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L–D dependence for supernova remnants and its connection with the Σ–*D* relation

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ABSTRACT

We discuss the *L–D* correlation (possible dependence of radio luminosity on linear diameter) for supernova remnants (SNRs) in order to see whether the Σ–*D* relation actually exists and whether determination of SNR distances on the basis of the Σ–*D* relation is possible. We do not find any significant correlation, except for the M82 starburst galaxy for which a good Σ–*D* relation does exist. Finally, we suggest that a similar relation might exist for Galactic SNRs associated with large molecular clouds, indicating once again that the density of the SNR environment is probably the crucial parameter in SNR evolution.

Key words: methods: statistical – supernova remnants – galaxies: individual: M82 – radio continuum: ISM.

1 INTRODUCTION

Even though we are mainly unable to observe the evolution of an individual supernova remnant (SNR) over a very long period of time, we can still try to trace it by analysing properties of different remnants at different stages of evolution, assuming that all SNRs follow similar evolutionary paths. One attempt to understand this evolution better is to study the surface brightness to diameter relation (Σ–*D* relation) in the radio domain. The first theoretical Σ–*D* relation was derived by Shklovsky (1960a) in the form of

$$\Sigma_\nu = AD^{-\beta}. \tag{1}$$

Shklovsky (1960b) was also the first to propose the use of this relation as a method for determining SNR distances. In the subsequent four decades many authors tried to improve the theoretical Σ–*D* relation, and also to update the empirical relation using new observational data [for a detailed review of Σ–*D* relations see Urošević (2002)]. Nevertheless, there are some doubts when it comes to using the Σ–*D* relation for determining distances to individual remnants. Evolutionary paths may differ substantially from remnant to remnant because of the potentially wide range of intrinsic properties of supernova (SN) explosions (or progenitor stars) and the interstellar media (ISM) into which they expand, and a variety of selection effects may be present in the data samples both Galactic and extragalactic (see Green 1984, 1991).

Our goal was to examine the utilization of the Σ–*D* relation for estimating distances to SNRs by discussing the possible dependence of the radio luminosity on linear diameter (*L–D* dependence). Using appropriate definitions of flux density and angular diameter, the following equation may be obtained:

$$\Sigma_\nu \propto S_\nu \theta^{-2} \propto L_\nu D^{-2}, \tag{2}$$

where L_ν is the radio luminosity of the remnant per unit frequency. To emphasize the dependence of luminosity on linear diameter, the Σ–*D* relation may be rewritten as

$$\Sigma_\nu = AD^{-2+\delta}. \tag{3}$$

Recent studies of Σ–*D* relations (excluding very young remnants) indicate that $\delta \approx 0$, which leads to a radio luminosity that is independent of diameter, $L_\nu =$ constant, and to the existence of the so-called trivial relation, $\Sigma_\nu \propto D^{-2}$. It is important to notice, however, that even if the flux S_ν (or luminosity $L_\nu \propto S_\nu d^2$; d being the distance to the remnant) is random, i.e. no relation at all, we can still get $\Sigma_\nu \propto D^{-2}$ simply because the inverse square dependence on D is implied by the definition of surface brightness.

The criterion that we have established in our analysis is as follows: if the *L–D* relation is obtained then the Σ–*D* relation exists and it may be used for the estimation of SNR distances. Otherwise, we must take two possibilities into consideration:

(i) $L_\nu = f(D) =$ constant;
(ii) $L_\nu \neq f(D)$.[1]

In the case of (i), luminosity is functionally independent of D and we get the trivial relation for the radio evolution of SNRs (and the probable change in the slope of the Σ–*D* relation for older SNRs, as we shall see later). In the case of (ii), there is a severe scattering

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[1] In the first case the luminosity is a function of diameter ($\exists f : D \rightarrow L_\nu$), while in the second case luminosity is not uniquely defined so we cannot speak of any specific function ($\nexists f : D \rightarrow L_\nu$).

of SNRs in the *LD* plane (luminosity is statistically independent of *D*) so the *L–D* and thereby the Σ–*D* relation cannot be claimed to exist (or, in other words, the trivial Σ–*D* relation that is produced in this case is not physically justified).

1.1 Objectives

This paper examines the Σ–*D* relations of Galactic and extragalactic SNRs for the purpose of

(i) establishing valid Σ–*D* relations that can be used for determination of distances to SNRs, and
(ii) identifying evolutionary tracks linked to a specific theory predicting them.

