TOPOLOGY OF THE LIOUVILLE FOLIATION IN THE INTEGRABLE CASE OF GORYACHEV IN THE PROBLEM OF MOTION OF A RIGID BODY IN FLUID

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The generalized Kirchhoff equations of rigid body motion in fluid have the form

$$\dot{s} = s \times \frac{\partial H}{\partial s} + r \times \frac{\partial H}{\partial r}, \;\; \dot{r} = r \times \frac{\partial H}{\partial s}, \tag{1}$$

where $s, r \in \mathbb{R}^3$ are the impulse moment and the impulse force respectively, $H = H(s, r)$ is the total energy. This system of equations always possesses the geometric integral $f_1 = r_1^2 + r_2^2 + r_3^2$, the area integral $f_2 = s_1 r_1 + s_2 r_2 + s_3 r_3$, and the energy integral H. At the common level set $\{f_1 = a^2,\ f_2 = g\}$ the system is Hamiltonian. In [1] D. N. Goryachev found an integrable case where

$$H = \frac{1}{2}(s_1^2 + s_2^2 + 2s_3^2) + \frac{1}{2}c(r_1^2 - r_2^2) + \frac{b}{2r_3^2}.$$

In [2], on the basis of Boolean functions method of M. P. Kharlamov [3], P. E. Ryabov obtained the real seraration of variables for the Goryachev case which allowed to study phase topology of the system.

For the partial case $b = 0$ integrability of the system (1) was proved by S. A. Chaplygin in [4]. In the case $g = 0$ he found an additional integral and also reduced the problem to elliptic quadratures. In [5] topology of the Liouville foliation in the case $b = 0$ was investigated (topological type of energy surfaces, bifurcation sets, bifurcations of Liouville tori). In the present talk for the Chaplygin case we calculate the Fomenko-Zieschang invariant which is known to be a complete invariant for the Liouville equivalence (see [6]).

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