

# LOCAL RIGIDITY OF SYMPLECTIC PAIRS AND FOUR-DIMENSIONAL GENERALIZED SYMMETRIC SPACES

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A symplectic pair on a four-dimensional manifold  $M$  is a pair of nontrivial closed two-forms  $(\omega, \eta)$  of constant and complementary ranks, for which  $\omega$  (resp.,  $\eta$ ) restricts to the leaves of the kernel foliation of  $\eta$  (resp.,  $\omega$ ) as a symplectic form [2]. Symplectic pairs are naturally associated to pairs of almost Kähler structures on four-manifolds. We show that indefinite strictly almost Kähler and opposite Kähler structures  $(J, J')$  on a four-dimensional manifold with  $J$ -invariant Ricci operator are rigid, thus extending a previous result of Apostolov, Armstrong and Drăghici from the positive definite case to the indefinite one [1, 4]. In contrast to this, examples of nonhomogeneous four-dimensional manifolds which admit strictly almost paraKähler and opposite paraKähler structures  $(J, J)$  with  $J$ -invariant Ricci operator are shown. As an application, some geometric characterizations of four-dimensional generalized symmetric spaces are given [3, 5].

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