These are very important astrophysical objectives, because

(i) the methods for determining distances to SNRs are very uncertain, especially for the SNRs identified only in the radio, and
(ii) for a better understanding of the SNR radio brightness evolution, we need confirmation of the Duric & Seaquist (1986) theory which predicts a break in the Σ–*D* relation.

2 ANALYSIS AND RESULTS

2.1 The *L–D* dependence

In order to investigate the *L–D* correlation, we analysed both Galactic and extragalactic SNRs. For the Galactic sample we used all SNRs (regardless of type) from Green's catalogue (Green 2001) for which independently determined distances were available. If there was more than one distance for an individual remnant (distance determination was based on different methods giving different results), we used the average value. If only a lower limit for distance was estimated, we used this as the actual distance. Flux densities and angular diameters were also taken from this catalogue. We included four main Galactic radio loops (Berkhuijsen 1986) in our Galactic sample, although it does not change the principal result. As for the extragalactic sample, we used all the data available (i.e. surface brightnesses and linear diameters) in the catalogue of Urošević et al. [see appendix in Urošević, Pannuti & Duric (2003c), and references therein].[2] Flux densities, surface brightnesses and the radio luminosities derived refer to 1 GHz.

Table 1 summarizes the results. As can be seen, a good *L–D* dependence is obtained only in the case of 21 SNRs from the M82 starburst galaxy (Huang et al. 1994; McDonald et al. 2002), while in all other cases there seems to be no correlation. The M82 *L–D* dependence is explicitly

$$L_{1\mathrm{GHz}} = 2.4^{+0.5}_{-0.5} \times 10^{26} D^{-1.4\pm0.3}\ \mathrm{erg\ s^{-1}\ Hz^{-1}} \quad (4)$$

with a fit quality of 64 per cent[3] (Fig. 1).

In order to study better the correlation in the *LD* plane, we interchanged the variables so that we could check the *D–L* dependence. Fit statistics for this situation are summarized in Table 2. If $\delta = 0$, then we would expect $1/\delta \rightarrow \infty$. However, we obtain $1/\delta \approx 0$ again. This shows that we are dealing with case (ii), previously stated. Of course, these are just quantitative results for what is obvious from

[2] The catalogue is also available on the World Wide Web at http://astro.matf.bg.ac.yu/dejanurosevic/catalogue/index.html
[3] In our previous study (Stanković, Tešić & Urošević 2003), with a sample of 11 SNRs, we obtained $L_{1\,\mathrm{GHz}} = 2.7 \times 10^{26} D^{-2.1\pm0.4}$ erg s^{-1} Hz^{-1} with 76 per cent fit quality.

Table 1. Coefficient δ, correlation coefficient and fit quality of the Galactic and extragalactic *L–D* relations. MW stands for Milky Way, while LMC and SMC stand for the Large and Small Magellanic Cloud, respectively.

Galaxy	δ	Correlation coefficient r	Fit quality r^2 (per cent)	Number of SNRs n
MW	−0.1 ± 0.2	−0.06	0	62
LMC	−0.3 ± 0.4	−0.14	2	25
SMC	−0.3 ± 0.6	−0.24	6	7
M31	0.3 ± 0.3	0.24	6	30
M33	0.2 ± 0.2	0.16	3	51
M82	−1.4 ± 0.3	−0.80	64	21

Table 2. Coefficient 1/δ and correlation coefficients in the case of interchanged variables *L* and *D* (the *D–L* dependence).

Galaxy	1/δ	Correlation coefficient r
MW	−0.04 ± 0.09	−0.06
LMC	−0.1 ± 0.1	−0.14
SMC	−0.2 ± 0.4	−0.24
M31	0.2 ± 0.2	0.24
M33	0.1 ± 0.2	0.16
M82	−0.46 ± 0.08	−0.80

