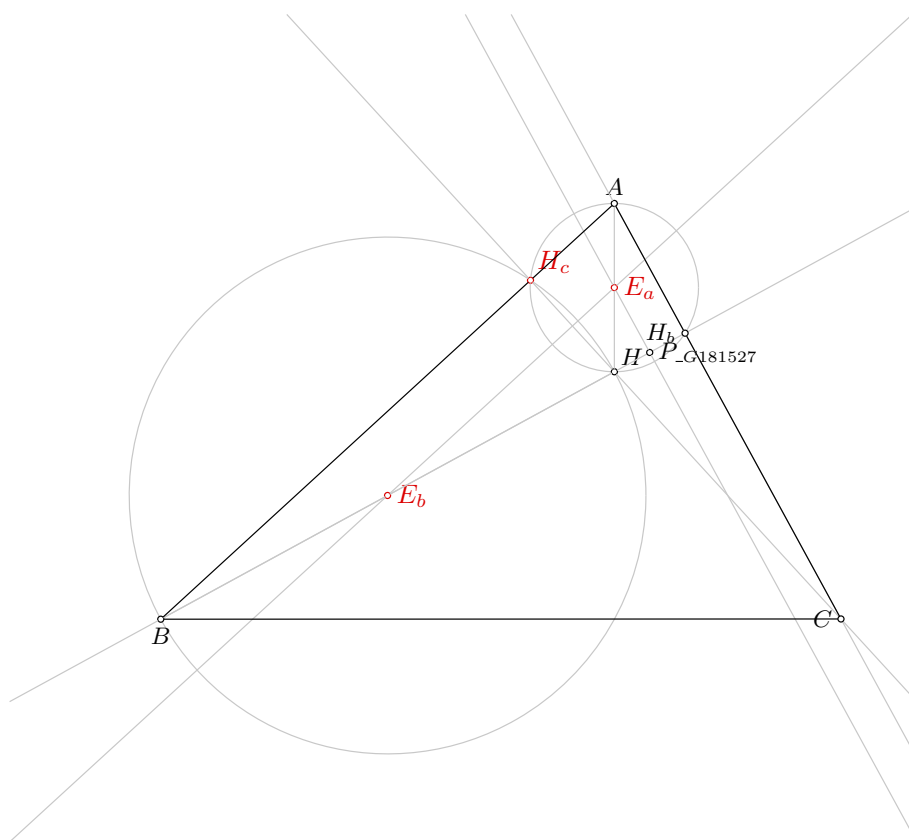


Figure 1: Illustration of the problem 0752

4.1.3 Proving $H_c = \neg H_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $E_b = \neg E_b$

Proving failed

4.2.3 Proving $H_c = \neg H_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $E_b = \neg E_b$

Proving failed

4.3.3 Proving $H_c = \neg H_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $E_b = \neg E_b$

Proving failed

4.4.3 Proving $H_c = \neg H_c$

Proving failed

Problem 753

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 753: Given a point E_a , a point E_b and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 754

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 754: Given a point E_a , a point M_a and a point E_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point M_a , construct a line $m(H_bH_c)$ (rule W02); % DET: points E_a and M_a are not the same;
2. Using the point E_a and the point M_a , construct a line $m(E_aM_a)$ (rule W14); % DET: points E_a and M_a are not the same;
3. Using the line $m(E_aM_a)$ and the line $m(H_bH_c)$, construct a point N (rule W03); % NDG: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not parallel % DET: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not the same;
4. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
5. Choose freely a point E_b on the circle $k(N, M_a)$ (rule W0ncircle);
6. Using the point E_b and the point N , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and N are not the same;
7. Using the circle $k(N, M_a)$, the line $m(H_aH_c)$, the point N and the point E_b , construct a point M_b (rule W05a); % NDG: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect % DET: points E_b and M_b must be different;
8. Choose freely a point A (rule free);
9. Using the point A and the point E_a , construct a point H (rule W01); ;
10. Using the point A and the point M_b , construct a point C (rule W01); ;
11. Using the point E_b and the point H , construct a point B (rule W01); .

Non-degenerate conditions: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect; points E_a and N are not the same; lines $m(E_aM_a)$ and $m(H_bH_c)$ are not parallel.

Determination conditions: points E_b and M_b must be different; points E_b and N are not the same; lines $m(E_aM_a)$ and $m(H_bH_c)$ are not the same; points E_a and M_a are not the same; points E_a and M_a are not the same.

Rules used: [W01,W02,W03,W05a,W06,W14,WOncircle1,free]

Lemmas used: [D22,D28,D29,D32,GD02,GL01,GL03,GL04,L17,L19,L20,L21,L22,L23,L38,L39,L44,L45,L47,L48]

Solving time: 200.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point M_{a} 65 40
```

```
point E_{b} 50 56.36
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_r M_{a}
```

```
cmark_r E_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{a} and M_{a} are not the same
```

```
% Constructing a line m(H_{b}H_{c}) which passes through point E_{a} and point M_{a}
```

```
line m(H_{b}H_{c}) E_{a} M_{a}
```

```
color 200 200 200
```

```
drawline m(H_{b}H_{c})
```

```
color 0 0 0
```

```
% DET: points E_{a} and M_{a} are not the same
```

```
% Constructing bisector m(E_{a}M_{a}) of the segment E_{a}M_{a}
```

```
med m(E_{a}M_{a}) E_{a} M_{a}
```

```
color 200 200 200
```

```
drawline m(E_{a}M_{a})
```

```
color 0 0 0
```

```
color 200 200 200
```

```
drawsegment E_{a} M_{a}
```

```
color 0 0 0
```

```
% NDG: lines m(E_{a}M_{a}) and m(H_{b}H_{c}) are not parallel% DET: lines m(E_{a}M_{a}) and m(H_{b}H_{c}) are not the same
```

```
% Constructing a point N which belongs to line m(E_{a}M_{a}) and line m(H_{b}H_{c})
```

```
intersec N m(E_{a}M_{a}) m(H_{b}H_{c})
```

```
cmark_r N
```

```

% NDG: points  $E_{\{a\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{a\}}$ 
circle k(N, M_{\{a\}}) N E_{\{a\}}

color 200 200 200
drawcircle k(N, M_{\{a\}})
color 0 0 0

% Choosing randomly a point  $E_{\{b\}}$  on the circle with center  $N$  through point  $E_{\{a\}}$ 
oncircle E_{\{b\}} N E_{\{a\}}
cmark_r E_{\{b\}}
color 200 200 200
drawcircle N E_{\{a\}}
color 0 0 0

% DET: points  $E_{\{b\}}$  and  $N$  are not the same
% Constructing a line  $m(H_{\{a\}}H_{\{c\}})$  which passes through point  $E_{\{b\}}$  and point  $N$ 
line m(H_{\{a\}}H_{\{c\}}) E_{\{b\}} N

color 200 200 200
drawline m(H_{\{a\}}H_{\{c\}})
color 0 0 0

% NDG: line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{b\}}$  and  $M_{\{b\}}$  must be
different
% Constructing a point  $M_{\{b\}}$  which is an image of the point  $E_{\{b\}}$  in the symmetry to point/line  $N$ 
sim M_{\{b\}} N E_{\{b\}}
cmark_lt M_{\{b\}}

% Constructing a free point  $A$ 
point A 80 95

cmark_t A

% Constructing a point  $H$  such that  $AH/AE_{\{a\}}=2$ 
towards H A E_{\{a\}} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% Constructing a point  $C$  such that  $AC/AM_{\{b\}}=2$ 

```



```

towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

```

```

% Constructing a point B such that  $E_{\{b\}B/E_{\{b\}H}=-1$ 
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

% Non-degenerate conditions: line $m(H_{\{a\}H_{\{c\}}})$ and circle $k(N, M_{\{a\}})$ intersect; points $E_{\{a\}}$ and N are not the same; lines $m(E_{\{a\}M_{\{a\}}})$ and $m(H_{\{b\}H_{\{c\}}})$ are not parallel

% Determination conditions: points $E_{\{b\}}$ and $M_{\{b\}}$ must be different; points $E_{\{b\}}$ and N are not the same; lines $m(E_{\{a\}M_{\{a\}}})$ and $m(H_{\{b\}H_{\{c\}}})$ are not the same; points $E_{\{a\}}$ and $M_{\{a\}}$ are not the same; points $E_{\{a\}}$ and $M_{\{a\}}$ are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 15 terms.

Time Complexity: Time spent by the prover is 0.32 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $E_b = \neg E_b$

Proving failed

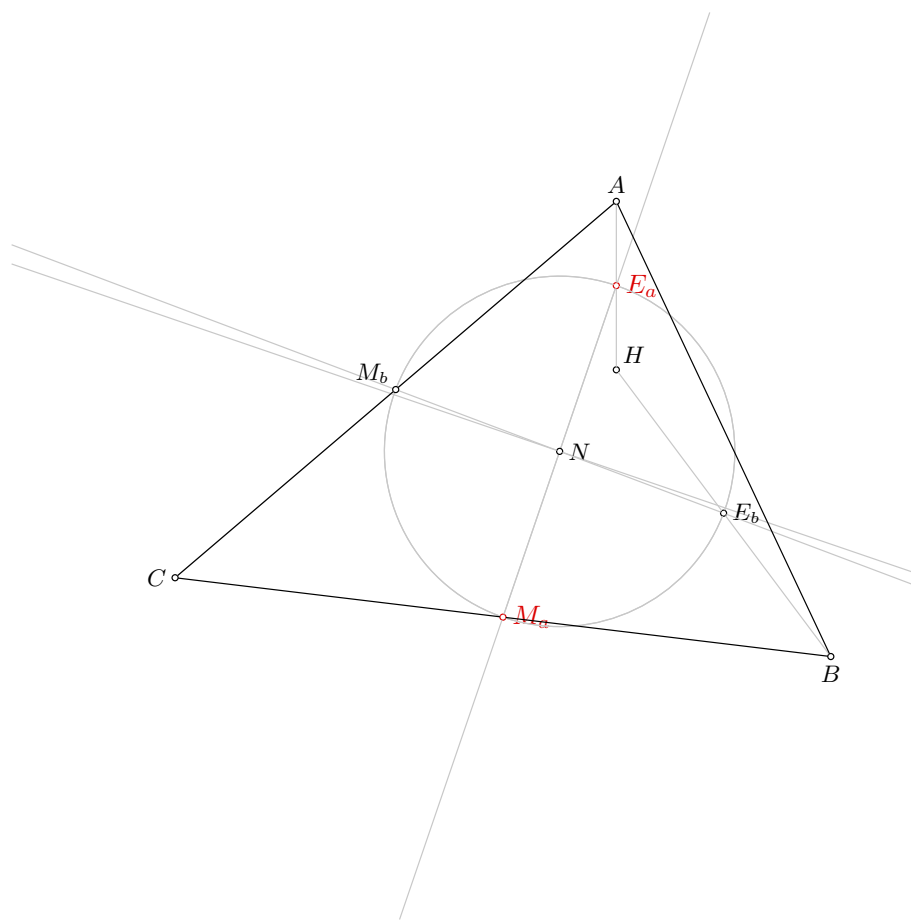


Figure 1: Illustration of the problem 0754

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $M_a = \neg M_a$

Proving failed

4.2.3 Proving $E_b = \neg E_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $M_a = \neg M_a$

Proving failed

4.3.3 Proving $E_b = \neg E_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $M_a = \neg M_a$

Proving failed

4.4.3 Proving $E_b = \neg E_b$

Proving failed

Problem 755

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 755: Given a point E_b , a point M_b and a point E_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point M_b , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and M_b are not the same;
2. Using the point E_b and the point M_b , construct a line $m(E_bM_b)$ (rule W14); % DET: points E_b and M_b are not the same;
3. Using the line $m(E_bM_b)$ and the line $m(H_aH_c)$, construct a point N (rule W03); % NDG: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel % DET: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same;
4. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
5. Choose freely a point E_a on the circle $k(N, M_a)$ (rule WOncircle);
6. Choose freely a point A (rule free);
7. Using the point A and the point E_a , construct a point H (rule W01); ;
8. Using the point A and the point M_b , construct a point C (rule W01); ;
9. Using the point E_b and the point H , construct a point B (rule W01); .

Non-degenerate conditions: points E_b and N are not the same; lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel.

Determination conditions: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same; points E_b and M_b are not the same; points E_b and M_b are not the same.

Rules used: [W01,W02,W03,W06,W14,WOncircle1,free]

Lemmas used: [D22,D28,D29,D32,GD02,GL01,GL03,GL04,L17,L19,L21,L23,L44,L45,L50,L51]

Solving time: 198.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{b} 50 56.36
point M_{b} 95 67.5
point E_{a} 80 83.86

color 220 0 0
fontsize 9

cmark_r E_{b}
cmark_lt M_{b}
cmark_r E_{a}
color 0 0 0
fontsize 8

% DET: points E_{b} and M_{b} are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point M_{b}
line m(H_{a}H_{c}) E_{b} M_{b}

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

% DET: points E_{b} and M_{b} are not the same
% Constructing bisector m(E_{b}M_{b}) of the segment E_{b}M_{b}
med m(E_{b}M_{b}) E_{b} M_{b}

color 200 200 200
drawline m(E_{b}M_{b})
color 0 0 0

color 200 200 200
drawsegment E_{b} M_{b}
color 0 0 0

% NDG: lines m(E_{b}M_{b}) and m(H_{a}H_{c}) are not parallel% DET: lines m(E_{b}M_{b}) and m(H_{a}H_{c}) are not the same
% Constructing a point N which belongs to line m(E_{b}M_{b}) and line m(H_{a}H_{c})
intersec N m(E_{b}M_{b}) m(H_{a}H_{c})
cmark_r N

% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```

% Choosing randomly a point  $E_{\{a\}}$  on the circle with center  $N$  through point  $E_{\{b\}}$ 
oncircle  $E_{\{a\}}$   $N$   $E_{\{b\}}$ 
cmark_r  $E_{\{a\}}$ 
color 200 200 200
drawcircle  $N$   $E_{\{b\}}$ 
color 0 0 0

% Constructing a free point  $A$ 
point  $A$  80 95

cmark_t  $A$ 

% Constructing a point  $H$  such that  $AH/AE_{\{a\}}=2$ 
towards  $H$   $A$   $E_{\{a\}}$  2
cmark_rt  $H$ 
color 200 200 200
drawsegment  $A$   $H$ 
color 0 0 0

% Constructing a point  $C$  such that  $AC/AM_{\{b\}}=2$ 
towards  $C$   $A$   $M_{\{b\}}$  2
cmark_l  $C$ 
color 200 200 200
drawsegment  $A$   $C$ 
color 0 0 0

% Constructing a point  $B$  such that  $E_{\{b\}}B/E_{\{b\}}H=-1$ 
towards  $B$   $E_{\{b\}}$   $H$  -1
cmark_b  $B$ 
color 200 200 200
drawsegment  $H$   $B$ 
color 0 0 0

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: points  $E_{\{b\}}$  and  $N$  are not the same; lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not parallel
% Determination conditions: lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not the same; points  $E_{\{b\}}$  and  $M_{\{b\}}$  are not the same; points  $E_{\{b\}}$  and  $M_{\{b\}}$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

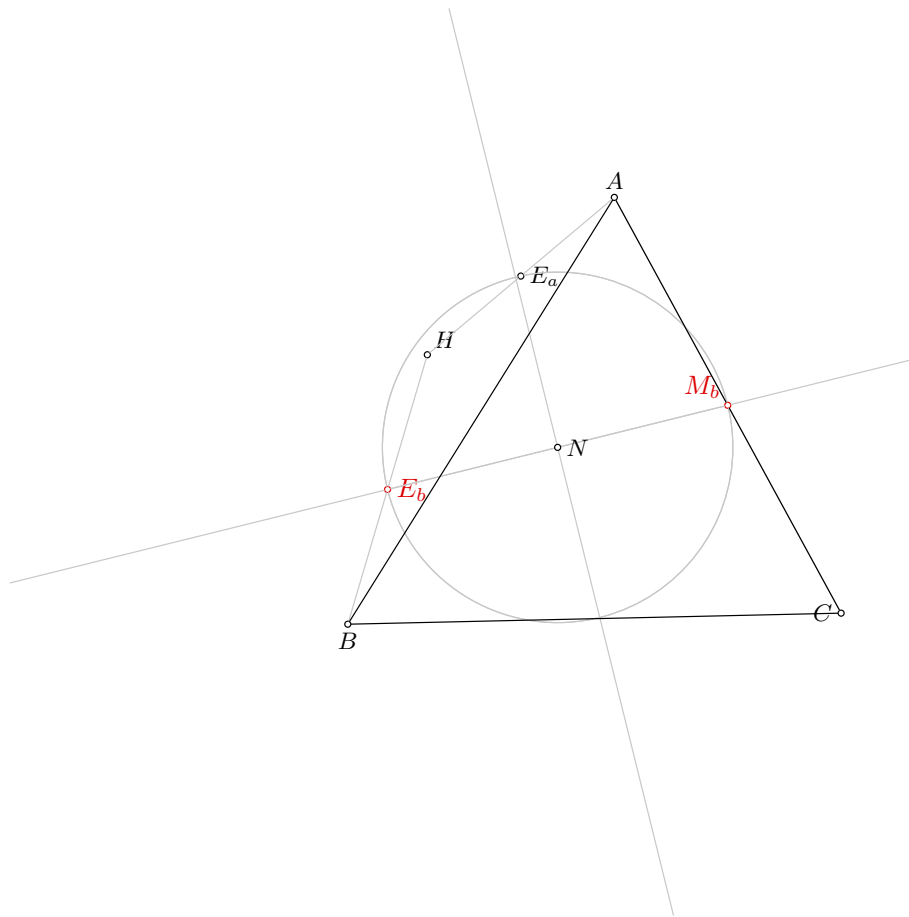


Figure 1: Illustration of the problem 0755

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.049 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $E_a = \neg E_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{M_{m(E_b M_b)}^0}^{E_b M_b} \neq S_{T_{m(E_b M_b)}^1}^{E_b M_b}$ i.e., lines $M_{m(E_b M_b)}^0$ and $T_{m(E_b M_b)}^1$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2 BF_{\neg h_b}^3}$ i.e., lines $AF_{\neg h_a}^2$ and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

4.2.3 Proving $E_a = \neg E_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $M_b = \neg M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 19 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.3.3 Proving $E_a = \neg E_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $M_b = \neg M_b$

Proving failed

4.4.3 Proving $E_a = \neg E_a$

Proving failed

Problem 756

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 756: Given a point E_a , a point E_b and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point M_c , construct a line $m(BH_a)$ (rule W02); % DET: points E_b and M_c are not the same;
2. Using the point E_a and the point E_b , construct a line $m(E_aE_b)$ (rule W14); % DET: points E_a and E_b are not the same;
3. Using the point E_a and the point M_c , construct a line $m(E_aM_c)$ (rule W14); % DET: points E_a and M_c are not the same;
4. Using the line $m(E_aM_c)$ and the line $m(E_aE_b)$, construct a point N (rule W03); % NDG: lines $m(E_aM_c)$ and $m(E_aE_b)$ are not parallel % DET: lines $m(E_aM_c)$ and $m(E_aE_b)$ are not the same;
5. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
6. Using the point E_a and the line $m(BH_a)$, construct a line h_a (rule W16); ;
7. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
8. Using the point H_a and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_a and E_b are not the same;
9. Using the circle $k(E_b, B)$, the line h_a , the point E_b and the point H_a , construct a point H (rule W05); % NDG: line h_a and circle $k(E_b, B)$ intersect % DET: points H_a and H must be different;

10. Using the point H and the point E_a , construct a point A (rule W01); ;
11. Using the point E_b and the point H , construct a point B (rule W01); ;
12. Using the point N and the point H , construct a point G (rule W01); ;
13. Using the point M_c and the point G , construct a point C (rule W01); .

Non-degenerate conditions: line h_a and circle $k(E_b, B)$ intersect; points H_a and E_b are not the same; line h_a and circle $k(N, M_a)$ intersect; points E_a and N are not the same; lines $m(E_a M_c)$ and $m(E_a E_b)$ are not parallel.

Determination conditions: points H_a and H must be different; points E_a and H_a must be different; lines $m(E_a M_c)$ and $m(E_a E_b)$ are not the same; points E_a and M_c are not the same; points E_a and E_b are not the same; points E_b and M_c are not the same.

Rules used: [W01,W02,W03,W05,W06,W14,W16]

Lemmas used: [D28,D29,D3,D32,D5,D8,GD02,GL01,GL03,GL04,GL09,L16,L18,L19,L22,L23,L40,L42,L49,L50]

Solving time: 25.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
point E_{b} 50 56.36
point M_{c} 50 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{a}
cmark_r E_{b}
cmark_lt M_{c}
color 0 0 0
fontsize 8
```

```
% DET: points E_{b} and M_{c} are not the same
% Constructing a line m(BH_{a}) which passes through point E_{b} and point M_{c}
line m(BH_{a}) E_{b} M_{c}
```

```
color 200 200 200
drawline m(BH_{a})
color 0 0 0
```

```
% DET: points E_{a} and E_{b} are not the same
% Constructing bisector m(E_{a}E_{b}) of the segment E_{a}E_{b}
med m(E_{a}E_{b}) E_{a} E_{b}
```

```
color 200 200 200
drawline m(E_{a}E_{b})
color 0 0 0
```

```

color 200 200 200
drawsegment E_{a} E_{b}
color 0 0 0

% DET: points E_{a} and M_{c} are not the same
% Constructing bisector m(E_{a}M_{c}) of the segment E_{a}M_{c}
med m(E_{a}M_{c}) E_{a} M_{c}

color 200 200 200
drawline m(E_{a}M_{c})
color 0 0 0

color 200 200 200
drawsegment E_{a} M_{c}
color 0 0 0

% NDG: lines m(E_{a}M_{c}) and m(E_{a}E_{b}) are not parallel% DET: lines m(E_{a}M_{c}) and m(E_{a}
E_{b}) are not the same
% Constructing a point N which belongs to line m(E_{a}M_{c}) and line m(E_{a}E_{b})
intersec N m(E_{a}M_{c}) m(E_{a}E_{b})
cmark_r N

% NDG: points E_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{a}
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Constructing a line h_{a} which contains the point E_{a} and is parallel to the line m(BH_{a})
parallel h_{a} E_{a} m(BH_{a})

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G49750} which is a foot of the point N on the line h_{a}
foot P_{\_G49750} N h_{a}
cmark_r P_{\_G49750}
color 200 200 200
drawline N P_{\_G49750}
color 0 0 0

% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
\_G49750}
sim H_{a} P_{\_G49750} E_{a}

```

```

cmark_r H_{a}

% NDG: points H_{a} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H
_{a}
circle k(E_{b},B) E_{b} H_{a}

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line h_{a} and circle k(E_{b},B) intersect% DET: points H_{a} and H must be different
% Constructing a point P_{\_G50054} which is a foot of the point E_{b} on the line h_{a}
foot P_{\_G50054} E_{b} h_{a}
cmark_r P_{\_G50054}
color 200 200 200
drawline E_{b} P_{\_G50054}
color 0 0 0

% Constructing a point H which is an image of the point H_{a} in the symmetry to point/line P_{\_G
50054}
sim H P_{\_G50054} H_{a}
cmark_rt H

% Constructing a point A such that HA/HE_{a}=2
towards A H E_{a} 2
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

% Constructing a point B such that E_{b}B/E_{b}H=-1
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

% Constructing a line L_{\_G50316} which passes through point N and point H
line L_{\_G50316} N H

color 200 200 200
drawline L_{\_G50316}
color 0 0 0

% Constructing a point P_{\_G50417} with coordinates (0,0)

```

```

point P_{\_G50417} 0 0
cmark_r P_{\_G50417}

% Constructing a point P_{\_G50341} such that NP_{\_G50341}/NP_{\_G50417}=-1
towards P_{\_G50341} N P_{\_G50417} -1
cmark_r P_{\_G50341}
color 200 200 200
drawsegment P_{\_G50417} P_{\_G50341}
color 0 0 0

% Constructing a point P_{\_G50386} such that NP_{\_G50386}/NP_{\_G50417}=3
towards P_{\_G50386} N P_{\_G50417} 3
cmark_r P_{\_G50386}
color 200 200 200
drawsegment N P_{\_G50386}
color 0 0 0

% Constructing a line L_{\_G50347} which passes through point H and point P_{\_G50386}
line L_{\_G50347} H P_{\_G50386}

color 200 200 200
drawline L_{\_G50347}
color 0 0 0

% Constructing a line L_{\_G50310} which contains the point P_{\_G50341} and is parallel to the
line L_{\_G50347}
parallel L_{\_G50310} P_{\_G50341} L_{\_G50347}

color 200 200 200
drawline L_{\_G50310}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G50310} and line L_{\_G50316}
intersec G L_{\_G50310} L_{\_G50316}
cmark_t G

% Constructing a point C such that M_{c}C/M_{c}G=3
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line h_{a} and circle k(E_{b},B) intersect; points H_{a} and E_{b} are
not the same; line h_{a} and circle k(N,M_{a}) intersect; points E_{a} and N are not the same;
lines m(E_{a}M_{c}) and m(E_{a}E_{b}) are not parallel

```

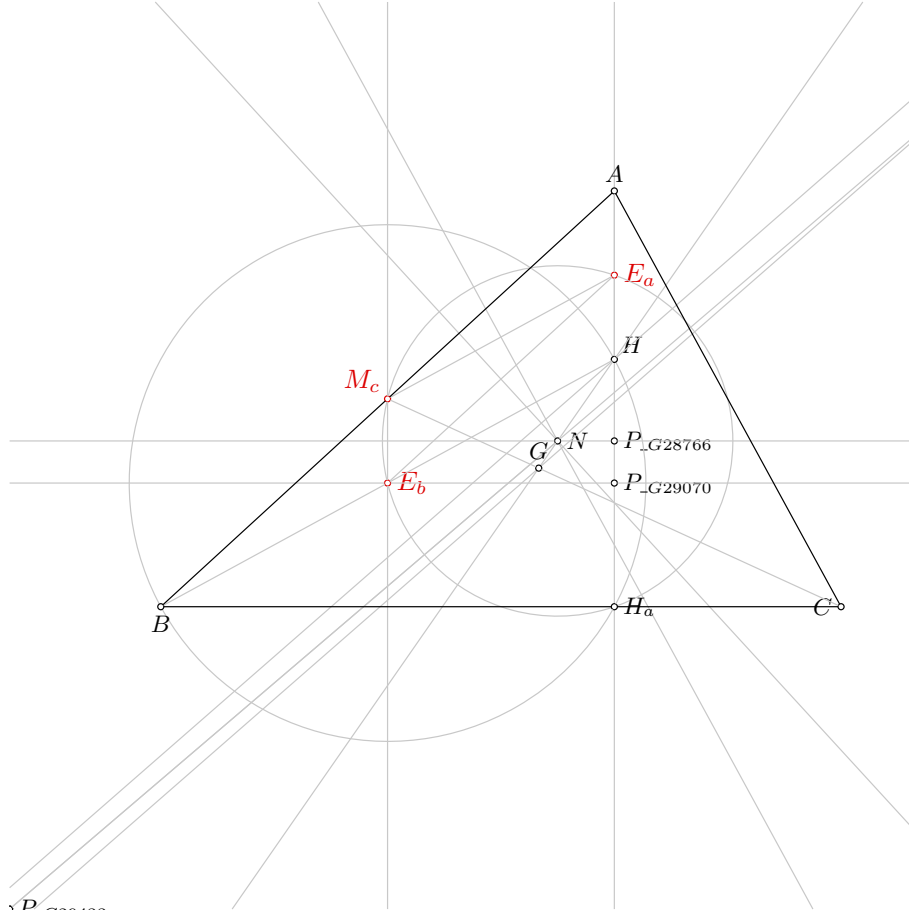


Figure 1: Illustration of the problem 0756

% Determination conditions: points $H_{\{a\}}$ and H must be different; points $E_{\{a\}}$ and $H_{\{a\}}$ must be different; lines $m(E_{\{a\}}M_{\{c\}})$ and $m(E_{\{a\}}E_{\{b\}})$ are not the same; points $E_{\{a\}}$ and $M_{\{c\}}$ are not the same; points $E_{\{a\}}$ and $E_{\{b\}}$ are not the same; points $E_{\{b\}}$ and $M_{\{c\}}$ are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = E_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 5665 terms.

Time Complexity: Time spent by the prover is 14.713 seconds.

NDG conditions Points M_c and E_b are not identical

Points M_c and E_b are not identical

Points M_c and E_b are not identical

Line through points M_c and P_{G41616} is not perpendicular to line through points P_{G41616} and E_b

Points E_b , P_{G41616} and E_a are not collinear

Points A , B and C are not collinear

Line through points B and E_b is not perpendicular to line through points E_b and C

4.1.2 Proving $E_b = E_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 5618 terms.

Time Complexity: Time spent by the prover is 16.115 seconds.

NDG conditions Points M_c and E_b are not identical

Points M_c and E_b are not identical

Points M_c and E_b are not identical

Line through points M_c and E_a is not perpendicular to line through points E_a and E_b

Points E_a and P_{G43666} are not identical

Points A , B and C are not collinear

Line through points B and E_b is not perpendicular to line through points E_b and C

4.1.3 Proving $M_c = M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 17 terms.

Time Complexity: Time spent by the prover is 1.111 seconds.

NDG conditions Points M_c and E_b are not identical

Points M_c and E_b are not identical

Points M_c and E_b are not identical

Line through points M_c and P_{G45716} is not perpendicular to line through points P_{G45716} and E_b

Points P_{G45716} and E_a are not identical

4.2 GCLC - Area method

4.2.1 Proving $E_a = E_a$

Proving failed

4.2.2 Proving $E_b = E_b$

Proving failed

4.2.3 Proving $M_c = M_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = E_a$

Proving failed

4.3.2 Proving $E_b = \neg E_b$

Proving failed

4.3.3 Proving $M_c = \neg M_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $E_b = \neg E_b$

Proving failed

4.4.3 Proving $M_c = \neg M_c$

Proving failed

Problem 757

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 757: Given a point E_b , a point N and a point E_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point N , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and N are not the same;
2. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
3. Choose freely a point E_a on the circle $k(N, M_a)$ (rule WOncircle);
4. Using the circle $k(N, M_a)$, the line $m(H_aH_c)$, the point N and the point E_b , construct a point M_b (rule W05a); % NDG: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect % DET: points E_b and M_b must be different;
5. Choose freely a point A (rule free);
6. Using the point A and the point E_a , construct a point H (rule W01); ;
7. Using the point A and the point M_b , construct a point C (rule W01); ;
8. Using the point E_b and the point H , construct a point B (rule W01); .

Non-degenerate conditions: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect; points E_b and N are not the same.

Determination conditions: points E_b and M_b must be different; points E_b and N are not the same.

Rules used: [W01,W02,W05a,W06,WOncircle1,free]

Lemmas used: [D22,D28,D29,D32,GD02,GL01,GL03,GL04,L17,L19,L21,L23,L44,L45,L50,L51]

Solving time: 155.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{b} 50 56.36
point N 72.5 61.93
point E_{a} 80 83.86

color 220 0 0
fontsize 9

cmark_r E_{b}
cmark_r N
cmark_r E_{a}
color 0 0 0
fontsize 8

% DET: points E_{b} and N are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point N
line m(H_{a}H_{c}) E_{b} N

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point E_{a} on the circle with center N through point E_{b}
oncircle E_{a} N E_{b}
cmark_r E_{a}
color 200 200 200
drawcircle N E_{b}
color 0 0 0

% NDG: line m(H_{a}H_{c}) and circle k(N,M_{a}) intersect% DET: points E_{b} and M_{b} must be
different
% Constructing a point M_{b} which is an image of the point E_{b} in the symmetry to point/line N
sim M_{b} N E_{b}
cmark_lt M_{b}

% Constructing a free point A
point A 80 95
```

```
cmark_t A
```

```
% Constructing a point H such that  $AH/AE_{\{a\}}=2$ 
towards H A  $E_{\{a\}}$  2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0
```

```
% Constructing a point C such that  $AC/AM_{\{b\}}=2$ 
towards C A  $M_{\{b\}}$  2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0
```

```
% Constructing a point B such that  $E_{\{b\}}B/E_{\{b\}}H=-1$ 
towards B  $E_{\{b\}}$  H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect; points  $E_{\{b\}}$  and  $N$ 
are not the same
% Determination conditions: points  $E_{\{b\}}$  and  $M_{\{b\}}$  must be different; points  $E_{\{b\}}$  and  $N$  are not
the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = _E_b$

Proving failed

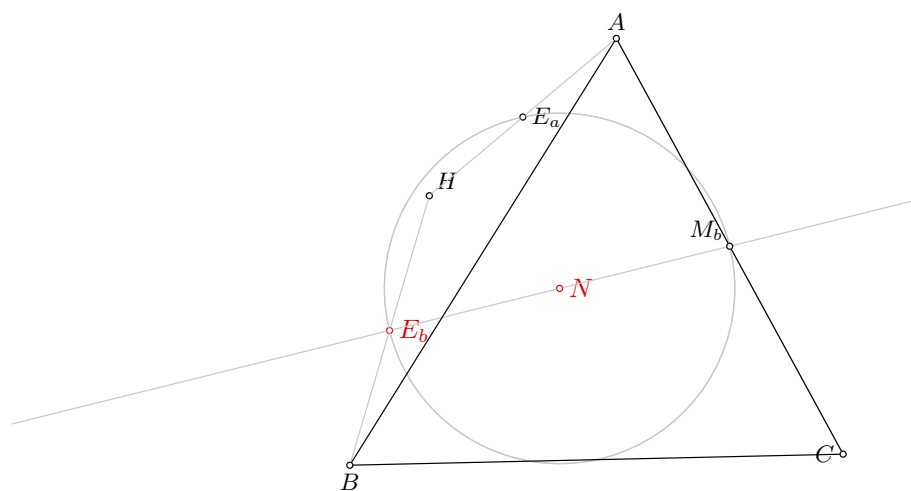


Figure 1: Illustration of the problem 0757

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $E_a=_Ea$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b=_Eb$

Proving failed

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $E_a=_Ea$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b=_Eb$

Proving failed

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $E_a=_Ea$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b=_Eb$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $E_a=_Ea$

Proving failed

Problem 758

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 758: Given a point E_a , a point E_b and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 759

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 759: Given a point E_a , a point E_b and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 760

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 760: Given a point E_a , a point E_b and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 761

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 761: Given a point E_a , a point E_b and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 762

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 762: Given a point E_a , a point E_c and a point G , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 763

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 763: Given a point E_a , a point E_c and a point H , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H , construct a point A (rule W01); ;
2. Using the point E_c and the point H , construct a point C (rule W01); ;
3. Using the point E_c and the point H , construct a line h_c (rule W02); % DET: points E_c and H are not the same;
4. Using the point A and the point C , construct a line b (rule W02); % DET: points A and C are not the same;
5. Using the point H and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H and E_a are not the same;
6. Using the circle $k(E_a, A)$, the line h_c , the point E_a and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_a, A)$ intersect % DET: points H and H_c must be different;
7. Using the point H_c and the point A , construct a line c (rule W02); % DET: points H_c and A are not the same;
8. Using the circle $k(E_a, A)$, the line b , the point E_a and the point A , construct a point H_b (rule W05); % NDG: line b and circle $k(E_a, A)$ intersect % DET: points A and H_b must be different;
9. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
10. Using the line c and the line h_b , construct a point B (rule W03); % NDG: lines c and h_b are not parallel % DET: lines c and h_b are not the same.

Non-degenerate conditions: lines c and h_b are not parallel; line b and circle $k(E_a, A)$ intersect; line h_c and circle $k(E_a, A)$ intersect; points H and E_a are not the same.

Determination conditions: lines c and h_b are not the same; points H_b and H are not the same; points A and H_b must be different; points H_c and A are not the same; points H and H_c must be different; points A and C are not the same; points E_c and H are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D28,D3,D30,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L46,L47,L48]

Solving time: 12.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point E_{c} 95 56.36
```

```
point H 80 72.73
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_r E_{c}
```

```
cmark_rt H
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point A such that E_{a}A/E_{a}H=-1
```

```
towards A E_{a} H -1
```

```
cmark_t A
```

```
color 200 200 200
```

```
drawsegment H A
```

```
color 0 0 0
```

```
% Constructing a point C such that E_{c}C/E_{c}H=-1
```

```
towards C E_{c} H -1
```

```
cmark_l C
```

```
color 200 200 200
```

```
drawsegment H C
```

```
color 0 0 0
```

```
% DET: points E_{c} and H are not the same
```

```
% Constructing a line h_{c} which passes through point E_{c} and point H
```

```
line h_{c} E_{c} H
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```

% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C

color 200 200 200
drawline b
color 0 0 0

% NDG: points H and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H
circle k(E_{a},A) E_{a} H

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: line h_{c} and circle k(E_{a},A) intersect% DET: points H and H_{c} must be different
% Constructing a point P_{\_G129738} which is a foot of the point E_{a} on the line h_{c}
foot P_{\_G129738} E_{a} h_{c}
cmark_r P_{\_G129738}
color 200 200 200
drawline E_{a} P_{\_G129738}
color 0 0 0

% Constructing a point H_{c} which is an image of the point H in the symmetry to point/line P_{\_G
129738}
sim H_{c} P_{\_G129738} H
cmark_rt H_{c}

% DET: points H_{c} and A are not the same
% Constructing a line c which passes through point H_{c} and point A
line c H_{c} A

color 200 200 200
drawline c
color 0 0 0

% NDG: line b and circle k(E_{a},A) intersect% DET: points A and H_{b} must be different
% Constructing a point P_{\_G129976} which is a foot of the point E_{a} on the line b
foot P_{\_G129976} E_{a} b
cmark_r P_{\_G129976}
color 200 200 200
drawline E_{a} P_{\_G129976}
color 0 0 0

% Constructing a point H_{b} which is an image of the point A in the symmetry to point/line P_{\_G
129976}
sim H_{b} P_{\_G129976} A
cmark_l H_{b}

```

```

% DET: points  $H_{\{b\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $H_{\{b\}}$  and point  $H$ 
line  $h_{\{b\}}$   $H_{\{b\}}$   $H$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: lines  $c$  and  $h_{\{b\}}$  are not parallel% DET: lines  $c$  and  $h_{\{b\}}$  are not the same
% Constructing a point  $B$  which belongs to line  $c$  and line  $h_{\{b\}}$ 
intersec  $B$   $c$   $h_{\{b\}}$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $c$  and  $h_{\{b\}}$  are not parallel; line  $b$  and circle  $k(E_{\{a\}},A)$ 
% intersect; line  $h_{\{c\}}$  and circle  $k(E_{\{a\}},A)$  intersect; points  $H$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $c$  and  $h_{\{b\}}$  are not the same; points  $H_{\{b\}}$  and  $H$  are not the same
% ; points  $A$  and  $H_{\{b\}}$  must be different; points  $H_{\{c\}}$  and  $A$  are not the same; points  $H$  and  $H_{\{c\}}$ 
% must be different; points  $A$  and  $C$  are not the same; points  $E_{\{c\}}$  and  $H$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $E_c = \neg E_c$

Proving failed

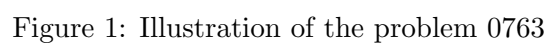
4.1.3 Proving $H = \neg H$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed



4.2.2 Proving $E_c = E_c$

Proving failed

4.2.3 Proving $H = H$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = E_a$

Proving failed

4.3.2 Proving $E_c = E_c$

Proving failed

4.3.3 Proving $H = H$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 2174 terms.

Time Complexity: Time spent by the prover is 1.800 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = E_a$

Proving failed

4.4.2 Proving $E_c = E_c$

Proving failed

4.4.3 Proving $H = H$

Proving failed

Problem 764

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 764: Given a point E_a , a point E_c and a point H_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;
2. Using the point H_a and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_a and E_c are not the same;
3. Using the circle $k(E_c, C)$, the line h_a , the point E_c and the point H_a , construct a point H (rule W05); % NDG: line h_a and circle $k(E_c, C)$ intersect % DET: points H_a and H must be different;
4. Using the point H and the point E_a , construct a point A (rule W01); ;
5. Using the point E_c and the point H , construct a point C (rule W01); ;
6. Using the point H_a and the point C , construct a line a (rule W02); % DET: points H_a and C are not the same;
7. Using the point A and the point C , construct a line b (rule W02); % DET: points A and C are not the same;
8. Using the circle $k(E_c, C)$, the line b , the point E_c and the point C , construct a point H_b (rule W05); % NDG: line b and circle $k(E_c, C)$ intersect % DET: points C and H_b must be different;
9. Using the point H and the point H_b , construct a line h_b (rule W02); % DET: points H and H_b are not the same;
10. Using the line a and the line h_b , construct a point B (rule W03); % NDG: lines a and h_b are not parallel % DET: lines a and h_b are not the same.

Non-degenerate conditions: lines a and h_b are not parallel; line b and circle $k(E_c, C)$ intersect; line h_a and circle $k(E_c, C)$ intersect; points H_a and E_c are not the same.

Determination conditions: lines a and h_b are not the same; points H and H_b are not the same; points C and H_b must be different; points A and C are not the same; points H_a and C are not the same; points H_a and H must be different; points E_a and H_a are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D28,D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,GL04,GL09,L52,L53,L54]

Solving time: 9.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point E_{c} 95 56.36
```

```
point H_{a} 80 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_r E_{c}
```

```
cmark_r H_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{a} and H_{a} are not the same
```

```
% Constructing a line h_{a} which passes through point E_{a} and point H_{a}
```

```
line h_{a} E_{a} H_{a}
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```
% NDG: points H_{a} and E_{c} are not the same
```

```
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H_{a}
```

```
circle k(E_{c},C) E_{c} H_{a}
```

```
color 200 200 200
```

```
drawcircle k(E_{c},C)
```

```
color 0 0 0
```

```
% NDG: line h_{a} and circle k(E_{c},C) intersect% DET: points H_{a} and H must be different
```

```
% Constructing a point P_{\_G159129} which is a foot of the point E_{c} on the line h_{a}
```

```
foot P_{\_G159129} E_{c} h_{a}
```

```
cmark_r P_{\_G159129}
```

```
color 200 200 200
```

```
drawline E_{c} P_{\_G159129}
```

```
color 0 0 0
```

```

% Constructing a point H which is an image of the point H_{a} in the symmetry to point/line P_{\_G
159129}
sim H P_{\_G159129} H_{a}
cmark_rt H

% Constructing a point A such that HA/HE_{a}=2
towards A H E_{a} 2
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

% DET: points H_{a} and C are not the same
% Constructing a line a which passes through point H_{a} and point C
line a H_{a} C

color 200 200 200
drawline a
color 0 0 0

% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(E_{c},C) intersect% DET: points C and H_{b} must be different
% Constructing a point P_{\_G159566} which is a foot of the point E_{c} on the line b
foot P_{\_G159566} E_{c} b
cmark_r P_{\_G159566}
color 200 200 200
drawline E_{c} P_{\_G159566}
color 0 0 0

% Constructing a point H_{b} which is an image of the point C in the symmetry to point/line P_{\_G
159566}
sim H_{b} P_{\_G159566} C

```

```

cmark_l H_{b}

% DET: points H and H_{b} are not the same
% Constructing a line h_{b} which passes through point H and point H_{b}
line h_{b} H H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines a and h_{b} are not parallel% DET: lines a and h_{b} are not the same
% Constructing a point B which belongs to line a and line h_{b}
intersec B a h_{b}
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and h_{b} are not parallel; line b and circle k(E_{c},C)
% intersect; line h_{a} and circle k(E_{c},C) intersect; points H_{a} and E_{c} are not the same
% Determination conditions: lines a and h_{b} are not the same; points H and H_{b} are not the same
% ; points C and H_{b} must be different; points A and C are not the same; points H_{a} and C are
% not the same; points H_{a} and H must be different; points E_{a} and H_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $E_c = \neg E_c$

Proving failed

4.1.3 Proving $H_a = \neg H_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

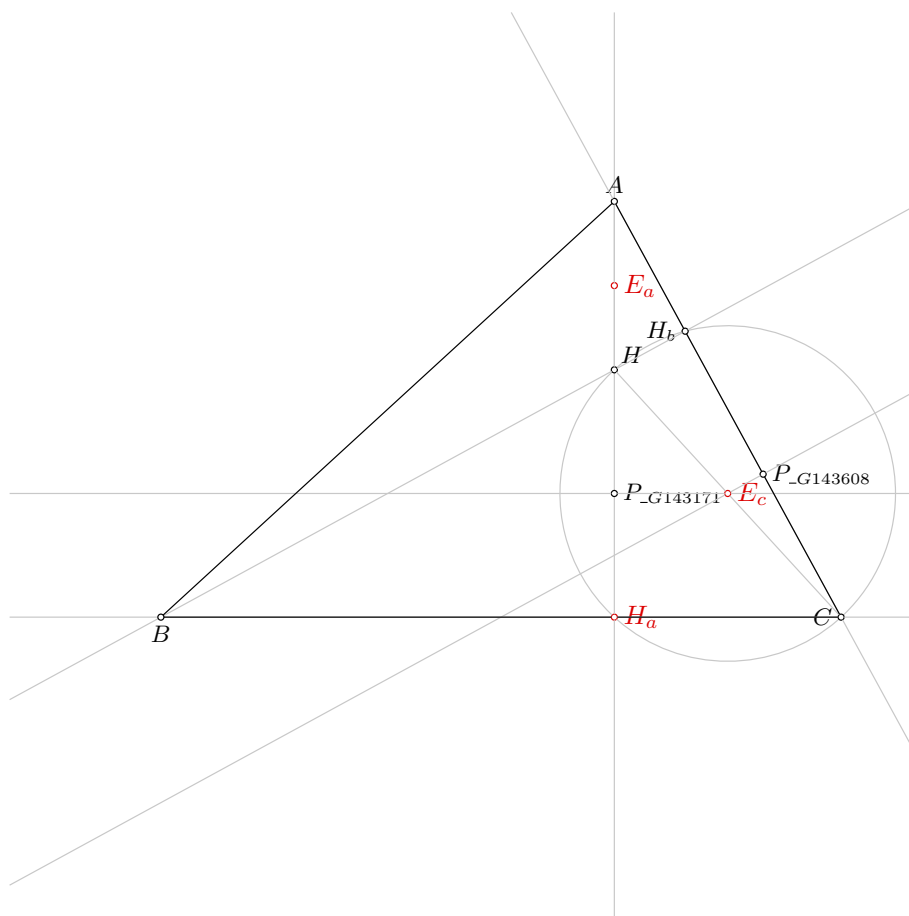


Figure 1: Illustration of the problem 0764

4.2.2 Proving $E_c = E_c$

Proving failed

4.2.3 Proving $H_a = H_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = E_a$

Proving failed

4.3.2 Proving $E_c = E_c$

Proving failed

4.3.3 Proving $H_a = H_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = E_a$

Proving failed

4.4.2 Proving $E_c = E_c$

Proving failed

4.4.3 Proving $H_a = H_a$

Proving failed

Problem 765

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 765: Given a point E_a , a point E_c and a point H_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H_b and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_b and E_a are not the same;
2. Using the point H_b and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_b and E_c are not the same;
3. Using the circle $k(E_a, A)$, the circle $k(E_c, C)$, the point H_b , the point E_a and the point E_c , construct a point H (rule W08); % NDG: circles $k(E_a, A)$ and $k(E_c, C)$ intersect % DET: circles $k(E_a, A)$ and $k(E_c, C)$ are not the same; points H_b and H must be different;
4. Using the point H and the point E_a , construct a point A (rule W01); ;
5. Using the point E_c and the point H , construct a point C (rule W01); ;
6. Using the point E_c and the point H , construct a line h_c (rule W02); % DET: points E_c and H are not the same;
7. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
8. Using the circle $k(E_a, A)$, the line h_c , the point E_a and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_a, A)$ intersect % DET: points H and H_c must be different;
9. Using the point A and the point H_c , construct a line c (rule W02); % DET: points A and H_c are not the same;
10. Using the line h_b and the line c , construct a point B (rule W03); % NDG: lines h_b and c are not parallel % DET: lines h_b and c are not the same.

Non-degenerate conditions: lines h_b and c are not parallel; line h_c and circle $k(E_a, A)$ intersect; circles $k(E_a, A)$ and $k(E_c, C)$ intersect; points H_b and E_c are not the same; points H_b and E_a are not the same.

Determination conditions: lines h_b and c are not the same; points A and H_c are not the same; points H and H_c must be different; points H_b and H are not the same; points E_c and H are not the same; circles $k(E_a, A)$ and $k(E_c, C)$ are not the same; points H_b and H must be different.

Rules used: [W01,W02,W03,W05,W06,W08]

Lemmas used: [D10,D28,D3,D30,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L46,L47,L48,L52,L53]

Solving time: 10.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
point E_{c} 95 56.36
point H_{b} 89.36 77.83
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{a}
cmark_r E_{c}
cmark_l H_{b}
color 0 0 0
fontsize 8
```

```
% NDG: points H_{b} and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H
_{b}
circle k(E_{a},A) E_{a} H_{b}
```

```
color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0
```

```
% NDG: points H_{b} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H
_{b}
circle k(E_{c},C) E_{c} H_{b}
```

```
color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0
```

```
% NDG: circles k(E_{a},A) and k(E_{c},C) intersect% DET: circles k(E_{a},A) and k(E_{c},C) are not
the same; points H_{b} and H must be different
% Constructing a line L_{\_G189475} which passes through point E_{a} and point E_{c}
line L_{\_G189475} E_{a} E_{c}
```

```

color 200 200 200
drawline L_{\_G189475}
color 0 0 0

% Constructing a point H which is an image of the point H_{b} in the symmetry to point/line L_{\_G
189475}
sim H L_{\_G189475} H_{b}
cmark_rt H

% Constructing a point A such that HA/HE_{a}=2
towards A H E_{a} 2
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

% DET: points E_{c} and H are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H
line h_{c} E_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{c} and circle k(E_{a},A) intersect% DET: points H and H_{c} must be different
% Constructing a point P_{\_G189892} which is a foot of the point E_{a} on the line h_{c}
foot P_{\_G189892} E_{a} h_{c}
cmark_r P_{\_G189892}
color 200 200 200
drawline E_{a} P_{\_G189892}

```

```

color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $H$  in the symmetry to point/line  $P_{\{\backslash\_G$ 
189892}
sim  $H_{\{c\}}$   $P_{\{\backslash\_G189892\}}$   $H$ 
cmark_rt  $H_{\{c\}}$ 

% DET: points  $A$  and  $H_{\{c\}}$  are not the same
% Constructing a line  $c$  which passes through point  $A$  and point  $H_{\{c\}}$ 
line c  $A$   $H_{\{c\}}$ 

color 200 200 200
drawline c
color 0 0 0

% NDG: lines  $h_{\{b\}}$  and  $c$  are not parallel% DET: lines  $h_{\{b\}}$  and  $c$  are not the same
% Constructing a point  $B$  which belongs to line  $h_{\{b\}}$  and line  $c$ 
intersec B  $h_{\{b\}}$  c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{b\}}$  and  $c$  are not parallel; line  $h_{\{c\}}$  and circle  $k(E_{\{a\}},A)$ 
intersect; circles  $k(E_{\{a\}},A)$  and  $k(E_{\{c\}},C)$  intersect; points  $H_{\{b\}}$  and  $E_{\{c\}}$  are not the same
; points  $H_{\{b\}}$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $h_{\{b\}}$  and  $c$  are not the same; points  $A$  and  $H_{\{c\}}$  are not the same
; points  $H$  and  $H_{\{c\}}$  must be different; points  $H_{\{b\}}$  and  $H$  are not the same; points  $E_{\{c\}}$  and  $H$ 
are not the same; circles  $k(E_{\{a\}},A)$  and  $k(E_{\{c\}},C)$  are not the same; points  $H_{\{b\}}$  and  $H$  must
be different

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $E_c = \neg E_c$

Proving failed

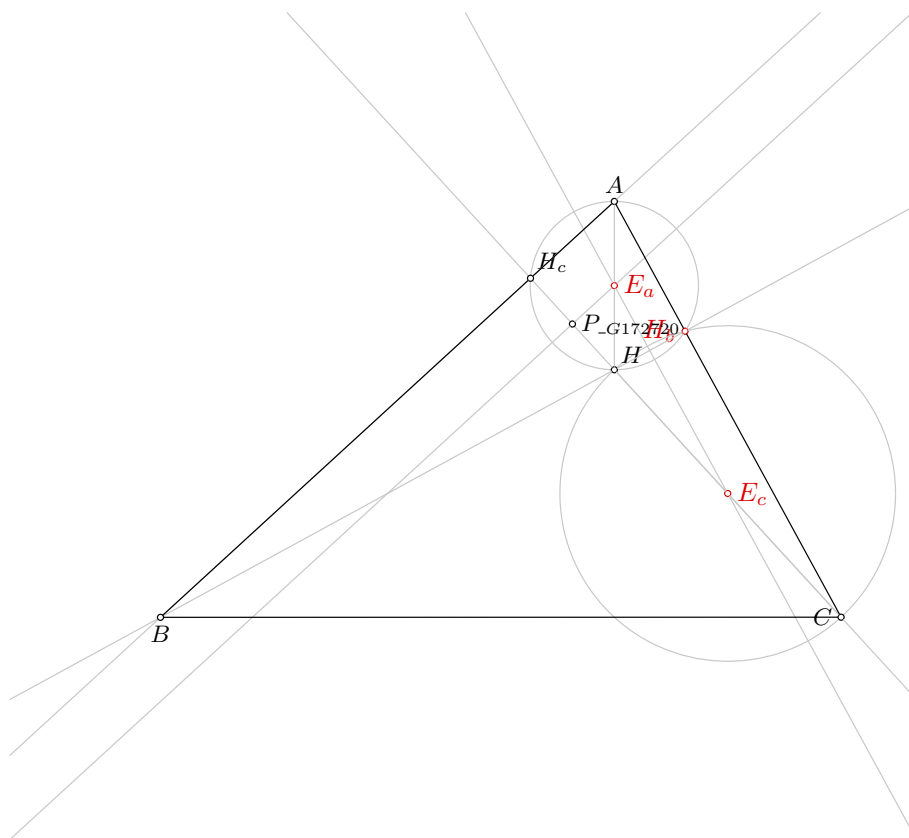


Figure 1: Illustration of the problem 0765

4.1.3 Proving $H_b = \neg H_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $E_c = \neg E_c$

Proving failed

4.2.3 Proving $H_b = \neg H_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $E_c = \neg E_c$

Proving failed

4.3.3 Proving $H_b = \neg H_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $E_c = \neg E_c$

Proving failed

4.4.3 Proving $H_b = \neg H_b$

Proving failed

Problem 766

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 766: Given a point E_a , a point E_c and a point H_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;
2. Using the point H_c and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_c and E_a are not the same;
3. Using the circle $k(E_a, A)$, the line h_c , the point E_a and the point H_c , construct a point H (rule W05); % NDG: line h_c and circle $k(E_a, A)$ intersect % DET: points H_c and H must be different;
4. Using the point H and the point E_a , construct a point A (rule W01); ;
5. Using the point E_c and the point H , construct a point C (rule W01); ;
6. Using the point H_c and the point A , construct a line c (rule W02); % DET: points H_c and A are not the same;
7. Using the point A and the point C , construct a line b (rule W02); % DET: points A and C are not the same;
8. Using the circle $k(E_a, A)$, the line b , the point E_a and the point A , construct a point H_b (rule W05); % NDG: line b and circle $k(E_a, A)$ intersect % DET: points A and H_b must be different;
9. Using the point H and the point H_b , construct a line h_b (rule W02); % DET: points H and H_b are not the same;
10. Using the line c and the line h_b , construct a point B (rule W03); % NDG: lines c and h_b are not parallel % DET: lines c and h_b are not the same.

Non-degenerate conditions: lines c and h_b are not parallel; line b and circle $k(E_a, A)$ intersect; line h_c and circle $k(E_a, A)$ intersect; points H_c and E_a are not the same.

Determination conditions: lines c and h_b are not the same; points H and H_b are not the same; points A and H_b must be different; points A and C are not the same; points H_c and A are not the same; points H_c and H must be different; points E_c and H_c are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D28,D3,D30,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L46,L47,L48]

Solving time: 9.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
point E_{c} 95 56.36
point H_{c} 68.91 84.83
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{a}
cmark_r E_{c}
cmark_rt H_{c}
color 0 0 0
fontsize 8
```

```
% DET: points E_{c} and H_{c} are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H_{c}
line h_{c} E_{c} H_{c}
```

```
color 200 200 200
drawline h_{c}
color 0 0 0
```

```
% NDG: points H_{c} and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H_{c}
circle k(E_{a},A) E_{a} H_{c}
```

```
color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0
```

```
% NDG: line h_{c} and circle k(E_{a},A) intersect% DET: points H_{c} and H must be different
% Constructing a point P_{\_G218523} which is a foot of the point E_{a} on the line h_{c}
foot P_{\_G218523} E_{a} h_{c}
cmark_r P_{\_G218523}
color 200 200 200
drawline E_{a} P_{\_G218523}
color 0 0 0
```

```

% Constructing a point H which is an image of the point H_{c} in the symmetry to point/line P_{\_G
218523}
sim H P_{\_G218523} H_{c}
cmark_rt H

% Constructing a point A such that HA/HE_{a}=2
towards A H E_{a} 2
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

% DET: points H_{c} and A are not the same
% Constructing a line c which passes through point H_{c} and point A
line c H_{c} A

color 200 200 200
drawline c
color 0 0 0

% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(E_{a},A) intersect% DET: points A and H_{b} must be different
% Constructing a point P_{\_G218960} which is a foot of the point E_{a} on the line b
foot P_{\_G218960} E_{a} b
cmark_r P_{\_G218960}
color 200 200 200
drawline E_{a} P_{\_G218960}
color 0 0 0

% Constructing a point H_{b} which is an image of the point A in the symmetry to point/line P_{\_G
218960}
sim H_{b} P_{\_G218960} A

```



```

cmark_l H_{b}

% DET: points H and H_{b} are not the same
% Constructing a line h_{b} which passes through point H and point H_{b}
line h_{b} H H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines c and h_{b} are not parallel% DET: lines c and h_{b} are not the same
% Constructing a point B which belongs to line c and line h_{b}
intersec B c h_{b}
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and h_{b} are not parallel; line b and circle k(E_{a},A)
% intersect; line h_{c} and circle k(E_{a},A) intersect; points H_{c} and E_{a} are not the same
% Determination conditions: lines c and h_{b} are not the same; points H and H_{b} are not the same
% ; points A and H_{b} must be different; points A and C are not the same; points H_{c} and A are
% not the same; points H_{c} and H must be different; points E_{c} and H_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $E_c = \neg E_c$

Proving failed

4.1.3 Proving $H_c = \neg H_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

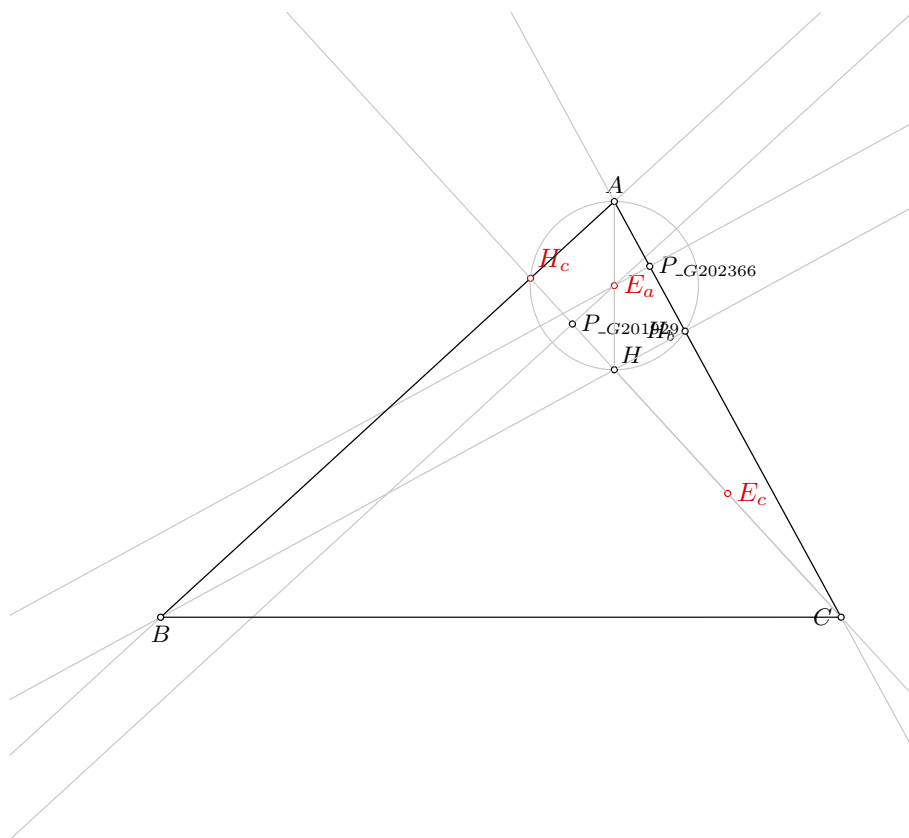


Figure 1: Illustration of the problem 0766

4.2.2 Proving $E_c = E_c$

Proving failed

4.2.3 Proving $H_c = H_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = E_a$

Proving failed

4.3.2 Proving $E_c = E_c$

Proving failed

4.3.3 Proving $H_c = H_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = E_a$

Proving failed

4.4.2 Proving $E_c = E_c$

Proving failed

4.4.3 Proving $H_c = H_c$

Proving failed

Problem 767

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 767: Given a point E_a , a point E_c and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 768

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 768: Given a point E_a , a point M_a and a point E_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point M_a , construct a line $m(H_bH_c)$ (rule W02); % DET: points E_a and M_a are not the same;
2. Using the point E_a and the point M_a , construct a line $m(E_aM_a)$ (rule W14); % DET: points E_a and M_a are not the same;
3. Using the line $m(E_aM_a)$ and the line $m(H_bH_c)$, construct a point N (rule W03); % NDG: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not parallel % DET: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not the same;
4. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
5. Choose freely a point E_c on the circle $k(N, M_a)$ (rule W0ncircle);
6. Using the point E_c and the point N , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and N are not the same;
7. Using the circle $k(N, M_a)$, the line $m(H_bH_a)$, the point N and the point E_c , construct a point M_c (rule W05a); % NDG: line $m(H_bH_a)$ and circle $k(N, M_a)$ intersect % DET: points E_c and M_c must be different;
8. Choose freely a point A (rule free);
9. Using the point A and the point E_a , construct a point H (rule W01); ;
10. Using the point A and the point M_c , construct a point B (rule W01); ;
11. Using the point E_c and the point H , construct a point C (rule W01); .

Non-degenerate conditions: line $m(H_bH_a)$ and circle $k(N, M_a)$ intersect; points E_a and N are not the same; lines $m(E_aM_a)$ and $m(H_bH_c)$ are not parallel.

Determination conditions: points E_c and M_c must be different; points E_c and N are not the same; lines $m(E_aM_a)$ and $m(H_bH_c)$ are not the same; points E_a and M_a are not the same; points E_a and M_a are not the same.

Rules used: [W01,W02,W03,W05a,W06,W14,WOncircle1,free]

Lemmas used: [D20,D28,D30,D32,GD02,GL01,GL03,GL04,L18,L19,L20,L21,L22,L24,L38,L39,L41,L42,L47,L48]

Solving time: 199.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point M_{a} 65 40
```

```
point E_{c} 95 56.36
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_r M_{a}
```

```
cmark_r E_{c}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{a} and M_{a} are not the same
```

```
% Constructing a line m(H_{b}H_{c}) which passes through point E_{a} and point M_{a}
```

```
line m(H_{b}H_{c}) E_{a} M_{a}
```

```
color 200 200 200
```

```
drawline m(H_{b}H_{c})
```

```
color 0 0 0
```

```
% DET: points E_{a} and M_{a} are not the same
```

```
% Constructing bisector m(E_{a}M_{a}) of the segment E_{a}M_{a}
```

```
med m(E_{a}M_{a}) E_{a} M_{a}
```

```
color 200 200 200
```

```
drawline m(E_{a}M_{a})
```

```
color 0 0 0
```

```
color 200 200 200
```

```
drawsegment E_{a} M_{a}
```

```
color 0 0 0
```

```
% NDG: lines m(E_{a}M_{a}) and m(H_{b}H_{c}) are not parallel% DET: lines m(E_{a}M_{a}) and m(H_{b}H_{c}) are not the same
```

```
% Constructing a point N which belongs to line m(E_{a}M_{a}) and line m(H_{b}H_{c})
```

```
intersec N m(E_{a}M_{a}) m(H_{b}H_{c})
```

```
cmark_r N
```

```

% NDG: points  $E_{\{a\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{a\}}$ 
circle k(N, M_{\{a\}}) N E_{\{a\}}

color 200 200 200
drawcircle k(N, M_{\{a\}})
color 0 0 0

% Choosing randomly a point  $E_{\{c\}}$  on the circle with center  $N$  through point  $E_{\{a\}}$ 
oncircle E_{\{c\}} N E_{\{a\}}
cmark_r E_{\{c\}}
color 200 200 200
drawcircle N E_{\{a\}}
color 0 0 0

% DET: points  $E_{\{c\}}$  and  $N$  are not the same
% Constructing a line  $m(H_{\{b\}}H_{\{a\}})$  which passes through point  $E_{\{c\}}$  and point  $N$ 
line m(H_{\{b\}}H_{\{a\}}) E_{\{c\}} N

color 200 200 200
drawline m(H_{\{b\}}H_{\{a\}})
color 0 0 0

% NDG: line  $m(H_{\{b\}}H_{\{a\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{c\}}$  and  $M_{\{c\}}$  must be
different
% Constructing a point  $M_{\{c\}}$  which is an image of the point  $E_{\{c\}}$  in the symmetry to point/line  $N$ 
sim M_{\{c\}} N E_{\{c\}}
cmark_lt M_{\{c\}}

% Constructing a free point  $A$ 
point A 80 95

cmark_t A

% Constructing a point  $H$  such that  $AH/AE_{\{a\}}=2$ 
towards H A E_{\{a\}} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% Constructing a point  $B$  such that  $AB/AM_{\{c\}}=2$ 

```

```

towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

```

```

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

% Non-degenerate conditions: line $m(H_{\{b\}}H_{\{a\}})$ and circle $k(N, M_{\{a\}})$ intersect; points $E_{\{a\}}$ and N are not the same; lines $m(E_{\{a\}}M_{\{a\}})$ and $m(H_{\{b\}}H_{\{c\}})$ are not parallel

% Determination conditions: points $E_{\{c\}}$ and $M_{\{c\}}$ must be different; points $E_{\{c\}}$ and N are not the same; lines $m(E_{\{a\}}M_{\{a\}})$ and $m(H_{\{b\}}H_{\{c\}})$ are not the same; points $E_{\{a\}}$ and $M_{\{a\}}$ are not the same; points $E_{\{a\}}$ and $M_{\{a\}}$ are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = E_a$

Proving failed

4.1.2 Proving $M_a = M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 15 terms.

Time Complexity: Time spent by the prover is 0.326 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $E_c = E_c$

Proving failed

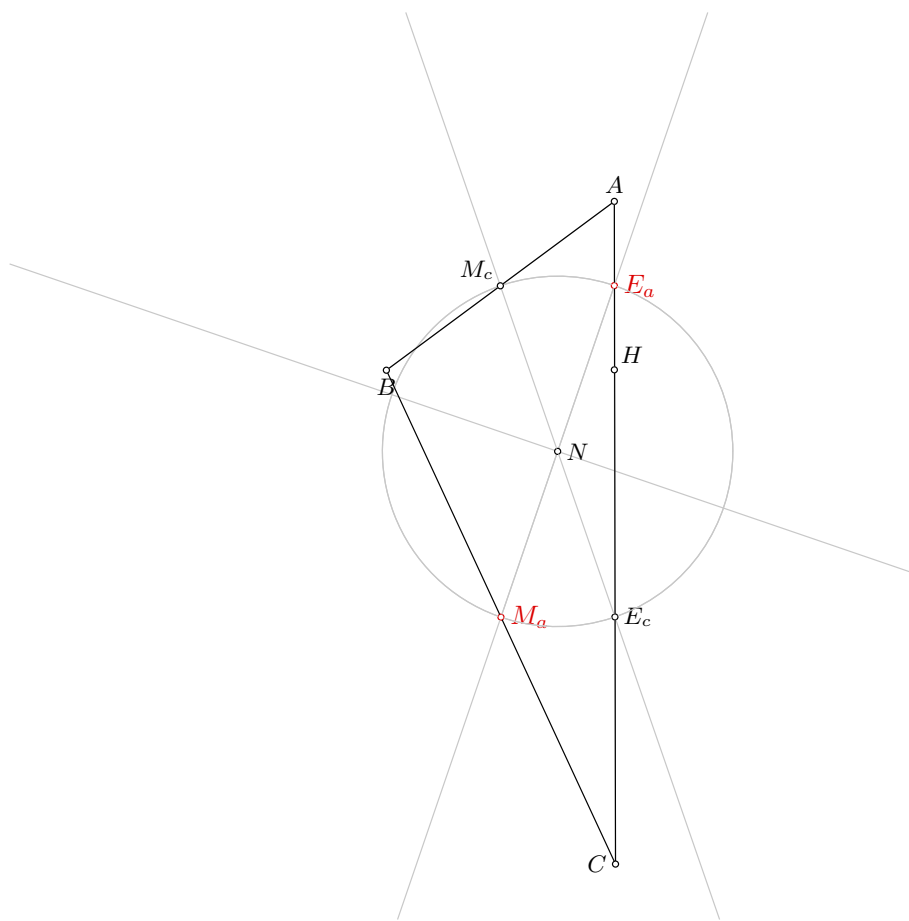


Figure 1: Illustration of the problem 0768

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $M_a = \neg M_a$

Proving failed

4.2.3 Proving $E_c = \neg E_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $M_a = \neg M_a$

Proving failed

4.3.3 Proving $E_c = \neg E_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $M_a = \neg M_a$

Proving failed

4.4.3 Proving $E_c = \neg E_c$

Proving failed

Problem 769

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 769: Given a point E_a , a point E_c and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point M_b , construct a line $m(CH_a)$ (rule W02); % DET: points E_c and M_b are not the same;
2. Using the point E_a and the point E_c , construct a line $m(E_aE_c)$ (rule W14); % DET: points E_a and E_c are not the same;
3. Using the point E_a and the point M_b , construct a line $m(E_aM_b)$ (rule W14); % DET: points E_a and M_b are not the same;
4. Using the line $m(E_aM_b)$ and the line $m(E_aE_c)$, construct a point N (rule W03); % NDG: lines $m(E_aM_b)$ and $m(E_aE_c)$ are not parallel % DET: lines $m(E_aM_b)$ and $m(E_aE_c)$ are not the same;
5. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
6. Using the point E_a and the line $m(CH_a)$, construct a line h_a (rule W16); ;
7. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
8. Using the point H_a and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_a and E_c are not the same;
9. Using the circle $k(E_c, C)$, the line h_a , the point E_c and the point H_a , construct a point H (rule W05); % NDG: line h_a and circle $k(E_c, C)$ intersect % DET: points H_a and H must be different;

10. Using the point H and the point E_a , construct a point A (rule W01); ;
11. Using the point E_c and the point H , construct a point C (rule W01); ;
12. Using the point N and the point H , construct a point G (rule W01); ;
13. Using the point M_b and the point G , construct a point B (rule W01); .

Non-degenerate conditions: line h_a and circle $k(E_c, C)$ intersect; points H_a and E_c are not the same; line h_a and circle $k(N, M_a)$ intersect; points E_a and N are not the same; lines $m(E_a M_b)$ and $m(E_a E_c)$ are not parallel.

Determination conditions: points H_a and H must be different; points E_a and H_a must be different; lines $m(E_a M_b)$ and $m(E_a E_c)$ are not the same; points E_a and M_b are not the same; points E_a and E_c are not the same; points E_c and M_b are not the same.

Rules used: [W01,W02,W03,W05,W06,W14,W16]

Lemmas used: [D28,D3,D30,D32,D5,D8,GD02,GL01,GL03,GL04,GL09,L16,L17,L19,L22,L24,L44,L52,L54,L56]

Solving time: 25.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
point E_{c} 95 56.36
point M_{b} 95 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{a}
cmark_r E_{c}
cmark_lt M_{b}
color 0 0 0
fontsize 8
```

```
% DET: points E_{c} and M_{b} are not the same
% Constructing a line m(CH_{a}) which passes through point E_{c} and point M_{b}
line m(CH_{a}) E_{c} M_{b}
```

```
color 200 200 200
drawline m(CH_{a})
color 0 0 0
```

```
% DET: points E_{a} and E_{c} are not the same
% Constructing bisector m(E_{a}E_{c}) of the segment E_{a}E_{c}
med m(E_{a}E_{c}) E_{a} E_{c}
```

```
color 200 200 200
drawline m(E_{a}E_{c})
color 0 0 0
```

```

color 200 200 200
drawsegment E_{a} E_{c}
color 0 0 0

% DET: points E_{a} and M_{b} are not the same
% Constructing bisector m(E_{a}M_{b}) of the segment E_{a}M_{b}
med m(E_{a}M_{b}) E_{a} M_{b}

color 200 200 200
drawline m(E_{a}M_{b})
color 0 0 0

color 200 200 200
drawsegment E_{a} M_{b}
color 0 0 0

% NDG: lines m(E_{a}M_{b}) and m(E_{a}E_{c}) are not parallel% DET: lines m(E_{a}M_{b}) and m(E_{a}
E_{c}) are not the same
% Constructing a point N which belongs to line m(E_{a}M_{b}) and line m(E_{a}E_{c})
intersec N m(E_{a}M_{b}) m(E_{a}E_{c})
cmark_r N

% NDG: points E_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{a}
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Constructing a line h_{a} which contains the point E_{a} and is parallel to the line m(CH_{a})
parallel h_{a} E_{a} m(CH_{a})

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G50558} which is a foot of the point N on the line h_{a}
foot P_{\_G50558} N h_{a}
cmark_r P_{\_G50558}
color 200 200 200
drawline N P_{\_G50558}
color 0 0 0

% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
\_G50558}
sim H_{a} P_{\_G50558} E_{a}

```

```

cmark_r H_{a}

% NDG: points H_{a} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H
_{a}
circle k(E_{c},C) E_{c} H_{a}

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% NDG: line h_{a} and circle k(E_{c},C) intersect% DET: points H_{a} and H must be different
% Constructing a point P_{\_G50862} which is a foot of the point E_{c} on the line h_{a}
foot P_{\_G50862} E_{c} h_{a}
cmark_r P_{\_G50862}
color 200 200 200
drawline E_{c} P_{\_G50862}
color 0 0 0

% Constructing a point H which is an image of the point H_{a} in the symmetry to point/line P_{\_G
50862}
sim H P_{\_G50862} H_{a}
cmark_rt H

% Constructing a point A such that HA/HE_{a}=2
towards A H E_{a} 2
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

% Constructing a line L_{\_G51124} which passes through point N and point H
line L_{\_G51124} N H

color 200 200 200
drawline L_{\_G51124}
color 0 0 0

% Constructing a point P_{\_G51225} with coordinates (0,0)

```

```

point P_{\_G51225} 0 0
cmark_r P_{\_G51225}

% Constructing a point P_{\_G51149} such that NP_{\_G51149}/NP_{\_G51225}=-1
towards P_{\_G51149} N P_{\_G51225} -1
cmark_r P_{\_G51149}
color 200 200 200
drawsegment P_{\_G51225} P_{\_G51149}
color 0 0 0

% Constructing a point P_{\_G51194} such that NP_{\_G51194}/NP_{\_G51225}=3
towards P_{\_G51194} N P_{\_G51225} 3
cmark_r P_{\_G51194}
color 200 200 200
drawsegment N P_{\_G51194}
color 0 0 0

% Constructing a line L_{\_G51155} which passes through point H and point P_{\_G51194}
line L_{\_G51155} H P_{\_G51194}

color 200 200 200
drawline L_{\_G51155}
color 0 0 0

% Constructing a line L_{\_G51118} which contains the point P_{\_G51149} and is parallel to the
line L_{\_G51155}
parallel L_{\_G51118} P_{\_G51149} L_{\_G51155}

color 200 200 200
drawline L_{\_G51118}
color 0 0 0

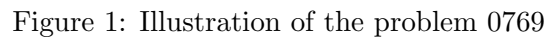
% Constructing a point G which belongs to line L_{\_G51118} and line L_{\_G51124}
intersec G L_{\_G51118} L_{\_G51124}
cmark_t G

% Constructing a point B such that M_{b}B/M_{b}G=3
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line h_{a} and circle k(E_{c},C) intersect; points H_{a} and E_{c} are
not the same; line h_{a} and circle k(N,M_{a}) intersect; points E_{a} and N are not the same;
lines m(E_{a}M_{b}) and m(E_{a}E_{c}) are not parallel

```



3.3 Illustration

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = -E_a$

NDG conditions Points M_b and E_c are not identical

Points M_b and E_c are not identical

Points M_b and E_c are not identical

Line through points M_b and P_{G42424} is not perpendicular to line through points P_{G42424} and E_c

Points E_c , P_{G42424} and E_a are not collinear

Points A , B and C are not collinear

Line through points E_c and B is not perpendicular to line through points B and E_a

4.1.2 Proving $E_c = E_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 5618 terms.

Time Complexity: Time spent by the prover is 15.75 seconds.

NDG conditions Points M_b and E_c are not identical

Points M_b and E_c are not identical

Points M_b and E_c are not identical

Line through points P_{G44474} and M_b is not perpendicular to line through points M_b and E_a

Points P_{G44474} and E_a are not identical

Points A , B and C are not collinear

Line through points E_c and B is not perpendicular to line through points B and E_a

4.1.3 Proving $M_b = M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 17 terms.

Time Complexity: Time spent by the prover is 1.147 seconds.

NDG conditions Points M_b and E_c are not identical

Points M_b and E_c are not identical

Points M_b and E_c are not identical

Line through points M_b and P_{G46524} is not perpendicular to line through points P_{G46524} and E_c

Points E_c , P_{G46524} and E_a are not collinear

4.2 GCLC - Area method

4.2.1 Proving $E_a = E_a$

Proving failed

4.2.2 Proving $E_c = E_c$

Proving failed

4.2.3 Proving $M_b = M_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = E_a$

Proving failed

4.3.2 Proving $E_c = \neg E_c$

Proving failed

4.3.3 Proving $M_b = \neg M_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $E_c = \neg E_c$

Proving failed

4.4.3 Proving $M_b = \neg M_b$

Proving failed

Problem 770

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 770: Given a point E_c , a point M_c and a point E_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point M_c , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and M_c are not the same;
2. Using the point E_c and the point M_c , construct a line $m(E_cM_c)$ (rule W14); % DET: points E_c and M_c are not the same;
3. Using the line $m(E_cM_c)$ and the line $m(H_bH_a)$, construct a point N (rule W03); % NDG: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel % DET: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same;
4. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
5. Choose freely a point E_a on the circle $k(N, M_a)$ (rule WOncircle);
6. Choose freely a point A (rule free);
7. Using the point A and the point E_a , construct a point H (rule W01); ;
8. Using the point A and the point M_c , construct a point B (rule W01); ;
9. Using the point E_c and the point H , construct a point C (rule W01); .

Non-degenerate conditions: points E_c and N are not the same; lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel.

Determination conditions: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same; points E_c and M_c are not the same; points E_c and M_c are not the same.

Rules used: [W01,W02,W03,W06,W14,WOncircle1,free]

Lemmas used: [D20,D28,D30,D32,GD02,GL01,GL03,GL04,L18,L19,L20,L24,L41,L42,L53,L54]

Solving time: 200.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{c} 95 56.36
point M_{c} 50 67.5
point E_{a} 80 83.86

color 220 0 0
fontsize 9

cmark_r E_{c}
cmark_lt M_{c}
cmark_r E_{a}
color 0 0 0
fontsize 8

% DET: points E_{c} and M_{c} are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point M_{c}
line m(H_{b}H_{a}) E_{c} M_{c}

color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0

% DET: points E_{c} and M_{c} are not the same
% Constructing bisector m(E_{c}M_{c}) of the segment E_{c}M_{c}
med m(E_{c}M_{c}) E_{c} M_{c}

color 200 200 200
drawline m(E_{c}M_{c})
color 0 0 0

color 200 200 200
drawsegment E_{c} M_{c}
color 0 0 0

% NDG: lines m(E_{c}M_{c}) and m(H_{b}H_{a}) are not parallel% DET: lines m(E_{c}M_{c}) and m(H_{b}H_{a}) are not the same
% Constructing a point N which belongs to line m(E_{c}M_{c}) and line m(H_{b}H_{a})
intersec N m(E_{c}M_{c}) m(H_{b}H_{a})
cmark_r N

% NDG: points E_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{c}
circle k(N,M_{a}) N E_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```

% Choosing randomly a point E_{a} on the circle with center N through point E_{c}
oncircle E_{a} N E_{c}
cmark_r E_{a}
color 200 200 200
drawcircle N E_{c}
color 0 0 0

% Constructing a free point A
point A 80 95

cmark_t A

% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: points E_{c} and N are not the same; lines m(E_{c}M_{c}) and m(H_{b}H_{a}) are not parallel
% Determination conditions: lines m(E_{c}M_{c}) and m(H_{b}H_{a}) are not the same; points E_{c} and M_{c} are not the same; points E_{c} and M_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

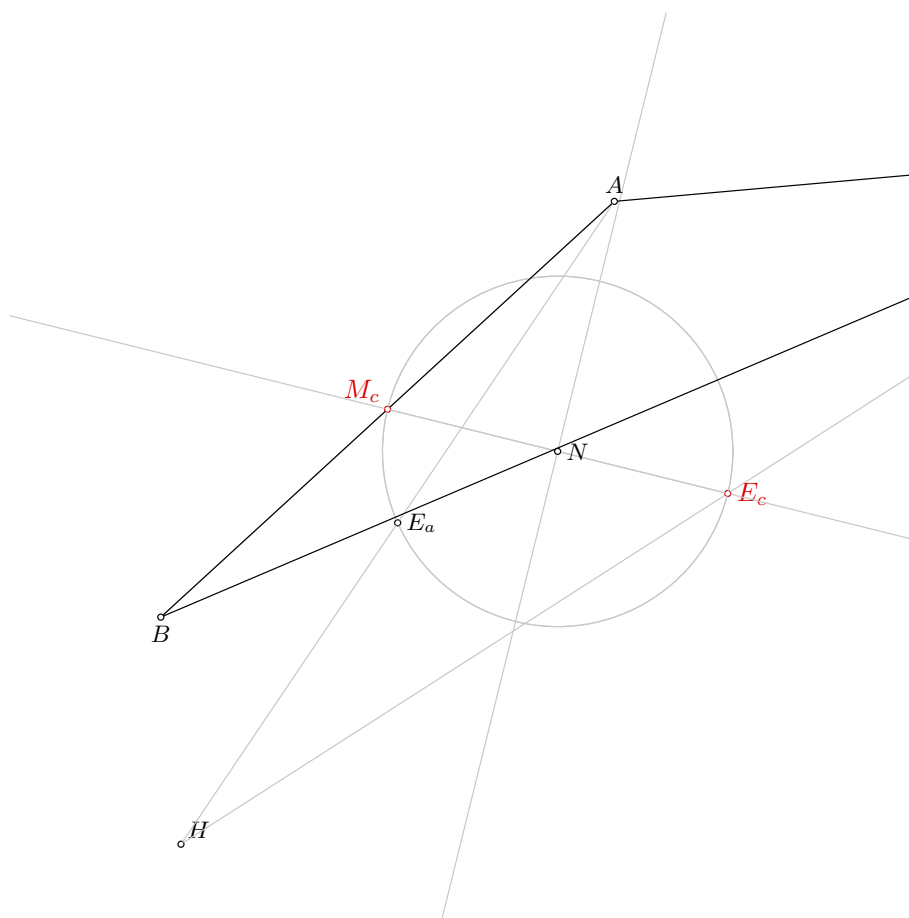


Figure 1: Illustration of the problem 0770

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $M_c = \neg M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.051 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $E_a = \neg E_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{m(E_c M_c) E_c M_c}^0 \neq S_{m(E_c M_c) E_c M_c}^1$ i.e., lines $M_{m(E_c M_c)}^0 T_{m(E_c M_c)}^1$ and $E_c M_c$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^3} \neq S_{F_{h_a}^2 BF_{h_b}^3}$ i.e., lines $AF_{h_a}^2$ and $BF_{h_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.2.3 Proving $E_a = \neg E_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $M_c = \neg M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 19 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.3.3 Proving $E_a = E_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = E_c$

Proving failed

4.4.2 Proving $M_c = M_c$

Proving failed

4.4.3 Proving $E_a = E_a$

Proving failed

Problem 771

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 771: Given a point E_c , a point N and a point E_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point N , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and N are not the same;
2. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
3. Choose freely a point E_a on the circle $k(N, M_a)$ (rule WOncircle);
4. Using the circle $k(N, M_a)$, the line $m(H_bH_a)$, the point N and the point E_c , construct a point M_c (rule W05a); % NDG: line $m(H_bH_a)$ and circle $k(N, M_a)$ intersect % DET: points E_c and M_c must be different;
5. Choose freely a point A (rule free);
6. Using the point A and the point E_a , construct a point H (rule W01); ;
7. Using the point A and the point M_c , construct a point B (rule W01); ;
8. Using the point E_c and the point H , construct a point C (rule W01); .

Non-degenerate conditions: line $m(H_bH_a)$ and circle $k(N, M_a)$ intersect; points E_c and N are not the same.

Determination conditions: points E_c and M_c must be different; points E_c and N are not the same.

Rules used: [W01,W02,W05a,W06,WOncircle1,free]

Lemmas used: [D20,D28,D30,D32,GD02,GL01,GL03,GL04,L18,L19,L20,L24,L41,L42,L53,L54]

Solving time: 148.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{c} 95 56.36
point N 72.5 61.93
point E_{a} 80 83.86

color 220 0 0
fontsize 9

cmark_r E_{c}
cmark_r N
cmark_r E_{a}
color 0 0 0
fontsize 8

% DET: points E_{c} and N are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point N
line m(H_{b}H_{a}) E_{c} N

color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0

% NDG: points E_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{c}
circle k(N,M_{a}) N E_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point E_{a} on the circle with center N through point E_{c}
oncircle E_{a} N E_{c}
cmark_r E_{a}
color 200 200 200
drawcircle N E_{c}
color 0 0 0

% NDG: line m(H_{b}H_{a}) and circle k(N,M_{a}) intersect% DET: points E_{c} and M_{c} must be
different
% Constructing a point M_{c} which is an image of the point E_{c} in the symmetry to point/line N
sim M_{c} N E_{c}
cmark_lt M_{c}

% Constructing a free point A
point A 80 95
```

```
cmark_t A
```

```
% Constructing a point H such that  $AH/AE_{\{a\}}=2$ 
towards H A  $E_{\{a\}}$  2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0
```

```
% Constructing a point B such that  $AB/AM_{\{c\}}=2$ 
towards B A  $M_{\{c\}}$  2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0
```

```
% Constructing a point C such that  $E_{\{c\}}C/E_{\{c\}}H=-1$ 
towards C  $E_{\{c\}}$  H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line  $m(H_{\{b\}}H_{\{a\}})$  and circle  $k(N, M_{\{a\}})$  intersect; points  $E_{\{c\}}$  and  $N$ 
are not the same
% Determination conditions: points  $E_{\{c\}}$  and  $M_{\{c\}}$  must be different; points  $E_{\{c\}}$  and  $N$  are not
the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

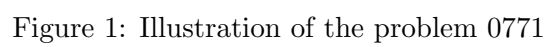
3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = _E_c$

Proving failed



4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $E_a=_Ea$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c=_Ec$

Proving failed

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $E_a=_Ea$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c=_Ec$

Proving failed

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $E_a=_Ea$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c=_Ec$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $E_a=_Ea$

Proving failed

Problem 772

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 772: Given a point E_a , a point E_c and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 773

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 773: Given a point E_a , a point E_c and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 774

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 774: Given a point E_a , a point E_c and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 775

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 775: Given a point E_a , a point E_c and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 776

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 776: Given a point E_a , a point G and a point H , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H , construct a point A (rule W01); ;
2. Using the point G and the point H , construct a point N (rule W01); ;
3. Using the point G and the point H , construct a point O (rule W01); ;
4. Using the point G and the point A , construct a point M_a (rule W01); ;
5. Using the point E_a and the point H , construct a line h_a (rule W02); % DET: points E_a and H are not the same;
6. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
8. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
9. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; line h_a and circle $k(N, M_a)$ intersect; points E_a and N are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points E_a and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L16,L19,L22,L55,L58]

Solving time: 7.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point G 70 58.33
```

```
point H 80 72.73
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_t G
```

```
cmark_rt H
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point A such that E_{a}A/E_{a}H=-1
```

```
towards A E_{a} H -1
```

```
cmark_t A
```

```
color 200 200 200
```

```
drawsegment H A
```

```
color 0 0 0
```

```
% Constructing a point N such that GN/GH=0.25
```

```
towards N G H 0.25
```

```
cmark_r N
```

```
color 200 200 200
```

```
drawsegment G H
```

```
color 0 0 0
```

```
% Constructing a point O such that GO/GH=-0.5
```

```
towards O G H -0.5
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment H O
```

```
color 0 0 0
```

```
% Constructing a point M_{a} such that GM_{a}/GA=-0.5
```

```

towards M_{a} G A -0.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% DET: points E_{a} and H are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H
line h_{a} E_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points E_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{a}
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G152129} which is a foot of the point N on the line h_{a}
foot P_{\_G152129} N h_{a}
cmark_r P_{\_G152129}
color 200 200 200
drawline N P_{\_G152129}
color 0 0 0

% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
\_G152129}
sim H_{a} P_{\_G152129} E_{a}
cmark_r H_{a}

% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200

```

```
drawcircle k(O,C)
color 0 0 0
```

```
% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% line h_{a} and circle k(N,M_{a}) intersect; points E_{a} and N are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
% different; points E_{a} and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = _E_a$

Proving failed

4.1.2 Proving $G = _G$

Proving failed

4.1.3 Proving $H = _H$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = _E_a$

Proving failed

4.2.2 Proving $G = _G$

Proving failed

4.2.3 Proving $H = _H$

Proving failed

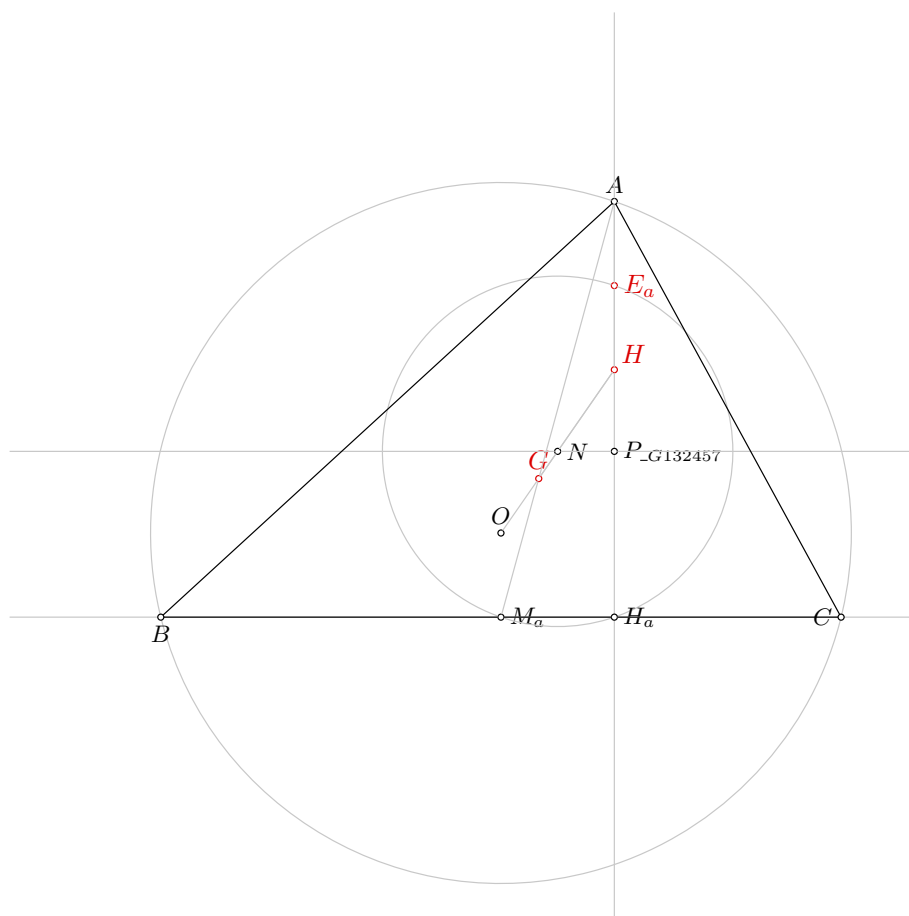


Figure 1: Illustration of the problem 0776

4.3 GCLC - Wu method

4.3.1 Proving $E_a = _E_a$

Proving failed

4.3.2 Proving $G = _G$

Proving failed

4.3.3 Proving $H = _H$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = _E_a$

Proving failed

4.4.2 Proving $G = _G$

Proving failed

4.4.3 Proving $H = _H$

Proving failed

Problem 777

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 777: Given a point E_a , a point G and a point H_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;
2. Using the point H_a and the line h_a , construct a line a (rule W10a); ;
3. Using the point G , the line h_a and the point E_a , construct a line $h_{G,-1/2}(h_a)$ (rule W15); ;
4. Using the line $h_{G,-1/2}(h_a)$ and the line a , construct a point M_a (rule W03); % NDG: lines $h_{G,-1/2}(h_a)$ and a are not parallel % DET: lines $h_{G,-1/2}(h_a)$ and a are not the same;
5. Using the point M_a and the point G , construct a point A (rule W01); ;
6. Using the point E_a and the point A , construct a point H (rule W01); ;
7. Using the point G and the point H , construct a point O (rule W01); ;
8. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
9. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; lines $h_{G,-1/2}(h_a)$ and a are not parallel.

Determination conditions: lines $h_{G,-1/2}(h_a)$ and a are not the same; points E_a and H_a are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W15]

Lemmas used: [D21,D26,D28,D3,D5,D8,GD01,GL02,GL03,GL04,GL09,L11,L12,L55,L58]

Solving time: 4.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{a} 80 83.86
point G 70 58.33
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_r E_{a}
cmark_t G
cmark_r H_{a}
color 0 0 0
fontsize 8

% DET: points E_{a} and H_{a} are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H_{a}
line h_{a} E_{a} H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
perp a H_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0

% Constructing a point P_{\_G178374} such that GP_{\_G178374}/GE_{a}=-0.5
towards P_{\_G178374} G E_{a} -0.5
cmark_r P_{\_G178374}
color 200 200 200
drawsegment E_{a} P_{\_G178374}
color 0 0 0

% Constructing a line h_{G,-1/2}(h_{a}) which contains the point P_{\_G178374} and is parallel to
the line h_{a}
parallel h_{G,-1/2}(h_{a}) P_{\_G178374} h_{a}

color 200 200 200
drawline h_{G,-1/2}(h_{a})
color 0 0 0
```

```

% NDG: lines  $h_{\{G,-1/2\}}(h_{\{a\}})$  and  $a$  are not parallel% DET: lines  $h_{\{G,-1/2\}}(h_{\{a\}})$  and  $a$  are not
the same
% Constructing a point  $M_{\{a\}}$  which belongs to line  $h_{\{G,-1/2\}}(h_{\{a\}})$  and line  $a$ 
intersec  $M_{\{a\}}$   $h_{\{G,-1/2\}}(h_{\{a\}})$   $a$ 
cmark_r  $M_{\{a\}}$ 

% Constructing a point  $A$  such that  $M_{\{a\}}A/M_{\{a\}}G=3$ 
towards A  $M_{\{a\}}$  G 3
cmark_t A
color 200 200 200
drawsegment  $M_{\{a\}}$  A
color 0 0 0

% Constructing a point  $H$  such that  $E_{\{a\}}H/E_{\{a\}}A=-1$ 
towards H  $E_{\{a\}}$  A -1
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% Constructing a point  $O$  such that  $GO/GH=-0.5$ 
towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% NDG: points  $A$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $A$ 
circle  $k(O,C)$  O A

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $a$  and circle  $k(O,C)$  intersect
% Constructing points  $C$  and  $B$  which are in intersection of  $k(O,C)$  and  $a$ 
intersec2 C B  $k(O,C)$   $a$ 
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```

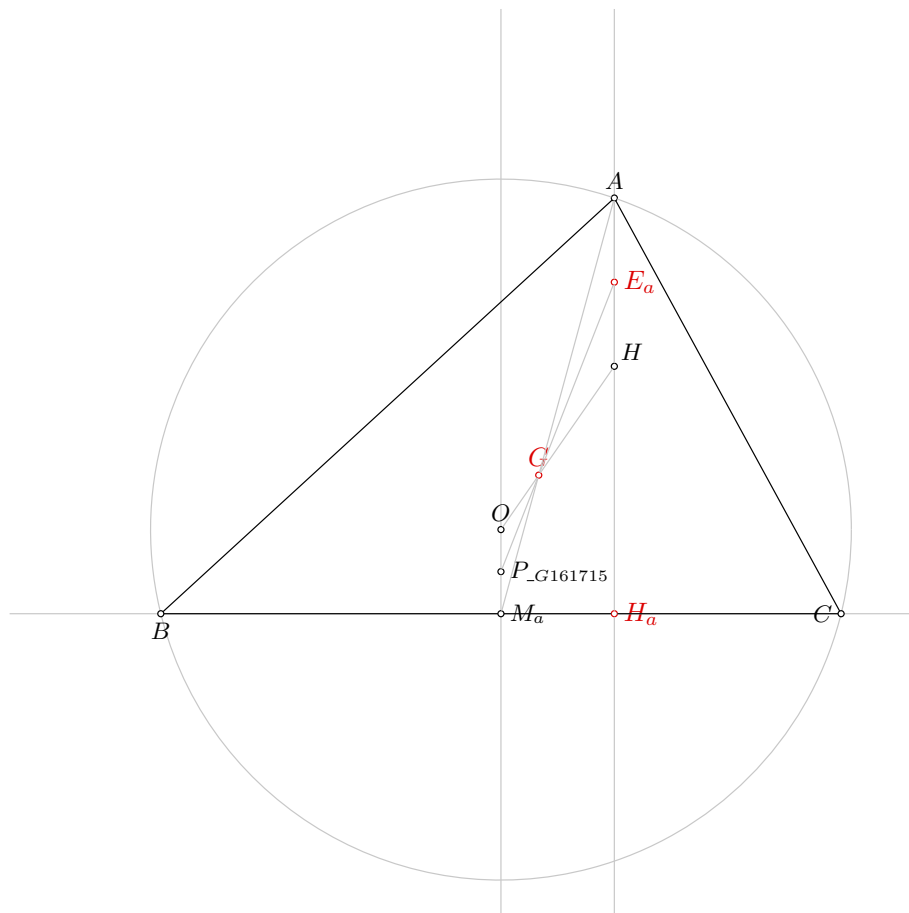


Figure 1: Illustration of the problem 0777

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% lines h_{G,-1/2}(h_{a}) and a are not parallel
% Determination conditions: lines h_{G,-1/2}(h_{a}) and a are not the same; points E_{a} and H_{a}
% are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = -E_a$

Proving failed

4.1.2 Proving $G=_G$

Proving failed

4.1.3 Proving $H_a=_H H_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a=_E E_a$

Proving failed

4.2.2 Proving $G=_G$

Proving failed

4.2.3 Proving $H_a=_H H_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a=_E E_a$

Proving failed

4.3.2 Proving $G=_G$

Proving failed

4.3.3 Proving $H_a=_H H_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a=_E E_a$

Proving failed

4.4.2 Proving $G=_G$

Proving failed

4.4.3 Proving $H_a=_H H_a$

Proving failed

Problem 778

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 778: Given a point E_a , a point G and a point H_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 779

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 779: Given a point E_a , a point G and a point H_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 780

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 780: Given a point E_a , a point G and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 781

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 781: Given a point E_a , a point G and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point G and the point M_a , construct a point A (rule W01); ;
2. Using the point E_a and the point A , construct a point H (rule W01); ;
3. Using the point G and the point H , construct a point N (rule W01); ;
4. Using the point G and the point H , construct a point O (rule W01); ;
5. Using the point E_a and the point A , construct a line h_a (rule W02); % DET: points E_a and A are not the same;
6. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
8. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
9. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; line h_a and circle $k(N, M_a)$ intersect; points E_a and N are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points E_a and A are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L16,L19,L22,L55,L58]

Solving time: 7.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{a} 80 83.86
point G 70 58.33
point M_{a} 65 40

color 220 0 0
fontsize 9

cmark_r E_{a}
cmark_t G
cmark_r M_{a}
color 0 0 0
fontsize 8

% Constructing a point A such that GA/GM_{a}=-2
towards A G M_{a} -2
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

% Constructing a point H such that E_{a}H/E_{a}A=-1
towards H E_{a} A -1
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% Constructing a point N such that GN/GH=0.25
towards N G H 0.25
cmark_r N
color 200 200 200
drawsegment G H
color 0 0 0

% Constructing a point O such that GO/GH=-0.5
```

```

towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% DET: points  $E_{\{a\}}$  and  $A$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $E_{\{a\}}$  and point  $A$ 
line  $h_{\{a\}}$   $E_{\{a\}}$   $A$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: points  $E_{\{a\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{a\}}$ 
circle  $k(N, M_{\{a\}})$   $N$   $E_{\{a\}}$ 

color 200 200 200
drawcircle  $k(N, M_{\{a\}})$ 
color 0 0 0

% NDG: line  $h_{\{a\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{a\}}$  and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\_G215460\}}$  which is a foot of the point  $N$  on the line  $h_{\{a\}}$ 
foot  $P_{\{\_G215460\}}$   $N$   $h_{\{a\}}$ 
cmark_r  $P_{\{\_G215460\}}$ 
color 200 200 200
drawline  $N$   $P_{\{\_G215460\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $E_{\{a\}}$  in the symmetry to point/line  $P_{\{\_G215460\}}$ 
sim  $H_{\{a\}}$   $P_{\{\_G215460\}}$   $E_{\{a\}}$ 
cmark_r  $H_{\{a\}}$ 

% DET: points  $H_{\{a\}}$  and  $M_{\{a\}}$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $M_{\{a\}}$ 
line  $a$   $H_{\{a\}}$   $M_{\{a\}}$ 

color 200 200 200
drawline  $a$ 
color 0 0 0

% NDG: points  $A$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $A$ 
circle  $k(O, C)$   $O$   $A$ 

color 200 200 200

```

```
drawcircle k(O,C)
color 0 0 0
```

```
% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% line h_{a} and circle k(N,M_{a}) intersect; points E_{a} and N are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
% different; points E_{a} and A are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = _E_a$

Proving failed

4.1.2 Proving $G = _G$

Proving failed

4.1.3 Proving $M_a = _M_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = _E_a$

Proving failed

4.2.2 Proving $G = _G$

Proving failed

4.2.3 Proving $M_a = _M_a$

Proving failed

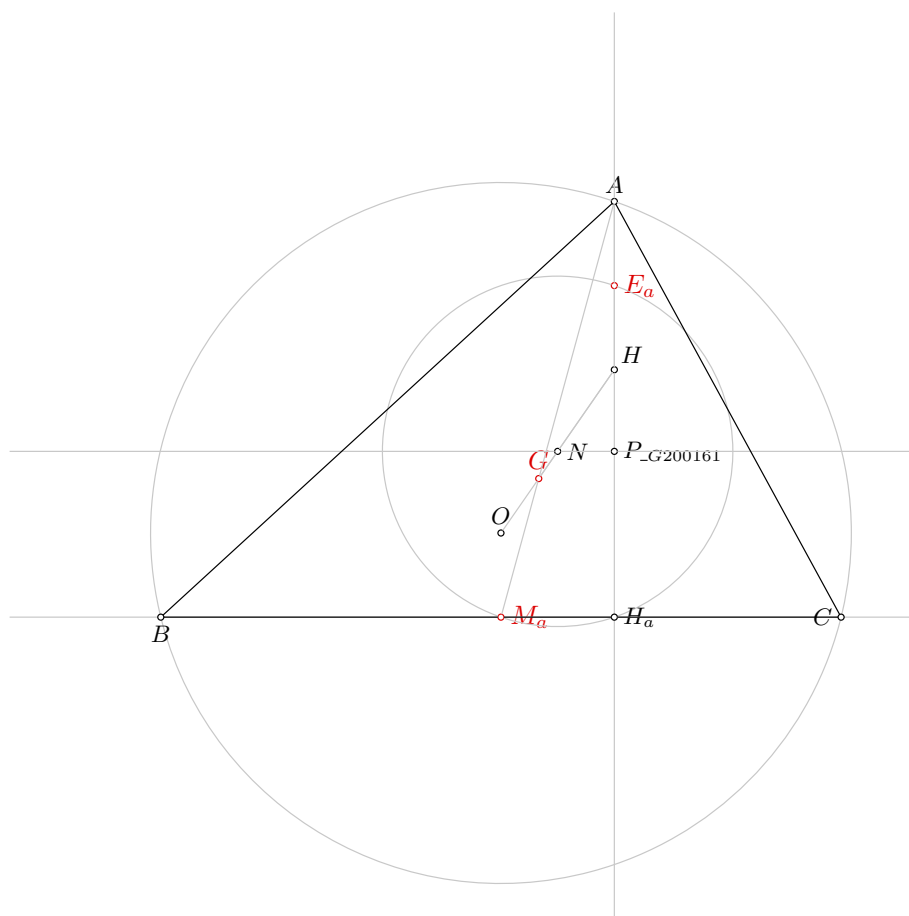


Figure 1: Illustration of the problem 0781

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $G = \neg G$

Proving failed

4.3.3 Proving $M_a = \neg M_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $G = \neg G$

Proving failed

4.4.3 Proving $M_a = \neg M_a$

Proving failed

Problem 782

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 782: Given a point E_a , a point G and a point M_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 783

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 783: Given a point E_a , a point G and a point M_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 784

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 784: Given a point E_a , a point G and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point G and the point N , construct a point H (rule W01); ;
2. Using the point E_a and the point H , construct a point A (rule W01); ;
3. Using the point G and the point N , construct a point O (rule W01); ;
4. Using the point G and the point A , construct a point M_a (rule W01); ;
5. Using the point E_a and the point H , construct a line h_a (rule W02); % DET: points E_a and H are not the same;
6. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
8. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
9. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; line h_a and circle $k(N, M_a)$ intersect; points E_a and N are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points E_a and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L15,L16,L19,L22,L55]

Solving time: 7.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point G 70 58.33
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_t G
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point H such that GH/GN=4
```

```
towards H G N 4
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment G H
```

```
color 0 0 0
```

```
% Constructing a point A such that E_{a}A/E_{a}H=-1
```

```
towards A E_{a} H -1
```

```
cmark_t A
```

```
color 200 200 200
```

```
drawsegment H A
```

```
color 0 0 0
```

```
% Constructing a point O such that GO/GN=-2
```

```
towards O G N -2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment N O
```

```
color 0 0 0
```

```
% Constructing a point M_{a} such that GM_{a}/GA=-0.5
```

```

towards M_{a} G A -0.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% DET: points E_{a} and H are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H
line h_{a} E_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points E_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{a}
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G20467} which is a foot of the point N on the line h_{a}
foot P_{\_G20467} N h_{a}
cmark_r P_{\_G20467}
color 200 200 200
drawline N P_{\_G20467}
color 0 0 0

% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
\_G20467}
sim H_{a} P_{\_G20467} E_{a}
cmark_r H_{a}

% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200

```

```
drawcircle k(O,C)
color 0 0 0
```

```
% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% line h_{a} and circle k(N,M_{a}) intersect; points E_{a} and N are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
% different; points E_{a} and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $G = \neg G$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $G = \neg G$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

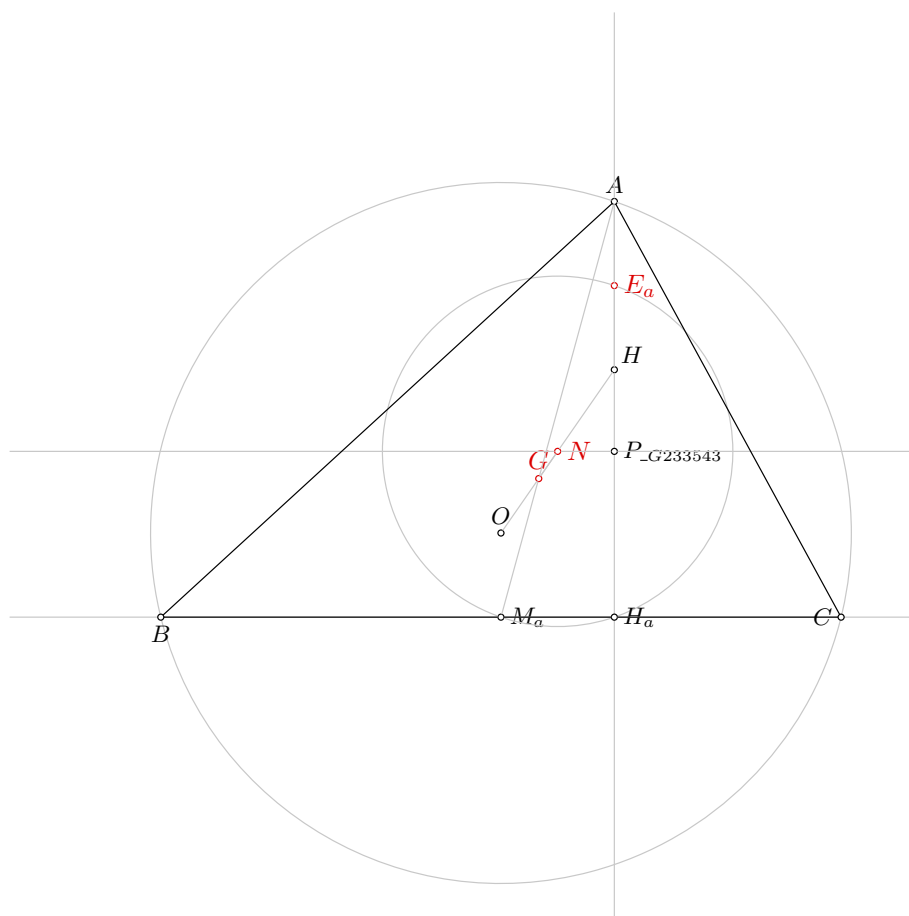


Figure 1: Illustration of the problem 0784

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $G = \neg G$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $G = \neg G$

Proving failed

4.4.3 Proving $N = \neg N$

Proving failed

Problem 785

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 785: Given a point E_a , a point G and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point G and the point O , construct a point N (rule W01); ;
2. Using the point G and the point O , construct a point H (rule W01); ;
3. Using the point E_a and the point H , construct a point A (rule W01); ;
4. Using the point G and the point A , construct a point M_a (rule W01); ;
5. Using the point E_a and the point H , construct a line h_a (rule W02); % DET: points E_a and H are not the same;
6. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
8. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
9. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; line h_a and circle $k(N, M_a)$ intersect; points E_a and N are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points E_a and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L15,L19,L22,L55,L58]

Solving time: 7.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point G 70 58.33
```

```
point O 65 51.14
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_t G
```

```
cmark_t O
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point N such that GN/GO=-0.5
```

```
towards N G O -0.5
```

```
cmark_r N
```

```
color 200 200 200
```

```
drawsegment O N
```

```
color 0 0 0
```

```
% Constructing a point H such that GH/GO=-2
```

```
towards H G O -2
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment O H
```

```
color 0 0 0
```

```
% Constructing a point A such that E_{a}A/E_{a}H=-1
```

```
towards A E_{a} H -1
```

```
cmark_t A
```

```
color 200 200 200
```

```
drawsegment H A
```

```
color 0 0 0
```

```
% Constructing a point M_{a} such that GM_{a}/GA=-0.5
```

```

towards M_{a} G A -0.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% DET: points E_{a} and H are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H
line h_{a} E_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points E_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{a}
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G50952} which is a foot of the point N on the line h_{a}
foot P_{\_G50952} N h_{a}
cmark_r P_{\_G50952}
color 200 200 200
drawline N P_{\_G50952}
color 0 0 0

% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
\_G50952}
sim H_{a} P_{\_G50952} E_{a}
cmark_r H_{a}

% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200

```



```
drawcircle k(O,C)
color 0 0 0
```

```
% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% line h_{a} and circle k(N,M_{a}) intersect; points E_{a} and N are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
% different; points E_{a} and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $G = \neg G$

Proving failed

4.1.3 Proving $O = \neg O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $G = \neg G$

Proving failed

4.2.3 Proving $O = \neg O$

Proving failed

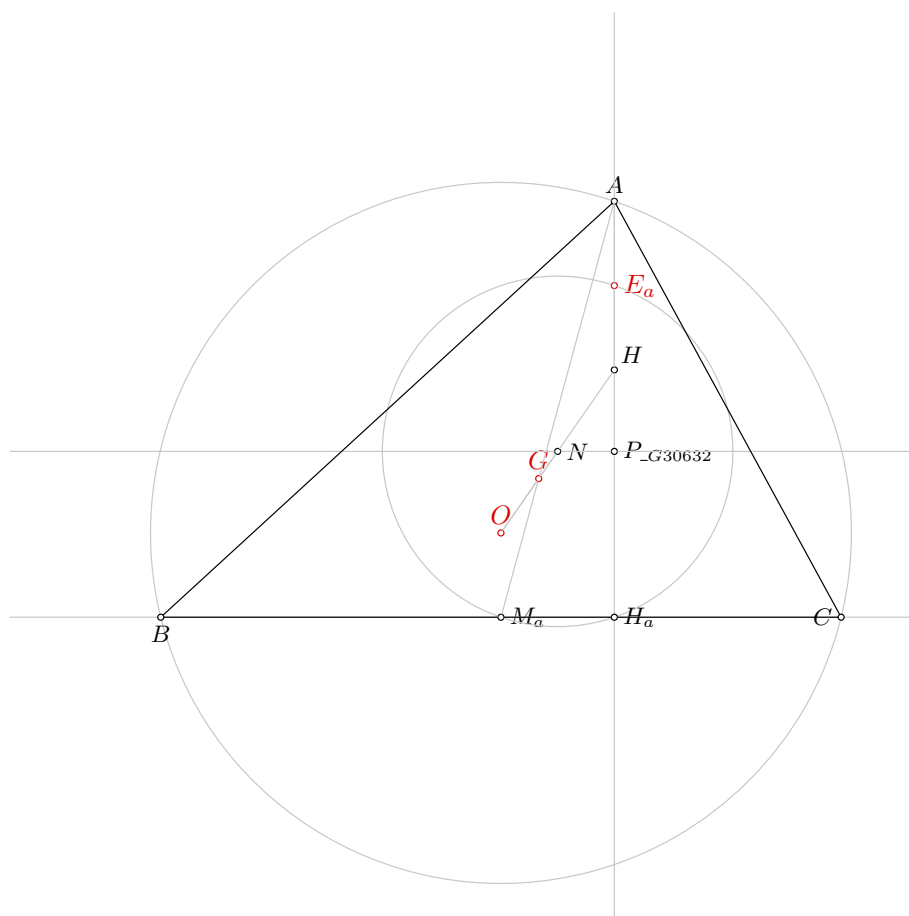


Figure 1: Illustration of the problem 0785

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $G = \neg G$

Proving failed

4.3.3 Proving $O = \neg O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $G = \neg G$

Proving failed

4.4.3 Proving $O = \neg O$

Proving failed

Problem 786

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 786: Given a point E_a , a point G and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 787

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 787: Given a point E_a , a point G and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 788

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 788: Given a point E_a , a point G and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 789

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 789: Given a point H , a point H_a and a point E_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H and the point H_a , construct a line h_a (rule W02); % DET: points H and H_a are not the same;
2. Choose freely a point E_a on the line h_a (rule WOnline1) ;
3. Using the point E_a and the point H , construct a point A (rule W01); ;
4. Using the point H and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H and E_a are not the same;
5. Using the point H_a and the line h_a , construct a line a (rule W10a); ;
6. Choose freely a point B on the line a (rule WOnline2);
7. Using the point B and the point H , construct a line h_b (rule W02); % DET: points B and H are not the same;
8. Using the circle $k(E_a, A)$, the line h_b , the point E_a and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_a, A)$ intersect % DET: points H and H_b must be different;
9. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;
10. Using the line a and the line b , construct a point C (rule W03); % NDG: lines a and b are not parallel % DET: lines a and b are not the same.

Non-degenerate conditions: lines a and b are not parallel; line h_b and circle $k(E_a, A)$ intersect; points H and E_a are not the same.

Determination conditions: lines a and b are not the same; points A and H_b are not the same; points H and H_b must be different; points B and H are not the same; points H and H_a are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D28,D3,D5,D6,D8,D9,GD01,GD02,GL03,GL04,L46,L47]

Solving time: 169.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point H 80 72.73
point H_{a} 80 40
point E_{a} 80 83.86

color 220 0 0
fontsize 9

cmark_rt H
cmark_r H_{a}
cmark_r E_{a}
color 0 0 0
fontsize 8

% DET: points H and H_{a} are not the same
% Constructing a line h_{a} which passes through point H and point H_{a}
line h_{a} H H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% Choosing randomly a point E_{a} on the line HH_{a}
online E_{a} H H_{a}
cmark_r E_{a}
color 200 200 200
drawline H H_{a}
color 0 0 0

% Constructing a point A such that E_{a}A/E_{a}H=-1
towards A E_{a} H -1
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0
```



```

% NDG: points H and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H
circle k(E_{a},A) E_{a} H

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
perp a H_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0

% Generating random value V[_G85554]
random V[_G85554]

% Calculating value V[_G85575] using formula V[_G85554]*20
expression V[_G85575] { V[_G85554]*20 }

% Constructing a point B which is a point for which holds  $H_{a}B = V[_G85575]$  and angle  $E_{a}H_{a}B = 90$ 
turtle B E_{a} H_{a} 90 V[_G85575]
cmark_b B

% DET: points B and H are not the same
% Constructing a line h_{b} which passes through point B and point H
line h_{b} B H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{b} and circle k(E_{a},A) intersect% DET: points H and H_{b} must be different
% Constructing a point P_{\_G85847} which is a foot of the point E_{a} on the line h_{b}
foot P_{\_G85847} E_{a} h_{b}
cmark_r P_{\_G85847}
color 200 200 200
drawline E_{a} P_{\_G85847}
color 0 0 0

% Constructing a point H_{b} which is an image of the point H in the symmetry to point/line P_{\_G85847}
sim H_{b} P_{\_G85847} H

```

```

cmark_l H_{b}

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and b are not parallel; line h_{b} and circle k(E_{a},A)
% intersect; points H and E_{a} are not the same
% Determination conditions: lines a and b are not the same; points A and H_{b} are not the same;
% points H and H_{b} must be different; points B and H are not the same; points H and H_{a} are
% not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H = H$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 61 terms.

Time Complexity: Time spent by the prover is 0.534 seconds.

NDG conditions Points B and H are not identical

Points B and H are not identical

Line through points H_b and H is not perpendicular to line through points H and A

Line through points C and B is not perpendicular to line through points B and H_a

Line through points A and H is not perpendicular to line through points H and B

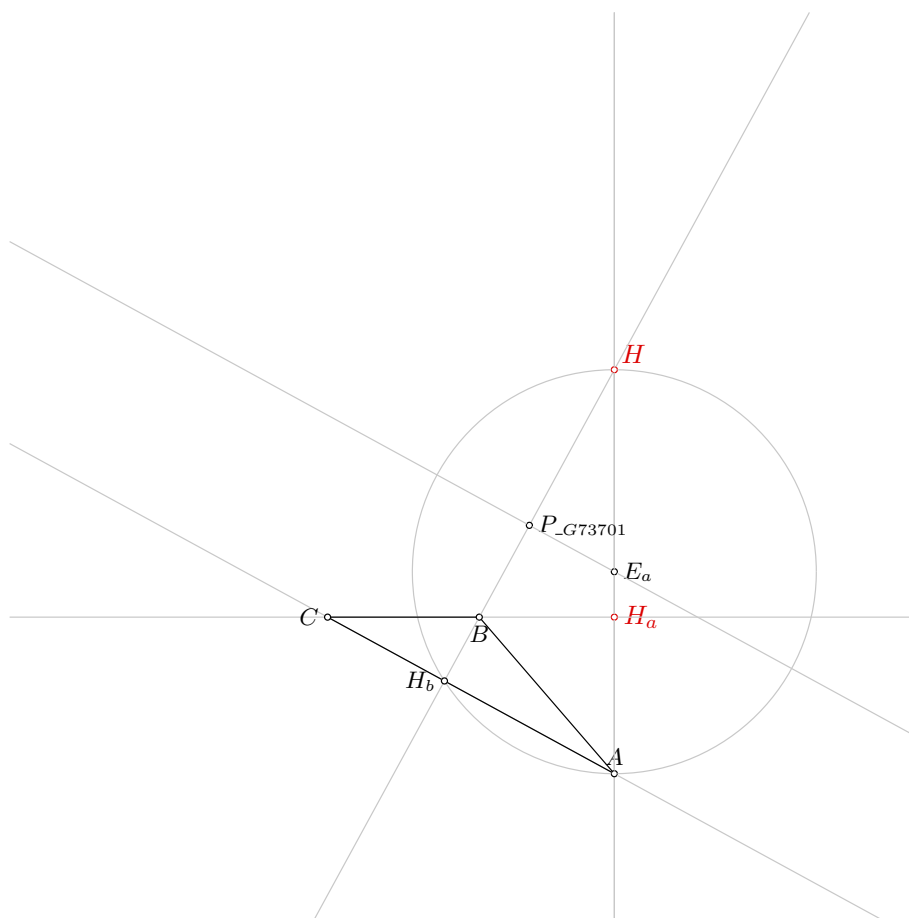


Figure 1: Illustration of the problem 0789

4.1.2 Proving $H_a = \neg H_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

Time Complexity: Time spent by the prover is 0.152 seconds.

NDG conditions Points B and H are not identical

Points B and H are not identical

Line through points H_b and H is not perpendicular to line through points H and A

Line through points C and B is not perpendicular to line through points B and H_a

4.1.3 Proving $E_a = \neg E_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H = \neg H$

Proving failed

4.2.2 Proving $H_a = \neg H_a$

Proving failed

4.2.3 Proving $E_a = \neg E_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H = \neg H$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 9 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $H_a = \neg H_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 16 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.3.3 Proving $E_a = \neg E_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 13 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $H = \neg H$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.4.2 Proving $H_a = \neg H_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.080 seconds. There are no ndg conditions.

4.4.3 Proving $E_a = \neg E_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.090 seconds. There are no ndg conditions.

Problem 790

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 790: Given a point H , a point H_b and a point E_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H and the point H_b , construct a line h_b (rule W02); % DET: points H and H_b are not the same;
2. Using the point H_b and the line h_b , construct a line b (rule W10a); ;
3. Using the point H , the line b and the point H_b , construct a line $h_{H,1/2}(b)$ (rule W15); ;
4. Choose freely a point E_a on the line $h_{H,1/2}(b)$ (rule WOnline4);
5. Using the point E_a and the point H , construct a point A (rule W01); ;
6. Using the point H and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H and E_a are not the same;
7. Choose freely a point B on the line h_b (rule WOnline1) ;
8. Using the point B and the point A , construct a line c (rule W02); % DET: points B and A are not the same;
9. Using the circle $k(E_a, A)$, the line c , the point E_a and the point A , construct a point H_c (rule W05); % NDG: line c and circle $k(E_a, A)$ intersect % DET: points A and H_c must be different;
10. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
11. Using the line b and the line h_c , construct a point C (rule W03); % NDG: lines b and h_c are not parallel % DET: lines b and h_c are not the same.

Non-degenerate conditions: lines b and h_c are not parallel; line c and circle $k(E_a, A)$ intersect; points H and E_a are not the same.

Determination conditions: lines b and h_c are not the same; points H_c and H are not the same; points A and H_c must be different; points B and A are not the same; points H and H_b are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,W15,WOnline1,WOnline4]

Lemmas used: [D10,D28,D3,D6,D7,D9,GD01,GD02,GL03,GL04,L3,L46,L48]

Solving time: 682.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H 80 72.73
```

```
point H_{b} 89.36 77.83
```

```
point E_{a} 80 83.86
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H
```

```
cmark_l H_{b}
```

```
cmark_r E_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H and H_{b} are not the same
```

```
% Constructing a line h_{b} which passes through point H and point H_{b}
```

```
line h_{b} H H_{b}
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```
% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}  
perp b H_{b} h_{b}
```

```
color 200 200 200
```

```
drawline b
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G113122} such that HP_{\_G113122}/HH_{b}=0.5
```

```
towards P_{\_G113122} H H_{b} 0.5
```

```
cmark_r P_{\_G113122}
```

```
color 200 200 200
```

```
drawsegment H H_{b}
```

```
color 0 0 0
```

```

% Constructing a line  $h_{H,1/2}(b)$  which contains the point  $P_{\backslash\_G113122}$  and is parallel to the
line b
parallel  $h_{H,1/2}(b)$   $P_{\backslash\_G113122}$  b

color 200 200 200
drawline  $h_{H,1/2}(b)$ 
color 0 0 0

% Constructing a point  $P_{\backslash\_G113429}$  such that  $HP_{\backslash\_G113429}/HH_{\{b\}}=0.5$ 
towards  $P_{\backslash\_G113429}$  H  $H_{\{b\}}$  0.5
cmark_r  $P_{\backslash\_G113429}$ 
color 200 200 200
drawsegment H  $H_{\{b\}}$ 
color 0 0 0

% Generating random value  $V_{\_G113368}$ 
random  $V_{\_G113368}$ 

% Calculating value  $V_{\_G113389}$  using formula  $V_{\_G113368}*20$ 
expression  $V_{\_G113389}$  {  $V_{\_G113368}*20$  }

% Constructing a point  $E_{\{a\}}$  which is a point for which holds  $P_{\backslash\_G113429}E_{\{a\}} = V_{\_G113389}$  and
angle  $HP_{\backslash\_G113429}E_{\{a\}} = 90$ 
turtle  $E_{\{a\}}$  H  $P_{\backslash\_G113429}$  90  $V_{\_G113389}$ 
cmark_r  $E_{\{a\}}$ 

% Constructing a point A such that  $E_{\{a\}}A/E_{\{a\}}H=-1$ 
towards A  $E_{\{a\}}$  H -1
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

% NDG: points H and  $E_{\{a\}}$  are not the same
% Constructing a circle  $k(E_{\{a\}},A)$  whose center is at point  $E_{\{a\}}$  and which passes through point H
circle  $k(E_{\{a\}},A)$   $E_{\{a\}}$  H

color 200 200 200
drawcircle  $k(E_{\{a\}},A)$ 
color 0 0 0

% Choosing randomly a point B on the line  $HH_{\{b\}}$ 
online B H  $H_{\{b\}}$ 
cmark_b B
color 200 200 200

```



```

drawline H H_{b}
color 0 0 0

% DET: points B and A are not the same
% Constructing a line c which passes through point B and point A
line c B A

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(E_{a},A) intersect% DET: points A and H_{c} must be different
% Constructing a point P_{\_G113994} which is a foot of the point E_{a} on the line c
foot P_{\_G113994} E_{a} c
cmark_r P_{\_G113994}
color 200 200 200
drawline E_{a} P_{\_G113994}
color 0 0 0

% Constructing a point H_{c} which is an image of the point A in the symmetry to point/line P_{\_G
113994}
sim H_{c} P_{\_G113994} A
cmark_rt H_{c}

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines b and h_{c} are not parallel% DET: lines b and h_{c} are not the same
% Constructing a point C which belongs to line b and line h_{c}
intersec C b h_{c}
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and h_{c} are not parallel; line c and circle k(E_{a},A)
intersect; points H and E_{a} are not the same
% Determination conditions: lines b and h_{c} are not the same; points H_{c} and H are not the same
; points A and H_{c} must be different; points B and A are not the same; points H and H_{b} are
not the same

```

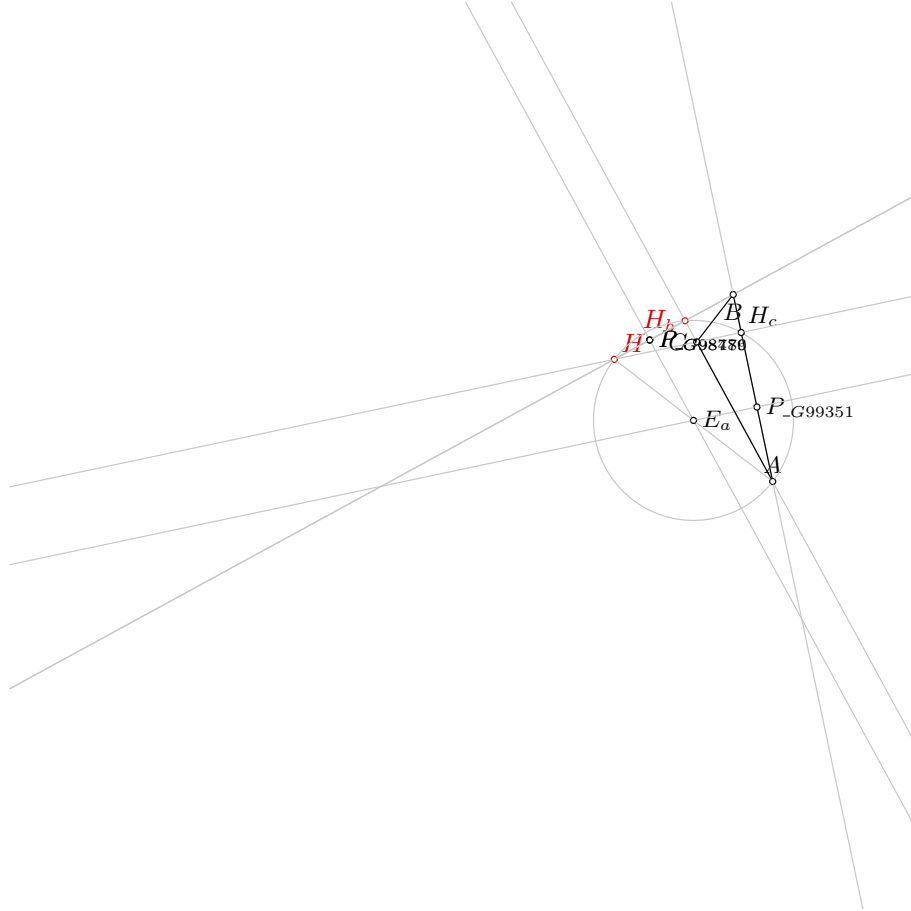


Figure 1: Illustration of the problem 0790

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H=_H$

Proving failed

4.1.2 Proving $H_b=_H H_b$

Proving failed

4.1.3 Proving $E_a=_E E_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H=_H$

Proving failed

4.2.2 Proving $H_b=_Hb$

Proving failed

4.2.3 Proving $E_a=_Ea$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H=_H$

Proving failed

4.3.2 Proving $H_b=_Hb$

Proving failed

4.3.3 Proving $E_a=_Ea$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H=_H$

Proving failed

4.4.2 Proving $H_b=_Hb$

Proving failed

4.4.3 Proving $E_a=_Ea$

Proving failed

Problem 791

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 791: Given a point H , a point H_c and a point E_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H and the point H_c , construct a line h_c (rule W02); % DET: points H and H_c are not the same;
2. Using the point H_c and the line h_c , construct a line c (rule W10a); ;
3. Using the point H , the line c and the point H_c , construct a line $h_{H,1/2}(c)$ (rule W15); ;
4. Choose freely a point E_a on the line $h_{H,1/2}(c)$ (rule WOnline4);
5. Using the point E_a and the point H , construct a point A (rule W01); ;
6. Using the point H and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H and E_a are not the same;
7. Choose freely a point B on the line c (rule WOnline1) ;
8. Using the point B and the point H , construct a line h_b (rule W02); % DET: points B and H are not the same;
9. Using the circle $k(E_a, A)$, the line h_b , the point E_a and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_a, A)$ intersect % DET: points H and H_b must be different;
10. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;
11. Using the line h_c and the line b , construct a point C (rule W03); % NDG: lines h_c and b are not parallel % DET: lines h_c and b are not the same.

Non-degenerate conditions: lines h_c and b are not parallel; line h_b and circle $k(E_a, A)$ intersect; points H and E_a are not the same.

Determination conditions: lines h_c and b are not the same; points A and H_b are not the same; points H and H_b must be different; points B and H are not the same; points H and H_c are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,W15,WOnline1,WOnline4]

Lemmas used: [D10,D28,D3,D6,D7,D9,GD01,GD02,GL03,GL04,L3,L46,L47]

Solving time: 695.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H 80 72.73
```

```
point H_{c} 68.91 84.83
```

```
point E_{a} 80 83.86
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H
```

```
cmark_rt H_{c}
```

```
cmark_r E_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H and H_{c} are not the same
```

```
% Constructing a line h_{c} which passes through point H and point H_{c}
```

```
line h_{c} H H_{c}
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```
% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
```

```
perp c H_{c} h_{c}
```

```
color 200 200 200
```

```
drawline c
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G142117} such that HP_{\_G142117}/HH_{c}=0.5
```

```
towards P_{\_G142117} H H_{c} 0.5
```

```
cmark_r P_{\_G142117}
```

```
color 200 200 200
```

```
drawsegment H H_{c}
```

```
color 0 0 0
```

```

% Constructing a line  $h_{\{H,1/2\}}(c)$  which contains the point  $P_{\{\backslash\_G142117\}}$  and is parallel to the
    line c
parallel  $h_{\{H,1/2\}}(c)$   $P_{\{\backslash\_G142117\}}$  c

color 200 200 200
drawline  $h_{\{H,1/2\}}(c)$ 
color 0 0 0

% Constructing a point  $P_{\{\backslash\_G142424\}}$  such that  $HP_{\{\backslash\_G142424\}}/HH_{\{c\}}=0.5$ 
towards  $P_{\{\backslash\_G142424\}}$  H  $H_{\{c\}}$  0.5
cmark_r  $P_{\{\backslash\_G142424\}}$ 
color 200 200 200
drawsegment H  $H_{\{c\}}$ 
color 0 0 0

% Generating random value  $V[_{G142363}]$ 
random  $V[_{G142363}]$ 

% Calculating value  $V[_{G142384}]$  using formula  $V[_{G142363}]*20$ 
expression  $V[_{G142384}]$  {  $V[_{G142363}]*20$  }

% Constructing a point  $E_{\{a\}}$  which is a point for which holds  $P_{\{\backslash\_G142424\}}E_{\{a\}} = V[_{G142384}]$  and
    angle  $HP_{\{\backslash\_G142424\}}E_{\{a\}} = 90$ 
turtle  $E_{\{a\}}$  H  $P_{\{\backslash\_G142424\}}$  90  $V[_{G142384}]$ 
cmark_r  $E_{\{a\}}$ 

% Constructing a point A such that  $E_{\{a\}}A/E_{\{a\}}H=-1$ 
towards A  $E_{\{a\}}$  H -1
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

% NDG: points H and  $E_{\{a\}}$  are not the same
% Constructing a circle  $k(E_{\{a\}},A)$  whose center is at point  $E_{\{a\}}$  and which passes through point H
circle  $k(E_{\{a\}},A)$   $E_{\{a\}}$  H

color 200 200 200
drawcircle  $k(E_{\{a\}},A)$ 
color 0 0 0

% Choosing randomly a point B on the line  $H_{\{c\}}A$ 
online B  $H_{\{c\}}$  A
cmark_b B
color 200 200 200

```

```

drawline H_{c} A
color 0 0 0

% DET: points B and H are not the same
% Constructing a line h_{b} which passes through point B and point H
line h_{b} B H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{b} and circle k(E_{a},A) intersect% DET: points H and H_{b} must be different
% Constructing a point P_{\_G142989} which is a foot of the point E_{a} on the line h_{b}
foot P_{\_G142989} E_{a} h_{b}
cmark_r P_{\_G142989}
color 200 200 200
drawline E_{a} P_{\_G142989}
color 0 0 0

% Constructing a point H_{b} which is an image of the point H in the symmetry to point/line P_{\_G
142989}
sim H_{b} P_{\_G142989} H
cmark_l H_{b}

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines h_{c} and b are not parallel% DET: lines h_{c} and b are not the same
% Constructing a point C which belongs to line h_{c} and line b
intersec C h_{c} b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and b are not parallel; line h_{b} and circle k(E_{a},A)
intersect; points H and E_{a} are not the same
% Determination conditions: lines h_{c} and b are not the same; points A and H_{b} are not the same
; points H and H_{b} must be different; points B and H are not the same; points H and H_{c} are
not the same

```

Figure 1: Illustration of the problem 0791

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H = _H$

Construction steps:

- Free point H
- Free point H_c
- Line h_c through two points H and H_c
- Line c through point H_c perpendicular to line h_c
- Segment division point $P_{G137153}$ of segment HH_c with division coefficient 1.0

- Line $h_{H,1/2}(c)$ through point $P_{G137153}$ parallel with line c
- Segment division point $P_{G137285}$ of segment HH_c with division coefficient -0.5
- Line $L_{G137288}$ through point $P_{G137285}$ parallel with line $h_{H,1/2}(c)$
- Random point E_a from line $L_{G137288}$
- Segment division point A of segment E_aH with division coefficient -0.5
- Circle $k(E_a, A)$ with center E_a and one point H
- Random point B from line c
- Line h_b through two points B and H
- Line footPointPerpLine118 through point E_a perpendicular to line h_b
- Intersection point $P_{G137547}$ of point sets footPointPerpLine118 and h_b
- Cental symmetric point H_b of point H with respect to center of symmetry $P_{G137547}$
- Line b through two points A and H_b
- Intersection point C of point sets h_c and b
- Line $_b$ through two points A and C
- Line $_a$ through two points B and C
- Line $_h_a$ through point A perpendicular to line $_a$
- Line $_h_b$ through point B perpendicular to line $_b$
- Intersection point $_H$ of point sets $_h_a$ and $_h_b$
- Line $_c$ through two points A and B
- Line $_h_c$ through point C perpendicular to line $_c$
- Intersection point $_H_c$ of point sets $_c$ and $_h_c$

Theorem statement:

- Points H and $_H$ are identical

Info: Attempting to add the construction of new random point tempPoint-447 $h_{H,1/2}(c)$ necessary for completion of construction of line $L_{G137288}$

Warning: Generated new random point tempPoint-447 $h_{H,1/2}(c)$ on line $h_{H,1/2}(c)$ in order to complete the construction of parallel line $L_{G137288}$

Validation result: Theorem protocol is valid.

NDG conditions Point H has been assigned following coordinates: (0, 0)

4.1.2 Proving $H_c = _Hc$

Construction steps:

- Free point H
- Free point H_c
- Line h_c through two points H and H_c
- Line c through point H_c perpendicular to line h_c
- Segment division point $P_{G138448}$ of segment HH_c with division coefficient 1.0
- Line $h_{H,1/2}(c)$ through point $P_{G138448}$ parallel with line c
- Segment division point $P_{G138580}$ of segment HH_c with division coefficient -0.5
- Line $L_{G138583}$ through point $P_{G138580}$ parallel with line $h_{H,1/2}(c)$
- Random point E_a from line $L_{G138583}$

- Segment division point A of segment E_aH with division coefficient -0.5
- Circle $k(E_a, A)$ with center E_a and one point H
- Random point B from line c
- Line h_b through two points B and H
- Line footPointPerpLine609 through point E_a perpendicular to line h_b
- Intersection point $P_{G138842}$ of point sets footPointPerpLine609 and h_b
- Cental symmetric point H_b of point H with respect to center of symmetry $P_{G138842}$
- Line b through two points A and H_b
- Intersection point C of point sets h_c and b
- Line $_b$ through two points A and C
- Line $_a$ through two points B and C
- Line $_h_a$ through point A perpendicular to line $_a$
- Line $_h_b$ through point B perpendicular to line $_b$
- Intersection point $_H$ of point sets $_h_a$ and $_h_b$
- Line $_c$ through two points A and B
- Line $_h_c$ through point C perpendicular to line $_c$
- Intersection point $_H_c$ of point sets $_c$ and $_h_c$

Theorem statement:

- Points H_c and $_H_c$ are identical

Info: Attempting to add the construction of new random point tempPoint-639 $h_{H,1/2}(c)$ necessary for completion of construction of line $L_{G138583}$

Warning: Generated new random point tempPoint-639 $h_{H,1/2}(c)$ on line $h_{H,1/2}(c)$ in order to complete the construction of parallel line $L_{G138583}$

Validation result: Theorem protocol is valid.

NDG conditions Point H has been assigned following coordinates: $(0, 0)$

4.1.3 Proving $E_a = _E_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H = _H$

Proving failed

4.2.2 Proving $H_c = _H_c$

Proving failed

4.2.3 Proving $E_a = _E_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H = _H$

Proving failed

4.3.2 Proving $H_c=_H H_c$

Proving failed

4.3.3 Proving $E_a=_E E_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H=_H H$

Proving failed

4.4.2 Proving $H_c=_H H_c$

Proving failed

4.4.3 Proving $E_a=_E E_a$

Proving failed

Problem 792

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 792: Given a point E_a , a point H and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 793

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 793: Given a point E_a , a point H and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H , construct a point A (rule W01); ;
2. Using the point M_a and the point A , construct a point G (rule W01); ;
3. Using the point H and the point G , construct a point N (rule W01); ;
4. Using the point H and the point G , construct a point O (rule W01); ;
5. Using the point E_a and the point H , construct a line h_a (rule W02); % DET: points E_a and H are not the same;
6. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
8. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
9. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; line h_a and circle $k(N, M_a)$ intersect; points E_a and N are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points E_a and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L16,L19,L22,L55,L58]

Solving time: 6.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point H 80 72.73
```

```
point M_{a} 65 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_rt H
```

```
cmark_r M_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point A such that E_{a}A/E_{a}H=-1
```

```
towards A E_{a} H -1
```

```
cmark_t A
```

```
color 200 200 200
```

```
drawsegment H A
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G174241} which passes through point M_{a} and point A
```

```
line L_{\_G174241} M_{a} A
```

```
color 200 200 200
```

```
drawline L_{\_G174241}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G174342} with coordinates (0,0)
```

```
point P_{\_G174342} 0 0
```

```
cmark_r P_{\_G174342}
```

```
% Constructing a point P_{\_G174266} such that M_{a}P_{\_G174266}/M_{a}P_{\_G174342}=1
```

```
towards P_{\_G174266} M_{a} P_{\_G174342} 1
```

```
cmark_r P_{\_G174266}
```

```
color 200 200 200
```

```
drawsegment M_{a} P_{\_G174266}
```

```
color 0 0 0
```

```

% Constructing a point P_{\_G174311} such that M_{a}P_{\_G174311}/M_{a}P_{\_G174342}=3
towards P_{\_G174311} M_{a} P_{\_G174342} 3
cmark_r P_{\_G174311}
color 200 200 200
drawsegment M_{a} P_{\_G174311}
color 0 0 0

% Constructing a line L_{\_G174272} which passes through point A and point P_{\_G174311}
line L_{\_G174272} A P_{\_G174311}

color 200 200 200
drawline L_{\_G174272}
color 0 0 0

% Constructing a line L_{\_G174235} which contains the point P_{\_G174266} and is parallel to the
line L_{\_G174272}
parallel L_{\_G174235} P_{\_G174266} L_{\_G174272}

color 200 200 200
drawline L_{\_G174235}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G174235} and line L_{\_G174241}
intersec G L_{\_G174235} L_{\_G174241}
cmark_t G

% Constructing a point N such that HN/HG=0.75
towards N H G 0.75
cmark_r N
color 200 200 200
drawsegment H G
color 0 0 0

% Constructing a point O such that HO/HG=1.5
towards O H G 1.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% DET: points E_{a} and H are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H
line h_{a} E_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

```

```

% NDG: points E_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{a}
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G175261} which is a foot of the point N on the line h_{a}
foot P_{\_G175261} N h_{a}
cmark_r P_{\_G175261}
color 200 200 200
drawline N P_{\_G175261}
color 0 0 0

% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
\_G175261}
sim H_{a} P_{\_G175261} E_{a}
cmark_r H_{a}

% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```

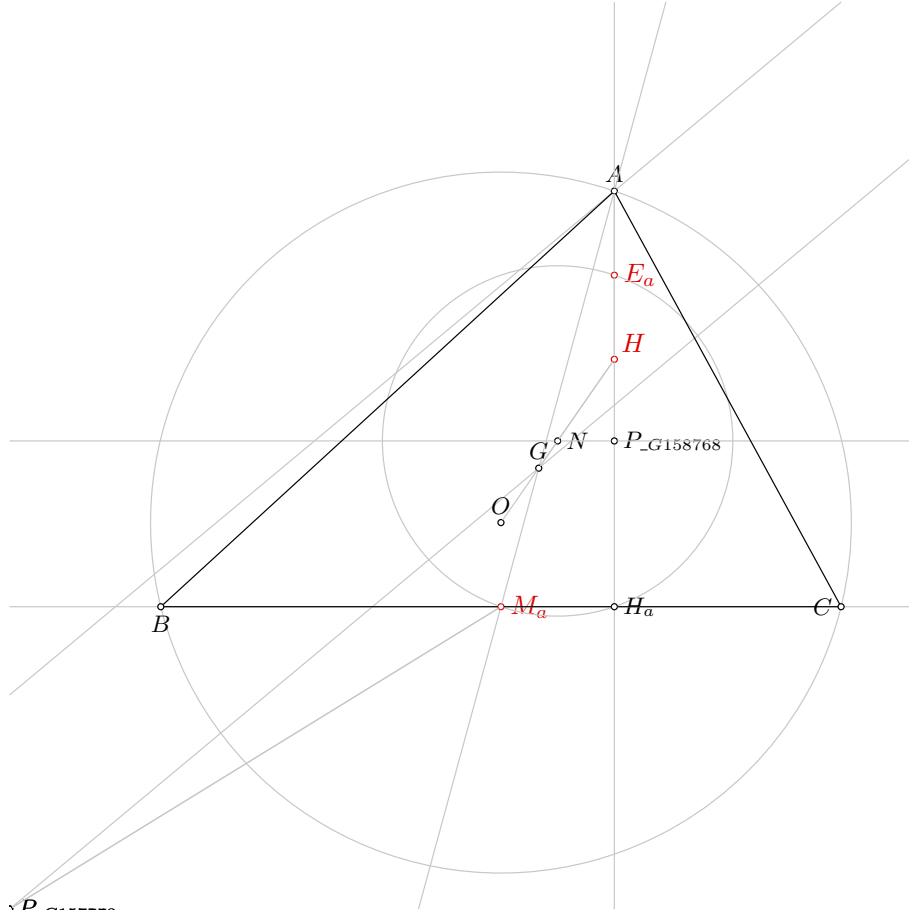



Figure 1: Illustration of the problem 0793

*% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
line h_{a} and circle k(N,M_{a}) intersect; points E_{a} and N are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
different; points E_{a} and H are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = E_a$

Proving failed

4.1.2 Proving $H=_H$

Proving failed

4.1.3 Proving $M_a=_M a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a=_E a$

Proving failed

4.2.2 Proving $H=_H$

Proving failed

4.2.3 Proving $M_a=_M a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a=_E a$

Proving failed

4.3.2 Proving $H=_H$

Proving failed

4.3.3 Proving $M_a=_M a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a=_E a$

Proving failed

4.4.2 Proving $H=_H$

Proving failed

4.4.3 Proving $M_a=_M a$

Proving failed

Problem 794

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 794: Given a point E_a , a point H and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H , construct a point A (rule W01); ;
2. Using the point M_b and the point A , construct a point C (rule W01); ;
3. Using the point H and the point C , construct a line h_c (rule W02); % DET: points H and C are not the same;
4. Using the point M_b and the point A , construct a line b (rule W02); % DET: points M_b and A are not the same;
5. Using the point H and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H and E_a are not the same;
6. Using the circle $k(E_a, A)$, the line h_c , the point E_a and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_a, A)$ intersect % DET: points H and H_c must be different;
7. Using the point H_c and the point A , construct a line c (rule W02); % DET: points H_c and A are not the same;
8. Using the circle $k(E_a, A)$, the line b , the point E_a and the point A , construct a point H_b (rule W05); % NDG: line b and circle $k(E_a, A)$ intersect % DET: points A and H_b must be different;
9. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
10. Using the line c and the line h_b , construct a point B (rule W03); % NDG: lines c and h_b are not parallel % DET: lines c and h_b are not the same.

Non-degenerate conditions: lines c and h_b are not parallel; line b and circle $k(E_a, A)$ intersect; line h_c and circle $k(E_a, A)$ intersect; points H and E_a are not the same.

Determination conditions: lines c and h_b are not the same; points H_b and H are not the same; points A and H_b must be different; points H_c and A are not the same; points H and H_c must be different; points M_b and A are not the same; points H and C are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D22,D28,D3,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L46,L47,L48]

Solving time: 12.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point H 80 72.73
```

```
point M_{b} 95 67.5
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_rt H
```

```
cmark_lt M_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point A such that E_{a}A/E_{a}H=-1
```

```
towards A E_{a} H -1
```

```
cmark_t A
```

```
color 200 200 200
```

```
drawsegment H A
```

```
color 0 0 0
```

```
% Constructing a point C such that M_{b}C/M_{b}A=-1
```

```
towards C M_{b} A -1
```

```
cmark_l C
```

```
color 200 200 200
```

```
drawsegment A C
```

```
color 0 0 0
```

```
% DET: points H and C are not the same
```

```
% Constructing a line h_{c} which passes through point H and point C
```

```
line h_{c} H C
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```

% DET: points  $M_{\{b\}}$  and  $A$  are not the same
% Constructing a line  $b$  which passes through point  $M_{\{b\}}$  and point  $A$ 
line b  $M_{\{b\}}$  A

color 200 200 200
drawline b
color 0 0 0

% NDG: points  $H$  and  $E_{\{a\}}$  are not the same
% Constructing a circle  $k(E_{\{a\}}, A)$  whose center is at point  $E_{\{a\}}$  and which passes through point  $H$ 
circle k( $E_{\{a\}}$ ,  $A$ )  $E_{\{a\}}$   $H$ 

color 200 200 200
drawcircle k( $E_{\{a\}}$ ,  $A$ )
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(E_{\{a\}}, A)$  intersect% DET: points  $H$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\_G201375\}}$  which is a foot of the point  $E_{\{a\}}$  on the line  $h_{\{c\}}$ 
foot  $P_{\{\_G201375\}}$   $E_{\{a\}}$   $h_{\{c\}}$ 
cmark_r  $P_{\{\_G201375\}}$ 
color 200 200 200
drawline  $E_{\{a\}}$   $P_{\{\_G201375\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $H$  in the symmetry to point/line  $P_{\{\_G201375\}}$ 
sim  $H_{\{c\}}$   $P_{\{\_G201375\}}$   $H$ 
cmark_rt  $H_{\{c\}}$ 

% DET: points  $H_{\{c\}}$  and  $A$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $A$ 
line c  $H_{\{c\}}$  A

color 200 200 200
drawline c
color 0 0 0

% NDG: line  $b$  and circle  $k(E_{\{a\}}, A)$  intersect% DET: points  $A$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\_G201613\}}$  which is a foot of the point  $E_{\{a\}}$  on the line  $b$ 
foot  $P_{\{\_G201613\}}$   $E_{\{a\}}$   $b$ 
cmark_r  $P_{\{\_G201613\}}$ 
color 200 200 200
drawline  $E_{\{a\}}$   $P_{\{\_G201613\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $A$  in the symmetry to point/line  $P_{\{\_G201613\}}$ 
sim  $H_{\{b\}}$   $P_{\{\_G201613\}}$   $A$ 
cmark_l  $H_{\{b\}}$ 

```

```

% DET: points  $H_{\{b\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $H_{\{b\}}$  and point  $H$ 
line  $h_{\{b\}}$   $H_{\{b\}}$   $H$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: lines  $c$  and  $h_{\{b\}}$  are not parallel% DET: lines  $c$  and  $h_{\{b\}}$  are not the same
% Constructing a point  $B$  which belongs to line  $c$  and line  $h_{\{b\}}$ 
intersec  $B$   $c$   $h_{\{b\}}$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $c$  and  $h_{\{b\}}$  are not parallel; line  $b$  and circle  $k(E_{\{a\}}, A)$ 
% intersect; line  $h_{\{c\}}$  and circle  $k(E_{\{a\}}, A)$  intersect; points  $H$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $c$  and  $h_{\{b\}}$  are not the same; points  $H_{\{b\}}$  and  $H$  are not the same
% ; points  $A$  and  $H_{\{b\}}$  must be different; points  $H_{\{c\}}$  and  $A$  are not the same; points  $H$  and  $H_{\{c\}}$ 
% must be different; points  $M_{\{b\}}$  and  $A$  are not the same; points  $H$  and  $C$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $H = \neg H$

Proving failed

4.1.3 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 8 terms.

Time Complexity: Time spent by the prover is 0.105 seconds.

NDG conditions There are no NDG conditions for this theorem

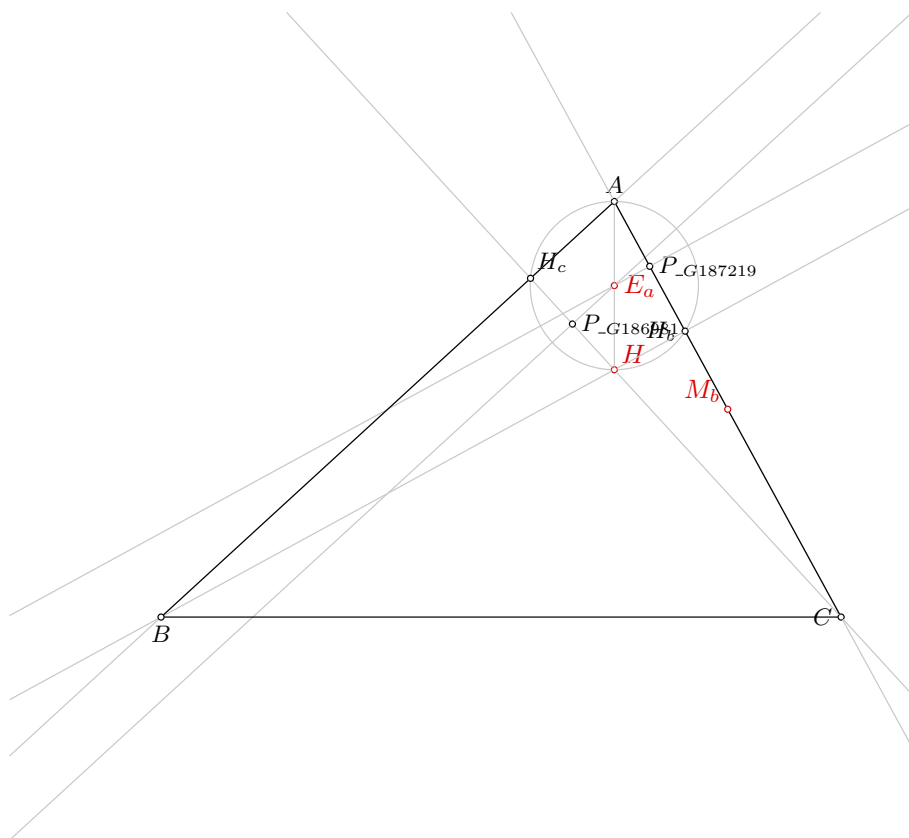


Figure 1: Illustration of the problem 0794

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $H = \neg H$

Proving failed

4.2.3 Proving $M_b = \neg M_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 1656 terms.

Time Complexity: Time spent by the prover is 10.570 seconds. There are no ndg conditions.

4.3.2 Proving $H = \neg H$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 333 terms.

Time Complexity: Time spent by the prover is 0.600 seconds. There are no ndg conditions.

4.3.3 Proving $M_b = \neg M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 19 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $H = \neg H$

Proving failed

4.4.3 Proving $M_b = \neg M_b$

Proving failed

Problem 795

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 795: Given a point E_a , a point H and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H , construct a point A (rule W01); ;
2. Using the point M_c and the point A , construct a point B (rule W01); ;
3. Using the point H and the point B , construct a line h_b (rule W02); % DET: points H and B are not the same;
4. Using the point M_c and the point A , construct a line c (rule W02); % DET: points M_c and A are not the same;
5. Using the point H and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H and E_a are not the same;
6. Using the circle $k(E_a, A)$, the line h_b , the point E_a and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_a, A)$ intersect % DET: points H and H_b must be different;
7. Using the point H_b and the point A , construct a line b (rule W02); % DET: points H_b and A are not the same;
8. Using the circle $k(E_a, A)$, the line c , the point E_a and the point A , construct a point H_c (rule W05); % NDG: line c and circle $k(E_a, A)$ intersect % DET: points A and H_c must be different;
9. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
10. Using the line b and the line h_c , construct a point C (rule W03); % NDG: lines b and h_c are not parallel % DET: lines b and h_c are not the same.

Non-degenerate conditions: lines b and h_c are not parallel; line c and circle $k(E_a, A)$ intersect; line h_b and circle $k(E_a, A)$ intersect; points H and E_a are not the same.

Determination conditions: lines b and h_c are not the same; points H_c and H are not the same; points A and H_c must be different; points H_b and A are not the same; points H and H_b must be different; points M_c and A are not the same; points H and B are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D20,D28,D3,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L46,L47,L48]

Solving time: 12.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point H 80 72.73
```

```
point M_{c} 50 67.5
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_rt H
```

```
cmark_lt M_{c}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point A such that E_{a}A/E_{a}H=-1
```

```
towards A E_{a} H -1
```

```
cmark_t A
```

```
color 200 200 200
```

```
drawsegment H A
```

```
color 0 0 0
```

```
% Constructing a point B such that M_{c}B/M_{c}A=-1
```

```
towards B M_{c} A -1
```

```
cmark_b B
```

```
color 200 200 200
```

```
drawsegment A B
```

```
color 0 0 0
```

```
% DET: points H and B are not the same
```

```
% Constructing a line h_{b} which passes through point H and point B
```

```
line h_{b} H B
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```

% DET: points  $M_{\{c\}}$  and  $A$  are not the same
% Constructing a line  $c$  which passes through point  $M_{\{c\}}$  and point  $A$ 
line c  $M_{\{c\}}$  A

color 200 200 200
drawline c
color 0 0 0

% NDG: points  $H$  and  $E_{\{a\}}$  are not the same
% Constructing a circle  $k(E_{\{a\}}, A)$  whose center is at point  $E_{\{a\}}$  and which passes through point  $H$ 
circle k( $E_{\{a\}}$ ,  $A$ )  $E_{\{a\}}$   $H$ 

color 200 200 200
drawcircle k( $E_{\{a\}}$ ,  $A$ )
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(E_{\{a\}}, A)$  intersect% DET: points  $H$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\_G227720\}}$  which is a foot of the point  $E_{\{a\}}$  on the line  $h_{\{b\}}$ 
foot  $P_{\{\_G227720\}}$   $E_{\{a\}}$   $h_{\{b\}}$ 
cmark_r  $P_{\{\_G227720\}}$ 
color 200 200 200
drawline  $E_{\{a\}}$   $P_{\{\_G227720\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $H$  in the symmetry to point/line  $P_{\{\_G227720\}}$ 
sim  $H_{\{b\}}$   $P_{\{\_G227720\}}$   $H$ 
cmark_l  $H_{\{b\}}$ 

% DET: points  $H_{\{b\}}$  and  $A$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $A$ 
line b  $H_{\{b\}}$  A

color 200 200 200
drawline b
color 0 0 0

% NDG: line  $c$  and circle  $k(E_{\{a\}}, A)$  intersect% DET: points  $A$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\_G227958\}}$  which is a foot of the point  $E_{\{a\}}$  on the line  $c$ 
foot  $P_{\{\_G227958\}}$   $E_{\{a\}}$   $c$ 
cmark_r  $P_{\{\_G227958\}}$ 
color 200 200 200
drawline  $E_{\{a\}}$   $P_{\{\_G227958\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $A$  in the symmetry to point/line  $P_{\{\_G227958\}}$ 
sim  $H_{\{c\}}$   $P_{\{\_G227958\}}$   $A$ 
cmark_rt  $H_{\{c\}}$ 

```

```

% DET: points  $H_{\{c\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $H_{\{c\}}$  and point  $H$ 
line  $h_{\{c\}}$   $H_{\{c\}}$   $H$ 

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: lines  $b$  and  $h_{\{c\}}$  are not parallel% DET: lines  $b$  and  $h_{\{c\}}$  are not the same
% Constructing a point  $C$  which belongs to line  $b$  and line  $h_{\{c\}}$ 
intersec  $C$   $b$   $h_{\{c\}}$ 
cmark_l  $C$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $b$  and  $h_{\{c\}}$  are not parallel; line  $c$  and circle  $k(E_{\{a\}}, A)$ 
% intersect; line  $h_{\{b\}}$  and circle  $k(E_{\{a\}}, A)$  intersect; points  $H$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $b$  and  $h_{\{c\}}$  are not the same; points  $H_{\{c\}}$  and  $H$  are not the same
% ; points  $A$  and  $H_{\{c\}}$  must be different; points  $H_{\{b\}}$  and  $A$  are not the same; points  $H$  and  $H_{\{b\}}$ 
% must be different; points  $M_{\{c\}}$  and  $A$  are not the same; points  $H$  and  $B$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = _E_a$

Proving failed

4.1.2 Proving $H = _H$

Proving failed

4.1.3 Proving $M_c = _M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 8 terms.

Time Complexity: Time spent by the prover is 0.101 seconds.

NDG conditions There are no NDG conditions for this theorem

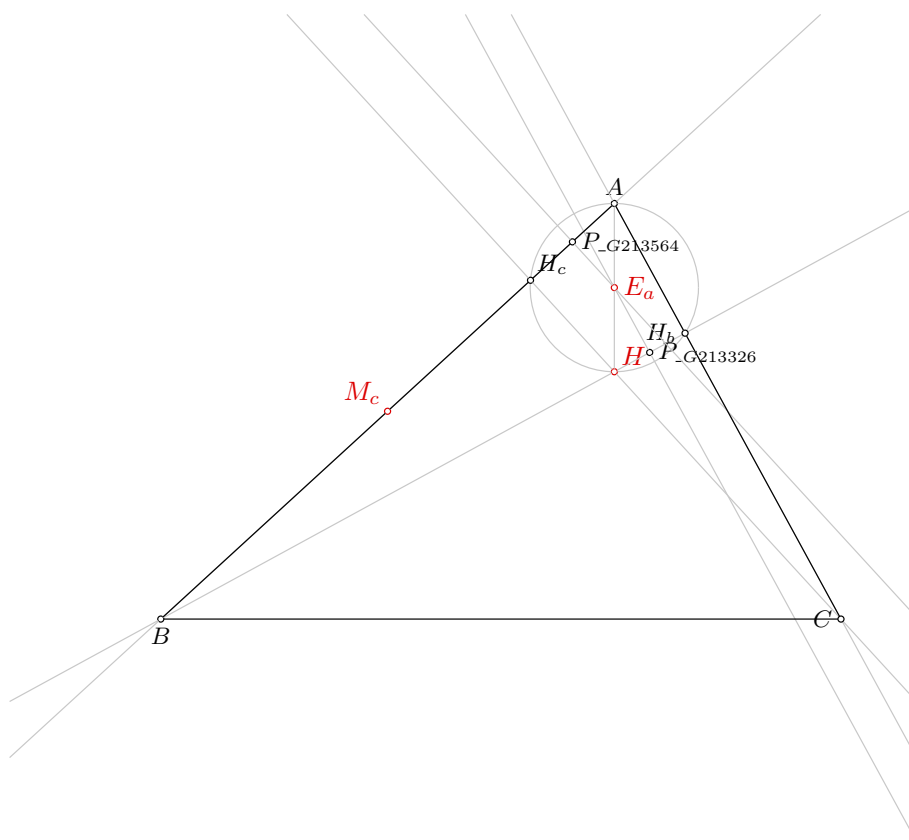


Figure 1: Illustration of the problem 0795

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $H = \neg H$

Proving failed

4.2.3 Proving $M_c = \neg M_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 2602 terms.

Time Complexity: Time spent by the prover is 14.030 seconds. There are no ndg conditions.

4.3.2 Proving $H = \neg H$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 494 terms.

Time Complexity: Time spent by the prover is 0.690 seconds. There are no ndg conditions.

