FOREWORD

The report studies the dynamical aspects of the artificial satellites of the Earth from the point of view of whole population. The probability of collision and properties of the orbits after a break-up of a satellite are studied.

I. THE STABILITY OF SATELLITE ORBITS

1. INTRODUCTION

A satellite in orbit about an isolated spherical planet with no atmosphere would follow the same elliptic orbit, without variation, for thousands of revolutions. For the real Earth this simple picture is greatly altered, because of three different perturbing forces:

1. The variation of the Earth’s gravitational attraction resulting from the flattening of the Earth at the poles, and other departures from spherical symmetry, such as the ”pear-shape” effect.

2. The air drag, caused by the rapid movement of the satellite through the upper atmosphere.

3. The forces due to the Sun and Moon - mainly their gravitational attraction, but also the effects of solar radiation pressure.

For most satellites, these are the three types of force which produce major changes in the orbits. Many other perturbations exist, but do not normally produce large changes, and will be ignored here, in providing a basic description. These neglected perturbations include those due to: upper-atmosphere winds; solar radiation reflected from the Earth; the Earth tides and ocean tides; the precession of the Earth’s axis in space; resonance with the Earth’s gravitational field; and relativity effects. Although these perturbations will be ignored, it should be remembered that they can be important for special satellites. For example, balloon satellites are affected by the pressure of Earth-reflected radiation, and a resonance with the Earth’s gravitational field can cause some considerable changes in special orbits.

The motion of satellite orbit, which will be further described, must be considered from a point of view of space, in which the Earth also rotates (i.e. like inside the satellite orbit). So that the trace of the orbit on the surface of the Earth arises as a superposition of those both motions.

The three main perturbations will now be described in turn.

2. THE MAIN PERTURBATIONS OF SATELLITE ORBITS

2.1 The elliptic orbit: definitions

The orbit of a satellite about an isolated spherical planet would be an ellipse with one focus at the center of the planet. The size and shape of the ellipse are defined by its semi-major axis, \( a \), and eccentricity, \( e \), as shown in Fig 1. As the satellite, \( S \), moves round the orbit, its nearest approach to the Earth is at perigee, \( P \), where the distance from the Earth’s center, \( C \), is \( a(l - e) \). The furthest distance of the satellite from the Earth is at apogee, \( A \), where the distance from the Earth’s center is \( a(l + e) \).
The orientation of the orbital plane in space is defined by the right ascension of the ascending node, Ω, and the inclination to the equator, \(i\), as shown in Fig 2, relative to a sphere with center at the Earth’s center, \(C\). The first point of Aries, \(\gamma\), is a fixed point in the sky used as a reference for measuring star positions, and \(\Omega\) is the angle measured positive eastwards along the equator from the first point of Aries to the ascending node, \(N\), the point where the satellite track crosses the equator going north. The inclination, \(i\), is the angle between the equatorial plane and the orbital plane, measured positive northwards as shown in Fig 2.

Finally, the orientation of the orbital ellipse within the orbital plane is specified by the argument of perigee, \(\omega\), which is the angle \(\angle NC P\) in Fig 2. Thus if \(\omega = 0^\circ\), 

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the perigee is on the equator going north; if $\omega = 90^\circ$, perigee is at maximum latitude north; if $\omega = 180^\circ$, perigee is again on the equator, this time as the satellite is going south; if $\omega = 270^\circ$, perigee is at maximum latitude south.

These five parameters, together with a sixth parameter, which specifies the angular travel of the satellite around its orbits, completely describe the motion in an unperturbed elliptic orbit. The angular position can be specified by the angle $PCS$ in Fig 1, denoted by $\theta$, and known as the true anomaly. Other angular variables are sometimes used, however.

The five parameters $a$, $e$, $i$, $\Omega$, $\omega$, together with the angular travel parameter, are known as the orbital elements.

### 2.2 Perturbation due to the Earth’s gravitational field

The Earth is not exactly spherical, being appreciably flattened at the poles: the equatorial radius is 6378.14 km, while the polar radius is only 6356.78 km. Because of this flattening of more than 21 km, the gravitational attraction of the real Earth is slightly different from that of a sphere, and as a result the elliptic orbit of a satellite suffers some changes. Although its size and shape remain almost the same (the changes in $a$, $e$ and $i$ being small), the orbit is no longer fixed in space. The orbital plane rotates about the Earth’s axis, with the inclination remaining constant. If the satellite is heading eastwards, the orbital plane swings to the west, as shown in Fig 3, and the right ascension of the node, $\Omega$, steadily decreases.

These effects are quite large for a satellite close to the Earth (see App. 1).

The rate of change of $\Omega$ depends on the inclination of satellite orbit, declining with growing inclination. Most satellites have inclinations less than 90° and travel eastwards. The orbital plane then swings from east to west, at a rate of about $8^\circ$/day for a near-equatorial orbit, $4^\circ$/day for an orbit at inclination 60°, or zero for a polar orbit.

In addition to this movement of the orbital plane, the orientation of the orbit within the plane also changes, so that the perigee latitude is continually moving. For a near-equatorial orbit, the perigee moves in the same direction as the satellite is travelling, at a rate of about $16^\circ$/day; for a polar orbit the perigee moves in the opposite direction to the satellite at about $4^\circ$/day. For an orbit at an inclination of 63.4°, which is often considered the critical inclination, perigee remains at a fixed latitude. Because of these two effects, a satellite may move to any point within a toroidal volume bounded by its perigee and apogee heights, and a maximum latitude north and south, which is equal to the inclination, as shown in Fig 4 and 5. The cross-section of such a toroid is shown on Fig 5.

Although the Earth’s flattening is its greatest departure from a sphere, there is also a slight asymmetry between the northern and southern hemispheres, and this leads to a north/south difference in the gravitational field, which produces appreciable orbital perturbations. This asymmetry is usually called the pear-shape effect, and it means that sea level at the north pole is about 44 m further from the equator than is sea level at the south pole. The resulting asymmetry in the Earth’s gravitational attraction causes small changes in the shape of the orbit of a satellite, producing an oscillation in the orbital eccentricity $e$, which leads to an oscillation in the perigee distance $a(1 - e)$, although the semi-major axis $a$ remains constant. The period of the oscillation in perigee distance depends on the motion of perigee and the amplitudes vary correspondingly (see App. 1).

This oscillation due to the effect of the Earth’s north/south asymmetry can obviously have an important effect on the perigee height, especially for satellites at inclinations between 60° and 90°. As the period of the oscillation is the same as that of $\omega$, it is between 25 days, for a near-equatorial orbit, and several years, for an orbit near 63° inclination.
Earth rotates Eastwards

Orbit swings Westwards

The gravitational pull of the Earth’s equatorial bulge makes the orbital plane of an eastbound satellite swings westward.

Fig. 3

The toroidal volume traversed by a satellite

Fig. 4
The effects already discussed are the most important perturbations for close Earth satellites. However, for satellites in distant orbits, for example geosyn-
chronous orbits at a height of 36000 km, the effect of the Earth’s oblateness is very much smaller, and the variations of the gravitational field with longitude rather than latitude become important. See section 3.

2.3 The effects of air drag

The upper atmosphere is extremely rarefied, having a density of only about 10 grams per cubic kilometer at a height near 350 km. However, a satellite travels at nearly $8\text{ km per second}$, and its collisions with the air molecules are frequent enough to create an appreciable drag force. If the orbit is non-circular, the air drag is much greater at perigee than at apogee, and the effect of drag on the orbit is to retard the satellite at perigee, so that it does not fly out so far at apogee. The orbit contracts and becomes more nearly circular, as shown diagrammatically in Fig 6. But the perigee height decreases only very slowly. In terms of the orbital elements $a, e, i, \Omega$ and $\omega$, air drag reduces both $a$ and $e$, but $a(1-e)$ decreases only very slowly. The effect of air drag on the other three elements is only indirect; for example, the rate of change of $\Omega$ is dependent on $a$ and increases as $a$ decreases.

If the orbit is initially circular, air drag acts all round the orbit, and its effect is to reduce the height of the satellite gradually, so that the satellite slowly spirals inwards.

![Fig. 7 Eccentricity of Cosmos 462](image_url)

For both circular and elliptic orbits, the drag rapidly increases as perigee height decreases and, when perigee descends to $80 – 100\text{ km}$, the satellite can no longer remain in orbit, and begins its final plunge into the lower atmosphere. It should be noted that although air drag retards the satellite at perigee, the overall effect of air drag is to make the satellite travel faster. Its orbital period decreases as its lifetime proceeds, and the final decay usually occurs when the orbital period has fallen to about 87 minutes. An example of the decrease in the orbital eccentricity is shown in Fig 7, for the satellite Cosmos 462, which was in orbit from December 1971.
to April 1975 (Ref 2) The decrease in eccentricity tends to be more rapid as the satellite approaches the end of its life, but there are many irregularities resulting from the wide variations in air density, which are discussed in more detail in section 4.

Cosmos 462 was at an inclination of $66^\circ, 0$, where there is only a small oscillation in eccentricity due to the north/south asymmetry of the Earth’s gravitational field. This oscillation has an amplitude too small to be visible in Fig 6, and the corresponding oscillation in perigee distance is only about 2 km.

The perigee height above the Earth’s surface during the life of Cosmos 462 is shown in Fig 8 (Ref. 3). Although the distance of the perigee from the Earth’s center varies only slowly, there is an oscillation in Fig 7, with an amplitude of about 10 km, which results from the change in the radius of the Earth due to the Earth’s flattening, with the height above the Earth’s surface reaching a minimum every time the perigee crosses the equator. This oscillation is superposed upon a slow but increasing tendency for the perigee height to fall as a result of the effect of air drag. At the end of the life this decrease due to air drag becomes dominant, and the perigee plunges to heights below 100 km.

