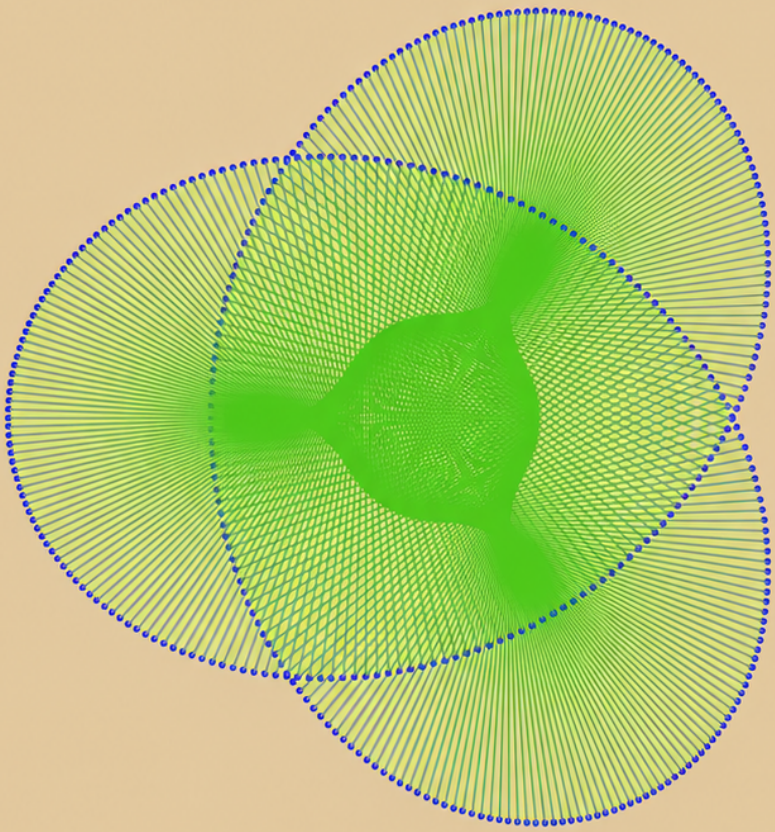
# XXIII Geometrical seminar

## *Book of Abstracts*



Zlatibor, 30.8. - 5.9.2026.

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Telekom Srbija

**XXIII Geometrical Seminar**
**Zlatibor, 30.8.-5.9.2026.**

# BOOK OF ABSTRACTS

Organized by

Faculty of Mathematics Belgrade, Serbia

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## Pointed Gromov-Hausdorff distance between $l^p$-wedge sums of metric spaces

Emin Abdullaev

*Lomonosov Moscow State University, Russia*
[x-west@hotmail.com]

This paper investigates $l^p$-wedge sums of pointed metric spaces and provides bilateral estimates for the pointed Gromov-Hausdorff distance between them. These estimates are based on the methods previously developed by the author for $l^p$-products of metric spaces. The case of linear $l^p$-wedge sums is considered separately, and an improved estimate is obtained for this specific class.

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## Gromov-Hausdorff distances, Borsuk-Ulam theorems, and Vietoris-Rips complexes

Henry Adams

*University of Florida, United States*
[henryhughadams@gmail.com]

The Gromov-Hausdorff distance is a notion of dissimilarity between two datasets or between two metric spaces. It is an important tool in geometry, but notoriously difficult to compute. I will show how to provide new lower bounds on the Gromov-Hausdorff distance between unit spheres of different dimensions by combining Borsuk-Ulam theorems with Vietoris-Rips complexes. This joint work with 15 coauthors is available at https://arxiv.org/abs/2301.00246. Many questions remain open.

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## Symmetric-like properties and the Laplace-Beltrami operator on Riemannian manifolds

Teresa Arias-Marco

*Department of Mathematics, University of Extremadura, Spain*
[ariasmarco@unex.es]

The main objective of the talk is to present the latest advances made in the determination of a symmetric-like property using the Laplace-Beltrami operator on Riemannian manifolds.

Symmetric-like properties are those which generalize the local symmetry on Riemannian manifolds. In this context, we focus in the existence of an operator which intertwines the Laplace-Beltrami operator of two Riemannian manifolds with certain symmetric-like properties.

Joint work with Jos'e Manuel Fernández-Barroso.

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## Clifford modules, associated homogeneous geometry and their applications

Dmitri Alekseevsky

*Higher School of Modern Mathematics MIPT, Russia*
[d.alekseevsky@gmail.com]

We recall the classification of Clifford algebras and Clifford modules over a pseudo-Euclidean vector space V. We then consider several homogeneous pseudo-Riemannian geometries associated with Clifford modules (such as Vinberg cones and Kaplan H-spaces) and discuss their applications to classical and quantum information, quantum physics, and quantum error-correcting codes.

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# Generalized nearly Kaehler f-structures on homogeneous spaces and Lie groups

Vitaly Balashchenko

*Belarusian State University, Minsk, Belarus*
[vitaly.balashchenko@gmail.com]

There are the following strict inclusions between the following special classes of metric $f$-structures by K.Yano ($f^3 + f = 0$): $\mathbf{Kill\,f} \subset \mathbf{NKf} \subset \mathbf{GNKf}$ (V.Kirichenko, A.Gritsans, V.Balashchenko). Note that in the particular case $f = J$ ($J^2 = -1$) all these classes coincide and give the well-known remarkable class **NK** of nearly Kähler structures ($(\nabla_X(J)(X) = 0)$).

Here we concentrate on the wide class **GNKf** of generalized nearly Kähler $f$-structures ($f\nabla_{fX}(f)(fX) = 0$). For naturally reductive homogeneous spaces the algebraic criteria for invariant $f$-structures of the class **GNKf** was obtained. Using the canonical $f$-structures on homogeneous $k$-symmetric spaces a welth of examples of generalized nearly Kähler $f$-structures was presented. Specifically, some examples on homogeneous 6-symmetric spaces were considered in more detail. As to the classes **Kill f** and **NKf**, the related wealth of examples was previously described in many papers (V.Balashchenko, Yu.Churbanov, D.Vylegzhanin, A.Sakovich, A.Samsonov, P.Dubovik).

We also investigate left-invariant metric $f$-structures on specific solvable Lie groups in dimension 3 and 4, where the left-invariant Riemannian metric is not naturally reductive. Some of the left-invariant metric $f$-structures on these Lie groups also belong to the class of generalized nearly Kähler $f$-structures (see [1], [2]).

1. Balashchenko V.V., Kunitsa V.N. Invariant $f$-structures on the four- dimensional oscillator group Journal of the Belarusian State University. Mathematics and Informatics. 2025. No. 1. P. 51–57 (in Russian).
2. Balashchenko V.V., Kunitsa V.N. Left-invariant metric $f$-structures on three- dimensional solvable Lie groups Proceedings of the National Academy of Sciences of Belarus. Physics and Mathematics series. 2025. V. 61, no. 2. P. 95–105 (in Russian).

## Submanifolds of codimension 2 of paracosymplectic manifolds with normal Reeb vector field

Cornelia-Livia Bejan

*retired, Romania*
[bejanliv@yahoo.com]

This study is devoted to a submanifold $M$ of codimension 2 of an almost paracontact metric manifold $\bar{M}$, for which the Reeb vector field of the ambient manifold is normal. Some sufficient conditions for the existence of $M$ are given. When $\bar{M}$ is paracosymplectic, then some necessary and sufficient conditions are established for $M$ to fall in one of the following classes of almost paracontact metric manifolds according to the classification given by S. Zamkovoy and G. Nakova: normal, paracontact metric, para-Sasakian, K-paracontact, quasi-para-Sasakian, respectively. When in addition, $M$ is para-Sasakian and $\bar{M}$ is paracosymplectic, some characterization results are obtained for $M$ to be totally umbilical, as well as a nonexistence result for $M$ to be totally geodesic is provided. The case when $\bar{M}$ is of a constant sectional curvature is analysed and an example is constructed.

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## Restoring a compact using its exponent

Semeon Bogatyy

*Moscow State University, Russia*
[bogatyi@inbox.ru]

Miloslav Marjanović showed that there are finitely many distinct exponents (hyperspaces) of zero-dimensional metric compact spaces. We show that the complement of a metric compactum in its exponent often uniquely determines the compactum itself.

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# Special curvature tensors in four-dimensional manifolds of neutral signature

Burcu Cındık[1], Bahar Kırık Rácz[2]

[1] *Marmara University, Institute of Pure and Applied Sciences, Department of Mathematics, Turkey*
[burcucindik@marun.edu.tr]

[2] *Marmara University, Faculty of Science, Department of Mathematics, Turkey*
[bahar.kirik@marmara.edu.tr]

The aim of this study is to investigate various properties of special curvature tensors in 4-dimensional manifolds endowed with a signature $(+,+,-,-)$ referred to as neutral signature. In this context, concircular, projective and conharmonic curvature tensors are discussed within the framework of holonomy theory for these manifolds. Regarding specific holonomy subalgebras of $o(2,2)$, some geometric properties of these tensor fields are investigated, and their eigenbivector structures are determined. By calculating the Riemann and Ricci tensors and the scalar curvature, the components of these tensor fields are identified according to holonomy types. Afterwards, their eigenbivector structures are analyzed to derive a number of results. On the other hand, considering the parallel and recurrent vector fields that arise in holonomy theory, several features of these tensor fields are also revealed. Finally, a collection of explicit examples is constructed to support our findings.
Acknowledgements: This work is based on the first author's (BC) ongoing PhD thesis, supervised by the second author (BKR). BC gratefully thanks the funding provided by the TÜB.ITAK-B.IDEB 2211-E National PhD Scholarship Program for Former Undergraduate and MSc/MA Scholars.

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# On hypersurfaces admitting generalized solitons in Riemannian warped products

Ahmet Umut Çorapli

*Istanbul Technical University, Turkey*
[corapli18@itu.edu.tr]

Let $R^4(\epsilon, f) = I \times_f \mathbb{Q}^3_\epsilon$ denote the four-dimensional Riemannian warped product manifold with $\epsilon \in \{-1, 0, 1\}$, equipped with the metric tensor

$$\widetilde{g} = dt^2 + f(t)^2 g_\epsilon,$$

where $g_\epsilon$ denotes the metric tensor on $\mathbb{Q}^3_\epsilon$ with

$$\mathbb{Q}^3_\epsilon = \begin{cases} \mathbb{S}^3, & \text{if } \epsilon = 1, \\ \mathbb{H}^3, & \text{if } \epsilon = -1, \\ \mathbb{E}^3, & \text{if } \epsilon = 0, \end{cases}$$

$I$ is an open interval and $f\colon I \to (0, \infty)$ is a smooth warping function.

