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Thermal emission at radio frequencies from supernova remnants and a modified theoretical Σ–D relation

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Abstract

In this paper, we discuss known discrepancies between theoretically derived and empirically measured relations between the radio surface brightness Σ and the diameter D of supernova remnants (SNRs): these relations are commonly known as the Σ–D relations. We argue that these discrepancies may be at least partially explained by taking into account thermal emission at radio frequencies from SNRs at particular evolutionary stages and located in particular environments. The major contributions of this paper may be summarized as follows: (i) we consider thermal emission at radio frequencies from SNRs in the following scenarios: a relatively young SNR evolving in a dense molecular cloud environment ($n \sim 100$–$1000\ \mathrm{cm}^{-3}$) and an extremely evolved SNR expanding in a dense warm medium ($n \sim 1$–$10\ \mathrm{cm}^{-3}$). Both of these SNRs are assumed to be in the adiabatic phase of evolution. We develop models of the radio emission from both of these types of SNRs and each of these models demonstrate that through the thermal bremsstrahlung process significant thermal emission at radio frequencies is expected from both types of SNR. Based on a literature search, we claim that thermal absorption or emission at radio frequencies has been detected for one evolved Galactic SNR and four young Galactic SNRs with similar properties to our modelled evolved and young SNRs. (ii) We construct artificial radio spectra for both of these two types of SNRs: in particular, we discuss our simulated spectrum for the evolved Galactic SNR OA 184. By including thermal emission in our simulated spectra, we obtain different slopes in Σ–D relations: these new slopes are in closer agreement to empirically obtained relations than the theoretically derived relations which do not take thermal emission into account. (iii) Lastly, we present an additional modification to the theoretical Σ–D relation for SNRs in the adiabatic expansion phase. This modification is based on the convolution of the synchrotron emissivity with the emissivity derived in this paper for thermal bremsstrahlung emission from an ionized gas cloud (that is, a theoretical construct of an SNR).
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1. Introduction

The relation between the surface brightness Σ and the diameter D of supernova remnants (SNRs)—the so-called Σ–D relation—provides a convenient way to investigate the radio brightness evolution of SNRs. The Σ–D relation was first presented and described by Shklovsky [29] in the course of a theoretical analysis of synchrotron radiation from an adiabatically expanding spherical nebula (that is, a theoretical construct to describe an SNR). The relation derived by Shklovsky [29] has the form:

$$\Sigma = AD^{-\beta}, \tag{1}$$

where the slope derived in that paper was $\beta = 6$. In this derivation (as in subsequent derivations of this relation), the assumed value for the radio spectral index α of the SNRs was $\alpha = 0.5$ (where α has been defined such that the flux density S_ν is proportional to $\nu^{-\alpha}$): this value corresponds to the average spectral index for SNRs. Lequeux [19] generalized the Σ–D relation to the case of shell-type SNRs to include the prototypical shell-type SNR Cas A: the re-derived relation presented in that work featured a slope ($\beta = 5.8$) which gave a superior approximation than the relation derived by Shklovsky [29] to empirical relations. As inspired by the work of van der Laan [36], Poveda and Woltjer [23] described a modification to the original derivation presented by Shklovsky [29], namely that the magnetic field of the SNR was assumed to remain constant as the SNR expands. The Σ–D relation derived by Poveda and Woltjer [23] in this manner featured a slope with $\beta = 3$, which closely matched an empirical Σ–D relation presented in the same paper. In addition, Kesteven [15] derived a relation with slope $\beta = 4.5$ for a shell-type SNR assuming that the thickness of the shell of the SNR remains constant as the SNR expands. Despite the work of Poveda and Woltjer [23] and Kesteven [15], however, significant inconsistencies between empirical and theoretical Σ–D relations remained. Duric and Seaquist [8] derived a Σ–D relation based on a theoretical interpretation that paralleled the work of Shklovsky [29]: specifically, Duric and Seaquist [8] adopted both the version of Fermi's accelerating mechanism presented by Bell [2,3] and the magnetic field model described by Gull [12] and Fedorenko [9]. An updated derivation of the theoretical Σ–D relation was derived and presented by Berezhko and Völk [4] who used the time-dependent non-linear kinetic theory for cosmic-ray acceleration in SNRs and obtained a slope of $\beta = 4.25$.

From the earliest studies of the Σ–D relation, significant differences between theoretical and empirical relations were obtained with Green [10] showing that the established calibrators were too scattered in the Σ–D space to derive a valid empirical relation. However, Case and Bhattacharya [6] and Urošević [31,32] derived the empirical Σ–D relations with much flatter slopes than those seen in earlier works. We believe that the discrepancies between theoretical and empirical Σ–D relations may be at least partially explained by considering thermal bremsstrahlung emission from SNRs at radio frequencies. In this paper, we present a new derivation of the Σ–D relation which takes into account (for the first time) this thermal emission at radio frequencies, and we show that the inclusion of this emission helps decrease the discrepancy between theoretical and empirical Σ–D relations.