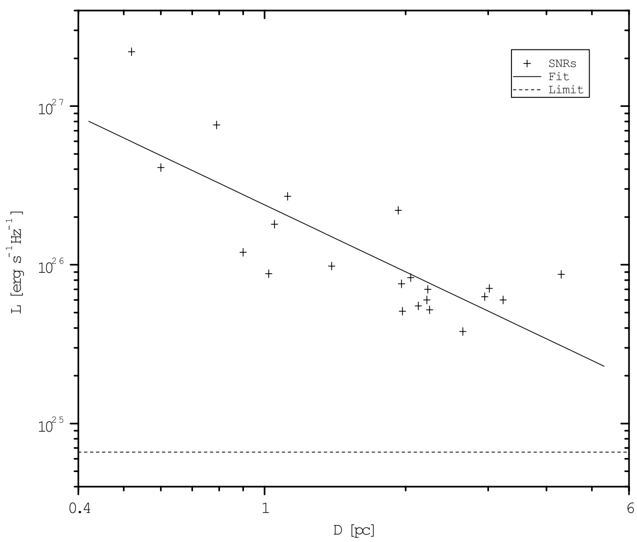

Figure 1. The *L–D* dependence for the M82 galaxy. The SNRs are marked with crosses, the solid line is a least-squares fit, while the dashed line represents the sensitivity limit.

Fig. 2. Nevertheless, we have also studied the distribution of luminosity alone for the Galactic, Large Magellanic Cloud and Small Magellanic Cloud together, M31 and M33 SNRs (Fig. 3). Once again, the standard deviation is too large to speak of a well-defined constant mean luminosity. What this means is that we can perform the power-law fit with an arbitrary fixed power δ and get equally

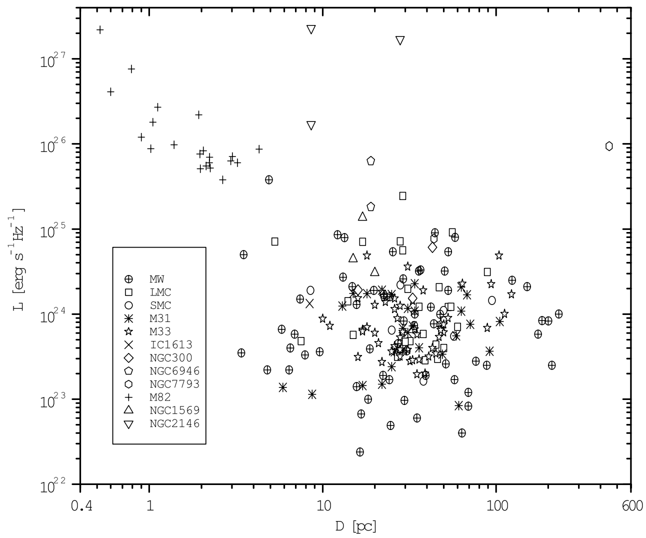

Figure 2. Figure shows all SNRs [including remnants of IC 1613 (1) and NGC 300 (3), 6946 (2), 7793 (2), 1569 (3) and 2146 (3)] in our sample. There is no obvious correlation between luminosity and linear diameter, except for the M82 SNRs.

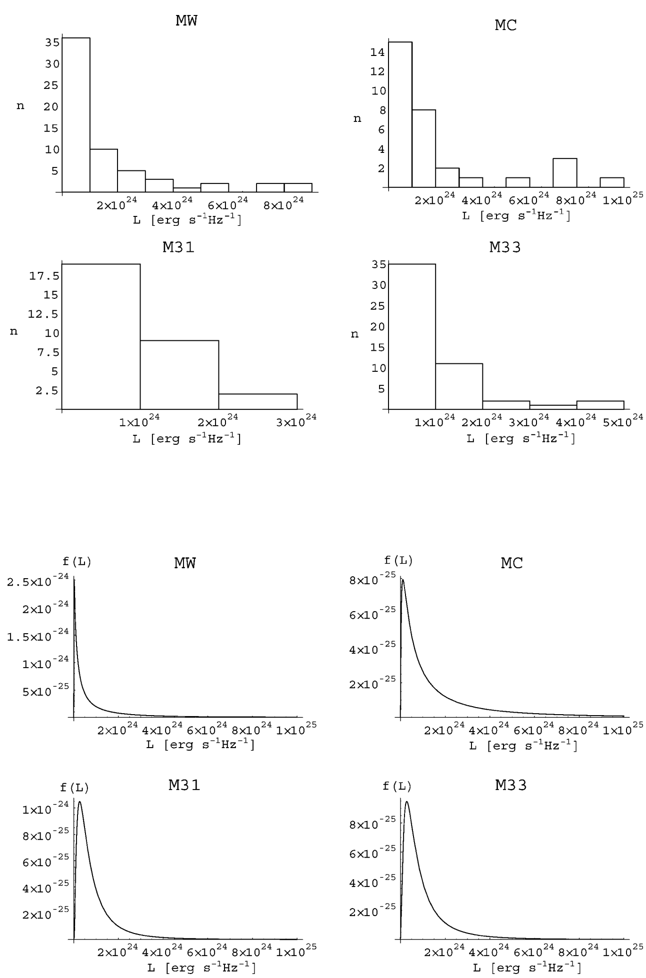

Figure 3. Frequency count for equidistant intervals of 10^{24} erg s^{-1} Hz^{-1} (up) and fitted lognormal probability density functions $f(L_\nu)$, which arise when many independent random variables (such as SN energy, ISM density and magnetic field strength) are combined in a multiplicative fashion (down). Cas A and B0525 − 696 in LMC are excluded from the plot.