The tuple $(g, \xi, \eta, \beta, \gamma, \delta)$ defines a generalized soliton on a Riemannian manifold $(M, g)$ if

$$\frac{1}{2}\mathcal{L}_\xi g + \beta \cdot \mathrm{Ric} = \gamma \cdot g + \delta \cdot \eta \otimes \eta,$$

where $\beta$, $\gamma$ and $\delta$ are smooth functions on $M$. Here, it is assumed that $\eta := \xi^\flat$ is the dual 1-form of the potential vector field $\xi$, [1]. In this definition, according to the choice of the smooth functions $\beta$, $\gamma$, and $\delta$, a generalized soliton reduces to a Ricci soliton, an almost Ricci soliton, a Yamabe soliton, or an almost Yamabe soliton.

In this work, we investigate hypersurfaces of the Riemannian warped product $R^4(\epsilon, f)$ admitting generalized solitons whose potential vector field is the tangential component $T$ of the unit vertical vector field $\partial_t$. We classify them according to whether they have three distinct principal curvatures, two distinct principal curvatures, or are totally umbilical.

**Acknowledgement:** This work is supported by The Scientific and Technological Research Council of Turkey (TUBITAK) under Project 124F501 and this work forms part of the author's Ph.D. thesis, joint work with Prof. Nurettin Cenk TURGAY and Assoc. Prof. Burcu Bektaş DEMIRCI.

1. A. M. Blaga and B. Y. Chen, *Harmonic Forms and Generalized Solitons*, Results Math. **79** (2024), 16.

# Some pseudosymmetry type curvature conditions

Ryszard Deszcz[1], Małgorzata Głogowska[2], Jan Jełowicki[3], Miroslava Petrović-Torgašev[4], Katarzyna Sawicz[5], and Georges Zafindratafa[6]

[1] *Department of Mathematics, Wrocław University of Environmental and Life Sciences, Wrocław, Poland*
[ryszard.deszcz@upwr.edu.pl]

[2] *Department of Mathematics, Wrocław University of Environmental and Life Sciences, Wrocław, Poland*
[malgorzata.glogowska@upwr.edu.pl]

[3] *Department of Mathematics, Wrocław University of Environmental and Life Sciences, Wrocław, Poland*
[jan.jelowicki@upwr.edu.pl]

[4] *Department of Sciences and Mathematics, State University of Novi Pazar, Novi Pazar, Serbia*
[mirapt@kg.ac.rs]

[5] *Department of Mathematics, Silesian University of Technology, Gliwice, Poland*
[katarzyna.sawicz@polsl.pl]

[6] *Laboratoire de Mathématiques pour l'Ingénieur (LMI), Université Polytechnique Hauts-de-France, Valenciennes, France*
[georges.zafindratafa@uphf.fr]

The $(0,6)$-tensor $R\cdot C - C\cdot R$, called the derivation-commutator of the Riemann-Christoffel curvature tensor $R$ and the Weyl conformal curvature tensor $C$, of certain semi-Riemannian manifolds $(M,g)$, $\dim M = n \geq 4$, is a linear combination of Tachibana tensors. In this talk we prove that some quasi-Einstein, as well as non-quasi-Einstein manifolds have this property. We also present examples of such manifolds. Our talk is based on [1] and [2].

1. R. Deszcz, M. Głogowska, J. Jełowicki, M. Petrović-Torgašev, K. Sawicz, and G. Zafindratafa, *Some pseudosymmetry type curvature conditions*, to appear.
2. R. Deszcz, M. Głogowska, J. Jełowicki, M. Petrović-Torgašev, and G. Zafindratafa, *Curvature properties of pseudosymmetry type of some 2-quasi-Einstein manifolds*, arXiv: 2405.18865 v3 [math.DG] 1 Apr 2025, p. 46.

# A simple model of nonlocal de Sitter gravity $\sqrt{dS}$

Ivan Dimitrijević

*University of Belgrade, Faculty of Mathematics, Belgrade, Serbia*
[ivan.dimitrijevic@matf.bg.ac.rs]

Nonlocal de Sitter $\sqrt{dS}$ gravity model is defined by the action

$$S = \frac{1}{16\pi G}\int \left(R - 2\Lambda + \sqrt{R-2\Lambda}F(\Box)\sqrt{R-2\Lambda}\right)\sqrt{-g}\mathrm{d}^4x$$

where $F(\Box)$ is an analytic function of the dâ€™Alembert-Beltrami operator $\Box$. By this way, nonlocal operator $F(\Box)$ is dimensionless. The corresponding equations of motion for the metric tensor $g_{\mu\nu}$ are presented.

We present and discuss several exact cosmological solutions for homogeneous and isotropic universe. One of these solutions $a(t) = t^{\frac{2}{3}}\exp(\frac{\Lambda}{14}t^2)$ have properties similar to ones that are usually assigned to dark matter and dark energy. These results are discussed in the context of $H_0$ tension. Some solutions are examples of the nonsingular bounce ones in flat, closed and open universe. All these cosmological solutions are a result of nonlocality and do not exist in the local de Sitter case.

This talk is based on joint work with Branko Dragovich, Zoran Rakić and Jelena Stanković.

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# A Universe with hybrid nonlocality

Branko Dragovich

*Institute of Physics, University of Belgrade, Pregrevica 118, Belgrade (Retired), Serbia*
[dragovich@ipb.ac.rs]

I will present a model of the universe in which gravity and matter are nonlocal. The gravitational field is described by a very simple nonlocal de Sitter model. The role of matter is played by a scalar field that has its origin in the p-adic theory of open strings. The connection of this scalar field with the nature of the cosmological constant will be discussed. This presentation is largely based on the work: B.D., A Closed Universe with Hybrid Nonlocality, International Journal of Geometric Methods in Modern Physics, Vol. 21, No. 10, 2440012 (2024).

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# Isoperiodic deformations of meromorphic differentials on Riemann surfaces and soliton theory

Vladimir Dragović

*The University of Texas at Dallas/MISANU*
[vladad@turing.mi.sanu.ac.rs]

We study deformations of elliptic and hyperelliptic Riemann surfaces with an Abelian differential of the second or third kind, which preserve the periods of the differential with respect to a chosen canonical homology basis of the surface. We derive differential equations with rational coefficients governing the deformations. We apply these results to soliton theory, e.g. to the Boussinesq, KdV, and sine-Gordon equations, and to the Toda lattice, as well as to SU(N) Seiberg-Witten theories. The talk is based on new joint results with Vasilisa Shramchenko:

1. V. Dragović, V. Shramchenko, Isoperiodic deformations of Toda curves and chains, the difference Korteweg -de Vries equation, and SU(N) Seiberg-Witten theories, arxiv:2512.21441

2. V. Dragović, V. Shramchenko, Deformations of the Hill curves and isoperiodicity in the KdV and the sine-Gordon equations, arxiv:2512.06168

3. V. Dragović, V. Shramchenko, Isoperiodic deformation of Abelian differentials of the second kind over elliptic curves and the Boussinesq equation, arXiv:2512.06162, SIAM Journal on Mathematical Analysis, accepted

4. V. Dragović, V. Shramchenko, Isoharmonic deformations and constrained Schlesinger systems, Advances in Mathematics, Vol. 499, July 2026

## Geodesic orbit Finsler metrics

Zdeněk Dušek

*Institute of Technology and Business in Ceske Budejovice, Czech Republic*
[zdusek@vste.cz]

The geodesic orbit condition for Finsler $(\alpha, \beta)$ metrics and a newly defined class of Finsler $\alpha_i$-type metrics based on Riemannian geodesic orbit metrics will be investigated. Interesting examples of $(\alpha, \beta)$ metrics are found such that $(G/H, \alpha)$ is Riemannian geodesic orbit space, but for the geodesic orbit property of $(G/H, F)$ the isometry group has to be extended. For a broad family of positively related Riemannian geodesic orbit metrics, it is shown that the derived Finsler $\alpha_i$-type metrics have also geodesic orbit property. Examples of these metrics on spheres will be presented with corresponding geodesic graphs. The natural reductivity in Finsler geometry will be also discussed, with an explicit construction of purely Finsler naturally reductive metrics.

Joint work with Teresa Arias Marco.

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## Novikov type Landau-Ginzburg model

Huijun Fan

*School of Mathematics and Statistics , China*
[fanhj@whu.edu.cn]

Landau-Ginzburg (LG) models arise naturally in the study of mirror symmetry, and a prominent conjecture "the Landau-Ginzburg/Calabi-Yau correspondence "links them to Calabi-Yau geometry. Consequently, a geometric analysis of LG models is of fundamental importance. In this talk, I introduce a class of Novikov-type Landau-Ginzburg models (NLGMs), exemplified by hyperplane arrangements including Yang-Yang function. For well-tamed NLGMs, we establish a Hodge decomposition theorem. The deformation theory of NLGMs is also discussed. This is a joint work with Jieran Wang.

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## Minimal ideal triangulations of 3-manifolds

Evgeny Fominykh

*St. Petersburg University, Russia*
[efominykh@gmail.com]

The triangulation complexity of a 3-manifold with boundary is the minimal number of tetrahedra in any its ideal triangulation. Recent developments in the theory of complexity for 3-manifolds are reviewed. New methods for computing complexity, based on calculation of the Turaev-Viro invariants and homologies of 3-manifolds, are described.

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## Higher-dimensional generalizations of the heavy rigid body motion with gyroscope

Borislav Gajić

*Mathematical Institute SANU, Serbia*
[gajab@mi.sanu.ac.rs]

Starting from the multidimensional integrable generalizations of the Euler top, the Lagrange top, the Lagrange bitop, and the totally symmetric case, we add to each of them a gyroscope. We provide a matrix Lax representation and we prove Liouville integrability for each of obtained systems. This is joint paper with Vladimir Dragović and Božidar Jovanović.

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## Nonlinear preserver problems on matrix domains

Ilja Gogic

*University of Zagreb, Croatia*
[ilja@math.hr]

This talk asks a simple preserver question for continuous maps on matrix domains: when does spectral containment already force spectral preservation, and when does commutativity preservation force a Jordan form? The linear background is the famous Kaplansky–Aupetit conjecture, while the nonlinear model is Šemrl's theorem, which characterizes Jordan automorphisms of $M_n(\mathbb{C})$, $n \geq 3$, as the continuous self-maps preserving spectrum and commutativity.

We study continuous maps $\varphi : X \to M_m(\mathbb{C})$, where $X \subseteq M_n(\mathbb{C})$ is a matrix domain, such that the spectrum of $\varphi(x)$ is contained in the spectrum of $x$ for every $x \in X$. For the full matrix algebra, the matrix Lie groups $\mathrm{GL}(n)$, $\mathrm{SL}(n)$ and $\mathrm{U}(n)$, the normal matrices, and the corresponding diagonalizable domains, such maps exist exactly when $n$ divides $m$; in that case the spectral containment is automatically an equality. Combined with commutativity preservation, this gives Šemrl-type preserver results: the maps extend to Jordan automorphisms of $M_n(\mathbb{C})$. We also treat singular rank domains, where injectivity is unavoidable. The proof combines algebraic, topological, and operator-theoretic arguments, with the Fundamental Theorem of Projective Geometry entering at the classification stage.