2. Models of thermal emission from SNRs

We now present and discuss models of thermal emission at radio frequencies from SNRs. There are two basic criteria for the production of a significant amount of radio emission through the thermal bremsstrahlung process from an SNR in the adiabatic phase of evolution: the SNR must be evolving in a denser than average environment and its temperature must be lower than average (but always greater than the recombination temperature).

2.1. Thermal radiation from an evolved SNR in the adiabatic phase

We first consider an evolved SNR with a diameter $D = 200$ pc, a surface brightness of $\Sigma = 10^{-22}$ W m^{-2} Hz^{-1} sr^{-1} at 1 GHz and a synchrotron shell with a thickness of 10 pc, representing 5% of the SNR diameter. Our adopted values for these properties correspond to those measured or indicated by observations of several evolved Galactic SNRs, such as the four radio loops observed by Spoelstra [30] and the SNR OA 184, as observed by Routledge et al. [26]. For the evolved SNR considered here, the assumed surface brightness corresponds to an emissivity $\varepsilon_{1\ \mathrm{GHz}} = 1.1 \times 10^{-38}$ ergs cm^{-3} s^{-1} Hz^{-1}. Based on these equipartition equations Pacholczyk [22], for this SNR we calculate a magnetic field strength of $H_{\mathrm{min}} = 1.3 \times 10^{-5}$ G and a minimum energy of $E_{\mathrm{min}} = 1.9 \times 10^{50}$ erg. This minimum energy corresponds to the sum of the energy in the magnetic field, the energy of the relativistic electrons and the energy of the heavier particles: we note that the energy calculated assuming equipartition is less than the canonical value of the SNR output energy, that is $E_0 = 10^{51}$ ergs. For very large and old SNRs, the total energy input must have been higher than the calculated minimum values [16].

We now estimate the amount of thermal bremsstrahlung radiation from this evolved SNR. For a density of $n \approx 1$ cm^{-3} and a temperature $T \approx 10^4$ K, thermal bremsstrahlung provides 10% of the emission ($\varepsilon_{1\ \mathrm{GHz}} \approx 10^{-39}$ ergs cm^{-3} s^{-1} Hz^{-1}) at 1 GHz produced by the synchrotron mechanism at that frequency. We therefore argue that thermal bremsstrahlung emission should represent a significant portion of the total radiation produced by specific types of extremely evolved SNRs. To illustrate this point, in Fig. 1 we have plotted the ratio of the thermal bremsstrahlung emissivity $\varepsilon_{\mathrm{therm}}$ to the synchrotron emissivity $\varepsilon_{\mathrm{synch}}$ (that is, the ratio of thermal to non-thermal emission) for the evolved SNR described here as a function of temperature at a frequency of 1 GHz for a range of values of gas density (0.1, 1.0 and 10 cm^{-3}).

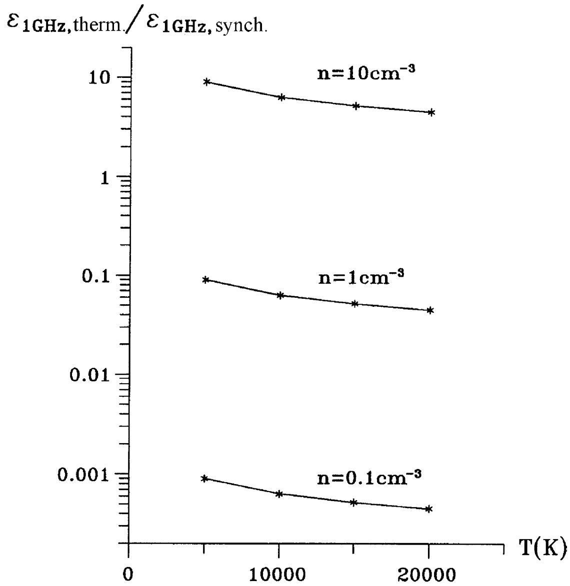

Fig. 1. The ratios between the thermal and non-thermal (synchrotron) emissivities at 1 GHz as a function of temperature for the case of a warm ISM. The ratios are plotted for constant gas densities of $n = 0.1$, 1 and 10 cm^{-3}.

As the radio frequency increases, the amount of synchrotron radiation from an SNR decreases and the amount of thermal bremsstrahlung emission becomes more significant. This trend is illustrated in Fig. 2, where we have again plotted the ratio of $\varepsilon_{\mathrm{therm}}$ to $\varepsilon_{\mathrm{synch}}$ as a function of frequency for two values of the gas density, 1 and 10 cm^{-3} (a temperature of 10^4 K has been assumed in both cases). If we define the Σ–D relation at 100 GHz, the thermal bremsstrahlung component of the total SNR radio emission at this frequency certainly influences the slope β because approximately half of the energy produced by the evolved SNR has a thermal origin.

