# LEVI-PARALLEL CONTACT RIEMANNIAN MANIFOLDS

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An *associated metric* on a contact manifold $(M, \eta)$ is a Riemannian metric $g$ such that the Reeb vector field $\xi$ is of unit length and orthogonal to the contact distribution $D := Ker(\eta)$, and also, denoting by $L_\eta$ the Levi-Tanaka form, the endomorphism $J := (L_\eta)^\sharp : D \to D$ is a partial complex structure, i.e. $J^2 = -I$. If we only require the former condition, we define $g$ an *admissible metric*.

In the talk I will show some recent results obtained in a joint work with A. Lotta, concerning the Riemannian geometry of a special class of contact manifolds $(M, \eta, g)$ equipped with an admissible metric: we ask the Levi-Tanaka form to be parallel with respect to a canonical connection with torsion, parallelizing both $\eta$ and $g$. For the special case of associated metrics, this condition of Levi-parallelism is equivalent to the integrability condition of $J$, so that the canonical connection we deal with plays the role of the Tanaka-Webster connection in pseudohermitian geometry.

To each Levi-parallel contact Riemannian manifold $(M, \eta, g)$ one can associate in a canonical way a finite set of negative numbers $\mathcal{S}$, called its *spectrum*, determined by the eigenvalues of the symmetric operator $J^2 : D \to D$. First we study Einstein metrics, providing a sufficient condition ensuring $(M, \eta, g)$ to be of Sasakian type, namely $\xi$ be Killing. Finally we show that, under suitable curvature assumptions on $g$, the spectrum $\mathcal{S}$ must actually be trivial, and we obtain some rigidity theorems, including a classification result for locally symmetric spaces.