This is joint work with Alexandru Chirvasitu (University at Buffalo) and Mateo Tomašević (University of Zagreb).

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## Conformal Fedosov structures on compact Riemannian spaces with non-positive sectional curvature

Nadezhda Guseva

*Moscow State Pedagogical University, Moscow, Russia*
[ngus12@mail.ru]

Join lecture with Chakibek Almazbekov, Bekbolot Kanetov, Irena Hinterleitner and Josef Mikeš. This lecture is devoted to the study of $n$-dimensional Riemannian spaces with conformally Fedosov structures ($n \geq 4$). We prove that no conformally Fedosov structures exist on a compact Riemannian space with non-positive sectional curvatures. This result also holds when the space has a boundary and the structure satisfies certain conditions on that boundary-conditions close to those of generalized Hermitian structures. Furthermore, it is shown that there are no conformally Fedosov structures that are also generalized Hermitian.

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## Formula for the lengths of the height and edges of a right-angled (n+1)-hedron

Irena Hinterleitner

*Brno University of Technology, Czech Republic*
[hinterleitner.i@fce.vutbr.cz]

Joint lecture with Josef Mikeš and Lucie Zrůstová. An interesting relationship formula for the lengths of the height and edges of a right-angled (n+1)-hedron in n-dimensional Euclidean spaces is proven. A formula generalizing Pythagoras and Faulhaberas formulas is found for dimension n.

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## A problem of intersection of balls in normed space

Daniil Iliukhin

*Lomonosov Moscow State University, Russia*
[daniil.ilukhin@math.msu.ru]

This talk investigates the topological properties of intersections of balls in finite-dimensional normed spaces—a problem that naturally arises when constructing covers for estimating the Gromov–Hausdorff distance. We study the topology of a set obtained by removing a large closed ball from a finite intersection of small open balls. It is proved that in an arbitrary normed plane, such a set is always contractible, provided that it is non-empty.

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## Some classification of oriented graph-links

Denis Ilyutko

*Lomonosov Moscow State University, Russia*
[ilyutko@yandex.ru]

A graph-link is defined as an equivalence class of simple labeled graphs with respect to special transformations which play the role of Reidemeister moves. This theory naturally describes classical and virtual links up to mutation. The aim of the talk is to define oriented graph-links and give some classification of them depending on the minimal number of vertices of representatives

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# Minimal fillings of finite metric spaces

Alexander Ivanov[1,2,3]

[1] *Lomonosov Moscow State University, Moscow, Russia*
[2] *Moscow Center for Fundamental and Applied Mathematics, Moscow, Russia*
[3] *Bauman Moscow State Technical University, Moscow, Russia*
[alexandr.o.ivanov@gmail.com]

Let $M$ be a finite metric space, and $G = (V, E)$ be a *graph connecting* $M$, that is, $G$ is connected and $M \subset V$. A non-negative *weight function* $\omega$ given on edges of $G$ induces a peudo-metric $\rho_\omega$ on $V$ as follows: $\rho_\omega(x, y) = \min \omega(\gamma_{xy})$, where the minimum is taken over all paths $\gamma_{xy}$ connecting $x$ and $y$, and $\omega(\gamma_{xy})$ is the total edges' weight of $\gamma_{xy}$. A weighted graph $(G, \omega)$ is called a *filling of* $M$ if $|mm'| \leq \rho_\omega(m, m')$ for any $m, m' \in M$ (here $|mm'|$ stands for the distance in $M$). The graph $G$ is called a *type of the filling*.

The *weight of a minimal filling of* $M$ is $\inf \omega(G)$, where the infimum is taken over all fillings of $M$. A filling which this infimum is attained on is a *minimal filling of* $M$. If a type $G$ of filling is fixed and the infimum is taken only over the weight functions $\omega$ that makes $G$ a filling of $M$, then one obtains a concepts of *parametric minimal fillings of type* $G$.

The problem of a minimal filling of a finite metric space was posed in [1]. Besides existence theorems, it was shown that as a minimal filling of the space $M$ one can always choose a tree whose vertex degrees equal 1 or 3, where $M$ coincides with the set of vertices of degree 1 (such trees are called *binary*). It was also proved that the length of a shortest tree of $M$ in a metric space $X$ is bounded from below by the weight of minimal filling of $M \subset X$.

To calculate the weight of minimal fillings it is useful to consider trees with arbitrary (not necessarily non-negative) weights. Such fillings are called *generalized*. Although the weight of a minimal parametric generalized filling (MPGF) of type $G$ may be less than the weight of an ordinary minimal parametric filling of the same type $G$, the weight of a minimal filling coincides with the weight of a generalized minimal filling, see [2]. A formula for the weight of MPGF of a given type $G$ of $M$ was obtained by A. Eremin [3] in terms of so-called *multitours of* $G$. A *multitour of multiplicity* $k$ *of* $G$ is a cyclic route $\mu$ in the complete graph $K(M)$ on the vertex set $M$ that passes through each vertex exactly $k$ times and is consistent with the tree $G$ in the following sense: for each partition of $M$ generated by an edge of $G$, exactly $2k$ edges of the route $\mu$ connect vertices from different subsets of this partition. The *semiperimeter* $|\mu|$ of a multitour $\mu$ of multiplicity $k$ is the sum of the lengths of all its edges divided by $2k$. According to the formula, the weight of a MPGF of type $G$ is $\sup |\mu|$, where the supremum is taken over

all multitours of $G$. A. Eremin also showed that it suffices to consider only the so-called *irreducible* multitours, whose set is finite.

The minimal parametric filling problem can be formulated as a linear programming problem (LLP), see [1]. It is useful to consider the dual problem, see [4]. The feasible region of the direct LPP depends as on the type $G$ of the filling, so as on the distances in $M$, but the feasible region $W_G$ of the dual LPP is a multidimensional polyhedron that does not depend of the distances and is defined by the type $G$ only. The weight of MPGF of a given type $G$ can be found as a maximal value of the dual objective function (that depends on the distances in $M$) on the vertex set of the polyhedron $W_G$. Using this approach explicit formulas for the weights of MPGF were obtained for the case of metric spaces consisting of at most 6 points, see [4]. Also it was shown that rational points of $W_G$ correspond to multitours of $G$, and an estimate for a necessary number of multitours to verify was obtained. Later, O. Shcherbakov noticed that the vertices of the polyhedron $W_G$ correspond exactly to irreducible multitours, thus the weight formulas from [4] and [3] are equivalent.

To find the minimum filling, one must enumerate all possible types of trees $G$ connecting $M$. It turns out that the following general result holds.

**Theorem 1.** *Let $M$ be a finite set. The union of the vertex sets of all polyhedra $W_G$, where the union is taken over all possible binary trees $G$ connecting $M$, is the vertex set of a convex polyhedron depending on $|M|$ only.*


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3. Eremin A.Yu., "A formula for the weight of a minimal filling of a finite metric space", *Sbornik Math.*, vol. 204, no. 9, pp. 1285–1306, 2013.
4. Ivanov A., Tuzhilin A., "Dual Linear Programming Problem and One-Dimensional Gromov Minimal Fillings of Finite Metric Space", in: *Differential Equations on Manifolds and Mathematical Physics*, ed. by: V. Manuilov, A. Mishchenko, V. Nazaikinskii, B.-W. Schulze, and W. Zhang, Trends in Math. Birkhauser, Cham. 2021, pp. 165–182.

## On the quantum phase operator

Nikolay Ivanov

*Sofia University, Bulgaria*
[nikolay.antonov.ivanov@gmail.com]

I will make some comments about the Quantum Phase Operator coming from the Harmonic Oscillator.

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## Spin(7) manifolds, instanton curvature and parallel torsion

Stefan Ivanov

*Sofia University St. Kliment Ohridski - FMI and IMI-BAS, Bulgaria*
[mira@matf.bg.ac.rs]

We observe that on a compact $Spin(7)$ manifold with $\nabla$-closed torsion 3-form the torsion connection $\nabla$ is a $Spin(7)$ instanton if and only if the torsion 3-form is parallel with respect to the torsion connection. We show that on a compact Spin(7) manifold with closed torsion 3-form the torsion connection is a Spin(7)-instanton exactly when the torsion 3-form is parallel with respect to the torsion connection.

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## $C^1$ and $C^0$ Hamiltonian systems

Božidar Jovanović

*Mathematical Institute SANU, Serbia*
[bozaj@mi.sanu.ac.rs]

We construct classical mechanical models with almost everywhere $C^\infty$-Hamiltonian functions that are of the class $C^1$ and $C^0$ on a given hypersurface $\Delta$. By restricting the continuous in $t$ Hamiltonian dynamics, to a discrete dynamics on $\Delta$, we introduce and describe a return map, an analog of a billiard map. As a motivating example, we derive and analyze models of bungee jumping (joint result with Vladimir Dragović and Borislav Gajić).

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# Strict equivalence of multi-virtual linkoids

Louis Kauffman

*Department of Mathematics, Statistics and Computer Science, University of Illinois at Chicago, USA*
[kauffman@math.uic.edu]

We utilize multi-virtual knot theory where there are a multiplicity of virtual crossings to study Strict Virtual Linkoids. In strict virtual linkoid theory, local moves define all virtual moves and Reidemeister moves.In the strict equivalence, no moves, classical or virtual, can transfer an arc across a linkoid endpoint. By taking closures of strict virtual linkoids that are multi-virtual knots and links, we obtain new invariants for strict virtual linkoids. Generalized bracket polynomial invariants and generalized loop bracket polynomial invariants (for planar strict virtual linkoids) are studied in this context. The talk defines Virtual Polar Links where there are degree two nodes in virtual link diagrams across which isotopies are forbidden. The talk shows how multi-virtual theory and its concepts can be applied to obtain invariants for polar virtual links. The talk is self-contained and we will begin the talk by explaining how the multi-virtual theory is motivated by coloring problems in graph theory and by extensions of the Penrose chromatic evaluations of trivalent graphs

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# Hypersurfaces of the sphere $\mathbb{S}^6(1)$ with four-dimensional nullity distribution

Djordje Kocić

*University of Belgrade, Faculty of Mathematics, Belgrade, Serbia*
[djordje.kocic@matf.bg.ac.rs]

This talk is based on a joint work [2] with Miroslava Antić. In this article, hypersurfaces of the nearly Kähler sphere $\mathbb{S}^6(1)$ are studied with respect to the dimension of their nullity distribution. The nullity distribution of a hypersurface $M$ of the sphere $\mathbb{S}^6$ is the distribution

$$\mathcal{D}_p = \{X \in T_pM \mid h(X,Y) = 0,\ \forall Y \in T_pM\}.$$

Since totally geodesic hyperspheres $\mathbb{S}^5$ trivially have a five-dimensional nullity distribution, a natural question of interest is whether there exist hypersurfaces with a four-dimensional nullity distribution and what their properties are. A local moving frame was constructed, compatible with the condition that the hypersurface has a four-dimensional nullity distribution, and the structural equations were derived using the vector fields of that frame. Their geometric properties were analyzed, among other things showing that they are product manifolds of special types. Furthermore, their classification was given, along with an explicit construction based on a sphere curve and a vector field along that curve.