We will now show that such an SNR would have aged enough for the X-ray emitting gas associated with the SNR to cool to the stable warm phase of the interstellar medium, even though the SNR itself has remained in the adiabatic phase. In Fig. 3, we present a schematic diagram of this

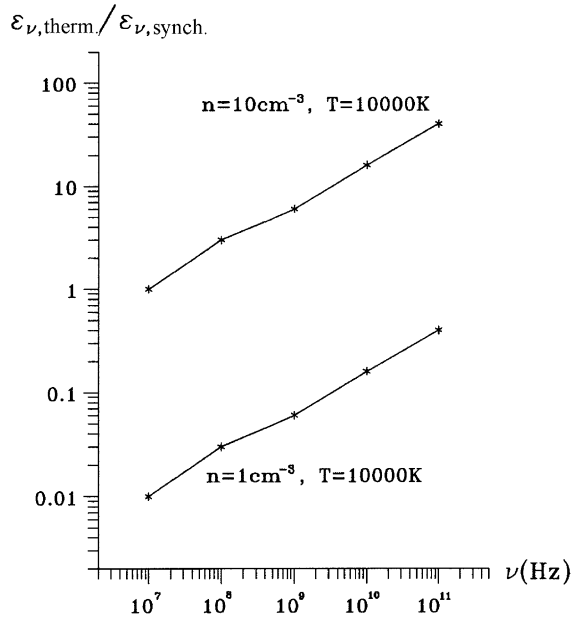

Fig. 2. The ratios between thermal and non-thermal (synchrotron) emissivities as a function of frequency in the radio domain. In both cases, the temperature of the medium of $T = 10^4$ K, and the densities are $n = 1$ and $10\ \mathrm{cm}^{-3}$.

SNR: notice that a double shell morphology is seen with a shell of X-ray emission located interior to a synchrotron-emitting shell. Note that thermal emission from the SNR at all frequencies is produced within these two shells. Magnier et al. [20] developed a theoretical model for the Σ_X–D relation for SNRs based on the Sedov similarity solution [28] (where Σ_X is the surface brightness of the SNR in the X-ray), deriving the following relationship between diameter and temperature for an SNR in the adiabatic phase:

$$T = 4900 E_{51} D^{-3} n_0^{-1}. \tag{2}$$

Here, T is the temperature (in units of keV/k, where k is Boltzmann's constant), E is the initial kinetic energy of the supernova explosion in 10^{51} ergs, D is in pc and n_0 is in cm^{-3}. Using this equation and assuming values of 200, 1 and 1 for D, n_0 and E_{51}, respectively, we calculate a temperature $T \approx 7000$ K. This value corresponds to the warm component of the interstellar medium and represents a thermodynamically stable phase of the interstellar medium (based on our assumed density of $n = 1\ \mathrm{cm}^{-3}$). This result for T indicates that an evolved SNR with a diameter $D = 200$ pc has aged long enough for the X-ray emitting gas to cool to the stable warm phase while the SNR itself has remained in the adiabatic phase. The interior of the SNR has a higher temperature because the SNR had a higher expansion velocity earlier in its evolution.

We have conducted a literature search for an evolved SNR which, based on radio observations, appears to be emitting a significant amount of thermal bremsstrahlung emission at radio frequencies. Through a multi-frequency radio study of the evolved SNR HB9 ($D \approx 150$ pc and $\Sigma_{1\,\mathrm{GHz}} \approx 10^{-21}\ \mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}$), Leahy et al. [17] noticed a spectral flattening in the emission from this source, which may be explained by thermal absorption and generally occurs on the rim of the SNR (at the interface between a molecular cloud and the SNR shock) at frequencies below 232 MHz.

2.2. Thermal radiation from a relatively young SNR in the adiabatic phase

We now consider relatively young SNRs in the adiabatic phase of evolution and estimate the amount of thermal bremsstrahlung emission expected from these sources at radio frequencies. Observations have already detected thermal bremsstrahlung absorption or emission at radio wavelengths from four relatively young SNRs: γ Cygni [38], the Cygnus Loop [18], HB21 [39] and 3C 391 [5]. The typical diameters of these SNRs are 20 pc, the mean thicknesses of their synchrotron shells are 1 pc (that is, about 5% of the SNR diameter) and their average surface brightnesses at 1 GHz are $\sim 10^{-20}\ \mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}$. If we consider a relatively young SNR with these typical values and assume that the SNR is evolving within a dense molecular cloud with a density $n = 300\ \mathrm{cm}^{-3}$ (observations of such SNRs suggest a range of densities from 100 to $1000\ \mathrm{cm}^{-3}$—see [18,5]), the synchrotron emissivity and the thermal bremsstrahlung emissivity are approximately the same ($\varepsilon_{1\,\mathrm{GHz}} \approx 10^{-35}\ \mathrm{ergs\,cm^{-3}\,s^{-1}\,Hz^{-1}}$) if we