4.3.3 Proving $M_c = \neg M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 19 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $H = \neg H$

Proving failed

4.4.3 Proving $M_c = \neg M_c$

Proving failed

Problem 796

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 796: Given a point E_a , a point H and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H , construct a point A (rule W01); ;
2. Using the point H and the point N , construct a point O (rule W01); ;
3. Using the point H and the point N , construct a point G (rule W01); ;
4. Using the point A and the point G , construct a point M_a (rule W01); ;
5. Using the point E_a and the point H , construct a line h_a (rule W02); % DET: points E_a and H are not the same;
6. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
8. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
9. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; line h_a and circle $k(N, M_a)$ intersect; points E_a and N are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points E_a and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D31,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L16,L19,L22,L55]

Solving time: 7.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point H 80 72.73
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_rt H
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point A such that E_{a}A/E_{a}H=-1
```

```
towards A E_{a} H -1
```

```
cmark_t A
```

```
color 200 200 200
```

```
drawsegment H A
```

```
color 0 0 0
```

```
% Constructing a point O such that HO/HN=2
```

```
towards O H N 2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment H O
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G28409} which passes through point H and point N
```

```
line L_{\_G28409} H N
```

```
color 200 200 200
```

```
drawline L_{\_G28409}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G28510} with coordinates (0,0)
```

```
point P_{\_G28510} 0 0
```



```

cmark_r P_{\_G28510}

% Constructing a point P_{\_G28434} such that HP_{\_G28434}/HP_{\_G28510}=4
towards P_{\_G28434} H P_{\_G28510} 4
cmark_r P_{\_G28434}
color 200 200 200
drawsegment H P_{\_G28434}
color 0 0 0

% Constructing a point P_{\_G28479} such that HP_{\_G28479}/HP_{\_G28510}=3
towards P_{\_G28479} H P_{\_G28510} 3
cmark_r P_{\_G28479}
color 200 200 200
drawsegment H P_{\_G28479}
color 0 0 0

% Constructing a line L_{\_G28440} which passes through point N and point P_{\_G28479}
line L_{\_G28440} N P_{\_G28479}

color 200 200 200
drawline L_{\_G28440}
color 0 0 0

% Constructing a line L_{\_G28403} which contains the point P_{\_G28434} and is parallel to the
line L_{\_G28440}
parallel L_{\_G28403} P_{\_G28434} L_{\_G28440}

color 200 200 200
drawline L_{\_G28403}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G28403} and line L_{\_G28409}
intersec G L_{\_G28403} L_{\_G28409}
cmark_t G

% Constructing a point M_{a} such that AM_{a}/AG=1.5
towards M_{a} A G 1.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% DET: points E_{a} and H are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H
line h_{a} E_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

```

```

% NDG: points  $E_{\{a\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{a\}}$ 
circle k(N, M_{\{a\}}) N E_{\{a\}}

color 200 200 200
drawcircle k(N, M_{\{a\}})
color 0 0 0

% NDG: line  $h_{\{a\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{a\}}$  and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\_G29346\}}$  which is a foot of the point  $N$  on the line  $h_{\{a\}}$ 
foot P_{\_G29346} N h_{\{a\}}
cmark_r P_{\_G29346}
color 200 200 200
drawline N P_{\_G29346}
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $E_{\{a\}}$  in the symmetry to point/line  $P_{\{\_G29346\}}$ 
sim H_{\{a\}} P_{\_G29346} E_{\{a\}}
cmark_r H_{\{a\}}

% DET: points  $H_{\{a\}}$  and  $M_{\{a\}}$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $M_{\{a\}}$ 
line a H_{\{a\}} M_{\{a\}}

color 200 200 200
drawline a
color 0 0 0

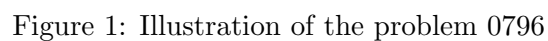
% NDG: points  $A$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $A$ 
circle k(O, C) O A

color 200 200 200
drawcircle k(O, C)
color 0 0 0

% NDG: line  $a$  and circle  $k(O, C)$  intersect
% Constructing points  $C$  and  $B$  which are in intersection of  $k(O, C)$  and  $a$ 
intersec2 C B k(O, C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```



```
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
    line h_{a} and circle k(N,M_{a}) intersect; points E_{a} and N are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
    different; points E_{a} and H are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = -E_a$

Proving failed

4.1.2 Proving $H=_H$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a=_Ea$

Proving failed

4.2.2 Proving $H=_H$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a=_Ea$

Proving failed

4.3.2 Proving $H=_H$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a=_Ea$

Proving failed

4.4.2 Proving $H=_H$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 797

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 797: Given a point E_a , a point H and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H , construct a point A (rule W01); ;
2. Using the point H and the point O , construct a point N (rule W01); ;
3. Using the point H and the point O , construct a point G (rule W01); ;
4. Using the point A and the point G , construct a point M_a (rule W01); ;
5. Using the point E_a and the point H , construct a line h_a (rule W02); % DET: points E_a and H are not the same;
6. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
8. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
9. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; line h_a and circle $k(N, M_a)$ intersect; points E_a and N are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points E_a and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D31,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L19,L22,L55,L58]

Solving time: 7.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point H 80 72.73
```

```
point O 65 51.14
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_rt H
```

```
cmark_t O
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point A such that E_{a}A/E_{a}H=-1
```

```
towards A E_{a} H -1
```

```
cmark_t A
```

```
color 200 200 200
```

```
drawsegment H A
```

```
color 0 0 0
```

```
% Constructing a point N such that HN/HO=0.5
```

```
towards N H O 0.5
```

```
cmark_r N
```

```
color 200 200 200
```

```
drawsegment H O
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G60697} which passes through point H and point O
```

```
line L_{\_G60697} H O
```

```
color 200 200 200
```

```
drawline L_{\_G60697}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G60798} with coordinates (0,0)
```

```
point P_{\_G60798} 0 0
```

```

cmark_r P_{\_G60798}

% Constructing a point P_{\_G60722} such that HP_{\_G60722}/HP_{\_G60798}=2
towards P_{\_G60722} H P_{\_G60798} 2
cmark_r P_{\_G60722}
color 200 200 200
drawsegment H P_{\_G60722}
color 0 0 0

% Constructing a point P_{\_G60767} such that HP_{\_G60767}/HP_{\_G60798}=3
towards P_{\_G60767} H P_{\_G60798} 3
cmark_r P_{\_G60767}
color 200 200 200
drawsegment H P_{\_G60767}
color 0 0 0

% Constructing a line L_{\_G60728} which passes through point O and point P_{\_G60767}
line L_{\_G60728} O P_{\_G60767}

color 200 200 200
drawline L_{\_G60728}
color 0 0 0

% Constructing a line L_{\_G60691} which contains the point P_{\_G60722} and is parallel to the
line L_{\_G60728}
parallel L_{\_G60691} P_{\_G60722} L_{\_G60728}

color 200 200 200
drawline L_{\_G60691}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G60691} and line L_{\_G60697}
intersec G L_{\_G60691} L_{\_G60697}
cmark_t G

% Constructing a point M_{a} such that AM_{a}/AG=1.5
towards M_{a} A G 1.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% DET: points E_{a} and H are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H
line h_{a} E_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

```

```

% NDG: points  $E_{\{a\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{a\}}$ 
circle k(N, M_{\{a\}}) N E_{\{a\}}

color 200 200 200
drawcircle k(N, M_{\{a\}})
color 0 0 0

% NDG: line  $h_{\{a\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{a\}}$  and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\_G61634\}}$  which is a foot of the point  $N$  on the line  $h_{\{a\}}$ 
foot P_{\_G61634} N h_{\{a\}}
cmark_r P_{\_G61634}
color 200 200 200
drawline N P_{\_G61634}
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $E_{\{a\}}$  in the symmetry to point/line  $P_{\_G61634}$ 
sim H_{\{a\}} P_{\_G61634} E_{\{a\}}
cmark_r H_{\{a\}}

% DET: points  $H_{\{a\}}$  and  $M_{\{a\}}$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $M_{\{a\}}$ 
line a H_{\{a\}} M_{\{a\}}

color 200 200 200
drawline a
color 0 0 0

% NDG: points  $A$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $A$ 
circle k(O, C) O A

color 200 200 200
drawcircle k(O, C)
color 0 0 0

% NDG: line  $a$  and circle  $k(O, C)$  intersect
% Constructing points  $C$  and  $B$  which are in intersection of  $k(O, C)$  and  $a$ 
intersec2 C B k(O, C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```

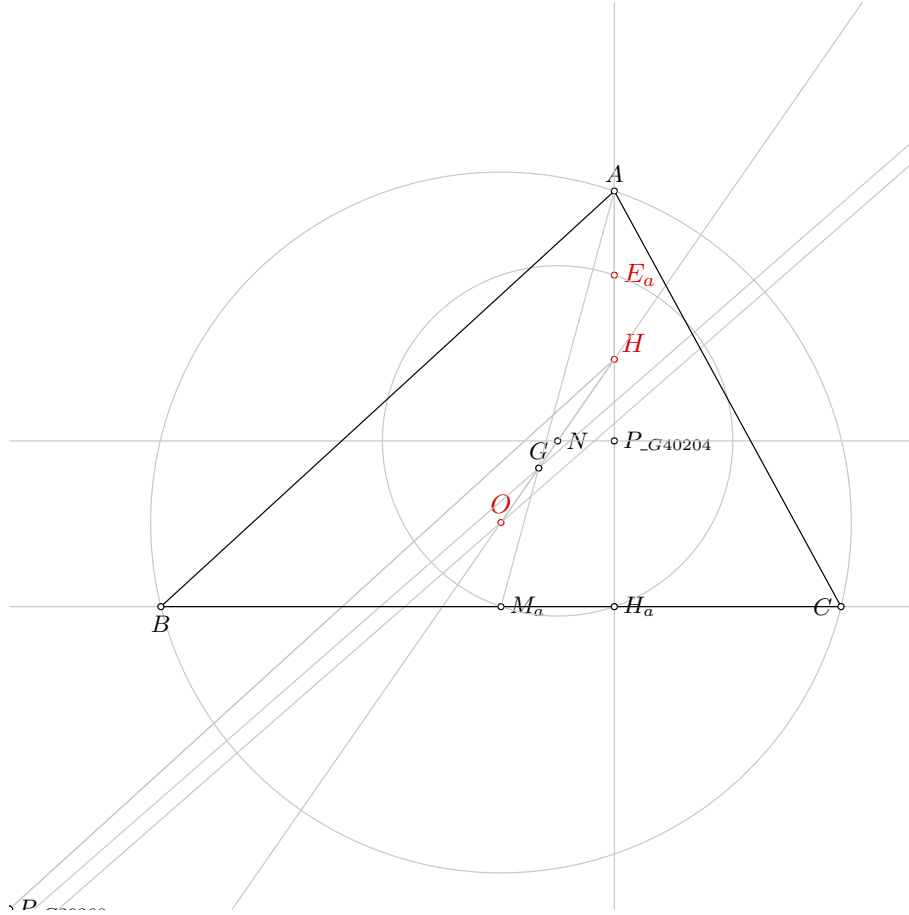



Figure 1: Illustration of the problem 0797

*% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
 line h_{a} and circle k(N,M_{a}) intersect; points E_{a} and N are not the same
 % Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
 different; points E_{a} and H are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = E_a$

Proving failed

4.1.2 Proving $H=_H$

Proving failed

4.1.3 Proving $O=_O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a=_Ea$

Proving failed

4.2.2 Proving $H=_H$

Proving failed

4.2.3 Proving $O=_O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a=_Ea$

Proving failed

4.3.2 Proving $H=_H$

Proving failed

4.3.3 Proving $O=_O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a=_Ea$

Proving failed

4.4.2 Proving $H=_H$

Proving failed

4.4.3 Proving $O=_O$

Proving failed

Problem 798

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 798: Given a point E_a , a point H and a point T_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H , construct a point A (rule W01); ;
2. Using the point E_a and the point H , construct a line h_a (rule W02); % DET: points E_a and H are not the same;
3. Using the point T_a and the point A , construct a line s_a (rule W02); % DET: points T_a and A are not the same;
4. Using the point T_a and the line h_a , construct a line a (rule W10a); ;
5. Using the line a and the line h_a , construct a point H_a (rule W03); % NDG: lines a and h_a are not parallel % DET: lines a and h_a are not the same;
6. Using the point H and the point H_a , construct a line $m(HH_a)$ (rule W14); % DET: points H and H_a are not the same;
7. Using the point E_a , the point A , the point T_a , the line s_a and the line h_a , construct a line AO (rule W17); % NDG: points A and T_a are not the same; points E_a and A are not the same % DET: points A and T_a are not the same;
8. Using the point E_a and the line AO , construct a line $m(H_bH_c)$ (rule W16); ;
9. Using the line $m(H_bH_c)$ and the line a , construct a point M_a (rule W03); % NDG: lines $m(H_bH_c)$ and a are not parallel % DET: lines $m(H_bH_c)$ and a are not the same;
10. Using the point M_a and the point A , construct a point G (rule W01); ;
11. Using the point H and the point G , construct a point N (rule W01); ;

12. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
13. Using the circle $k(N, M_a)$ and the line $m(HH_a)$, construct a point E_b and a point E_c (rule W04); % NDG: line $m(HH_a)$ and circle $k(N, M_a)$ intersect;
14. Using the point E_b and the point H , construct a point B (rule W01); ;
15. Using the point H and the point E_c , construct a point C (rule W01); .

Non-degenerate conditions: line $m(HH_a)$ and circle $k(N, M_a)$ intersect; points E_a and N are not the same; lines $m(H_bH_c)$ and a are not parallel; points A and T_a are not the same; points E_a and A are not the same; lines a and h_a are not parallel.

Determination conditions: lines $m(H_bH_c)$ and a are not the same; points A and T_a are not the same; points H and H_a are not the same; lines a and h_a are not the same; points T_a and A are not the same; points E_a and H are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W14,W16,W17]

Lemmas used: [D21,D23,D28,D29,D3,D30,D32,D5,D8,GD01,GD02,GL01,GL03,GL04,GL09,GL17,L104,L16,L2]

Solving time: 126.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point H 80 72.73
```

```
point T_{a} 70.86 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_rt H
```

```
cmark_rb T_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point A such that E_{a}A/E_{a}H=-1
```

```
towards A E_{a} H -1
```

```
cmark_t A
```

```
color 200 200 200
```

```
drawsegment H A
```

```
color 0 0 0
```

```
% DET: points E_{a} and H are not the same
```

```
% Constructing a line h_{a} which passes through point E_{a} and point H
```

```
line h_{a} E_{a} H
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```

% DET: points T_{a} and A are not the same
% Constructing a line s_{a} which passes through point T_{a} and point A
line s_{a} T_{a} A

color 200 200 200
drawline s_{a}
color 0 0 0

% Constructing a line a which is perpendicular to line h_{a} and which passes through point T_{a}
perp a T_{a} h_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines a and h_{a} are not parallel% DET: lines a and h_{a} are not the same
% Constructing a point H_{a} which belongs to line a and line h_{a}
intersec H_{a} a h_{a}
cmark_r H_{a}

% DET: points H and H_{a} are not the same
% Constructing bisector m(HH_{a}) of the segment HH_{a}
med m(HH_{a}) H H_{a}

color 200 200 200
drawline m(HH_{a})
color 0 0 0

color 200 200 200
drawsegment H H_{a}
color 0 0 0

% NDG: points A and T_{a} are not the same; points E_{a} and A are not the same% DET: points A and
T_{a} are not the same
% Constructing an angle V[_G100231] which is equal to the angle E_{a}AT_{a}
angle_o V[_G100231] E_{a} A T_{a}

% Calculating value angle[_G100310] using formula  $1/\text{pow}(2,0)*V[_G100231]+0/\text{pow}(2,0)*180$ 
expression angle[_G100310] {  $1/\text{pow}(2,0)*V[_G100231]+0/\text{pow}(2,0)*180$  }

% Constructing a point P_{\_G100307} which is an image of the point T_{a} in a rotation around the
point A for the angle  $1/\text{pow}(2,0)*V[_G100231]+0/\text{pow}(2,0)*180$ 
rotate P_{\_G100307} A angle[_G100310] T_{a}
cmark_r P_{\_G100307}

```

```

color 200 200 200
drawarc_p A T_{a} angle[_G100310]
color 0 0 0

% Constructing a line AO which passes through point A and point P_{\_G100307}
line AO A P_{\_G100307}

color 200 200 200
drawline AO
color 0 0 0

% Constructing a line m(H_{b}H_{c}) which contains the point E_{a} and is parallel to the line AO
parallel m(H_{b}H_{c}) E_{a} AO

color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0

% NDG: lines m(H_{b}H_{c}) and a are not parallel% DET: lines m(H_{b}H_{c}) and a are not the same
% Constructing a point M_{a} which belongs to line m(H_{b}H_{c}) and line a
intersec M_{a} m(H_{b}H_{c}) a
cmark_r M_{a}

% Constructing a line L_{\_G100832} which passes through point M_{a} and point A
line L_{\_G100832} M_{a} A

color 200 200 200
drawline L_{\_G100832}
color 0 0 0

% Constructing a point P_{\_G100933} with coordinates (0,0)
point P_{\_G100933} 0 0
cmark_r P_{\_G100933}

% Constructing a point P_{\_G100857} such that M_{a}P_{\_G100857}/M_{a}P_{\_G100933}=1
towards P_{\_G100857} M_{a} P_{\_G100933} 1
cmark_r P_{\_G100857}
color 200 200 200
drawsegment M_{a} P_{\_G100857}
color 0 0 0

% Constructing a point P_{\_G100902} such that M_{a}P_{\_G100902}/M_{a}P_{\_G100933}=3
towards P_{\_G100902} M_{a} P_{\_G100933} 3
cmark_r P_{\_G100902}
color 200 200 200
drawsegment M_{a} P_{\_G100902}
color 0 0 0

```

```

% Constructing a line  $L_{\{G100863\}}$  which passes through point A and point  $P_{\{G100902\}}$ 
line  $L_{\{G100863\}}$  A  $P_{\{G100902\}}$ 

color 200 200 200
drawline  $L_{\{G100863\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G100826\}}$  which contains the point  $P_{\{G100857\}}$  and is parallel to the
line  $L_{\{G100863\}}$ 
parallel  $L_{\{G100826\}}$   $P_{\{G100857\}}$   $L_{\{G100863\}}$ 

color 200 200 200
drawline  $L_{\{G100826\}}$ 
color 0 0 0

% Constructing a point G which belongs to line  $L_{\{G100826\}}$  and line  $L_{\{G100832\}}$ 
intersec G  $L_{\{G100826\}}$   $L_{\{G100832\}}$ 
cmark_t G

% Constructing a point N such that  $HN/HG=0.75$ 
towards N H G 0.75
cmark_r N
color 200 200 200
drawsegment H G
color 0 0 0

% NDG: points  $E_{\{a\}}$  and N are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point N and which passes through point  $E_{\{a\}}$ 
circle  $k(N, M_{\{a\}})$  N  $E_{\{a\}}$ 

color 200 200 200
drawcircle  $k(N, M_{\{a\}})$ 
color 0 0 0

% NDG: line  $m(HH_{\{a\}})$  and circle  $k(N, M_{\{a\}})$  intersect
% Constructing points  $E_{\{b\}}$  and  $E_{\{c\}}$  which are in intersection of  $k(N, M_{\{a\}})$  and  $m(HH_{\{a\}})$ 
intersec2  $E_{\{b\}}$   $E_{\{c\}}$   $k(N, M_{\{a\}})$   $m(HH_{\{a\}})$ 
cmark_r  $E_{\{b\}}$ 
cmark_r  $E_{\{c\}}$ 

% Constructing a point B such that  $E_{\{b\}}B/E_{\{b\}}H=-1$ 
towards B  $E_{\{b\}}$  H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

```

```

% Constructing a point C such that HC/HE_{c}=2
towards C H E_{c} 2
cmark_1 C
color 200 200 200
drawsegment H C
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions: line m(HH_{a}) and circle k(N,M_{a}) intersect; points E_{a} and N are
% not the same; lines m(H_{b}H_{c}) and a are not parallel; points A and T_{a} are not the same;
% points E_{a} and A are not the same; lines a and h_{a} are not parallel
% Determination conditions: lines m(H_{b}H_{c}) and a are not the same; points A and T_{a} are not
% the same; points H and H_{a} are not the same; lines a and h_{a} are not the same; points T_{a}
% and A are not the same; points E_{a} and H are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 374 terms.

Time Complexity: Time spent by the prover is 2.654 seconds.

NDG conditions Points T_a and H are not identical

Line through points E_a and A is not perpendicular to line through points A and T_a

Line through points A and H is not perpendicular to line through points H and P_{G90148}

Points A , M_a and P_{G90546} are not collinear

Points A , H_a and P_{G90546} are not collinear

Points H and M_a are not identical

Points H and M_a are not identical

Points A , B and C are not collinear

Line through points B and H is not perpendicular to line through points H and C

4.1.2 Proving $H = \neg H$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 326 terms.

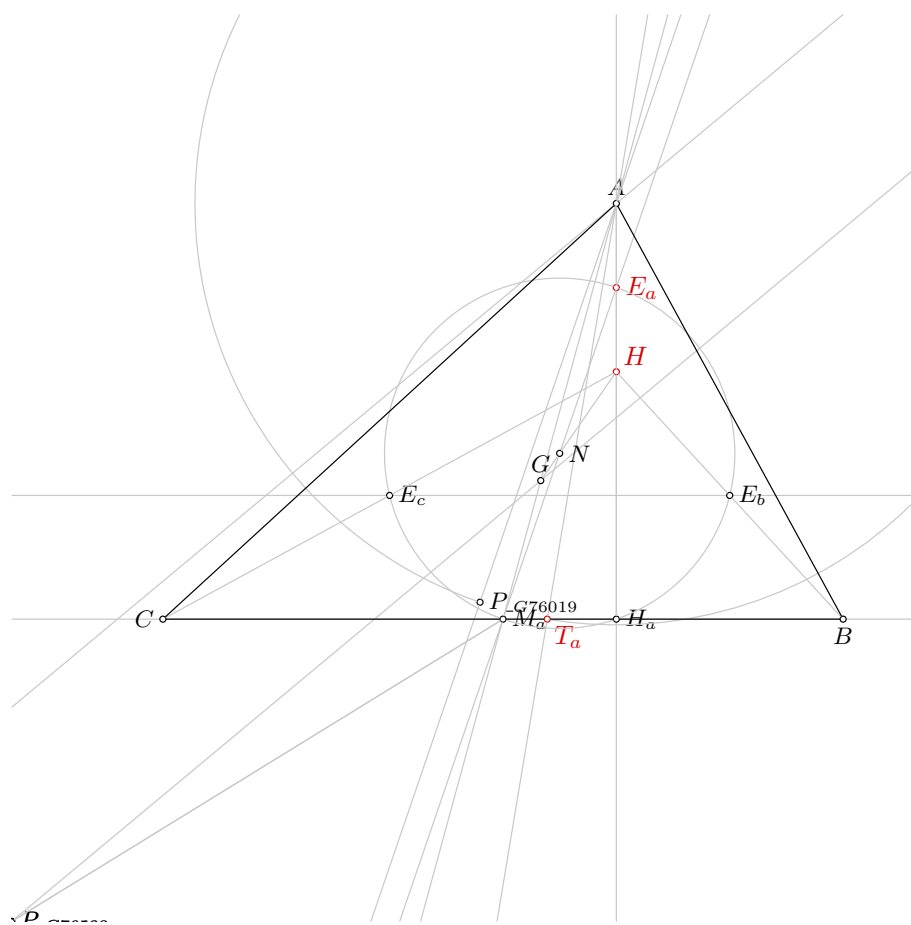


Figure 1: Illustration of the problem 0798

Time Complexity: Time spent by the prover is 2.48 seconds.

NDG conditions Points T_a and H are not identical

Line through points E_a and A is not perpendicular to line through points A and T_a

Line through points A and H is not perpendicular to line through points H and P_{G92921}

Points A , P_{G93319} and M_a are not collinear

Points P_{G93319} and E_a are not identical

Points H and M_a are not identical

Points H and M_a are not identical

Points A , B and C are not collinear

Line through points B and H is not perpendicular to line through points H and C

4.1.3 Proving $T_a = T_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = E_a$

Proving failed

4.2.2 Proving $H = H$

Proving failed

4.2.3 Proving $T_a = T_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = E_a$

Proving failed

4.3.2 Proving $H = H$

Proving failed

4.3.3 Proving $T_a = T_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = E_a$

Proving failed

4.4.2 Proving $H = H$

Proving failed

4.4.3 Proving $T_a = T_a$

Proving failed

Problem 799

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 799: Given a point E_a , a point H and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 800

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 800: Given a point E_a , a point H and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 801

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 801: Given a point E_a , a point H_a and a point H_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;
2. Using the point H_b and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_b and E_a are not the same;
3. Using the circle $k(E_a, A)$ and the line h_a , construct a point A and a point H (rule W04); % NDG: line h_a and circle $k(E_a, A)$ intersect;
4. Using the point H_b and the point A , construct a line b (rule W02); % DET: points H_b and A are not the same;
5. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
6. Using the point H_a and the line h_a , construct a line a (rule W10a); ;
7. Using the line a and the line b , construct a point C (rule W03); % NDG: lines a and b are not parallel % DET: lines a and b are not the same;
8. Using the line h_b and the line a , construct a point B (rule W03); % NDG: lines h_b and a are not parallel % DET: lines h_b and a are not the same.

Non-degenerate conditions: lines h_b and a are not parallel; lines a and b are not parallel; line h_a and circle $k(E_a, A)$ intersect; points H_b and E_a are not the same.

Determination conditions: lines h_b and a are not the same; lines a and b are not the same; points H_b and H are not the same; points H_b and A are not the same; points E_a and H_a are not the same.

Rules used: [W02,W03,W04,W06,W10a]
 Lemmas used: [D28,D3,D5,D6,D8,D9,GD01,GD02,GL09,L46,L47]
 Solving time: 6.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{a} 80 83.86
point H_{a} 80 40
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_r E_{a}
cmark_r H_{a}
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points E_{a} and H_{a} are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H_{a}
line h_{a} E_{a} H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points H_{b} and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H_{b}
circle k(E_{a},A) E_{a} H_{b}

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: line h_{a} and circle k(E_{a},A) intersect
% Constructing points A and H which are in intersection of k(E_{a},A) and h_{a}
intersec2 A H k(E_{a},A) h_{a}
cmark_t A
cmark_rt H

% DET: points H_{b} and A are not the same
% Constructing a line b which passes through point H_{b} and point A
line b H_{b} A

color 200 200 200
drawline b
```

```

color 0 0 0

% DET: points  $H_{\{b\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $H_{\{b\}}$  and point  $H$ 
line  $h_{\{b\}}$   $H_{\{b\}}$   $H$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% Constructing a line  $a$  which is perpendicular to line  $h_{\{a\}}$  and which passes through point  $H_{\{a\}}$ 
perp a  $H_{\{a\}}$   $h_{\{a\}}$ 

color 200 200 200
drawline a
color 0 0 0

% NDG: lines  $a$  and  $b$  are not parallel% DET: lines  $a$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $a$  and line  $b$ 
intersec C a b
cmark_l C

% NDG: lines  $h_{\{b\}}$  and  $a$  are not parallel% DET: lines  $h_{\{b\}}$  and  $a$  are not the same
% Constructing a point  $B$  which belongs to line  $h_{\{b\}}$  and line  $a$ 
intersec B  $h_{\{b\}}$  a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{b\}}$  and  $a$  are not parallel; lines  $a$  and  $b$  are not parallel;
% line  $h_{\{a\}}$  and circle  $k(E_{\{a\}}, A)$  intersect; points  $H_{\{b\}}$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $h_{\{b\}}$  and  $a$  are not the same; lines  $a$  and  $b$  are not the same;
% points  $H_{\{b\}}$  and  $H$  are not the same; points  $H_{\{b\}}$  and  $A$  are not the same; points  $E_{\{a\}}$  and  $H_{\{a\}}$ 
% are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

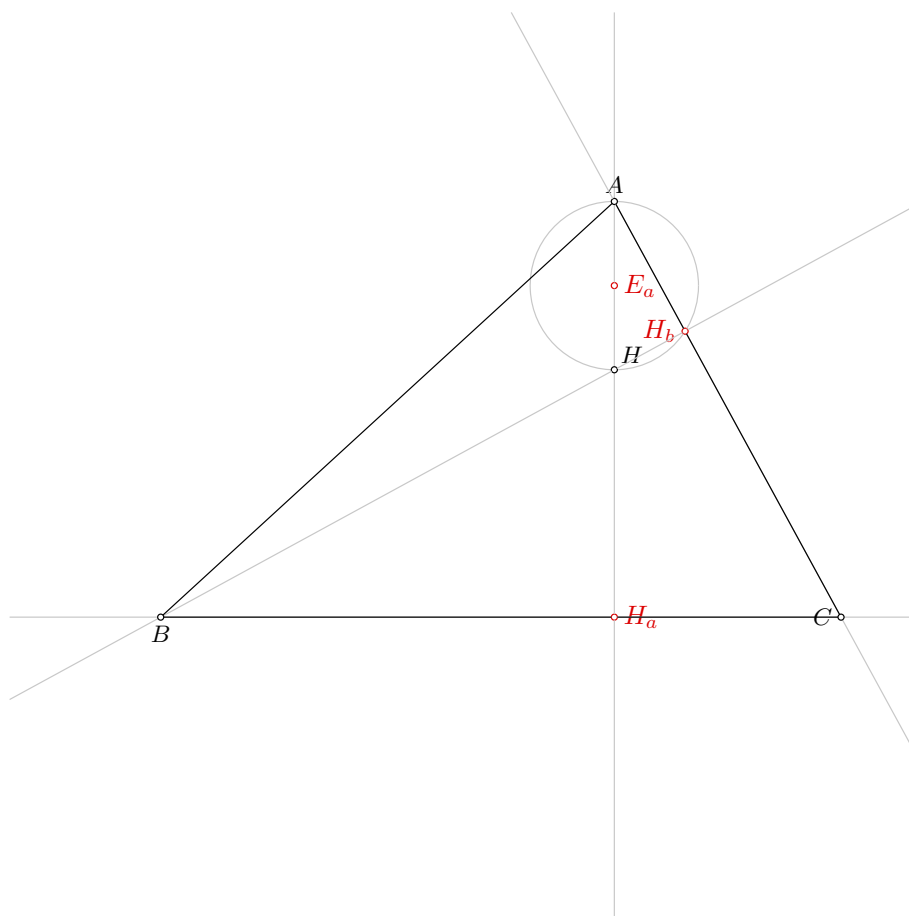


Figure 1: Illustration of the problem 0801

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 42 terms.

Time Complexity: Time spent by the prover is 0.387 seconds.

NDG conditions Line through points H_b and H_a is not perpendicular to line through points H_a and H

Line through points B and C is not parallel with line through points H_a and E_a

Line through points A and H_a is not perpendicular to line through points H_a and C

4.1.2 Proving $H_a = \neg H_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

Time Complexity: Time spent by the prover is 0.101 seconds.

NDG conditions Line through points H_b and H_a is not perpendicular to line through points H_a and H

Line through points B and C is not parallel with line through points H_a and E_a

4.1.3 Proving $H_b = \neg H_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $H_a = \neg H_a$

Proving failed

4.2.3 Proving $H_b = \neg H_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 65 terms.

Time Complexity: Time spent by the prover is 0.050 seconds. There are no ndg conditions.

4.3.2 Proving $H_a = \neg H_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.3 Proving $H_b = \neg H_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 904 terms.

Time Complexity: Time spent by the prover is 1.000 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $H_a = \neg H_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.3 Proving $H_b = \neg H_b$

Proving failed

Problem 802

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 802: Given a point E_a , a point H_a and a point H_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;
2. Using the point H_c and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_c and E_a are not the same;
3. Using the circle $k(E_a, A)$ and the line h_a , construct a point A and a point H (rule W04); % NDG: line h_a and circle $k(E_a, A)$ intersect;
4. Using the point H_c and the point A , construct a line c (rule W02); % DET: points H_c and A are not the same;
5. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
6. Using the point H_a and the line h_a , construct a line a (rule W10a); ;
7. Using the line a and the line c , construct a point B (rule W03); % NDG: lines a and c are not parallel % DET: lines a and c are not the same;
8. Using the line h_c and the line a , construct a point C (rule W03); % NDG: lines h_c and a are not parallel % DET: lines h_c and a are not the same.

Non-degenerate conditions: lines h_c and a are not parallel; lines a and c are not parallel; line h_a and circle $k(E_a, A)$ intersect; points H_c and E_a are not the same.

Determination conditions: lines h_c and a are not the same; lines a and c are not the same; points H_c and H are not the same; points H_c and A are not the same; points E_a and H_a are not the same.

Rules used: [W02,W03,W04,W06,W10a]

Lemmas used: [D10,D28,D3,D5,D7,D8,GD01,GD02,GL09,L3,L46,L48]

Solving time: 6.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
point H_{a} 80 40
point H_{c} 68.91 84.83
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{a}
cmark_r H_{a}
cmark_rt H_{c}
color 0 0 0
fontsize 8
```

```
% DET: points E_{a} and H_{a} are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H_{a}
line h_{a} E_{a} H_{a}
```

```
color 200 200 200
drawline h_{a}
color 0 0 0
```

```
% NDG: points H_{c} and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H_{c}
circle k(E_{a},A) E_{a} H_{c}
```

```
color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0
```

```
% NDG: line h_{a} and circle k(E_{a},A) intersect
% Constructing points A and H which are in intersection of k(E_{a},A) and h_{a}
intersec2 A H k(E_{a},A) h_{a}
cmark_t A
cmark_rt H
```

```
% DET: points H_{c} and A are not the same
% Constructing a line c which passes through point H_{c} and point A
line c H_{c} A
```

```
color 200 200 200
drawline c
```

```
color 0 0 0
```

```
% DET: points  $H_{\{c\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $H_{\{c\}}$  and point  $H$ 
line  $h_{\{c\}}$   $H_{\{c\}}$   $H$ 
```

```
color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0
```

```
% Constructing a line  $a$  which is perpendicular to line  $h_{\{a\}}$  and which passes through point  $H_{\{a\}}$ 
perp  $a$   $H_{\{a\}}$   $h_{\{a\}}$ 
```

```
color 200 200 200
drawline  $a$ 
color 0 0 0
```

```
% NDG: lines  $a$  and  $c$  are not parallel% DET: lines  $a$  and  $c$  are not the same
% Constructing a point  $B$  which belongs to line  $a$  and line  $c$ 
intersec  $B$   $a$   $c$ 
cmark_b  $B$ 
```

```
% NDG: lines  $h_{\{c\}}$  and  $a$  are not parallel% DET: lines  $h_{\{c\}}$  and  $a$  are not the same
% Constructing a point  $C$  which belongs to line  $h_{\{c\}}$  and line  $a$ 
intersec  $C$   $h_{\{c\}}$   $a$ 
cmark_l  $C$ 
```

```
drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 
```

```
% Non-degenerate conditions: lines  $h_{\{c\}}$  and  $a$  are not parallel; lines  $a$  and  $c$  are not parallel;
line  $h_{\{a\}}$  and circle  $k(E_{\{a\}}, A)$  intersect; points  $H_{\{c\}}$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $h_{\{c\}}$  and  $a$  are not the same; lines  $a$  and  $c$  are not the same;
points  $H_{\{c\}}$  and  $H$  are not the same; points  $H_{\{c\}}$  and  $A$  are not the same; points  $E_{\{a\}}$  and  $H_{\{a\}}$ 
are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

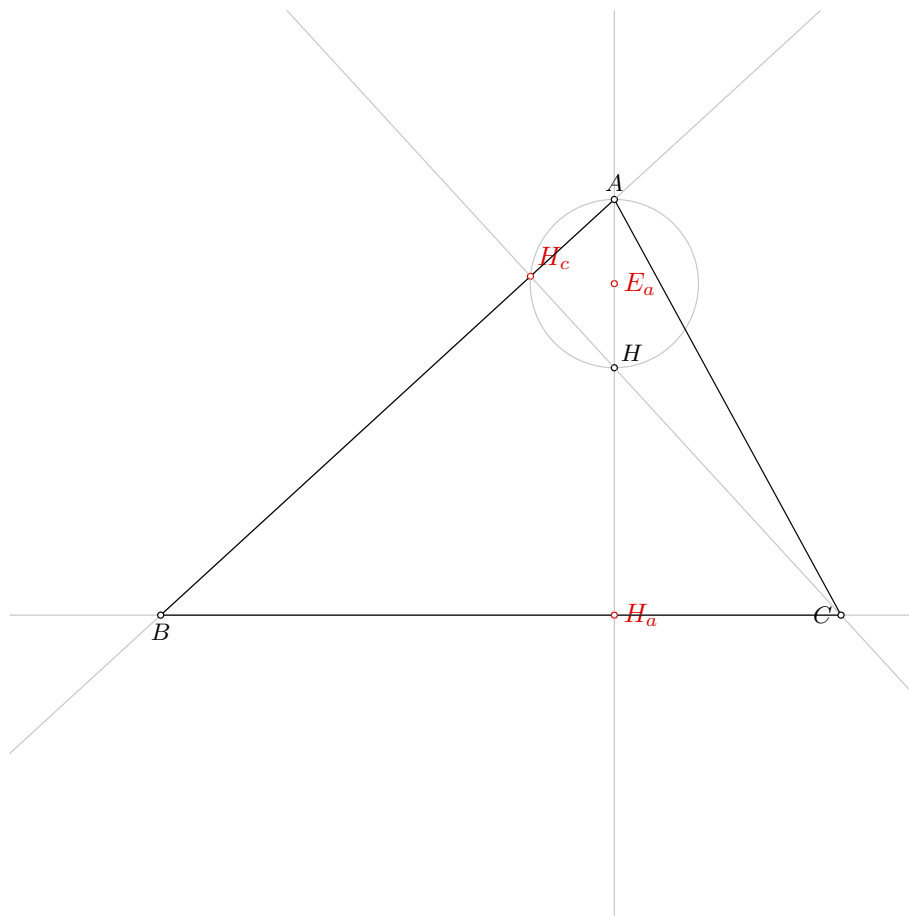


Figure 1: Illustration of the problem 0802

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 42 terms.

Time Complexity: Time spent by the prover is 0.453 seconds.

NDG conditions Point H is not the midpoint of segment with endpoints A and H_c

Line through points B and C is not parallel with line through points H_a and E_a

Line through points A and H_a is not perpendicular to line through points H_a and C

4.1.2 Proving $H_a = \neg H_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

Time Complexity: Time spent by the prover is 0.107 seconds.

NDG conditions Point H is not the midpoint of segment with endpoints A and H_c

Line through points B and C is not parallel with line through points H_a and E_a

4.1.3 Proving $H_c = \neg H_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $H_a = \neg H_a$

Proving failed

4.2.3 Proving $H_c = \neg H_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 65 terms.

Time Complexity: Time spent by the prover is 0.020 seconds. There are no ndg conditions.

4.3.2 Proving $H_a = \neg H_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.3 Proving $H_c = \neg H_c$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 395 terms.

Time Complexity: Time spent by the prover is 0.530 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $H_a = \neg H_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.3 Proving $H_c = \neg H_c$

Proving failed

Problem 803

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 803: Given a point E_a , a point H_a and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 804

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 804: Given a point H_a , a point M_a and a point E_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
2. Using the point H_a and the line a , construct a line h_a (rule W10b); ;
3. Choose freely a point E_a on the line h_a (rule WOnline2);
4. Using the point E_a and the point M_a , construct a line $m(H_bH_c)$ (rule W02); % DET: points E_a and M_a are not the same;
5. Using the point E_a and the point H_a , construct a line $m(E_aH_a)$ (rule W14); % DET: points E_a and H_a are not the same;
6. Using the line $m(E_aH_a)$ and the line $m(H_bH_c)$, construct a point N (rule W03); % NDG: lines $m(E_aH_a)$ and $m(H_bH_c)$ are not parallel % DET: lines $m(E_aH_a)$ and $m(H_bH_c)$ are not the same;
7. Choose freely a point A on the line h_a (rule WOnline1) ;
8. Using the point A and the point E_a , construct a point H (rule W01); ;
9. Using the point N and the point H , construct a point O (rule W01); ;
10. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
11. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; lines $m(E_a H_a)$ and $m(H_b H_c)$ are not parallel.

Determination conditions: lines $m(E_a H_a)$ and $m(H_b H_c)$ are not the same; points E_a and H_a are not the same; points E_a and M_a are not the same; points H_a and M_a are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14,WOnline1,WOnline2]

Lemmas used: [D21,D26,D28,D31,D32,D5,D8,GD01,GD02,GL01,GL03,GL04,GL09,L11,L12,L19,L20,L21,L22,

Solving time: 676.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{a} 80 40
point M_{a} 65 40
point E_{a} 80 83.86
```

```
color 220 0 0
fontsize 9
```

```
cmark_r H_{a}
cmark_r M_{a}
cmark_r E_{a}
color 0 0 0
fontsize 8
```

```
% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a
```

```
color 200 200 200
drawline h_{a}
color 0 0 0
```

```
% Generating random value V[_G179168]
random V[_G179168]
```

```
% Calculating value V[_G179189] using formula V[_G179168]*20
expression V[_G179189] { V[_G179168]*20 }
```

```

% Constructing a point E_{a} which is a point for which holds  $H_{a}E_{a} = V[_G179189]$  and angle  $M_{a}H_{a}E_{a} = 90$ 
turtle E_{a} M_{a} H_{a} 90 V[_G179189]
cmark_r E_{a}

% DET: points E_{a} and M_{a} are not the same
% Constructing a line  $m(H_{b}H_{c})$  which passes through point E_{a} and point M_{a}
line m(H_{b}H_{c}) E_{a} M_{a}

color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0

% DET: points E_{a} and H_{a} are not the same
% Constructing bisector  $m(E_{a}H_{a})$  of the segment  $E_{a}H_{a}$ 
med m(E_{a}H_{a}) E_{a} H_{a}

color 200 200 200
drawline m(E_{a}H_{a})
color 0 0 0

color 200 200 200
drawsegment E_{a} H_{a}
color 0 0 0

% NDG: lines  $m(E_{a}H_{a})$  and  $m(H_{b}H_{c})$  are not parallel% DET: lines  $m(E_{a}H_{a})$  and  $m(H_{b}H_{c})$  are not the same
% Constructing a point N which belongs to line  $m(E_{a}H_{a})$  and line  $m(H_{b}H_{c})$ 
intersec N m(E_{a}H_{a}) m(H_{b}H_{c})
cmark_r N

% Choosing randomly a point A on the line  $E_{a}H_{a}$ 
online A E_{a} H_{a}
cmark_t A
color 200 200 200
drawline E_{a} H_{a}
color 0 0 0

% Constructing a point H such that  $AH/AE_{a}=2$ 
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% Constructing a point O such that  $NO/NH=-1$ 

```

Figure 1: Illustration of the problem 0804

```

towards O N H -1
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% lines m(E_{a}H_{a}) and m(H_{b}H_{c}) are not parallel
% Determination conditions: lines m(E_{a}H_{a}) and m(H_{b}H_{c}) are not the same; points E_{a}
% and H_{a} are not the same; points E_{a} and M_{a} are not the same; points H_{a} and M_{a} are
% not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_a = H_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 4 terms.

Time Complexity: Time spent by the prover is 0.128 seconds.

NDG conditions Points H and M_a are not identical

Line through points A and H_a is not parallel with line through points B and C

4.1.2 Proving $M_a = \neg M_a$

Proving failed

4.1.3 Proving $E_a = \neg E_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_a = \neg H_a$

Proving failed

4.2.2 Proving $M_a = \neg M_a$

Proving failed

4.2.3 Proving $E_a = \neg E_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_a = \neg H_a$

Proving failed

4.3.2 Proving $M_a = \neg M_a$

Proving failed

4.3.3 Proving $E_a = \neg E_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_a = \neg H_a$

Proving failed

4.4.2 Proving $M_a = \neg M_a$

Proving failed

4.4.3 Proving $E_a = \neg E_a$

Proving failed

Problem 805

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 805: Given a point E_a , a point H_a and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;
2. Using the point H_a and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points H_a and M_b are not the same;
3. Using the circle $k(M_b, C)$, the line h_a , the point M_b and the point H_a , construct a point A (rule W05); % NDG: line h_a and circle $k(M_b, C)$ intersect % DET: points H_a and A must be different;
4. Using the point A and the point E_a , construct a point H (rule W01); ;
5. Using the point M_b and the point A , construct a point C (rule W01); ;
6. Using the point H_a and the point C , construct a line a (rule W02); % DET: points H_a and C are not the same;
7. Using the point H and the point C , construct a line h_c (rule W02); % DET: points H and C are not the same;
8. Using the circle $k(M_b, C)$, the line h_c , the point M_b and the point C , construct a point H_c (rule W05); % NDG: line h_c and circle $k(M_b, C)$ intersect % DET: points C and H_c must be different;
9. Using the point A and the point H_c , construct a line c (rule W02); % DET: points A and H_c are not the same;
10. Using the line a and the line c , construct a point B (rule W03); % NDG: lines a and c are not parallel % DET: lines a and c are not the same.

Non-degenerate conditions: lines a and c are not parallel; line h_c and circle $k(M_b, C)$ intersect; line h_a and circle $k(M_b, C)$ intersect; points H_a and M_b are not the same.

Determination conditions: lines a and c are not the same; points A and H_c are not the same; points C and H_c must be different; points H and C are not the same; points H_a and C are not the same; points H_a and A must be different; points E_a and H_a are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D22,D28,D3,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L43,L44,L45]

Solving time: 8.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point H_{a} 80 40
```

```
point M_{b} 95 67.5
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_r H_{a}
```

```
cmark_lt M_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{a} and H_{a} are not the same
```

```
% Constructing a line h_{a} which passes through point E_{a} and point H_{a}
```

```
line h_{a} E_{a} H_{a}
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```
% NDG: points H_{a} and M_{b} are not the same
```

```
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{a}
```

```
circle k(M_{b},C) M_{b} H_{a}
```

```
color 200 200 200
```

```
drawcircle k(M_{b},C)
```

```
color 0 0 0
```

```
% NDG: line h_{a} and circle k(M_{b},C) intersect% DET: points H_{a} and A must be different
```

```
% Constructing a point P_{\_G203423} which is a foot of the point M_{b} on the line h_{a}
```

```
foot P_{\_G203423} M_{b} h_{a}
```

```
cmark_r P_{\_G203423}
```

```
color 200 200 200
```

```
drawline M_{b} P_{\_G203423}
```

```
color 0 0 0
```



```

% Constructing a point A which is an image of the point H_{a} in the symmetry to point/line P_{\_G
203423}
sim A P_{\_G203423} H_{a}
cmark_t A

% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% Constructing a point C such that M_{b}C/M_{b}A=-1
towards C M_{b} A -1
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points H_{a} and C are not the same
% Constructing a line a which passes through point H_{a} and point C
line a H_{a} C

color 200 200 200
drawline a
color 0 0 0

% DET: points H and C are not the same
% Constructing a line h_{c} which passes through point H and point C
line h_{c} H C

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: line h_{c} and circle k(M_{b},C) intersect% DET: points C and H_{c} must be different
% Constructing a point P_{\_G203860} which is a foot of the point M_{b} on the line h_{c}
foot P_{\_G203860} M_{b} h_{c}
cmark_r P_{\_G203860}
color 200 200 200
drawline M_{b} P_{\_G203860}
color 0 0 0

% Constructing a point H_{c} which is an image of the point C in the symmetry to point/line P_{\_G
203860}
sim H_{c} P_{\_G203860} C

```

```

cmark_rt H_{c}

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines a and c are not parallel% DET: lines a and c are not the same
% Constructing a point B which belongs to line a and line c
intersec B a c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and c are not parallel; line h_{c} and circle k(M_{b},C)
% intersect; line h_{a} and circle k(M_{b},C) intersect; points H_{a} and M_{b} are not the same
% Determination conditions: lines a and c are not the same; points A and H_{c} are not the same;
% points C and H_{c} must be different; points H and C are not the same; points H_{a} and C are
% not the same; points H_{a} and A must be different; points E_{a} and H_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $H_a = \neg H_a$

Proving failed

4.1.3 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 8 terms.

Time Complexity: Time spent by the prover is 0.12 seconds.

NDG conditions There are no NDG conditions for this theorem

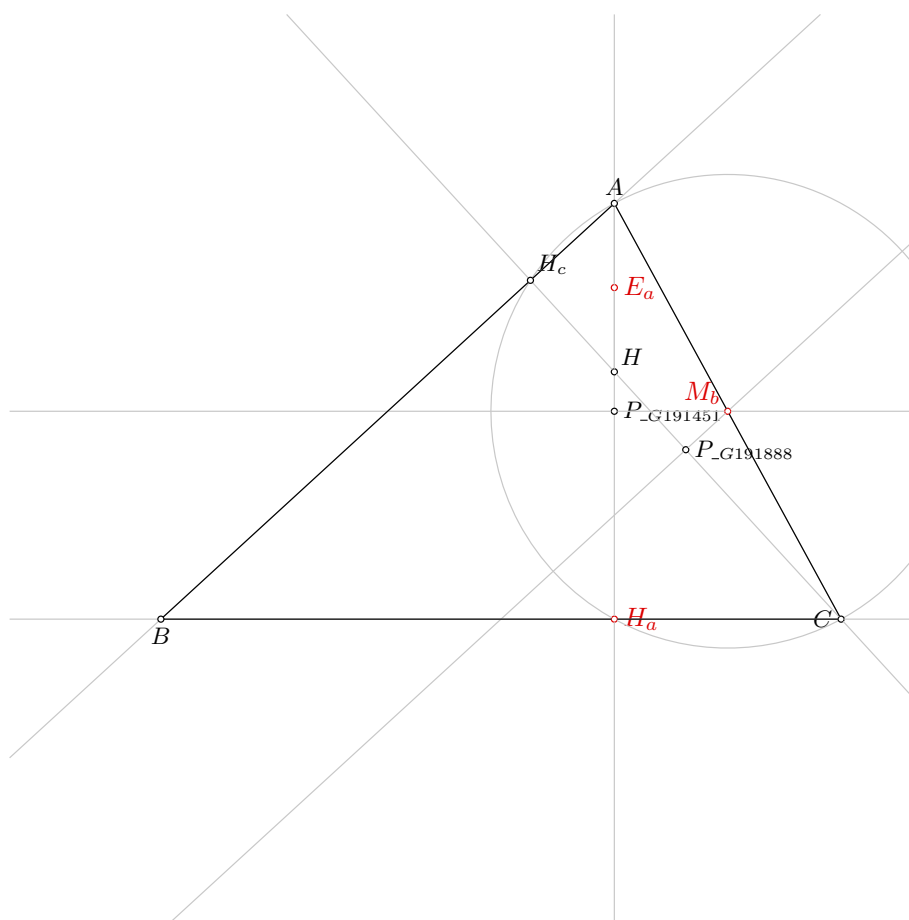


Figure 1: Illustration of the problem 0805

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $H_a = \neg H_a$

Proving failed

4.2.3 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{H_a A H_c} \neq S_{C A H_c}$ i.e., lines $H_a C$ and $A H_c$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{\neg h_b}} \neq S_{F^0_{\neg h_a} B F^1_{\neg h_b}}$ i.e., lines $AF^0_{\neg h_a}$ and $BF^1_{\neg h_b}$ are not parallel (construction based assumption)

$S_{BAF^0_{\neg h_a}} \neq S_{CA F^0_{\neg h_a}}$ i.e., lines BC and $AF^0_{\neg h_a}$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $H_a = \neg H_a$

Proving failed

4.3.3 Proving $M_b = \neg M_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $H_a = \neg H_a$

Proving failed

4.4.3 Proving $M_b = \neg M_b$

Proving failed

Problem 806

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 806: Given a point E_a , a point H_a and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;
2. Using the point H_a and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points H_a and M_c are not the same;
3. Using the circle $k(M_c, A)$, the line h_a , the point M_c and the point H_a , construct a point A (rule W05); % NDG: line h_a and circle $k(M_c, A)$ intersect % DET: points H_a and A must be different;
4. Using the point A and the point E_a , construct a point H (rule W01); ;
5. Using the point M_c and the point A , construct a point B (rule W01); ;
6. Using the point H_a and the point B , construct a line a (rule W02); % DET: points H_a and B are not the same;
7. Using the point H and the point B , construct a line h_b (rule W02); % DET: points H and B are not the same;
8. Using the circle $k(M_c, A)$, the line h_b , the point M_c and the point B , construct a point H_b (rule W05); % NDG: line h_b and circle $k(M_c, A)$ intersect % DET: points B and H_b must be different;
9. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;
10. Using the line a and the line b , construct a point C (rule W03); % NDG: lines a and b are not parallel % DET: lines a and b are not the same.

Non-degenerate conditions: lines a and b are not parallel; line h_b and circle $k(M_c, A)$ intersect; line h_a and circle $k(M_c, A)$ intersect; points H_a and M_c are not the same.

Determination conditions: lines a and b are not the same; points A and H_b are not the same; points B and H_b must be different; points H and B are not the same; points H_a and B are not the same; points H_a and A must be different; points E_a and H_a are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D20,D28,D3,D5,D6,D8,D9,GD01,GD02,GL03,GL04,GL09,L40,L41,L42]

Solving time: 8.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point H_{a} 80 40
```

```
point M_{c} 50 67.5
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_r H_{a}
```

```
cmark_lt M_{c}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{a} and H_{a} are not the same
```

```
% Constructing a line h_{a} which passes through point E_{a} and point H_{a}
```

```
line h_{a} E_{a} H_{a}
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```
% NDG: points H_{a} and M_{c} are not the same
```

```
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{a}
```

```
circle k(M_{c},A) M_{c} H_{a}
```

```
color 200 200 200
```

```
drawcircle k(M_{c},A)
```

```
color 0 0 0
```

```
% NDG: line h_{a} and circle k(M_{c},A) intersect% DET: points H_{a} and A must be different
```

```
% Constructing a point P_{\_G227404} which is a foot of the point M_{c} on the line h_{a}
```

```
foot P_{\_G227404} M_{c} h_{a}
```

```
cmark_r P_{\_G227404}
```

```
color 200 200 200
```

```
drawline M_{c} P_{\_G227404}
```

```
color 0 0 0
```

```

% Constructing a point A which is an image of the point H_{a} in the symmetry to point/line P_{\_G
227404}
sim A P_{\_G227404} H_{a}
cmark_t A

% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% Constructing a point B such that M_{c}B/M_{c}A=-1
towards B M_{c} A -1
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% DET: points H and B are not the same
% Constructing a line h_{b} which passes through point H and point B
line h_{b} H B

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{b} and circle k(M_{c},A) intersect% DET: points B and H_{b} must be different
% Constructing a point P_{\_G227841} which is a foot of the point M_{c} on the line h_{b}
foot P_{\_G227841} M_{c} h_{b}
cmark_r P_{\_G227841}
color 200 200 200
drawline M_{c} P_{\_G227841}
color 0 0 0

% Constructing a point H_{b} which is an image of the point B in the symmetry to point/line P_{\_G
227841}
sim H_{b} P_{\_G227841} B

```

```

cmark_l H_{b}

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines a and b are not parallel% DET: lines a and b are not the same
% Constructing a point C which belongs to line a and line b
intersec C a b
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and b are not parallel; line h_{b} and circle k(M_{c},A)
% intersect; line h_{a} and circle k(M_{c},A) intersect; points H_{a} and M_{c} are not the same
% Determination conditions: lines a and b are not the same; points A and H_{b} are not the same;
% points B and H_{b} must be different; points H and B are not the same; points H_{a} and B are
% not the same; points H_{a} and A must be different; points E_{a} and H_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $H_a = \neg H_a$

Proving failed

4.1.3 Proving $M_c = \neg M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 8 terms.

Time Complexity: Time spent by the prover is 0.112 seconds.

NDG conditions There are no NDG conditions for this theorem

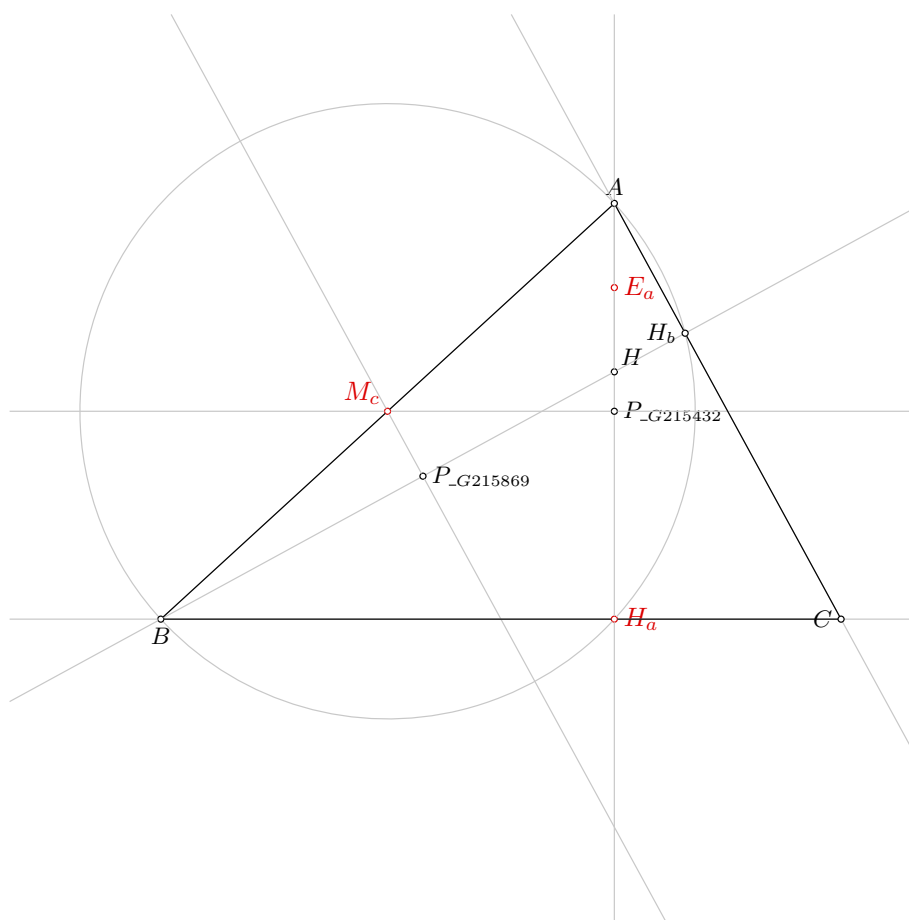


Figure 1: Illustration of the problem 0806

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $H_a = \neg H_a$

Proving failed

4.2.3 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{H_a A H_b} \neq S_{B A H_b}$ i.e., lines $H_a B$ and $A H_b$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{\neg h_b}} \neq S_{F^0_{\neg h_a} B F^1_{\neg h_b}}$ i.e., lines $AF^0_{\neg h_a}$ and $BF^1_{\neg h_b}$ are not parallel (construction based assumption)

$S_{BAF^0_{\neg h_a}} \neq S_{CA F^0_{\neg h_a}}$ i.e., lines BC and $AF^0_{\neg h_a}$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $H_a = \neg H_a$

Proving failed

4.3.3 Proving $M_c = \neg M_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $H_a = \neg H_a$

Proving failed

4.4.3 Proving $M_c = \neg M_c$

Proving failed

Problem 807

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 807: Given a point H_a , a point N and a point E_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_a and N are not the same;
2. Choose freely a point E_a on the circle $k(N, M_a)$ (rule W0ncircle);
3. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;
4. Using the point E_a and the point N , construct a line $m(H_bH_c)$ (rule W02); % DET: points E_a and N are not the same;
5. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point E_a , construct a point M_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points E_a and M_a must be different;
6. Using the point M_a and the point H_a , construct a line a (rule W02); % DET: points M_a and H_a are not the same;
7. Choose freely a point A on the line h_a (rule W0online1) ;
8. Using the point A and the point E_a , construct a point H (rule W01); ;
9. Using the point N and the point H , construct a point O (rule W01); ;
10. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
11. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; points H_a and N are not the same.

Determination conditions: points M_a and H_a are not the same; points E_a and M_a must be different; points E_a and N are not the same; points E_a and H_a are not the same.

Rules used: [W01,W02,W04,W05a,W06,WOncircle1,WOnline1]

Lemmas used: [D21,D26,D28,D3,D31,D32,D5,D8,GD01,GD02,GL01,GL03,GL04,GL09,L11,L12,L19,L20,L21,L

Solving time: 568.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{a} 80 40
point N 72.5 61.93
point E_{a} 80 83.86
```

```
color 220 0 0
fontsize 9
```

```
cmark_r H_{a}
cmark_r N
cmark_r E_{a}
color 0 0 0
fontsize 8
```

```
% NDG: points H_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{a}
circle k(N,M_{a}) N H_{a}
```

```
color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```
% Choosing randomly a point E_{a} on the circle with center N through point H_{a}
oncircle E_{a} N H_{a}
cmark_r E_{a}
color 200 200 200
drawcircle N H_{a}
color 0 0 0
```

```
% DET: points E_{a} and H_{a} are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H_{a}
line h_{a} E_{a} H_{a}
```

```
color 200 200 200
drawline h_{a}
color 0 0 0
```

```

% DET: points  $E_{\{a\}}$  and  $N$  are not the same
% Constructing a line  $m(H_{\{b\}}H_{\{c\}})$  which passes through point  $E_{\{a\}}$  and point  $N$ 
line m( $H_{\{b\}}H_{\{c\}}$ )  $E_{\{a\}}$  N

color 200 200 200
drawline m( $H_{\{b\}}H_{\{c\}}$ )
color 0 0 0

% NDG: line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{a\}}$  and  $M_{\{a\}}$  must be
different
% Constructing a point  $M_{\{a\}}$  which is an image of the point  $E_{\{a\}}$  in the symmetry to point/line  $N$ 
sim  $M_{\{a\}}$  N  $E_{\{a\}}$ 
cmark_r  $M_{\{a\}}$ 

% DET: points  $M_{\{a\}}$  and  $H_{\{a\}}$  are not the same
% Constructing a line  $a$  which passes through point  $M_{\{a\}}$  and point  $H_{\{a\}}$ 
line a  $M_{\{a\}}$   $H_{\{a\}}$ 

color 200 200 200
drawline a
color 0 0 0

% Choosing randomly a point  $A$  on the line  $E_{\{a\}}H_{\{a\}}$ 
online A  $E_{\{a\}}$   $H_{\{a\}}$ 
cmark_t A
color 200 200 200
drawline  $E_{\{a\}}$   $H_{\{a\}}$ 
color 0 0 0

% Constructing a point  $H$  such that  $AH/AE_{\{a\}}=2$ 
towards H A  $E_{\{a\}}$  2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% Constructing a point  $O$  such that  $NO/NH=-1$ 
towards O N H -1
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% NDG: points  $A$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $A$ 

```

Figure 1: Illustration of the problem 0807

```

circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% line m(H_{b}H_{c}) and circle k(N,M_{a}) intersect; points H_{a} and N are not the same
% Determination conditions: points M_{a} and H_{a} are not the same; points E_{a} and M_{a} must be
% different; points E_{a} and N are not the same; points E_{a} and H_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_a = _H_a$

Proving failed

4.1.2 Proving $N = _N$

Proving failed

4.1.3 Proving $E_a = _E_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_a = _H_a$

Proving failed

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $E_a=_E E_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_a=_H H_a$

Proving failed

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $E_a=_E E_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_a=_H H_a$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $E_a=_E E_a$

Proving failed

Problem 808

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 808: Given a point E_a , a point H_a and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;
2. Using the point H_a and the line h_a , construct a line a (rule W10a); ;
3. Using the point O and the line a , construct a line m_a (rule W10b); ;
4. Using the line m_a and the line a , construct a point M_a (rule W03); % NDG: lines m_a and a are not parallel % DET: lines m_a and a are not the same;
5. Using the point M_a and the point E_a , construct a line $m(H_bH_c)$ (rule W02); % DET: points M_a and E_a are not the same;
6. Using the point E_a and the point H_a , construct a line $m(E_aH_a)$ (rule W14); % DET: points E_a and H_a are not the same;
7. Using the line $m(E_aH_a)$ and the line $m(H_bH_c)$, construct a point N (rule W03); % NDG: lines $m(E_aH_a)$ and $m(H_bH_c)$ are not parallel % DET: lines $m(E_aH_a)$ and $m(H_bH_c)$ are not the same;
8. Using the point O and the point N , construct a point H (rule W01); ;
9. Using the point E_a and the point H , construct a point A (rule W01); ;
10. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
11. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; lines $m(E_a H_a)$ and $m(H_b H_c)$ are not parallel; lines m_a and a are not parallel.

Determination conditions: lines $m(E_a H_a)$ and $m(H_b H_c)$ are not the same; points E_a and H_a are not the same; points M_a and E_a are not the same; lines m_a and a are not the same; points E_a and H_a are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W10b,W14]

Lemmas used: [D1,D11,D21,D26,D28,D3,D31,D32,D5,D8,GD01,GD02,GL01,GL03,GL04,GL09,L11,L12,L19,L

Solving time: 3.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point H_{a} 80 40
```

```
point O 65 51.14
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_r H_{a}
```

```
cmark_t O
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{a} and H_{a} are not the same
```

```
% Constructing a line h_{a} which passes through point E_{a} and point H_{a}
```

```
line h_{a} E_{a} H_{a}
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```
% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
```

```
perp a H_{a} h_{a}
```

```
color 200 200 200
```

```
drawline a
```

```
color 0 0 0
```

```
% Constructing a line m_{a} which is perpendicular to line a and which passes through point O
```

```
perp m_{a} O a
```

```
color 200 200 200
```

```
drawline m_{a}
```

```
color 0 0 0
```

```

% NDG: lines  $m_{\{a\}}$  and  $a$  are not parallel% DET: lines  $m_{\{a\}}$  and  $a$  are not the same
% Constructing a point  $M_{\{a\}}$  which belongs to line  $m_{\{a\}}$  and line  $a$ 
intersec  $M_{\{a\}}$   $m_{\{a\}}$   $a$ 
cmark_r  $M_{\{a\}}$ 

% DET: points  $M_{\{a\}}$  and  $E_{\{a\}}$  are not the same
% Constructing a line  $m(H_{\{b\}}H_{\{c\}})$  which passes through point  $M_{\{a\}}$  and point  $E_{\{a\}}$ 
line  $m(H_{\{b\}}H_{\{c\}})$   $M_{\{a\}}$   $E_{\{a\}}$ 

color 200 200 200
drawline  $m(H_{\{b\}}H_{\{c\}})$ 
color 0 0 0

% DET: points  $E_{\{a\}}$  and  $H_{\{a\}}$  are not the same
% Constructing bisector  $m(E_{\{a\}}H_{\{a\}})$  of the segment  $E_{\{a\}}H_{\{a\}}$ 
med  $m(E_{\{a\}}H_{\{a\}})$   $E_{\{a\}}$   $H_{\{a\}}$ 

color 200 200 200
drawline  $m(E_{\{a\}}H_{\{a\}})$ 
color 0 0 0

color 200 200 200
drawsegment  $E_{\{a\}}$   $H_{\{a\}}$ 
color 0 0 0

% NDG: lines  $m(E_{\{a\}}H_{\{a\}})$  and  $m(H_{\{b\}}H_{\{c\}})$  are not parallel% DET: lines  $m(E_{\{a\}}H_{\{a\}})$  and  $m(H_{\{b\}}$ 
 $H_{\{c\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{a\}}H_{\{a\}})$  and line  $m(H_{\{b\}}H_{\{c\}})$ 
intersec  $N$   $m(E_{\{a\}}H_{\{a\}})$   $m(H_{\{b\}}H_{\{c\}})$ 
cmark_r  $N$ 

% Constructing a point  $H$  such that  $OH/ON=2$ 
towards  $H$   $O$   $N$  2
cmark_rt  $H$ 
color 200 200 200
drawsegment  $O$   $H$ 
color 0 0 0

% Constructing a point  $A$  such that  $E_{\{a\}}A/E_{\{a\}}H=-1$ 
towards  $A$   $E_{\{a\}}$   $H$  -1
cmark_t  $A$ 
color 200 200 200
drawsegment  $H$   $A$ 
color 0 0 0

```

```

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% lines m(E_{a}H_{a}) and m(H_{b}H_{c}) are not parallel; lines m_{a} and a are not parallel
% Determination conditions: lines m(E_{a}H_{a}) and m(H_{b}H_{c}) are not the same; points E_{a}
% and H_{a} are not the same; points M_{a} and E_{a} are not the same; lines m_{a} and a are not
% the same; points E_{a} and H_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = _E_a$

Proving failed

4.1.2 Proving $H_a = _H_a$

Proving failed

4.1.3 Proving $O = _O$

Status: Theorem has been proved.

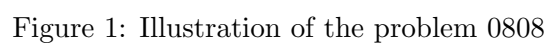
Space Complexity: The biggest polynomial obtained during prover execution contains 130 terms.

Time Complexity: Time spent by the prover is 0.717 seconds.

NDG conditions Points $_M_a$ and E_a are not identical

Points B , C and E_a are not collinear

Point H_a is not the midpoint of segment with endpoints A and B



4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $H_a = \neg H_a$

Proving failed

4.2.3 Proving $O = \neg O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 311 terms.

Time Complexity: Time spent by the prover is 0.520 seconds. There are no ndg conditions.

4.3.2 Proving $H_a = \neg H_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 15 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.3.3 Proving $O = \neg O$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 1065 terms.

Time Complexity: Time spent by the prover is 2.280 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $H_a = \neg H_a$

Proving failed

4.4.3 Proving $O = \neg O$

Proving failed

Problem 809

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 809: Given a point H_a , a point T_a and a point E_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_a and the point T_a , construct a line a (rule W02); % DET: points H_a and T_a are not the same;
2. Using the point H_a and the line a , construct a line h_a (rule W10b); ;
3. Choose freely a point E_a on the line h_a (rule WOnline2);
4. Choose freely a point A on the line h_a (rule WOnline1) ;
5. Using the point A and the point E_a , construct a point H (rule W01); ;
6. Using the point A and the point T_a , construct a line s_a (rule W02); % DET: points A and T_a are not the same;
7. Using the point E_a , the point A , the point T_a , the line s_a and the line h_a , construct a line AO (rule W17); % NDG: points A and T_a are not the same; points E_a and A are not the same % DET: points A and T_a are not the same;
8. Using the point E_a and the line AO , construct a line $m(H_bH_c)$ (rule W16); ;
9. Using the line $m(H_bH_c)$ and the line a , construct a point M_a (rule W03); % NDG: lines $m(H_bH_c)$ and a are not parallel % DET: lines $m(H_bH_c)$ and a are not the same;
10. Using the point M_a and the point A , construct a point G (rule W01); ;
11. Using the point H and the point G , construct a point O (rule W01); ;
12. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;

13. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); %
NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; lines $m(H_b H_c)$ and a are not parallel; points A and T_a are not the same; points E_a and A are not the same.

Determination conditions: lines $m(H_b H_c)$ and a are not the same; points A and T_a are not the same; points A and T_a are not the same; points H_a and T_a are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W16,W17,WOnline1,WOnline2]

Lemmas used: [D21,D23,D26,D28,D3,D5,D8,GD01,GD02,GL01,GL03,GL04,GL09,GL17,L104,L11,L12,L38,L39]

Solving time: 54.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{a} 80 40
point T_{a} 70.86 40
point E_{a} 80 83.86
```

```
color 220 0 0
fontsize 9
```

```
cmark_r H_{a}
cmark_rb T_{a}
cmark_r E_{a}
color 0 0 0
fontsize 8
```

```
% DET: points H_{a} and T_{a} are not the same
% Constructing a line a which passes through point H_{a} and point T_{a}
line a H_{a} T_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% Constructing a line h_{a} which is perpendicular to line a and which passes through point H_{a}
perp h_{a} H_{a} a
```

```
color 200 200 200
drawline h_{a}
color 0 0 0
```

```
% Generating random value V[_G78381]
random V[_G78381]
```

```

% Calculating value V[_G78402] using formula V[_G78381]*20
expression V[_G78402] { V[_G78381]*20 }

% Constructing a point E_{a} which is a point for which holds  $H_{\{a\}}E_{\{a\}} = V[_G78402]$  and angle  $T_{\{a\}}H_{\{a\}}E_{\{a\}} = 90$ 
turtle E_{a} T_{a} H_{a} 90 V[_G78402]
cmark_r E_{a}

% Choosing randomly a point A on the line E_{a}H_{a}
online A E_{a} H_{a}
cmark_t A
color 200 200 200
drawline E_{a} H_{a}
color 0 0 0

% Constructing a point H such that  $AH/AE_{\{a\}}=2$ 
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% DET: points A and T_{a} are not the same
% Constructing a line s_{a} which passes through point A and point T_{a}
line s_{a} A T_{a}

color 200 200 200
drawline s_{a}
color 0 0 0

% NDG: points A and T_{a} are not the same; points E_{a} and A are not the same% DET: points A and T_{a} are not the same
% Constructing an angle V[_G78764] which is equal to the angle  $E_{\{a\}}AT_{\{a\}}$ 
angle_o V[_G78764] E_{a} A T_{a}

% Calculating value angle[_G78843] using formula  $1/\text{pow}(2,0)*V[_G78764]+0/\text{pow}(2,0)*180$ 
expression angle[_G78843] { 1/\text{pow}(2,0)*V[_G78764]+0/\text{pow}(2,0)*180 }

% Constructing a point P_{\_G78840} which is an image of the point T_{a} in a rotation around the point A for the angle  $1/\text{pow}(2,0)*V[_G78764]+0/\text{pow}(2,0)*180$ 
rotate P_{\_G78840} A angle[_G78843] T_{a}
cmark_r P_{\_G78840}
color 200 200 200
drawarc_p A T_{a} angle[_G78843]
color 0 0 0

```



```
% Constructing a line AO which passes through point A and point P_{\_G78840}
line AO A P_{\_G78840}
```

```
color 200 200 200
drawline AO
color 0 0 0
```

```
% Constructing a line m(H_{b}H_{c}) which contains the point E_{a} and is parallel to the line AO
parallel m(H_{b}H_{c}) E_{a} AO
```

```
color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0
```

```
% NDG: lines m(H_{b}H_{c}) and a are not parallel% DET: lines m(H_{b}H_{c}) and a are not the same
% Constructing a point M_{a} which belongs to line m(H_{b}H_{c}) and line a
intersec M_{a} m(H_{b}H_{c}) a
cmark_r M_{a}
```

```
% Constructing a line L_{\_G79359} which passes through point M_{a} and point A
line L_{\_G79359} M_{a} A
```

```
color 200 200 200
drawline L_{\_G79359}
color 0 0 0
```

```
% Constructing a point P_{\_G79460} with coordinates (0,0)
point P_{\_G79460} 0 0
cmark_r P_{\_G79460}
```

```
% Constructing a point P_{\_G79384} such that M_{a}P_{\_G79384}/M_{a}P_{\_G79460}=1
towards P_{\_G79384} M_{a} P_{\_G79460} 1
cmark_r P_{\_G79384}
color 200 200 200
drawsegment M_{a} P_{\_G79384}
color 0 0 0
```

```
% Constructing a point P_{\_G79429} such that M_{a}P_{\_G79429}/M_{a}P_{\_G79460}=3
towards P_{\_G79429} M_{a} P_{\_G79460} 3
cmark_r P_{\_G79429}
color 200 200 200
drawsegment M_{a} P_{\_G79429}
color 0 0 0
```

```
% Constructing a line L_{\_G79390} which passes through point A and point P_{\_G79429}
line L_{\_G79390} A P_{\_G79429}
```

```

color 200 200 200
drawline L_{\_G79390}
color 0 0 0

% Constructing a line L_{\_G79353} which contains the point P_{\_G79384} and is parallel to the
  line L_{\_G79390}
parallel L_{\_G79353} P_{\_G79384} L_{\_G79390}

color 200 200 200
drawline L_{\_G79353}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G79353} and line L_{\_G79359}
intersec G L_{\_G79353} L_{\_G79359}
cmark_t G

% Constructing a point O such that HO/HG=1.5
towards O H G 1.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
  lines  $m(H_{\{b\}}H_{\{c\}})$  and a are not parallel; points A and  $T_{\{a\}}$  are not the same; points  $E_{\{a\}}$ 
  and A are not the same
% Determination conditions: lines  $m(H_{\{b\}}H_{\{c\}})$  and a are not the same; points A and  $T_{\{a\}}$  are not
  the same; points A and  $T_{\{a\}}$  are not the same; points  $H_{\{a\}}$  and  $T_{\{a\}}$  are not the same

```

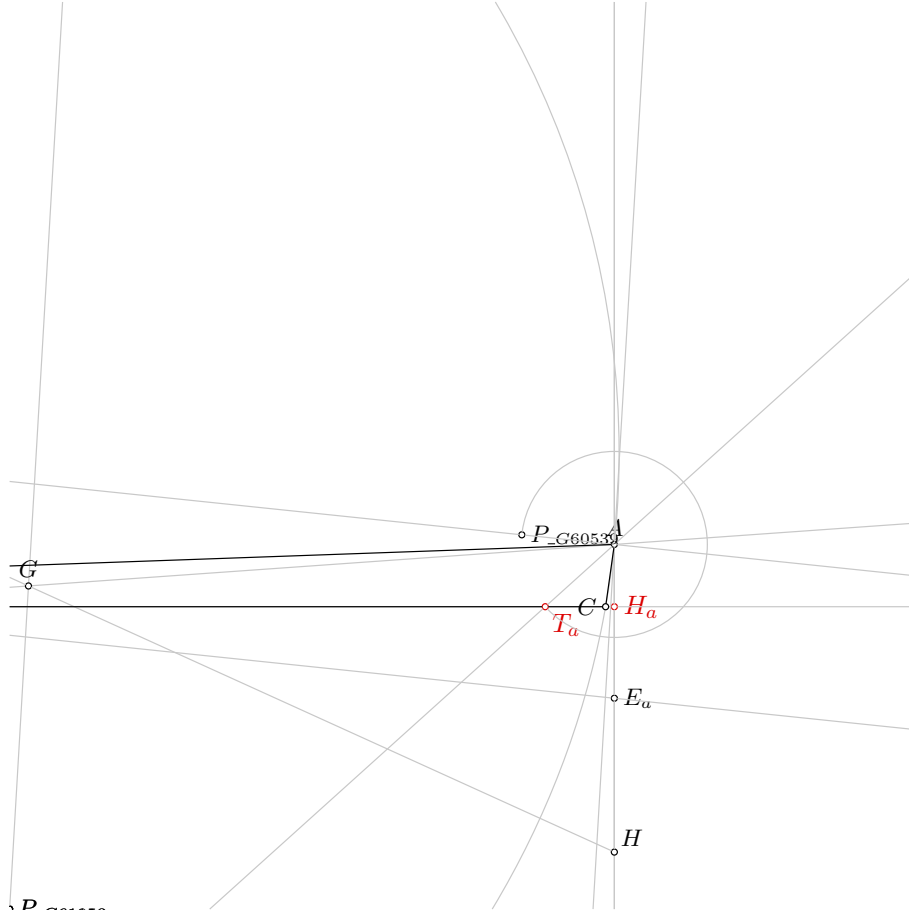


Figure 1: Illustration of the problem 0809

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_a = H_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 8 terms.

Time Complexity: Time spent by the prover is 0.214 seconds.

NDG conditions Points T_a and A are not identical

Points A and H are not identical

Points A , M_a and P_{G73321} are not collinear

Line through points H_a and A is not perpendicular to line through points A and P_{G73321}

Line through points A and H_a is not parallel with line through points B and C

4.1.2 Proving $T_a = \neg T_a$

Proving failed

4.1.3 Proving $E_a = \neg E_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_a = \neg H_a$

Proving failed

4.2.2 Proving $T_a = \neg T_a$

Proving failed

4.2.3 Proving $E_a = \neg E_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_a = \neg H_a$

Proving failed

4.3.2 Proving $T_a = \neg T_a$

Proving failed

4.3.3 Proving $E_a = \neg E_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_a = \neg H_a$

Proving failed

4.4.2 Proving $T_a = \neg T_a$

Proving failed

4.4.3 Proving $E_a = \neg E_a$

Proving failed

Problem 810

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 810: Given a point E_a , a point H_a and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 811

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 811: Given a point E_a , a point H_a and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 812

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 812: Given a point E_a , a point H_c and a point H_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_c and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_c and E_a are not the same;
2. Choose freely a point H_b on the circle $k(E_a, A)$ (rule W0ncircle);
3. Using the point H_b and the point E_a , construct a line $m(H_bE_a)$ (rule W14); % DET: points H_b and E_a are not the same;
4. Using the point H_b and the point H_c , construct a line $m(H_bH_c)$ (rule W14); % DET: points H_b and H_c are not the same;
5. Using the line $m(H_bH_c)$ and the line $m(H_bE_a)$, construct a point N (rule W03); % NDG: lines $m(H_bH_c)$ and $m(H_bE_a)$ are not parallel % DET: lines $m(H_bH_c)$ and $m(H_bE_a)$ are not the same;
6. Using the point H_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_b and N are not the same;
7. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point E_a , construct a point M_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points E_a and M_a must be different;
8. Choose freely a point A on the circle $k(E_a, A)$ (rule W0ncircle);
9. Using the point A and the point E_a , construct a point H (rule W01); ;
10. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;

11. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
12. Using the line b and the line h_c , construct a point C (rule W03); % NDG: lines b and h_c are not parallel % DET: lines b and h_c are not the same;
13. Using the point M_a and the point C , construct a point B (rule W01); .

Non-degenerate conditions: lines b and h_c are not parallel; line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect; points H_b and N are not the same; lines $m(H_bH_c)$ and $m(H_bE_a)$ are not parallel; points H_c and E_a are not the same.

Determination conditions: lines b and h_c are not the same; points H_c and H are not the same; points A and H_b are not the same; points E_a and M_a must be different; lines $m(H_bH_c)$ and $m(H_bE_a)$ are not the same; points H_b and H_c are not the same; points H_b and E_a are not the same.

Rules used: [W01,W02,W03,W05a,W06,W14,WOncircle1]

Lemmas used: [D10,D21,D28,D32,D6,D7,GD01,GD02,GL01,GL03,GL04,L20,L21,L22,L3,L38,L39,L47,L48]

Solving time: 65.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
point H_{c} 68.91 84.83
point H_{b} 89.36 77.83
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{a}
cmark_rt H_{c}
cmark_l H_{b}
color 0 0 0
fontsize 8
```

```
% NDG: points H_{c} and E_{a} are not the same
```

```
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H_{c}
```

```
circle k(E_{a},A) E_{a} H_{c}
```

```
color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0
```

```
% Choosing randomly a point H_{b} on the circle with center E_{a} through point H_{c}
```

```
oncircle H_{b} E_{a} H_{c}
cmark_l H_{b}
color 200 200 200
drawcircle E_{a} H_{c}
color 0 0 0
```



```

% DET: points  $H_{\{b\}}$  and  $E_{\{a\}}$  are not the same
% Constructing bisector  $m(H_{\{b\}}E_{\{a\}})$  of the segment  $H_{\{b\}}E_{\{a\}}$ 
med m( $H_{\{b\}}E_{\{a\}}$ )  $H_{\{b\}}$   $E_{\{a\}}$ 

color 200 200 200
drawline m( $H_{\{b\}}E_{\{a\}}$ )
color 0 0 0

color 200 200 200
drawsegment  $H_{\{b\}}$   $E_{\{a\}}$ 
color 0 0 0

% DET: points  $H_{\{b\}}$  and  $H_{\{c\}}$  are not the same
% Constructing bisector  $m(H_{\{b\}}H_{\{c\}})$  of the segment  $H_{\{b\}}H_{\{c\}}$ 
med m( $H_{\{b\}}H_{\{c\}}$ )  $H_{\{b\}}$   $H_{\{c\}}$ 

color 200 200 200
drawline m( $H_{\{b\}}H_{\{c\}}$ )
color 0 0 0

color 200 200 200
drawsegment  $H_{\{b\}}$   $H_{\{c\}}$ 
color 0 0 0

% NDG: lines  $m(H_{\{b\}}H_{\{c\}})$  and  $m(H_{\{b\}}E_{\{a\}})$  are not parallel% DET: lines  $m(H_{\{b\}}H_{\{c\}})$  and  $m(H_{\{b\}}$ 
 $E_{\{a\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(H_{\{b\}}H_{\{c\}})$  and line  $m(H_{\{b\}}E_{\{a\}})$ 
intersec N m( $H_{\{b\}}H_{\{c\}}$ ) m( $H_{\{b\}}E_{\{a\}}$ )
cmark_r N

% NDG: points  $H_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N,M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $H_{\{b\}}$ 
circle k( $N,M_{\{a\}}$ )  $N$   $H_{\{b\}}$ 

color 200 200 200
drawcircle k( $N,M_{\{a\}}$ )
color 0 0 0

% NDG: line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $E_{\{a\}}$  and  $M_{\{a\}}$  must be
different
% Constructing a point  $M_{\{a\}}$  which is an image of the point  $E_{\{a\}}$  in the symmetry to point/line  $N$ 
sim  $M_{\{a\}}$   $N$   $E_{\{a\}}$ 
cmark_r  $M_{\{a\}}$ 

% Choosing randomly a point  $A$  on the circle with center  $E_{\{a\}}$  through point  $H_{\{b\}}$ 
oncircle A  $E_{\{a\}}$   $H_{\{b\}}$ 
cmark_t A

```

```

color 200 200 200
drawcircle E_{a} H_{b}
color 0 0 0

```

```

% Constructing a point H such that  $AH/AE_{a}=2$ 
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

```

```

% DET: points A and  $H_{b}$  are not the same
% Constructing a line b which passes through point A and point  $H_{b}$ 
line b A H_{b}

```

```

color 200 200 200
drawline b
color 0 0 0

```

```

% DET: points  $H_{c}$  and H are not the same
% Constructing a line  $h_{c}$  which passes through point  $H_{c}$  and point H
line h_{c} H_{c} H

```

```

color 200 200 200
drawline h_{c}
color 0 0 0

```

```

% NDG: lines b and  $h_{c}$  are not parallel% DET: lines b and  $h_{c}$  are not the same
% Constructing a point C which belongs to line b and line  $h_{c}$ 
intersec C b h_{c}
cmark_l C

```

```

% Constructing a point B such that  $M_{a}B/M_{a}C=-1$ 
towards B M_{a} C -1
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

```

```

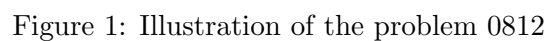
drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions: lines b and  $h_{c}$  are not parallel; line  $m(H_{b}H_{c})$  and circle  $k(N, M_{a})$  intersect; points  $H_{b}$  and N are not the same; lines  $m(H_{b}H_{c})$  and  $m(H_{b}E_{a})$ 

```



```

are not parallel; points  $H_{\{c\}}$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $b$  and  $h_{\{c\}}$  are not the same; points  $H_{\{c\}}$  and  $H$  are not the same
; points  $A$  and  $H_{\{b\}}$  are not the same; points  $E_{\{a\}}$  and  $M_{\{a\}}$  must be different; lines  $m(H_{\{b\}}H_{\{c\}})$  and  $m(H_{\{b\}}E_{\{a\}})$  are not the same; points  $H_{\{b\}}$  and  $H_{\{c\}}$  are not the same; points  $H_{\{b\}}$  and  $E_{\{a\}}$  are not the same

```

Illustration of the constructed figure is given in Figure 1

4 Correctness proof

4.1.1 Proving $E_a = -E_a$

Proving failed

4.1.2 Proving $H_c = \neg H_c$

Proving failed

4.1.3 Proving $H_b = \neg H_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $H_c = \neg H_c$

Proving failed

4.2.3 Proving $H_b = \neg H_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $H_c = \neg H_c$

Proving failed

4.3.3 Proving $H_b = \neg H_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $H_c = \neg H_c$

Proving failed

4.4.3 Proving $H_b = \neg H_b$

Proving failed

Problem 813

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 813: Given a point E_a , a point H_b and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 814

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 814: Given a point E_a , a point M_a and a point H_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point M_a , construct a line $m(H_bH_c)$ (rule W02); % DET: points E_a and M_a are not the same;
2. Using the point E_a and the point M_a , construct a line $m(E_aM_a)$ (rule W14); % DET: points E_a and M_a are not the same;
3. Using the line $m(E_aM_a)$ and the line $m(H_bH_c)$, construct a point N (rule W03); % NDG: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not parallel % DET: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not the same;
4. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
5. Choose freely a point H_b on the circle $k(N, M_a)$ (rule WOncircle);
6. Using the point H_b and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_b and E_a are not the same;
7. Using the circle $k(N, M_a)$, the circle $k(E_a, A)$, the point H_b , the point N and the point E_a , construct a point H_c (rule W08); % NDG: circles $k(N, M_a)$ and $k(E_a, A)$ intersect % DET: circles $k(N, M_a)$ and $k(E_a, A)$ are not the same; points H_b and H_c must be different;
8. Choose freely a point A on the circle $k(E_a, A)$ (rule WOncircle);
9. Using the point A and the point E_a , construct a point H (rule W01); ;
10. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;

11. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
12. Using the line b and the line h_c , construct a point C (rule W03); % NDG: lines b and h_c are not parallel % DET: lines b and h_c are not the same;
13. Using the point M_a and the point C , construct a point B (rule W01); .

Non-degenerate conditions: lines b and h_c are not parallel; circles $k(N, M_a)$ and $k(E_a, A)$ intersect; points H_b and E_a are not the same; points E_a and N are not the same; lines $m(E_a M_a)$ and $m(H_b H_c)$ are not parallel.

Determination conditions: lines b and h_c are not the same; points H_c and H are not the same; points A and H_b are not the same; circles $k(N, M_a)$ and $k(E_a, A)$ are not the same; points H_b and H_c must be different; lines $m(E_a M_a)$ and $m(H_b H_c)$ are not the same; points E_a and M_a are not the same; points E_a and M_a are not the same.

Rules used: [W01,W02,W03,W06,W08,W14,WOncircle1]

Lemmas used: [D10,D21,D28,D32,D6,D7,GD01,GD02,GL01,GL03,GL04,L20,L21,L22,L3,L38,L39,L47,L48]

Solving time: 66.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
point M_{a} 65 40
point H_{b} 89.36 77.83
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{a}
cmark_r M_{a}
cmark_l H_{b}
color 0 0 0
fontsize 8
```

```
% DET: points E_{a} and M_{a} are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point E_{a} and point M_{a}
line m(H_{b}H_{c}) E_{a} M_{a}
```

```
color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0
```

```
% DET: points E_{a} and M_{a} are not the same
% Constructing bisector m(E_{a}M_{a}) of the segment E_{a}M_{a}
med m(E_{a}M_{a}) E_{a} M_{a}
```

```
color 200 200 200
drawline m(E_{a}M_{a})
```

```

color 0 0 0

color 200 200 200
drawsegment E_{a} M_{a}
color 0 0 0

% NDG: lines  $m(E_{\{a\}}M_{\{a\}})$  and  $m(H_{\{b\}}H_{\{c\}})$  are not parallel% DET: lines  $m(E_{\{a\}}M_{\{a\}})$  and  $m(H_{\{b\}}H_{\{c\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{a\}}M_{\{a\}})$  and line  $m(H_{\{b\}}H_{\{c\}})$ 
intersec N m(E_{a}M_{a}) m(H_{b}H_{c})
cmark_r N

% NDG: points  $E_{\{a\}}$  and  $N$  are not the same
% Constructing a circle  $k(N,M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{a\}}$ 
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point  $H_{\{b\}}$  on the circle with center  $N$  through point  $E_{\{a\}}$ 
oncircle H_{b} N E_{a}
cmark_l H_{b}
color 200 200 200
drawcircle N E_{a}
color 0 0 0

% NDG: points  $H_{\{b\}}$  and  $E_{\{a\}}$  are not the same
% Constructing a circle  $k(E_{\{a\}},A)$  whose center is at point  $E_{\{a\}}$  and which passes through point  $H_{\{b\}}$ 
circle k(E_{a},A) E_{a} H_{b}

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: circles  $k(N,M_{\{a\}})$  and  $k(E_{\{a\}},A)$  intersect% DET: circles  $k(N,M_{\{a\}})$  and  $k(E_{\{a\}},A)$  are not the same; points  $H_{\{b\}}$  and  $H_{\{c\}}$  must be different
% Constructing a line  $L_{\{\backslash\_G149808\}}$  which passes through point  $N$  and point  $E_{\{a\}}$ 
line L_{\_G149808} N E_{a}

color 200 200 200
drawline L_{\_G149808}
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $H_{\{b\}}$  in the symmetry to point/line  $L_{\{\backslash\_G149808\}}$ 

```



```

sim H_{c} L_{\_G149808} H_{b}
cmark_rt H_{c}

```

```

% Choosing randomly a point A on the circle with center E_{a} through point H_{b}
oncircle A E_{a} H_{b}
cmark_t A
color 200 200 200
drawcircle E_{a} H_{b}
color 0 0 0

```

```

% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

```

```

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

```

```

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

```

```

% NDG: lines b and h_{c} are not parallel% DET: lines b and h_{c} are not the same
% Constructing a point C which belongs to line b and line h_{c}
intersec C b h_{c}
cmark_l C

```

```

% Constructing a point B such that M_{a}B/M_{a}C=-1
towards B M_{a} C -1
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

```

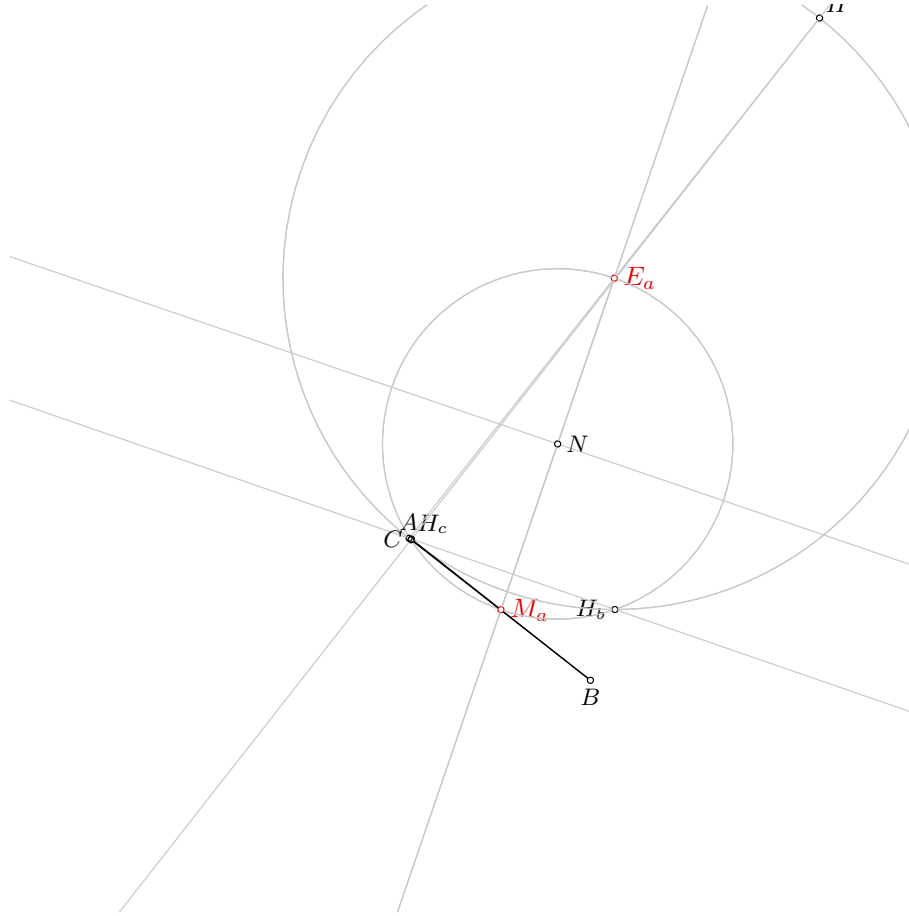


Figure 1: Illustration of the problem 0814

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines b and h_{c} are not parallel; circles k(N,M_{a}) and k(E_{a},A)
% intersect; points H_{b} and E_{a} are not the same; points E_{a} and N are not the same; lines
% m(E_{a}M_{a}) and m(H_{b}H_{c}) are not parallel
% Determination conditions: lines b and h_{c} are not the same; points H_{c} and H are not the same
% ; points A and H_{b} are not the same; circles k(N,M_{a}) and k(E_{a},A) are not the same;
% points H_{b} and H_{c} must be different; lines m(E_{a}M_{a}) and m(H_{b}H_{c}) are not the
% same; points E_{a} and M_{a} are not the same; points E_{a} and M_{a} are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 12 terms.

Time Complexity: Time spent by the prover is 0.247 seconds.

NDG conditions Points N and E_a are not identical

Line through points H_b and A is not parallel with line through points H_c and H

Points H_b and A are not identical

4.1.3 Proving $H_b = \neg H_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{M_{m(E_a M_a)}^0 E_a M_a} \neq S_{T_{m(E_a M_a)}^1 E_a M_a}$ i.e., lines $M_{m(E_a M_a)}^0 T_{m(E_a M_a)}^1$ and $E_a M_a$ are not parallel (construction based assumption)

$S_{AH_c H} \neq S_{H_b H_c H}$ i.e., lines AH_b and $H_c H$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{H_b}^3} \neq S_{F_{H_a}^2 BF_{H_b}^3}$ i.e., lines $AF_{H_a}^2$ and $BF_{H_b}^3$ are not parallel (construction based assumption)

$S_{ABF_{H_b}^3} \neq S_{CBF_{H_b}^3}$ i.e., lines AC and $BF_{H_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.2.3 Proving $H_b = \neg H_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $M_a = \neg M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 19 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.3.3 Proving $H_b = H_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = E_a$

Proving failed

4.4.2 Proving $M_a = M_a$

Proving failed

4.4.3 Proving $H_b = H_b$

Proving failed

Problem 815

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 815: Given a point E_a , a point H_b and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
2. Using the point H_b and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_b and E_a are not the same;
3. Using the circle $k(E_a, A)$, the line b , the point E_a and the point H_b , construct a point A (rule W05); % NDG: line b and circle $k(E_a, A)$ intersect % DET: points H_b and A must be different;
4. Using the point A and the point E_a , construct a point H (rule W01); ;
5. Using the point M_b and the point A , construct a point C (rule W01); ;
6. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
7. Using the point H and the point C , construct a line h_c (rule W02); % DET: points H and C are not the same;
8. Using the circle $k(E_a, A)$, the line h_c , the point E_a and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_a, A)$ intersect % DET: points H and H_c must be different;
9. Using the point A and the point H_c , construct a line c (rule W02); % DET: points A and H_c are not the same;
10. Using the line h_b and the line c , construct a point B (rule W03); % NDG: lines h_b and c are not parallel % DET: lines h_b and c are not the same.

Non-degenerate conditions: lines h_b and c are not parallel; line h_c and circle $k(E_a, A)$ intersect; line b and circle $k(E_a, A)$ intersect; points H_b and E_a are not the same.

Determination conditions: lines h_b and c are not the same; points A and H_c are not the same; points H and H_c must be different; points H and C are not the same; points H_b and H are not the same; points H_b and A must be different; points H_b and M_b are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D22,D28,D3,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L46,L47,L48]

Solving time: 9.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point H_{b} 89.36 77.83
```

```
point M_{b} 95 67.5
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_l H_{b}
```

```
cmark_lt M_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{b} and M_{b} are not the same
```

```
% Constructing a line b which passes through point H_{b} and point M_{b}
```

```
line b H_{b} M_{b}
```

```
color 200 200 200
```

```
drawline b
```

```
color 0 0 0
```

```
% NDG: points H_{b} and E_{a} are not the same
```

```
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H_{b}
```

```
circle k(E_{a},A) E_{a} H_{b}
```

```
color 200 200 200
```

```
drawcircle k(E_{a},A)
```

```
color 0 0 0
```

```
% NDG: line b and circle k(E_{a},A) intersect% DET: points H_{b} and A must be different
```

```
% Constructing a point P_{\_G174150} which is a foot of the point E_{a} on the line b
```

```
foot P_{\_G174150} E_{a} b
```

```
cmark_r P_{\_G174150}
```

```
color 200 200 200
```

```
drawline E_{a} P_{\_G174150}
```

```
color 0 0 0
```

```

% Constructing a point A which is an image of the point H_{b} in the symmetry to point/line P_{\_G
174150}
sim A P_{\_G174150} H_{b}
cmark_t A

% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% Constructing a point C such that M_{b}C/M_{b}A=-1
towards C M_{b} A -1
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% DET: points H and C are not the same
% Constructing a line h_{c} which passes through point H and point C
line h_{c} H C

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: line h_{c} and circle k(E_{a},A) intersect% DET: points H and H_{c} must be different
% Constructing a point P_{\_G174587} which is a foot of the point E_{a} on the line h_{c}
foot P_{\_G174587} E_{a} h_{c}
cmark_r P_{\_G174587}
color 200 200 200
drawline E_{a} P_{\_G174587}
color 0 0 0

% Constructing a point H_{c} which is an image of the point H in the symmetry to point/line P_{\_G
174587}
sim H_{c} P_{\_G174587} H

```

```

cmark_rt H_{c}

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines h_{b} and c are not parallel% DET: lines h_{b} and c are not the same
% Constructing a point B which belongs to line h_{b} and line c
intersec B h_{b} c
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{b} and c are not parallel; line h_{c} and circle k(E_{a},A)
% intersect; line b and circle k(E_{a},A) intersect; points H_{b} and E_{a} are not the same
% Determination conditions: lines h_{b} and c are not the same; points A and H_{c} are not the same
% ; points H and H_{c} must be different; points H and C are not the same; points H_{b} and H are
% not the same; points H_{b} and A must be different; points H_{b} and M_{b} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $H_b = \neg H_b$

Proving failed

4.1.3 Proving $M_b = \neg M_b$

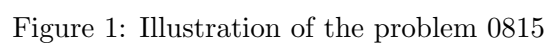
Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 11 terms.

Time Complexity: Time spent by the prover is 0.193 seconds.

NDG conditions Points M_b and H_b are not identical

Points M_b and H_b are not identical



4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $H_b = \neg H_b$

Proving failed

4.2.3 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{H_bAH_c} \neq S_{HAH_c}$ i.e., lines H_bH and AH_c are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{\neg h_b}} \neq S_{F^0_{\neg h_a}BF^1_{\neg h_b}}$ i.e., lines $AF^0_{\neg h_a}$ and $BF^1_{\neg h_b}$ are not parallel (construction based assumption)

$S_{ABF^1_{\neg h_b}} \neq S_{CBF^1_{\neg h_b}}$ i.e., lines AC and $BF^1_{\neg h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $H_b = \neg H_b$

Proving failed

4.3.3 Proving $M_b = \neg M_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $H_b = \neg H_b$

Proving failed

4.4.3 Proving $M_b = \neg M_b$

Proving failed

Problem 816

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 816: Given a point E_a , a point H_b and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H_b and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_b and E_a are not the same;
2. Using the point H_b and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points H_b and M_c are not the same;
3. Using the circle $k(E_a, A)$, the circle $k(M_c, A)$, the point H_b , the point E_a and the point M_c , construct a point A (rule W08); % NDG: circles $k(E_a, A)$ and $k(M_c, A)$ intersect % DET: circles $k(E_a, A)$ and $k(M_c, A)$ are not the same; points H_b and A must be different;
4. Using the point A and the point E_a , construct a point H (rule W01); ;
5. Using the point M_c and the point A , construct a point B (rule W01); ;
6. Using the point H_b and the point A , construct a line b (rule W02); % DET: points H_b and A are not the same;
7. Using the point M_c and the point A , construct a line c (rule W02); % DET: points M_c and A are not the same;
8. Using the circle $k(E_a, A)$, the line c , the point E_a and the point A , construct a point H_c (rule W05); % NDG: line c and circle $k(E_a, A)$ intersect % DET: points A and H_c must be different;
9. Using the point H and the point H_c , construct a line h_c (rule W02); % DET: points H and H_c are not the same;
10. Using the line b and the line h_c , construct a point C (rule W03); % NDG: lines b and h_c are not parallel % DET: lines b and h_c are not the same.

Non-degenerate conditions: lines b and h_c are not parallel; line c and circle $k(E_a, A)$ intersect; circles $k(E_a, A)$ and $k(M_c, A)$ intersect; points H_b and M_c are not the same; points H_b and E_a are not the same.

Determination conditions: lines b and h_c are not the same; points H and H_c are not the same; points A and H_c must be different; points M_c and A are not the same; points H_b and A are not the same; circles $k(E_a, A)$ and $k(M_c, A)$ are not the same; points H_b and A must be different.

Rules used: [W01,W02,W03,W05,W06,W08]

Lemmas used: [D10,D20,D28,D6,D7,GD01,GD02,GL03,GL04,GL09,L3,L41,L47,L48]

Solving time: 9.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
point H_{b} 89.36 77.83
point M_{c} 50 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{a}
cmark_l H_{b}
cmark_lt M_{c}
color 0 0 0
fontsize 8
```

```
% NDG: points H_{b} and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H_{b}
circle k(E_{a},A) E_{a} H_{b}
```

```
color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0
```

```
% NDG: points H_{b} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{b}
circle k(M_{c},A) M_{c} H_{b}
```

```
color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0
```

```
% NDG: circles k(E_{a},A) and k(M_{c},A) intersect% DET: circles k(E_{a},A) and k(M_{c},A) are not
the same; points H_{b} and A must be different
% Constructing a line L_{G200432} which passes through point E_{a} and point M_{c}
line L_{G200432} E_{a} M_{c}
```

```

color 200 200 200
drawline L_{\_G200432}
color 0 0 0

% Constructing a point A which is an image of the point H_{b} in the symmetry to point/line L_{\_G
200432}
sim A L_{\_G200432} H_{b}
cmark_t A

% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% Constructing a point B such that M_{c}B/M_{c}A=-1
towards B M_{c} A -1
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points H_{b} and A are not the same
% Constructing a line b which passes through point H_{b} and point A
line b H_{b} A

color 200 200 200
drawline b
color 0 0 0

% DET: points M_{c} and A are not the same
% Constructing a line c which passes through point M_{c} and point A
line c M_{c} A

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(E_{a},A) intersect% DET: points A and H_{c} must be different
% Constructing a point P_{\_G200849} which is a foot of the point E_{a} on the line c
foot P_{\_G200849} E_{a} c
cmark_r P_{\_G200849}
color 200 200 200
drawline E_{a} P_{\_G200849}

```

```

color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $A$  in the symmetry to point/line  $P_{\{G200849\}}$ 
sim  $H_{\{c\}}$   $P_{\{G200849\}}$   $A$ 
cmark_rt  $H_{\{c\}}$ 

% DET: points  $H$  and  $H_{\{c\}}$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $H$  and point  $H_{\{c\}}$ 
line  $h_{\{c\}}$   $H$   $H_{\{c\}}$ 

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: lines  $b$  and  $h_{\{c\}}$  are not parallel% DET: lines  $b$  and  $h_{\{c\}}$  are not the same
% Constructing a point  $C$  which belongs to line  $b$  and line  $h_{\{c\}}$ 
intersec  $C$   $b$   $h_{\{c\}}$ 
cmark_l  $C$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $b$  and  $h_{\{c\}}$  are not parallel; line  $c$  and circle  $k(E_{\{a\}},A)$ 
% intersect; circles  $k(E_{\{a\}},A)$  and  $k(M_{\{c\}},A)$  intersect; points  $H_{\{b\}}$  and  $M_{\{c\}}$  are not the same
% ; points  $H_{\{b\}}$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $b$  and  $h_{\{c\}}$  are not the same; points  $H$  and  $H_{\{c\}}$  are not the same
% ; points  $A$  and  $H_{\{c\}}$  must be different; points  $M_{\{c\}}$  and  $A$  are not the same; points  $H_{\{b\}}$  and  $A$ 
% are not the same; circles  $k(E_{\{a\}},A)$  and  $k(M_{\{c\}},A)$  are not the same; points  $H_{\{b\}}$  and  $A$  must
% be different

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $H_b = \neg H_b$

Proving failed

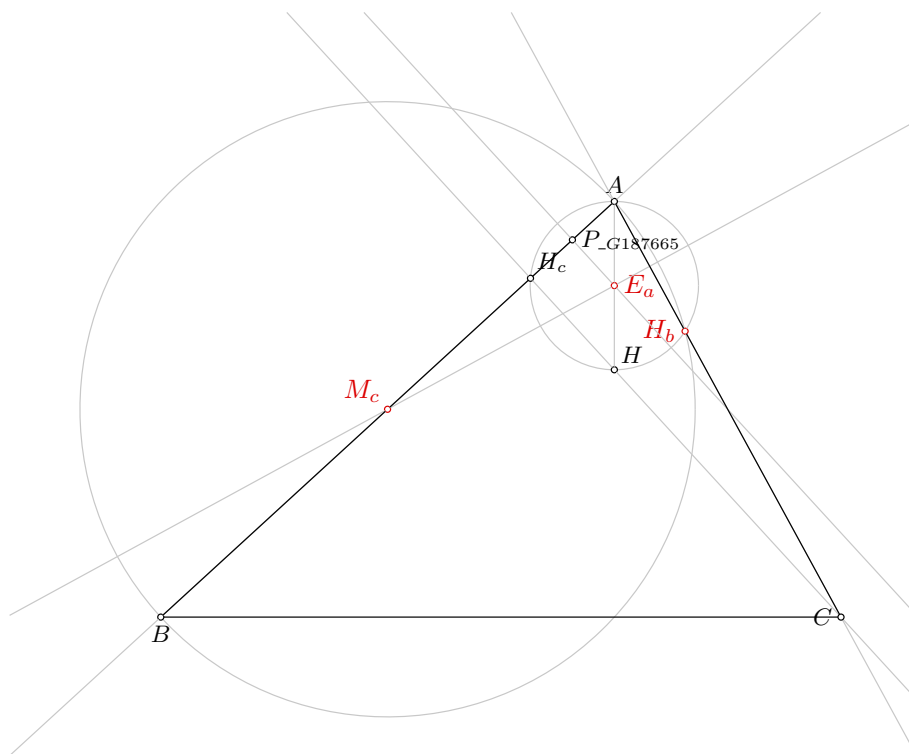


Figure 1: Illustration of the problem 0816

4.1.3 Proving $M_c = M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 11 terms.

Time Complexity: Time spent by the prover is 0.175 seconds.

NDG conditions Points M_c and E_a are not identical

Points M_c and E_a are not identical

4.2 GCLC - Area method

4.2.1 Proving $E_a = E_a$

Proving failed

4.2.2 Proving $H_b = H_b$

Proving failed

4.2.3 Proving $M_c = M_c$

NDG conditions are:

$S_{H_b H H_c} \neq S_{A H H_c}$ i.e., lines $H_b A$ and $H H_c$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a} B F^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

$S_{ABF^1_{-h_b}} \neq S_{CBF^1_{-h_b}}$ i.e., lines AC and $BF^1_{-h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_a = E_a$

Proving failed

4.3.2 Proving $H_b = H_b$

Proving failed

4.3.3 Proving $M_c = M_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = E_a$

Proving failed

4.4.2 Proving $H_b = H_b$

Proving failed

4.4.3 Proving $M_c = M_c$

Proving failed

Problem 817

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 817: Given a point H_b , a point N and a point E_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_b and N are not the same;
2. Choose freely a point E_a on the circle $k(N, M_a)$ (rule WOncircle);
3. Using the point E_a and the point N , construct a line $m(H_b H_c)$ (rule W02); % DET: points E_a and N are not the same;
4. Using the circle $k(N, M_a)$, the line $m(H_b H_c)$, the point N and the point E_a , construct a point M_a (rule W05a); % NDG: line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect % DET: points E_a and M_a must be different;
5. Using the point H_b and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_b and E_a are not the same;
6. Using the circle $k(N, M_a)$, the circle $k(E_a, A)$, the point H_b , the point N and the point E_a , construct a point H_c (rule W08); % NDG: circles $k(N, M_a)$ and $k(E_a, A)$ intersect % DET: circles $k(N, M_a)$ and $k(E_a, A)$ are not the same; points H_b and H_c must be different;
7. Choose freely a point A on the circle $k(E_a, A)$ (rule WOncircle);
8. Using the point A and the point E_a , construct a point H (rule W01); ;
9. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;
10. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;

11. Using the line b and the line h_c , construct a point C (rule W03); % NDG: lines b and h_c are not parallel % DET: lines b and h_c are not the same;
12. Using the point M_a and the point C , construct a point B (rule W01); .

Non-degenerate conditions: lines b and h_c are not parallel; circles $k(N, M_a)$ and $k(E_a, A)$ intersect; points H_b and E_a are not the same; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; points H_b and N are not the same.

Determination conditions: lines b and h_c are not the same; points H_c and H are not the same; points A and H_b are not the same; circles $k(N, M_a)$ and $k(E_a, A)$ are not the same; points H_b and H_c must be different; points E_a and M_a must be different; points E_a and N are not the same.

Rules used: [W01,W02,W03,W05a,W06,W08,WOncircle1]

Lemmas used: [D10,D21,D28,D32,D6,D7,GD01,GD02,GL01,GL03,GL04,L20,L21,L22,L3,L38,L39,L47,L48]

Solving time: 60.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{b} 89.36 77.83
point N 72.5 61.93
point E_{a} 80 83.86
```

```
color 220 0 0
fontsize 9
```

```
cmark_l H_{b}
cmark_r N
cmark_r E_{a}
color 0 0 0
fontsize 8
```

```
% NDG: points H_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{b}
circle k(N,M_{a}) N H_{b}
```

```
color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```
% Choosing randomly a point E_{a} on the circle with center N through point H_{b}
oncircle E_{a} N H_{b}
cmark_r E_{a}
color 200 200 200
drawcircle N H_{b}
color 0 0 0
```

```
% DET: points E_{a} and N are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point E_{a} and point N
```

```

line m(H_{b}H_{c}) E_{a} N

color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0

% NDG: line m(H_{b}H_{c}) and circle k(N,M_{a}) intersect% DET: points E_{a} and M_{a} must be
different
% Constructing a point M_{a} which is an image of the point E_{a} in the symmetry to point/line N
sim M_{a} N E_{a}
cmark_r M_{a}

% NDG: points H_{b} and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H
_{b}
circle k(E_{a},A) E_{a} H_{b}

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(E_{a},A) intersect% DET: circles k(N,M_{a}) and k(E_{a},A) are not
the same; points H_{b} and H_{c} must be different
% Constructing a line L_{\_G238080} which passes through point N and point E_{a}
line L_{\_G238080} N E_{a}

color 200 200 200
drawline L_{\_G238080}
color 0 0 0

% Constructing a point H_{c} which is an image of the point H_{b} in the symmetry to point/line L
_{\_G238080}
sim H_{c} L_{\_G238080} H_{b}
cmark_rt H_{c}

% Choosing randomly a point A on the circle with center E_{a} through point H_{b}
oncircle A E_{a} H_{b}
cmark_t A
color 200 200 200
drawcircle E_{a} H_{b}
color 0 0 0

% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200

```

```

drawsegment A H
color 0 0 0

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines b and h_{c} are not parallel% DET: lines b and h_{c} are not the same
% Constructing a point C which belongs to line b and line h_{c}
intersec C b h_{c}
cmark_l C

% Constructing a point B such that M_{a}B/M_{a}C=-1
towards B M_{a} C -1
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and h_{c} are not parallel; circles k(N,M_{a}) and k(E_{a},A)
% intersect; points H_{b} and E_{a} are not the same; line m(H_{b}H_{c}) and circle k(N,M_{a})
% intersect; points H_{b} and N are not the same
% Determination conditions: lines b and h_{c} are not the same; points H_{c} and H are not the same
% ; points A and H_{b} are not the same; circles k(N,M_{a}) and k(E_{a},A) are not the same;
% points H_{b} and H_{c} must be different; points E_{a} and M_{a} must be different; points E_{a}
% and N are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

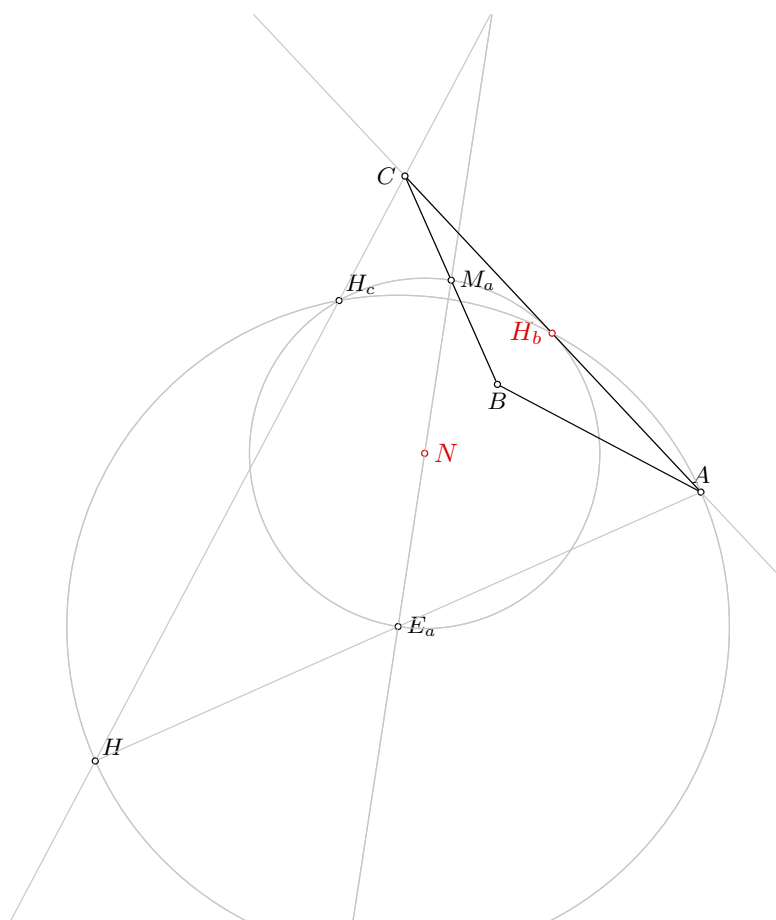


Figure 1: Illustration of the problem 0817

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_b = \neg H_b$

Proving failed

4.1.2 Proving $N = \neg N$

Proving failed

4.1.3 Proving $E_a = \neg E_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_b = \neg H_b$

Proving failed

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $E_a = \neg E_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_b = \neg H_b$

Proving failed

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $E_a = \neg E_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_b = \neg H_b$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $E_a = E_a$

Proving failed

Problem 818

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 818: Given a point E_a , a point H_b and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 819

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 819: Given a point E_a , a point H_b and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 820

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 820: Given a point E_a , a point H_b and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 821

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 821: Given a point E_a , a point H_b and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 822

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 822: Given a point E_a , a point H_c and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 823

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 823: Given a point E_a , a point M_a and a point H_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point M_a , construct a line $m(H_bH_c)$ (rule W02); % DET: points E_a and M_a are not the same;
2. Using the point E_a and the point M_a , construct a line $m(E_aM_a)$ (rule W14); % DET: points E_a and M_a are not the same;
3. Using the line $m(E_aM_a)$ and the line $m(H_bH_c)$, construct a point N (rule W03); % NDG: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not parallel % DET: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not the same;
4. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
5. Choose freely a point H_c on the circle $k(N, M_a)$ (rule WOncircle);
6. Using the point H_c and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_c and E_a are not the same;
7. Using the circle $k(N, M_a)$, the circle $k(E_a, A)$, the point H_c , the point N and the point E_a , construct a point H_b (rule W08); % NDG: circles $k(N, M_a)$ and $k(E_a, A)$ intersect % DET: circles $k(N, M_a)$ and $k(E_a, A)$ are not the same; points H_c and H_b must be different;
8. Choose freely a point A on the circle $k(E_a, A)$ (rule WOncircle);
9. Using the point A and the point E_a , construct a point H (rule W01); ;
10. Using the point A and the point H_c , construct a line c (rule W02); % DET: points A and H_c are not the same;

11. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
12. Using the line c and the line h_b , construct a point B (rule W03); % NDG: lines c and h_b are not parallel % DET: lines c and h_b are not the same;
13. Using the point M_a and the point B , construct a point C (rule W01); .

Non-degenerate conditions: lines c and h_b are not parallel; circles $k(N, M_a)$ and $k(E_a, A)$ intersect; points H_c and E_a are not the same; points E_a and N are not the same; lines $m(E_a M_a)$ and $m(H_b H_c)$ are not parallel.

Determination conditions: lines c and h_b are not the same; points H_b and H are not the same; points A and H_c are not the same; circles $k(N, M_a)$ and $k(E_a, A)$ are not the same; points H_c and H_b must be different; lines $m(E_a M_a)$ and $m(H_b H_c)$ are not the same; points E_a and M_a are not the same; points E_a and M_a are not the same.

Rules used: [W01,W02,W03,W06,W08,W14,WOncircle1]

Lemmas used: [D21,D28,D3,D32,D6,D7,D9,GD01,GD02,GL01,GL03,GL04,L20,L21,L22,L38,L39,L47,L48]

Solving time: 66.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
point M_{a} 65 40
point H_{c} 68.91 84.83
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{a}
cmark_r M_{a}
cmark_rt H_{c}
color 0 0 0
fontsize 8
```

```
% DET: points E_{a} and M_{a} are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point E_{a} and point M_{a}
line m(H_{b}H_{c}) E_{a} M_{a}
```

```
color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0
```

```
% DET: points E_{a} and M_{a} are not the same
% Constructing bisector m(E_{a}M_{a}) of the segment E_{a}M_{a}
med m(E_{a}M_{a}) E_{a} M_{a}
```

```
color 200 200 200
drawline m(E_{a}M_{a})
```

```

color 0 0 0

color 200 200 200
drawsegment E_{a} M_{a}
color 0 0 0

% NDG: lines  $m(E_{\{a\}}M_{\{a\}})$  and  $m(H_{\{b\}}H_{\{c\}})$  are not parallel% DET: lines  $m(E_{\{a\}}M_{\{a\}})$  and  $m(H_{\{b\}}H_{\{c\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{a\}}M_{\{a\}})$  and line  $m(H_{\{b\}}H_{\{c\}})$ 
intersec N m(E_{a}M_{a}) m(H_{b}H_{c})
cmark_r N

% NDG: points  $E_{\{a\}}$  and  $N$  are not the same
% Constructing a circle  $k(N,M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{a\}}$ 
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point  $H_{\{c\}}$  on the circle with center  $N$  through point  $E_{\{a\}}$ 
oncircle H_{c} N E_{a}
cmark_rt H_{c}
color 200 200 200
drawcircle N E_{a}
color 0 0 0

% NDG: points  $H_{\{c\}}$  and  $E_{\{a\}}$  are not the same
% Constructing a circle  $k(E_{\{a\}},A)$  whose center is at point  $E_{\{a\}}$  and which passes through point  $H_{\{c\}}$ 
circle k(E_{a},A) E_{a} H_{c}

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: circles  $k(N,M_{\{a\}})$  and  $k(E_{\{a\}},A)$  intersect% DET: circles  $k(N,M_{\{a\}})$  and  $k(E_{\{a\}},A)$  are not the same; points  $H_{\{c\}}$  and  $H_{\{b\}}$  must be different
% Constructing a line  $L_{\{\backslash\_G46021\}}$  which passes through point  $N$  and point  $E_{\{a\}}$ 
line L_{\_G46021} N E_{a}

color 200 200 200
drawline L_{\_G46021}
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $H_{\{c\}}$  in the symmetry to point/line  $L_{\{\backslash\_G46021\}}$ 

```

```

sim H_{b} L_{\G46021} H_{c}
cmark_l H_{b}

```

```

% Choosing randomly a point A on the circle with center E_{a} through point H_{c}
oncircle A E_{a} H_{c}
cmark_t A
color 200 200 200
drawcircle E_{a} H_{c}
color 0 0 0

```

```

% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

```

```

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0

```

```

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

```

```

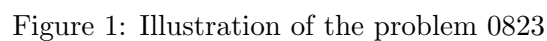
% NDG: lines c and h_{b} are not parallel% DET: lines c and h_{b} are not the same
% Constructing a point B which belongs to line c and line h_{b}
intersec B c h_{b}
cmark_b B

```

```

% Constructing a point C such that M_{a}C/M_{a}B=-1
towards C M_{a} B -1
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

```

```
% Non-degenerate conditions: lines c and h_{b} are not parallel; circles k(N,M_{a}) and k(E_{a},A)
% intersect; points H_{c} and E_{a} are not the same; points E_{a} and N are not the same; lines
% m(E_{a}M_{a}) and m(H_{b}H_{c}) are not parallel
% Determination conditions: lines c and h_{b} are not the same; points H_{b} and H are not the same
% ; points A and H_{c} are not the same; circles k(N,M_{a}) and k(E_{a},A) are not the same;
% points H_{c} and H_{b} must be different; lines m(E_{a}M_{a}) and m(H_{b}H_{c}) are not the
% same; points E_{a} and M_{a} are not the same; points E_{a} and M_{a} are not the same
```

Illustration of the constructed figure is given in Figure 1

1217

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 12 terms.

Time Complexity: Time spent by the prover is 0.246 seconds.

NDG conditions Points N and E_a are not identical

Line through points H_b and H is not parallel with line through points A and H_c

Points A and H_c are not identical

4.1.3 Proving $H_c = \neg H_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{M_{m(E_a M_a)}^0}^{E_a M_a} \neq S_{T_{m(E_a M_a)}^1}^{E_a M_a}$ i.e., lines $M_{m(E_a M_a)}^0 T_{m(E_a M_a)}^1$ and $E_a M_a$ are not parallel (construction based assumption)

$S_{AH_b H} \neq S_{H_c H_b H}$ i.e., lines AH_c and $H_b H$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{F_{-h_a}^2 BF_{-h_b}^3}$ i.e., lines $AF_{-h_a}^2$ and $BF_{-h_b}^3$ are not parallel (construction based assumption)

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{-h_c}^4} \neq S_{BCF_{-h_c}^4}$ i.e., lines AB and $CF_{-h_c}^4$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.2.3 Proving $H_c = \neg H_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $M_a = \neg M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 19 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.3.3 Proving $H_c = \neg H_c$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 4478 terms.

Time Complexity: Time spent by the prover is 4.370 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $M_a = \neg M_a$

Proving failed

4.4.3 Proving $H_c = \neg H_c$

Proving failed

Problem 824

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 824: Given a point E_a , a point H_c and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H_c and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_c and E_a are not the same;
2. Using the point H_c and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points H_c and M_b are not the same;
3. Using the circle $k(E_a, A)$, the circle $k(M_b, C)$, the point H_c , the point E_a and the point M_b , construct a point A (rule W08); % NDG: circles $k(E_a, A)$ and $k(M_b, C)$ intersect % DET: circles $k(E_a, A)$ and $k(M_b, C)$ are not the same; points H_c and A must be different;
4. Using the point A and the point E_a , construct a point H (rule W01); ;
5. Using the point M_b and the point A , construct a point C (rule W01); ;
6. Using the point H_c and the point A , construct a line c (rule W02); % DET: points H_c and A are not the same;
7. Using the point M_b and the point A , construct a line b (rule W02); % DET: points M_b and A are not the same;
8. Using the circle $k(E_a, A)$, the line b , the point E_a and the point A , construct a point H_b (rule W05); % NDG: line b and circle $k(E_a, A)$ intersect % DET: points A and H_b must be different;
9. Using the point H and the point H_b , construct a line h_b (rule W02); % DET: points H and H_b are not the same;
10. Using the line c and the line h_b , construct a point B (rule W03); % NDG: lines c and h_b are not parallel % DET: lines c and h_b are not the same.

Non-degenerate conditions: lines c and h_b are not parallel; line b and circle $k(E_a, A)$ intersect; circles $k(E_a, A)$ and $k(M_b, C)$ intersect; points H_c and M_b are not the same; points H_c and E_a are not the same.

Determination conditions: lines c and h_b are not the same; points H and H_b are not the same; points A and H_b must be different; points M_b and A are not the same; points H_c and A are not the same; circles $k(E_a, A)$ and $k(M_b, C)$ are not the same; points H_c and A must be different.

Rules used: [W01,W02,W03,W05,W06,W08]

Lemmas used: [D22,D28,D3,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L43,L45,L47,L48]

Solving time: 9.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{a} 80 83.86
point H_{c} 68.91 84.83
point M_{b} 95 67.5

color 220 0 0
fontsize 9

cmark_r E_{a}
cmark_rt H_{c}
cmark_lt M_{b}
color 0 0 0
fontsize 8

% NDG: points H_{c} and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H
%_{c}
circle k(E_{a},A) E_{a} H_{c}

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: points H_{c} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H
%_{c}
circle k(M_{b},C) M_{b} H_{c}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: circles k(E_{a},A) and k(M_{b},C) intersect% DET: circles k(E_{a},A) and k(M_{b},C) are not
% the same; points H_{c} and A must be different
% Constructing a line L_{\_G72953} which passes through point E_{a} and point M_{b}
line L_{\_G72953} E_{a} M_{b}
```

```

color 200 200 200
drawline L_{\_G72953}
color 0 0 0

```

```

% Constructing a point A which is an image of the point H_{c} in the symmetry to point/line L_{\_G
72953}
sim A L_{\_G72953} H_{c}
cmark_t A

```

```

% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

```

```

% Constructing a point C such that M_{b}C/M_{b}A=-1
towards C M_{b} A -1
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

```

```

% DET: points H_{c} and A are not the same
% Constructing a line c which passes through point H_{c} and point A
line c H_{c} A

```

```

color 200 200 200
drawline c
color 0 0 0

```

```

% DET: points M_{b} and A are not the same
% Constructing a line b which passes through point M_{b} and point A
line b M_{b} A

```

```

color 200 200 200
drawline b
color 0 0 0

```

```

% NDG: line b and circle k(E_{a},A) intersect% DET: points A and H_{b} must be different
% Constructing a point P_{\_G73370} which is a foot of the point E_{a} on the line b
foot P_{\_G73370} E_{a} b
cmark_r P_{\_G73370}
color 200 200 200
drawline E_{a} P_{\_G73370}

```

```

color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $A$  in the symmetry to point/line  $P_{\{\backslash\_G73370\}}$ 
sim  $H_{\{b\}}$   $P_{\{\backslash\_G73370\}}$   $A$ 
cmark_l  $H_{\{b\}}$ 

% DET: points  $H$  and  $H_{\{b\}}$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $H$  and point  $H_{\{b\}}$ 
line  $h_{\{b\}}$   $H$   $H_{\{b\}}$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: lines  $c$  and  $h_{\{b\}}$  are not parallel% DET: lines  $c$  and  $h_{\{b\}}$  are not the same
% Constructing a point  $B$  which belongs to line  $c$  and line  $h_{\{b\}}$ 
intersec  $B$   $c$   $h_{\{b\}}$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $c$  and  $h_{\{b\}}$  are not parallel; line  $b$  and circle  $k(E_{\{a\}},A)$ 
intersect; circles  $k(E_{\{a\}},A)$  and  $k(M_{\{b\}},C)$  intersect; points  $H_{\{c\}}$  and  $M_{\{b\}}$  are not the same
; points  $H_{\{c\}}$  and  $E_{\{a\}}$  are not the same
% Determination conditions: lines  $c$  and  $h_{\{b\}}$  are not the same; points  $H$  and  $H_{\{b\}}$  are not the same
; points  $A$  and  $H_{\{b\}}$  must be different; points  $M_{\{b\}}$  and  $A$  are not the same; points  $H_{\{c\}}$  and  $A$ 
are not the same; circles  $k(E_{\{a\}},A)$  and  $k(M_{\{b\}},C)$  are not the same; points  $H_{\{c\}}$  and  $A$  must
be different

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $H_c = \neg H_c$

Proving failed

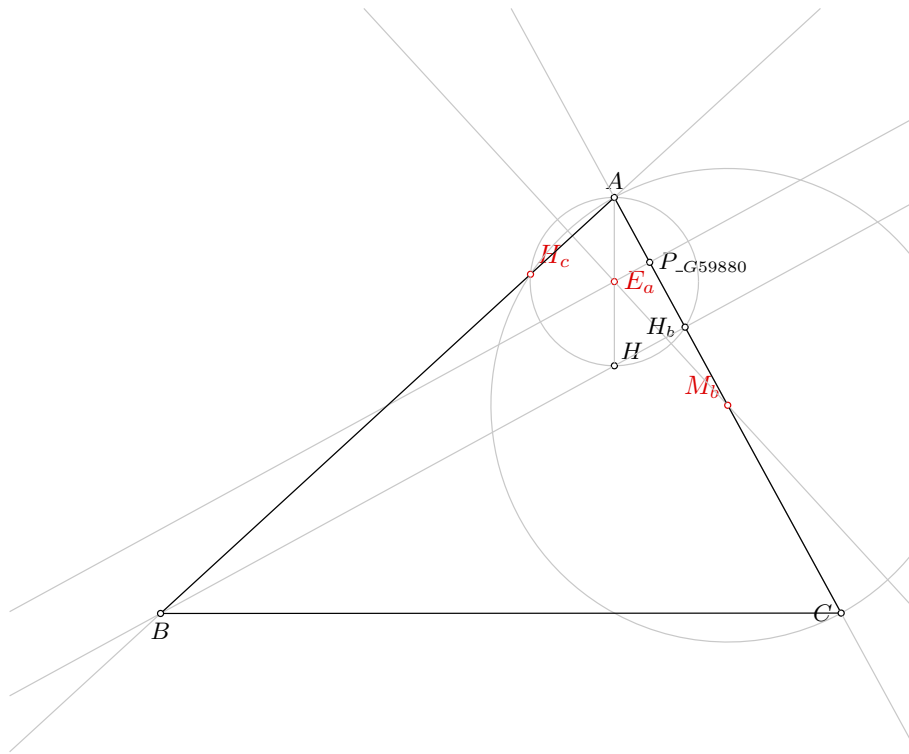


Figure 1: Illustration of the problem 0824

4.1.3 Proving $M_b = M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 11 terms.

Time Complexity: Time spent by the prover is 0.178 seconds.

NDG conditions Points M_b , H_c and E_a are not collinear

Points M_b , H_c and E_a are not collinear

4.2 GCLC - Area method

4.2.1 Proving $E_a = E_a$

Proving failed

4.2.2 Proving $H_c = H_c$

Proving failed

4.2.3 Proving $M_b = M_b$

NDG conditions are:

$S_{H_c H H_b} \neq S_{A H H_b}$ i.e., lines $H_c A$ and $H H_b$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{H_b}} \neq S_{F^0_{H_a} B F^1_{H_b}}$ i.e., lines $AF^0_{H_a}$ and $BF^1_{H_b}$ are not parallel (construction based assumption)

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF^2_{H_c}} \neq S_{BCF^2_{H_c}}$ i.e., lines AB and $CF^2_{H_c}$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_a = E_a$

Proving failed

4.3.2 Proving $H_c = H_c$

Proving failed

4.3.3 Proving $M_b = M_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = E_a$

Proving failed

4.4.2 Proving $H_c = H_c$

Proving failed

4.4.3 Proving $M_b = M_b$

Proving failed

Problem 825

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 825: Given a point E_a , a point H_c and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
2. Using the point H_c and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_c and E_a are not the same;
3. Using the circle $k(E_a, A)$, the line c , the point E_a and the point H_c , construct a point A (rule W05); % NDG: line c and circle $k(E_a, A)$ intersect % DET: points H_c and A must be different;
4. Using the point A and the point E_a , construct a point H (rule W01); ;
5. Using the point M_c and the point A , construct a point B (rule W01); ;
6. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
7. Using the point H and the point B , construct a line h_b (rule W02); % DET: points H and B are not the same;
8. Using the circle $k(E_a, A)$, the line h_b , the point E_a and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_a, A)$ intersect % DET: points H and H_b must be different;
9. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;
10. Using the line h_c and the line b , construct a point C (rule W03); % NDG: lines h_c and b are not parallel % DET: lines h_c and b are not the same.

Non-degenerate conditions: lines h_c and b are not parallel; line h_b and circle $k(E_a, A)$ intersect; line c and circle $k(E_a, A)$ intersect; points H_c and E_a are not the same.

Determination conditions: lines h_c and b are not the same; points A and H_b are not the same; points H and H_b must be different; points H and B are not the same; points H_c and H are not the same; points H_c and A must be different; points H_c and M_c are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D20,D28,D3,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L46,L47,L48]

Solving time: 9.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point H_{c} 68.91 84.83
```

```
point M_{c} 50 67.5
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_rt H_{c}
```

```
cmark_lt M_{c}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{c} and M_{c} are not the same
```

```
% Constructing a line c which passes through point H_{c} and point M_{c}
```

```
line c H_{c} M_{c}
```

```
color 200 200 200
```

```
drawline c
```

```
color 0 0 0
```

```
% NDG: points H_{c} and E_{a} are not the same
```

```
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H_{c}
```

```
circle k(E_{a},A) E_{a} H_{c}
```

```
color 200 200 200
```

```
drawcircle k(E_{a},A)
```

```
color 0 0 0
```

```
% NDG: line c and circle k(E_{a},A) intersect% DET: points H_{c} and A must be different
```

```
% Constructing a point P_{\_G98048} which is a foot of the point E_{a} on the line c
```

```
foot P_{\_G98048} E_{a} c
```

```
cmark_r P_{\_G98048}
```

```
color 200 200 200
```

```
drawline E_{a} P_{\_G98048}
```

```
color 0 0 0
```

```

% Constructing a point A which is an image of the point  $H_{\{c\}}$  in the symmetry to point/line  $P_{\{\backslash\_G98048\}}$ 
sim A  $P_{\{\backslash\_G98048\}}$   $H_{\{c\}}$ 
cmark_t A

% Constructing a point H such that  $AH/AE_{\{a\}}=2$ 
towards H A  $E_{\{a\}}$  2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% Constructing a point B such that  $M_{\{c\}}B/M_{\{c\}}A=-1$ 
towards B  $M_{\{c\}}$  A -1
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points  $H_{\{c\}}$  and H are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $H_{\{c\}}$  and point H
line  $h_{\{c\}}$   $H_{\{c\}}$  H

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% DET: points H and B are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point H and point B
line  $h_{\{b\}}$  H B

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(E_{\{a\}}, A)$  intersect% DET: points H and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G98485\}}$  which is a foot of the point  $E_{\{a\}}$  on the line  $h_{\{b\}}$ 
foot  $P_{\{\backslash\_G98485\}}$   $E_{\{a\}}$   $h_{\{b\}}$ 
cmark_r  $P_{\{\backslash\_G98485\}}$ 
color 200 200 200
drawline  $E_{\{a\}}$   $P_{\{\backslash\_G98485\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point H in the symmetry to point/line  $P_{\{\backslash\_G98485\}}$ 
sim  $H_{\{b\}}$   $P_{\{\backslash\_G98485\}}$  H

```

```

cmark_1 H_{b}

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines h_{c} and b are not parallel% DET: lines h_{c} and b are not the same
% Constructing a point C which belongs to line h_{c} and line b
intersec C h_{c} b
cmark_1 C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and b are not parallel; line h_{b} and circle k(E_{a},A)
% intersect; line c and circle k(E_{a},A) intersect; points H_{c} and E_{a} are not the same
% Determination conditions: lines h_{c} and b are not the same; points A and H_{b} are not the same
% ; points H and H_{b} must be different; points H and B are not the same; points H_{c} and H are
% not the same; points H_{c} and A must be different; points H_{c} and M_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = _E_a$

Proving failed

4.1.2 Proving $H_c = _H_c$

Proving failed

4.1.3 Proving $M_c = _M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 11 terms.

Time Complexity: Time spent by the prover is 0.176 seconds.

NDG conditions Points H_c , M_c and E_a are not collinear

Points H_c , M_c and E_a are not collinear

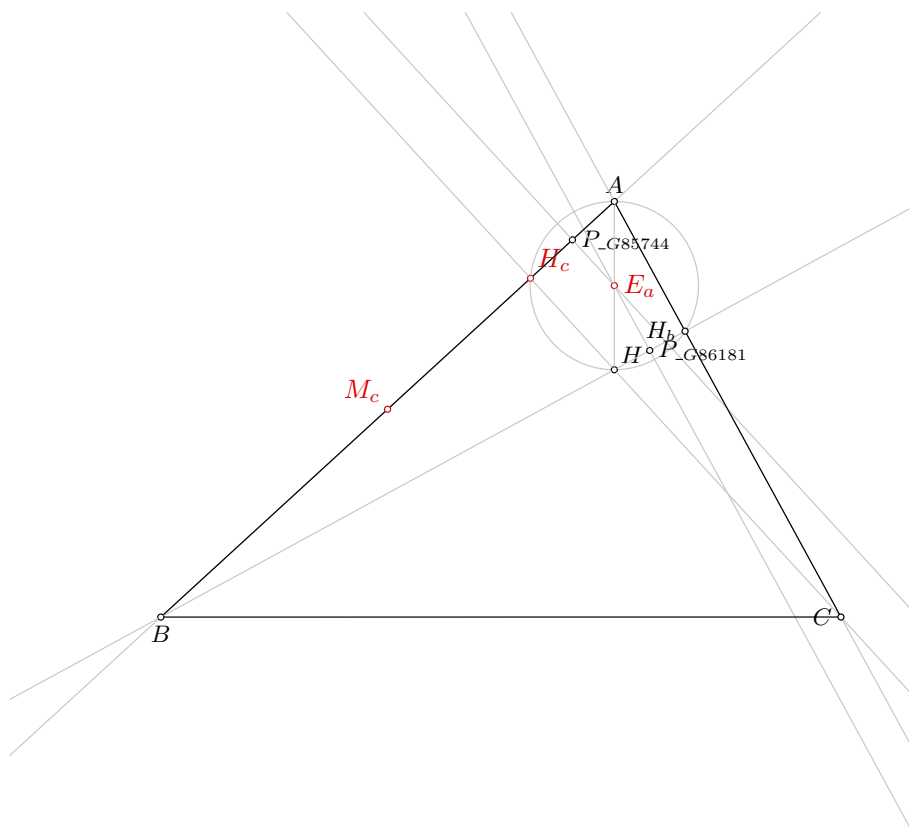


Figure 1: Illustration of the problem 0825

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $H_c = \neg H_c$

Proving failed

4.2.3 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{H_c A H_b} \neq S_{H A H_b}$ i.e., lines $H_c H$ and $A H_b$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{AB F^1_{\neg h_b}} \neq S_{F^0_{\neg h_a} B F^1_{\neg h_b}}$ i.e., lines $A F^0_{\neg h_a}$ and $B F^1_{\neg h_b}$ are not parallel (construction based assumption)

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{AC F^2_{\neg h_c}} \neq S_{BC F^2_{\neg h_c}}$ i.e., lines AB and $CF^2_{\neg h_c}$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $H_c = \neg H_c$

Proving failed

4.3.3 Proving $M_c = \neg M_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $H_c = \neg H_c$

Proving failed

4.4.3 Proving $M_c = \neg M_c$

Proving failed

Problem 826

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 826: Given a point H_c , a point N and a point E_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_c and N are not the same;
2. Choose freely a point E_a on the circle $k(N, M_a)$ (rule WOncircle);
3. Using the point E_a and the point N , construct a line $m(H_b H_c)$ (rule W02); % DET: points E_a and N are not the same;
4. Using the circle $k(N, M_a)$, the line $m(H_b H_c)$, the point N and the point E_a , construct a point M_a (rule W05a); % NDG: line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect % DET: points E_a and M_a must be different;
5. Using the point H_c and the point E_a , construct a circle $k(E_a, A)$ (rule W06); % NDG: points H_c and E_a are not the same;
6. Using the circle $k(N, M_a)$, the circle $k(E_a, A)$, the point H_c , the point N and the point E_a , construct a point H_b (rule W08); % NDG: circles $k(N, M_a)$ and $k(E_a, A)$ intersect % DET: circles $k(N, M_a)$ and $k(E_a, A)$ are not the same; points H_c and H_b must be different;
7. Choose freely a point A on the circle $k(E_a, A)$ (rule WOncircle);
8. Using the point A and the point E_a , construct a point H (rule W01); ;
9. Using the point A and the point H_c , construct a line c (rule W02); % DET: points A and H_c are not the same;
10. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;

11. Using the line c and the line h_b , construct a point B (rule W03); % NDG: lines c and h_b are not parallel % DET: lines c and h_b are not the same;
12. Using the point M_a and the point B , construct a point C (rule W01); .

Non-degenerate conditions: lines c and h_b are not parallel; circles $k(N, M_a)$ and $k(E_a, A)$ intersect; points H_c and E_a are not the same; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; points H_c and N are not the same.

Determination conditions: lines c and h_b are not the same; points H_b and H are not the same; points A and H_c are not the same; circles $k(N, M_a)$ and $k(E_a, A)$ are not the same; points H_c and H_b must be different; points E_a and M_a must be different; points E_a and N are not the same.

Rules used: [W01,W02,W03,W05a,W06,W08,WOncircle1]

Lemmas used: [D21,D28,D3,D32,D6,D7,D9,GD01,GD02,GL01,GL03,GL04,L20,L21,L22,L38,L39,L47,L48]

Solving time: 60.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{c} 68.91 84.83
point N 72.5 61.93
point E_{a} 80 83.86
```

```
color 220 0 0
fontsize 9
```

```
cmark_rt H_{c}
cmark_r N
cmark_r E_{a}
color 0 0 0
fontsize 8
```

```
% NDG: points H_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{c}
circle k(N,M_{a}) N H_{c}
```

```
color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```
% Choosing randomly a point E_{a} on the circle with center N through point H_{c}
oncircle E_{a} N H_{c}
cmark_r E_{a}
color 200 200 200
drawcircle N H_{c}
color 0 0 0
```

```
% DET: points E_{a} and N are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point E_{a} and point N
```

```

line m(H_{b}H_{c}) E_{a} N

color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0

% NDG: line m(H_{b}H_{c}) and circle k(N,M_{a}) intersect% DET: points E_{a} and M_{a} must be
different
% Constructing a point M_{a} which is an image of the point E_{a} in the symmetry to point/line N
sim M_{a} N E_{a}
cmark_r M_{a}

% NDG: points H_{c} and E_{a} are not the same
% Constructing a circle k(E_{a},A) whose center is at point E_{a} and which passes through point H
_{c}
circle k(E_{a},A) E_{a} H_{c}

color 200 200 200
drawcircle k(E_{a},A)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(E_{a},A) intersect% DET: circles k(N,M_{a}) and k(E_{a},A) are not
the same; points H_{c} and H_{b} must be different
% Constructing a line L_{\_G136441} which passes through point N and point E_{a}
line L_{\_G136441} N E_{a}

color 200 200 200
drawline L_{\_G136441}
color 0 0 0

% Constructing a point H_{b} which is an image of the point H_{c} in the symmetry to point/line L
_{\_G136441}
sim H_{b} L_{\_G136441} H_{c}
cmark_l H_{b}

% Choosing randomly a point A on the circle with center E_{a} through point H_{c}
oncircle A E_{a} H_{c}
cmark_t A
color 200 200 200
drawcircle E_{a} H_{c}
color 0 0 0

% Constructing a point H such that AH/AE_{a}=2
towards H A E_{a} 2
cmark_rt H
color 200 200 200

```

```

drawsegment A H
color 0 0 0

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines c and h_{b} are not parallel% DET: lines c and h_{b} are not the same
% Constructing a point B which belongs to line c and line h_{b}
intersec B c h_{b}
cmark_b B

% Constructing a point C such that M_{a}C/M_{a}B=-1
towards C M_{a} B -1
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and h_{b} are not parallel; circles k(N,M_{a}) and k(E_{a},A)
% intersect; points H_{c} and E_{a} are not the same; line m(H_{b}H_{c}) and circle k(N,M_{a})
% intersect; points H_{c} and N are not the same
% Determination conditions: lines c and h_{b} are not the same; points H_{b} and H are not the same
% ; points A and H_{c} are not the same; circles k(N,M_{a}) and k(E_{a},A) are not the same;
% points H_{c} and H_{b} must be different; points E_{a} and M_{a} must be different; points E_{a}
% and N are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

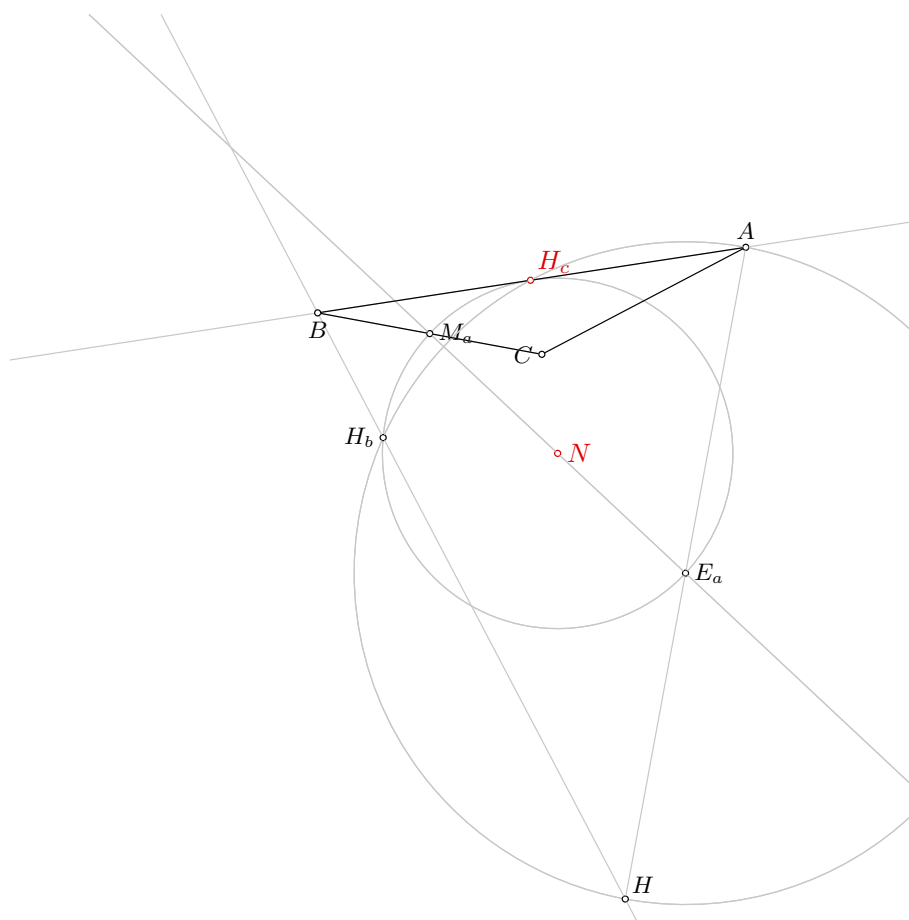


Figure 1: Illustration of the problem 0826

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_c = \neg H_c$

Proving failed

4.1.2 Proving $N = \neg N$

Proving failed

4.1.3 Proving $E_a = \neg E_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_c = \neg H_c$

Proving failed

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $E_a = \neg E_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_c = \neg H_c$

Proving failed

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $E_a = \neg E_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_c = \neg H_c$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $E_a = E_a$

Proving failed

Problem 827

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 827: Given a point E_a , a point H_c and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 828

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 828: Given a point E_a , a point H_c and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 829

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 829: Given a point E_a , a point H_c and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 830

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 830: Given a point E_a , a point H_c and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 831

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 831: Given a point E_a , a point I and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point M_a , construct a line $m(H_bH_c)$ (rule W02); % DET: points E_a and M_a are not the same;
2. Using the point I and the point M_a , construct a line IM_a (rule W02); % DET: points I and M_a are not the same;
3. Using the point I and the point M_a , construct a circle $k_{over}(I, M_a)$ (rule W09); % NDG: points I and M_a are not the same;
4. Using the point E_a and the point M_a , construct a line $m(E_aM_a)$ (rule W14); % DET: points E_a and M_a are not the same;
5. Using the line $m(E_aM_a)$ and the line $m(H_bH_c)$, construct a point N (rule W03); % NDG: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not parallel % DET: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not the same;
6. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
7. Using the point I , the circle $k(N, M_a)$, the point N and the point E_a , construct a circle $k(I, P_a)$ (rule W22); % NDG: point I is inside the circle $k(N, M_a)$; points I and N are not the same;
8. Using the circle $k(I, P_a)$ and the circle $k_{over}(I, M_a)$, construct a point P_a and a point A_{fi} (rule W07); % NDG: circles $k(I, P_a)$ and $k_{over}(I, M_a)$ intersect % DET: circles $k(I, P_a)$ and $k_{over}(I, M_a)$ are not the same;
9. Using the point P_a and the point M_a , construct a point P'_a (rule W01); ;

10. Using the circle $k(I, P_a)$, the point M_a and the point I , construct a line x_1 and a line a (rule W12); % NDG: point M_a is outside the circle $k(I, P_a)$;
11. Using the circle $k(N, M_a)$, the line a , the point N and the point M_a , construct a point H_a (rule W05); % NDG: line a and circle $k(N, M_a)$ intersect % DET: points M_a and H_a must be different;
12. Using the point H_a and the point E_a , construct a line h_a (rule W02); % DET: points H_a and E_a are not the same;
13. Using the point M_a and the line a , construct a line m_a (rule W10b); ;
14. Using the point P'_a and the line IM_a , construct a line AP'_a (rule W16); ;
15. Using the line AP'_a and the line h_a , construct a point A (rule W03); % NDG: lines AP'_a and h_a are not parallel % DET: lines AP'_a and h_a are not the same;
16. Using the point I and the point A , construct a line s_a (rule W02); % DET: points I and A are not the same;
17. Using the line m_a and the line s_a , construct a point N_a (rule W03); % NDG: lines m_a and s_a are not parallel % DET: lines m_a and s_a are not the same;
18. Using the point I and the point N_a , construct a circle $k(N_a, C)$ (rule W06); % NDG: points I and N_a are not the same;
19. Using the circle $k(N_a, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(N_a, C)$ intersect.

Non-degenerate conditions: line a and circle $k(N_a, C)$ intersect; points I and N_a are not the same; lines m_a and s_a are not parallel; lines AP'_a and h_a are not parallel; line a and circle $k(N, M_a)$ intersect; point M_a is outside the circle $k(I, P_a)$; circles $k(I, P_a)$ and $k_{over}(I, M_a)$ intersect; point I is inside the circle $k(N, M_a)$; points I and N are not the same; points E_a and N are not the same; lines $m(E_a M_a)$ and $m(H_b H_c)$ are not parallel; points I and M_a are not the same.

Determination conditions: lines m_a and s_a are not the same; points I and A are not the same; lines AP'_a and h_a are not the same; points H_a and E_a are not the same; points M_a and H_a must be different; circles $k(I, P_a)$ and $k_{over}(I, M_a)$ are not the same; lines $m(E_a M_a)$ and $m(H_b H_c)$ are not the same; points E_a and M_a are not the same; points I and M_a are not the same; points E_a and M_a are not the same.

Rules used: [W01, W02, W03, W04, W05, W06, W07, W09, W10b, W12, W14, W16, W22]

Lemmas used: [D11, D2, D21, D27, D28, D3, D32, D47, D5, D65, D8, D85, GD01, GD02, GL01, GL03, GL09, L119, L19, L20]

Solving time: 51.8 seconds.

3.2 Construction in GCLC language

dim 120 120

point E_{a} 80 83.86

point I 74.37 61.15

point M_{a} 65 40

```

color 220 0 0
fontsize 9

cmark_r E_{a}
cmark_b I
cmark_r M_{a}
color 0 0 0
fontsize 8

% DET: points E_{a} and M_{a} are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point E_{a} and point M_{a}
line m(H_{b}H_{c}) E_{a} M_{a}

color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0

% DET: points I and M_{a} are not the same
% Constructing a line IM_{a} which passes through point I and point M_{a}
line IM_{a} I M_{a}

color 200 200 200
drawline IM_{a}
color 0 0 0

% NDG: points I and M_{a} are not the same
% Constructing midpoint P_{\_G201308} of the segment IM_{a}
midpoint P_{\_G201308} I M_{a}
cmark_r P_{\_G201308}

% Constructing a circle k_{over(I,M_{a})} whose center is at point P_{\_G201308} and which passes
    through point I
circle k_{over(I,M_{a})} P_{\_G201308} I

color 200 200 200
drawcircle k_{over(I,M_{a})}
color 0 0 0

% DET: points E_{a} and M_{a} are not the same
% Constructing bisector m(E_{a}M_{a}) of the segment E_{a}M_{a}
med m(E_{a}M_{a}) E_{a} M_{a}

color 200 200 200
drawline m(E_{a}M_{a})
color 0 0 0

color 200 200 200
drawsegment E_{a} M_{a}
color 0 0 0

```

```

% NDG: lines  $m(E_{\{a\}}M_{\{a\}})$  and  $m(H_{\{b\}}H_{\{c\}})$  are not parallel% DET: lines  $m(E_{\{a\}}M_{\{a\}})$  and  $m(H_{\{b\}}H_{\{c\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{a\}}M_{\{a\}})$  and line  $m(H_{\{b\}}H_{\{c\}})$ 
intersec N m(E_{a}M_{a}) m(H_{b}H_{c})
cmark_r N

% NDG: points  $E_{\{a\}}$  and  $N$  are not the same
% Constructing a circle  $k(N,M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{a\}}$ 
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: point  $I$  is inside the circle  $k(N,M_{\{a\}})$ ; points  $I$  and  $N$  are not the same
% Calculating distance  $V[_G201863]$  from point  $N$  to point  $E_{\{a\}}$ 
distance V[_G201863] N E_{a}

% Calculating distance  $V[_G201887]$  from point  $N$  to point  $I$ 
distance V[_G201887] N I

% Calculating value  $V[_G201908]$  using formula  $V[_G201863]/V[_G201887]$ 
expression V[_G201908] { V[_G201863]/V[_G201887] }

% Constructing a point  $P_{\{\_G201939\}}$  such that  $NP_{\{\_G201939\}}/NI=V[_G201863]/V[_G201887]$ 
towards P_{\_G201939} N I V[_G201908]
cmark_r P_{\_G201939}

% Constructing a circle  $k(I,P_{\{a\}})$  whose center is at point  $I$  and which passes through point  $P_{\{\_G201939\}}$ 
circle k(I,P_{a}) I P_{\_G201939}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: circles  $k(I,P_{\{a\}})$  and  $k_{\text{over}}(I,M_{\{a\}})$  intersect% DET: circles  $k(I,P_{\{a\}})$  and  $k_{\text{over}}(I,M_{\{a\}})$  are not the same
% Constructing points  $P_{\{a\}}$  and  $A_{\{fi\}}$  which are in intersection of  $k(I,P_{\{a\}})$  and  $k_{\text{over}}(I,M_{\{a\}})$ 
intersec2 P_{a} A_{fi} k(I,P_{a}) k_{over}(I,M_{a})
cmark_r P_{a}
cmark_r A_{fi}

% Constructing a point  $P'_{\{a\}}$  such that  $P_{\{a\}}P'_{\{a\}}/P_{\{a\}}M_{\{a\}}=2$ 

```

```

towards P'_{a} P_{a} M_{a} 2
cmark_r P'_{a}
color 200 200 200
drawsegment P_{a} P'_{a}
color 0 0 0

% NDG: point M_{a} is outside the circle k(I,P_{a})
% Constructing midpoint P_{\_G202545} of the segment M_{a}I
midpoint P_{\_G202545} M_{a} I
cmark_r P_{\_G202545}

% Constructing a circle C_{\_G202548} whose center is at point P_{\_G202545} and which passes
    through point M_{a}
circle C_{\_G202548} P_{\_G202545} M_{a}

color 200 200 200
drawcircle C_{\_G202548}
color 0 0 0

% Constructing points P_{\_G202551} and P_{\_G202554} which are in intersection of C_{\_G202548}
    and k(I,P_{a})
intersec2 P_{\_G202551} P_{\_G202554} C_{\_G202548} k(I,P_{a})
cmark_r P_{\_G202551}
cmark_r P_{\_G202554}

% Constructing a line x1 which passes through point M_{a} and point P_{\_G202551}
line x1 M_{a} P_{\_G202551}

color 200 200 200
drawline x1
color 0 0 0

% Constructing a line a which passes through point M_{a} and point P_{\_G202554}
line a M_{a} P_{\_G202554}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(N,M_{a}) intersect% DET: points M_{a} and H_{a} must be different
% Constructing a point P_{\_G202888} which is a foot of the point N on the line a
foot P_{\_G202888} N a
cmark_r P_{\_G202888}
color 200 200 200
drawline N P_{\_G202888}
color 0 0 0

% Constructing a point H_{a} which is an image of the point M_{a} in the symmetry to point/line P
    _{\_G202888}
sim H_{a} P_{\_G202888} M_{a}
cmark_r H_{a}

```

```

% DET: points  $H_{\{a\}}$  and  $E_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $H_{\{a\}}$  and point  $E_{\{a\}}$ 
line  $h_{\{a\}}$   $H_{\{a\}}$   $E_{\{a\}}$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% Constructing a line  $m_{\{a\}}$  which is perpendicular to line  $a$  and which passes through point  $M_{\{a\}}$ 
perp  $m_{\{a\}}$   $M_{\{a\}}$   $a$ 

color 200 200 200
drawline  $m_{\{a\}}$ 
color 0 0 0

% Constructing a line  $AP'_{\{a\}}$  which contains the point  $P'_{\{a\}}$  and is parallel to the line  $IM_{\{a\}}$ 
parallel  $AP'_{\{a\}}$   $P'_{\{a\}}$   $IM_{\{a\}}$ 

color 200 200 200
drawline  $AP'_{\{a\}}$ 
color 0 0 0

% NDG: lines  $AP'_{\{a\}}$  and  $h_{\{a\}}$  are not parallel% DET: lines  $AP'_{\{a\}}$  and  $h_{\{a\}}$  are not the same
% Constructing a point  $A$  which belongs to line  $AP'_{\{a\}}$  and line  $h_{\{a\}}$ 
intersec  $A$   $AP'_{\{a\}}$   $h_{\{a\}}$ 
cmark_t  $A$ 

% DET: points  $I$  and  $A$  are not the same
% Constructing a line  $s_{\{a\}}$  which passes through point  $I$  and point  $A$ 
line  $s_{\{a\}}$   $I$   $A$ 

color 200 200 200
drawline  $s_{\{a\}}$ 
color 0 0 0

% NDG: lines  $m_{\{a\}}$  and  $s_{\{a\}}$  are not parallel% DET: lines  $m_{\{a\}}$  and  $s_{\{a\}}$  are not the same
% Constructing a point  $N_{\{a\}}$  which belongs to line  $m_{\{a\}}$  and line  $s_{\{a\}}$ 
intersec  $N_{\{a\}}$   $m_{\{a\}}$   $s_{\{a\}}$ 
cmark_b  $N_{\{a\}}$ 

% NDG: points  $I$  and  $N_{\{a\}}$  are not the same
% Constructing a circle  $k(N_{\{a\}}, C)$  whose center is at point  $N_{\{a\}}$  and which passes through point  $I$ 

```


Figure 1: Illustration of the problem 0831

```

circle k(N_{a},C) N_{a} I

color 200 200 200
drawcircle k(N_{a},C)
color 0 0 0

% NDG: line a and circle k(N_{a},C) intersect
% Constructing points C and B which are in intersection of k(N_{a},C) and a
intersec2 C B k(N_{a},C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(N_{a},C) intersect; points I and N_{a} are not the
same; lines m_{a} and s_{a} are not parallel; lines AP'_{a} and h_{a} are not parallel; line a
and circle k(N,M_{a}) intersect; point M_{a} is outside the circle k(I,P_{a}); circles k(I,P_{a})
and k_{over}(I,M_{a}) intersect; point I is inside the circle k(N,M_{a}); points I and N are
not the same; points E_{a} and N are not the same; lines m(E_{a}M_{a}) and m(H_{b}H_{c}) are
not parallel; points I and M_{a} are not the same
% Determination conditions: lines m_{a} and s_{a} are not the same; points I and A are not the same
; lines AP'_{a} and h_{a} are not the same; points H_{a} and E_{a} are not the same; points M_{a}
and H_{a} must be different; circles k(I,P_{a}) and k_{over}(I,M_{a}) are not the same; lines
m(E_{a}M_{a}) and m(H_{b}H_{c}) are not the same; points E_{a} and M_{a} are not the same;
points I and M_{a} are not the same; points E_{a} and M_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $I = \neg I$

Proving failed

4.1.3 Proving $M_a = \neg M_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $I = \neg I$

Proving failed

4.2.3 Proving $M_a = \neg M_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $I = \neg I$

Proving failed

4.3.3 Proving $M_a = \neg M_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $I = \neg I$

Proving failed

4.4.3 Proving $M_a = \neg M_a$

Proving failed

Problem 832

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 832: Given a point E_a , a point I and a point M_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 833

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 833: Given a point E_a , a point I and a point M_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 834

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 834: Given a point E_a , a point I and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point N , construct a line $m(H_bH_c)$ (rule W02); % DET: points E_a and N are not the same;
2. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
3. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point E_a , construct a point M_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points E_a and M_a must be different;
4. Using the point M_a and the point I , construct a line IM_a (rule W02); % DET: points M_a and I are not the same;
5. Using the point I and the point M_a , construct a circle $k_{over}(I, M_a)$ (rule W09); % NDG: points I and M_a are not the same;
6. Using the point I , the circle $k(N, M_a)$, the point N and the point E_a , construct a circle $k(I, P_a)$ (rule W22); % NDG: point I is inside the circle $k(N, M_a)$; points I and N are not the same;
7. Using the circle $k(I, P_a)$ and the circle $k_{over}(I, M_a)$, construct a point P_a and a point A_{fi} (rule W07); % NDG: circles $k(I, P_a)$ and $k_{over}(I, M_a)$ intersect % DET: circles $k(I, P_a)$ and $k_{over}(I, M_a)$ are not the same;
8. Using the point P_a and the point M_a , construct a point P'_a (rule W01); ;
9. Using the circle $k(I, P_a)$, the point M_a and the point I , construct a line $x1$ and a line a (rule W12); % NDG: point M_a is outside the circle $k(I, P_a)$;

10. Using the circle $k(N, M_a)$, the line a , the point N and the point M_a , construct a point H_a (rule W05); % NDG: line a and circle $k(N, M_a)$ intersect % DET: points M_a and H_a must be different;
11. Using the point H_a and the point E_a , construct a line h_a (rule W02); % DET: points H_a and E_a are not the same;
12. Using the point M_a and the line a , construct a line m_a (rule W10b); ;
13. Using the point P'_a and the line IM_a , construct a line AP'_a (rule W16); ;
14. Using the line AP'_a and the line h_a , construct a point A (rule W03); % NDG: lines AP'_a and h_a are not parallel % DET: lines AP'_a and h_a are not the same;
15. Using the point I and the point A , construct a line s_a (rule W02); % DET: points I and A are not the same;
16. Using the line m_a and the line s_a , construct a point N_a (rule W03); % NDG: lines m_a and s_a are not parallel % DET: lines m_a and s_a are not the same;
17. Using the point I and the point N_a , construct a circle $k(N_a, C)$ (rule W06); % NDG: points I and N_a are not the same;
18. Using the circle $k(N_a, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(N_a, C)$ intersect.

Non-degenerate conditions: line a and circle $k(N_a, C)$ intersect; points I and N_a are not the same; lines m_a and s_a are not parallel; lines AP'_a and h_a are not parallel; line a and circle $k(N, M_a)$ intersect; point M_a is outside the circle $k(I, P_a)$; circles $k(I, P_a)$ and $k_{over}(I, M_a)$ intersect; point I is inside the circle $k(N, M_a)$; points I and N are not the same; points I and M_a are not the same; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; points E_a and N are not the same.

Determination conditions: lines m_a and s_a are not the same; points I and A are not the same; lines AP'_a and h_a are not the same; points H_a and E_a are not the same; points M_a and H_a must be different; circles $k(I, P_a)$ and $k_{over}(I, M_a)$ are not the same; points M_a and I are not the same; points E_a and M_a must be different; points E_a and N are not the same.

Rules used: [W01, W02, W03, W04, W05, W05a, W06, W07, W09, W10b, W12, W16, W22]

Lemmas used: [D11, D2, D21, D27, D28, D3, D32, D47, D5, D65, D8, D85, GD01, GD02, GL01, GL03, GL09, L119, L19, L

Solving time: 52.7 seconds.

3.2 Construction in GCLC language

dim 120 120

point E_{a} 80 83.86
point I 74.37 61.15
point N 72.5 61.93

color 220 0 0
fontsize 9

```

cmark_r E_{a}
cmark_b I
cmark_r N
color 0 0 0
fontsize 8

% DET: points E_{a} and N are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point E_{a} and point N
line m(H_{b}H_{c}) E_{a} N

color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0

% NDG: points E_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{a}
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(H_{b}H_{c}) and circle k(N,M_{a}) intersect% DET: points E_{a} and M_{a} must be
different
% Constructing a point M_{a} which is an image of the point E_{a} in the symmetry to point/line N
sim M_{a} N E_{a}
cmark_r M_{a}

% DET: points M_{a} and I are not the same
% Constructing a line IM_{a} which passes through point M_{a} and point I
line IM_{a} M_{a} I

color 200 200 200
drawline IM_{a}
color 0 0 0

% NDG: points I and M_{a} are not the same
% Constructing midpoint P_{\_G36633} of the segment IM_{a}
midpoint P_{\_G36633} I M_{a}
cmark_r P_{\_G36633}

% Constructing a circle k_{over}(I,M_{a}) whose center is at point P_{\_G36633} and which passes
through point I
circle k_{over}(I,M_{a}) P_{\_G36633} I

color 200 200 200
drawcircle k_{over}(I,M_{a})
color 0 0 0

```

```

% NDG: point I is inside the circle k(N,M_{a}); points I and N are not the same
% Calculating distance V[_G36819] from point N to point E_{a}
distance V[_G36819] N E_{a}

% Calculating distance V[_G36843] from point N to point I
distance V[_G36843] N I

% Calculating value V[_G36864] using formula V[_G36819]/V[_G36843]
expression V[_G36864] { V[_G36819]/V[_G36843] }

% Constructing a point P_{\_G36895} such that NP_{\_G36895}/NI=V[_G36819]/V[_G36843]
towards P_{\_G36895} N I V[_G36864]
cmark_r P_{\_G36895}

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
36895}
circle k(I,P_{a}) I P_{\_G36895}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: circles k(I,P_{a}) and k_{over}(I,M_{a}) intersect% DET: circles k(I,P_{a}) and k_{over}(I,M_{a}
}) are not the same
% Constructing points P_{a} and A_{fi} which are in intersection of k(I,P_{a}) and k_{over}(I,M_{a})
intersec2 P_{a} A_{fi} k(I,P_{a}) k_{over}(I,M_{a})
cmark_r P_{a}
cmark_r A_{fi}

% Constructing a point P'_{a} such that P_{a}P'_{a}/P_{a}M_{a}=2
towards P'_{a} P_{a} M_{a} 2
cmark_r P'_{a}
color 200 200 200
drawsegment P_{a} P'_{a}
color 0 0 0

% NDG: point M_{a} is outside the circle k(I,P_{a})
% Constructing midpoint P_{\_G37500} of the segment M_{a}I
midpoint P_{\_G37500} M_{a} I
cmark_r P_{\_G37500}

% Constructing a circle C_{\_G37503} whose center is at point P_{\_G37500} and which passes through
point M_{a}
circle C_{\_G37503} P_{\_G37500} M_{a}

color 200 200 200

```



```

drawcircle C_{\_G37503}
color 0 0 0

% Constructing points P_{\_G37506} and P_{\_G37509} which are in intersection of C_{\_G37503} and k
(I,P_{a})
intersec2 P_{\_G37506} P_{\_G37509} C_{\_G37503} k(I,P_{a})
cmark_r P_{\_G37506}
cmark_r P_{\_G37509}

% Constructing a line x1 which passes through point M_{a} and point P_{\_G37506}
line x1 M_{a} P_{\_G37506}

color 200 200 200
drawline x1
color 0 0 0

% Constructing a line a which passes through point M_{a} and point P_{\_G37509}
line a M_{a} P_{\_G37509}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(N,M_{a}) intersect% DET: points M_{a} and H_{a} must be different
% Constructing a point P_{\_G37843} which is a foot of the point N on the line a
foot P_{\_G37843} N a
cmark_r P_{\_G37843}
color 200 200 200
drawline N P_{\_G37843}
color 0 0 0

% Constructing a point H_{a} which is an image of the point M_{a} in the symmetry to point/line P
_{\_G37843}
sim H_{a} P_{\_G37843} M_{a}
cmark_r H_{a}

% DET: points H_{a} and E_{a} are not the same
% Constructing a line h_{a} which passes through point H_{a} and point E_{a}
line h_{a} H_{a} E_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% Constructing a line m_{a} which is perpendicular to line a and which passes through point M_{a}
perp m_{a} M_{a} a

color 200 200 200

```

```

drawline m_{a}
color 0 0 0

% Constructing a line AP'_{a} which contains the point P'_{a} and is parallel to the line IM_{a}
parallel AP'_{a} P'_{a} IM_{a}

color 200 200 200
drawline AP'_{a}
color 0 0 0

% NDG: lines AP'_{a} and h_{a} are not parallel% DET: lines AP'_{a} and h_{a} are not the same
% Constructing a point A which belongs to line AP'_{a} and line h_{a}
intersec A AP'_{a} h_{a}
cmark_t A

% DET: points I and A are not the same
% Constructing a line s_{a} which passes through point I and point A
line s_{a} I A

color 200 200 200
drawline s_{a}
color 0 0 0

% NDG: lines m_{a} and s_{a} are not parallel% DET: lines m_{a} and s_{a} are not the same
% Constructing a point N_{a} which belongs to line m_{a} and line s_{a}
intersec N_{a} m_{a} s_{a}
cmark_b N_{a}

% NDG: points I and N_{a} are not the same
% Constructing a circle k(N_{a},C) whose center is at point N_{a} and which passes through point I
circle k(N_{a},C) N_{a} I

color 200 200 200
drawcircle k(N_{a},C)
color 0 0 0

% NDG: line a and circle k(N_{a},C) intersect
% Constructing points C and B which are in intersection of k(N_{a},C) and a
intersec2 C B k(N_{a},C) a
cmark_l C
cmark_b B

drawsegment A B

```

Figure 1: Illustration of the problem 0834

```
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(N_{a},C) intersect; points I and N_{a} are not the
    same; lines m_{a} and s_{a} are not parallel; lines AP'_{a} and h_{a} are not parallel; line a
    and circle k(N,M_{a}) intersect; point M_{a} is outside the circle k(I,P_{a}); circles k(I,P_{
a}) and k_{over}(I,M_{a}) intersect; point I is inside the circle k(N,M_{a}); points I and N are
    not the same; points I and M_{a} are not the same; line m(H_{b}H_{c}) and circle k(N,M_{a})
    intersect; points E_{a} and N are not the same
% Determination conditions: lines m_{a} and s_{a} are not the same; points I and A are not the same
    ; lines AP'_{a} and h_{a} are not the same; points H_{a} and E_{a} are not the same; points M_{
a} and H_{a} must be different; circles k(I,P_{a}) and k_{over}(I,M_{a}) are not the same; points
    M_{a} and I are not the same; points E_{a} and M_{a} must be different; points E_{a} and N are
    not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = _E_a$

Proving failed

4.1.2 Proving $I = _I$

Proving failed

4.1.3 Proving $N = _N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = _E_a$

Proving failed

4.2.2 Proving $I = _I$

Proving failed

4.2.3 Proving $N = _N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $I = \neg I$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $I = \neg I$

Proving failed

4.4.3 Proving $N = \neg N$

Proving failed

Problem 835

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 835: Given a point E_a , a point I and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 836

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 836: Given a point E_a , a point I and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 837

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 837: Given a point E_a , a point I and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 838

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 838: Given a point E_a , a point I and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 839

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 839: Given a point E_a , a point M_a and a point M_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point M_a , construct a line $m(H_bH_c)$ (rule W02); % DET: points E_a and M_a are not the same;
2. Using the point E_a and the point M_a , construct a line $m(E_aM_a)$ (rule W14); % DET: points E_a and M_a are not the same;
3. Using the line $m(E_aM_a)$ and the line $m(H_bH_c)$, construct a point N (rule W03); % NDG: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not parallel % DET: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not the same;
4. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
5. Choose freely a point M_b on the circle $k(N, M_a)$ (rule WOncircle);
6. Choose freely a point A (rule free);
7. Using the point A and the point M_b , construct a point C (rule W01); ;
8. Using the point A and the point M_a , construct a point G (rule W01); ;
9. Using the point M_b and the point G , construct a point B (rule W01); .

Non-degenerate conditions: points E_a and N are not the same; lines $m(E_aM_a)$ and $m(H_bH_c)$ are not parallel.

Determination conditions: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not the same; points E_a and M_a are not the same; points E_a and M_a are not the same.

Rules used: [W01,W02,W03,W06,W14,WOncircle1,free]

Lemmas used: [D22,D32,GD02,GL01,GL03,GL04,L20,L21,L22,L38,L39,L47,L48,L55,L56]

Solving time: 199.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{a} 80 83.86
point M_{a} 65 40
point M_{b} 95 67.5

color 220 0 0
fontsize 9

cmark_r E_{a}
cmark_r M_{a}
cmark_lt M_{b}
color 0 0 0
fontsize 8

% DET: points E_{a} and M_{a} are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point E_{a} and point M_{a}
line m(H_{b}H_{c}) E_{a} M_{a}

color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0

% DET: points E_{a} and M_{a} are not the same
% Constructing bisector m(E_{a}M_{a}) of the segment E_{a}M_{a}
med m(E_{a}M_{a}) E_{a} M_{a}

color 200 200 200
drawline m(E_{a}M_{a})
color 0 0 0

color 200 200 200
drawsegment E_{a} M_{a}
color 0 0 0

% NDG: lines m(E_{a}M_{a}) and m(H_{b}H_{c}) are not parallel% DET: lines m(E_{a}M_{a}) and m(H_{b}
H_{c}) are not the same
% Constructing a point N which belongs to line m(E_{a}M_{a}) and line m(H_{b}H_{c})
intersec N m(E_{a}M_{a}) m(H_{b}H_{c})
cmark_r N

% NDG: points E_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{a}
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```

% Choosing randomly a point  $M_{\{b\}}$  on the circle with center  $N$  through point  $E_{\{a\}}$ 
oncircle  $M_{\{b\}}$   $N$   $E_{\{a\}}$ 
cmark_lt  $M_{\{b\}}$ 
color 200 200 200
drawcircle  $N$   $E_{\{a\}}$ 
color 0 0 0

% Constructing a free point  $A$ 
point  $A$  80 95

cmark_t  $A$ 

% Constructing a point  $C$  such that  $AC/AM_{\{b\}}=2$ 
towards  $C$   $A$   $M_{\{b\}}$  2
cmark_l  $C$ 
color 200 200 200
drawsegment  $A$   $C$ 
color 0 0 0

% Constructing a line  $L_{\{G79842\}}$  which passes through point  $A$  and point  $M_{\{a\}}$ 
line  $L_{\{G79842\}}$   $A$   $M_{\{a\}}$ 

color 200 200 200
drawline  $L_{\{G79842\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G79943\}}$  with coordinates  $(0,0)$ 
point  $P_{\{G79943\}}$  0 0
cmark_r  $P_{\{G79943\}}$ 

% Constructing a point  $P_{\{G79867\}}$  such that  $AP_{\{G79867\}}/AP_{\{G79943\}}=2$ 
towards  $P_{\{G79867\}}$   $A$   $P_{\{G79943\}}$  2
cmark_r  $P_{\{G79867\}}$ 
color 200 200 200
drawsegment  $A$   $P_{\{G79867\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G79912\}}$  such that  $AP_{\{G79912\}}/AP_{\{G79943\}}=3$ 
towards  $P_{\{G79912\}}$   $A$   $P_{\{G79943\}}$  3
cmark_r  $P_{\{G79912\}}$ 
color 200 200 200
drawsegment  $A$   $P_{\{G79912\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G79873\}}$  which passes through point  $M_{\{a\}}$  and point  $P_{\{G79912\}}$ 
line  $L_{\{G79873\}}$   $M_{\{a\}}$   $P_{\{G79912\}}$ 

```

```

color 200 200 200
drawline L_{\_G79873}
color 0 0 0

% Constructing a line L_{\_G79836} which contains the point P_{\_G79867} and is parallel to the
  line L_{\_G79873}
parallel L_{\_G79836} P_{\_G79867} L_{\_G79873}

color 200 200 200
drawline L_{\_G79836}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G79836} and line L_{\_G79842}
intersec G L_{\_G79836} L_{\_G79842}
cmark_t G

% Constructing a point B such that M_{b}B/M_{b}G=3
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: points E_{a} and N are not the same; lines m(E_{a}M_{a}) and m(H_{b}H_{c})
  are not parallel
% Determination conditions: lines m(E_{a}M_{a}) and m(H_{b}H_{c}) are not the same; points E_{a}
  and M_{a} are not the same; points E_{a} and M_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

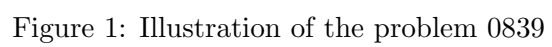
4.1.1 Proving $E_a = _E_a$

Proving failed

4.1.2 Proving $M_a = _M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 97 terms.



Time Complexity: Time spent by the prover is 1.075 seconds.

NDG conditions Points A , M_a and P_{G76115} are not collinear

Points E_a and P_{G76115} are not identical

4.1.3 Proving $M_b = M_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = E_a$

Proving failed

4.2.2 Proving $M_a = M_a$

Proving failed

4.2.3 Proving $M_b = M_b$

NDG conditions are:

$S_{M_{m(E_a M_a)}^0 E_a M_a} \neq S_{T_{m(E_a M_a)}^1 E_a M_a}$ i.e., lines $M_{m(E_a M_a)}^0 T_{m(E_a M_a)}^1$ and $E_a M_a$ are not parallel (construction based assumption)

$S_{P_{G72738} AM_a} \neq S_{P_{L_{G72707}}^2 AM_a}$ i.e., lines $P_{G72738} P_{L_{G72707}}^2$ and AM_a are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^4} \neq S_{F_{-h_a}^3 BF_{-h_b}^4}$ i.e., lines $AF_{-h_a}^3$ and $BF_{-h_b}^4$ are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_a = E_a$

Proving failed

4.3.2 Proving $M_a = M_a$

Proving failed

4.3.3 Proving $M_b = M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 28 terms.

Time Complexity: Time spent by the prover is 0.070 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = E_a$

Proving failed

4.4.2 Proving $M_a = M_a$

Proving failed

4.4.3 Proving $M_b = -M_b$

Proving failed

Problem 840

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 840: Given a point E_a , a point M_a and a point M_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point M_a , construct a line $m(H_bH_c)$ (rule W02); % DET: points E_a and M_a are not the same;
2. Using the point E_a and the point M_a , construct a line $m(E_aM_a)$ (rule W14); % DET: points E_a and M_a are not the same;
3. Using the line $m(E_aM_a)$ and the line $m(H_bH_c)$, construct a point N (rule W03); % NDG: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not parallel % DET: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not the same;
4. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
5. Choose freely a point M_c on the circle $k(N, M_a)$ (rule WOncircle);
6. Choose freely a point A (rule free);
7. Using the point A and the point M_c , construct a point B (rule W01); ;
8. Using the point A and the point M_a , construct a point G (rule W01); ;
9. Using the point M_c and the point G , construct a point C (rule W01); .

Non-degenerate conditions: points E_a and N are not the same; lines $m(E_aM_a)$ and $m(H_bH_c)$ are not parallel.

Determination conditions: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not the same; points E_a and M_a are not the same; points E_a and M_a are not the same.

Rules used: [W01,W02,W03,W06,W14,WOncircle1,free]

Lemmas used: [D20,D32,GD02,GL01,GL03,GL04,L20,L21,L22,L38,L39,L47,L48,L55,L57]

Solving time: 198.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{a} 80 83.86
point M_{a} 65 40
point M_{c} 50 67.5

color 220 0 0
fontsize 9

cmark_r E_{a}
cmark_r M_{a}
cmark_lt M_{c}
color 0 0 0
fontsize 8

% DET: points E_{a} and M_{a} are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point E_{a} and point M_{a}
line m(H_{b}H_{c}) E_{a} M_{a}

color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0

% DET: points E_{a} and M_{a} are not the same
% Constructing bisector m(E_{a}M_{a}) of the segment E_{a}M_{a}
med m(E_{a}M_{a}) E_{a} M_{a}

color 200 200 200
drawline m(E_{a}M_{a})
color 0 0 0

color 200 200 200
drawsegment E_{a} M_{a}
color 0 0 0

% NDG: lines m(E_{a}M_{a}) and m(H_{b}H_{c}) are not parallel% DET: lines m(E_{a}M_{a}) and m(H_{b}
H_{c}) are not the same
% Constructing a point N which belongs to line m(E_{a}M_{a}) and line m(H_{b}H_{c})
intersec N m(E_{a}M_{a}) m(H_{b}H_{c})
cmark_r N

% NDG: points E_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{a}
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```

% Choosing randomly a point  $M_{\{c\}}$  on the circle with center  $N$  through point  $E_{\{a\}}$ 
oncircle  $M_{\{c\}}$   $N$   $E_{\{a\}}$ 
cmark_lt  $M_{\{c\}}$ 
color 200 200 200
drawcircle  $N$   $E_{\{a\}}$ 
color 0 0 0

% Constructing a free point  $A$ 
point  $A$  80 95

cmark_t  $A$ 

% Constructing a point  $B$  such that  $AB/AM_{\{c\}}=2$ 
towards  $B$   $A$   $M_{\{c\}}$  2
cmark_b  $B$ 
color 200 200 200
drawsegment  $A$   $B$ 
color 0 0 0

% Constructing a line  $L_{\{G104210\}}$  which passes through point  $A$  and point  $M_{\{a\}}$ 
line  $L_{\{G104210\}}$   $A$   $M_{\{a\}}$ 

color 200 200 200
drawline  $L_{\{G104210\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G104311\}}$  with coordinates  $(0,0)$ 
point  $P_{\{G104311\}}$  0 0
cmark_r  $P_{\{G104311\}}$ 

% Constructing a point  $P_{\{G104235\}}$  such that  $AP_{\{G104235\}}/AP_{\{G104311\}}=2$ 
towards  $P_{\{G104235\}}$   $A$   $P_{\{G104311\}}$  2
cmark_r  $P_{\{G104235\}}$ 
color 200 200 200
drawsegment  $A$   $P_{\{G104235\}}$ 
color 0 0 0

% Constructing a point  $P_{\{G104280\}}$  such that  $AP_{\{G104280\}}/AP_{\{G104311\}}=3$ 
towards  $P_{\{G104280\}}$   $A$   $P_{\{G104311\}}$  3
cmark_r  $P_{\{G104280\}}$ 
color 200 200 200
drawsegment  $A$   $P_{\{G104280\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G104241\}}$  which passes through point  $M_{\{a\}}$  and point  $P_{\{G104280\}}$ 
line  $L_{\{G104241\}}$   $M_{\{a\}}$   $P_{\{G104280\}}$ 

```

```

color 200 200 200
drawline L_{\_G104241}
color 0 0 0

% Constructing a line L_{\_G104204} which contains the point P_{\_G104235} and is parallel to the
  line L_{\_G104241}
parallel L_{\_G104204} P_{\_G104235} L_{\_G104241}

color 200 200 200
drawline L_{\_G104204}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G104204} and line L_{\_G104210}
intersec G L_{\_G104204} L_{\_G104210}
cmark_t G

% Constructing a point C such that M_{c}C/M_{c}G=3
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: points E_{a} and N are not the same; lines m(E_{a}M_{a}) and m(H_{b}H_{c})
  are not parallel
% Determination conditions: lines m(E_{a}M_{a}) and m(H_{b}H_{c}) are not the same; points E_{a}
  and M_{a} are not the same; points E_{a} and M_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = _E_a$

Proving failed

4.1.2 Proving $M_a = _M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 97 terms.

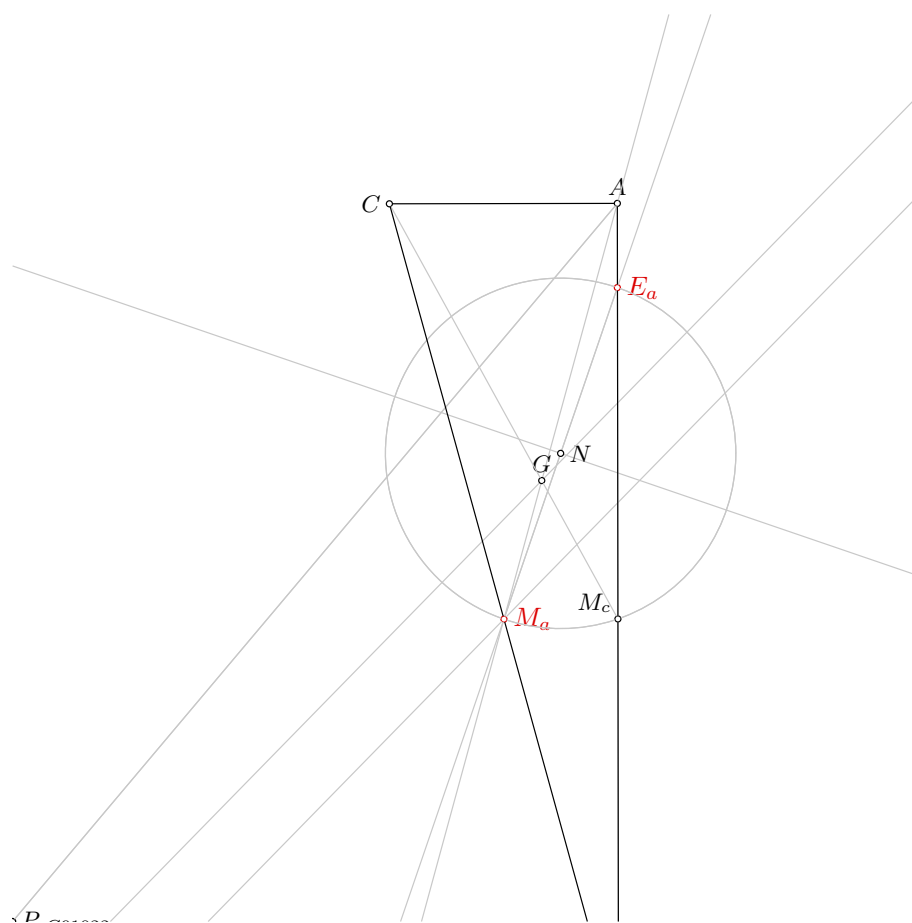


Figure 1: Illustration of the problem 0840

Time Complexity: Time spent by the prover is 1.021 seconds.

NDG conditions Points $P_{G100483}$, A and M_a are not collinear

Points $P_{G100483}$, N and E_a are not collinear

4.1.3 Proving $M_c = M_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = E_a$

Proving failed

4.2.2 Proving $M_a = M_a$

Proving failed

4.2.3 Proving $M_c = M_c$

NDG conditions are:

$S_{M_{m(E_a M_a)}^0 E_a M_a} \neq S_{T_{m(E_a M_a)}^1 E_a M_a}$ i.e., lines $M_{m(E_a M_a)}^0 T_{m(E_a M_a)}^1$ and $E_a M_a$ are not parallel (construction based assumption)

$S_{P_{G97106} A M_a} \neq S_{P_{L_{G97075}}^2 A M_a}$ i.e., lines $P_{G97106} P_{L_{G97075}}^2$ and $A M_a$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^4} \neq S_{F_{-h_a}^3 BF_{-h_b}^4}$ i.e., lines $AF_{-h_a}^3$ and $BF_{-h_b}^4$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_a = E_a$

Proving failed

4.3.2 Proving $M_a = M_a$

Proving failed

4.3.3 Proving $M_c = M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 28 terms.

Time Complexity: Time spent by the prover is 0.070 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = E_a$

Proving failed

4.4.2 Proving $M_a = M_a$

Proving failed

4.4.3 Proving $M_c = -M_c$

Proving failed

Problem 841

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 841: Given a point M_a , a point N and a point E_a , construct the triangle ABC .

2 Status

Problem is redundant.

3 Solution

3.1 Construction in natural language form

1. Using the point M_a and the point N , construct a line $m(H_bH_c)$ (rule W02); % DET: points M_a and N are not the same;
2. Using the point M_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_a and N are not the same;
3. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point M_a , construct a point E_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_a and E_a must be different;
4. Choose freely a point A (rule free);
5. Using the point A and the point M_a , construct a point G (rule W01); ;
6. Using the point N and the point G , construct a point O (rule W01); ;
7. Using the point A and the point E_a , construct a line h_a (rule W02); % DET: points A and E_a are not the same;
8. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
9. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
10. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;

11. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); %
NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; line h_a and circle $k(N, M_a)$ intersect; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; points M_a and N are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points A and E_a are not the same; points M_a and E_a must be different; points M_a and N are not the same.

Rules used: [W01,W02,W04,W05,W05a,W06,free]

Lemmas used: [D21,D26,D28,D3,D32,D5,D8,GD01,GD02,GL01,GL03,GL09,L11,L12,L15,L19,L20,L21,L22,L38]

Solving time: 6.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40
point N 72.5 61.93
point E_{a} 80 83.86
```

```
color 220 0 0
fontsize 9
```

```
cmark_r M_{a}
cmark_r N
cmark_r E_{a}
color 0 0 0
fontsize 8
```

```
% DET: points M_{a} and N are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point M_{a} and point N
line m(H_{b}H_{c}) M_{a} N
```

```
color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0
```

```
% NDG: points M_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{a}
circle k(N,M_{a}) N M_{a}
```

```
color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```
% NDG: line m(H_{b}H_{c}) and circle k(N,M_{a}) intersect% DET: points M_{a} and E_{a} must be
different
% Constructing a point E_{a} which is an image of the point M_{a} in the symmetry to point/line N
```



```

sim E_{a} N M_{a}
cmark_r E_{a}

% Constructing a free point A
point A 80 95

cmark_t A

% Constructing a line L_{\_G140096} which passes through point A and point M_{a}
line L_{\_G140096} A M_{a}

color 200 200 200
drawline L_{\_G140096}
color 0 0 0

% Constructing a point P_{\_G140197} with coordinates (0,0)
point P_{\_G140197} 0 0
cmark_r P_{\_G140197}

% Constructing a point P_{\_G140121} such that  $AP_{\_G140121}/AP_{\_G140197}=2$ 
towards P_{\_G140121} A P_{\_G140197} 2
cmark_r P_{\_G140121}
color 200 200 200
drawsegment A P_{\_G140121}
color 0 0 0

% Constructing a point P_{\_G140166} such that  $AP_{\_G140166}/AP_{\_G140197}=3$ 
towards P_{\_G140166} A P_{\_G140197} 3
cmark_r P_{\_G140166}
color 200 200 200
drawsegment A P_{\_G140166}
color 0 0 0

% Constructing a line L_{\_G140127} which passes through point M_{a} and point P_{\_G140166}
line L_{\_G140127} M_{a} P_{\_G140166}

color 200 200 200
drawline L_{\_G140127}
color 0 0 0

% Constructing a line L_{\_G140090} which contains the point P_{\_G140121} and is parallel to the
line L_{\_G140127}
parallel L_{\_G140090} P_{\_G140121} L_{\_G140127}

color 200 200 200
drawline L_{\_G140090}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G140090} and line L_{\_G140096}

```

```

intersec G L_{\_G140090} L_{\_G140096}
cmark_t G

```

```

% Constructing a point O such that NO/NG=3
towards O N G 3
cmark_t O
color 200 200 200
drawsegment N O
color 0 0 0

```

```

% DET: points A and E_{a} are not the same
% Constructing a line h_{a} which passes through point A and point E_{a}
line h_{a} A E_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

```

```

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G140973} which is a foot of the point N on the line h_{a}
foot P_{\_G140973} N h_{a}
cmark_r P_{\_G140973}
color 200 200 200
drawline N P_{\_G140973}
color 0 0 0

```

```

% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
_{\_G140973}
sim H_{a} P_{\_G140973} E_{a}
cmark_r H_{a}

```

```

% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0

```

```

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

```

```

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% line h_{a} and circle k(N,M_{a}) intersect; line m(H_{b}H_{c}) and circle k(N,M_{a}) intersect;
% points M_{a} and N are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
% different; points A and E_{a} are not the same; points M_{a} and E_{a} must be different;
% points M_{a} and N are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_a = _M_a$

Proving failed

4.1.2 Proving $N = _N$

Proving failed

4.1.3 Proving $E_a = _E_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $M_a = _M_a$

Proving failed

4.2.2 Proving $N = _N$

Proving failed

4.2.3 Proving $E_a = _E_a$

Proving failed

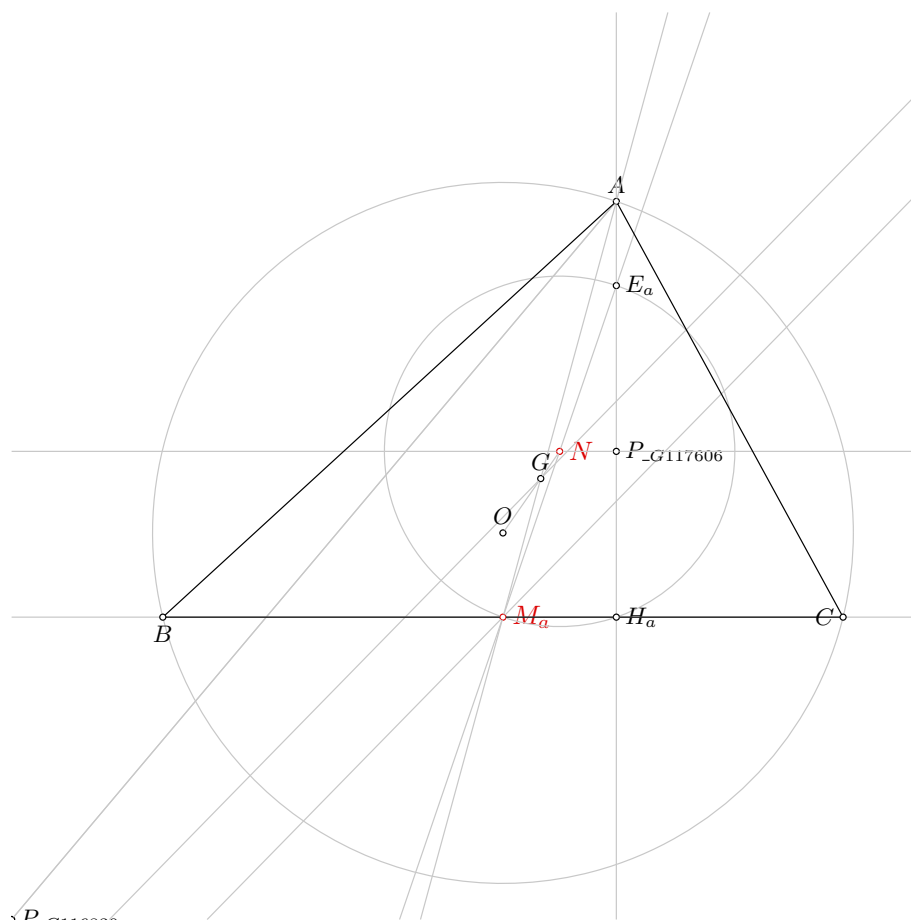


Figure 1: Illustration of the problem 0841

4.3 GCLC - Wu method

4.3.1 Proving $M_a = \neg M_a$

Proving failed

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $E_a = \neg E_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_a = \neg M_a$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $E_a = \neg E_a$

Proving failed

Problem 842

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 842: Given a point E_a , a point M_a and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point M_a , construct a line $m(H_bH_c)$ (rule W02); % DET: points E_a and M_a are not the same;
2. Using the point M_a and the point O , construct a line m_a (rule W02); % DET: points M_a and O are not the same;
3. Using the point M_a and the line m_a , construct a line a (rule W10a); ;
4. Using the point E_a and the point M_a , construct a line $m(E_aM_a)$ (rule W14); % DET: points E_a and M_a are not the same;
5. Using the line $m(E_aM_a)$ and the line $m(H_bH_c)$, construct a point N (rule W03); % NDG: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not parallel % DET: lines $m(E_aM_a)$ and $m(H_bH_c)$ are not the same;
6. Using the point N and the point O , construct a point G (rule W01); ;
7. Using the point M_a and the point G , construct a point A (rule W01); ;
8. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
9. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; lines $m(E_aM_a)$ and $m(H_bH_c)$ are not parallel.

Determination conditions: lines $m(E_a M_a)$ and $m(H_b H_c)$ are not the same; points E_a and M_a are not the same; points M_a and O are not the same; points E_a and M_a are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W14]

Lemmas used: [D1,D11,D21,D26,D32,GD01,GD02,GL01,GL03,GL04,GL09,L11,L12,L15,L20,L21,L22,L38,L39,

Solving time: 6.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
point M_{a} 65 40
point O 65 51.14
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{a}
cmark_r M_{a}
cmark_t O
color 0 0 0
fontsize 8
```

```
% DET: points E_{a} and M_{a} are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point E_{a} and point M_{a}
line m(H_{b}H_{c}) E_{a} M_{a}
```

```
color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0
```

```
% DET: points M_{a} and O are not the same
% Constructing a line m_{a} which passes through point M_{a} and point O
line m_{a} M_{a} O
```

```
color 200 200 200
drawline m_{a}
color 0 0 0
```

```
% Constructing a line a which is perpendicular to line m_{a} and which passes through point M_{a}
perp a M_{a} m_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% DET: points E_{a} and M_{a} are not the same
```

```

% Constructing bisector  $m(E_{\{a\}M_{\{a\}}})$  of the segment  $E_{\{a\}M_{\{a\}}}$ 
med m( $E_{\{a\}M_{\{a\}}}$ )  $E_{\{a\}M_{\{a\}}}$ 

color 200 200 200
drawline m( $E_{\{a\}M_{\{a\}}}$ )
color 0 0 0

color 200 200 200
drawsegment  $E_{\{a\}M_{\{a\}}}$ 
color 0 0 0

% NDG: lines  $m(E_{\{a\}M_{\{a\}}})$  and  $m(H_{\{b\}H_{\{c\}}})$  are not parallel% DET: lines  $m(E_{\{a\}M_{\{a\}}})$  and  $m(H_{\{b\}H_{\{c\}}})$ 
%  $H_{\{c\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{a\}M_{\{a\}}})$  and line  $m(H_{\{b\}H_{\{c\}}})$ 
intersec N m( $E_{\{a\}M_{\{a\}}}$ ) m( $H_{\{b\}H_{\{c\}}}$ )
cmark_r N

% Constructing a line  $L_{\{\_G170833\}}$  which passes through point  $N$  and point  $O$ 
line  $L_{\{\_G170833\}}$  N O

color 200 200 200
drawline  $L_{\{\_G170833\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\_G170934\}}$  with coordinates  $(0,0)$ 
point  $P_{\{\_G170934\}}$  0 0
cmark_r  $P_{\{\_G170934\}}$ 

% Constructing a point  $P_{\{\_G170858\}}$  such that  $NP_{\{\_G170858\}}/NP_{\{\_G170934\}}=1$ 
towards  $P_{\{\_G170858\}}$  N  $P_{\{\_G170934\}}$  1
cmark_r  $P_{\{\_G170858\}}$ 
color 200 200 200
drawsegment N  $P_{\{\_G170858\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\_G170903\}}$  such that  $NP_{\{\_G170903\}}/NP_{\{\_G170934\}}=3$ 
towards  $P_{\{\_G170903\}}$  N  $P_{\{\_G170934\}}$  3
cmark_r  $P_{\{\_G170903\}}$ 
color 200 200 200
drawsegment N  $P_{\{\_G170903\}}$ 
color 0 0 0

% Constructing a line  $L_{\{\_G170864\}}$  which passes through point  $O$  and point  $P_{\{\_G170903\}}$ 
line  $L_{\{\_G170864\}}$  O  $P_{\{\_G170903\}}$ 

color 200 200 200
drawline  $L_{\{\_G170864\}}$ 
color 0 0 0

% Constructing a line  $L_{\{\_G170827\}}$  which contains the point  $P_{\{\_G170858\}}$  and is parallel to the
line  $L_{\{\_G170864\}}$ 

```



```

parallel L_{\_G170827} P_{\_G170858} L_{\_G170864}

color 200 200 200
drawline L_{\_G170827}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G170827} and line L_{\_G170833}
intersec G L_{\_G170827} L_{\_G170833}
cmark_t G

% Constructing a point A such that  $M_{\{a\}}A/M_{\{a\}}G=3$ 
towards A M_{\{a\}} G 3
cmark_t A
color 200 200 200
drawsegment M_{\{a\}} A
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle  $k(O,C)$  intersect
% Constructing points C and B which are in intersection of  $k(O,C)$  and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle  $k(O,C)$  intersect; points A and O are not the same;
% lines  $m(E_{\{a\}}M_{\{a\}})$  and  $m(H_{\{b\}}H_{\{c\}})$  are not parallel
% Determination conditions: lines  $m(E_{\{a\}}M_{\{a\}})$  and  $m(H_{\{b\}}H_{\{c\}})$  are not the same; points  $E_{\{a\}}$ 
% and  $M_{\{a\}}$  are not the same; points  $M_{\{a\}}$  and O are not the same; points  $E_{\{a\}}$  and  $M_{\{a\}}$  are not
% the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

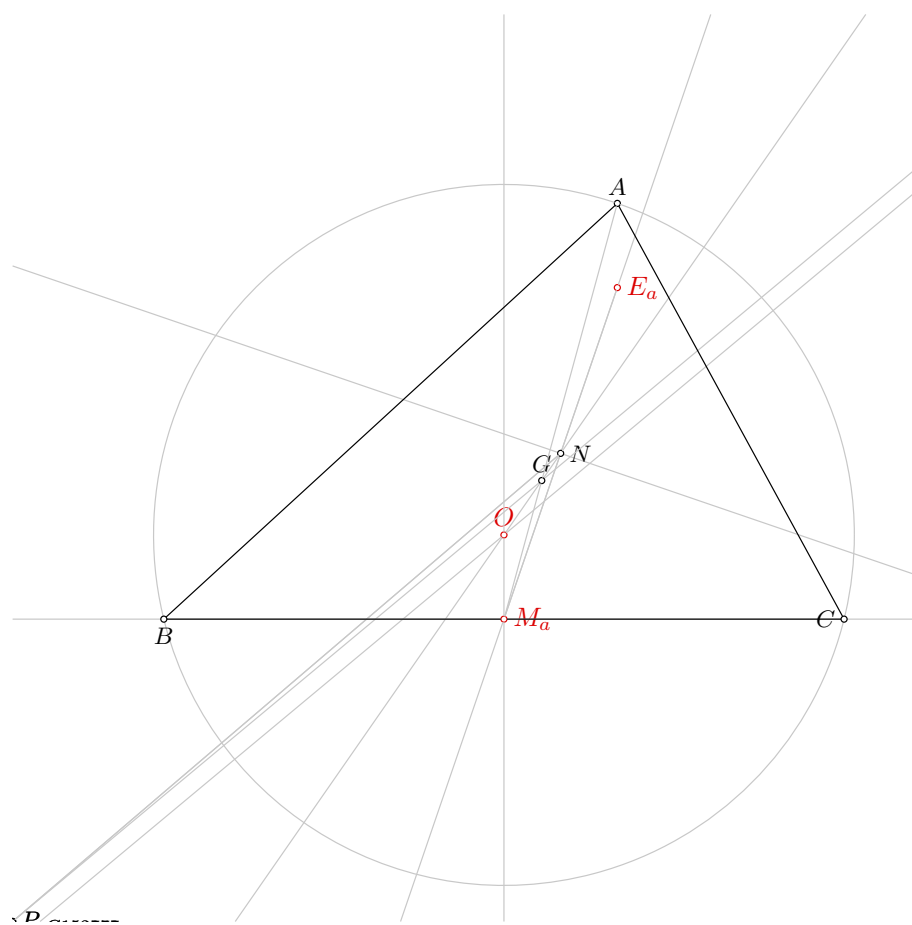


Figure 1: Illustration of the problem 0842

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $M_a = \neg M_a$

Proving failed

4.1.3 Proving $O = \neg O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $M_a = \neg M_a$

Proving failed

4.2.3 Proving $O = \neg O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = \neg E_a$

Proving failed

4.3.2 Proving $M_a = \neg M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 923 terms.

Time Complexity: Time spent by the prover is 1.990 seconds. There are no ndg conditions.

4.3.3 Proving $O = \neg O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $M_a = \neg M_a$

Proving failed

4.4.3 Proving $O=_O$

Proving failed

Problem 843

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 843: Given a point E_a , a point M_a and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 844

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 844: Given a point E_a , a point M_a and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 845

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 845: Given a point E_a , a point M_a and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 846

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 846: Given a point E_a , a point M_b and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point M_b , construct a line $m(E_aM_b)$ (rule W14); % DET: points E_a and M_b are not the same;
2. Using the point E_a and the point M_c , construct a line $m(E_aM_c)$ (rule W14); % DET: points E_a and M_c are not the same;
3. Using the line $m(E_aM_c)$ and the line $m(E_aM_b)$, construct a point N (rule W03); % NDG: lines $m(E_aM_c)$ and $m(E_aM_b)$ are not parallel % DET: lines $m(E_aM_c)$ and $m(E_aM_b)$ are not the same;
4. Using the point M_b and the point N , construct a line $m(H_aH_c)$ (rule W02); % DET: points M_b and N are not the same;
5. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
6. Using the circle $k(N, M_a)$, the line $m(H_aH_c)$, the point N and the point M_b , construct a point E_b (rule W05a); % NDG: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_b and E_b must be different;
7. Using the point M_c and the point E_b , construct a line $m(BH_a)$ (rule W02); % DET: points M_c and E_b are not the same;
8. Using the point E_a and the line $m(BH_a)$, construct a line h_a (rule W16); ;
9. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;

10. Using the point H_a and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points H_a and M_b are not the same;
11. Using the circle $k(M_b, C)$, the line h_a , the point M_b and the point H_a , construct a point A (rule W05); % NDG: line h_a and circle $k(M_b, C)$ intersect % DET: points H_a and A must be different;
12. Using the point M_b and the point A , construct a point C (rule W01); ;
13. Using the point M_c and the point A , construct a point B (rule W01); .

Non-degenerate conditions: line h_a and circle $k(M_b, C)$ intersect; points H_a and M_b are not the same; line h_a and circle $k(N, M_a)$ intersect; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; points E_a and N are not the same; lines $m(E_a M_c)$ and $m(E_a M_b)$ are not parallel.

Determination conditions: points H_a and A must be different; points E_a and H_a must be different; points M_c and E_b are not the same; points M_b and E_b must be different; points M_b and N are not the same; lines $m(E_a M_c)$ and $m(E_a M_b)$ are not the same; points E_a and M_c are not the same; points E_a and M_b are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W14,W16]

Lemmas used: [D20,D22,D28,D3,D32,D5,D8,GD02,GL01,GL03,GL04,GL09,L17,L18,L19,L21,L22,L23,L40,L42]

Solving time: 25.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point M_{b} 95 67.5
```

```
point M_{c} 50 67.5
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_lt M_{b}
```

```
cmark_lt M_{c}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{a} and M_{b} are not the same
```

```
% Constructing bisector m(E_{a}M_{b}) of the segment E_{a}M_{b}
```

```
med m(E_{a}M_{b}) E_{a} M_{b}
```

```
color 200 200 200
```

```
drawline m(E_{a}M_{b})
```

```
color 0 0 0
```

```
color 200 200 200
```

```
drawsegment E_{a} M_{b}
```

```
color 0 0 0
```

```

% DET: points  $E_{\{a\}}$  and  $M_{\{c\}}$  are not the same
% Constructing bisector  $m(E_{\{a\}}M_{\{c\}})$  of the segment  $E_{\{a\}}M_{\{c\}}$ 
med m( $E_{\{a\}}M_{\{c\}}$ )  $E_{\{a\}}$   $M_{\{c\}}$ 

color 200 200 200
drawline m( $E_{\{a\}}M_{\{c\}}$ )
color 0 0 0

color 200 200 200
drawsegment  $E_{\{a\}}$   $M_{\{c\}}$ 
color 0 0 0

% NDG: lines  $m(E_{\{a\}}M_{\{c\}})$  and  $m(E_{\{a\}}M_{\{b\}})$  are not parallel% DET: lines  $m(E_{\{a\}}M_{\{c\}})$  and  $m(E_{\{a\}}$ 
 $M_{\{b\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{a\}}M_{\{c\}})$  and line  $m(E_{\{a\}}M_{\{b\}})$ 
intersec N m( $E_{\{a\}}M_{\{c\}}$ ) m( $E_{\{a\}}M_{\{b\}}$ )
cmark_r N

% DET: points  $M_{\{b\}}$  and  $N$  are not the same
% Constructing a line  $m(H_{\{a\}}H_{\{c\}})$  which passes through point  $M_{\{b\}}$  and point  $N$ 
line m( $H_{\{a\}}H_{\{c\}}$ )  $M_{\{b\}}$  N

color 200 200 200
drawline m( $H_{\{a\}}H_{\{c\}}$ )
color 0 0 0

% NDG: points  $E_{\{a\}}$  and  $N$  are not the same
% Constructing a circle  $k(N,M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{a\}}$ 
circle k( $N,M_{\{a\}}$ ) N  $E_{\{a\}}$ 

color 200 200 200
drawcircle k( $N,M_{\{a\}}$ )
color 0 0 0

% NDG: line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $M_{\{b\}}$  and  $E_{\{b\}}$  must be
different
% Constructing a point  $E_{\{b\}}$  which is an image of the point  $M_{\{b\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{b\}}$  N  $M_{\{b\}}$ 
cmark_r  $E_{\{b\}}$ 

% DET: points  $M_{\{c\}}$  and  $E_{\{b\}}$  are not the same
% Constructing a line  $m(BH_{\{a\}})$  which passes through point  $M_{\{c\}}$  and point  $E_{\{b\}}$ 
line m( $BH_{\{a\}}$ )  $M_{\{c\}}$   $E_{\{b\}}$ 

color 200 200 200
drawline m( $BH_{\{a\}}$ )
color 0 0 0

```

```

% Constructing a line  $h_{\{a\}}$  which contains the point  $E_{\{a\}}$  and is parallel to the line  $m(BH_{\{a\}})$ 
parallel  $h_{\{a\}}$   $E_{\{a\}}$   $m(BH_{\{a\}})$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: line  $h_{\{a\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{a\}}$  and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\_G217692\}}$  which is a foot of the point  $N$  on the line  $h_{\{a\}}$ 
foot  $P_{\{\_G217692\}}$   $N$   $h_{\{a\}}$ 
cmark_r  $P_{\{\_G217692\}}$ 
color 200 200 200
drawline  $N$   $P_{\{\_G217692\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $E_{\{a\}}$  in the symmetry to point/line  $P_{\{\_G217692\}}$ 
sim  $H_{\{a\}}$   $P_{\{\_G217692\}}$   $E_{\{a\}}$ 
cmark_r  $H_{\{a\}}$ 

% NDG: points  $H_{\{a\}}$  and  $M_{\{b\}}$  are not the same
% Constructing a circle  $k(M_{\{b\}}, C)$  whose center is at point  $M_{\{b\}}$  and which passes through point  $H_{\{a\}}$ 
circle  $k(M_{\{b\}}, C)$   $M_{\{b\}}$   $H_{\{a\}}$ 

color 200 200 200
drawcircle  $k(M_{\{b\}}, C)$ 
color 0 0 0

% NDG: line  $h_{\{a\}}$  and circle  $k(M_{\{b\}}, C)$  intersect% DET: points  $H_{\{a\}}$  and  $A$  must be different
% Constructing a point  $P_{\{\_G217996\}}$  which is a foot of the point  $M_{\{b\}}$  on the line  $h_{\{a\}}$ 
foot  $P_{\{\_G217996\}}$   $M_{\{b\}}$   $h_{\{a\}}$ 
cmark_r  $P_{\{\_G217996\}}$ 
color 200 200 200
drawline  $M_{\{b\}}$   $P_{\{\_G217996\}}$ 
color 0 0 0

% Constructing a point  $A$  which is an image of the point  $H_{\{a\}}$  in the symmetry to point/line  $P_{\{\_G217996\}}$ 
sim  $A$   $P_{\{\_G217996\}}$   $H_{\{a\}}$ 
cmark_t  $A$ 

% Constructing a point  $C$  such that  $M_{\{b\}}C/M_{\{b\}}A = -1$ 
towards  $C$   $M_{\{b\}}$   $A$  -1
cmark_l  $C$ 
color 200 200 200
drawsegment  $A$   $C$ 