1. Antić, M. (2017) Four-dimensional CR submanifolds of the sphere $S^6(1)$ with two-dimensional nullity distribution. *J. Math. Anal. Appl.*, **445**, 1–12.
2. Antić, M., Kocić, Dj. (2025) Hypersurfaces of the sphere $\mathbb{S}^6(1)$ with four-dimensional nullity distribution. *Journal of Geometry and Physics*, **213**, no. 105493.
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# Banach space geometry, $c_0$ and $\ell^1$ in operator-valued integration

Mihailo Krstić

*University of Belgrade, Faculty of Mathematics, Belgrade, Serbia*
[mihailo.krstic@matf.bg.ac.rs]

We study how the geometric structure of Banach spaces influences the integration of operator-valued functions. In particular, we consider strongly integrable mappings with values in spaces of bounded linear operators $B(X,Y)$ between Banach spaces $X$ and $Y$ and investigate the countable additivity of the vector measures generated by their integrals. A central role is played by classical geometric properties related to the presence of isomorphic copies of $c_0$ and $\ell^1$.

We show that if the dual space $X^*$ does not contain an isomorphic copy of $c_0$ then a strongly integrable operator-valued function generates a countably additive operator-valued measure in the operator norm. We also study the compactness of such integrals and show that, under the assumption that $X$ does not contain an isomorphic copy of $\ell^1$, the integral of a strongly integrable compact-operator-valued function is compact.

As an application, we obtain an estimate for the spectral radius of the integral of a strongly integrable $\mu$-measurable function $A : \Omega \to B(X)$, assuming that $X^*$ does not contain an isomorphic copy of $c_0$ and that the operators $A_t$, $t \in \Omega$, are mutually commuting:

$$r\left(\int_\Omega A, d\mu\right) \leqslant \int_\Omega r(A_t)\, d\mu(t).$$

These results illustrate the interplay between the geometry of Banach spaces, operator-valued integration, compactness and spectral theory.

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# Symplectic classification of parabolic singularities and cuspidal tori in integrable Hamiltonian systems

Elena Kudryavtseva

*Moscow State University, Russia*
[ekudr@list.ru]

We study singularities of the Lagrangian foliation given by a completely integrable system. A general problem of the theory of singularities of integrable systems is to describe the topology of singular fibers and their saturated neighborhoods (and a similar problem can be stated for singular orbits). In this talk, we discuss just one particular type of singularitites, namely parabolic orbits and cuspidal tori (speaking informally, a cuspidal torus is a compact singular fiber that contains one parabolic orbit and no other singular points). We assume that the integrable system is real-analytic. An important property of parabolic orbits is their stability under small integrable perturbations (informally speaking, a singularity is called structurally stable if the topology of the fibration is preserved after any (small enough) real-analytic integrable perturbations of the system). This is one of the reasons why such orbits can be observed in many examples of integrable Hamiltonian systems (e.g. Kovalevskaya top and other integrable cases in rigid body dynamics). We give a classification, up to real-analytic symplectic equivalence, of the Lagrangian foliations in a neighbourhood of a cuspidal torus (and similarly for a parabolic orbit). We also extend this result to swallowtail singularities in any dimension. The work is supported by the Russian Science Foundation, project 24-71-10100.

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## Noncompact homogeneous Einstein spaces and extended Poincare algebras

Aleksei Lavrov

*Moscow Institute of Physics and Technology, Russia*
[lavrov.an@mipt.ru]

For any pseudo-Euclidean vector space $V = R^{3,q}$ and module W over the even Clifford algebra, V. Cortes has associated noncompact homogeneous quaternionic Kahler manifold. It happens that generalization of this construction to the case $V = R^{p,q}$, $p = 3mod4$, leads to noncompact Riemannian homogenous spaces which are not quaternionic, but still Einstein. So we obtain new examples of noncompact homogeneous Einstein spaces and corresponding nilsolitons. The talk is devoted to discussion of these results.

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## The basic functions of three dimensions of FRLW metric

Nenad Dj. Lazarov

*Department of Theoretical Physics and Condensed Matter Physics (020), Vinca Institute of Nuclear Sciences - National Institute of the Republic of Serbia, University of Belgrade, P.O. Box 522, 11001 Belgrade, Republic of Serbia, Serbia*
[lazarov@vinca.rs]

In this work, the velocity and radius will be presented as a function of time in two space dimensions and one time dimension FRLW metric. Geodesic equations will be given in two space dimensions and one time dimension. The geodesic equations have been solved, and the basic properties of the three dimensions of the cosmos will be presented. The theoretical results will be discussed using special cases of $f(R)$ gravity to determine the cosmic scaling parameter $a(t)$.

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# Neighborly Murai spheres and the simplicial Steinitz problem

Ivan Limonchenko

*Mathematical Institute SASA, Serbia*
[ilimonchenko@gmail.com]

In 1992, Thomas Bier introduced a beautiful and powerful construction, which yields a simplicial sphere starting from an abstract simplicial complex different from a simplex. Namely, a Bier sphere equals the deleted join of a complex and its combinatorial Alexander dual. In 2011, Satoshi Murai generalized this construction to multicomplexes. The classical Simplicial Steinitz Problem asks if a given simplicial sphere can be realized as a boundary of a convex simplicial polytope. It is known by an implicit argument that almost all Bier spheres are non-polytopal when the number of vertices tends to infinity, although no particular example of a non-polytopal Bier sphere has been constructed so far.

In this talk, we consider the Simplicial Steinitz Problem in the class of Murai spheres. Since this class is much bigger than that of Bier spheres, we can expect that it might be easier to construct an example of a non-polytopal Murai sphere than that of a Bier sphere. As a first step towards this goal, we identify a large class of polytopal Murai spheres, classify all neighborly Murai spheres and show that all of them belong to that class. As a byproduct of our technique, we prove that any simplicial sphere whose number of vertices and dimension differ not greater than by four is simplicially isomorphic to a Murai sphere.

The talk is based on joint works with Aleš Vavpetič (University of Ljubljana).

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## Noncommutative integrability in cosymplectic geometry

Katarina Lukić

*University of Belgrade, Faculty of Mathematics, Belgrade, Serbia*
[katarina.lukic@matf.bg.ac.rs]

Recently, a general setup for integrability that includes symplectics, contacts and Dirac structures has been performed by Zung (see [3]). In [1], motivated by time-dependent Hamiltonian dynamics, we consider some specific aspects of the integrability within the framework of cosymplectic geometry, which are not covered in [3]. We extend the notion of Arnold-Liouville and noncommutative integrability of Hamiltonian systems on symplectic manifolds (see [2]) to that on cosymplectic manifolds. Furthermore, we give a variant of noncommutative integrability for evaluation and Reeb vector fields on cosymplectic manifolds and a construction of cosymplectic action-angle variables.

Joint work with Božidar Jovanović.

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## Concircularly semi-symmetric metric connection and application to Lorentzian manifolds

Miroslav Maksimović

*Faculty of Sciences and Mathematics, University of Priština in Kosovska Mitrovica, Serbia*
[miroslav.maksimovic@pr.ac.rs]

We consider curvature properties of a concircularly semi-symmetric metric connection on pseudo-Riemannian manifolds. Applying this connection to Lorentzian manifolds leads us to GRW space-time.

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# Characterization of Π-manifolds by tensors generated by the pair of metrics interconnected by the structure

Mancho Manev

*Paisii Hilendarski University of Plovdiv , Bulgaria*
[mmanev@uni-plovdiv.bg]

We give an alternative characterization of the classes of almost paracontact almost paracomplex manifolds with a pair of Riemannian and pseudo-Riemannian metrics, $g$ and $\tilde{g}$, which are related to each other by structure. To this end, we use two approaches to the studied manifolds (shortly Π-manifolds). First, we make a classification with respect to $\tilde{g}$ in contrast to the known one for $g$. Second, we give a characterization by the difference of the two Levi-Civita connections generated by the pair of metrics. We comment on the mutual relations between these characterizations. We provide clear examples to illustrate the results.

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# Schwarz lemma for harmonic maps and minimal surfaces

Miodrag Mateljević

*University of Belgrade, Faculty of Mathematics, Belgrade, Serbia*
[miodrag@matf.bg.ac.rs]

Let $n \geq 2$, let $F : \mathbb{D} \to H$ (in particular $F : \mathbb{D} \to \mathbb{B}^n$) be harmonic, and write $F = P[\Phi]$, where $\Phi \in h^\infty(\mathbb{D}, H)$ ($\Phi : \mathbb{T} \to \overline{\mathbb{B}^n}$) is the radial boundary function. Suppose that $F(0) = p$ and that $dF_0$ is conformal, meaning that $F_x(0)$ and $F_y(0)$ are orthogonal and have the same Euclidean norm. If $dF_0 = 0$, the desired estimate is trivial. Otherwise let

$$\Lambda = dF_0(\mathbb{R}^2)$$

be the image plane, and let $\lambda > 0$ be the conformal factor. The plane $\Lambda$ is a linear two-plane through the origin in the ambient space $\mathbb{R}^n$. The point $p = F(0)$ need not lie in $\Lambda$; throughout this section, $\pi_\Lambda p$ denotes the orthogonal projection of the position vector $p$ onto this direction plane.

Write

$$p = p_\Lambda + p_\perp, \qquad p_\Lambda = \pi_\Lambda p \in \Lambda, \qquad p_\perp \perp \Lambda.$$

Choose an orthonormal basis $e_1, e_2$ of $\Lambda$ with $p_\Lambda = ae_1$, where $a = |p_\Lambda|$; if $a = 0$, choose $e_1$ arbitrarily. Since $dF_0$ is a similarity onto $\Lambda$, a rotation of the source variable gives the normalization

$$F_x(0) = \lambda e_1, \qquad F_y(0) = \lambda e_2. \tag{1}$$

Put $b = |p_\perp|$, and if $F : \mathbb{D} \to \mathbb{B}^n$ set

$$c = \sqrt{1 - b^2}, \qquad s = \frac{a}{c}. \tag{2}$$

Consider the standard Möbius transformation $\varphi_s(z) = \frac{s+z}{1+sz}$ for $|z| = 1$, $0 \leq s < 1$, and define the boundary-support vector field

$$E(z) = E_s^c(z) = \big(c\varphi_s(z), p_\perp\big) \in \Lambda \oplus \Lambda^\perp, \quad c > 0, \tag{3}$$

where the first component is understood as a vector in $\Lambda$. Define the total weighted support integral:

$$I = I(\Phi) = \langle \Phi, E \rangle_w = \int_{\mathbb{T}} w(z) \langle \Phi(z), E(z) \rangle \, dm(z). \tag{4}$$

Note that $E_s^c(0) = p$ if and only if $cs = a$.