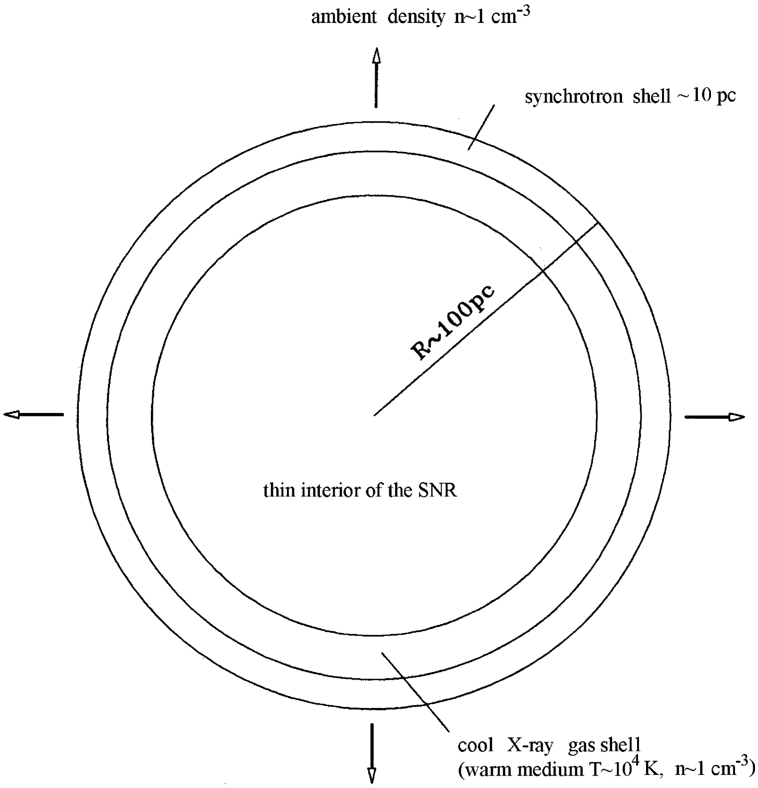

Fig. 3. A two-dimensional model of an evolved SNR (approximately 10^5–10^6 years old) in the adiabatic phase of expansion, with a radius $R = 100$ pc and synchrotron shell thickness 10 pc, evolving into a dense medium $n \sim 1\ \mathrm{cm}^{-3}$. This SNR produces a significant amount of thermal radiation in the radio and X-ray domains. A shell of X-ray-emitting gas is located behind a synchrotron-emitting shell: all of the thermal emission from the SNR at radio and X-ray frequencies is emitted from these two shells. The temperatures of the two shells are $T \sim 10^4$ K with densities of approximately $n \sim 1\ \mathrm{cm}^{-3}$. In comparison, the density of the interior of the SNR is lower and the temperature is higher.

assume a temperature of $T \approx 10^6$ K.[1] At 1 GHz, the relatively young SNR is optically thin for $n \sim 1000\ \mathrm{cm}^{-3}$ and $T \sim 10^6$ K and radio emission may be detected from the entire shell of the source. Note that this medium (with $n \approx 100$–$1000\ \mathrm{cm}^{-3}$ and $T \approx 10^6$ K) is unstable, and this instability leads to a very rapid evolution by the SNR into the adiabatic phase.

Recently, observation evidence that young SNRs can indeed produce significant amounts of thermal bremsstrahlung emission at radio frequencies was presented by Rodriguez-Rico et al. [24]. Those authors detected radio recombination lines from three compact sources that have been classified as young SNRs in the starburst galaxy M82.

[1] The canonical value for temperature of young SNRs.

2.3. Simulated spectra for possible thermally active SNRs

2.3.1. Simulated spectrum of OA 184

In this section we argue that a significant fraction of the detected radio emission from the SNR OA 184 has a thermal bremsstrahlung origin. The characteristics of this SNR closely resemble the modeled evolved SNR discussed earlier in this paper. Based on HI observations, Routledge et al. [26] concluded that OA 184 is associated with a molecular cloud and that its evolution is governed by a dense ambient medium. The diameter of this SNR is 175 pc, the synchrotron-emitting shell is 5% of the total diameter and the surface brightness is $\Sigma_{1\,\mathrm{GHz}} = 2.6 \times 10^{-22}$ W m^{-2} Hz^{-1} sr^{-1}. The regions of highest radio and optical emission from the SNR are coincident with regions of enhanced HI emission: these regions are located within the interior of the SNR and the implied density of the gas is 10 cm^{-3}. Optical line spectra of this SNR indicate that this source is similar to an HII region: because most of the radio emission from HII regions is thermal bremsstrahlung, we suspect that OA 184 also emits a significant amount of thermal emission at radio frequencies. Routledge et al. [26] concluded that OA 184 is in the late adiabatic phase of evolution and argued that the large amount of HI seen toward this SNR could be explained by recombination and a weakened SNR shock. Since OA 184 emits in both the optical and radio domains, it contains enough ionized gas to emit a significant amount of thermal bremsstrahlung, and the estimated density is appropriate for a high thermal radio flux. Therefore, we claim that observations of this SNR will detect an appreciable amount of thermal emission at radio frequencies.