```

```
color 0 0 0
```

```
% Constructing a point B such that  $M_{\{c\}}B/M_{\{c\}}A=-1$ 
towards B  $M_{\{c\}}$  A -1
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line  $h_{\{a\}}$  and circle  $k(M_{\{b\}}, C)$  intersect; points  $H_{\{a\}}$  and  $M_{\{b\}}$  are
not the same; line  $h_{\{a\}}$  and circle  $k(N, M_{\{a\}})$  intersect; line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$ 
intersect; points  $E_{\{a\}}$  and  $N$  are not the same; lines  $m(E_{\{a\}}M_{\{c\}})$  and  $m(E_{\{a\}}M_{\{b\}})$  are
not parallel
% Determination conditions: points  $H_{\{a\}}$  and  $A$  must be different; points  $E_{\{a\}}$  and  $H_{\{a\}}$  must be
different; points  $M_{\{c\}}$  and  $E_{\{b\}}$  are not the same; points  $M_{\{b\}}$  and  $E_{\{b\}}$  must be different;
points  $M_{\{b\}}$  and  $N$  are not the same; lines  $m(E_{\{a\}}M_{\{c\}})$  and  $m(E_{\{a\}}M_{\{b\}})$  are not the same;
points  $E_{\{a\}}$  and  $M_{\{c\}}$  are not the same; points  $E_{\{a\}}$  and  $M_{\{b\}}$  are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = -E_a$

Proving failed

4.1.2 Proving $M_b = -M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 9 terms.

Time Complexity: Time spent by the prover is 0.308 seconds.

NDG conditions Points M_b and M_c are not identical

Points M_b and M_c are not identical

Line through points M_b and E_a is not parallel with line through points M_c and E_b

Points tempPoint-256 h_a and E_a are not identical

Points M_b and tempPoint-256 h_a are not identical

Points $P_{G212710}$ and E_a are not identical

Points $P_{G212710}$ and E_a are not identical

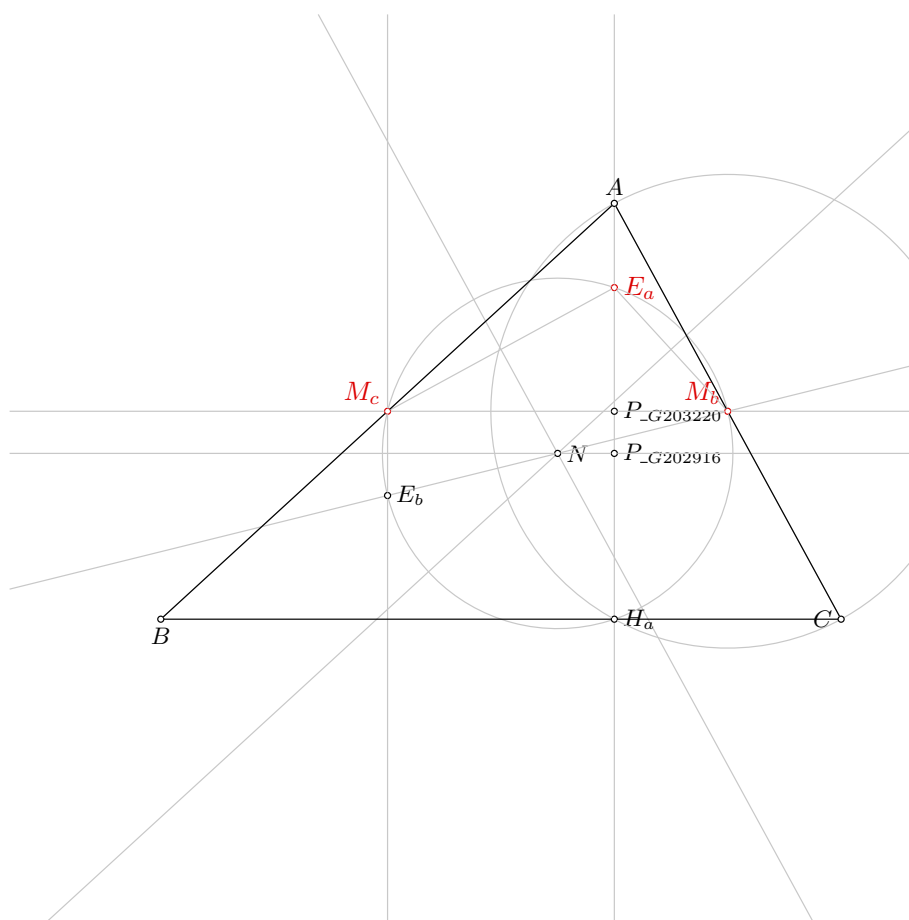


Figure 1: Illustration of the problem 0846

4.1.3 Proving $M_c = M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 11 terms.

Time Complexity: Time spent by the prover is 0.28 seconds.

NDG conditions Points M_b and M_c are not identical

Points M_b and M_c are not identical

Line through points M_b and E_a is not parallel with line through points M_c and E_b

Points tempPoint-76 h_a and E_a are not identical

Points M_b and tempPoint-76 h_a are not identical

Points E_a and $P_{G214046}$ are not identical

Points M_b , E_a and $P_{G214046}$ are not collinear

4.2 GCLC - Area method

4.2.1 Proving $E_a = E_a$

Proving failed

4.2.2 Proving $M_b = M_b$

NDG conditions are:

$S_{M^2_{m(E_a M_c)} M^0_{m(E_a M_b)} T^1_{m(E_a M_b)}} \neq S_{T^3_{m(E_a M_c)} M^0_{m(E_a M_b)} T^1_{m(E_a M_b)}}$ i.e., lines $M^2_{m(E_a M_c)} T^3_{m(E_a M_c)}$ and $M^0_{m(E_a M_b)} T^1_{m(E_a M_b)}$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^6_{h_b}} \neq S_{F^5_{h_a} BF^6_{h_b}}$ i.e., lines $AF^5_{h_a}$ and $BF^6_{h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.2.3 Proving $M_c = M_c$

NDG conditions are:

$S_{M^2_{m(E_a M_c)} M^0_{m(E_a M_b)} T^1_{m(E_a M_b)}} \neq S_{T^3_{m(E_a M_c)} M^0_{m(E_a M_b)} T^1_{m(E_a M_b)}}$ i.e., lines $M^2_{m(E_a M_c)} T^3_{m(E_a M_c)}$ and $M^0_{m(E_a M_b)} T^1_{m(E_a M_b)}$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^6_{h_b}} \neq S_{F^5_{h_a} BF^6_{h_b}}$ i.e., lines $AF^5_{h_a}$ and $BF^6_{h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_a = E_a$

Proving failed

4.3.2 Proving $M_b = M_b$

Proving failed

4.3.3 Proving $M_c = M_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = \neg E_a$

Proving failed

4.4.2 Proving $M_b = \neg M_b$

Proving failed

4.4.3 Proving $M_c = \neg M_c$

Proving failed

Problem 847

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 847: Given a point M_b , a point N and a point E_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point M_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_b and N are not the same;
2. Choose freely a point E_a on the circle $k(N, M_a)$ (rule WOncircle);
3. Using the point E_a and the point N , construct a line $m(H_b H_c)$ (rule W02); % DET: points E_a and N are not the same;
4. Using the circle $k(N, M_a)$, the line $m(H_b H_c)$, the point N and the point E_a , construct a point M_a (rule W05a); % NDG: line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect % DET: points E_a and M_a must be different;
5. Choose freely a point A (rule free);
6. Using the point A and the point M_b , construct a point C (rule W01); ;
7. Using the point A and the point M_a , construct a point G (rule W01); ;
8. Using the point M_b and the point G , construct a point B (rule W01); .

Non-degenerate conditions: line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; points M_b and N are not the same.

Determination conditions: points E_a and M_a must be different; points E_a and N are not the same.

Rules used: [W01,W02,W05a,W06,WOncircle1,free]

Lemmas used: [D22,D32,GD02,GL01,GL03,GL04,L17,L20,L21,L22,L38,L39,L47,L48,L55,L56]

Solving time: 149.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point N 72.5 61.93
point E_{a} 80 83.86

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_r N
cmark_r E_{a}
color 0 0 0
fontsize 8

% NDG: points M_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{b}
circle k(N,M_{a}) N M_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point E_{a} on the circle with center N through point M_{b}
oncircle E_{a} N M_{b}
cmark_r E_{a}
color 200 200 200
drawcircle N M_{b}
color 0 0 0

% DET: points E_{a} and N are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point E_{a} and point N
line m(H_{b}H_{c}) E_{a} N

color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0

% NDG: line m(H_{b}H_{c}) and circle k(N,M_{a}) intersect% DET: points E_{a} and M_{a} must be
different
% Constructing a point M_{a} which is an image of the point E_{a} in the symmetry to point/line N
sim M_{a} N E_{a}
cmark_r M_{a}

% Constructing a free point A
point A 80 95
```

```
cmark_t A
```

```
% Constructing a point C such that  $AC/AM_{\{b\}}=2$   
towards C A  $M_{\{b\}}$  2  
cmark_l C  
color 200 200 200  
drawsegment A C  
color 0 0 0
```

```
% Constructing a line  $L_{\{G249580\}}$  which passes through point A and point  $M_{\{a\}}$   
line  $L_{\{G249580\}}$  A  $M_{\{a\}}$   
  
color 200 200 200  
drawline  $L_{\{G249580\}}$   
color 0 0 0
```

```
% Constructing a point  $P_{\{G249681\}}$  with coordinates (0,0)  
point  $P_{\{G249681\}}$  0 0  
cmark_r  $P_{\{G249681\}}$ 
```

```
% Constructing a point  $P_{\{G249605\}}$  such that  $AP_{\{G249605\}}/AP_{\{G249681\}}=2$   
towards  $P_{\{G249605\}}$  A  $P_{\{G249681\}}$  2  
cmark_r  $P_{\{G249605\}}$   
color 200 200 200  
drawsegment A  $P_{\{G249605\}}$   
color 0 0 0
```

```
% Constructing a point  $P_{\{G249650\}}$  such that  $AP_{\{G249650\}}/AP_{\{G249681\}}=3$   
towards  $P_{\{G249650\}}$  A  $P_{\{G249681\}}$  3  
cmark_r  $P_{\{G249650\}}$   
color 200 200 200  
drawsegment A  $P_{\{G249650\}}$   
color 0 0 0
```

```
% Constructing a line  $L_{\{G249611\}}$  which passes through point  $M_{\{a\}}$  and point  $P_{\{G249650\}}$   
line  $L_{\{G249611\}}$   $M_{\{a\}}$   $P_{\{G249650\}}$   
  
color 200 200 200  
drawline  $L_{\{G249611\}}$   
color 0 0 0
```

```
% Constructing a line  $L_{\{G249574\}}$  which contains the point  $P_{\{G249605\}}$  and is parallel to the  
line  $L_{\{G249611\}}$   
parallel  $L_{\{G249574\}}$   $P_{\{G249605\}}$   $L_{\{G249611\}}$   
  
color 200 200 200  
drawline  $L_{\{G249574\}}$   
color 0 0 0
```

```

% Constructing a point G which belongs to line L_{\_G249574} and line L_{\_G249580}
intersec G L_{\_G249574} L_{\_G249580}
cmark_t G

% Constructing a point B such that M_{b}B/M_{b}G=3
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line m(H_{b}H_{c}) and circle k(N,M_{a}) intersect; points M_{b} and N
% are not the same
% Determination conditions: points E_{a} and M_{a} must be different; points E_{a} and N are not
% the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_b = _M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.032 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $N = _N$

Proving failed

4.1.3 Proving $E_a = _E_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $M_b = _M_b$

NDG conditions are:

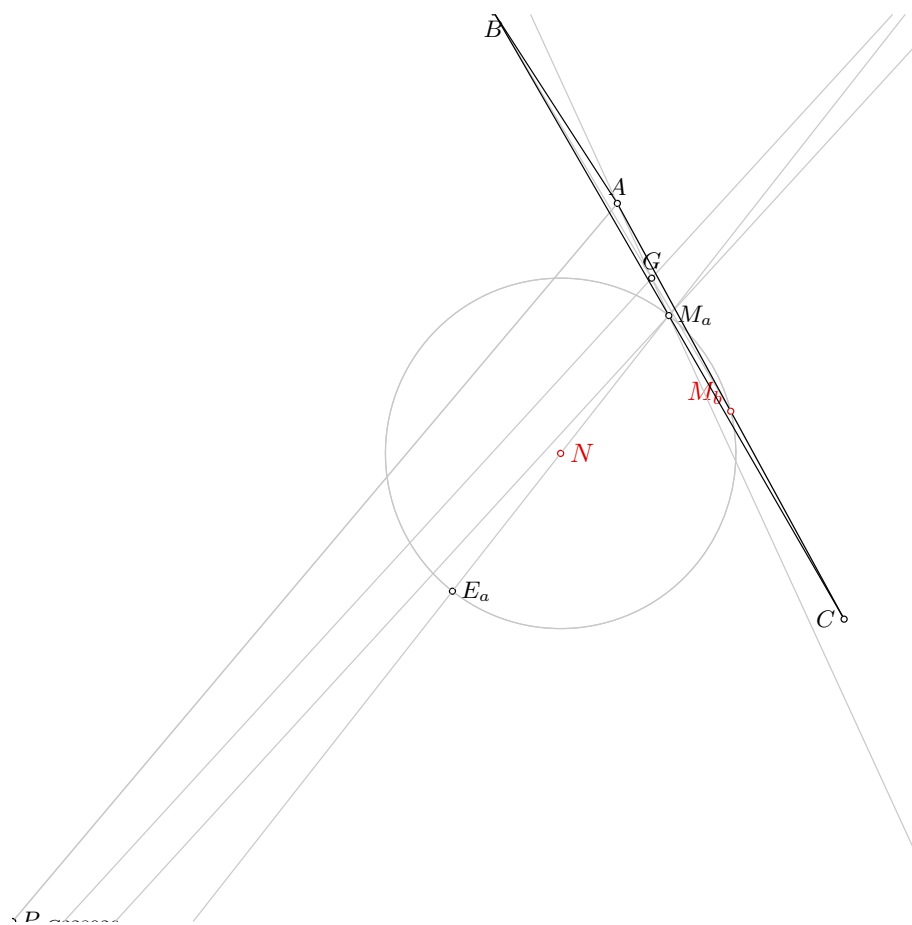


Figure 1: Illustration of the problem 0847

$S_{P_{G229195}AM_a} \neq S_{P_{L_{G229164}}^0 AM_a}$ i.e., lines $P_{G229195}P_{L_{G229164}}^0$ and AM_a are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points M_a, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points M_b, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^4} \neq S_{F_{-h_a}^3 BF_{-h_b}^4}$ i.e., lines $AF_{-h_a}^3$ and $BF_{-h_b}^4$ are not parallel (construction based assumption)

$S_{M_aM_bF_{-m_b}^2} \neq S_{F_{-m_a}^1 M_bF_{-m_b}^2}$ i.e., lines $M_aF_{-m_a}^1$ and $M_bF_{-m_b}^2$ are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $E_a=_Ea$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_b=_M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.050 seconds. There are no ndg conditions.

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $E_a=_Ea$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_b=_M_b$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $E_a=_Ea$

Proving failed

Problem 848

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 848: Given a point E_a , a point M_b and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point M_b , construct a line $m(AH_c)$ (rule W02); % DET: points E_a and M_b are not the same;
2. Using the point M_b and the point O , construct a line m_b (rule W02); % DET: points M_b and O are not the same;
3. Using the point M_b and the line m_b , construct a line b (rule W10a); ;
4. Using the point E_a and the line m_b , construct a line $m(AH_b)$ (rule W16); ;
5. Using the point O and the line $m(AH_c)$, construct a line m_c (rule W16); ;
6. Using the line m_c and the line $m(AH_b)$, construct a point M_c (rule W03); % NDG: lines m_c and $m(AH_b)$ are not parallel % DET: lines m_c and $m(AH_b)$ are not the same;
7. Using the point M_c and the line m_c , construct a line c (rule W10a); ;
8. Using the line c and the line b , construct a point A (rule W03); % NDG: lines c and b are not parallel % DET: lines c and b are not the same;
9. Using the point A and the point E_a , construct a point H (rule W01); ;
10. Using the point M_b and the point A , construct a point C (rule W01); ;
11. Using the point O and the point H , construct a point G (rule W01); ;
12. Using the point M_b and the point G , construct a point B (rule W01); .

Non-degenerate conditions: lines c and b are not parallel; lines m_c and $m(AH_b)$ are not parallel.
Determination conditions: lines c and b are not the same; lines m_c and $m(AH_b)$ are not the same; points M_b and O are not the same; points E_a and M_b are not the same.

Rules used: [W01,W02,W03,W10a,W16]

Lemmas used: [D1,D12,D13,D20,D22,D28,GD01,GD02,GL01,GL03,GL04,GL09,L1,L41,L43,L45,L47,L48,L56,L57]

Solving time: 3.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point M_{b} 95 67.5
```

```
point O 65 51.14
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_lt M_{b}
```

```
cmark_t O
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{a} and M_{b} are not the same
```

```
% Constructing a line m(AH_{c}) which passes through point E_{a} and point M_{b}
```

```
line m(AH_{c}) E_{a} M_{b}
```

```
color 200 200 200
```

```
drawline m(AH_{c})
```

```
color 0 0 0
```

```
% DET: points M_{b} and O are not the same
```

```
% Constructing a line m_{b} which passes through point M_{b} and point O
```

```
line m_{b} M_{b} O
```

```
color 200 200 200
```

```
drawline m_{b}
```

```
color 0 0 0
```

```
% Constructing a line b which is perpendicular to line m_{b} and which passes through point M_{b}
```

```
perp b M_{b} m_{b}
```

```
color 200 200 200
```

```
drawline b
```

```
color 0 0 0
```

```
% Constructing a line  $m(AH_{\{b\}})$  which contains the point  $E_{\{a\}}$  and is parallel to the line  $m_{\{b\}}$ 
parallel m(AH_{b}) E_{a} m_{b}
```

```
color 200 200 200
drawline m(AH_{b})
color 0 0 0
```

```
% Constructing a line  $m_{\{c\}}$  which contains the point  $O$  and is parallel to the line  $m(AH_{\{c\}})$ 
parallel m_{c} O m(AH_{c})
```

```
color 200 200 200
drawline m_{c}
color 0 0 0
```

```
% NDG: lines  $m_{\{c\}}$  and  $m(AH_{\{b\}})$  are not parallel% DET: lines  $m_{\{c\}}$  and  $m(AH_{\{b\}})$  are not the same
% Constructing a point  $M_{\{c\}}$  which belongs to line  $m_{\{c\}}$  and line  $m(AH_{\{b\}})$ 
intersec M_{c} m_{c} m(AH_{b})
cmark_lt M_{c}
```

```
% Constructing a line  $c$  which is perpendicular to line  $m_{\{c\}}$  and which passes through point  $M_{\{c\}}$ 
perp c M_{c} m_{c}
```

```
color 200 200 200
drawline c
color 0 0 0
```

```
% NDG: lines  $c$  and  $b$  are not parallel% DET: lines  $c$  and  $b$  are not the same
% Constructing a point  $A$  which belongs to line  $c$  and line  $b$ 
intersec A c b
cmark_t A
```

```
% Constructing a point  $H$  such that  $AH/AE_{\{a\}}=2$ 
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0
```

```
% Constructing a point  $C$  such that  $M_{\{b\}}C/M_{\{b\}}A=-1$ 
towards C M_{b} A -1
cmark_l C
color 200 200 200
```



```

drawsegment A C
color 0 0 0

% Constructing a line  $L_{\{ \_G44746 \}}$  which passes through point O and point H
line  $L_{\{ \_G44746 \}}$  O H

color 200 200 200
drawline  $L_{\{ \_G44746 \}}$ 
color 0 0 0

% Constructing a point  $P_{\{ \_G44847 \}}$  with coordinates (0,0)
point  $P_{\{ \_G44847 \}}$  0 0
cmark_r  $P_{\{ \_G44847 \}}$ 

% Constructing a point  $P_{\{ \_G44771 \}}$  such that  $OP_{\{ \_G44771 \}}/OP_{\{ \_G44847 \}}=1$ 
towards  $P_{\{ \_G44771 \}}$  0  $P_{\{ \_G44847 \}}$  1
cmark_r  $P_{\{ \_G44771 \}}$ 
color 200 200 200
drawsegment 0  $P_{\{ \_G44771 \}}$ 
color 0 0 0

% Constructing a point  $P_{\{ \_G44816 \}}$  such that  $OP_{\{ \_G44816 \}}/OP_{\{ \_G44847 \}}=3$ 
towards  $P_{\{ \_G44816 \}}$  0  $P_{\{ \_G44847 \}}$  3
cmark_r  $P_{\{ \_G44816 \}}$ 
color 200 200 200
drawsegment 0  $P_{\{ \_G44816 \}}$ 
color 0 0 0

% Constructing a line  $L_{\{ \_G44777 \}}$  which passes through point H and point  $P_{\{ \_G44816 \}}$ 
line  $L_{\{ \_G44777 \}}$  H  $P_{\{ \_G44816 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G44777 \}}$ 
color 0 0 0

% Constructing a line  $L_{\{ \_G44740 \}}$  which contains the point  $P_{\{ \_G44771 \}}$  and is parallel to the
line  $L_{\{ \_G44777 \}}$ 
parallel  $L_{\{ \_G44740 \}}$   $P_{\{ \_G44771 \}}$   $L_{\{ \_G44777 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G44740 \}}$ 
color 0 0 0

% Constructing a point G which belongs to line  $L_{\{ \_G44740 \}}$  and line  $L_{\{ \_G44746 \}}$ 
intersec G  $L_{\{ \_G44740 \}}$   $L_{\{ \_G44746 \}}$ 
cmark_t G

% Constructing a point B such that  $M_{\{ b \}}B/M_{\{ b \}}G=3$ 
towards B  $M_{\{ b \}}$  G 3

```

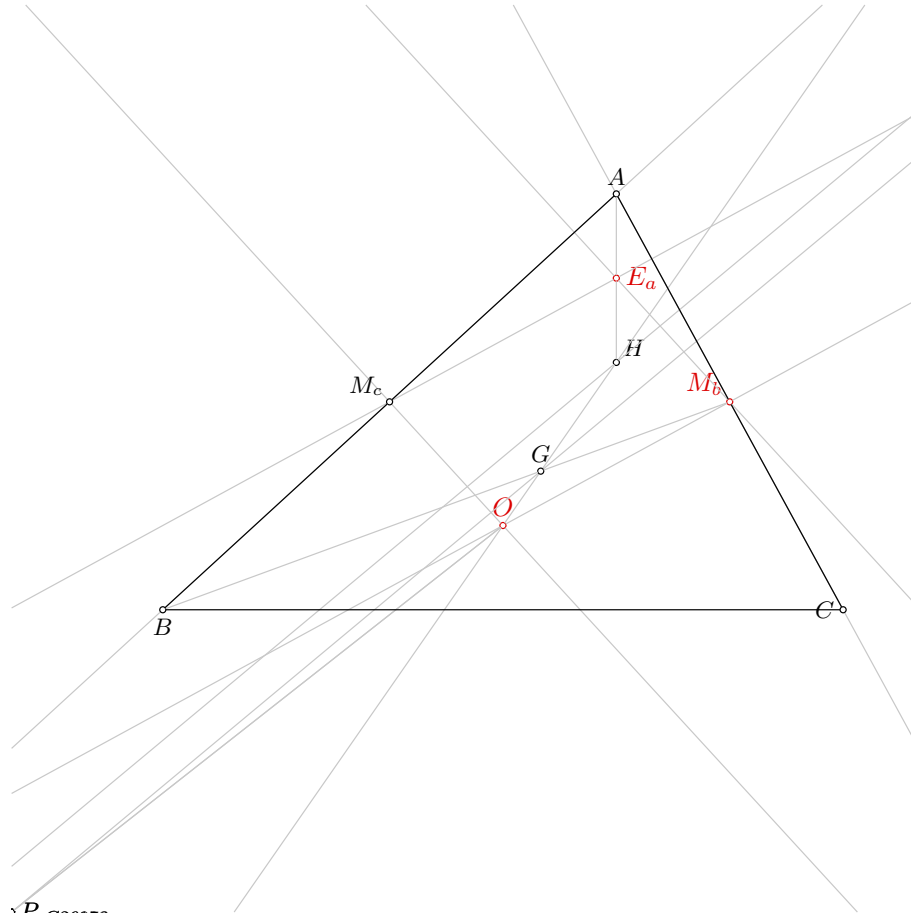


Figure 1: Illustration of the problem 0848

```

cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

% Non-degenerate conditions: lines c and b are not parallel; lines $m_{\{c\}}$ and $m(AH_{\{b\}})$ are not parallel

% Determination conditions: lines c and b are not the same; lines $m_{\{c\}}$ and $m(AH_{\{b\}})$ are not the same; points $M_{\{b\}}$ and O are not the same; points $E_{\{a\}}$ and $M_{\{b\}}$ are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 9 terms.

Time Complexity: Time spent by the prover is 0.18 seconds.

NDG conditions Points E_a and O are not identical

Line through points M_b and M_c is not perpendicular to line through points M_c and E_a

4.1.3 Proving $O = \neg O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{OE_a P_m^1(AH_b)} \neq S_{P_m^2 E_a P_m^1(AH_b)}$ i.e., lines OP_m^2 and $E_a P_m^1$ are not parallel (construction based assumption)
 $S_{M_c OP_m^2} \neq 0$ i.e., points M_c , O and P_m^2 are not collinear (foot is not the point itself; construction based assumption)

$S_{M_c M_b T_b^0} \neq S_{F_c^3 M_b T_b^0}$ i.e., lines $M_c F_c^3$ and $M_b T_b^0$ are not parallel (construction based assumption)

$S_{P_{-G30876} OH} \neq S_{P_{L_{-G30845}}^4 OH}$ i.e., lines $P_{-G30876} P_{L_{-G30845}}^4$ and OH are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^6} \neq S_{F_{-h_a}^5 BF_{-h_b}^6}$ i.e., lines $AF_{-h_a}^5$ and $BF_{-h_b}^6$ are not parallel (construction based assumption)

$S_{\neg M_a BC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_b AC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_a \neg M_b F_{-m_b}^8} \neq S_{F_{-m_a}^7 \neg M_b F_{-m_b}^8}$ i.e., lines $\neg M_a F_{-m_a}^7$ and $\neg M_b F_{-m_b}^8$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.2.3 Proving $O = \neg O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = E_a$

Proving failed

4.3.2 Proving $M_b = M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.090 seconds. There are no ndg conditions.

4.3.3 Proving $O = O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = E_a$

Proving failed

4.4.2 Proving $M_b = M_b$

Proving failed

4.4.3 Proving $O = O$

Proving failed

Problem 849

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 849: Given a point E_a , a point M_b and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 850

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 850: Given a point E_a , a point M_b and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 851

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 851: Given a point E_a , a point M_b and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 852

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 852: Given a point M_c , a point N and a point E_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point M_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_c and N are not the same;
2. Choose freely a point E_a on the circle $k(N, M_a)$ (rule WOncircle);
3. Using the point E_a and the point N , construct a line $m(H_bH_c)$ (rule W02); % DET: points E_a and N are not the same;
4. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point E_a , construct a point M_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points E_a and M_a must be different;
5. Choose freely a point A (rule free);
6. Using the point A and the point M_c , construct a point B (rule W01); ;
7. Using the point A and the point M_a , construct a point G (rule W01); ;
8. Using the point M_c and the point G , construct a point C (rule W01); .

Non-degenerate conditions: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect; points M_c and N are not the same.

Determination conditions: points E_a and M_a must be different; points E_a and N are not the same.

Rules used: [W01,W02,W05a,W06,WOncircle1,free]

Lemmas used: [D20,D32,GD02,GL01,GL03,GL04,L18,L20,L21,L22,L38,L39,L47,L48,L55,L57]

Solving time: 150.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point N 72.5 61.93
point E_{a} 80 83.86

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_r N
cmark_r E_{a}
color 0 0 0
fontsize 8

% NDG: points M_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{c}
circle k(N,M_{a}) N M_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point E_{a} on the circle with center N through point M_{c}
oncircle E_{a} N M_{c}
cmark_r E_{a}
color 200 200 200
drawcircle N M_{c}
color 0 0 0

% DET: points E_{a} and N are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point E_{a} and point N
line m(H_{b}H_{c}) E_{a} N

color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0

% NDG: line m(H_{b}H_{c}) and circle k(N,M_{a}) intersect% DET: points E_{a} and M_{a} must be
different
% Constructing a point M_{a} which is an image of the point E_{a} in the symmetry to point/line N
sim M_{a} N E_{a}
cmark_r M_{a}

% Constructing a free point A
point A 80 95
```

```
cmark_t A
```

```
% Constructing a point B such that  $AB/AM_{\{c\}}=2$ 
```

```
towards B A M_{\{c\}} 2
```

```
cmark_b B
```

```
color 200 200 200
```

```
drawsegment A B
```

```
color 0 0 0
```

```
% Constructing a line  $L_{\{\_G90526\}}$  which passes through point A and point  $M_{\{a\}}$ 
```

```
line L_{\_G90526} A M_{\{a\}}
```

```
color 200 200 200
```

```
drawline L_{\_G90526}
```

```
color 0 0 0
```

```
% Constructing a point  $P_{\{\_G90627\}}$  with coordinates (0,0)
```

```
point P_{\_G90627} 0 0
```

```
cmark_r P_{\_G90627}
```

```
% Constructing a point  $P_{\{\_G90551\}}$  such that  $AP_{\{\_G90551\}}/AP_{\{\_G90627\}}=2$ 
```

```
towards P_{\_G90551} A P_{\_G90627} 2
```

```
cmark_r P_{\_G90551}
```

```
color 200 200 200
```

```
drawsegment A P_{\_G90551}
```

```
color 0 0 0
```

```
% Constructing a point  $P_{\{\_G90596\}}$  such that  $AP_{\{\_G90596\}}/AP_{\{\_G90627\}}=3$ 
```

```
towards P_{\_G90596} A P_{\_G90627} 3
```

```
cmark_r P_{\_G90596}
```

```
color 200 200 200
```

```
drawsegment A P_{\_G90596}
```

```
color 0 0 0
```

```
% Constructing a line  $L_{\{\_G90557\}}$  which passes through point  $M_{\{a\}}$  and point  $P_{\{\_G90596\}}$ 
```

```
line L_{\_G90557} M_{\{a\}} P_{\_G90596}
```

```
color 200 200 200
```

```
drawline L_{\_G90557}
```

```
color 0 0 0
```

```
% Constructing a line  $L_{\{\_G90520\}}$  which contains the point  $P_{\{\_G90551\}}$  and is parallel to the  
line  $L_{\{\_G90557\}}$ 
```

```
parallel L_{\_G90520} P_{\_G90551} L_{\_G90557}
```

```
color 200 200 200
```

```
drawline L_{\_G90520}
```

```
color 0 0 0
```

```

% Constructing a point G which belongs to line L_{\_G90520} and line L_{\_G90526}
intersec G L_{\_G90520} L_{\_G90526}
cmark_t G

```

```

% Constructing a point C such that M_{c}C/M_{c}G=3
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions: line m(H_{b}H_{c}) and circle k(N,M_{a}) intersect; points M_{c} and N
are not the same
% Determination conditions: points E_{a} and M_{a} must be different; points E_{a} and N are not
the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_c = M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.032 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $N = N$

Proving failed

4.1.3 Proving $E_a = E_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $M_c = M_c$

NDG conditions are:

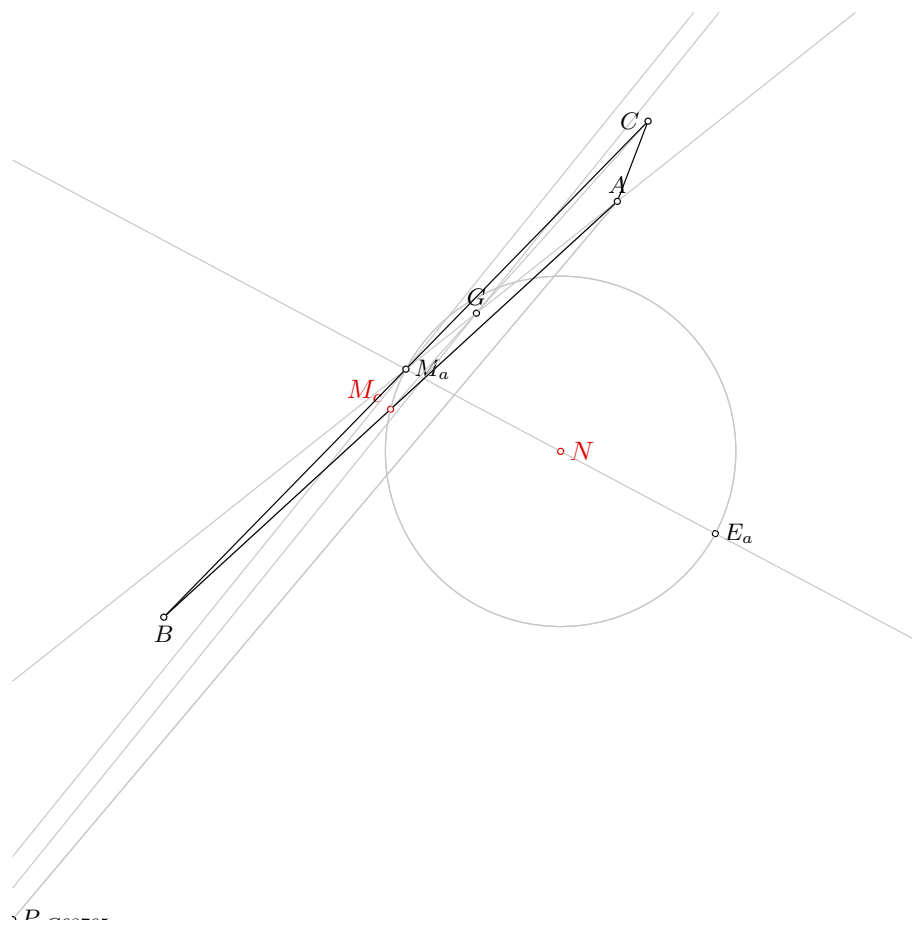


Figure 1: Illustration of the problem 0852

$S_{P_{-G69934}AM_a} \neq S_{P_{L_{-G69903}}^0 AM_a}$ i.e., lines $P_{-G69934}P_{L_{-G69903}}^0$ and AM_a are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points M_a, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points M_b, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^4} \neq S_{F_{-h_a}^3 BF_{-h_b}^4}$ i.e., lines $AF_{-h_a}^3$ and $BF_{-h_b}^4$ are not parallel (construction based assumption)

$S_{M_aM_bF_{-m_b}^2} \neq S_{F_{-m_a}^1 M_bF_{-m_b}^2}$ i.e., lines $M_aF_{-m_a}^1$ and $M_bF_{-m_b}^2$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $E_a=_Ea$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_c=_M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.070 seconds. There are no ndg conditions.

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $E_a=_Ea$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_c=_M_c$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $E_a=_Ea$

Proving failed

Problem 853

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 853: Given a point E_a , a point M_c and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_a and the point M_c , construct a line $m(AH_b)$ (rule W02); % DET: points E_a and M_c are not the same;
2. Using the point M_c and the point O , construct a line m_c (rule W02); % DET: points M_c and O are not the same;
3. Using the point M_c and the line m_c , construct a line c (rule W10a); ;
4. Using the point E_a and the line m_c , construct a line $m(AH_c)$ (rule W16); ;
5. Using the point O and the line $m(AH_b)$, construct a line m_b (rule W16); ;
6. Using the line m_b and the line $m(AH_c)$, construct a point M_b (rule W03); % NDG: lines m_b and $m(AH_c)$ are not parallel % DET: lines m_b and $m(AH_c)$ are not the same;
7. Using the point M_b and the line m_b , construct a line b (rule W10a); ;
8. Using the line b and the line c , construct a point A (rule W03); % NDG: lines b and c are not parallel % DET: lines b and c are not the same;
9. Using the point A and the point E_a , construct a point H (rule W01); ;
10. Using the point M_c and the point A , construct a point B (rule W01); ;
11. Using the point O and the point H , construct a point G (rule W01); ;
12. Using the point M_c and the point G , construct a point C (rule W01); .

Non-degenerate conditions: lines b and c are not parallel; lines m_b and $m(AH_c)$ are not parallel.
Determination conditions: lines b and c are not the same; lines m_b and $m(AH_c)$ are not the same; points M_c and O are not the same; points E_a and M_c are not the same.

Rules used: [W01,W02,W03,W10a,W16]

Lemmas used: [D1,D12,D13,D20,D22,D28,GD01,GD02,GL01,GL03,GL04,GL09,L1,L41,L43,L45,L47,L48,L57,L58]

Solving time: 3.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{a} 80 83.86
point M_{c} 50 67.5
point O 65 51.14

color 220 0 0
fontsize 9

cmark_r E_{a}
cmark_lt M_{c}
cmark_t O
color 0 0 0
fontsize 8

% DET: points E_{a} and M_{c} are not the same
% Constructing a line m(AH_{b}) which passes through point E_{a} and point M_{c}
line m(AH_{b}) E_{a} M_{c}

color 200 200 200
drawline m(AH_{b})
color 0 0 0

% DET: points M_{c} and O are not the same
% Constructing a line m_{c} which passes through point M_{c} and point O
line m_{c} M_{c} O

color 200 200 200
drawline m_{c}
color 0 0 0

% Constructing a line c which is perpendicular to line m_{c} and which passes through point M_{c}
perp c M_{c} m_{c}

color 200 200 200
drawline c
color 0 0 0
```

```
% Constructing a line  $m(AH_{\{c\}})$  which contains the point  $E_{\{a\}}$  and is parallel to the line  $m_{\{c\}}$ 
parallel m(AH_{c}) E_{a} m_{c}
```

```
color 200 200 200
drawline m(AH_{c})
color 0 0 0
```

```
% Constructing a line  $m_{\{b\}}$  which contains the point  $O$  and is parallel to the line  $m(AH_{\{b\}})$ 
parallel m_{b} O m(AH_{b})
```

```
color 200 200 200
drawline m_{b}
color 0 0 0
```

```
% NDG: lines  $m_{\{b\}}$  and  $m(AH_{\{c\}})$  are not parallel% DET: lines  $m_{\{b\}}$  and  $m(AH_{\{c\}})$  are not the same
% Constructing a point  $M_{\{b\}}$  which belongs to line  $m_{\{b\}}$  and line  $m(AH_{\{c\}})$ 
intersec M_{b} m_{b} m(AH_{c})
cmark_lt M_{b}
```

```
% Constructing a line  $b$  which is perpendicular to line  $m_{\{b\}}$  and which passes through point  $M_{\{b\}}$ 
perp b M_{b} m_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% NDG: lines  $b$  and  $c$  are not parallel% DET: lines  $b$  and  $c$  are not the same
% Constructing a point  $A$  which belongs to line  $b$  and line  $c$ 
intersec A b c
cmark_t A
```

```
% Constructing a point  $H$  such that  $AH/AE_{\{a\}}=2$ 
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0
```

```
% Constructing a point  $B$  such that  $M_{\{c\}}B/M_{\{c\}}A=-1$ 
towards B M_{c} A -1
cmark_b B
color 200 200 200
```



```

drawsegment A B
color 0 0 0

% Constructing a line  $L_{\setminus\_G125055}$  which passes through point O and point H
line  $L_{\setminus\_G125055}$  O H

color 200 200 200
drawline  $L_{\setminus\_G125055}$ 
color 0 0 0

% Constructing a point  $P_{\setminus\_G125156}$  with coordinates (0,0)
point  $P_{\setminus\_G125156}$  0 0
cmark_r  $P_{\setminus\_G125156}$ 

% Constructing a point  $P_{\setminus\_G125080}$  such that  $OP_{\setminus\_G125080}/OP_{\setminus\_G125156}=1$ 
towards  $P_{\setminus\_G125080}$  O  $P_{\setminus\_G125156}$  1
cmark_r  $P_{\setminus\_G125080}$ 
color 200 200 200
drawsegment O  $P_{\setminus\_G125080}$ 
color 0 0 0

% Constructing a point  $P_{\setminus\_G125125}$  such that  $OP_{\setminus\_G125125}/OP_{\setminus\_G125156}=3$ 
towards  $P_{\setminus\_G125125}$  O  $P_{\setminus\_G125156}$  3
cmark_r  $P_{\setminus\_G125125}$ 
color 200 200 200
drawsegment O  $P_{\setminus\_G125125}$ 
color 0 0 0

% Constructing a line  $L_{\setminus\_G125086}$  which passes through point H and point  $P_{\setminus\_G125125}$ 
line  $L_{\setminus\_G125086}$  H  $P_{\setminus\_G125125}$ 

color 200 200 200
drawline  $L_{\setminus\_G125086}$ 
color 0 0 0

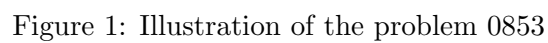
% Constructing a line  $L_{\setminus\_G125049}$  which contains the point  $P_{\setminus\_G125080}$  and is parallel to the
line  $L_{\setminus\_G125086}$ 
parallel  $L_{\setminus\_G125049}$   $P_{\setminus\_G125080}$   $L_{\setminus\_G125086}$ 

color 200 200 200
drawline  $L_{\setminus\_G125049}$ 
color 0 0 0

% Constructing a point G which belongs to line  $L_{\setminus\_G125049}$  and line  $L_{\setminus\_G125055}$ 
intersec G  $L_{\setminus\_G125049}$   $L_{\setminus\_G125055}$ 
cmark_t G

% Constructing a point C such that  $M_{\setminus\_c}C/M_{\setminus\_c}G=3$ 
towards C  $M_{\setminus\_c}$  G 3

```



% Determination conditions: lines b and c are not the same; lines $m_{\{b\}}$ and $m(AH_{\{c\}})$ are not the same; points $M_{\{c\}}$ and O are not the same; points $E_{\{a\}}$ and $M_{\{c\}}$ are not the same

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = \neg E_a$

Proving failed

4.1.2 Proving $M_c = \neg M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 9 terms.

Time Complexity: Time spent by the prover is 0.191 seconds.

NDG conditions Points M_c and O are not identical

Line through points M_b and M_c is not perpendicular to line through points M_c and O

4.1.3 Proving $O = \neg O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a = \neg E_a$

Proving failed

4.2.2 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{OE_a P_m^1(AH_c)} \neq S_{P_{m_b}^2 E_a P_m^1(AH_c)}$ i.e., lines $OP_{m_b}^2$ and $E_a P_m^1(AH_c)$ are not parallel (construction based assumption)
 $S_{M_b OP_{m_b}^2} \neq 0$ i.e., points M_b , O and $P_{m_b}^2$ are not collinear (foot is not the point itself; construction based assumption)

$S_{M_b M_c T_c^0} \neq S_{F_b^3 M_c T_c^0}$ i.e., lines $M_b F_b^3$ and $M_c T_c^0$ are not parallel (construction based assumption)

$S_{P_{-G110900} OH} \neq S_{P_{L_{-G110869}}^4 OH}$ i.e., lines $P_{-G110900} P_{L_{-G110869}}^4$ and OH are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^6} \neq S_{F_{-h_a}^5 BF_{-h_b}^6}$ i.e., lines $AF_{-h_a}^5$ and $BF_{-h_b}^6$ are not parallel (construction based assumption)

$S_{\neg M_a BC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_b AC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_a \neg M_b F_{-m_b}^8} \neq S_{F_{-m_a}^7 \neg M_b F_{-m_b}^8}$ i.e., lines $\neg M_a F_{-m_a}^7$ and $\neg M_b F_{-m_b}^8$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.2.3 Proving $O = \neg O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a = E_a$

Proving failed

4.3.2 Proving $M_c = M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.100 seconds. There are no ndg conditions.

4.3.3 Proving $O = O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a = E_a$

Proving failed

4.4.2 Proving $M_c = M_c$

Proving failed

4.4.3 Proving $O = O$

Proving failed

Problem 854

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 854: Given a point E_a , a point M_c and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 855

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 855: Given a point E_a , a point M_c and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 856

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 856: Given a point E_a , a point M_c and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 857

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 857: Given a point E_a , a point N and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point N and the point O , construct a point G (rule W01); ;
2. Using the point N and the point O , construct a point H (rule W01); ;
3. Using the point E_a and the point H , construct a point A (rule W01); ;
4. Using the point G and the point A , construct a point M_a (rule W01); ;
5. Using the point E_a and the point H , construct a line h_a (rule W02); % DET: points E_a and H are not the same;
6. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
8. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
9. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; line h_a and circle $k(N, M_a)$ intersect; points E_a and N are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points E_a and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D31,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L15,L19,L22,L55]

Solving time: 7.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{a} 80 83.86
```

```
point N 72.5 61.93
```

```
point O 65 51.14
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{a}
```

```
cmark_r N
```

```
cmark_t O
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a line L_{\_G182669} which passes through point N and point O
```

```
line L_{\_G182669} N O
```

```
color 200 200 200
```

```
drawline L_{\_G182669}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G182770} with coordinates (0,0)
```

```
point P_{\_G182770} 0 0
```

```
cmark_r P_{\_G182770}
```

```
% Constructing a point P_{\_G182694} such that NP_{\_G182694}/NP_{\_G182770}=1
```

```
towards P_{\_G182694} N P_{\_G182770} 1
```

```
cmark_r P_{\_G182694}
```

```
color 200 200 200
```

```
drawsegment N P_{\_G182694}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G182739} such that NP_{\_G182739}/NP_{\_G182770}=3
```

```
towards P_{\_G182739} N P_{\_G182770} 3
```

```
cmark_r P_{\_G182739}
```

```
color 200 200 200
```

```
drawsegment N P_{\_G182739}
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G182700} which passes through point O and point P_{\_G182739}
```

```
line L_{\_G182700} O P_{\_G182739}
```

```

color 200 200 200
drawline L_{\_G182700}
color 0 0 0

% Constructing a line L_{\_G182663} which contains the point P_{\_G182694} and is parallel to the
line L_{\_G182700}
parallel L_{\_G182663} P_{\_G182694} L_{\_G182700}

color 200 200 200
drawline L_{\_G182663}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G182663} and line L_{\_G182669}
intersec G L_{\_G182663} L_{\_G182669}
cmark_t G

% Constructing a point H such that NH/NO=-1
towards H N 0 -1
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0

% Constructing a point A such that E_{a}A/E_{a}H=-1
towards A E_{a} H -1
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

% Constructing a point M_{a} such that GM_{a}/GA=-0.5
towards M_{a} G A -0.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% DET: points E_{a} and H are not the same
% Constructing a line h_{a} which passes through point E_{a} and point H
line h_{a} E_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

```

```

% NDG: points E_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{a}
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G183754} which is a foot of the point N on the line h_{a}
foot P_{\_G183754} N h_{a}
cmark_r P_{\_G183754}
color 200 200 200
drawline N P_{\_G183754}
color 0 0 0

% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
\_G183754}
sim H_{a} P_{\_G183754} E_{a}
cmark_r H_{a}

% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```

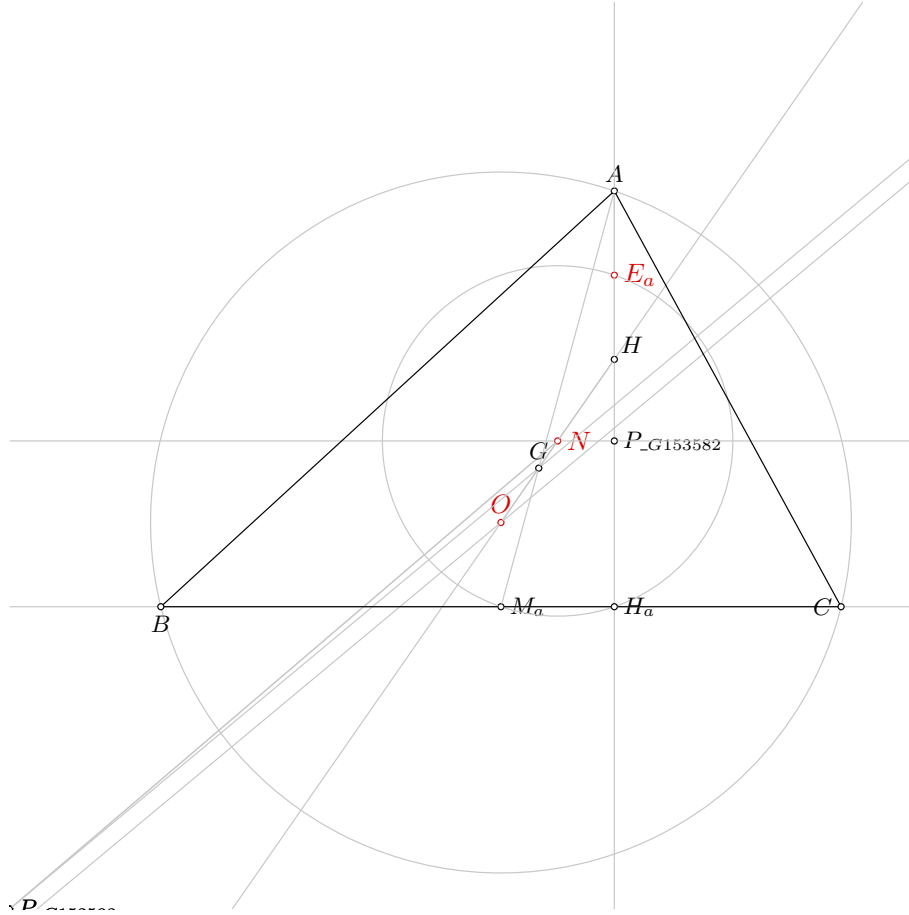


Figure 1: Illustration of the problem 0857

*% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
line h_{a} and circle k(N,M_{a}) intersect; points E_{a} and N are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
different; points E_{a} and H are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_a = E_a$

Proving failed

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $O=_O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_a=_Ea$

Proving failed

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $O=_O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_a=_Ea$

Proving failed

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $O=_O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_a=_Ea$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $O=_O$

Proving failed

Problem 858

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 858: Given a point E_a , a point N and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 859

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 859: Given a point E_a , a point N and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 860

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 860: Given a point E_a , a point N and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 861

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 861: Given a point E_a , a point O and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 862

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 862: Given a point E_a , a point O and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 863

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 863: Given a point E_a , a point O and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 864

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 864: Given a point E_a , a point T_a and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 865

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 865: Given a point E_a , a point T_a and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 866

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 866: Given a point E_a , a point T_b and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 867

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 867: Given a point E_b , a point E_c and a point G , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 868

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 868: Given a point E_b , a point E_c and a point H , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H , construct a point B (rule W01); ;
2. Using the point E_c and the point H , construct a point C (rule W01); ;
3. Using the point E_c and the point H , construct a line h_c (rule W02); % DET: points E_c and H are not the same;
4. Using the point B and the point C , construct a line a (rule W02); % DET: points B and C are not the same;
5. Using the point H and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H and E_b are not the same;
6. Using the circle $k(E_b, B)$, the line h_c , the point E_b and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_b, B)$ intersect % DET: points H and H_c must be different;
7. Using the point H_c and the point B , construct a line c (rule W02); % DET: points H_c and B are not the same;
8. Using the circle $k(E_b, B)$, the line a , the point E_b and the point B , construct a point H_a (rule W05); % NDG: line a and circle $k(E_b, B)$ intersect % DET: points B and H_a must be different;
9. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
10. Using the line c and the line h_a , construct a point A (rule W03); % NDG: lines c and h_a are not parallel % DET: lines c and h_a are not the same.

Non-degenerate conditions: lines c and h_a are not parallel; line a and circle $k(E_b, B)$ intersect; line h_c and circle $k(E_b, B)$ intersect; points H and E_b are not the same.

Determination conditions: lines c and h_a are not the same; points H_a and H are not the same; points B and H_a must be different; points H_c and B are not the same; points H and H_c must be different; points B and C are not the same; points E_c and H are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D29,D3,D30,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L49,L50,L51]

Solving time: 12.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
```

```
point E_{c} 95 56.36
```

```
point H 80 72.73
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{b}
```

```
cmark_r E_{c}
```

```
cmark_rt H
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point B such that E_{b}B/E_{b}H=-1
```

```
towards B E_{b} H -1
```

```
cmark_b B
```

```
color 200 200 200
```

```
drawsegment H B
```

```
color 0 0 0
```

```
% Constructing a point C such that E_{c}C/E_{c}H=-1
```

```
towards C E_{c} H -1
```

```
cmark_l C
```

```
color 200 200 200
```

```
drawsegment H C
```

```
color 0 0 0
```

```
% DET: points E_{c} and H are not the same
```

```
% Constructing a line h_{c} which passes through point E_{c} and point H
```

```
line h_{c} E_{c} H
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```

% DET: points B and C are not the same
% Constructing a line a which passes through point B and point C
line a B C

color 200 200 200
drawline a
color 0 0 0

% NDG: points H and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H
circle k(E_{b},B) E_{b} H

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line h_{c} and circle k(E_{b},B) intersect% DET: points H and H_{c} must be different
% Constructing a point P_{\_G29383} which is a foot of the point E_{b} on the line h_{c}
foot P_{\_G29383} E_{b} h_{c}
cmark_r P_{\_G29383}
color 200 200 200
drawline E_{b} P_{\_G29383}
color 0 0 0

% Constructing a point H_{c} which is an image of the point H in the symmetry to point/line P_{\_G
29383}
sim H_{c} P_{\_G29383} H
cmark_rt H_{c}

% DET: points H_{c} and B are not the same
% Constructing a line c which passes through point H_{c} and point B
line c H_{c} B

color 200 200 200
drawline c
color 0 0 0

% NDG: line a and circle k(E_{b},B) intersect% DET: points B and H_{a} must be different
% Constructing a point P_{\_G29621} which is a foot of the point E_{b} on the line a
foot P_{\_G29621} E_{b} a
cmark_r P_{\_G29621}
color 200 200 200
drawline E_{b} P_{\_G29621}
color 0 0 0

% Constructing a point H_{a} which is an image of the point B in the symmetry to point/line P_{\_G
29621}
sim H_{a} P_{\_G29621} B
cmark_r H_{a}

```

```

% DET: points  $H_{\{a\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $H_{\{a\}}$  and point  $H$ 
line  $h_{\{a\}}$   $H_{\{a\}}$   $H$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: lines  $c$  and  $h_{\{a\}}$  are not parallel% DET: lines  $c$  and  $h_{\{a\}}$  are not the same
% Constructing a point  $A$  which belongs to line  $c$  and line  $h_{\{a\}}$ 
intersec  $A$   $c$   $h_{\{a\}}$ 
cmark_t  $A$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $c$  and  $h_{\{a\}}$  are not parallel; line  $a$  and circle  $k(E_{\{b\}}, B)$ 
% intersect; line  $h_{\{c\}}$  and circle  $k(E_{\{b\}}, B)$  intersect; points  $H$  and  $E_{\{b\}}$  are not the same
% Determination conditions: lines  $c$  and  $h_{\{a\}}$  are not the same; points  $H_{\{a\}}$  and  $H$  are not the same
% ; points  $B$  and  $H_{\{a\}}$  must be different; points  $H_{\{c\}}$  and  $B$  are not the same; points  $H$  and  $H_{\{c\}}$ 
% must be different; points  $B$  and  $C$  are not the same; points  $E_{\{c\}}$  and  $H$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $E_c = \neg E_c$

Proving failed

4.1.3 Proving $H = \neg H$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

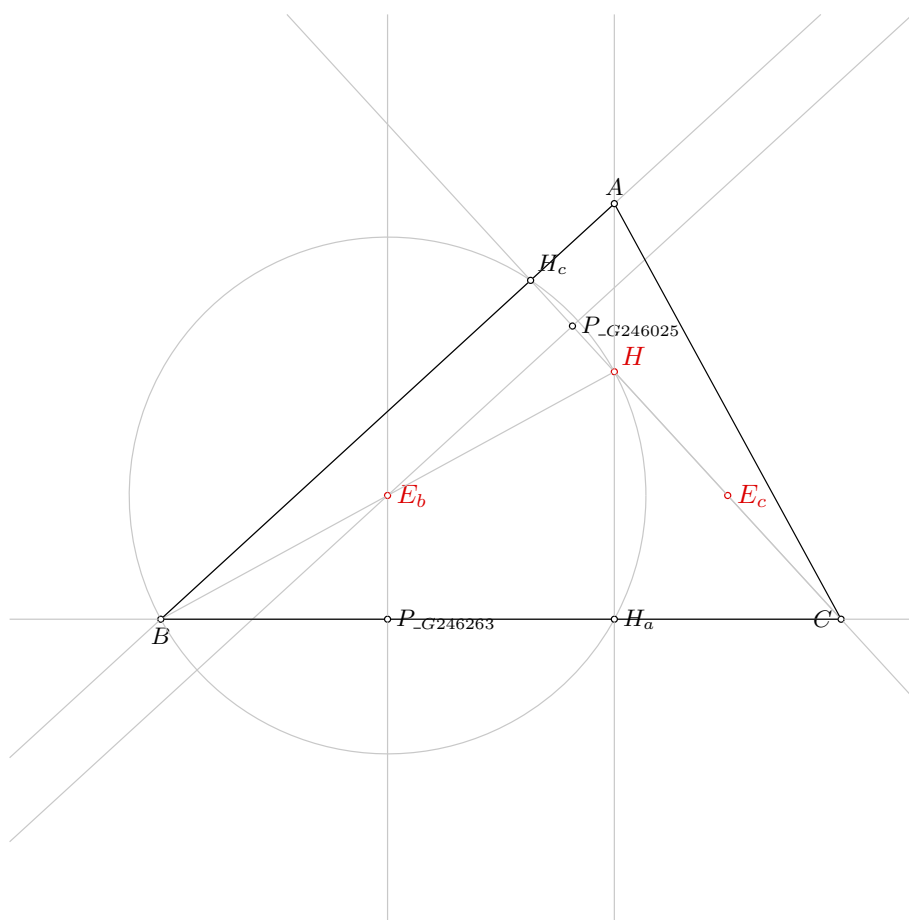


Figure 1: Illustration of the problem 0868

4.2.2 Proving $E_c = E_c$

Proving failed

4.2.3 Proving $H = H$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = E_b$

Proving failed

4.3.2 Proving $E_c = E_c$

Proving failed

4.3.3 Proving $H = H$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 2174 terms.

Time Complexity: Time spent by the prover is 1.880 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = E_b$

Proving failed

4.4.2 Proving $E_c = E_c$

Proving failed

4.4.3 Proving $H = H$

Proving failed

Problem 869

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 869: Given a point E_b , a point E_c and a point H_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H_a and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_a and E_b are not the same;
2. Using the point H_a and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_a and E_c are not the same;
3. Using the circle $k(E_b, B)$, the circle $k(E_c, C)$, the point H_a , the point E_b and the point E_c , construct a point H (rule W08); % NDG: circles $k(E_b, B)$ and $k(E_c, C)$ intersect % DET: circles $k(E_b, B)$ and $k(E_c, C)$ are not the same; points H_a and H must be different;
4. Using the point H and the point E_b , construct a point B (rule W01); ;
5. Using the point E_c and the point H , construct a point C (rule W01); ;
6. Using the point E_c and the point H , construct a line h_c (rule W02); % DET: points E_c and H are not the same;
7. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
8. Using the circle $k(E_b, B)$, the line h_c , the point E_b and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_b, B)$ intersect % DET: points H and H_c must be different;
9. Using the point B and the point H_c , construct a line c (rule W02); % DET: points B and H_c are not the same;
10. Using the line h_a and the line c , construct a point A (rule W03); % NDG: lines h_a and c are not parallel % DET: lines h_a and c are not the same.

Non-degenerate conditions: lines h_a and c are not parallel; line h_c and circle $k(E_b, B)$ intersect; circles $k(E_b, B)$ and $k(E_c, C)$ intersect; points H_a and E_c are not the same; points H_a and E_b are not the same.

Determination conditions: lines h_a and c are not the same; points B and H_c are not the same; points H and H_c must be different; points H_a and H are not the same; points E_c and H are not the same; circles $k(E_b, B)$ and $k(E_c, C)$ are not the same; points H_a and H must be different.

Rules used: [W01,W02,W03,W05,W06,W08]

Lemmas used: [D10,D29,D3,D30,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L49,L50,L51,L52,L54]

Solving time: 10.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
point E_{c} 95 56.36
point H_{a} 80 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{b}
cmark_r E_{c}
cmark_r H_{a}
color 0 0 0
fontsize 8
```

```
% NDG: points H_{a} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H
_{a}
circle k(E_{b},B) E_{b} H_{a}
```

```
color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0
```

```
% NDG: points H_{a} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H
_{a}
circle k(E_{c},C) E_{c} H_{a}
```

```
color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0
```

```
% NDG: circles k(E_{b},B) and k(E_{c},C) intersect% DET: circles k(E_{b},B) and k(E_{c},C) are not
the same; points H_{a} and H must be different
% Constructing a line L_{\_G62152} which passes through point E_{b} and point E_{c}
line L_{\_G62152} E_{b} E_{c}
```

```

color 200 200 200
drawline L_{\_G62152}
color 0 0 0

% Constructing a point H which is an image of the point H_{a} in the symmetry to point/line L_{\_G
62152}
sim H L_{\_G62152} H_{a}
cmark_rt H

% Constructing a point B such that HB/HE_{b}=2
towards B H E_{b} 2
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

% DET: points E_{c} and H are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H
line h_{c} E_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{c} and circle k(E_{b},B) intersect% DET: points H and H_{c} must be different
% Constructing a point P_{\_G62569} which is a foot of the point E_{b} on the line h_{c}
foot P_{\_G62569} E_{b} h_{c}
cmark_r P_{\_G62569}
color 200 200 200
drawline E_{b} P_{\_G62569}

```



```

color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $H$  in the symmetry to point/line  $P_{\{\backslash\_G62569\}}$ 
sim  $H_{\{c\}}$   $P_{\{\backslash\_G62569\}}$   $H$ 
cmark_rt  $H_{\{c\}}$ 

% DET: points  $B$  and  $H_{\{c\}}$  are not the same
% Constructing a line  $c$  which passes through point  $B$  and point  $H_{\{c\}}$ 
line c B  $H_{\{c\}}$ 

color 200 200 200
drawline c
color 0 0 0

% NDG: lines  $h_{\{a\}}$  and  $c$  are not parallel% DET: lines  $h_{\{a\}}$  and  $c$  are not the same
% Constructing a point  $A$  which belongs to line  $h_{\{a\}}$  and line  $c$ 
intersec A  $h_{\{a\}}$  c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{a\}}$  and  $c$  are not parallel; line  $h_{\{c\}}$  and circle  $k(E_{\{b\}},B)$ 
intersect; circles  $k(E_{\{b\}},B)$  and  $k(E_{\{c\}},C)$  intersect; points  $H_{\{a\}}$  and  $E_{\{c\}}$  are not the same
; points  $H_{\{a\}}$  and  $E_{\{b\}}$  are not the same
% Determination conditions: lines  $h_{\{a\}}$  and  $c$  are not the same; points  $B$  and  $H_{\{c\}}$  are not the same
; points  $H$  and  $H_{\{c\}}$  must be different; points  $H_{\{a\}}$  and  $H$  are not the same; points  $E_{\{c\}}$  and  $H$ 
are not the same; circles  $k(E_{\{b\}},B)$  and  $k(E_{\{c\}},C)$  are not the same; points  $H_{\{a\}}$  and  $H$  must
be different

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

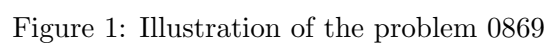
4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $E_c = \neg E_c$

Proving failed



4.1.3 Proving $H_a = \neg H_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $E_c = \neg E_c$

Proving failed

4.2.3 Proving $H_a = \neg H_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $E_c = \neg E_c$

Proving failed

4.3.3 Proving $H_a = \neg H_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $E_c = \neg E_c$

Proving failed

4.4.3 Proving $H_a = \neg H_a$

Proving failed

Problem 870

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 870: Given a point E_b , a point E_c and a point H_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;
2. Using the point H_b and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_b and E_c are not the same;
3. Using the circle $k(E_c, C)$, the line h_b , the point E_c and the point H_b , construct a point H (rule W05); % NDG: line h_b and circle $k(E_c, C)$ intersect % DET: points H_b and H must be different;
4. Using the point H and the point E_b , construct a point B (rule W01); ;
5. Using the point E_c and the point H , construct a point C (rule W01); ;
6. Using the point H_b and the point C , construct a line b (rule W02); % DET: points H_b and C are not the same;
7. Using the point B and the point C , construct a line a (rule W02); % DET: points B and C are not the same;
8. Using the circle $k(E_c, C)$, the line a , the point E_c and the point C , construct a point H_a (rule W05); % NDG: line a and circle $k(E_c, C)$ intersect % DET: points C and H_a must be different;
9. Using the point H and the point H_a , construct a line h_a (rule W02); % DET: points H and H_a are not the same;
10. Using the line b and the line h_a , construct a point A (rule W03); % NDG: lines b and h_a are not parallel % DET: lines b and h_a are not the same.

Non-degenerate conditions: lines b and h_a are not parallel; line a and circle $k(E_c, C)$ intersect; line h_b and circle $k(E_c, C)$ intersect; points H_b and E_c are not the same.

Determination conditions: lines b and h_a are not the same; points H and H_a are not the same; points C and H_a must be different; points B and C are not the same; points H_b and C are not the same; points H_b and H must be different; points E_b and H_b are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D29,D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,GL04,GL09,L52,L53,L54]

Solving time: 9.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
```

```
point E_{c} 95 56.36
```

```
point H_{b} 89.36 77.83
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{b}
```

```
cmark_r E_{c}
```

```
cmark_l H_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{b} and H_{b} are not the same
```

```
% Constructing a line h_{b} which passes through point E_{b} and point H_{b}
```

```
line h_{b} E_{b} H_{b}
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```
% NDG: points H_{b} and E_{c} are not the same
```

```
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H_{b}
```

```
circle k(E_{c},C) E_{c} H_{b}
```

```
color 200 200 200
```

```
drawcircle k(E_{c},C)
```

```
color 0 0 0
```

```
% NDG: line h_{b} and circle k(E_{c},C) intersect% DET: points H_{b} and H must be different
```

```
% Constructing a point P_{\_G91827} which is a foot of the point E_{c} on the line h_{b}
```

```
foot P_{\_G91827} E_{c} h_{b}
```

```
cmark_r P_{\_G91827}
```

```
color 200 200 200
```

```
drawline E_{c} P_{\_G91827}
```

```
color 0 0 0
```

```

% Constructing a point H which is an image of the point  $H_{\{b\}}$  in the symmetry to point/line  $P_{\{\backslash\_G91827\}}$ 
sim H  $P_{\{\backslash\_G91827\}}$   $H_{\{b\}}$ 
cmark_rt H

% Constructing a point B such that  $HB/HE_{\{b\}}=2$ 
towards B H  $E_{\{b\}}$  2
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

% Constructing a point C such that  $E_{\{c\}}C/E_{\{c\}}H=-1$ 
towards C  $E_{\{c\}}$  H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

% DET: points  $H_{\{b\}}$  and C are not the same
% Constructing a line b which passes through point  $H_{\{b\}}$  and point C
line b  $H_{\{b\}}$  C

color 200 200 200
drawline b
color 0 0 0

% DET: points B and C are not the same
% Constructing a line a which passes through point B and point C
line a B C

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle  $k(E_{\{c\}}, C)$  intersect% DET: points C and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G92264\}}$  which is a foot of the point  $E_{\{c\}}$  on the line a
foot  $P_{\{\backslash\_G92264\}}$   $E_{\{c\}}$  a
cmark_r  $P_{\{\backslash\_G92264\}}$ 
color 200 200 200
drawline  $E_{\{c\}}$   $P_{\{\backslash\_G92264\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point C in the symmetry to point/line  $P_{\{\backslash\_G92264\}}$ 
sim  $H_{\{a\}}$   $P_{\{\backslash\_G92264\}}$  C

```

```

cmark_r H_{a}

% DET: points H and H_{a} are not the same
% Constructing a line h_{a} which passes through point H and point H_{a}
line h_{a} H H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: lines b and h_{a} are not parallel% DET: lines b and h_{a} are not the same
% Constructing a point A which belongs to line b and line h_{a}
intersec A b h_{a}
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and h_{a} are not parallel; line a and circle k(E_{c},C)
% intersect; line h_{b} and circle k(E_{c},C) intersect; points H_{b} and E_{c} are not the same
% Determination conditions: lines b and h_{a} are not the same; points H and H_{a} are not the same
% ; points C and H_{a} must be different; points B and C are not the same; points H_{b} and C are
% not the same; points H_{b} and H must be different; points E_{b} and H_{b} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $E_c = \neg E_c$

Proving failed

4.1.3 Proving $H_b = \neg H_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

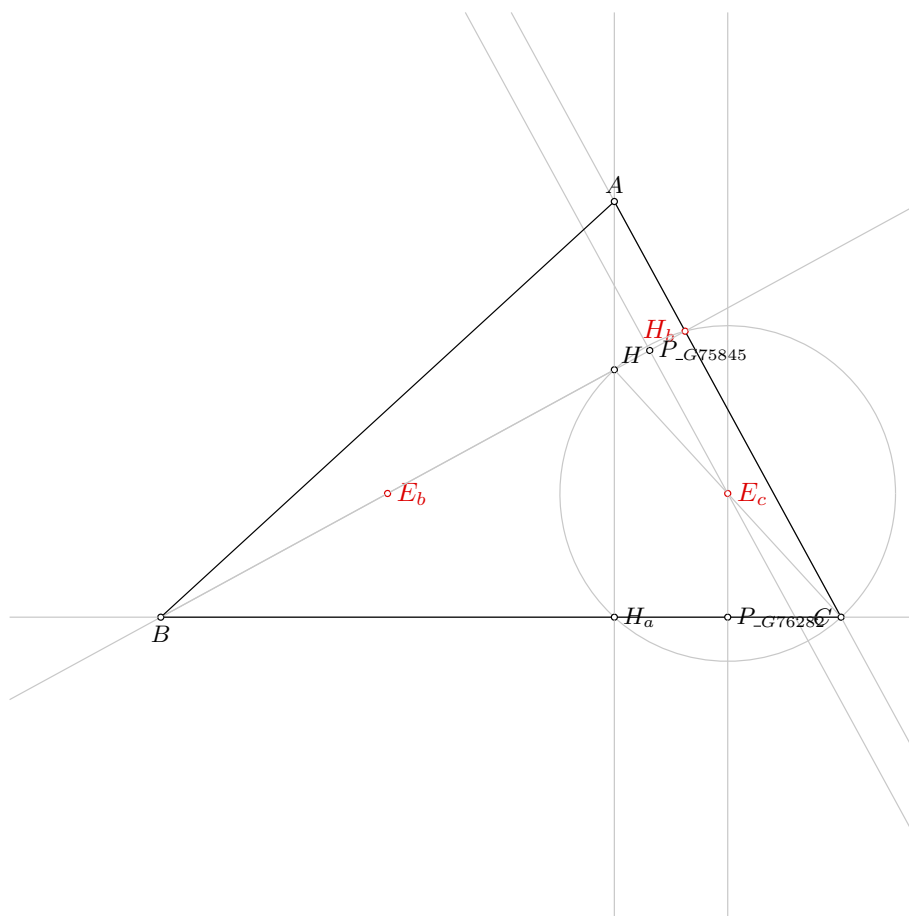


Figure 1: Illustration of the problem 0870

4.2.2 Proving $E_c = E_c$

Proving failed

4.2.3 Proving $H_b = H_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = E_b$

Proving failed

4.3.2 Proving $E_c = E_c$

Proving failed

4.3.3 Proving $H_b = H_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = E_b$

Proving failed

4.4.2 Proving $E_c = E_c$

Proving failed

4.4.3 Proving $H_b = H_b$

Proving failed

Problem 871

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 871: Given a point E_b , a point E_c and a point H_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;
2. Using the point H_c and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_c and E_b are not the same;
3. Using the circle $k(E_b, B)$, the line h_c , the point E_b and the point H_c , construct a point H (rule W05); % NDG: line h_c and circle $k(E_b, B)$ intersect % DET: points H_c and H must be different;
4. Using the point H and the point E_b , construct a point B (rule W01); ;
5. Using the point E_c and the point H , construct a point C (rule W01); ;
6. Using the point H_c and the point B , construct a line c (rule W02); % DET: points H_c and B are not the same;
7. Using the point B and the point C , construct a line a (rule W02); % DET: points B and C are not the same;
8. Using the circle $k(E_b, B)$, the line a , the point E_b and the point B , construct a point H_a (rule W05); % NDG: line a and circle $k(E_b, B)$ intersect % DET: points B and H_a must be different;
9. Using the point H and the point H_a , construct a line h_a (rule W02); % DET: points H and H_a are not the same;
10. Using the line c and the line h_a , construct a point A (rule W03); % NDG: lines c and h_a are not parallel % DET: lines c and h_a are not the same.

Non-degenerate conditions: lines c and h_a are not parallel; line a and circle $k(E_b, B)$ intersect; line h_c and circle $k(E_b, B)$ intersect; points H_c and E_b are not the same.

Determination conditions: lines c and h_a are not the same; points H and H_a are not the same; points B and H_a must be different; points B and C are not the same; points H_c and B are not the same; points H_c and H must be different; points E_c and H_c are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D29,D3,D30,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L49,L50,L51]

Solving time: 9.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{b} 50 56.36
point E_{c} 95 56.36
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_r E_{b}
cmark_r E_{c}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points E_{c} and H_{c} are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H_{c}
line h_{c} E_{c} H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points H_{c} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H_{c}
circle k(E_{b},B) E_{b} H_{c}

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line h_{c} and circle k(E_{b},B) intersect% DET: points H_{c} and H must be different
% Constructing a point P_{\_G122182} which is a foot of the point E_{b} on the line h_{c}
foot P_{\_G122182} E_{b} h_{c}
cmark_r P_{\_G122182}
color 200 200 200
drawline E_{b} P_{\_G122182}
color 0 0 0
```

```

% Constructing a point H which is an image of the point H_{c} in the symmetry to point/line P_{\_G
122182}
sim H P_{\_G122182} H_{c}
cmark_rt H

% Constructing a point B such that HB/HE_{b}=2
towards B H E_{b} 2
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

% DET: points H_{c} and B are not the same
% Constructing a line c which passes through point H_{c} and point B
line c H_{c} B

color 200 200 200
drawline c
color 0 0 0

% DET: points B and C are not the same
% Constructing a line a which passes through point B and point C
line a B C

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(E_{b},B) intersect% DET: points B and H_{a} must be different
% Constructing a point P_{\_G122619} which is a foot of the point E_{b} on the line a
foot P_{\_G122619} E_{b} a
cmark_r P_{\_G122619}
color 200 200 200
drawline E_{b} P_{\_G122619}
color 0 0 0

% Constructing a point H_{a} which is an image of the point B in the symmetry to point/line P_{\_G
122619}
sim H_{a} P_{\_G122619} B

```

```

cmark_r H_{a}

% DET: points H and H_{a} are not the same
% Constructing a line h_{a} which passes through point H and point H_{a}
line h_{a} H H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: lines c and h_{a} are not parallel% DET: lines c and h_{a} are not the same
% Constructing a point A which belongs to line c and line h_{a}
intersec A c h_{a}
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and h_{a} are not parallel; line a and circle k(E_{b},B)
% intersect; line h_{c} and circle k(E_{b},B) intersect; points H_{c} and E_{b} are not the same
% Determination conditions: lines c and h_{a} are not the same; points H and H_{a} are not the same
% ; points B and H_{a} must be different; points B and C are not the same; points H_{c} and B are
% not the same; points H_{c} and H must be different; points E_{c} and H_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $E_c = \neg E_c$

Proving failed

4.1.3 Proving $H_c = \neg H_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

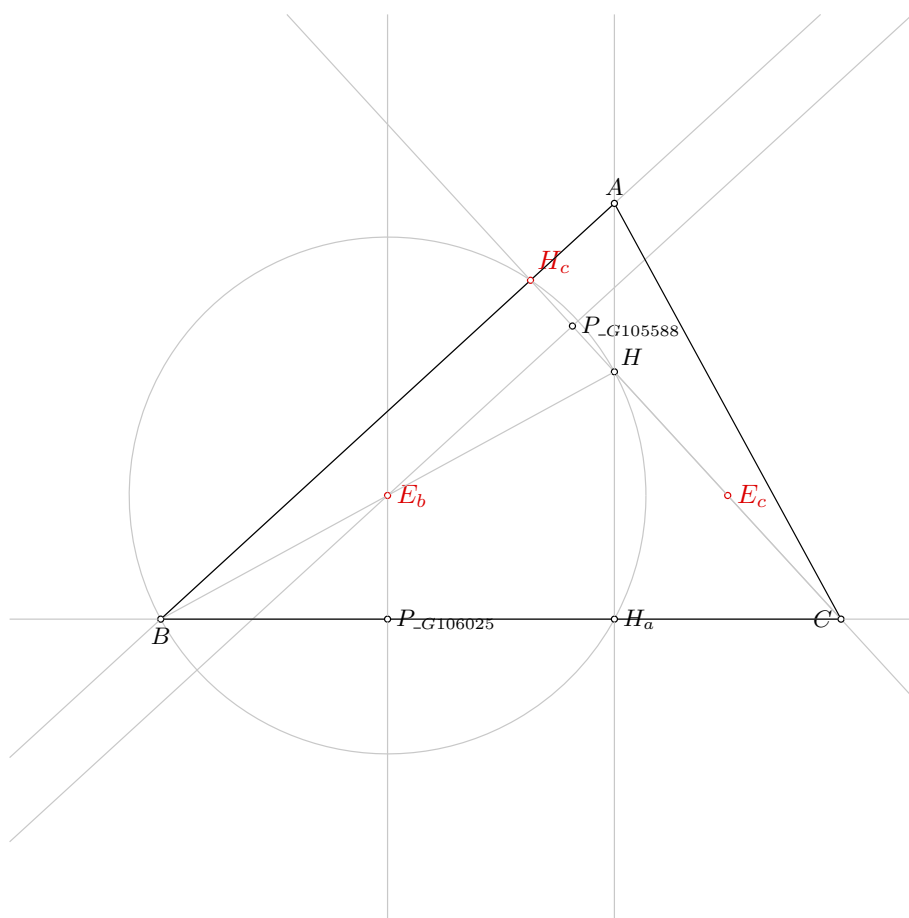


Figure 1: Illustration of the problem 0871

4.2.2 Proving $E_c = E_c$

Proving failed

4.2.3 Proving $H_c = H_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = E_b$

Proving failed

4.3.2 Proving $E_c = E_c$

Proving failed

4.3.3 Proving $H_c = H_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = E_b$

Proving failed

4.4.2 Proving $E_c = E_c$

Proving failed

4.4.3 Proving $H_c = H_c$

Proving failed

Problem 872

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 872: Given a point E_b , a point E_c and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 873

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 873: Given a point E_b , a point E_c and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point M_a , construct a line $m(CH_b)$ (rule W02); % DET: points E_c and M_a are not the same;
2. Using the point E_b and the point E_c , construct a line $m(E_bE_c)$ (rule W14); % DET: points E_b and E_c are not the same;
3. Using the point E_b and the point M_a , construct a line $m(E_bM_a)$ (rule W14); % DET: points E_b and M_a are not the same;
4. Using the line $m(E_bM_a)$ and the line $m(E_bE_c)$, construct a point N (rule W03); % NDG: lines $m(E_bM_a)$ and $m(E_bE_c)$ are not parallel % DET: lines $m(E_bM_a)$ and $m(E_bE_c)$ are not the same;
5. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
6. Using the point E_b and the line $m(CH_b)$, construct a line h_b (rule W16); ;
7. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
8. Using the point H_b and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_b and E_c are not the same;
9. Using the circle $k(E_c, C)$, the line h_b , the point E_c and the point H_b , construct a point H (rule W05); % NDG: line h_b and circle $k(E_c, C)$ intersect % DET: points H_b and H must be different;

10. Using the point H and the point E_b , construct a point B (rule W01); ;
11. Using the point E_c and the point H , construct a point C (rule W01); ;
12. Using the point N and the point H , construct a point G (rule W01); ;
13. Using the point M_a and the point G , construct a point A (rule W01); .

Non-degenerate conditions: line h_b and circle $k(E_c, C)$ intersect; points H_b and E_c are not the same; line h_b and circle $k(N, M_a)$ intersect; points E_b and N are not the same; lines $m(E_b M_a)$ and $m(E_b E_c)$ are not parallel.

Determination conditions: points H_b and H must be different; points E_b and H_b must be different; lines $m(E_b M_a)$ and $m(E_b E_c)$ are not the same; points E_b and M_a are not the same; points E_b and E_c are not the same; points E_c and M_a are not the same.

Rules used: [W01,W02,W03,W05,W06,W14,W16]

Lemmas used: [D29,D3,D30,D32,D6,D9,GD02,GL01,GL03,GL04,GL09,L16,L20,L23,L24,L37,L38,L52,L53,L55]

Solving time: 25.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
point E_{c} 95 56.36
point M_{a} 65 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{b}
cmark_r E_{c}
cmark_r M_{a}
color 0 0 0
fontsize 8
```

```
% DET: points E_{c} and M_{a} are not the same
% Constructing a line m(CH_{b}) which passes through point E_{c} and point M_{a}
line m(CH_{b}) E_{c} M_{a}
```

```
color 200 200 200
drawline m(CH_{b})
color 0 0 0
```

```
% DET: points E_{b} and E_{c} are not the same
% Constructing bisector m(E_{b}E_{c}) of the segment E_{b}E_{c}
med m(E_{b}E_{c}) E_{b} E_{c}
```

```
color 200 200 200
drawline m(E_{b}E_{c})
color 0 0 0
```

```

color 200 200 200
drawsegment E_{b} E_{c}
color 0 0 0

% DET: points E_{b} and M_{a} are not the same
% Constructing bisector m(E_{b}M_{a}) of the segment E_{b}M_{a}
med m(E_{b}M_{a}) E_{b} M_{a}

color 200 200 200
drawline m(E_{b}M_{a})
color 0 0 0

color 200 200 200
drawsegment E_{b} M_{a}
color 0 0 0

% NDG: lines m(E_{b}M_{a}) and m(E_{b}E_{c}) are not parallel% DET: lines m(E_{b}M_{a}) and m(E_{b}
E_{c}) are not the same
% Constructing a point N which belongs to line m(E_{b}M_{a}) and line m(E_{b}E_{c})
intersec N m(E_{b}M_{a}) m(E_{b}E_{c})
cmark_r N

% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Constructing a line h_{b} which contains the point E_{b} and is parallel to the line m(CH_{b})
parallel h_{b} E_{b} m(CH_{b})

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{b} and circle k(N,M_{a}) intersect% DET: points E_{b} and H_{b} must be different
% Constructing a point P_{\_G169415} which is a foot of the point N on the line h_{b}
foot P_{\_G169415} N h_{b}
cmark_r P_{\_G169415}
color 200 200 200
drawline N P_{\_G169415}
color 0 0 0

% Constructing a point H_{b} which is an image of the point E_{b} in the symmetry to point/line P
\_G169415}
sim H_{b} P_{\_G169415} E_{b}

```

```

cmark_l H_{b}

% NDG: points H_{b} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H
_{b}
circle k(E_{c},C) E_{c} H_{b}

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% NDG: line h_{b} and circle k(E_{c},C) intersect% DET: points H_{b} and H must be different
% Constructing a point P_{\_G169719} which is a foot of the point E_{c} on the line h_{b}
foot P_{\_G169719} E_{c} h_{b}
cmark_r P_{\_G169719}
color 200 200 200
drawline E_{c} P_{\_G169719}
color 0 0 0

% Constructing a point H which is an image of the point H_{b} in the symmetry to point/line P_{\_G
169719}
sim H P_{\_G169719} H_{b}
cmark_rt H

% Constructing a point B such that HB/HE_{b}=2
towards B H E_{b} 2
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

% Constructing a line L_{\_G169981} which passes through point N and point H
line L_{\_G169981} N H

color 200 200 200
drawline L_{\_G169981}
color 0 0 0

% Constructing a point P_{\_G170082} with coordinates (0,0)

```

```

point P_{\_G170082} 0 0
cmark_r P_{\_G170082}

% Constructing a point P_{\_G170006} such that NP_{\_G170006}/NP_{\_G170082}=-1
towards P_{\_G170006} N P_{\_G170082} -1
cmark_r P_{\_G170006}
color 200 200 200
drawsegment P_{\_G170082} P_{\_G170006}
color 0 0 0

% Constructing a point P_{\_G170051} such that NP_{\_G170051}/NP_{\_G170082}=3
towards P_{\_G170051} N P_{\_G170082} 3
cmark_r P_{\_G170051}
color 200 200 200
drawsegment N P_{\_G170051}
color 0 0 0

% Constructing a line L_{\_G170012} which passes through point H and point P_{\_G170051}
line L_{\_G170012} H P_{\_G170051}

color 200 200 200
drawline L_{\_G170012}
color 0 0 0

% Constructing a line L_{\_G169975} which contains the point P_{\_G170006} and is parallel to the
line L_{\_G170012}
parallel L_{\_G169975} P_{\_G170006} L_{\_G170012}

color 200 200 200
drawline L_{\_G169975}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G169975} and line L_{\_G169981}
intersec G L_{\_G169975} L_{\_G169981}
cmark_t G

% Constructing a point A such that M_{a}A/M_{a}G=3
towards A M_{a} G 3
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line h_{b} and circle k(E_{c},C) intersect; points H_{b} and E_{c} are
not the same; line h_{b} and circle k(N,M_{a}) intersect; points E_{b} and N are not the same;
lines m(E_{b}M_{a}) and m(E_{b}E_{c}) are not parallel

```

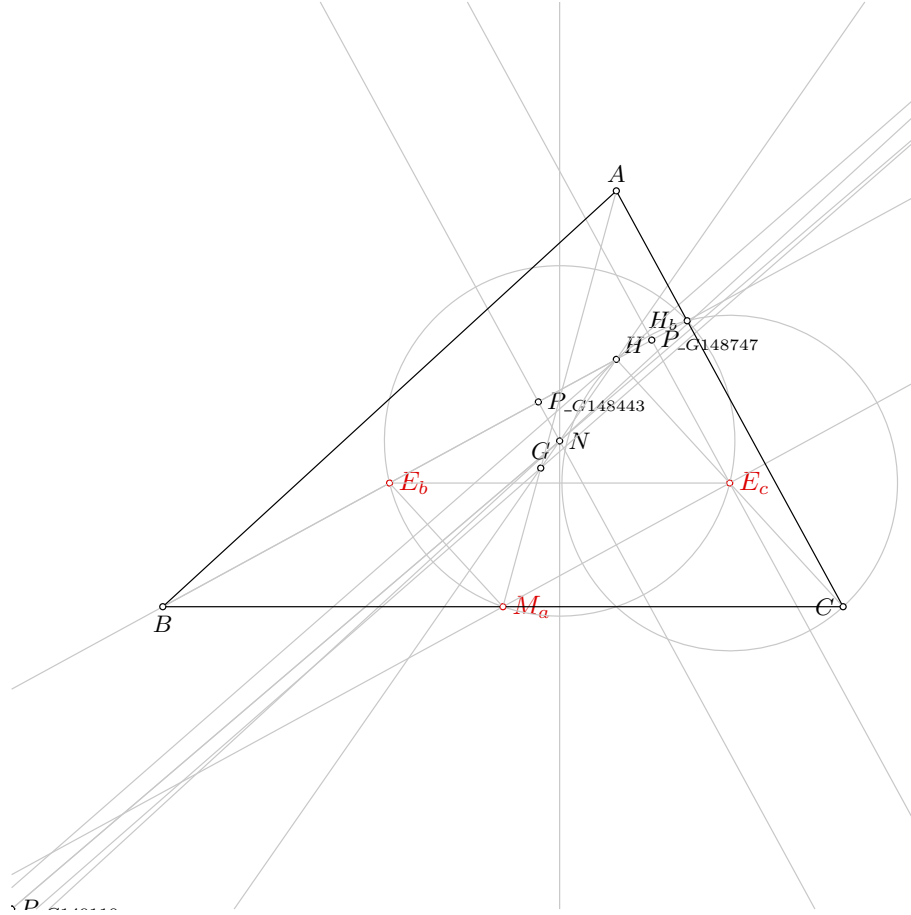


Figure 1: Illustration of the problem 0873

% Determination conditions: points $H_{\{b\}}$ and H must be different; points $E_{\{b\}}$ and $H_{\{b\}}$ must be different; lines $m(E_{\{b\}}M_{\{a\}})$ and $m(E_{\{b\}}E_{\{c\}})$ are not the same; points $E_{\{b\}}$ and $M_{\{a\}}$ are not the same; points $E_{\{b\}}$ and $E_{\{c\}}$ are not the same; points $E_{\{c\}}$ and $M_{\{a\}}$ are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = E_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2812 terms.

Time Complexity: Time spent by the prover is 9.422 seconds.

NDG conditions Points E_b and M_a are not identical

Points E_b and M_a are not identical
 Points E_c , E_b and M_a are not collinear
 Line through points E_c and E_b is not perpendicular to line through points E_b and M_a
 Points E_b and $P_{G161284}$ are not identical
 Points A , B and C are not collinear
 Line through points E_c and B is not perpendicular to line through points B and E_b

4.1.2 Proving $E_c = E_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2820 terms.

Time Complexity: Time spent by the prover is 9.817 seconds.

NDG conditions Points E_b and M_a are not identical

Points E_b and M_a are not identical
 Points E_c , E_b and M_a are not collinear
 Line through points E_c and $P_{G163334}$ is not perpendicular to line through points $P_{G163334}$ and M_a
 Points E_c , $P_{G163334}$ and E_b are not collinear
 Points A , B and C are not collinear
 Line through points E_c and B is not perpendicular to line through points B and E_b

4.1.3 Proving $M_a = M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 17 terms.

Time Complexity: Time spent by the prover is 1.151 seconds.

NDG conditions Points E_b and M_a are not identical

Points E_b and M_a are not identical
 Points E_c , E_b and M_a are not collinear
 Line through points E_c and E_b is not perpendicular to line through points E_b and M_a
 Points E_b and $P_{G165384}$ are not identical

4.2 GCLC - Area method

4.2.1 Proving $E_b = E_b$

Proving failed

4.2.2 Proving $E_c = E_c$

Proving failed

4.2.3 Proving $M_a = M_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = E_b$

Proving failed

4.3.2 Proving $E_c = E_c$

Proving failed

4.3.3 Proving $M_a = M_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = E_b$

Proving failed

4.4.2 Proving $E_c = E_c$

Proving failed

4.4.3 Proving $M_a = M_a$

Proving failed

Problem 874

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 874: Given a point E_b , a point M_b and a point E_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point M_b , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and M_b are not the same;
2. Using the point E_b and the point M_b , construct a line $m(E_bM_b)$ (rule W14); % DET: points E_b and M_b are not the same;
3. Using the line $m(E_bM_b)$ and the line $m(H_aH_c)$, construct a point N (rule W03); % NDG: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel % DET: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same;
4. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
5. Choose freely a point E_c on the circle $k(N, M_a)$ (rule WOncircle);
6. Using the point E_c and the point N , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and N are not the same;
7. Using the circle $k(N, M_a)$, the line $m(H_bH_a)$, the point N and the point E_c , construct a point M_c (rule W05a); % NDG: line $m(H_bH_a)$ and circle $k(N, M_a)$ intersect % DET: points E_c and M_c must be different;
8. Choose freely a point A (rule free);
9. Using the point A and the point M_b , construct a point C (rule W01); ;
10. Using the point A and the point M_c , construct a point B (rule W01); .

Non-degenerate conditions: line $m(H_bH_a)$ and circle $k(N, M_a)$ intersect; points E_b and N are not the same; lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel.

Determination conditions: points E_c and M_c must be different; points E_c and N are not the same; lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same; points E_b and M_b are not the same; points E_b and M_b are not the same.

Rules used: [W01,W02,W03,W05a,W06,W14,WOncircle1,free]

Lemmas used: [D20,D22,D32,GD02,GL01,GL03,GL04,L17,L18,L19,L20,L21,L23,L24,L41,L42,L44,L45,L50,L51]

Solving time: 199.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
```

```
point M_{b} 95 67.5
```

```
point E_{c} 95 56.36
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{b}
```

```
cmark_lt M_{b}
```

```
cmark_r E_{c}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{b} and M_{b} are not the same
```

```
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point M_{b}
```

```
line m(H_{a}H_{c}) E_{b} M_{b}
```

```
color 200 200 200
```

```
drawline m(H_{a}H_{c})
```

```
color 0 0 0
```

```
% DET: points E_{b} and M_{b} are not the same
```

```
% Constructing bisector m(E_{b}M_{b}) of the segment E_{b}M_{b}
```

```
med m(E_{b}M_{b}) E_{b} M_{b}
```

```
color 200 200 200
```

```
drawline m(E_{b}M_{b})
```

```
color 0 0 0
```

```
color 200 200 200
```

```
drawsegment E_{b} M_{b}
```

```
color 0 0 0
```

```
% NDG: lines m(E_{b}M_{b}) and m(H_{a}H_{c}) are not parallel% DET: lines m(E_{b}M_{b}) and m(H_{a}H_{c}) are not the same
```

```
% Constructing a point N which belongs to line m(E_{b}M_{b}) and line m(H_{a}H_{c})
```

```
intersec N m(E_{b}M_{b}) m(H_{a}H_{c})
```

```
cmark_r N
```

```

% NDG: points  $E_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{b\}}$ 
circle k(N, M_{a}) N E_{b}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% Choosing randomly a point  $E_{\{c\}}$  on the circle with center  $N$  through point  $E_{\{b\}}$ 
oncircle E_{c} N E_{b}
cmark_r E_{c}
color 200 200 200
drawcircle N E_{b}
color 0 0 0

% DET: points  $E_{\{c\}}$  and  $N$  are not the same
% Constructing a line  $m(H_{\{b\}}H_{\{a\}})$  which passes through point  $E_{\{c\}}$  and point  $N$ 
line m(H_{b}H_{a}) E_{c} N

color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0

% NDG: line  $m(H_{\{b\}}H_{\{a\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{c\}}$  and  $M_{\{c\}}$  must be
different
% Constructing a point  $M_{\{c\}}$  which is an image of the point  $E_{\{c\}}$  in the symmetry to point/line  $N$ 
sim M_{c} N E_{c}
cmark_lt M_{c}

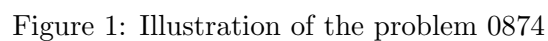
% Constructing a free point  $A$ 
point A 80 95

cmark_t A

% Constructing a point  $C$  such that  $AC/AM_{\{b\}}=2$ 
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% Constructing a point  $B$  such that  $AB/AM_{\{c\}}=2$ 

```



```
% Non-degenerate conditions: line  $m(H_{\{b\}H_{\{a\}}})$  and circle  $k(N,M_{\{a\}})$  intersect; points  $E_{\{b\}}$  and  $N$ 
are not the same; lines  $m(E_{\{b\}M_{\{b\}}})$  and  $m(H_{\{a\}H_{\{c\}}})$  are not parallel
% Determination conditions: points  $E_{\{c\}}$  and  $M_{\{c\}}$  must be different; points  $E_{\{c\}}$  and  $N$  are not
the same; lines  $m(E_{\{b\}M_{\{b\}}})$  and  $m(H_{\{a\}H_{\{c\}}})$  are not the same; points  $E_{\{b\}}$  and  $M_{\{b\}}$  are
not the same; points  $E_{\{b\}}$  and  $M_{\{b\}}$  are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.054 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $E_c = \neg E_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{M_{m(E_b M_b)}^0}^{E_b M_b} \neq S_{T_{m(E_b M_b)}^1}^{E_b M_b}$ i.e., lines $M_{m(E_b M_b)}^0$ and $T_{m(E_b M_b)}^1$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2 BF_{\neg h_b}^3}$ i.e., lines $AF_{\neg h_a}^2$ and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

4.2.3 Proving $E_c = \neg E_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $M_b = \neg M_b$

Proving failed

4.3.3 Proving $E_c = \neg E_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $M_b = \neg M_b$

Proving failed

4.4.3 Proving $E_c = \neg E_c$

Proving failed

Problem 875

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 875: Given a point E_c , a point M_c and a point E_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point M_c , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and M_c are not the same;
2. Using the point E_c and the point M_c , construct a line $m(E_cM_c)$ (rule W14); % DET: points E_c and M_c are not the same;
3. Using the line $m(E_cM_c)$ and the line $m(H_bH_a)$, construct a point N (rule W03); % NDG: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel % DET: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same;
4. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
5. Choose freely a point E_b on the circle $k(N, M_a)$ (rule WOncircle);
6. Using the point E_b and the point N , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and N are not the same;
7. Using the circle $k(N, M_a)$, the line $m(H_aH_c)$, the point N and the point E_b , construct a point M_b (rule W05a); % NDG: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect % DET: points E_b and M_b must be different;
8. Choose freely a point A (rule free);
9. Using the point A and the point M_c , construct a point B (rule W01); ;
10. Using the point A and the point M_b , construct a point C (rule W01); .

Non-degenerate conditions: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect; points E_c and N are not the same; lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel.

Determination conditions: points E_b and M_b must be different; points E_b and N are not the same; lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same; points E_c and M_c are not the same; points E_c and M_c are not the same.

Rules used: [W01,W02,W03,W05a,W06,W14,WOncircle1,free]

Lemmas used: [D20,D22,D32,GD02,GL01,GL03,GL04,L17,L18,L19,L20,L21,L23,L24,L41,L42,L44,L45,L50,L51]

Solving time: 199.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
```

```
point M_{c} 50 67.5
```

```
point E_{b} 50 56.36
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{c}
```

```
cmark_lt M_{c}
```

```
cmark_r E_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{c} and M_{c} are not the same
```

```
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point M_{c}
```

```
line m(H_{b}H_{a}) E_{c} M_{c}
```

```
color 200 200 200
```

```
drawline m(H_{b}H_{a})
```

```
color 0 0 0
```

```
% DET: points E_{c} and M_{c} are not the same
```

```
% Constructing bisector m(E_{c}M_{c}) of the segment E_{c}M_{c}
```

```
med m(E_{c}M_{c}) E_{c} M_{c}
```

```
color 200 200 200
```

```
drawline m(E_{c}M_{c})
```

```
color 0 0 0
```

```
color 200 200 200
```

```
drawsegment E_{c} M_{c}
```

```
color 0 0 0
```

```
% NDG: lines m(E_{c}M_{c}) and m(H_{b}H_{a}) are not parallel% DET: lines m(E_{c}M_{c}) and m(H_{b}H_{a}) are not the same
```

```
% Constructing a point N which belongs to line m(E_{c}M_{c}) and line m(H_{b}H_{a})
```

```
intersec N m(E_{c}M_{c}) m(H_{b}H_{a})
```

```
cmark_r N
```



```

% NDG: points  $E_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{c\}}$ 
circle k(N, M_{a}) N E_{c}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% Choosing randomly a point  $E_{\{b\}}$  on the circle with center  $N$  through point  $E_{\{c\}}$ 
oncircle E_{b} N E_{c}
cmark_r E_{b}
color 200 200 200
drawcircle N E_{c}
color 0 0 0

% DET: points  $E_{\{b\}}$  and  $N$  are not the same
% Constructing a line  $m(H_{\{a\}}H_{\{c\}})$  which passes through point  $E_{\{b\}}$  and point  $N$ 
line m(H_{a}H_{c}) E_{b} N

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

% NDG: line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{b\}}$  and  $M_{\{b\}}$  must be
different
% Constructing a point  $M_{\{b\}}$  which is an image of the point  $E_{\{b\}}$  in the symmetry to point/line  $N$ 
sim M_{b} N E_{b}
cmark_lt M_{b}

% Constructing a free point  $A$ 
point A 80 95

cmark_t A

% Constructing a point  $B$  such that  $AB/AM_{\{c\}}=2$ 
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% Constructing a point  $C$  such that  $AC/AM_{\{b\}}=2$ 

```

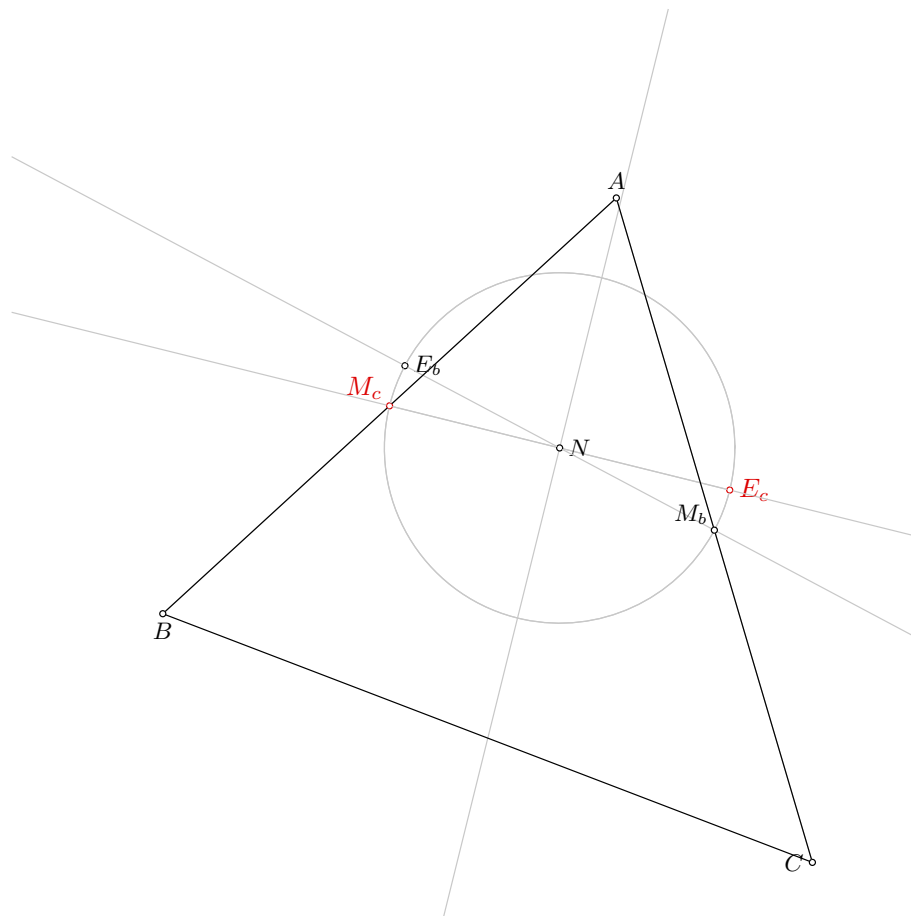


Figure 1: Illustration of the problem 0875

```
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

% Non-degenerate conditions: line $m(H_{\{a\}}H_{\{c\}})$ and circle $k(N,M_{\{a\}})$ intersect; points $E_{\{c\}}$ and N are not the same; lines $m(E_{\{c\}}M_{\{c\}})$ and $m(H_{\{b\}}H_{\{a\}})$ are not parallel
% Determination conditions: points $E_{\{b\}}$ and $M_{\{b\}}$ must be different; points $E_{\{b\}}$ and N are not the same; lines $m(E_{\{c\}}M_{\{c\}})$ and $m(H_{\{b\}}H_{\{a\}})$ are not the same; points $E_{\{c\}}$ and $M_{\{c\}}$ are not the same; points $E_{\{c\}}$ and $M_{\{c\}}$ are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $M_c = \neg M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.051 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $E_b = \neg E_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{m(E_c M_c) E_c M_c}^0 \neq S_{m(E_c M_c) E_c M_c}^1$ i.e., lines $M_{m(E_c M_c)}^0 T_{m(E_c M_c)}^1$ and $E_c M_c$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2 BF_{\neg h_b}^3}$ i.e., lines $AF_{\neg h_a}^2$ and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.2.3 Proving $E_b = \neg E_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $M_c = \neg M_c$

Proving failed

4.3.3 Proving $E_b = \neg E_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = E_c$

Proving failed

4.4.2 Proving $M_c = M_c$

Proving failed

4.4.3 Proving $E_b = E_b$

Proving failed

Problem 876

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 876: Given a point E_c , a point N and a point E_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point N , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and N are not the same;
2. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
3. Choose freely a point E_b on the circle $k(N, M_a)$ (rule W0ncircle);
4. Using the point E_b and the point N , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and N are not the same;
5. Using the circle $k(N, M_a)$, the line $m(H_bH_a)$, the point N and the point E_c , construct a point M_c (rule W05a); % NDG: line $m(H_bH_a)$ and circle $k(N, M_a)$ intersect % DET: points E_c and M_c must be different;
6. Using the circle $k(N, M_a)$, the line $m(H_aH_c)$, the point N and the point E_b , construct a point M_b (rule W05a); % NDG: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect % DET: points E_b and M_b must be different;
7. Choose freely a point A (rule free);
8. Using the point A and the point M_c , construct a point B (rule W01); ;
9. Using the point A and the point M_b , construct a point C (rule W01); .

Non-degenerate conditions: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect; line $m(H_bH_a)$ and circle $k(N, M_a)$ intersect; points E_c and N are not the same.

Determination conditions: points E_b and M_b must be different; points E_c and M_c must be different; points E_b and N are not the same; points E_c and N are not the same.

Rules used: [W01,W02,W05a,W06,WOncircle1,free]

Lemmas used: [D20,D22,D32,GD02,GL01,GL03,GL04,L17,L18,L19,L20,L21,L23,L24,L41,L42,L44,L45,L50,L51]

Solving time: 150.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
point N 72.5 61.93
point E_{b} 50 56.36
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{c}
cmark_r N
cmark_r E_{b}
color 0 0 0
fontsize 8
```

```
% DET: points E_{c} and N are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point N
line m(H_{b}H_{a}) E_{c} N
```

```
color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0
```

```
% NDG: points E_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{c}
circle k(N,M_{a}) N E_{c}
```

```
color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```
% Choosing randomly a point E_{b} on the circle with center N through point E_{c}
oncircle E_{b} N E_{c}
cmark_r E_{b}
color 200 200 200
drawcircle N E_{c}
color 0 0 0
```

```
% DET: points E_{b} and N are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point N
```

```

line m(H_{a}H_{c}) E_{b} N

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

% NDG: line m(H_{b}H_{a}) and circle k(N,M_{a}) intersect% DET: points E_{c} and M_{c} must be
different
% Constructing a point M_{c} which is an image of the point E_{c} in the symmetry to point/line N
sim M_{c} N E_{c}
cmark_lt M_{c}

% NDG: line m(H_{a}H_{c}) and circle k(N,M_{a}) intersect% DET: points E_{b} and M_{b} must be
different
% Constructing a point M_{b} which is an image of the point E_{b} in the symmetry to point/line N
sim M_{b} N E_{b}
cmark_lt M_{b}

% Constructing a free point A
point A 80 95

cmark_t A

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line m(H_{a}H_{c}) and circle k(N,M_{a}) intersect; line m(H_{b}H_{a})
and circle k(N,M_{a}) intersect; points E_{c} and N are not the same
% Determination conditions: points E_{b} and M_{b} must be different; points E_{c} and M_{c} must
be different; points E_{b} and N are not the same; points E_{c} and N are not the same

```

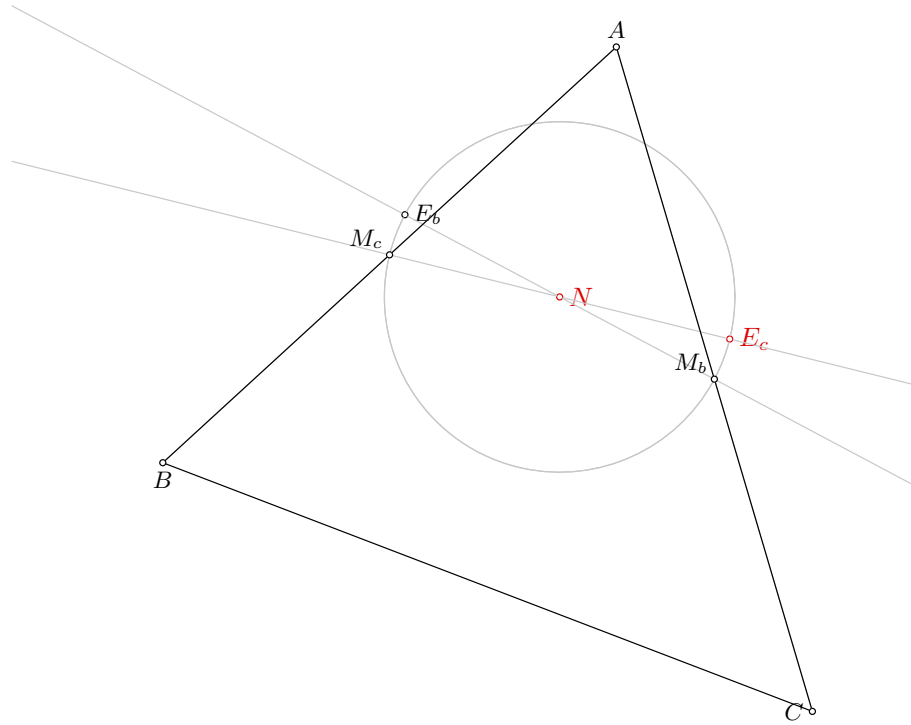


Figure 1: Illustration of the problem 0876

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = E_c$

Proving failed

4.1.2 Proving $N = N$

Proving failed

4.1.3 Proving $E_b = E_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $E_b = \neg E_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $E_b = \neg E_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $E_b = \neg E_b$

Proving failed

Problem 877

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 877: Given a point E_b , a point E_c and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 878

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 878: Given a point E_b , a point E_c and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 879

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 879: Given a point E_b , a point E_c and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 880

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 880: Given a point E_b , a point E_c and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 881

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 881: Given a point E_b , a point G and a point H , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H , construct a point B (rule W01); ;
2. Using the point G and the point H , construct a point N (rule W01); ;
3. Using the point G and the point H , construct a point O (rule W01); ;
4. Using the point G and the point B , construct a point M_b (rule W01); ;
5. Using the point E_b and the point H , construct a line h_b (rule W02); % DET: points E_b and H are not the same;
6. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
8. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
9. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; line h_b and circle $k(N, M_a)$ intersect; points E_b and N are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points E_b and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L16,L20,L23,L56,L58]

Solving time: 7.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{b} 50 56.36
point G 70 58.33
point H 80 72.73

color 220 0 0
fontsize 9

cmark_r E_{b}
cmark_t G
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a point B such that E_{b}B/E_{b}H=-1
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

% Constructing a point N such that GN/GH=0.25
towards N G H 0.25
cmark_r N
color 200 200 200
drawsegment G H
color 0 0 0

% Constructing a point O such that GO/GH=-0.5
towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% Constructing a point M_{b} such that GM_{b}/GB=-0.5
```

```

towards M_{b} G B -0.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% DET: points E_{b} and H are not the same
% Constructing a line h_{b} which passes through point E_{b} and point H
line h_{b} E_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{b} and circle k(N,M_{a}) intersect% DET: points E_{b} and H_{b} must be different
% Constructing a point P_{\_G70721} which is a foot of the point N on the line h_{b}
foot P_{\_G70721} N h_{b}
cmark_r P_{\_G70721}
color 200 200 200
drawline N P_{\_G70721}
color 0 0 0

% Constructing a point H_{b} which is an image of the point E_{b} in the symmetry to point/line P
\_G70721}
sim H_{b} P_{\_G70721} E_{b}
cmark_l H_{b}

% DET: points H_{b} and M_{b} are not the same
% Constructing a line b which passes through point H_{b} and point M_{b}
line b H_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200

```



```
drawcircle k(O,C)
color 0 0 0
```

```
% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;
% line h_{b} and circle k(N,M_{a}) intersect; points E_{b} and N are not the same
% Determination conditions: points H_{b} and M_{b} are not the same; points E_{b} and H_{b} must be
% different; points E_{b} and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = _E_b$

Proving failed

4.1.2 Proving $G = _G$

Proving failed

4.1.3 Proving $H = _H$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = _E_b$

Proving failed

4.2.2 Proving $G = _G$

Proving failed

4.2.3 Proving $H = _H$

Proving failed

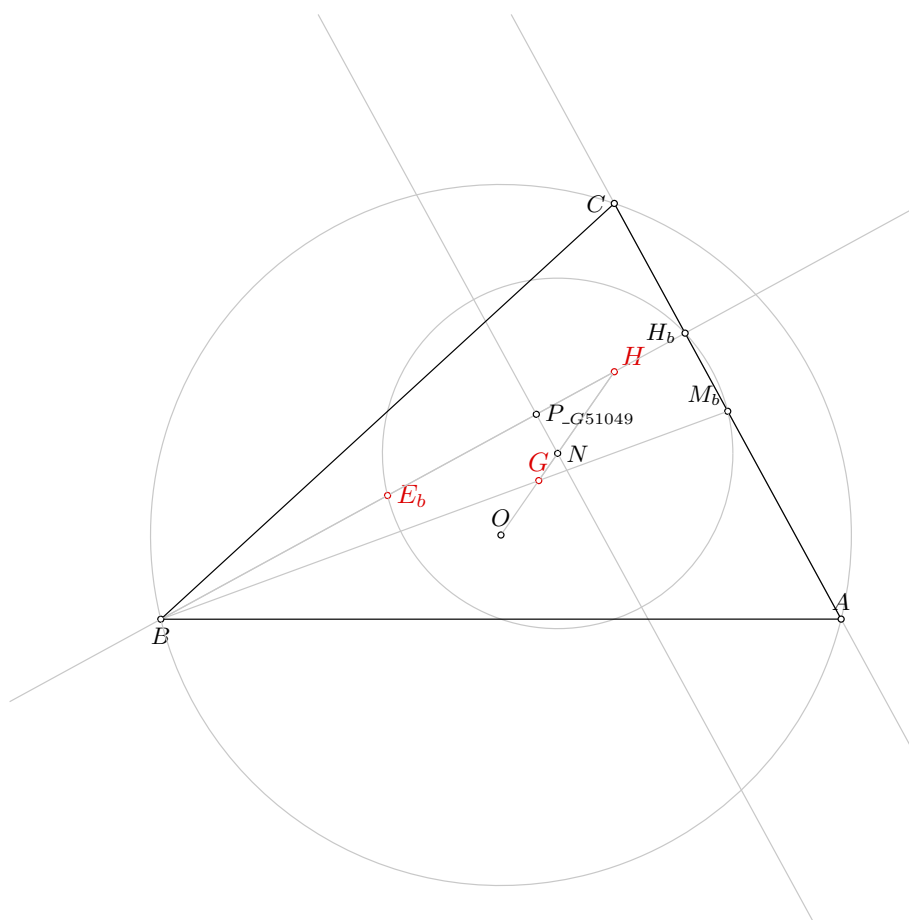


Figure 1: Illustration of the problem 0881

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $G = \neg G$

Proving failed

4.3.3 Proving $H = \neg H$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $G = \neg G$

Proving failed

4.4.3 Proving $H = \neg H$

Proving failed

Problem 882

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 882: Given a point E_b , a point G and a point H_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 883

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 883: Given a point E_b , a point G and a point H_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;
2. Using the point H_b and the line h_b , construct a line b (rule W10a); ;
3. Using the point G , the line h_b and the point E_b , construct a line $h_{G,-1/2}(h_b)$ (rule W15); ;
4. Using the line $h_{G,-1/2}(h_b)$ and the line b , construct a point M_b (rule W03); % NDG: lines $h_{G,-1/2}(h_b)$ and b are not parallel % DET: lines $h_{G,-1/2}(h_b)$ and b are not the same;
5. Using the point M_b and the point G , construct a point B (rule W01); ;
6. Using the point E_b and the point B , construct a point H (rule W01); ;
7. Using the point G and the point H , construct a point O (rule W01); ;
8. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
9. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; lines $h_{G,-1/2}(h_b)$ and b are not parallel.

Determination conditions: lines $h_{G,-1/2}(h_b)$ and b are not the same; points E_b and H_b are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W15]

Lemmas used: [D22,D26,D29,D3,D6,D9,GD01,GL02,GL03,GL04,GL09,L11,L12,L56,L58]

Solving time: 4.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{b} 50 56.36
point G 70 58.33
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_r E_{b}
cmark_t G
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points E_{b} and H_{b} are not the same
% Constructing a line h_{b} which passes through point E_{b} and point H_{b}
line h_{b} E_{b} H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
perp b H_{b} h_{b}

color 200 200 200
drawline b
color 0 0 0

% Constructing a point P_{\_G103345} such that GP_{\_G103345}/GE_{b}=-0.5
towards P_{\_G103345} G E_{b} -0.5
cmark_r P_{\_G103345}
color 200 200 200
drawsegment E_{b} P_{\_G103345}
color 0 0 0

% Constructing a line h_{G,-1/2}(h_{b}) which contains the point P_{\_G103345} and is parallel to
the line h_{b}
parallel h_{G,-1/2}(h_{b}) P_{\_G103345} h_{b}

color 200 200 200
drawline h_{G,-1/2}(h_{b})
color 0 0 0
```

```

% NDG: lines  $h_{\{G,-1/2\}}(h_{\{b\}})$  and  $b$  are not parallel% DET: lines  $h_{\{G,-1/2\}}(h_{\{b\}})$  and  $b$  are not
the same
% Constructing a point  $M_{\{b\}}$  which belongs to line  $h_{\{G,-1/2\}}(h_{\{b\}})$  and line  $b$ 
intersec  $M_{\{b\}}$   $h_{\{G,-1/2\}}(h_{\{b\}})$   $b$ 
cmark_lt  $M_{\{b\}}$ 

% Constructing a point  $B$  such that  $M_{\{b\}}B/M_{\{b\}}G=3$ 
towards B  $M_{\{b\}}$  G 3
cmark_b B
color 200 200 200
drawsegment  $M_{\{b\}}$  B
color 0 0 0

% Constructing a point  $H$  such that  $E_{\{b\}}H/E_{\{b\}}B=-1$ 
towards H  $E_{\{b\}}$  B -1
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0

% Constructing a point  $O$  such that  $GO/GH=-0.5$ 
towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

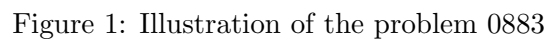
% NDG: points  $B$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $B$ 
circle  $k(O,C)$  O B

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $b$  and circle  $k(O,C)$  intersect
% Constructing points  $C$  and  $A$  which are in intersection of  $k(O,C)$  and  $b$ 
intersec2 C A  $k(O,C)$   $b$ 
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

```



3.3 Illustration

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = -E_b$

1416

4.1.2 Proving $G=_G$

Proving failed

4.1.3 Proving $H_b=_H H_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b=_E E_b$

Proving failed

4.2.2 Proving $G=_G$

Proving failed

4.2.3 Proving $H_b=_H H_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b=_E E_b$

Proving failed

4.3.2 Proving $G=_G$

Proving failed

4.3.3 Proving $H_b=_H H_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b=_E E_b$

Proving failed

4.4.2 Proving $G=_G$

Proving failed

4.4.3 Proving $H_b=_H H_b$

Proving failed

Problem 884

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 884: Given a point E_b , a point G and a point H_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 885

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 885: Given a point E_b , a point G and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 886

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 886: Given a point E_b , a point G and a point M_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 887

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 887: Given a point E_b , a point G and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point G and the point M_b , construct a point B (rule W01); ;
2. Using the point E_b and the point B , construct a point H (rule W01); ;
3. Using the point G and the point H , construct a point N (rule W01); ;
4. Using the point G and the point H , construct a point O (rule W01); ;
5. Using the point E_b and the point B , construct a line h_b (rule W02); % DET: points E_b and B are not the same;
6. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
8. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
9. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; line h_b and circle $k(N, M_a)$ intersect; points E_b and N are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points E_b and B are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L16,L20,L23,L56,L58]

Solving time: 7.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{b} 50 56.36
point G 70 58.33
point M_{b} 95 67.5

color 220 0 0
fontsize 9

cmark_r E_{b}
cmark_t G
cmark_lt M_{b}
color 0 0 0
fontsize 8

% Constructing a point B such that GB/GM_{b}=-2
towards B G M_{b} -2
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

% Constructing a point H such that E_{b}H/E_{b}B=-1
towards H E_{b} B -1
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0

% Constructing a point N such that GN/GH=0.25
towards N G H 0.25
cmark_r N
color 200 200 200
drawsegment G H
color 0 0 0

% Constructing a point O such that GO/GH=-0.5
```

```

towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% DET: points  $E_{\{b\}}$  and  $B$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $E_{\{b\}}$  and point  $B$ 
line  $h_{\{b\}}$   $E_{\{b\}}$  B

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: points  $E_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{b\}}$ 
circle  $k(N, M_{\{a\}})$  N  $E_{\{b\}}$ 

color 200 200 200
drawcircle  $k(N, M_{\{a\}})$ 
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{b\}}$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G146870\}}$  which is a foot of the point  $N$  on the line  $h_{\{b\}}$ 
foot  $P_{\{\backslash\_G146870\}}$  N  $h_{\{b\}}$ 
cmark_r  $P_{\{\backslash\_G146870\}}$ 
color 200 200 200
drawline N  $P_{\{\backslash\_G146870\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $E_{\{b\}}$  in the symmetry to point/line  $P_{\{\backslash\_G146870\}}$ 
sim  $H_{\{b\}}$   $P_{\{\backslash\_G146870\}}$   $E_{\{b\}}$ 
cmark_l  $H_{\{b\}}$ 

% DET: points  $H_{\{b\}}$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $M_{\{b\}}$ 
line b  $H_{\{b\}}$   $M_{\{b\}}$ 

color 200 200 200
drawline b
color 0 0 0

% NDG: points  $B$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $B$ 
circle  $k(O, C)$  O B

color 200 200 200

```

```
drawcircle k(O,C)
color 0 0 0
```

```
% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;
% line h_{b} and circle k(N,M_{a}) intersect; points E_{b} and N are not the same
% Determination conditions: points H_{b} and M_{b} are not the same; points E_{b} and H_{b} must be
% different; points E_{b} and B are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $G = \neg G$

Proving failed

4.1.3 Proving $M_b = \neg M_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

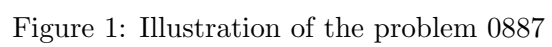
Proving failed

4.2.2 Proving $G = \neg G$

Proving failed

4.2.3 Proving $M_b = \neg M_b$

Proving failed



4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $G = \neg G$

Proving failed

4.3.3 Proving $M_b = \neg M_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $G = \neg G$

Proving failed

4.4.3 Proving $M_b = \neg M_b$

Proving failed

Problem 888

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 888: Given a point E_b , a point G and a point M_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 889

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 889: Given a point E_b , a point G and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point G and the point N , construct a point H (rule W01); ;
2. Using the point E_b and the point H , construct a point B (rule W01); ;
3. Using the point G and the point N , construct a point O (rule W01); ;
4. Using the point G and the point B , construct a point M_b (rule W01); ;
5. Using the point E_b and the point H , construct a line h_b (rule W02); % DET: points E_b and H are not the same;
6. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
8. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
9. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; line h_b and circle $k(N, M_a)$ intersect; points E_b and N are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points E_b and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L15,L16,L20,L23,L56]

Solving time: 7.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{b} 50 56.36
point G 70 58.33
point N 72.5 61.93

color 220 0 0
fontsize 9

cmark_r E_{b}
cmark_t G
cmark_r N
color 0 0 0
fontsize 8

% Constructing a point H such that GH/GN=4
towards H G N 4
cmark_rt H
color 200 200 200
drawsegment G H
color 0 0 0

% Constructing a point B such that E_{b}B/E_{b}H=-1
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

% Constructing a point O such that GO/GN=-2
towards O G N -2
cmark_t O
color 200 200 200
drawsegment N O
color 0 0 0

% Constructing a point M_{b} such that GM_{b}/GB=-0.5
```

```

towards M_{b} G B -0.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% DET: points E_{b} and H are not the same
% Constructing a line h_{b} which passes through point E_{b} and point H
line h_{b} E_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{b} and circle k(N,M_{a}) intersect% DET: points E_{b} and H_{b} must be different
% Constructing a point P_{\_G193713} which is a foot of the point N on the line h_{b}
foot P_{\_G193713} N h_{b}
cmark_r P_{\_G193713}
color 200 200 200
drawline N P_{\_G193713}
color 0 0 0

% Constructing a point H_{b} which is an image of the point E_{b} in the symmetry to point/line P
\_G193713}
sim H_{b} P_{\_G193713} E_{b}
cmark_l H_{b}

% DET: points H_{b} and M_{b} are not the same
% Constructing a line b which passes through point H_{b} and point M_{b}
line b H_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200

```

```
drawcircle k(O,C)
color 0 0 0
```

```
% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;
% line h_{b} and circle k(N,M_{a}) intersect; points E_{b} and N are not the same
% Determination conditions: points H_{b} and M_{b} are not the same; points E_{b} and H_{b} must be
% different; points E_{b} and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $G = \neg G$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

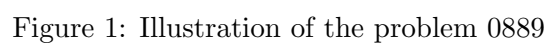
Proving failed

4.2.2 Proving $G = \neg G$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed



4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $G = \neg G$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $G = \neg G$

Proving failed

4.4.3 Proving $N = \neg N$

Proving failed

Problem 890

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 890: Given a point E_b , a point G and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point G and the point O , construct a point N (rule W01); ;
2. Using the point G and the point O , construct a point H (rule W01); ;
3. Using the point E_b and the point H , construct a point B (rule W01); ;
4. Using the point G and the point B , construct a point M_b (rule W01); ;
5. Using the point E_b and the point H , construct a line h_b (rule W02); % DET: points E_b and H are not the same;
6. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
8. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
9. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; line h_b and circle $k(N, M_a)$ intersect; points E_b and N are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points E_b and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L15,L20,L23,L56,L58]

Solving time: 7.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
```

```
point G 70 58.33
```

```
point O 65 51.14
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{b}
```

```
cmark_t G
```

```
cmark_t O
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point N such that GN/GO=-0.5
```

```
towards N G O -0.5
```

```
cmark_r N
```

```
color 200 200 200
```

```
drawsegment O N
```

```
color 0 0 0
```

```
% Constructing a point H such that GH/GO=-2
```

```
towards H G O -2
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment O H
```

```
color 0 0 0
```

```
% Constructing a point B such that E_{b}B/E_{b}H=-1
```

```
towards B E_{b} H -1
```

```
cmark_b B
```

```
color 200 200 200
```

```
drawsegment H B
```

```
color 0 0 0
```

```
% Constructing a point M_{b} such that GM_{b}/GB=-0.5
```

```

towards M_{b} G B -0.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% DET: points E_{b} and H are not the same
% Constructing a line h_{b} which passes through point E_{b} and point H
line h_{b} E_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{b} and circle k(N,M_{a}) intersect% DET: points E_{b} and H_{b} must be different
% Constructing a point P_{\_G225880} which is a foot of the point N on the line h_{b}
foot P_{\_G225880} N h_{b}
cmark_r P_{\_G225880}
color 200 200 200
drawline N P_{\_G225880}
color 0 0 0

% Constructing a point H_{b} which is an image of the point E_{b} in the symmetry to point/line P
\_G225880}
sim H_{b} P_{\_G225880} E_{b}
cmark_l H_{b}

% DET: points H_{b} and M_{b} are not the same
% Constructing a line b which passes through point H_{b} and point M_{b}
line b H_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200

```

```
drawcircle k(O,C)
color 0 0 0
```

```
% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;
% line h_{b} and circle k(N,M_{a}) intersect; points E_{b} and N are not the same
% Determination conditions: points H_{b} and M_{b} are not the same; points E_{b} and H_{b} must be
% different; points E_{b} and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $G = \neg G$

Proving failed

4.1.3 Proving $O = \neg O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $G = \neg G$

Proving failed

4.2.3 Proving $O = \neg O$

Proving failed

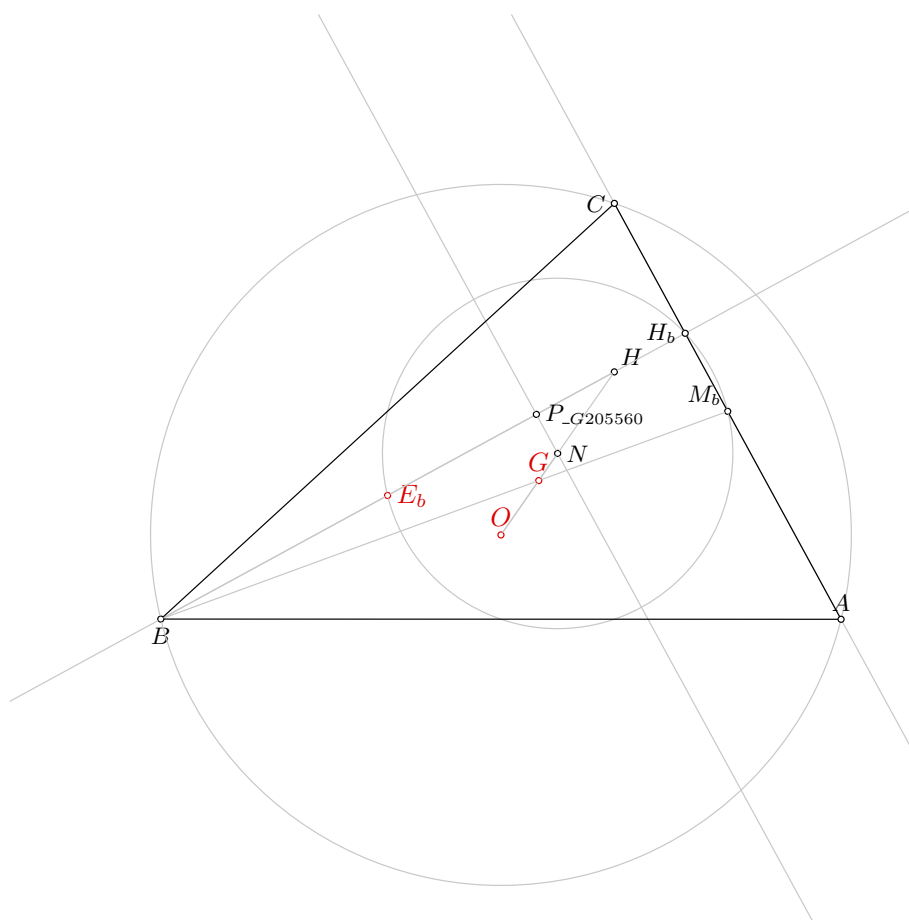


Figure 1: Illustration of the problem 0890

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $G = \neg G$

Proving failed

4.3.3 Proving $O = \neg O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $G = \neg G$

Proving failed

4.4.3 Proving $O = \neg O$

Proving failed

Problem 891

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 891: Given a point E_b , a point G and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 892

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 892: Given a point E_b , a point G and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 893

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 893: Given a point E_b , a point G and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 894

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 894: Given a point H , a point H_a and a point E_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H and the point H_a , construct a line h_a (rule W02); % DET: points H and H_a are not the same;
2. Using the point H_a and the line h_a , construct a line a (rule W10a); ;
3. Using the point H , the line a and the point H_a , construct a line $h_{H,1/2}(a)$ (rule W15); ;
4. Choose freely a point E_b on the line $h_{H,1/2}(a)$ (rule WOnline4);
5. Using the point E_b and the point H , construct a point B (rule W01); ;
6. Using the point H and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H and E_b are not the same;
7. Choose freely a point A on the line h_a (rule WOnline1) ;
8. Using the point A and the point B , construct a line c (rule W02); % DET: points A and B are not the same;
9. Using the circle $k(E_b, B)$, the line c , the point E_b and the point B , construct a point H_c (rule W05); % NDG: line c and circle $k(E_b, B)$ intersect % DET: points B and H_c must be different;
10. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
11. Using the line a and the line h_c , construct a point C (rule W03); % NDG: lines a and h_c are not parallel % DET: lines a and h_c are not the same.

Non-degenerate conditions: lines a and h_c are not parallel; line c and circle $k(E_b, B)$ intersect; points H and E_b are not the same.

Determination conditions: lines a and h_c are not the same; points H_c and H are not the same; points B and H_c must be different; points A and B are not the same; points H and H_a are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,W15,WOnline1,WOnline4]

Lemmas used: [D10,D29,D3,D5,D7,D8,GD01,GD02,GL03,GL04,L3,L49,L51]

Solving time: 689.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H 80 72.73
```

```
point H_{a} 80 40
```

```
point E_{b} 50 56.36
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H
```

```
cmark_r H_{a}
```

```
cmark_r E_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H and H_{a} are not the same
```

```
% Constructing a line h_{a} which passes through point H and point H_{a}
```

```
line h_{a} H H_{a}
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```
% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
```

```
perp a H_{a} h_{a}
```

```
color 200 200 200
```

```
drawline a
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G37215} such that HP_{\_G37215}/HH_{a}=0.5
```

```
towards P_{\_G37215} H H_{a} 0.5
```

```
cmark_r P_{\_G37215}
```

```
color 200 200 200
```

```
drawsegment H H_{a}
```

```
color 0 0 0
```

```

% Constructing a line  $h_{\{H,1/2\}}(a)$  which contains the point  $P_{\{\backslash\_G37215\}}$  and is parallel to the
line a
parallel  $h_{\{H,1/2\}}(a)$   $P_{\{\backslash\_G37215\}}$  a

```

```

color 200 200 200
drawline  $h_{\{H,1/2\}}(a)$ 
color 0 0 0

```

```

% Constructing a point  $P_{\{\backslash\_G37522\}}$  such that  $HP_{\{\backslash\_G37522\}}/HH_{\{a\}}=0.5$ 
towards  $P_{\{\backslash\_G37522\}}$  H  $H_{\{a\}}$  0.5
cmark_r  $P_{\{\backslash\_G37522\}}$ 
color 200 200 200
drawsegment H  $H_{\{a\}}$ 
color 0 0 0

```

```

% Generating random value  $V[_G37461]$ 
random  $V[_G37461]$ 

```

```

% Calculating value  $V[_G37482]$  using formula  $V[_G37461]*20$ 
expression  $V[_G37482]$  {  $V[_G37461]*20$  }

```

```

% Constructing a point  $E_{\{b\}}$  which is a point for which holds  $P_{\{\backslash\_G37522\}}E_{\{b\}} = V[_G37482]$  and
angle  $HP_{\{\backslash\_G37522\}}E_{\{b\}} = 90$ 
turtle  $E_{\{b\}}$  H  $P_{\{\backslash\_G37522\}}$  90  $V[_G37482]$ 
cmark_r  $E_{\{b\}}$ 

```

```

% Constructing a point B such that  $E_{\{b\}}B/E_{\{b\}}H=-1$ 
towards B  $E_{\{b\}}$  H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

```

```

% NDG: points H and  $E_{\{b\}}$  are not the same
% Constructing a circle  $k(E_{\{b\}},B)$  whose center is at point  $E_{\{b\}}$  and which passes through point H
circle  $k(E_{\{b\}},B)$   $E_{\{b\}}$  H

```

```

color 200 200 200
drawcircle  $k(E_{\{b\}},B)$ 
color 0 0 0

```

```

% Choosing randomly a point A on the line  $HH_{\{a\}}$ 
online A H  $H_{\{a\}}$ 
cmark_t A
color 200 200 200

```

```

drawline H H_{a}
color 0 0 0

% DET: points A and B are not the same
% Constructing a line c which passes through point A and point B
line c A B

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(E_{b},B) intersect% DET: points B and H_{c} must be different
% Constructing a point P_{\_G38087} which is a foot of the point E_{b} on the line c
foot P_{\_G38087} E_{b} c
cmark_r P_{\_G38087}
color 200 200 200
drawline E_{b} P_{\_G38087}
color 0 0 0

% Constructing a point H_{c} which is an image of the point B in the symmetry to point/line P_{\_G
38087}
sim H_{c} P_{\_G38087} B
cmark_rt H_{c}

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines a and h_{c} are not parallel% DET: lines a and h_{c} are not the same
% Constructing a point C which belongs to line a and line h_{c}
intersec C a h_{c}
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines a and h_{c} are not parallel; line c and circle k(E_{b},B)
intersect; points H and E_{b} are not the same
% Determination conditions: lines a and h_{c} are not the same; points H_{c} and H are not the same
; points B and H_{c} must be different; points A and B are not the same; points H and H_{a} are
not the same

```

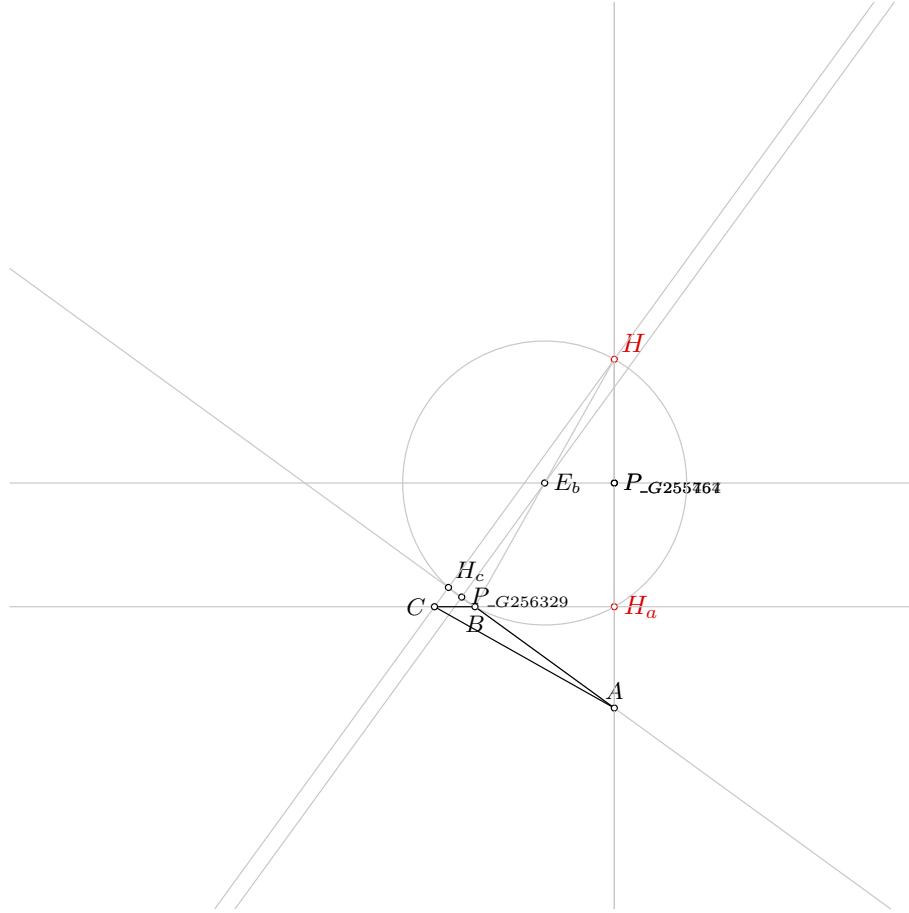


Figure 1: Illustration of the problem 0894

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H = H$

Proving failed

4.1.2 Proving $H_a = H_a$

Proving failed

4.1.3 Proving $E_b = E_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H=_H$

Proving failed

4.2.2 Proving $H_a=_H H_a$

Proving failed

4.2.3 Proving $E_b=_E E_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H=_H$

Proving failed

4.3.2 Proving $H_a=_H H_a$

Proving failed

4.3.3 Proving $E_b=_E E_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H=_H$

Proving failed

4.4.2 Proving $H_a=_H H_a$

Proving failed

4.4.3 Proving $E_b=_E E_b$

Proving failed

Problem 895

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 895: Given a point H , a point H_b and a point E_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H and the point H_b , construct a line h_b (rule W02); % DET: points H and H_b are not the same;
2. Choose freely a point E_b on the line h_b (rule WOnline1) ;
3. Using the point E_b and the point H , construct a point B (rule W01); ;
4. Using the point H and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H and E_b are not the same;
5. Using the point H_b and the line h_b , construct a line b (rule W10a); ;
6. Choose freely a point A on the line b (rule WOnline2);
7. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
8. Using the circle $k(E_b, B)$, the line h_a , the point E_b and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_b, B)$ intersect % DET: points H and H_a must be different;
9. Using the point B and the point H_a , construct a line a (rule W02); % DET: points B and H_a are not the same;
10. Using the line b and the line a , construct a point C (rule W03); % NDG: lines b and a are not parallel % DET: lines b and a are not the same.

Non-degenerate conditions: lines b and a are not parallel; line h_a and circle $k(E_b, B)$ intersect; points H and E_b are not the same.

Determination conditions: lines b and a are not the same; points B and H_a are not the same; points H and H_a must be different; points A and H are not the same; points H and H_b are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D29,D3,D5,D6,D8,D9,GD01,GD02,GL03,GL04,L49,L50]

Solving time: 171.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point H 80 72.73
point H_{b} 89.36 77.83
point E_{b} 50 56.36

color 220 0 0
fontsize 9

cmark_rt H
cmark_l H_{b}
cmark_r E_{b}
color 0 0 0
fontsize 8

% DET: points H and H_{b} are not the same
% Constructing a line h_{b} which passes through point H and point H_{b}
line h_{b} H H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% Choosing randomly a point E_{b} on the line HH_{b}
online E_{b} H H_{b}
cmark_r E_{b}
color 200 200 200
drawline H H_{b}
color 0 0 0

% Constructing a point B such that E_{b}B/E_{b}H=-1
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0
```

```

% NDG: points H and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H
circle k(E_{b},B) E_{b} H

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
perp b H_{b} h_{b}

color 200 200 200
drawline b
color 0 0 0

% Generating random value V[_G63840]
random V[_G63840]

% Calculating value V[_G63861] using formula V[_G63840]*20
expression V[_G63861] { V[_G63840]*20 }

% Constructing a point A which is a point for which holds  $H_{b}A = V[_G63861]$  and angle  $E_{b}H_{b}A = 90$ 
turtle A E_{b} H_{b} 90 V[_G63861]
cmark_t A

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{a} and circle k(E_{b},B) intersect% DET: points H and H_{a} must be different
% Constructing a point P_{\_G64133} which is a foot of the point E_{b} on the line h_{a}
foot P_{\_G64133} E_{b} h_{a}
cmark_r P_{\_G64133}
color 200 200 200
drawline E_{b} P_{\_G64133}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H in the symmetry to point/line P_{\_G64133}
sim H_{a} P_{\_G64133} H

```

```

cmark_r H_{a}

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and a are not parallel; line h_{a} and circle k(E_{b},B)
% intersect; points H and E_{b} are not the same
% Determination conditions: lines b and a are not the same; points B and H_{a} are not the same;
% points H and H_{a} must be different; points A and H are not the same; points H and H_{b} are
% not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H = H$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 61 terms.

Time Complexity: Time spent by the prover is 0.567 seconds.

NDG conditions Points A and B are not identical

Points A and B are not identical

Line through points H_b and B is not perpendicular to line through points B and H

Line through points A and C is not parallel with line through points E_b and H

Point C is not the midpoint of segment with endpoints H_b and B

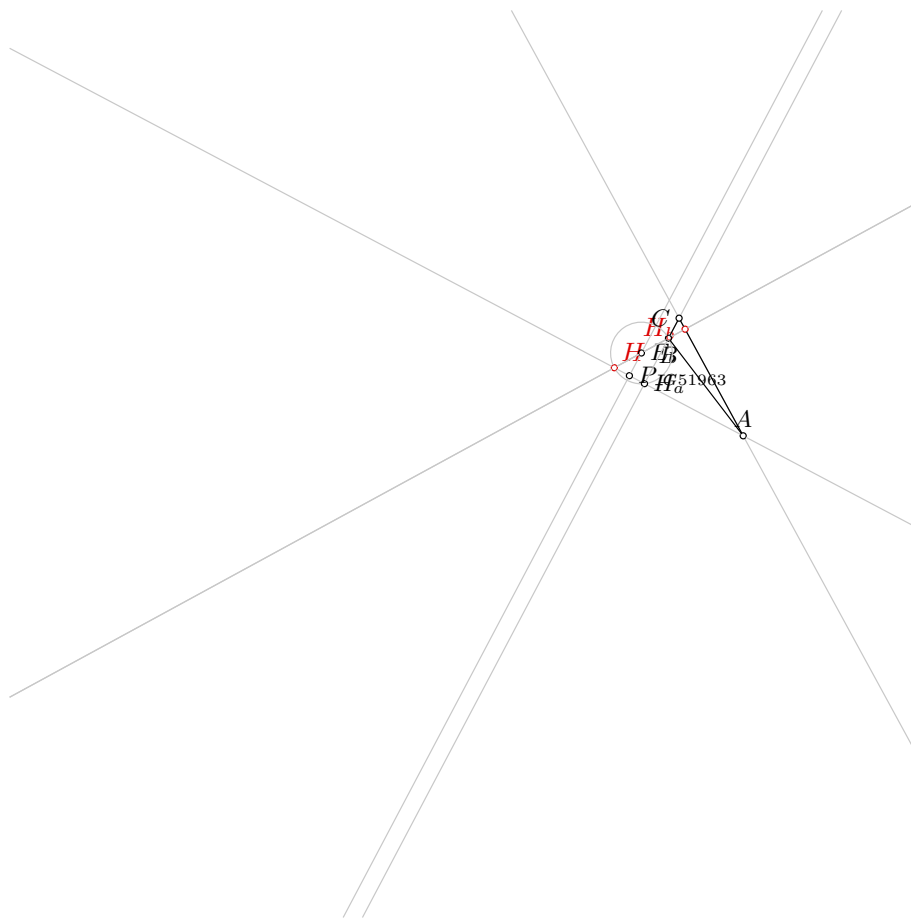


Figure 1: Illustration of the problem 0895

4.1.2 Proving $H_b = \neg H_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

Time Complexity: Time spent by the prover is 0.15 seconds.

NDG conditions Points A and B are not identical

Points A and B are not identical

Line through points H_b and B is not perpendicular to line through points B and H

Line through points A and C is not parallel with line through points E_b and H

4.1.3 Proving $E_b = \neg E_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H = \neg H$

Proving failed

4.2.2 Proving $H_b = \neg H_b$

Proving failed

4.2.3 Proving $E_b = \neg E_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H = \neg H$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 9 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $H_b = \neg H_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 16 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.3.3 Proving $E_b = \neg E_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 13 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $H = \neg H$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.090 seconds. There are no ndg conditions.

4.4.2 Proving $H_b = \neg H_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.080 seconds. There are no ndg conditions.

4.4.3 Proving $E_b = \neg E_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.090 seconds. There are no ndg conditions.

Problem 896

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 896: Given a point H , a point H_c and a point E_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H and the point H_c , construct a line h_c (rule W02); % DET: points H and H_c are not the same;
2. Using the point H_c and the line h_c , construct a line c (rule W10a); ;
3. Using the point H , the line c and the point H_c , construct a line $h_{H,1/2}(c)$ (rule W15); ;
4. Choose freely a point E_b on the line $h_{H,1/2}(c)$ (rule WOnline4);
5. Using the point E_b and the point H , construct a point B (rule W01); ;
6. Using the point H and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H and E_b are not the same;
7. Choose freely a point A on the line c (rule WOnline1) ;
8. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
9. Using the circle $k(E_b, B)$, the line h_a , the point E_b and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_b, B)$ intersect % DET: points H and H_a must be different;
10. Using the point B and the point H_a , construct a line a (rule W02); % DET: points B and H_a are not the same;
11. Using the line h_c and the line a , construct a point C (rule W03); % NDG: lines h_c and a are not parallel % DET: lines h_c and a are not the same.

Non-degenerate conditions: lines h_c and a are not parallel; line h_a and circle $k(E_b, B)$ intersect; points H and E_b are not the same.

Determination conditions: lines h_c and a are not the same; points B and H_a are not the same; points H and H_a must be different; points A and H are not the same; points H and H_c are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,W15,WOnline1,WOnline4]

Lemmas used: [D10,D29,D3,D5,D7,D8,GD01,GD02,GL03,GL04,L3,L49,L50]

Solving time: 687.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H 80 72.73
```

```
point H_{c} 68.91 84.83
```

```
point E_{b} 50 56.36
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H
```

```
cmark_rt H_{c}
```

```
cmark_r E_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H and H_{c} are not the same
```

```
% Constructing a line h_{c} which passes through point H and point H_{c}
```

```
line h_{c} H H_{c}
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```
% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
```

```
perp c H_{c} h_{c}
```

```
color 200 200 200
```

```
drawline c
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G93791} such that HP_{\_G93791}/HH_{c}=0.5
```

```
towards P_{\_G93791} H H_{c} 0.5
```

```
cmark_r P_{\_G93791}
```

```
color 200 200 200
```

```
drawsegment H H_{c}
```

```
color 0 0 0
```

```

% Constructing a line  $h_{\{H,1/2\}}(c)$  which contains the point  $P_{\{\backslash\_G93791\}}$  and is parallel to the
line c
parallel  $h_{\{H,1/2\}}(c)$   $P_{\{\backslash\_G93791\}}$  c

color 200 200 200
drawline  $h_{\{H,1/2\}}(c)$ 
color 0 0 0

% Constructing a point  $P_{\{\backslash\_G94098\}}$  such that  $HP_{\{\backslash\_G94098\}}/HH_{\{c\}}=0.5$ 
towards  $P_{\{\backslash\_G94098\}}$  H  $H_{\{c\}}$  0.5
cmark_r  $P_{\{\backslash\_G94098\}}$ 
color 200 200 200
drawsegment H  $H_{\{c\}}$ 
color 0 0 0

% Generating random value  $V[_{G94037}]$ 
random  $V[_{G94037}]$ 

% Calculating value  $V[_{G94058}]$  using formula  $V[_{G94037}]*20$ 
expression  $V[_{G94058}]$  {  $V[_{G94037}]*20$  }

% Constructing a point  $E_{\{b\}}$  which is a point for which holds  $P_{\{\backslash\_G94098\}}E_{\{b\}} = V[_{G94058}]$  and
angle  $HP_{\{\backslash\_G94098\}}E_{\{b\}} = 90$ 
turtle  $E_{\{b\}}$  H  $P_{\{\backslash\_G94098\}}$  90  $V[_{G94058}]$ 
cmark_r  $E_{\{b\}}$ 

% Constructing a point B such that  $E_{\{b\}}B/E_{\{b\}}H=-1$ 
towards B  $E_{\{b\}}$  H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

% NDG: points H and  $E_{\{b\}}$  are not the same
% Constructing a circle  $k(E_{\{b\}},B)$  whose center is at point  $E_{\{b\}}$  and which passes through point H
circle  $k(E_{\{b\}},B)$   $E_{\{b\}}$  H

color 200 200 200
drawcircle  $k(E_{\{b\}},B)$ 
color 0 0 0

% Choosing randomly a point A on the line  $H_{\{c\}}B$ 
online A  $H_{\{c\}}$  B
cmark_t A
color 200 200 200

```

```

drawline H_{c} B
color 0 0 0

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{a} and circle k(E_{b},B) intersect% DET: points H and H_{a} must be different
% Constructing a point P_{\_G94663} which is a foot of the point E_{b} on the line h_{a}
foot P_{\_G94663} E_{b} h_{a}
cmark_r P_{\_G94663}
color 200 200 200
drawline E_{b} P_{\_G94663}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H in the symmetry to point/line P_{\_G
94663}
sim H_{a} P_{\_G94663} H
cmark_r H_{a}

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines h_{c} and a are not parallel% DET: lines h_{c} and a are not the same
% Constructing a point C which belongs to line h_{c} and line a
intersec C h_{c} a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and a are not parallel; line h_{a} and circle k(E_{b},B)
intersect; points H and E_{b} are not the same
% Determination conditions: lines h_{c} and a are not the same; points B and H_{a} are not the same
; points H and H_{a} must be different; points A and H are not the same; points H and H_{c} are
not the same

```

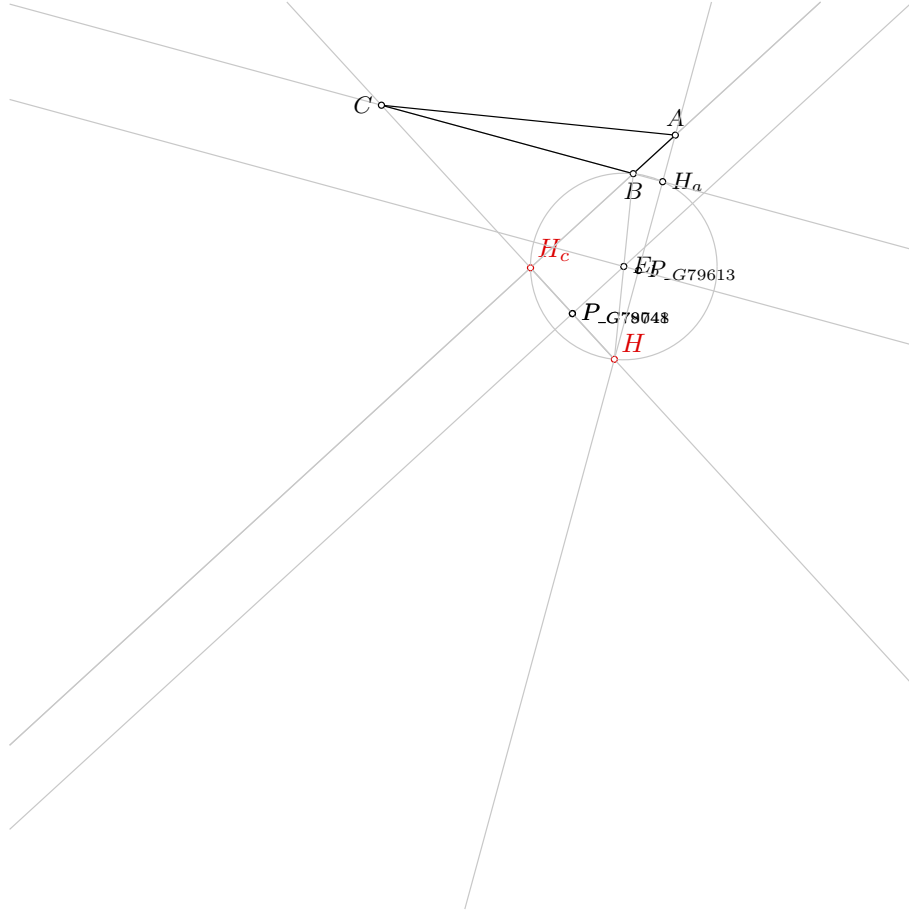


Figure 1: Illustration of the problem 0896

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H = H$

Construction steps:

- Free point H
- Free point H_c
- Line h_c through two points H and H_c
- Line c through point H_c perpendicular to line h_c
- Segment division point P_{G8827} of segment HH_c with division coefficient 1.0

- Line $h_{H,1/2}(c)$ through point P_{G88827} parallel with line c
- Segment division point P_{G88959} of segment HH_c with division coefficient -0.5
- Line L_{G88962} through point P_{G88959} parallel with line $h_{H,1/2}(c)$
- Random point E_b from line L_{G88962}
- Segment division point B of segment E_bH with division coefficient -0.5
- Circle $k(E_b, B)$ with center E_b and one point H
- Random point A from line c
- Line h_a through two points A and H
- Line footPointPerpLine650 through point E_b perpendicular to line h_a
- Intersection point P_{G89221} of point sets footPointPerpLine650 and h_a
- Cental symmetric point H_a of point H with respect to center of symmetry P_{G89221}
- Line a through two points B and H_a
- Intersection point C of point sets h_c and a
- Line $_b$ through two points A and C
- Line $_a$ through two points B and C
- Line $_h_a$ through point A perpendicular to line $_a$
- Line $_h_b$ through point B perpendicular to line $_b$
- Intersection point $_H$ of point sets $_h_a$ and $_h_b$
- Line $_c$ through two points A and B
- Line $_h_c$ through point C perpendicular to line $_c$
- Intersection point $_H_c$ of point sets $_c$ and $_h_c$

Theorem statement:

- Points H and $_H$ are identical

Info: Attempting to add the construction of new random point tempPoint-537 $h_{H,1/2}(c)$ necessary for completion of construction of line L_{G88962}

Warning: Generated new random point tempPoint-537 $h_{H,1/2}(c)$ on line $h_{H,1/2}(c)$ in order to complete the construction of parallel line L_{G88962}

Validation result: Theorem protocol is valid.

NDG conditions Point H has been assigned following coordinates: (0, 0)

4.1.2 Proving $H_c = _Hc$

Construction steps:

- Free point H
- Free point H_c
- Line h_c through two points H and H_c
- Line c through point H_c perpendicular to line h_c
- Segment division point P_{G90122} of segment HH_c with division coefficient 1.0
- Line $h_{H,1/2}(c)$ through point P_{G90122} parallel with line c
- Segment division point P_{G90254} of segment HH_c with division coefficient -0.5
- Line L_{G90257} through point P_{G90254} parallel with line $h_{H,1/2}(c)$
- Random point E_b from line L_{G90257}

- Segment division point B of segment E_bH with division coefficient -0.5
- Circle $k(E_b, B)$ with center E_b and one point H
- Random point A from line c
- Line h_a through two points A and H
- Line footPointPerpLine693 through point E_b perpendicular to line h_a
- Intersection point P_{G90516} of point sets footPointPerpLine693 and h_a
- Cental symmetric point H_a of point H with respect to center of symmetry P_{G90516}
- Line a through two points B and H_a
- Intersection point C of point sets h_c and a
- Line $_b$ through two points A and C
- Line $_a$ through two points B and C
- Line $_h_a$ through point A perpendicular to line $_a$
- Line $_h_b$ through point B perpendicular to line $_b$
- Intersection point $_H$ of point sets $_h_a$ and $_h_b$
- Line $_c$ through two points A and B
- Line $_h_c$ through point C perpendicular to line $_c$
- Intersection point $_H_c$ of point sets $_c$ and $_h_c$

Theorem statement:

- Points H_c and $_H_c$ are identical

Info: Attempting to add the construction of new random point tempPoint-329 $h_{H,1/2}(c)$ necessary for completion of construction of line L_{G90257}

Warning: Generated new random point tempPoint-329 $h_{H,1/2}(c)$ on line $h_{H,1/2}(c)$ in order to complete the construction of parallel line L_{G90257}

Validation result: Theorem protocol is valid.

NDG conditions Point H has been assigned following coordinates: $(0, 0)$

4.1.3 Proving $E_b=_E_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H=_H$

Proving failed

4.2.2 Proving $H_c=_H_c$

Proving failed

4.2.3 Proving $E_b=_E_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H=_H$

Proving failed

4.3.2 Proving $H_c = \neg H_c$

Proving failed

4.3.3 Proving $E_b = \neg E_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H = \neg H$

Proving failed

4.4.2 Proving $H_c = \neg H_c$

Proving failed

4.4.3 Proving $E_b = \neg E_b$

Proving failed

Problem 897

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 897: Given a point E_b , a point H and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 898

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 898: Given a point E_b , a point H and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H , construct a point B (rule W01); ;
2. Using the point M_a and the point B , construct a point C (rule W01); ;
3. Using the point H and the point C , construct a line h_c (rule W02); % DET: points H and C are not the same;
4. Using the point M_a and the point B , construct a line a (rule W02); % DET: points M_a and B are not the same;
5. Using the point H and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H and E_b are not the same;
6. Using the circle $k(E_b, B)$, the line h_c , the point E_b and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_b, B)$ intersect % DET: points H and H_c must be different;
7. Using the point H_c and the point B , construct a line c (rule W02); % DET: points H_c and B are not the same;
8. Using the circle $k(E_b, B)$, the line a , the point E_b and the point B , construct a point H_a (rule W05); % NDG: line a and circle $k(E_b, B)$ intersect % DET: points B and H_a must be different;
9. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
10. Using the line c and the line h_a , construct a point A (rule W03); % NDG: lines c and h_a are not parallel % DET: lines c and h_a are not the same.

Non-degenerate conditions: lines c and h_a are not parallel; line a and circle $k(E_b, B)$ intersect; line h_c and circle $k(E_b, B)$ intersect; points H and E_b are not the same.

Determination conditions: lines c and h_a are not the same; points H_a and H are not the same; points B and H_a must be different; points H_c and B are not the same; points H and H_c must be different; points M_a and B are not the same; points H and C are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D21,D29,D3,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L49,L50,L51]

Solving time: 12.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
```

```
point H 80 72.73
```

```
point M_{a} 65 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{b}
```

```
cmark_rt H
```

```
cmark_r M_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point B such that E_{b}B/E_{b}H=-1
```

```
towards B E_{b} H -1
```

```
cmark_b B
```

```
color 200 200 200
```

```
drawsegment H B
```

```
color 0 0 0
```

```
% Constructing a point C such that M_{a}C/M_{a}B=-1
```

```
towards C M_{a} B -1
```

```
cmark_l C
```

```
color 200 200 200
```

```
drawsegment B C
```

```
color 0 0 0
```

```
% DET: points H and C are not the same
```

```
% Constructing a line h_{c} which passes through point H and point C
```

```
line h_{c} H C
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```

% DET: points  $M_{\{a\}}$  and  $B$  are not the same
% Constructing a line  $a$  which passes through point  $M_{\{a\}}$  and point  $B$ 
line a  $M_{\{a\}}$  B

color 200 200 200
drawline a
color 0 0 0

% NDG: points  $H$  and  $E_{\{b\}}$  are not the same
% Constructing a circle  $k(E_{\{b\}}, B)$  whose center is at point  $E_{\{b\}}$  and which passes through point  $H$ 
circle k( $E_{\{b\}}$ ,  $B$ )  $E_{\{b\}}$   $H$ 

color 200 200 200
drawcircle k( $E_{\{b\}}$ ,  $B$ )
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(E_{\{b\}}, B)$  intersect% DET: points  $H$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\_G128645\}}$  which is a foot of the point  $E_{\{b\}}$  on the line  $h_{\{c\}}$ 
foot  $P_{\{\_G128645\}}$   $E_{\{b\}}$   $h_{\{c\}}$ 
cmark_r  $P_{\{\_G128645\}}$ 
color 200 200 200
drawline  $E_{\{b\}}$   $P_{\{\_G128645\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $H$  in the symmetry to point/line  $P_{\{\_G128645\}}$ 
sim  $H_{\{c\}}$   $P_{\{\_G128645\}}$   $H$ 
cmark_rt  $H_{\{c\}}$ 

% DET: points  $H_{\{c\}}$  and  $B$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $B$ 
line c  $H_{\{c\}}$  B

color 200 200 200
drawline c
color 0 0 0

% NDG: line  $a$  and circle  $k(E_{\{b\}}, B)$  intersect% DET: points  $B$  and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\_G128883\}}$  which is a foot of the point  $E_{\{b\}}$  on the line  $a$ 
foot  $P_{\{\_G128883\}}$   $E_{\{b\}}$   $a$ 
cmark_r  $P_{\{\_G128883\}}$ 
color 200 200 200
drawline  $E_{\{b\}}$   $P_{\{\_G128883\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $B$  in the symmetry to point/line  $P_{\{\_G128883\}}$ 
sim  $H_{\{a\}}$   $P_{\{\_G128883\}}$   $B$ 
cmark_r  $H_{\{a\}}$ 

```

```

% DET: points  $H_{\{a\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $H_{\{a\}}$  and point  $H$ 
line  $h_{\{a\}}$   $H_{\{a\}}$   $H$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: lines  $c$  and  $h_{\{a\}}$  are not parallel% DET: lines  $c$  and  $h_{\{a\}}$  are not the same
% Constructing a point  $A$  which belongs to line  $c$  and line  $h_{\{a\}}$ 
intersec  $A$   $c$   $h_{\{a\}}$ 
cmark_t  $A$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $c$  and  $h_{\{a\}}$  are not parallel; line  $a$  and circle  $k(E_{\{b\}}, B)$ 
% intersect; line  $h_{\{c\}}$  and circle  $k(E_{\{b\}}, B)$  intersect; points  $H$  and  $E_{\{b\}}$  are not the same
% Determination conditions: lines  $c$  and  $h_{\{a\}}$  are not the same; points  $H_{\{a\}}$  and  $H$  are not the same
% ; points  $B$  and  $H_{\{a\}}$  must be different; points  $H_{\{c\}}$  and  $B$  are not the same; points  $H$  and  $H_{\{c\}}$ 
% must be different; points  $M_{\{a\}}$  and  $B$  are not the same; points  $H$  and  $C$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $H = \neg H$

Proving failed

4.1.3 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 8 terms.

Time Complexity: Time spent by the prover is 0.108 seconds.

NDG conditions There are no NDG conditions for this theorem

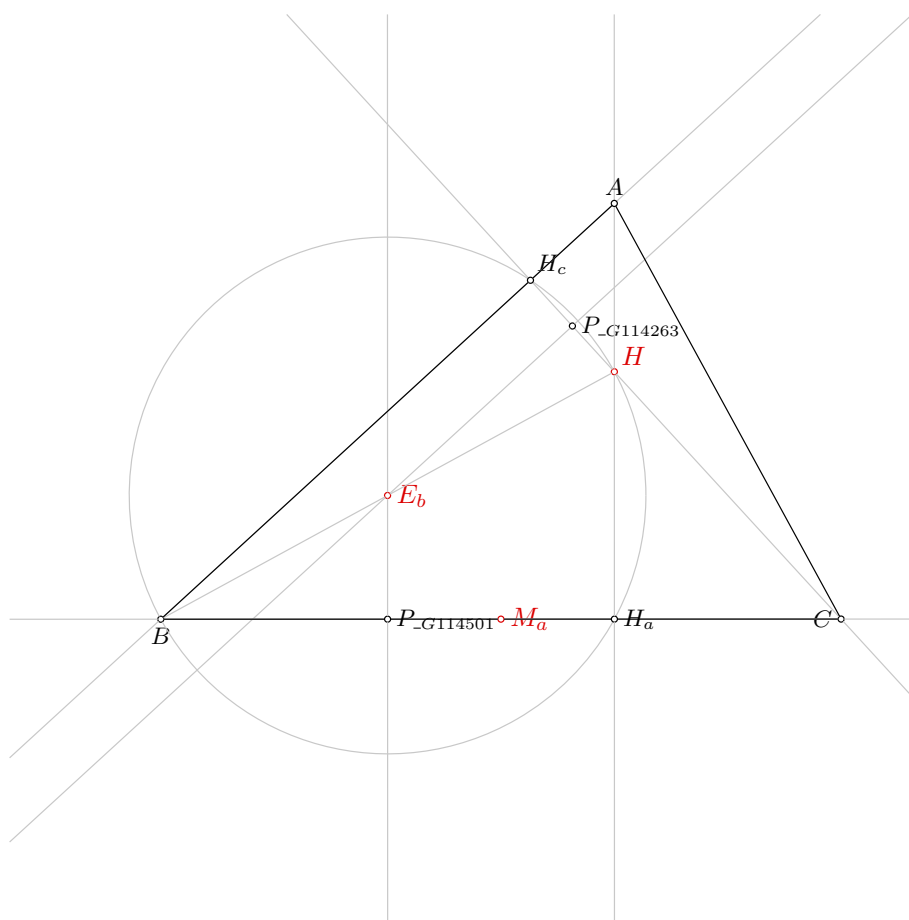


Figure 1: Illustration of the problem 0898

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $H = \neg H$

Proving failed

4.2.3 Proving $M_a = \neg M_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 3075 terms.

Time Complexity: Time spent by the prover is 21.650 seconds. There are no ndg conditions.

4.3.2 Proving $H = \neg H$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 570 terms.

Time Complexity: Time spent by the prover is 0.780 seconds. There are no ndg conditions.

4.3.3 Proving $M_a = \neg M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 19 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $H = \neg H$

Proving failed

4.4.3 Proving $M_a = \neg M_a$

Proving failed

Problem 899

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 899: Given a point E_b , a point H and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H , construct a point B (rule W01); ;
2. Using the point M_b and the point B , construct a point G (rule W01); ;
3. Using the point H and the point G , construct a point N (rule W01); ;
4. Using the point H and the point G , construct a point O (rule W01); ;
5. Using the point E_b and the point H , construct a line h_b (rule W02); % DET: points E_b and H are not the same;
6. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
8. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
9. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; line h_b and circle $k(N, M_a)$ intersect; points E_b and N are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points E_b and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L16,L20,L23,L56,L58]

Solving time: 6.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
```

```
point H 80 72.73
```

```
point M_{b} 95 67.5
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{b}
```

```
cmark_rt H
```

```
cmark_lt M_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point B such that E_{b}B/E_{b}H=-1
```

```
towards B E_{b} H -1
```

```
cmark_b B
```

```
color 200 200 200
```

```
drawsegment H B
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G156767} which passes through point M_{b} and point B
```

```
line L_{\_G156767} M_{b} B
```

```
color 200 200 200
```

```
drawline L_{\_G156767}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G156868} with coordinates (0,0)
```

```
point P_{\_G156868} 0 0
```

```
cmark_r P_{\_G156868}
```

```
% Constructing a point P_{\_G156792} such that M_{b}P_{\_G156792}/M_{b}P_{\_G156868}=1
```

```
towards P_{\_G156792} M_{b} P_{\_G156868} 1
```

```
cmark_r P_{\_G156792}
```

```
color 200 200 200
```

```
drawsegment M_{b} P_{\_G156792}
```

```
color 0 0 0
```



```

% Constructing a point P_{\_G156837} such that M_{b}P_{\_G156837}/M_{b}P_{\_G156868}=3
towards P_{\_G156837} M_{b} P_{\_G156868} 3
cmark_r P_{\_G156837}
color 200 200 200
drawsegment M_{b} P_{\_G156837}
color 0 0 0

% Constructing a line L_{\_G156798} which passes through point B and point P_{\_G156837}
line L_{\_G156798} B P_{\_G156837}

color 200 200 200
drawline L_{\_G156798}
color 0 0 0

% Constructing a line L_{\_G156761} which contains the point P_{\_G156792} and is parallel to the
line L_{\_G156798}
parallel L_{\_G156761} P_{\_G156792} L_{\_G156798}

color 200 200 200
drawline L_{\_G156761}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G156761} and line L_{\_G156767}
intersec G L_{\_G156761} L_{\_G156767}
cmark_t G

% Constructing a point N such that HN/HG=0.75
towards N H G 0.75
cmark_r N
color 200 200 200
drawsegment H G
color 0 0 0

% Constructing a point O such that HO/HG=1.5
towards O H G 1.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% DET: points E_{b} and H are not the same
% Constructing a line h_{b} which passes through point E_{b} and point H
line h_{b} E_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

```

```

% NDG: points  $E_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{b\}}$ 
circle k(N, M_{a}) N E_{b}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{b\}}$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\_G157787\}}$  which is a foot of the point  $N$  on the line  $h_{\{b\}}$ 
foot P_{\_G157787} N h_{b}
cmark_r P_{\_G157787}
color 200 200 200
drawline N P_{\_G157787}
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $E_{\{b\}}$  in the symmetry to point/line  $P_{\{\_G157787\}}$ 
sim H_{b} P_{\_G157787} E_{b}
cmark_l H_{b}

% DET: points  $H_{\{b\}}$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $M_{\{b\}}$ 
line b H_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0

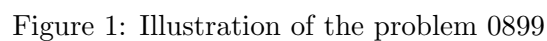
% NDG: points  $B$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $B$ 
circle k(O, C) O B

color 200 200 200
drawcircle k(O, C)
color 0 0 0

% NDG: line  $b$  and circle  $k(O, C)$  intersect
% Constructing points  $C$  and  $A$  which are in intersection of  $k(O, C)$  and  $b$ 
intersec2 C A k(O, C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

```



% Non-degenerate conditions: line b and circle $k(O,C)$ intersect; points B and O are not the same;
line $h_{\{b\}}$ and circle $k(N,M_{\{a\}})$ intersect; points $E_{\{b\}}$ and N are not the same
% Determination conditions: points $H_{\{b\}}$ and $M_{\{b\}}$ are not the same; points $E_{\{b\}}$ and $H_{\{b\}}$ must be
different; points $E_{\{b\}}$ and H are not the same

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = -E_b$

Proving failed

4.1.2 Proving $H=_H$

Proving failed

4.1.3 Proving $M_b=_M M_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b=_E E_b$

Proving failed

4.2.2 Proving $H=_H$

Proving failed

4.2.3 Proving $M_b=_M M_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b=_E E_b$

Proving failed

4.3.2 Proving $H=_H$

Proving failed

4.3.3 Proving $M_b=_M M_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b=_E E_b$

Proving failed

4.4.2 Proving $H=_H$

Proving failed

4.4.3 Proving $M_b=_M M_b$

Proving failed

Problem 900

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 900: Given a point E_b , a point H and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H , construct a point B (rule W01); ;
2. Using the point M_c and the point B , construct a point A (rule W01); ;
3. Using the point H and the point A , construct a line h_a (rule W02); % DET: points H and A are not the same;
4. Using the point M_c and the point B , construct a line c (rule W02); % DET: points M_c and B are not the same;
5. Using the point H and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H and E_b are not the same;
6. Using the circle $k(E_b, B)$, the line h_a , the point E_b and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_b, B)$ intersect % DET: points H and H_a must be different;
7. Using the point H_a and the point B , construct a line a (rule W02); % DET: points H_a and B are not the same;
8. Using the circle $k(E_b, B)$, the line c , the point E_b and the point B , construct a point H_c (rule W05); % NDG: line c and circle $k(E_b, B)$ intersect % DET: points B and H_c must be different;
9. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
10. Using the line a and the line h_c , construct a point C (rule W03); % NDG: lines a and h_c are not parallel % DET: lines a and h_c are not the same.

Non-degenerate conditions: lines a and h_c are not parallel; line c and circle $k(E_b, B)$ intersect; line h_a and circle $k(E_b, B)$ intersect; points H and E_b are not the same.

Determination conditions: lines a and h_c are not the same; points H_c and H are not the same; points B and H_c must be different; points H_a and B are not the same; points H and H_a must be different; points M_c and B are not the same; points H and A are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D20,D29,D3,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L49,L50,L51]

Solving time: 12.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
```

```
point H 80 72.73
```

```
point M_{c} 50 67.5
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{b}
```

```
cmark_rt H
```

```
cmark_lt M_{c}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point B such that E_{b}B/E_{b}H=-1
```

```
towards B E_{b} H -1
```

```
cmark_b B
```

```
color 200 200 200
```

```
drawsegment H B
```

```
color 0 0 0
```

```
% Constructing a point A such that M_{c}A/M_{c}B=-1
```

```
towards A M_{c} B -1
```

```
cmark_t A
```

```
color 200 200 200
```

```
drawsegment B A
```

```
color 0 0 0
```

```
% DET: points H and A are not the same
```

```
% Constructing a line h_{a} which passes through point H and point A
```

```
line h_{a} H A
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```

% DET: points  $M_{\{c\}}$  and  $B$  are not the same
% Constructing a line  $c$  which passes through point  $M_{\{c\}}$  and point  $B$ 
line c  $M_{\{c\}}$  B

color 200 200 200
drawline c
color 0 0 0

% NDG: points  $H$  and  $E_{\{b\}}$  are not the same
% Constructing a circle  $k(E_{\{b\}}, B)$  whose center is at point  $E_{\{b\}}$  and which passes through point  $H$ 
circle k( $E_{\{b\}}$ ,  $B$ )  $E_{\{b\}}$   $H$ 

color 200 200 200
drawcircle k( $E_{\{b\}}$ ,  $B$ )
color 0 0 0

% NDG: line  $h_{\{a\}}$  and circle  $k(E_{\{b\}}, B)$  intersect% DET: points  $H$  and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\_G185433\}}$  which is a foot of the point  $E_{\{b\}}$  on the line  $h_{\{a\}}$ 
foot  $P_{\{\_G185433\}}$   $E_{\{b\}}$   $h_{\{a\}}$ 
cmark_r  $P_{\{\_G185433\}}$ 
color 200 200 200
drawline  $E_{\{b\}}$   $P_{\{\_G185433\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $H$  in the symmetry to point/line  $P_{\{\_G185433\}}$ 
sim  $H_{\{a\}}$   $P_{\{\_G185433\}}$   $H$ 
cmark_r  $H_{\{a\}}$ 

% DET: points  $H_{\{a\}}$  and  $B$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $B$ 
line a  $H_{\{a\}}$  B

color 200 200 200
drawline a
color 0 0 0

% NDG: line  $c$  and circle  $k(E_{\{b\}}, B)$  intersect% DET: points  $B$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\_G185671\}}$  which is a foot of the point  $E_{\{b\}}$  on the line  $c$ 
foot  $P_{\{\_G185671\}}$   $E_{\{b\}}$   $c$ 
cmark_r  $P_{\{\_G185671\}}$ 
color 200 200 200
drawline  $E_{\{b\}}$   $P_{\{\_G185671\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $B$  in the symmetry to point/line  $P_{\{\_G185671\}}$ 
sim  $H_{\{c\}}$   $P_{\{\_G185671\}}$   $B$ 
cmark_rt  $H_{\{c\}}$ 

```

```

% DET: points  $H_{\{c\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $H_{\{c\}}$  and point  $H$ 
line  $h_{\{c\}}$   $H_{\{c\}}$   $H$ 

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: lines  $a$  and  $h_{\{c\}}$  are not parallel% DET: lines  $a$  and  $h_{\{c\}}$  are not the same
% Constructing a point  $C$  which belongs to line  $a$  and line  $h_{\{c\}}$ 
intersec  $C$   $a$   $h_{\{c\}}$ 
cmark_l  $C$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $a$  and  $h_{\{c\}}$  are not parallel; line  $c$  and circle  $k(E_{\{b\}}, B)$ 
% intersect; line  $h_{\{a\}}$  and circle  $k(E_{\{b\}}, B)$  intersect; points  $H$  and  $E_{\{b\}}$  are not the same
% Determination conditions: lines  $a$  and  $h_{\{c\}}$  are not the same; points  $H_{\{c\}}$  and  $H$  are not the same
% ; points  $B$  and  $H_{\{c\}}$  must be different; points  $H_{\{a\}}$  and  $B$  are not the same; points  $H$  and  $H_{\{a\}}$ 
% must be different; points  $M_{\{c\}}$  and  $B$  are not the same; points  $H$  and  $A$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = _E_b$

Proving failed

4.1.2 Proving $H = _H$

Proving failed

4.1.3 Proving $M_c = _M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 8 terms.

Time Complexity: Time spent by the prover is 0.109 seconds.

NDG conditions There are no NDG conditions for this theorem

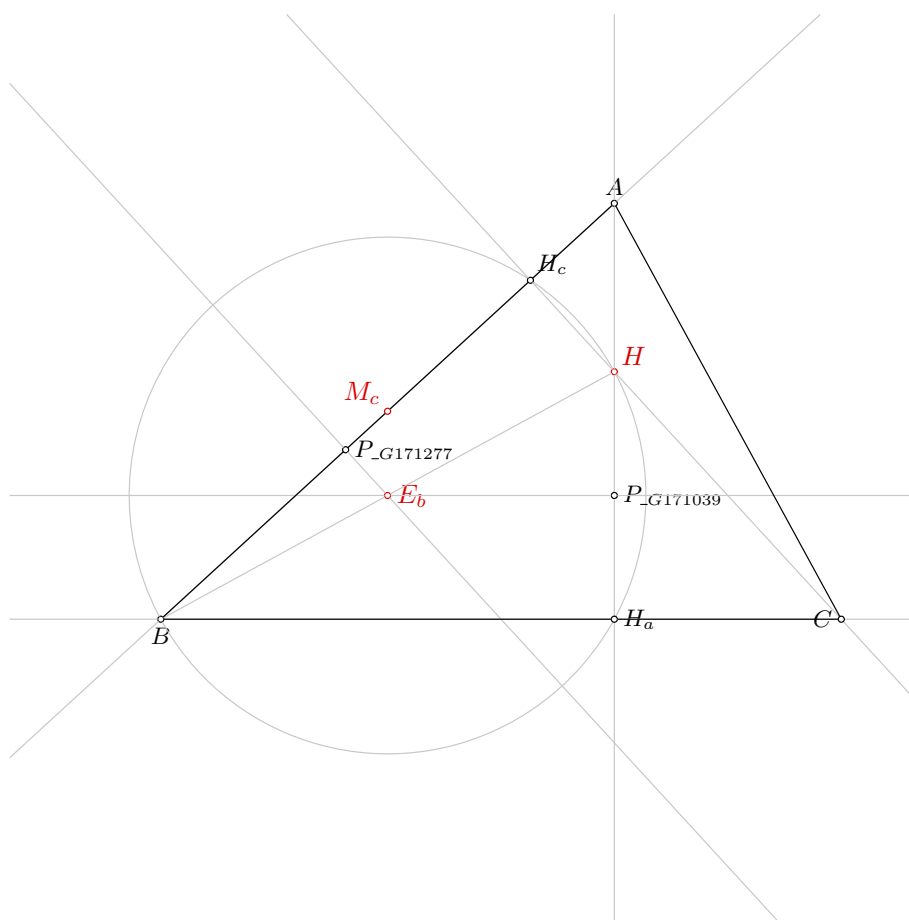


Figure 1: Illustration of the problem 0900

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $H = \neg H$

Proving failed

4.2.3 Proving $M_c = \neg M_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 3075 terms.

Time Complexity: Time spent by the prover is 21.640 seconds. There are no ndg conditions.

4.3.2 Proving $H = \neg H$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 570 terms.

Time Complexity: Time spent by the prover is 0.790 seconds. There are no ndg conditions.

4.3.3 Proving $M_c = \neg M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 19 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $H = \neg H$

Proving failed

4.4.3 Proving $M_c = \neg M_c$

Proving failed

Problem 901

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 901: Given a point E_b , a point H and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H , construct a point B (rule W01); ;
2. Using the point H and the point N , construct a point O (rule W01); ;
3. Using the point H and the point N , construct a point G (rule W01); ;
4. Using the point B and the point G , construct a point M_b (rule W01); ;
5. Using the point E_b and the point H , construct a line h_b (rule W02); % DET: points E_b and H are not the same;
6. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
8. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
9. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; line h_b and circle $k(N, M_a)$ intersect; points E_b and N are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points E_b and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D31,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L16,L20,L23,L56]

Solving time: 7.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
```

```
point H 80 72.73
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{b}
```

```
cmark_rt H
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point B such that E_{b}B/E_{b}H=-1
```

```
towards B E_{b} H -1
```

```
cmark_b B
```

```
color 200 200 200
```

```
drawsegment H B
```

```
color 0 0 0
```

```
% Constructing a point O such that HO/HN=2
```

```
towards O H N 2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment H O
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G227457} which passes through point H and point N
```

```
line L_{\_G227457} H N
```

```
color 200 200 200
```

```
drawline L_{\_G227457}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G227558} with coordinates (0,0)
```

```
point P_{\_G227558} 0 0
```

```

cmark_r P_{\_G227558}

% Constructing a point P_{\_G227482} such that HP_{\_G227482}/HP_{\_G227558}=4
towards P_{\_G227482} H P_{\_G227558} 4
cmark_r P_{\_G227482}
color 200 200 200
drawsegment H P_{\_G227482}
color 0 0 0

% Constructing a point P_{\_G227527} such that HP_{\_G227527}/HP_{\_G227558}=3
towards P_{\_G227527} H P_{\_G227558} 3
cmark_r P_{\_G227527}
color 200 200 200
drawsegment H P_{\_G227527}
color 0 0 0

% Constructing a line L_{\_G227488} which passes through point N and point P_{\_G227527}
line L_{\_G227488} N P_{\_G227527}

color 200 200 200
drawline L_{\_G227488}
color 0 0 0

% Constructing a line L_{\_G227451} which contains the point P_{\_G227482} and is parallel to the
line L_{\_G227488}
parallel L_{\_G227451} P_{\_G227482} L_{\_G227488}

color 200 200 200
drawline L_{\_G227451}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G227451} and line L_{\_G227457}
intersec G L_{\_G227451} L_{\_G227457}
cmark_t G

% Constructing a point M_{b} such that BM_{b}/BG=1.5
towards M_{b} B G 1.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% DET: points E_{b} and H are not the same
% Constructing a line h_{b} which passes through point E_{b} and point H
line h_{b} E_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

```

```

% NDG: points  $E_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{b\}}$ 
circle k(N, M_{a}) N E_{b}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{b\}}$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\_G228394\}}$  which is a foot of the point  $N$  on the line  $h_{\{b\}}$ 
foot P_{\_G228394} N h_{b}
cmark_r P_{\_G228394}
color 200 200 200
drawline N P_{\_G228394}
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $E_{\{b\}}$  in the symmetry to point/line  $P_{\{\_G228394\}}$ 
sim H_{b} P_{\_G228394} E_{b}
cmark_l H_{b}

% DET: points  $H_{\{b\}}$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $M_{\{b\}}$ 
line b H_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0

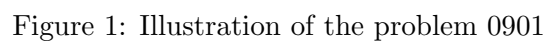
% NDG: points  $B$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $B$ 
circle k(O, C) O B

color 200 200 200
drawcircle k(O, C)
color 0 0 0

% NDG: line  $b$  and circle  $k(O, C)$  intersect
% Constructing points  $C$  and  $A$  which are in intersection of  $k(O, C)$  and  $b$ 
intersec2 C A k(O, C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

```



% Non-degenerate conditions: line b and circle $k(O,C)$ intersect; points B and O are not the same;
line $h_{\{b\}}$ and circle $k(N,M_{\{a\}})$ intersect; points $E_{\{b\}}$ and N are not the same
% Determination conditions: points $H_{\{b\}}$ and $M_{\{b\}}$ are not the same; points $E_{\{b\}}$ and $H_{\{b\}}$ must be
different; points $E_{\{b\}}$ and H are not the same

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = -E_b$

Proving failed

4.1.2 Proving $H=_H$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b=_E E_b$

Proving failed

4.2.2 Proving $H=_H$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b=_E E_b$

Proving failed

4.3.2 Proving $H=_H$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b=_E E_b$

Proving failed

4.4.2 Proving $H=_H$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 902

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 902: Given a point E_b , a point H and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H , construct a point B (rule W01); ;
2. Using the point H and the point O , construct a point N (rule W01); ;
3. Using the point H and the point O , construct a point G (rule W01); ;
4. Using the point B and the point G , construct a point M_b (rule W01); ;
5. Using the point E_b and the point H , construct a line h_b (rule W02); % DET: points E_b and H are not the same;
6. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
8. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
9. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; line h_b and circle $k(N, M_a)$ intersect; points E_b and N are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points E_b and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D31,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L20,L23,L56,L58]

Solving time: 7.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
```

```
point H 80 72.73
```

```
point O 65 51.14
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{b}
```

```
cmark_rt H
```

```
cmark_t O
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point B such that E_{b}B/E_{b}H=-1
```

```
towards B E_{b} H -1
```

```
cmark_b B
```

```
color 200 200 200
```

```
drawsegment H B
```

```
color 0 0 0
```

```
% Constructing a point N such that HN/HO=0.5
```

```
towards N H O 0.5
```

```
cmark_r N
```

```
color 200 200 200
```

```
drawsegment H O
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G261328} which passes through point H and point O
```

```
line L_{\_G28210} H O
```

```
color 200 200 200
```

```
drawline L_{\_G28210}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G261429} with coordinates (0,0)
```

```
point P_{\_G28214} 0 0
```

```

cmark_r P_{\_G28214}

% Constructing a point P_{\_G261353} such that HP_{\_G261353}/HP_{\_G261429}=2
towards P_{\_G28211} H P_{\_G28214} 2
cmark_r P_{\_G28211}
color 200 200 200
drawsegment H P_{\_G28211}
color 0 0 0

% Constructing a point P_{\_G261398} such that HP_{\_G261398}/HP_{\_G261429}=3
towards P_{\_G28213} H P_{\_G28214} 3
cmark_r P_{\_G28213}
color 200 200 200
drawsegment H P_{\_G28213}
color 0 0 0

% Constructing a line L_{\_G261359} which passes through point O and point P_{\_G261398}
line L_{\_G28212} O P_{\_G28213}

color 200 200 200
drawline L_{\_G28212}
color 0 0 0

% Constructing a line L_{\_G261322} which contains the point P_{\_G261353} and is parallel to the
line L_{\_G261359}
parallel L_{\_G28209} P_{\_G28211} L_{\_G28212}

color 200 200 200
drawline L_{\_G28209}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G261322} and line L_{\_G261328}
intersec G L_{\_G28209} L_{\_G28210}
cmark_t G

% Constructing a point M_{b} such that BM_{b}/BG=1.5
towards M_{b} B G 1.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% DET: points E_{b} and H are not the same
% Constructing a line h_{b} which passes through point E_{b} and point H
line h_{b} E_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

```

```

% NDG: points  $E_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{b\}}$ 
circle k(N, M_{a}) N E_{b}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{b\}}$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\_G28951\}}$  which is a foot of the point  $N$  on the line  $h_{\{b\}}$ 
foot P_{\_G28951} N h_{b}
cmark_r P_{\_G28951}
color 200 200 200
drawline N P_{\_G28951}
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $E_{\{b\}}$  in the symmetry to point/line  $P_{\{\_G28951\}}$ 
sim H_{b} P_{\_G28951} E_{b}
cmark_l H_{b}

% DET: points  $H_{\{b\}}$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $M_{\{b\}}$ 
line b H_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points  $B$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $B$ 
circle k(O, C) O B

color 200 200 200
drawcircle k(O, C)
color 0 0 0

% NDG: line  $b$  and circle  $k(O, C)$  intersect
% Constructing points  $C$  and  $A$  which are in intersection of  $k(O, C)$  and  $b$ 
intersec2 C A k(O, C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

```

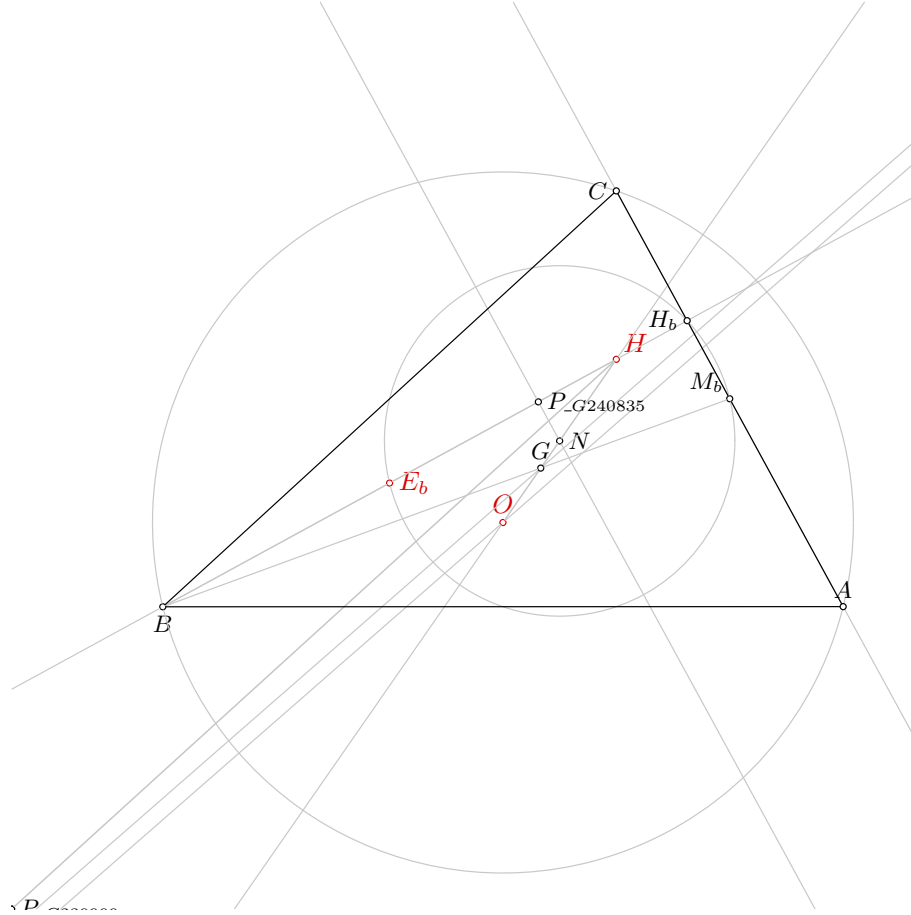


Figure 1: Illustration of the problem 0902

*% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;
 line h_{b} and circle k(N,M_{a}) intersect; points E_{b} and N are not the same
 % Determination conditions: points H_{b} and M_{b} are not the same; points E_{b} and H_{b} must be
 different; points E_{b} and H are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = E_b$

Proving failed

4.1.2 Proving $H=_H$

Proving failed

4.1.3 Proving $O=_O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b=_E_b$

Proving failed

4.2.2 Proving $H=_H$

Proving failed

4.2.3 Proving $O=_O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b=_E_b$

Proving failed

4.3.2 Proving $H=_H$

Proving failed

4.3.3 Proving $O=_O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b=_E_b$

Proving failed

4.4.2 Proving $H=_H$

Proving failed

4.4.3 Proving $O=_O$

Proving failed

Problem 903

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 903: Given a point E_b , a point H and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 904

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 904: Given a point E_b , a point H and a point T_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H , construct a point B (rule W01); ;
2. Using the point E_b and the point H , construct a line h_b (rule W02); % DET: points E_b and H are not the same;
3. Using the point T_b and the point B , construct a line s_b (rule W02); % DET: points T_b and B are not the same;
4. Using the point T_b and the line h_b , construct a line b (rule W10a); ;
5. Using the line b and the line h_b , construct a point H_b (rule W03); % NDG: lines b and h_b are not parallel % DET: lines b and h_b are not the same;
6. Using the point H and the point H_b , construct a line $m(HH_b)$ (rule W14); % DET: points H and H_b are not the same;
7. Using the point E_b , the point B , the point T_b , the line s_b and the line h_b , construct a line BO (rule W17); % NDG: points B and T_b are not the same; points E_b and B are not the same % DET: points B and T_b are not the same;
8. Using the point E_b and the line BO , construct a line $m(H_aH_c)$ (rule W16); ;
9. Using the line $m(H_aH_c)$ and the line b , construct a point M_b (rule W03); % NDG: lines $m(H_aH_c)$ and b are not parallel % DET: lines $m(H_aH_c)$ and b are not the same;
10. Using the point M_b and the point B , construct a point G (rule W01); ;
11. Using the point H and the point G , construct a point N (rule W01); ;

12. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
13. Using the circle $k(N, M_a)$ and the line $m(HH_b)$, construct a point E_a and a point E_c (rule W04); % NDG: line $m(HH_b)$ and circle $k(N, M_a)$ intersect;
14. Using the point E_a and the point H , construct a point A (rule W01); ;
15. Using the point H and the point E_c , construct a point C (rule W01); .