2.4 Lunisolar perturbations

2.4.1 Lunisolar gravity

Both the Sun and Moon exert a small gravitational attraction on all satellites, and they perturb satellite orbits because the attraction is greater when the satellite is on the side of the Earth nearer to the Sun (or Moon) than when it is on the other side. In general, lunar gravity is about twice as effective as solar gravity in perturbing satellite orbits. For a satellite close to the Earth, lunisolar gravitational perturbations produce small oscillatory changes in all the orbital elements except the semi-major axis. For most close Earth satellites the effects are quite small, usually less than about 2 km, and take the form of oscillations which have periods ranging from about 10 days to about a year, or sometimes more. An example of the lunisolar gravitational perturbations for a close satellite is shown in Fig 9, where the change in inclination is shown for Skylab 1 rocket, which was in a nearly circular orbit at a height near 400 km, at an inclination of $50^\circ$. It is clear that
several oscillations of different period are present, and over a time interval of 40 days the perturbation amounts to about 200m. This is quite typical for a satellite near the Earth.

![Fig. 9 Lunisolar gravitational perturbations to the orbital inclination of Skylab 1 rocket (1973-27B)](image)

However, the complete lunisolar perturbation is made up of many different terms, each with its own period. With so many possibilities, it is quite likely that one or more of these periods may be very long, perhaps several years. The orbit is then near-resonant and the perturbation may build up for several years, becoming much larger than normal, because it acts in the same direction for such a long time. For example, the perturbation may be only 10 meters per day, but if it continues for 500 days, it will amount to 5km.

For satellites at greater distances from the Earth, the lunisolar perturbations are greater, being approximately proportional to the orbital period, for a given eccentricity and inclination. For synchronous orbits, with period over 1400 minutes, the effects are very important: see section 3.
An increase in eccentricity also increases the effects of lunisolar perturbations, and these are particularly severe for the many Molniya satellites having eccentricities near 0.7 and orbital periods near 720 minutes. Fig. 9 shows the variation of perigee height for Molniya 2B. The main oscillation has an amplitude of several hundred kilometers, and in the end forces the perigee to descend to such a low altitude that the satellite decay (Ref 4.).

2.4.2 Solar radiation pressure

The pressure of solar radiation is very small \( (4.6 \times 10^{-6} \text{ newtons/m}^2) \), but its effect on satellite orbits can be appreciable. For balloon satellites the effects of solar radiation pressure produce the largest perturbations of all.

The acceleration of a satellite produced by solar radiation pressure is directly proportional to the satellites cross-sectional area divided by its mass – the area/mass ratio, as it is usually called. A normal satellite, constructed of metal, usually has an area/mass ratio near 0.01\(m^2/\text{kg} \), and the perturbations due to solar radiation pressure are usually very small, less than 1km.

For a balloon satellite, however, the area/mass ratio and the perturbations can be a thousand times greater. For the Echo 1 balloon, the area/mass was about 10\(m^2/\text{kg} \), and the main solar radiation pressure perturbation had a long period, about 20 months, with the result that the perigee height of Echo 1 oscillated with an amplitude of 500 km, as shown in Fig 10. The perturbations were so great that the perigee often became converted into the apogee. Even for satellites of normal construction, the effects of solar radiation pressure can be greater than the effects of air drag at heights above 500 km, though both effects are small.

**Fig. 10 Effect of lunisolar perturbations on the perigee height of Molniya 2B**
Fig. 11 Effect of solar radiation pressure on the perigee (or apogee) height of Echo 1

Fig. 12 Perigee height of the balloon satellite Dash 2 (1963-30D) forced down by resonant solar radiation pressure perturbations
Resonance is possible for solar radiation pressure in the same way as for the lunisolar gravitational perturbations, and the most striking example is a balloon satellite in a polar orbit, which can suffer a continuing decrease in perigee height, which eventually causes the satellite to decay in the Earth’s atmosphere. An example is the small balloon satellite Dash 2 (1963-30D), for which the variation in perigee height from 1968 to 1971 is shown in Fig 12: the perigee height decreases from 2000 km in 1968 to 300 km just before decay in April 1971. Initially, this satellite had an inclination of 88.4°, but the inclination also suffered near-resonant perturbations, and by the end of the life had been reduced to 84°.4 (Ref. 5).

3. GEOSYNCHRONOUS (24-HOUR) ORBITS

3.1 Introduction

A satellite in an eastbound equatorial orbit at a height of 36000 km travels at the same angular rate as the rotating Earth, and completes an orbit in one day. This is a most useful orbit for communications satellites, because the satellite, as seen from the Earth, appears at a fixed point in the sky, apart from perturbations.

For a synchronous satellite, the perturbations due to air drag and the Earth’s flattening and pear-shape are negligible, and the chief perturbations are those due to (a) the variation in the Earth’s gravity field with longitude, and (b) lunisolar gravitational perturbations.

3.2 Perturbations due to the Earth’s gravitational field

The variations of gravity with longitude are important because a geosynchronous orbit is another example of a resonant orbit. Any variations of gravity with longitude will produce small forces tending to alter the satellite’s longitude, and since the satellite continues in position near that longitude, those forces will operate day after day until eventually the satellite is forced away from the original longitude (Ref. 6).

Because of the variations of gravity with longitude, the Earth’s equator is not an exact circle. If the Earth was cut in half through the equator, the cross-section of the sea level surface would be seen to depart from a circle by up to about 100m. Although the detailed shape is complex, an ellipse provides a helpful first approximation, and the major axis of the ellipse points towards longitudes of 30°W (mid Atlantic) and 150°E (near New Guinea), with the minor axis pointing towards longitude 60°E (south-west of India) and 120°W (south of Los Angeles). A synchronous satellite tends to drift away from the major axis towards the minor axis. Thus synchronous satellites situated at longitudes near the Atlantic or New Guinea tend to be unstable, and would drift considerably in longitude over a period of a year or more; whereas those placed south-west of India and south of Los Angeles tend to be stable. In practice, all synchronous satellites are provided with small thrusting rockets to correct orbital errors, and the satellites can be kept at the required longitude with very little expenditure of fuel.

3.3 Lunisolar perturbations

A synchronous satellite in orbit about the Earth travels in the Earth’s equatorial plane, because the Earth exerts the strongest attraction on it. However, the Sun and Moon also attract it, and they move in a different plane: they travel (on average) in the plane of the ecliptic, which makes an angle of 23° with the plane of the Earth’s equator. The long-term effect of the Sun and Moon on the equatorial synchronous orbit can be regarded as similar to the effect of a ring of matter in the ecliptic plane. The effect of this lunisolar attraction is for the pole of the orbital plane to precess about the pole of the ecliptic (Ref. 7).
The orbital plane of a synchronous satellite, initially equatorial, rotates at a rate of a little less than 1 degree per year about a direction in the equatorial plane, nearly at right angles to its intersection with the ecliptic. If the perturbation is allowed to continue freely, the inclination of the orbit to the equator would gradually increase to about 15°, and then decrease to nearly zero after a time interval of just over 50 years. A small change in inclination is not very important for a communications satellite, but since the satellites have capability of manoeuvring, the inclination can be corrected at the same time as the longitude, although the thrust required is considerably greater than for the correction of the effects of the Earth’s gravitational field. Thus the lunisolar perturbations are much stronger than those of the Earth’s gravitational field, but less damaging over time intervals of up to about 5 years.

These perturbations do not significantly alter the distance of the satellite from the Earth’s center. Any danger of collision with communications satellites which are no longer operational can therefore be avoided if such satellites are moved into slightly higher orbits when their useful life for communication has been completed.

4. PREDICTING THE DECAY SATELLITES

As explained in section 2.3, most satellite orbits contract slowly under the influence of air drag, and the drag depends on the density of the upper atmosphere, which varies widely and irregularly, and depends in particular on the future variations of solar activity, which are at present unpredictable in detail. There are 4600 satellites in orbit now being tracked, and these are decaying at the rate of about 10 per week. The majority of these satellites are small fragments which burn up in the lower atmosphere, but an important proportion (about 2 per week) are large objects having a mass in orbit of more than a ton, and fragments up to 10 kg in weight may reach the ground when these large objects decay.

The procedure for predicting decay is first to assume that the air density will remain constant during the rest of the life of the satellite, and to calculate, either from theory or by numerical integration, the date at which the perigee will descend below 100 km (Ref. 4). The observed current rate of decay of orbital period is used as a measure of the air drag. Then adjustments are made to this basic calculation to allow for the variations in air density. In the absence of such variations in air density the lifetime could be accurately predicted if the satellite retained a constant cross-sectional area. But in practice air density always changes, and lifetime estimates having an accuracy of ±10% are about the best that can consistently be achieved, although occasional good luck will give an apparently more accurate result.

If the perigee height is greater than about 500 km, the orbital lifetime is usually 20 years or more. But most satellites have perigee heights between 200 and 500 km, and for these the orbital lifetime may be only a few days, or a few months, or a few years, depending on the exact orbit and the area/mass ratio of the satellite.

The lifetime is inversely proportional to the air density, and the wide variations in air density are indicated in Fig 13, which shows the variation of density with height, for heights between 100 an 1000 km, with the density on a logarithmic scale in nanograms per cubic meter (or grams per cubic kilometer) (Ref. 8). The scale in Fig 12 runs from $10^{-3}$ to $10^3$, so the density at a height of 150 km is about a million times greater than at a height of 1000 km. In predicting orbital lifetimes, the important feature of Fig 12 is the great variation in density at a given height when the solar activity changes from sunspot maximum, as in 1969 and again in 1979, to sunspot minimum, as in 1976. At a height of 500 km the density is about 10 times greater at sunspot maximum than at sunspot minimum. So a satellite in a circular orbit at a height of 500 km might have a lifetime of 5 years if launched a little before solar minimum, but a lifetime of 6 months if launched at solar maximum. Fig. 13
also shows that there is a large variation in density between day and night, the density being much greater by day than by night, by a factor of up to about 5 at a height of 500km. This variation must also be allowed for in predicting lifetime.

