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MODIFIED EQUIPARTITION CALCULATION FOR SUPERNOVA REMNANTS. CASES $\alpha = 0.5$ AND $\alpha = 1$

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ABSTRACT

The equipartition or minimum energy calculation is a well-known procedure for estimating the magnetic field strength and the total energy in the magnetic field and cosmic ray particles by using only the radio synchrotron emission. In one of our previous papers, we have offered a modified equipartition calculation for supernova remnants (SNRs) with spectral indices $0.5 < \alpha < 1$. Here we extend the analysis to SNRs with $\alpha = 0.5$ and $\alpha = 1$.

Key words: ISM: magnetic fields – ISM: supernova remnants – radio continuum: general

1. INTRODUCTION

The equipartition or minimum energy calculation is a well-known procedure for estimating the magnetic field strength and the total energy in the magnetic field and cosmic ray (CR) particles from the radio synchrotron emission of a source. It is often used when no other methods are available. Details of classical equipartition and revised equipartition calculations for radio sources in general are available in Pacholczyk (1970) and Beck & Krause (2005), respectively. Lacki & Beck (2013) derived an equipartition formula for starburst galaxies. In one of our previous papers (Arbutina et al. 2012, hereafter Paper I), we have offered a modified equipartition calculation for supernova remnants (SNRs) with spectral indices $0.5 < \alpha < 1$. Spectral index is defined via the equation $S_\nu \propto \nu^{-\alpha}$, where S_ν is the flux density. Our approach was similar to that of Beck & Krause (2005). However, rather than introducing a break in the power-law energy distribution, we assumed power-law spectra and integrated over momentum to obtain energy densities of particles. We further took into account different ion species (not just protons and electrons), used flux density at a given frequency, assumed an isotropic distribution of the pitch angles for the remnant as a whole, and incorporated the dependence on shock velocity v_s via injection energy $E_{\rm inj} \sim m_p v_s^2$.

For simplicity, in Paper I we assumed that CRs' momentum/energy spectrum extends to infinity. This introduces only a small error in the final result due to the power-law dependence of the momentum distribution function $f(p) = kp^{-(\gamma+2)}$. However, if γ is exactly 2, the integral for CR energy density diverges and one must set an upper limit for particle energy. This is unfortunate since $\alpha = 0.5$ ($\gamma = 2\alpha + 1$) is considered to be a typical spectral index for SNRs. Out of the 40 SNRs for which we can estimate the magnetic field from equipartition calculations, 22 have spectral indices $\alpha = 0.5$ (Pavlović et al. 2013). This value comes directly from the theory of diffusive shock acceleration (DSA) in the case of strong shocks with a compression ratio $r = 4$; $\gamma = (r+2)/(r-1)$ (Bell 1978a). Urošević et al. (2012) tried to overcome the problem by setting a fixed upper limit (10^{15} eV for protons and 10^{12} eV for electrons). In reality, however, the maximum energy of CRs will also depend on the magnetic field (and v_s), which makes the equipartition, i.e., minimum-energy calculation, more complicated.

A similar situation arises in the case $\alpha = 1$—the integral for CR energy density diverges unless a lower limit for particle energy, i.e., $E_{\rm inj}$, is set. SNRs generally have spectral indices lower than one. Still, SN 1987A remnant, for example, had $\alpha = 1$ and higher between days ∼2000 and 3000 since explosion (Zanardo et al. 2010). In the next section, we will consider special cases corresponding to $\alpha = 0.5$ and $\alpha = 1$.

2. ANALYSIS AND RESULTS

All designations in this section are taken from Paper I.

