TACHIBANA, KILLING AND PLANAR R-NUMBERS

Sergey Stepanov

Finance university under the Government of the Russian Federation, Dept.of Mathematics 49-55, Leningradsky Prospect, Moscow 125468, Russian Federation
`[s.e.stepanov@mail.ru]`

In our report we define the r-th Tachibana number, the r-th Killing number and r-th planar number of an n-dimensional closed and oriented Riemannian manifold (M, g) as dimensions of spaces of all conformal Killing r-forms, all coclosed conformal Killing r-forms and all closed conformal Killing r-forms, consequently for all $r = 1, 2, \ldots, n-1$. We show that the Tachibana r-number is a conformal scalar invariant, while the Killing r-number and the planar r-number are projective scalar invariants of (M, g). We prove duality properties of these numbers as an analogue to duality properties of the well known r-th Betti numbers of (M, g). In our report we determine a sharp upper bound on the r-th Tachibana number and sharp upper bounds of the r-th Tachibana number, the r-th Killing number and the r-th planar number. This result is a corollary of our result which was published in the paper entitled "The Killing-Yano tensor" (see Theoretical and Mathematical Physics, 2003, Vol. 134, No. 3, 333-338). Moreover this result is a generalization of well known results on of sharp upper bounds of dimensions of vector spaces of conformal Killing, Killing and concircular vector fields. In conclusion of our report we prove the vanishing theorem for the Tachibana numbers on an n-dimensional compact oriented Riemannian manifold (M, g) with negative curvature operator. And also we show that the r-th Tachibana number for are nonzero on unit n-dimensional Euclidian sphere.