Fig. 13 Density versus height from 150 to 1000 km for low and high solar activity

Unfortunately, the progress of the solar cycle can at present not be predicted at all accurately, and the Sun also has vigorous variations from day to day which are also partially unpredictable. Normally there is a fairly steady outflow of plasma from the Sun, but this is disrupted by shock waves when a solar disturbance occurs. When the shock waves reach the Earth the upper atmosphere responds strongly. At heights near 600km the density may increase by a factor of up to 8 at the time of a solar storm, and even as low as 180km, the density may be doubled in a few hours. So predictions of a satellite decaying in a week or 10 days can be seriously upset if an unpredicted solar disturbance occurs on the next day.

The unpredictability of solar activity is the main difficulty in predicting lifetimes of several years or a few days. With lifetimes of intermediate length, between one month and one year, another partially unpredictable variation becomes most important: this is the semi-annual variation in density. Fig 14 shows the variations in density at a fixed height near 250km averaged over the years 1972-5, with the effects of solar activity removed and the day-to-day irregularities smoothed out (Ref. 3).
There is an oscillation approximately every 6 months, with maxima in April and early November, and minima in January and July. The density changes by a factor of 1.7, so lifetime predictions can be in error by up to 30% if no allowance is made for the semi-annual variation. Unfortunately, the semi-annual variation itself changes appreciably from year to year, and Fig 14 shows the different variations in the individual years 1972-4. The future variation of the semi-annual effect is not predictable in the present state of knowledge of upper atmosphere physics. In the 1960s the semi-annual variation was appreciably weaker than shown in Fig 14, and also changed considerably in strength from year to year, as well as changing its shape slightly each year.

To summarize, it can be said that the prediction of satellite lifetimes is likely to remain an inaccurate procedure. For lifetimes of between 1 year and 20 years, the long-term forecasts of solar activity during a sunspot cycle are inadequate. For lifetimes between 1 month and 1 year, solar activity is still a major source of error (except near sunspot minimum), and so is the future course of the semi-annual variation in density. For lifetimes less than 1 month, the day-to-day irregularities – in density resulting from short-term solar disturbances are the major source of error. To achieve a prediction accurate to ±10%, in face of these problems, calls for considerable skill and experience, taking into account the interaction of many factors - for example the future variations of perigee height due to both geometrical and dynamical factors (see Fig. 8, 10 and 11), and the synchronization of such variations with the changing solar activity.

Although most satellites decay because their orbits steadily contract under the action of air drag, as shown in Fig 6, a small but important proportion of satellites have their orbital lifetimes brought to an end because the perigee is forced down into the lower atmosphere by lunisolar perturbations. Two examples have already been
given: a high eccentricity orbit \((e = 0.7)\), shown in Fig 10; and a balloon satellite in a resonant orbit, shown in Fig 12. If, at the end of the life, the perturbations are still driving the perigee rapidly downwards, the decay may be so rapid that it can be called catastrophic. For example, a satellite with an orbital period of 2 days and eccentricity 0.9 may have its perigee height reduced by as much as 30 km per revolution, by lunisolar perturbations. A passage of perigee with a perigee height of 120 km would reduce the speed at perigee only slightly, but the next revolution, with perigee height 90 km, would probably cause immediate decay. An example of such a satellite was Hawkeye (1974-40A), which decayed on 30 April 1978.

Since the lunisolar perturbations are accurately predictable and drag may not have an appreciable effect until the last few revolutions, the decay of such orbits is in principle quite accurately predictable. Unfortunately, however, such satellites are usually very difficult to track, because they move out to very great distances from the Earth. Consequently, their orbits are not well-determined, and the accuracy of the predictions is usually limited by the accuracy of the orbits available. Occasionally however, as with Hawkeye, which carried a radio transmitter, accurate predictions can be made for the decay of satellites which suffer strong perturbations in perigee height.

II. DISTRIBUTION OF SATELLITES IN SPACE

1. INTRODUCTION

In the following we shall try to analyze statistically the orbits of bodies orbiting the Earth. We concentrated our effort on bodies of sufficient size to be tracked. We call them generally "bodies" or "objects". Among them we have larger bodies, mostly proper satellites or rockets, the basic orbital elements of which are more or less regularly computed and published.

There are also smaller bodies (international or incidental) which are capable of tracking but of no special importance for regular orbit determination. In the following, we shall call them "fragments".

Without any doubt, the Earth neighborhood is contaminated by still smaller bodies of masses measurable in grams, which escape tracking. It would be very troublesome work to try to estimate their number; to say something about their orbits will be still more difficult. Therefore, those bodies, which we shall generally call "debris", were left unattended in our further analysis. Only by analogy with the faith of fragments, we feel that their number is slowly diminishing - as it was during 1979, at least.

Chapter II is supplied by a "Supplement" which is merely a machine output giving the formal distribution of orbiting bodies as listed in (Ref. 1), according to several technical criteria. This is intended for rough information only.

2. NUMBER OF BODIES

The number of bodies orbiting the Earth is a time-variable value - old bodies decay and new are launched. The probably most authoritative lists of objects which are tracked are issued regularly by NASA, as "Satellite Situation Report". Those publications give numbers of objects in orbits (not necessarily geocentric); a smooth curve through those values is on Fig. 14a. Probably, the most striking feature of that curve is the decline of the number of bodies, during 1979. Whereas the list of 28. 02. 1979 gives 4658, the date 30. 04. 1979 quotes 4633 and 30. 06. 1979. 4620 objects in orbit.
Of all bodies, the fragments make about one half. For further treatment, we took the data from "Satellite Situation Report" of April 30, 1979 (Ref. 1). There, the fragments count 1995. Most of the following statistical results are made both for all objects and for objects without fragments. First, we considered the fragments to be of substantially smaller masses and sizes and secondly, the orbital elements of those smaller bodies were only estimated. We took for the inclination of a fragment the inclination of the parent body; the perigee height was taken as 101 km in cases when some fragments of common origin already decayed. Otherwise, we based our estimation on the elements of the parent body.

The number of bodies in our list is 4516 whereas the original "Satellite Situation Report" (Ref. 1) quotes 4433 objects in geocentric orbits to that date. The difference is, in our opinion, due to the fact that we used the list of decayed objects from 31. 12. 1978.

However, some of the fragments decayed again till April 30, 1979; this would explain the difference of 83.

3. DISTRIBUTION OF BODIES IN ORBITAL PLANES

The number of bodies with respect to the inclinations of their orbital planes is plotted on Fig. 15.
The families of fragments make most of the "singularities" of the picture. Nevertheless, even without fragments (Fig. 16) there are some heavily populated planes: $i = 74^\circ$ (368 bodies), $i = 82^\circ$ (139) and $i = 83^\circ$ (121). This is due to the presence of Soviet satellites of "Cosmos", series which are being launched from the northern launch site at Plesetsk on 63° latitude.
Fig. 16 Distribution of bodies in terms of orbital inclination, if the number of bodies in a plane is greater than 3. Without fragments
The division of inclinations considered on Fig. 15 is $1^\circ$; the planes with at most 3 bodies are piled around the zero-point. The list of the orbital inclinations with at most 3 bodies is given in app. 3.

4. DENSITY OF OBJECTS IN SPACE

The much discussed problem of the density of the artificial bodies orbiting the Earth is a very complicated problem. The difficulty does not lie in a great number of bodies, but on the contrary - the number of bodies is too low to make a good realistic statistical treatment. The lower number of bodies gives rise to very occasional local changes in density caused e. g. by great number of fragments originating from one body or by random piling of bodies of different orbits. Nevertheless, we tried to make an estimate of the density averaged over time long enough the positions of the orbital planes to be periodically changed (app. during one year). The data are again taken from Ref. 1.

As it was already said, a satellite can take during 21 such a long time all positions within a toroidal space (Figs 4 and 5). We shall estimate the average number of objects which can be found during a day in a specific toroidal belt bounded by two concentric (geocentric) spherical volumes and two planes of different inclina-
tions to the equator. Our reasoning is explained in detail in App. 2. The density is then merely the number found divided by the respective volume.

The results are shown on Fig. 17 (all bodies) and Fig. 18 (without fragments). On the latter figure, neglected bodies (fragments) make about 44% of the number of objects taken into account on Fig. 17. From that point of view the Fig. 18 does not correspond to a real density distribution too precisely: on the other hand, Fig. 18 is certainly based on bodies of greater size.

Fig. 18 Density of greater objects (fragments excluded) in percentage of maximum density, which is 1 body per 60 mil. km³ in the region of 900-1000 km height and 0-10 degrees inclination

On the figures, the horizontal scale gives the distances from the Earth surface in km; the Earth is there depicted as a point (it would nor be able to make a picture in proper scale taking into account the dimensions of the Earth).