2.1. Case $\alpha = 0.5$

Let us start with energy density of a CR species with $\gamma = 2$ ($\alpha = 0.5$):

$$\begin{aligned}\epsilon &= \int_{p_{\rm inj}}^{p_\infty} 4\pi k p^{-2}\left(\sqrt{p^2c^2 + m^2c^4} - mc^2\right)dp \\ &= K\int_{x_{\rm inj}}^{x_\infty} x^{-2}\left(\sqrt{x^2+1} - 1\right)dx, \quad x = \frac{p}{mc}, \quad K = 4\pi kc, \\ &= K\ I_2(x)|_{x_{\rm inj}}^{x_\infty} = K\left(\frac{1}{x} - \frac{\sqrt{x^2+1}}{x} + \operatorname{arcsinh} x\right)\Bigg|_{x_{\rm inj}}^{x_\infty},\end{aligned} \tag{1}$$

where k and K are the constants in the momentum and energy distribution functions, respectively. When $x \to 0$, $I_2(x) \approx x/2$, while for $x \to \infty$, $I_2(x) \approx \ln x$. The total CR energy density $\epsilon_{\rm CR} = \epsilon_{\rm e} + \epsilon_{\rm ion}$ is then

$$\begin{aligned}\epsilon_{\rm CR} &= K_e\left(I_2\left(\tfrac{p_\infty^e}{m_ec}\right) - I_2\left(\tfrac{p_{\rm inj}^e}{m_ec}\right)\right) \\ &+ \sum_i K_i\left(I_2\left(\tfrac{p_\infty^i}{m_ic}\right) - I_2\left(\tfrac{p_{\rm inj}^i}{m_ic}\right)\right) \\ &\approx K_e\Bigg\{\ln\left(\frac{E_\infty^e}{m_ec^2}\right) - I_2\left(\frac{\sqrt{E_{\rm inj}^2 + 2m_ec^2E_{\rm inj}}}{m_ec^2}\right) + \frac{1}{\sum_i Z_i\nu_i} \\ &\cdot \sqrt{\frac{2m_pc^2E_{\rm inj}}{E_{\rm inj}^2 + 2m_ec^2E_{\rm inj}}}\left(\sum_i\sqrt{A_i}\nu_i\ln\left(\frac{E_\infty^p}{m_pc^2}\right) - \sqrt{\frac{E_{\rm inj}}{2m_pc^2}}\right. \\ &+ \left.\sum_i\sqrt{A_i}\ln\left(\tfrac{Z_i}{A_i}\right)\nu_i\right)\Bigg\},\end{aligned} \tag{2}$$

where we followed Bell's (1978a, 1978b) DSA theory and his assumptions concerning the injection of particles into the acceleration process, used Equations (25) and (26) from Paper I, and assumed $E_{\rm inj} \ll m_pc^2$, $p_\infty \approx E_\infty/c$ for all CRs species. A_i and Z_i are mass and charge numbers, respectively, and ν_i represents ion abundances (for further details, see Paper I). Assuming Bohm diffusion and synchrotron losses for electrons, for the maximum electron energy we use $E_\infty^e = (3/8)(m_e^2 c^3 v_s/\sqrt{(2/3)e^3 B})$ (Zirakashvili & Aharonian 2007) and for ions $E_\infty^i = (3/8)(v_s/c)Z_i eBR$ (see Bell et al. 2013 and references therein), where R is SNR radius. Both formulae are in cgs units. In reality, of course, we do not expect a sharp break in the energy spectra, but rather some steepening, especially in the case of electrons (Blasi 2010).

For the total energy, we have $E = (4\pi/3)R^3 f(\epsilon_{\rm CR} + \epsilon_B)$, $\epsilon_B = (1/8\pi)B^2$. Determining the minimum energy with respect to B, $(dE/dB) = 0$ gives