Non-degenerate conditions: line $m(HH_b)$ and circle $k(N, M_a)$ intersect; points E_b and N are not the same; lines $m(H_aH_c)$ and b are not parallel; points B and T_b are not the same; points E_b and B are not the same; lines b and h_b are not parallel.

Determination conditions: lines $m(H_aH_c)$ and b are not the same; points B and T_b are not the same; points H and H_b are not the same; lines b and h_b are not the same; points T_b and B are not the same; points E_b and H are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W14,W16,W17]

Lemmas used: [D22,D24,D28,D29,D3,D30,D32,D6,D9,GD01,GD02,GL01,GL03,GL04,GL09,GL17,L105,L16,L2]

Solving time: 125.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
```

```
point H 80 72.73
```

```
point T_{b} 94.25 68.88
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{b}
```

```
cmark_rt H
```

```
cmark_t T_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point B such that E_{b}B/E_{b}H=-1
```

```
towards B E_{b} H -1
```

```
cmark_b B
```

```
color 200 200 200
```

```
drawsegment H B
```

```
color 0 0 0
```

```
% DET: points E_{b} and H are not the same
```

```
% Constructing a line h_{b} which passes through point E_{b} and point H
```

```
line h_{b} E_{b} H
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```

% DET: points T_{b} and B are not the same
% Constructing a line s_{b} which passes through point T_{b} and point B
line s_{b} T_{b} B

color 200 200 200
drawline s_{b}
color 0 0 0

% Constructing a line b which is perpendicular to line h_{b} and which passes through point T_{b}
perp b T_{b} h_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines b and h_{b} are not parallel% DET: lines b and h_{b} are not the same
% Constructing a point H_{b} which belongs to line b and line h_{b}
intersec H_{b} b h_{b}
cmark_l H_{b}

% DET: points H and H_{b} are not the same
% Constructing bisector m(HH_{b}) of the segment HH_{b}
med m(HH_{b}) H H_{b}

color 200 200 200
drawline m(HH_{b})
color 0 0 0

color 200 200 200
drawsegment H H_{b}
color 0 0 0

% NDG: points B and T_{b} are not the same; points E_{b} and B are not the same% DET: points B and
T_{b} are not the same
% Constructing an angle V[_G73894] which is equal to the angle E_{b}BT_{b}
angle_o V[_G73894] E_{b} B T_{b}

% Calculating value angle[_G73973] using formula  $1/\text{pow}(2,0)*V[_G73894]+0/\text{pow}(2,0)*180$ 
expression angle[_G73973] {  $1/\text{pow}(2,0)*V[_G73894]+0/\text{pow}(2,0)*180$  }

% Constructing a point P_{\_G73970} which is an image of the point T_{b} in a rotation around the
point B for the angle  $1/\text{pow}(2,0)*V[_G73894]+0/\text{pow}(2,0)*180$ 
rotate P_{\_G73970} B angle[_G73973] T_{b}
cmark_r P_{\_G73970}

```

```

color 200 200 200
drawarc_p B T_{b} angle[_G73973]
color 0 0 0

% Constructing a line BO which passes through point B and point P_{\_G73970}
line BO B P_{\_G73970}

color 200 200 200
drawline BO
color 0 0 0


% Constructing a line m(H_{a}H_{c}) which contains the point E_{b} and is parallel to the line BO
parallel m(H_{a}H_{c}) E_{b} BO

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0


% NDG: lines m(H_{a}H_{c}) and b are not parallel% DET: lines m(H_{a}H_{c}) and b are not the same
% Constructing a point M_{b} which belongs to line m(H_{a}H_{c}) and line b
intersec M_{b} m(H_{a}H_{c}) b
cmark_lt M_{b}


% Constructing a line L_{\_G74489} which passes through point M_{b} and point B
line L_{\_G74489} M_{b} B

color 200 200 200
drawline L_{\_G74489}
color 0 0 0


% Constructing a point P_{\_G74590} with coordinates (0,0)
point P_{\_G74590} 0 0
cmark_r P_{\_G74590}


% Constructing a point P_{\_G74514} such that M_{b}P_{\_G74514}/M_{b}P_{\_G74590}=1
towards P_{\_G74514} M_{b} P_{\_G74590} 1
cmark_r P_{\_G74514}
color 200 200 200
drawsegment M_{b} P_{\_G74514}
color 0 0 0


% Constructing a point P_{\_G74559} such that M_{b}P_{\_G74559}/M_{b}P_{\_G74590}=3
towards P_{\_G74559} M_{b} P_{\_G74590} 3
cmark_r P_{\_G74559}
color 200 200 200
drawsegment M_{b} P_{\_G74559}
color 0 0 0

```

```

% Constructing a line L_{\_G74520} which passes through point B and point P_{\_G74559}
line L_{\_G74520} B P_{\_G74559}

color 200 200 200
drawline L_{\_G74520}
color 0 0 0

% Constructing a line L_{\_G74483} which contains the point P_{\_G74514} and is parallel to the
line L_{\_G74520}
parallel L_{\_G74483} P_{\_G74514} L_{\_G74520}

color 200 200 200
drawline L_{\_G74483}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G74483} and line L_{\_G74489}
intersec G L_{\_G74483} L_{\_G74489}
cmark_t G

% Constructing a point N such that HN/HG=0.75
towards N H G 0.75
cmark_r N
color 200 200 200
drawsegment H G
color 0 0 0

% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(HH_{b}) and circle k(N,M_{a}) intersect
% Constructing points E_{a} and E_{c} which are in intersection of k(N,M_{a}) and m(HH_{b})
intersec2 E_{a} E_{c} k(N,M_{a}) m(HH_{b})
cmark_r E_{a}
cmark_r E_{c}

% Constructing a point A such that E_{a}A/E_{a}H=-1
towards A E_{a} H -1
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

```

```

% Constructing a point C such that HC/HE_{c}=2
towards C H E_{c} 2
cmark_1 C
color 200 200 200
drawsegment H C
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions: line m(HH_{b}) and circle k(N,M_{a}) intersect; points E_{b} and N are
% not the same; lines m(H_{a}H_{c}) and b are not parallel; points B and T_{b} are not the same;
% points E_{b} and B are not the same; lines b and h_{b} are not parallel
% Determination conditions: lines m(H_{a}H_{c}) and b are not the same; points B and T_{b} are not
% the same; points H and H_{b} are not the same; lines b and h_{b} are not the same; points T_{b}
% and B are not the same; points E_{b} and H are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = _E_b$

Proving failed

4.1.2 Proving $H = _H$

Proving failed

4.1.3 Proving $T_b = _T_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = _E_b$

Proving failed

4.2.2 Proving $H = _H$

Proving failed

4.2.3 Proving $T_b = _T_b$

Proving failed

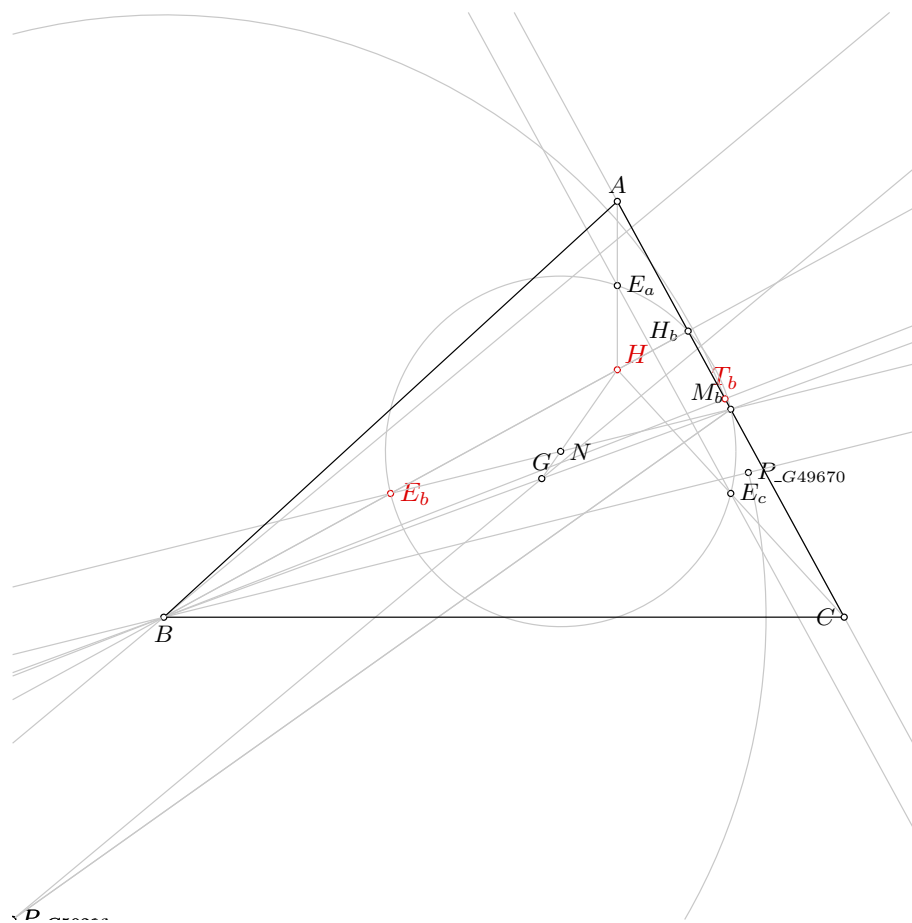


Figure 1: Illustration of the problem 0904

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $H = \neg H$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 974 terms.

Time Complexity: Time spent by the prover is 3.480 seconds. There are no ndg conditions.

4.3.3 Proving $T_b = \neg T_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $H = \neg H$

Proving failed

4.4.3 Proving $T_b = \neg T_b$

Proving failed

Problem 905

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 905: Given a point E_b , a point H and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 906

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 906: Given a point E_b , a point H_a and a point H_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;
2. Using the point H_a and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_a and E_b are not the same;
3. Using the circle $k(E_b, B)$ and the line h_b , construct a point B and a point H (rule W04); % NDG: line h_b and circle $k(E_b, B)$ intersect;
4. Using the point H_a and the point B , construct a line a (rule W02); % DET: points H_a and B are not the same;
5. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
6. Using the point H_b and the line h_b , construct a line b (rule W10a); ;
7. Using the line b and the line a , construct a point C (rule W03); % NDG: lines b and a are not parallel % DET: lines b and a are not the same;
8. Using the line h_a and the line b , construct a point A (rule W03); % NDG: lines h_a and b are not parallel % DET: lines h_a and b are not the same.

Non-degenerate conditions: lines h_a and b are not parallel; lines b and a are not parallel; line h_b and circle $k(E_b, B)$ intersect; points H_a and E_b are not the same.

Determination conditions: lines h_a and b are not the same; lines b and a are not the same; points H_a and H are not the same; points H_a and B are not the same; points E_b and H_b are not the same.

Rules used: [W02,W03,W04,W06,W10a]
 Lemmas used: [D29,D3,D5,D6,D8,D9,GD01,GD02,GL09,L49,L50]
 Solving time: 6.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{b} 50 56.36
point H_{a} 80 40
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_r E_{b}
cmark_r H_{a}
cmark_l H_{b}
color 0 0 0
fontsize 8

% DET: points E_{b} and H_{b} are not the same
% Constructing a line h_{b} which passes through point E_{b} and point H_{b}
line h_{b} E_{b} H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points H_{a} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H_{a}
circle k(E_{b},B) E_{b} H_{a}

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line h_{b} and circle k(E_{b},B) intersect
% Constructing points B and H which are in intersection of k(E_{b},B) and h_{b}
intersec2 B H k(E_{b},B) h_{b}
cmark_b B
cmark_rt H

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
```

```
color 0 0 0
```

```
% DET: points  $H_{\{a\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $H_{\{a\}}$  and point  $H$ 
line  $h_{\{a\}}$   $H_{\{a\}}$   $H$ 
```

```
color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0
```

```
% Constructing a line  $b$  which is perpendicular to line  $h_{\{b\}}$  and which passes through point  $H_{\{b\}}$ 
perp  $b$   $H_{\{b\}}$   $h_{\{b\}}$ 
```

```
color 200 200 200
drawline  $b$ 
color 0 0 0
```

```
% NDG: lines  $b$  and  $a$  are not parallel% DET: lines  $b$  and  $a$  are not the same
% Constructing a point  $C$  which belongs to line  $b$  and line  $a$ 
intersec  $C$   $b$   $a$ 
cmark_l  $C$ 
```

```
% NDG: lines  $h_{\{a\}}$  and  $b$  are not parallel% DET: lines  $h_{\{a\}}$  and  $b$  are not the same
% Constructing a point  $A$  which belongs to line  $h_{\{a\}}$  and line  $b$ 
intersec  $A$   $h_{\{a\}}$   $b$ 
cmark_t  $A$ 
```

```
drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 
```

```
% Non-degenerate conditions: lines  $h_{\{a\}}$  and  $b$  are not parallel; lines  $b$  and  $a$  are not parallel;
line  $h_{\{b\}}$  and circle  $k(E_{\{b\}}, B)$  intersect; points  $H_{\{a\}}$  and  $E_{\{b\}}$  are not the same
% Determination conditions: lines  $h_{\{a\}}$  and  $b$  are not the same; lines  $b$  and  $a$  are not the same;
points  $H_{\{a\}}$  and  $H$  are not the same; points  $H_{\{a\}}$  and  $B$  are not the same; points  $E_{\{b\}}$  and  $H_{\{b\}}$ 
are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

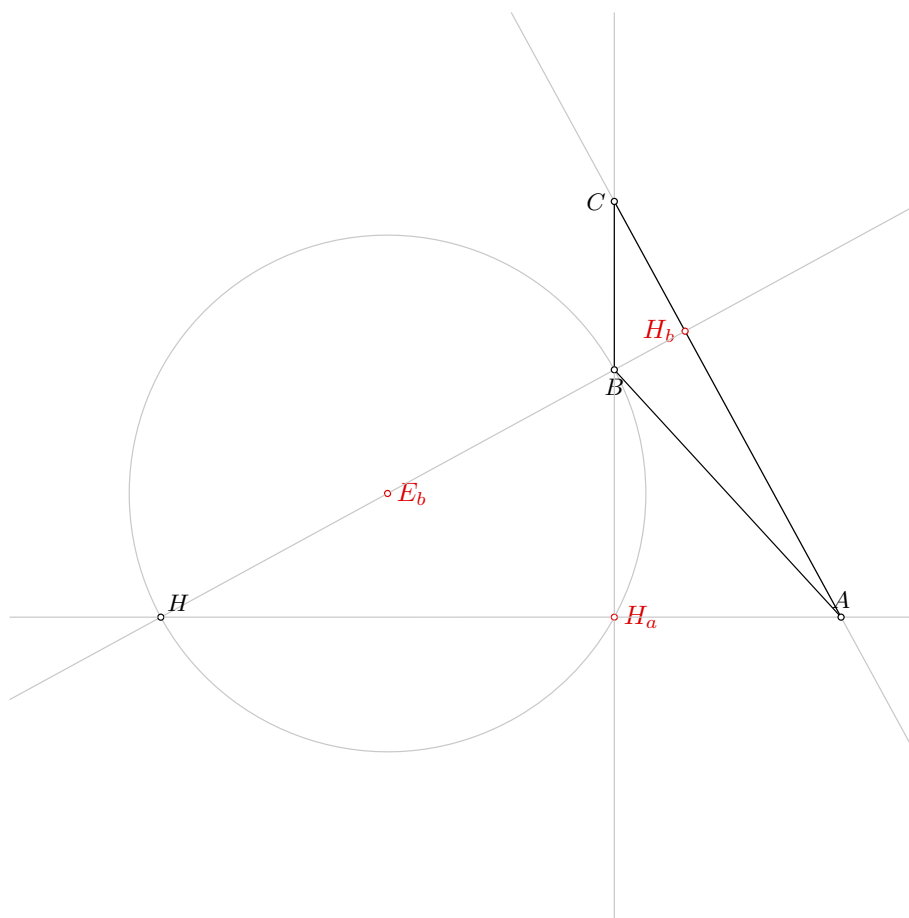


Figure 1: Illustration of the problem 0906

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $H_a = \neg H_a$

Proving failed

4.1.3 Proving $H_b = \neg H_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $H_a = \neg H_a$

Proving failed

4.2.3 Proving $H_b = \neg H_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $H_a = \neg H_a$

Proving failed

4.3.3 Proving $H_b = \neg H_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $H_a = \neg H_a$

Proving failed

4.4.3 Proving $H_b = \neg H_b$

Proving failed

Problem 907

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 907: Given a point E_b , a point H_c and a point H_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_c and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_c and E_b are not the same;
2. Choose freely a point H_a on the circle $k(E_b, B)$ (rule WOncircle);
3. Using the point H_a and the point E_b , construct a line $m(H_a E_b)$ (rule W14); % DET: points H_a and E_b are not the same;
4. Using the point H_a and the point H_c , construct a line $m(H_a H_c)$ (rule W14); % DET: points H_a and H_c are not the same;
5. Using the line $m(H_a H_c)$ and the line $m(H_a E_b)$, construct a point N (rule W03); % NDG: lines $m(H_a H_c)$ and $m(H_a E_b)$ are not parallel % DET: lines $m(H_a H_c)$ and $m(H_a E_b)$ are not the same;
6. Using the point H_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_a and N are not the same;
7. Using the circle $k(N, M_a)$, the line $m(H_a H_c)$, the point N and the point E_b , construct a point M_b (rule W05a); % NDG: line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect % DET: points E_b and M_b must be different;
8. Choose freely a point A on the circle $k(M_b, C)$ (rule WOncircle);
9. Using the point A and the point M_b , construct a point C (rule W01); ;
10. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;

11. Using the point H_c and the point C , construct a line h_c (rule W02); % DET: points H_c and C are not the same;
12. Using the line h_a and the line h_c , construct a point H (rule W03); % NDG: lines h_a and h_c are not parallel % DET: lines h_a and h_c are not the same;
13. Using the point E_b and the point H , construct a point B (rule W01); .

Non-degenerate conditions: lines h_a and h_c are not parallel; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; points H_a and N are not the same; lines $m(H_a H_c)$ and $m(H_a E_b)$ are not parallel; points H_c and E_b are not the same.

Determination conditions: lines h_a and h_c are not the same; points H_c and C are not the same; points A and H_a are not the same; points E_b and M_b must be different; lines $m(H_a H_c)$ and $m(H_a E_b)$ are not the same; points H_a and H_c are not the same; points H_a and E_b are not the same.

Rules used: [W01,W02,W03,W05a,W06,W14,WOncircle1]

Lemmas used: [D10,D22,D29,D3,D32,D5,D7,D8,GD02,GL01,GL03,GL04,L17,L19,L21,L23,L3,L44,L45,L50,L51]

Solving time: 64.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
point H_{c} 68.91 84.83
point H_{a} 80 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{b}
cmark_rt H_{c}
cmark_r H_{a}
color 0 0 0
fontsize 8
```

```
% NDG: points H_{c} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H_{c}
```

```
circle k(E_{b},B) E_{b} H_{c}
```

```
color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0
```

```
% Choosing randomly a point H_{a} on the circle with center E_{b} through point H_{c}
oncircle H_{a} E_{b} H_{c}
cmark_r H_{a}
color 200 200 200
```

```

drawcircle E_{b} H_{c}
color 0 0 0

% DET: points H_{a} and E_{b} are not the same
% Constructing bisector m(H_{a}E_{b}) of the segment H_{a}E_{b}
med m(H_{a}E_{b}) H_{a} E_{b}

color 200 200 200
drawline m(H_{a}E_{b})
color 0 0 0

color 200 200 200
drawsegment H_{a} E_{b}
color 0 0 0

% DET: points H_{a} and H_{c} are not the same
% Constructing bisector m(H_{a}H_{c}) of the segment H_{a}H_{c}
med m(H_{a}H_{c}) H_{a} H_{c}

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

color 200 200 200
drawsegment H_{a} H_{c}
color 0 0 0

% NDG: lines m(H_{a}H_{c}) and m(H_{a}E_{b}) are not parallel% DET: lines m(H_{a}H_{c}) and m(H_{a}
E_{b}) are not the same
% Constructing a point N which belongs to line m(H_{a}H_{c}) and line m(H_{a}E_{b})
intersec N m(H_{a}H_{c}) m(H_{a}E_{b})
cmark_r N

% NDG: points H_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{a}
circle k(N,M_{a}) N H_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(H_{a}H_{c}) and circle k(N,M_{a}) intersect% DET: points E_{b} and M_{b} must be
different
% Constructing a point M_{b} which is an image of the point E_{b} in the symmetry to point/line N
sim M_{b} N E_{b}
cmark_lt M_{b}

% Choosing randomly a point A on the circle with center M_{b} through point H_{a}

```



```

oncircle A M_{b} H_{a}
cmark_t A
color 200 200 200
drawcircle M_{b} H_{a}
color 0 0 0

% Constructing a point C such that  $AC/AM_{b}=2$ 
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% DET: points H_{c} and C are not the same
% Constructing a line h_{c} which passes through point H_{c} and point C
line h_{c} H_{c} C

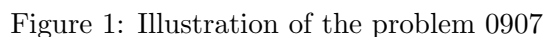
color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines h_{a} and h_{c} are not parallel% DET: lines h_{a} and h_{c} are not the same
% Constructing a point H which belongs to line h_{a} and line h_{c}
intersec H h_{a} h_{c}
cmark_rt H

% Constructing a point B such that  $E_{b}B/E_{b}H=-1$ 
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

```



3.3 Illustration

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1.1 Proving $E_b = -E_b$

1514

4.1.2 Proving $H_c = \neg H_c$

Proving failed

4.1.3 Proving $H_a = \neg H_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $H_c = \neg H_c$

Proving failed

4.2.3 Proving $H_a = \neg H_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $H_c = \neg H_c$

Proving failed

4.3.3 Proving $H_a = \neg H_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $H_c = \neg H_c$

Proving failed

4.4.3 Proving $H_a = \neg H_a$

Proving failed

Problem 908

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 908: Given a point E_b , a point H_a and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 909

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 909: Given a point E_b , a point H_a and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
2. Using the point H_a and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_a and E_b are not the same;
3. Using the circle $k(E_b, B)$, the line a , the point E_b and the point H_a , construct a point B (rule W05); % NDG: line a and circle $k(E_b, B)$ intersect % DET: points H_a and B must be different;
4. Using the point B and the point E_b , construct a point H (rule W01); ;
5. Using the point M_a and the point B , construct a point C (rule W01); ;
6. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
7. Using the point H and the point C , construct a line h_c (rule W02); % DET: points H and C are not the same;
8. Using the circle $k(E_b, B)$, the line h_c , the point E_b and the point H , construct a point H_c (rule W05); % NDG: line h_c and circle $k(E_b, B)$ intersect % DET: points H and H_c must be different;
9. Using the point B and the point H_c , construct a line c (rule W02); % DET: points B and H_c are not the same;
10. Using the line h_a and the line c , construct a point A (rule W03); % NDG: lines h_a and c are not parallel % DET: lines h_a and c are not the same.

Non-degenerate conditions: lines h_a and c are not parallel; line h_c and circle $k(E_b, B)$ intersect; line a and circle $k(E_b, B)$ intersect; points H_a and E_b are not the same.

Determination conditions: lines h_a and c are not the same; points B and H_c are not the same; points H and H_c must be different; points H and C are not the same; points H_a and H are not the same; points H_a and B must be different; points H_a and M_a are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D21,D29,D3,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L49,L50,L51]

Solving time: 9.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
```

```
point H_{a} 80 40
```

```
point M_{a} 65 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{b}
```

```
cmark_r H_{a}
```

```
cmark_r M_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{a} and M_{a} are not the same
```

```
% Constructing a line a which passes through point H_{a} and point M_{a}
```

```
line a H_{a} M_{a}
```

```
color 200 200 200
```

```
drawline a
```

```
color 0 0 0
```

```
% NDG: points H_{a} and E_{b} are not the same
```

```
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H_{a}
```

```
circle k(E_{b},B) E_{b} H_{a}
```

```
color 200 200 200
```

```
drawcircle k(E_{b},B)
```

```
color 0 0 0
```

```
% NDG: line a and circle k(E_{b},B) intersect% DET: points H_{a} and B must be different
```

```
% Constructing a point P_{\_G163776} which is a foot of the point E_{b} on the line a
```

```
foot P_{\_G163776} E_{b} a
```

```
cmark_r P_{\_G163776}
```

```
color 200 200 200
```

```
drawline E_{b} P_{\_G163776}
```

```
color 0 0 0
```

```

% Constructing a point B which is an image of the point H_{a} in the symmetry to point/line P_{\_G
163776}
sim B P_{\_G163776} H_{a}
cmark_b B

% Constructing a point H such that BH/BE_{b}=2
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0

% Constructing a point C such that M_{a}C/M_{a}B=-1
towards C M_{a} B -1
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% DET: points H and C are not the same
% Constructing a line h_{c} which passes through point H and point C
line h_{c} H C

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: line h_{c} and circle k(E_{b},B) intersect% DET: points H and H_{c} must be different
% Constructing a point P_{\_G164213} which is a foot of the point E_{b} on the line h_{c}
foot P_{\_G164213} E_{b} h_{c}
cmark_r P_{\_G164213}
color 200 200 200
drawline E_{b} P_{\_G164213}
color 0 0 0

% Constructing a point H_{c} which is an image of the point H in the symmetry to point/line P_{\_G
164213}
sim H_{c} P_{\_G164213} H

```

```

cmark_rt H_{c}

% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines h_{a} and c are not parallel% DET: lines h_{a} and c are not the same
% Constructing a point A which belongs to line h_{a} and line c
intersec A h_{a} c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{a} and c are not parallel; line h_{c} and circle k(E_{b},B)
% intersect; line a and circle k(E_{b},B) intersect; points H_{a} and E_{b} are not the same
% Determination conditions: lines h_{a} and c are not the same; points B and H_{c} are not the same
% ; points H and H_{c} must be different; points H and C are not the same; points H_{a} and H are
% not the same; points H_{a} and B must be different; points H_{a} and M_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $H_a = \neg H_a$

Proving failed

4.1.3 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 11 terms.

Time Complexity: Time spent by the prover is 0.182 seconds.

NDG conditions Points M_a and H_a are not identical

Points M_a and H_a are not identical

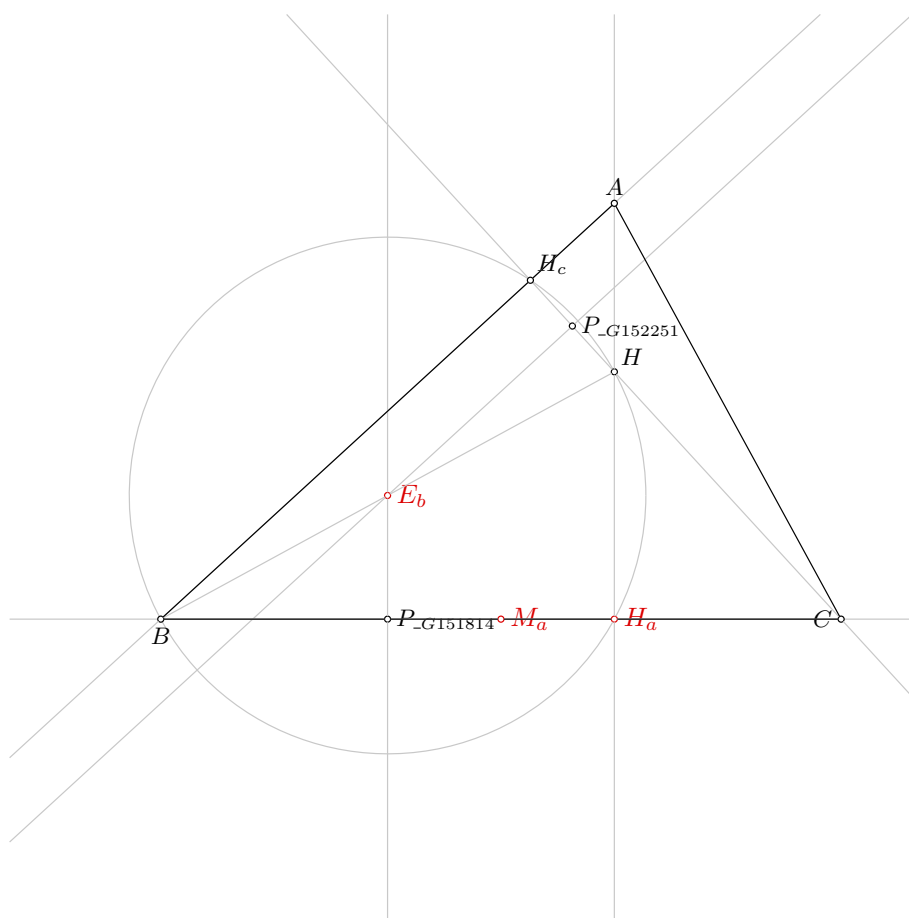


Figure 1: Illustration of the problem 0909

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $H_a = \neg H_a$

Proving failed

4.2.3 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{H_a B H_c} \neq S_{H B H_c}$ i.e., lines $H_a H$ and $B H_c$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{\neg h_b}} \neq S_{F^0_{\neg h_a} B F^1_{\neg h_b}}$ i.e., lines $AF^0_{\neg h_a}$ and $BF^1_{\neg h_b}$ are not parallel (construction based assumption)

$S_{BAF^0_{\neg h_a}} \neq S_{CA F^0_{\neg h_a}}$ i.e., lines BC and $AF^0_{\neg h_a}$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $H_a = \neg H_a$

Proving failed

4.3.3 Proving $M_a = \neg M_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $H_a = \neg H_a$

Proving failed

4.4.3 Proving $M_a = \neg M_a$

Proving failed

Problem 910

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 910: Given a point E_b , a point M_b and a point H_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point M_b , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and M_b are not the same;
2. Using the point E_b and the point M_b , construct a line $m(E_bM_b)$ (rule W14); % DET: points E_b and M_b are not the same;
3. Using the line $m(E_bM_b)$ and the line $m(H_aH_c)$, construct a point N (rule W03); % NDG: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel % DET: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same;
4. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
5. Choose freely a point H_a on the circle $k(N, M_a)$ (rule WOncircle);
6. Using the point H_a and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_a and E_b are not the same;
7. Using the circle $k(N, M_a)$, the circle $k(E_b, B)$, the point H_a , the point N and the point E_b , construct a point H_c (rule W08); % NDG: circles $k(N, M_a)$ and $k(E_b, B)$ intersect % DET: circles $k(N, M_a)$ and $k(E_b, B)$ are not the same; points H_a and H_c must be different;
8. Choose freely a point A on the circle $k(M_b, C)$ (rule WOncircle);
9. Using the point A and the point M_b , construct a point C (rule W01); ;
10. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;

11. Using the point H_c and the point C , construct a line h_c (rule W02); % DET: points H_c and C are not the same;
12. Using the line h_a and the line h_c , construct a point H (rule W03); % NDG: lines h_a and h_c are not parallel % DET: lines h_a and h_c are not the same;
13. Using the point E_b and the point H , construct a point B (rule W01); .

Non-degenerate conditions: lines h_a and h_c are not parallel; circles $k(N, M_a)$ and $k(E_b, B)$ intersect; points H_a and E_b are not the same; points E_b and N are not the same; lines $m(E_b M_b)$ and $m(H_a H_c)$ are not parallel.

Determination conditions: lines h_a and h_c are not the same; points H_c and C are not the same; points A and H_a are not the same; circles $k(N, M_a)$ and $k(E_b, B)$ are not the same; points H_a and H_c must be different; lines $m(E_b M_b)$ and $m(H_a H_c)$ are not the same; points E_b and M_b are not the same; points E_b and M_b are not the same.

Rules used: [W01,W02,W03,W06,W08,W14,WOncircle1]

Lemmas used: [D10,D22,D29,D3,D32,D5,D7,D8,GD02,GL01,GL03,GL04,L17,L19,L21,L23,L3,L44,L45,L50,L51]

Solving time: 64.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
```

```
point M_{b} 95 67.5
```

```
point H_{a} 80 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{b}
```

```
cmark_lt M_{b}
```

```
cmark_r H_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{b} and M_{b} are not the same
```

```
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point M_{b}
```

```
line m(H_{a}H_{c}) E_{b} M_{b}
```

```
color 200 200 200
```

```
drawline m(H_{a}H_{c})
```

```
color 0 0 0
```

```
% DET: points E_{b} and M_{b} are not the same
```

```
% Constructing bisector m(E_{b}M_{b}) of the segment E_{b}M_{b}
```

```
med m(E_{b}M_{b}) E_{b} M_{b}
```

```
color 200 200 200
```

```
drawline m(E_{b}M_{b})
```

```

color 0 0 0

color 200 200 200
drawsegment E_{b} M_{b}
color 0 0 0

% NDG: lines  $m(E_{b}M_{b})$  and  $m(H_{a}H_{c})$  are not parallel% DET: lines  $m(E_{b}M_{b})$  and  $m(H_{a}H_{c})$ 
%  $H_{c})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{b}M_{b})$  and line  $m(H_{a}H_{c})$ 
intersec N m(E_{b}M_{b}) m(H_{a}H_{c})
cmark_r N

% NDG: points  $E_{b}$  and  $N$  are not the same
% Constructing a circle  $k(N,M_{a})$  whose center is at point  $N$  and which passes through point  $E_{b}$ 
circle k(N,M_{a}) N E_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point  $H_{a}$  on the circle with center  $N$  through point  $E_{b}$ 
oncircle H_{a} N E_{b}
cmark_r H_{a}
color 200 200 200
drawcircle N E_{b}
color 0 0 0

% NDG: points  $H_{a}$  and  $E_{b}$  are not the same
% Constructing a circle  $k(E_{b},B)$  whose center is at point  $E_{b}$  and which passes through point  $H_{a}$ 
circle k(E_{b},B) E_{b} H_{a}

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: circles  $k(N,M_{a})$  and  $k(E_{b},B)$  intersect% DET: circles  $k(N,M_{a})$  and  $k(E_{b},B)$  are not
% the same; points  $H_{a}$  and  $H_{c}$  must be different
% Constructing a line  $L_{\backslash\_G193160}$  which passes through point  $N$  and point  $E_{b}$ 
line L_{\backslash\_G193160} N E_{b}

color 200 200 200
drawline L_{\backslash\_G193160}
color 0 0 0

% Constructing a point  $H_{c}$  which is an image of the point  $H_{a}$  in the symmetry to point/line  $L_{\backslash\_G193160}$ 

```

```

sim H_{c} L_{\backslash_G193160} H_{a}
cmark_rt H_{c}

```

```

% Choosing randomly a point A on the circle with center M_{b} through point H_{a}
oncircle A M_{b} H_{a}
cmark_t A
color 200 200 200
drawcircle M_{b} H_{a}
color 0 0 0

```

```

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

```

```

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

```

```

% DET: points H_{c} and C are not the same
% Constructing a line h_{c} which passes through point H_{c} and point C
line h_{c} H_{c} C

color 200 200 200
drawline h_{c}
color 0 0 0

```

```

% NDG: lines h_{a} and h_{c} are not parallel% DET: lines h_{a} and h_{c} are not the same
% Constructing a point H which belongs to line h_{a} and line h_{c}
intersec H h_{a} h_{c}
cmark_rt H

```

```

% Constructing a point B such that E_{b}B/E_{b}H=-1
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

```

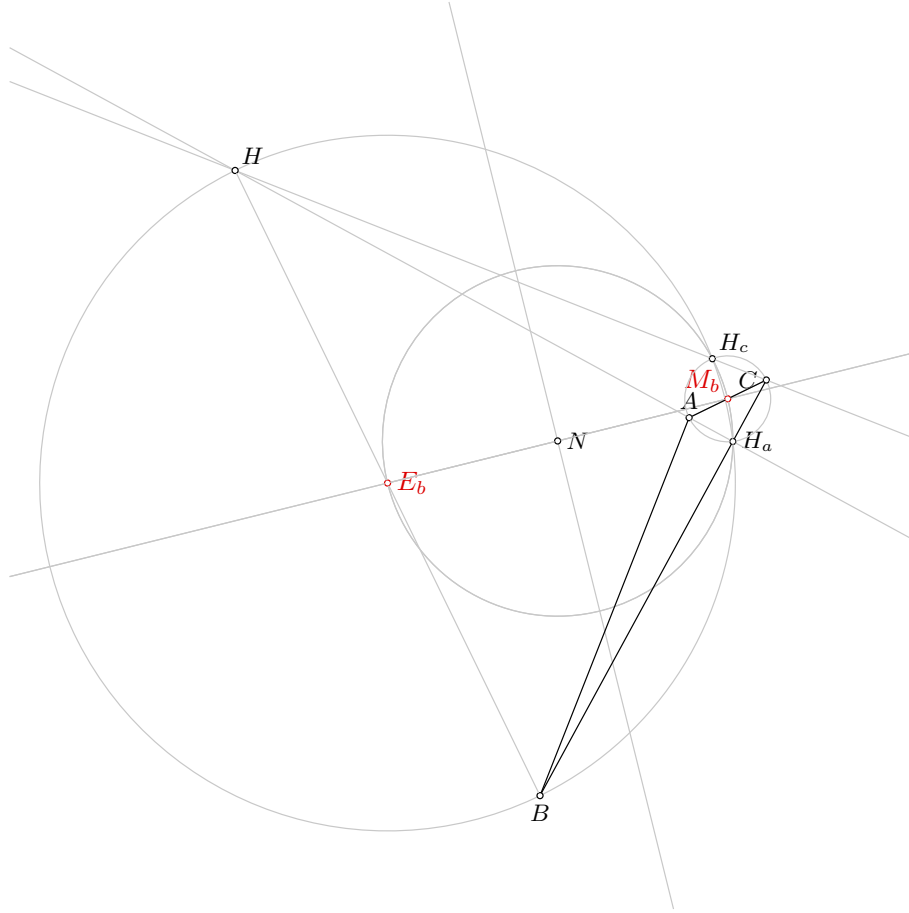


Figure 1: Illustration of the problem 0910

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines  $h_{\{a\}}$  and  $h_{\{c\}}$  are not parallel; circles  $k(N, M_{\{a\}})$  and  $k(E_{\{b\}}, B)$  intersect; points  $H_{\{a\}}$  and  $E_{\{b\}}$  are not the same; points  $E_{\{b\}}$  and  $N$  are not the same; lines  $m(E_{\{b\}}, M_{\{b\}})$  and  $m(H_{\{a\}}, H_{\{c\}})$  are not parallel
% Determination conditions: lines  $h_{\{a\}}$  and  $h_{\{c\}}$  are not the same; points  $H_{\{c\}}$  and  $C$  are not the same; points  $A$  and  $H_{\{a\}}$  are not the same; circles  $k(N, M_{\{a\}})$  and  $k(E_{\{b\}}, B)$  are not the same; points  $H_{\{a\}}$  and  $H_{\{c\}}$  must be different; lines  $m(E_{\{b\}}, M_{\{b\}})$  and  $m(H_{\{a\}}, H_{\{c\}})$  are not the same; points  $E_{\{b\}}$  and  $M_{\{b\}}$  are not the same; points  $E_{\{b\}}$  and  $M_{\{b\}}$  are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 8 terms.

Time Complexity: Time spent by the prover is 0.149 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $H_a = \neg H_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{M_{m(E_b M_b)}^0}^{E_b M_b} \neq S_{T_{m(E_b M_b)}^1}^{E_b M_b}$ i.e., lines $M_{m(E_b M_b)}^0 T_{m(E_b M_b)}^1$ and $E_b M_b$ are not parallel (construction based assumption)

$S_{AH_c C} \neq S_{H_a H_c C}$ i.e., lines AH_a and $H_c C$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2 BF_{\neg h_b}^3}$ i.e., lines $AF_{\neg h_a}^2$ and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

$S_{BAF_{\neg h_a}^2} \neq S_{CAF_{\neg h_a}^2}$ i.e., lines BC and $AF_{\neg h_a}^2$ are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

4.2.3 Proving $H_a = \neg H_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $M_b = \neg M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 19 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.3.3 Proving $H_a = \neg H_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $M_b = \neg M_b$

Proving failed

4.4.3 Proving $H_a = \neg H_a$

Proving failed

Problem 911

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 911: Given a point E_b , a point H_a and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H_a and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_a and E_b are not the same;
2. Using the point H_a and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points H_a and M_c are not the same;
3. Using the circle $k(E_b, B)$, the circle $k(M_c, A)$, the point H_a , the point E_b and the point M_c , construct a point B (rule W08); % NDG: circles $k(E_b, B)$ and $k(M_c, A)$ intersect % DET: circles $k(E_b, B)$ and $k(M_c, A)$ are not the same; points H_a and B must be different;
4. Using the point B and the point E_b , construct a point H (rule W01); ;
5. Using the point M_c and the point B , construct a point A (rule W01); ;
6. Using the point H_a and the point B , construct a line a (rule W02); % DET: points H_a and B are not the same;
7. Using the point M_c and the point B , construct a line c (rule W02); % DET: points M_c and B are not the same;
8. Using the circle $k(E_b, B)$, the line c , the point E_b and the point B , construct a point H_c (rule W05); % NDG: line c and circle $k(E_b, B)$ intersect % DET: points B and H_c must be different;
9. Using the point H and the point H_c , construct a line h_c (rule W02); % DET: points H and H_c are not the same;
10. Using the line a and the line h_c , construct a point C (rule W03); % NDG: lines a and h_c are not parallel % DET: lines a and h_c are not the same.

Non-degenerate conditions: lines a and h_c are not parallel; line c and circle $k(E_b, B)$ intersect; circles $k(E_b, B)$ and $k(M_c, A)$ intersect; points H_a and M_c are not the same; points H_a and E_b are not the same.

Determination conditions: lines a and h_c are not the same; points H and H_c are not the same; points B and H_c must be different; points M_c and B are not the same; points H_a and B are not the same; circles $k(E_b, B)$ and $k(M_c, A)$ are not the same; points H_a and B must be different.

Rules used: [W01,W02,W03,W05,W06,W08]

Lemmas used: [D10,D20,D29,D5,D7,GD01,GD02,GL03,GL04,GL09,L3,L40,L42,L50,L51]

Solving time: 9.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
point H_{a} 80 40
point M_{c} 50 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{b}
cmark_r H_{a}
cmark_lt M_{c}
color 0 0 0
fontsize 8
```

```
% NDG: points H_{a} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H
_{a}
circle k(E_{b},B) E_{b} H_{a}
```

```
color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0
```

```
% NDG: points H_{a} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H
_{a}
circle k(M_{c},A) M_{c} H_{a}
```

```
color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0
```

```
% NDG: circles k(E_{b},B) and k(M_{c},A) intersect% DET: circles k(E_{b},B) and k(M_{c},A) are not
the same; points H_{a} and B must be different
% Constructing a line L_{\_G220527} which passes through point E_{b} and point M_{c}
line L_{\_G220527} E_{b} M_{c}
```

```

color 200 200 200
drawline L_{\_G220527}
color 0 0 0

% Constructing a point B which is an image of the point H_{a} in the symmetry to point/line L_{\_G
220527}
sim B L_{\_G220527} H_{a}
cmark_b B

% Constructing a point H such that BH/BE_{b}=2
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0

% Constructing a point A such that M_{c}A/M_{c}B=-1
towards A M_{c} B -1
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% DET: points M_{c} and B are not the same
% Constructing a line c which passes through point M_{c} and point B
line c M_{c} B

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(E_{b},B) intersect% DET: points B and H_{c} must be different
% Constructing a point P_{\_G220944} which is a foot of the point E_{b} on the line c
foot P_{\_G220944} E_{b} c
cmark_r P_{\_G220944}
color 200 200 200
drawline E_{b} P_{\_G220944}

```

```

color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $B$  in the symmetry to point/line  $P_{\{\backslash\_G$ 
  220944}
sim  $H_{\{c\}}$   $P_{\{\backslash\_G220944\}}$   $B$ 
cmark_rt  $H_{\{c\}}$ 

% DET: points  $H$  and  $H_{\{c\}}$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $H$  and point  $H_{\{c\}}$ 
line  $h_{\{c\}}$   $H$   $H_{\{c\}}$ 

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: lines  $a$  and  $h_{\{c\}}$  are not parallel% DET: lines  $a$  and  $h_{\{c\}}$  are not the same
% Constructing a point  $C$  which belongs to line  $a$  and line  $h_{\{c\}}$ 
intersec  $C$   $a$   $h_{\{c\}}$ 
cmark_l  $C$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $a$  and  $h_{\{c\}}$  are not parallel; line  $c$  and circle  $k(E_{\{b\}},B)$ 
  intersect; circles  $k(E_{\{b\}},B)$  and  $k(M_{\{c\}},A)$  intersect; points  $H_{\{a\}}$  and  $M_{\{c\}}$  are not the same
  ; points  $H_{\{a\}}$  and  $E_{\{b\}}$  are not the same
% Determination conditions: lines  $a$  and  $h_{\{c\}}$  are not the same; points  $H$  and  $H_{\{c\}}$  are not the same
  ; points  $B$  and  $H_{\{c\}}$  must be different; points  $M_{\{c\}}$  and  $B$  are not the same; points  $H_{\{a\}}$  and  $B$ 
  are not the same; circles  $k(E_{\{b\}},B)$  and  $k(M_{\{c\}},A)$  are not the same; points  $H_{\{a\}}$  and  $B$  must
  be different

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $H_a = \neg H_a$

Proving failed

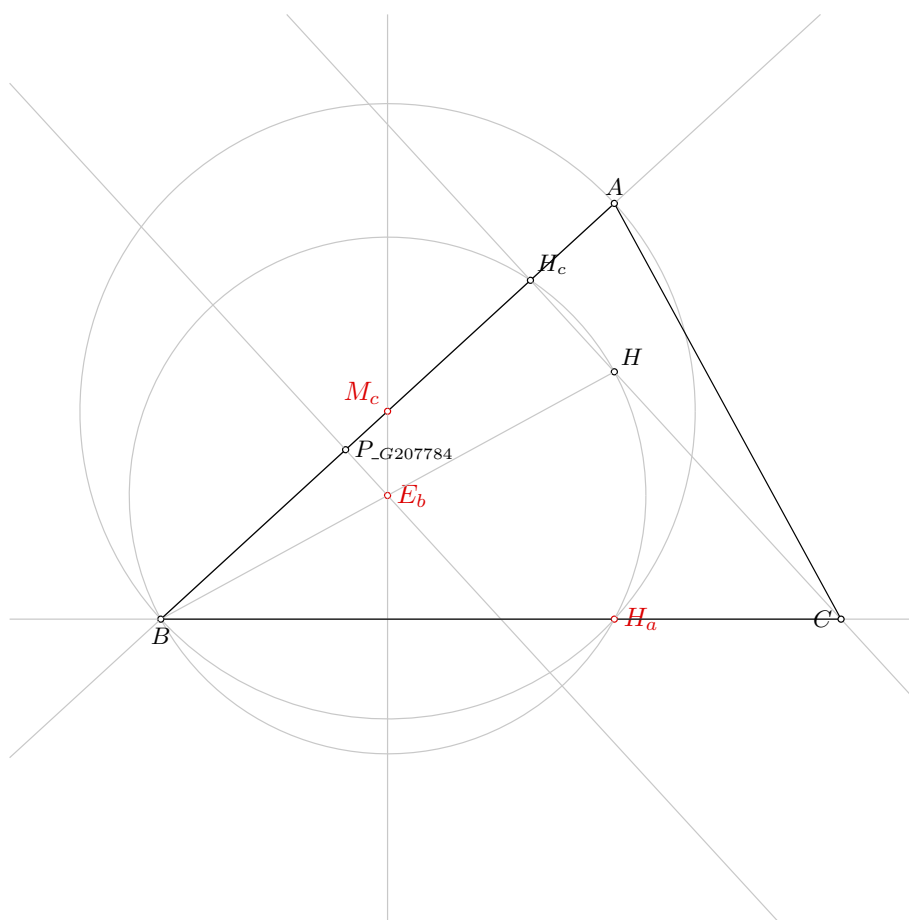


Figure 1: Illustration of the problem 0911

4.1.3 Proving $M_c = M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 11 terms.

Time Complexity: Time spent by the prover is 0.166 seconds.

NDG conditions Points M_c and E_b are not identical

Points M_c and E_b are not identical

4.2 GCLC - Area method

4.2.1 Proving $E_b = E_b$

Proving failed

4.2.2 Proving $H_a = H_a$

Proving failed

4.2.3 Proving $M_c = M_c$

NDG conditions are:

$S_{H_a H H_c} \neq S_{B H H_c}$ i.e., lines $H_a B$ and $H H_c$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{h_b}} \neq S_{F^0_{h_a} B F^1_{h_b}}$ i.e., lines $AF^0_{h_a}$ and $BF^1_{h_b}$ are not parallel (construction based assumption)

$S_{BAF^0_{h_a}} \neq S_{CAF^0_{h_a}}$ i.e., lines BC and $AF^0_{h_a}$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_b = E_b$

Proving failed

4.3.2 Proving $H_a = H_a$

Proving failed

4.3.3 Proving $M_c = M_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = E_b$

Proving failed

4.4.2 Proving $H_a = H_a$

Proving failed

4.4.3 Proving $M_c = M_c$

Proving failed

Problem 912

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 912: Given a point H_a , a point N and a point E_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_a and N are not the same;
2. Choose freely a point E_b on the circle $k(N, M_a)$ (rule WOncircle);
3. Using the point E_b and the point N , construct a line $m(H_a H_c)$ (rule W02); % DET: points E_b and N are not the same;
4. Using the circle $k(N, M_a)$, the line $m(H_a H_c)$, the point N and the point E_b , construct a point M_b (rule W05a); % NDG: line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect % DET: points E_b and M_b must be different;
5. Using the point H_a and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_a and E_b are not the same;
6. Using the circle $k(N, M_a)$, the circle $k(E_b, B)$, the point H_a , the point N and the point E_b , construct a point H_c (rule W08); % NDG: circles $k(N, M_a)$ and $k(E_b, B)$ intersect % DET: circles $k(N, M_a)$ and $k(E_b, B)$ are not the same; points H_a and H_c must be different;
7. Choose freely a point A on the circle $k(M_b, C)$ (rule WOncircle);
8. Using the point A and the point M_b , construct a point C (rule W01); ;
9. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
10. Using the point H_c and the point C , construct a line h_c (rule W02); % DET: points H_c and C are not the same;

11. Using the line h_a and the line h_c , construct a point H (rule W03); % NDG: lines h_a and h_c are not parallel % DET: lines h_a and h_c are not the same;
12. Using the point E_b and the point H , construct a point B (rule W01); .

Non-degenerate conditions: lines h_a and h_c are not parallel; circles $k(N, M_a)$ and $k(E_b, B)$ intersect; points H_a and E_b are not the same; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; points H_a and N are not the same.

Determination conditions: lines h_a and h_c are not the same; points H_c and C are not the same; points A and H_a are not the same; circles $k(N, M_a)$ and $k(E_b, B)$ are not the same; points H_a and H_c must be different; points E_b and M_b must be different; points E_b and N are not the same.

Rules used: [W01,W02,W03,W05a,W06,W08,WOncircle1]

Lemmas used: [D10,D22,D29,D3,D32,D5,D7,D8,GD02,GL01,GL03,GL04,L17,L19,L21,L23,L3,L44,L45,L50,L51]

Solving time: 57.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{a} 80 40
point N 72.5 61.93
point E_{b} 50 56.36
```

```
color 220 0 0
fontsize 9
```

```
cmark_r H_{a}
cmark_r N
cmark_r E_{b}
color 0 0 0
fontsize 8
```

```
% NDG: points H_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{a}
circle k(N,M_{a}) N H_{a}
```

```
color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```
% Choosing randomly a point E_{b} on the circle with center N through point H_{a}
oncircle E_{b} N H_{a}
cmark_r E_{b}
color 200 200 200
drawcircle N H_{a}
color 0 0 0
```

```
% DET: points E_{b} and N are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point N
```

```

line m(H_{a}H_{c}) E_{b} N

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

% NDG: line m(H_{a}H_{c}) and circle k(N,M_{a}) intersect% DET: points E_{b} and M_{b} must be
different
% Constructing a point M_{b} which is an image of the point E_{b} in the symmetry to point/line N
sim M_{b} N E_{b}
cmark_lt M_{b}

% NDG: points H_{a} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H
_{a}
circle k(E_{b},B) E_{b} H_{a}

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(E_{b},B) intersect% DET: circles k(N,M_{a}) and k(E_{b},B) are not
the same; points H_{a} and H_{c} must be different
% Constructing a line L_{\_G257976} which passes through point N and point E_{b}
line L_{\_G257976} N E_{b}

color 200 200 200
drawline L_{\_G257976}
color 0 0 0

% Constructing a point H_{c} which is an image of the point H_{a} in the symmetry to point/line L
_{\_G257976}
sim H_{c} L_{\_G257976} H_{a}
cmark_rt H_{c}

% Choosing randomly a point A on the circle with center M_{b} through point H_{a}
oncircle A M_{b} H_{a}
cmark_t A
color 200 200 200
drawcircle M_{b} H_{a}
color 0 0 0

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200

```

```

drawsegment A C
color 0 0 0

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% DET: points H_{c} and C are not the same
% Constructing a line h_{c} which passes through point H_{c} and point C
line h_{c} H_{c} C

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines h_{a} and h_{c} are not parallel% DET: lines h_{a} and h_{c} are not the same
% Constructing a point H which belongs to line h_{a} and line h_{c}
intersec H h_{a} h_{c}
cmark_rt H

% Constructing a point B such that E_{b}B/E_{b}H=-1
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

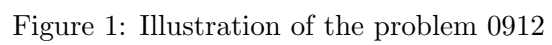
drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{a} and h_{c} are not parallel; circles k(N,M_{a}) and k(E_{b}
%,B) intersect; points H_{a} and E_{b} are not the same; line m(H_{a}H_{c}) and circle k(N,M_{a}
%) intersect; points H_{a} and N are not the same
% Determination conditions: lines h_{a} and h_{c} are not the same; points H_{c} and C are not the
% same; points A and H_{a} are not the same; circles k(N,M_{a}) and k(E_{b},B) are not the same;
% points H_{a} and H_{c} must be different; points E_{b} and M_{b} must be different; points E_{b}
% and N are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1



3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_a = \neg H_a$

Proving failed

4.1.2 Proving $N = \neg N$

Proving failed

4.1.3 Proving $E_b = \neg E_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_a = \neg H_a$

Proving failed

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $E_b = \neg E_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_a = \neg H_a$

Proving failed

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $E_b = \neg E_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_a = \neg H_a$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $E_b = E_b$

Proving failed

Problem 913

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 913: Given a point E_b , a point H_a and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 914

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 914: Given a point E_b , a point H_a and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 915

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 915: Given a point E_b , a point H_a and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 916

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 916: Given a point E_b , a point H_a and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 917

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 917: Given a point E_b , a point H_b and a point H_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;
2. Using the point H_c and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_c and E_b are not the same;
3. Using the circle $k(E_b, B)$ and the line h_b , construct a point B and a point H (rule W04); % NDG: line h_b and circle $k(E_b, B)$ intersect;
4. Using the point H_c and the point B , construct a line c (rule W02); % DET: points H_c and B are not the same;
5. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
6. Using the point H_b and the line h_b , construct a line b (rule W10a); ;
7. Using the line b and the line c , construct a point A (rule W03); % NDG: lines b and c are not parallel % DET: lines b and c are not the same;
8. Using the line h_c and the line b , construct a point C (rule W03); % NDG: lines h_c and b are not parallel % DET: lines h_c and b are not the same.

Non-degenerate conditions: lines h_c and b are not parallel; lines b and c are not parallel; line h_b and circle $k(E_b, B)$ intersect; points H_c and E_b are not the same.

Determination conditions: lines h_c and b are not the same; lines b and c are not the same; points H_c and H are not the same; points H_c and B are not the same; points E_b and H_b are not the same.

Rules used: [W02,W03,W04,W06,W10a]

Lemmas used: [D10,D29,D3,D6,D7,D9,GD01,GD02,GL09,L3,L49,L51]
Solving time: 6.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{b} 50 56.36
point H_{b} 89.36 77.83
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_r E_{b}
cmark_l H_{b}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points E_{b} and H_{b} are not the same
% Constructing a line h_{b} which passes through point E_{b} and point H_{b}
line h_{b} E_{b} H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points H_{c} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H_{c}
circle k(E_{b},B) E_{b} H_{c}

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line h_{b} and circle k(E_{b},B) intersect
% Constructing points B and H which are in intersection of k(E_{b},B) and h_{b}
intersec2 B H k(E_{b},B) h_{b}
cmark_b B
cmark_rt H

% DET: points H_{c} and B are not the same
% Constructing a line c which passes through point H_{c} and point B
line c H_{c} B

color 200 200 200
drawline c
color 0 0 0
```

```

% DET: points  $H_{\{c\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $H_{\{c\}}$  and point  $H$ 
line  $h_{\{c\}}$   $H_{\{c\}}$   $H$ 

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% Constructing a line  $b$  which is perpendicular to line  $h_{\{b\}}$  and which passes through point  $H_{\{b\}}$ 
perp  $b$   $H_{\{b\}}$   $h_{\{b\}}$ 

color 200 200 200
drawline  $b$ 
color 0 0 0

% NDG: lines  $b$  and  $c$  are not parallel% DET: lines  $b$  and  $c$  are not the same
% Constructing a point  $A$  which belongs to line  $b$  and line  $c$ 
intersec  $A$   $b$   $c$ 
cmark_t  $A$ 

% NDG: lines  $h_{\{c\}}$  and  $b$  are not parallel% DET: lines  $h_{\{c\}}$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $h_{\{c\}}$  and line  $b$ 
intersec  $C$   $h_{\{c\}}$   $b$ 
cmark_l  $C$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $h_{\{c\}}$  and  $b$  are not parallel; lines  $b$  and  $c$  are not parallel;
% line  $h_{\{b\}}$  and circle  $k(E_{\{b\}}, B)$  intersect; points  $H_{\{c\}}$  and  $E_{\{b\}}$  are not the same
% Determination conditions: lines  $h_{\{c\}}$  and  $b$  are not the same; lines  $b$  and  $c$  are not the same;
% points  $H_{\{c\}}$  and  $H$  are not the same; points  $H_{\{c\}}$  and  $B$  are not the same; points  $E_{\{b\}}$  and  $H_{\{b\}}$ 
% are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

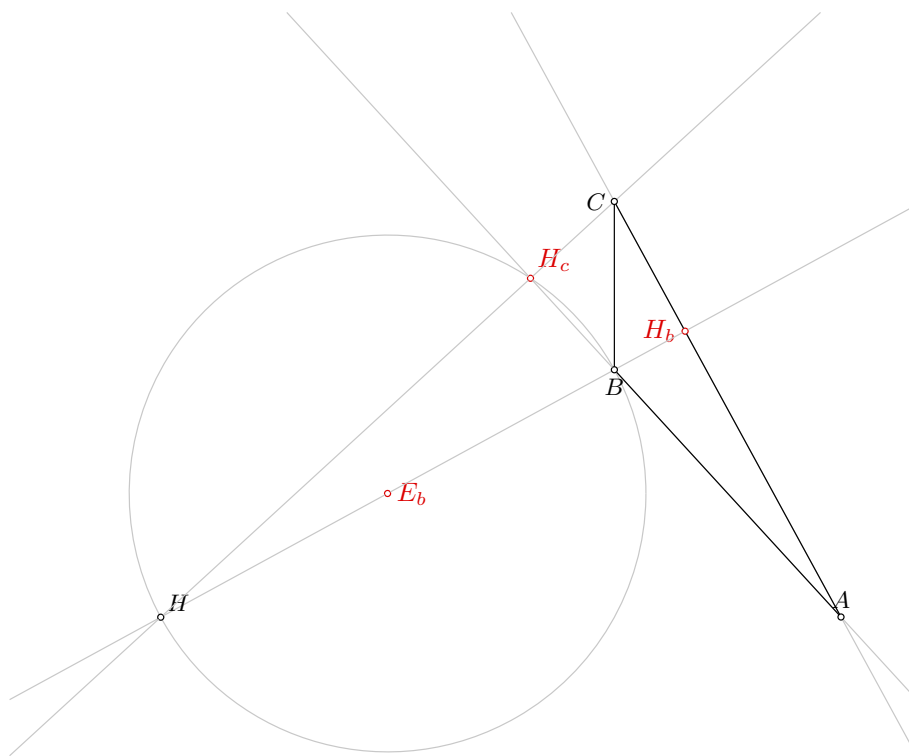


Figure 1: Illustration of the problem 0917

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 42 terms.

Time Complexity: Time spent by the prover is 0.38 seconds.

NDG conditions Point H is not the midpoint of segment with endpoints B and H_c

Points A , C and E_b are not collinear

Line through points B and H_b is not perpendicular to line through points H_b and E_b

4.1.2 Proving $H_b = \neg H_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

Time Complexity: Time spent by the prover is 0.099 seconds.

NDG conditions Point H is not the midpoint of segment with endpoints B and H_c

Points A , C and E_b are not collinear

4.1.3 Proving $H_c = \neg H_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $H_b = \neg H_b$

Proving failed

4.2.3 Proving $H_c = \neg H_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 65 terms.

Time Complexity: Time spent by the prover is 0.050 seconds. There are no ndg conditions.

4.3.2 Proving $H_b = \neg H_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.3 Proving $H_c = \neg H_c$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 395 terms.

Time Complexity: Time spent by the prover is 0.450 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $H_b = \neg H_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 0 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.3 Proving $H_c = \neg H_c$

Proving failed

Problem 918

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 918: Given a point E_b , a point H_b and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 919

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 919: Given a point E_b , a point H_b and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;
2. Using the point H_b and the point M_a , construct a circle $k(M_a, B)$ (rule W06); % NDG: points H_b and M_a are not the same;
3. Using the circle $k(M_a, B)$, the line h_b , the point M_a and the point H_b , construct a point B (rule W05); % NDG: line h_b and circle $k(M_a, B)$ intersect % DET: points H_b and B must be different;
4. Using the point B and the point E_b , construct a point H (rule W01); ;
5. Using the point M_a and the point B , construct a point C (rule W01); ;
6. Using the point H_b and the point C , construct a line b (rule W02); % DET: points H_b and C are not the same;
7. Using the point H and the point C , construct a line h_c (rule W02); % DET: points H and C are not the same;
8. Using the circle $k(M_a, B)$, the line h_c , the point M_a and the point C , construct a point H_c (rule W05); % NDG: line h_c and circle $k(M_a, B)$ intersect % DET: points C and H_c must be different;
9. Using the point B and the point H_c , construct a line c (rule W02); % DET: points B and H_c are not the same;
10. Using the line b and the line c , construct a point A (rule W03); % NDG: lines b and c are not parallel % DET: lines b and c are not the same.

Non-degenerate conditions: lines b and c are not parallel; line h_c and circle $k(M_a, B)$ intersect; line h_b and circle $k(M_a, B)$ intersect; points H_b and M_a are not the same.

Determination conditions: lines b and c are not the same; points B and H_c are not the same; points C and H_c must be different; points H and C are not the same; points H_b and C are not the same; points H_b and B must be different; points E_b and H_b are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D21,D29,D3,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L37,L38,L39]

Solving time: 8.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
```

```
point H_{b} 89.36 77.83
```

```
point M_{a} 65 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{b}
```

```
cmark_l H_{b}
```

```
cmark_r M_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{b} and H_{b} are not the same
```

```
% Constructing a line h_{b} which passes through point E_{b} and point H_{b}
```

```
line h_{b} E_{b} H_{b}
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```
% NDG: points H_{b} and M_{a} are not the same
```

```
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{b}
```

```
circle k(M_{a},B) M_{a} H_{b}
```

```
color 200 200 200
```

```
drawcircle k(M_{a},B)
```

```
color 0 0 0
```

```
% NDG: line h_{b} and circle k(M_{a},B) intersect% DET: points H_{b} and B must be different
```

```
% Constructing a point P_{\_G95861} which is a foot of the point M_{a} on the line h_{b}
```

```
foot P_{\_G95861} M_{a} h_{b}
```

```
cmark_r P_{\_G95861}
```

```
color 200 200 200
```

```
drawline M_{a} P_{\_G95861}
```

```
color 0 0 0
```

```

% Constructing a point B which is an image of the point H_{b} in the symmetry to point/line P_{\_G
95861}
sim B P_{\_G95861} H_{b}
cmark_b B

% Constructing a point H such that BH/BE_{b}=2
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0

% Constructing a point C such that M_{a}C/M_{a}B=-1
towards C M_{a} B -1
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% DET: points H_{b} and C are not the same
% Constructing a line b which passes through point H_{b} and point C
line b H_{b} C

color 200 200 200
drawline b
color 0 0 0

% DET: points H and C are not the same
% Constructing a line h_{c} which passes through point H and point C
line h_{c} H C

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: line h_{c} and circle k(M_{a},B) intersect% DET: points C and H_{c} must be different
% Constructing a point P_{\_G96298} which is a foot of the point M_{a} on the line h_{c}
foot P_{\_G96298} M_{a} h_{c}
cmark_r P_{\_G96298}
color 200 200 200
drawline M_{a} P_{\_G96298}
color 0 0 0

% Constructing a point H_{c} which is an image of the point C in the symmetry to point/line P_{\_G
96298}
sim H_{c} P_{\_G96298} C

```

```

cmark_rt H_{c}

% DET: points B and H_{c} are not the same
% Constructing a line c which passes through point B and point H_{c}
line c B H_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines b and c are not parallel% DET: lines b and c are not the same
% Constructing a point A which belongs to line b and line c
intersec A b c
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and c are not parallel; line h_{c} and circle k(M_{a},B)
% intersect; line h_{b} and circle k(M_{a},B) intersect; points H_{b} and M_{a} are not the same
% Determination conditions: lines b and c are not the same; points B and H_{c} are not the same;
% points C and H_{c} must be different; points H and C are not the same; points H_{b} and C are
% not the same; points H_{b} and B must be different; points E_{b} and H_{b} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $H_b = \neg H_b$

Proving failed

4.1.3 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 8 terms.

Time Complexity: Time spent by the prover is 0.12 seconds.

NDG conditions There are no NDG conditions for this theorem

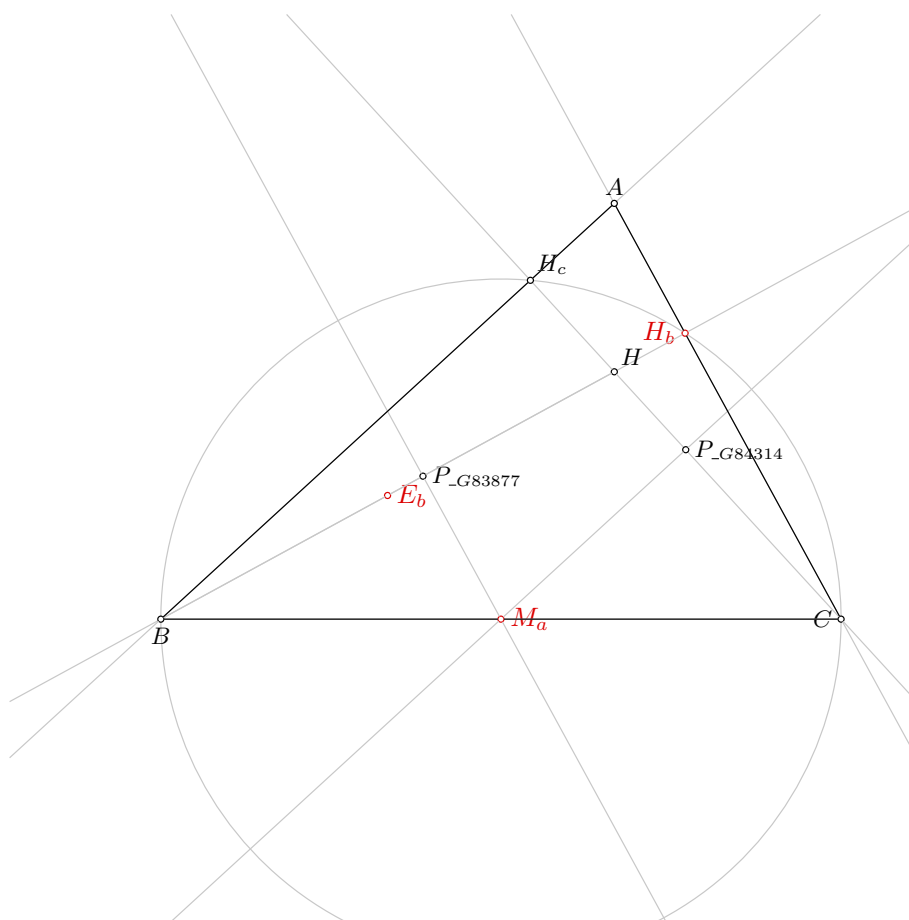


Figure 1: Illustration of the problem 0919

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $H_b = \neg H_b$

Proving failed

4.2.3 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{H_b B H_c} \neq S_{C B H_c}$ i.e., lines $H_b C$ and $B H_c$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^1} \neq S_{F_{\neg h_a}^0 B F_{\neg h_b}^1}$ i.e., lines $AF_{\neg h_a}^0$ and $BF_{\neg h_b}^1$ are not parallel (construction based assumption)

$S_{ABF_{\neg h_b}^1} \neq S_{CBF_{\neg h_b}^1}$ i.e., lines AC and $BF_{\neg h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $H_b = \neg H_b$

Proving failed

4.3.3 Proving $M_a = \neg M_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $H_b = \neg H_b$

Proving failed

4.4.3 Proving $M_a = \neg M_a$

Proving failed

Problem 920

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 920: Given a point H_b , a point M_b and a point E_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
2. Using the point H_b and the line b , construct a line h_b (rule W10b); ;
3. Choose freely a point E_b on the line h_b (rule WOnline2);
4. Using the point E_b and the point M_b , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and M_b are not the same;
5. Using the point E_b and the point H_b , construct a line $m(E_bH_b)$ (rule W14); % DET: points E_b and H_b are not the same;
6. Using the line $m(E_bH_b)$ and the line $m(H_aH_c)$, construct a point N (rule W03); % NDG: lines $m(E_bH_b)$ and $m(H_aH_c)$ are not parallel % DET: lines $m(E_bH_b)$ and $m(H_aH_c)$ are not the same;
7. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
8. Choose freely a point A on the line b (rule WOnline1) ;
9. Using the point A and the point M_b , construct a point C (rule W01); ;
10. Using the point A and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points A and M_b are not the same;

11. Using the circle $k(N, M_a)$ and the circle $k(M_b, C)$, construct a point H_a and a point H_c (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_b, C)$ intersect % DET: circles $k(N, M_a)$ and $k(M_b, C)$ are not the same;
12. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
13. Using the line h_b and the line h_a , construct a point H (rule W03); % NDG: lines h_b and h_a are not parallel % DET: lines h_b and h_a are not the same;
14. Using the point E_b and the point H , construct a point B (rule W01); .

Non-degenerate conditions: lines h_b and h_a are not parallel; circles $k(N, M_a)$ and $k(M_b, C)$ intersect; points A and M_b are not the same; points E_b and N are not the same; lines $m(E_b H_b)$ and $m(H_a H_c)$ are not parallel.

Determination conditions: lines h_b and h_a are not the same; points A and H_a are not the same; circles $k(N, M_a)$ and $k(M_b, C)$ are not the same; lines $m(E_b H_b)$ and $m(H_a H_c)$ are not the same; points E_b and H_b are not the same; points E_b and M_b are not the same; points H_b and M_b are not the same.

Rules used: [W01,W02,W03,W06,W07,W10b,W14,WOnline1,WOnline2]

Lemmas used: [D22,D29,D3,D32,D5,D6,D8,D9,GD01,GD02,GL01,GL03,GL04,GL09,L19,L20,L21,L23,L43,L44]

Solving time: 687.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{b} 89.36 77.83
```

```
point M_{b} 95 67.5
```

```
point E_{b} 50 56.36
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_l H_{b}
```

```
cmark_lt M_{b}
```

```
cmark_r E_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{b} and M_{b} are not the same
```

```
% Constructing a line b which passes through point H_{b} and point M_{b}
```

```
line b H_{b} M_{b}
```

```
color 200 200 200
```

```
drawline b
```

```
color 0 0 0
```

```
% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
```

```

perp h_{b} H_{b} b

color 200 200 200
drawline h_{b}
color 0 0 0

% Generating random value V[_G125648]
random V[_G125648]

% Calculating value V[_G125669] using formula V[_G125648]*20
expression V[_G125669] { V[_G125648]*20 }

% Constructing a point E_{b} which is a point for which holds  $H_{b}E_{b} = V[_G125669]$  and angle  $M_{b}H_{b}E_{b} = 90$ 
turtle E_{b} M_{b} H_{b} 90 V[_G125669]
cmark_r E_{b}

% DET: points E_{b} and M_{b} are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point M_{b}
line m(H_{a}H_{c}) E_{b} M_{b}

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

% DET: points E_{b} and H_{b} are not the same
% Constructing bisector m(E_{b}H_{b}) of the segment E_{b}H_{b}
med m(E_{b}H_{b}) E_{b} H_{b}

color 200 200 200
drawline m(E_{b}H_{b})
color 0 0 0

color 200 200 200
drawsegment E_{b} H_{b}
color 0 0 0

% NDG: lines m(E_{b}H_{b}) and m(H_{a}H_{c}) are not parallel% DET: lines m(E_{b}H_{b}) and m(H_{a}H_{c}) are not the same
% Constructing a point N which belongs to line m(E_{b}H_{b}) and line m(H_{a}H_{c})
intersec N m(E_{b}H_{b}) m(H_{a}H_{c})
cmark_r N

% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}

```

```

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point A on the line H_{b}M_{b}
online A H_{b} M_{b}
cmark_t A
color 200 200 200
drawline H_{b} M_{b}
color 0 0 0

% Constructing a point C such that  $AC/AM_{b}=2$ 
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% NDG: points A and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point A
circle k(M_{b},C) M_{b} A

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(M_{b},C) intersect% DET: circles k(N,M_{a}) and k(M_{b},C) are not
the same
% Constructing points H_{a} and H_{c} which are in intersection of k(N,M_{a}) and k(M_{b},C)
intersec2 H_{a} H_{c} k(N,M_{a}) k(M_{b},C)
cmark_r H_{a}
cmark_rt H_{c}

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: lines h_{b} and h_{a} are not parallel% DET: lines h_{b} and h_{a} are not the same
% Constructing a point H which belongs to line h_{b} and line h_{a}
intersec H h_{b} h_{a}

```

```
cmark_rt H
```

```
% Constructing a point B such that  $E_{\{b\}}B/E_{\{b\}}H=-1$ 
towards B  $E_{\{b\}}$  H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines  $h_{\{b\}}$  and  $h_{\{a\}}$  are not parallel; circles  $k(N, M_{\{a\}})$  and  $k(M_{\{b\}}, C)$  intersect; points  $A$  and  $M_{\{b\}}$  are not the same; points  $E_{\{b\}}$  and  $N$  are not the same; lines  $m(E_{\{b\}}H_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not parallel
% Determination conditions: lines  $h_{\{b\}}$  and  $h_{\{a\}}$  are not the same; points  $A$  and  $H_{\{a\}}$  are not the same; circles  $k(N, M_{\{a\}})$  and  $k(M_{\{b\}}, C)$  are not the same; lines  $m(E_{\{b\}}H_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not the same; points  $E_{\{b\}}$  and  $H_{\{b\}}$  are not the same; points  $E_{\{b\}}$  and  $M_{\{b\}}$  are not the same; points  $H_{\{b\}}$  and  $M_{\{b\}}$  are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_b = H_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 13 terms.

Time Complexity: Time spent by the prover is 0.213 seconds.

NDG conditions Points M_b and A are not identical

Points M_b and N are not identical

Line through points H_b and M_b is not perpendicular to line through points M_b and N

Line through points M_b and A is not perpendicular to line through points A and H_b

Points A , C and E_b are not collinear

4.1.2 Proving $M_b = M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.036 seconds.

NDG conditions There are no NDG conditions for this theorem

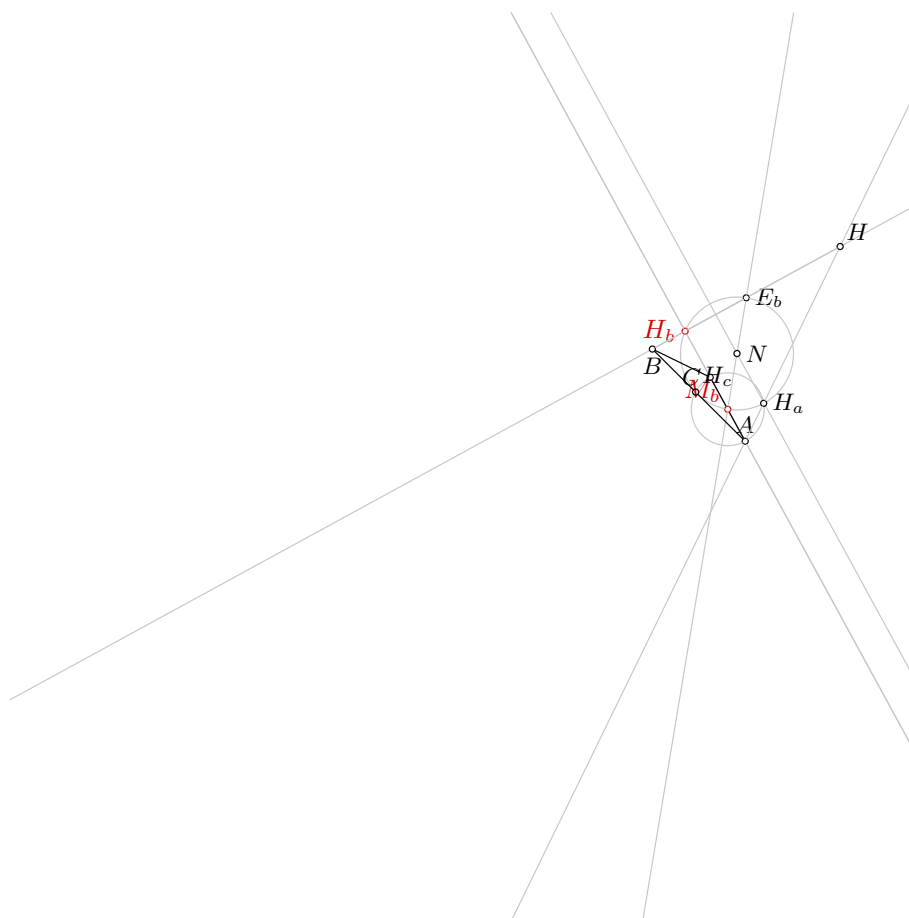


Figure 1: Illustration of the problem 0920

4.1.3 Proving $E_b = \neg E_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_b = \neg H_b$

Proving failed

4.2.2 Proving $M_b = \neg M_b$

Proving failed

4.2.3 Proving $E_b = \neg E_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_b = \neg H_b$

Proving failed

4.3.2 Proving $M_b = \neg M_b$

Proving failed

4.3.3 Proving $E_b = \neg E_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_b = \neg H_b$

Proving failed

4.4.2 Proving $M_b = \neg M_b$

Proving failed

4.4.3 Proving $E_b = \neg E_b$

Proving failed

Problem 921

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 921: Given a point E_b , a point H_b and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;
2. Using the point H_b and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points H_b and M_c are not the same;
3. Using the circle $k(M_c, A)$, the line h_b , the point M_c and the point H_b , construct a point B (rule W05); % NDG: line h_b and circle $k(M_c, A)$ intersect % DET: points H_b and B must be different;
4. Using the point B and the point E_b , construct a point H (rule W01); ;
5. Using the point M_c and the point B , construct a point A (rule W01); ;
6. Using the point H_b and the point A , construct a line b (rule W02); % DET: points H_b and A are not the same;
7. Using the point H and the point A , construct a line h_a (rule W02); % DET: points H and A are not the same;
8. Using the circle $k(M_c, A)$, the line h_a , the point M_c and the point A , construct a point H_a (rule W05); % NDG: line h_a and circle $k(M_c, A)$ intersect % DET: points A and H_a must be different;
9. Using the point B and the point H_a , construct a line a (rule W02); % DET: points B and H_a are not the same;
10. Using the line b and the line a , construct a point C (rule W03); % NDG: lines b and a are not parallel % DET: lines b and a are not the same.

Non-degenerate conditions: lines b and a are not parallel; line h_a and circle $k(M_c, A)$ intersect; line h_b and circle $k(M_c, A)$ intersect; points H_b and M_c are not the same.

Determination conditions: lines b and a are not the same; points B and H_a are not the same; points A and H_a must be different; points H and A are not the same; points H_b and A are not the same; points H_b and B must be different; points E_b and H_b are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D20,D29,D3,D5,D6,D8,D9,GD01,GD02,GL03,GL04,GL09,L40,L41,L42]

Solving time: 8.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
```

```
point H_{b} 89.36 77.83
```

```
point M_{c} 50 67.5
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{b}
```

```
cmark_l H_{b}
```

```
cmark_lt M_{c}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{b} and H_{b} are not the same
```

```
% Constructing a line h_{b} which passes through point E_{b} and point H_{b}
```

```
line h_{b} E_{b} H_{b}
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```
% NDG: points H_{b} and M_{c} are not the same
```

```
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{b}
```

```
circle k(M_{c},A) M_{c} H_{b}
```

```
color 200 200 200
```

```
drawcircle k(M_{c},A)
```

```
color 0 0 0
```

```
% NDG: line h_{b} and circle k(M_{c},A) intersect% DET: points H_{b} and B must be different
```

```
% Constructing a point P_{\_G151819} which is a foot of the point M_{c} on the line h_{b}
```

```
foot P_{\_G151819} M_{c} h_{b}
```

```
cmark_r P_{\_G151819}
```

```
color 200 200 200
```

```
drawline M_{c} P_{\_G151819}
```

```
color 0 0 0
```



```

% Constructing a point B which is an image of the point H_{b} in the symmetry to point/line P_{\_G
151819}
sim B P_{\_G151819} H_{b}
cmark_b B

% Constructing a point H such that BH/BE_{b}=2
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0

% Constructing a point A such that M_{c}A/M_{c}B=-1
towards A M_{c} B -1
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

% DET: points H_{b} and A are not the same
% Constructing a line b which passes through point H_{b} and point A
line b H_{b} A

color 200 200 200
drawline b
color 0 0 0

% DET: points H and A are not the same
% Constructing a line h_{a} which passes through point H and point A
line h_{a} H A

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{a} and circle k(M_{c},A) intersect% DET: points A and H_{a} must be different
% Constructing a point P_{\_G152256} which is a foot of the point M_{c} on the line h_{a}
foot P_{\_G152256} M_{c} h_{a}
cmark_r P_{\_G152256}
color 200 200 200
drawline M_{c} P_{\_G152256}
color 0 0 0

% Constructing a point H_{a} which is an image of the point A in the symmetry to point/line P_{\_G
152256}
sim H_{a} P_{\_G152256} A

```

```

cmark_r H_{a}

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and a are not parallel; line h_{a} and circle k(M_{c},A)
% intersect; line h_{b} and circle k(M_{c},A) intersect; points H_{b} and M_{c} are not the same
% Determination conditions: lines b and a are not the same; points B and H_{a} are not the same;
% points A and H_{a} must be different; points H and A are not the same; points H_{b} and A are
% not the same; points H_{b} and B must be different; points E_{b} and H_{b} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $H_b = \neg H_b$

Proving failed

4.1.3 Proving $M_c = \neg M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 8 terms.

Time Complexity: Time spent by the prover is 0.115 seconds.

NDG conditions There are no NDG conditions for this theorem

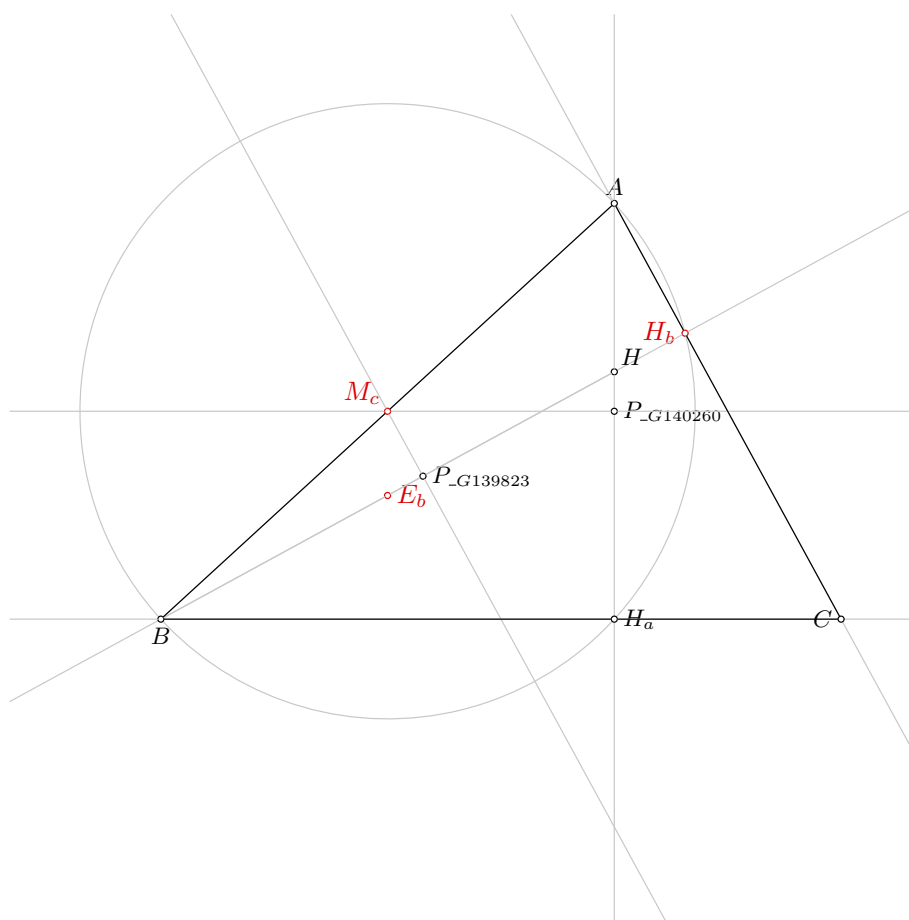


Figure 1: Illustration of the problem 0921

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $H_b = \neg H_b$

Proving failed

4.2.3 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{H_b B H_a} \neq S_{A B H_a}$ i.e., lines $H_b A$ and $B H_a$ are not parallel (construction based assumption)

$S_{A B C} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{B A C} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{A B F_{\neg h_b}^1} \neq S_{F_{\neg h_a}^0 B F_{\neg h_b}^1}$ i.e., lines $A F_{\neg h_a}^0$ and $B F_{\neg h_b}^1$ are not parallel (construction based assumption)

$S_{A B F_{\neg h_b}^1} \neq S_{C B F_{\neg h_b}^1}$ i.e., lines $A C$ and $B F_{\neg h_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $H_b = \neg H_b$

Proving failed

4.3.3 Proving $M_c = \neg M_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $H_b = \neg H_b$

Proving failed

4.4.3 Proving $M_c = \neg M_c$

Proving failed

Problem 922

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 922: Given a point H_b , a point N and a point E_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_b and N are not the same;
2. Choose freely a point E_b on the circle $k(N, M_a)$ (rule WOncircle);
3. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;
4. Using the point E_b and the point N , construct a line $m(H_a H_c)$ (rule W02); % DET: points E_b and N are not the same;
5. Using the circle $k(N, M_a)$, the line $m(H_a H_c)$, the point N and the point E_b , construct a point M_b (rule W05a); % NDG: line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect % DET: points E_b and M_b must be different;
6. Using the point M_b and the point H_b , construct a line b (rule W02); % DET: points M_b and H_b are not the same;
7. Choose freely a point A on the line b (rule WOnline1) ;
8. Using the point A and the point M_b , construct a point C (rule W01); ;
9. Using the point A and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points A and M_b are not the same;
10. Using the circle $k(N, M_a)$ and the circle $k(M_b, C)$, construct a point H_a and a point H_c (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_b, C)$ intersect % DET: circles $k(N, M_a)$ and $k(M_b, C)$ are not the same;

11. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
12. Using the line h_b and the line h_a , construct a point H (rule W03); % NDG: lines h_b and h_a are not parallel % DET: lines h_b and h_a are not the same;
13. Using the point E_b and the point H , construct a point B (rule W01); .

Non-degenerate conditions: lines h_b and h_a are not parallel; circles $k(N, M_a)$ and $k(M_b, C)$ intersect; points A and M_b are not the same; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; points H_b and N are not the same.

Determination conditions: lines h_b and h_a are not the same; points A and H_a are not the same; circles $k(N, M_a)$ and $k(M_b, C)$ are not the same; points M_b and H_b are not the same; points E_b and M_b must be different; points E_b and N are not the same; points E_b and H_b are not the same.

Rules used: [W01,W02,W03,W05a,W06,W07,WOncircle1,WOnline1]

Lemmas used: [D22,D29,D3,D32,D5,D6,D8,D9,GD01,GD02,GL01,GL03,GL04,GL09,L17,L19,L20,L21,L23,L43]

Solving time: 564.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{b} 89.36 77.83
point N 72.5 61.93
point E_{b} 50 56.36
```

```
color 220 0 0
fontsize 9
```

```
cmark_l H_{b}
cmark_r N
cmark_r E_{b}
color 0 0 0
fontsize 8
```

```
% NDG: points H_{b} and N are not the same
```

```
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{b}
circle k(N,M_{a}) N H_{b}
```

```
color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```
% Choosing randomly a point E_{b} on the circle with center N through point H_{b}
oncircle E_{b} N H_{b}
cmark_r E_{b}
color 200 200 200
drawcircle N H_{b}
color 0 0 0
```

```

% DET: points E_{b} and H_{b} are not the same
% Constructing a line h_{b} which passes through point E_{b} and point H_{b}
line h_{b} E_{b} H_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% DET: points E_{b} and N are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point N
line m(H_{a}H_{c}) E_{b} N

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

% NDG: line m(H_{a}H_{c}) and circle k(N,M_{a}) intersect% DET: points E_{b} and M_{b} must be
different
% Constructing a point M_{b} which is an image of the point E_{b} in the symmetry to point/line N
sim M_{b} N E_{b}
cmark_lt M_{b}

% DET: points M_{b} and H_{b} are not the same
% Constructing a line b which passes through point M_{b} and point H_{b}
line b M_{b} H_{b}

color 200 200 200
drawline b
color 0 0 0

% Choosing randomly a point A on the line H_{b}M_{b}
online A H_{b} M_{b}
cmark_t A
color 200 200 200
drawline H_{b} M_{b}
color 0 0 0

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

```

```

% NDG: points A and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point A
circle k(M_{b},C) M_{b} A

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(M_{b},C) intersect% DET: circles k(N,M_{a}) and k(M_{b},C) are not
the same
% Constructing points H_{a} and H_{c} which are in intersection of k(N,M_{a}) and k(M_{b},C)
intersec2 H_{a} H_{c} k(N,M_{a}) k(M_{b},C)
cmark_r H_{a}
cmark_rt H_{c}

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: lines h_{b} and h_{a} are not parallel% DET: lines h_{b} and h_{a} are not the same
% Constructing a point H which belongs to line h_{b} and line h_{a}
intersec H h_{b} h_{a}
cmark_rt H

% Constructing a point B such that E_{b}B/E_{b}H=-1
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{b} and h_{a} are not parallel; circles k(N,M_{a}) and k(M_{b},C) intersect; points A and M_{b} are not the same; line m(H_{a}H_{c}) and circle k(N,M_{a}) intersect; points H_{b} and N are not the same
% Determination conditions: lines h_{b} and h_{a} are not the same; points A and H_{a} are not the same; circles k(N,M_{a}) and k(M_{b},C) are not the same; points M_{b} and H_{b} are not the same; points E_{b} and M_{b} must be different; points E_{b} and N are not the same; points E_{b} and H_{b} are not the same

```

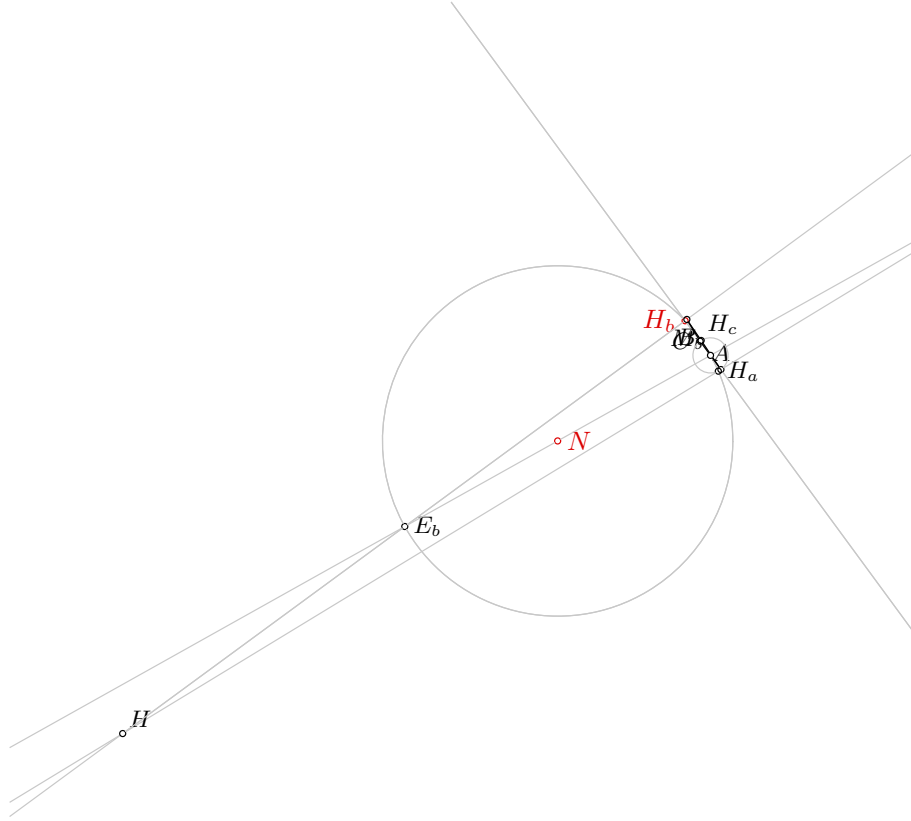



Figure 1: Illustration of the problem 0922

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_b = H_b$

Proving failed

4.1.2 Proving $N = N$

Proving failed

4.1.3 Proving $E_b = E_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_b = \neg H_b$

Proving failed

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $E_b = \neg E_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_b = \neg H_b$

Proving failed

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $E_b = \neg E_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_b = \neg H_b$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $E_b = \neg E_b$

Proving failed

Problem 923

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 923: Given a point E_b , a point H_b and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;
2. Using the point H_b and the line h_b , construct a line b (rule W10a); ;
3. Using the point O and the line b , construct a line m_b (rule W10b); ;
4. Using the line m_b and the line b , construct a point M_b (rule W03); % NDG: lines m_b and b are not parallel % DET: lines m_b and b are not the same;
5. Using the point M_b and the point E_b , construct a line $m(H_aH_c)$ (rule W02); % DET: points M_b and E_b are not the same;
6. Using the point E_b and the point H_b , construct a line $m(E_bH_b)$ (rule W14); % DET: points E_b and H_b are not the same;
7. Using the line $m(E_bH_b)$ and the line $m(H_aH_c)$, construct a point N (rule W03); % NDG: lines $m(E_bH_b)$ and $m(H_aH_c)$ are not parallel % DET: lines $m(E_bH_b)$ and $m(H_aH_c)$ are not the same;
8. Using the point O and the point N , construct a point H (rule W01); ;
9. Using the point E_b and the point H , construct a point B (rule W01); ;
10. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
11. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; lines $m(E_b H_b)$ and $m(H_a H_c)$ are not parallel; lines m_b and b are not parallel.

Determination conditions: lines $m(E_b H_b)$ and $m(H_a H_c)$ are not the same; points E_b and H_b are not the same; points M_b and E_b are not the same; lines m_b and b are not the same; points E_b and H_b are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W10b,W14]

Lemmas used: [D1,D12,D22,D26,D29,D3,D31,D32,D6,D9,GD01,GD02,GL01,GL03,GL04,GL09,L11,L12,L19,L

Solving time: 3.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
```

```
point H_{b} 89.36 77.83
```

```
point O 65 51.14
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{b}
```

```
cmark_l H_{b}
```

```
cmark_t O
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{b} and H_{b} are not the same
```

```
% Constructing a line h_{b} which passes through point E_{b} and point H_{b}
```

```
line h_{b} E_{b} H_{b}
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```
% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
```

```
perp b H_{b} h_{b}
```

```
color 200 200 200
```

```
drawline b
```

```
color 0 0 0
```

```
% Constructing a line m_{b} which is perpendicular to line b and which passes through point O
```

```
perp m_{b} O b
```

```
color 200 200 200
```

```
drawline m_{b}
```

```
color 0 0 0
```

```

% NDG: lines  $m_{\{b\}}$  and  $b$  are not parallel% DET: lines  $m_{\{b\}}$  and  $b$  are not the same
% Constructing a point  $M_{\{b\}}$  which belongs to line  $m_{\{b\}}$  and line  $b$ 
intersec  $M_{\{b\}}$   $m_{\{b\}}$   $b$ 
cmark_lt  $M_{\{b\}}$ 

% DET: points  $M_{\{b\}}$  and  $E_{\{b\}}$  are not the same
% Constructing a line  $m(H_{\{a\}}H_{\{c\}})$  which passes through point  $M_{\{b\}}$  and point  $E_{\{b\}}$ 
line  $m(H_{\{a\}}H_{\{c\}})$   $M_{\{b\}}$   $E_{\{b\}}$ 

color 200 200 200
drawline  $m(H_{\{a\}}H_{\{c\}})$ 
color 0 0 0

% DET: points  $E_{\{b\}}$  and  $H_{\{b\}}$  are not the same
% Constructing bisector  $m(E_{\{b\}}H_{\{b\}})$  of the segment  $E_{\{b\}}H_{\{b\}}$ 
med  $m(E_{\{b\}}H_{\{b\}})$   $E_{\{b\}}$   $H_{\{b\}}$ 

color 200 200 200
drawline  $m(E_{\{b\}}H_{\{b\}})$ 
color 0 0 0

color 200 200 200
drawsegment  $E_{\{b\}}$   $H_{\{b\}}$ 
color 0 0 0

% NDG: lines  $m(E_{\{b\}}H_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not parallel% DET: lines  $m(E_{\{b\}}H_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$ 
 $H_{\{c\}}$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{b\}}H_{\{b\}})$  and line  $m(H_{\{a\}}H_{\{c\}})$ 
intersec  $N$   $m(E_{\{b\}}H_{\{b\}})$   $m(H_{\{a\}}H_{\{c\}})$ 
cmark_r  $N$ 

% Constructing a point  $H$  such that  $OH/ON=2$ 
towards  $H$   $O$   $N$  2
cmark_rt  $H$ 
color 200 200 200
drawsegment  $O$   $H$ 
color 0 0 0

% Constructing a point  $B$  such that  $E_{\{b\}}B/E_{\{b\}}H=-1$ 
towards  $B$   $E_{\{b\}}$   $H$  -1
cmark_b  $B$ 
color 200 200 200
drawsegment  $H$   $B$ 
color 0 0 0

```

```

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;
% lines m(E_{b}H_{b}) and m(H_{a}H_{c}) are not parallel; lines m_{b} and b are not parallel
% Determination conditions: lines m(E_{b}H_{b}) and m(H_{a}H_{c}) are not the same; points E_{b}
% and H_{b} are not the same; points M_{b} and E_{b} are not the same; lines m_{b} and b are not
% the same; points E_{b} and H_{b} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = _E_b$

Proving failed

4.1.2 Proving $H_b = _H_b$

Proving failed

4.1.3 Proving $O = _O$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 120 terms.

Time Complexity: Time spent by the prover is 0.692 seconds.

NDG conditions Points H_b and A are not identical

Points A , C and E_b are not collinear

Point C is not the midpoint of segment with endpoints H_b and B

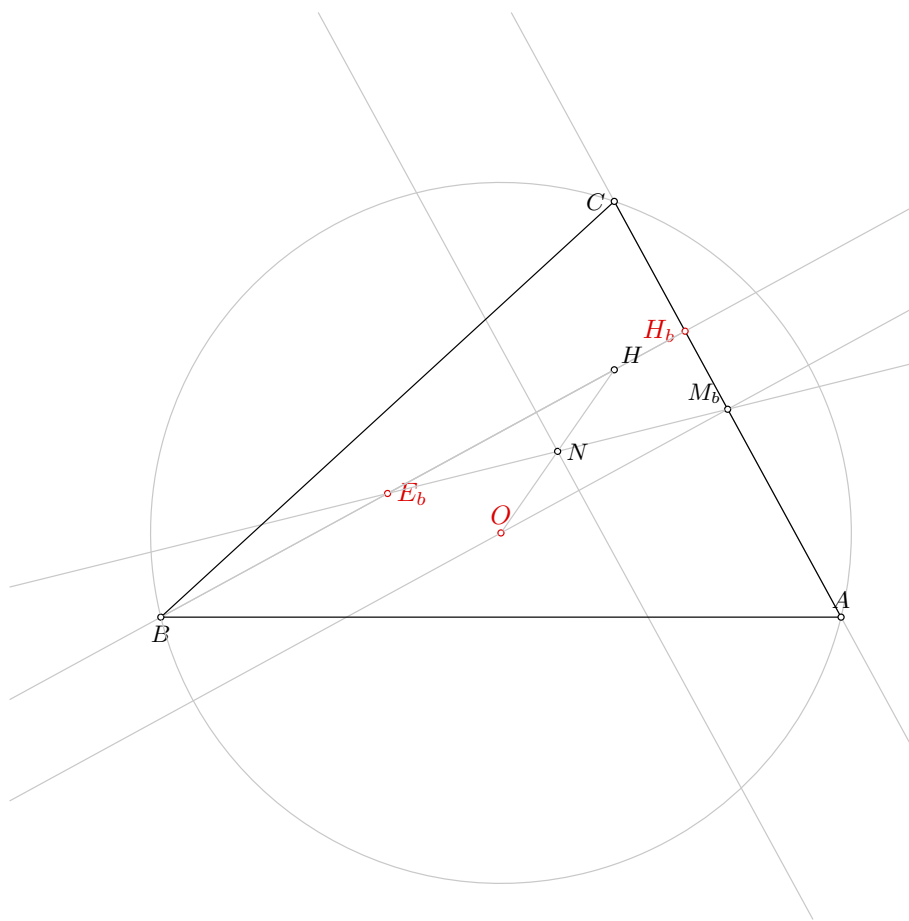


Figure 1: Illustration of the problem 0923

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $H_b = \neg H_b$

Proving failed

4.2.3 Proving $O = \neg O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 311 terms.

Time Complexity: Time spent by the prover is 0.360 seconds. There are no ndg conditions.

4.3.2 Proving $H_b = \neg H_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 15 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.3.3 Proving $O = \neg O$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 694 terms.

Time Complexity: Time spent by the prover is 0.860 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $H_b = \neg H_b$

Proving failed

4.4.3 Proving $O = \neg O$

Proving failed

Problem 924

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 924: Given a point E_b , a point H_b and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 925

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 925: Given a point H_b , a point T_b and a point E_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_b and the point T_b , construct a line b (rule W02); % DET: points H_b and T_b are not the same;
2. Using the point H_b and the line b , construct a line h_b (rule W10b); ;
3. Choose freely a point E_b on the line h_b (rule WOnline2);
4. Choose freely a point A on the line b (rule WOnline1) ;
5. Choose freely a point B on the line h_b (rule WOnline1) ;
6. Using the point B and the point E_b , construct a point H (rule W01); ;
7. Using the point B and the point A , construct a line c (rule W02); % DET: points B and A are not the same;
8. Using the point B and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points B and E_b are not the same;
9. Using the circle $k(E_b, B)$, the line c , the point E_b and the point B , construct a point H_c (rule W05); % NDG: line c and circle $k(E_b, B)$ intersect % DET: points B and H_c must be different;
10. Using the point H and the point H_c , construct a line h_c (rule W02); % DET: points H and H_c are not the same;
11. Using the line b and the line h_c , construct a point C (rule W03); % NDG: lines b and h_c are not parallel % DET: lines b and h_c are not the same.

Non-degenerate conditions: lines b and h_c are not parallel; line c and circle $k(E_b, B)$ intersect; points B and E_b are not the same.

Determination conditions: lines b and h_c are not the same; points H and H_c are not the same; points B and H_c must be different; points B and A are not the same; points H_b and T_b are not the same.

Rules used: [W01,W02,W03,W05,W06,W10b,WOnline1,WOnline2]

Lemmas used: [D10,D24,D29,D6,D7,D9,GD01,GD02,GL03,L3,L51]

Solving time: 215.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{b} 89.36 77.83
```

```
point T_{b} 94.25 68.88
```

```
point E_{b} 50 56.36
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_l H_{b}
```

```
cmark_t T_{b}
```

```
cmark_r E_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{b} and T_{b} are not the same
```

```
% Constructing a line b which passes through point H_{b} and point T_{b}
```

```
line b H_{b} T_{b}
```

```
color 200 200 200
```

```
drawline b
```

```
color 0 0 0
```

```
% Constructing a line h_{b} which is perpendicular to line b and which passes through point H_{b}
```

```
perp h_{b} H_{b} b
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```
% Generating random value V[_G251849]
```

```
random V[_G251849]
```

```
% Calculating value V[_G251870] using formula V[_G251849]*20
```

```
expression V[_G251870] { V[_G251849]*20 }
```

```

% Constructing a point E_{b} which is a point for which holds  $H_{\{b\}}E_{\{b\}} = V[_G251870]$  and angle  $T_{\{b\}}H_{\{b\}}E_{\{b\}} = 90$ 
turtle E_{b} T_{b} H_{b} 90 V[_G251870]
cmark_r E_{b}

% Choosing randomly a point A on the line  $H_{\{b\}}T_{\{b\}}$ 
online A H_{b} T_{b}
cmark_t A
color 200 200 200
drawline H_{b} T_{b}
color 0 0 0

% Choosing randomly a point B on the line  $E_{\{b\}}H_{\{b\}}$ 
online B E_{b} H_{b}
cmark_b B
color 200 200 200
drawline E_{b} H_{b}
color 0 0 0

% Constructing a point H such that  $BH/BE_{\{b\}}=2$ 
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0

% DET: points B and A are not the same
% Constructing a line c which passes through point B and point A
line c B A

color 200 200 200
drawline c
color 0 0 0

% NDG: points B and E_{b} are not the same
% Constructing a circle  $k(E_{\{b\}},B)$  whose center is at point  $E_{\{b\}}$  and which passes through point B
circle k(E_{b},B) E_{b} B

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line c and circle  $k(E_{\{b\}},B)$  intersect% DET: points B and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{G252447\}}$  which is a foot of the point  $E_{\{b\}}$  on the line c

```

```

foot P_{\_G252447} E_{b} c
cmark_r P_{\_G252447}
color 200 200 200
drawline E_{b} P_{\_G252447}
color 0 0 0

% Constructing a point H_{c} which is an image of the point B in the symmetry to point/line P_{\_G
252447}
sim H_{c} P_{\_G252447} B
cmark_rt H_{c}

% DET: points H and H_{c} are not the same
% Constructing a line h_{c} which passes through point H and point H_{c}
line h_{c} H H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines b and h_{c} are not parallel% DET: lines b and h_{c} are not the same
% Constructing a point C which belongs to line b and line h_{c}
intersec C b h_{c}
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and h_{c} are not parallel; line c and circle k(E_{b},B)
intersect; points B and E_{b} are not the same
% Determination conditions: lines b and h_{c} are not the same; points H and H_{c} are not the same
; points B and H_{c} must be different; points B and A are not the same; points H_{b} and T_{b}
are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_b = H_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.153 seconds.

NDG conditions Points A and B are not identical

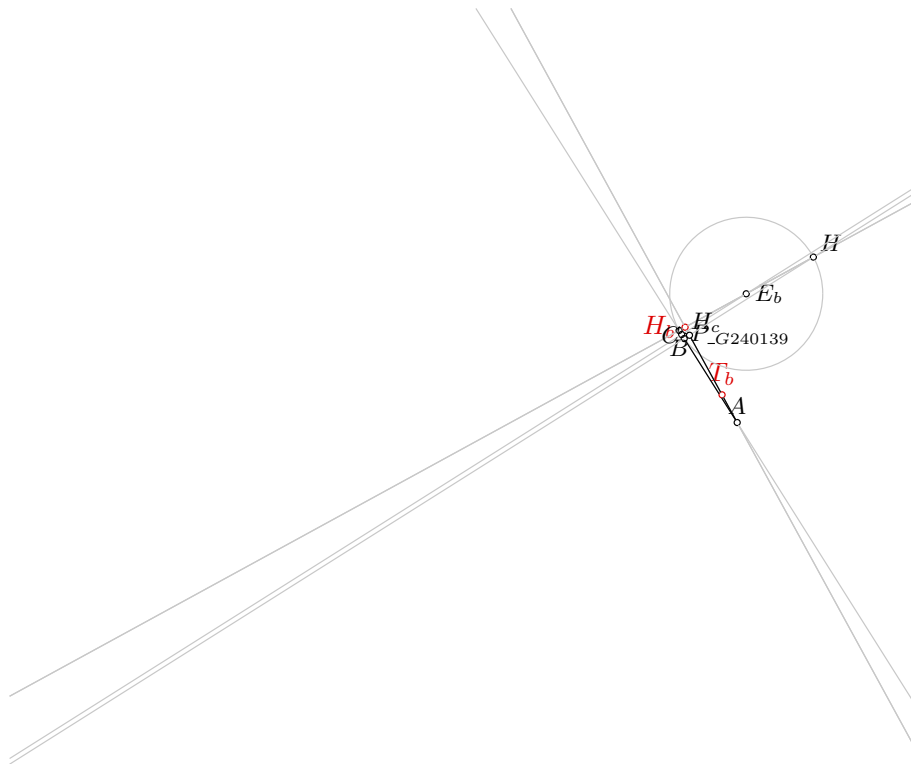


Figure 1: Illustration of the problem 0925

Points A and C are not identical

Line through points H_b and A is not parallel with line through points H_c and H

Line through points A and B is not perpendicular to line through points B and C

4.1.2 Proving $T_b = \neg T_b$

Proving failed

4.1.3 Proving $E_b = \neg E_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_b = \neg H_b$

Proving failed

4.2.2 Proving $T_b = \neg T_b$

Proving failed

4.2.3 Proving $E_b = \neg E_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_b = \neg H_b$

Proving failed

4.3.2 Proving $T_b = \neg T_b$

Proving failed

4.3.3 Proving $E_b = \neg E_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_b = \neg H_b$

Proving failed

4.4.2 Proving $T_b = \neg T_b$

Proving failed

4.4.3 Proving $E_b = \neg E_b$

Proving failed

Problem 926

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 926: Given a point E_b , a point H_b and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 927

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 927: Given a point E_b , a point H_c and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 928

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 928: Given a point E_b , a point H_c and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H_c and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_c and E_b are not the same;
2. Using the point H_c and the point M_a , construct a circle $k(M_a, B)$ (rule W06); % NDG: points H_c and M_a are not the same;
3. Using the circle $k(E_b, B)$, the circle $k(M_a, B)$, the point H_c , the point E_b and the point M_a , construct a point B (rule W08); % NDG: circles $k(E_b, B)$ and $k(M_a, B)$ intersect % DET: circles $k(E_b, B)$ and $k(M_a, B)$ are not the same; points H_c and B must be different;
4. Using the point B and the point E_b , construct a point H (rule W01); ;
5. Using the point M_a and the point B , construct a point C (rule W01); ;
6. Using the point H_c and the point B , construct a line c (rule W02); % DET: points H_c and B are not the same;
7. Using the point M_a and the point B , construct a line a (rule W02); % DET: points M_a and B are not the same;
8. Using the circle $k(E_b, B)$, the line a , the point E_b and the point B , construct a point H_a (rule W05); % NDG: line a and circle $k(E_b, B)$ intersect % DET: points B and H_a must be different;
9. Using the point H and the point H_a , construct a line h_a (rule W02); % DET: points H and H_a are not the same;
10. Using the line c and the line h_a , construct a point A (rule W03); % NDG: lines c and h_a are not parallel % DET: lines c and h_a are not the same.

Non-degenerate conditions: lines c and h_a are not parallel; line a and circle $k(E_b, B)$ intersect; circles $k(E_b, B)$ and $k(M_a, B)$ intersect; points H_c and M_a are not the same; points H_c and E_b are not the same.

Determination conditions: lines c and h_a are not the same; points H and H_a are not the same; points B and H_a must be different; points M_a and B are not the same; points H_c and B are not the same; circles $k(E_b, B)$ and $k(M_a, B)$ are not the same; points H_c and B must be different.

Rules used: [W01,W02,W03,W05,W06,W08]

Lemmas used: [D21,D29,D3,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L39,L50,L51]

Solving time: 9.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
point H_{c} 68.91 84.83
point M_{a} 65 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{b}
cmark_rt H_{c}
cmark_r M_{a}
color 0 0 0
fontsize 8
```

```
% NDG: points H_{c} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H
_{c}
circle k(E_{b},B) E_{b} H_{c}
```

```
color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0
```

```
% NDG: points H_{c} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H
_{c}
circle k(M_{a},B) M_{a} H_{c}
```

```
color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0
```

```
% NDG: circles k(E_{b},B) and k(M_{a},B) intersect% DET: circles k(E_{b},B) and k(M_{a},B) are not
the same; points H_{c} and B must be different
% Constructing a line L_{\_G57272} which passes through point E_{b} and point M_{a}
line L_{\_G57272} E_{b} M_{a}
```

```

color 200 200 200
drawline L_{\_G57272}
color 0 0 0

% Constructing a point B which is an image of the point H_{c} in the symmetry to point/line L_{\_G
57272}
sim B L_{\_G57272} H_{c}
cmark_b B

% Constructing a point H such that BH/BE_{b}=2
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0

% Constructing a point C such that M_{a}C/M_{a}B=-1
towards C M_{a} B -1
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

% DET: points H_{c} and B are not the same
% Constructing a line c which passes through point H_{c} and point B
line c H_{c} B

color 200 200 200
drawline c
color 0 0 0

% DET: points M_{a} and B are not the same
% Constructing a line a which passes through point M_{a} and point B
line a M_{a} B

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(E_{b},B) intersect% DET: points B and H_{a} must be different
% Constructing a point P_{\_G57689} which is a foot of the point E_{b} on the line a
foot P_{\_G57689} E_{b} a
cmark_r P_{\_G57689}
color 200 200 200
drawline E_{b} P_{\_G57689}

```

```

color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $B$  in the symmetry to point/line  $P_{\{\backslash\_G57689\}}$ 
sim  $H_{\{a\}}$   $P_{\{\backslash\_G57689\}}$   $B$ 
cmark_r  $H_{\{a\}}$ 

% DET: points  $H$  and  $H_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $H$  and point  $H_{\{a\}}$ 
line  $h_{\{a\}}$   $H$   $H_{\{a\}}$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: lines  $c$  and  $h_{\{a\}}$  are not parallel% DET: lines  $c$  and  $h_{\{a\}}$  are not the same
% Constructing a point  $A$  which belongs to line  $c$  and line  $h_{\{a\}}$ 
intersec  $A$   $c$   $h_{\{a\}}$ 
cmark_t  $A$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $c$  and  $h_{\{a\}}$  are not parallel; line  $a$  and circle  $k(E_{\{b\}},B)$ 
intersect; circles  $k(E_{\{b\}},B)$  and  $k(M_{\{a\}},B)$  intersect; points  $H_{\{c\}}$  and  $M_{\{a\}}$  are not the same
; points  $H_{\{c\}}$  and  $E_{\{b\}}$  are not the same
% Determination conditions: lines  $c$  and  $h_{\{a\}}$  are not the same; points  $H$  and  $H_{\{a\}}$  are not the same
; points  $B$  and  $H_{\{a\}}$  must be different; points  $M_{\{a\}}$  and  $B$  are not the same; points  $H_{\{c\}}$  and  $B$ 
are not the same; circles  $k(E_{\{b\}},B)$  and  $k(M_{\{a\}},B)$  are not the same; points  $H_{\{c\}}$  and  $B$  must
be different

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $H_c = \neg H_c$

Proving failed

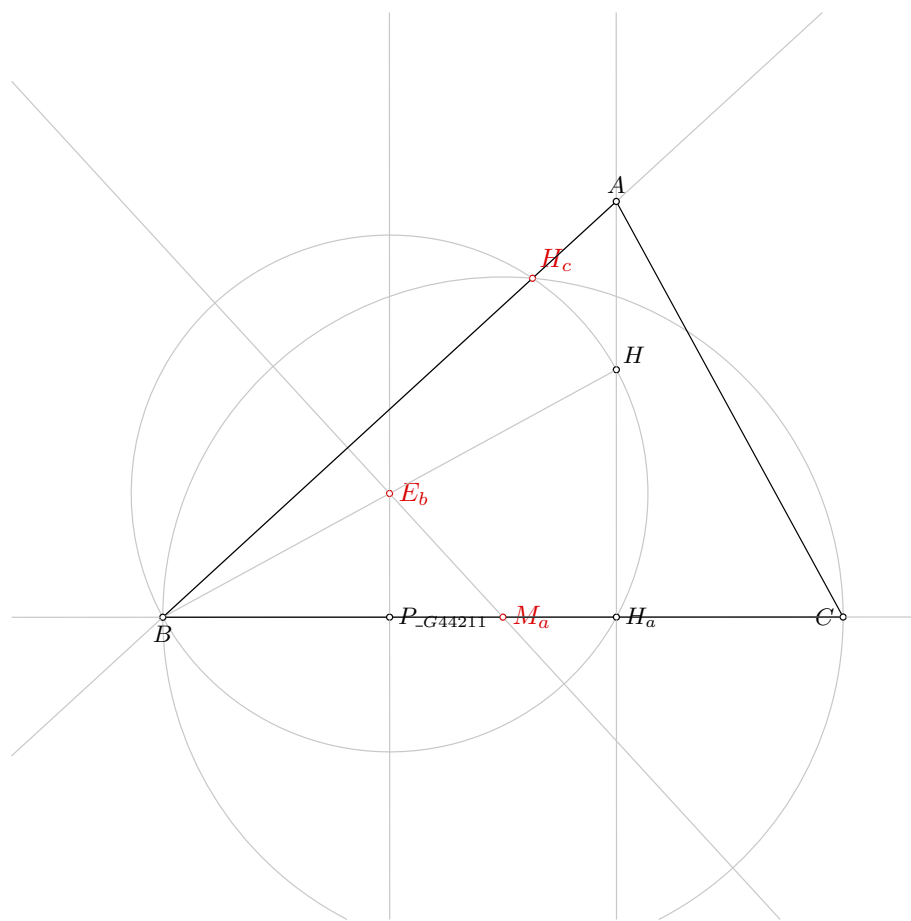


Figure 1: Illustration of the problem 0928

4.1.3 Proving $M_a = M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 11 terms.

Time Complexity: Time spent by the prover is 0.175 seconds.

NDG conditions Points E_b and M_a are not identical

Points E_b and M_a are not identical

4.2 GCLC - Area method

4.2.1 Proving $E_b = E_b$

Proving failed

4.2.2 Proving $H_c = H_c$

Proving failed

4.2.3 Proving $M_a = M_a$

NDG conditions are:

$S_{H_c H H_a} \neq S_{B H H_a}$ i.e., lines $H_c B$ and $H H_a$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^1} \neq S_{F_{-h_a}^0 BF_{-h_b}^1}$ i.e., lines $AF_{-h_a}^0$ and $BF_{-h_b}^1$ are not parallel (construction based assumption)

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{-h_c}^2} \neq S_{BCF_{-h_c}^2}$ i.e., lines AB and $CF_{-h_c}^2$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_b = E_b$

Proving failed

4.3.2 Proving $H_c = H_c$

Proving failed

4.3.3 Proving $M_a = M_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = E_b$

Proving failed

4.4.2 Proving $H_c = H_c$

Proving failed

4.4.3 Proving $M_a = M_a$

Proving failed

Problem 929

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 929: Given a point E_b , a point M_b and a point H_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point M_b , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and M_b are not the same;
2. Using the point E_b and the point M_b , construct a line $m(E_bM_b)$ (rule W14); % DET: points E_b and M_b are not the same;
3. Using the line $m(E_bM_b)$ and the line $m(H_aH_c)$, construct a point N (rule W03); % NDG: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel % DET: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same;
4. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
5. Choose freely a point H_c on the circle $k(N, M_a)$ (rule WOncircle);
6. Using the point H_c and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_c and E_b are not the same;
7. Using the circle $k(N, M_a)$, the circle $k(E_b, B)$, the point H_c , the point N and the point E_b , construct a point H_a (rule W08); % NDG: circles $k(N, M_a)$ and $k(E_b, B)$ intersect % DET: circles $k(N, M_a)$ and $k(E_b, B)$ are not the same; points H_c and H_a must be different;
8. Choose freely a point A on the circle $k(M_b, C)$ (rule WOncircle);
9. Using the point A and the point M_b , construct a point C (rule W01); ;
10. Using the point A and the point H_c , construct a line c (rule W02); % DET: points A and H_c are not the same;

11. Using the point H_a and the point C , construct a line a (rule W02); % DET: points H_a and C are not the same;
12. Using the line c and the line a , construct a point B (rule W03); % NDG: lines c and a are not parallel % DET: lines c and a are not the same.

Non-degenerate conditions: lines c and a are not parallel; circles $k(N, M_a)$ and $k(E_b, B)$ intersect; points H_c and E_b are not the same; points E_b and N are not the same; lines $m(E_b M_b)$ and $m(H_a H_c)$ are not parallel.

Determination conditions: lines c and a are not the same; points H_a and C are not the same; points A and H_c are not the same; circles $k(N, M_a)$ and $k(E_b, B)$ are not the same; points H_c and H_a must be different; lines $m(E_b M_b)$ and $m(H_a H_c)$ are not the same; points E_b and M_b are not the same; points E_b and M_b are not the same.

Rules used: [W01,W02,W03,W06,W08,W14,WOncircle1]

Lemmas used: [D22,D32,D5,D7,GD01,GD02,GL01,GL03,GL04,L17,L19,L21,L23,L44,L45,L50,L51]

Solving time: 64.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
point M_{b} 95 67.5
point H_{c} 68.91 84.83
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{b}
cmark_lt M_{b}
cmark_rt H_{c}
color 0 0 0
fontsize 8
```

```
% DET: points E_{b} and M_{b} are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point M_{b}
line m(H_{a}H_{c}) E_{b} M_{b}
```

```
color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0
```

```
% DET: points E_{b} and M_{b} are not the same
% Constructing bisector m(E_{b}M_{b}) of the segment E_{b}M_{b}
med m(E_{b}M_{b}) E_{b} M_{b}
```

```
color 200 200 200
drawline m(E_{b}M_{b})
color 0 0 0
```

```

color 200 200 200
drawsegment E_{b} M_{b}
color 0 0 0

% NDG: lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not parallel% DET: lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{b\}}M_{\{b\}})$  and line  $m(H_{\{a\}}H_{\{c\}})$ 
intersec N m(E_{b}M_{b}) m(H_{a}H_{c})
cmark_r N

% NDG: points  $E_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N,M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{b\}}$ 
circle k(N,M_{a}) N E_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point  $H_{\{c\}}$  on the circle with center  $N$  through point  $E_{\{b\}}$ 
oncircle H_{c} N E_{b}
cmark_rt H_{c}
color 200 200 200
drawcircle N E_{b}
color 0 0 0

% NDG: points  $H_{\{c\}}$  and  $E_{\{b\}}$  are not the same
% Constructing a circle  $k(E_{\{b\}},B)$  whose center is at point  $E_{\{b\}}$  and which passes through point  $H_{\{c\}}$ 
circle k(E_{b},B) E_{b} H_{c}

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: circles  $k(N,M_{\{a\}})$  and  $k(E_{\{b\}},B)$  intersect% DET: circles  $k(N,M_{\{a\}})$  and  $k(E_{\{b\}},B)$  are not the same; points  $H_{\{c\}}$  and  $H_{\{a\}}$  must be different
% Constructing a line  $L_{\{\_G85472\}}$  which passes through point  $N$  and point  $E_{\{b\}}$ 
line L_{\_G85472} N E_{b}

color 200 200 200
drawline L_{\_G85472}
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $H_{\{c\}}$  in the symmetry to point/line  $L_{\{\_G85472\}}$ 
sim H_{a} L_{\_G85472} H_{c}
cmark_r H_{a}

```

```

% Choosing randomly a point A on the circle with center  $M_{\{b\}}$  through point  $H_{\{c\}}$ 
oncircle A  $M_{\{b\}}$   $H_{\{c\}}$ 
cmark_t A
color 200 200 200
drawcircle  $M_{\{b\}}$   $H_{\{c\}}$ 
color 0 0 0

% Constructing a point C such that  $AC/AM_{\{b\}}=2$ 
towards C A  $M_{\{b\}}$  2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points A and  $H_{\{c\}}$  are not the same
% Constructing a line c which passes through point A and point  $H_{\{c\}}$ 
line c A  $H_{\{c\}}$ 

color 200 200 200
drawline c
color 0 0 0

% DET: points  $H_{\{a\}}$  and C are not the same
% Constructing a line a which passes through point  $H_{\{a\}}$  and point C
line a  $H_{\{a\}}$  C

color 200 200 200
drawline a
color 0 0 0

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and a are not parallel; circles  $k(N, M_{\{a\}})$  and  $k(E_{\{b\}}, B)$ 
% intersect; points  $H_{\{c\}}$  and  $E_{\{b\}}$  are not the same; points  $E_{\{b\}}$  and N are not the same; lines
%  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not parallel
% Determination conditions: lines c and a are not the same; points  $H_{\{a\}}$  and C are not the same;
% points A and  $H_{\{c\}}$  are not the same; circles  $k(N, M_{\{a\}})$  and  $k(E_{\{b\}}, B)$  are not the same; points

```

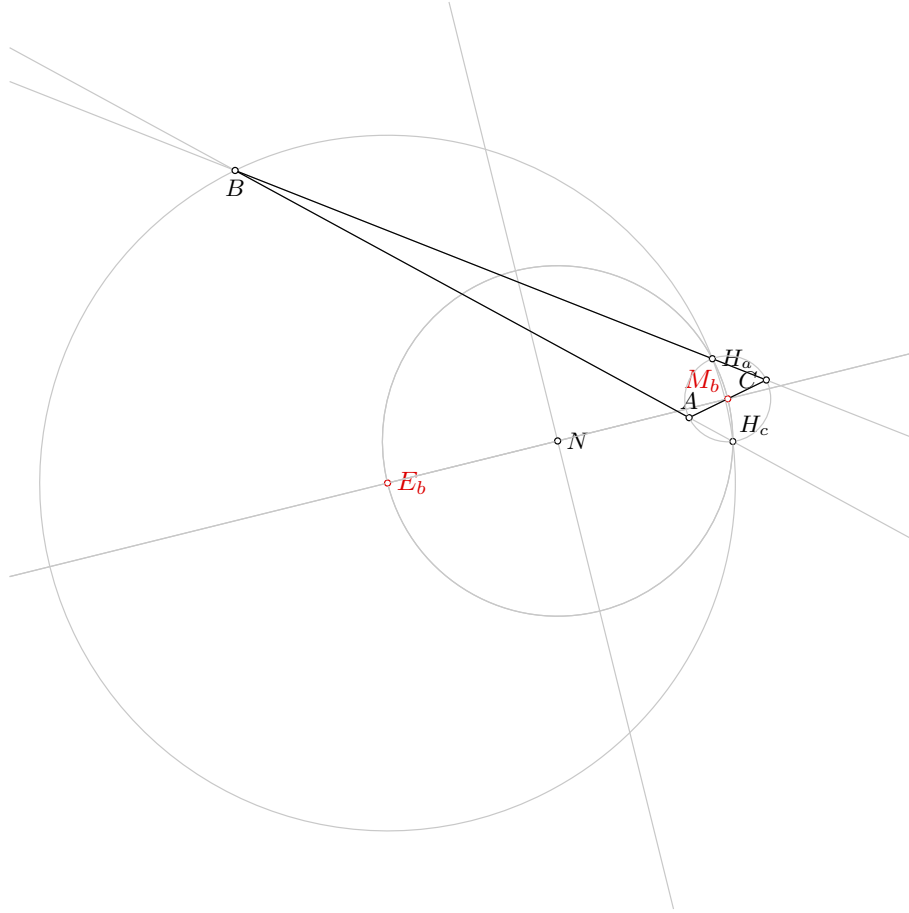


Figure 1: Illustration of the problem 0929

$H_{\{c\}}$ and $H_{\{a\}}$ must be different; lines $m(E_{\{b\}}M_{\{b\}})$ and $m(H_{\{a\}}H_{\{c\}})$ are not the same; points $E_{\{b\}}$ and $M_{\{b\}}$ are not the same; points $E_{\{b\}}$ and $M_{\{b\}}$ are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = E_b$

Proving failed

4.1.2 Proving $M_b = M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 8 terms.

Time Complexity: Time spent by the prover is 0.146 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $H_c = \neg H_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{M_{m(E_b M_b)} E_b M_b}^0 \neq S_{T_{m(E_b M_b)} E_b M_b}^1$ i.e., lines $M_{m(E_b M_b)}^0 T_{m(E_b M_b)}^1$ and $E_b M_b$ are not parallel (construction based assumption)

$S_{AH_a C} \neq S_{H_c H_a C}$ i.e., lines AH_c and $H_a C$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2 BF_{\neg h_b}^3}$ i.e., lines $AF_{\neg h_a}^2$ and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{\neg h_c}^4} \neq S_{BCF_{\neg h_c}^4}$ i.e., lines AB and $CF_{\neg h_c}^4$ are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

4.2.3 Proving $H_c = \neg H_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $M_b = \neg M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 19 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.3.3 Proving $H_c = \neg H_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 1148 terms.

Time Complexity: Time spent by the prover is 1.350 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $M_b = M_b$

Proving failed

4.4.3 Proving $H_c = H_c$

Proving failed

Problem 930

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 930: Given a point E_b , a point H_c and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
2. Using the point H_c and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_c and E_b are not the same;
3. Using the circle $k(E_b, B)$, the line c , the point E_b and the point H_c , construct a point B (rule W05); % NDG: line c and circle $k(E_b, B)$ intersect % DET: points H_c and B must be different;
4. Using the point B and the point E_b , construct a point H (rule W01); ;
5. Using the point M_c and the point B , construct a point A (rule W01); ;
6. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
7. Using the point H and the point A , construct a line h_a (rule W02); % DET: points H and A are not the same;
8. Using the circle $k(E_b, B)$, the line h_a , the point E_b and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_b, B)$ intersect % DET: points H and H_a must be different;
9. Using the point B and the point H_a , construct a line a (rule W02); % DET: points B and H_a are not the same;
10. Using the line h_c and the line a , construct a point C (rule W03); % NDG: lines h_c and a are not parallel % DET: lines h_c and a are not the same.

Non-degenerate conditions: lines h_c and a are not parallel; line h_a and circle $k(E_b, B)$ intersect; line c and circle $k(E_b, B)$ intersect; points H_c and E_b are not the same.

Determination conditions: lines h_c and a are not the same; points B and H_a are not the same; points H and H_a must be different; points H and A are not the same; points H_c and H are not the same; points H_c and B must be different; points H_c and M_c are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D20,D29,D3,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L49,L50,L51]

Solving time: 9.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{b} 50 56.36
point H_{c} 68.91 84.83
point M_{c} 50 67.5

color 220 0 0
fontsize 9

cmark_r E_{b}
cmark_rt H_{c}
cmark_lt M_{c}
color 0 0 0
fontsize 8

% DET: points H_{c} and M_{c} are not the same
% Constructing a line c which passes through point H_{c} and point M_{c}
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points H_{c} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H
_{c}
circle k(E_{b},B) E_{b} H_{c}

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: line c and circle k(E_{b},B) intersect% DET: points H_{c} and B must be different
% Constructing a point P_{\_G111239} which is a foot of the point E_{b} on the line c
foot P_{\_G111239} E_{b} c
cmark_r P_{\_G111239}
color 200 200 200
drawline E_{b} P_{\_G111239}
color 0 0 0
```



```

% Constructing a point B which is an image of the point H_{c} in the symmetry to point/line P_{\_G
111239}
sim B P_{\_G111239} H_{c}
cmark_b B

% Constructing a point H such that BH/BE_{b}=2
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0

% Constructing a point A such that M_{c}A/M_{c}B=-1
towards A M_{c} B -1
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% DET: points H and A are not the same
% Constructing a line h_{a} which passes through point H and point A
line h_{a} H A

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{a} and circle k(E_{b},B) intersect% DET: points H and H_{a} must be different
% Constructing a point P_{\_G111676} which is a foot of the point E_{b} on the line h_{a}
foot P_{\_G111676} E_{b} h_{a}
cmark_r P_{\_G111676}
color 200 200 200
drawline E_{b} P_{\_G111676}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H in the symmetry to point/line P_{\_G
111676}
sim H_{a} P_{\_G111676} H

```

```

cmark_r H_{a}

% DET: points B and H_{a} are not the same
% Constructing a line a which passes through point B and point H_{a}
line a B H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines h_{c} and a are not parallel% DET: lines h_{c} and a are not the same
% Constructing a point C which belongs to line h_{c} and line a
intersec C h_{c} a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and a are not parallel; line h_{a} and circle k(E_{b},B)
% intersect; line c and circle k(E_{b},B) intersect; points H_{c} and E_{b} are not the same
% Determination conditions: lines h_{c} and a are not the same; points B and H_{a} are not the same
% ; points H and H_{a} must be different; points H and A are not the same; points H_{c} and H are
% not the same; points H_{c} and B must be different; points H_{c} and M_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = E_b$

Proving failed

4.1.2 Proving $H_c = H_c$

Proving failed

4.1.3 Proving $M_c = M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 11 terms.

Time Complexity: Time spent by the prover is 0.181 seconds.

NDG conditions Points H_c and M_c are not identical

Points H_c and M_c are not identical

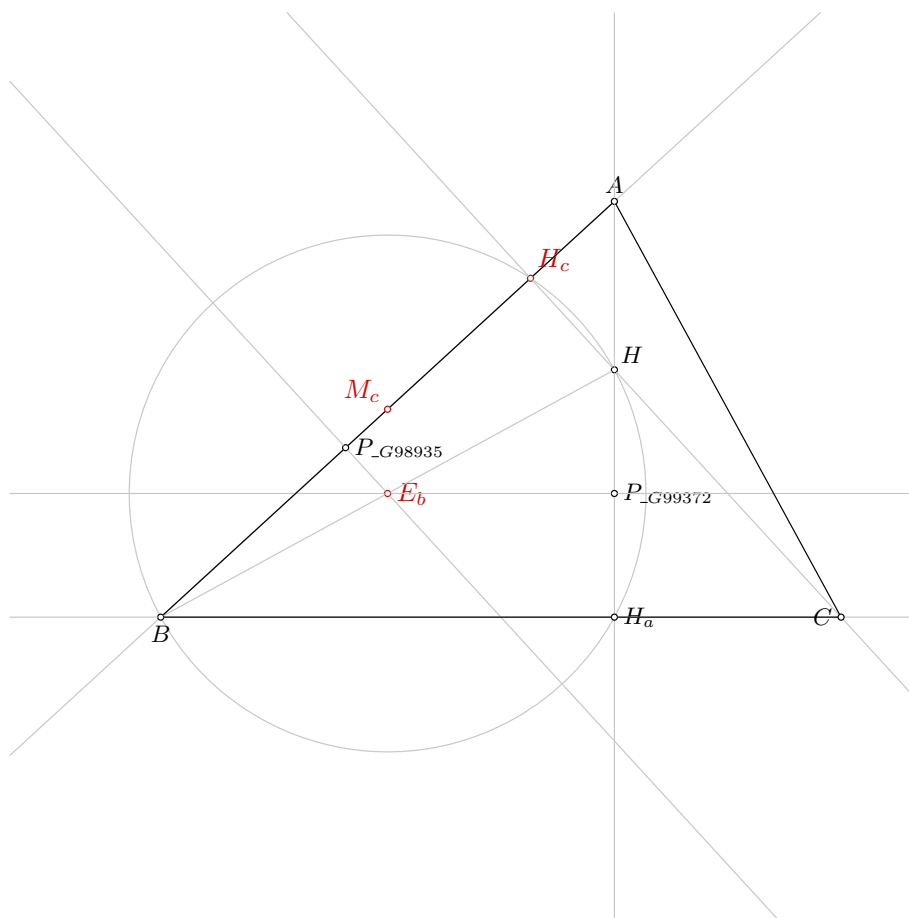


Figure 1: Illustration of the problem 0930

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $H_c = \neg H_c$

Proving failed

4.2.3 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{H_c B H_a} \neq S_{H B H_a}$ i.e., lines $H_c H$ and $B H_a$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{AB F^1_{\neg h_b}} \neq S_{F^0_{\neg h_a} B F^1_{\neg h_b}}$ i.e., lines $AF^0_{\neg h_a}$ and $BF^1_{\neg h_b}$ are not parallel (construction based assumption)

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{AC F^2_{\neg h_c}} \neq S_{BC F^2_{\neg h_c}}$ i.e., lines AB and $CF^2_{\neg h_c}$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $H_c = \neg H_c$

Proving failed

4.3.3 Proving $M_c = \neg M_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $H_c = \neg H_c$

Proving failed

4.4.3 Proving $M_c = \neg M_c$

Proving failed

Problem 931

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 931: Given a point H_c , a point N and a point E_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_c and N are not the same;
2. Choose freely a point E_b on the circle $k(N, M_a)$ (rule WOncircle);
3. Using the point E_b and the point N , construct a line $m(H_a H_c)$ (rule W02); % DET: points E_b and N are not the same;
4. Using the circle $k(N, M_a)$, the line $m(H_a H_c)$, the point N and the point E_b , construct a point M_b (rule W05a); % NDG: line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect % DET: points E_b and M_b must be different;
5. Using the point H_c and the point E_b , construct a circle $k(E_b, B)$ (rule W06); % NDG: points H_c and E_b are not the same;
6. Using the circle $k(N, M_a)$, the circle $k(E_b, B)$, the point H_c , the point N and the point E_b , construct a point H_a (rule W08); % NDG: circles $k(N, M_a)$ and $k(E_b, B)$ intersect % DET: circles $k(N, M_a)$ and $k(E_b, B)$ are not the same; points H_c and H_a must be different;
7. Choose freely a point A on the circle $k(M_b, C)$ (rule WOncircle);
8. Using the point A and the point M_b , construct a point C (rule W01); ;
9. Using the point A and the point H_c , construct a line c (rule W02); % DET: points A and H_c are not the same;
10. Using the point H_a and the point C , construct a line a (rule W02); % DET: points H_a and C are not the same;

11. Using the line c and the line a , construct a point B (rule W03); % NDG: lines c and a are not parallel % DET: lines c and a are not the same.

Non-degenerate conditions: lines c and a are not parallel; circles $k(N, M_a)$ and $k(E_b, B)$ intersect; points H_c and E_b are not the same; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; points H_c and N are not the same.

Determination conditions: lines c and a are not the same; points H_a and C are not the same; points A and H_c are not the same; circles $k(N, M_a)$ and $k(E_b, B)$ are not the same; points H_c and H_a must be different; points E_b and M_b must be different; points E_b and N are not the same.

Rules used: [W01,W02,W03,W05a,W06,W08,WOncircle1]

Lemmas used: [D22,D32,D5,D7,GD01,GD02,GL01,GL03,GL04,L17,L19,L21,L23,L44,L45,L50,L51]

Solving time: 57.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point H_{c} 68.91 84.83
point N 72.5 61.93
point E_{b} 50 56.36

color 220 0 0
fontsize 9

cmark_rt H_{c}
cmark_r N
cmark_r E_{b}
color 0 0 0
fontsize 8

% NDG: points H_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{c}
circle k(N,M_{a}) N H_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point E_{b} on the circle with center N through point H_{c}
oncircle E_{b} N H_{c}
cmark_r E_{b}
color 200 200 200
drawcircle N H_{c}
color 0 0 0

% DET: points E_{b} and N are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point N
line m(H_{a}H_{c}) E_{b} N
```

```

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

% NDG: line m(H_{a}H_{c}) and circle k(N,M_{a}) intersect% DET: points E_{b} and M_{b} must be
different
% Constructing a point M_{b} which is an image of the point E_{b} in the symmetry to point/line N
sim M_{b} N E_{b}
cmark_lt M_{b}

% NDG: points H_{c} and E_{b} are not the same
% Constructing a circle k(E_{b},B) whose center is at point E_{b} and which passes through point H
_{c}
circle k(E_{b},B) E_{b} H_{c}

color 200 200 200
drawcircle k(E_{b},B)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(E_{b},B) intersect% DET: circles k(N,M_{a}) and k(E_{b},B) are not
the same; points H_{c} and H_{a} must be different
% Constructing a line L_{\_G147802} which passes through point N and point E_{b}
line L_{\_G147802} N E_{b}

color 200 200 200
drawline L_{\_G147802}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H_{c} in the symmetry to point/line L
_{\_G147802}
sim H_{a} L_{\_G147802} H_{c}
cmark_r H_{a}

% Choosing randomly a point A on the circle with center M_{b} through point H_{c}
oncircle A M_{b} H_{c}
cmark_t A
color 200 200 200
drawcircle M_{b} H_{c}
color 0 0 0

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

```

```

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0

% DET: points H_{a} and C are not the same
% Constructing a line a which passes through point H_{a} and point C
line a H_{a} C

color 200 200 200
drawline a
color 0 0 0

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and a are not parallel; circles k(N,M_{a}) and k(E_{b},B)
% intersect; points H_{c} and E_{b} are not the same; line m(H_{a}H_{c}) and circle k(N,M_{a})
% intersect; points H_{c} and N are not the same
% Determination conditions: lines c and a are not the same; points H_{a} and C are not the same;
% points A and H_{c} are not the same; circles k(N,M_{a}) and k(E_{b},B) are not the same; points
% H_{c} and H_{a} must be different; points E_{b} and M_{b} must be different; points E_{b} and
% N are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_c = H_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2976 terms.

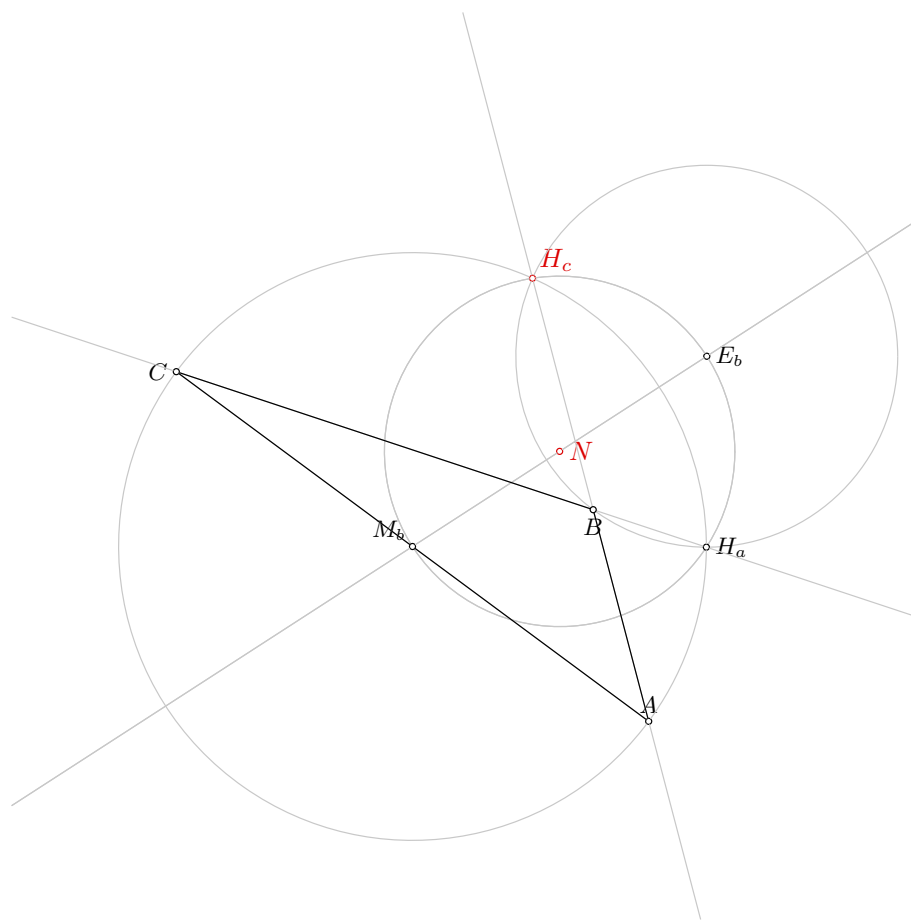


Figure 1: Illustration of the problem 0931

Time Complexity: Time spent by the prover is 13.175 seconds.

NDG conditions Points E_b and N are not identical

Line through points H_c and N is not perpendicular to line through points N and E_b

Line through points A and H_c is not parallel with line through points C and H_a

Points A , H_c and N are not collinear

Points A and B are not identical

Line through points A and B is not parallel with line through points H_c and N

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $E_b=_E E_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_c=_H H_c$

Proving failed

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $E_b=_E E_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_c=_H H_c$

Proving failed

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $E_b=_E E_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_c=_H H_c$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $E_b=_E E_b$

Proving failed

Problem 932

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 932: Given a point E_b , a point H_c and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 933

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 933: Given a point E_b , a point H_c and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 934

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 934: Given a point E_b , a point H_c and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 935

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 935: Given a point E_b , a point H_c and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 936

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 936: Given a point E_b , a point I and a point M_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 937

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 937: Given a point E_b , a point I and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point M_b , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and M_b are not the same;
2. Using the point I and the point M_b , construct a line IM_b (rule W02); % DET: points I and M_b are not the same;
3. Using the point I and the point M_b , construct a circle $k_{over}(I, M_b)$ (rule W09); % NDG: points I and M_b are not the same;
4. Using the point E_b and the point M_b , construct a line $m(E_bM_b)$ (rule W14); % DET: points E_b and M_b are not the same;
5. Using the line $m(E_bM_b)$ and the line $m(H_aH_c)$, construct a point N (rule W03); % NDG: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel % DET: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same;
6. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
7. Using the point I , the circle $k(N, M_a)$, the point N and the point E_b , construct a circle $k(I, P_a)$ (rule W22); % NDG: point I is inside the circle $k(N, M_a)$; points I and N are not the same;
8. Using the circle $k(I, P_a)$ and the circle $k_{over}(I, M_b)$, construct a point B_{fi} and a point P_b (rule W07); % NDG: circles $k(I, P_a)$ and $k_{over}(I, M_b)$ intersect % DET: circles $k(I, P_a)$ and $k_{over}(I, M_b)$ are not the same;
9. Using the point P_b and the point M_b , construct a point P'_b (rule W01); ;

10. Using the circle $k(I, P_a)$, the point M_b and the point I , construct a line x_2 and a line b (rule W12); % NDG: point M_b is outside the circle $k(I, P_a)$;
11. Using the circle $k(N, M_a)$, the line b , the point N and the point M_b , construct a point H_b (rule W05); % NDG: line b and circle $k(N, M_a)$ intersect % DET: points M_b and H_b must be different;
12. Using the point H_b and the point E_b , construct a line h_b (rule W02); % DET: points H_b and E_b are not the same;
13. Using the point M_b and the line b , construct a line m_b (rule W10b); ;
14. Using the point P'_b and the line IM_b , construct a line BP'_b (rule W16); ;
15. Using the line BP'_b and the line h_b , construct a point B (rule W03); % NDG: lines BP'_b and h_b are not parallel % DET: lines BP'_b and h_b are not the same;
16. Using the point I and the point B , construct a line s_b (rule W02); % DET: points I and B are not the same;
17. Using the line m_b and the line s_b , construct a point N_b (rule W03); % NDG: lines m_b and s_b are not parallel % DET: lines m_b and s_b are not the same;
18. Using the point I and the point N_b , construct a circle $k(N_b, A)$ (rule W06); % NDG: points I and N_b are not the same;
19. Using the circle $k(N_b, A)$ and the line b , construct a point A and a point C (rule W04); % NDG: line b and circle $k(N_b, A)$ intersect.

Non-degenerate conditions: line b and circle $k(N_b, A)$ intersect; points I and N_b are not the same; lines m_b and s_b are not parallel; lines BP'_b and h_b are not parallel; line b and circle $k(N, M_a)$ intersect; point M_b is outside the circle $k(I, P_a)$; circles $k(I, P_a)$ and $k_{over}(I, M_b)$ intersect; point I is inside the circle $k(N, M_a)$; points I and N are not the same; points E_b and N are not the same; lines $m(E_b M_b)$ and $m(H_a H_c)$ are not parallel; points I and M_b are not the same.

Determination conditions: lines m_b and s_b are not the same; points I and B are not the same; lines BP'_b and h_b are not the same; points H_b and E_b are not the same; points M_b and H_b must be different; circles $k(I, P_a)$ and $k_{over}(I, M_b)$ are not the same; lines $m(E_b M_b)$ and $m(H_a H_c)$ are not the same; points E_b and M_b are not the same; points I and M_b are not the same; points E_b and M_b are not the same.

Rules used: [W01,W02,W03,W04,W05,W06,W07,W09,W10b,W12,W14,W16,W22]

Lemmas used: [D12,D2,D22,D27,D29,D3,D32,D48,D6,D66,D86,D9,GD01,GD02,GL01,GL03,GL09,L119,L13,L

Solving time: 52.1 seconds.

3.2 Construction in GCLC language

dim 120 120

```
point E_{b} 50 56.36
point I 74.37 61.15
point M_{b} 95 67.5
```

```

color 220 0 0
fontsize 9

cmark_r E_{b}
cmark_b I
cmark_lt M_{b}
color 0 0 0
fontsize 8

% DET: points E_{b} and M_{b} are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point M_{b}
line m(H_{a}H_{c}) E_{b} M_{b}

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

% DET: points I and M_{b} are not the same
% Constructing a line IM_{b} which passes through point I and point M_{b}
line IM_{b} I M_{b}

color 200 200 200
drawline IM_{b}
color 0 0 0

% NDG: points I and M_{b} are not the same
% Constructing midpoint P_{\_G222054} of the segment IM_{b}
midpoint P_{\_G222054} I M_{b}
cmark_r P_{\_G222054}

% Constructing a circle k_{over}(I,M_{b}) whose center is at point P_{\_G222054} and which passes
    through point I
circle k_{over}(I,M_{b}) P_{\_G222054} I

color 200 200 200
drawcircle k_{over}(I,M_{b})
color 0 0 0

% DET: points E_{b} and M_{b} are not the same
% Constructing bisector m(E_{b}M_{b}) of the segment E_{b}M_{b}
med m(E_{b}M_{b}) E_{b} M_{b}

color 200 200 200
drawline m(E_{b}M_{b})
color 0 0 0

color 200 200 200
drawsegment E_{b} M_{b}
color 0 0 0

```

```

% NDG: lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not parallel% DET: lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{b\}}M_{\{b\}})$  and line  $m(H_{\{a\}}H_{\{c\}})$ 
intersec N m( $E_{\{b\}}M_{\{b\}}$ ) m( $H_{\{a\}}H_{\{c\}}$ )
cmark_r N

% NDG: points  $E_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N,M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{b\}}$ 
circle k( $N,M_{\{a\}}$ ) N  $E_{\{b\}}$ 

color 200 200 200
drawcircle k( $N,M_{\{a\}}$ )
color 0 0 0

% NDG: point  $I$  is inside the circle  $k(N,M_{\{a\}})$ ; points  $I$  and  $N$  are not the same
% Calculating distance  $V[_{G222609}]$  from point  $N$  to point  $E_{\{b\}}$ 
distance V[_{G222609}] N  $E_{\{b\}}$ 

% Calculating distance  $V[_{G222633}]$  from point  $N$  to point  $I$ 
distance V[_{G222633}] N I

% Calculating value  $V[_{G222654}]$  using formula  $V[_{G222609}]/V[_{G222633}]$ 
expression V[_{G222654}] { V[_{G222609}]/V[_{G222633}] }

% Constructing a point  $P_{\{\backslash_{G222685}\}}$  such that  $NP_{\{\backslash_{G222685}\}}/NI=V[_{G222609}]/V[_{G222633}]$ 
towards P_{\backslash_{G222685}} N I V[_{G222654}]
cmark_r P_{\backslash_{G222685}}

% Constructing a circle  $k(I,P_{\{a\}})$  whose center is at point  $I$  and which passes through point  $P_{\{\backslash_{G222685}\}}$ 
circle k( $I,P_{\{a\}}$ ) I P_{\backslash_{G222685}}

color 200 200 200
drawcircle k( $I,P_{\{a\}}$ )
color 0 0 0

% NDG: circles  $k(I,P_{\{a\}})$  and  $k_{\text{over}}(I,M_{\{b\}})$  intersect% DET: circles  $k(I,P_{\{a\}})$  and  $k_{\text{over}}(I,M_{\{b\}})$  are not the same
% Constructing points  $B_{\{fi\}}$  and  $P_{\{b\}}$  which are in intersection of  $k(I,P_{\{a\}})$  and  $k_{\text{over}}(I,M_{\{b\}})$ 
intersec2 B_{fi} P_{b} k( $I,P_{\{a\}}$ ) k_{\text{over}}( $I,M_{\{b\}}$ )
cmark_r B_{fi}
cmark_r P_{b}

% Constructing a point  $P'_{\{b\}}$  such that  $P_{\{b\}}P'_{\{b\}}/P_{\{b\}}M_{\{b\}}=2$ 

```

```

towards P'_{b} P_{b} M_{b} 2
cmark_r P'_{b}
color 200 200 200
drawsegment P_{b} P'_{b}
color 0 0 0

% NDG: point M_{b} is outside the circle k(I,P_{a})
% Constructing midpoint P_{\G223291} of the segment M_{b}I
midpoint P_{\G223291} M_{b} I
cmark_r P_{\G223291}

% Constructing a circle C_{\G223294} whose center is at point P_{\G223291} and which passes
    through point M_{b}
circle C_{\G223294} P_{\G223291} M_{b}

color 200 200 200
drawcircle C_{\G223294}
color 0 0 0

% Constructing points P_{\G223297} and P_{\G223300} which are in intersection of C_{\G223294}
    and k(I,P_{a})
intersec2 P_{\G223297} P_{\G223300} C_{\G223294} k(I,P_{a})
cmark_r P_{\G223297}
cmark_r P_{\G223300}

% Constructing a line x2 which passes through point M_{b} and point P_{\G223297}
line x2 M_{b} P_{\G223297}

color 200 200 200
drawline x2
color 0 0 0

% Constructing a line b which passes through point M_{b} and point P_{\G223300}
line b M_{b} P_{\G223300}

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(N,M_{a}) intersect% DET: points M_{b} and H_{b} must be different
% Constructing a point P_{\G223634} which is a foot of the point N on the line b
foot P_{\G223634} N b
cmark_r P_{\G223634}
color 200 200 200
drawline N P_{\G223634}
color 0 0 0

% Constructing a point H_{b} which is an image of the point M_{b} in the symmetry to point/line P
    _{\G223634}
sim H_{b} P_{\G223634} M_{b}
cmark_l H_{b}

```

```

% DET: points  $H_{\{b\}}$  and  $E_{\{b\}}$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $H_{\{b\}}$  and point  $E_{\{b\}}$ 
line  $h_{\{b\}}$   $H_{\{b\}}$   $E_{\{b\}}$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% Constructing a line  $m_{\{b\}}$  which is perpendicular to line  $b$  and which passes through point  $M_{\{b\}}$ 
perp  $m_{\{b\}}$   $M_{\{b\}}$   $b$ 

color 200 200 200
drawline  $m_{\{b\}}$ 
color 0 0 0

% Constructing a line  $BP'_{\{b\}}$  which contains the point  $P'_{\{b\}}$  and is parallel to the line  $IM_{\{b\}}$ 
parallel  $BP'_{\{b\}}$   $P'_{\{b\}}$   $IM_{\{b\}}$ 

color 200 200 200
drawline  $BP'_{\{b\}}$ 
color 0 0 0

% NDG: lines  $BP'_{\{b\}}$  and  $h_{\{b\}}$  are not parallel% DET: lines  $BP'_{\{b\}}$  and  $h_{\{b\}}$  are not the same
% Constructing a point  $B$  which belongs to line  $BP'_{\{b\}}$  and line  $h_{\{b\}}$ 
intersec  $B$   $BP'_{\{b\}}$   $h_{\{b\}}$ 
cmark_b  $B$ 

% DET: points  $I$  and  $B$  are not the same
% Constructing a line  $s_{\{b\}}$  which passes through point  $I$  and point  $B$ 
line  $s_{\{b\}}$   $I$   $B$ 

color 200 200 200
drawline  $s_{\{b\}}$ 
color 0 0 0

% NDG: lines  $m_{\{b\}}$  and  $s_{\{b\}}$  are not parallel% DET: lines  $m_{\{b\}}$  and  $s_{\{b\}}$  are not the same
% Constructing a point  $N_{\{b\}}$  which belongs to line  $m_{\{b\}}$  and line  $s_{\{b\}}$ 
intersec  $N_{\{b\}}$   $m_{\{b\}}$   $s_{\{b\}}$ 
cmark_rb  $N_{\{b\}}$ 

% NDG: points  $I$  and  $N_{\{b\}}$  are not the same
% Constructing a circle  $k(N_{\{b\}}, A)$  whose center is at point  $N_{\{b\}}$  and which passes through point  $I$ 

```

```

circle k(N_{b},A) N_{b} I

color 200 200 200
drawcircle k(N_{b},A)
color 0 0 0

% NDG: line b and circle k(N_{b},A) intersect
% Constructing points A and C which are in intersection of k(N_{b},A) and b
intersec2 A C k(N_{b},A) b
cmark_t A
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle k(N_{b},A) intersect; points I and N_{b} are not the
same; lines m_{b} and s_{b} are not parallel; lines BP'_{b} and h_{b} are not parallel; line b
and circle k(N,M_{a}) intersect; point M_{b} is outside the circle k(I,P_{a}); circles k(I,P_{a})
and k_{over}(I,M_{b}) intersect; point I is inside the circle k(N,M_{a}); points I and N are
not the same; points E_{b} and N are not the same; lines m(E_{b}M_{b}) and m(H_{a}H_{c}) are
not parallel; points I and M_{b} are not the same
% Determination conditions: lines m_{b} and s_{b} are not the same; points I and B are not the same
; lines BP'_{b} and h_{b} are not the same; points H_{b} and E_{b} are not the same; points M_{b}
and H_{b} must be different; circles k(I,P_{a}) and k_{over}(I,M_{b}) are not the same; lines
m(E_{b}M_{b}) and m(H_{a}H_{c}) are not the same; points E_{b} and M_{b} are not the same;
points I and M_{b} are not the same; points E_{b} and M_{b} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = _E_b$

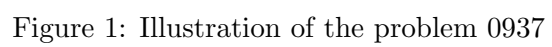
Proving failed

4.1.2 Proving $I = _I$

Proving failed

4.1.3 Proving $M_b = _M_b$

Proving failed



4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $I = \neg I$

Proving failed

4.2.3 Proving $M_b = \neg M_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $I = \neg I$

Proving failed

4.3.3 Proving $M_b = \neg M_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $I = \neg I$

Proving failed

4.4.3 Proving $M_b = \neg M_b$

Proving failed

Problem 938

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 938: Given a point E_b , a point I and a point M_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 939

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 939: Given a point E_b , a point I and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point N , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and N are not the same;
2. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
3. Using the circle $k(N, M_a)$, the line $m(H_aH_c)$, the point N and the point E_b , construct a point M_b (rule W05a); % NDG: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect % DET: points E_b and M_b must be different;
4. Using the point M_b and the point I , construct a line IM_b (rule W02); % DET: points M_b and I are not the same;
5. Using the point I and the point M_b , construct a circle $k_{over}(I, M_b)$ (rule W09); % NDG: points I and M_b are not the same;
6. Using the point I , the circle $k(N, M_a)$, the point N and the point E_b , construct a circle $k(I, P_a)$ (rule W22); % NDG: point I is inside the circle $k(N, M_a)$; points I and N are not the same;
7. Using the circle $k(I, P_a)$ and the circle $k_{over}(I, M_b)$, construct a point B_{fi} and a point P_b (rule W07); % NDG: circles $k(I, P_a)$ and $k_{over}(I, M_b)$ intersect % DET: circles $k(I, P_a)$ and $k_{over}(I, M_b)$ are not the same;
8. Using the point P_b and the point M_b , construct a point P'_b (rule W01); ;
9. Using the circle $k(I, P_a)$, the point M_b and the point I , construct a line x_2 and a line b (rule W12); % NDG: point M_b is outside the circle $k(I, P_a)$;

10. Using the circle $k(N, M_a)$, the line b , the point N and the point M_b , construct a point H_b (rule W05); % NDG: line b and circle $k(N, M_a)$ intersect % DET: points M_b and H_b must be different;
11. Using the point H_b and the point E_b , construct a line h_b (rule W02); % DET: points H_b and E_b are not the same;
12. Using the point M_b and the line b , construct a line m_b (rule W10b); ;
13. Using the point P'_b and the line IM_b , construct a line BP'_b (rule W16); ;
14. Using the line BP'_b and the line h_b , construct a point B (rule W03); % NDG: lines BP'_b and h_b are not parallel % DET: lines BP'_b and h_b are not the same;
15. Using the point I and the point B , construct a line s_b (rule W02); % DET: points I and B are not the same;
16. Using the line m_b and the line s_b , construct a point N_b (rule W03); % NDG: lines m_b and s_b are not parallel % DET: lines m_b and s_b are not the same;
17. Using the point I and the point N_b , construct a circle $k(N_b, A)$ (rule W06); % NDG: points I and N_b are not the same;
18. Using the circle $k(N_b, A)$ and the line b , construct a point A and a point C (rule W04); % NDG: line b and circle $k(N_b, A)$ intersect.

Non-degenerate conditions: line b and circle $k(N_b, A)$ intersect; points I and N_b are not the same; lines m_b and s_b are not parallel; lines BP'_b and h_b are not parallel; line b and circle $k(N, M_a)$ intersect; point M_b is outside the circle $k(I, P_a)$; circles $k(I, P_a)$ and $k_{over}(I, M_b)$ intersect; point I is inside the circle $k(N, M_a)$; points I and N are not the same; points I and M_b are not the same; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; points E_b and N are not the same.

Determination conditions: lines m_b and s_b are not the same; points I and B are not the same; lines BP'_b and h_b are not the same; points H_b and E_b are not the same; points M_b and H_b must be different; circles $k(I, P_a)$ and $k_{over}(I, M_b)$ are not the same; points M_b and I are not the same; points E_b and M_b must be different; points E_b and N are not the same.

Rules used: [W01,W02,W03,W04,W05,W05a,W06,W07,W09,W10b,W12,W16,W22]

Lemmas used: [D12,D2,D22,D27,D29,D3,D32,D48,D6,D66,D86,D9,GD01,GD02,GL01,GL03,GL09,L119,L13,L

Solving time: 53.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
point I 74.37 61.15
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```

cmark_r E_{b}
cmark_b I
cmark_r N
color 0 0 0
fontsize 8

% DET: points E_{b} and N are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point N
line m(H_{a}H_{c}) E_{b} N

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(H_{a}H_{c}) and circle k(N,M_{a}) intersect% DET: points E_{b} and M_{b} must be
different
% Constructing a point M_{b} which is an image of the point E_{b} in the symmetry to point/line N
sim M_{b} N E_{b}
cmark_lt M_{b}

% DET: points M_{b} and I are not the same
% Constructing a line IM_{b} which passes through point M_{b} and point I
line IM_{b} M_{b} I

color 200 200 200
drawline IM_{b}
color 0 0 0

% NDG: points I and M_{b} are not the same
% Constructing midpoint P_{\_G62020} of the segment IM_{b}
midpoint P_{\_G62020} I M_{b}
cmark_r P_{\_G62020}

% Constructing a circle k_{over}(I,M_{b}) whose center is at point P_{\_G62020} and which passes
through point I
circle k_{over}(I,M_{b}) P_{\_G62020} I

color 200 200 200
drawcircle k_{over}(I,M_{b})
color 0 0 0

```

```

% NDG: point I is inside the circle k(N,M_{a}); points I and N are not the same
% Calculating distance V[_G62206] from point N to point E_{b}
distance V[_G62206] N E_{b}

% Calculating distance V[_G62230] from point N to point I
distance V[_G62230] N I

% Calculating value V[_G62251] using formula V[_G62206]/V[_G62230]
expression V[_G62251] { V[_G62206]/V[_G62230] }

% Constructing a point P_{\_G62282} such that NP_{\_G62282}/NI=V[_G62206]/V[_G62230]
towards P_{\_G62282} N I V[_G62251]
cmark_r P_{\_G62282}

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
62282}
circle k(I,P_{a}) I P_{\_G62282}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: circles k(I,P_{a}) and k_{over}(I,M_{b}) intersect% DET: circles k(I,P_{a}) and k_{over}(I,M_{b}
}) are not the same
% Constructing points B_{fi} and P_{b} which are in intersection of k(I,P_{a}) and k_{over}(I,M_{b})
intersec2 B_{fi} P_{b} k(I,P_{a}) k_{over}(I,M_{b})
cmark_r B_{fi}
cmark_r P_{b}

% Constructing a point P'_{b} such that P_{b}P'_{b}/P_{b}M_{b}=2
towards P'_{b} P_{b} M_{b} 2
cmark_r P'_{b}
color 200 200 200
drawsegment P_{b} P'_{b}
color 0 0 0

% NDG: point M_{b} is outside the circle k(I,P_{a})
% Constructing midpoint P_{\_G62887} of the segment M_{b}I
midpoint P_{\_G62887} M_{b} I
cmark_r P_{\_G62887}

% Constructing a circle C_{\_G62890} whose center is at point P_{\_G62887} and which passes through
point M_{b}
circle C_{\_G62890} P_{\_G62887} M_{b}

color 200 200 200

```

```

drawcircle C_{\_G62890}
color 0 0 0

% Constructing points P_{\_G62893} and P_{\_G62896} which are in intersection of C_{\_G62890} and k
(I,P_{a})
intersec2 P_{\_G62893} P_{\_G62896} C_{\_G62890} k(I,P_{a})
cmark_r P_{\_G62893}
cmark_r P_{\_G62896}

% Constructing a line x2 which passes through point M_{b} and point P_{\_G62893}
line x2 M_{b} P_{\_G62893}

color 200 200 200
drawline x2
color 0 0 0

% Constructing a line b which passes through point M_{b} and point P_{\_G62896}
line b M_{b} P_{\_G62896}

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(N,M_{a}) intersect% DET: points M_{b} and H_{b} must be different
% Constructing a point P_{\_G63230} which is a foot of the point N on the line b
foot P_{\_G63230} N b
cmark_r P_{\_G63230}
color 200 200 200
drawline N P_{\_G63230}
color 0 0 0

% Constructing a point H_{b} which is an image of the point M_{b} in the symmetry to point/line P
_{\_G63230}
sim H_{b} P_{\_G63230} M_{b}
cmark_l H_{b}

% DET: points H_{b} and E_{b} are not the same
% Constructing a line h_{b} which passes through point H_{b} and point E_{b}
line h_{b} H_{b} E_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% Constructing a line m_{b} which is perpendicular to line b and which passes through point M_{b}
perp m_{b} M_{b} b

color 200 200 200

```

```

drawline m_{b}
color 0 0 0

% Constructing a line BP'_{b} which contains the point P'_{b} and is parallel to the line IM_{b}
parallel BP'_{b} P'_{b} IM_{b}

color 200 200 200
drawline BP'_{b}
color 0 0 0

% NDG: lines BP'_{b} and h_{b} are not parallel% DET: lines BP'_{b} and h_{b} are not the same
% Constructing a point B which belongs to line BP'_{b} and line h_{b}
intersec B BP'_{b} h_{b}
cmark_b B

% DET: points I and B are not the same
% Constructing a line s_{b} which passes through point I and point B
line s_{b} I B

color 200 200 200
drawline s_{b}
color 0 0 0

% NDG: lines m_{b} and s_{b} are not parallel% DET: lines m_{b} and s_{b} are not the same
% Constructing a point N_{b} which belongs to line m_{b} and line s_{b}
intersec N_{b} m_{b} s_{b}
cmark_rb N_{b}

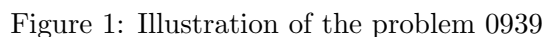
% NDG: points I and N_{b} are not the same
% Constructing a circle k(N_{b},A) whose center is at point N_{b} and which passes through point I
circle k(N_{b},A) N_{b} I

color 200 200 200
drawcircle k(N_{b},A)
color 0 0 0

% NDG: line b and circle k(N_{b},A) intersect
% Constructing points A and C which are in intersection of k(N_{b},A) and b
intersec2 A C k(N_{b},A) b
cmark_t A
cmark_l C

drawsegment A B

```



```
% Non-degenerate conditions: line b and circle k(N_{b},A) intersect; points I and N_{b} are not the
same; lines m_{b} and s_{b} are not parallel; lines BP'_{b} and h_{b} are not parallel; line b
and circle k(N,M_{a}) intersect; point M_{b} is outside the circle k(I,P_{a}); circles k(I,P_{a})
and k_{over}(I,M_{b}) intersect; point I is inside the circle k(N,M_{a}); points I and N are
not the same; points I and M_{b} are not the same; line m(H_{a}H_{c}) and circle k(N,M_{a})
intersect; points E_{b} and N are not the same
% Determination conditions: lines m_{b} and s_{b} are not the same; points I and B are not the same
; lines BP'_{b} and h_{b} are not the same; points H_{b} and E_{b} are not the same; points M_{b}
and H_{b} must be different; circles k(I,P_{a}) and k_{over}(I,M_{b}) are not the same; points
M_{b} and I are not the same; points E_{b} and M_{b} must be different; points E_{b} and N are
not the same
```

Illustration of the constructed figure is given in Figure 1

1640

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $I = \neg I$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $I = \neg I$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $I = \neg I$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $I = \neg I$

Proving failed

4.4.3 Proving $N = \neg N$

Proving failed

Problem 940

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 940: Given a point E_b , a point I and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 941

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 941: Given a point E_b , a point I and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 942

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 942: Given a point E_b , a point I and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 943

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 943: Given a point E_b , a point I and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 944

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 944: Given a point E_b , a point M_b and a point M_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point M_b , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and M_b are not the same;
2. Using the point E_b and the point M_b , construct a line $m(E_bM_b)$ (rule W14); % DET: points E_b and M_b are not the same;
3. Using the line $m(E_bM_b)$ and the line $m(H_aH_c)$, construct a point N (rule W03); % NDG: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel % DET: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same;
4. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
5. Choose freely a point M_a on the circle $k(N, M_a)$ (rule WOncircle);
6. Choose freely a point A (rule free);
7. Using the point A and the point M_b , construct a point C (rule W01); ;
8. Using the point M_a and the point C , construct a point B (rule W01); .

Non-degenerate conditions: points E_b and N are not the same; lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel.

Determination conditions: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same; points E_b and M_b are not the same; points E_b and M_b are not the same.

Rules used: [W01,W02,W03,W06,W14,WOncircle1,free]

Lemmas used: [D21,D22,D32,GD02,GL01,GL03,GL04,L17,L19,L21,L23,L44,L45,L50,L51]

Solving time: 200.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{b} 50 56.36
point M_{b} 95 67.5
point M_{a} 65 40

color 220 0 0
fontsize 9

cmark_r E_{b}
cmark_lt M_{b}
cmark_r M_{a}
color 0 0 0
fontsize 8

% DET: points E_{b} and M_{b} are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point M_{b}
line m(H_{a}H_{c}) E_{b} M_{b}

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

% DET: points E_{b} and M_{b} are not the same
% Constructing bisector m(E_{b}M_{b}) of the segment E_{b}M_{b}
med m(E_{b}M_{b}) E_{b} M_{b}

color 200 200 200
drawline m(E_{b}M_{b})
color 0 0 0

color 200 200 200
drawsegment E_{b} M_{b}
color 0 0 0

% NDG: lines m(E_{b}M_{b}) and m(H_{a}H_{c}) are not parallel% DET: lines m(E_{b}M_{b}) and m(H_{a}
H_{c}) are not the same
% Constructing a point N which belongs to line m(E_{b}M_{b}) and line m(H_{a}H_{c})
intersec N m(E_{b}M_{b}) m(H_{a}H_{c})
cmark_r N

% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```

% Choosing randomly a point  $M_{\{a\}}$  on the circle with center  $N$  through point  $E_{\{b\}}$ 
oncircle  $M_{\{a\}}$   $N$   $E_{\{b\}}$ 
cmark_r  $M_{\{a\}}$ 
color 200 200 200
drawcircle  $N$   $E_{\{b\}}$ 
color 0 0 0

% Constructing a free point  $A$ 
point  $A$  80 95

cmark_t  $A$ 

% Constructing a point  $C$  such that  $AC/AM_{\{b\}}=2$ 
towards  $C$   $A$   $M_{\{b\}}$  2
cmark_l  $C$ 
color 200 200 200
drawsegment  $A$   $C$ 
color 0 0 0

% Constructing a point  $B$  such that  $M_{\{a\}}B/M_{\{a\}}C=-1$ 
towards  $B$   $M_{\{a\}}$   $C$  -1
cmark_b  $B$ 
color 200 200 200
drawsegment  $C$   $B$ 
color 0 0 0

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: points  $E_{\{b\}}$  and  $N$  are not the same; lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not parallel
% Determination conditions: lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not the same; points  $E_{\{b\}}$  and  $M_{\{b\}}$  are not the same; points  $E_{\{b\}}$  and  $M_{\{b\}}$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

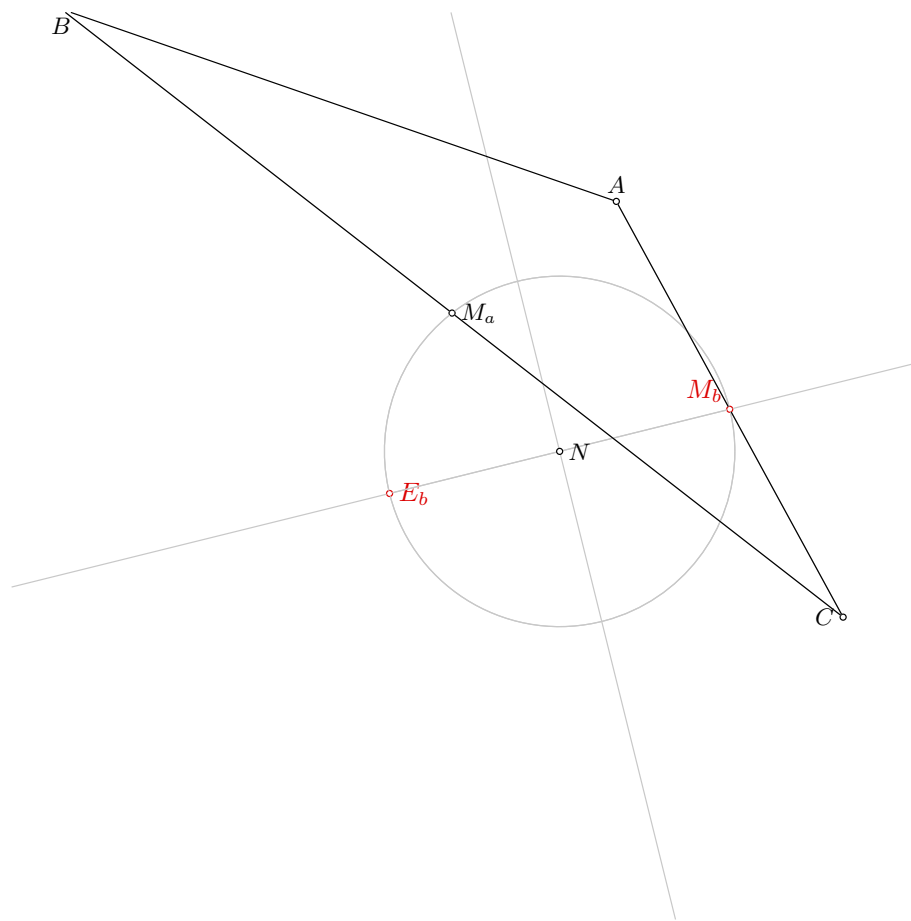


Figure 1: Illustration of the problem 0944

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.087 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $M_a = \neg M_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{m(E_b M_b) E_b M_b}^0 \neq S_{T_{m(E_b M_b) E_b M_b}^1}^1$ i.e., lines $M_{m(E_b M_b)}^0 T_{m(E_b M_b)}^1$ and $E_b M_b$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{F_{-h_a}^2 BF_{-h_b}^3}$ i.e., lines $AF_{-h_a}^2$ and $BF_{-h_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

4.2.3 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{m(E_b M_b) E_b M_b}^0 \neq S_{T_{m(E_b M_b) E_b M_b}^1}^1$ i.e., lines $M_{m(E_b M_b)}^0 T_{m(E_b M_b)}^1$ and $E_b M_b$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{F_{-h_a}^2 BF_{-h_b}^3}$ i.e., lines $AF_{-h_a}^2$ and $BF_{-h_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $M_b = \neg M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 19 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.3.3 Proving $M_a = \neg M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 28 terms.

Time Complexity: Time spent by the prover is 0.060 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $M_b = \neg M_b$

Proving failed

4.4.3 Proving $M_a = \neg M_a$

Proving failed

Problem 945

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 945: Given a point E_b , a point M_a and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point M_a , construct a line $m(E_bM_a)$ (rule W14); % DET: points E_b and M_a are not the same;
2. Using the point E_b and the point M_c , construct a line $m(E_bM_c)$ (rule W14); % DET: points E_b and M_c are not the same;
3. Using the line $m(E_bM_c)$ and the line $m(E_bM_a)$, construct a point N (rule W03); % NDG: lines $m(E_bM_c)$ and $m(E_bM_a)$ are not parallel % DET: lines $m(E_bM_c)$ and $m(E_bM_a)$ are not the same;
4. Using the point M_a and the point N , construct a line $m(H_bH_c)$ (rule W02); % DET: points M_a and N are not the same;
5. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
6. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point M_a , construct a point E_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_a and E_a must be different;
7. Using the point M_c and the point E_a , construct a line $m(AH_b)$ (rule W02); % DET: points M_c and E_a are not the same;
8. Using the point E_b and the line $m(AH_b)$, construct a line h_b (rule W16); ;
9. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;

10. Using the point H_b and the point M_a , construct a circle $k(M_a, B)$ (rule W06); % NDG: points H_b and M_a are not the same;
11. Using the circle $k(M_a, B)$, the line h_b , the point M_a and the point H_b , construct a point B (rule W05); % NDG: line h_b and circle $k(M_a, B)$ intersect % DET: points H_b and B must be different;
12. Using the point M_a and the point B , construct a point C (rule W01); ;
13. Using the point M_c and the point B , construct a point A (rule W01); .

Non-degenerate conditions: line h_b and circle $k(M_a, B)$ intersect; points H_b and M_a are not the same; line h_b and circle $k(N, M_a)$ intersect; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; points E_b and N are not the same; lines $m(E_b M_c)$ and $m(E_b M_a)$ are not parallel.

Determination conditions: points H_b and B must be different; points E_b and H_b must be different; points M_c and E_a are not the same; points M_a and E_a must be different; points M_a and N are not the same; lines $m(E_b M_c)$ and $m(E_b M_a)$ are not the same; points E_b and M_c are not the same; points E_b and M_a are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W14,W16]

Lemmas used: [D20,D21,D29,D3,D32,D6,D9,GD02,GL01,GL03,GL04,GL09,L18,L20,L21,L22,L23,L38,L39,L41]

Solving time: 25.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
point M_{a} 65 40
point M_{c} 50 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{b}
cmark_r M_{a}
cmark_lt M_{c}
color 0 0 0
fontsize 8
```

```
% DET: points E_{b} and M_{a} are not the same
% Constructing bisector m(E_{b}M_{a}) of the segment E_{b}M_{a}
med m(E_{b}M_{a}) E_{b} M_{a}
```

```
color 200 200 200
drawline m(E_{b}M_{a})
color 0 0 0
```

```
color 200 200 200
drawsegment E_{b} M_{a}
color 0 0 0
```

```

% DET: points  $E_{\{b\}}$  and  $M_{\{c\}}$  are not the same
% Constructing bisector  $m(E_{\{b\}}M_{\{c\}})$  of the segment  $E_{\{b\}}M_{\{c\}}$ 
med m( $E_{\{b\}}M_{\{c\}}$ )  $E_{\{b\}}$   $M_{\{c\}}$ 

color 200 200 200
drawline m( $E_{\{b\}}M_{\{c\}}$ )
color 0 0 0

color 200 200 200
drawsegment  $E_{\{b\}}$   $M_{\{c\}}$ 
color 0 0 0

% NDG: lines  $m(E_{\{b\}}M_{\{c\}})$  and  $m(E_{\{b\}}M_{\{a\}})$  are not parallel% DET: lines  $m(E_{\{b\}}M_{\{c\}})$  and  $m(E_{\{b\}}$ 
 $M_{\{a\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{b\}}M_{\{c\}})$  and line  $m(E_{\{b\}}M_{\{a\}})$ 
intersec N m( $E_{\{b\}}M_{\{c\}}$ ) m( $E_{\{b\}}M_{\{a\}}$ )
cmark_r N

% DET: points  $M_{\{a\}}$  and  $N$  are not the same
% Constructing a line  $m(H_{\{b\}}H_{\{c\}})$  which passes through point  $M_{\{a\}}$  and point  $N$ 
line m( $H_{\{b\}}H_{\{c\}}$ )  $M_{\{a\}}$  N

color 200 200 200
drawline m( $H_{\{b\}}H_{\{c\}}$ )
color 0 0 0

% NDG: points  $E_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{b\}}$ 
circle k( $N, M_{\{a\}}$ )  $N$   $E_{\{b\}}$ 

color 200 200 200
drawcircle k( $N, M_{\{a\}}$ )
color 0 0 0

% NDG: line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{a\}}$  and  $E_{\{a\}}$  must be
different
% Constructing a point  $E_{\{a\}}$  which is an image of the point  $M_{\{a\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{a\}}$  N  $M_{\{a\}}$ 
cmark_r  $E_{\{a\}}$ 

% DET: points  $M_{\{c\}}$  and  $E_{\{a\}}$  are not the same
% Constructing a line  $m(AH_{\{b\}})$  which passes through point  $M_{\{c\}}$  and point  $E_{\{a\}}$ 
line m( $AH_{\{b\}}$ )  $M_{\{c\}}$   $E_{\{a\}}$ 

color 200 200 200
drawline m( $AH_{\{b\}}$ )
color 0 0 0

```

```

% Constructing a line  $h_{\{b\}}$  which contains the point  $E_{\{b\}}$  and is parallel to the line  $m(AH_{\{b\}})$ 
parallel  $h_{\{b\}}$   $E_{\{b\}}$   $m(AH_{\{b\}})$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{b\}}$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G141182\}}$  which is a foot of the point  $N$  on the line  $h_{\{b\}}$ 
foot  $P_{\{\backslash\_G141182\}}$   $N$   $h_{\{b\}}$ 
cmark_r  $P_{\{\backslash\_G141182\}}$ 
color 200 200 200
drawline  $N$   $P_{\{\backslash\_G141182\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $E_{\{b\}}$  in the symmetry to point/line  $P_{\{\backslash\_G141182\}}$ 
sim  $H_{\{b\}}$   $P_{\{\backslash\_G141182\}}$   $E_{\{b\}}$ 
cmark_l  $H_{\{b\}}$ 

% NDG: points  $H_{\{b\}}$  and  $M_{\{a\}}$  are not the same
% Constructing a circle  $k(M_{\{a\}}, B)$  whose center is at point  $M_{\{a\}}$  and which passes through point  $H_{\{b\}}$ 
circle  $k(M_{\{a\}}, B)$   $M_{\{a\}}$   $H_{\{b\}}$ 

color 200 200 200
drawcircle  $k(M_{\{a\}}, B)$ 
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(M_{\{a\}}, B)$  intersect% DET: points  $H_{\{b\}}$  and  $B$  must be different
% Constructing a point  $P_{\{\backslash\_G141486\}}$  which is a foot of the point  $M_{\{a\}}$  on the line  $h_{\{b\}}$ 
foot  $P_{\{\backslash\_G141486\}}$   $M_{\{a\}}$   $h_{\{b\}}$ 
cmark_r  $P_{\{\backslash\_G141486\}}$ 
color 200 200 200
drawline  $M_{\{a\}}$   $P_{\{\backslash\_G141486\}}$ 
color 0 0 0

% Constructing a point  $B$  which is an image of the point  $H_{\{b\}}$  in the symmetry to point/line  $P_{\{\backslash\_G141486\}}$ 
sim  $B$   $P_{\{\backslash\_G141486\}}$   $H_{\{b\}}$ 
cmark_b  $B$ 

% Constructing a point  $C$  such that  $M_{\{a\}}C/M_{\{a\}}B=-1$ 
towards  $C$   $M_{\{a\}}$   $B$  -1
cmark_l  $C$ 
color 200 200 200
drawsegment  $B$   $C$ 

```

```
color 0 0 0
```

```
% Constructing a point A such that  $M_{\{c\}}A/M_{\{c\}}B=-1$ 
towards A  $M_{\{c\}}$  B -1
cmark_t A
color 200 200 200
drawsegment B A
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line  $h_{\{b\}}$  and circle  $k(M_{\{a\}}, B)$  intersect; points  $H_{\{b\}}$  and  $M_{\{a\}}$  are
not the same; line  $h_{\{b\}}$  and circle  $k(N, M_{\{a\}})$  intersect; line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$ 
intersect; points  $E_{\{b\}}$  and  $N$  are not the same; lines  $m(E_{\{b\}}M_{\{c\}})$  and  $m(E_{\{b\}}M_{\{a\}})$  are
not parallel
% Determination conditions: points  $H_{\{b\}}$  and  $B$  must be different; points  $E_{\{b\}}$  and  $H_{\{b\}}$  must be
different; points  $M_{\{c\}}$  and  $E_{\{a\}}$  are not the same; points  $M_{\{a\}}$  and  $E_{\{a\}}$  must be different;
points  $M_{\{a\}}$  and  $N$  are not the same; lines  $m(E_{\{b\}}M_{\{c\}})$  and  $m(E_{\{b\}}M_{\{a\}})$  are not the same;
points  $E_{\{b\}}$  and  $M_{\{c\}}$  are not the same; points  $E_{\{b\}}$  and  $M_{\{a\}}$  are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 9 terms.

Time Complexity: Time spent by the prover is 0.275 seconds.

NDG conditions Points M_c and E_b are not identical

Points M_c and E_b are not identical

Line through points M_c and E_a is not parallel with line through points E_b and M_a

Points tempPoint-815 h_b and E_b are not identical

Points tempPoint-815 h_b and E_b are not identical

Points E_b and $P_{G136203}$ are not identical

Points E_b , M_a and $P_{G136203}$ are not collinear

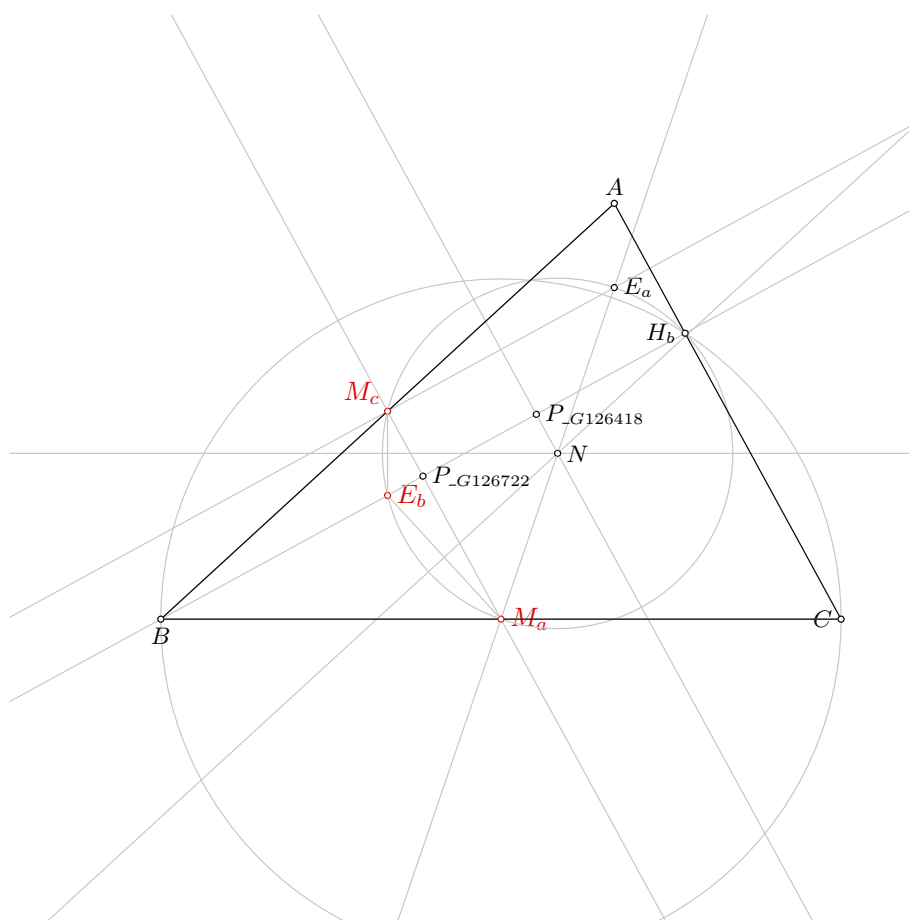


Figure 1: Illustration of the problem 0945

4.1.3 Proving $M_c = M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 11 terms.

Time Complexity: Time spent by the prover is 0.261 seconds.