The density distribution shows some peculiarities:

a) The density does not decline steadily with growing distance from the Earth surface. Although there is a sharp maximum (on Fig 17) in the region between 100 and 200 km density grows to another peak at 800 - 900 km. This maximum is about 2/3 of the first one. Then, the density diminishes but a gain it comes to a third peak (this time smaller) at about 1400 km. Only after that height, we have a steady decrease and in the distance of 3000 km there is density of only one thousandth of the main maximum.
b) Comparing Fig. 17 and 18 we see that the first maximum (150 km) is caused mainly by the fragments, approaching their decay; as mentioned previously, we assumed deliberately for every bunch of fragments of common origin the perigee height of 101 km in case some of them already decayed. Therefore this region (100 - 200 km) must not be necessarily as densely populated as shown.

c) The secondary maximum on Fig. 17 corresponds to the main one of Fig. 18 (about 900 km). Here, the effects of the atmosphere are already lower so that we can consider this region to be the mostly populated one, in a longer run.

d) As it could have been expected the region of high inclinations (between 80 and 100 degrees) does not belong to densely populated areas; the decline in density with growing distance goes here more rapidly.

e) Even if the density considered here should be understood rather in terms of probability of possible denser or looser piling of objects in a certain region of near-Earth space, it is still possible to make a numerical estimate of number of bodies. The maximum density in Fig. 17 occurs between $h = 100 - 200$ km and is about $0.6 \times 10^{-7}$ bodies/km$^3$/day, or one body per day in 17 millions km$^3$. The region of maximum density of Fig. 18 (900 km) has $0.16 \times 10^{-7}$ bodies/km$^3$/day, which is about 1 body per day in 60 mil. km$^3$.

![Fig. 19](image)

**Fig. 19** The density profile of the $i = 0 - 10^\circ$ region. The upper part gives
f) The density changes continuously. On Figs. 19 and 20 we have the "density profiles". i. e. the density changes along radial directions from the Earth. The 23 profiles concern the equatorial regions ($i = 0 - 10$ degrees). The vertical scale shows the density in terms of bodies/km$^3$/day. Again, Fig. 19 corresponds to all objects and Fig. 20 does not include fragments. For comparison, the upper parts of Figs. 19 and 20 shows the distribution of perigee heights bodies included in the statistics.

![Fig. 20 The density profile of the $i = 0 - 10^\circ$ region (no fragments). The upper part gives the distribution of perigee heights](image)

5. SPECIAL ORBITS

a) Geosynchronous orbits

There is a small augmentation of the density in the equatorial region, in the distance of 35780 km from the Earth surface. This is due to the presence of geosynchronous satellites. The most important of them are certainly the near geostationary ones, with circular orbits in the equatorial plane. They are more or less stable with respect to the Earth (see Chapter 1) so that in a reference frame fixed with the Earth we can have their distribution with relatively high precision. This is plotted in Fig. 21, where we introduced also the announced future missions. Data are taken from a list of Ref. 2.
b) Sun-synchronous orbits

In a geocentric reference system, in which we are considering the motion of a satellite, the Earth rotates and the satellite orbital plane moves, too (see Fig. 3). Furthermore, the revolution of the Earth causes an apparent motion of the Sun in that system, with a velocity $\approx 1$ degree per day. Therefore, if the satellite orbital plane rotates by 1 degree per day in opposite direction, the relative position of the plane and Sun remains fixed. This circumstance can be used for placing a satellite into an orbit from which Sun can be seen continuously.

Since the rotation of the satellite orbital plane is due chiefly to the Earth’s equatorial bulge we can use equation (1) of App. 1 to find such a combination of $a$, $e$ and $i$ to have $\Delta \Omega = 1^\circ$/day. Keeping $a$ and $e$ fixed there is just one orbital inclination for which the condition of Sun-synchronous orbit is fulfilled. This can be seen on Fig. 22, where we have the orbital inclination necessary to produce a sun-synchronous orbit in terms of perigee height and eccentricity. It can be seen that all Sun-synchronous orbits are retrograde (satellite moves from east to west, against the Earth’s rotation). There are limits above which the existence of a
Sun-synchronous orbit is impossible.

However, we can use the gravitational attraction of other celestial bodies to help us in establishing a special orbit; e. g. the Moon’s gravitational influence can be used to keep the orbit of a satellite in the Sun-synchronous condition. On Fig. 23 we see an orbit going beyond the Moon and proposed for the studies of the geomagnetic tail (Ref. 3). A double approach to the Moon forces the satellite to spend most of its lite within the Earth’s geomagnetic tail region (bounded by the "magnetopause").
Fig. 22 Inclination $i$ of Sun-synchronous orbit in terms of perigee height $h_p$ and eccentricity $e$

Fig. 23 Proposed Sun-synchronous orbit for geomagnetic tail mission
III. REMOVING OF INACTIVE SATELLITES FROM THEIR ORBITS

1. INTRODUCTION

As it was shown in the preceding sections, artificial satellites are moving around the Earth in orbits, which are relatively slowly changed by different natural perturbing effects (air drag, gravitational attraction of the Sun and Moon etc.). If we need to change the orbit abruptly in definite manner, we have to apply some artificial force. In outer space, the only possibility is to use a rocket propulsion, since it is independent of surrounding matter and therefore works in vacuo. It is not necessary to have a real "rocket motor" on board of the satellite, since more simple gas thrusters could also do the job as it will be shown later on (see Paragraph 3).

An active removal of the satellite from its orbit requires some of its systems to be operational at that time. The crucial ones are here communication systems and systems of satellite’s orientation and stabilization. The former systems are needed to provide all necessary commands and the latter to secure the proper direction of applied force. These conditions are satisfied for a few satellites so far. For example, all Soviet orbital stations of Salyut type and cargo-spacecraft of the Progress type are pushed into the Earth’s atmosphere before their active lifetime expires. Similarly, Intelsat communication satellites are removed from their geostationary positions when 8 newly launched satellite should replace their position in the global communication network.

Unfortunately, overwhelming majority of orbiting objects are inactive satellites (or debris) which could not be moved from their orbits without an external interference. Such satellites might be caught by remote manipulator of the space shuttle or some kind of maneuvering rocket powered unit should be attached to them before the start of actual removal from the orbit.

However, optimal methods of removal (with respect to energy) could be found independently of the purely technical problems. These methods will be discussed in the following paragraphs.

In principle, there might be some more or less rapid changes of the orbit using natural forces. E.g. the close encounters of a satellite with natural bodies might change the orbit profoundly (see Chapter II). Or, one can also change the effective area to mass ratio of a satellite by deploying some kind of large "wings". This would mean the greater influence of the non-gravitational forces (atmosphere, radiation pressure) which in turn may mean faster decay. But that does not seem to be very practical; moreover, the reentry into the atmosphere over uninhabited areas could not be guaranteed.

2. ORBITAL TRANSFERS

A satellite orbiting around the Earth has a constant energy, which is a sum of kinetic and potential energies. The satellite’s orbit is generally elliptical (circular one is only a special case). Moving from the farthest point of the orbit (called apogee) towards the lowest point (perigee), the potential energy of the satellite is decreasing, therefore its velocity (which is a measure of the kinetic energy) increases to make the sum of both energies constant. At the ascending half of the orbit, the situation is reverse - the velocity decreases with height. Consequently, the satellite’s velocity is maximal at perigee and minimal at apogee.

To change the orbit of the satellite, it is necessary to change its energy. The only possibility is to change the kinetic energy - because the potential one is determined by the instantaneous position of the satellite. Therefore, changing the velocity by a given amount, one can change the orbit of a satellite correspondingly.

The obvious condition for the optimization of orbital changes (or so called orbital transfers) is the minimization of the required amount of the necessary velocity...
change. This is because such a maneuver requires also minimum of propellant to be burned (see Paragraph 3).

For the purpose of this study, one can suppose that the velocity change is produced very quickly, i.e. by an impulse. Its duration is only few minutes compared with several hours of satellite’s orbital period (Ref.1). The resulting change in flight direction occurs practically instantaneously, so that the corresponding equations are not difficult to establish (see Appendix 4).

Dynamically, one can distinguish between a planar and a non-planar orbit change, depending on whether or not the inclination of the satellite’s orbit is altered. Since the velocity vector lies in the orbital plane, it is always difficult to change this plane (i.e. it requires great velocity impulse). Therefore, only planar orbital changes will be considered here.

To simplify the problem further, one can suppose that the initial orbit is circular. The optimizational analysis of orbital transfers between circular orbits was performed as early as in 1925 by a German scientist W. Hohmann. He proved, that the trajectory connecting two circular orbits in such manner that it is tangential to both of them (i.e. orbits are not crossing) will demand the minimum velocity change. During such a transfer, the satellite travels exactly half of the elliptical transfer orbit. This optimal transfer orbit is often called Hohmann orbit (for Hohmann transfers).

Transfer orbits, which are longer or shorter than Hohmann orbits demand greater velocity impulses. Similar, but a little bit more complicated analysis could be performed for orbital transfers involving two elliptical orbits (Ref 2) Generally speaking, it is preferable to perform a Hohmann transfer between the perigee of the initial orbit and the apogee of the final orbit. Since the mutual orientation of two ellipses can be different, more than two impulses might be necessary to accomplish such a transfer.

3. PROPELLANT CONSUMPTION

To realize a velocity change of a satellite, which is necessary to remove it from its orbit, one needs some kind of rocket propulsion. Only classical chemical rocket systems will be discussed here, since unconventional systems (e.g. ion engines, nuclear propulsion) are not generally used as yet.

The chemical rockets often use two components, fuel and oxidizer, burned in the propulsion chamber to provide the necessary rocket thrust. Before this chemical reaction ("burning") begins the propellant is usually in a liquid or solid state.

In the solid propellant motors, the propellant usually contains oxidizer and fuel which are nicely mixed and stored directly in the burning chamber. This simplifies the construction, increases reliability and ensures good storability in orbit. However, those motors are not as efficient as liquid propellant motors and it is very difficult to control the duration of the burning (i.e. the magnitude of the impulse). Consequently, they could be used only for (one) orbital maneuver whose magnitude is known before the launch of the satellite.

Retro-rockets for landing of the early types of manned spacecraft, or rocket stages build-in the construction of communication satellites to provide velocity kick necessary to attain circular geostationary orbit are good examples.