$$\frac{dK_e}{dB}\left[\ln\left(\frac{E_\infty^e}{m_e c^2}\right) - I\left(\frac{\sqrt{E_{\rm inj}^2+2m_e c^2 E_{\rm inj}}}{m_e c^2}\right) + \frac{1}{3} + \frac{1}{\sum_i Z_i\nu_i} \cdot \sqrt{\frac{2m_p c^2 E_{\rm inj}}{E_{\rm inj}^2+2m_e c^2 E_{\rm inj}}}\left(\sum_i \sqrt{A_i}\nu_i \ln\left(\frac{E_\infty^p}{m_p c^2}\right) - \sqrt{\frac{E_{\rm inj}}{2m_p c^2}} + \sum_i \sqrt{A_i}\ln\left(\tfrac{Z_i}{A_i}\right)\nu_i - \frac{2}{3}\sum_i\sqrt{A_i}\nu_i\right)\right] + \frac{1}{4\pi}B = 0, \quad (3)$$

where (using Equations (4)–(6) from Paper I)

$$\frac{dK_e}{dB} = -\frac{3}{2}\frac{K_e}{B} = -\frac{9}{4\pi}\frac{S_\nu}{f\theta^3 d}\frac{1}{c_5}\left(\frac{\nu}{2c_1}\right)^{1/2}\frac{\Gamma\left(\frac{9}{4}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{7}{4}\right)}B^{-5/2}. \quad (4)$$

In order to find magnetic field, Equation (3) must be solved numerically. When one finds B, minimum energy can be obtained from

$$E_{\rm min} = \left(1 + \frac{4}{3}\frac{\{\ldots\}}{[\ldots]}\right)E_B, \quad E_B = \frac{4\pi}{3}R^3 f\,\epsilon_B, \quad (5)$$

where $\{\ldots\}$ and $[\ldots]$ are expressions in the corresponding brackets in Equations (2) and (3), respectively.

2.2. *Case $\alpha = 1$*

In the situations when $\gamma = 3$ ($\alpha = 1$), the energy density of a CR species is

$$\epsilon = \int_{p_{\rm inj}}^{p_\infty} 4\pi k p^{-3}\left(\sqrt{p^2c^2 + m^2c^4} - mc^2\right)dp \quad (6)$$
$$= \frac{K}{mc^2} I_3(x)\Big|_{x_{\rm inj}}^{x_\infty}, \quad x = \frac{p}{mc}, \quad K = 4\pi k c^2,$$
$$= \frac{K}{mc^2}\left(\frac{1-\sqrt{x^2+1}}{2x^2} - \frac{1}{2}\ln\left(\frac{1+\sqrt{x^2+1}}{x}\right)\right)\Bigg|_{x_{\rm inj}}^{x_\infty}.$$

When $x \to 0$, $I_3(x) \approx (1/2)\ln x$ while for $x \to \infty$, $I_3(x) \approx -(1/2x)$. Total CR energy density is then

$$\epsilon_{\rm CR} = \frac{K_e}{m_e c^2}\left(I_3\left(\tfrac{p_\infty^e}{m_e c}\right) - I_3\left(\tfrac{p_{\rm inj}^e}{m_e c}\right)\right) + \sum_i \frac{K_i}{m_i c^2}\left(I_3\left(\tfrac{p_\infty^i}{m_i c}\right) - I_3\left(\tfrac{p_{\rm inj}^i}{m_i c}\right)\right)$$
$$\approx \frac{K_e}{m_e c^2}\left\{-\frac{m_e c^2}{2E_\infty^e} - I_3\left(\frac{\sqrt{E_{\rm inj}^2+2m_e c^2 E_{\rm inj}}}{m_e c^2}\right) - \frac{1}{\sum_i Z_i\nu_i} \cdot \frac{m_e c^2 E_{\rm inj}}{E_{\rm inj}^2+2m_e c^2 E_{\rm inj}}\left(\sum_i \frac{A_i}{Z_i}\frac{m_p c^2}{E_\infty^p}\nu_i + \frac{1}{2}\ln\left(\frac{2E_{\rm inj}}{m_p c^2}\right) - \frac{1}{2}\sum_i \ln(A_i)\nu_i\right)\right\}, \quad (7)$$

where we have used the same assumptions as in the derivation of Equation (2).