NDG conditions Points M_c and E_b are not identical

Points M_c and E_b are not identical

Line through points M_c and E_a is not parallel with line through points E_b and M_a

Points E_b and tempPoint-518 h_b are not identical

Points tempPoint-518 h_b and M_a are not identical

Points E_b and $P_{G137539}$ are not identical

Points E_b and $P_{G137539}$ are not identical

4.2 GCLC - Area method

4.2.1 Proving $E_b = E_b$

Proving failed

4.2.2 Proving $M_a = M_a$

NDG conditions are:

$S_{M^2_{m(E_b M_c)} M^0_{m(E_b M_a)} T^1_{m(E_b M_a)}} \neq S_{T^3_{m(E_b M_c)} M^0_{m(E_b M_a)} T^1_{m(E_b M_a)}}$ i.e., lines $M^2_{m(E_b M_c)} T^3_{m(E_b M_c)}$ and $M^0_{m(E_b M_a)} T^1_{m(E_b M_a)}$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^6_{h_b}} \neq S_{F^5_{h_a} BF^6_{h_b}}$ i.e., lines $AF^5_{h_a}$ and $BF^6_{h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.2.3 Proving $M_c = M_c$

NDG conditions are:

$S_{M^2_{m(E_b M_c)} M^0_{m(E_b M_a)} T^1_{m(E_b M_a)}} \neq S_{T^3_{m(E_b M_c)} M^0_{m(E_b M_a)} T^1_{m(E_b M_a)}}$ i.e., lines $M^2_{m(E_b M_c)} T^3_{m(E_b M_c)}$ and $M^0_{m(E_b M_a)} T^1_{m(E_b M_a)}$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^6_{h_b}} \neq S_{F^5_{h_a} BF^6_{h_b}}$ i.e., lines $AF^5_{h_a}$ and $BF^6_{h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_b = E_b$

Proving failed

4.3.2 Proving $M_a = M_a$

Proving failed

4.3.3 Proving $M_c = M_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $M_a = \neg M_a$

Proving failed

4.4.3 Proving $M_c = \neg M_c$

Proving failed

Problem 946

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 946: Given a point M_a , a point N and a point E_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point M_a and the point N , construct a line $m(H_bH_c)$ (rule W02); % DET: points M_a and N are not the same;
2. Using the point M_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_a and N are not the same;
3. Choose freely a point E_b on the circle $k(N, M_a)$ (rule WOncircle);
4. Using the point E_b and the point N , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and N are not the same;
5. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point M_a , construct a point E_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_a and E_a must be different;
6. Using the circle $k(N, M_a)$, the line $m(H_aH_c)$, the point N and the point E_b , construct a point M_b (rule W05a); % NDG: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect % DET: points E_b and M_b must be different;
7. Choose freely a point A (rule free);
8. Using the point A and the point E_a , construct a point H (rule W01); ;
9. Using the point A and the point M_b , construct a point C (rule W01); ;
10. Using the point E_b and the point H , construct a point B (rule W01); .

Non-degenerate conditions: line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; points M_a and N are not the same.

Determination conditions: points E_b and M_b must be different; points M_a and E_a must be different; points E_b and N are not the same; points M_a and N are not the same.

Rules used: [W01,W02,W05a,W06,WOncircle1,free]

Lemmas used: [D22,D28,D29,D32,GD02,GL01,GL03,GL04,L17,L19,L20,L21,L22,L23,L38,L39,L44,L45,L47,L48]

Solving time: 150.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40
```

```
point N 72.5 61.93
```

```
point E_{b} 50 56.36
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r M_{a}
```

```
cmark_r N
```

```
cmark_r E_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points M_{a} and N are not the same
```

```
% Constructing a line m(H_{b}H_{c}) which passes through point M_{a} and point N
```

```
line m(H_{b}H_{c}) M_{a} N
```

```
color 200 200 200
```

```
drawline m(H_{b}H_{c})
```

```
color 0 0 0
```

```
% NDG: points M_{a} and N are not the same
```

```
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{a}
```

```
circle k(N,M_{a}) N M_{a}
```

```
color 200 200 200
```

```
drawcircle k(N,M_{a})
```

```
color 0 0 0
```

```
% Choosing randomly a point E_{b} on the circle with center N through point M_{a}
```

```
oncircle E_{b} N M_{a}
```

```
cmark_r E_{b}
```

```
color 200 200 200
```

```
drawcircle N M_{a}
```

```
color 0 0 0
```

```

% DET: points  $E_{\{b\}}$  and  $N$  are not the same
% Constructing a line  $m(H_{\{a\}}H_{\{c\}})$  which passes through point  $E_{\{b\}}$  and point  $N$ 
line m( $H_{\{a\}}H_{\{c\}}$ )  $E_{\{b\}}$  N

color 200 200 200
drawline m( $H_{\{a\}}H_{\{c\}}$ )
color 0 0 0

% NDG: line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $M_{\{a\}}$  and  $E_{\{a\}}$  must be
different
% Constructing a point  $E_{\{a\}}$  which is an image of the point  $M_{\{a\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{a\}}$  N  $M_{\{a\}}$ 
cmark_r  $E_{\{a\}}$ 

% NDG: line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $E_{\{b\}}$  and  $M_{\{b\}}$  must be
different
% Constructing a point  $M_{\{b\}}$  which is an image of the point  $E_{\{b\}}$  in the symmetry to point/line  $N$ 
sim  $M_{\{b\}}$  N  $E_{\{b\}}$ 
cmark_lt  $M_{\{b\}}$ 

% Constructing a free point  $A$ 
point A 80 95

cmark_t A

% Constructing a point  $H$  such that  $AH/AE_{\{a\}}=2$ 
towards H A  $E_{\{a\}}$  2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% Constructing a point  $C$  such that  $AC/AM_{\{b\}}=2$ 
towards C A  $M_{\{b\}}$  2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% Constructing a point  $B$  such that  $E_{\{b\}}B/E_{\{b\}}H=-1$ 
towards B  $E_{\{b\}}$  H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

```

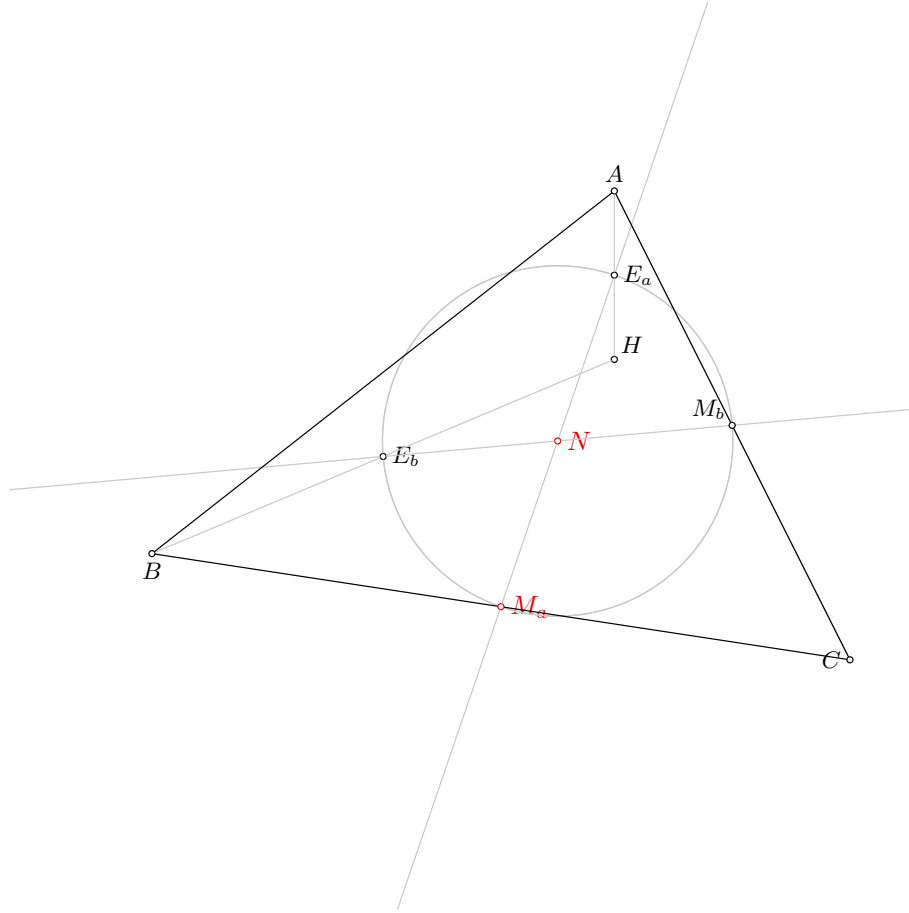


Figure 1: Illustration of the problem 0946

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line m(H_{a}H_{c}) and circle k(N,M_{a}) intersect; line m(H_{b}H_{c})
and circle k(N,M_{a}) intersect; points M_{a} and N are not the same
% Determination conditions: points E_{b} and M_{b} must be different; points M_{a} and E_{a} must
be different; points E_{b} and N are not the same; points M_{a} and N are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 15 terms.

Time Complexity: Time spent by the prover is 0.302 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $N = \neg N$

Proving failed

4.1.3 Proving $E_b = \neg E_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $M_a = \neg M_a$

Proving failed

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $E_b = \neg E_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_a = \neg M_a$

Proving failed

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $E_b = \neg E_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_a = \neg M_a$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $E_b = E_b$

Proving failed

Problem 947

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 947: Given a point E_b , a point M_a and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point M_a , construct a line $m(BH_c)$ (rule W02); % DET: points E_b and M_a are not the same;
2. Using the point M_a and the point O , construct a line m_a (rule W02); % DET: points M_a and O are not the same;
3. Using the point M_a and the line m_a , construct a line a (rule W10a); ;
4. Using the point E_b and the line m_a , construct a line $m(BH_a)$ (rule W16); ;
5. Using the point O and the line $m(BH_c)$, construct a line m_c (rule W16); ;
6. Using the line m_c and the line $m(BH_a)$, construct a point M_c (rule W03); % NDG: lines m_c and $m(BH_a)$ are not parallel % DET: lines m_c and $m(BH_a)$ are not the same;
7. Using the point M_c and the line m_c , construct a line c (rule W10a); ;
8. Using the line c and the line a , construct a point B (rule W03); % NDG: lines c and a are not parallel % DET: lines c and a are not the same;
9. Using the point B and the point E_b , construct a point H (rule W01); ;
10. Using the point M_a and the point B , construct a point C (rule W01); ;
11. Using the point O and the point H , construct a point G (rule W01); ;
12. Using the point M_a and the point G , construct a point A (rule W01); .

Non-degenerate conditions: lines c and a are not parallel; lines m_c and $m(BH_a)$ are not parallel.
Determination conditions: lines c and a are not the same; lines m_c and $m(BH_a)$ are not the same; points M_a and O are not the same; points E_b and M_a are not the same.

Rules used: [W01,W02,W03,W10a,W16]

Lemmas used: [D1,D11,D13,D20,D21,D29,GD01,GD02,GL01,GL03,GL04,GL09,L1,L39,L40,L42,L50,L51,L55,L56]

Solving time: 3.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{b} 50 56.36
point M_{a} 65 40
point O 65 51.14

color 220 0 0
fontsize 9

cmark_r E_{b}
cmark_r M_{a}
cmark_t O
color 0 0 0
fontsize 8

% DET: points E_{b} and M_{a} are not the same
% Constructing a line m(BH_{c}) which passes through point E_{b} and point M_{a}
line m(BH_{c}) E_{b} M_{a}

color 200 200 200
drawline m(BH_{c})
color 0 0 0

% DET: points M_{a} and O are not the same
% Constructing a line m_{a} which passes through point M_{a} and point O
line m_{a} M_{a} O

color 200 200 200
drawline m_{a}
color 0 0 0

% Constructing a line a which is perpendicular to line m_{a} and which passes through point M_{a}
perp a M_{a} m_{a}

color 200 200 200
drawline a
color 0 0 0
```

```
% Constructing a line  $m(BH_{\{a\}})$  which contains the point  $E_{\{b\}}$  and is parallel to the line  $m_{\{a\}}$ 
parallel m(BH_{a}) E_{b} m_{a}
```

```
color 200 200 200
drawline m(BH_{a})
color 0 0 0
```

```
% Constructing a line  $m_{\{c\}}$  which contains the point  $O$  and is parallel to the line  $m(BH_{\{c\}})$ 
parallel m_{c} O m(BH_{c})
```

```
color 200 200 200
drawline m_{c}
color 0 0 0
```

```
% NDG: lines  $m_{\{c\}}$  and  $m(BH_{\{a\}})$  are not parallel% DET: lines  $m_{\{c\}}$  and  $m(BH_{\{a\}})$  are not the same
% Constructing a point  $M_{\{c\}}$  which belongs to line  $m_{\{c\}}$  and line  $m(BH_{\{a\}})$ 
intersec M_{c} m_{c} m(BH_{a})
cmark_lt M_{c}
```

```
% Constructing a line  $c$  which is perpendicular to line  $m_{\{c\}}$  and which passes through point  $M_{\{c\}}$ 
perp c M_{c} m_{c}
```

```
color 200 200 200
drawline c
color 0 0 0
```

```
% NDG: lines  $c$  and  $a$  are not parallel% DET: lines  $c$  and  $a$  are not the same
% Constructing a point  $B$  which belongs to line  $c$  and line  $a$ 
intersec B c a
cmark_b B
```

```
% Constructing a point  $H$  such that  $BH/BE_{\{b\}}=2$ 
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0
```

```
% Constructing a point  $C$  such that  $M_{\{a\}}C/M_{\{a\}}B=-1$ 
towards C M_{a} B -1
cmark_l C
color 200 200 200
```

```

drawsegment B C
color 0 0 0

% Constructing a line  $L_{\{ \_G205340 \}}$  which passes through point O and point H
line  $L_{\{ \_G205340 \}}$  O H

color 200 200 200
drawline  $L_{\{ \_G205340 \}}$ 
color 0 0 0

% Constructing a point  $P_{\{ \_G205441 \}}$  with coordinates (0,0)
point  $P_{\{ \_G205441 \}}$  0 0
cmark_r  $P_{\{ \_G205441 \}}$ 

% Constructing a point  $P_{\{ \_G205365 \}}$  such that  $OP_{\{ \_G205365 \}}/OP_{\{ \_G205441 \}}=1$ 
towards  $P_{\{ \_G205365 \}}$  0  $P_{\{ \_G205441 \}}$  1
cmark_r  $P_{\{ \_G205365 \}}$ 
color 200 200 200
drawsegment 0  $P_{\{ \_G205365 \}}$ 
color 0 0 0

% Constructing a point  $P_{\{ \_G205410 \}}$  such that  $OP_{\{ \_G205410 \}}/OP_{\{ \_G205441 \}}=3$ 
towards  $P_{\{ \_G205410 \}}$  0  $P_{\{ \_G205441 \}}$  3
cmark_r  $P_{\{ \_G205410 \}}$ 
color 200 200 200
drawsegment 0  $P_{\{ \_G205410 \}}$ 
color 0 0 0

% Constructing a line  $L_{\{ \_G205371 \}}$  which passes through point H and point  $P_{\{ \_G205410 \}}$ 
line  $L_{\{ \_G205371 \}}$  H  $P_{\{ \_G205410 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G205371 \}}$ 
color 0 0 0

% Constructing a line  $L_{\{ \_G205334 \}}$  which contains the point  $P_{\{ \_G205365 \}}$  and is parallel to the
line  $L_{\{ \_G205371 \}}$ 
parallel  $L_{\{ \_G205334 \}}$   $P_{\{ \_G205365 \}}$   $L_{\{ \_G205371 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G205334 \}}$ 
color 0 0 0

% Constructing a point G which belongs to line  $L_{\{ \_G205334 \}}$  and line  $L_{\{ \_G205340 \}}$ 
intersec G  $L_{\{ \_G205334 \}}$   $L_{\{ \_G205340 \}}$ 
cmark_t G

% Constructing a point A such that  $M_{\{ a \}}A/M_{\{ a \}}G=3$ 
towards A  $M_{\{ a \}}$  G 3

```

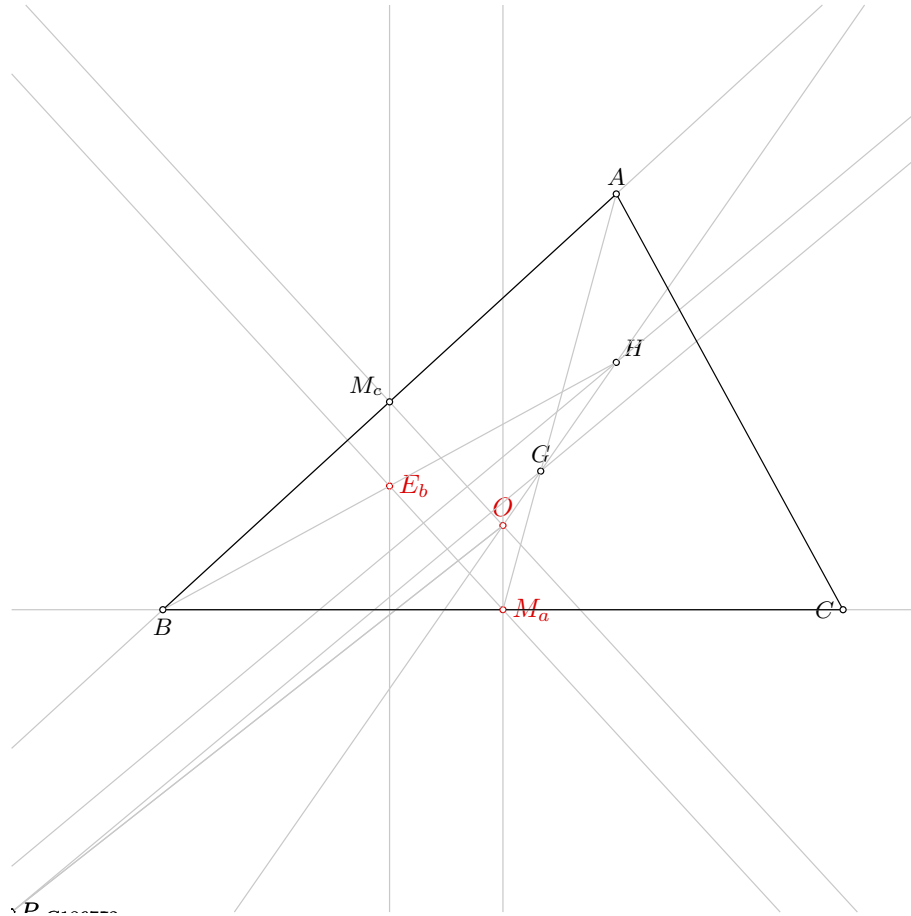


Figure 1: Illustration of the problem 0947

```

cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

% Non-degenerate conditions: lines c and a are not parallel; lines $m_{\{c\}}$ and $m(BH_{\{a\}})$ are not parallel

% Determination conditions: lines c and a are not the same; lines $m_{\{c\}}$ and $m(BH_{\{a\}})$ are not the same; points $M_{\{a\}}$ and O are not the same; points $E_{\{b\}}$ and $M_{\{a\}}$ are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 9 terms.

Time Complexity: Time spent by the prover is 0.191 seconds.

NDG conditions Points M_c and M_a are not identical

Line through points M_c and E_b is not perpendicular to line through points E_b and O

4.1.3 Proving $O = \neg O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{OE_bP_{m(BH_a)}^1} \neq S_{P_{m_c}^2E_bP_{m(BH_a)}^1}$ i.e., lines $OP_{m_c}^2$ and $E_bP_{m(BH_a)}^1$ are not parallel (construction based assumption)
 $S_{M_cOP_{m_c}^2} \neq 0$ i.e., points M_c , O and $P_{m_c}^2$ are not collinear (foot is not the point itself; construction based assumption)

$S_{M_cM_aT_a^0} \neq S_{F_c^3M_aT_a^0}$ i.e., lines $M_cF_c^3$ and $M_aT_a^0$ are not parallel (construction based assumption)
 $S_{P_{G191476}OH} \neq S_{P_{L_{G191445}}^4OH}$ i.e., lines $P_{G191476}P_{L_{G191445}}^4$ and OH are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^6} \neq S_{F_{h_a}^5BF_{h_b}^6}$ i.e., lines $AF_{h_a}^5$ and $BF_{h_b}^6$ are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points M_a , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points M_b , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aM_bF_{m_b}^8} \neq S_{F_{m_a}^7M_bF_{m_b}^8}$ i.e., lines $M_aF_{m_a}^7$ and $M_bF_{m_b}^8$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.2.3 Proving $O = \neg O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = E_b$

Proving failed

4.3.2 Proving $M_a = M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.080 seconds. There are no ndg conditions.

4.3.3 Proving $O = O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = E_b$

Proving failed

4.4.2 Proving $M_a = M_a$

Proving failed

4.4.3 Proving $O = O$

Proving failed

Problem 948

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 948: Given a point E_b , a point M_a and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 949

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 949: Given a point E_b , a point M_a and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 950

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 950: Given a point E_b , a point M_a and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 951

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 951: Given a point E_b , a point M_b and a point M_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point M_b , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and M_b are not the same;
2. Using the point E_b and the point M_b , construct a line $m(E_bM_b)$ (rule W14); % DET: points E_b and M_b are not the same;
3. Using the line $m(E_bM_b)$ and the line $m(H_aH_c)$, construct a point N (rule W03); % NDG: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel % DET: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same;
4. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
5. Choose freely a point M_c on the circle $k(N, M_a)$ (rule WOncircle);
6. Choose freely a point A (rule free);
7. Using the point A and the point M_c , construct a point B (rule W01); ;
8. Using the point A and the point M_b , construct a point C (rule W01); .

Non-degenerate conditions: points E_b and N are not the same; lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel.

Determination conditions: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same; points E_b and M_b are not the same; points E_b and M_b are not the same.

Rules used: [W01,W02,W03,W06,W14,WOncircle1,free]

Lemmas used: [D20,D22,D32,GD02,GL01,GL03,GL04,L17,L19,L21,L23,L44,L45,L50,L51]

Solving time: 203.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{b} 50 56.36
point M_{b} 95 67.5
point M_{c} 50 67.5

color 220 0 0
fontsize 9

cmark_r E_{b}
cmark_lt M_{b}
cmark_lt M_{c}
color 0 0 0
fontsize 8

% DET: points E_{b} and M_{b} are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point M_{b}
line m(H_{a}H_{c}) E_{b} M_{b}

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

% DET: points E_{b} and M_{b} are not the same
% Constructing bisector m(E_{b}M_{b}) of the segment E_{b}M_{b}
med m(E_{b}M_{b}) E_{b} M_{b}

color 200 200 200
drawline m(E_{b}M_{b})
color 0 0 0

color 200 200 200
drawsegment E_{b} M_{b}
color 0 0 0

% NDG: lines m(E_{b}M_{b}) and m(H_{a}H_{c}) are not parallel% DET: lines m(E_{b}M_{b}) and m(H_{a}
H_{c}) are not the same
% Constructing a point N which belongs to line m(E_{b}M_{b}) and line m(H_{a}H_{c})
intersec N m(E_{b}M_{b}) m(H_{a}H_{c})
cmark_r N

% NDG: points E_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{b}
circle k(N,M_{a}) N E_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```

% Choosing randomly a point  $M_{\{c\}}$  on the circle with center  $N$  through point  $E_{\{b\}}$ 
oncircle  $M_{\{c\}}$   $N$   $E_{\{b\}}$ 
cmark_lt  $M_{\{c\}}$ 
color 200 200 200
drawcircle  $N$   $E_{\{b\}}$ 
color 0 0 0

% Constructing a free point  $A$ 
point  $A$  80 95

cmark_t  $A$ 

% Constructing a point  $B$  such that  $AB/AM_{\{c\}}=2$ 
towards  $B$   $A$   $M_{\{c\}}$  2
cmark_b  $B$ 
color 200 200 200
drawsegment  $A$   $B$ 
color 0 0 0

% Constructing a point  $C$  such that  $AC/AM_{\{b\}}=2$ 
towards  $C$   $A$   $M_{\{b\}}$  2
cmark_l  $C$ 
color 200 200 200
drawsegment  $A$   $C$ 
color 0 0 0

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: points  $E_{\{b\}}$  and  $N$  are not the same; lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not parallel
% Determination conditions: lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not the same; points  $E_{\{b\}}$  and  $M_{\{b\}}$  are not the same; points  $E_{\{b\}}$  and  $M_{\{b\}}$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

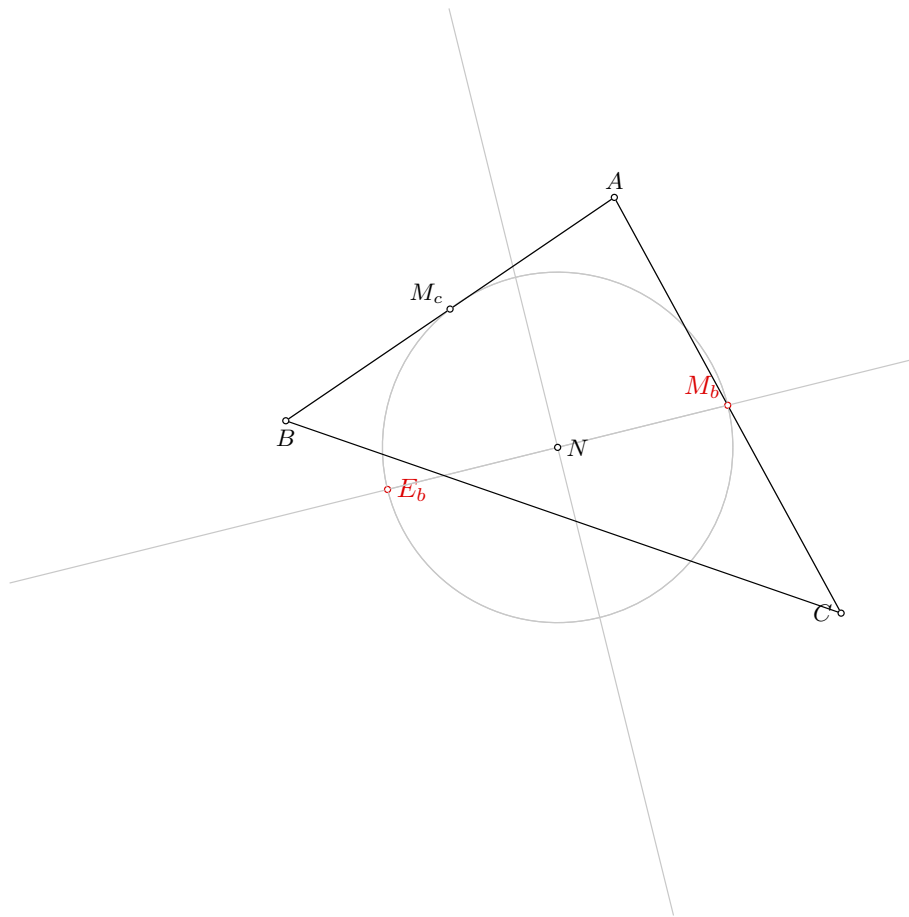


Figure 1: Illustration of the problem 0951

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.077 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $M_c = \neg M_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{m(E_b M_b) E_b M_b}^0 \neq S_{m(E_b M_b) E_b M_b}^1$ i.e., lines $M_{m(E_b M_b)}^0 T_{m(E_b M_b)}^1$ and $E_b M_b$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{F_{-h_a}^2 BF_{-h_b}^3}$ i.e., lines $AF_{-h_a}^2$ and $BF_{-h_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

4.2.3 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{m(E_b M_b) E_b M_b}^0 \neq S_{m(E_b M_b) E_b M_b}^1$ i.e., lines $M_{m(E_b M_b)}^0 T_{m(E_b M_b)}^1$ and $E_b M_b$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{F_{-h_a}^2 BF_{-h_b}^3}$ i.e., lines $AF_{-h_a}^2$ and $BF_{-h_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 11284 terms.

Time Complexity: Time spent by the prover is 47.170 seconds. There are no ndg conditions.

4.3.2 Proving $M_b = \neg M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 19 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.3.3 Proving $M_c = \neg M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 28 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $M_b = \neg M_b$

Proving failed

4.4.3 Proving $M_c = \neg M_c$

Proving failed

Problem 952

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 952: Given a point M_b , a point N and a point E_b , construct the triangle ABC .

2 Status

Problem is redundant.

3 Solution

3.1 Construction in natural language form

1. Using the point M_b and the point N , construct a line $m(H_aH_c)$ (rule W02); % DET: points M_b and N are not the same;
2. Using the point M_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_b and N are not the same;
3. Using the circle $k(N, M_a)$, the line $m(H_aH_c)$, the point N and the point M_b , construct a point E_b (rule W05a); % NDG: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_b and E_b must be different;
4. Choose freely a point A (rule free);
5. Using the point A and the point M_b , construct a point C (rule W01); ;
6. Using the point A and the point M_b , construct a line b (rule W02); % DET: points A and M_b are not the same;
7. Using the circle $k(N, M_a)$, the line b , the point N and the point M_b , construct a point H_b (rule W05); % NDG: line b and circle $k(N, M_a)$ intersect % DET: points M_b and H_b must be different;
8. Using the point H_b and the point E_b , construct a line h_b (rule W02); % DET: points H_b and E_b are not the same;
9. Using the point A and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points A and M_b are not the same;

10. Using the circle $k(N, M_a)$ and the circle $k(M_b, C)$, construct a point H_a and a point H_c (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_b, C)$ intersect % DET: circles $k(N, M_a)$ and $k(M_b, C)$ are not the same;
11. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
12. Using the line h_b and the line h_a , construct a point H (rule W03); % NDG: lines h_b and h_a are not parallel % DET: lines h_b and h_a are not the same;
13. Using the point N and the point H , construct a point G (rule W01); ;
14. Using the point M_b and the point G , construct a point B (rule W01); .

Non-degenerate conditions: lines h_b and h_a are not parallel; circles $k(N, M_a)$ and $k(M_b, C)$ intersect; points A and M_b are not the same; line b and circle $k(N, M_a)$ intersect; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; points M_b and N are not the same.

Determination conditions: lines h_b and h_a are not the same; points A and H_a are not the same; circles $k(N, M_a)$ and $k(M_b, C)$ are not the same; points H_b and E_b are not the same; points M_b and H_b must be different; points A and M_b are not the same; points M_b and E_b must be different; points M_b and N are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07,free]

Lemmas used: [D22,D29,D3,D32,D5,D6,D8,D9,GD01,GD02,GL01,GL03,GL04,GL09,L16,L17,L19,L20,L21,L23]

Solving time: 5.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{b} 95 67.5
point N 72.5 61.93
point E_{b} 50 56.36
```

```
color 220 0 0
fontsize 9
```

```
cmark_lt M_{b}
cmark_r N
cmark_r E_{b}
color 0 0 0
fontsize 8
```

```
% DET: points M_{b} and N are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point M_{b} and point N
line m(H_{a}H_{c}) M_{b} N
```

```
color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0
```

```

% NDG: points  $M_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $M_{\{b\}}$ 
circle k(N, M_{a}) N M_{b}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{b\}}$  and  $E_{\{b\}}$  must be
different
% Constructing a point  $E_{\{b\}}$  which is an image of the point  $M_{\{b\}}$  in the symmetry to point/line  $N$ 
sim E_{b} N M_{b}
cmark_r E_{b}

% Constructing a free point  $A$ 
point A 80 95

cmark_t A

% Constructing a point  $C$  such that  $AC/AM_{\{b\}}=2$ 
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points  $A$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $A$  and point  $M_{\{b\}}$ 
line b A M_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: line  $b$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{b\}}$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\_G54608\}}$  which is a foot of the point  $N$  on the line  $b$ 
foot P_{\_G54608} N b
cmark_r P_{\_G54608}
color 200 200 200
drawline N P_{\_G54608}
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $M_{\{b\}}$  in the symmetry to point/line  $P_{\{\_G54608\}}$ 
sim H_{b} P_{\_G54608} M_{b}
cmark_l H_{b}

```

```

% DET: points  $H_{\{b\}}$  and  $E_{\{b\}}$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $H_{\{b\}}$  and point  $E_{\{b\}}$ 
line  $h_{\{b\}}$   $H_{\{b\}}$   $E_{\{b\}}$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: points  $A$  and  $M_{\{b\}}$  are not the same
% Constructing a circle  $k(M_{\{b\}}, C)$  whose center is at point  $M_{\{b\}}$  and which passes through point  $A$ 
circle  $k(M_{\{b\}}, C)$   $M_{\{b\}}$   $A$ 

color 200 200 200
drawcircle  $k(M_{\{b\}}, C)$ 
color 0 0 0

% NDG: circles  $k(N, M_{\{a\}})$  and  $k(M_{\{b\}}, C)$  intersect% DET: circles  $k(N, M_{\{a\}})$  and  $k(M_{\{b\}}, C)$  are not
the same
% Constructing points  $H_{\{a\}}$  and  $H_{\{c\}}$  which are in intersection of  $k(N, M_{\{a\}})$  and  $k(M_{\{b\}}, C)$ 
intersec2  $H_{\{a\}}$   $H_{\{c\}}$   $k(N, M_{\{a\}})$   $k(M_{\{b\}}, C)$ 
cmark_r  $H_{\{a\}}$ 
cmark_rt  $H_{\{c\}}$ 

% DET: points  $A$  and  $H_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $A$  and point  $H_{\{a\}}$ 
line  $h_{\{a\}}$   $A$   $H_{\{a\}}$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: lines  $h_{\{b\}}$  and  $h_{\{a\}}$  are not parallel% DET: lines  $h_{\{b\}}$  and  $h_{\{a\}}$  are not the same
% Constructing a point  $H$  which belongs to line  $h_{\{b\}}$  and line  $h_{\{a\}}$ 
intersec  $H$   $h_{\{b\}}$   $h_{\{a\}}$ 
cmark_rt  $H$ 

% Constructing a line  $L_{\{\backslash\_G55152\}}$  which passes through point  $N$  and point  $H$ 
line  $L_{\{\backslash\_G55152\}}$   $N$   $H$ 

color 200 200 200
drawline  $L_{\{\backslash\_G55152\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\backslash\_G55253\}}$  with coordinates  $(0,0)$ 
point  $P_{\{\backslash\_G55253\}}$  0 0

```

```

cmark_r P_{\_G55253}

% Constructing a point P_{\_G55177} such that NP_{\_G55177}/NP_{\_G55253}=-1
towards P_{\_G55177} N P_{\_G55253} -1
cmark_r P_{\_G55177}
color 200 200 200
drawsegment P_{\_G55253} P_{\_G55177}
color 0 0 0

% Constructing a point P_{\_G55222} such that NP_{\_G55222}/NP_{\_G55253}=3
towards P_{\_G55222} N P_{\_G55253} 3
cmark_r P_{\_G55222}
color 200 200 200
drawsegment N P_{\_G55222}
color 0 0 0

% Constructing a line L_{\_G55183} which passes through point H and point P_{\_G55222}
line L_{\_G55183} H P_{\_G55222}

color 200 200 200
drawline L_{\_G55183}
color 0 0 0

% Constructing a line L_{\_G55146} which contains the point P_{\_G55177} and is parallel to the
line L_{\_G55183}
parallel L_{\_G55146} P_{\_G55177} L_{\_G55183}

color 200 200 200
drawline L_{\_G55146}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G55146} and line L_{\_G55152}
intersec G L_{\_G55146} L_{\_G55152}
cmark_t G

% Constructing a point B such that M_{b}B/M_{b}G=3
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{b} and h_{a} are not parallel; circles k(N,M_{a}) and k(M_{b}
,C) intersect; points A and M_{b} are not the same; line b and circle k(N,M_{a}) intersect;
line m(H_{a}H_{c}) and circle k(N,M_{a}) intersect; points M_{b} and N are not the same

```

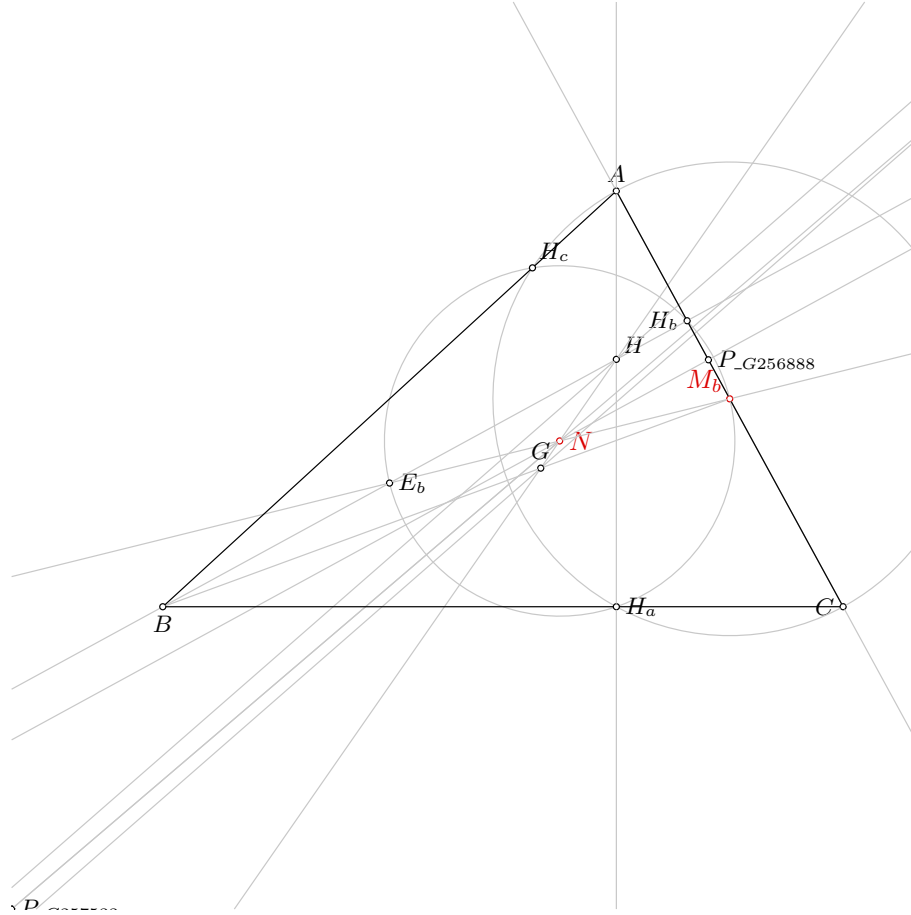


Figure 1: Illustration of the problem 0952

% Determination conditions: lines $h_{\{b\}}$ and $h_{\{a\}}$ are not the same; points A and $H_{\{a\}}$ are not the same; circles $k(N, M_{\{a\}})$ and $k(M_{\{b\}}, C)$ are not the same; points $H_{\{b\}}$ and $E_{\{b\}}$ are not the same; points $M_{\{b\}}$ and $H_{\{b\}}$ must be different; points A and $M_{\{b\}}$ are not the same; points $M_{\{b\}}$ and $E_{\{b\}}$ must be different; points $M_{\{b\}}$ and N are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_b = M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.036 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $E_b=_E$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $M_b=_M$

Proving failed

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $E_b=_E$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_b=_M$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.060 seconds. There are no ndg conditions.

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $E_b=_E$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_b=_M$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $E_b=_E$

Proving failed

Problem 953

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 953: Given a point E_b , a point M_b and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point M_b , construct a line $m(H_aH_c)$ (rule W02); % DET: points E_b and M_b are not the same;
2. Using the point M_b and the point O , construct a line m_b (rule W02); % DET: points M_b and O are not the same;
3. Using the point M_b and the line m_b , construct a line b (rule W10a); ;
4. Using the point E_b and the point M_b , construct a line $m(E_bM_b)$ (rule W14); % DET: points E_b and M_b are not the same;
5. Using the line $m(E_bM_b)$ and the line $m(H_aH_c)$, construct a point N (rule W03); % NDG: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel % DET: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same;
6. Using the point N and the point O , construct a point G (rule W01); ;
7. Using the point M_b and the point G , construct a point B (rule W01); ;
8. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
9. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; lines $m(E_bM_b)$ and $m(H_aH_c)$ are not parallel.

Determination conditions: lines $m(E_bM_b)$ and $m(H_aH_c)$ are not the same; points E_b and M_b are not the same; points M_b and O are not the same; points E_b and M_b are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W14]

Lemmas used: [D1,D12,D22,D26,D32,GD01,GD02,GL01,GL03,GL04,GL09,L11,L12,L15,L17,L19,L21,L23,L44,

Solving time: 6.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
point M_{b} 95 67.5
point O 65 51.14
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{b}
cmark_lt M_{b}
cmark_t O
color 0 0 0
fontsize 8
```

```
% DET: points E_{b} and M_{b} are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point M_{b}
line m(H_{a}H_{c}) E_{b} M_{b}
```

```
color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0
```

```
% DET: points M_{b} and O are not the same
% Constructing a line m_{b} which passes through point M_{b} and point O
line m_{b} M_{b} O
```

```
color 200 200 200
drawline m_{b}
color 0 0 0
```

```
% Constructing a line b which is perpendicular to line m_{b} and which passes through point M_{b}
perp b M_{b} m_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% DET: points E_{b} and M_{b} are not the same
```

```

% Constructing bisector  $m(E_{\{b\}}M_{\{b\}})$  of the segment  $E_{\{b\}}M_{\{b\}}$ 
med m( $E_{\{b\}}M_{\{b\}}$ )  $E_{\{b\}}$   $M_{\{b\}}$ 

color 200 200 200
drawline m( $E_{\{b\}}M_{\{b\}}$ )
color 0 0 0

color 200 200 200
drawsegment  $E_{\{b\}}$   $M_{\{b\}}$ 
color 0 0 0

% NDG: lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not parallel% DET: lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}$ 
 $H_{\{c\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{b\}}M_{\{b\}})$  and line  $m(H_{\{a\}}H_{\{c\}})$ 
intersec N m( $E_{\{b\}}M_{\{b\}}$ ) m( $H_{\{a\}}H_{\{c\}}$ )
cmark_r N

% Constructing a line  $L_{\{\_G86741\}}$  which passes through point  $N$  and point  $O$ 
line  $L_{\{\_G86741\}}$  N O

color 200 200 200
drawline  $L_{\{\_G86741\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\_G86842\}}$  with coordinates  $(0,0)$ 
point  $P_{\{\_G86842\}}$  0 0
cmark_r  $P_{\{\_G86842\}}$ 

% Constructing a point  $P_{\{\_G86766\}}$  such that  $NP_{\{\_G86766\}}/NP_{\{\_G86842\}}=1$ 
towards  $P_{\{\_G86766\}}$  N  $P_{\{\_G86842\}}$  1
cmark_r  $P_{\{\_G86766\}}$ 
color 200 200 200
drawsegment N  $P_{\{\_G86766\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\_G86811\}}$  such that  $NP_{\{\_G86811\}}/NP_{\{\_G86842\}}=3$ 
towards  $P_{\{\_G86811\}}$  N  $P_{\{\_G86842\}}$  3
cmark_r  $P_{\{\_G86811\}}$ 
color 200 200 200
drawsegment N  $P_{\{\_G86811\}}$ 
color 0 0 0

% Constructing a line  $L_{\{\_G86772\}}$  which passes through point  $O$  and point  $P_{\{\_G86811\}}$ 
line  $L_{\{\_G86772\}}$  O  $P_{\{\_G86811\}}$ 

color 200 200 200
drawline  $L_{\{\_G86772\}}$ 
color 0 0 0

% Constructing a line  $L_{\{\_G86735\}}$  which contains the point  $P_{\{\_G86766\}}$  and is parallel to the
line  $L_{\{\_G86772\}}$ 

```

```

parallel L_{\_G86735} P_{\_G86766} L_{\_G86772}

color 200 200 200
drawline L_{\_G86735}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G86735} and line L_{\_G86741}
intersec G L_{\_G86735} L_{\_G86741}
cmark_t G

% Constructing a point B such that  $M_{\{b\}B}/M_{\{b\}G}=3$ 
towards B M_{\{b\}} G 3
cmark_b B
color 200 200 200
drawsegment M_{\{b\}} B
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line b and circle  $k(O,C)$  intersect
% Constructing points C and A which are in intersection of  $k(O,C)$  and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line b and circle  $k(O,C)$  intersect; points B and O are not the same;
% lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not parallel
% Determination conditions: lines  $m(E_{\{b\}}M_{\{b\}})$  and  $m(H_{\{a\}}H_{\{c\}})$  are not the same; points  $E_{\{b\}}$ 
% and  $M_{\{b\}}$  are not the same; points  $M_{\{b\}}$  and O are not the same; points  $E_{\{b\}}$  and  $M_{\{b\}}$  are not
% the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

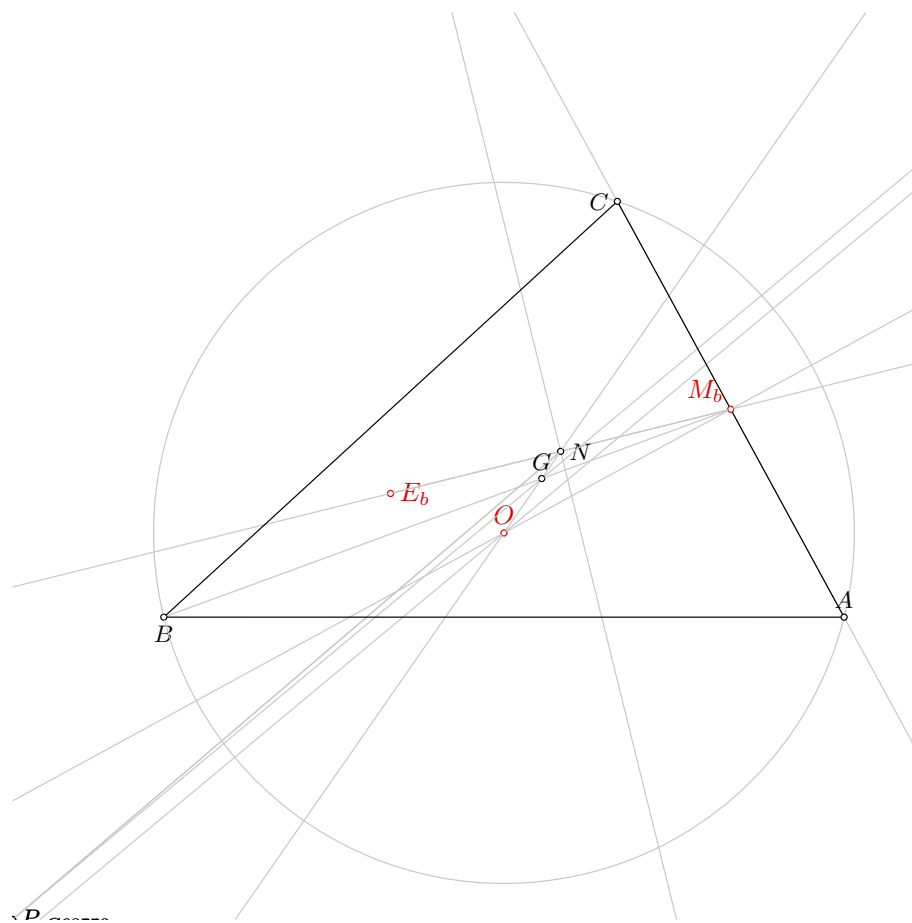


Figure 1: Illustration of the problem 0953

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $M_b = \neg M_b$

Proving failed

4.1.3 Proving $O = \neg O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $M_b = \neg M_b$

Proving failed

4.2.3 Proving $O = \neg O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = \neg E_b$

Proving failed

4.3.2 Proving $M_b = \neg M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 923 terms.

Time Complexity: Time spent by the prover is 2.050 seconds. There are no ndg conditions.

4.3.3 Proving $O = \neg O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = \neg E_b$

Proving failed

4.4.2 Proving $M_b = \neg M_b$

Proving failed

4.4.3 Proving $O=_O$

Proving failed

Problem 954

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 954: Given a point E_b , a point M_b and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 955

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 955: Given a point E_b , a point M_b and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 956

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 956: Given a point E_b , a point M_b and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 957

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 957: Given a point M_c , a point N and a point E_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point M_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_c and N are not the same;
2. Choose freely a point E_b on the circle $k(N, M_a)$ (rule WOncircle);
3. Using the point E_b and the point N , construct a line $m(H_a H_c)$ (rule W02); % DET: points E_b and N are not the same;
4. Using the circle $k(N, M_a)$, the line $m(H_a H_c)$, the point N and the point E_b , construct a point M_b (rule W05a); % NDG: line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect % DET: points E_b and M_b must be different;
5. Choose freely a point A (rule free);
6. Using the point A and the point M_c , construct a point B (rule W01); ;
7. Using the point A and the point M_b , construct a point C (rule W01); .

Non-degenerate conditions: line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; points M_c and N are not the same.

Determination conditions: points E_b and M_b must be different; points E_b and N are not the same.

Rules used: [W01,W02,W05a,W06,WOncircle1,free]

Lemmas used: [D20,D22,D32,GD02,GL01,GL03,GL04,L17,L18,L19,L21,L23,L44,L45,L50,L51]

Solving time: 150.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point N 72.5 61.93
point E_{b} 50 56.36

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_r N
cmark_r E_{b}
color 0 0 0
fontsize 8

% NDG: points M_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{c}
circle k(N,M_{a}) N M_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point E_{b} on the circle with center N through point M_{c}
oncircle E_{b} N M_{c}
cmark_r E_{b}
color 200 200 200
drawcircle N M_{c}
color 0 0 0

% DET: points E_{b} and N are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point E_{b} and point N
line m(H_{a}H_{c}) E_{b} N

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

% NDG: line m(H_{a}H_{c}) and circle k(N,M_{a}) intersect% DET: points E_{b} and M_{b} must be
different
% Constructing a point M_{b} which is an image of the point E_{b} in the symmetry to point/line N
sim M_{b} N E_{b}
cmark_lt M_{b}

% Constructing a free point A
point A 80 95
```

```
cmark_t A
```

```
% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0
```

```
% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line m(H_{a}H_{c}) and circle k(N,M_{a}) intersect; points M_{c} and N
are not the same
% Determination conditions: points E_{b} and M_{b} must be different; points E_{b} and N are not
the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_c = M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.029 seconds.

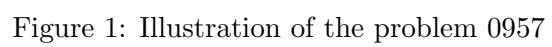
NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $N = N$

Proving failed

4.1.3 Proving $E_b = E_b$

Proving failed



4.2 GCLC - Area method

4.2.1 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{\neg M_a BC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_b AC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2 BF_{\neg h_b}^3}$ i.e., lines $AF_{\neg h_a}^2$ and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

$S_{\neg M_a \neg M_b F_{\neg m_b}^1} \neq S_{F_{\neg m_a}^0 \neg M_b F_{\neg m_b}^1}$ i.e., lines $\neg M_a F_{\neg m_a}^0$ and $\neg M_b F_{\neg m_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $E_b = \neg E_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_c = \neg M_c$

Proving failed

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $E_b = \neg E_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_c = \neg M_c$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $E_b = \neg E_b$

Proving failed

Problem 958

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 958: Given a point E_b , a point M_c and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_b and the point M_c , construct a line $m(BH_a)$ (rule W02); % DET: points E_b and M_c are not the same;
2. Using the point M_c and the point O , construct a line m_c (rule W02); % DET: points M_c and O are not the same;
3. Using the point M_c and the line m_c , construct a line c (rule W10a); ;
4. Using the point E_b and the line m_c , construct a line $m(BH_c)$ (rule W16); ;
5. Using the point O and the line $m(BH_a)$, construct a line m_a (rule W16); ;
6. Using the line m_a and the line $m(BH_c)$, construct a point M_a (rule W03); % NDG: lines m_a and $m(BH_c)$ are not parallel % DET: lines m_a and $m(BH_c)$ are not the same;
7. Using the point M_a and the line m_a , construct a line a (rule W10a); ;
8. Using the line a and the line c , construct a point B (rule W03); % NDG: lines a and c are not parallel % DET: lines a and c are not the same;
9. Using the point B and the point E_b , construct a point H (rule W01); ;
10. Using the point M_c and the point B , construct a point A (rule W01); ;
11. Using the point O and the point H , construct a point G (rule W01); ;
12. Using the point M_c and the point G , construct a point C (rule W01); .

Non-degenerate conditions: lines a and c are not parallel; lines m_a and $m(BH_c)$ are not parallel.
Determination conditions: lines a and c are not the same; lines m_a and $m(BH_c)$ are not the same; points M_c and O are not the same; points E_b and M_c are not the same.
Rules used: [W01,W02,W03,W10a,W16]
Lemmas used: [D1,D11,D13,D20,D21,D29,GD01,GD02,GL01,GL03,GL04,GL09,L1,L39,L40,L42,L50,L51,L57,L58]
Solving time: 3.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{b} 50 56.36
point M_{c} 50 67.5
point O 65 51.14

color 220 0 0
fontsize 9

cmark_r E_{b}
cmark_lt M_{c}
cmark_t O
color 0 0 0
fontsize 8

% DET: points E_{b} and M_{c} are not the same
% Constructing a line m(BH_{a}) which passes through point E_{b} and point M_{c}
line m(BH_{a}) E_{b} M_{c}

color 200 200 200
drawline m(BH_{a})
color 0 0 0

% DET: points M_{c} and O are not the same
% Constructing a line m_{c} which passes through point M_{c} and point O
line m_{c} M_{c} O

color 200 200 200
drawline m_{c}
color 0 0 0

% Constructing a line c which is perpendicular to line m_{c} and which passes through point M_{c}
perp c M_{c} m_{c}

color 200 200 200
drawline c
color 0 0 0
```

```
% Constructing a line  $m(BH_{\{c\}})$  which contains the point  $E_{\{b\}}$  and is parallel to the line  $m_{\{c\}}$ 
parallel m(BH_{c}) E_{b} m_{c}
```

```
color 200 200 200
drawline m(BH_{c})
color 0 0 0
```

```
% Constructing a line  $m_{\{a\}}$  which contains the point  $O$  and is parallel to the line  $m(BH_{\{a\}})$ 
parallel m_{a} O m(BH_{a})
```

```
color 200 200 200
drawline m_{a}
color 0 0 0
```

```
% NDG: lines  $m_{\{a\}}$  and  $m(BH_{\{c\}})$  are not parallel% DET: lines  $m_{\{a\}}$  and  $m(BH_{\{c\}})$  are not the same
% Constructing a point  $M_{\{a\}}$  which belongs to line  $m_{\{a\}}$  and line  $m(BH_{\{c\}})$ 
intersec M_{a} m_{a} m(BH_{c})
cmark_r M_{a}
```

```
% Constructing a line  $a$  which is perpendicular to line  $m_{\{a\}}$  and which passes through point  $M_{\{a\}}$ 
perp a M_{a} m_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% NDG: lines  $a$  and  $c$  are not parallel% DET: lines  $a$  and  $c$  are not the same
% Constructing a point  $B$  which belongs to line  $a$  and line  $c$ 
intersec B a c
cmark_b B
```

```
% Constructing a point  $H$  such that  $BH/BE_{\{b\}}=2$ 
towards H B E_{b} 2
cmark_rt H
color 200 200 200
drawsegment B H
color 0 0 0
```

```
% Constructing a point  $A$  such that  $M_{\{c\}}A/M_{\{c\}}B=-1$ 
towards A M_{c} B -1
cmark_t A
color 200 200 200
```

```
drawsegment B A
color 0 0 0
```

```
% Constructing a line  $L_{\{ \_G165768 \}}$  which passes through point O and point H
line L_{\_G165768} O H
```

```
color 200 200 200
drawline L_{\_G165768}
color 0 0 0
```

```
% Constructing a point  $P_{\{ \_G165869 \}}$  with coordinates (0,0)
point P_{\_G165869} 0 0
cmark_r P_{\_G165869}
```

```
% Constructing a point  $P_{\{ \_G165793 \}}$  such that  $OP_{\{ \_G165793 \}}/OP_{\{ \_G165869 \}}=1$ 
towards  $P_{\{ \_G165793 \}}$  O  $P_{\{ \_G165869 \}}$  1
cmark_r P_{\_G165793}
color 200 200 200
drawsegment O P_{\_G165793}
color 0 0 0
```

```
% Constructing a point  $P_{\{ \_G165838 \}}$  such that  $OP_{\{ \_G165838 \}}/OP_{\{ \_G165869 \}}=3$ 
towards  $P_{\{ \_G165838 \}}$  O  $P_{\{ \_G165869 \}}$  3
cmark_r P_{\_G165838}
color 200 200 200
drawsegment O P_{\_G165838}
color 0 0 0
```

```
% Constructing a line  $L_{\{ \_G165799 \}}$  which passes through point H and point  $P_{\{ \_G165838 \}}$ 
line L_{\_G165799} H P_{\_G165838}
```

```
color 200 200 200
drawline L_{\_G165799}
color 0 0 0
```

```
% Constructing a line  $L_{\{ \_G165762 \}}$  which contains the point  $P_{\{ \_G165793 \}}$  and is parallel to the
line  $L_{\{ \_G165799 \}}$ 
parallel L_{\_G165762} P_{\_G165793} L_{\_G165799}
```

```
color 200 200 200
drawline L_{\_G165762}
color 0 0 0
```

```
% Constructing a point G which belongs to line  $L_{\{ \_G165762 \}}$  and line  $L_{\{ \_G165768 \}}$ 
intersec G L_{\_G165762} L_{\_G165768}
cmark_t G
```

```
% Constructing a point C such that  $M_{\{ c \}}C/M_{\{ c \}}G=3$ 
towards C M_{c} G 3
```

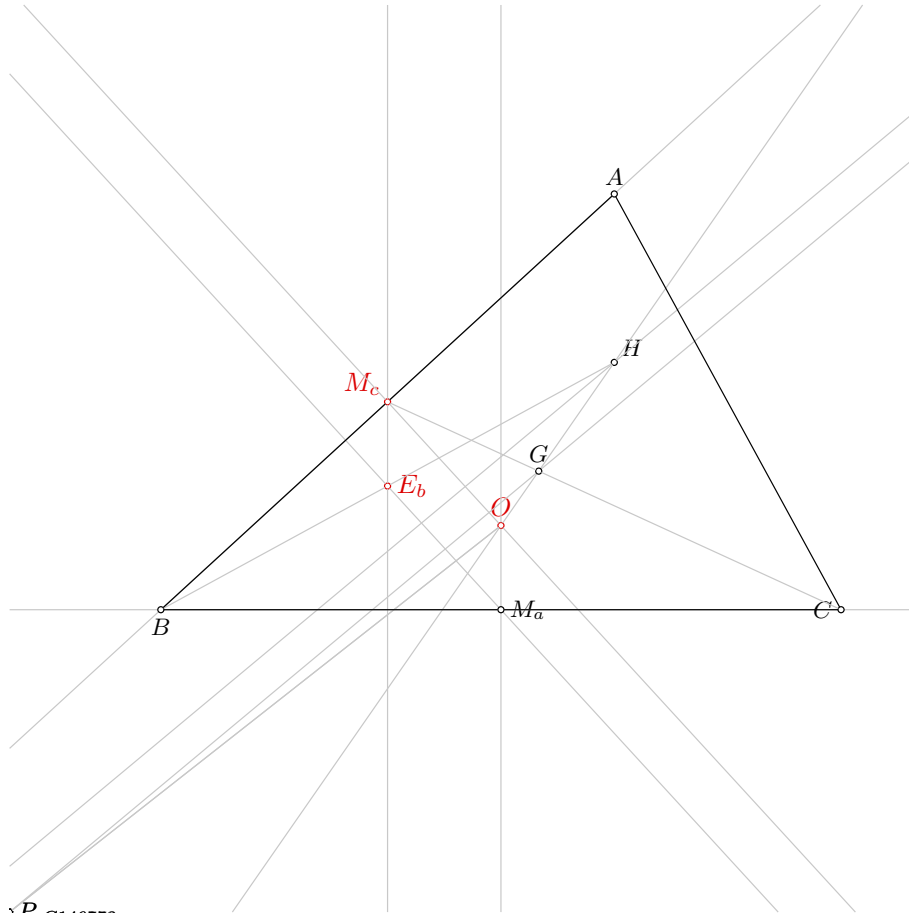


Figure 1: Illustration of the problem 0958

```

cmark_1 C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

% Non-degenerate conditions: lines a and c are not parallel; lines $m_{\{a\}}$ and $m(BH_{\{c\}})$ are not parallel

% Determination conditions: lines a and c are not the same; lines $m_{\{a\}}$ and $m(BH_{\{c\}})$ are not the same; points $M_{\{c\}}$ and O are not the same; points $E_{\{b\}}$ and $M_{\{c\}}$ are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = \neg E_b$

Proving failed

4.1.2 Proving $M_c = \neg M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 9 terms.

Time Complexity: Time spent by the prover is 0.194 seconds.

NDG conditions Points M_c and M_a are not identical

Line through points M_c and M_a is not perpendicular to line through points M_a and E_b

4.1.3 Proving $O = \neg O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b = \neg E_b$

Proving failed

4.2.2 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{OE_bP_{m(BH_c)}^1} \neq S_{P_{m_a}^2E_bP_{m(BH_c)}^1}$ i.e., lines $OP_{m_a}^2$ and $E_bP_{m(BH_c)}^1$ are not parallel (construction based assumption)
 $S_{M_aOP_{m_a}^2} \neq 0$ i.e., points M_a , O and $P_{m_a}^2$ are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aM_cT_c^0} \neq S_{F_a^3M_cT_c^0}$ i.e., lines $M_aF_a^3$ and $M_cT_c^0$ are not parallel (construction based assumption)
 $S_{P_{G151613}OH} \neq S_{P_{L_{G151582}}^4OH}$ i.e., lines $P_{G151613}P_{L_{G151582}}^4$ and OH are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{h_b}^6} \neq S_{F_{h_a}^5BF_{h_b}^6}$ i.e., lines $AF_{h_a}^5$ and $BF_{h_b}^6$ are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points M_a , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points M_b , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_aM_bF_{m_b}^8} \neq S_{F_{m_a}^7M_bF_{m_b}^8}$ i.e., lines $M_aF_{m_a}^7$ and $M_bF_{m_b}^8$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.2.3 Proving $O = \neg O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b = E_b$

Proving failed

4.3.2 Proving $M_c = M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.100 seconds. There are no ndg conditions.

4.3.3 Proving $O = O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b = E_b$

Proving failed

4.4.2 Proving $M_c = M_c$

Proving failed

4.4.3 Proving $O = O$

Proving failed

Problem 959

*Generated automatically by ArgoTriCS
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1 Problem

Problem 959: Given a point E_b , a point M_c and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 960

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 960: Given a point E_b , a point M_c and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 961

*Generated automatically by ArgoTriCS
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1 Problem

Problem 961: Given a point E_b , a point M_c and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 962

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 962: Given a point E_b , a point N and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point N and the point O , construct a point G (rule W01); ;
2. Using the point N and the point O , construct a point H (rule W01); ;
3. Using the point E_b and the point H , construct a point B (rule W01); ;
4. Using the point G and the point B , construct a point M_b (rule W01); ;
5. Using the point E_b and the point H , construct a line h_b (rule W02); % DET: points E_b and H are not the same;
6. Using the point E_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_b and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
8. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
9. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; line h_b and circle $k(N, M_a)$ intersect; points E_b and N are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points E_b and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D31,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L15,L20,L23,L56]

Solving time: 7.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{b} 50 56.36
```

```
point N 72.5 61.93
```

```
point O 65 51.14
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{b}
```

```
cmark_r N
```

```
cmark_t O
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a line L_{\_G225344} which passes through point N and point O
line L_{\_G225344} N O
```

```
color 200 200 200
```

```
drawline L_{\_G225344}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G225445} with coordinates (0,0)
```

```
point P_{\_G225445} 0 0
```

```
cmark_r P_{\_G225445}
```

```
% Constructing a point P_{\_G225369} such that NP_{\_G225369}/NP_{\_G225445}=1
```

```
towards P_{\_G225369} N P_{\_G225445} 1
```

```
cmark_r P_{\_G225369}
```

```
color 200 200 200
```

```
drawsegment N P_{\_G225369}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G225414} such that NP_{\_G225414}/NP_{\_G225445}=3
```

```
towards P_{\_G225414} N P_{\_G225445} 3
```

```
cmark_r P_{\_G225414}
```

```
color 200 200 200
```

```
drawsegment N P_{\_G225414}
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G225375} which passes through point O and point P_{\_G225414}
```

```
line L_{\_G225375} O P_{\_G225414}
```

```

color 200 200 200
drawline L_{\_G225375}
color 0 0 0

% Constructing a line L_{\_G225338} which contains the point P_{\_G225369} and is parallel to the
line L_{\_G225375}
parallel L_{\_G225338} P_{\_G225369} L_{\_G225375}

color 200 200 200
drawline L_{\_G225338}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G225338} and line L_{\_G225344}
intersec G L_{\_G225338} L_{\_G225344}
cmark_t G

% Constructing a point H such that  $NH/NO=-1$ 
towards H N 0 -1
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0

% Constructing a point B such that  $E_{\{b\}}B/E_{\{b\}}H=-1$ 
towards B E_{\{b\}} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

% Constructing a point M_{\{b\}} such that  $GM_{\{b\}}/GB=-0.5$ 
towards M_{\{b\}} G B -0.5
cmark_lt M_{\{b\}}
color 200 200 200
drawsegment B M_{\{b\}}
color 0 0 0

% DET: points E_{\{b\}} and H are not the same
% Constructing a line h_{\{b\}} which passes through point E_{\{b\}} and point H
line h_{\{b\}} E_{\{b\}} H

color 200 200 200
drawline h_{\{b\}}
color 0 0 0

```

```

% NDG: points  $E_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{b\}}$ 
circle k(N, M_{a}) N E_{b}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{b\}}$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\_G226429\}}$  which is a foot of the point  $N$  on the line  $h_{\{b\}}$ 
foot P_{\_G226429} N h_{b}
cmark_r P_{\_G226429}
color 200 200 200
drawline N P_{\_G226429}
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $E_{\{b\}}$  in the symmetry to point/line  $P_{\{\_G226429\}}$ 
sim H_{b} P_{\_G226429} E_{b}
cmark_l H_{b}

% DET: points  $H_{\{b\}}$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $M_{\{b\}}$ 
line b H_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0

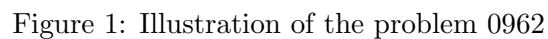
% NDG: points  $B$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $B$ 
circle k(O, C) O B

color 200 200 200
drawcircle k(O, C)
color 0 0 0

% NDG: line  $b$  and circle  $k(O, C)$  intersect
% Constructing points  $C$  and  $A$  which are in intersection of  $k(O, C)$  and  $b$ 
intersec2 C A k(O, C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

```



```
% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;
    line h_{b} and circle k(N,M_{a}) intersect; points E_{b} and N are not the same
% Determination conditions: points H_{b} and M_{b} are not the same; points E_{b} and H_{b} must be
    different; points E_{b} and H are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_b = -E_b$

Proving failed

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $O=_O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_b=_E E_b$

Proving failed

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $O=_O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_b=_E E_b$

Proving failed

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $O=_O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_b=_E E_b$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $O=_O$

Proving failed

Problem 963

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 963: Given a point E_b , a point N and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 964

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 964: Given a point E_b , a point N and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 965

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 965: Given a point E_b , a point N and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 966

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 966: Given a point E_b , a point O and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 967

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 967: Given a point E_b , a point O and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 968

*Generated automatically by ArgoTriCS
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1 Problem

Problem 968: Given a point E_b , a point O and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 969

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 969: Given a point E_b , a point T_a and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 970

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 970: Given a point E_b , a point T_a and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 971

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 971: Given a point E_b , a point T_b and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 972

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 972: Given a point E_c , a point G and a point H , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H , construct a point C (rule W01); ;
2. Using the point G and the point H , construct a point N (rule W01); ;
3. Using the point G and the point H , construct a point O (rule W01); ;
4. Using the point G and the point C , construct a point M_c (rule W01); ;
5. Using the point E_c and the point H , construct a line h_c (rule W02); % DET: points E_c and H are not the same;
6. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
8. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
9. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; line h_c and circle $k(N, M_a)$ intersect; points E_c and N are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points E_c and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L16,L21,L24,L3,L57,L58]

Solving time: 7.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{c} 95 56.36
point G 70 58.33
point H 80 72.73

color 220 0 0
fontsize 9

cmark_r E_{c}
cmark_t G
cmark_rt H
color 0 0 0
fontsize 8

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

% Constructing a point N such that GN/GH=0.25
towards N G H 0.25
cmark_r N
color 200 200 200
drawsegment G H
color 0 0 0

% Constructing a point O such that GO/GH=-0.5
towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% Constructing a point M_{c} such that GM_{c}/GC=-0.5
```

```

towards M_{c} G C -0.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% DET: points E_{c} and H are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H
line h_{c} E_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points E_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{c}
circle k(N,M_{a}) N E_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{c} and circle k(N,M_{a}) intersect% DET: points E_{c} and H_{c} must be different
% Constructing a point P_{\_G76401} which is a foot of the point N on the line h_{c}
foot P_{\_G76401} N h_{c}
cmark_r P_{\_G76401}
color 200 200 200
drawline N P_{\_G76401}
color 0 0 0

% Constructing a point H_{c} which is an image of the point E_{c} in the symmetry to point/line P
\_G76401}
sim H_{c} P_{\_G76401} E_{c}
cmark_rt H_{c}

% DET: points H_{c} and M_{c} are not the same
% Constructing a line c which passes through point H_{c} and point M_{c}
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200

```

```
drawcircle k(O,C)
color 0 0 0
```

```
% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
% line h_{c} and circle k(N,M_{a}) intersect; points E_{c} and N are not the same
% Determination conditions: points H_{c} and M_{c} are not the same; points E_{c} and H_{c} must be
% different; points E_{c} and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = _E_c$

Proving failed

4.1.2 Proving $G = _G$

Proving failed

4.1.3 Proving $H = _H$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = _E_c$

Proving failed

4.2.2 Proving $G = _G$

Proving failed

4.2.3 Proving $H = _H$

Proving failed

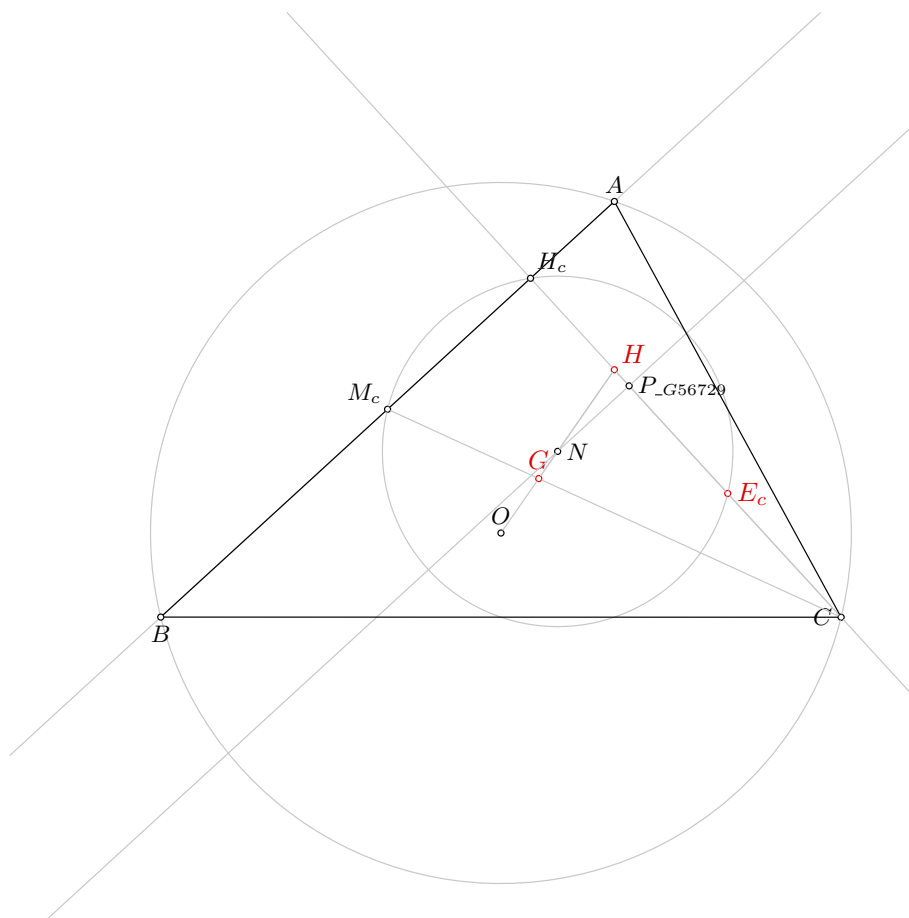


Figure 1: Illustration of the problem 0972

4.3 GCLC - Wu method

4.3.1 Proving $E_c =_c E_c$

Proving failed

4.3.2 Proving $G =_G G$

Proving failed

4.3.3 Proving $H =_H H$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c =_c E_c$

Proving failed

4.4.2 Proving $G =_G G$

Proving failed

4.4.3 Proving $H =_H H$

Proving failed

Problem 973

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 973: Given a point E_c , a point G and a point H_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 974

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 974: Given a point E_c , a point G and a point H_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 975

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 975: Given a point E_c , a point G and a point H_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;
2. Using the point H_c and the line h_c , construct a line c (rule W10a); ;
3. Using the point G , the line h_c and the point E_c , construct a line $h_{G,-1/2}(h_c)$ (rule W15); ;
4. Using the line $h_{G,-1/2}(h_c)$ and the line c , construct a point M_c (rule W03); % NDG: lines $h_{G,-1/2}(h_c)$ and c are not parallel % DET: lines $h_{G,-1/2}(h_c)$ and c are not the same;
5. Using the point M_c and the point G , construct a point C (rule W01); ;
6. Using the point E_c and the point C , construct a point H (rule W01); ;
7. Using the point G and the point H , construct a point O (rule W01); ;
8. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
9. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; lines $h_{G,-1/2}(h_c)$ and c are not parallel.

Determination conditions: lines $h_{G,-1/2}(h_c)$ and c are not the same; points E_c and H_c are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W15]

Lemmas used: [D10,D20,D26,D30,D7,GD01,GL02,GL03,GL04,GL09,L11,L12,L3,L57,L58]

Solving time: 4.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{c} 95 56.36
point G 70 58.33
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_r E_{c}
cmark_t G
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points E_{c} and H_{c} are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H_{c}
line h_{c} E_{c} H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0

% Constructing a point P_{\_G120124} such that GP_{\_G120124}/GE_{c}=-0.5
towards P_{\_G120124} G E_{c} -0.5
cmark_r P_{\_G120124}
color 200 200 200
drawsegment E_{c} P_{\_G120124}
color 0 0 0

% Constructing a line h_{G,-1/2}(h_{c}) which contains the point P_{\_G120124} and is parallel to
the line h_{c}
parallel h_{G,-1/2}(h_{c}) P_{\_G120124} h_{c}

color 200 200 200
drawline h_{G,-1/2}(h_{c})
color 0 0 0
```

```

% NDG: lines  $h_{\{G,-1/2\}}(h_{\{c\}})$  and  $c$  are not parallel% DET: lines  $h_{\{G,-1/2\}}(h_{\{c\}})$  and  $c$  are not
the same
% Constructing a point  $M_{\{c\}}$  which belongs to line  $h_{\{G,-1/2\}}(h_{\{c\}})$  and line  $c$ 
intersec  $M_{\{c\}}$   $h_{\{G,-1/2\}}(h_{\{c\}})$   $c$ 
cmark_lt  $M_{\{c\}}$ 

% Constructing a point  $C$  such that  $M_{\{c\}}C/M_{\{c\}}G=3$ 
towards C  $M_{\{c\}}$  G 3
cmark_l C
color 200 200 200
drawsegment  $M_{\{c\}}$  C
color 0 0 0

% Constructing a point  $H$  such that  $E_{\{c\}}H/E_{\{c\}}C=-1$ 
towards H  $E_{\{c\}}$  C -1
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0

% Constructing a point  $O$  such that  $GO/GH=-0.5$ 
towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

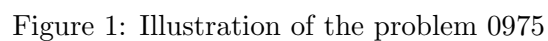
% NDG: points  $C$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $C$ 
circle  $k(O,C)$  O C

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $c$  and circle  $k(O,C)$  intersect
% Constructing points  $A$  and  $B$  which are in intersection of  $k(O,C)$  and  $c$ 
intersec2 A B  $k(O,C)$   $c$ 
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```



3.3 Illustration

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = -E_c$

1740

4.1.2 Proving $G=_G$

Proving failed

4.1.3 Proving $H_c=_Hc$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c=_Ee$

Proving failed

4.2.2 Proving $G=_G$

Proving failed

4.2.3 Proving $H_c=_Hc$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c=_Ee$

Proving failed

4.3.2 Proving $G=_G$

Proving failed

4.3.3 Proving $H_c=_Hc$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c=_Ee$

Proving failed

4.4.2 Proving $G=_G$

Proving failed

4.4.3 Proving $H_c=_Hc$

Proving failed

Problem 976

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 976: Given a point E_c , a point G and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 977

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 977: Given a point E_c , a point G and a point M_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 978

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 978: Given a point E_c , a point G and a point M_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 979

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 979: Given a point E_c , a point G and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point G and the point M_c , construct a point C (rule W01); ;
2. Using the point E_c and the point C , construct a point H (rule W01); ;
3. Using the point G and the point H , construct a point N (rule W01); ;
4. Using the point G and the point H , construct a point O (rule W01); ;
5. Using the point E_c and the point C , construct a line h_c (rule W02); % DET: points E_c and C are not the same;
6. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
8. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
9. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; line h_c and circle $k(N, M_a)$ intersect; points E_c and N are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points E_c and C are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L16,L21,L24,L3,L57,L58]

Solving time: 7.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{c} 95 56.36
point G 70 58.33
point M_{c} 50 67.5

color 220 0 0
fontsize 9

cmark_r E_{c}
cmark_t G
cmark_lt M_{c}
color 0 0 0
fontsize 8

% Constructing a point C such that GC/GM_{c}=-2
towards C G M_{c} -2
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

% Constructing a point H such that E_{c}H/E_{c}C=-1
towards H E_{c} C -1
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0

% Constructing a point N such that GN/GH=0.25
towards N G H 0.25
cmark_r N
color 200 200 200
drawsegment G H
color 0 0 0

% Constructing a point O such that GO/GH=-0.5
```

```

towards O G H -0.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% DET: points  $E_{\{c\}}$  and  $C$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $E_{\{c\}}$  and point  $C$ 
line  $h_{\{c\}}$   $E_{\{c\}}$  C

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: points  $E_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{c\}}$ 
circle  $k(N, M_{\{a\}})$  N  $E_{\{c\}}$ 

color 200 200 200
drawcircle  $k(N, M_{\{a\}})$ 
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{c\}}$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G170291\}}$  which is a foot of the point  $N$  on the line  $h_{\{c\}}$ 
foot  $P_{\{\backslash\_G170291\}}$  N  $h_{\{c\}}$ 
cmark_r  $P_{\{\backslash\_G170291\}}$ 
color 200 200 200
drawline N  $P_{\{\backslash\_G170291\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $E_{\{c\}}$  in the symmetry to point/line  $P_{\{\backslash\_G170291\}}$ 
sim  $H_{\{c\}}$   $P_{\{\backslash\_G170291\}}$   $E_{\{c\}}$ 
cmark_rt  $H_{\{c\}}$ 

% DET: points  $H_{\{c\}}$  and  $M_{\{c\}}$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $M_{\{c\}}$ 
line  $c$   $H_{\{c\}}$   $M_{\{c\}}$ 

color 200 200 200
drawline  $c$ 
color 0 0 0

% NDG: points  $C$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $C$ 
circle  $k(O, C)$  O C

color 200 200 200

```

```
drawcircle k(O,C)
color 0 0 0
```

```
% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
% line h_{c} and circle k(N,M_{a}) intersect; points E_{c} and N are not the same
% Determination conditions: points H_{c} and M_{c} are not the same; points E_{c} and H_{c} must be
% different; points E_{c} and C are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = _E_c$

Proving failed

4.1.2 Proving $G = _G$

Proving failed

4.1.3 Proving $M_c = _M_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = _E_c$

Proving failed

4.2.2 Proving $G = _G$

Proving failed

4.2.3 Proving $M_c = _M_c$

Proving failed

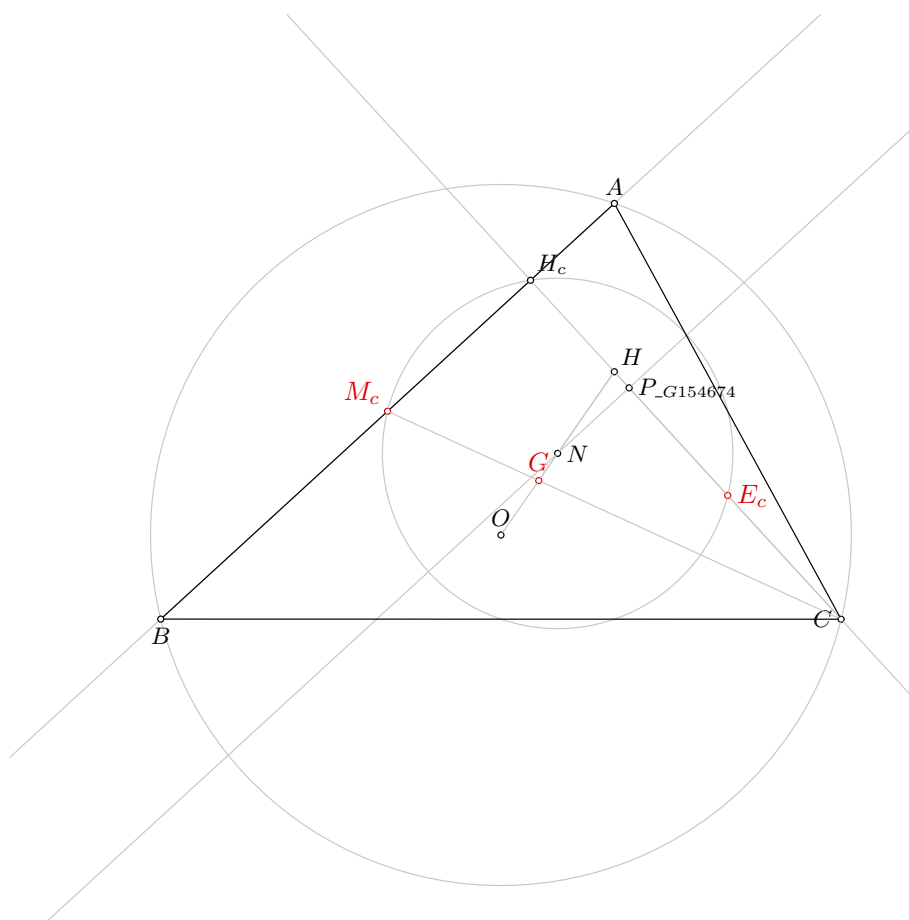


Figure 1: Illustration of the problem 0979

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $G = \neg G$

Proving failed

4.3.3 Proving $M_c = \neg M_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $G = \neg G$

Proving failed

4.4.3 Proving $M_c = \neg M_c$

Proving failed

Problem 980

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 980: Given a point E_c , a point G and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point G and the point N , construct a point H (rule W01); ;
2. Using the point E_c and the point H , construct a point C (rule W01); ;
3. Using the point G and the point N , construct a point O (rule W01); ;
4. Using the point G and the point C , construct a point M_c (rule W01); ;
5. Using the point E_c and the point H , construct a line h_c (rule W02); % DET: points E_c and H are not the same;
6. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
8. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
9. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; line h_c and circle $k(N, M_a)$ intersect; points E_c and N are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points E_c and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L15,L16,L21,L24,L3,L57]

Solving time: 7.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
```

```
point G 70 58.33
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{c}
```

```
cmark_t G
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point H such that GH/GN=4
```

```
towards H G N 4
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment G H
```

```
color 0 0 0
```

```
% Constructing a point C such that E_{c}C/E_{c}H=-1
```

```
towards C E_{c} H -1
```

```
cmark_l C
```

```
color 200 200 200
```

```
drawsegment H C
```

```
color 0 0 0
```

```
% Constructing a point O such that GO/GN=-2
```

```
towards O G N -2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment N O
```

```
color 0 0 0
```

```
% Constructing a point M_{c} such that GM_{c}/GC=-0.5
```



```

towards M_{c} G C -0.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% DET: points E_{c} and H are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H
line h_{c} E_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points E_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{c}
circle k(N,M_{a}) N E_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{c} and circle k(N,M_{a}) intersect% DET: points E_{c} and H_{c} must be different
% Constructing a point P_{\_G212889} which is a foot of the point N on the line h_{c}
foot P_{\_G212889} N h_{c}
cmark_r P_{\_G212889}
color 200 200 200
drawline N P_{\_G212889}
color 0 0 0

% Constructing a point H_{c} which is an image of the point E_{c} in the symmetry to point/line P
\_G212889}
sim H_{c} P_{\_G212889} E_{c}
cmark_rt H_{c}

% DET: points H_{c} and M_{c} are not the same
% Constructing a line c which passes through point H_{c} and point M_{c}
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200

```

```
drawcircle k(O,C)
color 0 0 0
```

```
% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
% line h_{c} and circle k(N,M_{a}) intersect; points E_{c} and N are not the same
% Determination conditions: points H_{c} and M_{c} are not the same; points E_{c} and H_{c} must be
% different; points E_{c} and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = _E_c$

Proving failed

4.1.2 Proving $G = _G$

Proving failed

4.1.3 Proving $N = _N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = _E_c$

Proving failed

4.2.2 Proving $G = _G$

Proving failed

4.2.3 Proving $N = _N$

Proving failed

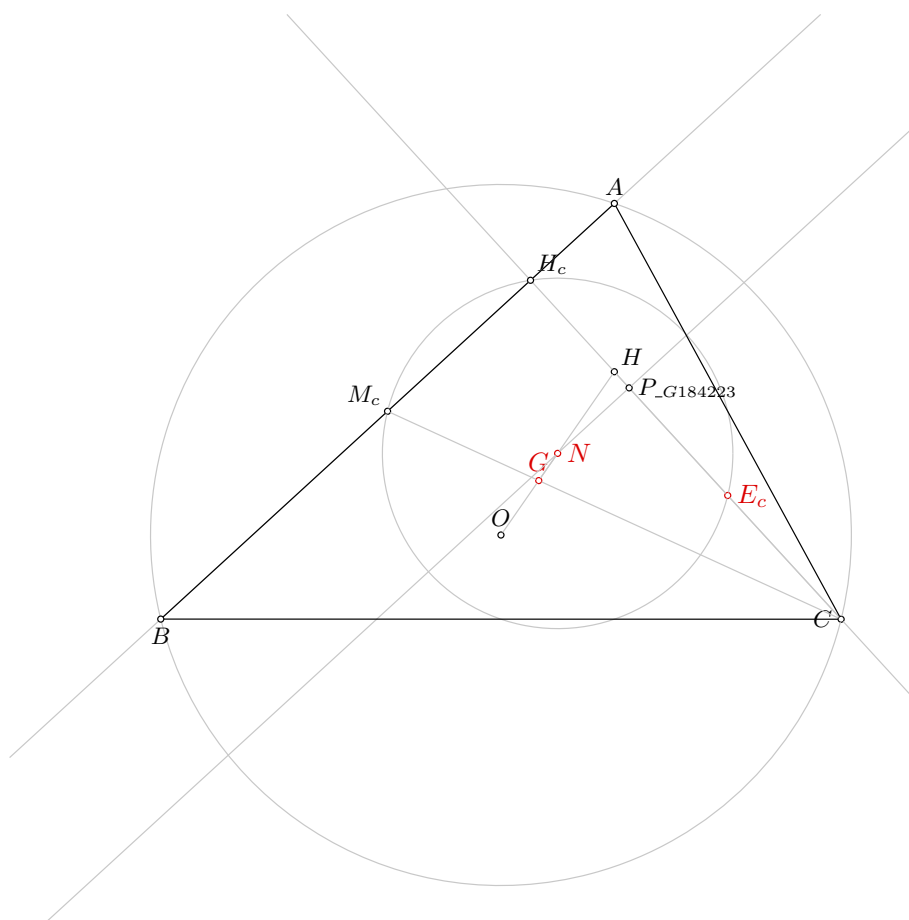


Figure 1: Illustration of the problem 0980

4.3 GCLC - Wu method

4.3.1 Proving $E_c =_c E_c$

Proving failed

4.3.2 Proving $G =_G G$

Proving failed

4.3.3 Proving $N =_N N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c =_c E_c$

Proving failed

4.4.2 Proving $G =_G G$

Proving failed

4.4.3 Proving $N =_N N$

Proving failed

Problem 981

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 981: Given a point E_c , a point G and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point G and the point O , construct a point N (rule W01); ;
2. Using the point G and the point O , construct a point H (rule W01); ;
3. Using the point E_c and the point H , construct a point C (rule W01); ;
4. Using the point G and the point C , construct a point M_c (rule W01); ;
5. Using the point E_c and the point H , construct a line h_c (rule W02); % DET: points E_c and H are not the same;
6. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
8. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
9. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; line h_c and circle $k(N, M_a)$ intersect; points E_c and N are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points E_c and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L15,L21,L24,L3,L57,L58]

Solving time: 7.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{c} 95 56.36
point G 70 58.33
point O 65 51.14

color 220 0 0
fontsize 9

cmark_r E_{c}
cmark_t G
cmark_t O
color 0 0 0
fontsize 8

% Constructing a point N such that GN/GO=-0.5
towards N G O -0.5
cmark_r N
color 200 200 200
drawsegment O N
color 0 0 0

% Constructing a point H such that GH/GO=-2
towards H G O -2
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

% Constructing a point M_{c} such that GM_{c}/GC=-0.5
```

```

towards M_{c} G C -0.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% DET: points E_{c} and H are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H
line h_{c} E_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points E_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{c}
circle k(N,M_{a}) N E_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{c} and circle k(N,M_{a}) intersect% DET: points E_{c} and H_{c} must be different
% Constructing a point P_{\_G247204} which is a foot of the point N on the line h_{c}
foot P_{\_G247204} N h_{c}
cmark_r P_{\_G247204}
color 200 200 200
drawline N P_{\_G247204}
color 0 0 0

% Constructing a point H_{c} which is an image of the point E_{c} in the symmetry to point/line P
\_G247204}
sim H_{c} P_{\_G247204} E_{c}
cmark_rt H_{c}

% DET: points H_{c} and M_{c} are not the same
% Constructing a line c which passes through point H_{c} and point M_{c}
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200

```

```
drawcircle k(O,C)
color 0 0 0
```

```
% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
% line h_{c} and circle k(N,M_{a}) intersect; points E_{c} and N are not the same
% Determination conditions: points H_{c} and M_{c} are not the same; points E_{c} and H_{c} must be
% different; points E_{c} and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $G = \neg G$

Proving failed

4.1.3 Proving $O = \neg O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $G = \neg G$

Proving failed

4.2.3 Proving $O = \neg O$

Proving failed

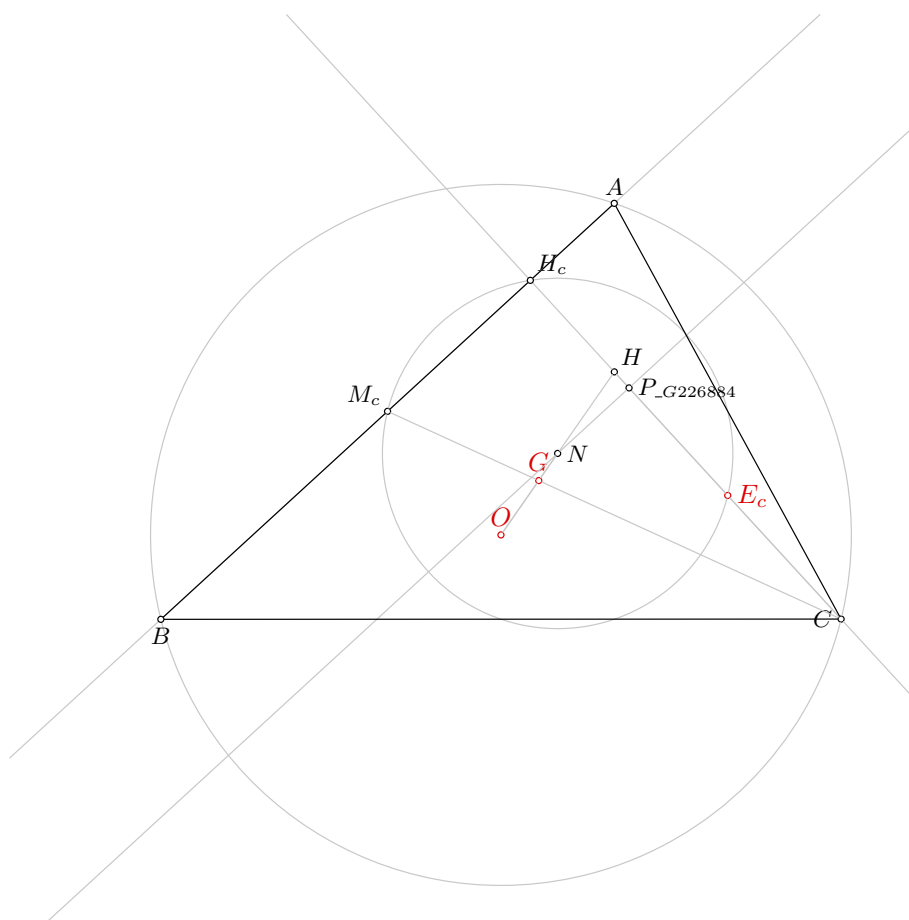


Figure 1: Illustration of the problem 0981

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $G = \neg G$

Proving failed

4.3.3 Proving $O = \neg O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $G = \neg G$

Proving failed

4.4.3 Proving $O = \neg O$

Proving failed

Problem 982

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 982: Given a point E_c , a point G and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 983

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 983: Given a point E_c , a point G and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 984

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 984: Given a point E_c , a point G and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 985

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 985: Given a point H , a point H_a and a point E_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H and the point H_a , construct a line h_a (rule W02); % DET: points H and H_a are not the same;
2. Using the point H_a and the line h_a , construct a line a (rule W10a); ;
3. Using the point H , the line a and the point H_a , construct a line $h_{H,1/2}(a)$ (rule W15); ;
4. Choose freely a point E_c on the line $h_{H,1/2}(a)$ (rule WOnline4);
5. Using the point E_c and the point H , construct a point C (rule W01); ;
6. Using the point H and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H and E_c are not the same;
7. Choose freely a point A on the line h_a (rule WOnline1) ;
8. Using the point A and the point C , construct a line b (rule W02); % DET: points A and C are not the same;
9. Using the circle $k(E_c, C)$, the line b , the point E_c and the point C , construct a point H_b (rule W05); % NDG: line b and circle $k(E_c, C)$ intersect % DET: points C and H_b must be different;
10. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
11. Using the line a and the line h_b , construct a point B (rule W03); % NDG: lines a and h_b are not parallel % DET: lines a and h_b are not the same.

Non-degenerate conditions: lines a and h_b are not parallel; line b and circle $k(E_c, C)$ intersect; points H and E_c are not the same.

Determination conditions: lines a and h_b are not the same; points H_b and H are not the same; points C and H_b must be different; points A and C are not the same; points H and H_a are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,W15,WOnline1,WOnline4]

Lemmas used: [D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,GL04,L52,L53]

Solving time: 691.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H 80 72.73
```

```
point H_{a} 80 40
```

```
point E_{c} 95 56.36
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H
```

```
cmark_r H_{a}
```

```
cmark_r E_{c}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H and H_{a} are not the same
```

```
% Constructing a line h_{a} which passes through point H and point H_{a}
```

```
line h_{a} H H_{a}
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```
% Constructing a line a which is perpendicular to line h_{a} and which passes through point H_{a}
```

```
perp a H_{a} h_{a}
```

```
color 200 200 200
```

```
drawline a
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G66145} such that HP_{\_G66145}/HH_{a}=0.5
```

```
towards P_{\_G66145} H H_{a} 0.5
```

```
cmark_r P_{\_G66145}
```

```
color 200 200 200
```

```
drawsegment H H_{a}
```

```
color 0 0 0
```

```

% Constructing a line  $h_{\{H,1/2\}}(a)$  which contains the point  $P_{\{\backslash\_G66145\}}$  and is parallel to the
line a
parallel  $h_{\{H,1/2\}}(a)$   $P_{\{\backslash\_G66145\}}$  a

```

```

color 200 200 200
drawline  $h_{\{H,1/2\}}(a)$ 
color 0 0 0

```

```

% Constructing a point  $P_{\{\backslash\_G66452\}}$  such that  $HP_{\{\backslash\_G66452\}}/HH_{\{a\}}=0.5$ 
towards  $P_{\{\backslash\_G66452\}}$  H  $H_{\{a\}}$  0.5
cmark_r  $P_{\{\backslash\_G66452\}}$ 
color 200 200 200
drawsegment H  $H_{\{a\}}$ 
color 0 0 0

```

```

% Generating random value  $V[_G66391]$ 
random  $V[_G66391]$ 

```

```

% Calculating value  $V[_G66412]$  using formula  $V[_G66391]*20$ 
expression  $V[_G66412]$  {  $V[_G66391]*20$  }

```

```

% Constructing a point  $E_{\{c\}}$  which is a point for which holds  $P_{\{\backslash\_G66452\}}E_{\{c\}} = V[_G66412]$  and
angle  $HP_{\{\backslash\_G66452\}}E_{\{c\}} = 90$ 
turtle  $E_{\{c\}}$  H  $P_{\{\backslash\_G66452\}}$  90  $V[_G66412]$ 
cmark_r  $E_{\{c\}}$ 

```

```

% Constructing a point C such that  $E_{\{c\}}C/E_{\{c\}}H=-1$ 
towards C  $E_{\{c\}}$  H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

```

```

% NDG: points H and  $E_{\{c\}}$  are not the same
% Constructing a circle  $k(E_{\{c\}},C)$  whose center is at point  $E_{\{c\}}$  and which passes through point H
circle  $k(E_{\{c\}},C)$   $E_{\{c\}}$  H

```

```

color 200 200 200
drawcircle  $k(E_{\{c\}},C)$ 
color 0 0 0

```

```

% Choosing randomly a point A on the line  $HH_{\{a\}}$ 
online A H  $H_{\{a\}}$ 
cmark_t A
color 200 200 200

```



```
drawline H H_{a}
color 0 0 0
```

```
% DET: points A and C are not the same
% Constructing a line b which passes through point A and point C
line b A C
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% NDG: line b and circle k(E_{c},C) intersect% DET: points C and H_{b} must be different
% Constructing a point P_{\_G67017} which is a foot of the point E_{c} on the line b
foot P_{\_G67017} E_{c} b
cmark_r P_{\_G67017}
color 200 200 200
drawline E_{c} P_{\_G67017}
color 0 0 0
```

```
% Constructing a point H_{b} which is an image of the point C in the symmetry to point/line P_{\_G
67017}
sim H_{b} P_{\_G67017} C
cmark_l H_{b}
```

```
% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H
```

```
color 200 200 200
drawline h_{b}
color 0 0 0
```

```
% NDG: lines a and h_{b} are not parallel% DET: lines a and h_{b} are not the same
% Constructing a point B which belongs to line a and line h_{b}
intersec B a h_{b}
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines a and h_{b} are not parallel; line b and circle k(E_{c},C)
intersect; points H and E_{c} are not the same
% Determination conditions: lines a and h_{b} are not the same; points H_{b} and H are not the same
; points C and H_{b} must be different; points A and C are not the same; points H and H_{a} are
not the same
```

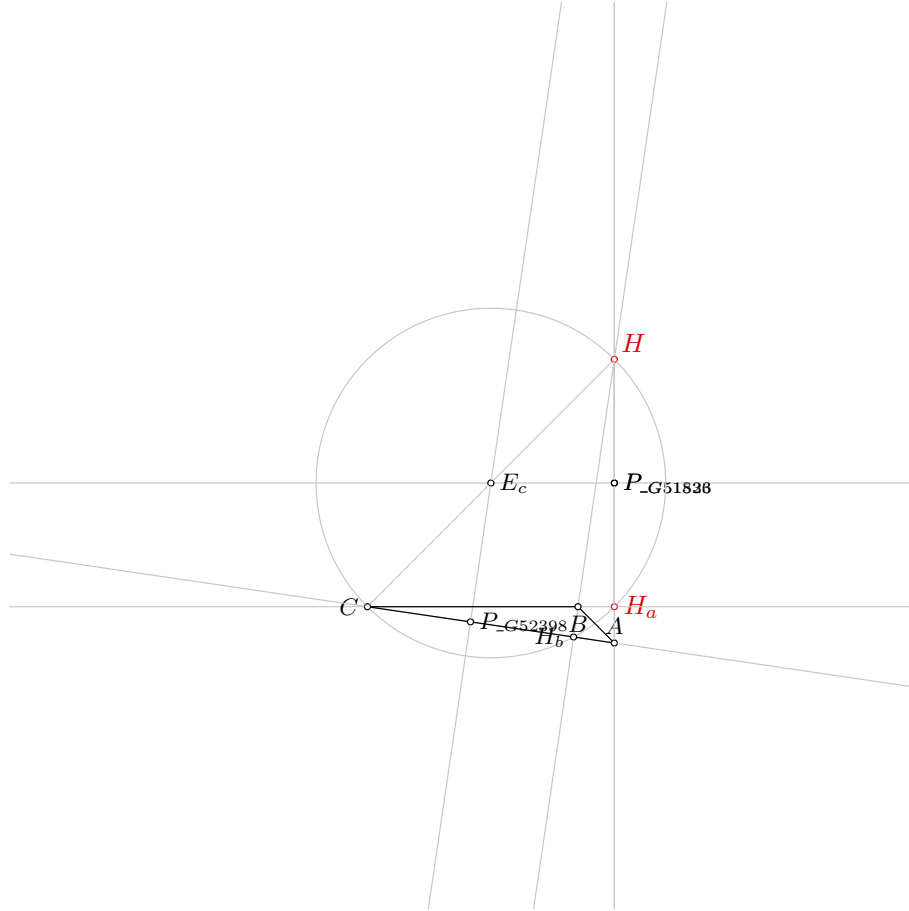


Figure 1: Illustration of the problem 0985

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H = H$

Proving failed

4.1.2 Proving $H_a = H_a$

Proving failed

4.1.3 Proving $E_c = E_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H=_H$

Proving failed

4.2.2 Proving $H_a=_H a$

Proving failed

4.2.3 Proving $E_c=_E c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H=_H$

Proving failed

4.3.2 Proving $H_a=_H a$

Proving failed

4.3.3 Proving $E_c=_E c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H=_H$

Proving failed

4.4.2 Proving $H_a=_H a$

Proving failed

4.4.3 Proving $E_c=_E c$

Proving failed

Problem 986

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 986: Given a point H , a point H_b and a point E_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H and the point H_b , construct a line h_b (rule W02); % DET: points H and H_b are not the same;
2. Using the point H_b and the line h_b , construct a line b (rule W10a); ;
3. Using the point H , the line b and the point H_b , construct a line $h_{H,1/2}(b)$ (rule W15); ;
4. Choose freely a point E_c on the line $h_{H,1/2}(b)$ (rule WOnline4);
5. Using the point E_c and the point H , construct a point C (rule W01); ;
6. Using the point H and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H and E_c are not the same;
7. Choose freely a point A on the line b (rule WOnline1) ;
8. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
9. Using the circle $k(E_c, C)$, the line h_a , the point E_c and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_c, C)$ intersect % DET: points H and H_a must be different;
10. Using the point C and the point H_a , construct a line a (rule W02); % DET: points C and H_a are not the same;
11. Using the line h_b and the line a , construct a point B (rule W03); % NDG: lines h_b and a are not parallel % DET: lines h_b and a are not the same.

Non-degenerate conditions: lines h_b and a are not parallel; line h_a and circle $k(E_c, C)$ intersect; points H and E_c are not the same.

Determination conditions: lines h_b and a are not the same; points C and H_a are not the same; points H and H_a must be different; points A and H are not the same; points H and H_b are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,W15,WOnline1,WOnline4]

Lemmas used: [D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,GL04,L52,L54]

Solving time: 697.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H 80 72.73
```

```
point H_{b} 89.36 77.83
```

```
point E_{c} 95 56.36
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H
```

```
cmark_l H_{b}
```

```
cmark_r E_{c}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H and H_{b} are not the same
```

```
% Constructing a line h_{b} which passes through point H and point H_{b}
```

```
line h_{b} H H_{b}
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```
% Constructing a line b which is perpendicular to line h_{b} and which passes through point H_{b}
```

```
perp b H_{b} h_{b}
```

```
color 200 200 200
```

```
drawline b
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G98364} such that HP_{\_G98364}/HH_{b}=0.5
```

```
towards P_{\_G98364} H H_{b} 0.5
```

```
cmark_r P_{\_G98364}
```

```
color 200 200 200
```

```
drawsegment H H_{b}
```

```
color 0 0 0
```

```

% Constructing a line  $h_{\{H,1/2\}}(b)$  which contains the point  $P_{\{\backslash\_G98364\}}$  and is parallel to the
line b
parallel  $h_{\{H,1/2\}}(b)$   $P_{\{\backslash\_G98364\}}$  b

```

```

color 200 200 200
drawline  $h_{\{H,1/2\}}(b)$ 
color 0 0 0

```

```

% Constructing a point  $P_{\{\backslash\_G98671\}}$  such that  $HP_{\{\backslash\_G98671\}}/HH_{\{b\}}=0.5$ 
towards  $P_{\{\backslash\_G98671\}}$  H  $H_{\{b\}}$  0.5
cmark_r  $P_{\{\backslash\_G98671\}}$ 
color 200 200 200
drawsegment H  $H_{\{b\}}$ 
color 0 0 0

```

```

% Generating random value  $V[_G98610]$ 
random  $V[_G98610]$ 

```

```

% Calculating value  $V[_G98631]$  using formula  $V[_G98610]*20$ 
expression  $V[_G98631]$  {  $V[_G98610]*20$  }

```

```

% Constructing a point  $E_{\{c\}}$  which is a point for which holds  $P_{\{\backslash\_G98671\}}E_{\{c\}} = V[_G98631]$  and
angle  $HP_{\{\backslash\_G98671\}}E_{\{c\}} = 90$ 
turtle  $E_{\{c\}}$  H  $P_{\{\backslash\_G98671\}}$  90  $V[_G98631]$ 
cmark_r  $E_{\{c\}}$ 

```

```

% Constructing a point C such that  $E_{\{c\}}C/E_{\{c\}}H=-1$ 
towards C  $E_{\{c\}}$  H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

```

```

% NDG: points H and  $E_{\{c\}}$  are not the same
% Constructing a circle  $k(E_{\{c\}},C)$  whose center is at point  $E_{\{c\}}$  and which passes through point H
circle  $k(E_{\{c\}},C)$   $E_{\{c\}}$  H

```

```

color 200 200 200
drawcircle  $k(E_{\{c\}},C)$ 
color 0 0 0

```

```

% Choosing randomly a point A on the line  $H_{\{b\}}C$ 
online A  $H_{\{b\}}$  C
cmark_t A
color 200 200 200

```

```

drawline H_{b} C
color 0 0 0

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{a} and circle k(E_{c},C) intersect% DET: points H and H_{a} must be different
% Constructing a point P_{\_G99236} which is a foot of the point E_{c} on the line h_{a}
foot P_{\_G99236} E_{c} h_{a}
cmark_r P_{\_G99236}
color 200 200 200
drawline E_{c} P_{\_G99236}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H in the symmetry to point/line P_{\_G
99236}
sim H_{a} P_{\_G99236} H
cmark_r H_{a}

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines h_{b} and a are not parallel% DET: lines h_{b} and a are not the same
% Constructing a point B which belongs to line h_{b} and line a
intersec B h_{b} a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{b} and a are not parallel; line h_{a} and circle k(E_{c},C)
intersect; points H and E_{c} are not the same
% Determination conditions: lines h_{b} and a are not the same; points C and H_{a} are not the same
; points H and H_{a} must be different; points A and H are not the same; points H and H_{b} are
not the same

```

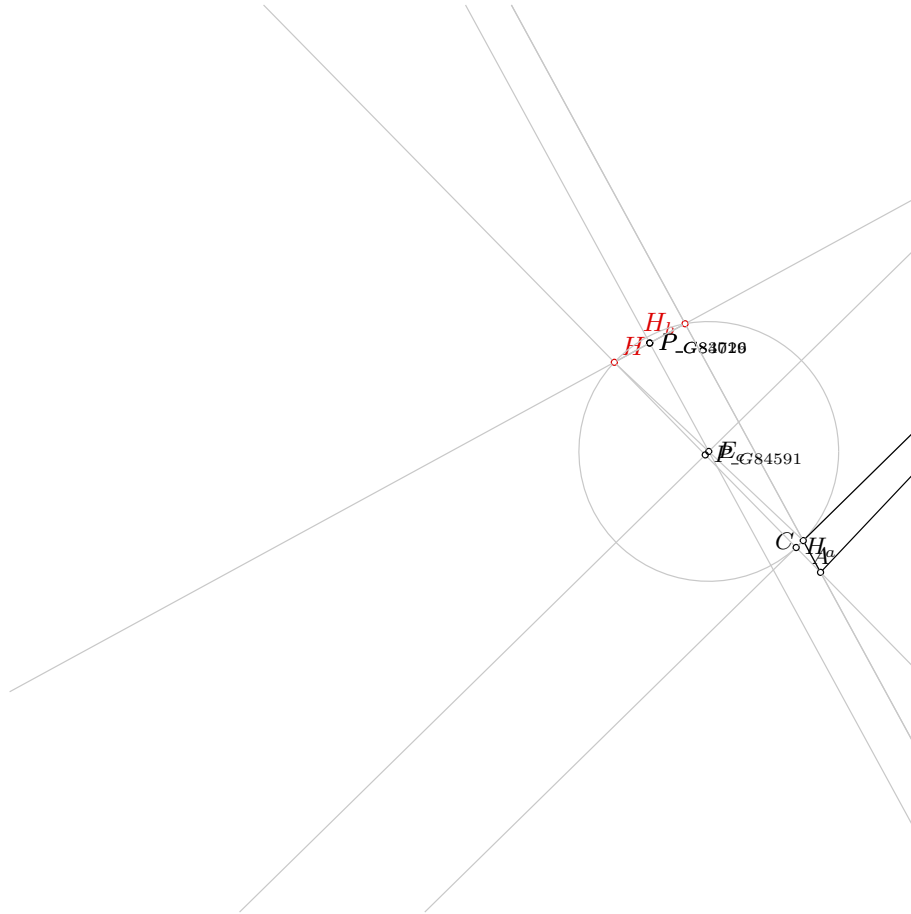


Figure 1: Illustration of the problem 0986

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H = H$

Construction steps:

- Free point H
- Free point H_b
- Line h_b through two points H and H_b
- Line b through point H_b perpendicular to line h_b
- Segment division point P_{G93517} of segment HH_b with division coefficient 1.0

- Line $h_{H,1/2}(b)$ through point P_{G93517} parallel with line b
- Segment division point P_{G93649} of segment HH_b with division coefficient -0.5
- Line L_{G93652} through point P_{G93649} parallel with line $h_{H,1/2}(b)$
- Random point E_c from line L_{G93652}
- Segment division point C of segment E_cH with division coefficient -0.5
- Circle $k(E_c, C)$ with center E_c and one point H
- Random point A from line b
- Line h_a through two points A and H
- Line footPointPerpLine586 through point E_c perpendicular to line h_a
- Intersection point P_{G93911} of point sets footPointPerpLine586 and h_a
- Cental symmetric point H_a of point H with respect to center of symmetry P_{G93911}
- Line a through two points C and H_a
- Intersection point B of point sets h_b and a
- Line $_b$ through two points A and C
- Line $_a$ through two points B and C
- Line $_h_a$ through point A perpendicular to line $_a$
- Line $_h_b$ through point B perpendicular to line $_b$
- Intersection point $_H$ of point sets $_h_a$ and $_h_b$
- Intersection point $_H_b$ of point sets $_b$ and $_h_b$

Theorem statement:

- Points H and $_H$ are identical

Info: Attempting to add the construction of new random point tempPoint-368 $h_{H,1/2}(b)$ necessary for completion of construction of line L_{G93652}

Warning: Generated new random point tempPoint-368 $h_{H,1/2}(b)$ on line $h_{H,1/2}(b)$ in order to complete the construction of parallel line L_{G93652}

Validation result: Theorem protocol is valid.

NDG conditions Point H has been assigned following coordinates: $(0, 0)$

4.1.2 Proving $H_b = _H_b$

Construction steps:

- Free point H
- Free point H_b
- Line h_b through two points H and H_b
- Line b through point H_b perpendicular to line h_b
- Segment division point P_{G94773} of segment HH_b with division coefficient 1.0
- Line $h_{H,1/2}(b)$ through point P_{G94773} parallel with line b
- Segment division point P_{G94905} of segment HH_b with division coefficient -0.5
- Line L_{G94908} through point P_{G94905} parallel with line $h_{H,1/2}(b)$
- Random point E_c from line L_{G94908}
- Segment division point C of segment E_cH with division coefficient -0.5
- Circle $k(E_c, C)$ with center E_c and one point H

- Random point A from line b
- Line h_a through two points A and H
- Line footPointPerpLine359 through point E_c perpendicular to line h_a
- Intersection point P_{G95167} of point sets footPointPerpLine359 and h_a
- Cental symmetric point H_a of point H with respect to center of symmetry P_{G95167}
- Line a through two points C and H_a
- Intersection point B of point sets h_b and a
- Line $_b$ through two points A and C
- Line $_a$ through two points B and C
- Line $_h_a$ through point A perpendicular to line $_a$
- Line $_h_b$ through point B perpendicular to line $_b$
- Intersection point $_H$ of point sets $_h_a$ and $_h_b$
- Intersection point $_H_b$ of point sets $_b$ and $_h_b$

Theorem statement:

- Points H_b and $_H_b$ are identical

Info: Attempting to add the construction of new random point tempPoint-206 $h_{H,1/2}(b)$ necessary for completion of construction of line L_{G94908}

Warning: Generated new random point tempPoint-206 $h_{H,1/2}(b)$ on line $h_{H,1/2}(b)$ in order to complete the construction of parallel line L_{G94908}

Validation result: Theorem protocol is valid.

NDG conditions Point H has been assigned following coordinates: $(0, 0)$

4.1.3 Proving $E_c = _E_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H = _H$

Proving failed

4.2.2 Proving $H_b = _H_b$

Proving failed

4.2.3 Proving $E_c = _E_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H = _H$

Proving failed

4.3.2 Proving $H_b = _H_b$

Proving failed

4.3.3 Proving $E_c = E_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H = H$

Proving failed

4.4.2 Proving $H_b = H_b$

Proving failed

4.4.3 Proving $E_c = E_c$

Proving failed

Problem 987

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 987: Given a point H , a point H_c and a point E_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H and the point H_c , construct a line h_c (rule W02); % DET: points H and H_c are not the same;
2. Choose freely a point E_c on the line h_c (rule WOnline1) ;
3. Using the point E_c and the point H , construct a point C (rule W01); ;
4. Using the point H and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H and E_c are not the same;
5. Using the point H_c and the line h_c , construct a line c (rule W10a); ;
6. Choose freely a point A on the line c (rule WOnline2);
7. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
8. Using the circle $k(E_c, C)$, the line h_a , the point E_c and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_c, C)$ intersect % DET: points H and H_a must be different;
9. Using the point C and the point H_a , construct a line a (rule W02); % DET: points C and H_a are not the same;
10. Using the line c and the line a , construct a point B (rule W03); % NDG: lines c and a are not parallel % DET: lines c and a are not the same.

Non-degenerate conditions: lines c and a are not parallel; line h_a and circle $k(E_c, C)$ intersect; points H and E_c are not the same.

Determination conditions: lines c and a are not the same; points C and H_a are not the same; points H and H_a must be different; points A and H are not the same; points H and H_c are not the same.

Rules used: [W01,W02,W03,W05,W06,W10a,WOnline1,WOnline2]

Lemmas used: [D10,D3,D30,D5,D7,D8,GD01,GD02,GL03,GL04,L3,L52,L54]

Solving time: 170.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point H 80 72.73
point H_{c} 68.91 84.83
point E_{c} 95 56.36

color 220 0 0
fontsize 9

cmark_rt H
cmark_rt H_{c}
cmark_r E_{c}
color 0 0 0
fontsize 8

% DET: points H and H_{c} are not the same
% Constructing a line h_{c} which passes through point H and point H_{c}
line h_{c} H H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% Choosing randomly a point E_{c} on the line HH_{c}
online E_{c} H H_{c}
cmark_r E_{c}
color 200 200 200
drawline H H_{c}
color 0 0 0

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0
```

```

% NDG: points H and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H
circle k(E_{c},C) E_{c} H

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
perp c H_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0

% Generating random value V[_G127647]
random V[_G127647]

% Calculating value V[_G127668] using formula V[_G127647]*20
expression V[_G127668] { V[_G127647]*20 }

% Constructing a point A which is a point for which holds  $H_{c}A = V[_G127668]$  and angle  $E_{c}H_{c}$ 
 $A = 90$ 
turtle A E_{c} H_{c} 90 V[_G127668]
cmark_t A

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{a} and circle k(E_{c},C) intersect% DET: points H and H_{a} must be different
% Constructing a point P_{\_G127940} which is a foot of the point E_{c} on the line h_{a}
foot P_{\_G127940} E_{c} h_{a}
cmark_r P_{\_G127940}
color 200 200 200
drawline E_{c} P_{\_G127940}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H in the symmetry to point/line P_{\_G127940}
sim H_{a} P_{\_G127940} H

```

```

cmark_r H_{a}

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and a are not parallel; line h_{a} and circle k(E_{c},C)
% intersect; points H and E_{c} are not the same
% Determination conditions: lines c and a are not the same; points C and H_{a} are not the same;
% points H and H_{a} must be different; points A and H are not the same; points H and H_{c} are
% not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H = H$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 91 terms.

Time Complexity: Time spent by the prover is 0.736 seconds.

NDG conditions Points A and H are not identical

Points B and H are not identical

Line through points H_c and C is not perpendicular to line through points C and H

Points A , B and C are not collinear

Line through points E_c and A is not perpendicular to line through points A and H

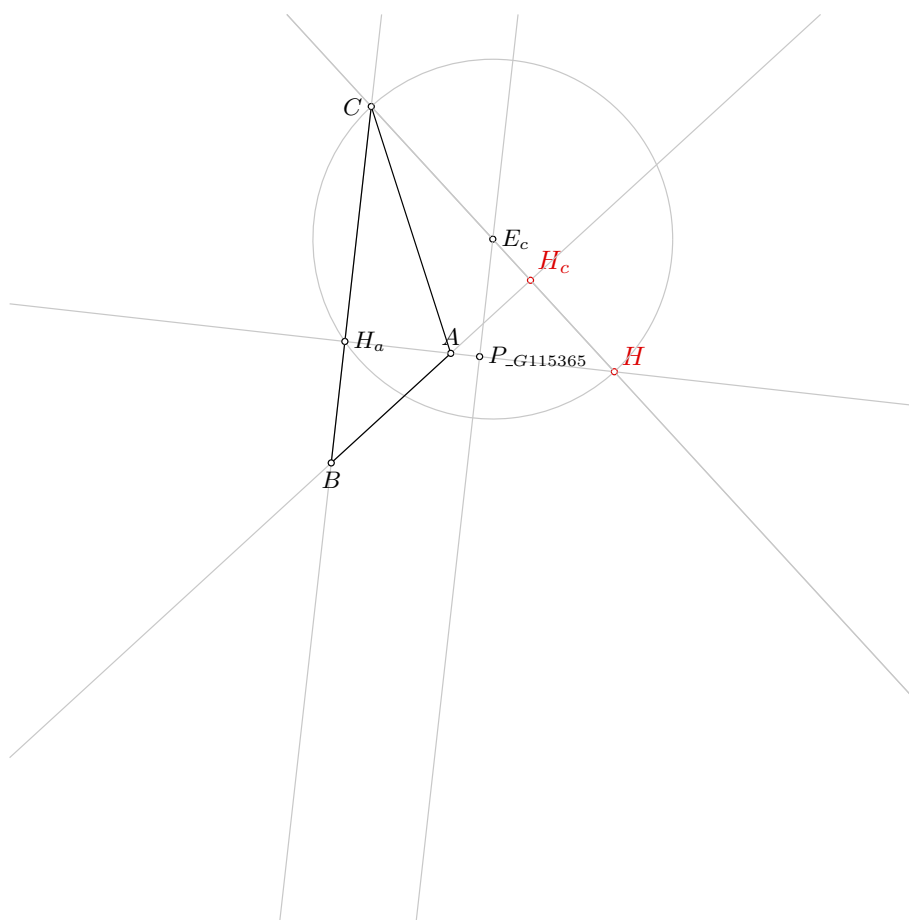


Figure 1: Illustration of the problem 0987

4.1.2 Proving $H_c = \neg H_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

Time Complexity: Time spent by the prover is 0.159 seconds.

NDG conditions Points A and H are not identical

Points B and H are not identical

Line through points H_c and C is not perpendicular to line through points C and H

Points A , B and H are not collinear

4.1.3 Proving $E_c = \neg E_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H = \neg H$

Proving failed

4.2.2 Proving $H_c = \neg H_c$

Proving failed

4.2.3 Proving $E_c = \neg E_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H = \neg H$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 6 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.2 Proving $H_c = \neg H_c$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 6 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.3.3 Proving $E_c = \neg E_c$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 13 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $H = \neg H$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 4 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

4.4.2 Proving $H_c = \neg H_c$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 5 terms.

Time Complexity: Time spent by the prover is 0.000 seconds. There are no ndg conditions.

4.4.3 Proving $E_c = \neg E_c$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 6 terms.

Time Complexity: Time spent by the prover is 0.010 seconds. There are no ndg conditions.

Problem 988

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 988: Given a point E_c , a point H and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 989

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 989: Given a point E_c , a point H and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H , construct a point C (rule W01); ;
2. Using the point M_a and the point C , construct a point B (rule W01); ;
3. Using the point H and the point B , construct a line h_b (rule W02); % DET: points H and B are not the same;
4. Using the point M_a and the point C , construct a line a (rule W02); % DET: points M_a and C are not the same;
5. Using the point H and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H and E_c are not the same;
6. Using the circle $k(E_c, C)$, the line h_b , the point E_c and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_c, C)$ intersect % DET: points H and H_b must be different;
7. Using the point H_b and the point C , construct a line b (rule W02); % DET: points H_b and C are not the same;
8. Using the circle $k(E_c, C)$, the line a , the point E_c and the point C , construct a point H_a (rule W05); % NDG: line a and circle $k(E_c, C)$ intersect % DET: points C and H_a must be different;
9. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
10. Using the line b and the line h_a , construct a point A (rule W03); % NDG: lines b and h_a are not parallel % DET: lines b and h_a are not the same.

Non-degenerate conditions: lines b and h_a are not parallel; line a and circle $k(E_c, C)$ intersect; line h_b and circle $k(E_c, C)$ intersect; points H and E_c are not the same.

Determination conditions: lines b and h_a are not the same; points H_a and H are not the same; points C and H_a must be different; points H_b and C are not the same; points H and H_b must be different; points M_a and C are not the same; points H and B are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D21,D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,GL04,GL09,L52,L53,L54]

Solving time: 12.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
```

```
point H 80 72.73
```

```
point M_{a} 65 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{c}
```

```
cmark_rt H
```

```
cmark_r M_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point C such that E_{c}C/E_{c}H=-1
```

```
towards C E_{c} H -1
```

```
cmark_l C
```

```
color 200 200 200
```

```
drawsegment H C
```

```
color 0 0 0
```

```
% Constructing a point B such that M_{a}B/M_{a}C=-1
```

```
towards B M_{a} C -1
```

```
cmark_b B
```

```
color 200 200 200
```

```
drawsegment C B
```

```
color 0 0 0
```

```
% DET: points H and B are not the same
```

```
% Constructing a line h_{b} which passes through point H and point B
```

```
line h_{b} H B
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```

% DET: points  $M_{\{a\}}$  and  $C$  are not the same
% Constructing a line  $a$  which passes through point  $M_{\{a\}}$  and point  $C$ 
line a  $M_{\{a\}}$  C

color 200 200 200
drawline a
color 0 0 0

% NDG: points  $H$  and  $E_{\{c\}}$  are not the same
% Constructing a circle  $k(E_{\{c\}}, C)$  whose center is at point  $E_{\{c\}}$  and which passes through point  $H$ 
circle k( $E_{\{c\}}$ ,  $C$ )  $E_{\{c\}}$  H

color 200 200 200
drawcircle k( $E_{\{c\}}$ ,  $C$ )
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(E_{\{c\}}, C)$  intersect% DET: points  $H$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G163950\}}$  which is a foot of the point  $E_{\{c\}}$  on the line  $h_{\{b\}}$ 
foot  $P_{\{\backslash\_G163950\}}$   $E_{\{c\}}$   $h_{\{b\}}$ 
cmark_r  $P_{\{\backslash\_G163950\}}$ 
color 200 200 200
drawline  $E_{\{c\}}$   $P_{\{\backslash\_G163950\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $H$  in the symmetry to point/line  $P_{\{\backslash\_G163950\}}$ 
sim  $H_{\{b\}}$   $P_{\{\backslash\_G163950\}}$  H
cmark_l  $H_{\{b\}}$ 

% DET: points  $H_{\{b\}}$  and  $C$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $C$ 
line b  $H_{\{b\}}$  C

color 200 200 200
drawline b
color 0 0 0

% NDG: line  $a$  and circle  $k(E_{\{c\}}, C)$  intersect% DET: points  $C$  and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\backslash\_G164188\}}$  which is a foot of the point  $E_{\{c\}}$  on the line  $a$ 
foot  $P_{\{\backslash\_G164188\}}$   $E_{\{c\}}$   $a$ 
cmark_r  $P_{\{\backslash\_G164188\}}$ 
color 200 200 200
drawline  $E_{\{c\}}$   $P_{\{\backslash\_G164188\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $C$  in the symmetry to point/line  $P_{\{\backslash\_G164188\}}$ 
sim  $H_{\{a\}}$   $P_{\{\backslash\_G164188\}}$  C
cmark_r  $H_{\{a\}}$ 

```

```

% DET: points  $H_{\{a\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $H_{\{a\}}$  and point  $H$ 
line  $h_{\{a\}}$   $H_{\{a\}}$   $H$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: lines  $b$  and  $h_{\{a\}}$  are not parallel% DET: lines  $b$  and  $h_{\{a\}}$  are not the same
% Constructing a point  $A$  which belongs to line  $b$  and line  $h_{\{a\}}$ 
intersec  $A$   $b$   $h_{\{a\}}$ 
cmark_t  $A$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $b$  and  $h_{\{a\}}$  are not parallel; line  $a$  and circle  $k(E_{\{c\}}, C)$ 
% intersect; line  $h_{\{b\}}$  and circle  $k(E_{\{c\}}, C)$  intersect; points  $H$  and  $E_{\{c\}}$  are not the same
% Determination conditions: lines  $b$  and  $h_{\{a\}}$  are not the same; points  $H_{\{a\}}$  and  $H$  are not the same
% ; points  $C$  and  $H_{\{a\}}$  must be different; points  $H_{\{b\}}$  and  $C$  are not the same; points  $H$  and  $H_{\{b\}}$ 
% must be different; points  $M_{\{a\}}$  and  $C$  are not the same; points  $H$  and  $B$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = _E_c$

Proving failed

4.1.2 Proving $H = _H$

Proving failed

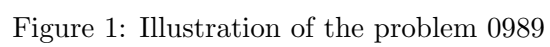
4.1.3 Proving $M_a = _M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 8 terms.

Time Complexity: Time spent by the prover is 0.109 seconds.

NDG conditions There are no NDG conditions for this theorem



4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $H = \neg H$

Proving failed

4.2.3 Proving $M_a = \neg M_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 2602 terms.

Time Complexity: Time spent by the prover is 14.230 seconds. There are no ndg conditions.

4.3.2 Proving $H = \neg H$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 494 terms.

Time Complexity: Time spent by the prover is 0.420 seconds. There are no ndg conditions.

4.3.3 Proving $M_a = \neg M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 19 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $H = \neg H$

Proving failed

4.4.3 Proving $M_a = \neg M_a$

Proving failed

Problem 990

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 990: Given a point E_c , a point H and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H , construct a point C (rule W01); ;
2. Using the point M_b and the point C , construct a point A (rule W01); ;
3. Using the point H and the point A , construct a line h_a (rule W02); % DET: points H and A are not the same;
4. Using the point M_b and the point C , construct a line b (rule W02); % DET: points M_b and C are not the same;
5. Using the point H and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H and E_c are not the same;
6. Using the circle $k(E_c, C)$, the line h_a , the point E_c and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_c, C)$ intersect % DET: points H and H_a must be different;
7. Using the point H_a and the point C , construct a line a (rule W02); % DET: points H_a and C are not the same;
8. Using the circle $k(E_c, C)$, the line b , the point E_c and the point C , construct a point H_b (rule W05); % NDG: line b and circle $k(E_c, C)$ intersect % DET: points C and H_b must be different;
9. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
10. Using the line a and the line h_b , construct a point B (rule W03); % NDG: lines a and h_b are not parallel % DET: lines a and h_b are not the same.

Non-degenerate conditions: lines a and h_b are not parallel; line b and circle $k(E_c, C)$ intersect; line h_a and circle $k(E_c, C)$ intersect; points H and E_c are not the same.

Determination conditions: lines a and h_b are not the same; points H_b and H are not the same; points C and H_b must be different; points H_a and C are not the same; points H and H_a must be different; points M_b and C are not the same; points H and A are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D22,D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,GL04,GL09,L52,L53,L54]

Solving time: 12.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
```

```
point H 80 72.73
```

```
point M_{b} 95 67.5
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{c}
```

```
cmark_rt H
```

```
cmark_lt M_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point C such that E_{c}C/E_{c}H=-1
```

```
towards C E_{c} H -1
```

```
cmark_l C
```

```
color 200 200 200
```

```
drawsegment H C
```

```
color 0 0 0
```

```
% Constructing a point A such that M_{b}A/M_{b}C=-1
```

```
towards A M_{b} C -1
```

```
cmark_t A
```

```
color 200 200 200
```

```
drawsegment C A
```

```
color 0 0 0
```

```
% DET: points H and A are not the same
```

```
% Constructing a line h_{a} which passes through point H and point A
```

```
line h_{a} H A
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```

% DET: points  $M_{\{b\}}$  and  $C$  are not the same
% Constructing a line  $b$  which passes through point  $M_{\{b\}}$  and point  $C$ 
line b  $M_{\{b\}}$  C

color 200 200 200
drawline b
color 0 0 0

% NDG: points  $H$  and  $E_{\{c\}}$  are not the same
% Constructing a circle  $k(E_{\{c\}}, C)$  whose center is at point  $E_{\{c\}}$  and which passes through point  $H$ 
circle k( $E_{\{c\}}$ ,  $C$ )  $E_{\{c\}}$  H

color 200 200 200
drawcircle k( $E_{\{c\}}$ ,  $C$ )
color 0 0 0

% NDG: line  $h_{\{a\}}$  and circle  $k(E_{\{c\}}, C)$  intersect% DET: points  $H$  and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\_G193955\}}$  which is a foot of the point  $E_{\{c\}}$  on the line  $h_{\{a\}}$ 
foot  $P_{\{\_G193955\}}$   $E_{\{c\}}$   $h_{\{a\}}$ 
cmark_r  $P_{\{\_G193955\}}$ 
color 200 200 200
drawline  $E_{\{c\}}$   $P_{\{\_G193955\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $H$  in the symmetry to point/line  $P_{\{\_G193955\}}$ 
sim  $H_{\{a\}}$   $P_{\{\_G193955\}}$  H
cmark_r  $H_{\{a\}}$ 

% DET: points  $H_{\{a\}}$  and  $C$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $C$ 
line a  $H_{\{a\}}$  C

color 200 200 200
drawline a
color 0 0 0

% NDG: line  $b$  and circle  $k(E_{\{c\}}, C)$  intersect% DET: points  $C$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\_G194193\}}$  which is a foot of the point  $E_{\{c\}}$  on the line  $b$ 
foot  $P_{\{\_G194193\}}$   $E_{\{c\}}$   $b$ 
cmark_r  $P_{\{\_G194193\}}$ 
color 200 200 200
drawline  $E_{\{c\}}$   $P_{\{\_G194193\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $C$  in the symmetry to point/line  $P_{\{\_G194193\}}$ 
sim  $H_{\{b\}}$   $P_{\{\_G194193\}}$  C
cmark_l  $H_{\{b\}}$ 

```

```

% DET: points  $H_{\{b\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $H_{\{b\}}$  and point  $H$ 
line  $h_{\{b\}}$   $H_{\{b\}}$   $H$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: lines  $a$  and  $h_{\{b\}}$  are not parallel% DET: lines  $a$  and  $h_{\{b\}}$  are not the same
% Constructing a point  $B$  which belongs to line  $a$  and line  $h_{\{b\}}$ 
intersec B a  $h_{\{b\}}$ 
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $a$  and  $h_{\{b\}}$  are not parallel; line  $b$  and circle  $k(E_{\{c\}}, C)$ 
% intersect; line  $h_{\{a\}}$  and circle  $k(E_{\{c\}}, C)$  intersect; points  $H$  and  $E_{\{c\}}$  are not the same
% Determination conditions: lines  $a$  and  $h_{\{b\}}$  are not the same; points  $H_{\{b\}}$  and  $H$  are not the same
% ; points  $C$  and  $H_{\{b\}}$  must be different; points  $H_{\{a\}}$  and  $C$  are not the same; points  $H$  and  $H_{\{a\}}$ 
% must be different; points  $M_{\{b\}}$  and  $C$  are not the same; points  $H$  and  $A$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $H = \neg H$

Proving failed

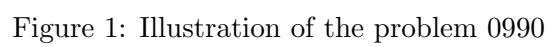
4.1.3 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 8 terms.

Time Complexity: Time spent by the prover is 0.117 seconds.

NDG conditions There are no NDG conditions for this theorem



4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $H = \neg H$

Proving failed

4.2.3 Proving $M_b = \neg M_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 1656 terms.

Time Complexity: Time spent by the prover is 10.840 seconds. There are no ndg conditions.

4.3.2 Proving $H = \neg H$

Status: The conjecture has been disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 333 terms.

Time Complexity: Time spent by the prover is 0.630 seconds. There are no ndg conditions.

4.3.3 Proving $M_b = \neg M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 19 terms.

Time Complexity: Time spent by the prover is 0.030 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $H = \neg H$

Proving failed

4.4.3 Proving $M_b = \neg M_b$

Proving failed

Problem 991

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 991: Given a point E_c , a point H and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H , construct a point C (rule W01); ;
2. Using the point M_c and the point C , construct a point G (rule W01); ;
3. Using the point H and the point G , construct a point N (rule W01); ;
4. Using the point H and the point G , construct a point O (rule W01); ;
5. Using the point E_c and the point H , construct a line h_c (rule W02); % DET: points E_c and H are not the same;
6. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
8. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
9. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; line h_c and circle $k(N, M_a)$ intersect; points E_c and N are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points E_c and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L16,L21,L24,L3,L57,L58]

Solving time: 7.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
```

```
point H 80 72.73
```

```
point M_{c} 50 67.5
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{c}
```

```
cmark_rt H
```

```
cmark_lt M_{c}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point C such that E_{c}C/E_{c}H=-1
```

```
towards C E_{c} H -1
```

```
cmark_l C
```

```
color 200 200 200
```

```
drawsegment H C
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G224262} which passes through point M_{c} and point C
```

```
line L_{\_G224262} M_{c} C
```

```
color 200 200 200
```

```
drawline L_{\_G224262}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G224363} with coordinates (0,0)
```

```
point P_{\_G224363} 0 0
```

```
cmark_r P_{\_G224363}
```

```
% Constructing a point P_{\_G224287} such that M_{c}P_{\_G224287}/M_{c}P_{\_G224363}=1
```

```
towards P_{\_G224287} M_{c} P_{\_G224363} 1
```

```
cmark_r P_{\_G224287}
```

```
color 200 200 200
```

```
drawsegment M_{c} P_{\_G224287}
```

```
color 0 0 0
```

```

% Constructing a point P_{\_G224332} such that M_{c}P_{\_G224332}/M_{c}P_{\_G224363}=3
towards P_{\_G224332} M_{c} P_{\_G224363} 3
cmark_r P_{\_G224332}
color 200 200 200
drawsegment M_{c} P_{\_G224332}
color 0 0 0

% Constructing a line L_{\_G224293} which passes through point C and point P_{\_G224332}
line L_{\_G224293} C P_{\_G224332}

color 200 200 200
drawline L_{\_G224293}
color 0 0 0

% Constructing a line L_{\_G224256} which contains the point P_{\_G224287} and is parallel to the
line L_{\_G224293}
parallel L_{\_G224256} P_{\_G224287} L_{\_G224293}

color 200 200 200
drawline L_{\_G224256}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G224256} and line L_{\_G224262}
intersec G L_{\_G224256} L_{\_G224262}
cmark_t G

% Constructing a point N such that HN/HG=0.75
towards N H G 0.75
cmark_r N
color 200 200 200
drawsegment H G
color 0 0 0

% Constructing a point O such that HO/HG=1.5
towards O H G 1.5
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% DET: points E_{c} and H are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H
line h_{c} E_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

```

```

% NDG: points  $E_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{c\}}$ 
circle k(N, M_{a}) N E_{c}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{c\}}$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\_G225282\}}$  which is a foot of the point  $N$  on the line  $h_{\{c\}}$ 
foot P_{\_G225282} N h_{c}
cmark_r P_{\_G225282}
color 200 200 200
drawline N P_{\_G225282}
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $E_{\{c\}}$  in the symmetry to point/line  $P_{\{\_G225282\}}$ 
sim H_{c} P_{\_G225282} E_{c}
cmark_rt H_{c}

% DET: points  $H_{\{c\}}$  and  $M_{\{c\}}$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $M_{\{c\}}$ 
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points  $C$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $C$ 
circle k(O, C) O C

color 200 200 200
drawcircle k(O, C)
color 0 0 0

% NDG: line  $c$  and circle  $k(O, C)$  intersect
% Constructing points  $A$  and  $B$  which are in intersection of  $k(O, C)$  and  $c$ 
intersec2 A B k(O, C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```

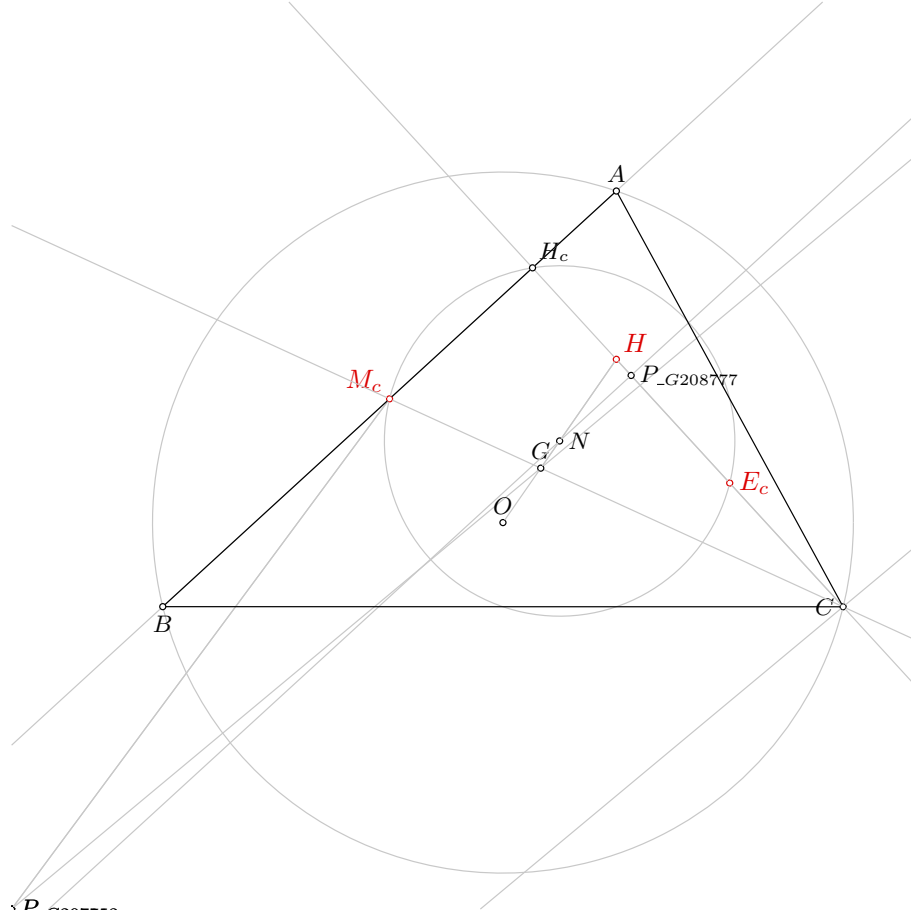


Figure 1: Illustration of the problem 0991

*% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
line h_{c} and circle k(N,M_{a}) intersect; points E_{c} and N are not the same
% Determination conditions: points H_{c} and M_{c} are not the same; points E_{c} and H_{c} must be
different; points E_{c} and H are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = E_c$

Proving failed

4.1.2 Proving $H=_H$

Proving failed

4.1.3 Proving $M_c=_M_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c=_E_c$

Proving failed

4.2.2 Proving $H=_H$

Proving failed

4.2.3 Proving $M_c=_M_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c=_E_c$

Proving failed

4.3.2 Proving $H=_H$

Proving failed

4.3.3 Proving $M_c=_M_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c=_E_c$

Proving failed

4.4.2 Proving $H=_H$

Proving failed

4.4.3 Proving $M_c=_M_c$

Proving failed

Problem 992

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 992: Given a point E_c , a point H and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H , construct a point C (rule W01); ;
2. Using the point H and the point N , construct a point O (rule W01); ;
3. Using the point H and the point N , construct a point G (rule W01); ;
4. Using the point C and the point G , construct a point M_c (rule W01); ;
5. Using the point E_c and the point H , construct a line h_c (rule W02); % DET: points E_c and H are not the same;
6. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
8. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
9. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; line h_c and circle $k(N, M_a)$ intersect; points E_c and N are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points E_c and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D31,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L16,L21,L24,L3,L57]

Solving time: 7.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
```

```
point H 80 72.73
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{c}
```

```
cmark_rt H
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point C such that E_{c}C/E_{c}H=-1
```

```
towards C E_{c} H -1
```

```
cmark_l C
```

```
color 200 200 200
```

```
drawsegment H C
```

```
color 0 0 0
```

```
% Constructing a point O such that HO/HN=2
```

```
towards O H N 2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment H O
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G41772} which passes through point H and point N
```

```
line L_{\_G41772} H N
```

```
color 200 200 200
```

```
drawline L_{\_G41772}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G41873} with coordinates (0,0)
```

```
point P_{\_G41873} 0 0
```

```

cmark_r P_{\_G41873}

% Constructing a point P_{\_G41797} such that  $HP_{\_G41797}/HP_{\_G41873}=4$ 
towards P_{\_G41797} H P_{\_G41873} 4
cmark_r P_{\_G41797}
color 200 200 200
drawsegment H P_{\_G41797}
color 0 0 0

% Constructing a point P_{\_G41842} such that  $HP_{\_G41842}/HP_{\_G41873}=3$ 
towards P_{\_G41842} H P_{\_G41873} 3
cmark_r P_{\_G41842}
color 200 200 200
drawsegment H P_{\_G41842}
color 0 0 0

% Constructing a line L_{\_G41803} which passes through point N and point P_{\_G41842}
line L_{\_G41803} N P_{\_G41842}

color 200 200 200
drawline L_{\_G41803}
color 0 0 0

% Constructing a line L_{\_G41766} which contains the point P_{\_G41797} and is parallel to the
line L_{\_G41803}
parallel L_{\_G41766} P_{\_G41797} L_{\_G41803}

color 200 200 200
drawline L_{\_G41766}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G41766} and line L_{\_G41772}
intersec G L_{\_G41766} L_{\_G41772}
cmark_t G

% Constructing a point M_{c} such that  $CM_{c}/CG=1.5$ 
towards M_{c} C G 1.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% DET: points E_{c} and H are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H
line h_{c} E_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

```



```

% NDG: points  $E_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{c\}}$ 
circle k(N, M_{a}) N E_{c}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{c\}}$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\_G42709\}}$  which is a foot of the point  $N$  on the line  $h_{\{c\}}$ 
foot P_{\_G42709} N h_{c}
cmark_r P_{\_G42709}
color 200 200 200
drawline N P_{\_G42709}
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $E_{\{c\}}$  in the symmetry to point/line  $P_{\{\_G42709\}}$ 
sim H_{c} P_{\_G42709} E_{c}
cmark_rt H_{c}

% DET: points  $H_{\{c\}}$  and  $M_{\{c\}}$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $M_{\{c\}}$ 
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

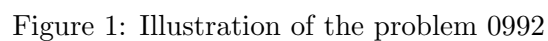
% NDG: points  $C$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $C$ 
circle k(O, C) O C

color 200 200 200
drawcircle k(O, C)
color 0 0 0

% NDG: line  $c$  and circle  $k(O, C)$  intersect
% Constructing points  $A$  and  $B$  which are in intersection of  $k(O, C)$  and  $c$ 
intersec2 A B k(O, C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```



```
% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
    line h_{c} and circle k(N,M_{a}) intersect; points E_{c} and N are not the same
% Determination conditions: points H_{c} and M_{c} are not the same; points E_{c} and H_{c} must be
    different; points E_{c} and H are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = -E_c$

Proving failed

4.1.2 Proving $H=_H$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c=_E_c$

Proving failed

4.2.2 Proving $H=_H$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c=_E_c$

Proving failed

4.3.2 Proving $H=_H$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c=_E_c$

Proving failed

4.4.2 Proving $H=_H$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 993

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 993: Given a point E_c , a point H and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H , construct a point C (rule W01); ;
2. Using the point H and the point O , construct a point N (rule W01); ;
3. Using the point H and the point O , construct a point G (rule W01); ;
4. Using the point C and the point G , construct a point M_c (rule W01); ;
5. Using the point E_c and the point H , construct a line h_c (rule W02); % DET: points E_c and H are not the same;
6. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
8. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
9. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; line h_c and circle $k(N, M_a)$ intersect; points E_c and N are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points E_c and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D31,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L21,L24,L3,L57,L58]

Solving time: 7.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
```

```
point H 80 72.73
```

```
point O 65 51.14
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{c}
```

```
cmark_rt H
```

```
cmark_t O
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point C such that E_{c}C/E_{c}H=-1
```

```
towards C E_{c} H -1
```

```
cmark_l C
```

```
color 200 200 200
```

```
drawsegment H C
```

```
color 0 0 0
```

```
% Constructing a point N such that HN/HO=0.5
```

```
towards N H O 0.5
```

```
cmark_r N
```

```
color 200 200 200
```

```
drawsegment H O
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G77794} which passes through point H and point O
```

```
line L_{\_G77794} H O
```

```
color 200 200 200
```

```
drawline L_{\_G77794}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G77895} with coordinates (0,0)
```

```
point P_{\_G77895} 0 0
```

```

cmark_r P_{\_G77895}

% Constructing a point P_{\_G77819} such that HP_{\_G77819}/HP_{\_G77895}=2
towards P_{\_G77819} H P_{\_G77895} 2
cmark_r P_{\_G77819}
color 200 200 200
drawsegment H P_{\_G77819}
color 0 0 0

% Constructing a point P_{\_G77864} such that HP_{\_G77864}/HP_{\_G77895}=3
towards P_{\_G77864} H P_{\_G77895} 3
cmark_r P_{\_G77864}
color 200 200 200
drawsegment H P_{\_G77864}
color 0 0 0

% Constructing a line L_{\_G77825} which passes through point O and point P_{\_G77864}
line L_{\_G77825} O P_{\_G77864}

color 200 200 200
drawline L_{\_G77825}
color 0 0 0

% Constructing a line L_{\_G77788} which contains the point P_{\_G77819} and is parallel to the
line L_{\_G77825}
parallel L_{\_G77788} P_{\_G77819} L_{\_G77825}

color 200 200 200
drawline L_{\_G77788}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G77788} and line L_{\_G77794}
intersec G L_{\_G77788} L_{\_G77794}
cmark_t G

% Constructing a point M_{c} such that CM_{c}/CG=1.5
towards M_{c} C G 1.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% DET: points E_{c} and H are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H
line h_{c} E_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

```

```

% NDG: points  $E_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{c\}}$ 
circle k(N, M_{a}) N E_{c}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{c\}}$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\_G78731\}}$  which is a foot of the point  $N$  on the line  $h_{\{c\}}$ 
foot P_{\_G78731} N h_{c}
cmark_r P_{\_G78731}
color 200 200 200
drawline N P_{\_G78731}
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $E_{\{c\}}$  in the symmetry to point/line  $P_{\{\_G78731\}}$ 
sim H_{c} P_{\_G78731} E_{c}
cmark_rt H_{c}

% DET: points  $H_{\{c\}}$  and  $M_{\{c\}}$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $M_{\{c\}}$ 
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points  $C$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $C$ 
circle k(O, C) O C

color 200 200 200
drawcircle k(O, C)
color 0 0 0

% NDG: line  $c$  and circle  $k(O, C)$  intersect
% Constructing points  $A$  and  $B$  which are in intersection of  $k(O, C)$  and  $c$ 
intersec2 A B k(O, C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```

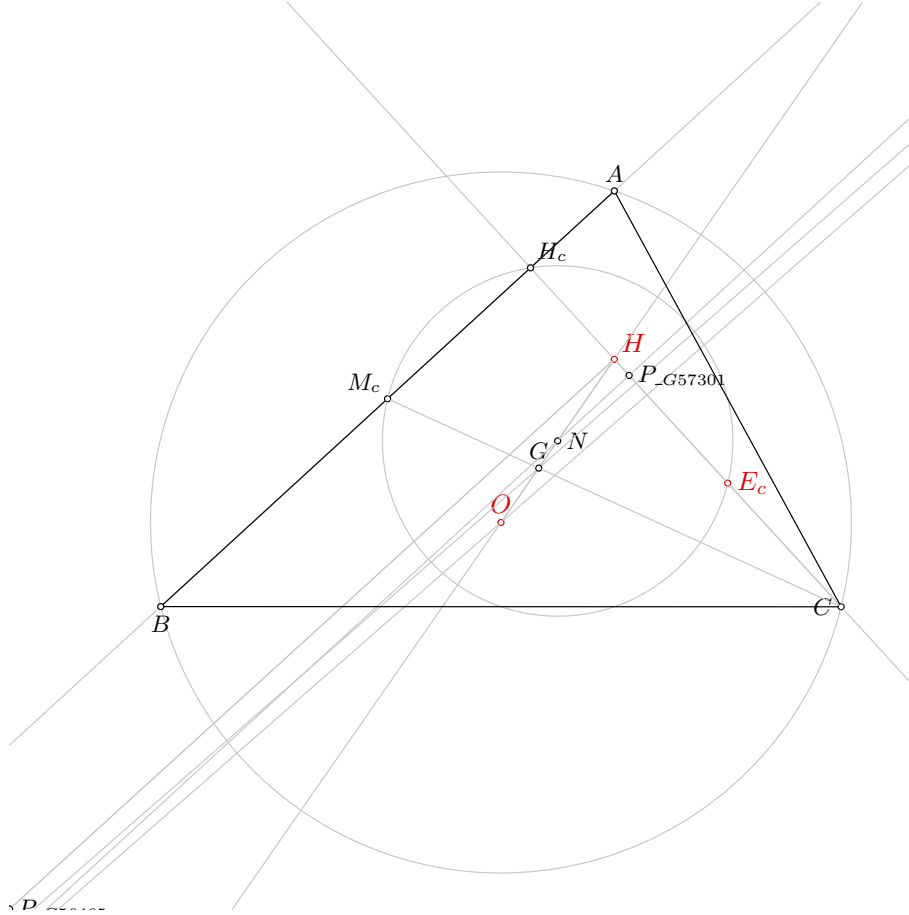


Figure 1: Illustration of the problem 0993

*% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
line h_{c} and circle k(N,M_{a}) intersect; points E_{c} and N are not the same
% Determination conditions: points H_{c} and M_{c} are not the same; points E_{c} and H_{c} must be
different; points E_{c} and H are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = E_c$

Proving failed

4.1.2 Proving $H=_H$

Proving failed

4.1.3 Proving $O=_O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c=_E_c$

Proving failed

4.2.2 Proving $H=_H$

Proving failed

4.2.3 Proving $O=_O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c=_E_c$

Proving failed

4.3.2 Proving $H=_H$

Proving failed

4.3.3 Proving $O=_O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c=_E_c$

Proving failed

4.4.2 Proving $H=_H$

Proving failed

4.4.3 Proving $O=_O$

Proving failed

Problem 994

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 994: Given a point E_c , a point H and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 995

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 995: Given a point E_c , a point H and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 996

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 996: Given a point E_c , a point H and a point T_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H , construct a point C (rule W01); ;
2. Using the point E_c and the point H , construct a line h_c (rule W02); % DET: points E_c and H are not the same;
3. Using the point T_c and the point C , construct a line s_c (rule W02); % DET: points T_c and C are not the same;
4. Using the point T_c and the line h_c , construct a line c (rule W10a); ;
5. Using the line c and the line h_c , construct a point H_c (rule W03); % NDG: lines c and h_c are not parallel % DET: lines c and h_c are not the same;
6. Using the point H and the point H_c , construct a line $m(HH_c)$ (rule W14); % DET: points H and H_c are not the same;
7. Using the point E_c , the point C , the point T_c , the line s_c and the line h_c , construct a line CO (rule W17); % NDG: points C and T_c are not the same; points E_c and C are not the same % DET: points C and T_c are not the same;
8. Using the point E_c and the line CO , construct a line $m(H_bH_a)$ (rule W16); ;
9. Using the line $m(H_bH_a)$ and the line c , construct a point M_c (rule W03); % NDG: lines $m(H_bH_a)$ and c are not parallel % DET: lines $m(H_bH_a)$ and c are not the same;
10. Using the point M_c and the point C , construct a point G (rule W01); ;
11. Using the point H and the point G , construct a point N (rule W01); ;

12. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
13. Using the circle $k(N, M_a)$ and the line $m(HH_c)$, construct a point E_a and a point E_b (rule W04); % NDG: line $m(HH_c)$ and circle $k(N, M_a)$ intersect;
14. Using the point E_a and the point H , construct a point A (rule W01); ;
15. Using the point H and the point E_b , construct a point B (rule W01); .

Non-degenerate conditions: line $m(HH_c)$ and circle $k(N, M_a)$ intersect; points E_c and N are not the same; lines $m(H_bH_a)$ and c are not parallel; points C and T_c are not the same; points E_c and C are not the same; lines c and h_c are not parallel.

Determination conditions: lines $m(H_bH_a)$ and c are not the same; points C and T_c are not the same; points H and H_c are not the same; lines c and h_c are not the same; points T_c and C are not the same; points E_c and H are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W14,W16,W17]

Lemmas used: [D10,D20,D25,D28,D29,D30,D32,D7,GD01,GD02,GL01,GL03,GL04,GL09,GL17,L106,L16,L22,

Solving time: 124.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
```

```
point H 80 72.73
```

```
point T_{c} 55.38 72.43
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{c}
```

```
cmark_rt H
```

```
cmark_rt T_{c}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point C such that E_{c}C/E_{c}H=-1
```

```
towards C E_{c} H -1
```

```
cmark_l C
```

```
color 200 200 200
```

```
drawsegment H C
```

```
color 0 0 0
```

```
% DET: points E_{c} and H are not the same
```

```
% Constructing a line h_{c} which passes through point E_{c} and point H
```

```
line h_{c} E_{c} H
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```

% DET: points T_{c} and C are not the same
% Constructing a line s_{c} which passes through point T_{c} and point C
line s_{c} T_{c} C

color 200 200 200
drawline s_{c}
color 0 0 0

% Constructing a line c which is perpendicular to line h_{c} and which passes through point T_{c}
perp c T_{c} h_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: lines c and h_{c} are not parallel% DET: lines c and h_{c} are not the same
% Constructing a point H_{c} which belongs to line c and line h_{c}
intersec H_{c} c h_{c}
cmark_rt H_{c}

% DET: points H and H_{c} are not the same
% Constructing bisector m(HH_{c}) of the segment HH_{c}
med m(HH_{c}) H H_{c}

color 200 200 200
drawline m(HH_{c})
color 0 0 0

color 200 200 200
drawsegment H H_{c}
color 0 0 0

% NDG: points C and T_{c} are not the same; points E_{c} and C are not the same% DET: points C and
T_{c} are not the same
% Constructing an angle V[_G133255] which is equal to the angle E_{c}CT_{c}
angle_o V[_G133255] E_{c} C T_{c}

% Calculating value angle[_G133334] using formula  $1/\text{pow}(2,0)*V[_G133255]+0/\text{pow}(2,0)*180$ 
expression angle[_G133334] {  $1/\text{pow}(2,0)*V[_G133255]+0/\text{pow}(2,0)*180$  }

% Constructing a point P_{\_G133331} which is an image of the point T_{c} in a rotation around the
point C for the angle  $1/\text{pow}(2,0)*V[_G133255]+0/\text{pow}(2,0)*180$ 
rotate P_{\_G133331} C angle[_G133334] T_{c}
cmark_r P_{\_G133331}

```

```

color 200 200 200
drawarc_p C T_{c} angle[_G133334]
color 0 0 0

% Constructing a line CD which passes through point C and point P_{\_G133331}
line CD C P_{\_G133331}

color 200 200 200
drawline CD
color 0 0 0


% Constructing a line m(H_{b}H_{a}) which contains the point E_{c} and is parallel to the line CD
parallel m(H_{b}H_{a}) E_{c} CD

color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0


% NDG: lines m(H_{b}H_{a}) and c are not parallel% DET: lines m(H_{b}H_{a}) and c are not the same
% Constructing a point M_{c} which belongs to line m(H_{b}H_{a}) and line c
intersec M_{c} m(H_{b}H_{a}) c
cmark_lt M_{c}


% Constructing a line L_{\_G133856} which passes through point M_{c} and point C
line L_{\_G133856} M_{c} C

color 200 200 200
drawline L_{\_G133856}
color 0 0 0


% Constructing a point P_{\_G133957} with coordinates (0,0)
point P_{\_G133957} 0 0
cmark_r P_{\_G133957}


% Constructing a point P_{\_G133881} such that M_{c}P_{\_G133881}/M_{c}P_{\_G133957}=1
towards P_{\_G133881} M_{c} P_{\_G133957} 1
cmark_r P_{\_G133881}
color 200 200 200
drawsegment M_{c} P_{\_G133881}
color 0 0 0


% Constructing a point P_{\_G133926} such that M_{c}P_{\_G133926}/M_{c}P_{\_G133957}=3
towards P_{\_G133926} M_{c} P_{\_G133957} 3
cmark_r P_{\_G133926}
color 200 200 200
drawsegment M_{c} P_{\_G133926}
color 0 0 0

```

```

% Constructing a line  $L_{\{G133887\}}$  which passes through point  $C$  and point  $P_{\{G133926\}}$ 
line  $L_{\{G133887\}}$  C  $P_{\{G133926\}}$ 

color 200 200 200
drawline  $L_{\{G133887\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G133850\}}$  which contains the point  $P_{\{G133881\}}$  and is parallel to the
line  $L_{\{G133887\}}$ 
parallel  $L_{\{G133850\}}$   $P_{\{G133881\}}$   $L_{\{G133887\}}$ 

color 200 200 200
drawline  $L_{\{G133850\}}$ 
color 0 0 0

% Constructing a point  $G$  which belongs to line  $L_{\{G133850\}}$  and line  $L_{\{G133856\}}$ 
intersec G  $L_{\{G133850\}}$   $L_{\{G133856\}}$ 
cmark_t G

% Constructing a point  $N$  such that  $HN/HG=0.75$ 
towards N H G 0.75
cmark_r N
color 200 200 200
drawsegment H G
color 0 0 0

% NDG: points  $E_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{c\}}$ 
circle  $k(N, M_{\{a\}})$  N  $E_{\{c\}}$ 

color 200 200 200
drawcircle  $k(N, M_{\{a\}})$ 
color 0 0 0

% NDG: line  $m(HH_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect
% Constructing points  $E_{\{a\}}$  and  $E_{\{b\}}$  which are in intersection of  $k(N, M_{\{a\}})$  and  $m(HH_{\{c\}})$ 
intersec2  $E_{\{a\}}$   $E_{\{b\}}$   $k(N, M_{\{a\}})$   $m(HH_{\{c\}})$ 
cmark_r  $E_{\{a\}}$ 
cmark_r  $E_{\{b\}}$ 

% Constructing a point  $A$  such that  $E_{\{a\}}A/E_{\{a\}}H=-1$ 
towards A  $E_{\{a\}}$  H -1
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

```



```

% Constructing a point B such that HB/HE_{b}=2
towards B H E_{b} 2
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions: line m(HH_{c}) and circle k(N,M_{a}) intersect; points E_{c} and N are
% not the same; lines m(H_{b}H_{a}) and c are not parallel; points C and T_{c} are not the same;
% points E_{c} and C are not the same; lines c and h_{c} are not parallel
% Determination conditions: lines m(H_{b}H_{a}) and c are not the same; points C and T_{c} are not
% the same; points H and H_{c} are not the same; lines c and h_{c} are not the same; points T_{c}
% and C are not the same; points E_{c} and H are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = _E_c$

Proving failed

4.1.2 Proving $H = _H$

Proving failed

4.1.3 Proving $T_c = _T_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = _E_c$

Proving failed

4.2.2 Proving $H = _H$

Proving failed

4.2.3 Proving $T_c = _T_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $H = \neg H$

Proving failed

4.3.3 Proving $T_c = \neg T_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $H = \neg H$

Proving failed

4.4.3 Proving $T_c = \neg T_c$

Proving failed

Problem 997

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 997: Given a point E_c , a point H_b and a point H_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_b and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_b and E_c are not the same;
2. Choose freely a point H_a on the circle $k(E_c, C)$ (rule W0ncircle);
3. Using the point H_a and the point E_c , construct a line $m(H_a E_c)$ (rule W14); % DET: points H_a and E_c are not the same;
4. Using the point H_a and the point H_b , construct a line $m(H_a H_b)$ (rule W14); % DET: points H_a and H_b are not the same;
5. Using the line $m(H_a H_b)$ and the line $m(H_a E_c)$, construct a point N (rule W03); % NDG: lines $m(H_a H_b)$ and $m(H_a E_c)$ are not parallel % DET: lines $m(H_a H_b)$ and $m(H_a E_c)$ are not the same;
6. Using the point H_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_a and N are not the same;
7. Using the circle $k(N, M_a)$, the line $m(H_a H_b)$, the point N and the point E_c , construct a point M_c (rule W05a); % NDG: line $m(H_a H_b)$ and circle $k(N, M_a)$ intersect % DET: points E_c and M_c must be different;
8. Choose freely a point A on the circle $k(M_c, A)$ (rule W0ncircle);
9. Using the point A and the point M_c , construct a point B (rule W01); ;
10. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;

11. Using the point H_b and the point B , construct a line h_b (rule W02); % DET: points H_b and B are not the same;
12. Using the line h_a and the line h_b , construct a point H (rule W03); % NDG: lines h_a and h_b are not parallel % DET: lines h_a and h_b are not the same;
13. Using the point E_c and the point H , construct a point C (rule W01); .

Non-degenerate conditions: lines h_a and h_b are not parallel; line $m(H_a H_b)$ and circle $k(N, M_a)$ intersect; points H_a and N are not the same; lines $m(H_a H_b)$ and $m(H_a E_c)$ are not parallel; points H_b and E_c are not the same.

Determination conditions: lines h_a and h_b are not the same; points H_b and B are not the same; points A and H_a are not the same; points E_c and M_c must be different; lines $m(H_a H_b)$ and $m(H_a E_c)$ are not the same; points H_a and H_b are not the same; points H_a and E_c are not the same.

Rules used: [W01,W02,W03,W05a,W06,W14,WOncircle1]

Lemmas used: [D20,D3,D30,D32,D5,D6,D8,D9,GD02,GL01,GL03,GL04,L18,L19,L20,L24,L41,L42,L53,L54]

Solving time: 64.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{c} 95 56.36
point H_{b} 89.36 77.83
point H_{a} 80 40

color 220 0 0
fontsize 9

cmark_r E_{c}
cmark_l H_{b}
cmark_r H_{a}
color 0 0 0
fontsize 8

% NDG: points H_{b} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H_{b}
circle k(E_{c},C) E_{c} H_{b}

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% Choosing randomly a point H_{a} on the circle with center E_{c} through point H_{b}
oncircle H_{a} E_{c} H_{b}
cmark_r H_{a}
color 200 200 200
```

```

drawcircle E_{c} H_{b}
color 0 0 0

% DET: points H_{a} and E_{c} are not the same
% Constructing bisector m(H_{a}E_{c}) of the segment H_{a}E_{c}
med m(H_{a}E_{c}) H_{a} E_{c}

color 200 200 200
drawline m(H_{a}E_{c})
color 0 0 0

color 200 200 200
drawsegment H_{a} E_{c}
color 0 0 0

% DET: points H_{a} and H_{b} are not the same
% Constructing bisector m(H_{a}H_{b}) of the segment H_{a}H_{b}
med m(H_{a}H_{b}) H_{a} H_{b}

color 200 200 200
drawline m(H_{a}H_{b})
color 0 0 0

color 200 200 200
drawsegment H_{a} H_{b}
color 0 0 0

% NDG: lines m(H_{a}H_{b}) and m(H_{a}E_{c}) are not parallel% DET: lines m(H_{a}H_{b}) and m(H_{a}
E_{c}) are not the same
% Constructing a point N which belongs to line m(H_{a}H_{b}) and line m(H_{a}E_{c})
intersec N m(H_{a}H_{b}) m(H_{a}E_{c})
cmark_r N

% NDG: points H_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{a}
circle k(N,M_{a}) N H_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(H_{a}H_{b}) and circle k(N,M_{a}) intersect% DET: points E_{c} and M_{c} must be
different
% Constructing a point M_{c} which is an image of the point E_{c} in the symmetry to point/line N
sim M_{c} N E_{c}
cmark_lt M_{c}

% Choosing randomly a point A on the circle with center M_{c} through point H_{a}

```

```

oncircle A M_{c} H_{a}
cmark_t A
color 200 200 200
drawcircle M_{c} H_{a}
color 0 0 0

% Constructing a point B such that  $AB/AM_c=2$ 
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points A and H_a are not the same
% Constructing a line h_a which passes through point A and point H_a
line h_a A H_a

color 200 200 200
drawline h_a
color 0 0 0

% DET: points H_b and B are not the same
% Constructing a line h_b which passes through point H_b and point B
line h_b H_b B

color 200 200 200
drawline h_b
color 0 0 0

% NDG: lines h_a and h_b are not parallel% DET: lines h_a and h_b are not the same
% Constructing a point H which belongs to line h_a and line h_b
intersec H h_a h_b
cmark_rt H

% Constructing a point C such that  $E_cC/E_cH=-1$ 
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

```

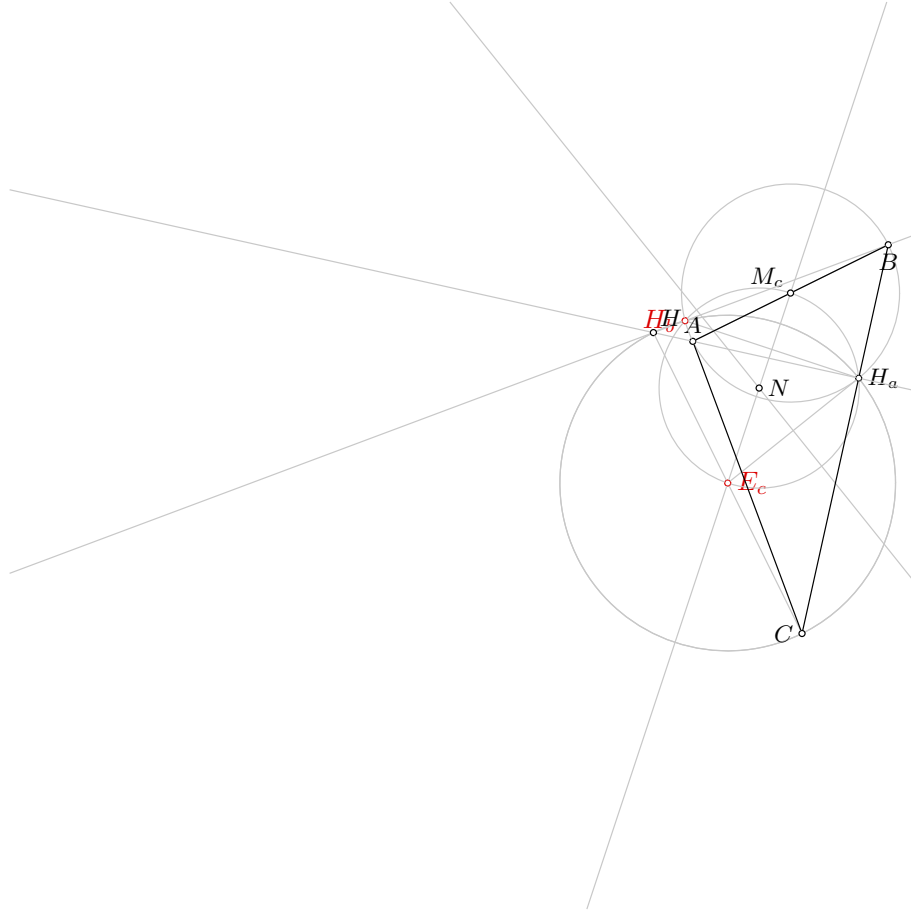


Figure 1: Illustration of the problem 0997

% Non-degenerate conditions: lines $h_{\{a\}}$ and $h_{\{b\}}$ are not parallel; line $m(H_{\{a\}}H_{\{b\}})$ and circle $k(N, M_{\{a\}})$ intersect; points $H_{\{a\}}$ and N are not the same; lines $m(H_{\{a\}}H_{\{b\}})$ and $m(H_{\{a\}}E_{\{c\}})$ are not parallel; points $H_{\{b\}}$ and $E_{\{c\}}$ are not the same
% Determination conditions: lines $h_{\{a\}}$ and $h_{\{b\}}$ are not the same; points $H_{\{b\}}$ and B are not the same; points A and $H_{\{a\}}$ are not the same; points $E_{\{c\}}$ and $M_{\{c\}}$ must be different; lines $m(H_{\{a\}}H_{\{b\}})$ and $m(H_{\{a\}}E_{\{c\}})$ are not the same; points $H_{\{a\}}$ and $H_{\{b\}}$ are not the same; points $H_{\{a\}}$ and $E_{\{c\}}$ are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = E_c$

Proving failed

4.1.2 Proving $H_b = \neg H_b$

Proving failed

4.1.3 Proving $H_a = \neg H_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $H_b = \neg H_b$

Proving failed

4.2.3 Proving $H_a = \neg H_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $H_b = \neg H_b$

Proving failed

4.3.3 Proving $H_a = \neg H_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $H_b = \neg H_b$

Proving failed

4.4.3 Proving $H_a = \neg H_a$

Proving failed

Problem 998

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 998: Given a point E_c , a point H_a and a point H_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;
2. Using the point H_a and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_a and E_c are not the same;
3. Using the circle $k(E_c, C)$ and the line h_c , construct a point C and a point H (rule W04); % NDG: line h_c and circle $k(E_c, C)$ intersect;
4. Using the point H_a and the point C , construct a line a (rule W02); % DET: points H_a and C are not the same;
5. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
6. Using the point H_c and the line h_c , construct a line c (rule W10a); ;
7. Using the line c and the line a , construct a point B (rule W03); % NDG: lines c and a are not parallel % DET: lines c and a are not the same;
8. Using the line h_a and the line c , construct a point A (rule W03); % NDG: lines h_a and c are not parallel % DET: lines h_a and c are not the same.

Non-degenerate conditions: lines h_a and c are not parallel; lines c and a are not parallel; line h_c and circle $k(E_c, C)$ intersect; points H_a and E_c are not the same.

Determination conditions: lines h_a and c are not the same; lines c and a are not the same; points H_a and H are not the same; points H_a and C are not the same; points E_c and H_c are not the same.

Rules used: [W02,W03,W04,W06,W10a]

Lemmas used: [D10,D3,D30,D5,D7,D8,GD01,GD02,GL09,L3,L52,L54]

Solving time: 6.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
point H_{a} 80 40
point H_{c} 68.91 84.83
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{c}
cmark_r H_{a}
cmark_rt H_{c}
color 0 0 0
fontsize 8
```

```
% DET: points E_{c} and H_{c} are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H_{c}
line h_{c} E_{c} H_{c}
```

```
color 200 200 200
drawline h_{c}
color 0 0 0
```

```
% NDG: points H_{a} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H_{a}
circle k(E_{c},C) E_{c} H_{a}
```

```
color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0
```

```
% NDG: line h_{c} and circle k(E_{c},C) intersect
% Constructing points C and H which are in intersection of k(E_{c},C) and h_{c}
intersec2 C H k(E_{c},C) h_{c}
cmark_l C
cmark_rt H
```

```
% DET: points H_{a} and C are not the same
% Constructing a line a which passes through point H_{a} and point C
line a H_{a} C
```

```
color 200 200 200
drawline a
```

```
color 0 0 0
```

```
% DET: points  $H_{\{a\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $H_{\{a\}}$  and point  $H$ 
line  $h_{\{a\}}$   $H_{\{a\}}$   $H$ 
```

```
color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0
```

```
% Constructing a line  $c$  which is perpendicular to line  $h_{\{c\}}$  and which passes through point  $H_{\{c\}}$ 
perp  $c$   $H_{\{c\}}$   $h_{\{c\}}$ 
```

```
color 200 200 200
drawline  $c$ 
color 0 0 0
```

```
% NDG: lines  $c$  and  $a$  are not parallel% DET: lines  $c$  and  $a$  are not the same
% Constructing a point  $B$  which belongs to line  $c$  and line  $a$ 
intersec  $B$   $c$   $a$ 
cmark_b  $B$ 
```

```
% NDG: lines  $h_{\{a\}}$  and  $c$  are not parallel% DET: lines  $h_{\{a\}}$  and  $c$  are not the same
% Constructing a point  $A$  which belongs to line  $h_{\{a\}}$  and line  $c$ 
intersec  $A$   $h_{\{a\}}$   $c$ 
cmark_t  $A$ 
```

```
drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 
```

```
% Non-degenerate conditions: lines  $h_{\{a\}}$  and  $c$  are not parallel; lines  $c$  and  $a$  are not parallel;
line  $h_{\{c\}}$  and circle  $k(E_{\{c\}}, C)$  intersect; points  $H_{\{a\}}$  and  $E_{\{c\}}$  are not the same
% Determination conditions: lines  $h_{\{a\}}$  and  $c$  are not the same; lines  $c$  and  $a$  are not the same;
points  $H_{\{a\}}$  and  $H$  are not the same; points  $H_{\{a\}}$  and  $C$  are not the same; points  $E_{\{c\}}$  and  $H_{\{c\}}$ 
are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

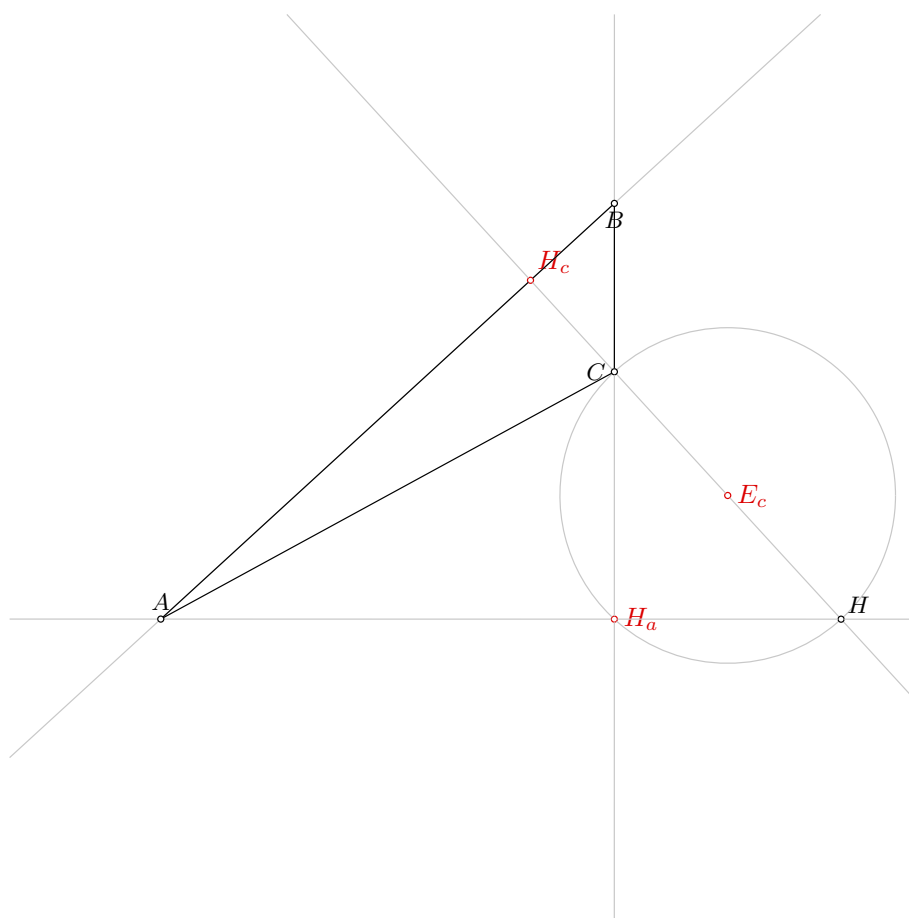


Figure 1: Illustration of the problem 0998

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $H_a = \neg H_a$

Proving failed

4.1.3 Proving $H_c = \neg H_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $H_a = \neg H_a$

Proving failed

4.2.3 Proving $H_c = \neg H_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $H_a = \neg H_a$

Proving failed

4.3.3 Proving $H_c = \neg H_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $H_a = \neg H_a$

Proving failed

4.4.3 Proving $H_c = \neg H_c$

Proving failed

Problem 999

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 999: Given a point E_c , a point H_a and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1000

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1000: Given a point E_c , a point H_a and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
2. Using the point H_a and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_a and E_c are not the same;
3. Using the circle $k(E_c, C)$, the line a , the point E_c and the point H_a , construct a point C (rule W05); % NDG: line a and circle $k(E_c, C)$ intersect % DET: points H_a and C must be different;
4. Using the point C and the point E_c , construct a point H (rule W01); ;
5. Using the point M_a and the point C , construct a point B (rule W01); ;
6. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
7. Using the point H and the point B , construct a line h_b (rule W02); % DET: points H and B are not the same;
8. Using the circle $k(E_c, C)$, the line h_b , the point E_c and the point H , construct a point H_b (rule W05); % NDG: line h_b and circle $k(E_c, C)$ intersect % DET: points H and H_b must be different;
9. Using the point C and the point H_b , construct a line b (rule W02); % DET: points C and H_b are not the same;
10. Using the line h_a and the line b , construct a point A (rule W03); % NDG: lines h_a and b are not parallel % DET: lines h_a and b are not the same.

Non-degenerate conditions: lines h_a and b are not parallel; line h_b and circle $k(E_c, C)$ intersect; line a and circle $k(E_c, C)$ intersect; points H_a and E_c are not the same.

Determination conditions: lines h_a and b are not the same; points C and H_b are not the same; points H and H_b must be different; points H and B are not the same; points H_a and H are not the same; points H_a and C must be different; points H_a and M_a are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D21,D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,GL04,GL09,L52,L53,L54]

Solving time: 9.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{c} 95 56.36
point H_{a} 80 40
point M_{a} 65 40

color 220 0 0
fontsize 9

cmark_r E_{c}
cmark_r H_{a}
cmark_r M_{a}
color 0 0 0
fontsize 8

% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points H_{a} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H_{a}
circle k(E_{c},C) E_{c} H_{a}

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% NDG: line a and circle k(E_{c},C) intersect% DET: points H_{a} and C must be different
% Constructing a point P_{\_G221884} which is a foot of the point E_{c} on the line a
foot P_{\_G221884} E_{c} a
cmark_r P_{\_G221884}
color 200 200 200
drawline E_{c} P_{\_G221884}
color 0 0 0
```

```

% Constructing a point C which is an image of the point H_{a} in the symmetry to point/line P_{\_G
221884}
sim C P_{\_G221884} H_{a}
cmark_l C

% Constructing a point H such that CH/CE_{c}=2
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0

% Constructing a point B such that M_{a}B/M_{a}C=-1
towards B M_{a} C -1
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% DET: points H and B are not the same
% Constructing a line h_{b} which passes through point H and point B
line h_{b} H B

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{b} and circle k(E_{c},C) intersect% DET: points H and H_{b} must be different
% Constructing a point P_{\_G222321} which is a foot of the point E_{c} on the line h_{b}
foot P_{\_G222321} E_{c} h_{b}
cmark_r P_{\_G222321}
color 200 200 200
drawline E_{c} P_{\_G222321}
color 0 0 0

% Constructing a point H_{b} which is an image of the point H in the symmetry to point/line P_{\_G
222321}
sim H_{b} P_{\_G222321} H

```

```

cmark_l H_{b}

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines h_{a} and b are not parallel% DET: lines h_{a} and b are not the same
% Constructing a point A which belongs to line h_{a} and line b
intersec A h_{a} b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{a} and b are not parallel; line h_{b} and circle k(E_{c},C)
% intersect; line a and circle k(E_{c},C) intersect; points H_{a} and E_{c} are not the same
% Determination conditions: lines h_{a} and b are not the same; points C and H_{b} are not the same
% ; points H and H_{b} must be different; points H and B are not the same; points H_{a} and H are
% not the same; points H_{a} and C must be different; points H_{a} and M_{a} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $H_a = \neg H_a$

Proving failed

4.1.3 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 11 terms.

Time Complexity: Time spent by the prover is 0.189 seconds.

NDG conditions Points M_a and H_a are not identical

Points M_a and H_a are not identical

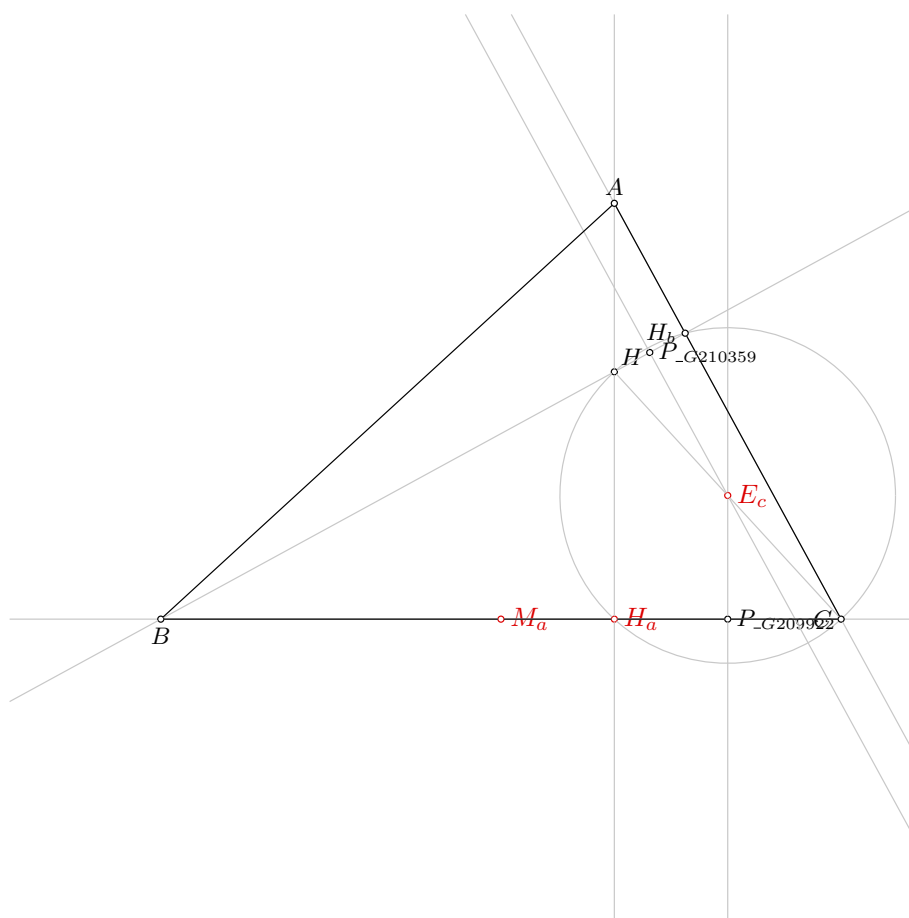


Figure 1: Illustration of the problem 1000

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $H_a = \neg H_a$

Proving failed

4.2.3 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{H_aCH_b} \neq S_{HCH_b}$ i.e., lines H_aH and CH_b are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{\neg h_b}} \neq S_{F^0_{\neg h_a}BF^1_{\neg h_b}}$ i.e., lines $AF^0_{\neg h_a}$ and $BF^1_{\neg h_b}$ are not parallel (construction based assumption)

$S_{BAF^0_{\neg h_a}} \neq S_{CAF^0_{\neg h_a}}$ i.e., lines BC and $AF^0_{\neg h_a}$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $H_a = \neg H_a$

Proving failed

4.3.3 Proving $M_a = \neg M_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $H_a = \neg H_a$

Proving failed

4.4.3 Proving $M_a = \neg M_a$

Proving failed

Problem 1001

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1001: Given a point E_c , a point H_a and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H_a and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_a and E_c are not the same;
2. Using the point H_a and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points H_a and M_b are not the same;
3. Using the circle $k(E_c, C)$, the circle $k(M_b, C)$, the point H_a , the point E_c and the point M_b , construct a point C (rule W08); % NDG: circles $k(E_c, C)$ and $k(M_b, C)$ intersect % DET: circles $k(E_c, C)$ and $k(M_b, C)$ are not the same; points H_a and C must be different;
4. Using the point C and the point E_c , construct a point H (rule W01); ;
5. Using the point M_b and the point C , construct a point A (rule W01); ;
6. Using the point H_a and the point C , construct a line a (rule W02); % DET: points H_a and C are not the same;
7. Using the point M_b and the point C , construct a line b (rule W02); % DET: points M_b and C are not the same;
8. Using the circle $k(E_c, C)$, the line b , the point E_c and the point C , construct a point H_b (rule W05); % NDG: line b and circle $k(E_c, C)$ intersect % DET: points C and H_b must be different;
9. Using the point H and the point H_b , construct a line h_b (rule W02); % DET: points H and H_b are not the same;
10. Using the line a and the line h_b , construct a point B (rule W03); % NDG: lines a and h_b are not parallel % DET: lines a and h_b are not the same.

Non-degenerate conditions: lines a and h_b are not parallel; line b and circle $k(E_c, C)$ intersect; circles $k(E_c, C)$ and $k(M_b, C)$ intersect; points H_a and M_b are not the same; points H_a and E_c are not the same.

Determination conditions: lines a and h_b are not the same; points H and H_b are not the same; points C and H_b must be different; points M_b and C are not the same; points H_a and C are not the same; circles $k(E_c, C)$ and $k(M_b, C)$ are not the same; points H_a and C must be different.

Rules used: [W01,W02,W03,W05,W06,W08]

Lemmas used: [D22,D3,D30,D5,D6,D9,GD01,GD02,GL03,GL04,GL09,L44,L53,L54]

Solving time: 9.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
point H_{a} 80 40
point M_{b} 95 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{c}
cmark_r H_{a}
cmark_lt M_{b}
color 0 0 0
fontsize 8
```

```
% NDG: points H_{a} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H
_{a}
circle k(E_{c},C) E_{c} H_{a}
```

```
color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0
```

```
% NDG: points H_{a} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H
_{a}
circle k(M_{b},C) M_{b} H_{a}
```

```
color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0
```

```
% NDG: circles k(E_{c},C) and k(M_{b},C) intersect% DET: circles k(E_{c},C) and k(M_{b},C) are not
the same; points H_{a} and C must be different
% Constructing a line L_{G249791} which passes through point E_{c} and point M_{b}
line L_{G249791} E_{c} M_{b}
```

```

color 200 200 200
drawline L_{\_G249791}
color 0 0 0

% Constructing a point C which is an image of the point H_{a} in the symmetry to point/line L_{\_G
249791}
sim C L_{\_G249791} H_{a}
cmark_l C

% Constructing a point H such that CH/CE_{c}=2
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0

% Constructing a point A such that M_{b}A/M_{b}C=-1
towards A M_{b} C -1
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

% DET: points H_{a} and C are not the same
% Constructing a line a which passes through point H_{a} and point C
line a H_{a} C

color 200 200 200
drawline a
color 0 0 0

% DET: points M_{b} and C are not the same
% Constructing a line b which passes through point M_{b} and point C
line b M_{b} C

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(E_{c},C) intersect% DET: points C and H_{b} must be different
% Constructing a point P_{\_G250208} which is a foot of the point E_{c} on the line b
foot P_{\_G250208} E_{c} b
cmark_r P_{\_G250208}
color 200 200 200
drawline E_{c} P_{\_G250208}

```



```

color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $C$  in the symmetry to point/line  $P_{\{\backslash\_G$ 
  250208}
sim  $H_{\{b\}}$   $P_{\{\backslash\_G250208\}}$   $C$ 
cmark_l  $H_{\{b\}}$ 

% DET: points  $H$  and  $H_{\{b\}}$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $H$  and point  $H_{\{b\}}$ 
line  $h_{\{b\}}$   $H$   $H_{\{b\}}$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% NDG: lines  $a$  and  $h_{\{b\}}$  are not parallel% DET: lines  $a$  and  $h_{\{b\}}$  are not the same
% Constructing a point  $B$  which belongs to line  $a$  and line  $h_{\{b\}}$ 
intersec  $B$   $a$   $h_{\{b\}}$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $a$  and  $h_{\{b\}}$  are not parallel; line  $b$  and circle  $k(E_{\{c\}},C)$ 
  intersect; circles  $k(E_{\{c\}},C)$  and  $k(M_{\{b\}},C)$  intersect; points  $H_{\{a\}}$  and  $M_{\{b\}}$  are not the same
  ; points  $H_{\{a\}}$  and  $E_{\{c\}}$  are not the same
% Determination conditions: lines  $a$  and  $h_{\{b\}}$  are not the same; points  $H$  and  $H_{\{b\}}$  are not the same
  ; points  $C$  and  $H_{\{b\}}$  must be different; points  $M_{\{b\}}$  and  $C$  are not the same; points  $H_{\{a\}}$  and  $C$ 
  are not the same; circles  $k(E_{\{c\}},C)$  and  $k(M_{\{b\}},C)$  are not the same; points  $H_{\{a\}}$  and  $C$  must
  be different

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $H_a = \neg H_a$

Proving failed

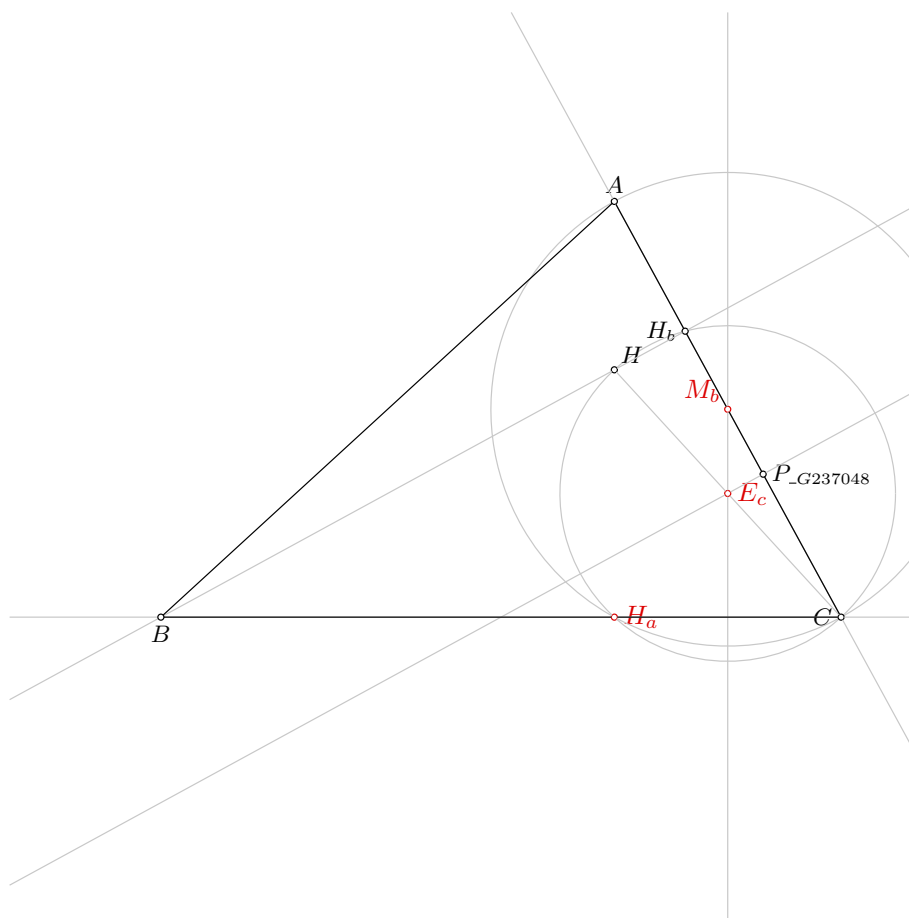


Figure 1: Illustration of the problem 1001

4.1.3 Proving $M_b = M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 11 terms.

Time Complexity: Time spent by the prover is 0.186 seconds.