Let $\mathbb{D}$ denote the unit disk and $\rho_{\mathbb{D}}$ the hyperbolic metric on it.

**Theorem 1.** *Text of Theorem 1*
Let $H$ be a real or complex Hilbert space, and let $F : \mathbb{D} \to H$ be a harmonic map belonging to the Hardy space $h^\infty(\mathbb{D}, H)$. Let $F(0) = p$, and suppose that the real differential $dF_0$ is conformal (or more generally, $K$-quasiconformal) and the tangent plane $T_p$ is parallel to $\mathbb{D}$. If $dF_0 \neq 0$, let $\Lambda = dF_0(\mathbb{R}^2)$ be the closed two-dimensional image plane in $H$, and let $\lambda = \|dF_0\|$ be the conformal factor and assume that $|\langle \Phi, E_s^c \rangle_w| \leq 1 + s^2$ (the support inequality w.r.t. Möbius transformation $E_s^c$). If in addition $E_s^c(0) = p$ (i.e. $cs = a$), then $E_s^c$ is the unique extremal, and we obtain:

$$\lambda \leq \lambda_0 = \frac{1 - |p|^2}{\sqrt{1 - |p|^2 + |\pi_\Lambda p|^2}}. \tag{5}$$

For $p \in \mathbb{B}^n$ and $v \in \mathbb{R}^n \simeq T_p\mathbb{B}^n$, define

$$CK_{\mathbb{B}^n}(p; v) = \frac{\sqrt{(1 - |p|^2)|v|^2 + \langle p, v \rangle^2}}{1 - |p|^2}. \tag{6}$$

This is the infinitesimal norm of the Beltrami–Klein model of real hyperbolic space. Let $\mathbb{B}_H^n$ denote the unit ball in Hilbert space $H$. We use this definition also for $p \in \mathbb{B}_H^n$ and $v \in H$, and by $CK$ we also denote the Cayley–Klein metric on $\mathbb{B}_H^n$.

**Theorem 2.** *Let $F : \mathbb{D} \to \mathbb{B}_H^n$ be harmonic, let $z_0 \in \mathbb{D}$, and suppose that $dF_{z_0}$ is $K$-quasiconformal at $z_0$. Then, for every $v \in T_{z_0}\mathbb{D}$,*

$$CK_{\mathbb{B}^n}(F(z_0); dF_{z_0} v) \leq K \frac{|v|}{1 - |z_0|^2}. \tag{7}$$

Equivalently, in the Schwarz–Pick normalization,

$$CK_{\mathbb{B}^n}(F(z_0); dF_{z_0} v) \leq \rho_{\mathbb{D}}(z_0; v). Geoseminar$$

From **Theorem 2** we derive:

**Theorem 3.** *Let $F : \mathbb{D} \to \mathbb{B}_H^n$ be a harmonic $K$-quasiconformal map, then*

$$CK_{\mathbb{B}^n}(F(z_0), F(z)) \leq K \rho_{\mathbb{D}}(z_0, z).$$

The part of the plan of communication is joint work with M. Knežević.

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2. M. Mateljević and M. Svetlik, Hyperbolic metric on the strip and the Schwarz lemma for HQR mappings, *Applicable Analysis and Discrete Mathematics* **14** (2020), no. 1, 150–168.
3. Mateljević, M., Mutavdžić, N. *The Boundary Schwarz Lemma for Harmonic and Pluriharmonic Mappings and Some Generalizations.* Bull. Malays. Math. Sci. Soc. 45, 3177–3195 (2022).
4. W. Zwonek, Complex geodesics in tube domains and their role in the study of harmonic mappings in the disc, *Anal. Math.* **48** (2022), no. 2, 601–617.

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# On Ricci-Yamabe solitons

Şemsi Eken Meriç

*Mersin University, Turkey*
[semsieken@mersin.edu.tr]

In this paper, we study Riemannian maps between Riemannian manifolds admitting a Ricci-Yamabe soliton. We establish necessary and sufficient conditions for the fiber and base manifolds of such maps to be Einstein or to admit a Ricci-Yamabe soliton structure.

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# Special vector fields: torqued and anti-torqued

Adela Mihai

*Technical University of Civil Engineering Bucharest and Transilvania University of Braşov, Romania*
[adela.mihai@utcb.ro, adela.mihai@unitbv.ro]

We first study the existence of torqued and anti-torqued vector fields on the hyperbolic ambient space $\mathbf{H}^n$ [1]. Although there are examples of proper torqued vector fields on open subsets on $\mathbf{H}^n$, we prove that there is no a proper torqued vector field globally defined on $\mathbf{H}^n$. Similarly, we have another non-existence result for anti-torqued vector fields, as long as their conformal scalar is a non-constant function. Moreover, when the conformal scalar is constant, some examples of anti-torqued vector fields are provided.

In the second part of the talk we study rectifying submanifolds of a Riemannian manifold endowed with an anti-torqued vector field [2]. For this, we first determine a necessary and sufficient condition for the ambient space to admit such a vector field. Then we characterize submanifolds for which an anti-torqued vector field is always assumed to be tangent or normal. A similar characterization is also done in the case of the torqued vector fields. We obtain that the rectifying submanifolds with anti-torqued axis are the warped products whose warping function is a first integration of the conformal scalar of the axis.

Other results obtained in recent papers [3] and [4] will be stated.

This is a joint work with Muhittin Evren AYDIN and Cihan ÖZGÜR under the TUBITAK Project Grant 123F451.

1. M. E. Aydin, A. Mihai, C. Özgür, *Torqued and anti-torqued vector fields on hyperbolic spaces*, Filomat **39(20)** (2025), 7043-7051.
2. M. E. Aydin, A. Mihai, C. Özgür, *Rectifying submanifolds of Riemannian manifolds with anti-torqued axis*, J. Korean Math. Soc. **62(5)** (2025), 1127-1142.
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4. M. E. Aydin, A. Mihai, C. Özgür, *Hypersurfaces in Riemannian manifolds with torse-forming axes*, accepted for publication in Turkish J. Math. https://arxiv.org/abs/2511.06370

# Geodesic mappings and their generalizations "on whole"

Josef Mikeš

*Palacky University in Olomouc, Czech Republic*
[josef.mikes@upol.cz]

The examples of geodesic, almost geodesic, holomorphically projective and rotary mappings on compact Riemannian spaces "on whole" are presented. We obtain the criterions of rigidity on the above mappings and transformations with cetain conditions of geodesics. Join lecture with N. Guseva, I. Hinterleitner, P. Peška, Ž. Radulović and L. Vítková.

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# Gromov–Hausdorff distances between unbounded metric spaces

Ivan Mikhailov

*Lomonosov Moscow State University, Russia*
[1hedgemih1@gmail.com]

The classical Gromov–Hausdorff distance between metric spaces $X$ and $Y$ is defined as the infinum of Hausdorff distances between the images $X'$ and $Y'$ of $X$ and $Y$ over all isometric embeddings $\varphi\colon X \to Z$ and $\psi\colon Y \to Z$ in some metric space $Z$. This distance was first introduced by David Edwards in 1975 and later became famous due to the work of Mikhail Gromov on groups of polynomial growth.

In the well-known monograph *Metric structures for riemannian and non-riemannian surfaces* Mikhail Gromov introduced classes of metric spaces at finite Gromov–Hausdorff distances from each other (later, in the work of S. A. Bogatyi and A. A. Tuzhilin, such classes were called *clouds*). The structure of the cloud of all bounded metric spaces is well known: it is a contractible cone with its vertex in a one-point metric space. However, until recently, almost nothing was known about the geometry of all other clouds defined by unbounded metric spaces.

In the talk, we discuss some recent results related to the calculation of Gromov–Hausdorff distances between some classes of unbounded metric spaces, i. e., normed spaces and $\varepsilon$-nets in them, metric trees and their sufficiently dense subsets, etc. Along the way we will see a number of elegant techniques of computing the Gromov–Hausdorff distances, including new applications of ultrametrization, asymptotic dimension, and simplicial complexes. We will also concern the geometry of the cloud of the real line, and construct geodesic lines of new type lying in this cloud.

# Invariant geometric objects of special types of mappings of generalized Riemannian spaces

Vladislava Milenković[1], Mića Stanković[2], Nenad Vesić[3],

[1] *Faculty of Technology, University of Niš, Serbia*
[vladislava@tf.ni.ac.rs]

[2] *Faculty of Sciences and Mathematics, University of Niš, Serbia*
[mica.stankovic@pmf.edu.rs]

[3] *Mathematical Institute of Serbian Academy of Sciences and Arts, Belgrade, Serbia*
[n.o.vesic@outlook.com]

The talk is devoted to the study of invariant geometric objects associated with special types of mappings of generalized Riemannian spaces. The main emphasis is placed on the construction of invariant geometric objects and the investigation of their fundamental properties. Several relations satisfied by these invariants are derived, providing a deeper insight into the geometric characteristics of the considered mappings.

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N. O. Vesić wishes to thank project O-40-26 of the Serbian Academy of Sciences and Arts, Branch in Niš, for supporting this research.

# Combinatorics of universal enveloping Lie algebras and some geometrical problems

Dmitry Millionschikov

*Lomonosov Moscow State University, Russia*
[dmitry.millionschikov@math.msu.ru]

The talk will be devoted to three topics related to applications of combinatorics of universal enveloping Lie algebras. We will discuss: 1) Virasoro singular vectors [1]; 2) higher symmetries of hyperbolic Klein-Gordon equations [2, 3, 4]; 3) canonical operator Van Vleck perturbation theory in high-resolution molecular laser spectroscopy [5, 6].

1. D. V. Millionshchikov, *Singular vectors of Verma modules over the Virasoro algebrs*, Funct. Analysis and its Appl. **50**:3 (2016), 66–72.
2. Dmitry Millionshchikov, *Lie Algebras of Slow Growth and Klein–Gordon PDE*, Algebras and Representation Theory, **21**:5 (2018), 1037–1069.
3. D.V. Millionshchikov, "Characteristic Lie algebras of the sine-Gordon and Tzitzeica equations," Uspekhi Mat. Nauk, **72**:6(438) (2017), 203–204.
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## Canonical parameters on marginally trapped surfaces in pseudo-Euclidean 4-spaces

Velichka Milousheva

*Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria*
[vmil@math.bas.bg]

We introduce special parameters (which we call canonical parameters) for some classes of surfaces in the Minkowski 4-space and the pseudo-Euclidean 4-space with neutral metric, namely: zero mean curvature surfaces, surfaces with parallel normalized mean curvature vector field, and marginally trapped (quasi-minimal) surfaces. These parameters allow us to describe these surfaces in terms of minimal number of invariant functions satisfying a natural system of partial differential equations. This solves the Lund-Regge problem for these classes of surfaces.

