RESULTS ON THREE DIMENSIONAL REAL HYPERSURFACES OF COMPLEX SPACE FORM

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Some of the following results are based on a joint work with Professor Philippos J. Xenos.

A real hypersurface M in a complex space form $M_n(c)$, is an immersed submanifold and its dimension is $2n-1$. An almost contact metric structure (ϕ, ξ, η, g) can be defined on M and it is induced from the Kaehler metric G and the complex structure J of $M_n(c)$. A real hypersurface is said to be *Hopf* if the structure vector field ξ is principal, i.e. $A\xi = \alpha\xi$, where A is the shape operator of M. The classification problem of real hypersurfaces in non-flat complex space form ($M_n(c)$, $c \neq 0$) is of great importance in the area of Differential Geometry. It was initiated by Takagi in 1973 (see [6], [7]), who classified the homogeneous real hypersurfaces of $\mathbb{C}P^n$. Further work on this area was done by Cecil and Ryan (see [2]) and finally Kimura in [3] gave the local classification of Hopf hypersurfaces with constant principal curvatures. In the case of $\mathbb{C}H^n$, Berndt in [1] classified Hopf hypersurfaces with constant principal curvatures.

The **structure Jacobi operator** of a real hypersurface M is denoted by l and is given by the relation $lX = R_\xi X = R(X,\xi)\xi$ and it plays an important role in the study of them. In [5] Perez and Santos classified real hypersurfaces in complex projective space $\mathbb{C}P^n$, $n \geq 3$, whose Lie derivative of the structure Jacobi operator with respect to ξ coincides with the covariant derivative of it in the same direction, i.e. $\mathcal{L}_\xi l = \nabla_\xi l$. Motivated by their work, in [4] the same condition is studied for real hypersurfaces in $\mathbb{C}P^2$ and $\mathbb{C}H^2$ and they are classified. Additionally, the case in which the Lie derivative of the structure Jacobi operator in the direction of $X \in \mathbb{D} = \ker(\eta)$ coincides with the covariant derivative in the same direction, i.e. $\mathcal{L}_X l = \nabla_X l$ will be presented. More precisely, real hypersurfaces in $\mathbb{C}P^2$ and $\mathbb{C}H^2$ equipped with structure Jacobi operator which satisfies the latter condition do not exist.

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[3] M. Kimura, *Real Hypersurfaces and Complex Submanifolds in complex projective spaces*, Trans. Amer. Math. Soc., **296** (1986), 137-149.

[4] K. Panagiotidou and Ph. J. Xenos, *Real hypersurfaces in $\mathbb{C}P^2$ and $\mathbb{C}H^2$ equipped with structure Jacobi operator satisfying $\mathcal{L}_\xi l = \nabla_\xi l$*, Advances in Pure Mathematics, **2** (2012), no. 1, 1-5.

[5] J. D. Perez and F. G. Santos, *Real hypersurfaces in complex projective space whose structure Jacobi operator satisfies $\mathcal{L}_\xi R_\xi = \nabla_\xi R_\xi$* , Rocky Mountain J. Math., **39** (2009), no. 4, 1293-1301.

[6] R. Takagi, *On homogeneous real hypersurfaces in a complex projective space*, Osaka J. Math., **10** (1973), 495-506.

[7] R. Takagi, *Real hypersurfacesin a complex prjective space with constant principal curvatures*, J. Math. Soc, Japan, **27** (1975), 43-53.