To determine whether OA 184 does indeed emit a significant amount of thermal bremsstrahlung emission at radio frequencies, we simulated an artificial spectrum for an SNR in which the thermal component accounts for 10% of the total emissivity. The measured flux density at 1 GHz of OA 184 is 11 Jy [13] and we assume that at this frequency the synchrotron mechanism produces 10 Jy of the observed flux density while thermal bremsstrahlung emission accounts for the remaining 1 Jy. Additionally, we assume the spectral indices of the synchrotron emission and the thermal bremsstrahlung emission to be 0.6 and 0.1, respectively. Using these parameters, we simulated a radio spectrum for this SNR at 200 frequencies uniformly distributed over the frequency range 10^7–10^{11} Hz as the sum of contributions from both synchrotron and thermal bremsstrahlung emission. The resultant spectrum is presented in Fig. 4 and plotted as a thick line while the dashed line represents a linear fit through the measured datapoints that correspond to real observed flux densities of OA 184 as presented by Routledge et al. [26] (and references therein). From inspection of this figure it is obvious that the inclusion of a thermal component produces a clear bend in the radio spectrum. The parameters for the plot depicted in Fig. 4 may be summarized as follows: a spectral index $\alpha = 0.506$ is obtained if a linear fit is used on simulated data and the corresponding coefficient of correlation is 0.999. The measured spectral index of OA 184 is 0.5 [11] which closely matches the value obtained using a linear fit through the plotted datapoints. Based on previous analysis and inspection of Fig. 4, we argue that the total radio spectrum of OA 184 (with a known spectral index of $\alpha = 0.5$) is best modelled as the sum of synchrotron ($\alpha = 0.6$) and thermal bremsstrahlung ($\alpha = 0.1$) components. The real measured spectrum (dominated by a linear synchrotron component) should be shallower than the pure synchrotron spectrum and we give both qualitative and quantitative descriptions of this effect here. Although the measured data-points are better fit with a curved rather than a linear spectrum (see Fig. 4), the uncertainties in the measured flux densities are larger than the differences between the curved model and the linear spectrum: therefore firm conclusions about this effect cannot be derived without

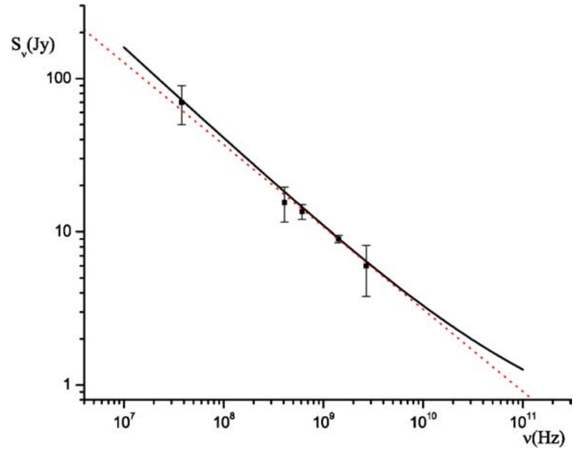

Fig. 4. The simulated spectrum with thermal emission included of the SNR OA 184 (thick line). For comparison, we have also plotted a linear fit through the measured datapoints (taken from Routledge et al. [26]): this linear fit is shown with a dashed line.

additional observations at higher radio frequencies (namely between 1 and 100 GHz).

2.3.2. Simulated spectra for thermal radio emission from young and evolved SNRs

For constructing simulated radio spectra of young and evolved SNRs which includes both synchrotron emission and thermal bremsstrahlung emission, we have used thorough this paper a similar procedure to the one presented previously to model the spectrum of OA 184. For "thermally active" evolved SNRs we used the parameters defined in Section 2.1 where the spectral indices of the synchrotron emission and the thermal bremsstrahlung emission were 0.5 and 0.1, respectively. Additionally, we assumed that the thermal emission accounts for 10% of the total observed emission.

For the case of a relatively young thermally active SNR the parameters defined in Section 2.2 were used. Once again, the spectral indices of the synchrotron and thermal bremsstrahlung components were assumed to be 0.5 and 0.1, respectively. We also assumed that the young SNR is expanding in a dense environment of molecular clouds with a typical ambient density of $n = 300\ \mathrm{cm}^{-3}$. For this density, the amount of thermal emission is actually comparable to the amount of synchrotron emission (see Section 2.2).