The talk is based on a joint work with Miroslav Maksimović.

The author is partially supported by the National Science Fund, Ministry of Education and Science of Bulgaria under contract KP-06-N82/6.

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## Billiard trajectories inside cones

Andrey Mironov

*Sobolev Institute of Mathematics, Russia*
[mironov@math.nsc.ru]

We discuss billiard trajectories inside an n-dimensional cone over a strictly convex closed manifold M. It is shown that if $M$ is a $C^3$- smooth manifold, then every trajectory has a finite number of reflections and, in this case, the billiard admits first integrals whose values uniquely determine all billiard trajectories. At the same time, there exists a $C^2$-smooth manifold $M$ and a billiard trajectory in the cone such that this trajectory has infinitely many reflections in finite time. The results were obtained with Siyao Yin.

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## Geometric properties of infinitesimally deformed dual curves

Marija Najdanović

*University of Priština in Kosovska Mitrovica, Faculty of Sciences and Mathematics, Serbia*
[marija.najdanovic@pr.ac.rs]

The classical Study correspondence establishes a one-to-one relation between oriented lines in Euclidean three-space and points of the dual unit sphere. Under this correspondence, a differentiable dual spherical curve determines a ruled surface, while the geometric invariants of the curve encode important properties of the associated surface. This provides an effective framework for studying deformations of ruled surfaces through the corresponding deformations of dual curves. We investigate infinitesimal bendings of curves in three-dimensional dual space. Such deformations are generated by dual vector fields and preserve the dual arc length up to the prescribed order of approximation. We derive geometric characterizations of the corresponding bending fields and analyze their influence on curvature-dependent quantities of dual curves. Several examples illustrate the construction of bending fields for dual spherical curves and the resulting infinitesimal deformations of their corresponding ruled surfaces. This is a joint work with Svetozar Rančić and Ljubica Velimirović.

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## Killing tensors on symmetric spaces

Yuri Nikolayevsky

*La Trobe University, Melbourne, Australia*
[y.nikolayevsky@latrobe.edu.au]

I will present new and old results on geometry and algebra of higher rank Killing tensor fields on Riemannian symmetric spaces, with a particular focus on the following question: Is any Killing tensor field on a symmetric space a polynomial in Killing vector fields? The talk will include some joint results with Vladimir Matveev (Germany) and Owen Dearricott and An Ky Nguyen (Australia).

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## Multicrossing homology and secant classes

Igor Nikonov

*Moscow State University, Russia*
[mira@matf.bg.ac.rs]

The multicrossing homology of a knot can be considered as a generalization of the homology of its fundamental quandle. In the talk we show that certain sums of knot secants define invariant classes in the multicrossing homology. The 2-secant class generalizes such invariants as linking number and quandle cocycle invariants.

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## Properties of plane angles of tetrahedra with a given base

Yurii Nikonorov

*Southern Mathematical Institute of the Vladikavkaz Scientific Center of the Russian Academy of Sciences, Russia*
[nikonorov2006@mail.ru]

This talk is based on the joint project with Evgenii Niktenko (Rubtsovsk Industrial Institute of Altai State Technical University after I. I. Polzunov, Rubtsovsk, Russia).

Let $\Omega(\triangle ABC)$ be the set of all tetrahedra $ABCD$ in three-dimensional Euclidean space with a given non-degenerate base $ABC$ and a vertex $D$ lying outside the plane $ABC$. Let us consider the set

$$\Sigma(\triangle ABC) = \{(\cos\alpha, \cos\beta, \cos\gamma) \in \mathbb{R}^3 | ABCD \in \Omega(\triangle ABC)\},$$

where $\alpha = \angle BDC$, $\beta = \angle ADC$, and $\gamma = \angle ADB$. The talk is devoted to describing the closure of the set $\Sigma(\triangle ABC)$ in $\mathbb{R}^3$.

The main results were obtained in the following paper:

E.V. Nikitenko, Yu.G. Nikonorov, *On face angles of tetrahedra with a given base.* Results in Mathematics, 81, article number 60 (2026), 50 pp., https://doi.org/10.1007/s00025-026-02610-x.

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## Image sharpening improvement via the geometrical coding approach

A.O. Ivanov, G.V. Nosovskiy[1], V. Kibkalo, D. Fedoseev, G. Begunov, G. Chernyshov, A. Leonenko

[1] *Moscow Lomonosov State University, Russia*
[gleb-sveta2013@yandex.ru]

Geometrical coding of color/grayscale images makes it possible to sufficiently improve image sharpening procedures, based on the unsharp mask (USM) approach. It is well-known that in order to improve USM procedures it is important to know contour configuration in the vicinity of the pixel. Main idea of USM modification by means of geometrical coding is to use certain geometry-based functions on coding surfaces in order to estimate distance to the nearest contour of the image.

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## Classification of the geodesic curves of a sub-Riemannian LR system on the Heisenberg group in dimension 5

Milan Pavlović

*University of Belgrade, Faculty of Mathematics, Belgrade, Serbia*
[milan.pavlovic@matf.bg.ac.rs]

We study sub-Riemannian structures obtained by the restriction of the standard left-invariant Riemannian metric to a right-invariant completely nonholonomic distribution on the second Heisenberg group. Integrability of such sub-Riemannian structure has been established in previous investigations. Motivated by these results, we turn our attention to the study of geodesic lines, whose analysis provides a natural continuation of this research and contributes to a more comprehensive understanding of the underlying sub-Riemannian geometry. Reduced Hamiltonian system is studied in hyperspherical coordinates. This coordinates allows us to classify geodesics according to the properties of $r(t)$. Further, $r(t)$ is computed as the first step of the quadrature. We conclude by presenting numerical examples that illustrate the theoretical results.

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## Non-existence of conformal Fedosov structures on Roter type manifolds

Patrik Peška

*Palacky University Olomouc, Czech Republic*
[patrik.peska@upol.cz]

We investigate conformal Fedosov structures on pseudo-Riemannian Roter type manifolds. Such a structure is determined by a non-degenerate two-form $\omega$ satisfying $\nabla\omega = \theta \otimes \omega$ with respect to the Levi–Civita connection, where $\theta$ is the Lee form. Using the Ricci identity, we prove that $\theta$ is closed and that, after a local conformal rescaling, $\omega$ becomes parallel. Consequently, the associated endomorphism $J = g^{-1}\omega$ commutes with the Ricci operator. Combining this integrability condition with the Roter decomposition of the curvature tensor, we show that in even dimensions $n \geq 6$ the existence of a conformal Fedosov structure forces the manifold to have constant sectional curvature. This demonstrates the rigidity of conformally Fedosov geometry within the Roter class.

Joint work with Marek Jukl and Josef Mikeš.

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## The almost Schur lemma and the positivity conditions in quaternionic contact geometry

Alexander Petkov

*Sofia University St. Kliment Ohridski, Bulgaria*
[a_petkov_fmi@abv.bg]

The main goal of this talk is to present quaternionic contact (qc) versions of the so called Almost Schur Lemma, which give estimations of the qc scalar curvature on a compact qc manifold to be a constant in terms of the norm of the [-1]-component and the norm of the trace-free part of the [3]-component of the horizontal qc Ricci tensor and the torsion endomorphism, under certain positivity conditions. The talk is based on a joint work with Stefan Ivanov.

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## Integrable billiards and refraction

Theodore Popelensky

*Moscow State Lomonosov University, Russia*
[popelens@gmail.com]

In a recent paper by Nikulin and the speaker (Sbornik: Mathematics, 2026), a new class of integrable systems was introduced in an ellipse filled with media of different optical densities. It is based on a new non-standard law of refraction. The report will describe this class of systems and provide a topological analysis of several specific ones.

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## Topological complexity of oriented Grassmann manifolds

Marko Radovanović

*Faculty of Mathematics, University of Belgrade, Serbia*
[marko.radovanovic@matf.bg.ac.rs]

By a result of Farber, for any field $K$, the topological complexity of a space $X$ is greater than $\mathrm{zcl}_K(X)$, where $\mathrm{zcl}_K(X)$ is the $K$-zero-divisor cup-length of $X$. In this talk we study $\mathrm{zcl}_{\mathbb{Z}_2}(\widetilde{G}_{n,3})$, where $\widetilde{G}_{n,3}$ is the Grassmann manifold of oriented 3-dimensional vector subspaces in $\mathbb{R}^n$. For all integers $n \geq 6$, we will shown a lower and an upper bound for this invariant that differ by at most 1. This leads to a lower bound for the topological complexity of Grassmannians $\widetilde{G}_{n,3}$.

This is joint work with Uroš Colović and Branislav Prvulović.

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# Weak metric structures and their applications

Vladimir Rovenski

*Department of Mathematics, University of Haifa, Israel*
[vrovenski@univ.haifa.ac.il]

Weak metric structures [1] (i.e., the complex structure on the contact distribution is replaced with a nonsingular skew-symmetric (1,1)-tensor) generalize the K. Yano's $f$-structure and almost contact metric structure, and allow us to find new applications of contact Riemannian geometry. A. Einstein considered the metricity condition $(\nabla_X G)(Y,Z) = -G(T(X,Y),Z)$ for a connection $\nabla$ with torsion $T$ on a manifold equipped with a nonsymmetric basic (0,2)-tensor $G = g + F$, where $g$ is a pseudo-Riemannian metric associated with gravity, and $F \neq 0$ is a skew-symmetric tensor associated with electromagnetism. The theory of weak metric structures is fully consistent with the skew-symmetric part of $G$.

In the lecture we discuss geometry and applications of weak metric structures to characterizations of K-contact and Sasakian manifolds, splitting of weak contact manifolds with curvature related conditions, Ricci-type solitons and Einstein like metrics on weak K-contact and weak Kenmotsu manifolds, and Einstein's connections of weak almost Hermitian and weak almost contact metric manifolds, see [1]–[6].

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# Four dimensional ancient solutions asymptotic to a bubble sheet

Nataša Šešum

*Rutgers University, USA*
[natasas@math.rutgers.edu]

We will discuss 4d ancient solutions asymptotic to a bubble sheet. We will discuss the behavior of scalar curvature at infinity on complete non-compact steady gradient Ricci solitons. In dimension four, we identify the two edges of the soliton and prove that the scalar curvature decays at a linear rate away from these edges. We show that if the scalar curvature vanishes at infinity, then the asymptotic cone is a ray. In particular, our results apply to the four-dimensional flying wings constructed by Lai. We will also mention deriving precise asymptotics for 4d symmetric ancient closed solutions asymptotic to a bubble sheet.