In the liquid propellant motors, an oxidizer and a fuel are usually stored in separate tanks and transported by a feed system to the reaction chamber. The initial ignition is made by some chemical or electrical agent.

However, there are some propellant combinations which ignite spontaneously when they come in contact with each other (so called hypergolic propellants). To provide a small reactive thrust, one component (monergol) could be used to escape the chamber without burning (e.g. hydrogen peroxide or nitrogen released under
Such small "control nozzles" are used mainly to control orientation and stabilization of the satellites.

Among the advantages of the liquid propellant motors, most important are their high efficiency, long operation time, controllable thrust direction, throttling and duration and possibility of restart. However, most efficient propellants (e.g. hydrogen-oxygen combination) are not storable, thus for our purpose (to remove the satellite from the orbit after prolonged time interval) they can not be used.

The efficiency of the rocket propulsion system can be expressed in terms of the specific impulse, which corresponds to the effective exhaust velocity of gases from the motor.

A typical two-component propellant used for orbital maneuvers is a combination of nitrogen tetroxide and unsymmetrical dimethylhydrazine UDMH (or monomethylhydrazine MMH), having the specific impulse of 2855 m/s. As for the monergol (monopropellants) propellants, hydrogen peroxide has only 1618 m/s and hydrazine released under pressure through a catalyst which causes its explosive decomposition has 1952 m/s. Using higher pressure in the burning chamber these values could be improved by approx. 10%. Specific impulse of solid propellant motors is not greater than 2500 m/s.

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**Fig. 28 Efficiency of three types of propellants used for orbital maneuvers**

The fundamental equation of the rocket propulsion (see Appendix 4) expresses the velocity of the vehicle in terms of specific impulse and the ratio of initial and final masses of the vehicle. Therefore, for a required change of velocity using a specific propellant, one can compute the necessary amount of the propellant. Ex-
amples are shown on Fig. 28. E.g., to change the velocity of a satellite by 100 m/s using nitrogen tetroxide and UDMH (curve A), 3.5% of the initial mass of the satellite must be the propellant itself. For hydrazine (curve B), the percentage is 4.9% and for hydrogen peroxide (curve C) as high as 6.0%. For a typical geostationary satellite of the Intelsat IV type (mass 700 kg according to Ref.3), this corresponds to 34 kg of hydrazine.

It is evident from Fig. 28 that great velocity changes demand lots of propellant which greatly increases the total mass of the satellite (not only the propellant, but also storage tanks and rocket motor construction are necessary). Therefore, a careful optimization analysis is necessary to minimize such maneuvers - this being the subject of following paragraphs.

4. PUSHING THE SATELLITE INTO THE ATMOSPHERE

Applying the retro-kick, the velocity of a satellite could be decreased by such an amount that it enters the Earth’s atmosphere. As a height at which a satellite can exists without burning in, a 100 km height above surface is used here.

Therefore, a question could be posed, how to put a satellite from a circular orbit of a given radius into orbit which is touching, or crossing below the 100 km level.

As it was shown in Paragraph 2, the optimal maneuver (in terms of energy) is to push the satellite into a Hohmann transfer orbit in which a satellite travels half of the revolution to reach the desired height (in our case 100 km). The decelerating kick is applied directly opposite to the direction of the orbital velocity (see Fig. 24) and its magnitude is a function of the radius of the initial orbit. The higher the orbit, the greater retro-kick is necessary (see Fig. 25). E.g. to force a satellite at 200 km circular orbit to decay, a 30 m/s impulse suffices, whereas for 500 km it raises to 115 m/s and for 1000 km is already 243 m/s.

Having a satellite on an elliptical (instead of circular) orbit changes the situation only slightly. The optimal retro-kick is to be applied at apogee and its magnitude is always smaller than for a circular orbit at the same height. The difference is greater if the perigee is lower (evidently, if the perigee is already at 100 km, we need no impulse...).

However, the usage of exactly Hohmann orbits is not a very safe method to remove a satellite from an orbit, since in this case it travels through upper atmosphere practically horizontally (an angle between the trajectory and the local horizon is zero). Consequently, it might happen, that the atmospheric drag is not sufficient to "catch" the satellite and it could perform several revolutions on the transfer orbit before actually decay. This situation is similar to ballistic reentry of the spacecraft with high velocities (e.g. from the lunar mission), where "atmospheric skip" occurs if the angle to the local horizon is less than 3°.

Therefore, when planning the reentry of the given satellite into the atmosphere, it is advantageous to use as much of the remaining propellant as possible. E.g., an increase of the retro-kick for low orbits (200-500 km) by only 3% (about 3 m/s) shortens the transfer trajectory by 20% (from original 180O to 1500) and increases the reentry angle from zero to 0°.5 which is quite sufficient to ensure the atmospheric capture.

Once in the atmosphere, the descending body is moving mainly according to the aerodynamical laws. Its final fate depends on many factors, the most important ones are the entry velocity, shape and area to mass ratio of the body and thermal and mechanical characteristics of its structure.

To discuss all these factors is out of the scope of the present study, but generally the body is destroyed at a certain altitude. Some of the resulting debris being capable to reach the Earth's surface. Till the end of 1979, from all known 11366
objects, 6733 already decayed, in only 16 cases the fragments surviving re-entry have been picked up on land (including the well-known case of Skylab space station)
Fig. 24 Three methods of satellite’s removal from orbit
The conclusion could be drawn, that for low orbits (see Paragraph 7 and Fig. 29), pushing of the satellite, into the Earth’s atmosphere is an effective means of its removal. Applying minimum necessary retro-kick results in long trajectories (near to the Holmann ones) and in small angles of the atmospheric entry. The atmospheric breaking is therefore relatively long, the body being destroyed mainly by long thermal loading.

Greater retro-kicks increase the entry angles and for some values (which are used for reentry of manned spacecraft) both thermal loading and overloading are relatively small; here we have highest probability of the body’s survival till hitting the Earth’s surface. For even greater velocities (and entry angles), overloading increases, destroying the body mainly by mechanical forces.

Timing the retro-firing carefully, a decay over uninhabited areas of the Pacific ocean could be easily secured. In this case, resulting fragments are not dangerous - even if they hit the Earth’s surface. The only problem might be an increasing concentration of the dust in the higher levels of the atmosphere over this area. This
could perhaps change the climatic conditions a little bit. Keeping in mind the low frequency of such reentries and small amount of material involved, such an effect seems to be negligible.

5. PUSHING THE SATELLITE OUT OF THE EARTH’S INFLUENCE
The gravitational acceleration produced by each celestial body decreases with the square of the distance, according to the famous Newton’s gravitation law. Consequently, the gravitational influence of a given body is important only till some limiting distance, where gravity of the other body starts to govern. Without going into further details (see e.g. Ref. 2), we can state, that the so called sphere of influence of the Earth has radius of approx. one million kilometers. At greater distances, Sun’s gravity prevails.

However, there is a "moving hole" in the Earth’s sphere of influence because at distances up to 66000 km around the Moon (which itself revolves at the distance 385 000 km from the Earth) the lunar gravity is principal.

Therefore, by increasing the velocity of the orbiting satellite sufficiently, we can increase the apogee of its orbit over one million kilometers and it will not orbit the Earth any more. In other words, we remove the satellite “to infinity". The trajectory (which in ideal case is a parabola) is often called escaping trajectory. The difference between escape and circular velocity as a function of the height above Earth’s surface is shown in Fig. 25 and labelled as "into ∞".

To change the circular orbit, a velocity kick is to be applied preferably in the direction of the orbital motion of the satellite. This increases the velocity, allowing the escape. For elliptical orbits, it is optimal to change the velocity always at perigee where the necessary velocity increase is smaller than for circular orbit at the same height. The difference (the velocity gain) is greater for higher apogees (for apogee over 1 million km we need no change of velocity at all...).

From the energetic point of view, one can use the gravity field of the Moon to pull the satellite during the close encounter out of the Earth-Moon system. However, it requires precise aiming of the trajectory at the encounter and therefore does not seem to be practical for removal of satellites with simple on-board orientation and stabilization systems. Also the velocity gain is not substantial.

What happens with the body after it leaves the Earth’s gravity influence? This depends on the residual velocity after escape. If this velocity is directed opposite to the Earth’s revolution around the Sun, than the body is orbiting around the Sun inside the Earth’s orbit (and touching it at the point of escape). If the residual velocity is directed along the Earth’s orbital velocity, the elliptical orbit of the body around the Sun is outside the Earth’s orbit and touches it again at the point of escape.

The probability of the repeated entry into the Earth’s sphere of influence is very low (several orders of magnitude lower than the probability of collision for satellites orbiting the Earth). There are even some projects to dispose the nuclear waste materials this way.

To be quite sure about the final fate of the removed body, it is possible to put it into a disposal orbit around the Sun which does not cross the Earth’s orbit, or push it out of the solar system completely. To change the 200 km high orbit around the Earth into an orbit escaping the solar system requires a velocity kick of 8.87 km/s. However, pushing the body directly into the Sun demands even greater impulse - 24.03 km/s ! Values for a simple escape (into the solar orbit near to the Earth’s path) are much smaller and are given in Fig. 25.

6. PUTTING THE SATELLITE INTO DISPOSAL ORBIT
The last method to remove the satellite from its initial orbit to be discussed here
is the method of disposal orbits. By disposal orbit we mean a circular orbit, which is generally higher than the original one and located at heights where no (or only a few) active satellites are revolving.

Using again results of the paragraph 2, we can characterize the necessary Hohmann transfers. To realize such a transfer between two circular orbits, two velocity kicks are necessary. First, to depart from the initial orbit, and second (half orbit later) to enter the final circular disposal orbit. Both kicks are to be parallel with the orbital velocity and increase it by a necessary amount. The lower kick (nearer to the center of gravity) is always larger than the higher one.