The parameters for the simulated spectrum of a young thermally active SNR (as well as the changes in slopes for Σ–D relations which correspond to the changes in spectral indices) are summarized in Table 1. These changes in slope were calculated using the Σ–D relation derived by Duric and Seaquist [8], that is,

Table 1
The parameters of the simulated spectra and corresponding changes in spectral index α and Σ–D slope β

	Simulated α (c.c., f.q.)	$\Delta\alpha$	$\Delta\beta$
OA 184	0.50 (−0.99865; 0.99729)	0.10	0.50
Evolved SNRs	0.43 (−0.99935; 0.99869)	0.07	0.35
Young SNRs	0.21 (−0.98857; 0.97727)	0.29	1.45

Note: c.c. and f.q. represent the correlation coefficient and the fit quality, respectively. The fit quality is based on the value of minimum Chi-squared (scatter of residuals relative to the best fit line).

$$\Sigma_{1\,\mathrm{GHz}} \propto D^{-(5\alpha+1)}. \tag{3}$$

Inspection of Table 1 reveals that by modelling the radio emission from SNRs with a thermal component as well as a non-thermal component, a clear change in the spectral index ($\Delta\alpha$) is seen. Most importantly, we emphasize that a change in the spectral index is also correlated with a change in the slope β of the Σ–D relation. In particular, a very dramatic change in the slope is seen when we consider the case of young SNRs expanding within a dense molecular cloud: in that scenario, we find that $\Delta\beta \approx 1.5$. Lastly, we comment that because the correlation coefficient and fit quality values for this scenario are very close to unity, a spectrum which actually is curved may appear to be linear.

3. Thermal radio emission from SNRs and a modified theoretical Σ–D relation

In this section, we present a method for modifying the theoretical Σ–D relation by taking into account thermal bremsstrahlung emission from SNRs at radio frequencies and using an analytical convolution procedure that involves both synchrotron-based and thermal bremsstrahlung-based Σ–D relations. We argue that perhaps the empirical–theoretical inconsistency can be at least partially explained by the omission of thermal bremsstrahlung emission at radio frequencies from SNRs in previous derivations of theoretical Σ–D relations. Discussions on the effects of this thermal radio emission on the Σ–D relation have already been presented by Urošević et al. [33,34]. In this section, the combined emissivity that we derive through the convolution method will yield a new Σ–D relation where the slope will be reduced and more closely approximate the empirical relations.

3.1. The Σ–D relation for thermal radiation from an ionized gas cloud: the case of constant temperature

For the derivation of the Σ–D relation based on thermal emission from an ionized gas cloud, we will apply an algorithm applied by Shklovsky [29] for the derivation of the relation based on synchrotron emission from SNRs. From the theory of the bremsstrahlung radiation applied to an ionized gas cloud, we adopt a volume emissivity of the following form [25]:

$$\varepsilon_\nu \propto N_i N_e T^{-1/2}, \tag{4}$$

where T is thermodynamic temperature of the medium and N_i and N_e are the volume concentrations of the ions and electrons, respectively.

We assume that the temperature and density of the particles does not change with changing distance from the center of an SNR. This is consistent with the model for the hot interstellar medium (HIM) described by McKee and Ostriker [21]. From Eq. (4) and defining $\Sigma_\nu = \varepsilon_\nu R/4\pi$, we obtain

$$\varepsilon_\nu = \text{constant} \quad \text{and} \quad \Sigma_\nu \propto R. \tag{5}$$

From inspection of this relation, we notice that as the size of the SNR increases, its surface brightness also increases: this result is consistent with our expectations for an optically thin medium.

3.2. The Σ–D relation for synchrotron radiation and thermal bremsstrahlung radiation from an ionized gas cloud: the case of constant temperature

The final result of the theory presented by Shklovsky [29] is $\varepsilon_\nu \propto D^{-7}$ (again assuming an average spectral index for SNRs of 0.5): this relation is scaled by the maximum value ε_{max} of the emissivity of SNRs at the outset of their evolution. We can therefore express the normalized emissivity ε_{norm} as

$$\varepsilon_{norm} = \frac{\zeta_1}{\varepsilon_{max}} R^{-7}, \tag{6}$$

where ζ_1 is a constant which contains the portion of the synchrotron emissivity which does not depend on R. The maximum value of the emissivity ε_{max} corresponds to the minimum radius of the SNR (R_{min}), while the minimum value of the emissivity corresponds to the maximum radius of the SNR (R_{max}) which in turn corresponds to an SNR at the end of its evolution (that is, the dissipation phase). If the emissivity from Eq. (5) is convolved with emissivity from Eq. (6), we obtain the following integral expression for ε as a function of time:

$$\varepsilon(t) = \frac{\zeta_1}{\varepsilon_{max}} \int_{R_{min}}^{R_{max}} \frac{\zeta_2}{(t-R)^7} \, dR. \tag{7}$$

