SOME GEOMETRIC PROPERTIES OF CERTAIN NATURALLY REDUCTIVE HOMOGENEOUS SPACES DERIVED THROUGH CURVATURE

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The theory of Jacobi fields has been widely developed in the mathematical literature. These vector fields are solutions of certain differential equations involving the Jacobi operator, and can be studied for any connection with or without torsion.

The solution of Jacobi equation is particularly simple for symmetric spaces Let us recall that locally symmetric spaces are characterized by the fact that the curvature tensor is parallel with respect to the Levi-Civita connection.

As it is well known, symmetric spaces are a particular case of homogeneous spaces, and, in particular, every symmetric space is both a naturally reductive homogeneous spaces and a g. o. space. In Ann. Scuola Norm. Sup. (1961), Berger classified the simply connected normal homogeneous spaces of rank one, proving that they consist of the symmetric spaces plus two new examples of dimensions 7 and 13. Berard-Bergery, in J. Math. P. Appl. (1961), showed that Berger had omitted one case. This corresponds with the example built by Wilking, Proc. Amer. Math. Soc. (1999), who proves that this 7-dimensional example coincides with one studied by Allof-Wallach, Bull. Amer. math. Soc. (1975).

Tsukada, Kodai Math. J. (1996), analyzes the properties of the osculating rank of the Jacobi operator, and it is also known that in the symmetric spaces all the Jacobi fields are isotropic. In Bull. Amer.Math. Soc. (1967) and Comm. Math. Helv. (1967) Chavel proves that Berger manifolds of dimensions 7 and 13 have anisotropic Jacobi fields.

In Monatsch. Math. (2008), along with Tarrío, we determine the oscilating rank for Berger mannifold of dimension 7. A similar result is obtained for Wilking manifold by Macías-Virgós, Tarrío and the author in C.R. Acad. Sci. Paris (2008), and by Arias Marco and the author, Publ. Math. Debrecen (2009), for the g.o. spaces (generalizing an example of Kaplan Bull. London Math. Soc. (1983)). Arias Marco also computed the rank of the flag manifold F^6, Arch. Math (Brno) (2009). In j. Diff. Geom. (1970), Chavel poses the following conjecture:

"If every conjugate point of a simply connected normal homogeneous Riemannian manifold G/K of rank one is isotropic, then G/K is isometric to a Riemannian symmetric space of rank one."

We prove, jointly with González-Dávila, that the Wilking manifold has anisotropic Jacobi fields, and, in general, the Chavel conjecture.