Velocities, necessary to increase the height of a given circular orbit by a particular increment are compared in Fig. 26. It can be seen that at greater heights even a small velocity kick could cause great orbital change (because the gravity field of the Earth is weak there).

![Fig. 26 Mass of propellant and velocity necessary to increase the height at a given circular orbit](image)

Somewhat arbitrary disposal heights are 2000 km, 3000 km, 5000 km, 10 000 km, 20 000 km and 50 000 km. However, these orbits cover practically all necessary
initial heights (50000 km level could be used for disposal of geostationary satellites). Results are shown on Fig. 27.

If the initial orbit is not circular, it is better to do the first kick at its perigee where its magnitude is smaller than for circular orbit at the same altitude. This is because the most efficient way of changing the apogee is to apply an impulse at perigee and the most efficient way of changing the perigee is by an impulse at apogee. If the apogee of the initial orbit is at the desired disposal altitude, only one (circulating the orbit) impulse suffices.

![Fig. 27 Mass of propellant and velocity necessary to push the satellite into different disposal orbits](image)

7. CONCLUSIONS
Results of all preceding paragraphs are summed up on Fig. 29, where the most efficient methods of satellite removal from its orbit are shown as function of altitude.

a) For low orbits (LEO) (up to 1500 km), pushing the satellite into the atmosphere is the most efficient way. The required velocity kick does not exceed 350 m/s which means that no more than 10% of the initial satellite mass is to
be devoted to the propellant.

b) For medium orbits (MEO) (between 1500-10000 km), either atmospheric entry or disposal orbit method could be used. Conveniently selected disposal orbits (not very far from the initial ones) demand sufficiently small magnitude of the velocity impulses. However, the sum of both impulses (initial and circulating ones) could reach in extreme cases as much as 1400 m/s.

c) For geostationary orbits (at 36000 km), the method of disposal orbit is preferable. Currently used orbit, which is 500 km above the geostationary one, requires only 18 m/s which is well within limits of the on-board propulsion systems of the communication satellites.

d) For extremely high orbits (over 40 000 km), pushing the satellite out of the Earth’s gravity influence ("to infinity") is clearly preferable.

Fig. 29 Proposed method of satellite’s removal from orbit as function of altitude

Therefore the design study of a particular future satellite should contain a part about the rocket propulsion system capable to remove the satellite from its orbit after fulfilment of its tasks. A standard propulsion system could be developed with variable capacity of fuel storage (depending on the number or on the diameter of the propellant tanks).

For small satellites and debris already in orbit without their own on-board propulsion systems, a special service should be organized. Space shuttle is very convenient for this purpose, since its crew could reach the satellite by remote manipulator arm and attach the maneuvering unit to it or lower the satellite into the shuttle cargo bay. The shuttle orbiter then could perform standard de-orbit maneuvers, enter the atmosphere and land with one or more satellites for possible refurbishment and reuse.

In the shuttle era, the number of inactive satellites will be certainly smaller than now, because the shuttle could also provide on-orbit service for malfunctioning satellites. Using the shuttle, delivering the satellites back to the Earth (or simply pushing them into atmosphere) is clearly preferable (compared to other described methods), since the shuttle itself must eventually return to the Earth and necessary maneuvers are integral part of its operational capabilities. Moreover, extensive use of a limited number of disposal orbits could lead to increasing density of fragments at those altitudes and to creation of an artificial ring around the Earth. However,
in the more distant future, such a concentration of high quality material orbiting the Earth might be used as a construction framework for a large multipurpose orbital stations.
IV. DISTRIBUTION AND ORBITS
OF INCIDENTAL SATELLITE DEBRIS

1. INCIDENTAL EXPLOSION

The approach to this problem has been deterministic and greatly simplified. Whatever the origin of the debris could be (explosion or collision with meteorite), we have assumed that the final velocity of a given point (such a debris) was obtained from the initial velocity (before the "accident") in adding an increment of $\Delta V$ in an arbitrary direction. The magnitude of $\Delta V$ will be one of the fundamental parameters in the following. Our other hypothesis are then:

- the initial orbit of the satellite is almost circular
- the Earth is assumed to be spherical, of radius $R$
- the accident occurs at distance $D_0$ greater than $R$, from the center of the Earth.

We want then to determine the minimum distance $d_{\text{min}}$ of the body (on its new trajectory) to the center of the Earth. The event will be called catastrophic when $d_{\text{min}}$ is less or equal to $R$. The involved parameters are shown on Fig. 30. (see Appendix 5). Results are summarized on Fig. 32.

![Graph showing $d_{\text{min}}$ vs. $R$ for different $\Delta V$ values.]

We limited ourselves to values of $\Delta V$ ranging from 0.0 to 1.0 km/s (to give an example, the initial velocity of a gun-bullet is between 0.5 and 1.3 km/s, usually) and to values of $D_0$ smaller than 7 Earth radii. Only a few curves are shown, which clearly demonstrate that the so-called "catastrophic line" can be often reached, for these "explosive" incremental velocities, for all objects orbiting the Earth between 200 and about 7000 km altitude.

2. COLLISION OF SATELLITES

Our study has been based on existing orbits of objects still orbiting the Earth as of June 1979. From all orbital elements listed in the NASA Satellite Situation Report, vol. 19, No.3, we derived a density distribution of orbits (see Appendix 6).
Our probabilistic approach enables us to work with all existing satellites at the same time and to design an algorithm for computing the collision probability at any epoch, considering that, due to various perturbations (as exposed in the first chapter) the distribution of these satellites in space will change.

We first show how this distribution function evolves with time, in general, and then in a simplified case (the very case we shall, then deal with). The collision probability function is described in Appendix 7. Results are briefly summarized in Fig. 34 and 35. The level curves have been drawn with step size of $10^{-6}$. The striking result is that this probability of collision remains always very small (less than $6 \times 10^{-6}$), and the pattern of the level curves does not change drastically with time.

APPENDIX 1

Perturbations due to the gravitational field

a) The influence of the Earth oblateness

The rate of change of $\Omega$ is

\[-9.964 \left( \frac{R}{a} \right)^{1.5} \left( \frac{R}{p} \right)^{2} \cos i\]
degrees per day, where \( p = a(1 - e^2) \) and \( R \) is the Earth’s equatorial radius. Thus for a close-Earth satellite in a near circular orbit at a height of 350 km, with \( R/a \) and \( R/p \) both having values near 0.95, \( \Omega \) changes at a rate of 

\[-8.33 \cos \deg/day.\]

The rate of change of argument of perigee, \( \omega \), is

\[4.982 \left( \frac{R}{a} \right)^{1.5} \left( \frac{R}{p} \right)^2 (5 \cos^2 i - 1)\]

degrees per day. Thus for a close satellite in nearly-circular orbit as specified previously, the perigee swings round at an angular rate of \( 4.16(5 \cos^2 i - 1) \) deg/day.

b) The influence of the Earth north-south asymmetry

The oscillation in perigee distance may be written as \( K \sin \omega \), where value of \( K \) depends on inclination and increase from zero at \( i = 0^\circ \), to 5 km at \( i = 4^\circ \), 10 km at \( i = 58^\circ \), and 50 km at \( i = 62.9^\circ \). For inclinations between 63\(^\circ\) and 64\(^\circ\), the perigee moves only very slowly, so the perturbation builds up for many years, and become extremely large. As the inclination increases further, \( K \) takes the value 

\[-10 \text{ km at } i = 65^\circ, \text{ zero at } i = 66.1^\circ, \text{ and } +10 \text{ km at } i = 90^\circ (\text{Ref. 1}).\]

APPENDIX 2

The density of artificial bodies orbiting the Earth

A satellite may move to any point within a toroidal volume (Figs 4 and 5), limited by a geocentric spherical surfaces of radiuses equal to the perigee and apogee radius-vector, respectively, and by cones of solid angles equal to \( 90^\circ - i \) (\( i = \text{inclination} \)). Dividing that volume by a concentric spherical surfaces with radiuses \( R_i \) into separate toroids, the probability that a satellite will be in a separate toroidal volume is proportional to the time spent there by a satellite; half-time (notice the symmetry of the orbit along the line of apsides) is given by

\[\Delta t_j = t_j - t_{j-1} = \sqrt{\frac{p^3}{GM}} \int_{v_{j-1}}^{v_j} \frac{dv}{(1 + e \cos v)^2},\]

from which

\[\sqrt{\frac{GM}{p^3}} \Delta t_j = (1 + \frac{3}{2} e^2 + \frac{15}{8} e^4)(v_j - v_{j-1}) - (2e + 3e^3)(\sin v_j - \sin v_{j-1}) + \]

\[+ \frac{1}{4}(3e^2 + 5e^4)(\sin 2v_j - \sin 2v_{j-1}) - \frac{1}{3} e^3(\sin 3v_j - \sin 3v_{j-1}) + \]

\[+ \frac{5}{32} e^4(\sin 4v_j - \sin 4v_{j-1}),\]

where \( GM \) is the geocentric (Earth) gravitational constant and \( v_j \) is the true anomaly of the point of intersection of the orbit with the spherical shell of radius \( R_j \):

\[\cos v_j = \frac{1}{e} \left( \frac{p}{R_j} - 1 \right).\]

The time spent in a specific toroidal surface is multiplied by the mean angular motion \( n \) to get a common time scale. The density \( D - j \) is then obtained as a sum of those quantities over all bodies, divided by a volume of a \( j \)-th toroid:
\[ D_j = \frac{\sum_k 2 \Delta t_j^{(k)} n_k}{\frac{4}{3} \pi (R_j^3 - R_{j-1}^3) \sin i}, \]

where we sum over \( k \) satellites.