Here, ζ_2 is another constant which contains the portion of the thermal bremsstrahlung emissivity which does not depend on R. For a qualitative analysis, this integral may be approximated as

$$\varepsilon(t) \approx \frac{\zeta_3}{\varepsilon_{max}} \int_0^\infty \frac{1}{(t-R)^7} \, dR. \tag{8}$$

Here ζ_3 is the product of the constants ζ_1 and ζ_2 and the integral is evaluated over the range of $R = 0$ through $R = +\infty$ to describe the expansion of the SNR from very small values (nearly zero) at the beginning of its evolution to very large values (limiting case is ∞) at the end of its lifetime. This integral has the following solution:

$$\varepsilon(t) \propto \int_0^\infty \frac{1}{(t-R)^7} \, dR \propto t^{-6}. \tag{9}$$

Using this equation we obtain:

$$\Sigma_\nu \propto D^{-5}, \tag{10}$$

Therefore, the introduction of the thermal component to the relation derived by Shklovsky [29] leads to a form of the Σ–D relation with a significantly flatter slope. This change is larger than illustrated in Table 1 and therefore this modification shows only qualitative trend of flattening.

3.3. The Σ–D relation for thermal bremsstrahlung radiation from an ionized gas cloud: the case of variable temperature

We now develop a Σ–D relation for thermal bremsstrahlung radiation from an ionized gas cloud in the case where the temperature varies throughout the cloud. Following the example of the derivation presented in Section 3.1, we assume that the particle density does not change with distance from the center of the cloud: this assumption is consistent with the model presented by McKee and Ostriker [21]. If we suppose that the interstellar medium is uniform (in the sense of the distributions of both the clouds and the intercloud matter), the swept-up mass increases as the SNR expands and therefore the density within the SNR remains constant. This situation is similar to an adiabatic expansion in which the continuous inflow of the interstellar matter provides constant density, but the system as a whole is cooled down because the strength of the shock wave decreases, leading to a decrease in the temperature. Some energy is lost as emission in the adiabatic phase (although this energy loss is negligible in comparison with the total kinetic energy, per definition of the adiabatic phase) and this phase ends when half of total kinetic energy of an SNR has dissipated [37]. This process may be considered to be quasi-adiabatic because some energy is exchanged with the ambient environment during the adiabatic phase, especially at the end of this phase. In the late phase of the adiabatic evolution when an SNR loses energy more rapidly, we can expect thermal radiation from the warm medium.

Since the SNR is assumed to be in the adiabatic phase (i.e., the SNR is cooling adiabatically as it expands), we start with the adiabatic equation, expressed as

$$TV^{\gamma-1} = \text{constant}. \tag{11}$$

In the case of a spherical cloud and assuming $\gamma = \frac{5}{3}$ (i.e., assuming that the gas in the SNR interior behaves like an ideal gas), we obtain the following dependence of temperature with respect to cloud radius:

$$T \propto R^{-2}. \tag{12}$$

Substituting Eq. (12) into Eq. (4), we may therefore express the emissivity as

$$\varepsilon_\nu \propto R. \tag{13}$$

Accordingly, we then have

$$\Sigma_\nu \propto R^2. \tag{14}$$

Since it is well-known that SNRs are associated with relativistic electrons which emit synchrotron radiation, based on the presence of these particles we may derive another constraint on the dependence of emissivity on radius. If the total energy of a particle is much greater than its rest mass, the rest mass may therefore be ignored when considering the particle's total energy. Similar to the case of an ideal gas, if we neglect relativistic corrections for temperatures $T \leqslant 10^6$ K [27] and set $\gamma = \frac{4}{3}$, we derive the following expression for emissivity with respect to cloud radius:

$$\varepsilon_\nu \propto R^{0.5}. \tag{15}$$

Therefore, following the model presented by McKee and Ostriker [21], these relations yield a Σ–D relation for thermal emission from SNRs of the following form:

$$\Sigma_\nu \propto D^{1.5 \leqslant -\beta \leqslant 2.0}. \tag{16}$$

3.4. The Σ–D relation for synchrotron radiation and thermal bremsstrahlung radiation from an ionized gas cloud: the case of variable temperature