Derivative of the total energy with respect to B gives

$$\frac{1}{m_e c^2}\frac{dK_e}{dB}\left[-\frac{3m_e c^2}{8E_\infty^e} - I_3\left(\frac{\sqrt{E_{\rm inj}^2+2m_e c^2 E_{\rm inj}}}{m_e c^2}\right) - \frac{1}{\sum_i Z_i\nu_i} \cdot \frac{m_e c^2 E_{\rm inj}}{E_{\rm inj}^2+2m_e c^2 E_{\rm inj}}\left(\frac{3}{2}\sum_i \frac{A_i}{Z_i}\frac{m_p c^2}{E_\infty^p}\nu_i + \frac{1}{2}\ln\left(\frac{2E_{\rm inj}}{m_p c^2}\right) - \frac{1}{2}\sum_i \ln(A_i)\nu_i\right)\right] + \frac{1}{4\pi}B = 0, \quad (8)$$

where (see Equations (4)–(6) from Paper I)

$$\frac{dK_e}{dB} = -2\frac{K_e}{B} = -\frac{9}{8\pi}\frac{S_\nu}{f\theta^3 d}\frac{1}{c_5}\frac{\nu}{c_1}B^{-3}. \quad (9)$$

To find the magnetic field more precisely, Equation (8) must be solved numerically. Nevertheless, unlike the case $\alpha = 0.5$, the solution will only weakly depend on the upper limits for energy, so the terms containing E_∞^e and E_∞^p could, in principle, be neglected. When one finds B, the minimum energy can be obtained from

$$E_{\rm min} = \left(1 + \frac{\{\ldots\}}{[\ldots]}\right)E_B, \quad E_B = \frac{4\pi}{3}R^3 f\,\epsilon_B, \quad (10)$$

where now $\{\ldots\}$ and $[\ldots]$ are expressions in the corresponding brackets in Equations (7) and (8), respectively.

3. CONCLUSIONS

In Paper I, we offered a modified equipartition calculation for SNRs with spectral indices $0.5 < \alpha < 1$. In this paper, we extend the analysis to SNRs with $\alpha = 0.5$ and $\alpha = 1$. Spectral indices higher than $\alpha = 1$ are rarely observed in SNRs and are more typical for some extended radio galaxies and active galactic nuclei (Kellermann & Owen 1988). Lower spectral indices $\alpha < 0.5$ can be expected in SNRs expanding in a low-β plasma (i.e., the dominant magnetic field, where β is the ratio of thermal to magnetic pressures; Schlickeiser & Fürst 1989). There are other possible explanations for SNRs with $\alpha < 0.5$, such as non-negligible thermal emission (Onić 2013). However, the assumption of "equipartition" is likely to no longer be valid in these cases. Case $\alpha = 0.5$ is particularly important since this value comes directly from the test-particle DSA theory in the case of strong shocks. Of course, the non-linear DSA theory does not provide simple power-law spectra, thus we are concerned about average spectral indices. In Table 1 we have calculated the magnetic field strengths for 22 SNRs by applying four different methods. Flux densities at 1 GHz, angular sizes, and distances are taken from Green (2009) and Pavlović et al. (2013). Our values for SNRs with $\alpha = 0.5$ are approximately 40% higher than those derived by using the classical approach of Pacholczyk (1970) and similar to those obtained by applying the Beck & Krause (2005) revised equipartition formula and the Urošević et al. (2012) approximation. Nevertheless, the derivation presented in this paper is more accurate than the latter two—in particular, the upper limit for particle energy, i.e., momentum, depends on the magnetic field itself and the upper