By the procedure described, a satellite with an orbit which lies completely within a specific toroidal belt (e.g. a geostationary satellite) contributes to the density by one unit.

**APPENDIX 3**

Inclinations of orbital planes with less than 4 bodies

<table>
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<tr>
<th>Inclination [deg]</th>
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<th>Inclination [deg]</th>
<th>Number of bodies</th>
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**APPENDIX 4**

Velocity impulses for orbital transfer

a) Impulses necessary for Hohmann transfer between circular orbit with radius \( r_1 \) and circular orbit with radius \( r_2 \) are computed as follows:

I. impulse (from circular orbit into elliptical transfer orbit)

\[ \Delta_1 = \sqrt{\frac{2 \cdot GM \cdot r_2}{r_1 \cdot (r_1 + r_2)}} - \sqrt{\frac{GM}{r_1}}, \]

where \( GM \) is Earth gravitational constant (\( GM = 398600 \ km^3/s^2 \))

II. impulse (from elliptical transfer orbit into circular orbit)

\[ \Delta_2 = \sqrt{\frac{GM}{r_2}} - \sqrt{\frac{2 \cdot GM \cdot r_1}{r_2 \cdot (r_1 + r_2)}}, \]
b) Total impulse is sum of both increments. For atmospheric re-entry, first impulse suffices and \( r_2 = (6378 + 100) \text{ km} \). For transfer "to infinity", first impulse suffices also, but \( r_2 \) is to be as great as possible (e.g. \( 10^6 \text{ km} \)).

c) For atmospheric reentry, more general formula could be used (Ref. 2):

\[
\begin{align*}
  r \cdot \Delta^2 \cos \alpha [\sin \alpha \sin \varphi + \cos \alpha (r - \cos \varphi)] - \\
  - r^{1/2} \Delta [\sin \alpha \cdot \sin \varphi + 2 \cos \alpha (r - \cos \varphi)] + r - 1 &= 0
\end{align*}
\]

where \( r \) is the initial radius of the orbit (in units of atmospheric entry radius \( 6378 + 100 \) km), \( \Delta \) is magnitude of velocity impulse (in units of circular velocity at unit radius), \( \alpha \) is the angle between direction of the impulse and local horizon and \( \varphi \) is angular distance travelled by the satellite on the transfer orbit. This formula reduces to the Hohmann formula if \( \alpha = 0^\circ \) and \( \varphi = 180^\circ \).

**Consumption of propellant**

d) The consumption of propellant for orbital transfers was computed from the formula

\[
\Delta = w_e \cdot \log \left( \frac{M_f + M_p}{M_p} \right)
\]

where \( \Delta \) is the velocity impulse, \( w_e \) is specific impulse of the given propellant (ideal), \( M_f \) is the mass of the satellite body without propellant and \( M_p \) is the mass of propellant.

**APPENDIX 5**

**Computation of the minimum distance \( d_{min} \) after explosion**

Velocity impulse \( \Delta \vec{V} \) is parameterized as:

\[
\Delta \vec{V} = \Delta V (\cos \alpha \vec{v} + \sin \alpha \vec{w})
\]

where \( \alpha \) is angle between \( 0^\circ \) and \( 360^\circ \) and \( \{\vec{v}, \vec{w}\} \) system of orthogonal unit vectors (shown on Fig. 31), we derive the minimum distance \( d_{min} \) as follows:

a) angular momentum per unit mass after the accident is \( D_0 V'_0, t \), where \( V'_0, t \) is the tangential component of the new velocity, that is \( D_0 (V_0 + \Delta V \cos \alpha) \)

b) let \( q \) be the quantity \((V_0 + \Delta V \cos \alpha)^2 - (2 \mu/D_0)\), where \( \mu \) is the product of the mass of the Earth by the newtonian constant (geocentric gravitational constant); then:
(1) if \( q > 0 \), the final trajectory is hyperbola
(2) if \( q = 0 \), the final trajectory is parabola
(3) if \( q > 0 \), the final trajectory is ellipse.

The most interesting cases are (1) and (3)-(2) being a limit case which practically can never happen. The semi-major axis of the resulting orbit is given by:

\[
a(\alpha) = \pm 1/ \left( \frac{q}{D_0} - \frac{V_0'^2}{\mu} \right),
\]

where the positive sign has to be taken in case (3). Of course, is computed as:

\[
V_0'^2 = V_0^2 + \Delta V^2 + 2\Delta V V_0 \cos \alpha
\]

where, according to our hypothesis, \( V_0 = \sqrt{\mu/D_0} \).

Similarly, the eccentricity can be computed as:

\[
e(\alpha) = \sqrt{1 + D_0^2(V_0'^2/\mu^2)} \left( V_0'^2 - \frac{2\mu}{D_0} \right)
\]

and, finally, the minimum distance is (in all cases):

\[
d_{\text{min}} = \inf \left\{ \frac{1 - \sqrt{1 + D_0^2(V_0'^2/\mu^2)} \left( V_0'^2 - \frac{2\mu}{D_0} \right)}{\frac{2}{D_0} - \frac{V_0'^2(\alpha)}{\mu}} \right\}.
\]

Appendix 6

Density distribution and its variation with time

a) Density distribution

\[
D_0(\vec{q}_0) = C \exp\left\{ -\sum_{i=1}^{6} k_i^0(q_i - \bar{q}_i^0)^2 \right\}
\]

where \( C \) is a constant so that:

\[
\int_{R}^{+\infty} \int_{0}^{1} da \int_{-1}^{1} de \int_{0}^{1} \cos idi \int_{0}^{2\pi} d\Omega \int_{0}^{2\pi} d\omega \int_{0}^{2\pi} D_0(\vec{q}_0) dM = 1
\]

and where: \( k_1^0 = A_0, k_2^0 = E_0, \ldots \) are six other constants. Obviously, \( \vec{q} \) designates the vector of the 6 orbital elements, \( \vec{q}_0 \) its value at epoch \( t_0 \) (June 30, 1979) for any satellite, \( q_i \) its components, in the usual order (\( q_1 = e, q_2 = i, q_3 = \Omega, q_4 = \omega, q_5 = M, q_6 = a \)), and \( \bar{q}_i^0 \) the initial mean values of these quantities.

b) Variation of density with time

We shall assume that all forces acting on the satellite are well known, so that any state vector \( \vec{q} \) behaves as:

\[
\frac{d\vec{q}}{dt} = \vec{F}(t, \vec{q}), \text{ with initial conditions } \vec{q}(t_0) = \vec{q}_0.
\]

The initial conditions have a density of probability \( D_0(\vec{q}_0) \), which is a function of six variables taking only real positive values, as obvious from its expressions.

We want to find the density of probability \( D(t, \vec{q}) \) at epoch \( t \). \( V_0 \) being the volume of all possible state vectors at \( t_0 \), and \( V \) the corresponding volume at \( t \), we state that:
\[ p(t, V) = p(t_0, V_0), \text{ equal probabilities,} \]

with

\[
p(t_0, V_0) = \int \int \ldots \int_{V_0} D_0(\vec{q}_0) d\tau(\vec{q}_0)
\]

\[
p(t, V) = \int \int \ldots \int_{V} D(t, \vec{q}) d\tau(\vec{q})
\]

where

\[
d\tau(\vec{q}_0) = dq_0^1 \land dq_0^2 \land \ldots \land dq_0^6
\]

\[
d\tau(\vec{q}) = dq^1 \land dq^2 \land \ldots \land dq^6.
\]

The conclusion is that:

\[
D(t, \vec{q}) = \text{det} \Phi(t_0, t, \vec{q}_0) D_0(\vec{q}_0)
\]

where \( \Phi(t_0, t, \vec{q}_0) \) is matrization (or state transition matrix) of the variational equations system:

\[
\frac{d\delta \vec{q}}{dt} = \frac{\partial \vec{F}}{\partial \vec{q}}(t, \vec{q}(t)) \delta \vec{q}
\]

with \( \delta \vec{q}(t_0) = \delta \vec{q}_0 \) the solution of which being:

\[
\delta \vec{q}(t) = \Phi(t, t_0, \vec{q}_0) \delta \vec{q}_0.
\]

For instance, in the Keplerian motion case, \( \Phi \) will be simply:

\[
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
-\frac{3}{2} \pi & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\]

But this assumption would be too simple...In the following, we shall consider that the tree first elements are almost constant, and that the tree last ones (\( \Omega, \omega, M \)) are only affected by secular perturbations. For establishing a criterion of collision, we then shall be obliged to replace a given orbit by a tube, in the satellite will move according to the perturbations which act on it, with amplitudes of a few kilometers; the tube itself will be a "precessing" mean ellipse.