NDG conditions Points M_b and H_a are not identical

Points M_b and H_a are not identical

4.2 GCLC - Area method

4.2.1 Proving $E_c = E_c$

Proving failed

4.2.2 Proving $H_a = H_a$

Proving failed

4.2.3 Proving $M_b = M_b$

NDG conditions are:

$S_{H_a H H_b} \neq S_{C H H_b}$ i.e., lines $H_a C$ and $H H_b$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{-h_b}} \neq S_{F^0_{-h_a} B F^1_{-h_b}}$ i.e., lines $AF^0_{-h_a}$ and $BF^1_{-h_b}$ are not parallel (construction based assumption)

$S_{BAF^0_{-h_a}} \neq S_{CAF^0_{-h_a}}$ i.e., lines BC and $AF^0_{-h_a}$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_c = E_c$

Proving failed

4.3.2 Proving $H_a = H_a$

Proving failed

4.3.3 Proving $M_b = M_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = E_c$

Proving failed

4.4.2 Proving $H_a = H_a$

Proving failed

4.4.3 Proving $M_b = M_b$

Proving failed

Problem 1002

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1002: Given a point E_c , a point M_c and a point H_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point M_c , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and M_c are not the same;
2. Using the point E_c and the point M_c , construct a line $m(E_cM_c)$ (rule W14); % DET: points E_c and M_c are not the same;
3. Using the line $m(E_cM_c)$ and the line $m(H_bH_a)$, construct a point N (rule W03); % NDG: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel % DET: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same;
4. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
5. Choose freely a point H_a on the circle $k(N, M_a)$ (rule WOncircle);
6. Using the point H_a and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_a and E_c are not the same;
7. Using the circle $k(N, M_a)$, the circle $k(E_c, C)$, the point H_a , the point N and the point E_c , construct a point H_b (rule W08); % NDG: circles $k(N, M_a)$ and $k(E_c, C)$ intersect % DET: circles $k(N, M_a)$ and $k(E_c, C)$ are not the same; points H_a and H_b must be different;
8. Choose freely a point A on the circle $k(M_c, A)$ (rule WOncircle);
9. Using the point A and the point M_c , construct a point B (rule W01); ;
10. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;

11. Using the point H_b and the point B , construct a line h_b (rule W02); % DET: points H_b and B are not the same;
12. Using the line h_a and the line h_b , construct a point H (rule W03); % NDG: lines h_a and h_b are not parallel % DET: lines h_a and h_b are not the same;
13. Using the point E_c and the point H , construct a point C (rule W01); .

Non-degenerate conditions: lines h_a and h_b are not parallel; circles $k(N, M_a)$ and $k(E_c, C)$ intersect; points H_a and E_c are not the same; points E_c and N are not the same; lines $m(E_c M_c)$ and $m(H_b H_a)$ are not parallel.

Determination conditions: lines h_a and h_b are not the same; points H_b and B are not the same; points A and H_a are not the same; circles $k(N, M_a)$ and $k(E_c, C)$ are not the same; points H_a and H_b must be different; lines $m(E_c M_c)$ and $m(H_b H_a)$ are not the same; points E_c and M_c are not the same; points E_c and M_c are not the same.

Rules used: [W01,W02,W03,W06,W08,W14,WOncircle1]

Lemmas used: [D20,D3,D30,D32,D5,D6,D8,D9,GD02,GL01,GL03,GL04,L18,L19,L20,L24,L41,L42,L53,L54]

Solving time: 65.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
```

```
point M_{c} 50 67.5
```

```
point H_{a} 80 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{c}
```

```
cmark_lt M_{c}
```

```
cmark_r H_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{c} and M_{c} are not the same
```

```
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point M_{c}
```

```
line m(H_{b}H_{a}) E_{c} M_{c}
```

```
color 200 200 200
```

```
drawline m(H_{b}H_{a})
```

```
color 0 0 0
```

```
% DET: points E_{c} and M_{c} are not the same
```

```
% Constructing bisector m(E_{c}M_{c}) of the segment E_{c}M_{c}
```

```
med m(E_{c}M_{c}) E_{c} M_{c}
```

```
color 200 200 200
```

```
drawline m(E_{c}M_{c})
```

```

color 0 0 0

color 200 200 200
drawsegment E_{c} M_{c}
color 0 0 0

% NDG: lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not parallel% DET: lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{c\}}M_{\{c\}})$  and line  $m(H_{\{b\}}H_{\{a\}})$ 
intersec N m(E_{c}M_{c}) m(H_{b}H_{a})
cmark_r N

% NDG: points  $E_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N,M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{c\}}$ 
circle k(N,M_{a}) N E_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point  $H_{\{a\}}$  on the circle with center  $N$  through point  $E_{\{c\}}$ 
oncircle H_{a} N E_{c}
cmark_r H_{a}
color 200 200 200
drawcircle N E_{c}
color 0 0 0

% NDG: points  $H_{\{a\}}$  and  $E_{\{c\}}$  are not the same
% Constructing a circle  $k(E_{\{c\}},C)$  whose center is at point  $E_{\{c\}}$  and which passes through point  $H_{\{a\}}$ 
circle k(E_{c},C) E_{c} H_{a}

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% NDG: circles  $k(N,M_{\{a\}})$  and  $k(E_{\{c\}},C)$  intersect% DET: circles  $k(N,M_{\{a\}})$  and  $k(E_{\{c\}},C)$  are not the same; points  $H_{\{a\}}$  and  $H_{\{b\}}$  must be different
% Constructing a line  $L_{\{\backslash\_G53141\}}$  which passes through point  $N$  and point  $E_{\{c\}}$ 
line L_{\_G53141} N E_{c}

color 200 200 200
drawline L_{\_G53141}
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $H_{\{a\}}$  in the symmetry to point/line  $L_{\{\backslash\_G53141\}}$ 

```

```

sim H_{b} L_{\_G53141} H_{a}
cmark_l H_{b}

```

```

% Choosing randomly a point A on the circle with center M_{c} through point H_{a}
oncircle A M_{c} H_{a}
cmark_t A
color 200 200 200
drawcircle M_{c} H_{a}
color 0 0 0

```

```

% Constructing a point B such that  $AB/AM_{c}=2$ 
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

```

```

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

```

```

% DET: points H_{b} and B are not the same
% Constructing a line h_{b} which passes through point H_{b} and point B
line h_{b} H_{b} B

color 200 200 200
drawline h_{b}
color 0 0 0

```

```

% NDG: lines h_{a} and h_{b} are not parallel% DET: lines h_{a} and h_{b} are not the same
% Constructing a point H which belongs to line h_{a} and line h_{b}
intersec H h_{a} h_{b}
cmark_rt H

```

```

% Constructing a point C such that  $E_{c}C/E_{c}H=-1$ 
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

```

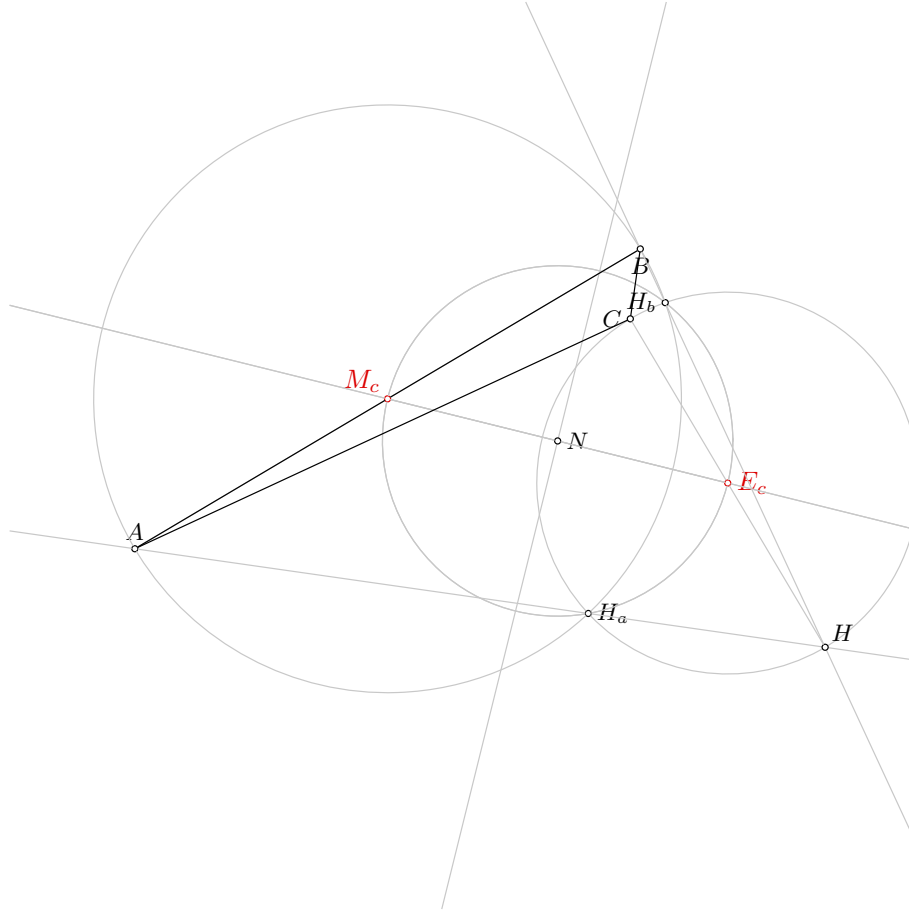


Figure 1: Illustration of the problem 1002

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines  $h_{\{a\}}$  and  $h_{\{b\}}$  are not parallel; circles  $k(N, M_{\{a\}})$  and  $k(E_{\{c\}}, C)$  intersect; points  $H_{\{a\}}$  and  $E_{\{c\}}$  are not the same; points  $E_{\{c\}}$  and  $N$  are not the same; lines  $m(E_{\{c\}}, M_{\{c\}})$  and  $m(H_{\{b\}}, H_{\{a\}})$  are not parallel
% Determination conditions: lines  $h_{\{a\}}$  and  $h_{\{b\}}$  are not the same; points  $H_{\{b\}}$  and  $B$  are not the same; points  $A$  and  $H_{\{a\}}$  are not the same; circles  $k(N, M_{\{a\}})$  and  $k(E_{\{c\}}, C)$  are not the same; points  $H_{\{a\}}$  and  $H_{\{b\}}$  must be different; lines  $m(E_{\{c\}}, M_{\{c\}})$  and  $m(H_{\{b\}}, H_{\{a\}})$  are not the same; points  $E_{\{c\}}$  and  $M_{\{c\}}$  are not the same; points  $E_{\{c\}}$  and  $M_{\{c\}}$  are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $M_c = \neg M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 8 terms.

Time Complexity: Time spent by the prover is 0.14 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $H_a = \neg H_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{m(E_c M_c) E_c M_c}^0 \neq S_{m(E_c M_c) E_c M_c}^1$ i.e., lines $M_{m(E_c M_c)}^0 T_{m(E_c M_c)}^1$ and $E_c M_c$ are not parallel (construction based assumption)

$S_{AH_b B} \neq S_{H_a H_b B}$ i.e., lines AH_a and $H_b B$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2 BF_{\neg h_b}^3}$ i.e., lines $AF_{\neg h_a}^2$ and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

$S_{BAF_{\neg h_a}^2} \neq S_{CAF_{\neg h_a}^2}$ i.e., lines BC and $AF_{\neg h_a}^2$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.2.3 Proving $H_a = \neg H_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $M_c = \neg M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 19 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.3.3 Proving $H_a = H_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = E_c$

Proving failed

4.4.2 Proving $M_c = M_c$

Proving failed

4.4.3 Proving $H_a = H_a$

Proving failed

Problem 1003

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1003: Given a point H_a , a point N and a point E_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_a and N are not the same;
2. Choose freely a point E_c on the circle $k(N, M_a)$ (rule WOncircle);
3. Using the point E_c and the point N , construct a line $m(H_b H_a)$ (rule W02); % DET: points E_c and N are not the same;
4. Using the circle $k(N, M_a)$, the line $m(H_b H_a)$, the point N and the point E_c , construct a point M_c (rule W05a); % NDG: line $m(H_b H_a)$ and circle $k(N, M_a)$ intersect % DET: points E_c and M_c must be different;
5. Using the point H_a and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_a and E_c are not the same;
6. Using the circle $k(N, M_a)$, the circle $k(E_c, C)$, the point H_a , the point N and the point E_c , construct a point H_b (rule W08); % NDG: circles $k(N, M_a)$ and $k(E_c, C)$ intersect % DET: circles $k(N, M_a)$ and $k(E_c, C)$ are not the same; points H_a and H_b must be different;
7. Choose freely a point A on the circle $k(M_c, A)$ (rule WOncircle);
8. Using the point A and the point M_c , construct a point B (rule W01); ;
9. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
10. Using the point H_b and the point B , construct a line h_b (rule W02); % DET: points H_b and B are not the same;

11. Using the line h_a and the line h_b , construct a point H (rule W03); % NDG: lines h_a and h_b are not parallel % DET: lines h_a and h_b are not the same;
12. Using the point E_c and the point H , construct a point C (rule W01); .

Non-degenerate conditions: lines h_a and h_b are not parallel; circles $k(N, M_a)$ and $k(E_c, C)$ intersect; points H_a and E_c are not the same; line $m(H_b H_a)$ and circle $k(N, M_a)$ intersect; points H_a and N are not the same.

Determination conditions: lines h_a and h_b are not the same; points H_b and B are not the same; points A and H_a are not the same; circles $k(N, M_a)$ and $k(E_c, C)$ are not the same; points H_a and H_b must be different; points E_c and M_c must be different; points E_c and N are not the same.

Rules used: [W01,W02,W03,W05a,W06,W08,WOncircle1]

Lemmas used: [D20,D3,D30,D32,D5,D6,D8,D9,GD02,GL01,GL03,GL04,L18,L19,L20,L24,L41,L42,L53,L54]

Solving time: 57.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{a} 80 40
point N 72.5 61.93
point E_{c} 95 56.36
```

```
color 220 0 0
fontsize 9
```

```
cmark_r H_{a}
cmark_r N
cmark_r E_{c}
color 0 0 0
fontsize 8
```

```
% NDG: points H_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{a}
circle k(N,M_{a}) N H_{a}
```

```
color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```
% Choosing randomly a point E_{c} on the circle with center N through point H_{a}
oncircle E_{c} N H_{a}
cmark_r E_{c}
color 200 200 200
drawcircle N H_{a}
color 0 0 0
```

```
% DET: points E_{c} and N are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point N
```

```

line m(H_{b}H_{a}) E_{c} N

color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0

% NDG: line m(H_{b}H_{a}) and circle k(N,M_{a}) intersect% DET: points E_{c} and M_{c} must be
different
% Constructing a point M_{c} which is an image of the point E_{c} in the symmetry to point/line N
sim M_{c} N E_{c}
cmark_lt M_{c}

% NDG: points H_{a} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H
_{a}
circle k(E_{c},C) E_{c} H_{a}

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(E_{c},C) intersect% DET: circles k(N,M_{a}) and k(E_{c},C) are not
the same; points H_{a} and H_{b} must be different
% Constructing a line L_{\_G90838} which passes through point N and point E_{c}
line L_{\_G90838} N E_{c}

color 200 200 200
drawline L_{\_G90838}
color 0 0 0

% Constructing a point H_{b} which is an image of the point H_{a} in the symmetry to point/line L
_{\_G90838}
sim H_{b} L_{\_G90838} H_{a}
cmark_l H_{b}

% Choosing randomly a point A on the circle with center M_{c} through point H_{a}
oncircle A M_{c} H_{a}
cmark_t A
color 200 200 200
drawcircle M_{c} H_{a}
color 0 0 0

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200

```

```

drawsegment A B
color 0 0 0

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% DET: points H_{b} and B are not the same
% Constructing a line h_{b} which passes through point H_{b} and point B
line h_{b} H_{b} B

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines h_{a} and h_{b} are not parallel% DET: lines h_{a} and h_{b} are not the same
% Constructing a point H which belongs to line h_{a} and line h_{b}
intersec H h_{a} h_{b}
cmark_rt H

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{a} and h_{b} are not parallel; circles k(N,M_{a}) and k(E_{c},C) intersect; points H_{a} and E_{c} are not the same; line m(H_{b}H_{a}) and circle k(N,M_{a}) intersect; points H_{a} and N are not the same
% Determination conditions: lines h_{a} and h_{b} are not the same; points H_{b} and B are not the same; points A and H_{a} are not the same; circles k(N,M_{a}) and k(E_{c},C) are not the same; points H_{a} and H_{b} must be different; points E_{c} and M_{c} must be different; points E_{c} and N are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

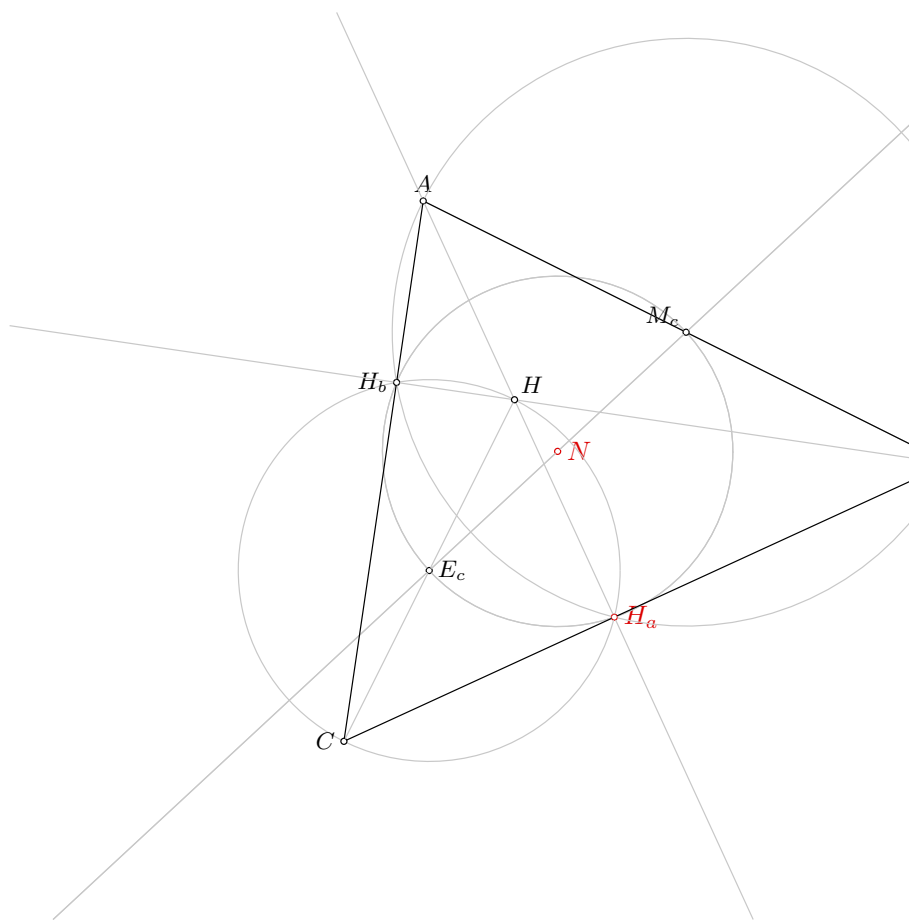


Figure 1: Illustration of the problem 1003

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_a = \neg H_a$

Proving failed

4.1.2 Proving $N = \neg N$

Proving failed

4.1.3 Proving $E_c = \neg E_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_a = \neg H_a$

Proving failed

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $E_c = \neg E_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_a = \neg H_a$

Proving failed

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $E_c = \neg E_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_a = \neg H_a$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $E_c = E_c$

Proving failed

Problem 1004

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1004: Given a point E_c , a point H_a and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1005

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1005: Given a point E_c , a point H_a and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1006

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1006: Given a point E_c , a point H_a and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1007

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1007: Given a point E_c , a point H_a and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1008

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1008: Given a point E_c , a point H_b and a point H_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;
2. Using the point H_b and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_b and E_c are not the same;
3. Using the circle $k(E_c, C)$ and the line h_c , construct a point C and a point H (rule W04); % NDG: line h_c and circle $k(E_c, C)$ intersect;
4. Using the point H_b and the point C , construct a line b (rule W02); % DET: points H_b and C are not the same;
5. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
6. Using the point H_c and the line h_c , construct a line c (rule W10a); ;
7. Using the line c and the line b , construct a point A (rule W03); % NDG: lines c and b are not parallel % DET: lines c and b are not the same;
8. Using the line h_b and the line c , construct a point B (rule W03); % NDG: lines h_b and c are not parallel % DET: lines h_b and c are not the same.

Non-degenerate conditions: lines h_b and c are not parallel; lines c and b are not parallel; line h_c and circle $k(E_c, C)$ intersect; points H_b and E_c are not the same.

Determination conditions: lines h_b and c are not the same; lines c and b are not the same; points H_b and H are not the same; points H_b and C are not the same; points E_c and H_c are not the same.

Rules used: [W02,W03,W04,W06,W10a]

Lemmas used: [D10,D3,D30,D6,D7,D9,GD01,GD02,GL09,L3,L52,L53]
Solving time: 6.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{c} 95 56.36
point H_{b} 89.36 77.83
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_r E_{c}
cmark_l H_{b}
cmark_rt H_{c}
color 0 0 0
fontsize 8

% DET: points E_{c} and H_{c} are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H_{c}
line h_{c} E_{c} H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points H_{b} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H_{b}
circle k(E_{c},C) E_{c} H_{b}

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% NDG: line h_{c} and circle k(E_{c},C) intersect
% Constructing points C and H which are in intersection of k(E_{c},C) and h_{c}
intersec2 C H k(E_{c},C) h_{c}
cmark_l C
cmark_rt H

% DET: points H_{b} and C are not the same
% Constructing a line b which passes through point H_{b} and point C
line b H_{b} C

color 200 200 200
drawline b
color 0 0 0
```

```

% DET: points  $H_{\{b\}}$  and  $H$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $H_{\{b\}}$  and point  $H$ 
line  $h_{\{b\}}$   $H_{\{b\}}$   $H$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

% Constructing a line  $c$  which is perpendicular to line  $h_{\{c\}}$  and which passes through point  $H_{\{c\}}$ 
perp  $c$   $H_{\{c\}}$   $h_{\{c\}}$ 

color 200 200 200
drawline  $c$ 
color 0 0 0

% NDG: lines  $c$  and  $b$  are not parallel% DET: lines  $c$  and  $b$  are not the same
% Constructing a point  $A$  which belongs to line  $c$  and line  $b$ 
intersec  $A$   $c$   $b$ 
cmark_t  $A$ 

% NDG: lines  $h_{\{b\}}$  and  $c$  are not parallel% DET: lines  $h_{\{b\}}$  and  $c$  are not the same
% Constructing a point  $B$  which belongs to line  $h_{\{b\}}$  and line  $c$ 
intersec  $B$   $h_{\{b\}}$   $c$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $h_{\{b\}}$  and  $c$  are not parallel; lines  $c$  and  $b$  are not parallel;
% line  $h_{\{c\}}$  and circle  $k(E_{\{c\}}, C)$  intersect; points  $H_{\{b\}}$  and  $E_{\{c\}}$  are not the same
% Determination conditions: lines  $h_{\{b\}}$  and  $c$  are not the same; lines  $c$  and  $b$  are not the same;
% points  $H_{\{b\}}$  and  $H$  are not the same; points  $H_{\{b\}}$  and  $C$  are not the same; points  $E_{\{c\}}$  and  $H_{\{c\}}$ 
% are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

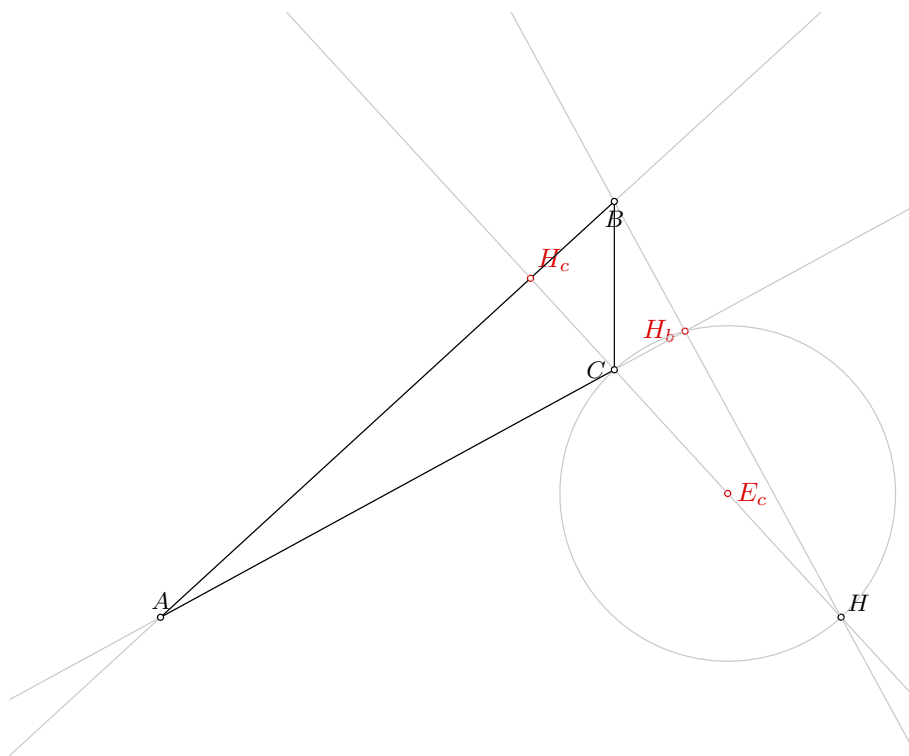


Figure 1: Illustration of the problem 1008

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $H_b = \neg H_b$

Proving failed

4.1.3 Proving $H_c = \neg H_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $H_b = \neg H_b$

Proving failed

4.2.3 Proving $H_c = \neg H_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $H_b = \neg H_b$

Proving failed

4.3.3 Proving $H_c = \neg H_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $H_b = \neg H_b$

Proving failed

4.4.3 Proving $H_c = \neg H_c$

Proving failed

Problem 1009

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1009: Given a point E_c , a point H_b and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1010

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1010: Given a point E_c , a point H_b and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H_b and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_b and E_c are not the same;
2. Using the point H_b and the point M_a , construct a circle $k(M_a, B)$ (rule W06); % NDG: points H_b and M_a are not the same;
3. Using the circle $k(E_c, C)$, the circle $k(M_a, B)$, the point H_b , the point E_c and the point M_a , construct a point C (rule W08); % NDG: circles $k(E_c, C)$ and $k(M_a, B)$ intersect % DET: circles $k(E_c, C)$ and $k(M_a, B)$ are not the same; points H_b and C must be different;
4. Using the point C and the point E_c , construct a point H (rule W01); ;
5. Using the point M_a and the point C , construct a point B (rule W01); ;
6. Using the point H_b and the point C , construct a line b (rule W02); % DET: points H_b and C are not the same;
7. Using the point M_a and the point C , construct a line a (rule W02); % DET: points M_a and C are not the same;
8. Using the circle $k(E_c, C)$, the line a , the point E_c and the point C , construct a point H_a (rule W05); % NDG: line a and circle $k(E_c, C)$ intersect % DET: points C and H_a must be different;
9. Using the point H and the point H_a , construct a line h_a (rule W02); % DET: points H and H_a are not the same;
10. Using the line b and the line h_a , construct a point A (rule W03); % NDG: lines b and h_a are not parallel % DET: lines b and h_a are not the same.

Non-degenerate conditions: lines b and h_a are not parallel; line a and circle $k(E_c, C)$ intersect; circles $k(E_c, C)$ and $k(M_a, B)$ intersect; points H_b and M_a are not the same; points H_b and E_c are not the same.

Determination conditions: lines b and h_a are not the same; points H and H_a are not the same; points C and H_a must be different; points M_a and C are not the same; points H_b and C are not the same; circles $k(E_c, C)$ and $k(M_a, B)$ are not the same; points H_b and C must be different.

Rules used: [W01,W02,W03,W05,W06,W08]

Lemmas used: [D21,D3,D30,D5,D6,D8,GD01,GD02,GL03,GL04,GL09,L37,L38,L53,L54]

Solving time: 9.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
point H_{b} 89.36 77.83
point M_{a} 65 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{c}
cmark_l H_{b}
cmark_r M_{a}
color 0 0 0
fontsize 8
```

```
% NDG: points H_{b} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H
_{b}
circle k(E_{c},C) E_{c} H_{b}
```

```
color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0
```

```
% NDG: points H_{b} and M_{a} are not the same
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H
_{b}
circle k(M_{a},B) M_{a} H_{b}
```

```
color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0
```

```
% NDG: circles k(E_{c},C) and k(M_{a},B) intersect% DET: circles k(E_{c},C) and k(M_{a},B) are not
the same; points H_{b} and C must be different
% Constructing a line L_{\_G174172} which passes through point E_{c} and point M_{a}
line L_{\_G174172} E_{c} M_{a}
```

```

color 200 200 200
drawline L_{\_G174172}
color 0 0 0

% Constructing a point C which is an image of the point H_{b} in the symmetry to point/line L_{\_G
174172}
sim C L_{\_G174172} H_{b}
cmark_l C

% Constructing a point H such that CH/CE_{c}=2
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0

% Constructing a point B such that M_{a}B/M_{a}C=-1
towards B M_{a} C -1
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% DET: points H_{b} and C are not the same
% Constructing a line b which passes through point H_{b} and point C
line b H_{b} C

color 200 200 200
drawline b
color 0 0 0

% DET: points M_{a} and C are not the same
% Constructing a line a which passes through point M_{a} and point C
line a M_{a} C

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(E_{c},C) intersect% DET: points C and H_{a} must be different
% Constructing a point P_{\_G174589} which is a foot of the point E_{c} on the line a
foot P_{\_G174589} E_{c} a
cmark_r P_{\_G174589}
color 200 200 200
drawline E_{c} P_{\_G174589}

```

```

color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $C$  in the symmetry to point/line  $P_{\{\backslash\_G$ 
174589}
sim  $H_{\{a\}}$   $P_{\{\backslash\_G174589\}}$   $C$ 
cmark_r  $H_{\{a\}}$ 

% DET: points  $H$  and  $H_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $H$  and point  $H_{\{a\}}$ 
line  $h_{\{a\}}$   $H$   $H_{\{a\}}$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: lines  $b$  and  $h_{\{a\}}$  are not parallel% DET: lines  $b$  and  $h_{\{a\}}$  are not the same
% Constructing a point  $A$  which belongs to line  $b$  and line  $h_{\{a\}}$ 
intersec  $A$   $b$   $h_{\{a\}}$ 
cmark_t  $A$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: lines  $b$  and  $h_{\{a\}}$  are not parallel; line  $a$  and circle  $k(E_{\{c\}},C)$ 
intersect; circles  $k(E_{\{c\}},C)$  and  $k(M_{\{a\}},B)$  intersect; points  $H_{\{b\}}$  and  $M_{\{a\}}$  are not the same
; points  $H_{\{b\}}$  and  $E_{\{c\}}$  are not the same
% Determination conditions: lines  $b$  and  $h_{\{a\}}$  are not the same; points  $H$  and  $H_{\{a\}}$  are not the same
; points  $C$  and  $H_{\{a\}}$  must be different; points  $M_{\{a\}}$  and  $C$  are not the same; points  $H_{\{b\}}$  and  $C$ 
are not the same; circles  $k(E_{\{c\}},C)$  and  $k(M_{\{a\}},B)$  are not the same; points  $H_{\{b\}}$  and  $C$  must
be different

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $H_b = \neg H_b$

Proving failed

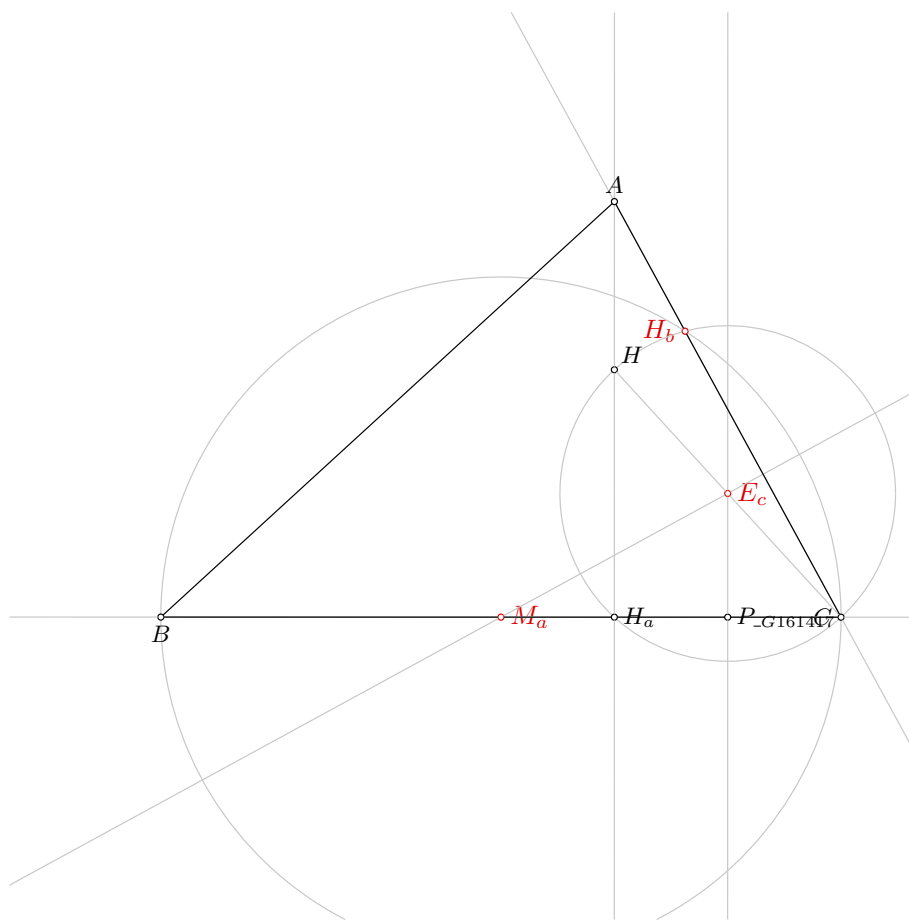


Figure 1: Illustration of the problem 1010

4.1.3 Proving $M_a = M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 11 terms.

Time Complexity: Time spent by the prover is 0.167 seconds.

NDG conditions Points H_b , E_c and M_a are not collinear

Points H_b , E_c and M_a are not collinear

4.2 GCLC - Area method

4.2.1 Proving $E_c = E_c$

Proving failed

4.2.2 Proving $H_b = H_b$

Proving failed

4.2.3 Proving $M_a = M_a$

NDG conditions are:

$S_{H_b H H_a} \neq S_{C H H_a}$ i.e., lines $H_b C$ and $H H_a$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{-H_b}} \neq S_{F^0_{-H_a} BF^1_{-H_b}}$ i.e., lines $AF^0_{-H_a}$ and $BF^1_{-H_b}$ are not parallel (construction based assumption)

$S_{ABF^1_{-H_b}} \neq S_{CBF^1_{-H_b}}$ i.e., lines AC and $BF^1_{-H_b}$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_c = E_c$

Proving failed

4.3.2 Proving $H_b = H_b$

Proving failed

4.3.3 Proving $M_a = M_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = E_c$

Proving failed

4.4.2 Proving $H_b = H_b$

Proving failed

4.4.3 Proving $M_a = M_a$

Proving failed

Problem 1011

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1011: Given a point E_c , a point H_b and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
2. Using the point H_b and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_b and E_c are not the same;
3. Using the circle $k(E_c, C)$, the line b , the point E_c and the point H_b , construct a point C (rule W05); % NDG: line b and circle $k(E_c, C)$ intersect % DET: points H_b and C must be different;
4. Using the point C and the point E_c , construct a point H (rule W01); ;
5. Using the point M_b and the point C , construct a point A (rule W01); ;
6. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
7. Using the point H and the point A , construct a line h_a (rule W02); % DET: points H and A are not the same;
8. Using the circle $k(E_c, C)$, the line h_a , the point E_c and the point H , construct a point H_a (rule W05); % NDG: line h_a and circle $k(E_c, C)$ intersect % DET: points H and H_a must be different;
9. Using the point C and the point H_a , construct a line a (rule W02); % DET: points C and H_a are not the same;
10. Using the line h_b and the line a , construct a point B (rule W03); % NDG: lines h_b and a are not parallel % DET: lines h_b and a are not the same.

Non-degenerate conditions: lines h_b and a are not parallel; line h_a and circle $k(E_c, C)$ intersect; line b and circle $k(E_c, C)$ intersect; points H_b and E_c are not the same.

Determination conditions: lines h_b and a are not the same; points C and H_a are not the same; points H and H_a must be different; points H and A are not the same; points H_b and H are not the same; points H_b and C must be different; points H_b and M_b are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D22,D3,D30,D5,D6,D8,D9,GD01,GD02,GL03,GL04,GL09,L52,L53,L54]

Solving time: 9.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{c} 95 56.36
point H_{b} 89.36 77.83
point M_{b} 95 67.5

color 220 0 0
fontsize 9

cmark_r E_{c}
cmark_l H_{b}
cmark_lt M_{b}
color 0 0 0
fontsize 8

% DET: points H_{b} and M_{b} are not the same
% Constructing a line b which passes through point H_{b} and point M_{b}
line b H_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points H_{b} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H_{b}
circle k(E_{c},C) E_{c} H_{b}

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% NDG: line b and circle k(E_{c},C) intersect% DET: points H_{b} and C must be different
% Constructing a point P_{\_G200293} which is a foot of the point E_{c} on the line b
foot P_{\_G200293} E_{c} b
cmark_r P_{\_G200293}
color 200 200 200
drawline E_{c} P_{\_G200293}
color 0 0 0
```

```

% Constructing a point C which is an image of the point H_{b} in the symmetry to point/line P_{\_G
200293}
sim C P_{\_G200293} H_{b}
cmark_l C

% Constructing a point H such that CH/CE_{c}=2
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0

% Constructing a point A such that M_{b}A/M_{b}C=-1
towards A M_{b} C -1
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% DET: points H and A are not the same
% Constructing a line h_{a} which passes through point H and point A
line h_{a} H A

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{a} and circle k(E_{c},C) intersect% DET: points H and H_{a} must be different
% Constructing a point P_{\_G200730} which is a foot of the point E_{c} on the line h_{a}
foot P_{\_G200730} E_{c} h_{a}
cmark_r P_{\_G200730}
color 200 200 200
drawline E_{c} P_{\_G200730}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H in the symmetry to point/line P_{\_G
200730}
sim H_{a} P_{\_G200730} H

```

```

cmark_r H_{a}

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines h_{b} and a are not parallel% DET: lines h_{b} and a are not the same
% Constructing a point B which belongs to line h_{b} and line a
intersec B h_{b} a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{b} and a are not parallel; line h_{a} and circle k(E_{c},C)
% intersect; line b and circle k(E_{c},C) intersect; points H_{b} and E_{c} are not the same
% Determination conditions: lines h_{b} and a are not the same; points C and H_{a} are not the same
% ; points H and H_{a} must be different; points H and A are not the same; points H_{b} and H are
% not the same; points H_{b} and C must be different; points H_{b} and M_{b} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = E_c$

Proving failed

4.1.2 Proving $H_b = H_b$

Proving failed

4.1.3 Proving $M_b = M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 11 terms.

Time Complexity: Time spent by the prover is 0.232 seconds.

NDG conditions Points M_b , H_b and E_c are not collinear

Points M_b , H_b and E_c are not collinear

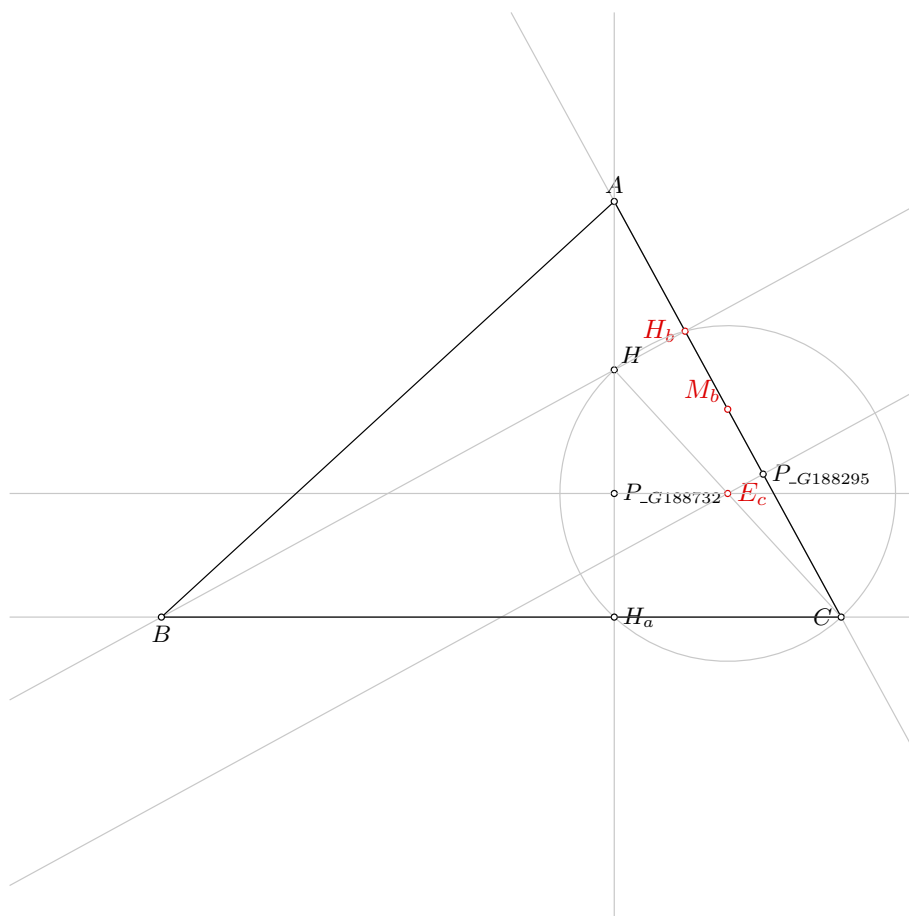


Figure 1: Illustration of the problem 1011

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $H_b = \neg H_b$

Proving failed

4.2.3 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{H_bCH_a} \neq S_{HCH_a}$ i.e., lines H_bH and CH_a are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{\neg h_b}} \neq S_{F^0_{\neg h_a}BF^1_{\neg h_b}}$ i.e., lines $AF^0_{\neg h_a}$ and $BF^1_{\neg h_b}$ are not parallel (construction based assumption)

$S_{ABF^1_{\neg h_b}} \neq S_{CBF^1_{\neg h_b}}$ i.e., lines AC and $BF^1_{\neg h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $H_b = \neg H_b$

Proving failed

4.3.3 Proving $M_b = \neg M_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $H_b = \neg H_b$

Proving failed

4.4.3 Proving $M_b = \neg M_b$

Proving failed

Problem 1012

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1012: Given a point E_c , a point M_c and a point H_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point M_c , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and M_c are not the same;
2. Using the point E_c and the point M_c , construct a line $m(E_cM_c)$ (rule W14); % DET: points E_c and M_c are not the same;
3. Using the line $m(E_cM_c)$ and the line $m(H_bH_a)$, construct a point N (rule W03); % NDG: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel % DET: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same;
4. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
5. Choose freely a point H_b on the circle $k(N, M_a)$ (rule WOncircle);
6. Using the point H_b and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_b and E_c are not the same;
7. Using the circle $k(N, M_a)$, the circle $k(E_c, C)$, the point H_b , the point N and the point E_c , construct a point H_a (rule W08); % NDG: circles $k(N, M_a)$ and $k(E_c, C)$ intersect % DET: circles $k(N, M_a)$ and $k(E_c, C)$ are not the same; points H_b and H_a must be different;
8. Choose freely a point A on the circle $k(M_c, A)$ (rule WOncircle);
9. Using the point A and the point M_c , construct a point B (rule W01); ;
10. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;

11. Using the point H_a and the point B , construct a line a (rule W02); % DET: points H_a and B are not the same;
12. Using the line b and the line a , construct a point C (rule W03); % NDG: lines b and a are not parallel % DET: lines b and a are not the same.

Non-degenerate conditions: lines b and a are not parallel; circles $k(N, M_a)$ and $k(E_c, C)$ intersect; points H_b and E_c are not the same; points E_c and N are not the same; lines $m(E_c M_c)$ and $m(H_b H_a)$ are not parallel.

Determination conditions: lines b and a are not the same; points H_a and B are not the same; points A and H_b are not the same; circles $k(N, M_a)$ and $k(E_c, C)$ are not the same; points H_b and H_a must be different; lines $m(E_c M_c)$ and $m(H_b H_a)$ are not the same; points E_c and M_c are not the same; points E_c and M_c are not the same.

Rules used: [W01,W02,W03,W06,W08,W14,WOncircle1]

Lemmas used: [D20,D32,D5,D6,GD01,GD02,GL01,GL03,L18,L19,L20,L24,L41,L42,L53,L54]

Solving time: 65.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
point M_{c} 50 67.5
point H_{b} 89.36 77.83
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{c}
cmark_lt M_{c}
cmark_l H_{b}
color 0 0 0
fontsize 8
```

```
% DET: points E_{c} and M_{c} are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point M_{c}
line m(H_{b}H_{a}) E_{c} M_{c}
```

```
color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0
```

```
% DET: points E_{c} and M_{c} are not the same
% Constructing bisector m(E_{c}M_{c}) of the segment E_{c}M_{c}
med m(E_{c}M_{c}) E_{c} M_{c}
```

```
color 200 200 200
drawline m(E_{c}M_{c})
color 0 0 0
```

```

color 200 200 200
drawsegment E_{c} M_{c}
color 0 0 0

% NDG: lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not parallel% DET: lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{c\}}M_{\{c\}})$  and line  $m(H_{\{b\}}H_{\{a\}})$ 
intersec N m(E_{c}M_{c}) m(H_{b}H_{a})
cmark_r N

% NDG: points  $E_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N,M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{c\}}$ 
circle k(N,M_{a}) N E_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point  $H_{\{b\}}$  on the circle with center  $N$  through point  $E_{\{c\}}$ 
oncircle H_{b} N E_{c}
cmark_l H_{b}
color 200 200 200
drawcircle N E_{c}
color 0 0 0

% NDG: points  $H_{\{b\}}$  and  $E_{\{c\}}$  are not the same
% Constructing a circle  $k(E_{\{c\}},C)$  whose center is at point  $E_{\{c\}}$  and which passes through point  $H_{\{b\}}$ 
circle k(E_{c},C) E_{c} H_{b}

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% NDG: circles  $k(N,M_{\{a\}})$  and  $k(E_{\{c\}},C)$  intersect% DET: circles  $k(N,M_{\{a\}})$  and  $k(E_{\{c\}},C)$  are not the same; points  $H_{\{b\}}$  and  $H_{\{a\}}$  must be different
% Constructing a line  $L_{\{\backslash\_G228720\}}$  which passes through point  $N$  and point  $E_{\{c\}}$ 
line L_{\_G228720} N E_{c}

color 200 200 200
drawline L_{\_G228720}
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $H_{\{b\}}$  in the symmetry to point/line  $L_{\{\backslash\_G228720\}}$ 
sim H_{a} L_{\_G228720} H_{b}
cmark_r H_{a}

```

```

% Choosing randomly a point A on the circle with center  $M_{\{c\}}$  through point  $H_{\{b\}}$ 
oncircle A  $M_{\{c\}}$   $H_{\{b\}}$ 
cmark_t A
color 200 200 200
drawcircle  $M_{\{c\}}$   $H_{\{b\}}$ 
color 0 0 0

% Constructing a point B such that  $AB/AM_{\{c\}}=2$ 
towards B A  $M_{\{c\}}$  2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points A and  $H_{\{b\}}$  are not the same
% Constructing a line b which passes through point A and point  $H_{\{b\}}$ 
line b A  $H_{\{b\}}$ 

color 200 200 200
drawline b
color 0 0 0

% DET: points  $H_{\{a\}}$  and B are not the same
% Constructing a line a which passes through point  $H_{\{a\}}$  and point B
line a  $H_{\{a\}}$  B

color 200 200 200
drawline a
color 0 0 0

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and a are not parallel; circles  $k(N, M_{\{a\}})$  and  $k(E_{\{c\}}, C)$ 
% intersect; points  $H_{\{b\}}$  and  $E_{\{c\}}$  are not the same; points  $E_{\{c\}}$  and N are not the same; lines
%  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not parallel
% Determination conditions: lines b and a are not the same; points  $H_{\{a\}}$  and B are not the same;
% points A and  $H_{\{b\}}$  are not the same; circles  $k(N, M_{\{a\}})$  and  $k(E_{\{c\}}, C)$  are not the same; points

```

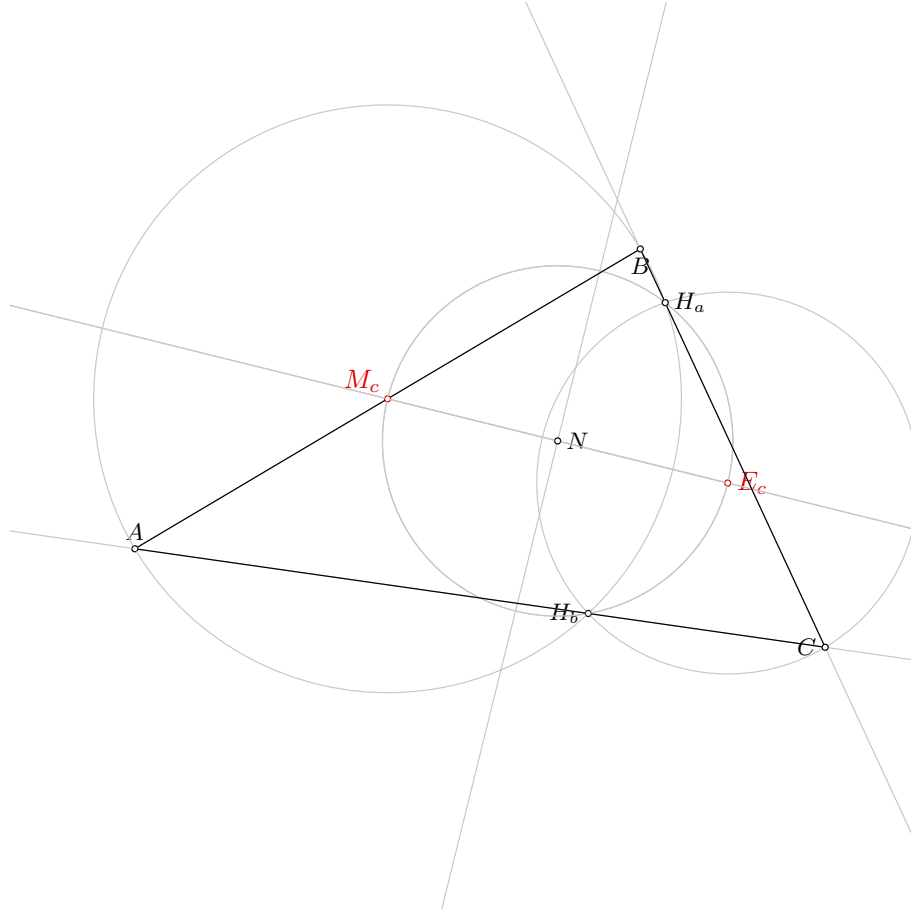


Figure 1: Illustration of the problem 1012

H_b and H_a must be different; lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same; points E_c and M_c are not the same; points E_c and M_c are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = E_c$

Proving failed

4.1.2 Proving $M_c = M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 8 terms.

Time Complexity: Time spent by the prover is 0.16 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $H_b = \neg H_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{M^0_{m(E_c M_c)} E_c M_c} \neq S_{T^1_{m(E_c M_c)} E_c M_c}$ i.e., lines $M^0_{m(E_c M_c)} T^1_{m(E_c M_c)}$ and $E_c M_c$ are not parallel (construction based assumption)

$S_{AH_a B} \neq S_{H_b H_a B}$ i.e., lines AH_b and $H_a B$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^3_{\neg h_b}} \neq S_{F^2_{\neg h_a} BF^3_{\neg h_b}}$ i.e., lines $AF^2_{\neg h_a}$ and $BF^3_{\neg h_b}$ are not parallel (construction based assumption)

$S_{ABF^3_{\neg h_b}} \neq S_{CBF^3_{\neg h_b}}$ i.e., lines AC and $BF^3_{\neg h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.2.3 Proving $H_b = \neg H_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $M_c = \neg M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 19 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.3.3 Proving $H_b = \neg H_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $M_c = M_c$

Proving failed

4.4.3 Proving $H_b = H_b$

Proving failed

Problem 1013

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1013: Given a point H_b , a point N and a point E_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_b and N are not the same;
2. Choose freely a point E_c on the circle $k(N, M_a)$ (rule WOncircle);
3. Using the point E_c and the point N , construct a line $m(H_b H_a)$ (rule W02); % DET: points E_c and N are not the same;
4. Using the circle $k(N, M_a)$, the line $m(H_b H_a)$, the point N and the point E_c , construct a point M_c (rule W05a); % NDG: line $m(H_b H_a)$ and circle $k(N, M_a)$ intersect % DET: points E_c and M_c must be different;
5. Using the point H_b and the point E_c , construct a circle $k(E_c, C)$ (rule W06); % NDG: points H_b and E_c are not the same;
6. Using the circle $k(N, M_a)$, the circle $k(E_c, C)$, the point H_b , the point N and the point E_c , construct a point H_a (rule W08); % NDG: circles $k(N, M_a)$ and $k(E_c, C)$ intersect % DET: circles $k(N, M_a)$ and $k(E_c, C)$ are not the same; points H_b and H_a must be different;
7. Choose freely a point A on the circle $k(M_c, A)$ (rule WOncircle);
8. Using the point A and the point M_c , construct a point B (rule W01); ;
9. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;
10. Using the point H_a and the point B , construct a line a (rule W02); % DET: points H_a and B are not the same;

11. Using the line b and the line a , construct a point C (rule W03); % NDG: lines b and a are not parallel % DET: lines b and a are not the same.

Non-degenerate conditions: lines b and a are not parallel; circles $k(N, M_a)$ and $k(E_c, C)$ intersect; points H_b and E_c are not the same; line $m(H_b H_a)$ and circle $k(N, M_a)$ intersect; points H_b and N are not the same.

Determination conditions: lines b and a are not the same; points H_a and B are not the same; points A and H_b are not the same; circles $k(N, M_a)$ and $k(E_c, C)$ are not the same; points H_b and H_a must be different; points E_c and M_c must be different; points E_c and N are not the same.

Rules used: [W01,W02,W03,W05a,W06,W08,WOncircle1]

Lemmas used: [D20,D32,D5,D6,GD01,GD02,GL01,GL03,L18,L19,L20,L24,L41,L42,L53,L54]

Solving time: 57.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point H_{b} 89.36 77.83
point N 72.5 61.93
point E_{c} 95 56.36

color 220 0 0
fontsize 9

cmark_l H_{b}
cmark_r N
cmark_r E_{c}
color 0 0 0
fontsize 8

% NDG: points H_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{b}
circle k(N,M_{a}) N H_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point E_{c} on the circle with center N through point H_{b}
oncircle E_{c} N H_{b}
cmark_r E_{c}
color 200 200 200
drawcircle N H_{b}
color 0 0 0

% DET: points E_{c} and N are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point N
line m(H_{b}H_{a}) E_{c} N
```



```

color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0

% NDG: line m(H_{b}H_{a}) and circle k(N,M_{a}) intersect% DET: points E_{c} and M_{c} must be
different
% Constructing a point M_{c} which is an image of the point E_{c} in the symmetry to point/line N
sim M_{c} N E_{c}
cmark_lt M_{c}

% NDG: points H_{b} and E_{c} are not the same
% Constructing a circle k(E_{c},C) whose center is at point E_{c} and which passes through point H
_{b}
circle k(E_{c},C) E_{c} H_{b}

color 200 200 200
drawcircle k(E_{c},C)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(E_{c},C) intersect% DET: circles k(N,M_{a}) and k(E_{c},C) are not
the same; points H_{b} and H_{a} must be different
% Constructing a line L_{\_G40337} which passes through point N and point E_{c}
line L_{\_G40337} N E_{c}

color 200 200 200
drawline L_{\_G40337}
color 0 0 0

% Constructing a point H_{a} which is an image of the point H_{b} in the symmetry to point/line L
_{\_G40337}
sim H_{a} L_{\_G40337} H_{b}
cmark_r H_{a}

% Choosing randomly a point A on the circle with center M_{c} through point H_{b}
oncircle A M_{c} H_{b}
cmark_t A
color 200 200 200
drawcircle M_{c} H_{b}
color 0 0 0

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

```

```

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points H_{a} and B are not the same
% Constructing a line a which passes through point H_{a} and point B
line a H_{a} B

color 200 200 200
drawline a
color 0 0 0

% NDG: lines b and a are not parallel% DET: lines b and a are not the same
% Constructing a point C which belongs to line b and line a
intersec C b a
cmark_l C

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and a are not parallel; circles k(N,M_{a}) and k(E_{c},C)
% intersect; points H_{b} and E_{c} are not the same; line m(H_{b}H_{a}) and circle k(N,M_{a})
% intersect; points H_{b} and N are not the same
% Determination conditions: lines b and a are not the same; points H_{a} and B are not the same;
% points A and H_{b} are not the same; circles k(N,M_{a}) and k(E_{c},C) are not the same; points
% H_{b} and H_{a} must be different; points E_{c} and M_{c} must be different; points E_{c} and
% N are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_b = _H_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 2976 terms.

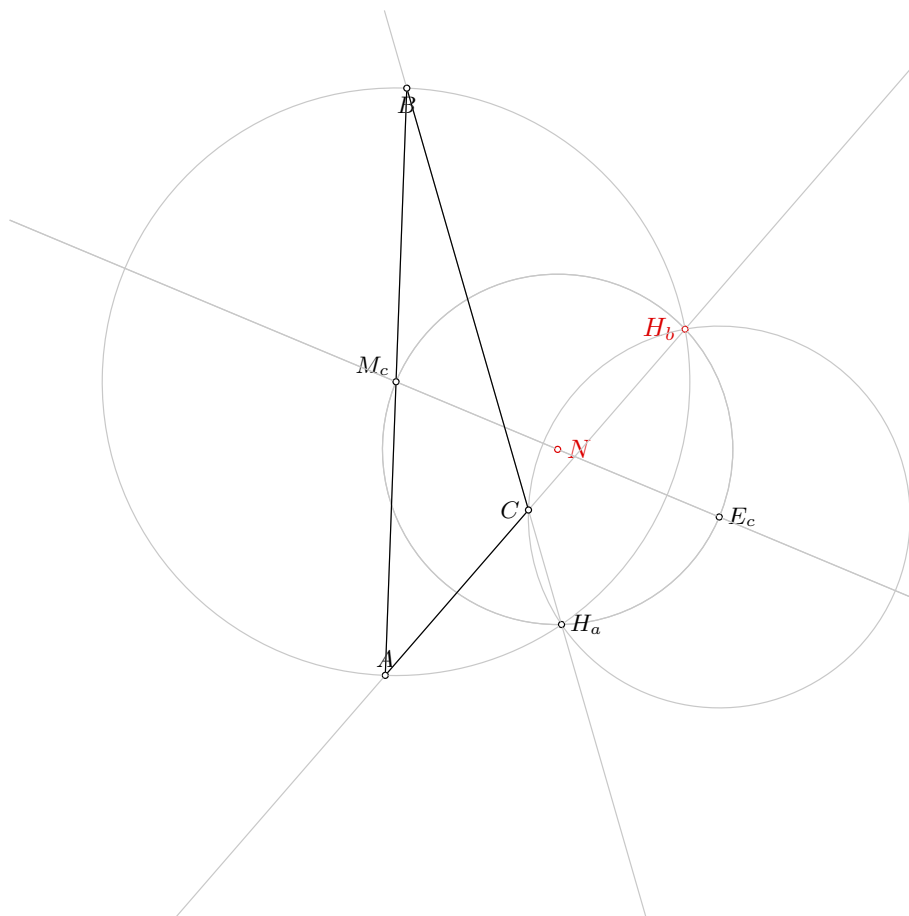


Figure 1: Illustration of the problem 1013

Time Complexity: Time spent by the prover is 12.568 seconds.

NDG conditions Points E_c and N are not identical

Line through points H_b and N is not perpendicular to line through points N and E_c

Line through points H_b and A is not parallel with line through points B and H_a

Points A and N are not identical

Points A and C are not identical

Line through points H_b and N is not parallel with line through points A and C

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $E_c=_E_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_b=_H_b$

Proving failed

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $E_c=_E_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_b=_H_b$

Proving failed

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $E_c=_E_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_b=_H_b$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $E_c=_E_c$

Proving failed

Problem 1014

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1014: Given a point E_c , a point H_b and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1015

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1015: Given a point E_c , a point H_b and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1016

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1016: Given a point E_c , a point H_b and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1017

*Generated automatically by ArgoTriCS
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1 Problem

Problem 1017: Given a point E_c , a point H_b and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1018

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1018: Given a point E_c , a point H_c and a point I , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1019

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1019: Given a point E_c , a point H_c and a point M_a , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;
2. Using the point H_c and the point M_a , construct a circle $k(M_a, B)$ (rule W06); % NDG: points H_c and M_a are not the same;
3. Using the circle $k(M_a, B)$, the line h_c , the point M_a and the point H_c , construct a point C (rule W05); % NDG: line h_c and circle $k(M_a, B)$ intersect % DET: points H_c and C must be different;
4. Using the point C and the point E_c , construct a point H (rule W01); ;
5. Using the point M_a and the point C , construct a point B (rule W01); ;
6. Using the point H_c and the point B , construct a line c (rule W02); % DET: points H_c and B are not the same;
7. Using the point H and the point B , construct a line h_b (rule W02); % DET: points H and B are not the same;
8. Using the circle $k(M_a, B)$, the line h_b , the point M_a and the point B , construct a point H_b (rule W05); % NDG: line h_b and circle $k(M_a, B)$ intersect % DET: points B and H_b must be different;
9. Using the point C and the point H_b , construct a line b (rule W02); % DET: points C and H_b are not the same;
10. Using the line c and the line b , construct a point A (rule W03); % NDG: lines c and b are not parallel % DET: lines c and b are not the same.

Non-degenerate conditions: lines c and b are not parallel; line h_b and circle $k(M_a, B)$ intersect; line h_c and circle $k(M_a, B)$ intersect; points H_c and M_a are not the same.

Determination conditions: lines c and b are not the same; points C and H_b are not the same; points B and H_b must be different; points H and B are not the same; points H_c and B are not the same; points H_c and C must be different; points E_c and H_c are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D21,D3,D30,D6,D7,D9,GD01,GD02,GL03,GL04,GL09,L3,L37,L38,L39]

Solving time: 8.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
```

```
point H_{c} 68.91 84.83
```

```
point M_{a} 65 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{c}
```

```
cmark_rt H_{c}
```

```
cmark_r M_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{c} and H_{c} are not the same
```

```
% Constructing a line h_{c} which passes through point E_{c} and point H_{c}
```

```
line h_{c} E_{c} H_{c}
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```
% NDG: points H_{c} and M_{a} are not the same
```

```
% Constructing a circle k(M_{a},B) whose center is at point M_{a} and which passes through point H_{c}
```

```
circle k(M_{a},B) M_{a} H_{c}
```

```
color 200 200 200
```

```
drawcircle k(M_{a},B)
```

```
color 0 0 0
```

```
% NDG: line h_{c} and circle k(M_{a},B) intersect% DET: points H_{c} and C must be different
```

```
% Constructing a point P_{\_G98111} which is a foot of the point M_{a} on the line h_{c}
```

```
foot P_{\_G98111} M_{a} h_{c}
```

```
cmark_r P_{\_G98111}
```

```
color 200 200 200
```

```
drawline M_{a} P_{\_G98111}
```

```
color 0 0 0
```

```

% Constructing a point C which is an image of the point H_{c} in the symmetry to point/line P_{\_G
98111}
sim C P_{\_G98111} H_{c}
cmark_l C

% Constructing a point H such that CH/CE_{c}=2
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0

% Constructing a point B such that M_{a}B/M_{a}C=-1
towards B M_{a} C -1
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

% DET: points H_{c} and B are not the same
% Constructing a line c which passes through point H_{c} and point B
line c H_{c} B

color 200 200 200
drawline c
color 0 0 0

% DET: points H and B are not the same
% Constructing a line h_{b} which passes through point H and point B
line h_{b} H B

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: line h_{b} and circle k(M_{a},B) intersect% DET: points B and H_{b} must be different
% Constructing a point P_{\_G98548} which is a foot of the point M_{a} on the line h_{b}
foot P_{\_G98548} M_{a} h_{b}
cmark_r P_{\_G98548}
color 200 200 200
drawline M_{a} P_{\_G98548}
color 0 0 0

% Constructing a point H_{b} which is an image of the point B in the symmetry to point/line P_{\_G
98548}
sim H_{b} P_{\_G98548} B

```

```

cmark_l H_{b}

% DET: points C and H_{b} are not the same
% Constructing a line b which passes through point C and point H_{b}
line b C H_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: lines c and b are not parallel% DET: lines c and b are not the same
% Constructing a point A which belongs to line c and line b
intersec A c b
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and b are not parallel; line h_{b} and circle k(M_{a},B)
% intersect; line h_{c} and circle k(M_{a},B) intersect; points H_{c} and M_{a} are not the same
% Determination conditions: lines c and b are not the same; points C and H_{b} are not the same;
% points B and H_{b} must be different; points H and B are not the same; points H_{c} and B are
% not the same; points H_{c} and C must be different; points E_{c} and H_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $H_c = \neg H_c$

Proving failed

4.1.3 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 8 terms.

Time Complexity: Time spent by the prover is 0.12 seconds.

NDG conditions There are no NDG conditions for this theorem

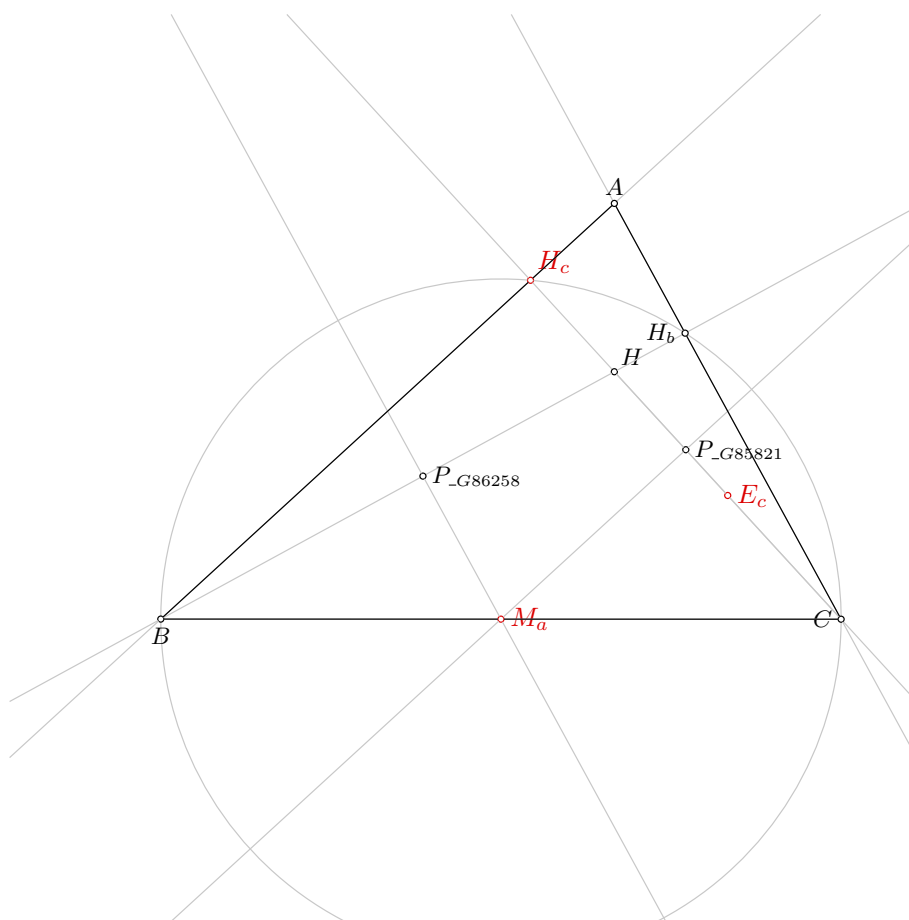


Figure 1: Illustration of the problem 1019

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $H_c = \neg H_c$

Proving failed

4.2.3 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{H_cCH_b} \neq S_{BCH_b}$ i.e., lines H_cB and CH_b are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{\neg h_b}} \neq S_{F^0_{\neg h_a}BF^1_{\neg h_b}}$ i.e., lines $AF^0_{\neg h_a}$ and $BF^1_{\neg h_b}$ are not parallel (construction based assumption)

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF^2_{\neg h_c}} \neq S_{BCF^2_{\neg h_c}}$ i.e., lines AB and $CF^2_{\neg h_c}$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $H_c = \neg H_c$

Proving failed

4.3.3 Proving $M_a = \neg M_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $H_c = \neg H_c$

Proving failed

4.4.3 Proving $M_a = \neg M_a$

Proving failed

Problem 1020

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1020: Given a point E_c , a point H_c and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;
2. Using the point H_c and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points H_c and M_b are not the same;
3. Using the circle $k(M_b, C)$, the line h_c , the point M_b and the point H_c , construct a point C (rule W05); % NDG: line h_c and circle $k(M_b, C)$ intersect % DET: points H_c and C must be different;
4. Using the point C and the point E_c , construct a point H (rule W01); ;
5. Using the point M_b and the point C , construct a point A (rule W01); ;
6. Using the point H_c and the point A , construct a line c (rule W02); % DET: points H_c and A are not the same;
7. Using the point H and the point A , construct a line h_a (rule W02); % DET: points H and A are not the same;
8. Using the circle $k(M_b, C)$, the line h_a , the point M_b and the point A , construct a point H_a (rule W05); % NDG: line h_a and circle $k(M_b, C)$ intersect % DET: points A and H_a must be different;
9. Using the point C and the point H_a , construct a line a (rule W02); % DET: points C and H_a are not the same;
10. Using the line c and the line a , construct a point B (rule W03); % NDG: lines c and a are not parallel % DET: lines c and a are not the same.

Non-degenerate conditions: lines c and a are not parallel; line h_a and circle $k(M_b, C)$ intersect; line h_c and circle $k(M_b, C)$ intersect; points H_c and M_b are not the same.

Determination conditions: lines c and a are not the same; points C and H_a are not the same; points A and H_a must be different; points H and A are not the same; points H_c and A are not the same; points H_c and C must be different; points E_c and H_c are not the same.

Rules used: [W01,W02,W03,W05,W06]

Lemmas used: [D10,D22,D3,D30,D5,D7,D8,GD01,GD02,GL03,GL04,GL09,L3,L43,L44,L45]

Solving time: 8.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
```

```
point H_{c} 68.91 84.83
```

```
point M_{b} 95 67.5
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{c}
```

```
cmark_rt H_{c}
```

```
cmark_lt M_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{c} and H_{c} are not the same
```

```
% Constructing a line h_{c} which passes through point E_{c} and point H_{c}
```

```
line h_{c} E_{c} H_{c}
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```
% NDG: points H_{c} and M_{b} are not the same
```

```
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{c}
```

```
circle k(M_{b},C) M_{b} H_{c}
```

```
color 200 200 200
```

```
drawcircle k(M_{b},C)
```

```
color 0 0 0
```

```
% NDG: line h_{c} and circle k(M_{b},C) intersect% DET: points H_{c} and C must be different
```

```
% Constructing a point P_{\_G124467} which is a foot of the point M_{b} on the line h_{c}
```

```
foot P_{\_G124467} M_{b} h_{c}
```

```
cmark_r P_{\_G124467}
```

```
color 200 200 200
```

```
drawline M_{b} P_{\_G124467}
```

```
color 0 0 0
```

```

% Constructing a point C which is an image of the point H_{c} in the symmetry to point/line P_{\_G
124467}
sim C P_{\_G124467} H_{c}
cmark_l C

% Constructing a point H such that CH/CE_{c}=2
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0

% Constructing a point A such that M_{b}A/M_{b}C=-1
towards A M_{b} C -1
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0

% DET: points H_{c} and A are not the same
% Constructing a line c which passes through point H_{c} and point A
line c H_{c} A

color 200 200 200
drawline c
color 0 0 0

% DET: points H and A are not the same
% Constructing a line h_{a} which passes through point H and point A
line h_{a} H A

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: line h_{a} and circle k(M_{b},C) intersect% DET: points A and H_{a} must be different
% Constructing a point P_{\_G124904} which is a foot of the point M_{b} on the line h_{a}
foot P_{\_G124904} M_{b} h_{a}
cmark_r P_{\_G124904}
color 200 200 200
drawline M_{b} P_{\_G124904}
color 0 0 0

% Constructing a point H_{a} which is an image of the point A in the symmetry to point/line P_{\_G
124904}
sim H_{a} P_{\_G124904} A

```

```

cmark_r H_{a}

% DET: points C and H_{a} are not the same
% Constructing a line a which passes through point C and point H_{a}
line a C H_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: lines c and a are not parallel% DET: lines c and a are not the same
% Constructing a point B which belongs to line c and line a
intersec B c a
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and a are not parallel; line h_{a} and circle k(M_{b},C)
% intersect; line h_{c} and circle k(M_{b},C) intersect; points H_{c} and M_{b} are not the same
% Determination conditions: lines c and a are not the same; points C and H_{a} are not the same;
% points A and H_{a} must be different; points H and A are not the same; points H_{c} and A are
% not the same; points H_{c} and C must be different; points E_{c} and H_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $H_c = \neg H_c$

Proving failed

4.1.3 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 8 terms.

Time Complexity: Time spent by the prover is 0.119 seconds.

NDG conditions There are no NDG conditions for this theorem

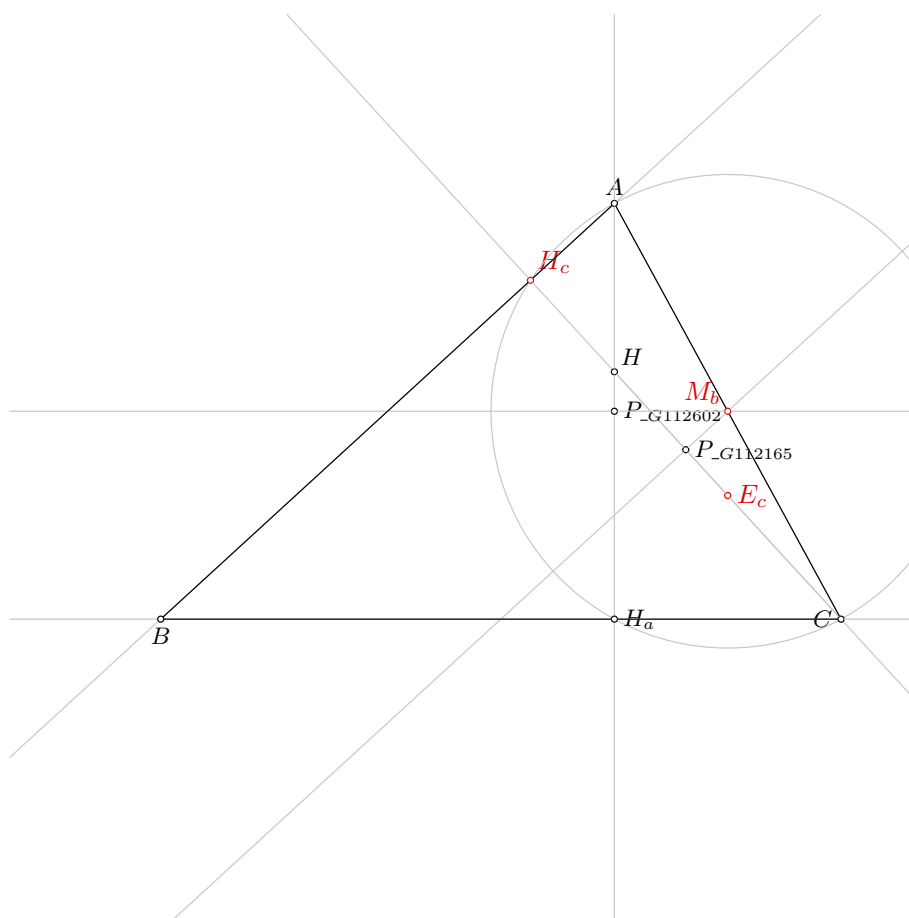


Figure 1: Illustration of the problem 1020

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $H_c = \neg H_c$

Proving failed

4.2.3 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{H_cCH_a} \neq S_{ACH_a}$ i.e., lines H_cA and CH_a are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^1_{\neg h_b}} \neq S_{F^0_{\neg h_a}BF^1_{\neg h_b}}$ i.e., lines $AF^0_{\neg h_a}$ and $BF^1_{\neg h_b}$ are not parallel (construction based assumption)

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF^2_{\neg h_c}} \neq S_{BCF^2_{\neg h_c}}$ i.e., lines AB and $CF^2_{\neg h_c}$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $H_c = \neg H_c$

Proving failed

4.3.3 Proving $M_b = \neg M_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $H_c = \neg H_c$

Proving failed

4.4.3 Proving $M_b = \neg M_b$

Proving failed

Problem 1021

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1021: Given a point H_c , a point M_c and a point E_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
2. Using the point H_c and the line c , construct a line h_c (rule W10b); ;
3. Choose freely a point E_c on the line h_c (rule WOnline2);
4. Using the point E_c and the point M_c , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and M_c are not the same;
5. Using the point E_c and the point H_c , construct a line $m(E_cH_c)$ (rule W14); % DET: points E_c and H_c are not the same;
6. Using the line $m(E_cH_c)$ and the line $m(H_bH_a)$, construct a point N (rule W03); % NDG: lines $m(E_cH_c)$ and $m(H_bH_a)$ are not parallel % DET: lines $m(E_cH_c)$ and $m(H_bH_a)$ are not the same;
7. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
8. Choose freely a point A on the line c (rule WOnline1) ;
9. Using the point A and the point M_c , construct a point B (rule W01); ;
10. Using the point A and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points A and M_c are not the same;

11. Using the circle $k(N, M_a)$ and the circle $k(M_c, A)$, construct a point H_a and a point H_b (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_c, A)$ intersect % DET: circles $k(N, M_a)$ and $k(M_c, A)$ are not the same;
12. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
13. Using the line h_c and the line h_a , construct a point H (rule W03); % NDG: lines h_c and h_a are not parallel % DET: lines h_c and h_a are not the same;
14. Using the point E_c and the point H , construct a point C (rule W01); .

Non-degenerate conditions: lines h_c and h_a are not parallel; circles $k(N, M_a)$ and $k(M_c, A)$ intersect; points A and M_c are not the same; points E_c and N are not the same; lines $m(E_c H_c)$ and $m(H_b H_a)$ are not parallel.

Determination conditions: lines h_c and h_a are not the same; points A and H_a are not the same; circles $k(N, M_a)$ and $k(M_c, A)$ are not the same; lines $m(E_c H_c)$ and $m(H_b H_a)$ are not the same; points E_c and H_c are not the same; points E_c and M_c are not the same; points H_c and M_c are not the same.

Rules used: [W01,W02,W03,W06,W07,W10b,W14,WOnline1,WOnline2]

Lemmas used: [D10,D20,D3,D30,D32,D5,D7,D8,GD01,GD02,GL01,GL03,GL04,GL09,L19,L20,L21,L24,L3,L41]

Solving time: 689.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{c} 68.91 84.83
```

```
point M_{c} 50 67.5
```

```
point E_{c} 95 56.36
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H_{c}
```

```
cmark_lt M_{c}
```

```
cmark_r E_{c}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points H_{c} and M_{c} are not the same
```

```
% Constructing a line c which passes through point H_{c} and point M_{c}
```

```
line c H_{c} M_{c}
```

```
color 200 200 200
```

```
drawline c
```

```
color 0 0 0
```

```
% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
```

```

perp h_{c} H_{c} c

color 200 200 200
drawline h_{c}
color 0 0 0

% Generating random value V[_G154872]
random V[_G154872]

% Calculating value V[_G154893] using formula V[_G154872]*20
expression V[_G154893] { V[_G154872]*20 }

% Constructing a point E_{c} which is a point for which holds  $H_{c}E_{c} = V[_G154893]$  and angle  $M_{c}H_{c}E_{c} = 90$ 
turtle E_{c} M_{c} H_{c} 90 V[_G154893]
cmark_r E_{c}

% DET: points E_{c} and M_{c} are not the same
% Constructing a line  $m(H_{b}H_{a})$  which passes through point E_{c} and point M_{c}
line m(H_{b}H_{a}) E_{c} M_{c}

color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0

% DET: points E_{c} and H_{c} are not the same
% Constructing bisector  $m(E_{c}H_{c})$  of the segment  $E_{c}H_{c}$ 
med m(E_{c}H_{c}) E_{c} H_{c}

color 200 200 200
drawline m(E_{c}H_{c})
color 0 0 0

color 200 200 200
drawsegment E_{c} H_{c}
color 0 0 0

% NDG: lines  $m(E_{c}H_{c})$  and  $m(H_{b}H_{a})$  are not parallel% DET: lines  $m(E_{c}H_{c})$  and  $m(H_{b}H_{a})$  are not the same
% Constructing a point N which belongs to line  $m(E_{c}H_{c})$  and line  $m(H_{b}H_{a})$ 
intersec N m(E_{c}H_{c}) m(H_{b}H_{a})
cmark_r N

% NDG: points E_{c} and N are not the same
% Constructing a circle  $k(N, M_{a})$  whose center is at point N and which passes through point E_{c}
circle k(N, M_{a}) N E_{c}

```



```

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point A on the line H_{c}M_{c}
online A H_{c} M_{c}
cmark_t A
color 200 200 200
drawline H_{c} M_{c}
color 0 0 0

% Constructing a point B such that  $AB/AM_{c}=2$ 
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% NDG: points A and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point A
circle k(M_{c},A) M_{c} A

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(M_{c},A) intersect% DET: circles k(N,M_{a}) and k(M_{c},A) are not
the same
% Constructing points H_{a} and H_{b} which are in intersection of k(N,M_{a}) and k(M_{c},A)
intersec2 H_{a} H_{b} k(N,M_{a}) k(M_{c},A)
cmark_r H_{a}
cmark_l H_{b}

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: lines h_{c} and h_{a} are not parallel% DET: lines h_{c} and h_{a} are not the same
% Constructing a point H which belongs to line h_{c} and line h_{a}
intersec H h_{c} h_{a}

```

```
cmark_rt H
```

```
% Constructing a point C such that  $E_{\{c\}}C/E_{\{c\}}H=-1$ 
towards C  $E_{\{c\}}$  H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines  $h_{\{c\}}$  and  $h_{\{a\}}$  are not parallel; circles  $k(N, M_{\{a\}})$  and  $k(M_{\{c\}}, A)$  intersect; points  $A$  and  $M_{\{c\}}$  are not the same; points  $E_{\{c\}}$  and  $N$  are not the same; lines  $m(E_{\{c\}}H_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not parallel
% Determination conditions: lines  $h_{\{c\}}$  and  $h_{\{a\}}$  are not the same; points  $A$  and  $H_{\{a\}}$  are not the same; circles  $k(N, M_{\{a\}})$  and  $k(M_{\{c\}}, A)$  are not the same; lines  $m(E_{\{c\}}H_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not the same; points  $E_{\{c\}}$  and  $H_{\{c\}}$  are not the same; points  $E_{\{c\}}$  and  $M_{\{c\}}$  are not the same; points  $H_{\{c\}}$  and  $M_{\{c\}}$  are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_c = H_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 13 terms.

Time Complexity: Time spent by the prover is 0.205 seconds.

NDG conditions Points A and E_c are not identical

Points M_c and N are not identical

Line through points N and M_c is not perpendicular to line through points M_c and H_c

Line through points E_c and H_c is not parallel with line through points A and H_a

Line through points A and M_c is not perpendicular to line through points M_c and B

4.1.2 Proving $M_c = M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.039 seconds.

NDG conditions There are no NDG conditions for this theorem

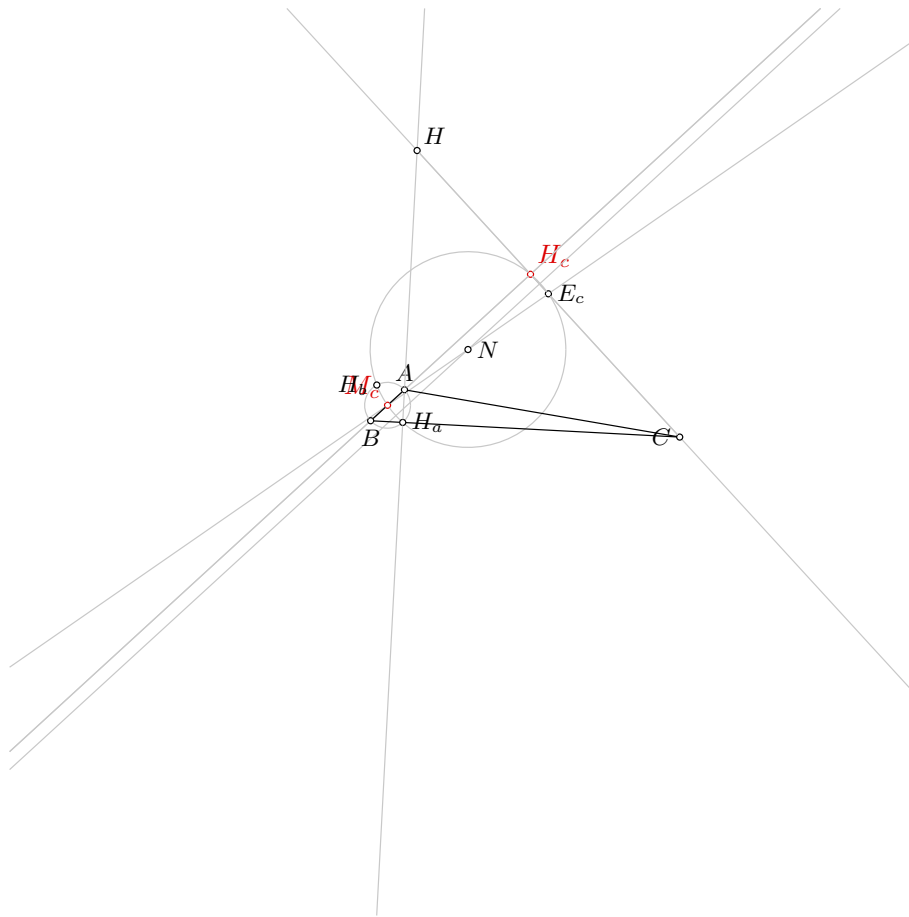


Figure 1: Illustration of the problem 1021

4.1.3 Proving $E_c = E_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_c = H_c$

Proving failed

4.2.2 Proving $M_c = M_c$

Proving failed

4.2.3 Proving $E_c = E_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_c = H_c$

Proving failed

4.3.2 Proving $M_c = M_c$

Proving failed

4.3.3 Proving $E_c = E_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_c = H_c$

Proving failed

4.4.2 Proving $M_c = M_c$

Proving failed

4.4.3 Proving $E_c = E_c$

Proving failed

Problem 1022

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1022: Given a point H_c , a point N and a point E_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_c and N are not the same;
2. Choose freely a point E_c on the circle $k(N, M_a)$ (rule W0ncircle);
3. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;
4. Using the point E_c and the point N , construct a line $m(H_b H_a)$ (rule W02); % DET: points E_c and N are not the same;
5. Using the circle $k(N, M_a)$, the line $m(H_b H_a)$, the point N and the point E_c , construct a point M_c (rule W05a); % NDG: line $m(H_b H_a)$ and circle $k(N, M_a)$ intersect % DET: points E_c and M_c must be different;
6. Using the point M_c and the point H_c , construct a line c (rule W02); % DET: points M_c and H_c are not the same;
7. Choose freely a point A on the line c (rule W0Online1) ;
8. Using the point A and the point M_c , construct a point B (rule W01); ;
9. Using the point A and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points A and M_c are not the same;
10. Using the circle $k(N, M_a)$ and the circle $k(M_c, A)$, construct a point H_a and a point H_b (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_c, A)$ intersect % DET: circles $k(N, M_a)$ and $k(M_c, A)$ are not the same;

11. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
12. Using the line h_c and the line h_a , construct a point H (rule W03); % NDG: lines h_c and h_a are not parallel % DET: lines h_c and h_a are not the same;
13. Using the point E_c and the point H , construct a point C (rule W01); .

Non-degenerate conditions: lines h_c and h_a are not parallel; circles $k(N, M_a)$ and $k(M_c, A)$ intersect; points A and M_c are not the same; line $m(H_b H_a)$ and circle $k(N, M_a)$ intersect; points H_c and N are not the same.

Determination conditions: lines h_c and h_a are not the same; points A and H_a are not the same; circles $k(N, M_a)$ and $k(M_c, A)$ are not the same; points M_c and H_c are not the same; points E_c and M_c must be different; points E_c and N are not the same; points E_c and H_c are not the same.

Rules used: [W01,W02,W03,W05a,W06,W07,WOncircle1,WOnline1]

Lemmas used: [D10,D20,D3,D30,D32,D5,D7,D8,GD01,GD02,GL01,GL03,GL04,GL09,L18,L19,L20,L21,L24,L3]

Solving time: 571.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{c} 68.91 84.83
point N 72.5 61.93
point E_{c} 95 56.36
```

```
color 220 0 0
fontsize 9
```

```
cmark_rt H_{c}
cmark_r N
cmark_r E_{c}
color 0 0 0
fontsize 8
```

```
% NDG: points H_{c} and N are not the same
```

```
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{c}
circle k(N,M_{a}) N H_{c}
```

```
color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```
% Choosing randomly a point E_{c} on the circle with center N through point H_{c}
oncircle E_{c} N H_{c}
cmark_r E_{c}
color 200 200 200
drawcircle N H_{c}
color 0 0 0
```

```

% DET: points E_{c} and H_{c} are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H_{c}
line h_{c} E_{c} H_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% DET: points E_{c} and N are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point N
line m(H_{b}H_{a}) E_{c} N

color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0

% NDG: line m(H_{b}H_{a}) and circle k(N,M_{a}) intersect% DET: points E_{c} and M_{c} must be
different
% Constructing a point M_{c} which is an image of the point E_{c} in the symmetry to point/line N
sim M_{c} N E_{c}
cmark_lt M_{c}

% DET: points M_{c} and H_{c} are not the same
% Constructing a line c which passes through point M_{c} and point H_{c}
line c M_{c} H_{c}

color 200 200 200
drawline c
color 0 0 0

% Choosing randomly a point A on the line H_{c}M_{c}
online A H_{c} M_{c}
cmark_t A
color 200 200 200
drawline H_{c} M_{c}
color 0 0 0

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

```

```

% NDG: points A and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point A
circle k(M_{c},A) M_{c} A

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(M_{c},A) intersect% DET: circles k(N,M_{a}) and k(M_{c},A) are not
the same
% Constructing points H_{a} and H_{b} which are in intersection of k(N,M_{a}) and k(M_{c},A)
intersec2 H_{a} H_{b} k(N,M_{a}) k(M_{c},A)
cmark_r H_{a}
cmark_l H_{b}

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: lines h_{c} and h_{a} are not parallel% DET: lines h_{c} and h_{a} are not the same
% Constructing a point H which belongs to line h_{c} and line h_{a}
intersec H h_{c} h_{a}
cmark_rt H

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and h_{a} are not parallel; circles k(N,M_{a}) and k(M_{c},A) intersect; points A and M_{c} are not the same; line m(H_{b}H_{a}) and circle k(N,M_{a}) intersect; points H_{c} and N are not the same
% Determination conditions: lines h_{c} and h_{a} are not the same; points A and H_{a} are not the same; circles k(N,M_{a}) and k(M_{c},A) are not the same; points M_{c} and H_{c} are not the same; points E_{c} and M_{c} must be different; points E_{c} and N are not the same; points E_{c} and H_{c} are not the same

```

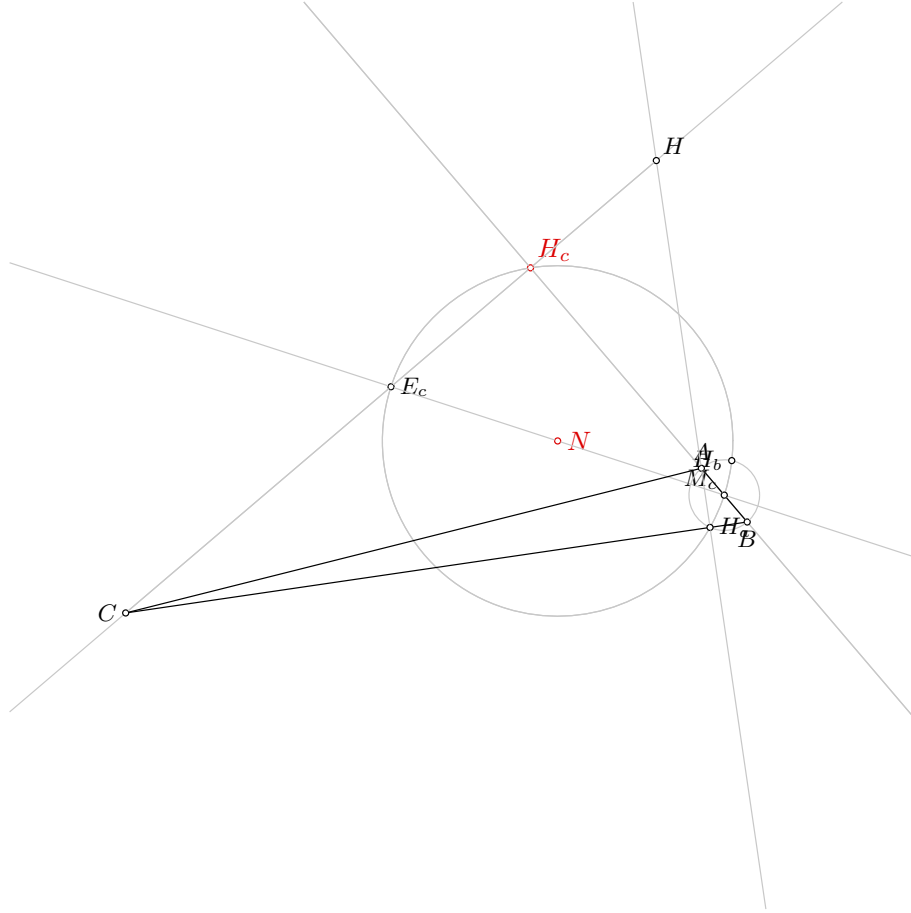



Figure 1: Illustration of the problem 1022

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_c = H_c$

Proving failed

4.1.2 Proving $N = N$

Proving failed

4.1.3 Proving $E_c = E_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_c = \neg H_c$

Proving failed

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $E_c = \neg E_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_c = \neg H_c$

Proving failed

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $E_c = \neg E_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_c = \neg H_c$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $E_c = \neg E_c$

Proving failed

Problem 1023

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1023: Given a point E_c , a point H_c and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;
2. Using the point H_c and the line h_c , construct a line c (rule W10a); ;
3. Using the point O and the line c , construct a line m_c (rule W10b); ;
4. Using the line m_c and the line c , construct a point M_c (rule W03); % NDG: lines m_c and c are not parallel % DET: lines m_c and c are not the same;
5. Using the point M_c and the point E_c , construct a line $m(H_bH_a)$ (rule W02); % DET: points M_c and E_c are not the same;
6. Using the point E_c and the point H_c , construct a line $m(E_cH_c)$ (rule W14); % DET: points E_c and H_c are not the same;
7. Using the line $m(E_cH_c)$ and the line $m(H_bH_a)$, construct a point N (rule W03); % NDG: lines $m(E_cH_c)$ and $m(H_bH_a)$ are not parallel % DET: lines $m(E_cH_c)$ and $m(H_bH_a)$ are not the same;
8. Using the point O and the point N , construct a point H (rule W01); ;
9. Using the point E_c and the point H , construct a point C (rule W01); ;
10. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
11. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; lines $m(E_c H_c)$ and $m(H_b H_a)$ are not parallel; lines m_c and c are not parallel.

Determination conditions: lines $m(E_c H_c)$ and $m(H_b H_a)$ are not the same; points E_c and H_c are not the same; points M_c and E_c are not the same; lines m_c and c are not the same; points E_c and H_c are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W10b,W14]

Lemmas used: [D10,D13,D20,D26,D30,D31,D32,D7,GD01,GD02,GL01,GL03,GL04,GL09,L1,L11,L12,L19,L20,

Solving time: 3.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
```

```
point H_{c} 68.91 84.83
```

```
point O 65 51.14
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{c}
```

```
cmark_rt H_{c}
```

```
cmark_t O
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{c} and H_{c} are not the same
```

```
% Constructing a line h_{c} which passes through point E_{c} and point H_{c}
```

```
line h_{c} E_{c} H_{c}
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```
% Constructing a line c which is perpendicular to line h_{c} and which passes through point H_{c}
```

```
perp c H_{c} h_{c}
```

```
color 200 200 200
```

```
drawline c
```

```
color 0 0 0
```

```
% Constructing a line m_{c} which is perpendicular to line c and which passes through point O
```

```
perp m_{c} O c
```

```
color 200 200 200
```

```
drawline m_{c}
```

```
color 0 0 0
```

```

% NDG: lines  $m_{\{c\}}$  and  $c$  are not parallel% DET: lines  $m_{\{c\}}$  and  $c$  are not the same
% Constructing a point  $M_{\{c\}}$  which belongs to line  $m_{\{c\}}$  and line  $c$ 
intersec  $M_{\{c\}}$   $m_{\{c\}}$   $c$ 
cmark_lt  $M_{\{c\}}$ 

% DET: points  $M_{\{c\}}$  and  $E_{\{c\}}$  are not the same
% Constructing a line  $m(H_{\{b\}}H_{\{a\}})$  which passes through point  $M_{\{c\}}$  and point  $E_{\{c\}}$ 
line  $m(H_{\{b\}}H_{\{a\}})$   $M_{\{c\}}$   $E_{\{c\}}$ 

color 200 200 200
drawline  $m(H_{\{b\}}H_{\{a\}})$ 
color 0 0 0

% DET: points  $E_{\{c\}}$  and  $H_{\{c\}}$  are not the same
% Constructing bisector  $m(E_{\{c\}}H_{\{c\}})$  of the segment  $E_{\{c\}}H_{\{c\}}$ 
med  $m(E_{\{c\}}H_{\{c\}})$   $E_{\{c\}}$   $H_{\{c\}}$ 

color 200 200 200
drawline  $m(E_{\{c\}}H_{\{c\}})$ 
color 0 0 0

color 200 200 200
drawsegment  $E_{\{c\}}$   $H_{\{c\}}$ 
color 0 0 0

% NDG: lines  $m(E_{\{c\}}H_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not parallel% DET: lines  $m(E_{\{c\}}H_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{c\}}H_{\{c\}})$  and line  $m(H_{\{b\}}H_{\{a\}})$ 
intersec  $N$   $m(E_{\{c\}}H_{\{c\}})$   $m(H_{\{b\}}H_{\{a\}})$ 
cmark_r  $N$ 

% Constructing a point  $H$  such that  $OH/ON=2$ 
towards  $H$   $O$   $N$  2
cmark_rt  $H$ 
color 200 200 200
drawsegment  $O$   $H$ 
color 0 0 0

% Constructing a point  $C$  such that  $E_{\{c\}}C/E_{\{c\}}H=-1$ 
towards  $C$   $E_{\{c\}}$   $H$  -1
cmark_l  $C$ 
color 200 200 200
drawsegment  $H$   $C$ 
color 0 0 0

```

```

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
% lines m(E_{c}H_{c}) and m(H_{b}H_{a}) are not parallel; lines m_{c} and c are not parallel
% Determination conditions: lines m(E_{c}H_{c}) and m(H_{b}H_{a}) are not the same; points E_{c}
% and H_{c} are not the same; points M_{c} and E_{c} are not the same; lines m_{c} and c are not
% the same; points E_{c} and H_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = _E_c$

Proving failed

4.1.2 Proving $H_c = _H_c$

Proving failed

4.1.3 Proving $O = _O$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 513 terms.

Time Complexity: Time spent by the prover is 1.126 seconds.

NDG conditions Points A and B are not identical

Points A , B and C are not collinear

Point M_c is not the midpoint of segment with endpoints B and C

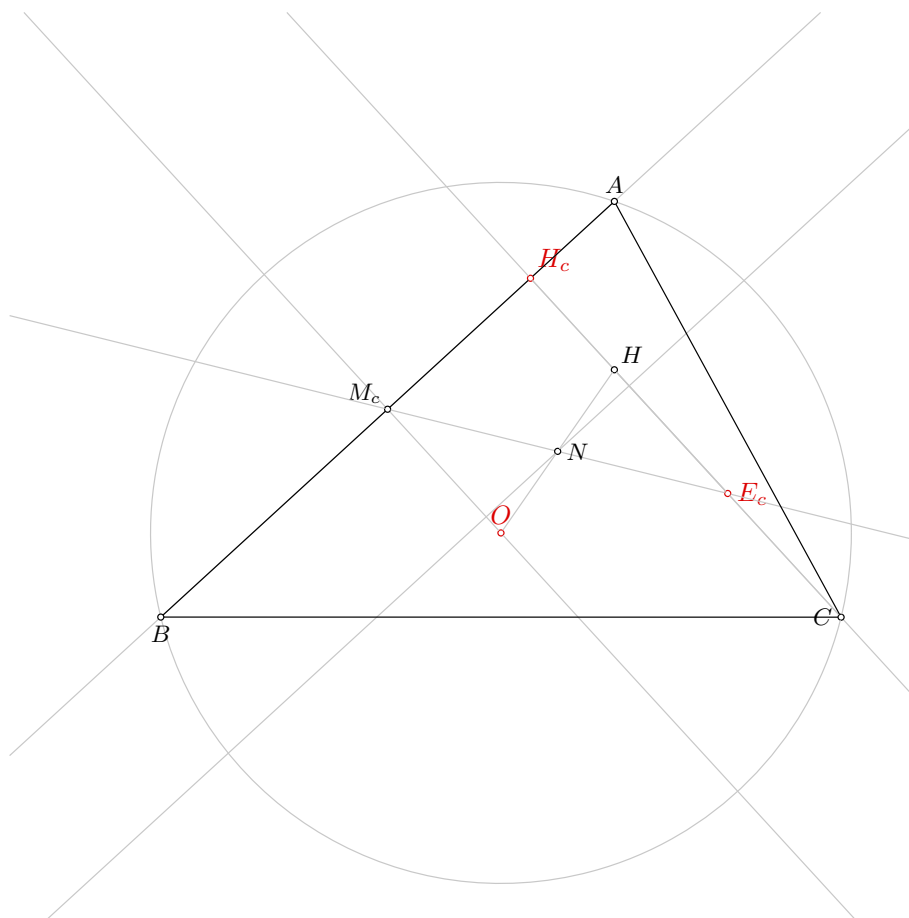


Figure 1: Illustration of the problem 1023

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $H_c = \neg H_c$

Proving failed

4.2.3 Proving $O = \neg O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 8016 terms.

Time Complexity: Time spent by the prover is 24.470 seconds. There are no ndg conditions.

4.3.2 Proving $H_c = \neg H_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 18 terms.

Time Complexity: Time spent by the prover is 0.050 seconds. There are no ndg conditions.

4.3.3 Proving $O = \neg O$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 1497 terms.

Time Complexity: Time spent by the prover is 3.590 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $H_c = \neg H_c$

Proving failed

4.4.3 Proving $O = \neg O$

Proving failed

Problem 1024

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1024: Given a point E_c , a point H_c and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1025

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1025: Given a point E_c , a point H_c and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1026

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1026: Given a point H_c , a point T_c and a point E_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_c and the point T_c , construct a line c (rule W02); % DET: points H_c and T_c are not the same;
2. Using the point H_c and the line c , construct a line h_c (rule W10b); ;
3. Choose freely a point E_c on the line h_c (rule WOnline2);
4. Using the point E_c and the point H_c , construct a line $m(E_cH_c)$ (rule W14); % DET: points E_c and H_c are not the same;
5. Choose freely a point A on the line c (rule WOnline1) ;
6. Using the point A and the point H_c , construct a line $m(AH_c)$ (rule W14); % DET: points A and H_c are not the same;
7. Choose freely a point B on the line c (rule WOnline1) ;
8. Using the point B and the point A , construct a point M_c (rule W01); ;
9. Using the point E_c and the point M_c , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and M_c are not the same;
10. Using the line $m(E_cH_c)$ and the line $m(H_bH_a)$, construct a point N (rule W03); % NDG: lines $m(E_cH_c)$ and $m(H_bH_a)$ are not parallel % DET: lines $m(E_cH_c)$ and $m(H_bH_a)$ are not the same;
11. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;

12. Using the circle $k(N, M_a)$ and the line $m(AH_c)$, construct a point M_b and a point E_a (rule W04); % NDG: line $m(AH_c)$ and circle $k(N, M_a)$ intersect;
13. Using the point A and the point M_b , construct a point C (rule W01); .

Non-degenerate conditions: line $m(AH_c)$ and circle $k(N, M_a)$ intersect; points E_c and N are not the same; lines $m(E_cH_c)$ and $m(H_bH_a)$ are not parallel.

Determination conditions: lines $m(E_cH_c)$ and $m(H_bH_a)$ are not the same; points E_c and M_c are not the same; points A and H_c are not the same; points E_c and H_c are not the same; points H_c and T_c are not the same.

Rules used: [W01,W02,W03,W04,W06,W10b,W14,WOnline1,WOnline2]

Lemmas used: [D10,D20,D22,D25,D32,D7,GD02,GL01,GL03,GL04,L17,L19,L20,L21,L22,L24,L41,L42,L43,L45]

Solving time: 209.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{c} 68.91 84.83
point T_{c} 55.38 72.43
point E_{c} 95 56.36
```

```
color 220 0 0
fontsize 9
```

```
cmark_rt H_{c}
cmark_rt T_{c}
cmark_r E_{c}
color 0 0 0
fontsize 8
```

```
% DET: points H_{c} and T_{c} are not the same
% Constructing a line c which passes through point H_{c} and point T_{c}
line c H_{c} T_{c}
```

```
color 200 200 200
drawline c
color 0 0 0
```

```
% Constructing a line h_{c} which is perpendicular to line c and which passes through point H_{c}
perp h_{c} H_{c} c
```

```
color 200 200 200
drawline h_{c}
color 0 0 0
```

```
% Generating random value V[_G44665]
```

```

random V[_G44665]

% Calculating value V[_G44686] using formula V[_G44665]*20
expression V[_G44686] { V[_G44665]*20 }

% Constructing a point E_{c} which is a point for which holds  $H_{c}E_{c} = V[_G44686]$  and angle  $T_{c}H_{c}E_{c} = 90$ 
turtle E_{c} T_{c} H_{c} 90 V[_G44686]
cmark_r E_{c}

% DET: points E_{c} and H_{c} are not the same
% Constructing bisector  $m(E_{c}H_{c})$  of the segment  $E_{c}H_{c}$ 
med m(E_{c}H_{c}) E_{c} H_{c}

color 200 200 200
drawline m(E_{c}H_{c})
color 0 0 0

color 200 200 200
drawsegment E_{c} H_{c}
color 0 0 0

% Choosing randomly a point A on the line  $H_{c}T_{c}$ 
online A H_{c} T_{c}
cmark_t A
color 200 200 200
drawline H_{c} T_{c}
color 0 0 0

% DET: points A and H_{c} are not the same
% Constructing bisector  $m(AH_{c})$  of the segment  $AH_{c}$ 
med m(AH_{c}) A H_{c}

color 200 200 200
drawline m(AH_{c})
color 0 0 0

color 200 200 200
drawsegment A H_{c}
color 0 0 0

% Choosing randomly a point B on the line  $AH_{c}$ 
online B A H_{c}
cmark_b B
color 200 200 200
drawline A H_{c}
color 0 0 0

```

```

% Constructing a point  $M_{\{c\}}$  such that  $BM_{\{c\}}/BA=0.5$ 
towards  $M_{\{c\}}$  B A 0.5
cmark_lt  $M_{\{c\}}$ 
color 200 200 200
drawsegment B A
color 0 0 0

% DET: points  $E_{\{c\}}$  and  $M_{\{c\}}$  are not the same
% Constructing a line  $m(H_{\{b\}}H_{\{a\}})$  which passes through point  $E_{\{c\}}$  and point  $M_{\{c\}}$ 
line  $m(H_{\{b\}}H_{\{a\}})$   $E_{\{c\}}$   $M_{\{c\}}$ 

color 200 200 200
drawline  $m(H_{\{b\}}H_{\{a\}})$ 
color 0 0 0

% NDG: lines  $m(E_{\{c\}}H_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not parallel% DET: lines  $m(E_{\{c\}}H_{\{c\}})$  and  $m(H_{\{b\}}$ 
 $H_{\{a\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{c\}}H_{\{c\}})$  and line  $m(H_{\{b\}}H_{\{a\}})$ 
intersec N  $m(E_{\{c\}}H_{\{c\}})$   $m(H_{\{b\}}H_{\{a\}})$ 
cmark_r N

% NDG: points  $E_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N,M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{c\}}$ 
circle  $k(N,M_{\{a\}})$  N  $E_{\{c\}}$ 

color 200 200 200
drawcircle  $k(N,M_{\{a\}})$ 
color 0 0 0

% NDG: line  $m(AH_{\{c\}})$  and circle  $k(N,M_{\{a\}})$  intersect
% Constructing points  $M_{\{b\}}$  and  $E_{\{a\}}$  which are in intersection of  $k(N,M_{\{a\}})$  and  $m(AH_{\{c\}})$ 
intersec2  $M_{\{b\}}$   $E_{\{a\}}$   $k(N,M_{\{a\}})$   $m(AH_{\{c\}})$ 
cmark_lt  $M_{\{b\}}$ 
cmark_r  $E_{\{a\}}$ 

% Constructing a point  $C$  such that  $AC/AM_{\{b\}}=2$ 
towards C A  $M_{\{b\}}$  2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

drawsegment A B
drawsegment A C

```

Figure 1: Illustration of the problem 1026

drawsegment B C

% Non-degenerate conditions: line $m(AH_{\{c\}})$ and circle $k(N, M_{\{a\}})$ intersect; points $E_{\{c\}}$ and N are not the same; lines $m(E_{\{c\}}H_{\{c\}})$ and $m(H_{\{b\}}H_{\{a\}})$ are not parallel
% Determination conditions: lines $m(E_{\{c\}}H_{\{c\}})$ and $m(H_{\{b\}}H_{\{a\}})$ are not the same; points $E_{\{c\}}$ and $M_{\{c\}}$ are not the same; points A and $H_{\{c\}}$ are not the same; points $E_{\{c\}}$ and $H_{\{c\}}$ are not the same; points $H_{\{c\}}$ and $T_{\{c\}}$ are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_c = H_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

Time Complexity: Time spent by the prover is 0.144 seconds.

NDG conditions Points A and E_c are not identical

4.1.2 Proving $T_c = T_c$

Proving failed

4.1.3 Proving $E_c = E_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_c = H_c$

Proving failed

4.2.2 Proving $T_c = T_c$

Proving failed

4.2.3 Proving $E_c = E_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_c = H_c$

Proving failed

4.3.2 Proving $T_c = \neg T_c$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 52 terms.

Time Complexity: Time spent by the prover is 0.050 seconds. There are no ndg conditions.

4.3.3 Proving $E_c = \neg E_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_c = \neg H_c$

Proving failed

4.4.2 Proving $T_c = \neg T_c$

Proving failed

4.4.3 Proving $E_c = \neg E_c$

Proving failed

Problem 1027

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1027: Given a point E_c , a point I and a point M_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1028

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1028: Given a point E_c , a point I and a point M_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1029

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1029: Given a point E_c , a point I and a point M_c , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point M_c , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and M_c are not the same;
2. Using the point I and the point M_c , construct a line IM_c (rule W02); % DET: points I and M_c are not the same;
3. Using the point I and the point M_c , construct a circle $k_{over}(I, M_c)$ (rule W09); % NDG: points I and M_c are not the same;
4. Using the point E_c and the point M_c , construct a line $m(E_cM_c)$ (rule W14); % DET: points E_c and M_c are not the same;
5. Using the line $m(E_cM_c)$ and the line $m(H_bH_a)$, construct a point N (rule W03); % NDG: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel % DET: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same;
6. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
7. Using the point I , the circle $k(N, M_a)$, the point N and the point E_c , construct a circle $k(I, P_a)$ (rule W22); % NDG: point I is inside the circle $k(N, M_a)$; points I and N are not the same;
8. Using the circle $k(I, P_a)$ and the circle $k_{over}(I, M_c)$, construct a point C_{fi} and a point P_c (rule W07); % NDG: circles $k(I, P_a)$ and $k_{over}(I, M_c)$ intersect % DET: circles $k(I, P_a)$ and $k_{over}(I, M_c)$ are not the same;
9. Using the point P_c and the point M_c , construct a point P'_c (rule W01); ;

10. Using the circle $k(I, P_a)$, the point M_c and the point I , construct a line x_3 and a line c (rule W12); % NDG: point M_c is outside the circle $k(I, P_a)$;
11. Using the circle $k(N, M_a)$, the line c , the point N and the point M_c , construct a point H_c (rule W05); % NDG: line c and circle $k(N, M_a)$ intersect % DET: points M_c and H_c must be different;
12. Using the point H_c and the point E_c , construct a line h_c (rule W02); % DET: points H_c and E_c are not the same;
13. Using the point M_c and the line c , construct a line m_c (rule W10b); ;
14. Using the point P'_c and the line IM_c , construct a line CP'_c (rule W16); ;
15. Using the line CP'_c and the line h_c , construct a point C (rule W03); % NDG: lines CP'_c and h_c are not parallel % DET: lines CP'_c and h_c are not the same;
16. Using the point I and the point C , construct a line s_c (rule W02); % DET: points I and C are not the same;
17. Using the line m_c and the line s_c , construct a point N_c (rule W03); % NDG: lines m_c and s_c are not parallel % DET: lines m_c and s_c are not the same;
18. Using the point I and the point N_c , construct a circle $k(N_c, B)$ (rule W06); % NDG: points I and N_c are not the same;
19. Using the circle $k(N_c, B)$ and the line c , construct a point B and a point A (rule W04); % NDG: line c and circle $k(N_c, B)$ intersect.

Non-degenerate conditions: line c and circle $k(N_c, B)$ intersect; points I and N_c are not the same; lines m_c and s_c are not parallel; lines CP'_c and h_c are not parallel; line c and circle $k(N, M_a)$ intersect; point M_c is outside the circle $k(I, P_a)$; circles $k(I, P_a)$ and $k_{over}(I, M_c)$ intersect; point I is inside the circle $k(N, M_a)$; points I and N are not the same; points E_c and N are not the same; lines $m(E_c M_c)$ and $m(H_b H_a)$ are not parallel; points I and M_c are not the same.

Determination conditions: lines m_c and s_c are not the same; points I and C are not the same; lines CP'_c and h_c are not the same; points H_c and E_c are not the same; points M_c and H_c must be different; circles $k(I, P_a)$ and $k_{over}(I, M_c)$ are not the same; lines $m(E_c M_c)$ and $m(H_b H_a)$ are not the same; points E_c and M_c are not the same; points I and M_c are not the same; points E_c and M_c are not the same.

Rules used: [W01, W02, W03, W04, W05, W06, W07, W09, W10b, W12, W14, W16, W22]

Lemmas used: [D10, D13, D20, D27, D30, D32, D49, D67, D7, D87, GD01, GD02, GL01, GL03, GL09, L119, L14, L18, L19]

Solving time: 52.2 seconds.

3.2 Construction in GCLC language

dim 120 120

```
point E_{c} 95 56.36
point I 74.37 61.15
point M_{c} 50 67.5
```

```

color 220 0 0
fontsize 9

cmark_r E_{c}
cmark_b I
cmark_lt M_{c}
color 0 0 0
fontsize 8

% DET: points E_{c} and M_{c} are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point M_{c}
line m(H_{b}H_{a}) E_{c} M_{c}

color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0

% DET: points I and M_{c} are not the same
% Constructing a line IM_{c} which passes through point I and point M_{c}
line IM_{c} I M_{c}

color 200 200 200
drawline IM_{c}
color 0 0 0

% NDG: points I and M_{c} are not the same
% Constructing midpoint P_{\_G106330} of the segment IM_{c}
midpoint P_{\_G106330} I M_{c}
cmark_r P_{\_G106330}

% Constructing a circle k_{over(I,M_{c})} whose center is at point P_{\_G106330} and which passes
    through point I
circle k_{over(I,M_{c})} P_{\_G106330} I

color 200 200 200
drawcircle k_{over(I,M_{c})}
color 0 0 0

% DET: points E_{c} and M_{c} are not the same
% Constructing bisector m(E_{c}M_{c}) of the segment E_{c}M_{c}
med m(E_{c}M_{c}) E_{c} M_{c}

color 200 200 200
drawline m(E_{c}M_{c})
color 0 0 0

color 200 200 200
drawsegment E_{c} M_{c}
color 0 0 0

```

```

% NDG: lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not parallel% DET: lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{c\}}M_{\{c\}})$  and line  $m(H_{\{b\}}H_{\{a\}})$ 
intersec N m( $E_{\{c\}}M_{\{c\}}$ ) m( $H_{\{b\}}H_{\{a\}}$ )
cmark_r N

% NDG: points  $E_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N,M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{c\}}$ 
circle k( $N,M_{\{a\}}$ ) N  $E_{\{c\}}$ 

color 200 200 200
drawcircle k( $N,M_{\{a\}}$ )
color 0 0 0

% NDG: point  $I$  is inside the circle  $k(N,M_{\{a\}})$ ; points  $I$  and  $N$  are not the same
% Calculating distance  $V[_{G106885}]$  from point  $N$  to point  $E_{\{c\}}$ 
distance V[_{G106885}] N  $E_{\{c\}}$ 

% Calculating distance  $V[_{G106909}]$  from point  $N$  to point  $I$ 
distance V[_{G106909}] N I

% Calculating value  $V[_{G106930}]$  using formula  $V[_{G106885}]/V[_{G106909}]$ 
expression V[_{G106930}] { V[_{G106885}]/V[_{G106909}] }

% Constructing a point  $P_{\{\backslash_{G106961}\}}$  such that  $NP_{\{\backslash_{G106961}\}}/NI=V[_{G106885}]/V[_{G106909}]$ 
towards P_{\backslash_{G106961}} N I V[_{G106930}]
cmark_r P_{\backslash_{G106961}}

% Constructing a circle  $k(I,P_{\{a\}})$  whose center is at point  $I$  and which passes through point  $P_{\{\backslash_{G106961}\}}$ 
circle k( $I,P_{\{a\}}$ ) I P_{\backslash_{G106961}}

color 200 200 200
drawcircle k( $I,P_{\{a\}}$ )
color 0 0 0

% NDG: circles  $k(I,P_{\{a\}})$  and  $k_{\text{over}}(I,M_{\{c\}})$  intersect% DET: circles  $k(I,P_{\{a\}})$  and  $k_{\text{over}}(I,M_{\{c\}})$  are not the same
% Constructing points  $C_{\{fi\}}$  and  $P_{\{c\}}$  which are in intersection of  $k(I,P_{\{a\}})$  and  $k_{\text{over}}(I,M_{\{c\}})$ 
intersec2 C_{fi} P_{c} k( $I,P_{\{a\}}$ ) k_{\text{over}}( $I,M_{\{c\}}$ )
cmark_r C_{fi}
cmark_r P_{c}

% Constructing a point  $P'_{\{c\}}$  such that  $P_{\{c\}}P'_{\{c\}}/P_{\{c\}}M_{\{c\}}=2$ 

```

```

towards P'_{c} P_{c} M_{c} 2
cmark_r P'_{c}
color 200 200 200
drawsegment P_{c} P'_{c}
color 0 0 0

% NDG: point M_{c} is outside the circle k(I,P_{a})
% Constructing midpoint P_{\_G107567} of the segment M_{c}I
midpoint P_{\_G107567} M_{c} I
cmark_r P_{\_G107567}

% Constructing a circle C_{\_G107570} whose center is at point P_{\_G107567} and which passes
    through point M_{c}
circle C_{\_G107570} P_{\_G107567} M_{c}

color 200 200 200
drawcircle C_{\_G107570}
color 0 0 0

% Constructing points P_{\_G107573} and P_{\_G107576} which are in intersection of C_{\_G107570}
    and k(I,P_{a})
intersec2 P_{\_G107573} P_{\_G107576} C_{\_G107570} k(I,P_{a})
cmark_r P_{\_G107573}
cmark_r P_{\_G107576}

% Constructing a line x3 which passes through point M_{c} and point P_{\_G107573}
line x3 M_{c} P_{\_G107573}

color 200 200 200
drawline x3
color 0 0 0

% Constructing a line c which passes through point M_{c} and point P_{\_G107576}
line c M_{c} P_{\_G107576}

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(N,M_{a}) intersect% DET: points M_{c} and H_{c} must be different
% Constructing a point P_{\_G107910} which is a foot of the point N on the line c
foot P_{\_G107910} N c
cmark_r P_{\_G107910}
color 200 200 200
drawline N P_{\_G107910}
color 0 0 0

% Constructing a point H_{c} which is an image of the point M_{c} in the symmetry to point/line P
    _{\_G107910}
sim H_{c} P_{\_G107910} M_{c}
cmark_rt H_{c}

```

```

% DET: points  $H_{\{c\}}$  and  $E_{\{c\}}$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $H_{\{c\}}$  and point  $E_{\{c\}}$ 
line  $h_{\{c\}}$   $H_{\{c\}}$   $E_{\{c\}}$ 

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% Constructing a line  $m_{\{c\}}$  which is perpendicular to line  $c$  and which passes through point  $M_{\{c\}}$ 
perp  $m_{\{c\}}$   $M_{\{c\}}$   $c$ 

color 200 200 200
drawline  $m_{\{c\}}$ 
color 0 0 0

% Constructing a line  $CP'_{\{c\}}$  which contains the point  $P'_{\{c\}}$  and is parallel to the line  $IM_{\{c\}}$ 
parallel  $CP'_{\{c\}}$   $P'_{\{c\}}$   $IM_{\{c\}}$ 

color 200 200 200
drawline  $CP'_{\{c\}}$ 
color 0 0 0

% NDG: lines  $CP'_{\{c\}}$  and  $h_{\{c\}}$  are not parallel% DET: lines  $CP'_{\{c\}}$  and  $h_{\{c\}}$  are not the same
% Constructing a point  $C$  which belongs to line  $CP'_{\{c\}}$  and line  $h_{\{c\}}$ 
intersec  $C$   $CP'_{\{c\}}$   $h_{\{c\}}$ 
cmark_l  $C$ 

% DET: points  $I$  and  $C$  are not the same
% Constructing a line  $s_{\{c\}}$  which passes through point  $I$  and point  $C$ 
line  $s_{\{c\}}$   $I$   $C$ 

color 200 200 200
drawline  $s_{\{c\}}$ 
color 0 0 0

% NDG: lines  $m_{\{c\}}$  and  $s_{\{c\}}$  are not parallel% DET: lines  $m_{\{c\}}$  and  $s_{\{c\}}$  are not the same
% Constructing a point  $N_{\{c\}}$  which belongs to line  $m_{\{c\}}$  and line  $s_{\{c\}}$ 
intersec  $N_{\{c\}}$   $m_{\{c\}}$   $s_{\{c\}}$ 
cmark_b  $N_{\{c\}}$ 

% NDG: points  $I$  and  $N_{\{c\}}$  are not the same
% Constructing a circle  $k(N_{\{c\}}, B)$  whose center is at point  $N_{\{c\}}$  and which passes through point  $I$ 

```



```

circle k(N_{c},B) N_{c} I

color 200 200 200
drawcircle k(N_{c},B)
color 0 0 0

% NDG: line c and circle k(N_{c},B) intersect
% Constructing points B and A which are in intersection of k(N_{c},B) and c
intersec2 B A k(N_{c},B) c
cmark_b B
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle k(N_{c},B) intersect; points I and N_{c} are not the
% same; lines m_{c} and s_{c} are not parallel; lines CP'_{c} and h_{c} are not parallel; line c
% and circle k(N,M_{a}) intersect; point M_{c} is outside the circle k(I,P_{a}); circles k(I,P_{a})
% and k_{over}(I,M_{c}) intersect; point I is inside the circle k(N,M_{a}); points I and N are
% not the same; points E_{c} and N are not the same; lines m(E_{c}M_{c}) and m(H_{b}H_{a}) are
% not parallel; points I and M_{c} are not the same
% Determination conditions: lines m_{c} and s_{c} are not the same; points I and C are not the same
% ; lines CP'_{c} and h_{c} are not the same; points H_{c} and E_{c} are not the same; points M_{c}
% and H_{c} must be different; circles k(I,P_{a}) and k_{over}(I,M_{c}) are not the same; lines
% m(E_{c}M_{c}) and m(H_{b}H_{a}) are not the same; points E_{c} and M_{c} are not the same;
% points I and M_{c} are not the same; points E_{c} and M_{c} are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = _E_c$

Proving failed

4.1.2 Proving $I = _I$

Proving failed

4.1.3 Proving $M_c = _M_c$

Proving failed

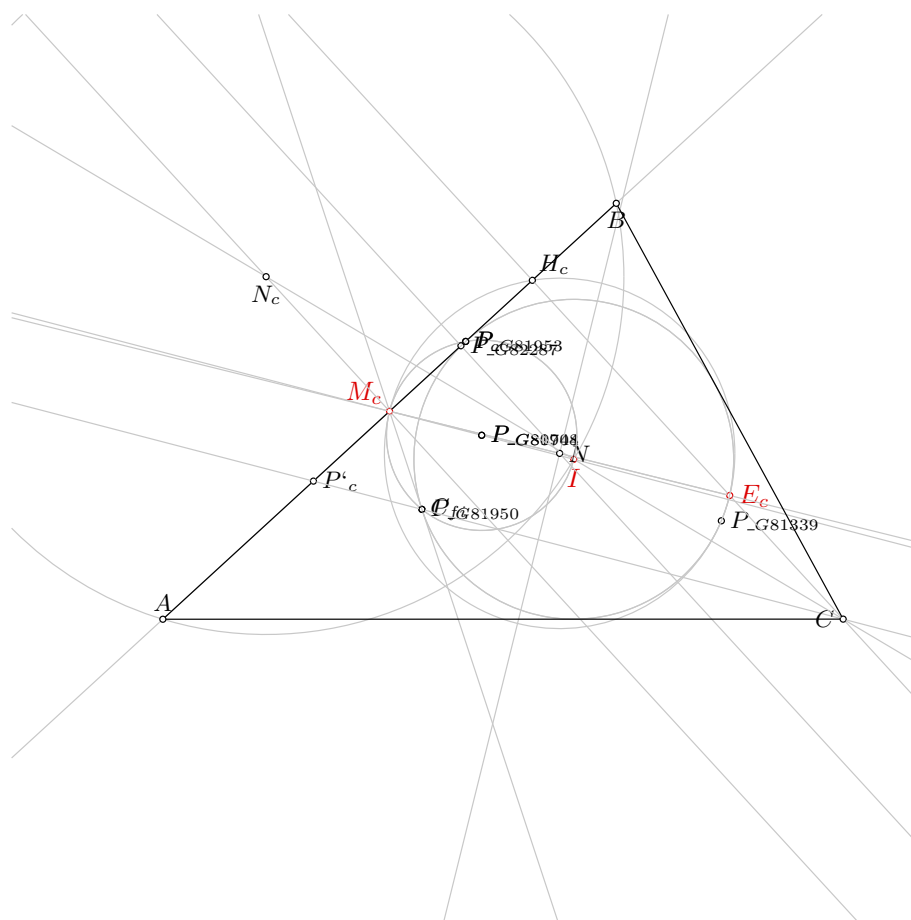


Figure 1: Illustration of the problem 1029

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $I = \neg I$

Proving failed

4.2.3 Proving $M_c = \neg M_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $I = \neg I$

Proving failed

4.3.3 Proving $M_c = \neg M_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $I = \neg I$

Proving failed

4.4.3 Proving $M_c = \neg M_c$

Proving failed

Problem 1030

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1030: Given a point E_c , a point I and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point N , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and N are not the same;
2. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
3. Using the circle $k(N, M_a)$, the line $m(H_bH_a)$, the point N and the point E_c , construct a point M_c (rule W05a); % NDG: line $m(H_bH_a)$ and circle $k(N, M_a)$ intersect % DET: points E_c and M_c must be different;
4. Using the point M_c and the point I , construct a line IM_c (rule W02); % DET: points M_c and I are not the same;
5. Using the point I and the point M_c , construct a circle $k_{over}(I, M_c)$ (rule W09); % NDG: points I and M_c are not the same;
6. Using the point I , the circle $k(N, M_a)$, the point N and the point E_c , construct a circle $k(I, P_a)$ (rule W22); % NDG: point I is inside the circle $k(N, M_a)$; points I and N are not the same;
7. Using the circle $k(I, P_a)$ and the circle $k_{over}(I, M_c)$, construct a point C_{fi} and a point P_c (rule W07); % NDG: circles $k(I, P_a)$ and $k_{over}(I, M_c)$ intersect % DET: circles $k(I, P_a)$ and $k_{over}(I, M_c)$ are not the same;
8. Using the point P_c and the point M_c , construct a point P'_c (rule W01); ;
9. Using the circle $k(I, P_a)$, the point M_c and the point I , construct a line $x3$ and a line c (rule W12); % NDG: point M_c is outside the circle $k(I, P_a)$;

10. Using the circle $k(N, M_a)$, the line c , the point N and the point M_c , construct a point H_c (rule W05); % NDG: line c and circle $k(N, M_a)$ intersect % DET: points M_c and H_c must be different;
11. Using the point H_c and the point E_c , construct a line h_c (rule W02); % DET: points H_c and E_c are not the same;
12. Using the point M_c and the line c , construct a line m_c (rule W10b); ;
13. Using the point P'_c and the line IM_c , construct a line CP'_c (rule W16); ;
14. Using the line CP'_c and the line h_c , construct a point C (rule W03); % NDG: lines CP'_c and h_c are not parallel % DET: lines CP'_c and h_c are not the same;
15. Using the point I and the point C , construct a line s_c (rule W02); % DET: points I and C are not the same;
16. Using the line m_c and the line s_c , construct a point N_c (rule W03); % NDG: lines m_c and s_c are not parallel % DET: lines m_c and s_c are not the same;
17. Using the point I and the point N_c , construct a circle $k(N_c, B)$ (rule W06); % NDG: points I and N_c are not the same;
18. Using the circle $k(N_c, B)$ and the line c , construct a point B and a point A (rule W04); % NDG: line c and circle $k(N_c, B)$ intersect.

Non-degenerate conditions: line c and circle $k(N_c, B)$ intersect; points I and N_c are not the same; lines m_c and s_c are not parallel; lines CP'_c and h_c are not parallel; line c and circle $k(N, M_a)$ intersect; point M_c is outside the circle $k(I, P_a)$; circles $k(I, P_a)$ and $k_{over}(I, M_c)$ intersect; point I is inside the circle $k(N, M_a)$; points I and N are not the same; points I and M_c are not the same; line $m(H_b H_a)$ and circle $k(N, M_a)$ intersect; points E_c and N are not the same.

Determination conditions: lines m_c and s_c are not the same; points I and C are not the same; lines CP'_c and h_c are not the same; points H_c and E_c are not the same; points M_c and H_c must be different; circles $k(I, P_a)$ and $k_{over}(I, M_c)$ are not the same; points M_c and I are not the same; points E_c and M_c must be different; points E_c and N are not the same.

Rules used: [W01,W02,W03,W04,W05,W05a,W06,W07,W09,W10b,W12,W16,W22]

Lemmas used: [D10,D13,D20,D27,D30,D32,D49,D67,D7,D87,GD01,GD02,GL01,GL03,GL09,L119,L14,L18,L19]

Solving time: 53.4 seconds.

3.2 Construction in GCLC language

dim 120 120

point E_{c} 95 56.36
point I 74.37 61.15
point N 72.5 61.93

color 220 0 0
fontsize 9

```

cmark_r E_{c}
cmark_b I
cmark_r N
color 0 0 0
fontsize 8

% DET: points E_{c} and N are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point N
line m(H_{b}H_{a}) E_{c} N

color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0

% NDG: points E_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{c}
circle k(N,M_{a}) N E_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(H_{b}H_{a}) and circle k(N,M_{a}) intersect% DET: points E_{c} and M_{c} must be
different
% Constructing a point M_{c} which is an image of the point E_{c} in the symmetry to point/line N
sim M_{c} N E_{c}
cmark_lt M_{c}

% DET: points M_{c} and I are not the same
% Constructing a line IM_{c} which passes through point M_{c} and point I
line IM_{c} M_{c} I

color 200 200 200
drawline IM_{c}
color 0 0 0

% NDG: points I and M_{c} are not the same
% Constructing midpoint P_{\_G169156} of the segment IM_{c}
midpoint P_{\_G169156} I M_{c}
cmark_r P_{\_G169156}

% Constructing a circle k_{over}(I,M_{c}) whose center is at point P_{\_G169156} and which passes
through point I
circle k_{over}(I,M_{c}) P_{\_G169156} I

color 200 200 200
drawcircle k_{over}(I,M_{c})
color 0 0 0

```

```

% NDG: point I is inside the circle k(N,M_{a}); points I and N are not the same
% Calculating distance V[_G169342] from point N to point E_{c}
distance V[_G169342] N E_{c}

% Calculating distance V[_G169366] from point N to point I
distance V[_G169366] N I

% Calculating value V[_G169387] using formula V[_G169342]/V[_G169366]
expression V[_G169387] { V[_G169342]/V[_G169366] }

% Constructing a point P_{\_G169418} such that NP_{\_G169418}/NI=V[_G169342]/V[_G169366]
towards P_{\_G169418} N I V[_G169387]
cmark_r P_{\_G169418}

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
169418}
circle k(I,P_{a}) I P_{\_G169418}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: circles k(I,P_{a}) and k_{over}(I,M_{c}) intersect% DET: circles k(I,P_{a}) and k_{over}(I,M_{c}
}) are not the same
% Constructing points C_{fi} and P_{c} which are in intersection of k(I,P_{a}) and k_{over}(I,M_{c})
intersec2 C_{fi} P_{c} k(I,P_{a}) k_{over}(I,M_{c})
cmark_r C_{fi}
cmark_r P_{c}

% Constructing a point P'_{c} such that P_{c}P'_{c}/P_{c}M_{c}=2
towards P'_{c} P_{c} M_{c} 2
cmark_r P'_{c}
color 200 200 200
drawsegment P_{c} P'_{c}
color 0 0 0

% NDG: point M_{c} is outside the circle k(I,P_{a})
% Constructing midpoint P_{\_G170024} of the segment M_{c}I
midpoint P_{\_G170024} M_{c} I
cmark_r P_{\_G170024}

% Constructing a circle C_{\_G170027} whose center is at point P_{\_G170024} and which passes
through point M_{c}
circle C_{\_G170027} P_{\_G170024} M_{c}

color 200 200 200

```

```

drawcircle C_{\_G170027}
color 0 0 0

% Constructing points P_{\_G170030} and P_{\_G170033} which are in intersection of C_{\_G170027}
and k(I,P_{a})
intersec2 P_{\_G170030} P_{\_G170033} C_{\_G170027} k(I,P_{a})
cmark_r P_{\_G170030}
cmark_r P_{\_G170033}

% Constructing a line x3 which passes through point M_{c} and point P_{\_G170030}
line x3 M_{c} P_{\_G170030}

color 200 200 200
drawline x3
color 0 0 0

% Constructing a line c which passes through point M_{c} and point P_{\_G170033}
line c M_{c} P_{\_G170033}

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(N,M_{a}) intersect% DET: points M_{c} and H_{c} must be different
% Constructing a point P_{\_G170367} which is a foot of the point N on the line c
foot P_{\_G170367} N c
cmark_r P_{\_G170367}
color 200 200 200
drawline N P_{\_G170367}
color 0 0 0

% Constructing a point H_{c} which is an image of the point M_{c} in the symmetry to point/line P
_{\_G170367}
sim H_{c} P_{\_G170367} M_{c}
cmark_rt H_{c}

% DET: points H_{c} and E_{c} are not the same
% Constructing a line h_{c} which passes through point H_{c} and point E_{c}
line h_{c} H_{c} E_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% Constructing a line m_{c} which is perpendicular to line c and which passes through point M_{c}
perp m_{c} M_{c} c

color 200 200 200

```



```

drawline m_{c}
color 0 0 0

% Constructing a line CP'_{c} which contains the point P'_{c} and is parallel to the line IM_{c}
parallel CP'_{c} P'_{c} IM_{c}

color 200 200 200
drawline CP'_{c}
color 0 0 0

% NDG: lines CP'_{c} and h_{c} are not parallel% DET: lines CP'_{c} and h_{c} are not the same
% Constructing a point C which belongs to line CP'_{c} and line h_{c}
intersec C CP'_{c} h_{c}
cmark_l C

% DET: points I and C are not the same
% Constructing a line s_{c} which passes through point I and point C
line s_{c} I C

color 200 200 200
drawline s_{c}
color 0 0 0

% NDG: lines m_{c} and s_{c} are not parallel% DET: lines m_{c} and s_{c} are not the same
% Constructing a point N_{c} which belongs to line m_{c} and line s_{c}
intersec N_{c} m_{c} s_{c}
cmark_b N_{c}

% NDG: points I and N_{c} are not the same
% Constructing a circle k(N_{c},B) whose center is at point N_{c} and which passes through point I
circle k(N_{c},B) N_{c} I

color 200 200 200
drawcircle k(N_{c},B)
color 0 0 0

% NDG: line c and circle k(N_{c},B) intersect
% Constructing points B and A which are in intersection of k(N_{c},B) and c
intersec2 B A k(N_{c},B) c
cmark_b B
cmark_t A

drawsegment A B

```

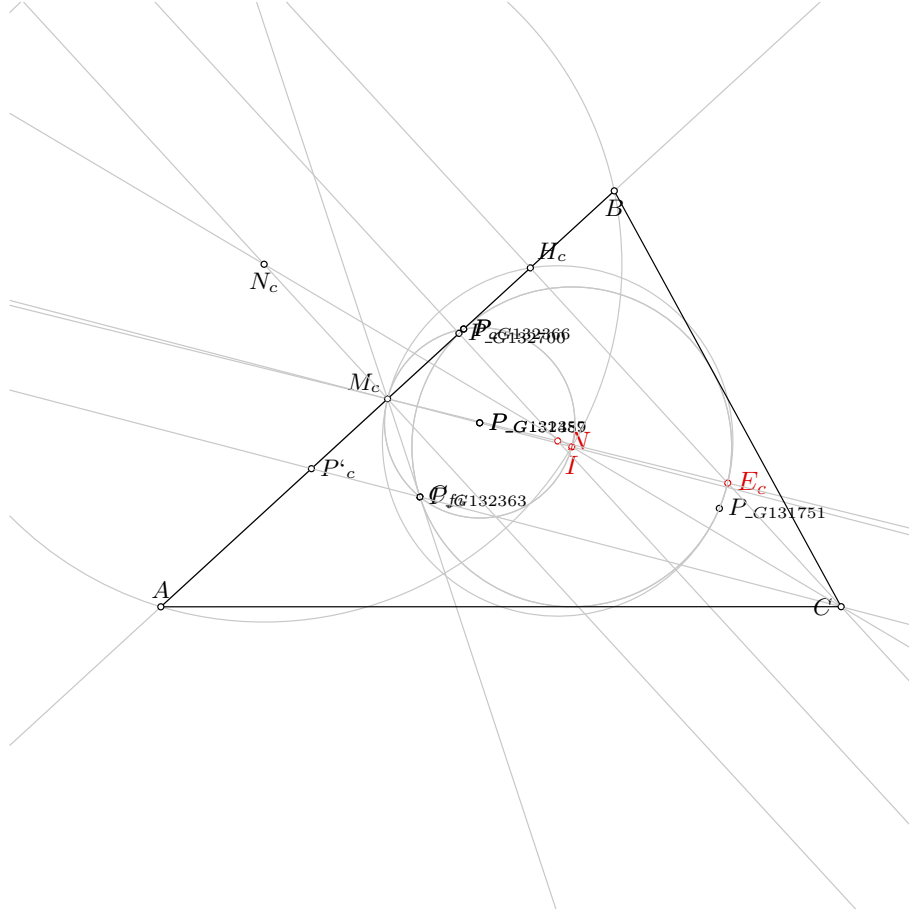


Figure 1: Illustration of the problem 1030

```
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line c and circle k(N_{c},B) intersect; points I and N_{c} are not the
same; lines m_{c} and s_{c} are not parallel; lines CP'_{c} and h_{c} are not parallel; line c
and circle k(N,M_{a}) intersect; point M_{c} is outside the circle k(I,P_{a}); circles k(I,P_{a})
and k_{over}(I,M_{c}) intersect; point I is inside the circle k(N,M_{a}); points I and N are
not the same; points I and M_{c} are not the same; line m(H_{b}H_{a}) and circle k(N,M_{a})
intersect; points E_{c} and N are not the same
% Determination conditions: lines m_{c} and s_{c} are not the same; points I and C are not the same
; lines CP'_{c} and h_{c} are not the same; points H_{c} and E_{c} are not the same; points M_{c}
and H_{c} must be different; circles k(I,P_{a}) and k_{over}(I,M_{c}) are not the same; points
M_{c} and I are not the same; points E_{c} and M_{c} must be different; points E_{c} and N are
not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $I = \neg I$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $I = \neg I$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $I = \neg I$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $I = \neg I$

Proving failed

4.4.3 Proving $N = \neg N$

Proving failed

Problem 1031

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1031: Given a point E_c , a point I and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1032

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1032: Given a point E_c , a point I and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1033

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1033: Given a point E_c , a point I and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1034

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1034: Given a point E_c , a point I and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1035

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1035: Given a point E_c , a point M_a and a point M_b , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point M_a , construct a line $m(E_cM_a)$ (rule W14); % DET: points E_c and M_a are not the same;
2. Using the point E_c and the point M_b , construct a line $m(E_cM_b)$ (rule W14); % DET: points E_c and M_b are not the same;
3. Using the line $m(E_cM_b)$ and the line $m(E_cM_a)$, construct a point N (rule W03); % NDG: lines $m(E_cM_b)$ and $m(E_cM_a)$ are not parallel % DET: lines $m(E_cM_b)$ and $m(E_cM_a)$ are not the same;
4. Using the point M_a and the point N , construct a line $m(H_bH_c)$ (rule W02); % DET: points M_a and N are not the same;
5. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
6. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point M_a , construct a point E_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_a and E_a must be different;
7. Using the point M_b and the point E_a , construct a line $m(AH_c)$ (rule W02); % DET: points M_b and E_a are not the same;
8. Using the point E_c and the line $m(AH_c)$, construct a line h_c (rule W16); ;
9. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;

10. Using the point H_c and the point M_a , construct a circle $k(M_a, B)$ (rule W06); % NDG: points H_c and M_a are not the same;
11. Using the circle $k(M_a, B)$, the line h_c , the point M_a and the point H_c , construct a point C (rule W05); % NDG: line h_c and circle $k(M_a, B)$ intersect % DET: points H_c and C must be different;
12. Using the point M_a and the point C , construct a point B (rule W01); ;
13. Using the point M_b and the point C , construct a point A (rule W01); .

Non-degenerate conditions: line h_c and circle $k(M_a, B)$ intersect; points H_c and M_a are not the same; line h_c and circle $k(N, M_a)$ intersect; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; points E_c and N are not the same; lines $m(E_c M_b)$ and $m(E_c M_a)$ are not parallel.

Determination conditions: points H_c and C must be different; points E_c and H_c must be different; points M_b and E_a are not the same; points M_a and E_a must be different; points M_a and N are not the same; lines $m(E_c M_b)$ and $m(E_c M_a)$ are not the same; points E_c and M_b are not the same; points E_c and M_a are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W14,W16]

Lemmas used: [D10,D21,D22,D30,D32,D7,GD02,GL01,GL03,GL04,GL09,L17,L20,L21,L22,L24,L3,L37,L38,L39]

Solving time: 26.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
point M_{a} 65 40
point M_{b} 95 67.5
```

```
color 220 0 0
fontsize 9
```

```
cmark_r E_{c}
cmark_r M_{a}
cmark_lt M_{b}
color 0 0 0
fontsize 8
```

```
% DET: points E_{c} and M_{a} are not the same
% Constructing bisector m(E_{c}M_{a}) of the segment E_{c}M_{a}
med m(E_{c}M_{a}) E_{c} M_{a}
```

```
color 200 200 200
drawline m(E_{c}M_{a})
color 0 0 0
```

```
color 200 200 200
drawsegment E_{c} M_{a}
color 0 0 0
```

```

% DET: points  $E_{\{c\}}$  and  $M_{\{b\}}$  are not the same
% Constructing bisector  $m(E_{\{c\}}M_{\{b\}})$  of the segment  $E_{\{c\}}M_{\{b\}}$ 
med m( $E_{\{c\}}M_{\{b\}}$ )  $E_{\{c\}}$   $M_{\{b\}}$ 

color 200 200 200
drawline m( $E_{\{c\}}M_{\{b\}}$ )
color 0 0 0

color 200 200 200
drawsegment  $E_{\{c\}}$   $M_{\{b\}}$ 
color 0 0 0

% NDG: lines  $m(E_{\{c\}}M_{\{b\}})$  and  $m(E_{\{c\}}M_{\{a\}})$  are not parallel% DET: lines  $m(E_{\{c\}}M_{\{b\}})$  and  $m(E_{\{c\}}$ 
 $M_{\{a\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{c\}}M_{\{b\}})$  and line  $m(E_{\{c\}}M_{\{a\}})$ 
intersec N m( $E_{\{c\}}M_{\{b\}}$ ) m( $E_{\{c\}}M_{\{a\}}$ )
cmark_r N

% DET: points  $M_{\{a\}}$  and  $N$  are not the same
% Constructing a line  $m(H_{\{b\}}H_{\{c\}})$  which passes through point  $M_{\{a\}}$  and point  $N$ 
line m( $H_{\{b\}}H_{\{c\}}$ )  $M_{\{a\}}$  N

color 200 200 200
drawline m( $H_{\{b\}}H_{\{c\}}$ )
color 0 0 0

% NDG: points  $E_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{c\}}$ 
circle k( $N, M_{\{a\}}$ ) N  $E_{\{c\}}$ 

color 200 200 200
drawcircle k( $N, M_{\{a\}}$ )
color 0 0 0

% NDG: line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{a\}}$  and  $E_{\{a\}}$  must be
different
% Constructing a point  $E_{\{a\}}$  which is an image of the point  $M_{\{a\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{a\}}$  N  $M_{\{a\}}$ 
cmark_r  $E_{\{a\}}$ 

% DET: points  $M_{\{b\}}$  and  $E_{\{a\}}$  are not the same
% Constructing a line  $m(AH_{\{c\}})$  which passes through point  $M_{\{b\}}$  and point  $E_{\{a\}}$ 
line m( $AH_{\{c\}}$ )  $M_{\{b\}}$   $E_{\{a\}}$ 

color 200 200 200
drawline m( $AH_{\{c\}}$ )
color 0 0 0

```

```

% Constructing a line  $h_{\{c\}}$  which contains the point  $E_{\{c\}}$  and is parallel to the line  $m(AH_{\{c\}})$ 
parallel  $h_{\{c\}}$   $E_{\{c\}}$   $m(AH_{\{c\}})$ 

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{c\}}$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\_G234725\}}$  which is a foot of the point  $N$  on the line  $h_{\{c\}}$ 
foot  $P_{\{\_G234725\}}$   $N$   $h_{\{c\}}$ 
cmark_r  $P_{\{\_G234725\}}$ 
color 200 200 200
drawline  $N$   $P_{\{\_G234725\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $E_{\{c\}}$  in the symmetry to point/line  $P_{\{\_G234725\}}$ 
sim  $H_{\{c\}}$   $P_{\{\_G234725\}}$   $E_{\{c\}}$ 
cmark_rt  $H_{\{c\}}$ 

% NDG: points  $H_{\{c\}}$  and  $M_{\{a\}}$  are not the same
% Constructing a circle  $k(M_{\{a\}}, B)$  whose center is at point  $M_{\{a\}}$  and which passes through point  $H_{\{c\}}$ 
circle  $k(M_{\{a\}}, B)$   $M_{\{a\}}$   $H_{\{c\}}$ 

color 200 200 200
drawcircle  $k(M_{\{a\}}, B)$ 
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(M_{\{a\}}, B)$  intersect% DET: points  $H_{\{c\}}$  and  $C$  must be different
% Constructing a point  $P_{\{\_G235029\}}$  which is a foot of the point  $M_{\{a\}}$  on the line  $h_{\{c\}}$ 
foot  $P_{\{\_G235029\}}$   $M_{\{a\}}$   $h_{\{c\}}$ 
cmark_r  $P_{\{\_G235029\}}$ 
color 200 200 200
drawline  $M_{\{a\}}$   $P_{\{\_G235029\}}$ 
color 0 0 0

% Constructing a point  $C$  which is an image of the point  $H_{\{c\}}$  in the symmetry to point/line  $P_{\{\_G235029\}}$ 
sim  $C$   $P_{\{\_G235029\}}$   $H_{\{c\}}$ 
cmark_l  $C$ 

% Constructing a point  $B$  such that  $M_{\{a\}}B/M_{\{a\}}C = -1$ 
towards  $B$   $M_{\{a\}}$   $C$  -1
cmark_b  $B$ 
color 200 200 200
drawsegment  $C$   $B$ 

```

```
color 0 0 0
```

```
% Constructing a point A such that  $M_{\{b\}}A/M_{\{b\}}C=-1$ 
towards A  $M_{\{b\}}$  C -1
cmark_t A
color 200 200 200
drawsegment C A
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line  $h_{\{c\}}$  and circle  $k(M_{\{a\}}, B)$  intersect; points  $H_{\{c\}}$  and  $M_{\{a\}}$  are
not the same; line  $h_{\{c\}}$  and circle  $k(N, M_{\{a\}})$  intersect; line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$ 
intersect; points  $E_{\{c\}}$  and  $N$  are not the same; lines  $m(E_{\{c\}}M_{\{b\}})$  and  $m(E_{\{c\}}M_{\{a\}})$  are
not parallel
% Determination conditions: points  $H_{\{c\}}$  and  $C$  must be different; points  $E_{\{c\}}$  and  $H_{\{c\}}$  must be
different; points  $M_{\{b\}}$  and  $E_{\{a\}}$  are not the same; points  $M_{\{a\}}$  and  $E_{\{a\}}$  must be different;
points  $M_{\{a\}}$  and  $N$  are not the same; lines  $m(E_{\{c\}}M_{\{b\}})$  and  $m(E_{\{c\}}M_{\{a\}})$  are not the same;
points  $E_{\{c\}}$  and  $M_{\{b\}}$  are not the same; points  $E_{\{c\}}$  and  $M_{\{a\}}$  are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 9 terms.

Time Complexity: Time spent by the prover is 0.27 seconds.

NDG conditions Points M_b , E_c and M_a are not collinear

Points M_b , E_c and M_a are not collinear

Line through points M_b and E_a is not parallel with line through points E_c and M_a

Points E_c and tempPoint-961 h_c are not identical

Points E_c , tempPoint-961 h_c and M_a are not collinear

Points E_c and $P_{G229746}$ are not identical

Points E_c and $P_{G229746}$ are not identical

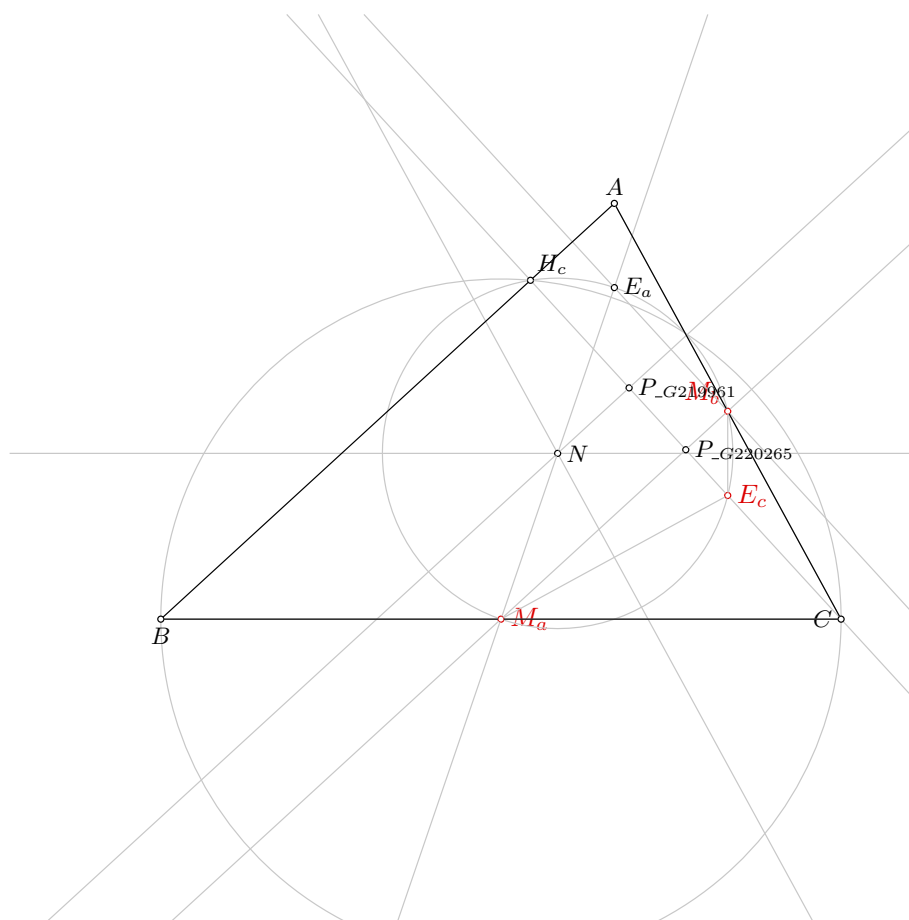


Figure 1: Illustration of the problem 1035

4.1.3 Proving $M_b = M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 11 terms.

Time Complexity: Time spent by the prover is 0.278 seconds.

NDG conditions Points M_b , E_c and M_a are not collinear

Points M_b , E_c and M_a are not collinear

Line through points M_b and E_a is not parallel with line through points E_c and M_a

Points E_c and tempPoint-450 h_c are not identical

Points E_c , tempPoint-450 h_c and M_a are not collinear

Points $P_{G231082}$ and E_c are not identical

Points $P_{G231082}$ and E_c are not identical

4.2 GCLC - Area method

4.2.1 Proving $E_c = E_c$

Proving failed

4.2.2 Proving $M_a = M_a$

NDG conditions are:

$S_{M^2_{m(E_c M_b)} M^0_{m(E_c M_a)} T^1_{m(E_c M_a)}} \neq S_{T^3_{m(E_c M_b)} M^0_{m(E_c M_a)} T^1_{m(E_c M_a)}}$ i.e., lines $M^2_{m(E_c M_b)} T^3_{m(E_c M_b)}$ and $M^0_{m(E_c M_a)} T^1_{m(E_c M_a)}$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^6_{h_b}} \neq S_{F^5_{h_a} BF^6_{h_b}}$ i.e., lines $AF^5_{h_a}$ and $BF^6_{h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.2.3 Proving $M_b = M_b$

NDG conditions are:

$S_{M^2_{m(E_c M_b)} M^0_{m(E_c M_a)} T^1_{m(E_c M_a)}} \neq S_{T^3_{m(E_c M_b)} M^0_{m(E_c M_a)} T^1_{m(E_c M_a)}}$ i.e., lines $M^2_{m(E_c M_b)} T^3_{m(E_c M_b)}$ and $M^0_{m(E_c M_a)} T^1_{m(E_c M_a)}$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^6_{h_b}} \neq S_{F^5_{h_a} BF^6_{h_b}}$ i.e., lines $AF^5_{h_a}$ and $BF^6_{h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_c = E_c$

Proving failed

4.3.2 Proving $M_a = M_a$

Proving failed

4.3.3 Proving $M_b = M_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $M_a = \neg M_a$

Proving failed

4.4.3 Proving $M_b = \neg M_b$

Proving failed

Problem 1036

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1036: Given a point E_c , a point M_c and a point M_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point M_c , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and M_c are not the same;
2. Using the point E_c and the point M_c , construct a line $m(E_cM_c)$ (rule W14); % DET: points E_c and M_c are not the same;
3. Using the line $m(E_cM_c)$ and the line $m(H_bH_a)$, construct a point N (rule W03); % NDG: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel % DET: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same;
4. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
5. Choose freely a point M_a on the circle $k(N, M_a)$ (rule WOncircle);
6. Choose freely a point A (rule free);
7. Using the point A and the point M_c , construct a point B (rule W01); ;
8. Using the point M_a and the point B , construct a point C (rule W01); .

Non-degenerate conditions: points E_c and N are not the same; lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel.

Determination conditions: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same; points E_c and M_c are not the same; points E_c and M_c are not the same.

Rules used: [W01,W02,W03,W06,W14,WOncircle1,free]

Lemmas used: [D20,D21,D32,GD02,GL01,GL03,GL04,L18,L19,L20,L24,L41,L42,L53,L54]

Solving time: 203.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{c} 95 56.36
point M_{c} 50 67.5
point M_{a} 65 40

color 220 0 0
fontsize 9

cmark_r E_{c}
cmark_lt M_{c}
cmark_r M_{a}
color 0 0 0
fontsize 8

% DET: points E_{c} and M_{c} are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point M_{c}
line m(H_{b}H_{a}) E_{c} M_{c}

color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0

% DET: points E_{c} and M_{c} are not the same
% Constructing bisector m(E_{c}M_{c}) of the segment E_{c}M_{c}
med m(E_{c}M_{c}) E_{c} M_{c}

color 200 200 200
drawline m(E_{c}M_{c})
color 0 0 0

color 200 200 200
drawsegment E_{c} M_{c}
color 0 0 0

% NDG: lines m(E_{c}M_{c}) and m(H_{b}H_{a}) are not parallel% DET: lines m(E_{c}M_{c}) and m(H_{b}
H_{a}) are not the same
% Constructing a point N which belongs to line m(E_{c}M_{c}) and line m(H_{b}H_{a})
intersec N m(E_{c}M_{c}) m(H_{b}H_{a})
cmark_r N

% NDG: points E_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{c}
circle k(N,M_{a}) N E_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```

% Choosing randomly a point  $M_{\{a\}}$  on the circle with center  $N$  through point  $E_{\{c\}}$ 
oncircle M_{a} N E_{c}
cmark_r M_{a}
color 200 200 200
drawcircle N E_{c}
color 0 0 0

% Constructing a free point  $A$ 
point A 80 95

cmark_t A

% Constructing a point  $B$  such that  $AB/AM_{\{c\}}=2$ 
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% Constructing a point  $C$  such that  $M_{\{a\}}C/M_{\{a\}}B=-1$ 
towards C M_{a} B -1
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: points  $E_{\{c\}}$  and  $N$  are not the same; lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not parallel
% Determination conditions: lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not the same; points  $E_{\{c\}}$  and  $M_{\{c\}}$  are not the same; points  $E_{\{c\}}$  and  $M_{\{c\}}$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

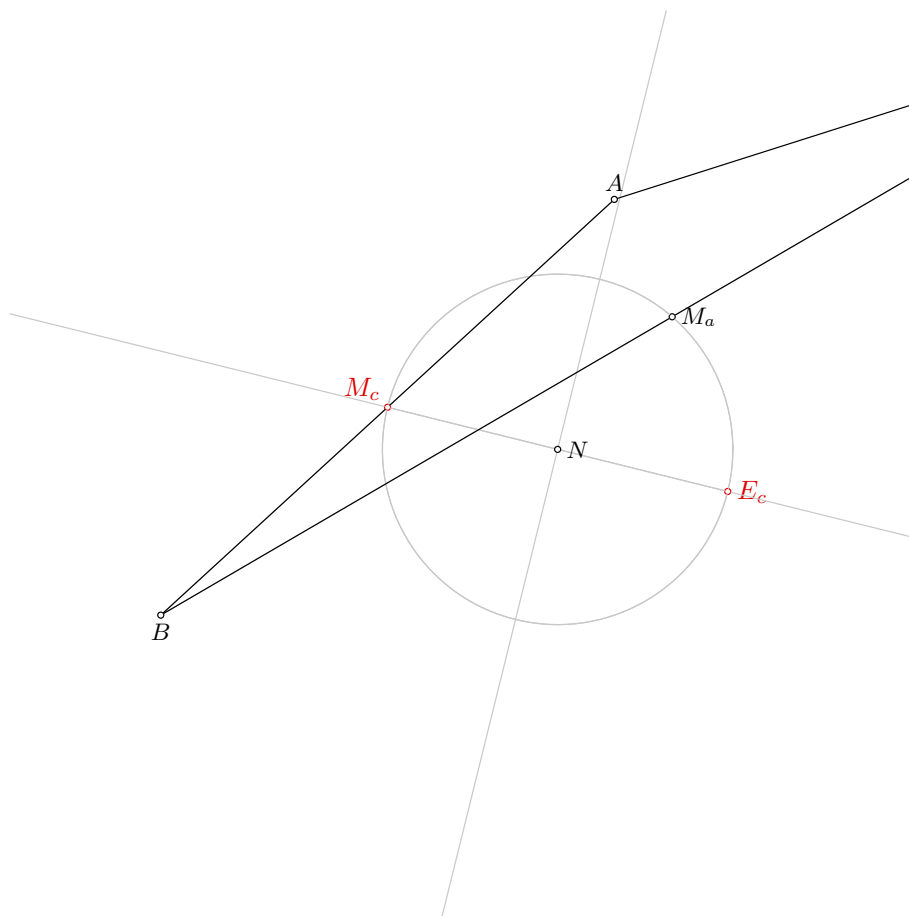


Figure 1: Illustration of the problem 1036

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $M_c = \neg M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.068 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $M_a = \neg M_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{m(E_c M_c) E_c M_c}^0 \neq S_{m(E_c M_c) E_c M_c}^1$ i.e., lines $M_{m(E_c M_c)}^0 T_{m(E_c M_c)}^1$ and $E_c M_c$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{F_{-h_a}^2 BF_{-h_b}^3}$ i.e., lines $AF_{-h_a}^2$ and $BF_{-h_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.2.3 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{M_{m(E_c M_c) E_c M_c}^0} \neq S_{M_{m(E_c M_c) E_c M_c}^1}$ i.e., lines $M_{m(E_c M_c)}^0 T_{m(E_c M_c)}^1$ and $E_c M_c$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{F_{-h_a}^2 BF_{-h_b}^3}$ i.e., lines $AF_{-h_a}^2$ and $BF_{-h_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $M_c = \neg M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 19 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.3.3 Proving $M_a = \neg M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 28 terms.

Time Complexity: Time spent by the prover is 0.050 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $M_c = \neg M_c$

Proving failed

4.4.3 Proving $M_a = \neg M_a$

Proving failed

Problem 1037

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1037: Given a point M_a , a point N and a point E_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point M_a and the point N , construct a line $m(H_bH_c)$ (rule W02); % DET: points M_a and N are not the same;
2. Using the point M_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_a and N are not the same;
3. Choose freely a point E_c on the circle $k(N, M_a)$ (rule WOncircle);
4. Using the point E_c and the point N , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and N are not the same;
5. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point M_a , construct a point E_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_a and E_a must be different;
6. Using the circle $k(N, M_a)$, the line $m(H_bH_a)$, the point N and the point E_c , construct a point M_c (rule W05a); % NDG: line $m(H_bH_a)$ and circle $k(N, M_a)$ intersect % DET: points E_c and M_c must be different;
7. Choose freely a point A (rule free);
8. Using the point A and the point E_a , construct a point H (rule W01); ;
9. Using the point A and the point M_c , construct a point B (rule W01); ;
10. Using the point E_c and the point H , construct a point C (rule W01); .

Non-degenerate conditions: line $m(H_bH_a)$ and circle $k(N, M_a)$ intersect; line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect; points M_a and N are not the same.

Determination conditions: points E_c and M_c must be different; points M_a and E_a must be different; points E_c and N are not the same; points M_a and N are not the same.