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# Equitorsion holomorphically projective mappings of generalized Kähler spaces of the first, second, and third kind

Mića S. Stanković[1] and Vladislava M. Milenković[2]

[1] *Faculty of Sciences and Mathematics, University of Niš, Serbia*
[mica.stankovic@pmf.edu.rs]

[2] *Faculty of Technology, University of Niš, Serbia*
[vladislava@tf.ni.ac.rs]

We consider generalized Kähler spaces of the first, second, and third kind, $G\underset{\theta}{K}_N$, $\theta = 1, 2, 3$. These spaces represent a natural generalization of classical Kähler geometry and provide a suitable framework for studying mappings that preserve important geometric structures.

In particular, we investigate holomorphically projective mappings with invariant complex structure. Such mappings preserve the complex structure while transforming generalized geodesics in a projective manner, making them an important class of transformations in generalized Kähler geometry.

Furthermore, we study equitorsion holomorphically projective mappings between two generalized Kähler spaces of the same kind, $G\underset{\theta}{K}_N$ and $G\underset{\theta}{\overline{K}}_N$, $\theta = 1, 2, 3$, under the assumption that the torsion tensor remains unchanged. Special attention is devoted to the construction and investigation of geometric objects that remain invariant under such mappings. These invariant geometric objects provide effective criteria for characterizing equitorsion holomorphically projective mappings and contribute to a deeper understanding of the geometry of generalized Kähler spaces.

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# Half-symmetric connections of generalized Riemannian spaces

Marko Stefanović

*Faculty of Sciences and Mathematics, University of Niš, Niš, Serbia and Faculty of Information Technology, Metropolitan University, Serbia*
[marko.stefanovic@pmf.edu.rs]

In this talk, we investigate a special class of half-symmetric affine connections on almost Hermitian spaces. Starting from a canonical half-symmetric affine connection generated by the almost complex structure, we analyze its symmetric and dual associated connections and study their fundamental geometric properties. Necessary and sufficient conditions are established for these associated connections to be metric connections and F-connections. We further characterize the vanishing of the torsion tensor and relate it to the geometry of almost Hermitian, Hermitian, almost Tachibana, and almost semi-Kähler spaces. Several new equivalence conditions are obtained, providing geometric interpretations of the introduced half-symmetric connection. The second part of the talk is devoted to geometric objects defined with respect to this connection. We derive relations for Killing, affine Killing, conformal Killing, projective Killing, harmonic, and almost analytic vector fields and compare them with their classical counterparts. In addition, curvature tensors corresponding to the symmetric part of the introduced connection are determined explicitly. These results extend the theory of half-symmetric affine connections from general affine spaces to almost Hermitian geometry and provide new characterizations of important classes of almost Hermitian manifolds through the properties of the introduced connection.

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# A curvature-dependent inequality between Dirichlet energy and bienergy with rigidity

Sergey Stepanov

*Russian Institute for Scientific and Technical Information of the Russian Academy of Sciences, Moscow, Russia; and Finance University, Moscow, Russia*
[s.e.stepanov@mail.ru]

Geometric inequalities constitute a fundamental component of modern Riemannian geometry, providing quantitative relationships between curvature bounds, variation-al functionals, and rigidity phenomena. In differential geometry and the calculus of variations, the Dirichlet energy $E_1(f)$ and the bienergy $E_2(f)$ quantify the smoothness of maps $f : (M, g) \to \left(\hat{M}, \hat{g}\right)$ between Riemannian manifolds (see [1]; [2]). Assume that $(M, g)$ is a compact, connected Riemannian manifold whose Ricci curvature admits a global minimum $\mathrm{Ric}_{\min}$, and that the target manifold $(\hat{M}, \hat{g})$ has non-positive sectional curvature along $f(M) \subset \hat{M}$ (see [3]). We prove that under these assumptions, both functionals satisfy the inequality:

$$E_2(f) \geq \mathrm{Ric}_{\min} E_1(f).$$

We further analyze the equality case and obtain a rigidity result: equality holds if and only if $f$ is totally geodesic and has constant rank. Applications to maps into Hadamard manifolds are also presented. Finally, we consider a functional combining the Dirichlet energy and the bienergy, which arises naturally in the theory of sesqui-harmonic maps. To the best of our knowledge, this is the first geometric inequality directly relating the Dirichlet energy and the bienergy of smooth maps. The result establishes a direct connection between the Ricci curvature of the domain and higher-order variational energies.

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## Computing tensor logarithmic norm: explicit forms and spectral properties

Jelena Stojanov

*University of Novi Sad, Technical faculty "Mihajlo Pupin" Zrenjanin, Serbia*
[jelena.stojanov@uns.ac.rs]

Logarithmic norms are a classical tool for studying growth estimates and stability properties of matrix problems. Corresponding results for higher-order tensors remain incomplete, despite the increasing importance of tensor methods. Tensor logarithmic norms will be presented and discussed. Logarithmic norms are distinguished for tensor treated as hypermatrix and as multilinear operator, concerning their explicit forms and computational characteristics. Spectral properties of tensor logarithmic norms are determined by associated tensor eigenvalue problems. Some of the results are based on joint work with M. Žigić and D. Risteski.

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## Cone-type triangulations of hypercube

Milica Stojanović

*University of Belgrade, Faculty of Organizational Sciences, Serbia*
[milica.stojanovic@fon.bg.ac.rs]

The 4-triangulation of a 4-polytope is its division into 4-simplices (hypertetrahedra) without adding new vertices and without gaps or overlaps. Here we consider a special type of 4-triangulation of the hypercube where all hypertetrahedra have a common vertex V. There are more possibilities for cone-type triangulations, because there are more 3-triangulations of cubes opposite the vertex V.

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## New advancements in knot and 3-manifold invariants

Marko Stošić

*Technical University of Lisbon,Portugal*
[mstosic@isr.ist.utl.pt]

In this talk I will present some of the recent results concerning quantum knot invariants, like colored Jones and colored HOMFLY-PT invariants, and $\hat{Z}$ invariants of 3-manifolds. Mainly this will be done through the prism of knots-quivers correspondence, including appropriate variations and generalizations.

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# Canonical form of the inertia tensor of a rigid body in pseudo-Euclidean space and on the Lobachevsky plane

Anastasiia Shubert

*Lomonosov Moscow State University, Russia*
[anastasiia.shubert@math.msu.ru]

The talk concerns the inertia tensor of a rigid body in a three-dimensional pseudo-Euclidean vector space and the related motion of a plate on the Lobachevsky plane or on a pseudosphere. The configuration space is identified with the Lie group of automorphisms, similarly to the configuration space of a rigid body with a fixed point.

The kinetic energy is a left-invariant pseudo-Riemannian metric on this configuration space [1, 2], determined by a quadratic form on its Lie algebra. The associated symmetric operator is the *covariant inertia tensor* [2]. Using a pseudo-Euclidean analogue of the vector product, we define the *inertia operator* and relate it to the structure of the Lie algebra and the Killing–Cartan form [3].

We describe the canonical form of the inertia operator, including all nondiagonalizable cases [3]. and state realization theorems for possible triples of principal moments of inertia and centers of mass. For plates on a pseudosphere, realizability is characterized by the moments of inertia, while the sign of the trace determines the type of pseudosphere [4].

Applications include realizable pseudo-Euclidean Euler and Lagrange tops [4]; the Kovalevskaya analogue is shown to be nonrealizable within this construction. Several open problems are formulated.

The speaker is a BASIS Foundation scholarship recipient.

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# Central extensions of Lie algebras, dynamical systems, and symplectic nilmanifolds

Iskander Taimanov

*Sobolev Institute of Mathematics, Novosibirsk, Russia*
[taimanov@math.nsc.ru]

We describe connections between Euler's equations on central extensions of Lie algebras and Euler's equations on the inital algebras. We also discuss the special infinite sequence of central extensions of nilpotent Lie algebras constructed from the Lie algebra of formal vector fields on the line and describe the orbits of coadjoint representations for them. By using the compact nilmanifolds constructed from these algebras by I.K. Babenko and the author, it is shown that covering Lie groups for symplectic nilmanifolds can have any rank as solvable Lie groups.

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# Smooth manifolds in $G_{n,2}$ and $G_{n,2}/T^n$ assigned to the canonical $T^n$-action

Svjetlana Terzić

*University of Montenegro, Montenegro*
[sterzic@ucg.ac.me]

We discuss problems related to the canonical $T^n$-action on the complex Grassmann manifolds $G_{n,2}$, which consist of two-dimensional complex vector subspaces in $\mathbb{C}^n$.

Many structures on $G_{n,2}$ can be assigned to these torus actions. There is the standard moment map $\mu : G_{n,2} \to \Delta_{n,2}$ for the hypersimplex $\Delta_{n,2} \subset \mathbb{R}^n$. A regular value, in the classical sense, of the map $\mu$ is known [1] to be a point $x \in \Delta_{n,2}$ such that the stabilizer of any $y \in \mu^{-1}(x)$, for the $T^n$-action on $G_{n,2}$, is the diagonal circle $S^1$. The preimage $\mu^{-1}(x) \subset G_{n,2}$ is a smooth submanifold, and the orbit space $\mu^{-1}(x)/T^n$ is a symplectic manifold, known as a symplectic reduction for the given $T^n$-action.

In this talk, we discuss our results from [3] related to the problem of describing smooth manifolds $\mu^{-1}(x)$ for the Grassmannians $G_{n,2}$. We study in detail the Grassmann manifold $G_{4,2}$ and prove that $\mu^{-1}(x) \cong S^3 \times T^2$ for any regular value $x \in \Delta_{4,2}$.

In addition, since in this case, by [2], [4] any of the smooth manifolds $\mu^{-1}(x)/T^n$ is a moduli space of weighted pointed stable genus zero curves, we discuss such moduli spaces which can be obtained by the symplectic reduction on $G_{n,2}$. In this context, we prove that the Deligne-Mumford spaces $\overline{\mathcal{M}}_{0,n}$ and Losev-Manin spaces $\bar{L}_{0,n}$ are symplectic reductions only for $n = 4, 5$.

The talk is based on the joint work with Victor M. Buchstaber.

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# Research on geometry of gravitation by distinguishing the spatial and temporal part of spherically metric

Kostadin Trenčevski[1], Emilija Celakoska[2]

[1] *Ss Cyril and Methodius University in Skopje*
[kostatre@pmf.ukim.mk]

[2] *Ss Cyril and Methodius University in Skopje*
[emilija.celakoska@mf.ukim.mk]

We prove a theorem which states that in case of spherically symmetric metric the curvature tensor of the spatial metric (by ignoring the temporal part) must be 0, because otherwise it leads to a paradox. So, we consider gravitation, assuming that a point-mass body with mass $M$ radially translates each point by a 3-vector with magnitude $GM/c^2$. Then, the Newton's acceleration is deduced to be $\mathbf{g} = -\frac{GM}{r^2(r+GM/c^2)}\mathbf{r}$. The temporal metric coefficient $g_{44}$ is uniquely determined by $\mathbf{g}$ and then the Riemann curvature tensor is non-zero. The experimental tests apply and the basic results remain the same as in General Relativity. The obtained equations for $n$-body problem differ from the Einstein-Infeld-Hoffmann equations by Lorentz invariant addends. Compared with the General Relativity it differs by the following property: While in General Relativity the curvature scalar $\mathcal{R}$ is proportional to the density of matter, here $\mathcal{R}$ is proportional to the density of matter and also to the density of energy, considering the equality $E = mc^2$.