With such an hypothesis, the matrization can be well approximated in most cases by:

\[
\Phi(t, t_0, \vec{q}_0) = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
\frac{3}{2} \pi & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\]

where

\[
\text{EXP}(1, 4) \quad \text{EXP}(2, 4) \quad \text{EXP}(3, 4) \quad 1 \quad 0 \quad 0 \\
\text{EXP}(1, 5) \quad \text{EXP}(2, 5) \quad \text{EXP}(3, 5) \quad 0 \quad 1 \quad 0 \\
\text{EXP}(1, 6) \quad \text{EXP}(2, 6) \quad \text{EXP}(3, 6) \quad 0 \quad 0 \quad 1
\]
\[ EXP(1, 4) = -\frac{21n J_2 R^2 (5 \cos^2 i - 1)}{8 a} \frac{\Delta t}{(1 - e^2)^2 a^2} \]
\[ EXP(2, 4) = 3n \frac{J_2 R^2 e (5 \cos^2 i - 1)}{(1 - e^2)^3 a^2} \Delta t \]
\[ EXP(3, 4) = -\frac{15}{2} \frac{n J_2 R^2 \cos i \sin i}{(1 - e^2)^2 a^2} \Delta t \]
\[ EXP(1, 5) = \frac{21n J_2 R^2 \cos i}{4 a} \frac{\Delta t}{(1 - e^2)^2 a^2} \]
\[ EXP(2, 5) = -6n \frac{J_2 R^2 e \cos i}{(1 - e^2)^3 a^2} \Delta t \]
\[ EXP(3, 5) = \frac{3}{2} \frac{n J_2 R^2 \sin i}{(1 - e^2)^2 a^2} \Delta t \]
\[ EXP(1, 6) = -\frac{3n J_2 R^2 (3 \cos^2 i - 1)}{2 a} \frac{\Delta t}{(\sqrt{1 - e^2})^3 a^2} \]
\[ EXP(2, 6) = \frac{9}{4} n \frac{J_2 R^2 e (3 \cos^2 i - 1)}{(\sqrt{1 - e^2})^3 a^2} \Delta t \]
\[ EXP(3, 6) = -\frac{9}{2} \frac{n J_2 R^2 \cos i \sin i}{(\sqrt{1 - e^2})^3 a^2} \]

where \( \Delta t = t - t_0 \), \( R \) = Earth radius \((6378 \text{ km})\), \( J_2 \) = 2\textsuperscript{nd} zonal harmonic of the Earth’s gravity field, \( n = \sqrt{\mu/a^3} \).

**Appendix 7**

**Collision probability function**

The probability of collision will be computed locally around the Earth, precisely as a mean probability values between two given inclinations \((I_j, I_{j+1})\) and two distances \((a_i, a_{i+1})\), so as to generate a grid of numbers through which we can fit level curves or fixed probability. This approach will then give a clear picture, at a given time, of the risks of collision in space, according to the areas (Fig. 30).
We shall further assume that the satellite can be anywhere along its orbit (which results in an average over the mean anomaly $M$). The mean probability of collision over the area $[a_i, a_{i+1}] \times [I_j, I_{j+1}]$ is finally defined as:

$$
\omega_{ij}(t) = \frac{\int_{M'} \int_{\Omega'} \int_{\omega'} \int_{\ell'} \int_{\omega} \int_{I_j}^{I_{j+1}} \int_{a_i}^{a_{i+1}} D(t, \vec{q}) D(t, \vec{q}') \Delta(\vec{q}, \vec{q}') d\tau d\tau'}{\{\int_{M'} \int_{\Omega'} \int_{\omega'} \int_{\ell'} \int_{\omega} \int_{I_j}^{I_{j+1}} \int_{a_i}^{a_{i+1}} D(t, \vec{q}) \Delta(\vec{q}, \vec{q}') d\tau\}}
$$

Obviously, the first set of six integrals in the denominator equals to one.

The function $\Delta(\vec{q}, \vec{q}')$ is given as:

$$
\Delta(\vec{q}, \vec{q}') = \frac{1}{(1 + d/d_0)}
$$

where $d$ is the minimum distance between two given satellites with orbital elements $\vec{q}$ and $\vec{q}'$, averaged over $M$ and $M'$, computed numerically. Value of $d_0$ has been chosen equal to 5 km, according to the averaged amplitude of periodic perturbations ("size" of the tube), mostly of short period.

$D(t, \vec{q})$ and $D(t, \vec{q}')$ are computed as indicated, using the state transition matrix. The multiple integrals have been evaluated by Gauss quadrature formula of order eight.
Orbital Debris

In this assignment you will model the orbital debris collision hazard for spacecraft in Earth orbit. This will require you to program formulas representing the flux (impacts/ unit time/ unit area) of debris, numerically integrate these formulas forward in time, and perform an analysis where different inputs will affect the results (a parametric study).

Background

The flux of orbital debris on a spacecraft can be represented by a fairly simple formula (the boxed equation below) that depends on two sets of information: Information about the spacecraft such as altitude, inclination, and orientation; and, information about the environment such as particle sizes, growth rates, and the effects of the solar cycle (through changing atmospheric drag).

For a little perspective realize that while a 0.1 mm particle may cause serious surface erosion, a 1 mm particle can be very damaging. A 3 mm particle travelling at 10km/s has the kinetic energy of a bowling ball thrown at 100km/hr and a 1 cm particle would have the energy of a 180 kg safe thrown at the same speed. The U.S. space shuttles have already changed their orbits several times to avoid large debris. There has been pitting of tiles, and the loss of several panes of its multi-paned windshield due to impact with a small fleck of paint.

Quoting from Kessler, Reynolds, and Phillip, "NASA Technical Memorandum 100 471: Orbital Debris Environment for Spacecraft Designed to Operate in Low Earth Orbit":

The natural meteoroid environment has historically been a design consideration for spacecraft. Meteoroids are part of the interplanetary environment and sweep through Earth Orbital space at an average speed of 20km/s. At any one instant, a total of 200kg of meteoroid mass is within 2000km of the Earth’s surface. Most of this mass is concentrated in 0.1 mm meteoroids.

Within this same 2000km above the Earth’s surface, however, are an estimated 3,000,000kg of man-made orbiting objects. These objects are mostly in high inclination orbits and sweep past one another at an average speed of 10km/s. Most of the mass is concentrated in approximately 3000 spent rocket stages, inactive payloads and a few active payloads. A smaller amount of mass, approximately 40,000kg, is in the remaining 4000 objects currently being tracked by U.S. Space Command radars. Most of the objects are the result of more than 90 on-orbit satellite fragmentation. Recent ground telescope measurements of orbital debris combined with an analysis of hyper-velocity impact pits on the returned surfaces of the Solar Maximum Mission, (SMM), satellite indicate a total mass of approximately 2000kg for orbital debris of 1cm or smaller and approximately 300kg for orbital debris smaller than 1mm. This distribution of mass and relative velocity is sufficient to cause the orbital debris environment to be more hazardous than the meteoroid environment to most spacecraft orbiting below 2000km altitude.

While low-altitude debris will fall to the Earth due to atmospheric drag, it is quickly replenished by particles higher up and from collisions. It has been proposed that if too much debris gets into orbit, collisions could cause an increasing number of breakups, leading to an exponential growth in the number of particles. Such a catastrophic chain-reaction has been referred to as the "Kessler Syndrome". Even with the envisioned growth in launch rate, the growth of orbital debris can be greatly reduced by de-orbiting spent rocket stages and satellite at the end of their useful lifetimes. Better design and management can also reduce the likelihood of explosions (often related to propulsion systems) and reduce the amount of debris.
likely to be generated in a collision. Orbiting robots have also been proposed to scavenge for old satellites so they could be recycled before they disintegrate, perhaps to be used as small source of materials in future space development.

**Space Debris Flux Modelling**

\[
F = k \cdot \phi(h, S) \cdot \psi(t) \cdot [F_1(d) \cdot g_1(t) + F_2(d) \cdot g_2(t)],
\]

\(F\) = flux, in impacts per square meter of surface per year

\[
k = \begin{cases} 
1 & \text{for randomly tumbling surface} \\
0 & \text{In negative velocity direction.} \\
4 & \text{If normal vector is in direction of mono-directional flux,} \\
\end{cases}
\]

d = orbital debris diameter in cm

t = time in years, \(t_1\) start year, \(t_2\) ending year

\(h\) = altitude in km (\(h < 2000\) km)

\(S\) = 13-month smoothed 10.7 cm-wavelength solar flux in \(10^4\) Jy; lagged by 1 year from \(t\).

Use

\[
40 \left[ \cos \left( \frac{2 \cdot \pi}{11} \cdot (t - 1978) \right) \right] + 110
\]

to approximate solar flux for 11 year cycle, where \(t\) is year

\(i\) = inclination in degrees

Use 28.5 degrees, typical value for space shuttle

\[
\phi = \frac{\phi_1}{\phi_1 + 1}
\]

where

\[
\phi_1 = 10^{\left( \frac{h - 600}{100} - 1.5 \right)}
\]

\(p\) = the assumed annual growth rate of mass in orbit (\(p = 0.05\))

\[
F_1 = 1.05 \times 10^{-5} \cdot d^{-2.5}, \quad F_2 = 7.0 \times 10^{10} \cdot (d + 700)^{-6}
\]

\[
g_1 = (1 + 2p)^{(t - 1985)}, \quad g_2 = (1 + p)^{(t - 1985)}
\]

\(\psi\) = flux enhancement factor, inclination-dependent function that is a ratio of flux on spacecraft to the flux on the spacecraft from the current debris population’s average of 60°.

\(N\) = number of impacts on a spacecraft of surface area \(A\), between time interval \(t_1\) and \(t_2\)

\[
N = \int_{t_1}^{t_2} (F \cdot A)dt
\]

where

\(A\) = exposed area in square meters. Use \(A = 100\) square meters.
### Flux Enhancement Factor $\psi$

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### The Assignment

1. Prepare a multi-curve plot that illustrates the average number of impacts on a spacecraft ($y$-axis) vs. altitude in $km$ ($x$-axis) for particle sizes of 0.1, 0.2, and 0.3 cm. The range of heights should be 100km to 2,000km, and assume the 10-year time interval: $t_1 = 1985$, $t_2 = 1995$. Use the default values listed with the equations, i.e., $k = 1$; random tumbling surface; $I = 28.5$ degrees inclination, etc.

2. Make at least two other graphs of your choice, in each one varying the effect of altering one variable - change $k, S, t_2, i(\psi)$ or $p$. For comparison keep $d = 0.1, 0.2, 0.3cm$ or $d = 0.1cm$ depending on your graph. Explain your choices.

$k$ - represents type / attitude of spacecraft

$S$ - measure of solar cycle effects

$t_2$ - represents mission duration effects

$\psi, i$ - represents orbital inclination effects

$p$ - represents growth rate effects