Rules used: [W01,W02,W05a,W06,WOncircle1,free]

Lemmas used: [D20,D28,D30,D32,GD02,GL01,GL03,GL04,L18,L19,L20,L21,L22,L24,L38,L39,L41,L42,L47,L48]

Solving time: 150.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40
```

```
point N 72.5 61.93
```

```
point E_{c} 95 56.36
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r M_{a}
```

```
cmark_r N
```

```
cmark_r E_{c}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points M_{a} and N are not the same
```

```
% Constructing a line m(H_{b}H_{c}) which passes through point M_{a} and point N
```

```
line m(H_{b}H_{c}) M_{a} N
```

```
color 200 200 200
```

```
drawline m(H_{b}H_{c})
```

```
color 0 0 0
```

```
% NDG: points M_{a} and N are not the same
```

```
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{a}
```

```
circle k(N,M_{a}) N M_{a}
```

```
color 200 200 200
```

```
drawcircle k(N,M_{a})
```

```
color 0 0 0
```

```
% Choosing randomly a point E_{c} on the circle with center N through point M_{a}
```

```
oncircle E_{c} N M_{a}
```

```
cmark_r E_{c}
```

```
color 200 200 200
```

```
drawcircle N M_{a}
```

```
color 0 0 0
```

```

% DET: points  $E_{\{c\}}$  and  $N$  are not the same
% Constructing a line  $m(H_{\{b\}}H_{\{a\}})$  which passes through point  $E_{\{c\}}$  and point  $N$ 
line m( $H_{\{b\}}H_{\{a\}}$ )  $E_{\{c\}}$  N

color 200 200 200
drawline m( $H_{\{b\}}H_{\{a\}}$ )
color 0 0 0

% NDG: line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{a\}}$  and  $E_{\{a\}}$  must be
different
% Constructing a point  $E_{\{a\}}$  which is an image of the point  $M_{\{a\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{a\}}$  N  $M_{\{a\}}$ 
cmark_r  $E_{\{a\}}$ 

% NDG: line  $m(H_{\{b\}}H_{\{a\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{c\}}$  and  $M_{\{c\}}$  must be
different
% Constructing a point  $M_{\{c\}}$  which is an image of the point  $E_{\{c\}}$  in the symmetry to point/line  $N$ 
sim  $M_{\{c\}}$  N  $E_{\{c\}}$ 
cmark_lt  $M_{\{c\}}$ 

% Constructing a free point  $A$ 
point A 80 95

cmark_t A

% Constructing a point  $H$  such that  $AH/AE_{\{a\}}=2$ 
towards H A  $E_{\{a\}}$  2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0

% Constructing a point  $B$  such that  $AB/AM_{\{c\}}=2$ 
towards B A  $M_{\{c\}}$  2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% Constructing a point  $C$  such that  $E_{\{c\}}C/E_{\{c\}}H=-1$ 
towards C  $E_{\{c\}}$  H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

```

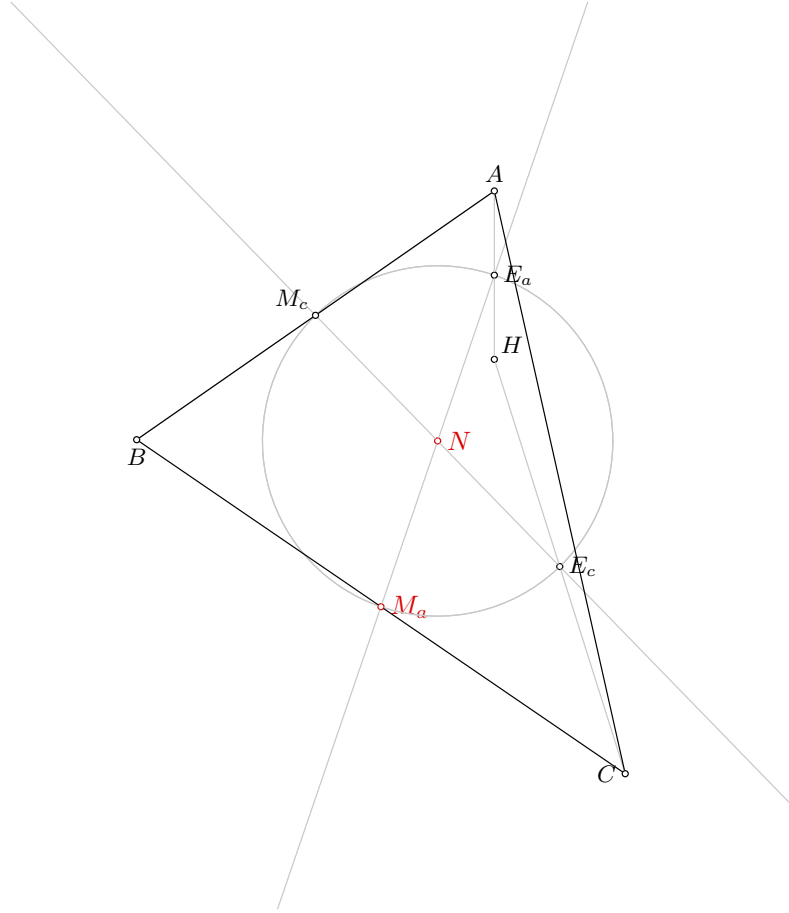



Figure 1: Illustration of the problem 1037

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line m(H_{b}H_{a}) and circle k(N,M_{a}) intersect; line m(H_{b}H_{c})
and circle k(N,M_{a}) intersect; points M_{a} and N are not the same
% Determination conditions: points E_{c} and M_{c} must be different; points M_{a} and E_{a} must
be different; points E_{c} and N are not the same; points M_{a} and N are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 15 terms.

Time Complexity: Time spent by the prover is 0.283 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $N = \neg N$

Proving failed

4.1.3 Proving $E_c = \neg E_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $M_a = \neg M_a$

Proving failed

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $E_c = \neg E_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_a = \neg M_a$

Proving failed

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $E_c = \neg E_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_a = \neg M_a$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $E_c = E_c$

Proving failed

Problem 1038

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1038: Given a point E_c , a point M_a and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point M_a , construct a line $m(CH_b)$ (rule W02); % DET: points E_c and M_a are not the same;
2. Using the point M_a and the point O , construct a line m_a (rule W02); % DET: points M_a and O are not the same;
3. Using the point M_a and the line m_a , construct a line a (rule W10a); ;
4. Using the point E_c and the line m_a , construct a line $m(CH_a)$ (rule W16); ;
5. Using the point O and the line $m(CH_b)$, construct a line m_b (rule W16); ;
6. Using the line m_b and the line $m(CH_a)$, construct a point M_b (rule W03); % NDG: lines m_b and $m(CH_a)$ are not parallel % DET: lines m_b and $m(CH_a)$ are not the same;
7. Using the point M_b and the line m_b , construct a line b (rule W10a); ;
8. Using the line b and the line a , construct a point C (rule W03); % NDG: lines b and a are not parallel % DET: lines b and a are not the same;
9. Using the point C and the point E_c , construct a point H (rule W01); ;
10. Using the point M_a and the point C , construct a point B (rule W01); ;
11. Using the point O and the point H , construct a point G (rule W01); ;
12. Using the point M_a and the point G , construct a point A (rule W01); .

Non-degenerate conditions: lines b and a are not parallel; lines m_b and $m(CH_a)$ are not parallel.

Determination conditions: lines b and a are not the same; lines m_b and $m(CH_a)$ are not the same; points M_a and O are not the same; points E_c and M_a are not the same.

Rules used: [W01,W02,W03,W10a,W16]

Lemmas used: [D1,D11,D12,D21,D22,D30,GD01,GD02,GL01,GL03,GL04,GL09,L37,L38,L44,L53,L54,L55,L58]

Solving time: 3.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
```

```
point M_{a} 65 40
```

```
point O 65 51.14
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{c}
```

```
cmark_r M_{a}
```

```
cmark_t O
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{c} and M_{a} are not the same
```

```
% Constructing a line m(CH_{b}) which passes through point E_{c} and point M_{a}
```

```
line m(CH_{b}) E_{c} M_{a}
```

```
color 200 200 200
```

```
drawline m(CH_{b})
```

```
color 0 0 0
```

```
% DET: points M_{a} and O are not the same
```

```
% Constructing a line m_{a} which passes through point M_{a} and point O
```

```
line m_{a} M_{a} O
```

```
color 200 200 200
```

```
drawline m_{a}
```

```
color 0 0 0
```

```
% Constructing a line a which is perpendicular to line m_{a} and which passes through point M_{a}  
perp a M_{a} m_{a}
```

```
color 200 200 200
```

```
drawline a
```

```
color 0 0 0
```

```
% Constructing a line  $m(CH_{\{a\}})$  which contains the point  $E_{\{c\}}$  and is parallel to the line  $m_{\{a\}}$ 
parallel m(CH_{a}) E_{c} m_{a}
```

```
color 200 200 200
drawline m(CH_{a})
color 0 0 0
```

```
% Constructing a line  $m_{\{b\}}$  which contains the point  $O$  and is parallel to the line  $m(CH_{\{b\}})$ 
parallel m_{b} O m(CH_{b})
```

```
color 200 200 200
drawline m_{b}
color 0 0 0
```

```
% NDG: lines  $m_{\{b\}}$  and  $m(CH_{\{a\}})$  are not parallel% DET: lines  $m_{\{b\}}$  and  $m(CH_{\{a\}})$  are not the same
% Constructing a point  $M_{\{b\}}$  which belongs to line  $m_{\{b\}}$  and line  $m(CH_{\{a\}})$ 
intersec M_{b} m_{b} m(CH_{a})
cmark_lt M_{b}
```

```
% Constructing a line  $b$  which is perpendicular to line  $m_{\{b\}}$  and which passes through point  $M_{\{b\}}$ 
perp b M_{b} m_{b}
```

```
color 200 200 200
drawline b
color 0 0 0
```

```
% NDG: lines  $b$  and  $a$  are not parallel% DET: lines  $b$  and  $a$  are not the same
% Constructing a point  $C$  which belongs to line  $b$  and line  $a$ 
intersec C b a
cmark_l C
```

```
% Constructing a point  $H$  such that  $CH/CE_{\{c\}}=2$ 
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0
```

```
% Constructing a point  $B$  such that  $M_{\{a\}}B/M_{\{a\}}C=-1$ 
towards B M_{a} C -1
cmark_b B
color 200 200 200
```

```

drawsegment C B
color 0 0 0

% Constructing a line  $L_{\{ \_G97165 \}}$  which passes through point O and point H
line  $L_{\{ \_G97165 \}}$  O H

color 200 200 200
drawline  $L_{\{ \_G97165 \}}$ 
color 0 0 0

% Constructing a point  $P_{\{ \_G97266 \}}$  with coordinates (0,0)
point  $P_{\{ \_G97266 \}}$  0 0
cmark_r  $P_{\{ \_G97266 \}}$ 

% Constructing a point  $P_{\{ \_G97190 \}}$  such that  $OP_{\{ \_G97190 \}}/OP_{\{ \_G97266 \}}=1$ 
towards  $P_{\{ \_G97190 \}}$  O  $P_{\{ \_G97266 \}}$  1
cmark_r  $P_{\{ \_G97190 \}}$ 
color 200 200 200
drawsegment O  $P_{\{ \_G97190 \}}$ 
color 0 0 0

% Constructing a point  $P_{\{ \_G97235 \}}$  such that  $OP_{\{ \_G97235 \}}/OP_{\{ \_G97266 \}}=3$ 
towards  $P_{\{ \_G97235 \}}$  O  $P_{\{ \_G97266 \}}$  3
cmark_r  $P_{\{ \_G97235 \}}$ 
color 200 200 200
drawsegment O  $P_{\{ \_G97235 \}}$ 
color 0 0 0

% Constructing a line  $L_{\{ \_G97196 \}}$  which passes through point H and point  $P_{\{ \_G97235 \}}$ 
line  $L_{\{ \_G97196 \}}$  H  $P_{\{ \_G97235 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G97196 \}}$ 
color 0 0 0

% Constructing a line  $L_{\{ \_G97159 \}}$  which contains the point  $P_{\{ \_G97190 \}}$  and is parallel to the
line  $L_{\{ \_G97196 \}}$ 
parallel  $L_{\{ \_G97159 \}}$   $P_{\{ \_G97190 \}}$   $L_{\{ \_G97196 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G97159 \}}$ 
color 0 0 0

% Constructing a point G which belongs to line  $L_{\{ \_G97159 \}}$  and line  $L_{\{ \_G97165 \}}$ 
intersec G  $L_{\{ \_G97159 \}}$   $L_{\{ \_G97165 \}}$ 
cmark_t G

% Constructing a point A such that  $M_{\{ a \}}A/M_{\{ a \}}G=3$ 
towards A  $M_{\{ a \}}$  G 3

```

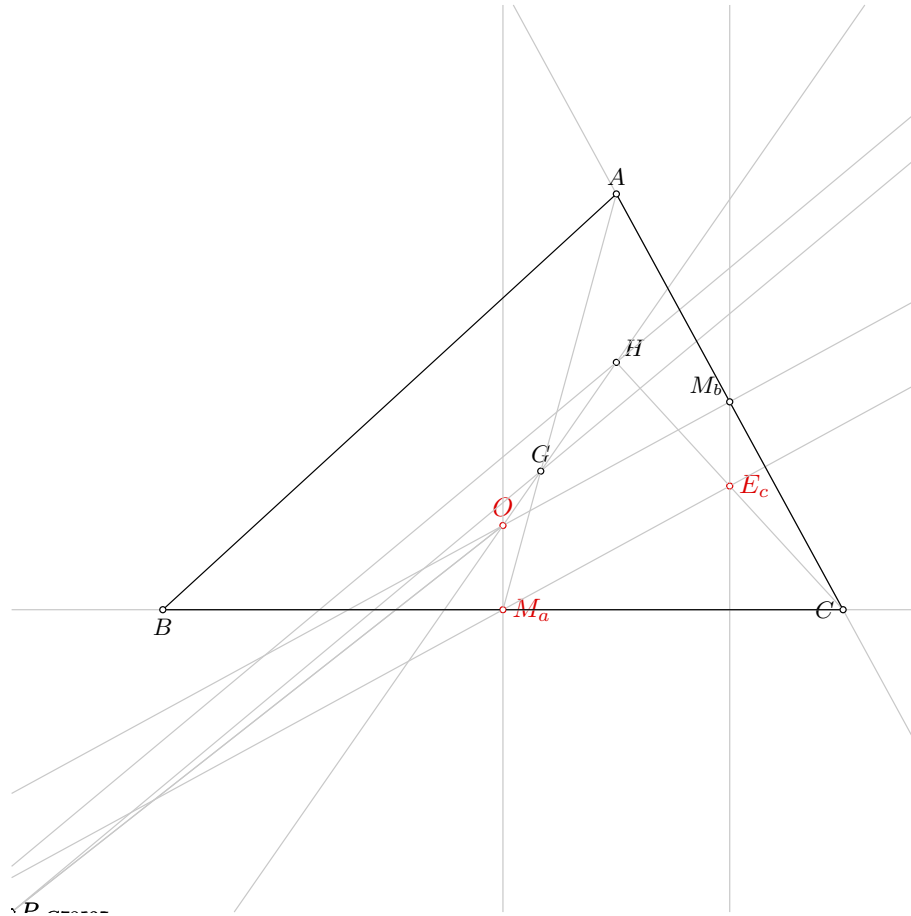


Figure 1: Illustration of the problem 1038

```

cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

% Non-degenerate conditions: lines b and a are not parallel; lines $m_{\{b\}}$ and $m(CH_{\{a\}})$ are not parallel

% Determination conditions: lines b and a are not the same; lines $m_{\{b\}}$ and $m(CH_{\{a\}})$ are not the same; points $M_{\{a\}}$ and O are not the same; points $E_{\{c\}}$ and $M_{\{a\}}$ are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 9 terms.

Time Complexity: Time spent by the prover is 0.188 seconds.

NDG conditions Points M_b , E_c and M_a are not collinear

Points M_b , M_a and O are not collinear

4.1.3 Proving $O = \neg O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{OE_c P_{m(CH_a)}^1} \neq S_{P_{m_b}^2 E_c P_{m(CH_a)}^1}$ i.e., lines $OP_{m_b}^2$ and $E_c P_{m(CH_a)}^1$ are not parallel (construction based assumption)

$S_{M_b OP_{m_b}^2} \neq 0$ i.e., points M_b , O and $P_{m_b}^2$ are not collinear (foot is not the point itself; construction based assumption)

$S_{M_b M_a T_a^0} \neq S_{F_b^3 M_a T_a^0}$ i.e., lines $M_b F_b^3$ and $M_a T_a^0$ are not parallel (construction based assumption)

$S_{P_{-G83301} OH} \neq S_{P_{L_{-G83270}}^4 OH}$ i.e., lines $P_{-G83301} P_{L_{-G83270}}^4$ and OH are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^6} \neq S_{F_{-h_a}^5 BF_{-h_b}^6}$ i.e., lines $AF_{-h_a}^5$ and $BF_{-h_b}^6$ are not parallel (construction based assumption)

$S_{\neg M_a BC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_b AC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_a \neg M_b F_{-m_b}^8} \neq S_{F_{-m_a}^7 \neg M_b F_{-m_b}^8}$ i.e., lines $\neg M_a F_{-m_a}^7$ and $\neg M_b F_{-m_b}^8$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.2.3 Proving $O = \neg O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = E_c$

Proving failed

4.3.2 Proving $M_a = M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.080 seconds. There are no ndg conditions.

4.3.3 Proving $O = O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = E_c$

Proving failed

4.4.2 Proving $M_a = M_a$

Proving failed

4.4.3 Proving $O = O$

Proving failed

Problem 1039

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1039: Given a point E_c , a point M_a and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1040

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1040: Given a point E_c , a point M_a and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1041

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1041: Given a point E_c , a point M_a and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1042

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1042: Given a point E_c , a point M_c and a point M_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point M_c , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and M_c are not the same;
2. Using the point E_c and the point M_c , construct a line $m(E_cM_c)$ (rule W14); % DET: points E_c and M_c are not the same;
3. Using the line $m(E_cM_c)$ and the line $m(H_bH_a)$, construct a point N (rule W03); % NDG: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel % DET: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same;
4. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
5. Choose freely a point M_b on the circle $k(N, M_a)$ (rule WOncircle);
6. Choose freely a point A (rule free);
7. Using the point A and the point M_b , construct a point C (rule W01); ;
8. Using the point A and the point M_c , construct a point B (rule W01); .

Non-degenerate conditions: points E_c and N are not the same; lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel.

Determination conditions: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same; points E_c and M_c are not the same; points E_c and M_c are not the same.

Rules used: [W01,W02,W03,W06,W14,WOncircle1,free]

Lemmas used: [D20,D22,D32,GD02,GL01,GL03,GL04,L18,L19,L20,L24,L41,L42,L53,L54]

Solving time: 202.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point E_{c} 95 56.36
point M_{c} 50 67.5
point M_{b} 95 67.5

color 220 0 0
fontsize 9

cmark_r E_{c}
cmark_lt M_{c}
cmark_lt M_{b}
color 0 0 0
fontsize 8

% DET: points E_{c} and M_{c} are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point M_{c}
line m(H_{b}H_{a}) E_{c} M_{c}

color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0

% DET: points E_{c} and M_{c} are not the same
% Constructing bisector m(E_{c}M_{c}) of the segment E_{c}M_{c}
med m(E_{c}M_{c}) E_{c} M_{c}

color 200 200 200
drawline m(E_{c}M_{c})
color 0 0 0

color 200 200 200
drawsegment E_{c} M_{c}
color 0 0 0

% NDG: lines m(E_{c}M_{c}) and m(H_{b}H_{a}) are not parallel% DET: lines m(E_{c}M_{c}) and m(H_{b}
H_{a}) are not the same
% Constructing a point N which belongs to line m(E_{c}M_{c}) and line m(H_{b}H_{a})
intersec N m(E_{c}M_{c}) m(H_{b}H_{a})
cmark_r N

% NDG: points E_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{c}
circle k(N,M_{a}) N E_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```

% Choosing randomly a point  $M_{\{b\}}$  on the circle with center  $N$  through point  $E_{\{c\}}$ 
oncircle  $M_{\{b\}}$   $N$   $E_{\{c\}}$ 
cmark_lt  $M_{\{b\}}$ 
color 200 200 200
drawcircle  $N$   $E_{\{c\}}$ 
color 0 0 0

% Constructing a free point  $A$ 
point  $A$  80 95

cmark_t  $A$ 

% Constructing a point  $C$  such that  $AC/AM_{\{b\}}=2$ 
towards  $C$   $A$   $M_{\{b\}}$  2
cmark_l  $C$ 
color 200 200 200
drawsegment  $A$   $C$ 
color 0 0 0

% Constructing a point  $B$  such that  $AB/AM_{\{c\}}=2$ 
towards  $B$   $A$   $M_{\{c\}}$  2
cmark_b  $B$ 
color 200 200 200
drawsegment  $A$   $B$ 
color 0 0 0

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

% Non-degenerate conditions: points  $E_{\{c\}}$  and  $N$  are not the same; lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not parallel
% Determination conditions: lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not the same; points  $E_{\{c\}}$  and  $M_{\{c\}}$  are not the same; points  $E_{\{c\}}$  and  $M_{\{c\}}$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

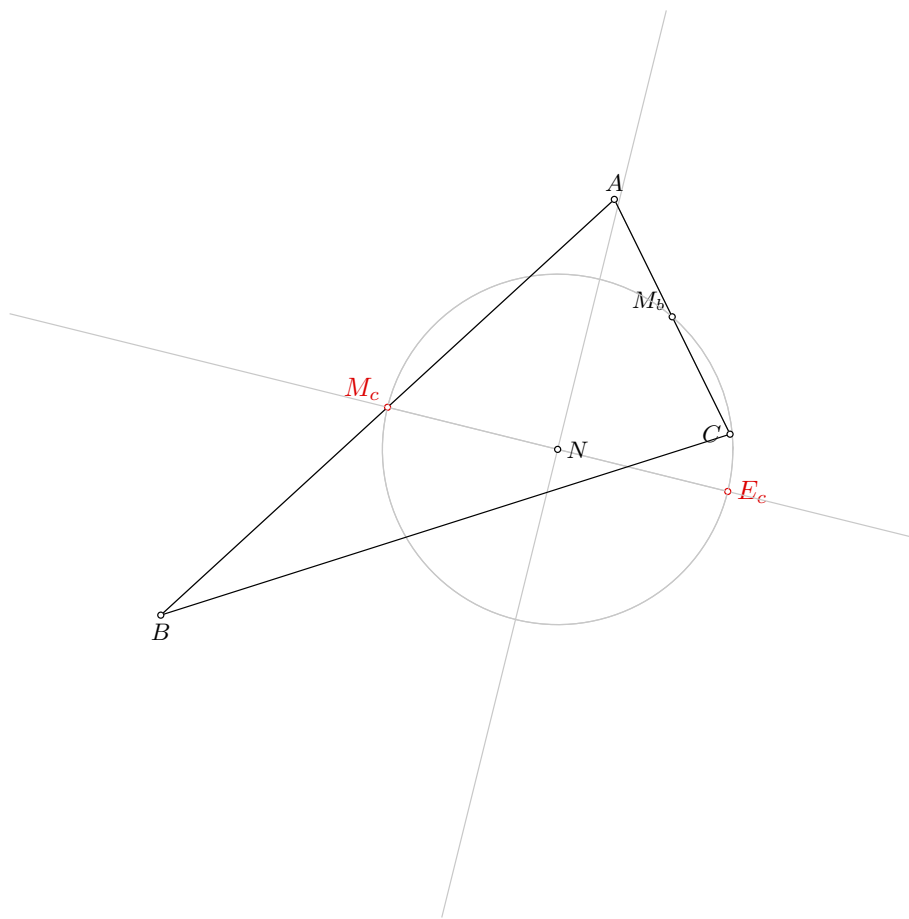


Figure 1: Illustration of the problem 1042

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $M_c = \neg M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.056 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.3 Proving $M_b = \neg M_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{m(E_c M_c) E_c M_c}^0 \neq S_{m(E_c M_c) E_c M_c}^1$ i.e., lines $M_{m(E_c M_c)}^0 T_{m(E_c M_c)}^1$ and $E_c M_c$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{F_{-h_a}^2 BF_{-h_b}^3}$ i.e., lines $AF_{-h_a}^2$ and $BF_{-h_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.2.3 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{m(E_c M_c) E_c M_c}^0 \neq S_{m(E_c M_c) E_c M_c}^1$ i.e., lines $M_{m(E_c M_c)}^0 T_{m(E_c M_c)}^1$ and $E_c M_c$ are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^3} \neq S_{F_{-h_a}^2 BF_{-h_b}^3}$ i.e., lines $AF_{-h_a}^2$ and $BF_{-h_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $M_c = \neg M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 19 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.3.3 Proving $M_b = \neg M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 28 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $M_c = \neg M_c$

Proving failed

4.4.3 Proving $M_b = \neg M_b$

Proving failed

Problem 1043

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1043: Given a point M_b , a point N and a point E_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point M_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_b and N are not the same;
2. Choose freely a point E_c on the circle $k(N, M_a)$ (rule WOncircle);
3. Using the point E_c and the point N , construct a line $m(H_b H_a)$ (rule W02); % DET: points E_c and N are not the same;
4. Using the circle $k(N, M_a)$, the line $m(H_b H_a)$, the point N and the point E_c , construct a point M_c (rule W05a); % NDG: line $m(H_b H_a)$ and circle $k(N, M_a)$ intersect % DET: points E_c and M_c must be different;
5. Choose freely a point A (rule free);
6. Using the point A and the point M_b , construct a point C (rule W01); ;
7. Using the point A and the point M_c , construct a point B (rule W01); .

Non-degenerate conditions: line $m(H_b H_a)$ and circle $k(N, M_a)$ intersect; points M_b and N are not the same.

Determination conditions: points E_c and M_c must be different; points E_c and N are not the same.

Rules used: [W01,W02,W05a,W06,WOncircle1,free]

Lemmas used: [D20,D22,D32,GD02,GL01,GL03,GL04,L17,L18,L19,L20,L24,L41,L42,L53,L54]

Solving time: 151.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point N 72.5 61.93
point E_{c} 95 56.36

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_r N
cmark_r E_{c}
color 0 0 0
fontsize 8

% NDG: points M_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{b}
circle k(N,M_{a}) N M_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point E_{c} on the circle with center N through point M_{b}
oncircle E_{c} N M_{b}
cmark_r E_{c}
color 200 200 200
drawcircle N M_{b}
color 0 0 0

% DET: points E_{c} and N are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point N
line m(H_{b}H_{a}) E_{c} N

color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0

% NDG: line m(H_{b}H_{a}) and circle k(N,M_{a}) intersect% DET: points E_{c} and M_{c} must be
different
% Constructing a point M_{c} which is an image of the point E_{c} in the symmetry to point/line N
sim M_{c} N E_{c}
cmark_lt M_{c}

% Constructing a free point A
point A 80 95
```

```
cmark_t A
```

```
% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0
```

```
% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line m(H_{b}H_{a}) and circle k(N,M_{a}) intersect; points M_{b} and N
are not the same
% Determination conditions: points E_{c} and M_{c} must be different; points E_{c} and N are not
the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_b = M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.055 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $N = N$

Proving failed

4.1.3 Proving $E_c = E_c$

Proving failed

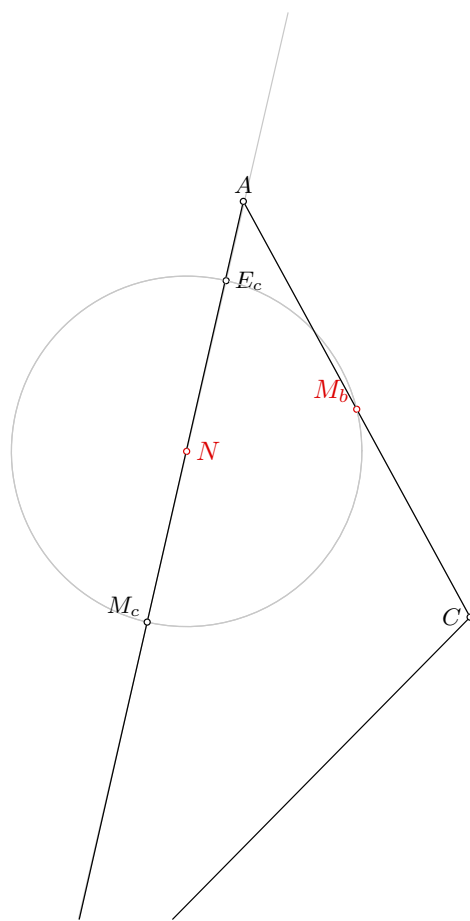


Figure 1: Illustration of the problem 1043

4.2 GCLC - Area method

4.2.1 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{\neg M_a BC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_b AC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2 BF_{\neg h_b}^3}$ i.e., lines $AF_{\neg h_a}^2$ and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

$S_{\neg M_a \neg M_b F_{\neg m_b}^1} \neq S_{F_{\neg m_a}^0 \neg M_b F_{\neg m_b}^1}$ i.e., lines $\neg M_a F_{\neg m_a}^0$ and $\neg M_b F_{\neg m_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $E_c = \neg E_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_b = \neg M_b$

Proving failed

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $E_c = \neg E_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_b = \neg M_b$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $E_c = \neg E_c$

Proving failed

Problem 1044

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1044: Given a point E_c , a point M_b and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point M_b , construct a line $m(CH_a)$ (rule W02); % DET: points E_c and M_b are not the same;
2. Using the point M_b and the point O , construct a line m_b (rule W02); % DET: points M_b and O are not the same;
3. Using the point M_b and the line m_b , construct a line b (rule W10a); ;
4. Using the point E_c and the line m_b , construct a line $m(CH_b)$ (rule W16); ;
5. Using the point O and the line $m(CH_a)$, construct a line m_a (rule W16); ;
6. Using the line m_a and the line $m(CH_b)$, construct a point M_a (rule W03); % NDG: lines m_a and $m(CH_b)$ are not parallel % DET: lines m_a and $m(CH_b)$ are not the same;
7. Using the point M_a and the line m_a , construct a line a (rule W10a); ;
8. Using the line a and the line b , construct a point C (rule W03); % NDG: lines a and b are not parallel % DET: lines a and b are not the same;
9. Using the point C and the point E_c , construct a point H (rule W01); ;
10. Using the point M_b and the point C , construct a point A (rule W01); ;
11. Using the point O and the point H , construct a point G (rule W01); ;
12. Using the point M_b and the point G , construct a point B (rule W01); .

Non-degenerate conditions: lines a and b are not parallel; lines m_a and $m(CH_b)$ are not parallel.

Determination conditions: lines a and b are not the same; lines m_a and $m(CH_b)$ are not the same; points M_b and O are not the same; points E_c and M_b are not the same.

Rules used: [W01,W02,W03,W10a,W16]

Lemmas used: [D1,D11,D12,D21,D22,D30,GD01,GD02,GL01,GL03,GL04,GL09,L37,L38,L44,L53,L54,L56,L58]

Solving time: 3.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
```

```
point M_{b} 95 67.5
```

```
point O 65 51.14
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{c}
```

```
cmark_lt M_{b}
```

```
cmark_t O
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{c} and M_{b} are not the same
```

```
% Constructing a line m(CH_{a}) which passes through point E_{c} and point M_{b}
```

```
line m(CH_{a}) E_{c} M_{b}
```

```
color 200 200 200
```

```
drawline m(CH_{a})
```

```
color 0 0 0
```

```
% DET: points M_{b} and O are not the same
```

```
% Constructing a line m_{b} which passes through point M_{b} and point O
```

```
line m_{b} M_{b} O
```

```
color 200 200 200
```

```
drawline m_{b}
```

```
color 0 0 0
```

```
% Constructing a line b which is perpendicular to line m_{b} and which passes through point M_{b}
```

```
perp b M_{b} m_{b}
```

```
color 200 200 200
```

```
drawline b
```

```
color 0 0 0
```

```
% Constructing a line  $m(CH_{\{b\}})$  which contains the point  $E_{\{c\}}$  and is parallel to the line  $m_{\{b\}}$ 
parallel m(CH_{b}) E_{c} m_{b}
```

```
color 200 200 200
drawline m(CH_{b})
color 0 0 0
```

```
% Constructing a line  $m_{\{a\}}$  which contains the point  $O$  and is parallel to the line  $m(CH_{\{a\}})$ 
parallel m_{a} O m(CH_{a})
```

```
color 200 200 200
drawline m_{a}
color 0 0 0
```

```
% NDG: lines  $m_{\{a\}}$  and  $m(CH_{\{b\}})$  are not parallel% DET: lines  $m_{\{a\}}$  and  $m(CH_{\{b\}})$  are not the same
% Constructing a point  $M_{\{a\}}$  which belongs to line  $m_{\{a\}}$  and line  $m(CH_{\{b\}})$ 
intersec M_{a} m_{a} m(CH_{b})
cmark_r M_{a}
```

```
% Constructing a line  $a$  which is perpendicular to line  $m_{\{a\}}$  and which passes through point  $M_{\{a\}}$ 
perp a M_{a} m_{a}
```

```
color 200 200 200
drawline a
color 0 0 0
```

```
% NDG: lines  $a$  and  $b$  are not parallel% DET: lines  $a$  and  $b$  are not the same
% Constructing a point  $C$  which belongs to line  $a$  and line  $b$ 
intersec C a b
cmark_l C
```

```
% Constructing a point  $H$  such that  $CH/CE_{\{c\}}=2$ 
towards H C E_{c} 2
cmark_rt H
color 200 200 200
drawsegment C H
color 0 0 0
```

```
% Constructing a point  $A$  such that  $M_{\{b\}}A/M_{\{b\}}C=-1$ 
towards A M_{b} C -1
cmark_t A
color 200 200 200
```

```

drawsegment C A
color 0 0 0

% Constructing a line  $L_{\{ \_G200292 \}}$  which passes through point O and point H
line  $L_{\{ \_G200292 \}}$  O H

color 200 200 200
drawline  $L_{\{ \_G200292 \}}$ 
color 0 0 0

% Constructing a point  $P_{\{ \_G200393 \}}$  with coordinates (0,0)
point  $P_{\{ \_G200393 \}}$  0 0
cmark_r  $P_{\{ \_G200393 \}}$ 

% Constructing a point  $P_{\{ \_G200317 \}}$  such that  $OP_{\{ \_G200317 \}}/OP_{\{ \_G200393 \}}=1$ 
towards  $P_{\{ \_G200317 \}}$  O  $P_{\{ \_G200393 \}}$  1
cmark_r  $P_{\{ \_G200317 \}}$ 
color 200 200 200
drawsegment O  $P_{\{ \_G200317 \}}$ 
color 0 0 0

% Constructing a point  $P_{\{ \_G200362 \}}$  such that  $OP_{\{ \_G200362 \}}/OP_{\{ \_G200393 \}}=3$ 
towards  $P_{\{ \_G200362 \}}$  O  $P_{\{ \_G200393 \}}$  3
cmark_r  $P_{\{ \_G200362 \}}$ 
color 200 200 200
drawsegment O  $P_{\{ \_G200362 \}}$ 
color 0 0 0

% Constructing a line  $L_{\{ \_G200323 \}}$  which passes through point H and point  $P_{\{ \_G200362 \}}$ 
line  $L_{\{ \_G200323 \}}$  H  $P_{\{ \_G200362 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G200323 \}}$ 
color 0 0 0

% Constructing a line  $L_{\{ \_G200286 \}}$  which contains the point  $P_{\{ \_G200317 \}}$  and is parallel to the
line  $L_{\{ \_G200323 \}}$ 
parallel  $L_{\{ \_G200286 \}}$   $P_{\{ \_G200317 \}}$   $L_{\{ \_G200323 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G200286 \}}$ 
color 0 0 0

% Constructing a point G which belongs to line  $L_{\{ \_G200286 \}}$  and line  $L_{\{ \_G200292 \}}$ 
intersec G  $L_{\{ \_G200286 \}}$   $L_{\{ \_G200292 \}}$ 
cmark_t G

% Constructing a point B such that  $M_{\{ b \}}B/M_{\{ b \}}G=3$ 
towards B  $M_{\{ b \}}$  G 3

```

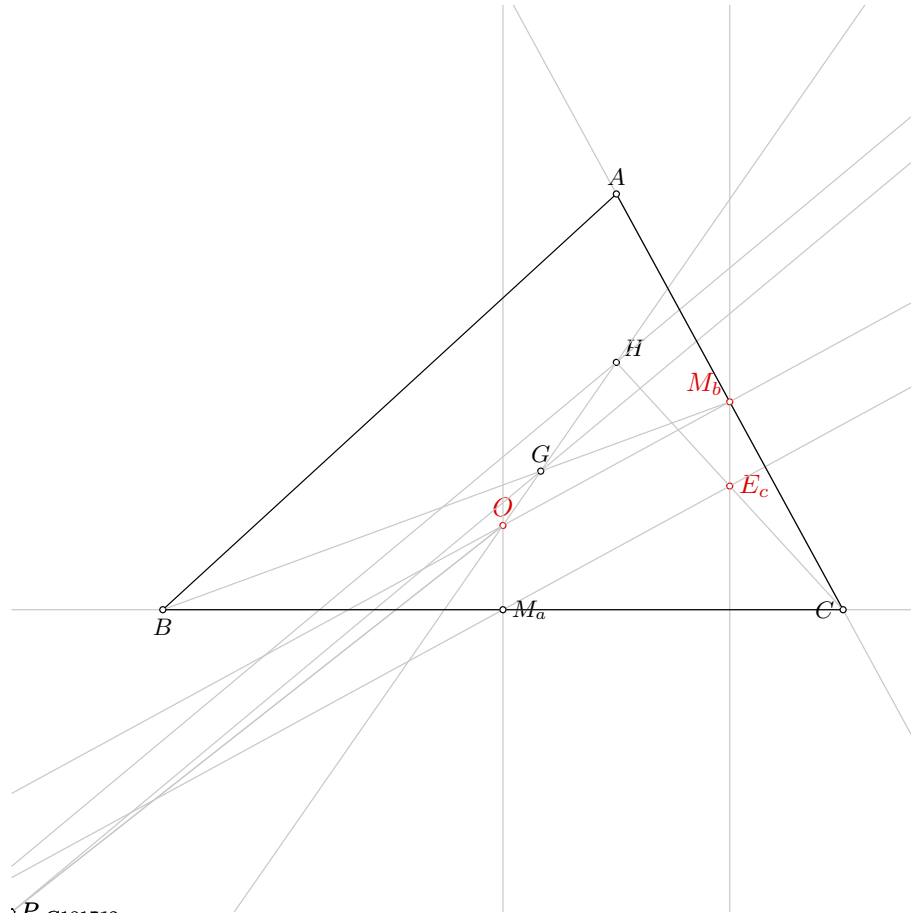


Figure 1: Illustration of the problem 1044

```

cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

% Non-degenerate conditions: lines a and b are not parallel; lines $m_{\{a\}}$ and $m(CH_{\{b\}})$ are not parallel

% Determination conditions: lines a and b are not the same; lines $m_{\{a\}}$ and $m(CH_{\{b\}})$ are not the same; points $M_{\{b\}}$ and O are not the same; points $E_{\{c\}}$ and $M_{\{b\}}$ are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 9 terms.

Time Complexity: Time spent by the prover is 0.186 seconds.

NDG conditions Points M_b and O are not identical

Points M_b , M_a and O are not collinear

4.1.3 Proving $O = \neg O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{OE_c P_{m(CH_b)}^1} \neq S_{P_{m_a}^2 E_c P_{m(CH_b)}^1}$ i.e., lines $OP_{m_a}^2$ and $E_c P_{m(CH_b)}^1$ are not parallel (construction based assumption)
 $S_{M_a OP_{m_a}^2} \neq 0$ i.e., points M_a , O and $P_{m_a}^2$ are not collinear (foot is not the point itself; construction based assumption)

$S_{M_a M_b T_b^0} \neq S_{F_a^3 M_b T_b^0}$ i.e., lines $M_a F_a^3$ and $M_b T_b^0$ are not parallel (construction based assumption)

$S_{P_{-G186422} OH} \neq S_{P_{L_{-G186391}}^4 OH}$ i.e., lines $P_{-G186422} P_{L_{-G186391}}^4$ and OH are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{-h_b}^6} \neq S_{F_{-h_a}^5 BF_{-h_b}^6}$ i.e., lines $AF_{-h_a}^5$ and $BF_{-h_b}^6$ are not parallel (construction based assumption)

$S_{\neg M_a BC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_b AC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_a \neg M_b F_{\neg m_b}^8} \neq S_{F_{\neg m_a}^7 \neg M_b F_{\neg m_b}^8}$ i.e., lines $\neg M_a F_{\neg m_a}^7$ and $\neg M_b F_{\neg m_b}^8$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.2.3 Proving $O = \neg O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = E_c$

Proving failed

4.3.2 Proving $M_b = M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.080 seconds. There are no ndg conditions.

4.3.3 Proving $O = O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = E_c$

Proving failed

4.4.2 Proving $M_b = M_b$

Proving failed

4.4.3 Proving $O = O$

Proving failed

Problem 1045

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1045: Given a point E_c , a point M_b and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1046

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1046: Given a point E_c , a point M_b and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1047

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1047: Given a point E_c , a point M_b and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1048

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1048: Given a point M_c , a point N and a point E_c , construct the triangle ABC .

2 Status

Problem is redundant.

3 Solution

3.1 Construction in natural language form

1. Using the point M_c and the point N , construct a line $m(H_bH_a)$ (rule W02); % DET: points M_c and N are not the same;
2. Using the point M_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_c and N are not the same;
3. Using the circle $k(N, M_a)$, the line $m(H_bH_a)$, the point N and the point M_c , construct a point E_c (rule W05a); % NDG: line $m(H_bH_a)$ and circle $k(N, M_a)$ intersect % DET: points M_c and E_c must be different;
4. Choose freely a point A (rule free);
5. Using the point A and the point M_c , construct a point B (rule W01); ;
6. Using the point A and the point M_c , construct a line c (rule W02); % DET: points A and M_c are not the same;
7. Using the circle $k(N, M_a)$, the line c , the point N and the point M_c , construct a point H_c (rule W05); % NDG: line c and circle $k(N, M_a)$ intersect % DET: points M_c and H_c must be different;
8. Using the point H_c and the point E_c , construct a line h_c (rule W02); % DET: points H_c and E_c are not the same;
9. Using the point A and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points A and M_c are not the same;

10. Using the circle $k(N, M_a)$ and the circle $k(M_c, A)$, construct a point H_a and a point H_b (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_c, A)$ intersect % DET: circles $k(N, M_a)$ and $k(M_c, A)$ are not the same;
11. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
12. Using the line h_c and the line h_a , construct a point H (rule W03); % NDG: lines h_c and h_a are not parallel % DET: lines h_c and h_a are not the same;
13. Using the point N and the point H , construct a point G (rule W01); ;
14. Using the point M_c and the point G , construct a point C (rule W01); .

Non-degenerate conditions: lines h_c and h_a are not parallel; circles $k(N, M_a)$ and $k(M_c, A)$ intersect; points A and M_c are not the same; line c and circle $k(N, M_a)$ intersect; line $m(H_b H_a)$ and circle $k(N, M_a)$ intersect; points M_c and N are not the same.

Determination conditions: lines h_c and h_a are not the same; points A and H_a are not the same; circles $k(N, M_a)$ and $k(M_c, A)$ are not the same; points H_c and E_c are not the same; points M_c and H_c must be different; points A and M_c are not the same; points M_c and E_c must be different; points M_c and N are not the same.

Rules used: [W01,W02,W03,W05,W05a,W06,W07,free]

Lemmas used: [D10,D20,D3,D30,D32,D5,D7,D8,GD01,GD02,GL01,GL03,GL04,GL09,L16,L18,L19,L20,L21,L22]

Solving time: 5.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{c} 50 67.5
point N 72.5 61.93
point E_{c} 95 56.36
```

```
color 220 0 0
fontsize 9
```

```
cmark_lt M_{c}
cmark_r N
cmark_r E_{c}
color 0 0 0
fontsize 8
```

```
% DET: points M_{c} and N are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point M_{c} and point N
line m(H_{b}H_{a}) M_{c} N
```

```
color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0
```

```

% NDG: points  $M_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $M_{\{c\}}$ 
circle k(N, M_{a}) N M_{c}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line  $m(H_{\{b\}}H_{\{a\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{c\}}$  and  $E_{\{c\}}$  must be
different
% Constructing a point  $E_{\{c\}}$  which is an image of the point  $M_{\{c\}}$  in the symmetry to point/line  $N$ 
sim E_{c} N M_{c}
cmark_r E_{c}

% Constructing a free point  $A$ 
point A 80 95

cmark_t A

% Constructing a point  $B$  such that  $AB/AM_{\{c\}}=2$ 
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points  $A$  and  $M_{\{c\}}$  are not the same
% Constructing a line  $c$  which passes through point  $A$  and point  $M_{\{c\}}$ 
line c A M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: line  $c$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{c\}}$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\_G260322\}}$  which is a foot of the point  $N$  on the line  $c$ 
foot P_{\_G260322} N c
cmark_r P_{\_G260322}
color 200 200 200
drawline N P_{\_G260322}
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $M_{\{c\}}$  in the symmetry to point/line  $P_{\{\_G260322\}}$ 
sim H_{c} P_{\_G260322} M_{c}
cmark_rt H_{c}

```

```

% DET: points  $H_{\{c\}}$  and  $E_{\{c\}}$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $H_{\{c\}}$  and point  $E_{\{c\}}$ 
line  $h_{\{c\}}$   $H_{\{c\}}$   $E_{\{c\}}$ 

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: points  $A$  and  $M_{\{c\}}$  are not the same
% Constructing a circle  $k(M_{\{c\}}, A)$  whose center is at point  $M_{\{c\}}$  and which passes through point  $A$ 
circle  $k(M_{\{c\}}, A)$   $M_{\{c\}}$   $A$ 

color 200 200 200
drawcircle  $k(M_{\{c\}}, A)$ 
color 0 0 0

% NDG: circles  $k(N, M_{\{a\}})$  and  $k(M_{\{c\}}, A)$  intersect% DET: circles  $k(N, M_{\{a\}})$  and  $k(M_{\{c\}}, A)$  are not
the same
% Constructing points  $H_{\{a\}}$  and  $H_{\{b\}}$  which are in intersection of  $k(N, M_{\{a\}})$  and  $k(M_{\{c\}}, A)$ 
intersec2  $H_{\{a\}}$   $H_{\{b\}}$   $k(N, M_{\{a\}})$   $k(M_{\{c\}}, A)$ 
cmark_r  $H_{\{a\}}$ 
cmark_l  $H_{\{b\}}$ 

% DET: points  $A$  and  $H_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $A$  and point  $H_{\{a\}}$ 
line  $h_{\{a\}}$   $A$   $H_{\{a\}}$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% NDG: lines  $h_{\{c\}}$  and  $h_{\{a\}}$  are not parallel% DET: lines  $h_{\{c\}}$  and  $h_{\{a\}}$  are not the same
% Constructing a point  $H$  which belongs to line  $h_{\{c\}}$  and line  $h_{\{a\}}$ 
intersec  $H$   $h_{\{c\}}$   $h_{\{a\}}$ 
cmark_rt  $H$ 

% Constructing a line  $L_{\{\backslash\_G260866\}}$  which passes through point  $N$  and point  $H$ 
line  $L_{\{\backslash\_G260866\}}$   $N$   $H$ 

color 200 200 200
drawline  $L_{\{\backslash\_G260866\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\backslash\_G260967\}}$  with coordinates  $(0,0)$ 
point  $P_{\{\backslash\_G260967\}}$  0 0

```

```

cmark_r P_{\_G260967}

% Constructing a point P_{\_G260891} such that NP_{\_G260891}/NP_{\_G260967}=-1
towards P_{\_G260891} N P_{\_G260967} -1
cmark_r P_{\_G260891}
color 200 200 200
drawsegment P_{\_G260967} P_{\_G260891}
color 0 0 0

% Constructing a point P_{\_G260936} such that NP_{\_G260936}/NP_{\_G260967}=3
towards P_{\_G260936} N P_{\_G260967} 3
cmark_r P_{\_G260936}
color 200 200 200
drawsegment N P_{\_G260936}
color 0 0 0

% Constructing a line L_{\_G260897} which passes through point H and point P_{\_G260936}
line L_{\_G260897} H P_{\_G260936}

color 200 200 200
drawline L_{\_G260897}
color 0 0 0

% Constructing a line L_{\_G260860} which contains the point P_{\_G260891} and is parallel to the
line L_{\_G260897}
parallel L_{\_G260860} P_{\_G260891} L_{\_G260897}

color 200 200 200
drawline L_{\_G260860}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G260860} and line L_{\_G260866}
intersec G L_{\_G260860} L_{\_G260866}
cmark_t G

% Constructing a point C such that M_{c}C/M_{c}G=3
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines h_{c} and h_{a} are not parallel; circles k(N,M_{a}) and k(M_{c}
,A) intersect; points A and M_{c} are not the same; line c and circle k(N,M_{a}) intersect;
line m(H_{b}H_{a}) and circle k(N,M_{a}) intersect; points M_{c} and N are not the same

```

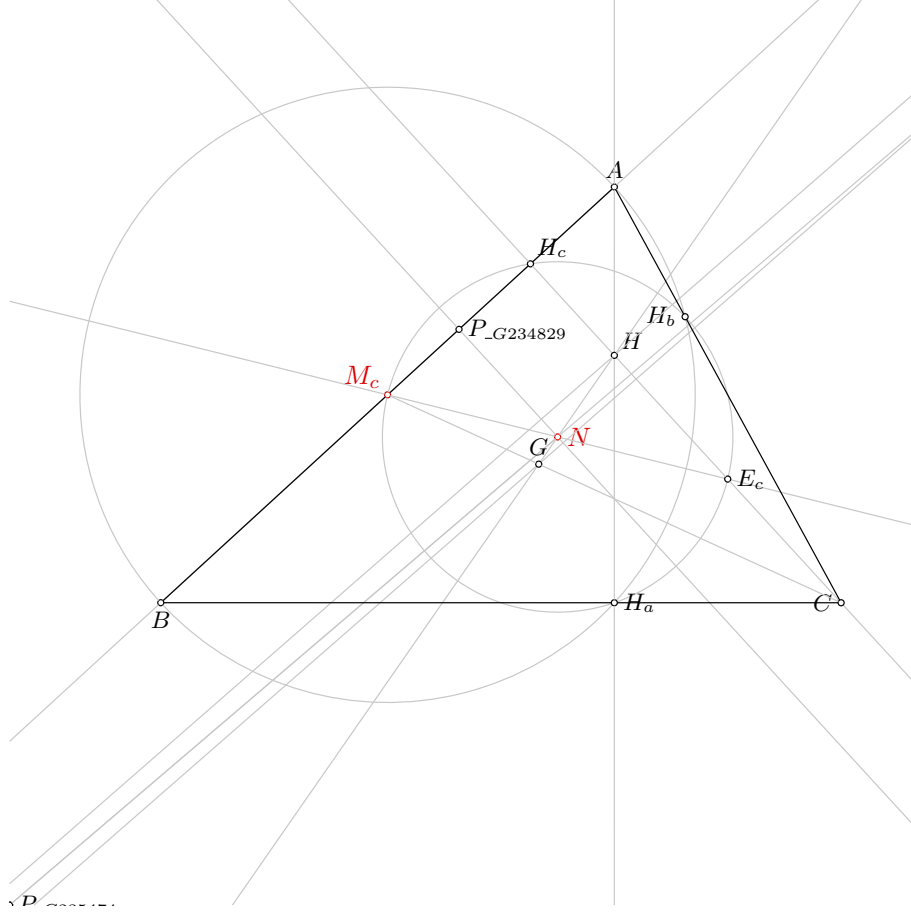


Figure 1: Illustration of the problem 1048

% Determination conditions: lines $h_{\{c\}}$ and $h_{\{a\}}$ are not the same; points A and $H_{\{a\}}$ are not the same; circles $k(N, M_{\{a\}})$ and $k(M_{\{c\}}, A)$ are not the same; points $H_{\{c\}}$ and $E_{\{c\}}$ are not the same; points $M_{\{c\}}$ and $H_{\{c\}}$ must be different; points A and $M_{\{c\}}$ are not the same; points $M_{\{c\}}$ and $E_{\{c\}}$ must be different; points $M_{\{c\}}$ and N are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_c = M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.036 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $E_c=_E E_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $M_c=_M M_c$

Proving failed

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $E_c=_E E_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_c=_M M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.080 seconds. There are no ndg conditions.

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $E_c=_E E_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_c=_M M_c$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $E_c=_E E_c$

Proving failed

Problem 1049

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1049: Given a point E_c , a point M_c and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point E_c and the point M_c , construct a line $m(H_bH_a)$ (rule W02); % DET: points E_c and M_c are not the same;
2. Using the point M_c and the point O , construct a line m_c (rule W02); % DET: points M_c and O are not the same;
3. Using the point M_c and the line m_c , construct a line c (rule W10a); ;
4. Using the point E_c and the point M_c , construct a line $m(E_cM_c)$ (rule W14); % DET: points E_c and M_c are not the same;
5. Using the line $m(E_cM_c)$ and the line $m(H_bH_a)$, construct a point N (rule W03); % NDG: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel % DET: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same;
6. Using the point N and the point O , construct a point G (rule W01); ;
7. Using the point M_c and the point G , construct a point C (rule W01); ;
8. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
9. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; lines $m(E_cM_c)$ and $m(H_bH_a)$ are not parallel.

Determination conditions: lines $m(E_cM_c)$ and $m(H_bH_a)$ are not the same; points E_c and M_c are not the same; points M_c and O are not the same; points E_c and M_c are not the same.

Rules used: [W01,W02,W03,W04,W06,W10a,W14]

Lemmas used: [D13,D20,D26,D32,GD01,GD02,GL01,GL03,GL04,GL09,L1,L11,L12,L15,L18,L19,L20,L24,L41,

Solving time: 6.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
```

```
point M_{c} 50 67.5
```

```
point O 65 51.14
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{c}
```

```
cmark_lt M_{c}
```

```
cmark_t O
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points E_{c} and M_{c} are not the same
```

```
% Constructing a line m(H_{b}H_{a}) which passes through point E_{c} and point M_{c}
```

```
line m(H_{b}H_{a}) E_{c} M_{c}
```

```
color 200 200 200
```

```
drawline m(H_{b}H_{a})
```

```
color 0 0 0
```

```
% DET: points M_{c} and O are not the same
```

```
% Constructing a line m_{c} which passes through point M_{c} and point O
```

```
line m_{c} M_{c} O
```

```
color 200 200 200
```

```
drawline m_{c}
```

```
color 0 0 0
```

```
% Constructing a line c which is perpendicular to line m_{c} and which passes through point M_{c}
```

```
perp c M_{c} m_{c}
```

```
color 200 200 200
```

```
drawline c
```

```
color 0 0 0
```

```
% DET: points E_{c} and M_{c} are not the same
```

```

% Constructing bisector  $m(E_{\{c\}}M_{\{c\}})$  of the segment  $E_{\{c\}}M_{\{c\}}$ 
med m( $E_{\{c\}}M_{\{c\}}$ )  $E_{\{c\}}$   $M_{\{c\}}$ 

color 200 200 200
drawline m( $E_{\{c\}}M_{\{c\}}$ )
color 0 0 0

color 200 200 200
drawsegment  $E_{\{c\}}$   $M_{\{c\}}$ 
color 0 0 0

% NDG: lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not parallel% DET: lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}$ 
 $H_{\{a\}})$  are not the same
% Constructing a point  $N$  which belongs to line  $m(E_{\{c\}}M_{\{c\}})$  and line  $m(H_{\{b\}}H_{\{a\}})$ 
intersec N m( $E_{\{c\}}M_{\{c\}}$ ) m( $H_{\{b\}}H_{\{a\}}$ )
cmark_r N

% Constructing a line  $L_{\{\_G73528\}}$  which passes through point  $N$  and point  $O$ 
line  $L_{\{\_G73528\}}$  N O

color 200 200 200
drawline  $L_{\{\_G73528\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\_G73629\}}$  with coordinates  $(0,0)$ 
point  $P_{\{\_G73629\}}$  0 0
cmark_r  $P_{\{\_G73629\}}$ 

% Constructing a point  $P_{\{\_G73553\}}$  such that  $NP_{\{\_G73553\}}/NP_{\{\_G73629\}}=1$ 
towards  $P_{\{\_G73553\}}$  N  $P_{\{\_G73629\}}$  1
cmark_r  $P_{\{\_G73553\}}$ 
color 200 200 200
drawsegment N  $P_{\{\_G73553\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\_G73598\}}$  such that  $NP_{\{\_G73598\}}/NP_{\{\_G73629\}}=3$ 
towards  $P_{\{\_G73598\}}$  N  $P_{\{\_G73629\}}$  3
cmark_r  $P_{\{\_G73598\}}$ 
color 200 200 200
drawsegment N  $P_{\{\_G73598\}}$ 
color 0 0 0

% Constructing a line  $L_{\{\_G73559\}}$  which passes through point  $O$  and point  $P_{\{\_G73598\}}$ 
line  $L_{\{\_G73559\}}$  O  $P_{\{\_G73598\}}$ 

color 200 200 200
drawline  $L_{\{\_G73559\}}$ 
color 0 0 0

% Constructing a line  $L_{\{\_G73522\}}$  which contains the point  $P_{\{\_G73553\}}$  and is parallel to the
line  $L_{\{\_G73559\}}$ 

```

```

parallel L_{\_G73522} P_{\_G73553} L_{\_G73559}

color 200 200 200
drawline L_{\_G73522}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G73522} and line L_{\_G73528}
intersec G L_{\_G73522} L_{\_G73528}
cmark_t G

% Constructing a point C such that  $M_{\{c\}}C/M_{\{c\}}G=3$ 
towards C M_{\{c\}} G 3
cmark_l C
color 200 200 200
drawsegment M_{\{c\}} C
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle  $k(O,C)$  whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line c and circle  $k(O,C)$  intersect
% Constructing points A and B which are in intersection of  $k(O,C)$  and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line c and circle  $k(O,C)$  intersect; points C and O are not the same;
% lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not parallel
% Determination conditions: lines  $m(E_{\{c\}}M_{\{c\}})$  and  $m(H_{\{b\}}H_{\{a\}})$  are not the same; points  $E_{\{c\}}$ 
% and  $M_{\{c\}}$  are not the same; points  $M_{\{c\}}$  and O are not the same; points  $E_{\{c\}}$  and  $M_{\{c\}}$  are not
% the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

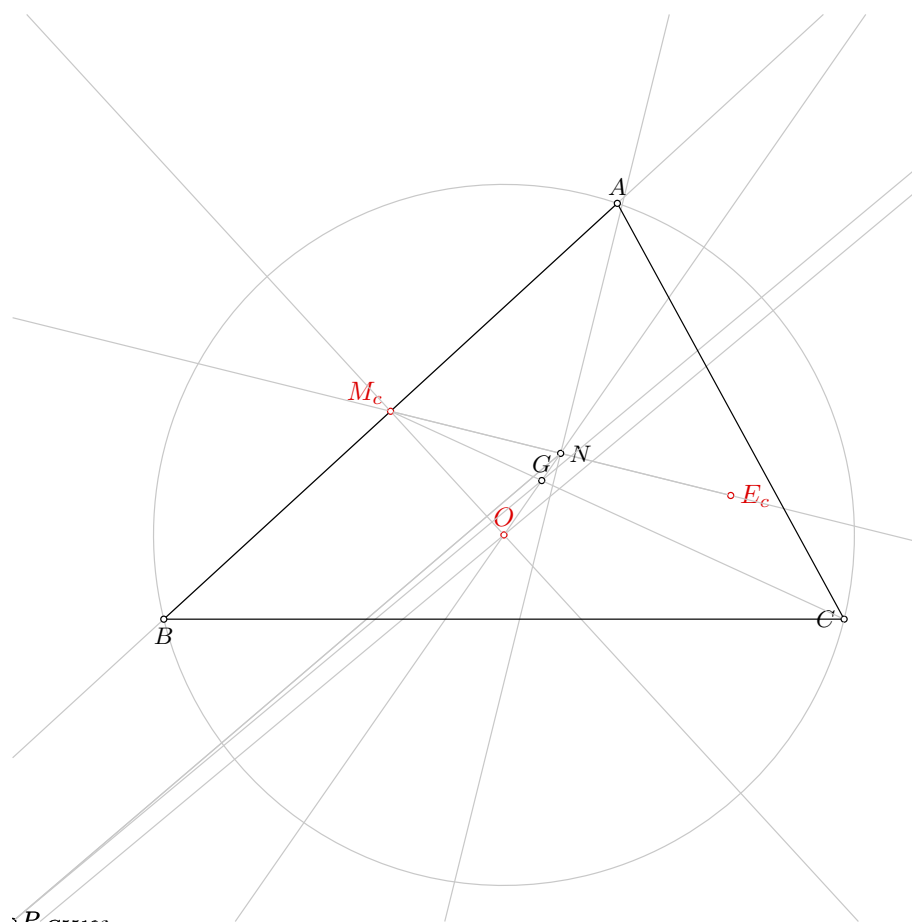


Figure 1: Illustration of the problem 1049

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = \neg E_c$

Proving failed

4.1.2 Proving $M_c = \neg M_c$

Proving failed

4.1.3 Proving $O = \neg O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c = \neg E_c$

Proving failed

4.2.2 Proving $M_c = \neg M_c$

Proving failed

4.2.3 Proving $O = \neg O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c = \neg E_c$

Proving failed

4.3.2 Proving $M_c = \neg M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 923 terms.

Time Complexity: Time spent by the prover is 1.960 seconds. There are no ndg conditions.

4.3.3 Proving $O = \neg O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c = \neg E_c$

Proving failed

4.4.2 Proving $M_c = \neg M_c$

Proving failed

4.4.3 Proving $O=_O$

Proving failed

Problem 1050

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1050: Given a point E_c , a point M_c and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1051

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1051: Given a point E_c , a point M_c and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1052

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1052: Given a point E_c , a point M_c and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1053

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1053: Given a point E_c , a point N and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point N and the point O , construct a point G (rule W01); ;
2. Using the point N and the point O , construct a point H (rule W01); ;
3. Using the point E_c and the point H , construct a point C (rule W01); ;
4. Using the point G and the point C , construct a point M_c (rule W01); ;
5. Using the point E_c and the point H , construct a line h_c (rule W02); % DET: points E_c and H are not the same;
6. Using the point E_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_c and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
8. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
9. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; line h_c and circle $k(N, M_a)$ intersect; points E_c and N are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points E_c and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D31,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L15,L21,L24,L3,L57]

Solving time: 7.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point E_{c} 95 56.36
```

```
point N 72.5 61.93
```

```
point O 65 51.14
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r E_{c}
```

```
cmark_r N
```

```
cmark_t O
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a line L_{\_G134783} which passes through point N and point O
```

```
line L_{\_G134783} N O
```

```
color 200 200 200
```

```
drawline L_{\_G134783}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G134884} with coordinates (0,0)
```

```
point P_{\_G134884} 0 0
```

```
cmark_r P_{\_G134884}
```

```
% Constructing a point P_{\_G134808} such that NP_{\_G134808}/NP_{\_G134884}=1
```

```
towards P_{\_G134808} N P_{\_G134884} 1
```

```
cmark_r P_{\_G134808}
```

```
color 200 200 200
```

```
drawsegment N P_{\_G134808}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G134853} such that NP_{\_G134853}/NP_{\_G134884}=3
```

```
towards P_{\_G134853} N P_{\_G134884} 3
```

```
cmark_r P_{\_G134853}
```

```
color 200 200 200
```

```
drawsegment N P_{\_G134853}
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G134814} which passes through point O and point P_{\_G134853}
```

```
line L_{\_G134814} O P_{\_G134853}
```

```

color 200 200 200
drawline L_{\_G134814}
color 0 0 0

% Constructing a line L_{\_G134777} which contains the point P_{\_G134808} and is parallel to the
line L_{\_G134814}
parallel L_{\_G134777} P_{\_G134808} L_{\_G134814}

color 200 200 200
drawline L_{\_G134777}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G134777} and line L_{\_G134783}
intersec G L_{\_G134777} L_{\_G134783}
cmark_t G

% Constructing a point H such that NH/NO=-1
towards H N 0 -1
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

% Constructing a point M_{c} such that GM_{c}/GC=-0.5
towards M_{c} G C -0.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% DET: points E_{c} and H are not the same
% Constructing a line h_{c} which passes through point E_{c} and point H
line h_{c} E_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

```

```

% NDG: points  $E_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $E_{\{c\}}$ 
circle k(N, M_{a}) N E_{c}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{c\}}$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\_G135868\}}$  which is a foot of the point  $N$  on the line  $h_{\{c\}}$ 
foot P_{\_G135868} N h_{c}
cmark_r P_{\_G135868}
color 200 200 200
drawline N P_{\_G135868}
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $E_{\{c\}}$  in the symmetry to point/line  $P_{\{\_G135868\}}$ 
sim H_{c} P_{\_G135868} E_{c}
cmark_rt H_{c}

% DET: points  $H_{\{c\}}$  and  $M_{\{c\}}$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $M_{\{c\}}$ 
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points  $C$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $C$ 
circle k(O, C) O C

color 200 200 200
drawcircle k(O, C)
color 0 0 0

% NDG: line  $c$  and circle  $k(O, C)$  intersect
% Constructing points  $A$  and  $B$  which are in intersection of  $k(O, C)$  and  $c$ 
intersec2 A B k(O, C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```

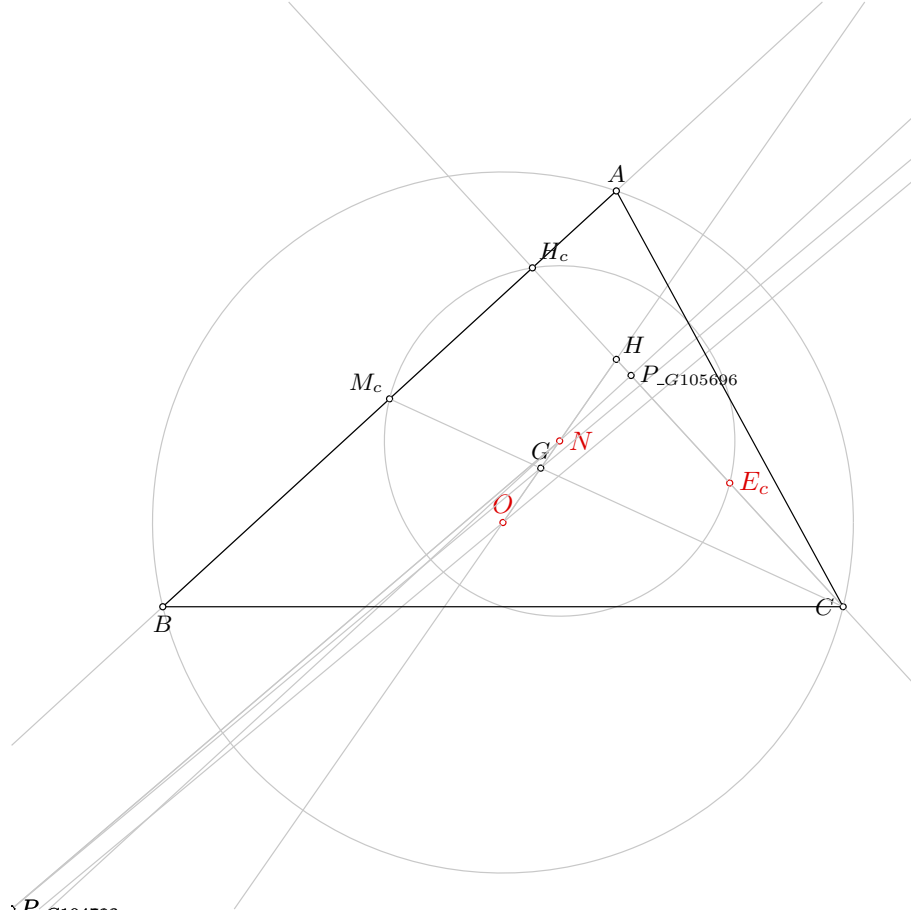


Figure 1: Illustration of the problem 1053

*% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
 line h_{c} and circle k(N,M_{a}) intersect; points E_{c} and N are not the same
 % Determination conditions: points H_{c} and M_{c} are not the same; points E_{c} and H_{c} must be
 different; points E_{c} and H are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $E_c = E_c$

Proving failed

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $O=_O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $E_c=_E$

Proving failed

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $O=_O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $E_c=_E$

Proving failed

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $O=_O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $E_c=_E$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $O=_O$

Proving failed

Problem 1054

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1054: Given a point E_c , a point N and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1055

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1055: Given a point E_c , a point N and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1056

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1056: Given a point E_c , a point N and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1057

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1057: Given a point E_c , a point O and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1058

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1058: Given a point E_c , a point O and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1059

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1059: Given a point E_c , a point O and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1060

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1060: Given a point E_c , a point T_a and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1061

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1061: Given a point E_c , a point T_a and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1062

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1062: Given a point E_c , a point T_b and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1063

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1063: Given a point H , a point N and a point G , construct the triangle ABC .

2 Status

Problem is redundant.

3 Solution

3.1 Construction in natural language form

1. Using the point H and the point N , construct a point O (rule W01); ;
2. Using the point H and the point N , construct a point G (rule W01); ;
3. Choose freely a point A (rule free);
4. Using the point A and the point H , construct a point E_a (rule W01); ;
5. Using the point A and the point G , construct a point M_a (rule W01); ;
6. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
7. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
8. Using the point E_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points E_a and N are not the same;
9. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
10. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
11. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; line h_a and circle $k(N, M_a)$ intersect; points E_a and N are not the same; points A and O are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points A and H are not the same.

Rules used: [W01,W02,W04,W05,W06,free]

Lemmas used: [D21,D26,D28,D3,D31,D32,D5,D8,GD01,GL03,GL09,L11,L12,L16,L19,L22,L55]

Solving time: 8.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H 80 72.73
```

```
point N 72.5 61.93
```

```
point G 70 58.33
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H
```

```
cmark_r N
```

```
cmark_t G
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point O such that HO/HN=2
```

```
towards O H N 2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment H O
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G234721} which passes through point H and point N
```

```
line L_{\_G234721} H N
```

```
color 200 200 200
```

```
drawline L_{\_G234721}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G234822} with coordinates (0,0)
```

```
point P_{\_G234822} 0 0
```

```
cmark_r P_{\_G234822}
```

```
% Constructing a point P_{\_G234746} such that HP_{\_G234746}/HP_{\_G234822}=4
```

```
towards P_{\_G234746} H P_{\_G234822} 4
```

```
cmark_r P_{\_G234746}
```

```
color 200 200 200
```

```
drawsegment H P_{\_G234746}
```

```
color 0 0 0
```

```

% Constructing a point  $P_{\{G234791\}}$  such that  $HP_{\{G234791\}}/HP_{\{G234822\}}=3$ 
towards  $P_{\{G234791\}}$  H  $P_{\{G234822\}}$  3
cmark_r  $P_{\{G234791\}}$ 
color 200 200 200
drawsegment H  $P_{\{G234791\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G234752\}}$  which passes through point N and point  $P_{\{G234791\}}$ 
line  $L_{\{G234752\}}$  N  $P_{\{G234791\}}$ 

color 200 200 200
drawline  $L_{\{G234752\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G234715\}}$  which contains the point  $P_{\{G234746\}}$  and is parallel to the
line  $L_{\{G234752\}}$ 
parallel  $L_{\{G234715\}}$   $P_{\{G234746\}}$   $L_{\{G234752\}}$ 

color 200 200 200
drawline  $L_{\{G234715\}}$ 
color 0 0 0

% Constructing a point G which belongs to line  $L_{\{G234715\}}$  and line  $L_{\{G234721\}}$ 
intersec G  $L_{\{G234715\}}$   $L_{\{G234721\}}$ 
cmark_t G

% Constructing a free point A
point A 80 95

cmark_t A

% Constructing a point  $E_{\{a\}}$  such that  $AE_{\{a\}}/AH=0.5$ 
towards  $E_{\{a\}}$  A H 0.5
cmark_r  $E_{\{a\}}$ 
color 200 200 200
drawsegment A H
color 0 0 0

% Constructing a point  $M_{\{a\}}$  such that  $AM_{\{a\}}/AG=1.5$ 
towards  $M_{\{a\}}$  A G 1.5
cmark_r  $M_{\{a\}}$ 
color 200 200 200
drawsegment A  $M_{\{a\}}$ 
color 0 0 0

% DET: points A and H are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point A and point H

```

```

line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points E_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point E_{a}
circle k(N,M_{a}) N E_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G235825} which is a foot of the point N on the line h_{a}
foot P_{\_G235825} N h_{a}
cmark_r P_{\_G235825}
color 200 200 200
drawline N P_{\_G235825}
color 0 0 0

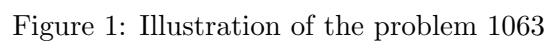
% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
\_G235825}
sim H_{a} P_{\_G235825} E_{a}
cmark_r H_{a}

% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C

```



```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; line h_{a} and circle k(N,M_{a})
% intersect; points E_{a} and N are not the same; points A and O are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
% different; points A and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H=_H$

Proving failed

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $G=_G$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H=_H$

Proving failed

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $G=_G$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H=_H$

Proving failed

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $G=_G$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H=_H$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $G=_G$

Proving failed

Problem 1064

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1064: Given a point G , a point H_a and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point G and the point N , construct a point H (rule W01); ;
2. Using the point G and the point N , construct a point O (rule W01); ;
3. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
4. Using the point H_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_a and N are not the same;
5. Using the circle $k(N, M_a)$, the line h_a , the point N and the point H_a , construct a point E_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points H_a and E_a must be different;
6. Using the point E_a and the point H , construct a point A (rule W01); ;
7. Using the point G and the point A , construct a point M_a (rule W01); ;
8. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
9. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; line h_a and circle $k(N, M_a)$ intersect; points H_a and N are not the same.

Determination conditions: points H_a and M_a are not the same; points H_a and E_a must be different; points H_a and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L15,L16,L19,L22,L55]

Solving time: 5.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point G 70 58.33
```

```
point H_{a} 80 40
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_t G
```

```
cmark_r H_{a}
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point H such that GH/GN=4
```

```
towards H G N 4
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment G H
```

```
color 0 0 0
```

```
% Constructing a point O such that GO/GN=-2
```

```
towards O G N -2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment N O
```

```
color 0 0 0
```

```
% DET: points H_{a} and H are not the same
```

```
% Constructing a line h_{a} which passes through point H_{a} and point H
```

```
line h_{a} H_{a} H
```

```
color 200 200 200
```

```
drawline h_{a}
```

```
color 0 0 0
```

```
% NDG: points H_{a} and N are not the same
```

```

% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $H_{\{a\}}$ 
circle k(N, M_{a}) N H_{a}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line  $h_{\{a\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $H_{\{a\}}$  and  $E_{\{a\}}$  must be different
% Constructing a point  $P_{\{\_G50813\}}$  which is a foot of the point  $N$  on the line  $h_{\{a\}}$ 
foot P_{\_G50813} N h_{a}
cmark_r P_{\_G50813}
color 200 200 200
drawline N P_{\_G50813}
color 0 0 0

% Constructing a point  $E_{\{a\}}$  which is an image of the point  $H_{\{a\}}$  in the symmetry to point/line  $P_{\{\_G50813\}}$ 
sim E_{a} P_{\_G50813} H_{a}
cmark_r E_{a}

% Constructing a point  $A$  such that  $E_{\{a\}}A/E_{\{a\}}H=-1$ 
towards A E_{a} H -1
cmark_t A
color 200 200 200
drawsegment H A
color 0 0 0

% Constructing a point  $M_{\{a\}}$  such that  $GM_{\{a\}}/GA=-0.5$ 
towards M_{a} G A -0.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% DET: points  $H_{\{a\}}$  and  $M_{\{a\}}$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $M_{\{a\}}$ 
line a H_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points  $A$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $A$ 
circle k(O, C) O A

color 200 200 200

```

```
drawcircle k(O,C)
color 0 0 0
```

```
% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% line h_{a} and circle k(N,M_{a}) intersect; points H_{a} and N are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points H_{a} and E_{a} must be
% different; points H_{a} and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $G = G$

Proving failed

4.1.2 Proving $H_a = H_a$

Proving failed

4.1.3 Proving $N = N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $G = G$

Proving failed

4.2.2 Proving $H_a = H_a$

Proving failed

4.2.3 Proving $N = N$

Proving failed

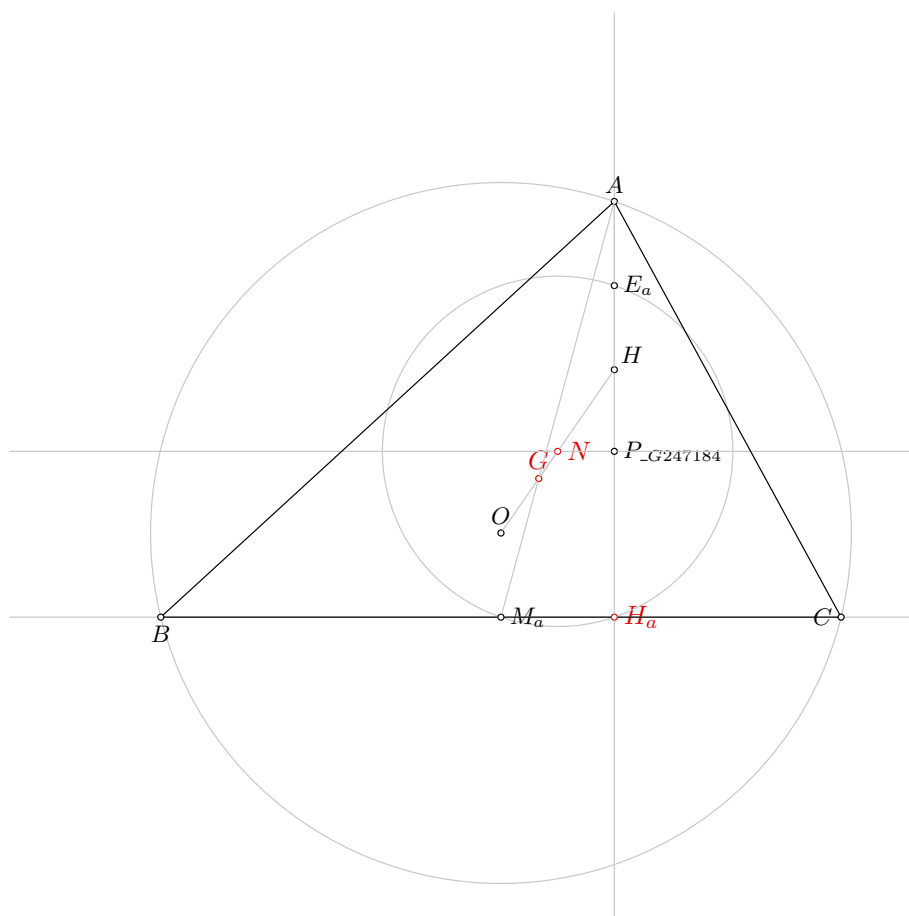


Figure 1: Illustration of the problem 1064

4.3 GCLC - Wu method

4.3.1 Proving $G=_G$

Proving failed

4.3.2 Proving $H_a=_H H_a$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $G=_G$

Proving failed

4.4.2 Proving $H_a=_H H_a$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 1065

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1065: Given a point G , a point H_b and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point G and the point N , construct a point H (rule W01); ;
2. Using the point G and the point N , construct a point O (rule W01); ;
3. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
4. Using the point H_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_b and N are not the same;
5. Using the circle $k(N, M_a)$, the line h_b , the point N and the point H_b , construct a point E_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points H_b and E_b must be different;
6. Using the point E_b and the point H , construct a point B (rule W01); ;
7. Using the point G and the point B , construct a point M_b (rule W01); ;
8. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
9. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; line h_b and circle $k(N, M_a)$ intersect; points H_b and N are not the same.

Determination conditions: points H_b and M_b are not the same; points H_b and E_b must be different; points H_b and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L15,L16,L20,L23,L56]

Solving time: 5.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point G 70 58.33
```

```
point H_{b} 89.36 77.83
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_t G
```

```
cmark_l H_{b}
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point H such that GH/GN=4
```

```
towards H G N 4
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment G H
```

```
color 0 0 0
```

```
% Constructing a point O such that GO/GN=-2
```

```
towards O G N -2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment N O
```

```
color 0 0 0
```

```
% DET: points H_{b} and H are not the same
```

```
% Constructing a line h_{b} which passes through point H_{b} and point H
```

```
line h_{b} H_{b} H
```

```
color 200 200 200
```

```
drawline h_{b}
```

```
color 0 0 0
```

```
% NDG: points H_{b} and N are not the same
```



```

% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $H_{\{b\}}$ 
circle k(N, M_{a}) N H_{b}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $H_{\{b\}}$  and  $E_{\{b\}}$  must be different
% Constructing a point  $P_{\{\_G87905\}}$  which is a foot of the point  $N$  on the line  $h_{\{b\}}$ 
foot P_{\_G87905} N h_{b}
cmark_r P_{\_G87905}
color 200 200 200
drawline N P_{\_G87905}
color 0 0 0

% Constructing a point  $E_{\{b\}}$  which is an image of the point  $H_{\{b\}}$  in the symmetry to point/line  $P_{\{\_G87905\}}$ 
sim E_{b} P_{\_G87905} H_{b}
cmark_r E_{b}

% Constructing a point  $B$  such that  $E_{\{b\}}B/E_{\{b\}}H=-1$ 
towards B E_{b} H -1
cmark_b B
color 200 200 200
drawsegment H B
color 0 0 0

% Constructing a point  $M_{\{b\}}$  such that  $GM_{\{b\}}/GB=-0.5$ 
towards M_{b} G B -0.5
cmark_lt M_{b}
color 200 200 200
drawsegment B M_{b}
color 0 0 0

% DET: points  $H_{\{b\}}$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $M_{\{b\}}$ 
line b H_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points  $B$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $B$ 
circle k(O, C) O B

color 200 200 200

```

```
drawcircle k(O,C)
color 0 0 0
```

```
% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;
% line h_{b} and circle k(N,M_{a}) intersect; points H_{b} and N are not the same
% Determination conditions: points H_{b} and M_{b} are not the same; points H_{b} and E_{b} must be
% different; points H_{b} and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $G=_G$

Proving failed

4.1.2 Proving $H_b=_H H_b$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $G=_G$

Proving failed

4.2.2 Proving $H_b=_H H_b$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

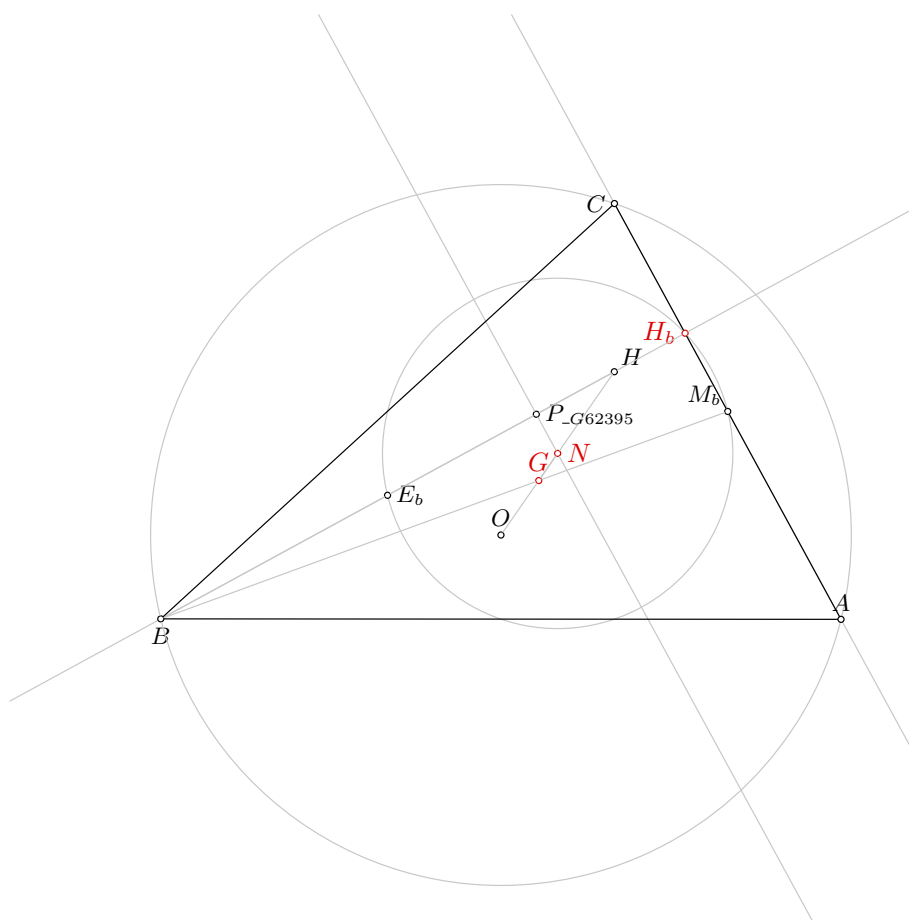


Figure 1: Illustration of the problem 1065

4.3 GCLC - Wu method

4.3.1 Proving $G=_G$

Proving failed

4.3.2 Proving $H_b=_H H_b$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $G=_G$

Proving failed

4.4.2 Proving $H_b=_H H_b$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 1066

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1066: Given a point G , a point H_c and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point G and the point N , construct a point H (rule W01); ;
2. Using the point G and the point N , construct a point O (rule W01); ;
3. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
4. Using the point H_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_c and N are not the same;
5. Using the circle $k(N, M_a)$, the line h_c , the point N and the point H_c , construct a point E_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points H_c and E_c must be different;
6. Using the point E_c and the point H , construct a point C (rule W01); ;
7. Using the point G and the point C , construct a point M_c (rule W01); ;
8. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
9. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; line h_c and circle $k(N, M_a)$ intersect; points H_c and N are not the same.

Determination conditions: points H_c and M_c are not the same; points H_c and E_c must be different; points H_c and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L15,L16,L21,L24,L3,L57]

Solving time: 5.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point G 70 58.33
```

```
point H_{c} 68.91 84.83
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_t G
```

```
cmark_rt H_{c}
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point H such that GH/GN=4
```

```
towards H G N 4
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment G H
```

```
color 0 0 0
```

```
% Constructing a point O such that GO/GN=-2
```

```
towards O G N -2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment N O
```

```
color 0 0 0
```

```
% DET: points H_{c} and H are not the same
```

```
% Constructing a line h_{c} which passes through point H_{c} and point H
```

```
line h_{c} H_{c} H
```

```
color 200 200 200
```

```
drawline h_{c}
```

```
color 0 0 0
```

```
% NDG: points H_{c} and N are not the same
```

```

% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $H_{\{c\}}$ 
circle k(N, M_{a}) N H_{c}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $H_{\{c\}}$  and  $E_{\{c\}}$  must be different
% Constructing a point  $P_{\{\_G126340\}}$  which is a foot of the point  $N$  on the line  $h_{\{c\}}$ 
foot P_{\_G126340} N h_{c}
cmark_r P_{\_G126340}
color 200 200 200
drawline N P_{\_G126340}
color 0 0 0

% Constructing a point  $E_{\{c\}}$  which is an image of the point  $H_{\{c\}}$  in the symmetry to point/line  $P_{\{\_G126340\}}$ 
sim E_{c} P_{\_G126340} H_{c}
cmark_r E_{c}

% Constructing a point  $C$  such that  $E_{\{c\}}C/E_{\{c\}}H=-1$ 
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

% Constructing a point  $M_{\{c\}}$  such that  $GM_{\{c\}}/GC=-0.5$ 
towards M_{c} G C -0.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% DET: points  $H_{\{c\}}$  and  $M_{\{c\}}$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $M_{\{c\}}$ 
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points  $C$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $C$ 
circle k(O, C) O C

color 200 200 200

```

```
drawcircle k(O,C)
color 0 0 0
```

```
% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
% line h_{c} and circle k(N,M_{a}) intersect; points H_{c} and N are not the same
% Determination conditions: points H_{c} and M_{c} are not the same; points H_{c} and E_{c} must be
% different; points H_{c} and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $G=_G$

Proving failed

4.1.2 Proving $H_c=_Hc$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $G=_G$

Proving failed

4.2.2 Proving $H_c=_Hc$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

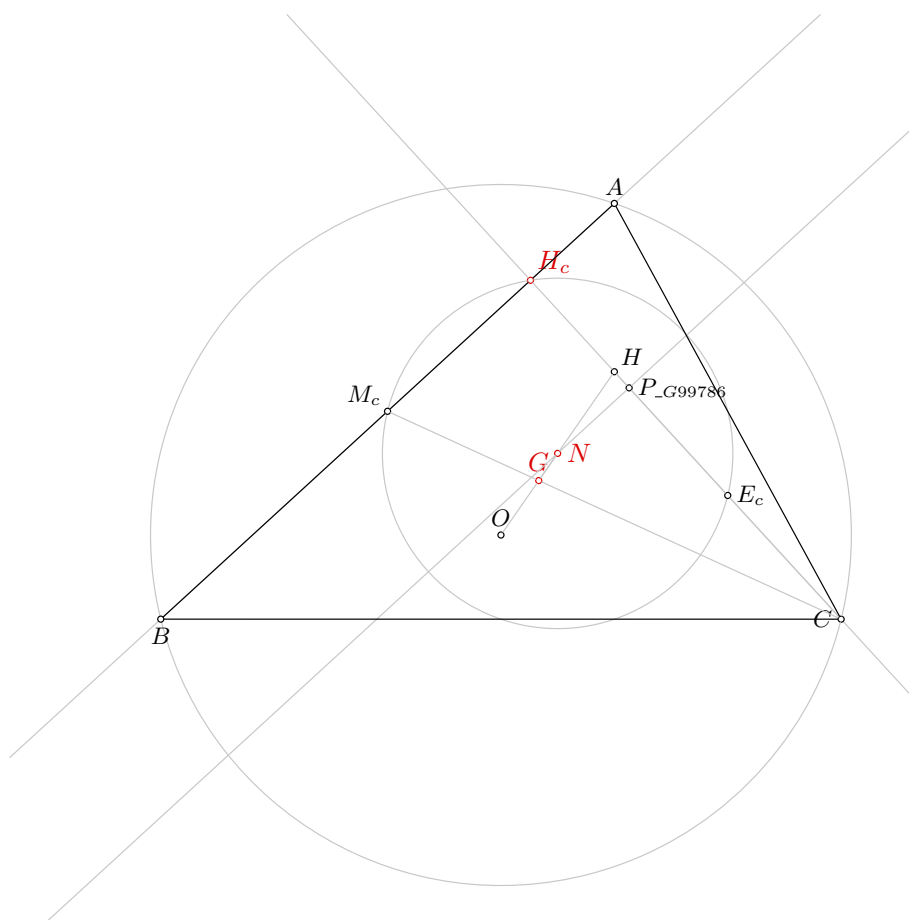


Figure 1: Illustration of the problem 1066

4.3 GCLC - Wu method

4.3.1 Proving $G=_G$

Proving failed

4.3.2 Proving $H_c=_H H_c$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $G=_G$

Proving failed

4.4.2 Proving $H_c=_H H_c$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 1067

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1067: Given a point G , a point I and a point N , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1068

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1068: Given a point G , a point M_a and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point G and the point M_a , construct a point A (rule W01); ;
2. Using the point G and the point N , construct a point H (rule W01); ;
3. Using the point G and the point N , construct a point O (rule W01); ;
4. Using the point A and the point H , construct a point E_a (rule W01); ;
5. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
6. Using the point M_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_a and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
8. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
9. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; line h_a and circle $k(N, M_a)$ intersect; points M_a and N are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points A and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L15,L16,L19,L22,L55]

Solving time: 7.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point G 70 58.33
```

```
point M_{a} 65 40
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_t G
```

```
cmark_r M_{a}
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point A such that GA/GM_{a}=-2
```

```
towards A G M_{a} -2
```

```
cmark_t A
```

```
color 200 200 200
```

```
drawsegment M_{a} A
```

```
color 0 0 0
```

```
% Constructing a point H such that GH/GN=4
```

```
towards H G N 4
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment G H
```

```
color 0 0 0
```

```
% Constructing a point O such that GO/GN=-2
```

```
towards O G N -2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment N O
```

```
color 0 0 0
```

```
% Constructing a point E_{a} such that AE_{a}/AH=0.5
```

```

towards E_{a} A H 0.5
cmark_r E_{a}
color 200 200 200
drawsegment A H
color 0 0 0

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H
line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points M_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{a}
circle k(N,M_{a}) N M_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G170151} which is a foot of the point N on the line h_{a}
foot P_{\_G170151} N h_{a}
cmark_r P_{\_G170151}
color 200 200 200
drawline N P_{\_G170151}
color 0 0 0

% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
\_G170151}
sim H_{a} P_{\_G170151} E_{a}
cmark_r H_{a}

% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200

```

```
drawcircle k(O,C)
color 0 0 0
```

```
% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% line h_{a} and circle k(N,M_{a}) intersect; points M_{a} and N are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
% different; points A and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $G=_G$

Proving failed

4.1.2 Proving $M_a=_M_a$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $G=_G$

Proving failed

4.2.2 Proving $M_a=_M_a$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

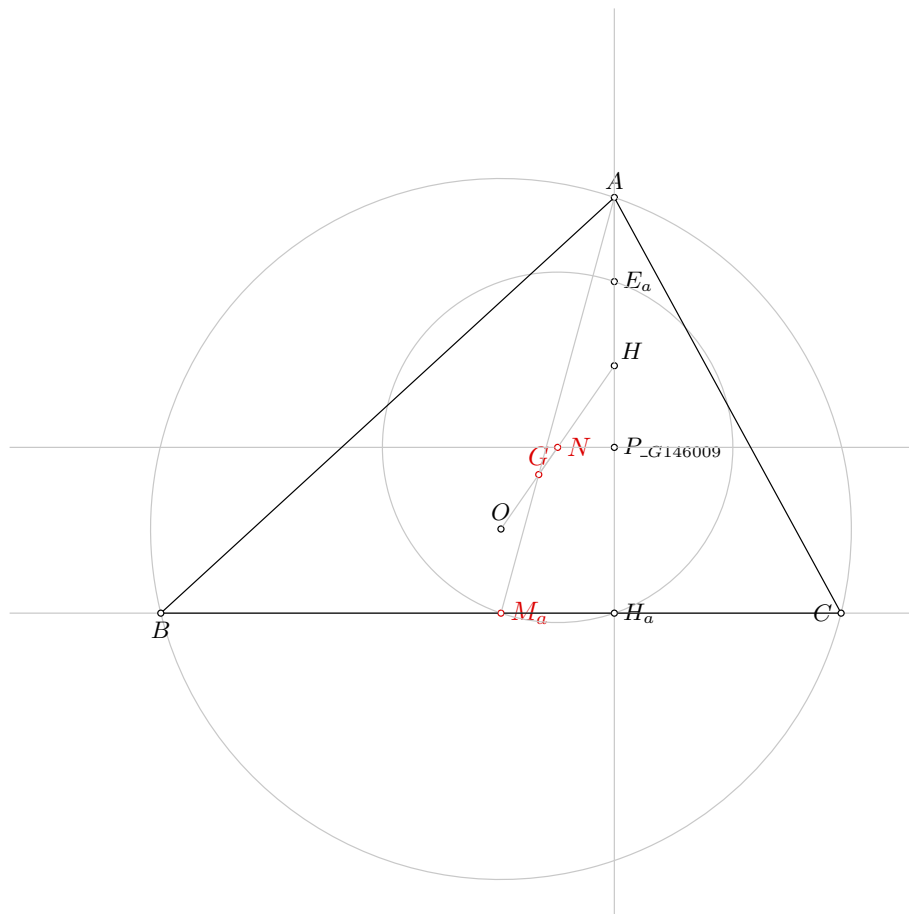


Figure 1: Illustration of the problem 1068

4.3 GCLC - Wu method

4.3.1 Proving $G=_G$

Proving failed

4.3.2 Proving $M_a=_M M_a$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 874 terms.

Time Complexity: Time spent by the prover is 3.580 seconds. There are no ndg conditions.

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $G=_G$

Proving failed

4.4.2 Proving $M_a=_M M_a$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 1069

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1069: Given a point G , a point M_b and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point G and the point M_b , construct a point B (rule W01); ;
2. Using the point G and the point N , construct a point H (rule W01); ;
3. Using the point G and the point N , construct a point O (rule W01); ;
4. Using the point B and the point H , construct a point E_b (rule W01); ;
5. Using the point B and the point H , construct a line h_b (rule W02); % DET: points B and H are not the same;
6. Using the point M_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_b and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
8. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
9. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; line h_b and circle $k(N, M_a)$ intersect; points M_b and N are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points B and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L15,L16,L17,L20,L23,L56]

Solving time: 7.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point G 70 58.33
```

```
point M_{b} 95 67.5
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_t G
```

```
cmark_lt M_{b}
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point B such that GB/GM_{b}=-2
```

```
towards B G M_{b} -2
```

```
cmark_b B
```

```
color 200 200 200
```

```
drawsegment M_{b} B
```

```
color 0 0 0
```

```
% Constructing a point H such that GH/GN=4
```

```
towards H G N 4
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment G H
```

```
color 0 0 0
```

```
% Constructing a point O such that GO/GN=-2
```

```
towards O G N -2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment N O
```

```
color 0 0 0
```

```
% Constructing a point E_{b} such that BE_{b}/BH=0.5
```

```

towards E_{b} B H 0.5
cmark_r E_{b}
color 200 200 200
drawsegment B H
color 0 0 0

% DET: points B and H are not the same
% Constructing a line h_{b} which passes through point B and point H
line h_{b} B H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points M_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{b}
circle k(N,M_{a}) N M_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{b} and circle k(N,M_{a}) intersect% DET: points E_{b} and H_{b} must be different
% Constructing a point P_{\_G206417} which is a foot of the point N on the line h_{b}
foot P_{\_G206417} N h_{b}
cmark_r P_{\_G206417}
color 200 200 200
drawline N P_{\_G206417}
color 0 0 0

% Constructing a point H_{b} which is an image of the point E_{b} in the symmetry to point/line P
\_G206417}
sim H_{b} P_{\_G206417} E_{b}
cmark_l H_{b}

% DET: points H_{b} and M_{b} are not the same
% Constructing a line b which passes through point H_{b} and point M_{b}
line b H_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points B and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point B
circle k(O,C) O B

color 200 200 200

```

```
drawcircle k(O,C)
color 0 0 0
```

```
% NDG: line b and circle k(O,C) intersect
% Constructing points C and A which are in intersection of k(O,C) and b
intersec2 C A k(O,C) b
cmark_l C
cmark_t A
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;
% line h_{b} and circle k(N,M_{a}) intersect; points M_{b} and N are not the same
% Determination conditions: points H_{b} and M_{b} are not the same; points E_{b} and H_{b} must be
% different; points B and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $G=_G$

Proving failed

4.1.2 Proving $M_b=_M M_b$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $G=_G$

Proving failed

4.2.2 Proving $M_b=_M M_b$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

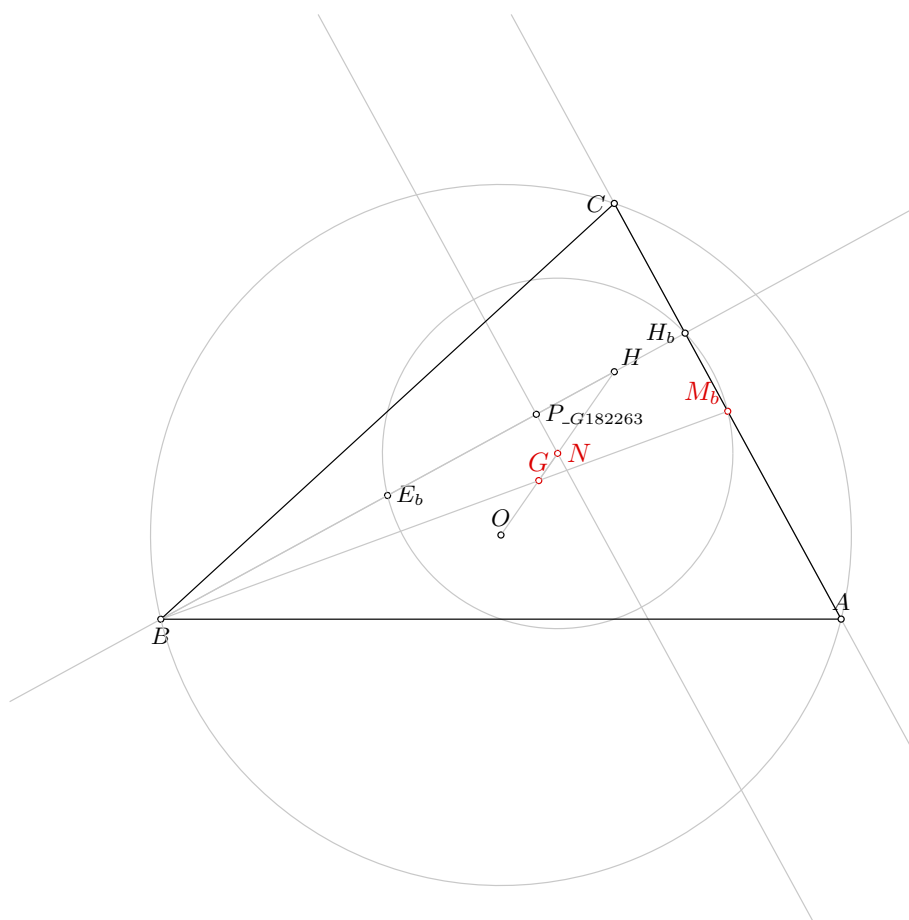


Figure 1: Illustration of the problem 1069

4.3 GCLC - Wu method

4.3.1 Proving $G=_G$

Proving failed

4.3.2 Proving $M_b=_M M_b$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 874 terms.

Time Complexity: Time spent by the prover is 3.480 seconds. There are no ndg conditions.

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $G=_G$

Proving failed

4.4.2 Proving $M_b=_M M_b$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 1070

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1070: Given a point G , a point M_c and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point G and the point M_c , construct a point C (rule W01); ;
2. Using the point G and the point N , construct a point H (rule W01); ;
3. Using the point G and the point N , construct a point O (rule W01); ;
4. Using the point C and the point H , construct a point E_c (rule W01); ;
5. Using the point C and the point H , construct a line h_c (rule W02); % DET: points C and H are not the same;
6. Using the point M_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_c and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
8. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
9. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; line h_c and circle $k(N, M_a)$ intersect; points M_c and N are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points C and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L15,L16,L18,L21,L24,L3,L57]

Solving time: 7.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point G 70 58.33
```

```
point M_{c} 50 67.5
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_t G
```

```
cmark_lt M_{c}
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point C such that GC/GM_{c}=-2
```

```
towards C G M_{c} -2
```

```
cmark_l C
```

```
color 200 200 200
```

```
drawsegment M_{c} C
```

```
color 0 0 0
```

```
% Constructing a point H such that GH/GN=4
```

```
towards H G N 4
```

```
cmark_rt H
```

```
color 200 200 200
```

```
drawsegment G H
```

```
color 0 0 0
```

```
% Constructing a point O such that GO/GN=-2
```

```
towards O G N -2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment N O
```

```
color 0 0 0
```

```
% Constructing a point E_{c} such that CE_{c}/CH=0.5
```

```

towards E_{c} C H 0.5
cmark_r E_{c}
color 200 200 200
drawsegment C H
color 0 0 0

% DET: points C and H are not the same
% Constructing a line h_{c} which passes through point C and point H
line h_{c} C H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points M_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{c}
circle k(N,M_{a}) N M_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{c} and circle k(N,M_{a}) intersect% DET: points E_{c} and H_{c} must be different
% Constructing a point P_{\_G243456} which is a foot of the point N on the line h_{c}
foot P_{\_G243456} N h_{c}
cmark_r P_{\_G243456}
color 200 200 200
drawline N P_{\_G243456}
color 0 0 0

% Constructing a point H_{c} which is an image of the point E_{c} in the symmetry to point/line P
\_G243456}
sim H_{c} P_{\_G243456} E_{c}
cmark_rt H_{c}

% DET: points H_{c} and M_{c} are not the same
% Constructing a line c which passes through point H_{c} and point M_{c}
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points C and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point C
circle k(O,C) O C

color 200 200 200

```

```
drawcircle k(O,C)
color 0 0 0
```

```
% NDG: line c and circle k(O,C) intersect
% Constructing points A and B which are in intersection of k(O,C) and c
intersec2 A B k(O,C) c
cmark_t A
cmark_b B
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
% line h_{c} and circle k(N,M_{a}) intersect; points M_{c} and N are not the same
% Determination conditions: points H_{c} and M_{c} are not the same; points E_{c} and H_{c} must be
% different; points C and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $G=_G$

Proving failed

4.1.2 Proving $M_c=_M_c$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $G=_G$

Proving failed

4.2.2 Proving $M_c=_M_c$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

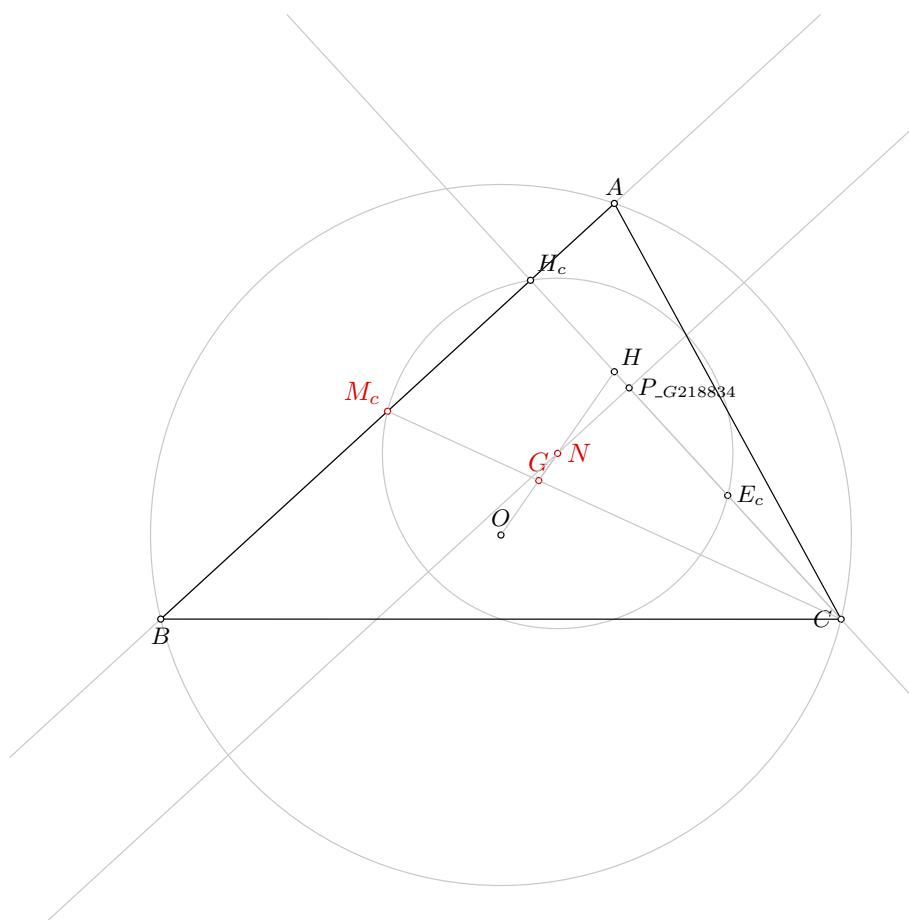


Figure 1: Illustration of the problem 1070

4.3 GCLC - Wu method

4.3.1 Proving $G=_G$

Proving failed

4.3.2 Proving $M_c=_M M_c$

Status: The conjecture has not been proved nor disproved.

Space Complexity: The biggest polynomial obtained during proof process contained 874 terms.

Time Complexity: Time spent by the prover is 3.350 seconds. There are no ndg conditions.

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $G=_G$

Proving failed

4.4.2 Proving $M_c=_M M_c$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 1071

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1071: Given a point N , a point O and a point G , construct the triangle ABC .

2 Status

Problem is redundant.

3 Solution

3.1 Construction in natural language form

1. Using the point N and the point O , construct a point G (rule W01); ;
2. Choose freely a point A (rule free);
3. Using the point A and the point G , construct a point M_a (rule W01); ;
4. Using the point N and the point O , construct a point H (rule W01); ;
5. Using the point A and the point H , construct a point E_a (rule W01); ;
6. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
7. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
8. Using the point M_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_a and N are not the same;
9. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
10. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
11. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; line h_a and circle $k(N, M_a)$ intersect; points M_a and N are not the same; points A and O are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points A and H are not the same.

Rules used: [W01,W02,W04,W05,W06,free]

Lemmas used: [D21,D26,D28,D3,D31,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L15,L19,L22,L55]

Solving time: 8.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point N 72.5 61.93
```

```
point O 65 51.14
```

```
point G 70 58.33
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r N
```

```
cmark_t O
```

```
cmark_t G
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a line L_{\_G64150} which passes through point N and point O
line L_{\_G64150} N O
```

```
color 200 200 200
```

```
drawline L_{\_G64150}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G64251} with coordinates (0,0)
```

```
point P_{\_G64251} 0 0
```

```
cmark_r P_{\_G64251}
```

```
% Constructing a point P_{\_G64175} such that NP_{\_G64175}/NP_{\_G64251}=1
```

```
towards P_{\_G64175} N P_{\_G64251} 1
```

```
cmark_r P_{\_G64175}
```

```
color 200 200 200
```

```
drawsegment N P_{\_G64175}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G64220} such that NP_{\_G64220}/NP_{\_G64251}=3
```

```
towards P_{\_G64220} N P_{\_G64251} 3
```

```
cmark_r P_{\_G64220}
```

```
color 200 200 200
```

```
drawsegment N P_{\_G64220}
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G64181} which passes through point O and point P_{\_G64220}
```

```
line L_{\_G64181} O P_{\_G64220}
```

```

color 200 200 200
drawline L_{\_G64181}
color 0 0 0

% Constructing a line L_{\_G64144} which contains the point P_{\_G64175} and is parallel to the
line L_{\_G64181}
parallel L_{\_G64144} P_{\_G64175} L_{\_G64181}

color 200 200 200
drawline L_{\_G64144}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G64144} and line L_{\_G64150}
intersec G L_{\_G64144} L_{\_G64150}
cmark_t G

% Constructing a free point A
point A 80 95

cmark_t A

% Constructing a point M_{a} such that  $AM_{a}/AG=1.5$ 
towards M_{a} A G 1.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% Constructing a point H such that  $NH/NO=-1$ 
towards H N 0 -1
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0

% Constructing a point E_{a} such that  $AE_{a}/AH=0.5$ 
towards E_{a} A H 0.5
cmark_r E_{a}
color 200 200 200
drawsegment A H
color 0 0 0

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H

```



```

line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0


% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0


% NDG: points M_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{a}
circle k(N,M_{a}) N M_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0


% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G65328} which is a foot of the point N on the line h_{a}
foot P_{\_G65328} N h_{a}
cmark_r P_{\_G65328}
color 200 200 200
drawline N P_{\_G65328}
color 0 0 0

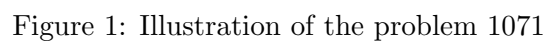

% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
\_G65328}
sim H_{a} P_{\_G65328} E_{a}
cmark_r H_{a}


% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0


% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C

```



```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; line h_{a} and circle k(N,M_{a})
% intersect; points M_{a} and N are not the same; points A and O are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
% different; points A and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $N=_N$

Proving failed

4.1.2 Proving $O=_O$

Proving failed

4.1.3 Proving $G=_G$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $N=_N$

Proving failed

4.2.2 Proving $O=_O$

Proving failed

4.2.3 Proving $G=_G$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $N=_N$

Proving failed

4.3.2 Proving $O=_O$

Proving failed

4.3.3 Proving $G=_G$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $N=_N$

Proving failed

4.4.2 Proving $O=_O$

Proving failed

4.4.3 Proving $G=_G$

Proving failed

Problem 1072

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1072: Given a point G , a point N and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1073

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1073: Given a point G , a point N and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1074

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1074: Given a point G , a point N and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1075

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1075: Given a point H , a point H_a and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H and the point N , construct a point O (rule W01); ;
2. Using the point H and the point N , construct a point G (rule W01); ;
3. Using the point H and the point H_a , construct a line h_a (rule W02); % DET: points H and H_a are not the same;
4. Using the point H_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_a and N are not the same;
5. Using the circle $k(N, M_a)$, the line h_a , the point N and the point H_a , construct a point E_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points H_a and E_a must be different;
6. Using the point E_a and the point H , construct a point A (rule W01); ;
7. Using the point G and the point A , construct a point M_a (rule W01); ;
8. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
9. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; line h_a and circle $k(N, M_a)$ intersect; points H_a and N are not the same.

Determination conditions: points H_a and M_a are not the same; points H_a and E_a must be different; points H and H_a are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D31,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L16,L19,L22,L55]

Solving time: 4.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point H 80 72.73
point H_{a} 80 40
point N 72.5 61.93

color 220 0 0
fontsize 9

cmark_rt H
cmark_r H_{a}
cmark_r N
color 0 0 0
fontsize 8

% Constructing a point O such that HO/HN=2
towards O H N 2
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0

% Constructing a line L_{\_G120721} which passes through point H and point N
line L_{\_G120721} H N

color 200 200 200
drawline L_{\_G120721}
color 0 0 0

% Constructing a point P_{\_G120822} with coordinates (0,0)
point P_{\_G120822} 0 0
cmark_r P_{\_G120822}

% Constructing a point P_{\_G120746} such that HP_{\_G120746}/HP_{\_G120822}=4
towards P_{\_G120746} H P_{\_G120822} 4
cmark_r P_{\_G120746}
color 200 200 200
drawsegment H P_{\_G120746}
color 0 0 0
```



```

% Constructing a point  $P_{\setminus G120791}$  such that  $HP_{\setminus G120791}/HP_{\setminus G120822}=3$ 
towards  $P_{\setminus G120791}$  H  $P_{\setminus G120822}$  3
cmark_r  $P_{\setminus G120791}$ 
color 200 200 200
drawsegment H  $P_{\setminus G120791}$ 
color 0 0 0

% Constructing a line  $L_{\setminus G120752}$  which passes through point N and point  $P_{\setminus G120791}$ 
line  $L_{\setminus G120752}$  N  $P_{\setminus G120791}$ 

color 200 200 200
drawline  $L_{\setminus G120752}$ 
color 0 0 0

% Constructing a line  $L_{\setminus G120715}$  which contains the point  $P_{\setminus G120746}$  and is parallel to the
line  $L_{\setminus G120752}$ 
parallel  $L_{\setminus G120715}$   $P_{\setminus G120746}$   $L_{\setminus G120752}$ 

color 200 200 200
drawline  $L_{\setminus G120715}$ 
color 0 0 0

% Constructing a point G which belongs to line  $L_{\setminus G120715}$  and line  $L_{\setminus G120721}$ 
intersec G  $L_{\setminus G120715}$   $L_{\setminus G120721}$ 
cmark_t G

% DET: points H and  $H_{\setminus a}$  are not the same
% Constructing a line  $h_{\setminus a}$  which passes through point H and point  $H_{\setminus a}$ 
line  $h_{\setminus a}$  H  $H_{\setminus a}$ 

color 200 200 200
drawline  $h_{\setminus a}$ 
color 0 0 0

% NDG: points  $H_{\setminus a}$  and N are not the same
% Constructing a circle  $k(N, M_{\setminus a})$  whose center is at point N and which passes through point  $H_{\setminus a}$ 
circle  $k(N, M_{\setminus a})$  N  $H_{\setminus a}$ 

color 200 200 200
drawcircle  $k(N, M_{\setminus a})$ 
color 0 0 0

% NDG: line  $h_{\setminus a}$  and circle  $k(N, M_{\setminus a})$  intersect% DET: points  $H_{\setminus a}$  and  $E_{\setminus a}$  must be different
% Constructing a point  $P_{\setminus G121575}$  which is a foot of the point N on the line  $h_{\setminus a}$ 
foot  $P_{\setminus G121575}$  N  $h_{\setminus a}$ 
cmark_r  $P_{\setminus G121575}$ 
color 200 200 200
drawline N  $P_{\setminus G121575}$ 
color 0 0 0

```

```

% Constructing a point  $E_{\{a\}}$  which is an image of the point  $H_{\{a\}}$  in the symmetry to point/line  $P$ 
_{\_G121575}
sim  $E_{\{a\}}$   $P_{\{\_G121575\}}$   $H_{\{a\}}$ 
cmark_r  $E_{\{a\}}$ 

% Constructing a point  $A$  such that  $E_{\{a\}}A/E_{\{a\}}H=-1$ 
towards  $A$   $E_{\{a\}}$   $H$  -1
cmark_t  $A$ 
color 200 200 200
drawsegment  $H$   $A$ 
color 0 0 0

% Constructing a point  $M_{\{a\}}$  such that  $GM_{\{a\}}/GA=-0.5$ 
towards  $M_{\{a\}}$   $G$   $A$  -0.5
cmark_r  $M_{\{a\}}$ 
color 200 200 200
drawsegment  $A$   $M_{\{a\}}$ 
color 0 0 0

% DET: points  $H_{\{a\}}$  and  $M_{\{a\}}$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $M_{\{a\}}$ 
line  $a$   $H_{\{a\}}$   $M_{\{a\}}$ 

color 200 200 200
drawline  $a$ 
color 0 0 0

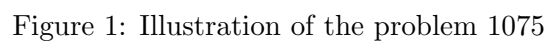
% NDG: points  $A$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $A$ 
circle  $k(O,C)$   $O$   $A$ 

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $a$  and circle  $k(O,C)$  intersect
% Constructing points  $C$  and  $B$  which are in intersection of  $k(O,C)$  and  $a$ 
intersec2  $C$   $B$   $k(O,C)$   $a$ 
cmark_l  $C$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

```



```
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
    line h_{a} and circle k(N,M_{a}) intersect; points H_{a} and N are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points H_{a} and E_{a} must be
    different; points H and H_{a} are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H = _H$

Proving failed

4.1.2 Proving $H_a = \neg H_a$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H = \neg H$

Proving failed

4.2.2 Proving $H_a = \neg H_a$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H = \neg H$

Proving failed

4.3.2 Proving $H_a = \neg H_a$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H = \neg H$

Proving failed

4.4.2 Proving $H_a = \neg H_a$

Proving failed

4.4.3 Proving $N = \neg N$

Proving failed

Problem 1076

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1076: Given a point H , a point H_b and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H and the point N , construct a point O (rule W01); ;
2. Using the point H and the point N , construct a point G (rule W01); ;
3. Using the point H and the point H_b , construct a line h_b (rule W02); % DET: points H and H_b are not the same;
4. Using the point H_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_b and N are not the same;
5. Using the circle $k(N, M_a)$, the line h_b , the point N and the point H_b , construct a point E_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points H_b and E_b must be different;
6. Using the point E_b and the point H , construct a point B (rule W01); ;
7. Using the point G and the point B , construct a point M_b (rule W01); ;
8. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
9. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; line h_b and circle $k(N, M_a)$ intersect; points H_b and N are not the same.

Determination conditions: points H_b and M_b are not the same; points H_b and E_b must be different; points H and H_b are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D31,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L16,L20,L23,L56]

Solving time: 4.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H 80 72.73
```

```
point H_{b} 89.36 77.83
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H
```

```
cmark_l H_{b}
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point O such that HO/HN=2
```

```
towards O H N 2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment H O
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G159484} which passes through point H and point N
```

```
line L_{\_G159484} H N
```

```
color 200 200 200
```

```
drawline L_{\_G159484}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G159585} with coordinates (0,0)
```

```
point P_{\_G159585} 0 0
```

```
cmark_r P_{\_G159585}
```

```
% Constructing a point P_{\_G159509} such that HP_{\_G159509}/HP_{\_G159585}=4
```

```
towards P_{\_G159509} H P_{\_G159585} 4
```

```
cmark_r P_{\_G159509}
```

```
color 200 200 200
```

```
drawsegment H P_{\_G159509}
```

```
color 0 0 0
```

```

% Constructing a point  $P_{\setminus\_G159554}$  such that  $HP_{\setminus\_G159554}/HP_{\setminus\_G159585}=3$ 
towards  $P_{\setminus\_G159554}$  H  $P_{\setminus\_G159585}$  3
cmark_r  $P_{\setminus\_G159554}$ 
color 200 200 200
drawsegment H  $P_{\setminus\_G159554}$ 
color 0 0 0

% Constructing a line  $L_{\setminus\_G159515}$  which passes through point N and point  $P_{\setminus\_G159554}$ 
line  $L_{\setminus\_G159515}$  N  $P_{\setminus\_G159554}$ 

color 200 200 200
drawline  $L_{\setminus\_G159515}$ 
color 0 0 0

% Constructing a line  $L_{\setminus\_G159478}$  which contains the point  $P_{\setminus\_G159509}$  and is parallel to the
line  $L_{\setminus\_G159515}$ 
parallel  $L_{\setminus\_G159478}$   $P_{\setminus\_G159509}$   $L_{\setminus\_G159515}$ 

color 200 200 200
drawline  $L_{\setminus\_G159478}$ 
color 0 0 0

% Constructing a point G which belongs to line  $L_{\setminus\_G159478}$  and line  $L_{\setminus\_G159484}$ 
intersec G  $L_{\setminus\_G159478}$   $L_{\setminus\_G159484}$ 
cmark_t G

% DET: points H and  $H_{\setminus\_b}$  are not the same
% Constructing a line  $h_{\setminus\_b}$  which passes through point H and point  $H_{\setminus\_b}$ 
line  $h_{\setminus\_b}$  H  $H_{\setminus\_b}$ 

color 200 200 200
drawline  $h_{\setminus\_b}$ 
color 0 0 0

% NDG: points  $H_{\setminus\_b}$  and N are not the same
% Constructing a circle  $k(N, M_{\setminus\_a})$  whose center is at point N and which passes through point  $H_{\setminus\_b}$ 
circle  $k(N, M_{\setminus\_a})$  N  $H_{\setminus\_b}$ 

color 200 200 200
drawcircle  $k(N, M_{\setminus\_a})$ 
color 0 0 0

% NDG: line  $h_{\setminus\_b}$  and circle  $k(N, M_{\setminus\_a})$  intersect% DET: points  $H_{\setminus\_b}$  and  $E_{\setminus\_b}$  must be different
% Constructing a point  $P_{\setminus\_G160338}$  which is a foot of the point N on the line  $h_{\setminus\_b}$ 
foot  $P_{\setminus\_G160338}$  N  $h_{\setminus\_b}$ 
cmark_r  $P_{\setminus\_G160338}$ 
color 200 200 200
drawline N  $P_{\setminus\_G160338}$ 
color 0 0 0

```

```

% Constructing a point  $E_{\{b\}}$  which is an image of the point  $H_{\{b\}}$  in the symmetry to point/line  $P$ 
_{\_G160338}
sim  $E_{\{b\}}$   $P_{\{\_G160338\}}$   $H_{\{b\}}$ 
cmark_r  $E_{\{b\}}$ 

% Constructing a point  $B$  such that  $E_{\{b\}}B/E_{\{b\}}H=-1$ 
towards  $B$   $E_{\{b\}}$   $H$  -1
cmark_b  $B$ 
color 200 200 200
drawsegment  $H$   $B$ 
color 0 0 0

% Constructing a point  $M_{\{b\}}$  such that  $GM_{\{b\}}/GB=-0.5$ 
towards  $M_{\{b\}}$   $G$   $B$  -0.5
cmark_lt  $M_{\{b\}}$ 
color 200 200 200
drawsegment  $B$   $M_{\{b\}}$ 
color 0 0 0

% DET: points  $H_{\{b\}}$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $M_{\{b\}}$ 
line  $b$   $H_{\{b\}}$   $M_{\{b\}}$ 

color 200 200 200
drawline  $b$ 
color 0 0 0

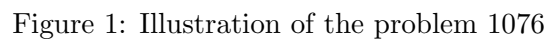
% NDG: points  $B$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $B$ 
circle  $k(O,C)$   $O$   $B$ 

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $b$  and circle  $k(O,C)$  intersect
% Constructing points  $C$  and  $A$  which are in intersection of  $k(O,C)$  and  $b$ 
intersec2  $C$   $A$   $k(O,C)$   $b$ 
cmark_l  $C$ 
cmark_t  $A$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

```

```
% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;
    line h_{b} and circle k(N,M_{a}) intersect; points H_{b} and N are not the same
% Determination conditions: points H_{b} and M_{b} are not the same; points H_{b} and E_{b} must be
    different; points H and H_{b} are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H = _H$

Proving failed

4.1.2 Proving $H_b = _H_b$

Proving failed

4.1.3 Proving $N = _N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H = _H$

Proving failed

4.2.2 Proving $H_b = _H_b$

Proving failed

4.2.3 Proving $N = _N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H = _H$

Proving failed

4.3.2 Proving $H_b = _H_b$

Proving failed

4.3.3 Proving $N = _N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H = _H$

Proving failed

4.4.2 Proving $H_b = _H_b$

Proving failed

4.4.3 Proving $N = _N$

Proving failed

Problem 1077

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1077: Given a point H , a point H_c and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H and the point N , construct a point O (rule W01); ;
2. Using the point H and the point N , construct a point G (rule W01); ;
3. Using the point H and the point H_c , construct a line h_c (rule W02); % DET: points H and H_c are not the same;
4. Using the point H_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_c and N are not the same;
5. Using the circle $k(N, M_a)$, the line h_c , the point N and the point H_c , construct a point E_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points H_c and E_c must be different;
6. Using the point E_c and the point H , construct a point C (rule W01); ;
7. Using the point G and the point C , construct a point M_c (rule W01); ;
8. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
9. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; line h_c and circle $k(N, M_a)$ intersect; points H_c and N are not the same.

Determination conditions: points H_c and M_c are not the same; points H_c and E_c must be different; points H and H_c are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D31,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L16,L21,L24,L3,L57]

Solving time: 4.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H 80 72.73
```

```
point H_{c} 68.91 84.83
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H
```

```
cmark_rt H_{c}
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point O such that HO/HN=2
```

```
towards O H N 2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment H O
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G199488} which passes through point H and point N
```

```
line L_{\_G199488} H N
```

```
color 200 200 200
```

```
drawline L_{\_G199488}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G199589} with coordinates (0,0)
```

```
point P_{\_G199589} 0 0
```

```
cmark_r P_{\_G199589}
```

```
% Constructing a point P_{\_G199513} such that HP_{\_G199513}/HP_{\_G199589}=4
```

```
towards P_{\_G199513} H P_{\_G199589} 4
```

```
cmark_r P_{\_G199513}
```

```
color 200 200 200
```

```
drawsegment H P_{\_G199513}
```

```
color 0 0 0
```

```

% Constructing a point  $P_{\{ \_G199558 \}}$  such that  $HP_{\{ \_G199558 \}}/HP_{\{ \_G199589 \}}=3$ 
towards  $P_{\{ \_G199558 \}}$  H  $P_{\{ \_G199589 \}}$  3
cmark_r  $P_{\{ \_G199558 \}}$ 
color 200 200 200
drawsegment H  $P_{\{ \_G199558 \}}$ 
color 0 0 0

% Constructing a line  $L_{\{ \_G199519 \}}$  which passes through point N and point  $P_{\{ \_G199558 \}}$ 
line  $L_{\{ \_G199519 \}}$  N  $P_{\{ \_G199558 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G199519 \}}$ 
color 0 0 0

% Constructing a line  $L_{\{ \_G199482 \}}$  which contains the point  $P_{\{ \_G199513 \}}$  and is parallel to the
line  $L_{\{ \_G199519 \}}$ 
parallel  $L_{\{ \_G199482 \}}$   $P_{\{ \_G199513 \}}$   $L_{\{ \_G199519 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G199482 \}}$ 
color 0 0 0

% Constructing a point G which belongs to line  $L_{\{ \_G199482 \}}$  and line  $L_{\{ \_G199488 \}}$ 
intersec G  $L_{\{ \_G199482 \}}$   $L_{\{ \_G199488 \}}$ 
cmark_t G

% DET: points H and  $H_{\{c\}}$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point H and point  $H_{\{c\}}$ 
line  $h_{\{c\}}$  H  $H_{\{c\}}$ 

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

% NDG: points  $H_{\{c\}}$  and N are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point N and which passes through point  $H_{\{c\}}$ 
circle k(N,  $M_{\{a\}}$ ) N  $H_{\{c\}}$ 

color 200 200 200
drawcircle k(N,  $M_{\{a\}}$ )
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $H_{\{c\}}$  and  $E_{\{c\}}$  must be different
% Constructing a point  $P_{\{ \_G200342 \}}$  which is a foot of the point N on the line  $h_{\{c\}}$ 
foot  $P_{\{ \_G200342 \}}$  N  $h_{\{c\}}$ 
cmark_r  $P_{\{ \_G200342 \}}$ 
color 200 200 200
drawline N  $P_{\{ \_G200342 \}}$ 
color 0 0 0

```

```

% Constructing a point E_{c} which is an image of the point H_{c} in the symmetry to point/line P
_{\_G200342}
sim E_{c} P_{\_G200342} H_{c}
cmark_r E_{c}

% Constructing a point C such that E_{c}C/E_{c}H=-1
towards C E_{c} H -1
cmark_l C
color 200 200 200
drawsegment H C
color 0 0 0

% Constructing a point M_{c} such that GM_{c}/GC=-0.5
towards M_{c} G C -0.5
cmark_lt M_{c}
color 200 200 200
drawsegment C M_{c}
color 0 0 0

% DET: points H_{c} and M_{c} are not the same
% Constructing a line c which passes through point H_{c} and point M_{c}
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points C and D are not the same
% Constructing a circle k(D,C) whose center is at point D and which passes through point C
circle k(D,C) D C

color 200 200 200
drawcircle k(D,C)
color 0 0 0

% NDG: line c and circle k(D,C) intersect
% Constructing points A and B which are in intersection of k(D,C) and c
intersec2 A B k(D,C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```

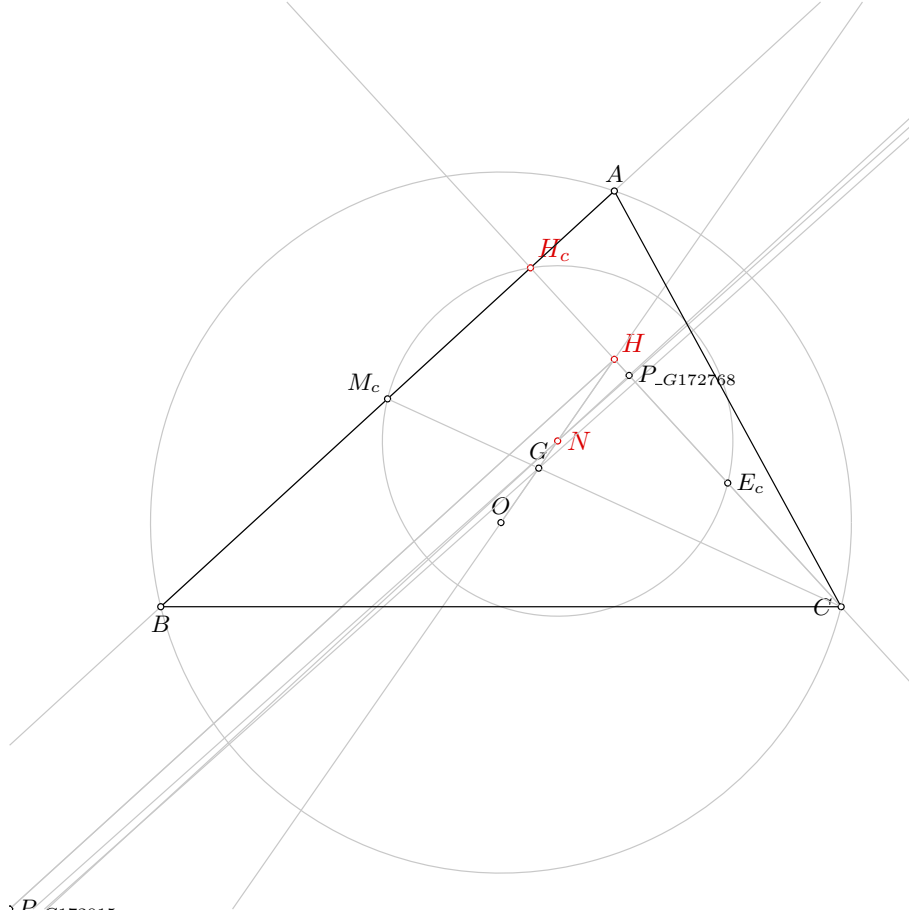


Figure 1: Illustration of the problem 1077

*% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
 line h_{c} and circle k(N,M_{a}) intersect; points H_{c} and N are not the same
 % Determination conditions: points H_{c} and M_{c} are not the same; points H_{c} and E_{c} must be
 different; points H and H_{c} are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H = H$

Proving failed

4.1.2 Proving $H_c = \neg H_c$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H = \neg H$

Proving failed

4.2.2 Proving $H_c = \neg H_c$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H = \neg H$

Proving failed

4.3.2 Proving $H_c = \neg H_c$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H = \neg H$

Proving failed

4.4.2 Proving $H_c = \neg H_c$

Proving failed

4.4.3 Proving $N = \neg N$

Proving failed

Problem 1078

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1078: Given a point H , a point I and a point N , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1079

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1079: Given a point H , a point M_a and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H and the point N , construct a point O (rule W01); ;
2. Using the point H and the point N , construct a point G (rule W01); ;
3. Using the point M_a and the point G , construct a point A (rule W01); ;
4. Using the point H and the point A , construct a point E_a (rule W01); ;
5. Using the point H and the point A , construct a line h_a (rule W02); % DET: points H and A are not the same;
6. Using the point M_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_a and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
8. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
9. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; line h_a and circle $k(N, M_a)$ intersect; points M_a and N are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points H and A are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D31,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L16,L19,L22,L55]

Solving time: 8.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H 80 72.73
```

```
point M_{a} 65 40
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H
```

```
cmark_r M_{a}
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point O such that HO/HN=2
```

```
towards O H N 2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment H O
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G244371} which passes through point H and point N
```

```
line L_{\_G244371} H N
```

```
color 200 200 200
```

```
drawline L_{\_G244371}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G244472} with coordinates (0,0)
```

```
point P_{\_G244472} 0 0
```

```
cmark_r P_{\_G244472}
```

```
% Constructing a point P_{\_G244396} such that HP_{\_G244396}/HP_{\_G244472}=4
```

```
towards P_{\_G244396} H P_{\_G244472} 4
```

```
cmark_r P_{\_G244396}
```

```
color 200 200 200
```

```
drawsegment H P_{\_G244396}
```

```
color 0 0 0
```

```

% Constructing a point P_{\_G244441} such that HP_{\_G244441}/HP_{\_G244472}=3
towards P_{\_G244441} H P_{\_G244472} 3
cmark_r P_{\_G244441}
color 200 200 200
drawsegment H P_{\_G244441}
color 0 0 0

% Constructing a line L_{\_G244402} which passes through point N and point P_{\_G244441}
line L_{\_G244402} N P_{\_G244441}

color 200 200 200
drawline L_{\_G244402}
color 0 0 0

% Constructing a line L_{\_G244365} which contains the point P_{\_G244396} and is parallel to the
line L_{\_G244402}
parallel L_{\_G244365} P_{\_G244396} L_{\_G244402}

color 200 200 200
drawline L_{\_G244365}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G244365} and line L_{\_G244371}
intersec G L_{\_G244365} L_{\_G244371}
cmark_t G

% Constructing a point A such that M_{a}A/M_{a}G=3
towards A M_{a} G 3
cmark_t A
color 200 200 200
drawsegment M_{a} A
color 0 0 0

% Constructing a point E_{a} such that HE_{a}/HA=0.5
towards E_{a} H A 0.5
cmark_r E_{a}
color 200 200 200
drawsegment H A
color 0 0 0

% DET: points H and A are not the same
% Constructing a line h_{a} which passes through point H and point A
line h_{a} H A

color 200 200 200
drawline h_{a}
color 0 0 0

```

```

% NDG: points  $M_{\{a\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $M_{\{a\}}$ 
circle k(N, M_{\{a\}}) N M_{\{a\}}

color 200 200 200
drawcircle k(N, M_{\{a\}})
color 0 0 0

% NDG: line  $h_{\{a\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{a\}}$  and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\_G245382\}}$  which is a foot of the point  $N$  on the line  $h_{\{a\}}$ 
foot P_{\_G245382} N h_{\{a\}}
cmark_r P_{\_G245382}
color 200 200 200
drawline N P_{\_G245382}
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $E_{\{a\}}$  in the symmetry to point/line  $P_{\{\_G245382\}}$ 
sim H_{\{a\}} P_{\_G245382} E_{\{a\}}
cmark_r H_{\{a\}}

% DET: points  $H_{\{a\}}$  and  $M_{\{a\}}$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $M_{\{a\}}$ 
line a H_{\{a\}} M_{\{a\}}

color 200 200 200
drawline a
color 0 0 0

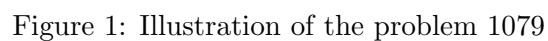
% NDG: points  $A$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $A$ 
circle k(O, C) O A

color 200 200 200
drawcircle k(O, C)
color 0 0 0

% NDG: line  $a$  and circle  $k(O, C)$  intersect
% Constructing points  $C$  and  $B$  which are in intersection of  $k(O, C)$  and  $a$ 
intersec2 C B k(O, C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```



```
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
    line h_{a} and circle k(N,M_{a}) intersect; points M_{a} and N are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
    different; points H and A are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H = _H$

Proving failed

4.1.2 Proving $M_a = \neg M_a$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H = \neg H$

Proving failed

4.2.2 Proving $M_a = \neg M_a$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H = \neg H$

Proving failed

4.3.2 Proving $M_a = \neg M_a$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H = \neg H$

Proving failed

4.4.2 Proving $M_a = \neg M_a$

Proving failed

4.4.3 Proving $N = \neg N$

Proving failed

Problem 1080

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1080: Given a point H , a point M_b and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H and the point N , construct a point O (rule W01); ;
2. Using the point H and the point N , construct a point G (rule W01); ;
3. Using the point M_b and the point G , construct a point B (rule W01); ;
4. Using the point H and the point B , construct a point E_b (rule W01); ;
5. Using the point H and the point B , construct a line h_b (rule W02); % DET: points H and B are not the same;
6. Using the point M_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_b and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
8. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
9. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; line h_b and circle $k(N, M_a)$ intersect; points M_b and N are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points H and B are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D31,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L16,L17,L20,L23,L56]

Solving time: 8.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H 80 72.73
point M_{b} 95 67.5
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```
cmark_rt H
cmark_lt M_{b}
cmark_r N
color 0 0 0
fontsize 8
```

```
% Constructing a point O such that HO/HN=2
towards O H N 2
cmark_t O
color 200 200 200
drawsegment H O
color 0 0 0
```

```
% Constructing a line L_{\_G61988} which passes through point H and point N
line L_{\_G61988} H N
```

```
color 200 200 200
drawline L_{\_G61988}
color 0 0 0
```

```
% Constructing a point P_{\_G62089} with coordinates (0,0)
point P_{\_G62089} 0 0
cmark_r P_{\_G62089}
```

```
% Constructing a point P_{\_G62013} such that HP_{\_G62013}/HP_{\_G62089}=4
towards P_{\_G62013} H P_{\_G62089} 4
cmark_r P_{\_G62013}
color 200 200 200
drawsegment H P_{\_G62013}
color 0 0 0
```

```

% Constructing a point P_{\_G62058} such that HP_{\_G62058}/HP_{\_G62089}=3
towards P_{\_G62058} H P_{\_G62089} 3
cmark_r P_{\_G62058}
color 200 200 200
drawsegment H P_{\_G62058}
color 0 0 0

% Constructing a line L_{\_G62019} which passes through point N and point P_{\_G62058}
line L_{\_G62019} N P_{\_G62058}

color 200 200 200
drawline L_{\_G62019}
color 0 0 0

% Constructing a line L_{\_G61982} which contains the point P_{\_G62013} and is parallel to the
line L_{\_G62019}
parallel L_{\_G61982} P_{\_G62013} L_{\_G62019}

color 200 200 200
drawline L_{\_G61982}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G61982} and line L_{\_G61988}
intersec G L_{\_G61982} L_{\_G61988}
cmark_t G

% Constructing a point B such that M_{b}B/M_{b}G=3
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

% Constructing a point E_{b} such that HE_{b}/HB=0.5
towards E_{b} H B 0.5
cmark_r E_{b}
color 200 200 200
drawsegment H B
color 0 0 0

% DET: points H and B are not the same
% Constructing a line h_{b} which passes through point H and point B
line h_{b} H B

color 200 200 200
drawline h_{b}
color 0 0 0

```

```

% NDG: points  $M_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $M_{\{b\}}$ 
circle k(N, M_{a}) N M_{b}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{b\}}$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\_G62999\}}$  which is a foot of the point  $N$  on the line  $h_{\{b\}}$ 
foot P_{\_G62999} N h_{b}
cmark_r P_{\_G62999}
color 200 200 200
drawline N P_{\_G62999}
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $E_{\{b\}}$  in the symmetry to point/line  $P_{\{\_G62999\}}$ 
sim H_{b} P_{\_G62999} E_{b}
cmark_l H_{b}

% DET: points  $H_{\{b\}}$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $M_{\{b\}}$ 
line b H_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0

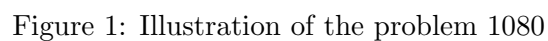
% NDG: points  $B$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $B$ 
circle k(O, C) O B

color 200 200 200
drawcircle k(O, C)
color 0 0 0

% NDG: line  $b$  and circle  $k(O, C)$  intersect
% Constructing points  $C$  and  $A$  which are in intersection of  $k(O, C)$  and  $b$ 
intersec2 C A k(O, C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

```



```
% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;
    line h_{b} and circle k(N,M_{a}) intersect; points M_{b} and N are not the same
% Determination conditions: points H_{b} and M_{b} are not the same; points E_{b} and H_{b} must be
    different; points H and B are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H = _H$

Proving failed

4.1.2 Proving $M_b = \neg M_b$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H = \neg H$

Proving failed

4.2.2 Proving $M_b = \neg M_b$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H = \neg H$

Proving failed

4.3.2 Proving $M_b = \neg M_b$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H = \neg H$

Proving failed

4.4.2 Proving $M_b = \neg M_b$

Proving failed

4.4.3 Proving $N = \neg N$

Proving failed

Problem 1081

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1081: Given a point H , a point M_c and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H and the point N , construct a point O (rule W01); ;
2. Using the point H and the point N , construct a point G (rule W01); ;
3. Using the point M_c and the point G , construct a point C (rule W01); ;
4. Using the point H and the point C , construct a point E_c (rule W01); ;
5. Using the point H and the point C , construct a line h_c (rule W02); % DET: points H and C are not the same;
6. Using the point M_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_c and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
8. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
9. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; line h_c and circle $k(N, M_a)$ intersect; points M_c and N are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points H and C are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D31,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L16,L18,L21,L24,L3,L57]

Solving time: 8.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H 80 72.73
```

```
point M_{c} 50 67.5
```

```
point N 72.5 61.93
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H
```

```
cmark_lt M_{c}
```

```
cmark_r N
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a point O such that HO/HN=2
```

```
towards O H N 2
```

```
cmark_t O
```

```
color 200 200 200
```

```
drawsegment H O
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G100220} which passes through point H and point N
```

```
line L_{\_G100220} H N
```

```
color 200 200 200
```

```
drawline L_{\_G100220}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G100321} with coordinates (0,0)
```

```
point P_{\_G100321} 0 0
```

```
cmark_r P_{\_G100321}
```

```
% Constructing a point P_{\_G100245} such that HP_{\_G100245}/HP_{\_G100321}=4
```

```
towards P_{\_G100245} H P_{\_G100321} 4
```

```
cmark_r P_{\_G100245}
```

```
color 200 200 200
```

```
drawsegment H P_{\_G100245}
```

```
color 0 0 0
```

```

% Constructing a point P_{\_G100290} such that HP_{\_G100290}/HP_{\_G100321}=3
towards P_{\_G100290} H P_{\_G100321} 3
cmark_r P_{\_G100290}
color 200 200 200
drawsegment H P_{\_G100290}
color 0 0 0

% Constructing a line L_{\_G100251} which passes through point N and point P_{\_G100290}
line L_{\_G100251} N P_{\_G100290}

color 200 200 200
drawline L_{\_G100251}
color 0 0 0

% Constructing a line L_{\_G100214} which contains the point P_{\_G100245} and is parallel to the
line L_{\_G100251}
parallel L_{\_G100214} P_{\_G100245} L_{\_G100251}

color 200 200 200
drawline L_{\_G100214}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G100214} and line L_{\_G100220}
intersec G L_{\_G100214} L_{\_G100220}
cmark_t G

% Constructing a point C such that M_{c}C/M_{c}G=3
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

% Constructing a point E_{c} such that HE_{c}/HC=0.5
towards E_{c} H C 0.5
cmark_r E_{c}
color 200 200 200
drawsegment H C
color 0 0 0

% DET: points H and C are not the same
% Constructing a line h_{c} which passes through point H and point C
line h_{c} H C

color 200 200 200
drawline h_{c}
color 0 0 0

```



```

% NDG: points  $M_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $M_{\{c\}}$ 
circle k(N, M_{a}) N M_{c}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{c\}}$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\_G101231\}}$  which is a foot of the point  $N$  on the line  $h_{\{c\}}$ 
foot P_{\_G101231} N h_{c}
cmark_r P_{\_G101231}
color 200 200 200
drawline N P_{\_G101231}
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $E_{\{c\}}$  in the symmetry to point/line  $P_{\{\_G101231\}}$ 
sim H_{c} P_{\_G101231} E_{c}
cmark_rt H_{c}

% DET: points  $H_{\{c\}}$  and  $M_{\{c\}}$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $M_{\{c\}}$ 
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

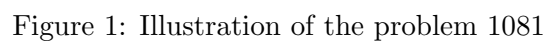
% NDG: points  $C$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $C$ 
circle k(O, C) O C

color 200 200 200
drawcircle k(O, C)
color 0 0 0

% NDG: line  $c$  and circle  $k(O, C)$  intersect
% Constructing points  $A$  and  $B$  which are in intersection of  $k(O, C)$  and  $c$ 
intersec2 A B k(O, C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```



```
% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
% line h_{c} and circle k(N,M_{a}) intersect; points M_{c} and N are not the same
% Determination conditions: points H_{c} and M_{c} are not the same; points E_{c} and H_{c} must be
% different; points H and C are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H = _H$

Proving failed

4.1.2 Proving $M_c = M_c$

Proving failed

4.1.3 Proving $N = N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H = H$

Proving failed

4.2.2 Proving $M_c = M_c$

Proving failed

4.2.3 Proving $N = N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H = H$

Proving failed

4.3.2 Proving $M_c = M_c$

Proving failed

4.3.3 Proving $N = N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H = H$

Proving failed

4.4.2 Proving $M_c = M_c$

Proving failed

4.4.3 Proving $N = N$

Proving failed

Problem 1082

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1082: Given a point N , a point O and a point H , construct the triangle ABC .

2 Status

Problem is redundant.

3 Solution

3.1 Construction in natural language form

1. Using the point N and the point O , construct a point G (rule W01); ;
2. Using the point N and the point O , construct a point H (rule W01); ;
3. Choose freely a point A (rule free);
4. Using the point A and the point G , construct a point M_a (rule W01); ;
5. Using the point A and the point H , construct a point E_a (rule W01); ;
6. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
7. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
8. Using the point M_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_a and N are not the same;
9. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
10. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
11. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; line h_a and circle $k(N, M_a)$ intersect; points M_a and N are not the same; points A and O are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points A and H are not the same.

Rules used: [W01,W02,W04,W05,W06,free]

Lemmas used: [D21,D26,D28,D3,D31,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L15,L19,L22,L55]

Solving time: 8.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point N 72.5 61.93
```

```
point O 65 51.14
```

```
point H 80 72.73
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_r N
```

```
cmark_t O
```

```
cmark_rt H
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a line L_{\_G142490} which passes through point N and point O
line L_{\_G142490} N O
```

```
color 200 200 200
```

```
drawline L_{\_G142490}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G142591} with coordinates (0,0)
```

```
point P_{\_G142591} 0 0
```

```
cmark_r P_{\_G142591}
```

```
% Constructing a point P_{\_G142515} such that NP_{\_G142515}/NP_{\_G142591}=1
```

```
towards P_{\_G142515} N P_{\_G142591} 1
```

```
cmark_r P_{\_G142515}
```

```
color 200 200 200
```

```
drawsegment N P_{\_G142515}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G142560} such that NP_{\_G142560}/NP_{\_G142591}=3
```

```
towards P_{\_G142560} N P_{\_G142591} 3
```

```
cmark_r P_{\_G142560}
```

```
color 200 200 200
```

```
drawsegment N P_{\_G142560}
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G142521} which passes through point O and point P_{\_G142560}
```

```
line L_{\_G142521} O P_{\_G142560}
```

```

color 200 200 200
drawline L_{\_G142521}
color 0 0 0

% Constructing a line L_{\_G142484} which contains the point P_{\_G142515} and is parallel to the
line L_{\_G142521}
parallel L_{\_G142484} P_{\_G142515} L_{\_G142521}

color 200 200 200
drawline L_{\_G142484}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G142484} and line L_{\_G142490}
intersec G L_{\_G142484} L_{\_G142490}
cmark_t G

% Constructing a point H such that NH/NO=-1
towards H N 0 -1
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0

% Constructing a free point A
point A 80 95

cmark_t A

% Constructing a point M_{a} such that AM_{a}/AG=1.5
towards M_{a} A G 1.5
cmark_r M_{a}
color 200 200 200
drawsegment A M_{a}
color 0 0 0

% Constructing a point E_{a} such that AE_{a}/AH=0.5
towards E_{a} A H 0.5
cmark_r E_{a}
color 200 200 200
drawsegment A H
color 0 0 0

% DET: points A and H are not the same
% Constructing a line h_{a} which passes through point A and point H

```

```

line h_{a} A H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points A and O are not the same
% Constructing a circle k(O,C) whose center is at point O and which passes through point A
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: points M_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{a}
circle k(N,M_{a}) N M_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points E_{a} and H_{a} must be different
% Constructing a point P_{\_G143668} which is a foot of the point N on the line h_{a}
foot P_{\_G143668} N h_{a}
cmark_r P_{\_G143668}
color 200 200 200
drawline N P_{\_G143668}
color 0 0 0

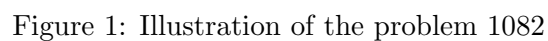
% Constructing a point H_{a} which is an image of the point E_{a} in the symmetry to point/line P
\_G143668}
sim H_{a} P_{\_G143668} E_{a}
cmark_r H_{a}

% DET: points H_{a} and M_{a} are not the same
% Constructing a line a which passes through point H_{a} and point M_{a}
line a H_{a} M_{a}

color 200 200 200
drawline a
color 0 0 0

% NDG: line a and circle k(O,C) intersect
% Constructing points C and B which are in intersection of k(O,C) and a
intersec2 C B k(O,C) a
cmark_l C

```



```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; line h_{a} and circle k(N,M_{a})
% intersect; points M_{a} and N are not the same; points A and O are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
% different; points A and H are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $N=_N$

Proving failed

4.1.2 Proving $O=_O$

Proving failed

4.1.3 Proving $H=_H$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $N=_N$

Proving failed

4.2.2 Proving $O=_O$

Proving failed

4.2.3 Proving $H=_H$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $N=_N$

Proving failed

4.3.2 Proving $O=_O$

Proving failed

4.3.3 Proving $H=_H$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $N=_N$

Proving failed

4.4.2 Proving $O=_O$

Proving failed

4.4.3 Proving $H=_H$

Proving failed

Problem 1083

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1083: Given a point H , a point N and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1084

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1084: Given a point H , a point N and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1085

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1085: Given a point H , a point N and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1086

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1086: Given a point H_b , a point N and a point H_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_b and N are not the same;
2. Choose freely a point H_a on the circle $k(N, M_a)$ (rule WOncircle);
3. Using the point H_a and the point H_b , construct a line $m(H_a H_b)$ (rule W14); % DET: points H_a and H_b are not the same;
4. Using the circle $k(N, M_a)$ and the line $m(H_a H_b)$, construct a point M_c and a point E_c (rule W04); % NDG: line $m(H_a H_b)$ and circle $k(N, M_a)$ intersect;
5. Choose freely a point A on the circle $k(M_c, A)$ (rule WOncircle);
6. Using the point A and the point M_c , construct a point B (rule W01); ;
7. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
8. Using the point H_b and the point B , construct a line h_b (rule W02); % DET: points H_b and B are not the same;
9. Using the line h_a and the line h_b , construct a point H (rule W03); % NDG: lines h_a and h_b are not parallel % DET: lines h_a and h_b are not the same;
10. Using the point N and the point H , construct a point G (rule W01); ;
11. Using the point M_c and the point G , construct a point C (rule W01); .

Non-degenerate conditions: lines h_a and h_b are not parallel; line $m(H_a H_b)$ and circle $k(N, M_a)$ intersect; points H_b and N are not the same.

Determination conditions: lines h_a and h_b are not the same; points H_b and B are not the same; points A and H_a are not the same; points H_a and H_b are not the same.

Rules used: [W01,W02,W03,W04,W06,W14,WOncircle1]

Lemmas used: [D20,D3,D32,D5,D6,D8,D9,GD02,GL01,GL03,GL04,L16,L18,L20,L24,L41,L42,L53,L54,L57]

Solving time: 58.0 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{b} 89.36 77.83
```

```
point N 72.5 61.93
```

```
point H_{a} 80 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_l H_{b}
```

```
cmark_r N
```

```
cmark_r H_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% NDG: points H_{b} and N are not the same
```

```
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{b}
```

```
circle k(N,M_{a}) N H_{b}
```

```
color 200 200 200
```

```
drawcircle k(N,M_{a})
```

```
color 0 0 0
```

```
% Choosing randomly a point H_{a} on the circle with center N through point H_{b}
```

```
oncircle H_{a} N H_{b}
```

```
cmark_r H_{a}
```

```
color 200 200 200
```

```
drawcircle N H_{b}
```

```
color 0 0 0
```

```
% DET: points H_{a} and H_{b} are not the same
```

```
% Constructing bisector m(H_{a}H_{b}) of the segment H_{a}H_{b}
```

```
med m(H_{a}H_{b}) H_{a} H_{b}
```

```
color 200 200 200
```

```
drawline m(H_{a}H_{b})
```

```
color 0 0 0
```

```
color 200 200 200
```

```
drawsegment H_{a} H_{b}
```

```

color 0 0 0

% NDG: line  $m(H_{\{a\}}H_{\{b\}})$  and circle  $k(N,M_{\{a\}})$  intersect
% Constructing points  $M_{\{c\}}$  and  $E_{\{c\}}$  which are in intersection of  $k(N,M_{\{a\}})$  and  $m(H_{\{a\}}H_{\{b\}})$ 
intersec2 M_{c} E_{c} k(N,M_{a}) m(H_{a}H_{b})
cmark_lt M_{c}
cmark_r E_{c}

% Choosing randomly a point A on the circle with center  $M_{\{c\}}$  through point  $H_{\{a\}}$ 
oncircle A M_{c} H_{a}
cmark_t A
color 200 200 200
drawcircle M_{c} H_{a}
color 0 0 0

% Constructing a point B such that  $AB/AM_{\{c\}}=2$ 
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% DET: points A and  $H_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point A and point  $H_{\{a\}}$ 
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% DET: points  $H_{\{b\}}$  and B are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $H_{\{b\}}$  and point B
line h_{b} H_{b} B

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines  $h_{\{a\}}$  and  $h_{\{b\}}$  are not parallel% DET: lines  $h_{\{a\}}$  and  $h_{\{b\}}$  are not the same
% Constructing a point H which belongs to line  $h_{\{a\}}$  and line  $h_{\{b\}}$ 
intersec H h_{a} h_{b}
cmark_rt H

% Constructing a line  $L_{\{G198116\}}$  which passes through point N and point H

```

```

line L_{\_G198116} N H

color 200 200 200
drawline L_{\_G198116}
color 0 0 0

% Constructing a point P_{\_G198217} with coordinates (0,0)
point P_{\_G198217} 0 0
cmark_r P_{\_G198217}

% Constructing a point P_{\_G198141} such that NP_{\_G198141}/NP_{\_G198217}=-1
towards P_{\_G198141} N P_{\_G198217} -1
cmark_r P_{\_G198141}
color 200 200 200
drawsegment P_{\_G198217} P_{\_G198141}
color 0 0 0

% Constructing a point P_{\_G198186} such that NP_{\_G198186}/NP_{\_G198217}=3
towards P_{\_G198186} N P_{\_G198217} 3
cmark_r P_{\_G198186}
color 200 200 200
drawsegment N P_{\_G198186}
color 0 0 0

% Constructing a line L_{\_G198147} which passes through point H and point P_{\_G198186}
line L_{\_G198147} H P_{\_G198186}

color 200 200 200
drawline L_{\_G198147}
color 0 0 0

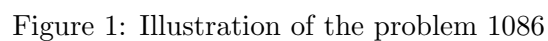
% Constructing a line L_{\_G198110} which contains the point P_{\_G198141} and is parallel to the
line L_{\_G198147}
parallel L_{\_G198110} P_{\_G198141} L_{\_G198147}

color 200 200 200
drawline L_{\_G198110}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G198110} and line L_{\_G198116}
intersec G L_{\_G198110} L_{\_G198116}
cmark_t G

% Constructing a point C such that M_{c}C/M_{c}G=3
towards C M_{c} G 3
cmark_l C
color 200 200 200
drawsegment M_{c} C
color 0 0 0

```

```
% Non-degenerate conditions: lines  $h_a$  and  $h_b$  are not parallel; line  $m(H_aH_b)$  and circle
%  $k(N, M_a)$  intersect; points  $H_b$  and  $N$  are not the same
% Determination conditions: lines  $h_a$  and  $h_b$  are not the same; points  $H_b$  and  $B$  are not the
% same; points  $A$  and  $H_a$  are not the same; points  $H_a$  and  $H_b$  are not the same
```

Illustration of the constructed figure is given in Figure 1

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4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_b = \neg H_b$

Proving failed

4.1.2 Proving $N = \neg N$

Proving failed

4.1.3 Proving $H_a = \neg H_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_b = \neg H_b$

Proving failed

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $H_a = \neg H_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_b = \neg H_b$

Proving failed

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $H_a = \neg H_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_b = \neg H_b$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $H_a = \neg H_a$

Proving failed

Problem 1087

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1087: Given a point H_c , a point N and a point H_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_c and N are not the same;
2. Choose freely a point H_a on the circle $k(N, M_a)$ (rule WOncircle);
3. Using the point H_a and the point H_c , construct a line $m(H_a H_c)$ (rule W14); % DET: points H_a and H_c are not the same;
4. Using the circle $k(N, M_a)$ and the line $m(H_a H_c)$, construct a point M_b and a point E_b (rule W04); % NDG: line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect;
5. Choose freely a point A on the circle $k(M_b, C)$ (rule WOncircle);
6. Using the point A and the point M_b , construct a point C (rule W01); ;
7. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
8. Using the point H_c and the point C , construct a line h_c (rule W02); % DET: points H_c and C are not the same;
9. Using the line h_a and the line h_c , construct a point H (rule W03); % NDG: lines h_a and h_c are not parallel % DET: lines h_a and h_c are not the same;
10. Using the point N and the point H , construct a point G (rule W01); ;
11. Using the point M_b and the point G , construct a point B (rule W01); .

Non-degenerate conditions: lines h_a and h_c are not parallel; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; points H_c and N are not the same.

Determination conditions: lines h_a and h_c are not the same; points H_c and C are not the same; points A and H_a are not the same; points H_a and H_c are not the same.

Rules used: [W01,W02,W03,W04,W06,W14,WOncircle1]

Lemmas used: [D10,D22,D3,D32,D5,D7,D8,GD02,GL01,GL03,GL04,L16,L17,L21,L23,L3,L44,L45,L50,L51,L56]

Solving time: 57.8 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{c} 68.91 84.83
```

```
point N 72.5 61.93
```

```
point H_{a} 80 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H_{c}
```

```
cmark_r N
```

```
cmark_r H_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% NDG: points H_{c} and N are not the same
```

```
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{c}
```

```
circle k(N,M_{a}) N H_{c}
```

```
color 200 200 200
```

```
drawcircle k(N,M_{a})
```

```
color 0 0 0
```

```
% Choosing randomly a point H_{a} on the circle with center N through point H_{c}
```

```
oncircle H_{a} N H_{c}
```

```
cmark_r H_{a}
```

```
color 200 200 200
```

```
drawcircle N H_{c}
```

```
color 0 0 0
```

```
% DET: points H_{a} and H_{c} are not the same
```

```
% Constructing bisector m(H_{a}H_{c}) of the segment H_{a}H_{c}
```

```
med m(H_{a}H_{c}) H_{a} H_{c}
```

```
color 200 200 200
```

```
drawline m(H_{a}H_{c})
```

```
color 0 0 0
```

```
color 200 200 200
```

```
drawsegment H_{a} H_{c}
```

```

color 0 0 0

% NDG: line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N,M_{\{a\}})$  intersect
% Constructing points  $M_{\{b\}}$  and  $E_{\{b\}}$  which are in intersection of  $k(N,M_{\{a\}})$  and  $m(H_{\{a\}}H_{\{c\}})$ 
intersec2 M_{b} E_{b} k(N,M_{a}) m(H_{a}H_{c})
cmark_lt M_{b}
cmark_r E_{b}

% Choosing randomly a point A on the circle with center  $M_{\{b\}}$  through point  $H_{\{a\}}$ 
oncircle A M_{b} H_{a}
cmark_t A
color 200 200 200
drawcircle M_{b} H_{a}
color 0 0 0

% Constructing a point C such that  $AC/AM_{\{b\}}=2$ 
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% DET: points A and  $H_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point A and point  $H_{\{a\}}$ 
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% DET: points  $H_{\{c\}}$  and C are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $H_{\{c\}}$  and point C
line h_{c} H_{c} C

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines  $h_{\{a\}}$  and  $h_{\{c\}}$  are not parallel% DET: lines  $h_{\{a\}}$  and  $h_{\{c\}}$  are not the same
% Constructing a point H which belongs to line  $h_{\{a\}}$  and line  $h_{\{c\}}$ 
intersec H h_{a} h_{c}
cmark_rt H

% Constructing a line  $L_{\{\backslash\_G235889\}}$  which passes through point N and point H

```

```

line L_{\_G235889} N H

color 200 200 200
drawline L_{\_G235889}
color 0 0 0

% Constructing a point P_{\_G235990} with coordinates (0,0)
point P_{\_G235990} 0 0
cmark_r P_{\_G235990}

% Constructing a point P_{\_G235914} such that NP_{\_G235914}/NP_{\_G235990}=-1
towards P_{\_G235914} N P_{\_G235990} -1
cmark_r P_{\_G235914}
color 200 200 200
drawsegment P_{\_G235990} P_{\_G235914}
color 0 0 0

% Constructing a point P_{\_G235959} such that NP_{\_G235959}/NP_{\_G235990}=3
towards P_{\_G235959} N P_{\_G235990} 3
cmark_r P_{\_G235959}
color 200 200 200
drawsegment N P_{\_G235959}
color 0 0 0

% Constructing a line L_{\_G235920} which passes through point H and point P_{\_G235959}
line L_{\_G235920} H P_{\_G235959}

color 200 200 200
drawline L_{\_G235920}
color 0 0 0

% Constructing a line L_{\_G235883} which contains the point P_{\_G235914} and is parallel to the
line L_{\_G235920}
parallel L_{\_G235883} P_{\_G235914} L_{\_G235920}

color 200 200 200
drawline L_{\_G235883}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G235883} and line L_{\_G235889}
intersec G L_{\_G235883} L_{\_G235889}
cmark_t G

% Constructing a point B such that M_{b}B/M_{b}G=3
towards B M_{b} G 3
cmark_b B
color 200 200 200
drawsegment M_{b} B
color 0 0 0

```

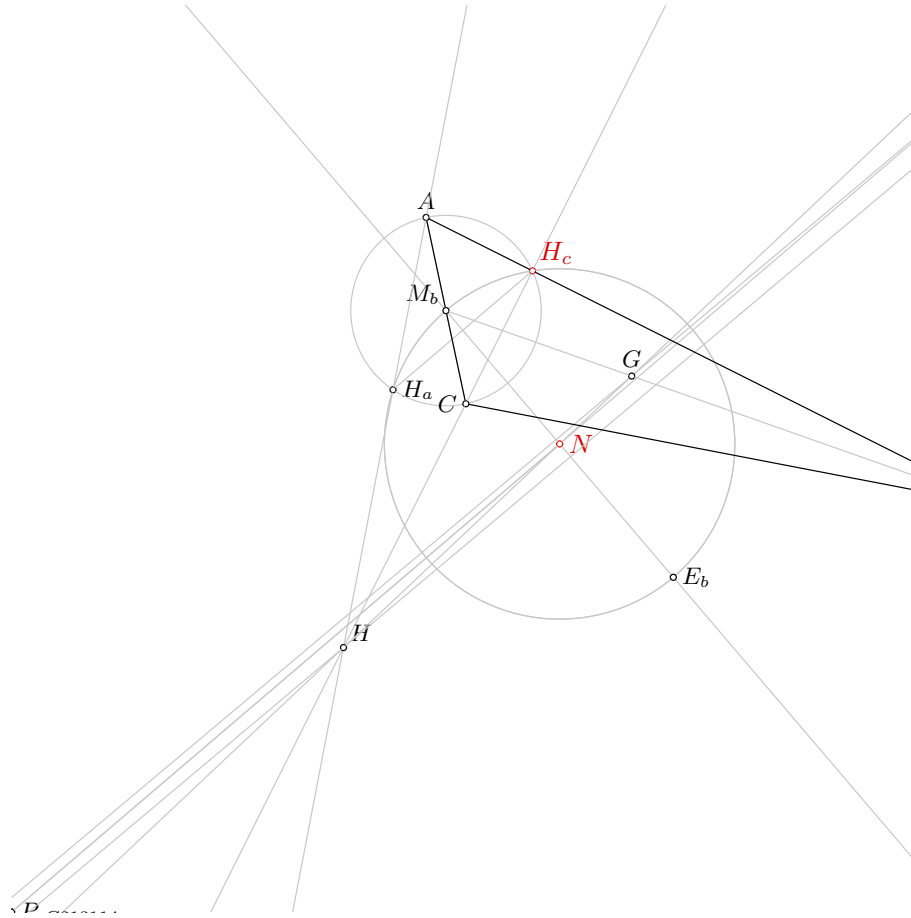


Figure 1: Illustration of the problem 1087

```
drawsegment A B
drawsegment A C
drawsegment B C
```

% Non-degenerate conditions: lines $h_{\{a\}}$ and $h_{\{c\}}$ are not parallel; line $m(H_{\{a\}}H_{\{c\}})$ and circle $k(N, M_{\{a\}})$ intersect; points $H_{\{c\}}$ and N are not the same

% Determination conditions: lines $h_{\{a\}}$ and $h_{\{c\}}$ are not the same; points $H_{\{c\}}$ and C are not the same; points A and $H_{\{a\}}$ are not the same; points $H_{\{a\}}$ and $H_{\{c\}}$ are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_c = \neg H_c$

Proving failed

4.1.2 Proving $N = \neg N$

Proving failed

4.1.3 Proving $H_a = \neg H_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_c = \neg H_c$

Proving failed

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $H_a = \neg H_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_c = \neg H_c$

Proving failed

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $H_a = \neg H_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_c = \neg H_c$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $H_a = \neg H_a$

Proving failed

Problem 1088

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1088: Given a point H_a , a point I and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_a and N are not the same;
2. Using the point I , the circle $k(N, M_a)$, the point N and the point H_a , construct a circle $k(I, P_a)$ (rule W22); % NDG: point I is inside the circle $k(N, M_a)$; points I and N are not the same;
3. Using the circle $k(I, P_a)$, the point H_a and the point I , construct a line y_1 and a line a (rule W12); % NDG: point H_a is outside the circle $k(I, P_a)$;
4. Using the circle $k(N, M_a)$, the line a , the point N and the point H_a , construct a point M_a (rule W05); % NDG: line a and circle $k(N, M_a)$ intersect % DET: points H_a and M_a must be different;
5. Using the point M_a and the point I , construct a line IM_a (rule W02); % DET: points M_a and I are not the same;
6. Using the point N and the point M_a , construct a line $m(H_bH_c)$ (rule W02); % DET: points N and M_a are not the same;
7. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point M_a , construct a point E_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_a and E_a must be different;
8. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;

9. Using the point I and the point M_a , construct a circle $k_{over}(I, M_a)$ (rule W09); % NDG: points I and M_a are not the same;
10. Using the circle $k_{over}(I, M_a)$ and the circle $k(I, P_a)$, construct a point A_{fi} and a point P_a (rule W07); % NDG: circles $k_{over}(I, M_a)$ and $k(I, P_a)$ intersect % DET: circles $k_{over}(I, M_a)$ and $k(I, P_a)$ are not the same;
11. Using the point P_a and the point M_a , construct a point P'_a (rule W01); ;
12. Using the point M_a and the line a , construct a line m_a (rule W10b); ;
13. Using the point P'_a and the line IM_a , construct a line AP'_a (rule W16); ;
14. Using the line AP'_a and the line h_a , construct a point A (rule W03); % NDG: lines AP'_a and h_a are not parallel % DET: lines AP'_a and h_a are not the same;
15. Using the point I and the point A , construct a line s_a (rule W02); % DET: points I and A are not the same;
16. Using the line m_a and the line s_a , construct a point N_a (rule W03); % NDG: lines m_a and s_a are not parallel % DET: lines m_a and s_a are not the same;
17. Using the point I and the point N_a , construct a circle $k(N_a, C)$ (rule W06); % NDG: points I and N_a are not the same;
18. Using the circle $k(N_a, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(N_a, C)$ intersect.

Non-degenerate conditions: line a and circle $k(N_a, C)$ intersect; points I and N_a are not the same; lines m_a and s_a are not parallel; lines AP'_a and h_a are not parallel; circles $k_{over}(I, M_a)$ and $k(I, P_a)$ intersect; points I and M_a are not the same; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; line a and circle $k(N, M_a)$ intersect; point H_a is outside the circle $k(I, P_a)$; point I is inside the circle $k(N, M_a)$; points I and N are not the same; points H_a and N are not the same.

Determination conditions: lines m_a and s_a are not the same; points I and A are not the same; lines AP'_a and h_a are not the same; circles $k_{over}(I, M_a)$ and $k(I, P_a)$ are not the same; points E_a and H_a are not the same; points M_a and E_a must be different; points N and M_a are not the same; points M_a and I are not the same; points H_a and M_a must be different.

Rules used: [W01,W02,W03,W04,W05,W05a,W06,W07,W09,W10b,W12,W16,W22]

Lemmas used: [D11,D2,D21,D27,D28,D3,D32,D47,D5,D65,D8,D88,GD01,GD02,GL01,GL03,GL09,L119,L19,L20]

Solving time: 41.6 seconds.

3.2 Construction in GCLC language

dim 120 120

point H_{a} 80 40
point I 74.37 61.15
point N 72.5 61.93

color 220 0 0

```

fontsize 9

cmark_r H_{a}
cmark_b I
cmark_r N
color 0 0 0
fontsize 8

% NDG: points H_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{a}
circle k(N,M_{a}) N H_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: point I is inside the circle k(N,M_{a}); points I and N are not the same
% Calculating distance V[_G76914] from point N to point H_{a}
distance V[_G76914] N H_{a}

% Calculating distance V[_G76938] from point N to point I
distance V[_G76938] N I

% Calculating value V[_G76959] using formula V[_G76914]/V[_G76938]
expression V[_G76959] { V[_G76914]/V[_G76938] }

% Constructing a point P_{\_G76990} such that NP_{\_G76990}/NI=V[_G76914]/V[_G76938]
towards P_{\_G76990} N I V[_G76959]
cmark_r P_{\_G76990}

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
76990}
circle k(I,P_{a}) I P_{\_G76990}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point H_{a} is outside the circle k(I,P_{a})
% Constructing midpoint P_{\_G77363} of the segment H_{a}I
midpoint P_{\_G77363} H_{a} I
cmark_r P_{\_G77363}

% Constructing a circle C_{\_G77366} whose center is at point P_{\_G77363} and which passes through
point H_{a}
circle C_{\_G77366} P_{\_G77363} H_{a}

color 200 200 200
drawcircle C_{\_G77366}

```

```

color 0 0 0

% Constructing points  $P_{\{ \_G77369 \}}$  and  $P_{\{ \_G77372 \}}$  which are in intersection of  $C_{\{ \_G77366 \}}$  and  $k(I, P_{\{ a \}})$ 
intersec2  $P_{\{ \_G77369 \}}$   $P_{\{ \_G77372 \}}$   $C_{\{ \_G77366 \}}$   $k(I, P_{\{ a \}})$ 
cmark_r  $P_{\{ \_G77369 \}}$ 
cmark_r  $P_{\{ \_G77372 \}}$ 

% Constructing a line  $y1$  which passes through point  $H_{\{ a \}}$  and point  $P_{\{ \_G77369 \}}$ 
line  $y1$   $H_{\{ a \}}$   $P_{\{ \_G77369 \}}$ 

color 200 200 200
drawline  $y1$ 
color 0 0 0

% Constructing a line  $a$  which passes through point  $H_{\{ a \}}$  and point  $P_{\{ \_G77372 \}}$ 
line  $a$   $H_{\{ a \}}$   $P_{\{ \_G77372 \}}$ 

color 200 200 200
drawline  $a$ 
color 0 0 0

% NDG: line  $a$  and circle  $k(N, M_{\{ a \}})$  intersect% DET: points  $H_{\{ a \}}$  and  $M_{\{ a \}}$  must be different
% Constructing a point  $P_{\{ \_G77706 \}}$  which is a foot of the point  $N$  on the line  $a$ 
foot  $P_{\{ \_G77706 \}}$   $N$   $a$ 
cmark_r  $P_{\{ \_G77706 \}}$ 
color 200 200 200
drawline  $N$   $P_{\{ \_G77706 \}}$ 
color 0 0 0

% Constructing a point  $M_{\{ a \}}$  which is an image of the point  $H_{\{ a \}}$  in the symmetry to point/line  $P_{\{ \_G77706 \}}$ 
sim  $M_{\{ a \}}$   $P_{\{ \_G77706 \}}$   $H_{\{ a \}}$ 
cmark_r  $M_{\{ a \}}$ 

% DET: points  $M_{\{ a \}}$  and  $I$  are not the same
% Constructing a line  $IM_{\{ a \}}$  which passes through point  $M_{\{ a \}}$  and point  $I$ 
line  $IM_{\{ a \}}$   $M_{\{ a \}}$   $I$ 

color 200 200 200
drawline  $IM_{\{ a \}}$ 
color 0 0 0

% DET: points  $N$  and  $M_{\{ a \}}$  are not the same
% Constructing a line  $m(H_{\{ b \}}H_{\{ c \}})$  which passes through point  $N$  and point  $M_{\{ a \}}$ 
line  $m(H_{\{ b \}}H_{\{ c \}})$   $N$   $M_{\{ a \}}$ 

color 200 200 200
drawline  $m(H_{\{ b \}}H_{\{ c \}})$ 

```

```
color 0 0 0
```

```
% NDG: line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $M_{\{a\}}$  and  $E_{\{a\}}$  must be
different
% Constructing a point  $E_{\{a\}}$  which is an image of the point  $M_{\{a\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{a\}}$   $N$   $M_{\{a\}}$ 
cmark_r  $E_{\{a\}}$ 
```

```
% DET: points  $E_{\{a\}}$  and  $H_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $E_{\{a\}}$  and point  $H_{\{a\}}$ 
line  $h_{\{a\}}$   $E_{\{a\}}$   $H_{\{a\}}$ 
```

```
color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0
```

```
% NDG: points  $I$  and  $M_{\{a\}}$  are not the same
% Constructing midpoint  $P_{\{\backslash\_G78174\}}$  of the segment  $IM_{\{a\}}$ 
midpoint  $P_{\{\backslash\_G78174\}}$   $I$   $M_{\{a\}}$ 
cmark_r  $P_{\{\backslash\_G78174\}}$ 
```

```
% Constructing a circle  $k_{\text{over}}(I,M_{\{a\}})$  whose center is at point  $P_{\{\backslash\_G78174\}}$  and which passes
through point  $I$ 
circle  $k_{\text{over}}(I,M_{\{a\}})$   $P_{\{\backslash\_G78174\}}$   $I$ 
```

```
color 200 200 200
drawcircle  $k_{\text{over}}(I,M_{\{a\}})$ 
color 0 0 0
```

```
% NDG: circles  $k_{\text{over}}(I,M_{\{a\}})$  and  $k(I,P_{\{a\}})$  intersect% DET: circles  $k_{\text{over}}(I,M_{\{a\}})$  and  $k(I,P_{\{a\}})$ 
are not the same
% Constructing points  $A_{\{fi\}}$  and  $P_{\{a\}}$  which are in intersection of  $k_{\text{over}}(I,M_{\{a\}})$  and  $k(I,P_{\{a\}})$ 
intersec2  $A_{\{fi\}}$   $P_{\{a\}}$   $k_{\text{over}}(I,M_{\{a\}})$   $k(I,P_{\{a\}})$ 
cmark_r  $A_{\{fi\}}$ 
cmark_r  $P_{\{a\}}$ 
```

```
% Constructing a point  $P'_{\{a\}}$  such that  $P_{\{a\}}P'_{\{a\}}/P_{\{a\}}M_{\{a\}}=2$ 
towards  $P'_{\{a\}}$   $P_{\{a\}}$   $M_{\{a\}}$  2
cmark_r  $P'_{\{a\}}$ 
color 200 200 200
drawsegment  $P_{\{a\}}$   $P'_{\{a\}}$ 
color 0 0 0
```

```
% Constructing a line  $m_{\{a\}}$  which is perpendicular to line  $a$  and which passes through point  $M_{\{a\}}$ 
perp  $m_{\{a\}}$   $M_{\{a\}}$   $a$ 
```

```

color 200 200 200
drawline m_{a}
color 0 0 0

% Constructing a line AP'_{a} which contains the point P'_{a} and is parallel to the line IM_{a}
parallel AP'_{a} P'_{a} IM_{a}

color 200 200 200
drawline AP'_{a}
color 0 0 0

% NDG: lines AP'_{a} and h_{a} are not parallel% DET: lines AP'_{a} and h_{a} are not the same
% Constructing a point A which belongs to line AP'_{a} and line h_{a}
intersec A AP'_{a} h_{a}
cmark_t A

% DET: points I and A are not the same
% Constructing a line s_{a} which passes through point I and point A
line s_{a} I A

color 200 200 200
drawline s_{a}
color 0 0 0

% NDG: lines m_{a} and s_{a} are not parallel% DET: lines m_{a} and s_{a} are not the same
% Constructing a point N_{a} which belongs to line m_{a} and line s_{a}
intersec N_{a} m_{a} s_{a}
cmark_b N_{a}

% NDG: points I and N_{a} are not the same
% Constructing a circle k(N_{a},C) whose center is at point N_{a} and which passes through point I
circle k(N_{a},C) N_{a} I

color 200 200 200
drawcircle k(N_{a},C)
color 0 0 0

% NDG: line a and circle k(N_{a},C) intersect
% Constructing points C and B which are in intersection of k(N_{a},C) and a
intersec2 C B k(N_{a},C) a
cmark_l C
cmark_b B

```

Figure 1: Illustration of the problem 1088

```

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: line a and circle k(N_{a},C) intersect; points I and N_{a} are not the
% same; lines m_{a} and s_{a} are not parallel; lines AP'_{a} and h_{a} are not parallel;
% circles k_{over}(I,M_{a}) and k(I,P_{a}) intersect; points I and M_{a} are not the same; line m(H
% _{b})H_{c}) and circle k(N,M_{a}) intersect; line a and circle k(N,M_{a}) intersect; point H_{a}
% is outside the circle k(I,P_{a}); point I is inside the circle k(N,M_{a}); points I and N are
% not the same; points H_{a} and N are not the same
% Determination conditions: lines m_{a} and s_{a} are not the same; points I and A are not the same
% ; lines AP'_{a} and h_{a} are not the same; circles k_{over}(I,M_{a}) and k(I,P_{a}) are not the
% same; points E_{a} and H_{a} are not the same; points M_{a} and E_{a} must be different; points
% N and M_{a} are not the same; points M_{a} and I are not the same; points H_{a} and M_{a} must
% be different

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_a = H_a$

Proving failed

4.1.2 Proving $I = I$

Proving failed

4.1.3 Proving $N = N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_a = H_a$

Proving failed

4.2.2 Proving $I = I$

Proving failed

4.2.3 Proving $N = N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_a =_L H_a$

Proving failed

4.3.2 Proving $I =_L I$

Proving failed

4.3.3 Proving $N =_L N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_a =_L H_a$

Proving failed

4.4.2 Proving $I =_L I$

Proving failed

4.4.3 Proving $N =_L N$

Proving failed

Problem 1089

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1089: Given a point M_a , a point N and a point H_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point M_a and the point N , construct a line $m(H_bH_c)$ (rule W02); % DET: points M_a and N are not the same;
2. Using the point M_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_a and N are not the same;
3. Choose freely a point H_a on the circle $k(N, M_a)$ (rule WOncircle);
4. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
5. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point M_a , construct a point E_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_a and E_a must be different;
6. Using the point E_a and the point H_a , construct a line h_a (rule W02); % DET: points E_a and H_a are not the same;
7. Choose freely a point A on the line h_a (rule WOnline1) ;
8. Using the point A and the point M_a , construct a point G (rule W01); ;
9. Using the point N and the point G , construct a point O (rule W01); ;
10. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
11. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; points M_a and N are not the same.

Determination conditions: points E_a and H_a are not the same; points M_a and E_a must be different; points H_a and M_a are not the same; points M_a and N are not the same.

Rules used: [W01,W02,W04,W05a,W06,WOncircle1,WOnline1]

Lemmas used: [D21,D26,D28,D3,D32,D5,D8,GD01,GD02,GL01,GL03,GL09,L11,L12,L15,L20,L21,L22,L38,L39]

Solving time: 571.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40
point N 72.5 61.93
point H_{a} 80 40
```

```
color 220 0 0
fontsize 9
```

```
cmark_r M_{a}
cmark_r N
cmark_r H_{a}
color 0 0 0
fontsize 8
```

```
% DET: points M_{a} and N are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point M_{a} and point N
line m(H_{b}H_{c}) M_{a} N
```

```
color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0
```

```
% NDG: points M_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{a}
circle k(N,M_{a}) N M_{a}
```

```
color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```
% Choosing randomly a point H_{a} on the circle with center N through point M_{a}
oncircle H_{a} N M_{a}
cmark_r H_{a}
color 200 200 200
drawcircle N M_{a}
color 0 0 0
```

```

% DET: points  $H_{\{a\}}$  and  $M_{\{a\}}$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $M_{\{a\}}$ 
line a  $H_{\{a\}}$   $M_{\{a\}}$ 

color 200 200 200
drawline a
color 0 0 0

% NDG: line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{a\}}$  and  $E_{\{a\}}$  must be
different
% Constructing a point  $E_{\{a\}}$  which is an image of the point  $M_{\{a\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{a\}}$   $N$   $M_{\{a\}}$ 
cmark_r  $E_{\{a\}}$ 

% DET: points  $E_{\{a\}}$  and  $H_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $E_{\{a\}}$  and point  $H_{\{a\}}$ 
line  $h_{\{a\}}$   $E_{\{a\}}$   $H_{\{a\}}$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% Choosing randomly a point  $A$  on the line  $H_{\{a\}}E_{\{a\}}$ 
online A  $H_{\{a\}}$   $E_{\{a\}}$ 
cmark_t A
color 200 200 200
drawline  $H_{\{a\}}$   $E_{\{a\}}$ 
color 0 0 0

% Constructing a line  $L_{\{\_G114874\}}$  which passes through point  $A$  and point  $M_{\{a\}}$ 
line  $L_{\{\_G114874\}}$  A  $M_{\{a\}}$ 

color 200 200 200
drawline  $L_{\{\_G114874\}}$ 
color 0 0 0

% Constructing a point  $P_{\{\_G114975\}}$  with coordinates  $(0,0)$ 
point  $P_{\{\_G114975\}}$  0 0
cmark_r  $P_{\{\_G114975\}}$ 

% Constructing a point  $P_{\{\_G114899\}}$  such that  $AP_{\{\_G114899\}}/AP_{\{\_G114975\}}=2$ 
towards  $P_{\{\_G114899\}}$  A  $P_{\{\_G114975\}}$  2
cmark_r  $P_{\{\_G114899\}}$ 
color 200 200 200
drawsegment A  $P_{\{\_G114899\}}$ 
color 0 0 0

```

```

% Constructing a point  $P_{\{ \_G114944 \}}$  such that  $AP_{\{ \_G114944 \}}/AP_{\{ \_G114975 \}}=3$ 
towards P_{\_G114944} A P_{\_G114975} 3
cmark_r P_{\_G114944}
color 200 200 200
drawsegment A P_{\_G114944}
color 0 0 0

% Constructing a line  $L_{\{ \_G114905 \}}$  which passes through point  $M_{\{ a \}}$  and point  $P_{\{ \_G114944 \}}$ 
line L_{\_G114905} M_{a} P_{\_G114944}

color 200 200 200
drawline L_{\_G114905}
color 0 0 0

% Constructing a line  $L_{\{ \_G114868 \}}$  which contains the point  $P_{\{ \_G114899 \}}$  and is parallel to the
line  $L_{\{ \_G114905 \}}$ 
parallel L_{\_G114868} P_{\_G114899} L_{\_G114905}

color 200 200 200
drawline L_{\_G114868}
color 0 0 0

% Constructing a point  $G$  which belongs to line  $L_{\{ \_G114868 \}}$  and line  $L_{\{ \_G114874 \}}$ 
intersec G L_{\_G114868} L_{\_G114874}
cmark_t G

% Constructing a point  $O$  such that  $NO/NG=3$ 
towards O N G 3
cmark_t O
color 200 200 200
drawsegment N O
color 0 0 0

% NDG: points  $A$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $A$ 
circle k(O,C) O A

color 200 200 200
drawcircle k(O,C)
color 0 0 0

% NDG: line  $a$  and circle  $k(O,C)$  intersect
% Constructing points  $C$  and  $B$  which are in intersection of  $k(O,C)$  and  $a$ 
intersec2 C B k(O,C) a
cmark_l C
cmark_b B

```

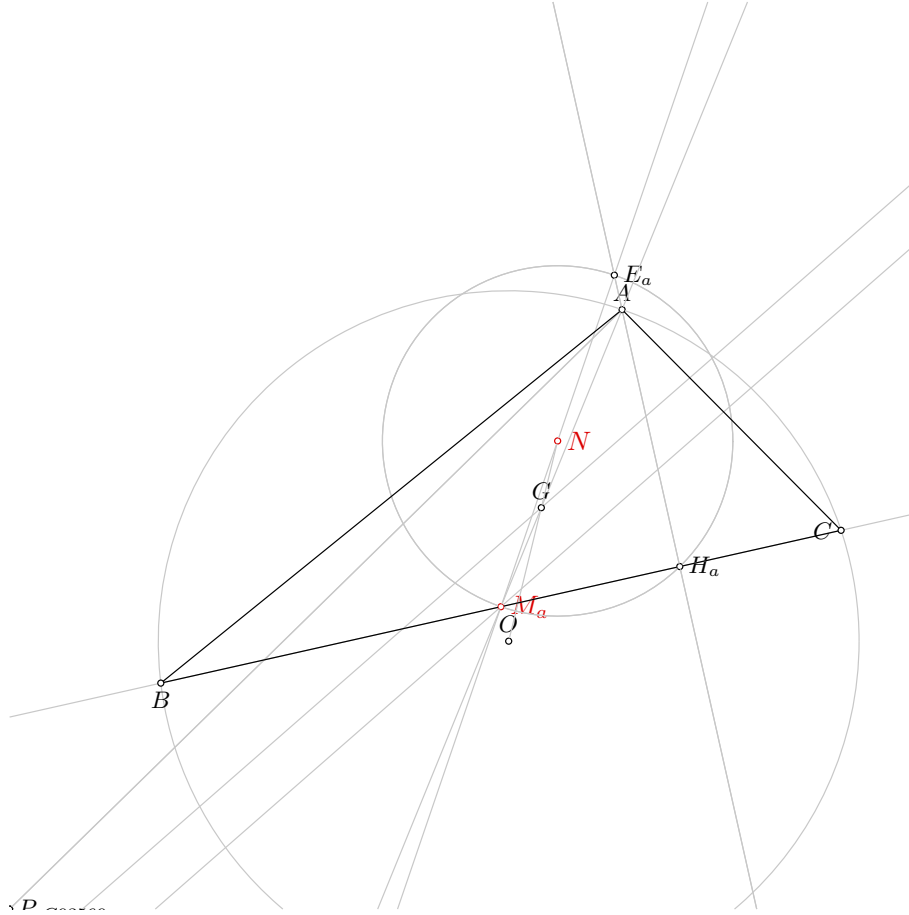


Figure 1: Illustration of the problem 1089

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
% line m(H_{b}H_{c}) and circle k(N,M_{a}) intersect; points M_{a} and N are not the same
% Determination conditions: points E_{a} and H_{a} are not the same; points M_{a} and E_{a} must be
% different; points H_{a} and M_{a} are not the same; points M_{a} and N are not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_a = \neg M_a$

Proving failed

4.1.2 Proving $N = \neg N$

Proving failed

4.1.3 Proving $H_a = \neg H_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $M_a = \neg M_a$

Proving failed

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $H_a = \neg H_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_a = \neg M_a$

Proving failed

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $H_a = \neg H_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_a = \neg M_a$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $H_a = \neg H_a$

Proving failed

Problem 1090

*Generated automatically by ArgoTriCS
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1 Problem

Problem 1090: Given a point M_b , a point N and a point H_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point M_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_b and N are not the same;
2. Choose freely a point H_a on the circle $k(N, M_a)$ (rule WOncircle);
3. Using the point H_a and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points H_a and M_b are not the same;
4. Using the circle $k(N, M_a)$, the circle $k(M_b, C)$, the point H_a , the point N and the point M_b , construct a point H_c (rule W08); % NDG: circles $k(N, M_a)$ and $k(M_b, C)$ intersect % DET: circles $k(N, M_a)$ and $k(M_b, C)$ are not the same; points H_a and H_c must be different;
5. Choose freely a point A on the circle $k(M_b, C)$ (rule WOncircle);
6. Using the point A and the point M_b , construct a point C (rule W01); ;
7. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
8. Using the point H_c and the point C , construct a line h_c (rule W02); % DET: points H_c and C are not the same;
9. Using the line h_a and the line h_c , construct a point H (rule W03); % NDG: lines h_a and h_c are not parallel % DET: lines h_a and h_c are not the same;
10. Using the point N and the point H , construct a point G (rule W01); ;
11. Using the point M_b and the point G , construct a point B (rule W01); .

Non-degenerate conditions: lines h_a and h_c are not parallel; circles $k(N, M_a)$ and $k(M_b, C)$ intersect; points H_a and M_b are not the same; points M_b and N are not the same.

Determination conditions: lines h_a and h_c are not the same; points H_c and C are not the same; points A and H_a are not the same; circles $k(N, M_a)$ and $k(M_b, C)$ are not the same; points H_a and H_c must be different.

Rules used: [W01,W02,W03,W06,W08,WOncircle1]

Lemmas used: [D10,D22,D3,D32,D5,D7,D8,GD02,GL03,GL04,L16,L17,L19,L21,L3,L44,L45,L56]

Solving time: 58.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{b} 95 67.5
```

```
point N 72.5 61.93
```

```
point H_{a} 80 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_lt M_{b}
```

```
cmark_r N
```

```
cmark_r H_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% NDG: points M_{b} and N are not the same
```

```
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{b}
```

```
circle k(N,M_{a}) N M_{b}
```

```
color 200 200 200
```

```
drawcircle k(N,M_{a})
```

```
color 0 0 0
```

```
% Choosing randomly a point H_{a} on the circle with center N through point M_{b}
```

```
oncircle H_{a} N M_{b}
```

```
cmark_r H_{a}
```

```
color 200 200 200
```

```
drawcircle N M_{b}
```

```
color 0 0 0
```

```
% NDG: points H_{a} and M_{b} are not the same
```

```
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{a}
```

```
circle k(M_{b},C) M_{b} H_{a}
```

```
color 200 200 200
```

```
drawcircle k(M_{b},C)
```

```
color 0 0 0
```



```

% NDG: circles  $k(N, M_{\{a\}})$  and  $k(M_{\{b\}}, C)$  intersect% DET: circles  $k(N, M_{\{a\}})$  and  $k(M_{\{b\}}, C)$  are not
the same; points  $H_{\{a\}}$  and  $H_{\{c\}}$  must be different
% Constructing a line  $L_{\{\backslash\_G152292\}}$  which passes through point  $N$  and point  $M_{\{b\}}$ 
line  $L_{\{\backslash\_G152292\}}$   $N$   $M_{\{b\}}$ 

color 200 200 200
drawline  $L_{\{\backslash\_G152292\}}$ 
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $H_{\{a\}}$  in the symmetry to point/line  $L_{\{\backslash\_G152292\}}$ 
sim  $H_{\{c\}}$   $L_{\{\backslash\_G152292\}}$   $H_{\{a\}}$ 
cmark_rt  $H_{\{c\}}$ 

% Choosing randomly a point  $A$  on the circle with center  $M_{\{b\}}$  through point  $H_{\{a\}}$ 
oncircle  $A$   $M_{\{b\}}$   $H_{\{a\}}$ 
cmark_t  $A$ 
color 200 200 200
drawcircle  $M_{\{b\}}$   $H_{\{a\}}$ 
color 0 0 0

% Constructing a point  $C$  such that  $AC/AM_{\{b\}}=2$ 
towards  $C$   $A$   $M_{\{b\}}$  2
cmark_l  $C$ 
color 200 200 200
drawsegment  $A$   $C$ 
color 0 0 0

% DET: points  $A$  and  $H_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $A$  and point  $H_{\{a\}}$ 
line  $h_{\{a\}}$   $A$   $H_{\{a\}}$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% DET: points  $H_{\{c\}}$  and  $C$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $H_{\{c\}}$  and point  $C$ 
line  $h_{\{c\}}$   $H_{\{c\}}$   $C$ 

color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0

```

```

% NDG: lines  $h_{\{a\}}$  and  $h_{\{c\}}$  are not parallel% DET: lines  $h_{\{a\}}$  and  $h_{\{c\}}$  are not the same
% Constructing a point  $H$  which belongs to line  $h_{\{a\}}$  and line  $h_{\{c\}}$ 
intersec H  $h_{\{a\}}$   $h_{\{c\}}$ 
cmark_rt H

```

```

% Constructing a line  $L_{\{\_G152676\}}$  which passes through point  $N$  and point  $H$ 
line  $L_{\{\_G152676\}}$  N H

```

```

color 200 200 200
drawline  $L_{\{\_G152676\}}$ 
color 0 0 0

```

```

% Constructing a point  $P_{\{\_G152777\}}$  with coordinates  $(0,0)$ 
point  $P_{\{\_G152777\}}$  0 0
cmark_r  $P_{\{\_G152777\}}$ 

```

```

% Constructing a point  $P_{\{\_G152701\}}$  such that  $NP_{\{\_G152701\}}/NP_{\{\_G152777\}}=-1$ 
towards  $P_{\{\_G152701\}}$  N  $P_{\{\_G152777\}}$  -1
cmark_r  $P_{\{\_G152701\}}$ 
color 200 200 200
drawsegment  $P_{\{\_G152777\}}$   $P_{\{\_G152701\}}$ 
color 0 0 0

```

```

% Constructing a point  $P_{\{\_G152746\}}$  such that  $NP_{\{\_G152746\}}/NP_{\{\_G152777\}}=3$ 
towards  $P_{\{\_G152746\}}$  N  $P_{\{\_G152777\}}$  3
cmark_r  $P_{\{\_G152746\}}$ 
color 200 200 200
drawsegment N  $P_{\{\_G152746\}}$ 
color 0 0 0

```

```

% Constructing a line  $L_{\{\_G152707\}}$  which passes through point  $H$  and point  $P_{\{\_G152746\}}$ 
line  $L_{\{\_G152707\}}$  H  $P_{\{\_G152746\}}$ 

```

```

color 200 200 200
drawline  $L_{\{\_G152707\}}$ 
color 0 0 0

```

```

% Constructing a line  $L_{\{\_G152670\}}$  which contains the point  $P_{\{\_G152701\}}$  and is parallel to the
line  $L_{\{\_G152707\}}$ 
parallel  $L_{\{\_G152670\}}$   $P_{\{\_G152701\}}$   $L_{\{\_G152707\}}$ 

```

```

color 200 200 200
drawline  $L_{\{\_G152670\}}$ 
color 0 0 0

```

```

% Constructing a point  $G$  which belongs to line  $L_{\{\_G152670\}}$  and line  $L_{\{\_G152676\}}$ 
intersec G  $L_{\{\_G152670\}}$   $L_{\{\_G152676\}}$ 
cmark_t G

```

```

% Constructing a point B such that  $M_{\{b\}B/M_{\{b\}G}=3$ 
towards B  $M_{\{b\}}$  G 3
cmark_b B
color 200 200 200
drawsegment  $M_{\{b\}}$  B
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions: lines  $h_{\{a\}}$  and  $h_{\{c\}}$  are not parallel; circles  $k(N, M_{\{a\}})$  and  $k(M_{\{b\}}, C)$  intersect; points  $H_{\{a\}}$  and  $M_{\{b\}}$  are not the same; points  $M_{\{b\}}$  and  $N$  are not the same
% Determination conditions: lines  $h_{\{a\}}$  and  $h_{\{c\}}$  are not the same; points  $H_{\{c\}}$  and  $C$  are not the same; points  $A$  and  $H_{\{a\}}$  are not the same; circles  $k(N, M_{\{a\}})$  and  $k(M_{\{b\}}, C)$  are not the same; points  $H_{\{a\}}$  and  $H_{\{c\}}$  must be different

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_b = _M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

Time Complexity: Time spent by the prover is 0.124 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $N = _N$

Proving failed

4.1.3 Proving $H_a = _H_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $M_b = _M_b$

NDG conditions are:

$S_{AH_cC} \neq S_{H_aH_cC}$ i.e., lines AH_a and H_cC are not parallel (construction based assumption)
 $S_{P_{-G132290}NH} \neq S_{P_{L_{-G132259}}^0NH}$ i.e., lines $P_{-G132290}P_{L_{-G132259}}^0$ and NH are not parallel (construction based assumption)
 $S_{_M_aBC} \neq 0$ i.e., points $_M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{_M_bAC} \neq 0$ i.e., points $_M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

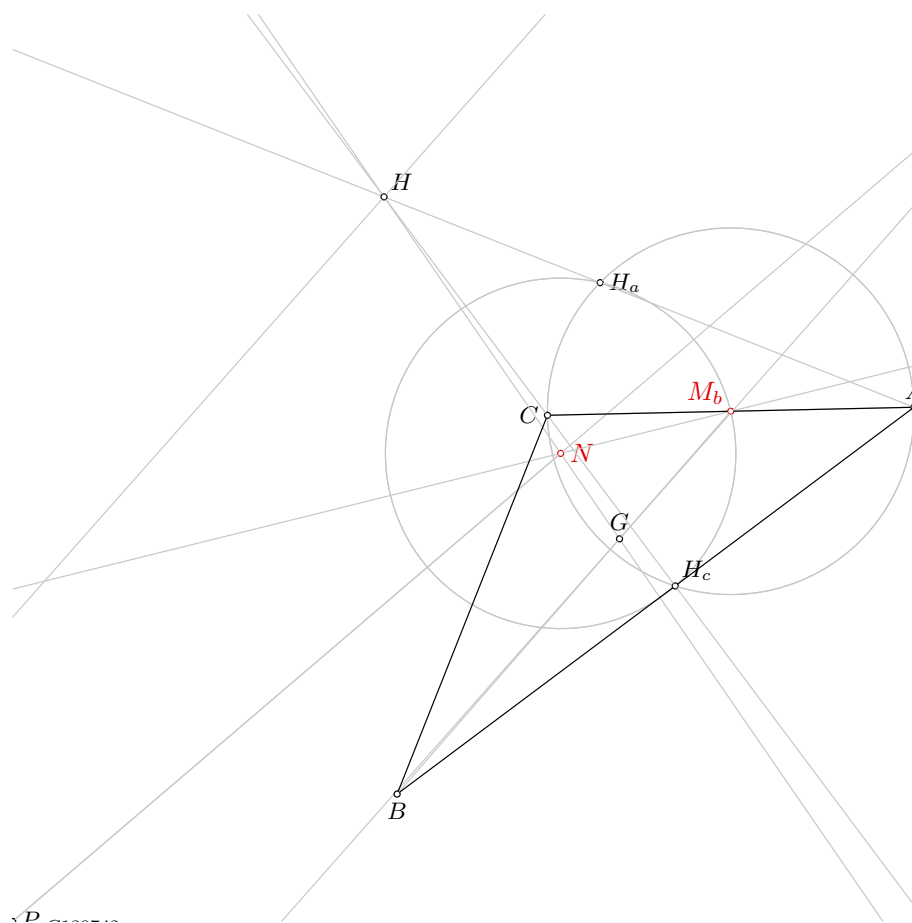


Figure 1: Illustration of the problem 1090

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF_{-h_b}^4} \neq S_{F_{-h_a}^3 BF_{-h_b}^4}$ i.e., lines $AF_{-h_a}^3$ and $BF_{-h_b}^4$ are not parallel (construction based assumption)
 $S_{-M_a -M_b F_{-m_b}^2} \neq S_{F_{-m_a}^1 -M_b F_{-m_b}^2}$ i.e., lines $-M_a F_{-m_a}^1$ and $-M_b F_{-m_b}^2$ are not parallel (construction based assumption)
 $S_{BAF_{-h_a}^3} \neq S_{CAF_{-h_a}^3}$ i.e., lines BC and $AF_{-h_a}^3$ are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

4.2.2 Proving $N = -N$

Proving failed

4.2.3 Proving $H_a = -H_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_b = -M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.070 seconds. There are no ndg conditions.

4.3.2 Proving $N = -N$

Proving failed

4.3.3 Proving $H_a = -H_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_b = -M_b$

Proving failed

4.4.2 Proving $N = -N$

Proving failed

4.4.3 Proving $H_a = -H_a$

Proving failed

Problem 1091

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1091: Given a point M_c , a point N and a point H_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point M_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_c and N are not the same;
2. Choose freely a point H_a on the circle $k(N, M_a)$ (rule WOncircle);
3. Using the point H_a and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points H_a and M_c are not the same;
4. Using the circle $k(N, M_a)$, the circle $k(M_c, A)$, the point H_a , the point N and the point M_c , construct a point H_b (rule W08); % NDG: circles $k(N, M_a)$ and $k(M_c, A)$ intersect % DET: circles $k(N, M_a)$ and $k(M_c, A)$ are not the same; points H_a and H_b must be different;
5. Choose freely a point A on the circle $k(M_c, A)$ (rule WOncircle);
6. Using the point A and the point M_c , construct a point B (rule W01); ;
7. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
8. Using the point H_b and the point B , construct a line h_b (rule W02); % DET: points H_b and B are not the same;
9. Using the line h_a and the line h_b , construct a point H (rule W03); % NDG: lines h_a and h_b are not parallel % DET: lines h_a and h_b are not the same;
10. Using the point N and the point H , construct a point G (rule W01); ;
11. Using the point M_c and the point G , construct a point C (rule W01); .

Non-degenerate conditions: lines h_a and h_b are not parallel; circles $k(N, M_a)$ and $k(M_c, A)$ intersect; points H_a and M_c are not the same; points M_c and N are not the same.

Determination conditions: lines h_a and h_b are not the same; points H_b and B are not the same; points A and H_a are not the same; circles $k(N, M_a)$ and $k(M_c, A)$ are not the same; points H_a and H_b must be different.

Rules used: [W01,W02,W03,W06,W08,WOncircle1]

Lemmas used: [D20,D3,D32,D5,D6,D8,D9,GD02,GL03,GL04,L16,L18,L19,L20,L41,L42,L57]

Solving time: 57.6 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{c} 50 67.5
```

```
point N 72.5 61.93
```

```
point H_{a} 80 40
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_lt M_{c}
```

```
cmark_r N
```

```
cmark_r H_{a}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% NDG: points M_{c} and N are not the same
```

```
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{c}
```

```
circle k(N,M_{a}) N M_{c}
```

```
color 200 200 200
```

```
drawcircle k(N,M_{a})
```

```
color 0 0 0
```

```
% Choosing randomly a point H_{a} on the circle with center N through point M_{c}
```

```
oncircle H_{a} N M_{c}
```

```
cmark_r H_{a}
```

```
color 200 200 200
```

```
drawcircle N M_{c}
```

```
color 0 0 0
```

```
% NDG: points H_{a} and M_{c} are not the same
```

```
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{a}
```

```
circle k(M_{c},A) M_{c} H_{a}
```

```
color 200 200 200
```

```
drawcircle k(M_{c},A)
```

```
color 0 0 0
```

```

% NDG: circles  $k(N, M_{\{a\}})$  and  $k(M_{\{c\}}, A)$  intersect% DET: circles  $k(N, M_{\{a\}})$  and  $k(M_{\{c\}}, A)$  are not
the same; points  $H_{\{a\}}$  and  $H_{\{b\}}$  must be different
% Constructing a line  $L_{\{\backslash\_G190215\}}$  which passes through point  $N$  and point  $M_{\{c\}}$ 
line  $L_{\{\backslash\_G190215\}}$   $N$   $M_{\{c\}}$ 

color 200 200 200
drawline  $L_{\{\backslash\_G190215\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $H_{\{a\}}$  in the symmetry to point/line  $L_{\{\backslash\_G190215\}}$ 
sim  $H_{\{b\}}$   $L_{\{\backslash\_G190215\}}$   $H_{\{a\}}$ 
cmark_l  $H_{\{b\}}$ 

% Choosing randomly a point  $A$  on the circle with center  $M_{\{c\}}$  through point  $H_{\{a\}}$ 
oncircle  $A$   $M_{\{c\}}$   $H_{\{a\}}$ 
cmark_t  $A$ 
color 200 200 200
drawcircle  $M_{\{c\}}$   $H_{\{a\}}$ 
color 0 0 0

% Constructing a point  $B$  such that  $AB/AM_{\{c\}}=2$ 
towards  $B$   $A$   $M_{\{c\}}$  2
cmark_b  $B$ 
color 200 200 200
drawsegment  $A$   $B$ 
color 0 0 0

% DET: points  $A$  and  $H_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $A$  and point  $H_{\{a\}}$ 
line  $h_{\{a\}}$   $A$   $H_{\{a\}}$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% DET: points  $H_{\{b\}}$  and  $B$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $H_{\{b\}}$  and point  $B$ 
line  $h_{\{b\}}$   $H_{\{b\}}$   $B$ 

color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0

```



```

% NDG: lines  $h_{\{a\}}$  and  $h_{\{b\}}$  are not parallel% DET: lines  $h_{\{a\}}$  and  $h_{\{b\}}$  are not the same
% Constructing a point  $H$  which belongs to line  $h_{\{a\}}$  and line  $h_{\{b\}}$ 
intersec H  $h_{\{a\}}$   $h_{\{b\}}$ 
cmark_rt H

```

```

% Constructing a line  $L_{\{G190599\}}$  which passes through point  $N$  and point  $H$ 
line  $L_{\{G190599\}}$  N H

```

```

color 200 200 200
drawline  $L_{\{G190599\}}$ 
color 0 0 0

```

```

% Constructing a point  $P_{\{G190700\}}$  with coordinates  $(0,0)$ 
point  $P_{\{G190700\}}$  0 0
cmark_r  $P_{\{G190700\}}$ 

```

```

% Constructing a point  $P_{\{G190624\}}$  such that  $NP_{\{G190624\}}/NP_{\{G190700\}}=-1$ 
towards  $P_{\{G190624\}}$  N  $P_{\{G190700\}}$  -1
cmark_r  $P_{\{G190624\}}$ 
color 200 200 200
drawsegment  $P_{\{G190700\}}$   $P_{\{G190624\}}$ 
color 0 0 0

```

```

% Constructing a point  $P_{\{G190669\}}$  such that  $NP_{\{G190669\}}/NP_{\{G190700\}}=3$ 
towards  $P_{\{G190669\}}$  N  $P_{\{G190700\}}$  3
cmark_r  $P_{\{G190669\}}$ 
color 200 200 200
drawsegment N  $P_{\{G190669\}}$ 
color 0 0 0

```

```

% Constructing a line  $L_{\{G190630\}}$  which passes through point  $H$  and point  $P_{\{G190669\}}$ 
line  $L_{\{G190630\}}$  H  $P_{\{G190669\}}$ 

```

```

color 200 200 200
drawline  $L_{\{G190630\}}$ 
color 0 0 0

```

```

% Constructing a line  $L_{\{G190593\}}$  which contains the point  $P_{\{G190624\}}$  and is parallel to the
line  $L_{\{G190630\}}$ 
parallel  $L_{\{G190593\}}$   $P_{\{G190624\}}$   $L_{\{G190630\}}$ 

```

```

color 200 200 200
drawline  $L_{\{G190593\}}$ 
color 0 0 0

```

```

% Constructing a point  $G$  which belongs to line  $L_{\{G190593\}}$  and line  $L_{\{G190599\}}$ 
intersec G  $L_{\{G190593\}}$   $L_{\{G190599\}}$ 
cmark_t G

```

```

% Constructing a point C such that  $M_{\{c\}}C/M_{\{c\}}G=3$ 
towards C  $M_{\{c\}}$  G 3
cmark_1 C
color 200 200 200
drawsegment  $M_{\{c\}}$  C
color 0 0 0

```

```

drawsegment A B
drawsegment A C
drawsegment B C

```

```

% Non-degenerate conditions: lines  $h_{\{a\}}$  and  $h_{\{b\}}$  are not parallel; circles  $k(N, M_{\{a\}})$  and  $k(M_{\{c\}}, A)$  intersect; points  $H_{\{a\}}$  and  $M_{\{c\}}$  are not the same; points  $M_{\{c\}}$  and  $N$  are not the same
% Determination conditions: lines  $h_{\{a\}}$  and  $h_{\{b\}}$  are not the same; points  $H_{\{b\}}$  and  $B$  are not the same; points  $A$  and  $H_{\{a\}}$  are not the same; circles  $k(N, M_{\{a\}})$  and  $k(M_{\{c\}}, A)$  are not the same; points  $H_{\{a\}}$  and  $H_{\{b\}}$  must be different

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_c = _M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

Time Complexity: Time spent by the prover is 0.118 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $N = _N$

Proving failed

4.1.3 Proving $H_a = _H_a$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $M_c = _M_c$

NDG conditions are:

$S_{AH_bB} \neq S_{H_aH_bB}$ i.e., lines AH_a and H_bB are not parallel (construction based assumption)
 $S_{P_{-G170006}NH} \neq S_{P_{L_{-G169975}}^0NH}$ i.e., lines $P_{-G170006}P_{L_{-G169975}}^0$ and NH are not parallel (construction based assumption)
 $S_{_M_aBC} \neq 0$ i.e., points $_M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{_M_bAC} \neq 0$ i.e., points $_M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)



Figure 1: Illustration of the problem 1091

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)
 $S_{ABF^4_{-h_b}} \neq S_{F^3_{-h_a}BF^4_{-h_b}}$ i.e., lines $AF^3_{-h_a}$ and $BF^4_{-h_b}$ are not parallel (construction based assumption)
 $S_{-M_a-M_bF^2_{-m_b}} \neq S_{F^1_{-m_a}-M_bF^2_{-m_b}}$ i.e., lines $-M_aF^1_{-m_a}$ and $-M_bF^2_{-m_b}$ are not parallel (construction based assumption)
 $S_{BAF^3_{-h_a}} \neq S_{CAF^3_{-h_a}}$ i.e., lines BC and $AF^3_{-h_a}$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.2.2 Proving $N = -N$

Proving failed

4.2.3 Proving $H_a = -H_a$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_c = -M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.080 seconds. There are no ndg conditions.

4.3.2 Proving $N = -N$

Proving failed

4.3.3 Proving $H_a = -H_a$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_c = -M_c$

Proving failed

4.4.2 Proving $N = -N$

Proving failed

4.4.3 Proving $H_a = -H_a$

Proving failed

Problem 1092

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1092: Given a point H_a , a point N and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point N and the point O , construct a point G (rule W01); ;
2. Using the point N and the point O , construct a point H (rule W01); ;
3. Using the point H_a and the point H , construct a line h_a (rule W02); % DET: points H_a and H are not the same;
4. Using the point H_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_a and N are not the same;
5. Using the circle $k(N, M_a)$, the line h_a , the point N and the point H_a , construct a point E_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points H_a and E_a must be different;
6. Using the point E_a and the point H , construct a point A (rule W01); ;
7. Using the point G and the point A , construct a point M_a (rule W01); ;
8. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
9. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; line h_a and circle $k(N, M_a)$ intersect; points H_a and N are not the same.