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# A hyperbolic optimization paradigm for orbital transfers: integrating Riemannian geometry and machine learning

Mustafa Deniz Türkoğlu

*Doğuş University Turkey*
[mturkoglu@dogus.edu.tr]

This study delineates orbital transfer dynamics outside the confines of classical Newtonian mechanics and reestablishes them within curved manifolds based on differential geometry. This theoretical framework for regular orbital resupply missions discards conventional Euclidean altitude variations; rather, it incorporates Riemannian and Poincare upper half-plane metrics to formulate geodesic transfer trajectories and elliptical arc lengths in tensorial representation. Analytical results demonstrate that hyperbolic space topology produces an asymptotic "geometric deviation multiplier" in contrast to scalar altitude variations. In the realm of systems engineering, these topological distances are integrated with the kinematic $\Delta v$ budget and operational capacity limitations to create a multivariate, deterministic feasibility (Go/No-Go) matrix. During the research's innovative stage, the acquired non-linear differential characteristics are simulated by machine learning frameworks; predictive assessments concerning maneuver expenses and mission optimization are corroborated with significant precision. This cross-disciplinary perspective uncovers the interplay between Riemannian geometry and artificial intelligence, introducing a novel hyperbolic optimization framework for computational astrophysics and deep space logistics.

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## Continuous Gromov-Hausdorff distance

Alexey Tuzhilin

*Lomonosov Moscow State University, Russia*
[tuz@mech.math.msu.su]

The famous Gromov-Hausdorff distance measures the degree of non-isometricity of metric spaces: for isometric spaces, the distance is zero, and the more dissimilar the spaces, the larger this distance. The Gromov-Hausdorff distance has many important applications, both in fundamental science and in applied fields, for example, in studying the growth rate of discrete groups, in computer graphics and computational geometry, in pattern recognition theory, in robotics, and even in cosmology. However, the classical definition of the Gromov-Hausdorff distance does not take into account additional structures that metric spaces may possess. Even the topology generated by the metric is ignored by this distance (the best "comparisons" of spaces do not have to be continuous). This talk will discuss several modifications of the Gromov-Hausdorff distance that take continuity into account, and will also highlight some differences and similarities with the classical distance.

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## Generalized Riemannian Geometry: An Application to Generative AI

Ana Velimirović

*Faculty of Sciences and Mathematics, University of Niš, Niš, Serbia*
[velimirovic018@gmail.com]

Motivated by Einstein's search for a unified field theory, the theory of nonsymmetric affine connection spaces and generalized Riemannian spaces $GR_N$ has been systematically developed in Minčić, Stanković and Zlatanović. In such spaces, the nonsymmetric fundamental tensor $g_{ij} \neq g_{ji}$ and a connection carrying nontrivial torsion give rise naturally to four kinds of covariant derivative and five linearly independent curvature tensors a classification without analogue in the classical Levi-Civita setting.

We briefly recall this framework, drawing on previous results of the author on equitorsion transformations in $GR_N$. The main focus is on an unexpected bridge to a rapidly developing area of contemporary machine learning: Riemannian Flow Matching (Chen and Lipman, 2024), a generative modeling framework driving state-of-the-art advances in protein and material design.

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# On the role of Riemannian spaces in the invariant theory of Riemannian and symmetric affine connection spaces

Nenad Vesić

*Mathematical institute of Serbian Academy of Sciences and Arts, Serbia*
[vesic.specijalac@gmail.com]

The core contribution of this work focuses on extending this methodology to derive general invariants for mappings of spaces with symmetric affine connection. Crucially, we demonstrate how these comprehensive affine invariants can be systematically reduced to the corresponding invariants for mappings of Riemannian spaces. To validate the physical significance of this reduction, we perform variational calculi on Lagrangian densities within Riemannian spaces. It is shown that while symmetric affine connection spaces can accommodate varied and complex manifold characteristics, the analysis of the resulting invariants and fields within the underlying Riemannian space is sufficient to capture the essential physics.

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# Geometrical invariants of thin structure deformation

Nenad Vesić

*Mathematical institute of Serbian Academy of Sciences and Arts, Serbia*
[vesic.specijalac@gmail.com]

Differential geometry is a powerful tool for describing deformation process of a structure from its initial, reference configuration to a deformed state of the current configuration, deriving general rules of mapping between them. However, these general results usually lack practical application examples so finding particular relations is often not a straightforward task and it requires considering some additional physical constraints for a given specific case. On the other hand, physical background and conditions of deformation of any particular structure are very consistently captured within the framework of continuum mechanics, that provides application-ready procedures for a vast variety of specific engineering situations. In this paper, the gap is bridged between the general and abstract framework of classical differential geometry on one side, and the physically grounded continuum mechanics models suitable for finding concrete numerical solutions to a given problem on the other, relating the well-known concepts from the continuum mechanics with general equations from differential geometry. Geometrical invariants will be derived which do not depend on a particular deformation case, and two illustrative numerical examples will be provided - deformation of a thin solid plate under purely mechanical loading, and thermal deformations of a thin bi-metal plate under varying temperature field.

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# Three-variable transcendental invariant of planar knotoids

Andrei Vesnin

*Sobolev Institute of Mathematics, Novosibirsk, Russia*
[vesnin@math.nsc.ru ]

The theory of knotoids was introduced in 2012 by V. Turaev [1]. He established basic properties of knotoids including several invariants of spherical and planar knotoids. It is known that planar knotoids are useful models of protein chains [2]. A three-variable transcendental invariant of planar knotoids was introduced in [3]. We will discuss properties of this invariant and demonstrate its application for calculating Gordian distance between planar knotoids. The talk is based on the joint work with W. Feng and F. Li, see [3].

1. V. Turaev, Knotoids. Osaka J. Math. 49:1 (2012), 195-223.
2. D. Goundaroulis et al. Studies of global and local entanglements of individual protein chains using the concept of knotoids. Scientific Reports. 7:1 (2017), 6309.
3. W. Feng, F. Li, A. Vesnin, A three-variable transcendental invariant of planar knotoids via Gauss diagrams. Mediterranean Journal of Mathematics 2025, 22(4), paper number 99, 24 pp.

# Modifications of the continuous Gromov–Hausdorff distance

Anton Vikhrov

*Lomonosov Moscow State University, Russia*
[vihrov.09@gmail.com]

The Gromov–Hausdorff distance was introduced to compare metric spaces up to isometry. It is defined as the infimum of the Hausdorff distances between isometric embeddings of the spaces into a common metric space.

The Gromov–Hausdorff distance does not take the topological structure of the spaces into account: the correspondences and maps arising in its definition and equivalent formulations may be discontinuous. There are known examples where the continuous Gromov–Hausdorff distance differs from the classical Gromov–Hausdorff distance.

In this talk, we introduce two new modifications of the continuous Gromov–Hausdorff distance: the smooth and the partially continuous distances. We discuss their main properties and establish relationships between the smooth and partially continuous Gromov–Hausdorff distances.

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# On geometry of screw motion

Srdjan Vukmirović

*University of Belgrade, Faculty of Mathematics, Belgrade, Serbia*
[srdjan.vukmirovic@matf.bg.ac.rs]

Any motion of a rigid body in Euclidean space can be realized as a screw motion. Elements of that screw motion are usually reconstructed by analyzing the group $SE(3) = SO(3) \ltimes \mathbb{R}^3$, which describes all possible motions in three-dimensional space. In computer graphics, robotics and 3D animation, there is a widely used mathematical concept for interpolation between orientations of rigid bodies called SLERP.

In this lecture, we present an alternative method to determine the elements of a screw motion representing given motion. To achieve this we use the action of group $SE(3)$ on the Plücker coordinates. We also compare SLERP interpolation and screw motion from geometrical point of view and prove that those interpolations coincide on orientations. This is joint work with J. Ormanović and M. Pavlović.

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# On classification of $\delta(2,2)$-ideal Lagrangian submanifolds of dimension-4

Handan Yıldırım

*Istanbul University, Turkey*
[handanyildirim@istanbul.edu.tr]

In 1990s, Chen defined new types of Riemannian invariants named $\delta$-invariants and constructed general optimal inequalities including $\delta$-invariants and the squared mean curvature of Riemannian submanifolds. Thus, he used $\delta$-invariants to determine an optimal lower bound for the mean curvature of these submanifolds. There have been a lot of works related to $\delta$-invariants and their applications (See [1] for the details.). For instance, $\delta$-invariants have been used for Lagrangian submanifolds of complex space forms.

Let $M^n$ $(n \geq 3)$ be an $n$-dimensional Lagrangian submanifold of a complex space form $\tilde{M}^n(4c)$. In [2], optimal inequalities related to $\delta$-invariants were given as a final solution for $M^n$ depending on whether $n_1+\cdots+n_k = n$ or $n_1+\cdots+n_k < n$ for $k$ $(k \geq 1)$ mutually orthogonal subspaces $L_i$ $(i = 1,\ldots,k)$ of the tangent space $T_pM^n$ at any point $p \in M^n$, where $\dim L_i = n_i$, $2 \leq n_1,\ldots,n_k \leq n-1$ and $n_1 + \cdots + n_k \leq n$. It is known that submanifolds satisfying the equality case of one of these inequalities are said to be ideal Lagrangian submanifolds (Cf. [1] and [2].).

In this talk which is based on the joint work with Bang-Yen Chen, Joeri Van der Veken and Luc Vrancken, taking into account [2] and [3], I mention about the classification of 4-dimensional $\delta(2,2)$-ideal Lagrangian submanifolds of a complex space form $\tilde{M}^4(4c)$.

1. B.-Y. Chen, *Pseudo-Riemannian geometry, $\delta$-invariants and Applications*, World Scientific, World Scientific Publ., Hackensack, NJ 2011.
2. B. Y. Chen, F. Dillen, J. Van der Veken and L. Vrancken, *Curvature inequalities for Lagrangian submanifolds: the final solution*, Differential Geometry and its Applications, **31** (2013), no. 6, 808–819.
3. H. Yıldırım and L. Vrancken, *$\delta^\sharp(2,2)$-ideal centroaffine hypersurfaces of dimension 4*, Taiwanese Journal of Mathematics, **27** (2023), no. 6, 1075-1104.