The theoretical model described by Duric and Seaquist [8] yields a Σ–D relation of the form $\Sigma \propto D^{-3.5}$ ($\varepsilon \propto D^{-4.5}$) for evolved SNRs and $\Sigma \propto D^{-5}$ ($\varepsilon \propto D^{-6}$) for young SNRs. As in the case considered in Section 3.2 (that is, where a shell model for the SNR was assumed), we can expect thermal flux from the shell. In this case, flux from the low density interior may be neglected because the particle concentration is higher in the shell, resulting in a greater efficiency of thermal radiation from the ionized gas cloud. Relativistic particles in the shell (and probably in the X-ray emitting region) will contribute, thereby introducing the thermal component to the total emissivity as shown in Eq. (15). The appropriate convolution integrals (in the cases of both evolved and young SNRs) are

$$\text{Evolved SNRs} \rightarrow \varepsilon(t) \propto \int_0^\infty \frac{R^{0.5}}{(t-R)^{4.5}} \mathrm{d}R \propto t^{-3}, \tag{17}$$

$$\text{Young SNRs} \rightarrow \varepsilon(t) \propto \int_0^\infty \frac{R^{0.5}}{(t-R)^{6}} \mathrm{d}R \propto t^{-4.5}. \tag{18}$$

Similar to the previous convolution, we obtain

$$\text{Evolved SNRs} \rightarrow \Sigma_\nu \propto D^{-2}, \tag{19}$$

$$\text{Young SNRs} \rightarrow \Sigma_\nu \propto D^{-3.5}. \tag{20}$$

If we once again assume an average spectral index for SNRs of $\alpha = 0.5$, the first relation has a value for β which is closest to the latest "shallower master'' empirical Σ–D relations [31,32]. This result should be taken with caution because our theoretical modification is not appropriate for SNRs which evolve in dilute media. However, the second relation yields a value for β which is closer to that derived for the very rich young radio SNR population found in M82 [14], that is, $\beta = 3.4$ [35], and for all SNRs which evolve in the dense molecular cloud environment [1]. For these SNRs located in such dense environments (e.g., $n \approx 1000\,\mathrm{cm}^{-3}$ in M82 [7]), a change in slope $\Delta\beta \approx 1.5$ predicted in this paper by a simulated spectrum (Table 1) has approximately the same values as the value derived through a convolution-modified relation.

4. Summary

The main results of this paper may be summarized as follows:

(i) We have considered the thermal emission at radio frequencies for two types of SNRs and we have included this emission in a model of the total radio emission from SNRs. We also developed two models describing relatively young and evolved SNRs in the adiabatic phase of evolution, respectively, and we have shown that both types of SNRs emit significant amounts of thermal bremsstrahlung emission at radio frequencies. For evolved SNRs, the necessary parameters for producing significant amounts of thermal radio emission via the bremsstrahlung process are $T \sim 10^4\,\mathrm{K}$ and $n \sim 1-10\,\mathrm{cm}^{-3}$ (that is, roughly the same parameters that describe a denser warm medium). For relatively young SNRs, the most important condition for producing significant amounts of thermal radio emission is that the SNR is evolving in a dense molecular cloud with $n = 100-1000\,\mathrm{cm}^{-3}$. Based on a literature search, we argue that thermal absorption or emission at radio frequencies was probably detected from the evolved SNR HB9: we also suspect that such emission was detected from another evolved SNR, OA 184, but the situation is less clear. Likewise, observations also appear to have detected thermal radio absorption or emission from the young SNRs (γ Cygni, Cygnus Loop, HB21 and 3C 391).

(ii) We have constructed an artificial radio spectrum for the Galactic SNR OA 184 in which thermal emission from this SNR has been modelled along with its well-known synchrotron emission. This combination artificial spectrum fits the observational data better than a spectrum that is based solely on synchrotron emission. We have also prepared simulated spectra for particular types of both young and evolved SNRs from which we expect significant amounts of thermal emission at radio frequencies. Our artificial spectra both feature spectral bending but with small curvature: therefore, a linear spectrum gives a comparable fit to a simulated spectrum with a bend. The flattening in spectral index given by simulations provides the significant flattening in the slope of the Σ–D relation ($\Delta\beta \approx 1.5$). This change provides better agreement between the theoretical and empirical derived slopes in the case of SNRs evolving in or near dense molecular clouds.

(iii) By modifying the theory presented by Shklovsky [29] through the introduction of the thermal bremsstrahlung mechanism to describe SNR evolution in the adiabatic phase, we have derived a Σ–D relation which is in closer agreement to the empirical results than previous theoretical models. The modified theoretical relation presented by Duric and Seaquist [8] (in the case of constant density and variable temperature) for evolved SNRs gives the best agreement with the updated ''flatter master'' empirical Σ–D relation obtained by Urošević [31,32]. This result should be taken with caution because our theoretical modification is not appropriate for SNRs which evolve in dilute media. In the case of young SNRs, the modified theoretical relation described by Duric and Seaquist [8] gives the best agreement with the updated Σ–D relation for the population of young radio SNRs in the starburst galaxy M82 [35], and for all SNRs which evolve in the dense molecular cloud environment [1].

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