Determination conditions: points H_a and M_a are not the same; points H_a and E_a must be different; points H_a and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D31,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L15,L19,L22,L55]

Solving time: 5.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{a} 80 40
point N 72.5 61.93
point O 65 51.14
```

```
color 220 0 0
fontsize 9
```

```
cmark_r H_{a}
cmark_r N
cmark_t O
color 0 0 0
fontsize 8
```

```
% Constructing a line L_{\_G232378} which passes through point N and point O
line L_{\_G232378} N O
```

```
color 200 200 200
drawline L_{\_G232378}
color 0 0 0
```

```
% Constructing a point P_{\_G232479} with coordinates (0,0)
point P_{\_G232479} 0 0
cmark_r P_{\_G232479}
```

```
% Constructing a point P_{\_G232403} such that NP_{\_G232403}/NP_{\_G232479}=1
towards P_{\_G232403} N P_{\_G232479} 1
cmark_r P_{\_G232403}
color 200 200 200
drawsegment N P_{\_G232403}
color 0 0 0
```

```
% Constructing a point P_{\_G232448} such that NP_{\_G232448}/NP_{\_G232479}=3
towards P_{\_G232448} N P_{\_G232479} 3
cmark_r P_{\_G232448}
color 200 200 200
drawsegment N P_{\_G232448}
color 0 0 0
```

```
% Constructing a line L_{\_G232409} which passes through point O and point P_{\_G232448}
line L_{\_G232409} O P_{\_G232448}
```

```

color 200 200 200
drawline L_{\_G232409}
color 0 0 0

% Constructing a line L_{\_G232372} which contains the point P_{\_G232403} and is parallel to the
line L_{\_G232409}
parallel L_{\_G232372} P_{\_G232403} L_{\_G232409}

color 200 200 200
drawline L_{\_G232372}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G232372} and line L_{\_G232378}
intersec G L_{\_G232372} L_{\_G232378}
cmark_t G

% Constructing a point H such that NH/NO=-1
towards H N 0 -1
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0

% DET: points H_{a} and H are not the same
% Constructing a line h_{a} which passes through point H_{a} and point H
line h_{a} H_{a} H

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: points H_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{a}
circle k(N,M_{a}) N H_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{a} and circle k(N,M_{a}) intersect% DET: points H_{a} and E_{a} must be different
% Constructing a point P_{\_G233306} which is a foot of the point N on the line h_{a}
foot P_{\_G233306} N h_{a}
cmark_r P_{\_G233306}
color 200 200 200
drawline N P_{\_G233306}
color 0 0 0

```

```

% Constructing a point  $E_{\{a\}}$  which is an image of the point  $H_{\{a\}}$  in the symmetry to point/line  $P$ 
_{\_G233306}
sim  $E_{\{a\}}$   $P_{\{\_G233306\}}$   $H_{\{a\}}$ 
cmark_r  $E_{\{a\}}$ 

```

```

% Constructing a point  $A$  such that  $E_{\{a\}}A/E_{\{a\}}H=-1$ 
towards  $A$   $E_{\{a\}}$   $H$  -1
cmark_t  $A$ 
color 200 200 200
drawsegment  $H$   $A$ 
color 0 0 0

```

```

% Constructing a point  $M_{\{a\}}$  such that  $GM_{\{a\}}/GA=-0.5$ 
towards  $M_{\{a\}}$   $G$   $A$  -0.5
cmark_r  $M_{\{a\}}$ 
color 200 200 200
drawsegment  $A$   $M_{\{a\}}$ 
color 0 0 0

```

```

% DET: points  $H_{\{a\}}$  and  $M_{\{a\}}$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $M_{\{a\}}$ 
line  $a$   $H_{\{a\}}$   $M_{\{a\}}$ 

color 200 200 200
drawline  $a$ 
color 0 0 0

```

```

% NDG: points  $A$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $A$ 
circle  $k(O,C)$   $O$   $A$ 

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

```

```

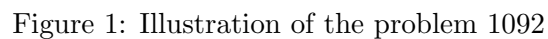
% NDG: line  $a$  and circle  $k(O,C)$  intersect
% Constructing points  $C$  and  $B$  which are in intersection of  $k(O,C)$  and  $a$ 
intersec2  $C$   $B$   $k(O,C)$   $a$ 
cmark_l  $C$ 
cmark_b  $B$ 

```

```

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

```

```
% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
    line h_{a} and circle k(N,M_{a}) intersect; points H_{a} and N are not the same
% Determination conditions: points H_{a} and M_{a} are not the same; points H_{a} and E_{a} must be
    different; points H_{a} and H are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_a = H_a$

Proving failed

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $O=_O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_a=_H_a$

Proving failed

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $O=_O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_a=_H_a$

Proving failed

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $O=_O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_a=_H_a$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $O=_O$

Proving failed

Problem 1093

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1093: Given a point H_a , a point N and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1094

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1094: Given a point H_a , a point N and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1095

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1095: Given a point H_a , a point N and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1096

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1096: Given a point H_c , a point N and a point H_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point H_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_c and N are not the same;
2. Choose freely a point H_b on the circle $k(N, M_a)$ (rule WOncircle);
3. Using the point H_b and the point H_c , construct a line $m(H_bH_c)$ (rule W14); % DET: points H_b and H_c are not the same;
4. Using the circle $k(N, M_a)$ and the line $m(H_bH_c)$, construct a point M_a and a point E_a (rule W04); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect;
5. Choose freely a point A on the circle $k(E_a, A)$ (rule WOncircle);
6. Using the point A and the point E_a , construct a point H (rule W01); ;
7. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;
8. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
9. Using the line b and the line h_c , construct a point C (rule W03); % NDG: lines b and h_c are not parallel % DET: lines b and h_c are not the same;
10. Using the point M_a and the point C , construct a point B (rule W01); .

Non-degenerate conditions: lines b and h_c are not parallel; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; points H_c and N are not the same.

Determination conditions: lines b and h_c are not the same; points H_c and H are not the same; points A and H_b are not the same; points H_b and H_c are not the same.

Rules used: [W01,W02,W03,W04,W06,W14,WOncircle1]

Lemmas used: [D10,D21,D28,D32,D6,D7,GD01,GD02,GL01,GL03,GL04,L21,L22,L3,L38,L39,L47,L48]

Solving time: 59.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point H_{c} 68.91 84.83
```

```
point N 72.5 61.93
```

```
point H_{b} 89.36 77.83
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_rt H_{c}
```

```
cmark_r N
```

```
cmark_l H_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% NDG: points H_{c} and N are not the same
```

```
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{c}
```

```
circle k(N,M_{a}) N H_{c}
```

```
color 200 200 200
```

```
drawcircle k(N,M_{a})
```

```
color 0 0 0
```

```
% Choosing randomly a point H_{b} on the circle with center N through point H_{c}
```

```
oncircle H_{b} N H_{c}
```

```
cmark_l H_{b}
```

```
color 200 200 200
```

```
drawcircle N H_{c}
```

```
color 0 0 0
```

```
% DET: points H_{b} and H_{c} are not the same
```

```
% Constructing bisector m(H_{b}H_{c}) of the segment H_{b}H_{c}
```

```
med m(H_{b}H_{c}) H_{b} H_{c}
```

```
color 200 200 200
```

```
drawline m(H_{b}H_{c})
```

```
color 0 0 0
```

```
color 200 200 200
```

```
drawsegment H_{b} H_{c}
```

```
color 0 0 0
```

```
% NDG: line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N,M_{\{a\}})$  intersect
% Constructing points  $M_{\{a\}}$  and  $E_{\{a\}}$  which are in intersection of  $k(N,M_{\{a\}})$  and  $m(H_{\{b\}}H_{\{c\}})$ 
intersec2 M_{a} E_{a} k(N,M_{a}) m(H_{b}H_{c})
cmark_r M_{a}
cmark_r E_{a}
```

```
% Choosing randomly a point A on the circle with center  $E_{\{a\}}$  through point  $H_{\{b\}}$ 
oncircle A E_{a} H_{b}
cmark_t A
color 200 200 200
drawcircle E_{a} H_{b}
color 0 0 0
```

```
% Constructing a point H such that  $AH/AE_{\{a\}}=2$ 
towards H A E_{a} 2
cmark_rt H
color 200 200 200
drawsegment A H
color 0 0 0
```

```
% DET: points A and  $H_{\{b\}}$  are not the same
% Constructing a line b which passes through point A and point  $H_{\{b\}}$ 
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0
```

```
% DET: points  $H_{\{c\}}$  and H are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $H_{\{c\}}$  and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0
```

```
% NDG: lines b and  $h_{\{c\}}$  are not parallel% DET: lines b and  $h_{\{c\}}$  are not the same
% Constructing a point C which belongs to line b and line  $h_{\{c\}}$ 
intersec C b h_{c}
cmark_l C
```

```
% Constructing a point B such that  $M_{\{a\}}B/M_{\{a\}}C=-1$ 
```

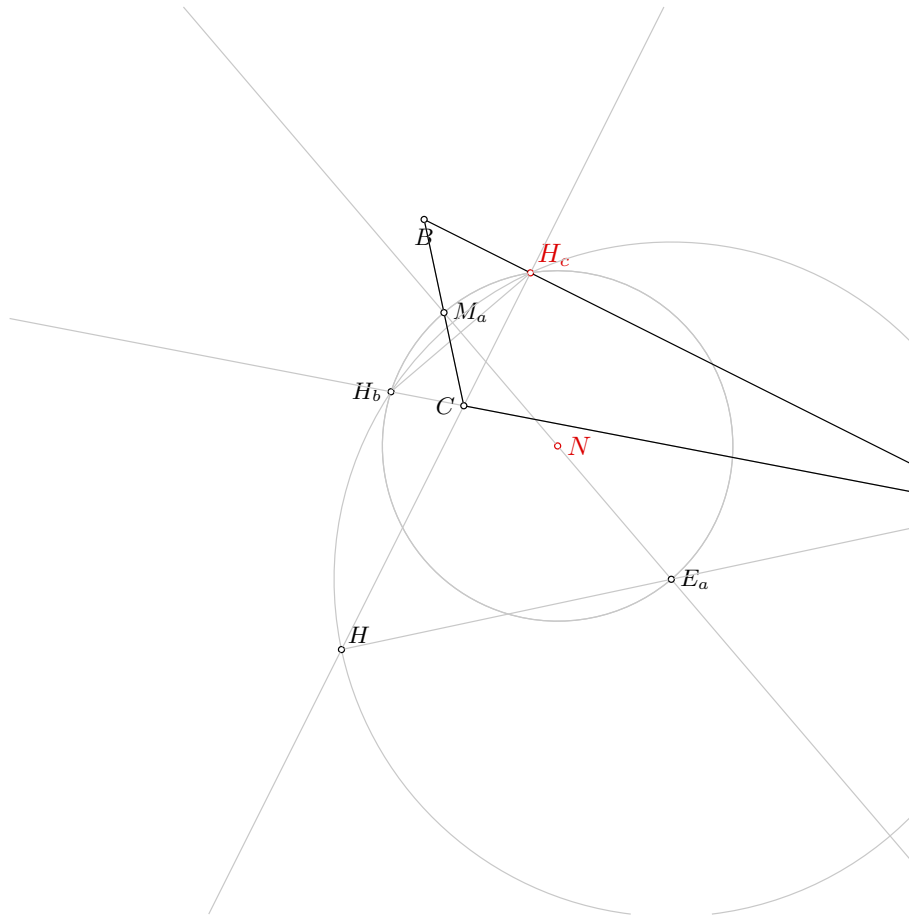



Figure 1: Illustration of the problem 1096

```
towards B M_{a} C -1
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

% Non-degenerate conditions: lines b and h_{c} are not parallel; line m(H_{b}H_{c}) and circle k(N, M_{a}) intersect; points H_{c} and N are not the same
% Determination conditions: lines b and h_{c} are not the same; points H_{c} and H are not the same
; points A and H_{b} are not the same; points H_{b} and H_{c} are not the same

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_c = \neg H_c$

Proving failed

4.1.2 Proving $N = \neg N$

Proving failed

4.1.3 Proving $H_b = \neg H_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_c = \neg H_c$

Proving failed

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $H_b = \neg H_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_c = \neg H_c$

Proving failed

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $H_b = \neg H_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_c = \neg H_c$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $H_b = H_b$

Proving failed

Problem 1097

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1097: Given a point H_b , a point I and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_b and N are not the same;
2. Using the point I , the circle $k(N, M_a)$, the point N and the point H_b , construct a circle $k(I, P_a)$ (rule W22); % NDG: point I is inside the circle $k(N, M_a)$; points I and N are not the same;
3. Using the circle $k(I, P_a)$, the point H_b and the point I , construct a line y_2 and a line b (rule W12); % NDG: point H_b is outside the circle $k(I, P_a)$;
4. Using the circle $k(N, M_a)$, the line b , the point N and the point H_b , construct a point M_b (rule W05); % NDG: line b and circle $k(N, M_a)$ intersect % DET: points H_b and M_b must be different;
5. Using the point M_b and the point I , construct a line IM_b (rule W02); % DET: points M_b and I are not the same;
6. Using the point N and the point M_b , construct a line $m(H_aH_c)$ (rule W02); % DET: points N and M_b are not the same;
7. Using the circle $k(N, M_a)$, the line $m(H_aH_c)$, the point N and the point M_b , construct a point E_b (rule W05a); % NDG: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_b and E_b must be different;
8. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;

9. Using the point I and the point M_b , construct a circle $k_{over}(I, M_b)$ (rule W09); % NDG: points I and M_b are not the same;
10. Using the circle $k_{over}(I, M_b)$ and the circle $k(I, P_a)$, construct a point B_{fi} and a point P_b (rule W07); % NDG: circles $k_{over}(I, M_b)$ and $k(I, P_a)$ intersect % DET: circles $k_{over}(I, M_b)$ and $k(I, P_a)$ are not the same;
11. Using the point P_b and the point M_b , construct a point P'_b (rule W01); ;
12. Using the point M_b and the line b , construct a line m_b (rule W10b); ;
13. Using the point P'_b and the line IM_b , construct a line BP'_b (rule W16); ;
14. Using the line BP'_b and the line h_b , construct a point B (rule W03); % NDG: lines BP'_b and h_b are not parallel % DET: lines BP'_b and h_b are not the same;
15. Using the point I and the point B , construct a line s_b (rule W02); % DET: points I and B are not the same;
16. Using the line m_b and the line s_b , construct a point N_b (rule W03); % NDG: lines m_b and s_b are not parallel % DET: lines m_b and s_b are not the same;
17. Using the point I and the point N_b , construct a circle $k(N_b, A)$ (rule W06); % NDG: points I and N_b are not the same;
18. Using the circle $k(N_b, A)$ and the line b , construct a point A and a point C (rule W04); % NDG: line b and circle $k(N_b, A)$ intersect.

Non-degenerate conditions: line b and circle $k(N_b, A)$ intersect; points I and N_b are not the same; lines m_b and s_b are not parallel; lines BP'_b and h_b are not parallel; circles $k_{over}(I, M_b)$ and $k(I, P_a)$ intersect; points I and M_b are not the same; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; line b and circle $k(N, M_a)$ intersect; point H_b is outside the circle $k(I, P_a)$; point I is inside the circle $k(N, M_a)$; points I and N are not the same; points H_b and N are not the same.

Determination conditions: lines m_b and s_b are not the same; points I and B are not the same; lines BP'_b and h_b are not the same; circles $k_{over}(I, M_b)$ and $k(I, P_a)$ are not the same; points E_b and H_b are not the same; points M_b and E_b must be different; points N and M_b are not the same; points M_b and I are not the same; points H_b and M_b must be different.

Rules used: [W01,W02,W03,W04,W05,W05a,W06,W07,W09,W10b,W12,W16,W22]

Lemmas used: [D12,D2,D22,D27,D29,D3,D32,D48,D6,D66,D89,D9,GD01,GD02,GL01,GL03,GL09,L119,L13,L

Solving time: 41.6 seconds.

3.2 Construction in GCLC language

dim 120 120

point H_{b} 89.36 77.83

point I 74.37 61.15

point N 72.5 61.93

color 220 0 0

```

fontsize 9

cmark_l H_{b}
cmark_b I
cmark_r N
color 0 0 0
fontsize 8

% NDG: points H_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{b}
circle k(N,M_{a}) N H_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: point I is inside the circle k(N,M_{a}); points I and N are not the same
% Calculating distance V[_G124541] from point N to point H_{b}
distance V[_G124541] N H_{b}

% Calculating distance V[_G124565] from point N to point I
distance V[_G124565] N I

% Calculating value V[_G124586] using formula V[_G124541]/V[_G124565]
expression V[_G124586] { V[_G124541]/V[_G124565] }

% Constructing a point P_{\_G124617} such that NP_{\_G124617}/NI=V[_G124541]/V[_G124565]
towards P_{\_G124617} N I V[_G124586]
cmark_r P_{\_G124617}

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
124617}
circle k(I,P_{a}) I P_{\_G124617}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point H_{b} is outside the circle k(I,P_{a})
% Constructing midpoint P_{\_G124991} of the segment H_{b}I
midpoint P_{\_G124991} H_{b} I
cmark_r P_{\_G124991}

% Constructing a circle C_{\_G124994} whose center is at point P_{\_G124991} and which passes
through point H_{b}
circle C_{\_G124994} P_{\_G124991} H_{b}

color 200 200 200
drawcircle C_{\_G124994}

```

```

color 0 0 0

% Constructing points  $P_{\backslash\_G124997}$  and  $P_{\backslash\_G125000}$  which are in intersection of  $C_{\backslash\_G124994}$ 
and  $k(I, P_{\{a\}})$ 
intersec2  $P_{\backslash\_G124997}$   $P_{\backslash\_G125000}$   $C_{\backslash\_G124994}$   $k(I, P_{\{a\}})$ 
cmark_r  $P_{\backslash\_G124997}$ 
cmark_r  $P_{\backslash\_G125000}$ 

% Constructing a line  $y2$  which passes through point  $H_{\{b\}}$  and point  $P_{\backslash\_G124997}$ 
line  $y2$   $H_{\{b\}}$   $P_{\backslash\_G124997}$ 

color 200 200 200
drawline  $y2$ 
color 0 0 0

% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $P_{\backslash\_G125000}$ 
line  $b$   $H_{\{b\}}$   $P_{\backslash\_G125000}$ 

color 200 200 200
drawline  $b$ 
color 0 0 0

% NDG: line  $b$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $H_{\{b\}}$  and  $M_{\{b\}}$  must be different
% Constructing a point  $P_{\backslash\_G125334}$  which is a foot of the point  $N$  on the line  $b$ 
foot  $P_{\backslash\_G125334}$   $N$   $b$ 
cmark_r  $P_{\backslash\_G125334}$ 
color 200 200 200
drawline  $N$   $P_{\backslash\_G125334}$ 
color 0 0 0

% Constructing a point  $M_{\{b\}}$  which is an image of the point  $H_{\{b\}}$  in the symmetry to point/line  $P_{\backslash\_G125334}$ 
sim  $M_{\{b\}}$   $P_{\backslash\_G125334}$   $H_{\{b\}}$ 
cmark_lt  $M_{\{b\}}$ 

% DET: points  $M_{\{b\}}$  and  $I$  are not the same
% Constructing a line  $IM_{\{b\}}$  which passes through point  $M_{\{b\}}$  and point  $I$ 
line  $IM_{\{b\}}$   $M_{\{b\}}$   $I$ 

color 200 200 200
drawline  $IM_{\{b\}}$ 
color 0 0 0

% DET: points  $N$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $m(H_{\{a\}}H_{\{c\}})$  which passes through point  $N$  and point  $M_{\{b\}}$ 
line  $m(H_{\{a\}}H_{\{c\}})$   $N$   $M_{\{b\}}$ 

color 200 200 200
drawline  $m(H_{\{a\}}H_{\{c\}})$ 

```

```
color 0 0 0
```

```
% NDG: line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $M_{\{b\}}$  and  $E_{\{b\}}$  must be
different
% Constructing a point  $E_{\{b\}}$  which is an image of the point  $M_{\{b\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{b\}}$   $N$   $M_{\{b\}}$ 
cmark_r  $E_{\{b\}}$ 
```

```
% DET: points  $E_{\{b\}}$  and  $H_{\{b\}}$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $E_{\{b\}}$  and point  $H_{\{b\}}$ 
line  $h_{\{b\}}$   $E_{\{b\}}$   $H_{\{b\}}$ 
```

```
color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0
```

```
% NDG: points  $I$  and  $M_{\{b\}}$  are not the same
% Constructing midpoint  $P_{\{\backslash\_G125802\}}$  of the segment  $IM_{\{b\}}$ 
midpoint  $P_{\{\backslash\_G125802\}}$   $I$   $M_{\{b\}}$ 
cmark_r  $P_{\{\backslash\_G125802\}}$ 
```

```
% Constructing a circle  $k_{\text{over}}(I,M_{\{b\}})$  whose center is at point  $P_{\{\backslash\_G125802\}}$  and which passes
through point  $I$ 
circle  $k_{\text{over}}(I,M_{\{b\}})$   $P_{\{\backslash\_G125802\}}$   $I$ 
```

```
color 200 200 200
drawcircle  $k_{\text{over}}(I,M_{\{b\}})$ 
color 0 0 0
```

```
% NDG: circles  $k_{\text{over}}(I,M_{\{b\}})$  and  $k(I,P_{\{a\}})$  intersect% DET: circles  $k_{\text{over}}(I,M_{\{b\}})$  and  $k(I,P_{\{a\}}$ 
 $\}$ ) are not the same
% Constructing points  $B_{\{fi\}}$  and  $P_{\{b\}}$  which are in intersection of  $k_{\text{over}}(I,M_{\{b\}})$  and  $k(I,P_{\{a\}})$ 
intersec2  $B_{\{fi\}}$   $P_{\{b\}}$   $k_{\text{over}}(I,M_{\{b\}})$   $k(I,P_{\{a\}})$ 
cmark_r  $B_{\{fi\}}$ 
cmark_r  $P_{\{b\}}$ 
```

```
% Constructing a point  $P'_{\{b\}}$  such that  $P_{\{b\}}P'_{\{b\}}/P_{\{b\}}M_{\{b\}}=2$ 
towards  $P'_{\{b\}}$   $P_{\{b\}}$   $M_{\{b\}}$  2
cmark_r  $P'_{\{b\}}$ 
color 200 200 200
drawsegment  $P_{\{b\}}$   $P'_{\{b\}}$ 
color 0 0 0
```

```
% Constructing a line  $m_{\{b\}}$  which is perpendicular to line  $b$  and which passes through point  $M_{\{b\}}$ 
perp  $m_{\{b\}}$   $M_{\{b\}}$   $b$ 
```



```

color 200 200 200
drawline m_{b}
color 0 0 0

% Constructing a line BP'_{b} which contains the point P'_{b} and is parallel to the line IM_{b}
parallel BP'_{b} P'_{b} IM_{b}

color 200 200 200
drawline BP'_{b}
color 0 0 0

% NDG: lines BP'_{b} and h_{b} are not parallel% DET: lines BP'_{b} and h_{b} are not the same
% Constructing a point B which belongs to line BP'_{b} and line h_{b}
intersec B BP'_{b} h_{b}
cmark_b B

% DET: points I and B are not the same
% Constructing a line s_{b} which passes through point I and point B
line s_{b} I B

color 200 200 200
drawline s_{b}
color 0 0 0

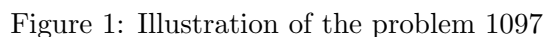
% NDG: lines m_{b} and s_{b} are not parallel% DET: lines m_{b} and s_{b} are not the same
% Constructing a point N_{b} which belongs to line m_{b} and line s_{b}
intersec N_{b} m_{b} s_{b}
cmark_rb N_{b}

% NDG: points I and N_{b} are not the same
% Constructing a circle k(N_{b},A) whose center is at point N_{b} and which passes through point I
circle k(N_{b},A) N_{b} I

color 200 200 200
drawcircle k(N_{b},A)
color 0 0 0

% NDG: line b and circle k(N_{b},A) intersect
% Constructing points A and C which are in intersection of k(N_{b},A) and b
intersec2 A C k(N_{b},A) b
cmark_t A
cmark_l C

```



% Non-degenerate conditions: line b and circle $k(N_{\{b\}}, A)$ intersect; points I and $N_{\{b\}}$ are not the same; lines $m_{\{b\}}$ and $s_{\{b\}}$ are not parallel; lines $BP'_{\{b\}}$ and $h_{\{b\}}$ are not parallel; circles $k_{\text{over}}(I, M_{\{b\}})$ and $k(I, P_{\{a\}})$ intersect; points I and $M_{\{b\}}$ are not the same; line $m(H_{\{a\}}H_{\{c\}})$ and circle $k(N, M_{\{a\}})$ intersect; line b and circle $k(N, M_{\{a\}})$ intersect; point $H_{\{b\}}$ is outside the circle $k(I, P_{\{a\}})$; point I is inside the circle $k(N, M_{\{a\}})$; points I and N are not the same; points $H_{\{b\}}$ and N are not the same

% Determination conditions: lines $m_{\{b\}}$ and $s_{\{b\}}$ are not the same; points I and B are not the same; lines $BP'_{\{b\}}$ and $h_{\{b\}}$ are not the same; circles $k_{\text{over}}(I, M_{\{b\}})$ and $k(I, P_{\{a\}})$ are not the same; points $E_{\{b\}}$ and $H_{\{b\}}$ are not the same; points $M_{\{b\}}$ and $E_{\{b\}}$ must be different; points N and $M_{\{b\}}$ are not the same; points $M_{\{b\}}$ and I are not the same; points $H_{\{b\}}$ and $M_{\{b\}}$ must be different

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_b = \neg H_b$

Proving failed

4.1.2 Proving $I = \neg I$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_b = \neg H_b$

Proving failed

4.2.2 Proving $I = \neg I$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_b = \neg H_b$

Proving failed

4.3.2 Proving $I = \neg I$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_b = \neg H_b$

Proving failed

4.4.2 Proving $I = \neg I$

Proving failed

4.4.3 Proving $N = _N$

Proving failed

Problem 1098

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1098: Given a point M_a , a point N and a point H_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point M_a and the point N , construct a line $m(H_bH_c)$ (rule W02); % DET: points M_a and N are not the same;
2. Using the point M_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_a and N are not the same;
3. Choose freely a point H_b on the circle $k(N, M_a)$ (rule WOncircle);
4. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point M_a , construct a point E_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_a and E_a must be different;
5. Using the point H_b and the point M_a , construct a circle $k(M_a, B)$ (rule W06); % NDG: points H_b and M_a are not the same;
6. Using the circle $k(N, M_a)$, the circle $k(M_a, B)$, the point H_b , the point N and the point M_a , construct a point H_c (rule W08); % NDG: circles $k(N, M_a)$ and $k(M_a, B)$ intersect % DET: circles $k(N, M_a)$ and $k(M_a, B)$ are not the same; points H_b and H_c must be different;
7. Choose freely a point A on the circle $k(E_a, A)$ (rule WOncircle);
8. Using the point A and the point E_a , construct a point H (rule W01); ;
9. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;
10. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;

11. Using the line b and the line h_c , construct a point C (rule W03); % NDG: lines b and h_c are not parallel % DET: lines b and h_c are not the same;
12. Using the point M_a and the point C , construct a point B (rule W01); .

Non-degenerate conditions: lines b and h_c are not parallel; circles $k(N, M_a)$ and $k(M_a, B)$ intersect; points H_b and M_a are not the same; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; points M_a and N are not the same.

Determination conditions: lines b and h_c are not the same; points H_c and H are not the same; points A and H_b are not the same; circles $k(N, M_a)$ and $k(M_a, B)$ are not the same; points H_b and H_c must be different; points M_a and E_a must be different; points M_a and N are not the same.

Rules used: [W01,W02,W03,W05a,W06,W08,WOncircle1]

Lemmas used: [D10,D21,D28,D32,D6,D7,GD01,GD02,GL01,GL03,GL04,L20,L21,L22,L3,L38,L39,L47,L48]

Solving time: 59.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40
point N 72.5 61.93
point H_{b} 89.36 77.83
```

```
color 220 0 0
fontsize 9
```

```
cmark_r M_{a}
cmark_r N
cmark_l H_{b}
color 0 0 0
fontsize 8
```

```
% DET: points M_{a} and N are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point M_{a} and point N
line m(H_{b}H_{c}) M_{a} N
```

```
color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0
```

```
% NDG: points M_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{a}
circle k(N,M_{a}) N M_{a}
```

```
color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```

% Choosing randomly a point  $H_{\{b\}}$  on the circle with center  $N$  through point  $M_{\{a\}}$ 
oncircle H_{b} N M_{a}
cmark_l H_{b}
color 200 200 200
drawcircle N M_{a}
color 0 0 0

% NDG: line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $M_{\{a\}}$  and  $E_{\{a\}}$  must be
different
% Constructing a point  $E_{\{a\}}$  which is an image of the point  $M_{\{a\}}$  in the symmetry to point/line  $N$ 
sim E_{a} N M_{a}
cmark_r E_{a}

% NDG: points  $H_{\{b\}}$  and  $M_{\{a\}}$  are not the same
% Constructing a circle  $k(M_{\{a\}},B)$  whose center is at point  $M_{\{a\}}$  and which passes through point  $H_{\{b\}}$ 
circle k(M_{a},B) M_{a} H_{b}

color 200 200 200
drawcircle k(M_{a},B)
color 0 0 0

% NDG: circles  $k(N,M_{\{a\}})$  and  $k(M_{\{a\}},B)$  intersect% DET: circles  $k(N,M_{\{a\}})$  and  $k(M_{\{a\}},B)$  are not
the same; points  $H_{\{b\}}$  and  $H_{\{c\}}$  must be different
% Constructing a line  $L_{\{\_G163597\}}$  which passes through point  $N$  and point  $M_{\{a\}}$ 
line L_{\_G163597} N M_{a}

color 200 200 200
drawline L_{\_G163597}
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $H_{\{b\}}$  in the symmetry to point/line  $L_{\{\_G163597\}}$ 
sim H_{c} L_{\_G163597} H_{b}
cmark_rt H_{c}

% Choosing randomly a point  $A$  on the circle with center  $E_{\{a\}}$  through point  $H_{\{b\}}$ 
oncircle A E_{a} H_{b}
cmark_t A
color 200 200 200
drawcircle E_{a} H_{b}
color 0 0 0

% Constructing a point  $H$  such that  $AH/AE_{\{a\}}=2$ 
towards H A E_{a} 2
cmark_rt H
color 200 200 200

```

```

drawsegment A H
color 0 0 0

% DET: points A and H_{b} are not the same
% Constructing a line b which passes through point A and point H_{b}
line b A H_{b}

color 200 200 200
drawline b
color 0 0 0

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: lines b and h_{c} are not parallel% DET: lines b and h_{c} are not the same
% Constructing a point C which belongs to line b and line h_{c}
intersec C b h_{c}
cmark_l C

% Constructing a point B such that M_{a}B/M_{a}C=-1
towards B M_{a} C -1
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines b and h_{c} are not parallel; circles k(N,M_{a}) and k(M_{a},B)
% intersect; points H_{b} and M_{a} are not the same; line m(H_{b}H_{c}) and circle k(N,M_{a})
% intersect; points M_{a} and N are not the same
% Determination conditions: lines b and h_{c} are not the same; points H_{c} and H are not the same
% ; points A and H_{b} are not the same; circles k(N,M_{a}) and k(M_{a},B) are not the same;
% points H_{b} and H_{c} must be different; points M_{a} and E_{a} must be different; points M_{a}
% and N are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

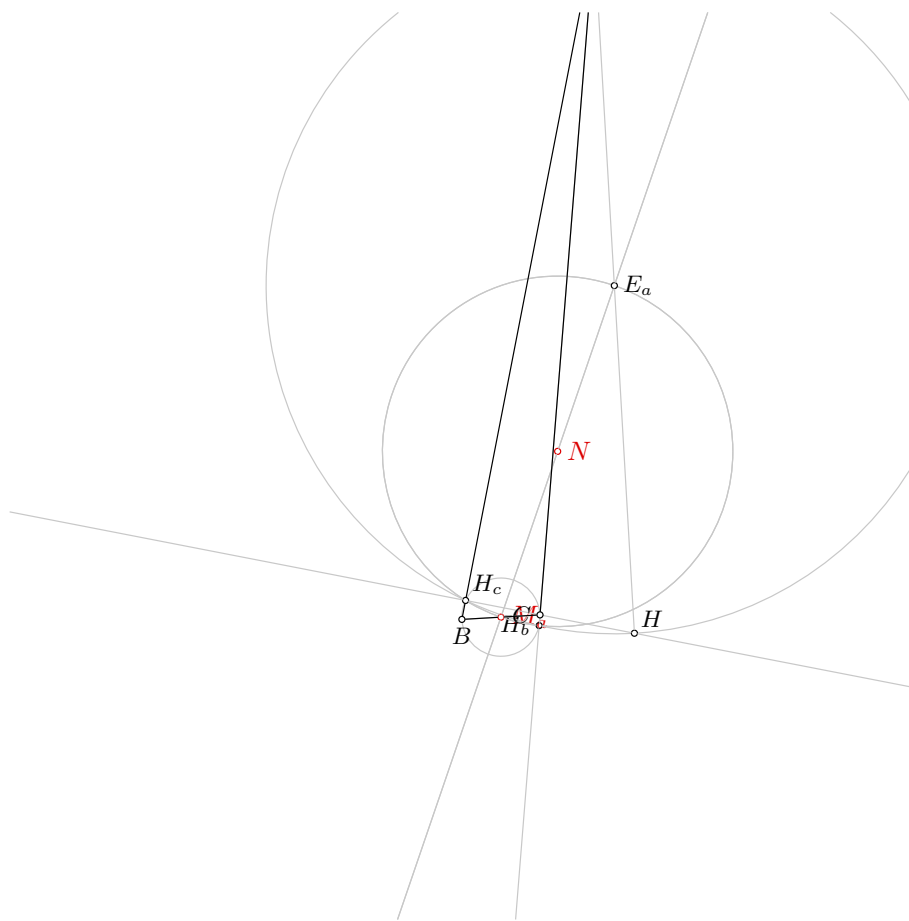


Figure 1: Illustration of the problem 1098

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 11 terms.

Time Complexity: Time spent by the prover is 0.201 seconds.

NDG conditions Line through points H_b and A is not parallel with line through points H_c and H
Points H_b and A are not identical

4.1.2 Proving $N = \neg N$

Proving failed

4.1.3 Proving $H_b = \neg H_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{AH_cH} \neq S_{H_bH_cH}$ i.e., lines AH_b and H_cH are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^3_{\neg h_b}} \neq S_{F^2_{\neg h_a}BF^3_{\neg h_b}}$ i.e., lines $AF^2_{\neg h_a}$ and $BF^3_{\neg h_b}$ are not parallel (construction based assumption)

$S_{\neg M_a \neg M_b F^1_{\neg m_b}} \neq S_{F^0_{\neg m_a} \neg M_b F^1_{\neg m_b}}$ i.e., lines $\neg M_a F^0_{\neg m_a}$ and $\neg M_b F^1_{\neg m_b}$ are not parallel (construction based assumption)

$S_{ABF^3_{\neg h_b}} \neq S_{CBF^3_{\neg h_b}}$ i.e., lines AC and $BF^3_{\neg h_b}$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $H_b = \neg H_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_a = \neg M_a$

Proving failed

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $H_b = \neg H_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_a = \neg M_a$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $H_b = \neg H_b$

Proving failed

Problem 1099

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1099: Given a point M_b , a point N and a point H_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point M_b and the point N , construct a line $m(H_aH_c)$ (rule W02); % DET: points M_b and N are not the same;
2. Using the point M_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_b and N are not the same;
3. Choose freely a point H_b on the circle $k(N, M_a)$ (rule WOncircle);
4. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
5. Using the circle $k(N, M_a)$, the line $m(H_aH_c)$, the point N and the point M_b , construct a point E_b (rule W05a); % NDG: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_b and E_b must be different;
6. Using the point E_b and the point H_b , construct a line h_b (rule W02); % DET: points E_b and H_b are not the same;
7. Choose freely a point A on the line b (rule WOnline1) ;
8. Using the point A and the point M_b , construct a point C (rule W01); ;
9. Using the point A and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points A and M_b are not the same;
10. Using the circle $k(N, M_a)$ and the circle $k(M_b, C)$, construct a point H_a and a point H_c (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_b, C)$ intersect % DET: circles $k(N, M_a)$ and $k(M_b, C)$ are not the same;

11. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
12. Using the line h_b and the line h_a , construct a point H (rule W03); % NDG: lines h_b and h_a are not parallel % DET: lines h_b and h_a are not the same;
13. Using the point N and the point H , construct a point G (rule W01); ;
14. Using the point M_b and the point G , construct a point B (rule W01); .

Non-degenerate conditions: lines h_b and h_a are not parallel; circles $k(N, M_a)$ and $k(M_b, C)$ intersect; points A and M_b are not the same; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; points M_b and N are not the same.

Determination conditions: lines h_b and h_a are not the same; points A and H_a are not the same; circles $k(N, M_a)$ and $k(M_b, C)$ are not the same; points E_b and H_b are not the same; points M_b and E_b must be different; points H_b and M_b are not the same; points M_b and N are not the same.

Rules used: [W01,W02,W03,W05a,W06,W07,WOncircle1,WOnline1]

Lemmas used: [D22,D29,D3,D32,D5,D6,D8,D9,GD01,GD02,GL01,GL03,GL04,GL09,L16,L17,L19,L21,L23,L43]

Solving time: 569.9 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{b} 95 67.5
```

```
point N 72.5 61.93
```

```
point H_{b} 89.36 77.83
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_lt M_{b}
```

```
cmark_r N
```

```
cmark_l H_{b}
```

```
color 0 0 0
```

```
fontsize 8
```

```
% DET: points M_{b} and N are not the same
```

```
% Constructing a line m(H_{a}H_{c}) which passes through point M_{b} and point N
```

```
line m(H_{a}H_{c}) M_{b} N
```

```
color 200 200 200
```

```
drawline m(H_{a}H_{c})
```

```
color 0 0 0
```

```
% NDG: points M_{b} and N are not the same
```

```
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{b}
```

```
circle k(N,M_{a}) N M_{b}
```

```
color 200 200 200
```

```
drawcircle k(N,M_{a})
```

```
color 0 0 0
```

```
% Choosing randomly a point  $H_{\{b\}}$  on the circle with center  $N$  through point  $M_{\{b\}}$ 
oncircle  $H_{\{b\}}$   $N$   $M_{\{b\}}$ 
cmark_l  $H_{\{b\}}$ 
color 200 200 200
drawcircle  $N$   $M_{\{b\}}$ 
color 0 0 0
```

```
% DET: points  $H_{\{b\}}$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $M_{\{b\}}$ 
line  $b$   $H_{\{b\}}$   $M_{\{b\}}$ 
```

```
color 200 200 200
drawline  $b$ 
color 0 0 0
```

```
% NDG: line  $m(H_{\{a\}}H_{\{c\}})$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $M_{\{b\}}$  and  $E_{\{b\}}$  must be
different
% Constructing a point  $E_{\{b\}}$  which is an image of the point  $M_{\{b\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{b\}}$   $N$   $M_{\{b\}}$ 
cmark_r  $E_{\{b\}}$ 
```

```
% DET: points  $E_{\{b\}}$  and  $H_{\{b\}}$  are not the same
% Constructing a line  $h_{\{b\}}$  which passes through point  $E_{\{b\}}$  and point  $H_{\{b\}}$ 
line  $h_{\{b\}}$   $E_{\{b\}}$   $H_{\{b\}}$ 
```

```
color 200 200 200
drawline  $h_{\{b\}}$ 
color 0 0 0
```

```
% Choosing randomly a point  $A$  on the line  $H_{\{b\}}M_{\{b\}}$ 
online  $A$   $H_{\{b\}}$   $M_{\{b\}}$ 
cmark_t  $A$ 
color 200 200 200
drawline  $H_{\{b\}}$   $M_{\{b\}}$ 
color 0 0 0
```

```
% Constructing a point  $C$  such that  $AC/AM_{\{b\}}=2$ 
towards  $C$   $A$   $M_{\{b\}}$  2
cmark_l  $C$ 
color 200 200 200
drawsegment  $A$   $C$ 
color 0 0 0
```

```

% NDG: points A and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point A
circle k(M_{b},C) M_{b} A

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(M_{b},C) intersect% DET: circles k(N,M_{a}) and k(M_{b},C) are not
the same
% Constructing points H_{a} and H_{c} which are in intersection of k(N,M_{a}) and k(M_{b},C)
intersec2 H_{a} H_{c} k(N,M_{a}) k(M_{b},C)
cmark_r H_{a}
cmark_rt H_{c}

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: lines h_{b} and h_{a} are not parallel% DET: lines h_{b} and h_{a} are not the same
% Constructing a point H which belongs to line h_{b} and line h_{a}
intersec H h_{b} h_{a}
cmark_rt H

% Constructing a line L_{\_G206131} which passes through point N and point H
line L_{\_G206131} N H

color 200 200 200
drawline L_{\_G206131}
color 0 0 0

% Constructing a point P_{\_G206232} with coordinates (0,0)
point P_{\_G206232} 0 0
cmark_r P_{\_G206232}

% Constructing a point P_{\_G206156} such that NP_{\_G206156}/NP_{\_G206232}=-1
towards P_{\_G206156} N P_{\_G206232} -1
cmark_r P_{\_G206156}
color 200 200 200
drawsegment P_{\_G206232} P_{\_G206156}
color 0 0 0

```

```

% Constructing a point  $P_{\{ \_G206201 \}}$  such that  $NP_{\{ \_G206201 \}}/NP_{\{ \_G206232 \}}=3$ 
towards  $P_{\{ \_G206201 \}}$  N  $P_{\{ \_G206232 \}}$  3
cmark_r  $P_{\{ \_G206201 \}}$ 
color 200 200 200
drawsegment N  $P_{\{ \_G206201 \}}$ 
color 0 0 0

% Constructing a line  $L_{\{ \_G206162 \}}$  which passes through point  $H$  and point  $P_{\{ \_G206201 \}}$ 
line  $L_{\{ \_G206162 \}}$  H  $P_{\{ \_G206201 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G206162 \}}$ 
color 0 0 0

% Constructing a line  $L_{\{ \_G206125 \}}$  which contains the point  $P_{\{ \_G206156 \}}$  and is parallel to the
line  $L_{\{ \_G206162 \}}$ 
parallel  $L_{\{ \_G206125 \}}$   $P_{\{ \_G206156 \}}$   $L_{\{ \_G206162 \}}$ 

color 200 200 200
drawline  $L_{\{ \_G206125 \}}$ 
color 0 0 0

% Constructing a point  $G$  which belongs to line  $L_{\{ \_G206125 \}}$  and line  $L_{\{ \_G206131 \}}$ 
intersec G  $L_{\{ \_G206125 \}}$   $L_{\{ \_G206131 \}}$ 
cmark_t G

% Constructing a point  $B$  such that  $M_{\{ b \}}B/M_{\{ b \}}G=3$ 
towards B  $M_{\{ b \}}$  G 3
cmark_b B
color 200 200 200
drawsegment  $M_{\{ b \}}$  B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{ b \}}$  and  $h_{\{ a \}}$  are not parallel; circles  $k(N, M_{\{ a \}})$  and  $k(M_{\{ b \}}, C)$  intersect; points  $A$  and  $M_{\{ b \}}$  are not the same; line  $m(H_{\{ a \}}H_{\{ c \}})$  and circle  $k(N, M_{\{ a \}})$  intersect; points  $M_{\{ b \}}$  and  $N$  are not the same
% Determination conditions: lines  $h_{\{ b \}}$  and  $h_{\{ a \}}$  are not the same; points  $A$  and  $H_{\{ a \}}$  are not the same; circles  $k(N, M_{\{ a \}})$  and  $k(M_{\{ b \}}, C)$  are not the same; points  $E_{\{ b \}}$  and  $H_{\{ b \}}$  are not the same; points  $M_{\{ b \}}$  and  $E_{\{ b \}}$  must be different; points  $H_{\{ b \}}$  and  $M_{\{ b \}}$  are not the same; points  $M_{\{ b \}}$  and  $N$  are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format



Figure 1: Illustration of the problem 1099

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

Time Complexity: Time spent by the prover is 0.114 seconds.

NDG conditions Points M_b and H_b are not identical

4.1.2 Proving $N = \neg N$

Proving failed

4.1.3 Proving $H_b = \neg H_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $M_b = \neg M_b$

Proving failed

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $H_b = \neg H_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_b = \neg M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.070 seconds. There are no ndg conditions.

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $H_b = \neg H_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_b = \neg M_b$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $H_b=_H H_b$

Proving failed

Problem 1100

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1100: Given a point M_c , a point N and a point H_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point M_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_c and N are not the same;
2. Choose freely a point H_b on the circle $k(N, M_a)$ (rule WOncircle);
3. Using the point H_b and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points H_b and M_c are not the same;
4. Using the circle $k(N, M_a)$, the circle $k(M_c, A)$, the point H_b , the point N and the point M_c , construct a point H_a (rule W08); % NDG: circles $k(N, M_a)$ and $k(M_c, A)$ intersect % DET: circles $k(N, M_a)$ and $k(M_c, A)$ are not the same; points H_b and H_a must be different;
5. Choose freely a point A on the circle $k(M_c, A)$ (rule WOncircle);
6. Using the point A and the point M_c , construct a point B (rule W01); ;
7. Using the point A and the point H_b , construct a line b (rule W02); % DET: points A and H_b are not the same;
8. Using the point H_a and the point B , construct a line a (rule W02); % DET: points H_a and B are not the same;
9. Using the line b and the line a , construct a point C (rule W03); % NDG: lines b and a are not parallel % DET: lines b and a are not the same.

Non-degenerate conditions: lines b and a are not parallel; circles $k(N, M_a)$ and $k(M_c, A)$ intersect; points H_b and M_c are not the same; points M_c and N are not the same.

Determination conditions: lines b and a are not the same; points H_a and B are not the same; points A and H_b are not the same; circles $k(N, M_a)$ and $k(M_c, A)$ are not the same; points H_b and H_a must be different.

Rules used: [W01,W02,W03,W06,W08,WOncircle1]

Lemmas used: [D20,D32,D5,D6,GD01,GD02,GL03,L18,L19,L20,L41,L42]

Solving time: 57.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point N 72.5 61.93
point H_{b} 89.36 77.83

color 220 0 0
fontsize 9

cmark_lt M_{c}
cmark_r N
cmark_l H_{b}
color 0 0 0
fontsize 8

% NDG: points M_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{c}
circle k(N,M_{a}) N M_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point H_{b} on the circle with center N through point M_{c}
oncircle H_{b} N M_{c}
cmark_l H_{b}
color 200 200 200
drawcircle N M_{c}
color 0 0 0

% NDG: points H_{b} and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point H_{b}
circle k(M_{c},A) M_{c} H_{b}

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0
```

```

% NDG: circles  $k(N, M_{\{a\}})$  and  $k(M_{\{c\}}, A)$  intersect% DET: circles  $k(N, M_{\{a\}})$  and  $k(M_{\{c\}}, A)$  are not
the same; points  $H_{\{b\}}$  and  $H_{\{a\}}$  must be different
% Constructing a line  $L_{\{\_G238759\}}$  which passes through point  $N$  and point  $M_{\{c\}}$ 
line  $L_{\{\_G238759\}}$   $N$   $M_{\{c\}}$ 

color 200 200 200
drawline  $L_{\{\_G238759\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $H_{\{b\}}$  in the symmetry to point/line  $L_{\{\_G238759\}}$ 
sim  $H_{\{a\}}$   $L_{\{\_G238759\}}$   $H_{\{b\}}$ 
cmark_r  $H_{\{a\}}$ 

% Choosing randomly a point  $A$  on the circle with center  $M_{\{c\}}$  through point  $H_{\{b\}}$ 
oncircle  $A$   $M_{\{c\}}$   $H_{\{b\}}$ 
cmark_t  $A$ 
color 200 200 200
drawcircle  $M_{\{c\}}$   $H_{\{b\}}$ 
color 0 0 0

% Constructing a point  $B$  such that  $AB/AM_{\{c\}}=2$ 
towards  $B$   $A$   $M_{\{c\}}$  2
cmark_b  $B$ 
color 200 200 200
drawsegment  $A$   $B$ 
color 0 0 0

% DET: points  $A$  and  $H_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $A$  and point  $H_{\{b\}}$ 
line  $b$   $A$   $H_{\{b\}}$ 

color 200 200 200
drawline  $b$ 
color 0 0 0

% DET: points  $H_{\{a\}}$  and  $B$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $B$ 
line  $a$   $H_{\{a\}}$   $B$ 

color 200 200 200
drawline  $a$ 
color 0 0 0

% NDG: lines  $b$  and  $a$  are not parallel% DET: lines  $b$  and  $a$  are not the same
% Constructing a point  $C$  which belongs to line  $b$  and line  $a$ 

```

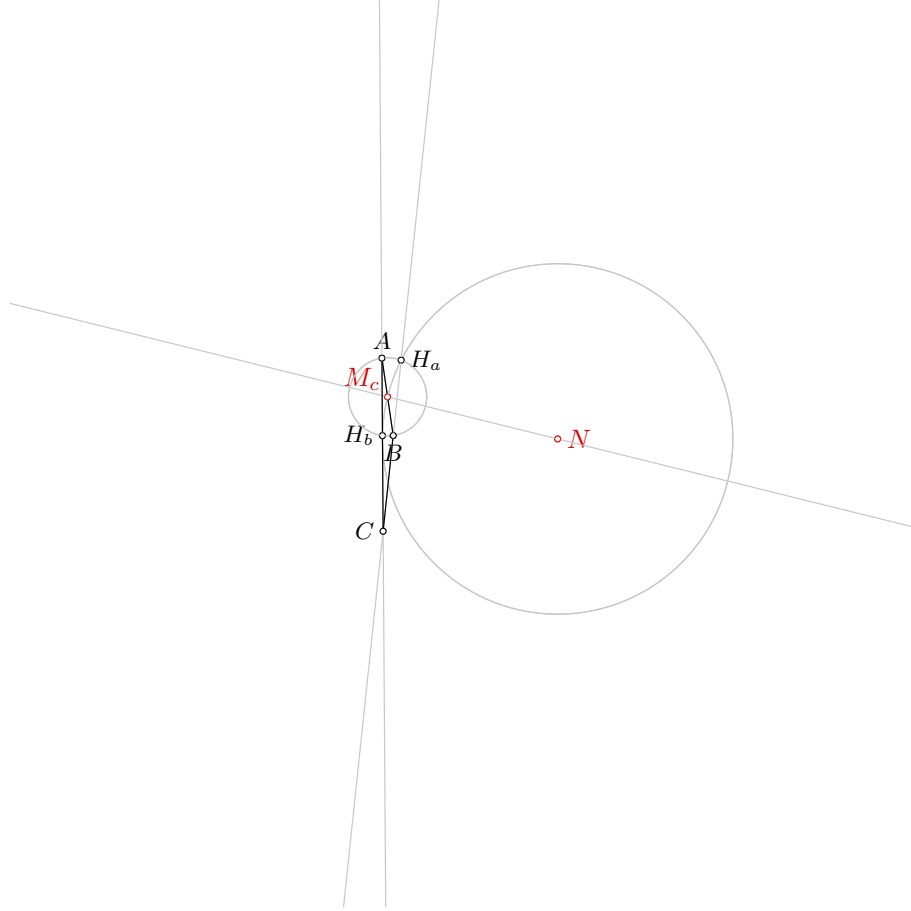


Figure 1: Illustration of the problem 1100

```
intersec C b a
cmark_1 C
```

```
drawsegment A B
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: lines b and a are not parallel; circles  $k(N, M_{\{a\}})$  and  $k(M_{\{c\}}, A)$ 
% intersect; points  $H_{\{b\}}$  and  $M_{\{c\}}$  are not the same; points  $M_{\{c\}}$  and  $N$  are not the same
% Determination conditions: lines b and a are not the same; points  $H_{\{a\}}$  and  $B$  are not the same;
% points  $A$  and  $H_{\{b\}}$  are not the same; circles  $k(N, M_{\{a\}})$  and  $k(M_{\{c\}}, A)$  are not the same; points
%  $H_{\{b\}}$  and  $H_{\{a\}}$  must be different
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_c = \neg M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

Time Complexity: Time spent by the prover is 0.117 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $N = \neg N$

Proving failed

4.1.3 Proving $H_b = \neg H_b$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{AH_aB} \neq S_{H_bH_aB}$ i.e., lines AH_b and H_aB are not parallel (construction based assumption)

$S_{\neg M_aBC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_bAC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2BF_{\neg h_b}^3}$ i.e., lines $AF_{\neg h_a}^2$ and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

$S_{\neg M_a\neg M_bF_{\neg m_b}^1} \neq S_{F_{\neg m_a}^0\neg M_bF_{\neg m_b}^1}$ i.e., lines $\neg M_aF_{\neg m_a}^0$ and $\neg M_bF_{\neg m_b}^1$ are not parallel (construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{CBF_{\neg h_b}^3}$ i.e., lines AC and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $H_b = \neg H_b$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_c = \neg M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.050 seconds. There are no ndg conditions.

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $H_b = H_b$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_c = M_c$

Proving failed

4.4.2 Proving $N = N$

Proving failed

4.4.3 Proving $H_b = H_b$

Proving failed

Problem 1101

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1101: Given a point H_b , a point N and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point N and the point O , construct a point G (rule W01); ;
2. Using the point N and the point O , construct a point H (rule W01); ;
3. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;
4. Using the point H_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_b and N are not the same;
5. Using the circle $k(N, M_a)$, the line h_b , the point N and the point H_b , construct a point E_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points H_b and E_b must be different;
6. Using the point E_b and the point H , construct a point B (rule W01); ;
7. Using the point G and the point B , construct a point M_b (rule W01); ;
8. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
9. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; line h_b and circle $k(N, M_a)$ intersect; points H_b and N are not the same.

Determination conditions: points H_b and M_b are not the same; points H_b and E_b must be different; points H_b and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D31,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L15,L20,L23,L56]

Solving time: 5.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point H_{b} 89.36 77.83
point N 72.5 61.93
point O 65 51.14

color 220 0 0
fontsize 9

cmark_l H_{b}
cmark_r N
cmark_t O
color 0 0 0
fontsize 8

% Constructing a line L_{\_G62350} which passes through point N and point O
line L_{\_G62350} N O

color 200 200 200
drawline L_{\_G62350}
color 0 0 0

% Constructing a point P_{\_G62451} with coordinates (0,0)
point P_{\_G62451} 0 0
cmark_r P_{\_G62451}

% Constructing a point P_{\_G62375} such that NP_{\_G62375}/NP_{\_G62451}=1
towards P_{\_G62375} N P_{\_G62451} 1
cmark_r P_{\_G62375}
color 200 200 200
drawsegment N P_{\_G62375}
color 0 0 0

% Constructing a point P_{\_G62420} such that NP_{\_G62420}/NP_{\_G62451}=3
towards P_{\_G62420} N P_{\_G62451} 3
cmark_r P_{\_G62420}
color 200 200 200
drawsegment N P_{\_G62420}
color 0 0 0

% Constructing a line L_{\_G62381} which passes through point O and point P_{\_G62420}
line L_{\_G62381} O P_{\_G62420}
```

```

color 200 200 200
drawline L_{\_G62381}
color 0 0 0

% Constructing a line L_{\_G62344} which contains the point P_{\_G62375} and is parallel to the
line L_{\_G62381}
parallel L_{\_G62344} P_{\_G62375} L_{\_G62381}

color 200 200 200
drawline L_{\_G62344}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G62344} and line L_{\_G62350}
intersec G L_{\_G62344} L_{\_G62350}
cmark_t G

% Constructing a point H such that NH/NO=-1
towards H N 0 -1
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: points H_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{b}
circle k(N,M_{a}) N H_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{b} and circle k(N,M_{a}) intersect% DET: points H_{b} and E_{b} must be different
% Constructing a point P_{\_G63278} which is a foot of the point N on the line h_{b}
foot P_{\_G63278} N h_{b}
cmark_r P_{\_G63278}
color 200 200 200
drawline N P_{\_G63278}
color 0 0 0

```

```

% Constructing a point  $E_{\{b\}}$  which is an image of the point  $H_{\{b\}}$  in the symmetry to point/line  $P_{\{\backslash\_G63278\}}$ 
sim  $E_{\{b\}}$   $P_{\{\backslash\_G63278\}}$   $H_{\{b\}}$ 
cmark_r  $E_{\{b\}}$ 

% Constructing a point  $B$  such that  $E_{\{b\}}B/E_{\{b\}}H=-1$ 
towards  $B$   $E_{\{b\}}$   $H$  -1
cmark_b  $B$ 
color 200 200 200
drawsegment  $H$   $B$ 
color 0 0 0

% Constructing a point  $M_{\{b\}}$  such that  $GM_{\{b\}}/GB=-0.5$ 
towards  $M_{\{b\}}$   $G$   $B$  -0.5
cmark_lt  $M_{\{b\}}$ 
color 200 200 200
drawsegment  $B$   $M_{\{b\}}$ 
color 0 0 0

% DET: points  $H_{\{b\}}$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $M_{\{b\}}$ 
line  $b$   $H_{\{b\}}$   $M_{\{b\}}$ 

color 200 200 200
drawline  $b$ 
color 0 0 0

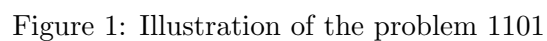
% NDG: points  $B$  and  $O$  are not the same
% Constructing a circle  $k(O,C)$  whose center is at point  $O$  and which passes through point  $B$ 
circle  $k(O,C)$   $O$   $B$ 

color 200 200 200
drawcircle  $k(O,C)$ 
color 0 0 0

% NDG: line  $b$  and circle  $k(O,C)$  intersect
% Constructing points  $C$  and  $A$  which are in intersection of  $k(O,C)$  and  $b$ 
intersec2  $C$   $A$   $k(O,C)$   $b$ 
cmark_l  $C$ 
cmark_t  $A$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

```



```
% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;
    line h_{b} and circle k(N,M_{a}) intersect; points H_{b} and N are not the same
% Determination conditions: points H_{b} and M_{b} are not the same; points H_{b} and E_{b} must be
    different; points H_{b} and H are not the same
```

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_b = \neg H_b$

Proving failed

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $O=_O$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_b=_H H_b$

Proving failed

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $O=_O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_b=_H H_b$

Proving failed

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $O=_O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_b=_H H_b$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $O=_O$

Proving failed

Problem 1102

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1102: Given a point H_b , a point N and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1103

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1103: Given a point H_b , a point N and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1104

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1104: Given a point H_b , a point N and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1105

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1105: Given a point H_c , a point I and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point H_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_c and N are not the same;
2. Using the point I , the circle $k(N, M_a)$, the point N and the point H_c , construct a circle $k(I, P_a)$ (rule W22); % NDG: point I is inside the circle $k(N, M_a)$; points I and N are not the same;
3. Using the circle $k(I, P_a)$, the point H_c and the point I , construct a line y_3 and a line c (rule W12); % NDG: point H_c is outside the circle $k(I, P_a)$;
4. Using the circle $k(N, M_a)$, the line c , the point N and the point H_c , construct a point M_c (rule W05); % NDG: line c and circle $k(N, M_a)$ intersect % DET: points H_c and M_c must be different;
5. Using the point M_c and the point I , construct a line IM_c (rule W02); % DET: points M_c and I are not the same;
6. Using the point N and the point M_c , construct a line $m(H_aH_b)$ (rule W02); % DET: points N and M_c are not the same;
7. Using the circle $k(N, M_a)$, the line $m(H_aH_b)$, the point N and the point M_c , construct a point E_c (rule W05a); % NDG: line $m(H_aH_b)$ and circle $k(N, M_a)$ intersect % DET: points M_c and E_c must be different;
8. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;

9. Using the point I and the point M_c , construct a circle $k_{over}(I, M_c)$ (rule W09); % NDG: points I and M_c are not the same;
10. Using the circle $k_{over}(I, M_c)$ and the circle $k(I, P_a)$, construct a point C_{fi} and a point P_c (rule W07); % NDG: circles $k_{over}(I, M_c)$ and $k(I, P_a)$ intersect % DET: circles $k_{over}(I, M_c)$ and $k(I, P_a)$ are not the same;
11. Using the point P_c and the point M_c , construct a point P'_c (rule W01); ;
12. Using the point M_c and the line c , construct a line m_c (rule W10b); ;
13. Using the point P'_c and the line IM_c , construct a line CP'_c (rule W16); ;
14. Using the line CP'_c and the line h_c , construct a point C (rule W03); % NDG: lines CP'_c and h_c are not parallel % DET: lines CP'_c and h_c are not the same;
15. Using the point I and the point C , construct a line s_c (rule W02); % DET: points I and C are not the same;
16. Using the line m_c and the line s_c , construct a point N_c (rule W03); % NDG: lines m_c and s_c are not parallel % DET: lines m_c and s_c are not the same;
17. Using the point I and the point N_c , construct a circle $k(N_c, B)$ (rule W06); % NDG: points I and N_c are not the same;
18. Using the circle $k(N_c, B)$ and the line c , construct a point B and a point A (rule W04); % NDG: line c and circle $k(N_c, B)$ intersect.

Non-degenerate conditions: line c and circle $k(N_c, B)$ intersect; points I and N_c are not the same; lines m_c and s_c are not parallel; lines CP'_c and h_c are not parallel; circles $k_{over}(I, M_c)$ and $k(I, P_a)$ intersect; points I and M_c are not the same; line $m(H_a H_b)$ and circle $k(N, M_a)$ intersect; line c and circle $k(N, M_a)$ intersect; point H_c is outside the circle $k(I, P_a)$; point I is inside the circle $k(N, M_a)$; points I and N are not the same; points H_c and N are not the same.

Determination conditions: lines m_c and s_c are not the same; points I and C are not the same; lines CP'_c and h_c are not the same; circles $k_{over}(I, M_c)$ and $k(I, P_a)$ are not the same; points E_c and H_c are not the same; points M_c and E_c must be different; points N and M_c are not the same; points M_c and I are not the same; points H_c and M_c must be different.

Rules used: [W01,W02,W03,W04,W05,W05a,W06,W07,W09,W10b,W12,W16,W22]

Lemmas used: [D10,D13,D20,D27,D30,D32,D49,D67,D7,D90,GD01,GD02,GL01,GL03,GL09,L119,L14,L18,L19]

Solving time: 41.6 seconds.

3.2 Construction in GCLC language

dim 120 120

point H_{c} 68.91 84.83

point I 74.37 61.15

point N 72.5 61.93

color 220 0 0

```

fontsize 9

cmark_rt H_{c}
cmark_b I
cmark_r N
color 0 0 0
fontsize 8

% NDG: points H_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{c}
circle k(N,M_{a}) N H_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: point I is inside the circle k(N,M_{a}); points I and N are not the same
% Calculating distance V[_G139297] from point N to point H_{c}
distance V[_G139297] N H_{c}

% Calculating distance V[_G139321] from point N to point I
distance V[_G139321] N I

% Calculating value V[_G139342] using formula V[_G139297]/V[_G139321]
expression V[_G139342] { V[_G139297]/V[_G139321] }

% Constructing a point P_{\_G139373} such that NP_{\_G139373}/NI=V[_G139297]/V[_G139321]
towards P_{\_G139373} N I V[_G139342]
cmark_r P_{\_G139373}

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
139373}
circle k(I,P_{a}) I P_{\_G139373}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: point H_{c} is outside the circle k(I,P_{a})
% Constructing midpoint P_{\_G139747} of the segment H_{c}I
midpoint P_{\_G139747} H_{c} I
cmark_r P_{\_G139747}

% Constructing a circle C_{\_G139750} whose center is at point P_{\_G139747} and which passes
through point H_{c}
circle C_{\_G139750} P_{\_G139747} H_{c}

color 200 200 200
drawcircle C_{\_G139750}

```

```

color 0 0 0

% Constructing points  $P_{\setminus G139753}$  and  $P_{\setminus G139756}$  which are in intersection of  $C_{\setminus G139750}$ 
and  $k(I, P_{\setminus a})$ 
intersec2  $P_{\setminus G139753}$   $P_{\setminus G139756}$   $C_{\setminus G139750}$   $k(I, P_{\setminus a})$ 
cmark_r  $P_{\setminus G139753}$ 
cmark_r  $P_{\setminus G139756}$ 

% Constructing a line  $y3$  which passes through point  $H_{\setminus c}$  and point  $P_{\setminus G139753}$ 
line  $y3$   $H_{\setminus c}$   $P_{\setminus G139753}$ 

color 200 200 200
drawline  $y3$ 
color 0 0 0

% Constructing a line  $c$  which passes through point  $H_{\setminus c}$  and point  $P_{\setminus G139756}$ 
line  $c$   $H_{\setminus c}$   $P_{\setminus G139756}$ 

color 200 200 200
drawline  $c$ 
color 0 0 0

% NDG: line  $c$  and circle  $k(N, M_{\setminus a})$  intersect% DET: points  $H_{\setminus c}$  and  $M_{\setminus c}$  must be different
% Constructing a point  $P_{\setminus G140090}$  which is a foot of the point  $N$  on the line  $c$ 
foot  $P_{\setminus G140090}$   $N$   $c$ 
cmark_r  $P_{\setminus G140090}$ 
color 200 200 200
drawline  $N$   $P_{\setminus G140090}$ 
color 0 0 0

% Constructing a point  $M_{\setminus c}$  which is an image of the point  $H_{\setminus c}$  in the symmetry to point/line  $P_{\setminus G140090}$ 
sim  $M_{\setminus c}$   $P_{\setminus G140090}$   $H_{\setminus c}$ 
cmark_lt  $M_{\setminus c}$ 

% DET: points  $M_{\setminus c}$  and  $I$  are not the same
% Constructing a line  $IM_{\setminus c}$  which passes through point  $M_{\setminus c}$  and point  $I$ 
line  $IM_{\setminus c}$   $M_{\setminus c}$   $I$ 

color 200 200 200
drawline  $IM_{\setminus c}$ 
color 0 0 0

% DET: points  $N$  and  $M_{\setminus c}$  are not the same
% Constructing a line  $m(H_{\setminus a}H_{\setminus b})$  which passes through point  $N$  and point  $M_{\setminus c}$ 
line  $m(H_{\setminus a}H_{\setminus b})$   $N$   $M_{\setminus c}$ 

color 200 200 200
drawline  $m(H_{\setminus a}H_{\setminus b})$ 

```

```
color 0 0 0
```

```
% NDG: line  $m(H_{\{a\}}H_{\{b\}})$  and circle  $k(N,M_{\{a\}})$  intersect% DET: points  $M_{\{c\}}$  and  $E_{\{c\}}$  must be
different
% Constructing a point  $E_{\{c\}}$  which is an image of the point  $M_{\{c\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{c\}}$   $N$   $M_{\{c\}}$ 
cmark_r  $E_{\{c\}}$ 
```

```
% DET: points  $E_{\{c\}}$  and  $H_{\{c\}}$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $E_{\{c\}}$  and point  $H_{\{c\}}$ 
line  $h_{\{c\}}$   $E_{\{c\}}$   $H_{\{c\}}$ 
```

```
color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0
```

```
% NDG: points  $I$  and  $M_{\{c\}}$  are not the same
% Constructing midpoint  $P_{\{\backslash\_G140558\}}$  of the segment  $IM_{\{c\}}$ 
midpoint  $P_{\{\backslash\_G140558\}}$   $I$   $M_{\{c\}}$ 
cmark_r  $P_{\{\backslash\_G140558\}}$ 
```

```
% Constructing a circle  $k_{\text{over}}(I,M_{\{c\}})$  whose center is at point  $P_{\{\backslash\_G140558\}}$  and which passes
through point  $I$ 
circle  $k_{\text{over}}(I,M_{\{c\}})$   $P_{\{\backslash\_G140558\}}$   $I$ 
```

```
color 200 200 200
drawcircle  $k_{\text{over}}(I,M_{\{c\}})$ 
color 0 0 0
```

```
% NDG: circles  $k_{\text{over}}(I,M_{\{c\}})$  and  $k(I,P_{\{a\}})$  intersect% DET: circles  $k_{\text{over}}(I,M_{\{c\}})$  and  $k(I,P_{\{a\}})$ 
are not the same
% Constructing points  $C_{\{fi\}}$  and  $P_{\{c\}}$  which are in intersection of  $k_{\text{over}}(I,M_{\{c\}})$  and  $k(I,P_{\{a\}})$ 
intersec2  $C_{\{fi\}}$   $P_{\{c\}}$   $k_{\text{over}}(I,M_{\{c\}})$   $k(I,P_{\{a\}})$ 
cmark_r  $C_{\{fi\}}$ 
cmark_r  $P_{\{c\}}$ 
```

```
% Constructing a point  $P'_{\{c\}}$  such that  $P_{\{c\}}P'_{\{c\}}/P_{\{c\}}M_{\{c\}}=2$ 
towards  $P'_{\{c\}}$   $P_{\{c\}}$   $M_{\{c\}}$  2
cmark_r  $P'_{\{c\}}$ 
color 200 200 200
drawsegment  $P_{\{c\}}$   $P'_{\{c\}}$ 
color 0 0 0
```

```
% Constructing a line  $m_{\{c\}}$  which is perpendicular to line  $c$  and which passes through point  $M_{\{c\}}$ 
perp  $m_{\{c\}}$   $M_{\{c\}}$   $c$ 
```

```

color 200 200 200
drawline m_{c}
color 0 0 0

% Constructing a line CP'_{c} which contains the point P'_{c} and is parallel to the line IM_{c}
parallel CP'_{c} P'_{c} IM_{c}

color 200 200 200
drawline CP'_{c}
color 0 0 0

% NDG: lines CP'_{c} and h_{c} are not parallel% DET: lines CP'_{c} and h_{c} are not the same
% Constructing a point C which belongs to line CP'_{c} and line h_{c}
intersec C CP'_{c} h_{c}
cmark_l C

% DET: points I and C are not the same
% Constructing a line s_{c} which passes through point I and point C
line s_{c} I C

color 200 200 200
drawline s_{c}
color 0 0 0

% NDG: lines m_{c} and s_{c} are not parallel% DET: lines m_{c} and s_{c} are not the same
% Constructing a point N_{c} which belongs to line m_{c} and line s_{c}
intersec N_{c} m_{c} s_{c}
cmark_b N_{c}

% NDG: points I and N_{c} are not the same
% Constructing a circle k(N_{c},B) whose center is at point N_{c} and which passes through point I
circle k(N_{c},B) N_{c} I

color 200 200 200
drawcircle k(N_{c},B)
color 0 0 0

% NDG: line c and circle k(N_{c},B) intersect
% Constructing points B and A which are in intersection of k(N_{c},B) and c
intersec2 B A k(N_{c},B) c
cmark_b B
cmark_t A

```

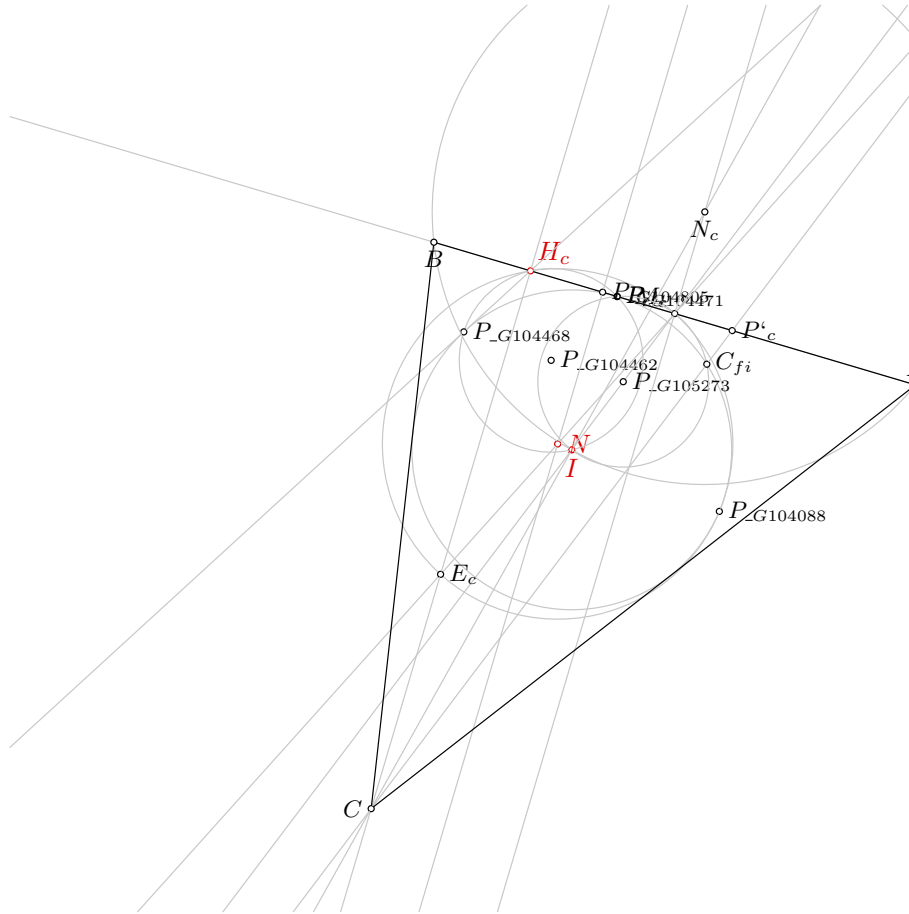



Figure 1: Illustration of the problem 1105

```
drawsegment A B
drawsegment A C
drawsegment B C
```

% Non-degenerate conditions: line c and circle $k(N_{\{c\}}, B)$ intersect; points I and $N_{\{c\}}$ are not the same; lines $m_{\{c\}}$ and $s_{\{c\}}$ are not parallel; lines $CP'_{\{c\}}$ and $h_{\{c\}}$ are not parallel; circles $k_{\text{over}}(I, M_{\{c\}})$ and $k(I, P_{\{a\}})$ intersect; points I and $M_{\{c\}}$ are not the same; line $m(H_{\{a\}}H_{\{b\}})$ and circle $k(N, M_{\{a\}})$ intersect; line c and circle $k(N, M_{\{a\}})$ intersect; point $H_{\{c\}}$ is outside the circle $k(I, P_{\{a\}})$; point I is inside the circle $k(N, M_{\{a\}})$; points I and N are not the same; points $H_{\{c\}}$ and N are not the same

% Determination conditions: lines $m_{\{c\}}$ and $s_{\{c\}}$ are not the same; points I and C are not the same; lines $CP'_{\{c\}}$ and $h_{\{c\}}$ are not the same; circles $k_{\text{over}}(I, M_{\{c\}})$ and $k(I, P_{\{a\}})$ are not the same; points $E_{\{c\}}$ and $H_{\{c\}}$ are not the same; points $M_{\{c\}}$ and $E_{\{c\}}$ must be different; points N and $M_{\{c\}}$ are not the same; points $M_{\{c\}}$ and I are not the same; points $H_{\{c\}}$ and $M_{\{c\}}$ must be different

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_c = \neg H_c$

Proving failed

4.1.2 Proving $I = \neg I$

Proving failed

4.1.3 Proving $N = \neg N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $H_c = \neg H_c$

Proving failed

4.2.2 Proving $I = \neg I$

Proving failed

4.2.3 Proving $N = \neg N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_c = \neg H_c$

Proving failed

4.3.2 Proving $I = \neg I$

Proving failed

4.3.3 Proving $N = \neg N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_c = \neg H_c$

Proving failed

4.4.2 Proving $I = \neg I$

Proving failed

4.4.3 Proving $N = _N$

Proving failed

Problem 1106

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1106: Given a point M_a , a point N and a point H_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point M_a and the point N , construct a line $m(H_bH_c)$ (rule W02); % DET: points M_a and N are not the same;
2. Using the point M_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_a and N are not the same;
3. Choose freely a point H_c on the circle $k(N, M_a)$ (rule WOncircle);
4. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point M_a , construct a point E_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_a and E_a must be different;
5. Using the point H_c and the point M_a , construct a circle $k(M_a, B)$ (rule W06); % NDG: points H_c and M_a are not the same;
6. Using the circle $k(N, M_a)$, the circle $k(M_a, B)$, the point H_c , the point N and the point M_a , construct a point H_b (rule W08); % NDG: circles $k(N, M_a)$ and $k(M_a, B)$ intersect % DET: circles $k(N, M_a)$ and $k(M_a, B)$ are not the same; points H_c and H_b must be different;
7. Choose freely a point A on the circle $k(E_a, A)$ (rule WOncircle);
8. Using the point A and the point E_a , construct a point H (rule W01); ;
9. Using the point A and the point H_c , construct a line c (rule W02); % DET: points A and H_c are not the same;
10. Using the point H_b and the point H , construct a line h_b (rule W02); % DET: points H_b and H are not the same;

11. Using the line c and the line h_b , construct a point B (rule W03); % NDG: lines c and h_b are not parallel % DET: lines c and h_b are not the same;
12. Using the point M_a and the point B , construct a point C (rule W01); .

Non-degenerate conditions: lines c and h_b are not parallel; circles $k(N, M_a)$ and $k(M_a, B)$ intersect; points H_c and M_a are not the same; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; points M_a and N are not the same.

Determination conditions: lines c and h_b are not the same; points H_b and H are not the same; points A and H_c are not the same; circles $k(N, M_a)$ and $k(M_a, B)$ are not the same; points H_c and H_b must be different; points M_a and E_a must be different; points M_a and N are not the same.

Rules used: [W01,W02,W03,W05a,W06,W08,WOncircle1]

Lemmas used: [D21,D28,D3,D32,D6,D7,D9,GD01,GD02,GL01,GL03,GL04,L20,L21,L22,L38,L39,L47,L48]

Solving time: 59.7 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40
point N 72.5 61.93
point H_{c} 68.91 84.83
```

```
color 220 0 0
fontsize 9
```

```
cmark_r M_{a}
cmark_r N
cmark_rt H_{c}
color 0 0 0
fontsize 8
```

```
% DET: points M_{a} and N are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point M_{a} and point N
line m(H_{b}H_{c}) M_{a} N
```

```
color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0
```

```
% NDG: points M_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{a}
circle k(N,M_{a}) N M_{a}
```

```
color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0
```

```

% Choosing randomly a point  $H_{\{c\}}$  on the circle with center  $N$  through point  $M_{\{a\}}$ 
oncircle  $H_{\{c\}}$   $N$   $M_{\{a\}}$ 
cmark_rt  $H_{\{c\}}$ 
color 200 200 200
drawcircle  $N$   $M_{\{a\}}$ 
color 0 0 0

% NDG: line  $m(H_{\{b\}}H_{\{c\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{a\}}$  and  $E_{\{a\}}$  must be
different
% Constructing a point  $E_{\{a\}}$  which is an image of the point  $M_{\{a\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{a\}}$   $N$   $M_{\{a\}}$ 
cmark_r  $E_{\{a\}}$ 

% NDG: points  $H_{\{c\}}$  and  $M_{\{a\}}$  are not the same
% Constructing a circle  $k(M_{\{a\}}, B)$  whose center is at point  $M_{\{a\}}$  and which passes through point  $H_{\{c\}}$ 
circle  $k(M_{\{a\}}, B)$   $M_{\{a\}}$   $H_{\{c\}}$ 

color 200 200 200
drawcircle  $k(M_{\{a\}}, B)$ 
color 0 0 0

% NDG: circles  $k(N, M_{\{a\}})$  and  $k(M_{\{a\}}, B)$  intersect% DET: circles  $k(N, M_{\{a\}})$  and  $k(M_{\{a\}}, B)$  are not
the same; points  $H_{\{c\}}$  and  $H_{\{b\}}$  must be different
% Constructing a line  $L_{\{\backslash\_G180912\}}$  which passes through point  $N$  and point  $M_{\{a\}}$ 
line  $L_{\{\backslash\_G180912\}}$   $N$   $M_{\{a\}}$ 

color 200 200 200
drawline  $L_{\{\backslash\_G180912\}}$ 
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $H_{\{c\}}$  in the symmetry to point/line  $L_{\{\backslash\_G180912\}}$ 
sim  $H_{\{b\}}$   $L_{\{\backslash\_G180912\}}$   $H_{\{c\}}$ 
cmark_l  $H_{\{b\}}$ 

% Choosing randomly a point  $A$  on the circle with center  $E_{\{a\}}$  through point  $H_{\{c\}}$ 
oncircle  $A$   $E_{\{a\}}$   $H_{\{c\}}$ 
cmark_t  $A$ 
color 200 200 200
drawcircle  $E_{\{a\}}$   $H_{\{c\}}$ 
color 0 0 0

% Constructing a point  $H$  such that  $AH/AE_{\{a\}}=2$ 
towards  $H$   $A$   $E_{\{a\}}$  2
cmark_rt  $H$ 
color 200 200 200

```

```

drawsegment A H
color 0 0 0

% DET: points A and H_{c} are not the same
% Constructing a line c which passes through point A and point H_{c}
line c A H_{c}

color 200 200 200
drawline c
color 0 0 0

% DET: points H_{b} and H are not the same
% Constructing a line h_{b} which passes through point H_{b} and point H
line h_{b} H_{b} H

color 200 200 200
drawline h_{b}
color 0 0 0

% NDG: lines c and h_{b} are not parallel% DET: lines c and h_{b} are not the same
% Constructing a point B which belongs to line c and line h_{b}
intersec B c h_{b}
cmark_b B

% Constructing a point C such that M_{a}C/M_{a}B=-1
towards C M_{a} B -1
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines c and h_{b} are not parallel; circles k(N,M_{a}) and k(M_{a},B)
% intersect; points H_{c} and M_{a} are not the same; line m(H_{b}H_{c}) and circle k(N,M_{a})
% intersect; points M_{a} and N are not the same
% Determination conditions: lines c and h_{b} are not the same; points H_{b} and H are not the same
% ; points A and H_{c} are not the same; circles k(N,M_{a}) and k(M_{a},B) are not the same;
% points H_{c} and H_{b} must be different; points M_{a} and E_{a} must be different; points M_{a}
% and N are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

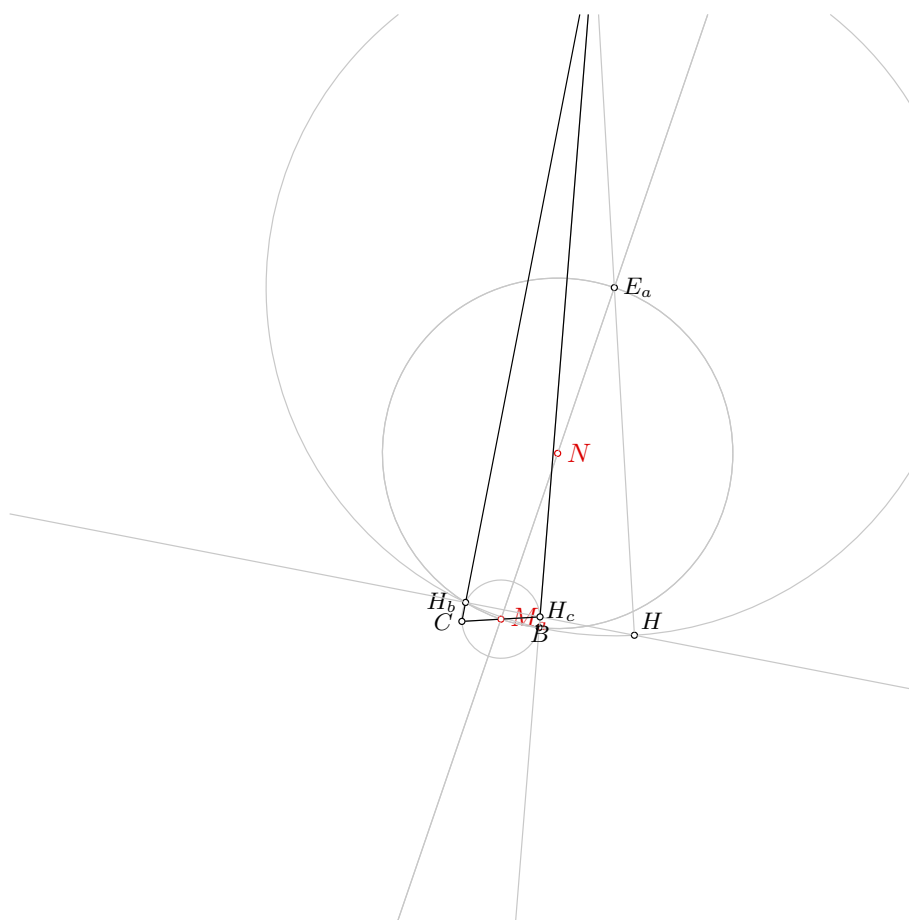


Figure 1: Illustration of the problem 1106

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_a = _M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 11 terms.

Time Complexity: Time spent by the prover is 0.196 seconds.

NDG conditions Line through points H_b and H is not parallel with line through points A and H_c

Points A , H_c and $P_{G171685}$ are not collinear

4.1.2 Proving $N = _N$

Proving failed

4.1.3 Proving $H_c = _H_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $M_a = _M_a$

NDG conditions are:

$S_{AH_bH} \neq S_{H_cH_bH}$ i.e., lines AH_c and H_bH are not parallel (construction based assumption)

$S_{M_aBC} \neq 0$ i.e., points $_M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{M_bAC} \neq 0$ i.e., points $_M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{_h_b}^3} \neq S_{F_{_h_a}^2BF_{_h_b}^3}$ i.e., lines $AF_{_h_a}^2$ and $BF_{_h_b}^3$ are not parallel (construction based assumption)

$S_{_M_a_-M_bF_{_m_b}^1} \neq S_{F_{_m_a}^0_-M_bF_{_m_b}^1}$ i.e., lines $_M_aF_{_m_a}^0$ and $_M_bF_{_m_b}^1$ are not parallel (construction based assumption)

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{_h_c}^4} \neq S_{BCF_{_h_c}^4}$ i.e., lines AB and $CF_{_h_c}^4$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.2.2 Proving $N = _N$

Proving failed

4.2.3 Proving $H_c = _H_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_a = _M_a$

Proving failed

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $H_c=_H H_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_a=_M M_a$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $H_c=_H H_c$

Proving failed

Problem 1107

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1107: Given a point M_b , a point N and a point H_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point M_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_b and N are not the same;
2. Choose freely a point H_c on the circle $k(N, M_a)$ (rule WOncircle);
3. Using the point H_c and the point M_b , construct a circle $k(M_b, C)$ (rule W06); % NDG: points H_c and M_b are not the same;
4. Using the circle $k(N, M_a)$, the circle $k(M_b, C)$, the point H_c , the point N and the point M_b , construct a point H_a (rule W08); % NDG: circles $k(N, M_a)$ and $k(M_b, C)$ intersect % DET: circles $k(N, M_a)$ and $k(M_b, C)$ are not the same; points H_c and H_a must be different;
5. Choose freely a point A on the circle $k(M_b, C)$ (rule WOncircle);
6. Using the point A and the point M_b , construct a point C (rule W01); ;
7. Using the point A and the point H_c , construct a line c (rule W02); % DET: points A and H_c are not the same;
8. Using the point H_a and the point C , construct a line a (rule W02); % DET: points H_a and C are not the same;
9. Using the line c and the line a , construct a point B (rule W03); % NDG: lines c and a are not parallel % DET: lines c and a are not the same.

Non-degenerate conditions: lines c and a are not parallel; circles $k(N, M_a)$ and $k(M_b, C)$ intersect; points H_c and M_b are not the same; points M_b and N are not the same.

Determination conditions: lines c and a are not the same; points H_a and C are not the same; points A and H_c are not the same; circles $k(N, M_a)$ and $k(M_b, C)$ are not the same; points H_c and H_a must be different.

Rules used: [W01,W02,W03,W06,W08,WOncircle1]

Lemmas used: [D22,D32,D5,D7,GD01,GD02,GL03,GL04,L17,L19,L21,L44,L45]

Solving time: 57.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point N 72.5 61.93
point H_{c} 68.91 84.83

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_r N
cmark_rt H_{c}
color 0 0 0
fontsize 8

% NDG: points M_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{b}
circle k(N,M_{a}) N M_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point H_{c} on the circle with center N through point M_{b}
oncircle H_{c} N M_{b}
cmark_rt H_{c}
color 200 200 200
drawcircle N M_{b}
color 0 0 0

% NDG: points H_{c} and M_{b} are not the same
% Constructing a circle k(M_{b},C) whose center is at point M_{b} and which passes through point H_{c}
circle k(M_{b},C) M_{b} H_{c}

color 200 200 200
drawcircle k(M_{b},C)
color 0 0 0
```

```

% NDG: circles  $k(N, M_{\{a\}})$  and  $k(M_{\{b\}}, C)$  intersect% DET: circles  $k(N, M_{\{a\}})$  and  $k(M_{\{b\}}, C)$  are not
the same; points  $H_{\{c\}}$  and  $H_{\{a\}}$  must be different
% Constructing a line  $L_{\{\backslash\_G216066\}}$  which passes through point  $N$  and point  $M_{\{b\}}$ 
line  $L_{\{\backslash\_G216066\}}$   $N$   $M_{\{b\}}$ 

color 200 200 200
drawline  $L_{\{\backslash\_G216066\}}$ 
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $H_{\{c\}}$  in the symmetry to point/line  $L_{\{\backslash\_G216066\}}$ 
sim  $H_{\{a\}}$   $L_{\{\backslash\_G216066\}}$   $H_{\{c\}}$ 
cmark_r  $H_{\{a\}}$ 

% Choosing randomly a point  $A$  on the circle with center  $M_{\{b\}}$  through point  $H_{\{c\}}$ 
oncircle  $A$   $M_{\{b\}}$   $H_{\{c\}}$ 
cmark_t  $A$ 
color 200 200 200
drawcircle  $M_{\{b\}}$   $H_{\{c\}}$ 
color 0 0 0

% Constructing a point  $C$  such that  $AC/AM_{\{b\}}=2$ 
towards  $C$   $A$   $M_{\{b\}}$  2
cmark_l  $C$ 
color 200 200 200
drawsegment  $A$   $C$ 
color 0 0 0

% DET: points  $A$  and  $H_{\{c\}}$  are not the same
% Constructing a line  $c$  which passes through point  $A$  and point  $H_{\{c\}}$ 
line  $c$   $A$   $H_{\{c\}}$ 

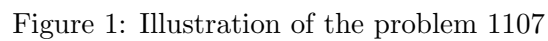
color 200 200 200
drawline  $c$ 
color 0 0 0

% DET: points  $H_{\{a\}}$  and  $C$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $C$ 
line  $a$   $H_{\{a\}}$   $C$ 

color 200 200 200
drawline  $a$ 
color 0 0 0

% NDG: lines  $c$  and  $a$  are not parallel% DET: lines  $c$  and  $a$  are not the same
% Constructing a point  $B$  which belongs to line  $c$  and line  $a$ 

```



```
% Non-degenerate conditions: lines c and a are not parallel; circles k(N,M_{a}) and k(M_{b},C)
% intersect; points H_{c} and M_{b} are not the same; points M_{b} and N are not the same
% Determination conditions: lines c and a are not the same; points H_{a} and C are not the same;
% points A and H_{c} are not the same; circles k(N,M_{a}) and k(M_{b},C) are not the same; points
% H_{c} and H_{a} must be different
```

Illustration of the constructed figure is given in Figure 1

2262

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

Time Complexity: Time spent by the prover is 0.136 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $N = \neg N$

Proving failed

4.1.3 Proving $H_c = \neg H_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{AH_aC} \neq S_{H_cH_aC}$ i.e., lines AH_c and H_aC are not parallel (construction based assumption)

$S_{\neg M_aBC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_bAC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2BF_{\neg h_b}^3}$ i.e., lines $AF_{\neg h_a}^2$ and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

$S_{\neg M_a\neg M_bF_{\neg m_b}^1} \neq S_{F_{\neg m_a}^0\neg M_bF_{\neg m_b}^1}$ i.e., lines $\neg M_aF_{\neg m_a}^0$ and $\neg M_bF_{\neg m_b}^1$ are not parallel (construction based assumption)

$S_{CAB} \neq 0$ i.e., points C , A and B are not collinear (foot is not the point itself; construction based assumption)

$S_{ACF_{\neg h_c}^4} \neq S_{BCF_{\neg h_c}^4}$ i.e., lines AB and $CF_{\neg h_c}^4$ are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $H_c = \neg H_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_b = \neg M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $H_c = \neg H_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 868 terms.

Time Complexity: Time spent by the prover is 1.020 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_b = \neg M_b$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $H_c = \neg H_c$

Proving failed

Problem 1108

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1108: Given a point M_c , a point N and a point H_c , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point M_c and the point N , construct a line $m(H_bH_a)$ (rule W02); % DET: points M_c and N are not the same;
2. Using the point M_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_c and N are not the same;
3. Choose freely a point H_c on the circle $k(N, M_a)$ (rule WOncircle);
4. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
5. Using the circle $k(N, M_a)$, the line $m(H_bH_a)$, the point N and the point M_c , construct a point E_c (rule W05a); % NDG: line $m(H_bH_a)$ and circle $k(N, M_a)$ intersect % DET: points M_c and E_c must be different;
6. Using the point E_c and the point H_c , construct a line h_c (rule W02); % DET: points E_c and H_c are not the same;
7. Choose freely a point A on the line c (rule WOnline1) ;
8. Using the point A and the point M_c , construct a point B (rule W01); ;
9. Using the point A and the point M_c , construct a circle $k(M_c, A)$ (rule W06); % NDG: points A and M_c are not the same;
10. Using the circle $k(N, M_a)$ and the circle $k(M_c, A)$, construct a point H_a and a point H_b (rule W07); % NDG: circles $k(N, M_a)$ and $k(M_c, A)$ intersect % DET: circles $k(N, M_a)$ and $k(M_c, A)$ are not the same;

11. Using the point A and the point H_a , construct a line h_a (rule W02); % DET: points A and H_a are not the same;
12. Using the line h_c and the line h_a , construct a point H (rule W03); % NDG: lines h_c and h_a are not parallel % DET: lines h_c and h_a are not the same;
13. Using the point N and the point H , construct a point G (rule W01); ;
14. Using the point M_c and the point G , construct a point C (rule W01); .

Non-degenerate conditions: lines h_c and h_a are not parallel; circles $k(N, M_a)$ and $k(M_c, A)$ intersect; points A and M_c are not the same; line $m(H_b H_a)$ and circle $k(N, M_a)$ intersect; points M_c and N are not the same.

Determination conditions: lines h_c and h_a are not the same; points A and H_a are not the same; circles $k(N, M_a)$ and $k(M_c, A)$ are not the same; points E_c and H_c are not the same; points M_c and E_c must be different; points H_c and M_c are not the same; points M_c and N are not the same.

Rules used: [W01,W02,W03,W05a,W06,W07,WOncircle1,WOnline1]

Lemmas used: [D10,D20,D3,D30,D32,D5,D7,D8,GD01,GD02,GL01,GL03,GL04,GL09,L16,L18,L19,L20,L24,L3]

Solving time: 568.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{c} 50 67.5
point N 72.5 61.93
point H_{c} 68.91 84.83
```

```
color 220 0 0
fontsize 9
```

```
cmark_lt M_{c}
cmark_r N
cmark_rt H_{c}
color 0 0 0
fontsize 8
```

```
% DET: points M_{c} and N are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point M_{c} and point N
line m(H_{b}H_{a}) M_{c} N
```

```
color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0
```

```
% NDG: points M_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{c}
circle k(N,M_{a}) N M_{c}
```

```
color 200 200 200
drawcircle k(N,M_{a})
```

```
color 0 0 0
```

```
% Choosing randomly a point  $H_{\{c\}}$  on the circle with center  $N$  through point  $M_{\{c\}}$ 
oncircle  $H_{\{c\}}$   $N$   $M_{\{c\}}$ 
cmark_rt  $H_{\{c\}}$ 
color 200 200 200
drawcircle  $N$   $M_{\{c\}}$ 
color 0 0 0
```

```
% DET: points  $H_{\{c\}}$  and  $M_{\{c\}}$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $M_{\{c\}}$ 
line  $c$   $H_{\{c\}}$   $M_{\{c\}}$ 
```

```
color 200 200 200
drawline  $c$ 
color 0 0 0
```

```
% NDG: line  $m(H_{\{b\}}H_{\{a\}})$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{c\}}$  and  $E_{\{c\}}$  must be
different
% Constructing a point  $E_{\{c\}}$  which is an image of the point  $M_{\{c\}}$  in the symmetry to point/line  $N$ 
sim  $E_{\{c\}}$   $N$   $M_{\{c\}}$ 
cmark_r  $E_{\{c\}}$ 
```

```
% DET: points  $E_{\{c\}}$  and  $H_{\{c\}}$  are not the same
% Constructing a line  $h_{\{c\}}$  which passes through point  $E_{\{c\}}$  and point  $H_{\{c\}}$ 
line  $h_{\{c\}}$   $E_{\{c\}}$   $H_{\{c\}}$ 
```

```
color 200 200 200
drawline  $h_{\{c\}}$ 
color 0 0 0
```

```
% Choosing randomly a point  $A$  on the line  $H_{\{c\}}M_{\{c\}}$ 
online  $A$   $H_{\{c\}}$   $M_{\{c\}}$ 
cmark_t  $A$ 
color 200 200 200
drawline  $H_{\{c\}}$   $M_{\{c\}}$ 
color 0 0 0
```

```
% Constructing a point  $B$  such that  $AB/AM_{\{c\}}=2$ 
towards  $B$   $A$   $M_{\{c\}}$  2
cmark_b  $B$ 
color 200 200 200
drawsegment  $A$   $B$ 
color 0 0 0
```

```

% NDG: points A and M_{c} are not the same
% Constructing a circle k(M_{c},A) whose center is at point M_{c} and which passes through point A
circle k(M_{c},A) M_{c} A

color 200 200 200
drawcircle k(M_{c},A)
color 0 0 0

% NDG: circles k(N,M_{a}) and k(M_{c},A) intersect% DET: circles k(N,M_{a}) and k(M_{c},A) are not
the same
% Constructing points H_{a} and H_{b} which are in intersection of k(N,M_{a}) and k(M_{c},A)
intersec2 H_{a} H_{b} k(N,M_{a}) k(M_{c},A)
cmark_r H_{a}
cmark_l H_{b}

% DET: points A and H_{a} are not the same
% Constructing a line h_{a} which passes through point A and point H_{a}
line h_{a} A H_{a}

color 200 200 200
drawline h_{a}
color 0 0 0

% NDG: lines h_{c} and h_{a} are not parallel% DET: lines h_{c} and h_{a} are not the same
% Constructing a point H which belongs to line h_{c} and line h_{a}
intersec H h_{c} h_{a}
cmark_rt H

% Constructing a line L_{\_G46118} which passes through point N and point H
line L_{\_G46118} N H

color 200 200 200
drawline L_{\_G46118}
color 0 0 0

% Constructing a point P_{\_G46219} with coordinates (0,0)
point P_{\_G46219} 0 0
cmark_r P_{\_G46219}

% Constructing a point P_{\_G46143} such that NP_{\_G46143}/NP_{\_G46219}=-1
towards P_{\_G46143} N P_{\_G46219} -1
cmark_r P_{\_G46143}
color 200 200 200
drawsegment P_{\_G46219} P_{\_G46143}
color 0 0 0

```

```

% Constructing a point  $P_{\{G46188\}}$  such that  $NP_{\{G46188\}}/NP_{\{G46219\}}=3$ 
towards  $P_{\{G46188\}}$  N  $P_{\{G46219\}}$  3
cmark_r  $P_{\{G46188\}}$ 
color 200 200 200
drawsegment N  $P_{\{G46188\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G46149\}}$  which passes through point H and point  $P_{\{G46188\}}$ 
line  $L_{\{G46149\}}$  H  $P_{\{G46188\}}$ 

color 200 200 200
drawline  $L_{\{G46149\}}$ 
color 0 0 0

% Constructing a line  $L_{\{G46112\}}$  which contains the point  $P_{\{G46143\}}$  and is parallel to the
line  $L_{\{G46149\}}$ 
parallel  $L_{\{G46112\}}$   $P_{\{G46143\}}$   $L_{\{G46149\}}$ 

color 200 200 200
drawline  $L_{\{G46112\}}$ 
color 0 0 0

% Constructing a point G which belongs to line  $L_{\{G46112\}}$  and line  $L_{\{G46118\}}$ 
intersec G  $L_{\{G46112\}}$   $L_{\{G46118\}}$ 
cmark_t G

% Constructing a point C such that  $M_{\{c\}}C/M_{\{c\}}G=3$ 
towards C  $M_{\{c\}}$  G 3
cmark_l C
color 200 200 200
drawsegment  $M_{\{c\}}$  C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: lines  $h_{\{c\}}$  and  $h_{\{a\}}$  are not parallel; circles  $k(N, M_{\{a\}})$  and  $k(M_{\{c\}}, A)$  intersect; points A and  $M_{\{c\}}$  are not the same; line  $m(H_{\{b\}}H_{\{a\}})$  and circle  $k(N, M_{\{a\}})$  intersect; points  $M_{\{c\}}$  and N are not the same
% Determination conditions: lines  $h_{\{c\}}$  and  $h_{\{a\}}$  are not the same; points A and  $H_{\{a\}}$  are not the same; circles  $k(N, M_{\{a\}})$  and  $k(M_{\{c\}}, A)$  are not the same; points  $E_{\{c\}}$  and  $H_{\{c\}}$  are not the same; points  $M_{\{c\}}$  and  $E_{\{c\}}$  must be different; points  $H_{\{c\}}$  and  $M_{\{c\}}$  are not the same; points  $M_{\{c\}}$  and N are not the same

```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

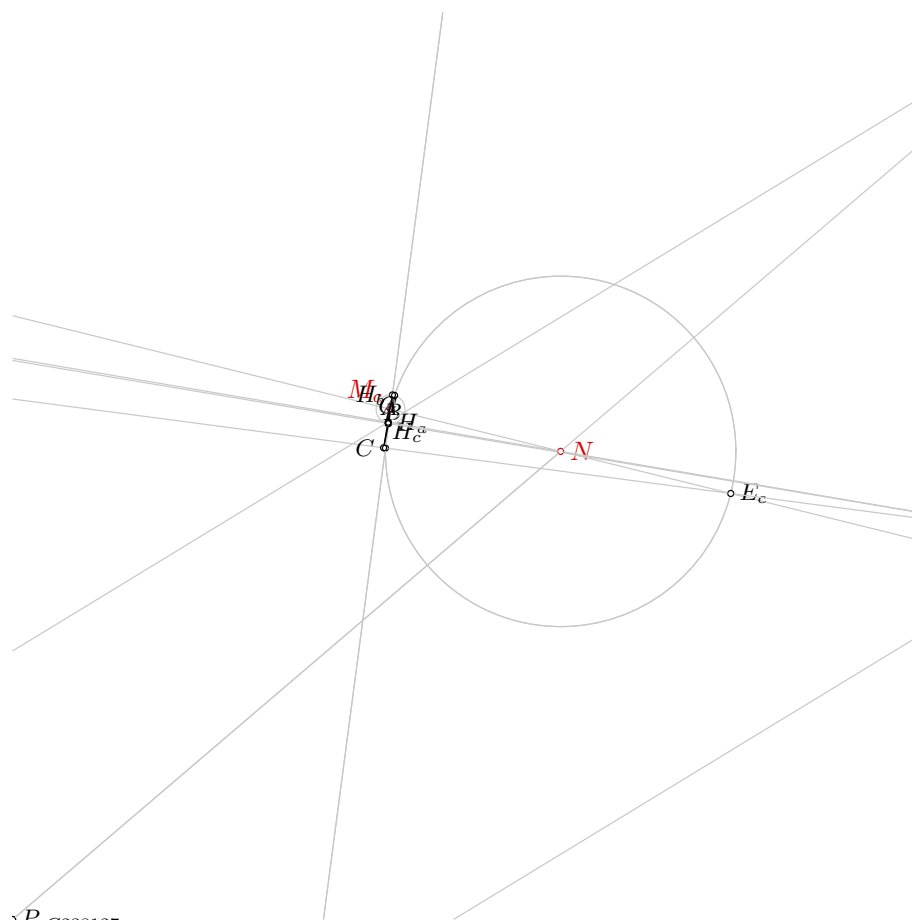


Figure 1: Illustration of the problem 1108

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_c = \neg M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 6 terms.

Time Complexity: Time spent by the prover is 0.112 seconds.

NDG conditions Points H_c and M_c are not identical

4.1.2 Proving $N = \neg N$

Proving failed

4.1.3 Proving $H_c = \neg H_c$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $M_c = \neg M_c$

Proving failed

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $H_c = \neg H_c$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_c = \neg M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.070 seconds. There are no ndg conditions.

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $H_c = \neg H_c$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_c = \neg M_c$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $H_c=_H H_c$

Proving failed

Problem 1109

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1109: Given a point H_c , a point N and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point N and the point O , construct a point G (rule W01); ;
2. Using the point N and the point O , construct a point H (rule W01); ;
3. Using the point H_c and the point H , construct a line h_c (rule W02); % DET: points H_c and H are not the same;
4. Using the point H_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points H_c and N are not the same;
5. Using the circle $k(N, M_a)$, the line h_c , the point N and the point H_c , construct a point E_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points H_c and E_c must be different;
6. Using the point E_c and the point H , construct a point C (rule W01); ;
7. Using the point G and the point C , construct a point M_c (rule W01); ;
8. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
9. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; line h_c and circle $k(N, M_a)$ intersect; points H_c and N are not the same.

Determination conditions: points H_c and M_c are not the same; points H_c and E_c must be different; points H_c and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D31,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L15,L21,L24,L3,L57]

Solving time: 5.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point H_{c} 68.91 84.83
point N 72.5 61.93
point O 65 51.14

color 220 0 0
fontsize 9

cmark_rt H_{c}
cmark_r N
cmark_t O
color 0 0 0
fontsize 8

% Constructing a line L_{\_G88896} which passes through point N and point O
line L_{\_G88896} N O

color 200 200 200
drawline L_{\_G88896}
color 0 0 0

% Constructing a point P_{\_G88997} with coordinates (0,0)
point P_{\_G88997} 0 0
cmark_r P_{\_G88997}

% Constructing a point P_{\_G88921} such that NP_{\_G88921}/NP_{\_G88997}=1
towards P_{\_G88921} N P_{\_G88997} 1
cmark_r P_{\_G88921}
color 200 200 200
drawsegment N P_{\_G88921}
color 0 0 0

% Constructing a point P_{\_G88966} such that NP_{\_G88966}/NP_{\_G88997}=3
towards P_{\_G88966} N P_{\_G88997} 3
cmark_r P_{\_G88966}
color 200 200 200
drawsegment N P_{\_G88966}
color 0 0 0

% Constructing a line L_{\_G88927} which passes through point O and point P_{\_G88966}
line L_{\_G88927} O P_{\_G88966}
```

```

color 200 200 200
drawline L_{\_G88927}
color 0 0 0

% Constructing a line L_{\_G88890} which contains the point P_{\_G88921} and is parallel to the
line L_{\_G88927}
parallel L_{\_G88890} P_{\_G88921} L_{\_G88927}

color 200 200 200
drawline L_{\_G88890}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G88890} and line L_{\_G88896}
intersec G L_{\_G88890} L_{\_G88896}
cmark_t G

% Constructing a point H such that NH/NO=-1
towards H N 0 -1
cmark_rt H
color 200 200 200
drawsegment 0 H
color 0 0 0

% DET: points H_{c} and H are not the same
% Constructing a line h_{c} which passes through point H_{c} and point H
line h_{c} H_{c} H

color 200 200 200
drawline h_{c}
color 0 0 0

% NDG: points H_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point H_{c}
circle k(N,M_{a}) N H_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line h_{c} and circle k(N,M_{a}) intersect% DET: points H_{c} and E_{c} must be different
% Constructing a point P_{\_G89824} which is a foot of the point N on the line h_{c}
foot P_{\_G89824} N h_{c}
cmark_r P_{\_G89824}
color 200 200 200
drawline N P_{\_G89824}
color 0 0 0

```

```

% Constructing a point  $E_{\{c\}}$  which is an image of the point  $H_{\{c\}}$  in the symmetry to point/line  $P_{\{\backslash\_G89824\}}$ 
sim  $E_{\{c\}}$   $P_{\{\backslash\_G89824\}}$   $H_{\{c\}}$ 
cmark_r  $E_{\{c\}}$ 

% Constructing a point  $C$  such that  $E_{\{c\}}C/E_{\{c\}}H=-1$ 
towards  $C$   $E_{\{c\}}$   $H$  -1
cmark_l  $C$ 
color 200 200 200
drawsegment  $H$   $C$ 
color 0 0 0

% Constructing a point  $M_{\{c\}}$  such that  $GM_{\{c\}}/GC=-0.5$ 
towards  $M_{\{c\}}$   $G$   $C$  -0.5
cmark_lt  $M_{\{c\}}$ 
color 200 200 200
drawsegment  $C$   $M_{\{c\}}$ 
color 0 0 0

% DET: points  $H_{\{c\}}$  and  $M_{\{c\}}$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $M_{\{c\}}$ 
line  $c$   $H_{\{c\}}$   $M_{\{c\}}$ 

color 200 200 200
drawline  $c$ 
color 0 0 0

% NDG: points  $C$  and  $D$  are not the same
% Constructing a circle  $k(D,C)$  whose center is at point  $D$  and which passes through point  $C$ 
circle  $k(D,C)$   $D$   $C$ 

color 200 200 200
drawcircle  $k(D,C)$ 
color 0 0 0

% NDG: line  $c$  and circle  $k(D,C)$  intersect
% Constructing points  $A$  and  $B$  which are in intersection of  $k(D,C)$  and  $c$ 
intersec2  $A$   $B$   $k(D,C)$   $c$ 
cmark_t  $A$ 
cmark_b  $B$ 

drawsegment  $A$   $B$ 
drawsegment  $A$   $C$ 
drawsegment  $B$   $C$ 

```

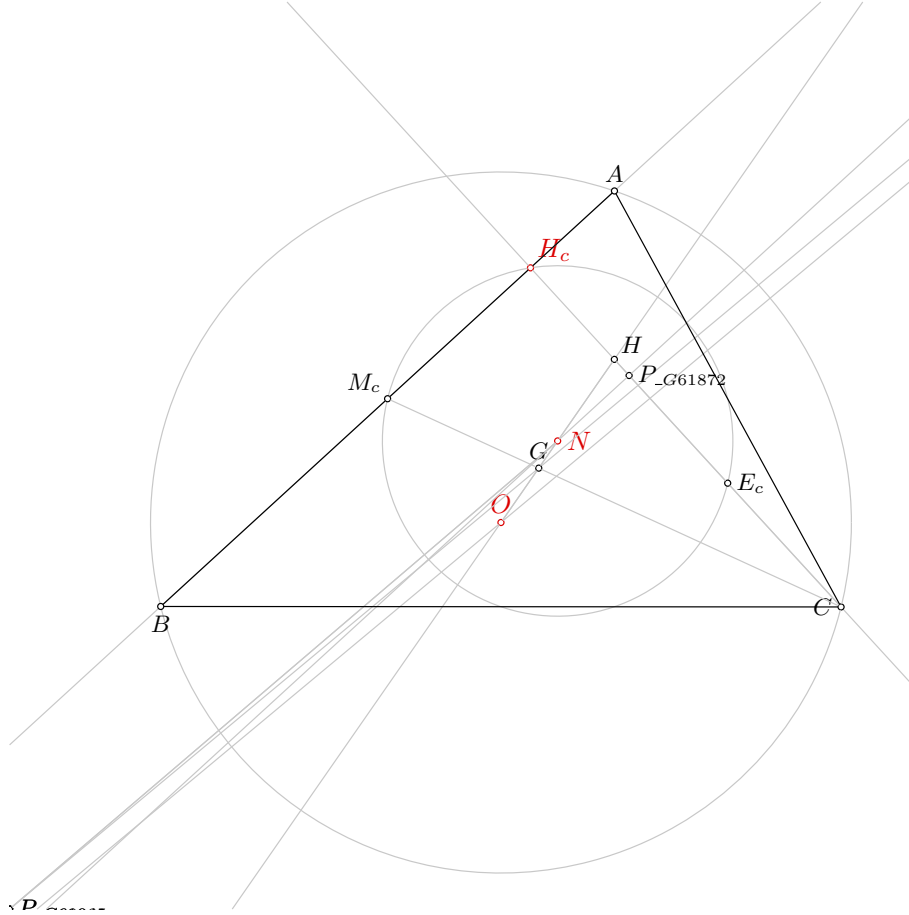


Figure 1: Illustration of the problem 1109

*% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
 line h_{c} and circle k(N,M_{a}) intersect; points H_{c} and N are not the same
 % Determination conditions: points H_{c} and M_{c} are not the same; points H_{c} and E_{c} must be
 different; points H_{c} and H are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $H_c = H_c$

Proving failed

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $O=_O$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3990 terms.

Time Complexity: Time spent by the prover is 9.077 seconds.

NDG conditions Points P_{G84194} , N and O are not collinear

Line through points H_c and N is not parallel with line through points P_{G84194} and O

Points H_c and H are not identical

Points H_c , N and H are not collinear

Points P_{G84695} , C and G are not collinear

Line through points H_c and N is not parallel with line through points C and P_{G84695}

Points H_c and M_c are not identical

Points H_c and M_c are not identical

Points A , B and C are not collinear

Line through points H_c and B is not perpendicular to line through points B and N

4.2 GCLC - Area method

4.2.1 Proving $H_c=_H H_c$

Proving failed

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $O=_O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $H_c=_H H_c$

Proving failed

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $O=_O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $H_c=_H H_c$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $O=_O$

Proving failed

Problem 1110

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1110: Given a point H_c , a point N and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1111

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1111: Given a point H_c , a point N and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1112

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1112: Given a point H_c , a point N and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1113

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1113: Given a point I , a point M_a and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point I and the point M_a , construct a line IM_a (rule W02); % DET: points I and M_a are not the same;
2. Using the point M_a and the point N , construct a line $m(H_bH_c)$ (rule W02); % DET: points M_a and N are not the same;
3. Using the point M_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_a and N are not the same;
4. Using the circle $k(N, M_a)$, the line $m(H_bH_c)$, the point N and the point M_a , construct a point E_a (rule W05a); % NDG: line $m(H_bH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_a and E_a must be different;
5. Using the point I and the point M_a , construct a circle $k_{over}(I, M_a)$ (rule W09); % NDG: points I and M_a are not the same;
6. Using the point I , the circle $k(N, M_a)$, the point N and the point M_a , construct a circle $k(I, P_a)$ (rule W22); % NDG: point I is inside the circle $k(N, M_a)$; points I and N are not the same;
7. Using the circle $k(I, P_a)$ and the circle $k_{over}(I, M_a)$, construct a point P_a and a point A_{fi} (rule W07); % NDG: circles $k(I, P_a)$ and $k_{over}(I, M_a)$ intersect % DET: circles $k(I, P_a)$ and $k_{over}(I, M_a)$ are not the same;
8. Using the point P_a and the point M_a , construct a point P'_a (rule W01); ;
9. Using the circle $k(I, P_a)$, the point M_a and the point I , construct a line $x1$ and a line a (rule W12); % NDG: point M_a is outside the circle $k(I, P_a)$;

10. Using the circle $k(N, M_a)$, the line a , the point N and the point M_a , construct a point H_a (rule W05); % NDG: line a and circle $k(N, M_a)$ intersect % DET: points M_a and H_a must be different;
11. Using the point H_a and the point E_a , construct a line h_a (rule W02); % DET: points H_a and E_a are not the same;
12. Using the point M_a and the line a , construct a line m_a (rule W10b); ;
13. Using the point P'_a and the line IM_a , construct a line AP'_a (rule W16); ;
14. Using the line AP'_a and the line h_a , construct a point A (rule W03); % NDG: lines AP'_a and h_a are not parallel % DET: lines AP'_a and h_a are not the same;
15. Using the point I and the point A , construct a line s_a (rule W02); % DET: points I and A are not the same;
16. Using the line m_a and the line s_a , construct a point N_a (rule W03); % NDG: lines m_a and s_a are not parallel % DET: lines m_a and s_a are not the same;
17. Using the point I and the point N_a , construct a circle $k(N_a, C)$ (rule W06); % NDG: points I and N_a are not the same;
18. Using the circle $k(N_a, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(N_a, C)$ intersect.

Non-degenerate conditions: line a and circle $k(N_a, C)$ intersect; points I and N_a are not the same; lines m_a and s_a are not parallel; lines AP'_a and h_a are not parallel; line a and circle $k(N, M_a)$ intersect; point M_a is outside the circle $k(I, P_a)$; circles $k(I, P_a)$ and $k_{over}(I, M_a)$ intersect; point I is inside the circle $k(N, M_a)$; points I and N are not the same; points I and M_a are not the same; line $m(H_b H_c)$ and circle $k(N, M_a)$ intersect; points M_a and N are not the same.

Determination conditions: lines m_a and s_a are not the same; points I and A are not the same; lines AP'_a and h_a are not the same; points H_a and E_a are not the same; points M_a and H_a must be different; circles $k(I, P_a)$ and $k_{over}(I, M_a)$ are not the same; points M_a and E_a must be different; points M_a and N are not the same; points I and M_a are not the same.

Rules used: [W01,W02,W03,W04,W05,W05a,W06,W07,W09,W10b,W12,W16,W22]

Lemmas used: [D11,D2,D21,D27,D28,D3,D32,D47,D5,D65,D8,D85,GD01,GD02,GL01,GL03,GL09,L119,L19,L

Solving time: 52.3 seconds.

3.2 Construction in GCLC language

dim 120 120

point I 74.37 61.15

point M_{a} 65 40

point N 72.5 61.93

color 220 0 0

fontsize 9

```

cmark_b I
cmark_r M_{a}
cmark_r N
color 0 0 0
fontsize 8

% DET: points I and M_{a} are not the same
% Constructing a line IM_{a} which passes through point I and point M_{a}
line IM_{a} I M_{a}

color 200 200 200
drawline IM_{a}
color 0 0 0

% DET: points M_{a} and N are not the same
% Constructing a line m(H_{b}H_{c}) which passes through point M_{a} and point N
line m(H_{b}H_{c}) M_{a} N

color 200 200 200
drawline m(H_{b}H_{c})
color 0 0 0

% NDG: points M_{a} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{a}
circle k(N,M_{a}) N M_{a}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(H_{b}H_{c}) and circle k(N,M_{a}) intersect% DET: points M_{a} and E_{a} must be
different
% Constructing a point E_{a} which is an image of the point M_{a} in the symmetry to point/line N
sim E_{a} N M_{a}
cmark_r E_{a}

% NDG: points I and M_{a} are not the same
% Constructing midpoint P_{\_G170192} of the segment IM_{a}
midpoint P_{\_G170192} I M_{a}
cmark_r P_{\_G170192}

% Constructing a circle k_{over}(I,M_{a}) whose center is at point P_{\_G170192} and which passes
through point I
circle k_{over}(I,M_{a}) P_{\_G170192} I

color 200 200 200
drawcircle k_{over}(I,M_{a})
color 0 0 0

```

```

% NDG: point I is inside the circle k(N,M_{a}); points I and N are not the same
% Calculating distance V[_G170378] from point N to point M_{a}
distance V[_G170378] N M_{a}

% Calculating distance V[_G170402] from point N to point I
distance V[_G170402] N I

% Calculating value V[_G170423] using formula V[_G170378]/V[_G170402]
expression V[_G170423] { V[_G170378]/V[_G170402] }

% Constructing a point P_{\_G170454} such that NP_{\_G170454}/NI=V[_G170378]/V[_G170402]
towards P_{\_G170454} N I V[_G170423]
cmark_r P_{\_G170454}

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
170454}
circle k(I,P_{a}) I P_{\_G170454}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: circles k(I,P_{a}) and k_{over}(I,M_{a}) intersect% DET: circles k(I,P_{a}) and k_{over}(I,M_{a}
}) are not the same
% Constructing points P_{a} and A_{fi} which are in intersection of k(I,P_{a}) and k_{over}(I,M_{a})
intersec2 P_{a} A_{fi} k(I,P_{a}) k_{over}(I,M_{a})
cmark_r P_{a}
cmark_r A_{fi}

% Constructing a point P'_{a} such that P_{a}P'_{a}/P_{a}M_{a}=2
towards P'_{a} P_{a} M_{a} 2
cmark_r P'_{a}
color 200 200 200
drawsegment P_{a} P'_{a}
color 0 0 0

% NDG: point M_{a} is outside the circle k(I,P_{a})
% Constructing midpoint P_{\_G171060} of the segment M_{a}I
midpoint P_{\_G171060} M_{a} I
cmark_r P_{\_G171060}

% Constructing a circle C_{\_G171063} whose center is at point P_{\_G171060} and which passes
through point M_{a}
circle C_{\_G171063} P_{\_G171060} M_{a}

color 200 200 200

```

```

drawcircle C_{\_G171063}
color 0 0 0

% Constructing points  $P_{\_G171066}$  and  $P_{\_G171069}$  which are in intersection of  $C_{\_G171063}$ 
and  $k(I, P_{\{a\}})$ 
intersec2 P_{\_G171066} P_{\_G171069} C_{\_G171063} k(I, P_{\{a\}})
cmark_r P_{\_G171066}
cmark_r P_{\_G171069}

% Constructing a line  $x1$  which passes through point  $M_{\{a\}}$  and point  $P_{\_G171066}$ 
line x1  $M_{\{a\}}$   $P_{\_G171066}$ 

color 200 200 200
drawline x1
color 0 0 0

% Constructing a line  $a$  which passes through point  $M_{\{a\}}$  and point  $P_{\_G171069}$ 
line a  $M_{\{a\}}$   $P_{\_G171069}$ 

color 200 200 200
drawline a
color 0 0 0

% NDG: line  $a$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $M_{\{a\}}$  and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\_G171403}$  which is a foot of the point  $N$  on the line  $a$ 
foot P_{\_G171403} N a
cmark_r P_{\_G171403}
color 200 200 200
drawline N P_{\_G171403}
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $M_{\{a\}}$  in the symmetry to point/line  $P_{\_G171403}$ 
sim  $H_{\{a\}}$  P_{\_G171403}  $M_{\{a\}}$ 
cmark_r  $H_{\{a\}}$ 

% DET: points  $H_{\{a\}}$  and  $E_{\{a\}}$  are not the same
% Constructing a line  $h_{\{a\}}$  which passes through point  $H_{\{a\}}$  and point  $E_{\{a\}}$ 
line  $h_{\{a\}}$   $H_{\{a\}}$   $E_{\{a\}}$ 

color 200 200 200
drawline  $h_{\{a\}}$ 
color 0 0 0

% Constructing a line  $m_{\{a\}}$  which is perpendicular to line  $a$  and which passes through point  $M_{\{a\}}$ 
perp  $m_{\{a\}}$   $M_{\{a\}}$  a

color 200 200 200

```

```

drawline m_{a}
color 0 0 0

% Constructing a line AP'_{a} which contains the point P'_{a} and is parallel to the line IM_{a}
parallel AP'_{a} P'_{a} IM_{a}

color 200 200 200
drawline AP'_{a}
color 0 0 0

% NDG: lines AP'_{a} and h_{a} are not parallel% DET: lines AP'_{a} and h_{a} are not the same
% Constructing a point A which belongs to line AP'_{a} and line h_{a}
intersec A AP'_{a} h_{a}
cmark_t A

% DET: points I and A are not the same
% Constructing a line s_{a} which passes through point I and point A
line s_{a} I A

color 200 200 200
drawline s_{a}
color 0 0 0

% NDG: lines m_{a} and s_{a} are not parallel% DET: lines m_{a} and s_{a} are not the same
% Constructing a point N_{a} which belongs to line m_{a} and line s_{a}
intersec N_{a} m_{a} s_{a}
cmark_b N_{a}

% NDG: points I and N_{a} are not the same
% Constructing a circle k(N_{a},C) whose center is at point N_{a} and which passes through point I
circle k(N_{a},C) N_{a} I

color 200 200 200
drawcircle k(N_{a},C)
color 0 0 0

% NDG: line a and circle k(N_{a},C) intersect
% Constructing points C and B which are in intersection of k(N_{a},C) and a
intersec2 C B k(N_{a},C) a
cmark_l C
cmark_b B

drawsegment A B

```


Figure 1: Illustration of the problem 1113

```
drawsegment A C
drawsegment B C
```

```
% Non-degenerate conditions: line a and circle k(N_{a},C) intersect; points I and N_{a} are not the
    same; lines m_{a} and s_{a} are not parallel; lines AP'_{a} and h_{a} are not parallel; line a
    and circle k(N,M_{a}) intersect; point M_{a} is outside the circle k(I,P_{a}); circles k(I,P_{
a}) and k_{over}(I,M_{a}) intersect; point I is inside the circle k(N,M_{a}); points I and N are
    not the same; points I and M_{a} are not the same; line m(H_{b}H_{c}) and circle k(N,M_{a})
    intersect; points M_{a} and N are not the same
% Determination conditions: lines m_{a} and s_{a} are not the same; points I and A are not the same
    ; lines AP'_{a} and h_{a} are not the same; points H_{a} and E_{a} are not the same; points M_{
a} and H_{a} must be different; circles k(I,P_{a}) and k_{over}(I,M_{a}) are not the same; points
    M_{a} and E_{a} must be different; points M_{a} and N are not the same; points I and M_{a} are
    not the same
```

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $I = I$

Proving failed

4.1.2 Proving $M_a = M_a$

Proving failed

4.1.3 Proving $N = N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $I = I$

Proving failed

4.2.2 Proving $M_a = M_a$

Proving failed

4.2.3 Proving $N = N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $I=_J$

Proving failed

4.3.2 Proving $M_a=_M_a$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $I=_J$

Proving failed

4.4.2 Proving $M_a=_M_a$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 1114

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1114: Given a point I , a point M_b and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point I and the point M_b , construct a line IM_b (rule W02); % DET: points I and M_b are not the same;
2. Using the point M_b and the point N , construct a line $m(H_aH_c)$ (rule W02); % DET: points M_b and N are not the same;
3. Using the point M_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_b and N are not the same;
4. Using the circle $k(N, M_a)$, the line $m(H_aH_c)$, the point N and the point M_b , construct a point E_b (rule W05a); % NDG: line $m(H_aH_c)$ and circle $k(N, M_a)$ intersect % DET: points M_b and E_b must be different;
5. Using the point I and the point M_b , construct a circle $k_{over}(I, M_b)$ (rule W09); % NDG: points I and M_b are not the same;
6. Using the point I , the circle $k(N, M_a)$, the point N and the point M_b , construct a circle $k(I, P_a)$ (rule W22); % NDG: point I is inside the circle $k(N, M_a)$; points I and N are not the same;
7. Using the circle $k(I, P_a)$ and the circle $k_{over}(I, M_b)$, construct a point B_{fi} and a point P_b (rule W07); % NDG: circles $k(I, P_a)$ and $k_{over}(I, M_b)$ intersect % DET: circles $k(I, P_a)$ and $k_{over}(I, M_b)$ are not the same;
8. Using the point P_b and the point M_b , construct a point P'_b (rule W01); ;
9. Using the circle $k(I, P_a)$, the point M_b and the point I , construct a line x_2 and a line b (rule W12); % NDG: point M_b is outside the circle $k(I, P_a)$;

10. Using the circle $k(N, M_a)$, the line b , the point N and the point M_b , construct a point H_b (rule W05); % NDG: line b and circle $k(N, M_a)$ intersect % DET: points M_b and H_b must be different;
11. Using the point H_b and the point E_b , construct a line h_b (rule W02); % DET: points H_b and E_b are not the same;
12. Using the point M_b and the line b , construct a line m_b (rule W10b); ;
13. Using the point P'_b and the line IM_b , construct a line BP'_b (rule W16); ;
14. Using the line BP'_b and the line h_b , construct a point B (rule W03); % NDG: lines BP'_b and h_b are not parallel % DET: lines BP'_b and h_b are not the same;
15. Using the point I and the point B , construct a line s_b (rule W02); % DET: points I and B are not the same;
16. Using the line m_b and the line s_b , construct a point N_b (rule W03); % NDG: lines m_b and s_b are not parallel % DET: lines m_b and s_b are not the same;
17. Using the point I and the point N_b , construct a circle $k(N_b, A)$ (rule W06); % NDG: points I and N_b are not the same;
18. Using the circle $k(N_b, A)$ and the line b , construct a point A and a point C (rule W04); % NDG: line b and circle $k(N_b, A)$ intersect.

Non-degenerate conditions: line b and circle $k(N_b, A)$ intersect; points I and N_b are not the same; lines m_b and s_b are not parallel; lines BP'_b and h_b are not parallel; line b and circle $k(N, M_a)$ intersect; point M_b is outside the circle $k(I, P_a)$; circles $k(I, P_a)$ and $k_{over}(I, M_b)$ intersect; point I is inside the circle $k(N, M_a)$; points I and N are not the same; points I and M_b are not the same; line $m(H_a H_c)$ and circle $k(N, M_a)$ intersect; points M_b and N are not the same.

Determination conditions: lines m_b and s_b are not the same; points I and B are not the same; lines BP'_b and h_b are not the same; points H_b and E_b are not the same; points M_b and H_b must be different; circles $k(I, P_a)$ and $k_{over}(I, M_b)$ are not the same; points M_b and E_b must be different; points M_b and N are not the same; points I and M_b are not the same.

Rules used: [W01,W02,W03,W04,W05,W05a,W06,W07,W09,W10b,W12,W16,W22]

Lemmas used: [D12,D2,D22,D27,D29,D3,D32,D48,D6,D66,D86,D9,GD01,GD02,GL01,GL03,GL09,L119,L13,L

Solving time: 52.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point I 74.37 61.15
point M_{b} 95 67.5
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```

cmark_b I
cmark_lt M_{b}
cmark_r N
color 0 0 0
fontsize 8

% DET: points I and M_{b} are not the same
% Constructing a line IM_{b} which passes through point I and point M_{b}
line IM_{b} I M_{b}

color 200 200 200
drawline IM_{b}
color 0 0 0

% DET: points M_{b} and N are not the same
% Constructing a line m(H_{a}H_{c}) which passes through point M_{b} and point N
line m(H_{a}H_{c}) M_{b} N

color 200 200 200
drawline m(H_{a}H_{c})
color 0 0 0

% NDG: points M_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{b}
circle k(N,M_{a}) N M_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(H_{a}H_{c}) and circle k(N,M_{a}) intersect% DET: points M_{b} and E_{b} must be
different
% Constructing a point E_{b} which is an image of the point M_{b} in the symmetry to point/line N
sim E_{b} N M_{b}
cmark_r E_{b}

% NDG: points I and M_{b} are not the same
% Constructing midpoint P_{\_G226156} of the segment IM_{b}
midpoint P_{\_G226156} I M_{b}
cmark_r P_{\_G226156}

% Constructing a circle k_{over}(I,M_{b}) whose center is at point P_{\_G226156} and which passes
through point I
circle k_{over}(I,M_{b}) P_{\_G226156} I

color 200 200 200
drawcircle k_{over}(I,M_{b})
color 0 0 0

```

```

% NDG: point I is inside the circle k(N,M_{a}); points I and N are not the same
% Calculating distance V[_G226342] from point N to point M_{b}
distance V[_G226342] N M_{b}

% Calculating distance V[_G226366] from point N to point I
distance V[_G226366] N I

% Calculating value V[_G226387] using formula V[_G226342]/V[_G226366]
expression V[_G226387] { V[_G226342]/V[_G226366] }

% Constructing a point P_{\_G226418} such that NP_{\_G226418}/NI=V[_G226342]/V[_G226366]
towards P_{\_G226418} N I V[_G226387]
cmark_r P_{\_G226418}

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
226418}
circle k(I,P_{a}) I P_{\_G226418}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: circles k(I,P_{a}) and k_{over}(I,M_{b}) intersect% DET: circles k(I,P_{a}) and k_{over}(I,M_{b}
)}) are not the same
% Constructing points B_{fi} and P_{b} which are in intersection of k(I,P_{a}) and k_{over}(I,M_{b})
intersec2 B_{fi} P_{b} k(I,P_{a}) k_{over}(I,M_{b})
cmark_r B_{fi}
cmark_r P_{b}

% Constructing a point P'_{b} such that P_{b}P'_{b}/P_{b}M_{b}=2
towards P'_{b} P_{b} M_{b} 2
cmark_r P'_{b}
color 200 200 200
drawsegment P_{b} P'_{b}
color 0 0 0

% NDG: point M_{b} is outside the circle k(I,P_{a})
% Constructing midpoint P_{\_G227024} of the segment M_{b}I
midpoint P_{\_G227024} M_{b} I
cmark_r P_{\_G227024}

% Constructing a circle C_{\_G227027} whose center is at point P_{\_G227024} and which passes
through point M_{b}
circle C_{\_G227027} P_{\_G227024} M_{b}

color 200 200 200

```

```

drawcircle C_{\_G227027}
color 0 0 0

% Constructing points P_{\_G227030} and P_{\_G227033} which are in intersection of C_{\_G227027}
and k(I,P_{a})
intersec2 P_{\_G227030} P_{\_G227033} C_{\_G227027} k(I,P_{a})
cmark_r P_{\_G227030}
cmark_r P_{\_G227033}

% Constructing a line x2 which passes through point M_{b} and point P_{\_G227030}
line x2 M_{b} P_{\_G227030}

color 200 200 200
drawline x2
color 0 0 0

% Constructing a line b which passes through point M_{b} and point P_{\_G227033}
line b M_{b} P_{\_G227033}

color 200 200 200
drawline b
color 0 0 0

% NDG: line b and circle k(N,M_{a}) intersect% DET: points M_{b} and H_{b} must be different
% Constructing a point P_{\_G227367} which is a foot of the point N on the line b
foot P_{\_G227367} N b
cmark_r P_{\_G227367}
color 200 200 200
drawline N P_{\_G227367}
color 0 0 0

% Constructing a point H_{b} which is an image of the point M_{b} in the symmetry to point/line P
_{\_G227367}
sim H_{b} P_{\_G227367} M_{b}
cmark_l H_{b}

% DET: points H_{b} and E_{b} are not the same
% Constructing a line h_{b} which passes through point H_{b} and point E_{b}
line h_{b} H_{b} E_{b}

color 200 200 200
drawline h_{b}
color 0 0 0

% Constructing a line m_{b} which is perpendicular to line b and which passes through point M_{b}
perp m_{b} M_{b} b

color 200 200 200

```

```

drawline m_{b}
color 0 0 0

% Constructing a line BP'_{b} which contains the point P'_{b} and is parallel to the line IM_{b}
parallel BP'_{b} P'_{b} IM_{b}

color 200 200 200
drawline BP'_{b}
color 0 0 0

% NDG: lines BP'_{b} and h_{b} are not parallel% DET: lines BP'_{b} and h_{b} are not the same
% Constructing a point B which belongs to line BP'_{b} and line h_{b}
intersec B BP'_{b} h_{b}
cmark_b B

% DET: points I and B are not the same
% Constructing a line s_{b} which passes through point I and point B
line s_{b} I B

color 200 200 200
drawline s_{b}
color 0 0 0

% NDG: lines m_{b} and s_{b} are not parallel% DET: lines m_{b} and s_{b} are not the same
% Constructing a point N_{b} which belongs to line m_{b} and line s_{b}
intersec N_{b} m_{b} s_{b}
cmark_rb N_{b}

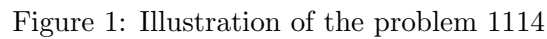
% NDG: points I and N_{b} are not the same
% Constructing a circle k(N_{b},A) whose center is at point N_{b} and which passes through point I
circle k(N_{b},A) N_{b} I

color 200 200 200
drawcircle k(N_{b},A)
color 0 0 0

% NDG: line b and circle k(N_{b},A) intersect
% Constructing points A and C which are in intersection of k(N_{b},A) and b
intersec2 A C k(N_{b},A) b
cmark_t A
cmark_l C

drawsegment A B

```

```
% Non-degenerate conditions: line b and circle k(N_{b},A) intersect; points I and N_{b} are not the
same; lines m_{b} and s_{b} are not parallel; lines BP'_{b} and h_{b} are not parallel; line b
and circle k(N,M_{a}) intersect; point M_{b} is outside the circle k(I,P_{a}); circles k(I,P_{a})
and k_{over}(I,M_{b}) intersect; point I is inside the circle k(N,M_{a}); points I and N are
not the same; points I and M_{b} are not the same; line m(H_{a}H_{c}) and circle k(N,M_{a})
intersect; points M_{b} and N are not the same
% Determination conditions: lines m_{b} and s_{b} are not the same; points I and B are not the same
; lines BP'_{b} and h_{b} are not the same; points H_{b} and E_{b} are not the same; points M_{b}
and H_{b} must be different; circles k(I,P_{a}) and k_{over}(I,M_{b}) are not the same; points
M_{b} and E_{b} must be different; points M_{b} and N are not the same; points I and M_{b} are
not the same
```

Illustration of the constructed figure is given in Figure 1

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4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $I=_I$

Proving failed

4.1.2 Proving $M_b=_M M_b$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $I=_I$

Proving failed

4.2.2 Proving $M_b=_M M_b$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $I=_I$

Proving failed

4.3.2 Proving $M_b=_M M_b$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $I=_I$

Proving failed

4.4.2 Proving $M_b=_M M_b$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 1115

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1115: Given a point I , a point M_c and a point N , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point I and the point M_c , construct a line IM_c (rule W02); % DET: points I and M_c are not the same;
2. Using the point M_c and the point N , construct a line $m(H_bH_a)$ (rule W02); % DET: points M_c and N are not the same;
3. Using the point M_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_c and N are not the same;
4. Using the circle $k(N, M_a)$, the line $m(H_bH_a)$, the point N and the point M_c , construct a point E_c (rule W05a); % NDG: line $m(H_bH_a)$ and circle $k(N, M_a)$ intersect % DET: points M_c and E_c must be different;
5. Using the point I and the point M_c , construct a circle $k_{over}(I, M_c)$ (rule W09); % NDG: points I and M_c are not the same;
6. Using the point I , the circle $k(N, M_a)$, the point N and the point M_c , construct a circle $k(I, P_a)$ (rule W22); % NDG: point I is inside the circle $k(N, M_a)$; points I and N are not the same;
7. Using the circle $k(I, P_a)$ and the circle $k_{over}(I, M_c)$, construct a point C_{fi} and a point P_c (rule W07); % NDG: circles $k(I, P_a)$ and $k_{over}(I, M_c)$ intersect % DET: circles $k(I, P_a)$ and $k_{over}(I, M_c)$ are not the same;
8. Using the point P_c and the point M_c , construct a point P'_c (rule W01); ;
9. Using the circle $k(I, P_a)$, the point M_c and the point I , construct a line $x3$ and a line c (rule W12); % NDG: point M_c is outside the circle $k(I, P_a)$;

10. Using the circle $k(N, M_a)$, the line c , the point N and the point M_c , construct a point H_c (rule W05); % NDG: line c and circle $k(N, M_a)$ intersect % DET: points M_c and H_c must be different;
11. Using the point H_c and the point E_c , construct a line h_c (rule W02); % DET: points H_c and E_c are not the same;
12. Using the point M_c and the line c , construct a line m_c (rule W10b); ;
13. Using the point P'_c and the line IM_c , construct a line CP'_c (rule W16); ;
14. Using the line CP'_c and the line h_c , construct a point C (rule W03); % NDG: lines CP'_c and h_c are not parallel % DET: lines CP'_c and h_c are not the same;
15. Using the point I and the point C , construct a line s_c (rule W02); % DET: points I and C are not the same;
16. Using the line m_c and the line s_c , construct a point N_c (rule W03); % NDG: lines m_c and s_c are not parallel % DET: lines m_c and s_c are not the same;
17. Using the point I and the point N_c , construct a circle $k(N_c, B)$ (rule W06); % NDG: points I and N_c are not the same;
18. Using the circle $k(N_c, B)$ and the line c , construct a point B and a point A (rule W04); % NDG: line c and circle $k(N_c, B)$ intersect.

Non-degenerate conditions: line c and circle $k(N_c, B)$ intersect; points I and N_c are not the same; lines m_c and s_c are not parallel; lines CP'_c and h_c are not parallel; line c and circle $k(N, M_a)$ intersect; point M_c is outside the circle $k(I, P_a)$; circles $k(I, P_a)$ and $k_{over}(I, M_c)$ intersect; point I is inside the circle $k(N, M_a)$; points I and N are not the same; points I and M_c are not the same; line $m(H_b H_a)$ and circle $k(N, M_a)$ intersect; points M_c and N are not the same.

Determination conditions: lines m_c and s_c are not the same; points I and C are not the same; lines CP'_c and h_c are not the same; points H_c and E_c are not the same; points M_c and H_c must be different; circles $k(I, P_a)$ and $k_{over}(I, M_c)$ are not the same; points M_c and E_c must be different; points M_c and N are not the same; points I and M_c are not the same.

Rules used: [W01,W02,W03,W04,W05,W05a,W06,W07,W09,W10b,W12,W16,W22]

Lemmas used: [D10,D13,D20,D27,D30,D32,D49,D67,D7,D87,GD01,GD02,GL01,GL03,GL09,L119,L14,L18,L19]

Solving time: 52.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point I 74.37 61.15
point M_{c} 50 67.5
point N 72.5 61.93
```

```
color 220 0 0
fontsize 9
```

```

cmark_b I
cmark_lt M_{c}
cmark_r N
color 0 0 0
fontsize 8

% DET: points I and M_{c} are not the same
% Constructing a line IM_{c} which passes through point I and point M_{c}
line IM_{c} I M_{c}

color 200 200 200
drawline IM_{c}
color 0 0 0

% DET: points M_{c} and N are not the same
% Constructing a line m(H_{b}H_{a}) which passes through point M_{c} and point N
line m(H_{b}H_{a}) M_{c} N

color 200 200 200
drawline m(H_{b}H_{a})
color 0 0 0

% NDG: points M_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{c}
circle k(N,M_{a}) N M_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% NDG: line m(H_{b}H_{a}) and circle k(N,M_{a}) intersect% DET: points M_{c} and E_{c} must be
different
% Constructing a point E_{c} which is an image of the point M_{c} in the symmetry to point/line N
sim E_{c} N M_{c}
cmark_r E_{c}

% NDG: points I and M_{c} are not the same
% Constructing midpoint P_{\_G69510} of the segment IM_{c}
midpoint P_{\_G69510} I M_{c}
cmark_r P_{\_G69510}

% Constructing a circle k_{over}(I,M_{c}) whose center is at point P_{\_G69510} and which passes
through point I
circle k_{over}(I,M_{c}) P_{\_G69510} I

color 200 200 200
drawcircle k_{over}(I,M_{c})
color 0 0 0

```

```

% NDG: point I is inside the circle k(N,M_{a}); points I and N are not the same
% Calculating distance V[_G69696] from point N to point M_{c}
distance V[_G69696] N M_{c}

% Calculating distance V[_G69720] from point N to point I
distance V[_G69720] N I

% Calculating value V[_G69741] using formula V[_G69696]/V[_G69720]
expression V[_G69741] { V[_G69696]/V[_G69720] }

% Constructing a point P_{\_G69772} such that NP_{\_G69772}/NI=V[_G69696]/V[_G69720]
towards P_{\_G69772} N I V[_G69741]
cmark_r P_{\_G69772}

% Constructing a circle k(I,P_{a}) whose center is at point I and which passes through point P_{\_G
69772}
circle k(I,P_{a}) I P_{\_G69772}

color 200 200 200
drawcircle k(I,P_{a})
color 0 0 0

% NDG: circles k(I,P_{a}) and k_{over}(I,M_{c}) intersect% DET: circles k(I,P_{a}) and k_{over}(I,M_{c
}) are not the same
% Constructing points C_{fi} and P_{c} which are in intersection of k(I,P_{a}) and k_{over}(I,M_{c})
intersec2 C_{fi} P_{c} k(I,P_{a}) k_{over}(I,M_{c})
cmark_r C_{fi}
cmark_r P_{c}

% Constructing a point P'_{c} such that P_{c}P'_{c}/P_{c}M_{c}=2
towards P'_{c} P_{c} M_{c} 2
cmark_r P'_{c}
color 200 200 200
drawsegment P_{c} P'_{c}
color 0 0 0

% NDG: point M_{c} is outside the circle k(I,P_{a})
% Constructing midpoint P_{\_G70377} of the segment M_{c}I
midpoint P_{\_G70377} M_{c} I
cmark_r P_{\_G70377}

% Constructing a circle C_{\_G70380} whose center is at point P_{\_G70377} and which passes through
point M_{c}
circle C_{\_G70380} P_{\_G70377} M_{c}

color 200 200 200

```

```

drawcircle C_{\_G70380}
color 0 0 0

% Constructing points P_{\_G70383} and P_{\_G70386} which are in intersection of C_{\_G70380} and k
(I,P_{a})
intersec2 P_{\_G70383} P_{\_G70386} C_{\_G70380} k(I,P_{a})
cmark_r P_{\_G70383}
cmark_r P_{\_G70386}

% Constructing a line x3 which passes through point M_{c} and point P_{\_G70383}
line x3 M_{c} P_{\_G70383}

color 200 200 200
drawline x3
color 0 0 0

% Constructing a line c which passes through point M_{c} and point P_{\_G70386}
line c M_{c} P_{\_G70386}

color 200 200 200
drawline c
color 0 0 0

% NDG: line c and circle k(N,M_{a}) intersect% DET: points M_{c} and H_{c} must be different
% Constructing a point P_{\_G70720} which is a foot of the point N on the line c
foot P_{\_G70720} N c
cmark_r P_{\_G70720}
color 200 200 200
drawline N P_{\_G70720}
color 0 0 0

% Constructing a point H_{c} which is an image of the point M_{c} in the symmetry to point/line P
_{\_G70720}
sim H_{c} P_{\_G70720} M_{c}
cmark_rt H_{c}

% DET: points H_{c} and E_{c} are not the same
% Constructing a line h_{c} which passes through point H_{c} and point E_{c}
line h_{c} H_{c} E_{c}

color 200 200 200
drawline h_{c}
color 0 0 0

% Constructing a line m_{c} which is perpendicular to line c and which passes through point M_{c}
perp m_{c} M_{c} c

color 200 200 200

```

```

drawline m_{c}
color 0 0 0

% Constructing a line CP'_{c} which contains the point P'_{c} and is parallel to the line IM_{c}
parallel CP'_{c} P'_{c} IM_{c}

color 200 200 200
drawline CP'_{c}
color 0 0 0

% NDG: lines CP'_{c} and h_{c} are not parallel% DET: lines CP'_{c} and h_{c} are not the same
% Constructing a point C which belongs to line CP'_{c} and line h_{c}
intersec C CP'_{c} h_{c}
cmark_l C

% DET: points I and C are not the same
% Constructing a line s_{c} which passes through point I and point C
line s_{c} I C

color 200 200 200
drawline s_{c}
color 0 0 0

% NDG: lines m_{c} and s_{c} are not parallel% DET: lines m_{c} and s_{c} are not the same
% Constructing a point N_{c} which belongs to line m_{c} and line s_{c}
intersec N_{c} m_{c} s_{c}
cmark_b N_{c}

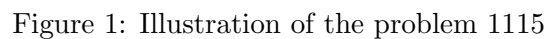
% NDG: points I and N_{c} are not the same
% Constructing a circle k(N_{c},B) whose center is at point N_{c} and which passes through point I
circle k(N_{c},B) N_{c} I

color 200 200 200
drawcircle k(N_{c},B)
color 0 0 0

% NDG: line c and circle k(N_{c},B) intersect
% Constructing points B and A which are in intersection of k(N_{c},B) and c
intersec2 B A k(N_{c},B) c
cmark_b B
cmark_t A

drawsegment A B

```

% Non-degenerate conditions: line c and circle $k(N_{\{c\}}, B)$ intersect; points I and $N_{\{c\}}$ are not the same; lines $m_{\{c\}}$ and $s_{\{c\}}$ are not parallel; lines $CP'_{\{c\}}$ and $h_{\{c\}}$ are not parallel; line c and circle $k(N, M_{\{a\}})$ intersect; point $M_{\{c\}}$ is outside the circle $k(I, P_{\{a\}})$; circles $k(I, P_{\{a\}})$ and $k_{\text{over}}(I, M_{\{c\}})$ intersect; point I is inside the circle $k(N, M_{\{a\}})$; points I and N are not the same; points I and $M_{\{c\}}$ are not the same; line $m(H_{\{b\}}H_{\{a\}})$ and circle $k(N, M_{\{a\}})$ intersect; points $M_{\{c\}}$ and N are not the same

% Determination conditions: lines $m_{\{c\}}$ and $s_{\{c\}}$ are not the same; points I and C are not the same; lines $CP'_{\{c\}}$ and $h_{\{c\}}$ are not the same; points $H_{\{c\}}$ and $E_{\{c\}}$ are not the same; points $M_{\{c\}}$ and $H_{\{c\}}$ must be different; circles $k(I, P_{\{a\}})$ and $k_{\text{over}}(I, M_{\{c\}})$ are not the same; points $M_{\{c\}}$ and $E_{\{c\}}$ must be different; points $M_{\{c\}}$ and N are not the same; points I and $M_{\{c\}}$ are not the same

Illustration of the constructed figure is given in Figure 1

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4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $I=_I$

Proving failed

4.1.2 Proving $M_c=_M_c$

Proving failed

4.1.3 Proving $N=_N$

Proving failed

4.2 GCLC - Area method

4.2.1 Proving $I=_I$

Proving failed

4.2.2 Proving $M_c=_M_c$

Proving failed

4.2.3 Proving $N=_N$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $I=_I$

Proving failed

4.3.2 Proving $M_c=_M_c$

Proving failed

4.3.3 Proving $N=_N$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $I=_I$

Proving failed

4.4.2 Proving $M_c=_M_c$

Proving failed

4.4.3 Proving $N=_N$

Proving failed

Problem 1116

*Generated automatically by ArgoTriCS
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1 Problem

Problem 1116: Given a point I , a point N and a point O , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1117

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1117: Given a point I , a point N and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1118

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1118: Given a point I , a point N and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1119

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1119: Given a point I , a point N and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1120

*Generated automatically by ArgoTriCS
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1 Problem

Problem 1120: Given a point M_b , a point N and a point M_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point M_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_b and N are not the same;
2. Choose freely a point M_a on the circle $k(N, M_a)$ (rule WOncircle);
3. Choose freely a point A (rule free);
4. Using the point A and the point M_b , construct a point C (rule W01); ;
5. Using the point M_a and the point C , construct a point B (rule W01); .

Non-degenerate conditions: points M_b and N are not the same.

Determination conditions: .

Rules used: [W01,W06,WOncircle1,free]

Lemmas used: [D21,D22,D32,GL03,GL04,L17]

Solving time: 149.1 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{b} 95 67.5  
point N 72.5 61.93  
point M_{a} 65 40
```

```
color 220 0 0  
fontsize 9
```

```

cmark_lt M_{b}
cmark_r N
cmark_r M_{a}
color 0 0 0
fontsize 8

% NDG: points M_{b} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{a}
circle k(N,M_{a}) N M_{b}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point M_{a} on the circle with center N through point M_{b}
oncircle M_{a} N M_{b}
cmark_r M_{a}
color 200 200 200
drawcircle N M_{b}
color 0 0 0

% Constructing a free point A
point A 80 95

cmark_t A

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% Constructing a point B such that M_{a}B/M_{a}C=-1
towards B M_{a} C -1
cmark_b B
color 200 200 200
drawsegment C B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: points M_{b} and N are not the same

```

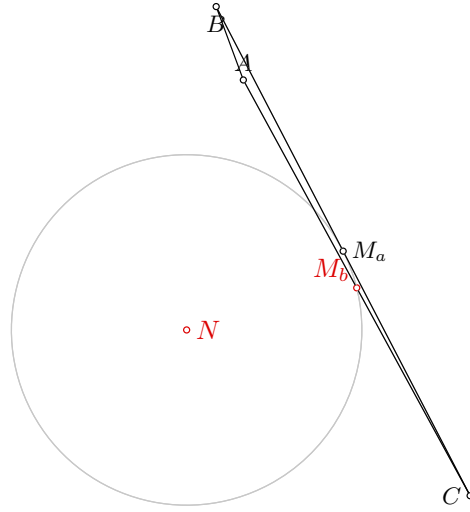



Figure 1: Illustration of the problem 1120

% Determination conditions:

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_b = M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.051 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $N = \neg N$

Proving failed

4.1.3 Proving $M_a = \neg M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 12 terms.

Time Complexity: Time spent by the prover is 0.148 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{\neg M_a BC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_b AC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2 BF_{\neg h_b}^3}$ i.e., lines $AF_{\neg h_a}^2$ and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

$S_{\neg M_a \neg M_b F_{\neg m_b}^1} \neq S_{F_{\neg m_a}^0 \neg M_b F_{\neg m_b}^1}$ i.e., lines $\neg M_a F_{\neg m_a}^0$ and $\neg M_b F_{\neg m_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $M_a = \neg M_a$

NDG conditions are:

$S_{\neg M_a BC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_b AC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2 BF_{\neg h_b}^3}$ i.e., lines $AF_{\neg h_a}^2$ and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

$S_{\neg M_a \neg M_b F_{\neg m_b}^1} \neq S_{F_{\neg m_a}^0 \neg M_b F_{\neg m_b}^1}$ i.e., lines $\neg M_a F_{\neg m_a}^0$ and $\neg M_b F_{\neg m_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 44

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $M_b = \neg M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $M_a = \neg M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 28 terms.

Time Complexity: Time spent by the prover is 0.060 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_b = \neg M_b$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $M_a = \neg M_a$

Proving failed

Problem 1121

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1121: Given a point M_c , a point N and a point M_a , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point M_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_c and N are not the same;
2. Choose freely a point M_a on the circle $k(N, M_a)$ (rule WOncircle);
3. Choose freely a point A (rule free);
4. Using the point A and the point M_c , construct a point B (rule W01); ;
5. Using the point M_a and the point B , construct a point C (rule W01); .

Non-degenerate conditions: points M_c and N are not the same.

Determination conditions: .

Rules used: [W01,W06,WOncircle1,free]

Lemmas used: [D20,D21,D32,GL03,GL04,L18]

Solving time: 150.5 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point M_{c} 50 67.5
point N 72.5 61.93
point M_{a} 65 40

color 220 0 0
fontsize 9
```

```

cmark_lt M_{c}
cmark_r N
cmark_r M_{a}
color 0 0 0
fontsize 8

% NDG: points M_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{c}
circle k(N,M_{a}) N M_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point M_{a} on the circle with center N through point M_{c}
oncircle M_{a} N M_{c}
cmark_r M_{a}
color 200 200 200
drawcircle N M_{c}
color 0 0 0

% Constructing a free point A
point A 80 95

cmark_t A

% Constructing a point B such that  $AB/AM_{c}=2$ 
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

% Constructing a point C such that  $M_{a}C/M_{a}B=-1$ 
towards C M_{a} B -1
cmark_l C
color 200 200 200
drawsegment B C
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: points M_{c} and N are not the same

```

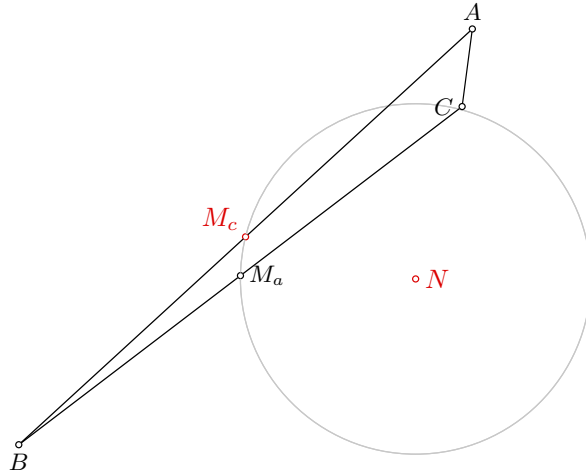


Figure 1: Illustration of the problem 1121

% Determination conditions:

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_c = M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.055 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $M_a=_M_a$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 12 terms.

Time Complexity: Time spent by the prover is 0.157 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $M_c=_M_c$

NDG conditions are:

$S_{_M_aBC} \neq 0$ i.e., points $_M_a, B$ and C are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$ i.e., points $_M_b, A$ and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^3_{_h_b}} \neq S_{F^2_{_h_a}BF^3_{_h_b}}$ i.e., lines $AF^2_{_h_a}$ and $BF^3_{_h_b}$ are not parallel (construction based assumption)

$S_{_M_a-M_bF^1_{_m_b}} \neq S_{F^0_{_m_a}-M_bF^1_{_m_b}}$ i.e., lines $_M_aF^0_{_m_a}$ and $_M_bF^1_{_m_b}$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $M_a=_M_a$

NDG conditions are:

$S_{_M_aBC} \neq 0$ i.e., points $_M_a, B$ and C are not collinear (foot is not the point itself; construction based assumption)

$S_{_M_bAC} \neq 0$ i.e., points $_M_b, A$ and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF^3_{_h_b}} \neq S_{F^2_{_h_a}BF^3_{_h_b}}$ i.e., lines $AF^2_{_h_a}$ and $BF^3_{_h_b}$ are not parallel (construction based assumption)

$S_{_M_a-M_bF^1_{_m_b}} \neq S_{F^0_{_m_a}-M_bF^1_{_m_b}}$ i.e., lines $_M_aF^0_{_m_a}$ and $_M_bF^1_{_m_b}$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $M_c=_M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $M_a = \neg M_a$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 28 terms.

Time Complexity: Time spent by the prover is 0.070 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_c = \neg M_c$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $M_a = \neg M_a$

Proving failed

Problem 1122

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1122: Given a point M_a , a point N and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point N and the point O , construct a point G (rule W01); ;
2. Using the point M_a and the point G , construct a point A (rule W01); ;
3. Using the point N and the point O , construct a point H (rule W01); ;
4. Using the point A and the point H , construct a point E_a (rule W01); ;
5. Using the point A and the point H , construct a line h_a (rule W02); % DET: points A and H are not the same;
6. Using the point M_a and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_a and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_a , the point N and the point E_a , construct a point H_a (rule W05); % NDG: line h_a and circle $k(N, M_a)$ intersect % DET: points E_a and H_a must be different;
8. Using the point H_a and the point M_a , construct a line a (rule W02); % DET: points H_a and M_a are not the same;
9. Using the point A and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points A and O are not the same;
10. Using the circle $k(O, C)$ and the line a , construct a point C and a point B (rule W04); % NDG: line a and circle $k(O, C)$ intersect.

Non-degenerate conditions: line a and circle $k(O, C)$ intersect; points A and O are not the same; line h_a and circle $k(N, M_a)$ intersect; points M_a and N are not the same.

Determination conditions: points H_a and M_a are not the same; points E_a and H_a must be different; points A and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D21,D26,D28,D3,D31,D32,D5,D8,GD01,GL03,GL04,GL09,L11,L12,L15,L19,L22,L55]

Solving time: 7.3 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{a} 65 40
point N 72.5 61.93
point O 65 51.14
```

```
color 220 0 0
fontsize 9
```

```
cmark_r M_{a}
cmark_r N
cmark_t O
color 0 0 0
fontsize 8
```

```
% Constructing a line L_{\_G188317} which passes through point N and point O
line L_{\_G188317} N O
```

```
color 200 200 200
drawline L_{\_G188317}
color 0 0 0
```

```
% Constructing a point P_{\_G188418} with coordinates (0,0)
point P_{\_G188418} 0 0
cmark_r P_{\_G188418}
```

```
% Constructing a point P_{\_G188342} such that NP_{\_G188342}/NP_{\_G188418}=1
towards P_{\_G188342} N P_{\_G188418} 1
cmark_r P_{\_G188342}
color 200 200 200
drawsegment N P_{\_G188342}
color 0 0 0
```

```
% Constructing a point P_{\_G188387} such that NP_{\_G188387}/NP_{\_G188418}=3
towards P_{\_G188387} N P_{\_G188418} 3
cmark_r P_{\_G188387}
color 200 200 200
drawsegment N P_{\_G188387}
color 0 0 0
```

```
% Constructing a line L_{\_G188348} which passes through point O and point P_{\_G188387}
line L_{\_G188348} O P_{\_G188387}
```

```

color 200 200 200
drawline L_{\_G188348}
color 0 0 0

% Constructing a line L_{\_G188311} which contains the point P_{\_G188342} and is parallel to the
  line L_{\_G188348}
parallel L_{\_G188311} P_{\_G188342} L_{\_G188348}

color 200 200 200
drawline L_{\_G188311}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G188311} and line L_{\_G188317}
intersec G L_{\_G188311} L_{\_G188317}
cmark_t G

% Constructing a point A such that  $M_{\{a\}}A/M_{\{a\}}G=3$ 
towards A M_{\{a\}} G 3
cmark_t A
color 200 200 200
drawsegment M_{\{a\}} A
color 0 0 0

% Constructing a point H such that  $NH/NO=-1$ 
towards H N 0 -1
cmark_rt H
color 200 200 200
drawsegment 0 H
color 0 0 0

% Constructing a point E_{\{a\}} such that  $AE_{\{a\}}/AH=0.5$ 
towards E_{\{a\}} A H 0.5
cmark_r E_{\{a\}}
color 200 200 200
drawsegment A H
color 0 0 0

% DET: points A and H are not the same
% Constructing a line h_{\{a\}} which passes through point A and point H
line h_{\{a\}} A H

color 200 200 200
drawline h_{\{a\}}
color 0 0 0

```

```

% NDG: points  $M_{\{a\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $M_{\{a\}}$ 
circle k(N,  $M_{\{a\}}$ ) N  $M_{\{a\}}$ 

color 200 200 200
drawcircle k(N,  $M_{\{a\}}$ )
color 0 0 0

% NDG: line  $h_{\{a\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{a\}}$  and  $H_{\{a\}}$  must be different
% Constructing a point  $P_{\{\_G189402\}}$  which is a foot of the point  $N$  on the line  $h_{\{a\}}$ 
foot P_{\_G189402} N  $h_{\{a\}}$ 
cmark_r P_{\_G189402}
color 200 200 200
drawline N P_{\_G189402}
color 0 0 0

% Constructing a point  $H_{\{a\}}$  which is an image of the point  $E_{\{a\}}$  in the symmetry to point/line  $P_{\{\_G189402\}}$ 
sim  $H_{\{a\}}$  P_{\_G189402}  $E_{\{a\}}$ 
cmark_r  $H_{\{a\}}$ 

% DET: points  $H_{\{a\}}$  and  $M_{\{a\}}$  are not the same
% Constructing a line  $a$  which passes through point  $H_{\{a\}}$  and point  $M_{\{a\}}$ 
line a  $H_{\{a\}}$   $M_{\{a\}}$ 

color 200 200 200
drawline a
color 0 0 0

% NDG: points  $A$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $A$ 
circle k(O, C) O A

color 200 200 200
drawcircle k(O, C)
color 0 0 0

% NDG: line  $a$  and circle  $k(O, C)$  intersect
% Constructing points  $C$  and  $B$  which are in intersection of  $k(O, C)$  and  $a$ 
intersec2 C B k(O, C) a
cmark_l C
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```

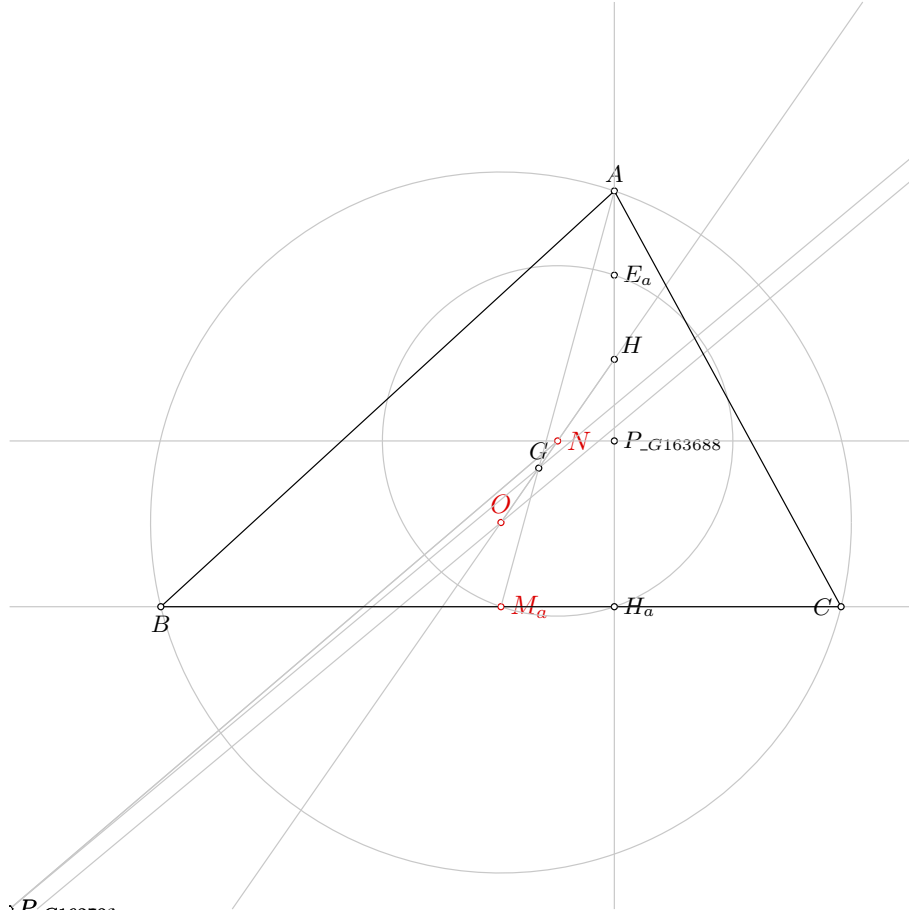


Figure 1: Illustration of the problem 1122

*% Non-degenerate conditions: line a and circle k(O,C) intersect; points A and O are not the same;
 line h_{a} and circle k(N,M_{a}) intersect; points M_{a} and N are not the same
 % Determination conditions: points H_{a} and M_{a} are not the same; points E_{a} and H_{a} must be
 different; points A and H are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_a = M_a$

Proving failed

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $O=_O$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 4927 terms.

Time Complexity: Time spent by the prover is 11.886 seconds.

NDG conditions Points $P_{G184115}$, N and O are not collinear

Line through points $P_{G184115}$ and O is not parallel with line through points N and M_a

Points A and H are not identical

Line through points A and N is not perpendicular to line through points N and H

Points M_a and H_a are not identical

Points M_a and H_a are not identical

Points A , B and C are not collinear

Line through points B and N is not perpendicular to line through points N and C

4.2 GCLC - Area method

4.2.1 Proving $M_a=_M M_a$

Proving failed

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $O=_O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_a=_M M_a$

Proving failed

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $O=_O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_a=_M M_a$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $O=_O$

Proving failed

Problem 1123

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1123: Given a point M_a , a point N and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1124

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1124: Given a point M_a , a point N and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1125

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1125: Given a point M_a , a point N and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1126

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1126: Given a point M_c , a point N and a point M_b , construct the triangle ABC .

2 Status

Problem is locus dependent.

3 Solution

3.1 Construction in natural language form

1. Using the point M_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_c and N are not the same;
2. Choose freely a point M_b on the circle $k(N, M_a)$ (rule WOncircle);
3. Choose freely a point A (rule free);
4. Using the point A and the point M_b , construct a point C (rule W01); ;
5. Using the point A and the point M_c , construct a point B (rule W01); .

Non-degenerate conditions: points M_c and N are not the same.

Determination conditions: .

Rules used: [W01,W06,WOncircle1,free]

Lemmas used: [D20,D22,D32,GL03,GL04,L18]

Solving time: 153.2 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{c} 50 67.5  
point N 72.5 61.93  
point M_{b} 95 67.5
```

```
color 220 0 0  
fontsize 9
```

```

cmark_lt M_{c}
cmark_r N
cmark_lt M_{b}
color 0 0 0
fontsize 8

% NDG: points M_{c} and N are not the same
% Constructing a circle k(N,M_{a}) whose center is at point N and which passes through point M_{c}
circle k(N,M_{a}) N M_{c}

color 200 200 200
drawcircle k(N,M_{a})
color 0 0 0

% Choosing randomly a point M_{b} on the circle with center N through point M_{c}
oncircle M_{b} N M_{c}
cmark_lt M_{b}
color 200 200 200
drawcircle N M_{c}
color 0 0 0

% Constructing a free point A
point A 80 95

cmark_t A

% Constructing a point C such that AC/AM_{b}=2
towards C A M_{b} 2
cmark_l C
color 200 200 200
drawsegment A C
color 0 0 0

% Constructing a point B such that AB/AM_{c}=2
towards B A M_{c} 2
cmark_b B
color 200 200 200
drawsegment A B
color 0 0 0

drawsegment A B
drawsegment A C
drawsegment B C

% Non-degenerate conditions: points M_{c} and N are not the same

```

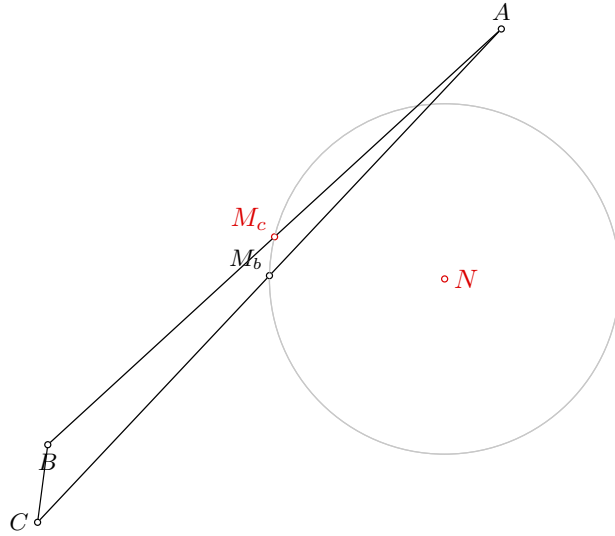


Figure 1: Illustration of the problem 1126

% Determination conditions:

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_c = M_c$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3 terms.

Time Complexity: Time spent by the prover is 0.048 seconds.

NDG conditions There are no NDG conditions for this theorem

4.1.2 Proving $N = \neg N$

Proving failed

4.1.3 Proving $M_b = \neg M_b$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 8 terms.

Time Complexity: Time spent by the prover is 0.137 seconds.

NDG conditions There are no NDG conditions for this theorem

4.2 GCLC - Area method

4.2.1 Proving $M_c = \neg M_c$

NDG conditions are:

$S_{\neg M_a BC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_b AC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2 BF_{\neg h_b}^3}$ i.e., lines $AF_{\neg h_a}^2$ and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

$S_{\neg M_a \neg M_b F_{\neg m_b}^1} \neq S_{F_{\neg m_a}^0 \neg M_b F_{\neg m_b}^1}$ i.e., lines $\neg M_a F_{\neg m_a}^0$ and $\neg M_b F_{\neg m_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 41

Time spent by the prover: 0.000 seconds

4.2.2 Proving $N = \neg N$

Proving failed

4.2.3 Proving $M_b = \neg M_b$

NDG conditions are:

$S_{\neg M_a BC} \neq 0$ i.e., points $\neg M_a$, B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{\neg M_b AC} \neq 0$ i.e., points $\neg M_b$, A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (foot is not the point itself; construction based assumption)

$S_{BAC} \neq 0$ i.e., points B , A and C are not collinear (foot is not the point itself; construction based assumption)

$S_{ABF_{\neg h_b}^3} \neq S_{F_{\neg h_a}^2 BF_{\neg h_b}^3}$ i.e., lines $AF_{\neg h_a}^2$ and $BF_{\neg h_b}^3$ are not parallel (construction based assumption)

$S_{\neg M_a \neg M_b F_{\neg m_b}^1} \neq S_{F_{\neg m_a}^0 \neg M_b F_{\neg m_b}^1}$ i.e., lines $\neg M_a F_{\neg m_a}^0$ and $\neg M_b F_{\neg m_b}^1$ are not parallel (construction based assumption)

Total number of proof steps: 42

Time spent by the prover: 0.000 seconds

4.3 GCLC - Wu method

4.3.1 Proving $M_c = \neg M_c$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 22 terms.

Time Complexity: Time spent by the prover is 0.040 seconds. There are no ndg conditions.

4.3.2 Proving $N = \neg N$

Proving failed

4.3.3 Proving $M_b = \neg M_b$

Status: The conjecture has been proved.

Space Complexity: The biggest polynomial obtained during proof process contained 28 terms.

Time Complexity: Time spent by the prover is 0.060 seconds. There are no ndg conditions.

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_c = \neg M_c$

Proving failed

4.4.2 Proving $N = \neg N$

Proving failed

4.4.3 Proving $M_b = \neg M_b$

Proving failed

Problem 1127

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1127: Given a point M_b , a point N and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point N and the point O , construct a point G (rule W01); ;
2. Using the point M_b and the point G , construct a point B (rule W01); ;
3. Using the point N and the point O , construct a point H (rule W01); ;
4. Using the point B and the point H , construct a point E_b (rule W01); ;
5. Using the point B and the point H , construct a line h_b (rule W02); % DET: points B and H are not the same;
6. Using the point M_b and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_b and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_b , the point N and the point E_b , construct a point H_b (rule W05); % NDG: line h_b and circle $k(N, M_a)$ intersect % DET: points E_b and H_b must be different;
8. Using the point H_b and the point M_b , construct a line b (rule W02); % DET: points H_b and M_b are not the same;
9. Using the point B and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points B and O are not the same;
10. Using the circle $k(O, C)$ and the line b , construct a point C and a point A (rule W04); % NDG: line b and circle $k(O, C)$ intersect.

Non-degenerate conditions: line b and circle $k(O, C)$ intersect; points B and O are not the same; line h_b and circle $k(N, M_a)$ intersect; points M_b and N are not the same.

Determination conditions: points H_b and M_b are not the same; points E_b and H_b must be different; points B and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D22,D26,D29,D3,D31,D32,D6,D9,GD01,GL03,GL04,GL09,L11,L12,L15,L17,L20,L23,L56]

Solving time: 7.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120

point M_{b} 95 67.5
point N 72.5 61.93
point O 65 51.14

color 220 0 0
fontsize 9

cmark_lt M_{b}
cmark_r N
cmark_t O
color 0 0 0
fontsize 8

% Constructing a line L_{\_G58374} which passes through point N and point O
line L_{\_G58374} N O

color 200 200 200
drawline L_{\_G58374}
color 0 0 0

% Constructing a point P_{\_G58475} with coordinates (0,0)
point P_{\_G58475} 0 0
cmark_r P_{\_G58475}

% Constructing a point P_{\_G58399} such that NP_{\_G58399}/NP_{\_G58475}=1
towards P_{\_G58399} N P_{\_G58475} 1
cmark_r P_{\_G58399}
color 200 200 200
drawsegment N P_{\_G58399}
color 0 0 0

% Constructing a point P_{\_G58444} such that NP_{\_G58444}/NP_{\_G58475}=3
towards P_{\_G58444} N P_{\_G58475} 3
cmark_r P_{\_G58444}
color 200 200 200
drawsegment N P_{\_G58444}
color 0 0 0

% Constructing a line L_{\_G58405} which passes through point O and point P_{\_G58444}
line L_{\_G58405} O P_{\_G58444}
```

```

color 200 200 200
drawline L_{\_G58405}
color 0 0 0

% Constructing a line L_{\_G58368} which contains the point P_{\_G58399} and is parallel to the
line L_{\_G58405}
parallel L_{\_G58368} P_{\_G58399} L_{\_G58405}

color 200 200 200
drawline L_{\_G58368}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G58368} and line L_{\_G58374}
intersec G L_{\_G58368} L_{\_G58374}
cmark_t G

% Constructing a point B such that  $M_{\{b\}}B/M_{\{b\}}G=3$ 
towards B M_{\{b\}} G 3
cmark_b B
color 200 200 200
drawsegment M_{\{b\}} B
color 0 0 0

% Constructing a point H such that  $NH/NO=-1$ 
towards H N 0 -1
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0

% Constructing a point E_{\{b\}} such that  $BE_{\{b\}}/BH=0.5$ 
towards E_{\{b\}} B H 0.5
cmark_r E_{\{b\}}
color 200 200 200
drawsegment B H
color 0 0 0

% DET: points B and H are not the same
% Constructing a line h_{\{b\}} which passes through point B and point H
line h_{\{b\}} B H

color 200 200 200
drawline h_{\{b\}}
color 0 0 0

```

```

% NDG: points  $M_{\{b\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $M_{\{b\}}$ 
circle k(N, M_{a}) N M_{b}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line  $h_{\{b\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{b\}}$  and  $H_{\{b\}}$  must be different
% Constructing a point  $P_{\{\_G59459\}}$  which is a foot of the point  $N$  on the line  $h_{\{b\}}$ 
foot P_{\_G59459} N h_{b}
cmark_r P_{\_G59459}
color 200 200 200
drawline N P_{\_G59459}
color 0 0 0

% Constructing a point  $H_{\{b\}}$  which is an image of the point  $E_{\{b\}}$  in the symmetry to point/line  $P_{\{\_G59459\}}$ 
sim H_{b} P_{\_G59459} E_{b}
cmark_l H_{b}

% DET: points  $H_{\{b\}}$  and  $M_{\{b\}}$  are not the same
% Constructing a line  $b$  which passes through point  $H_{\{b\}}$  and point  $M_{\{b\}}$ 
line b H_{b} M_{b}

color 200 200 200
drawline b
color 0 0 0

% NDG: points  $B$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $B$ 
circle k(O, C) O B

color 200 200 200
drawcircle k(O, C)
color 0 0 0

% NDG: line  $b$  and circle  $k(O, C)$  intersect
% Constructing points  $C$  and  $A$  which are in intersection of  $k(O, C)$  and  $b$ 
intersec2 C A k(O, C) b
cmark_l C
cmark_t A

drawsegment A B
drawsegment A C
drawsegment B C

```

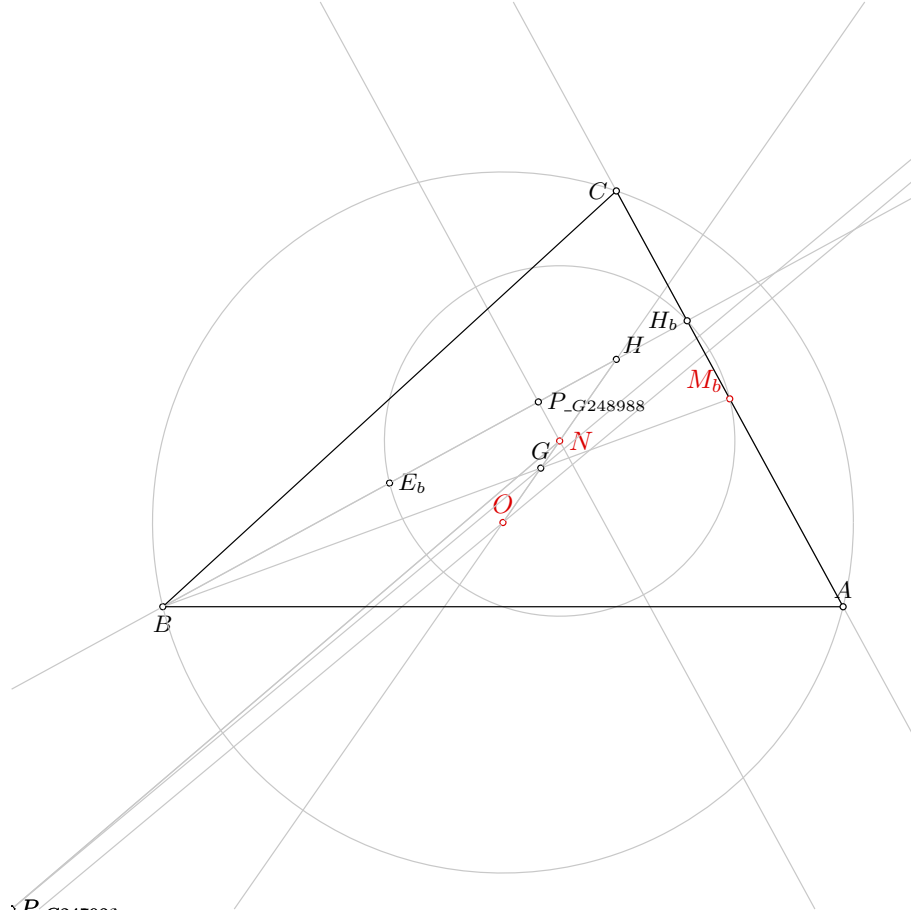


Figure 1: Illustration of the problem 1127

*% Non-degenerate conditions: line b and circle k(O,C) intersect; points B and O are not the same;
line h_{b} and circle k(N,M_{a}) intersect; points M_{b} and N are not the same
% Determination conditions: points H_{b} and M_{b} are not the same; points E_{b} and H_{b} must be
different; points B and H are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_b = M_b$

Proving failed

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $O=_O$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 1828 terms.

Time Complexity: Time spent by the prover is 5.436 seconds.

NDG conditions Points N , O and P_{G54169} are not collinear

Line through points M_b and N is not parallel with line through points O and P_{G54169}

Points B and H are not identical

Line through points M_b and B is not perpendicular to line through points B and N

Points M_b and H_b are not identical

Points M_b and H_b are not identical

Points A , B and C are not collinear

Line through points M_b and B is not perpendicular to line through points B and N

4.2 GCLC - Area method

4.2.1 Proving $M_b=_M M_b$

Proving failed

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $O=_O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_b=_M M_b$

Proving failed

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $O=_O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_b=_M M_b$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $O=_O$

Proving failed

Problem 1128

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1128: Given a point M_b , a point N and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1129

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1129: Given a point M_b , a point N and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1130

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1130: Given a point M_b , a point N and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1131

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1131: Given a point M_c , a point N and a point O , construct the triangle ABC .

2 Status

Problem is solvable.

3 Solution

3.1 Construction in natural language form

1. Using the point N and the point O , construct a point G (rule W01); ;
2. Using the point M_c and the point G , construct a point C (rule W01); ;
3. Using the point N and the point O , construct a point H (rule W01); ;
4. Using the point C and the point H , construct a point E_c (rule W01); ;
5. Using the point C and the point H , construct a line h_c (rule W02); % DET: points C and H are not the same;
6. Using the point M_c and the point N , construct a circle $k(N, M_a)$ (rule W06); % NDG: points M_c and N are not the same;
7. Using the circle $k(N, M_a)$, the line h_c , the point N and the point E_c , construct a point H_c (rule W05); % NDG: line h_c and circle $k(N, M_a)$ intersect % DET: points E_c and H_c must be different;
8. Using the point H_c and the point M_c , construct a line c (rule W02); % DET: points H_c and M_c are not the same;
9. Using the point C and the point O , construct a circle $k(O, C)$ (rule W06); % NDG: points C and O are not the same;
10. Using the circle $k(O, C)$ and the line c , construct a point A and a point B (rule W04); % NDG: line c and circle $k(O, C)$ intersect.

Non-degenerate conditions: line c and circle $k(O, C)$ intersect; points C and O are not the same; line h_c and circle $k(N, M_a)$ intersect; points M_c and N are not the same.

Determination conditions: points H_c and M_c are not the same; points E_c and H_c must be different; points C and H are not the same.

Rules used: [W01,W02,W04,W05,W06]

Lemmas used: [D10,D20,D26,D30,D31,D32,D7,GD01,GL03,GL04,GL09,L11,L12,L15,L18,L21,L24,L3,L57]

Solving time: 7.4 seconds.

3.2 Construction in GCLC language

```
dim 120 120
```

```
point M_{c} 50 67.5
```

```
point N 72.5 61.93
```

```
point O 65 51.14
```

```
color 220 0 0
```

```
fontsize 9
```

```
cmark_lt M_{c}
```

```
cmark_r N
```

```
cmark_t O
```

```
color 0 0 0
```

```
fontsize 8
```

```
% Constructing a line L_{\_G117879} which passes through point N and point O
```

```
line L_{\_G117879} N O
```

```
color 200 200 200
```

```
drawline L_{\_G117879}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G117980} with coordinates (0,0)
```

```
point P_{\_G117980} 0 0
```

```
cmark_r P_{\_G117980}
```

```
% Constructing a point P_{\_G117904} such that NP_{\_G117904}/NP_{\_G117980}=1
```

```
towards P_{\_G117904} N P_{\_G117980} 1
```

```
cmark_r P_{\_G117904}
```

```
color 200 200 200
```

```
drawsegment N P_{\_G117904}
```

```
color 0 0 0
```

```
% Constructing a point P_{\_G117949} such that NP_{\_G117949}/NP_{\_G117980}=3
```

```
towards P_{\_G117949} N P_{\_G117980} 3
```

```
cmark_r P_{\_G117949}
```

```
color 200 200 200
```

```
drawsegment N P_{\_G117949}
```

```
color 0 0 0
```

```
% Constructing a line L_{\_G117910} which passes through point O and point P_{\_G117949}
```

```
line L_{\_G117910} O P_{\_G117949}
```

```

color 200 200 200
drawline L_{\_G117910}
color 0 0 0

% Constructing a line L_{\_G117873} which contains the point P_{\_G117904} and is parallel to the
line L_{\_G117910}
parallel L_{\_G117873} P_{\_G117904} L_{\_G117910}

color 200 200 200
drawline L_{\_G117873}
color 0 0 0

% Constructing a point G which belongs to line L_{\_G117873} and line L_{\_G117879}
intersec G L_{\_G117873} L_{\_G117879}
cmark_t G

% Constructing a point C such that  $M_{\{c\}}C/M_{\{c\}}G=3$ 
towards C M_{\{c\}} G 3
cmark_l C
color 200 200 200
drawsegment M_{\{c\}} C
color 0 0 0

% Constructing a point H such that  $NH/NO=-1$ 
towards H N 0 -1
cmark_rt H
color 200 200 200
drawsegment O H
color 0 0 0

% Constructing a point E_{\{c\}} such that  $CE_{\{c\}}/CH=0.5$ 
towards E_{\{c\}} C H 0.5
cmark_r E_{\{c\}}
color 200 200 200
drawsegment C H
color 0 0 0

% DET: points C and H are not the same
% Constructing a line h_{\{c\}} which passes through point C and point H
line h_{\{c\}} C H

color 200 200 200
drawline h_{\{c\}}
color 0 0 0

```

```

% NDG: points  $M_{\{c\}}$  and  $N$  are not the same
% Constructing a circle  $k(N, M_{\{a\}})$  whose center is at point  $N$  and which passes through point  $M_{\{c\}}$ 
circle k(N, M_{a}) N M_{c}

color 200 200 200
drawcircle k(N, M_{a})
color 0 0 0

% NDG: line  $h_{\{c\}}$  and circle  $k(N, M_{\{a\}})$  intersect% DET: points  $E_{\{c\}}$  and  $H_{\{c\}}$  must be different
% Constructing a point  $P_{\{\_G118964\}}$  which is a foot of the point  $N$  on the line  $h_{\{c\}}$ 
foot P_{\_G118964} N h_{c}
cmark_r P_{\_G118964}
color 200 200 200
drawline N P_{\_G118964}
color 0 0 0

% Constructing a point  $H_{\{c\}}$  which is an image of the point  $E_{\{c\}}$  in the symmetry to point/line  $P_{\{\_G118964\}}$ 
sim H_{c} P_{\_G118964} E_{c}
cmark_rt H_{c}

% DET: points  $H_{\{c\}}$  and  $M_{\{c\}}$  are not the same
% Constructing a line  $c$  which passes through point  $H_{\{c\}}$  and point  $M_{\{c\}}$ 
line c H_{c} M_{c}

color 200 200 200
drawline c
color 0 0 0

% NDG: points  $C$  and  $O$  are not the same
% Constructing a circle  $k(O, C)$  whose center is at point  $O$  and which passes through point  $C$ 
circle k(O, C) O C

color 200 200 200
drawcircle k(O, C)
color 0 0 0

% NDG: line  $c$  and circle  $k(O, C)$  intersect
% Constructing points  $A$  and  $B$  which are in intersection of  $k(O, C)$  and  $c$ 
intersec2 A B k(O, C) c
cmark_t A
cmark_b B

drawsegment A B
drawsegment A C
drawsegment B C

```

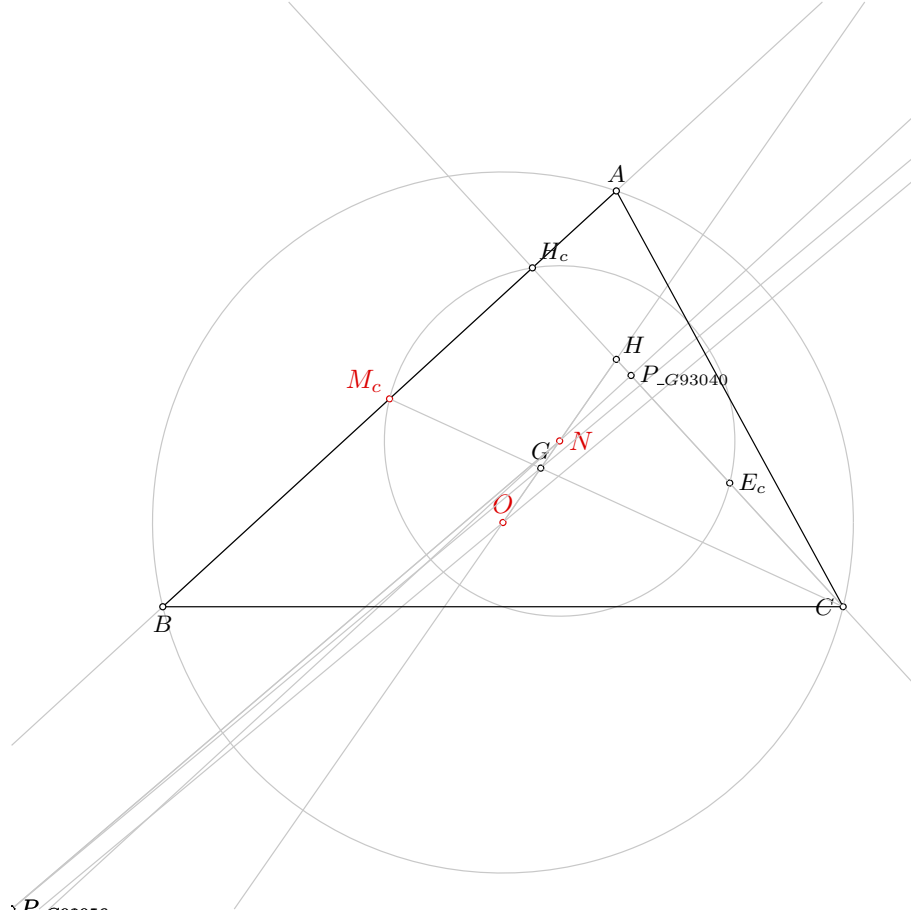


Figure 1: Illustration of the problem 1131

*% Non-degenerate conditions: line c and circle k(O,C) intersect; points C and O are not the same;
 line h_{c} and circle k(N,M_{a}) intersect; points M_{c} and N are not the same
 % Determination conditions: points H_{c} and M_{c} are not the same; points E_{c} and H_{c} must be
 different; points C and H are not the same*

3.3 Illustration

Illustration of the constructed figure is given in Figure 1

3.4 Construction in OpenGeoProver format

4 Correctness proof

4.1 OGP - Wu method

4.1.1 Proving $M_c = M_c$

Proving failed

4.1.2 Proving $N=_N$

Proving failed

4.1.3 Proving $O=_O$

Status: Theorem has been proved.

Space Complexity: The biggest polynomial obtained during prover execution contains 3990 terms.

Time Complexity: Time spent by the prover is 9.024 seconds.

NDG conditions Points $P_{G113641}$, N and O are not collinear

Line through points $P_{G113641}$ and O is not parallel with line through points M_c and N

Points C and H are not identical

Line through points C and M_c is not perpendicular to line through points M_c and H

Points H_c and M_c are not identical

Points H_c and M_c are not identical

Points A , B and C are not collinear

Line through points B and M_c is not perpendicular to line through points M_c and C

4.2 GCLC - Area method

4.2.1 Proving $M_c=_M_c$

Proving failed

4.2.2 Proving $N=_N$

Proving failed

4.2.3 Proving $O=_O$

Proving failed

4.3 GCLC - Wu method

4.3.1 Proving $M_c=_M_c$

Proving failed

4.3.2 Proving $N=_N$

Proving failed

4.3.3 Proving $O=_O$

Proving failed

4.4 GCLC - Grobner basis method

4.4.1 Proving $M_c=_M_c$

Proving failed

4.4.2 Proving $N=_N$

Proving failed

4.4.3 Proving $O=_O$

Proving failed

Problem 1132

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1132: Given a point M_c , a point N and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1133

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1133: Given a point M_c , a point N and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1134

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1134: Given a point M_c , a point N and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1135

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1135: Given a point N , a point O and a point T_a , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1136

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1136: Given a point N , a point O and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1137

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1137: Given a point N , a point O and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1138

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1138: Given a point N , a point T_a and a point T_b , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1139

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1139: Given a point N , a point T_a and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.

Problem 1140

*Generated automatically by ArgoTriCS
Developed by Vesna Marinković, University of Belgrade*

1 Problem

Problem 1140: Given a point N , a point T_b and a point T_c , construct the triangle ABC .

2 Status

Status of the problem is unknown.