AFFINE HYPERSURFACES WITH WARPED PRODUCT STRUCTURE

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The aim of this work is to classify all the strictly locally convex affine hypersurface M^{n+1}, $n \geq 2$ in R^{n+2} such that there exists an affine hypersphere M^n in R^{n+1} such that $M^{n+1} = I \times_\rho M^n$, where $I \subset R$ and the function ρ depends only on I, meaning M^{n+1} admits a warped product structure. If $\frac{\partial}{\partial t}$ and $\frac{\partial}{\partial x_i}, i = 1, \ldots, n$ span the tangent bundles of I and M^n, respectively, then there exist differentiable functions $\lambda_1, \lambda_2, \mu_1$ and μ_2 such that the difference tensor and the shape operator are of the following form

$$K(\frac{\partial}{\partial t}, \frac{\partial}{\partial t}) = \lambda_1 \frac{\partial}{\partial t}, \quad K(\frac{\partial}{\partial t}, \frac{\partial}{\partial x_i}) = \lambda_2 \frac{\partial}{\partial x_i},$$
$$S\frac{\partial}{\partial t} = \mu_1 \frac{\partial}{\partial t}, \quad S\frac{\partial}{\partial x_i} = \mu_2 \frac{\partial}{\partial x_i}, \quad \forall i = 1, \ldots, n.$$

Conversely, suppose M^{n+1} is a locally strongly convex hypersurface of the affine space R^{n+2} such that its tangent bundle is an orthogonal sum, with respect to the metric h, of two distributions, one-dimensional $\mathcal{D}_1$ spanned by unit vector field T and n-dimensional $\mathcal{D}_2$ $(n \geq 2)$, with orthonormal frame $X_1, X_2, \ldots, X_n$ such that

$$K(T, T) = \lambda_1 T, \quad K(T, X) = \lambda_2 X,$$
$$ST = \mu_1 T, \quad SX = \mu_2 X, \quad \forall X \in \mathcal{D}_2.$$

Let $\gamma_1, \gamma_2 : R \to R$ be functions such that $\gamma_1' \neq 0$, $\gamma_1'\gamma_2'' \neq \gamma_1''\gamma_2'$. Then M^{n+1} an affine hypersphere such that $K_T = 0$ or is affine congruent to one of the following immersions

1. $f(t, x_1, \ldots, x_n) = (\gamma_1(t), \gamma_2(t)g_2(x_1, \ldots, x_n))$, where $g_2 : R^n \to A^{n+1}$ is a proper affine hypersphere centered at the origin, for γ_1, γ_2 such that $\gamma_2 \neq 0$, $\gamma_1'\gamma_2 - \gamma_1\gamma_2' \neq 0$, and moreover, $sgn(\gamma_1'\gamma_2) = sgn(\gamma_1'\gamma_2 - \gamma_1\gamma_2') = sgn(\gamma_1'\gamma_2'' - \gamma_1''\gamma_2')$,
2. $f(t, x_1, \ldots, x_n) = \gamma_1(t)C(x_1, \ldots, x_n) + \gamma_2(t)e_{n+1}$, where $C : R^n \to A^{n+2}$ is an improper affine sphere given by

   $$C(x_1, \ldots, x_n) = (x_1, \ldots, x_n, p(x_1, \ldots, x_n), 1)$$

   , with the affine normal e_{n+1}, for γ_1, γ_2 such that $sgn(\frac{\gamma_1'\gamma_2'' - \gamma_1''\gamma_2'}{\gamma_1'}) = -sgn\gamma_1$,
3. $f(t, x_1, \ldots, x_n) = C(x_1, \ldots, x_n) + \gamma_2(t)e_{n+1} + \gamma_1(t)e_{n+2}$ where $C : R^n \to A^{n+2}$ is previously given improper affine sphere, for γ_1, γ_2 such that $sgn(\gamma_1'\gamma_2'' - \gamma_1''\gamma_2') = sgn\gamma_1'$.

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