CONTACTS OF SPACELIKE HYPERSURFACES IN THE LIGHTCONE WITH ϕ-HYPERBOLIC FLAT HYPERQUADRICS

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A simply connected Riemannian manifold N with $dimN \geq 3$ is conformally flat if and only if it can be embedded as a spacelike hypersurface in the lightcone, [3]. So, if an extrinsic differential geometry on spacelike hypersurfaces in the lightcone is studied, then the extrinsic invariants of conformally flat Riemannian manifolds may be found. This is one of the main motivations for the study of spacelike hypersurfaces in the lightcone. Since the induced metric on the lightcone is degenerate, the unit normal vector field for a spacelike hypersurface in the lightcone can not be defined by the ordinary arguments. In order to avoid this difficulty, a basic duality theorem for four Legendrian dualities related with the pseudo-spheres in Lorentz-Minkowski space was obtained in [4]. And also as an application of this basic duality theorem, a new extrinsic differential geometry on spacelike hypersurfaces in the lightcone was established in [4]. These Legendri an dualities have been extended in [5] for one-parameter families depending on a parameter $\phi \in [0, \pi/2]$ of pseudo-spheres in Lorentz-Minkowski space. Moreover, as an application of these extended Legendrian dualities, one-parameter families depending on this parameter $\phi \in [0, \pi/2]$ of new extrinsic differential geometries on spacelike hypersurfaces in the lightcone have been constructed in [6]. These give a lot of information about the spacelike hypersurfaces in the lightcone and include the results of [4] as a special case. We call these geometries slant geometry of spacelike hypersurfaces in the lightcone. In this talk, from a contact view point, slant geometry of spacelike hypersurfaces in the lightcone is mentioned about. For this aim, some models (ϕ-hyperbolic flat hyperquadrics) which are invariant through the Lorentzian group are introduced and their contacts with the considered hypersurfaces are investigated. These contacts are des cribed by the singularities of ϕ-hyperbolic height functions, [6].

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