

GENERALIZED S-SPACE-FORM

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It is an interesting problem to analyze what kind of Riemannian manifolds may be determined by special pointwise expressions for their curvatures. For instance, it is well known that the sectional curvatures of a Riemannian manifold determine the curvature tensor field completely.

Further, F. Tricerri and L. Vanhecke introduced the notion of a generalized complex-space-form, as a generalization of complex-space-forms and in the contact case, P. Alegre, D.E. Blair and A. Carriazo have studied generalized Sasakian-space forms as a generalization of Sasakian-space forms

In 1963, K. Yano introduced the notion of f -structure on a C^∞ $(2n+s)$ -dimensional manifold M , as a non-vanishing tensor field f of type $(1,1)$ on M which satisfies $f^3 + f = 0$ and has constant rank $r = 2n$. Almost complex ($s = 0$) and almost contact ($s = 1$) are well-known examples of f -structures. In this context, D.E. Blair defined K -manifolds (and particular cases of S -manifolds and C -manifolds) as the analogue of Kaehlerian manifolds in the almost complex geometry and of quasi-Sasakian manifolds in the almost contact geometry and he showed that the curvature of either S -manifolds or C -manifolds is completely determined by their f -sectional curvatures.

For these reasons it is interesting to introduce the notion of generalized S -space-forms on metric f -manifolds. This work was made by A. Carriazo, L.M. Fernandez and A.M. Fuentes for metric f -manifolds with two structure vector fields, giving some interesting examples.

So, in this communication we introduced and study generalized S -space forms, for any number of structure vector fields. We establish that the writing of the curvature tensor field is unique in terms of a family of differentiable functions on the manifold if and only if the dimension of the manifold is greater than $2 + s$, being s the number of structure vector fields. We also consider a different definition given by M. Falcitelli and A.M. Pastore and compare both definitions. Finally, we investigate generalized S -space-forms endowed with an additional structure and we obtain some obstructions for them to be S -manifolds.