Table 1
Calculated Magnetic Field Strengths for SNRs with $\alpha = 0.5$

Catalog Name	Other Name	Pacholczyk (1970) (μGa)	Beck & Krause (2005) (μGa)	Urošević et al. (2012) (μGa)	This Paper[a] (μGa)
G21.8−0.6	Kes 69	83.5	116.4	113.6	116.9
G23.3−0.3	W41	68.9	96.0	93.8	96.3
G33.6+0.1	Kes 79	100.3	139.7	136.4	139.7
G46.8−0.3	HC30	60.8	84.7	82.7	84.8
G54.4−0.3	HC40	41.0	57.1	55.8	56.2
G84.2−0.8		56.7	79.0	77.1	78.3
G96.0+2.0		15.2	21.2	20.7	20.5
G108.2−0.6		19.8	27.6	26.9	27.3
G109.1−1.0	CTB 109	51.9	72.3	70.6	71.7
G114.3+0.3		24.5	34.1	33.3	32.8
G116.5+1.1		23.2	32.4	31.6	31.7
G156.2+5.7		14.7	20.4	19.9	19.8
G205.5+0.5	Monoceros Nebula	20.7	28.8	28.1	28.7
G260.4−3.4	Puppis A	54.0	75.2	73.4	75.1
G292.2−0.5		42.9	59.7	58.3	59.6
G296.5+10.0	PKS 1209-51/52	30.9	43.1	42.0	42.8
G309.8+0.0		57.8	80.5	78.6	79.7
G332.4−0.4	RCW 103	135.6	188.9	184.4	186.8
G337.8−0.1	Kes 41	108.4	151.0	147.4	151.8
G344.7−0.1		55.5	77.3	75.5	76.2
G346.6−0.2		79.9	111.3	108.7	111.4
G349.7+0.2		274.7	382.7	373.7	375.2

Note. [a] For $\upsilon_s \sim 1000$ km s^{-1}.

and lower limits depend on shock velocity and by varying this last parameter one can obtain different magnetic field estimates.

The Web application for calculation of the magnetic field strength of SNRs is available at http://poincare.matf.bg.ac.rs/∼arbo/eqp/.

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REFERENCES

Arbutina, B., Urošević, D., Andjelić, M. M., Pavlović, M. Z., & Vukotić, B. 2012, ApJ, 746, 79 (Paper I)
Beck, R., & Krause, M. 2005, AN, 326, 414
Bell, A. R. 1978a, MNRAS, 182, 147
Bell, A. R. 1978b, MNRAS, 182, 443
Bell, A. R., Schure, K. M., Reville, B., & Giacinti, G. 2013, MNRAS, 431, 415
Blasi, P. 2010, MNRAS, 402, 2807
Green, D. A. 2009, A Catalogue of Galactic Supernova Remnants (2009 March version), Astrophysics Group, Cavendish Laboratory, Cambridge, United Kingdom (http://www.mrao.cam.ac.uk/surveys/snrs/)
Kellermann, I. K., & Owen, F. N. 1988, in Galactic and Extragalactic Radio Astronomy, ed. G. L. Verschuur & K. I. Kellermann (2nd ed.; Berlin: Springer), 563
Lacki, B. C., & Beck, R. 2013, MNRAS, 430, 3171
Onić, D. 2013, Ap&SS, 346, 3
Pacholczyk, A. G. 1970, Radio Astrophysics (San Francisco: Freeman)
Pavlović, M. Z., Urošević, D., Vukotić, B., Arbutina, B., & Göker, Ü. D. 2013, ApJS, 204, 4
Schlickeiser, R., & Fürst, E. 1989, A&A, 219, 192
Urošević, D., Pavlović, M. Z., Arbutina, B., & Dobardžić, A. 2012, in XXVIIth IAU General Assembly, SpS4, New era for studying interstellar and intergalactic magnetic fields, ed. M. Haverkorn & J. L. Han (Cambridge: Cambridge Univ. Press), in press
Zanardo, G., Staveley-Smith, L., Ball, L., et al. 2010, ApJ, 710, 1515
Zirakashvili, V. N., & Aharonian, F. 2007, A&A, 465, 695