Table 3. Quality of the power-law fit, $L_\nu = CD^\delta$, when δ is fixed (and equals −1.5 and −3, respectively).

Galaxy	Fit quality $r^2\|_{\delta=-1.5}$ (per cent)	Fit quality $r^2\|_{\delta=-3}$ (per cent)
MW	6.4	3.2
MC	1.2	1.7
M31	4.3	5.7
M33	0.25	0.05

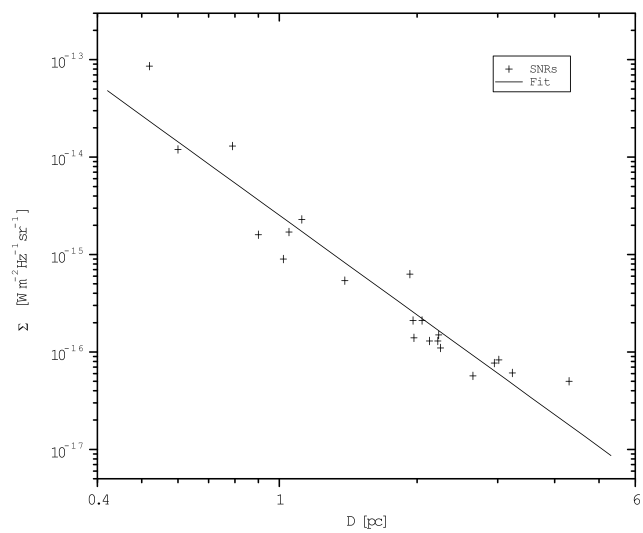

Figure 4. The Σ–*D* relation for M82: SNRs from our sample are marked with crosses, and the solid line is the least-squares fit.

good (or, rather bad) results (Table 3), and hence a definite choice of δ cannot be made.

2.2 The Σ–*D* relation

According to our criterion a true Σ–*D* relation exists only for SNRs in the M82 starburst galaxy. The relation is

$$\Sigma_{1\ \mathrm{GHz}} = 2.5^{+0.6}_{-0.4} \times 10^{-15} D^{-3.4\pm0.3}\ \mathrm{W\ m^{-2}\ Hz^{-1}\ sr^{-1}}, \quad (5)$$

with 91 per cent fit quality (Fig. 4). We define the fractional error

$$f = \left| \frac{d_{\mathrm{M82}} - d_\Sigma}{d_{\mathrm{M82}}} \right| \quad (6)$$

in order to get an estimate of the accuracy, should this relation be used for distance determination. d_{M82} is the distance to M82 [$d_{\mathrm{M82}} =$ 3.9 Mpc (Sakai & Madore 1999)] and the d_Σ is what this distance would be if it were derived (for every individual remnant) from the Σ–*D* relation. The maximum and average fractional errors are $f_{\max} = 0.33$ and $\bar{f} = 0.14$, respectively.

There are two main general properties of SNRs in the M82 sample that we believe are responsible for the good result obtained in constructing the Σ–*D* relation: the SNRs are all small (and presumably young), with linear diameters of a few parsecs, and very luminous. Both properties can be explained by a much higher (and presumably more uniform) mean density of the ISM in M82 (i.e. its central region) than is seen in other (normal) galaxies. It is interesting to

Table 4. Basic properties of the 14 Galactic SNRs associated with molecular clouds. Angular diameters and flux densities are taken from Green's catalogue. References for adopted distances are in the footnote. $d_{\Sigma'}$ and $d_{\Sigma''}$ are distances derived from Galactic molecular clouds and the M82 Σ–*D* relation, respectively.

Catalogue name	Other name	Angular diameter θ (arcsec)	Flux density $S_{1\,\mathrm{GHz}}$ (Jy)	Surface brightness $\Sigma_{1\,\mathrm{GHz}}$ ($\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}$)	Distance d_{I} (kpc)	Linear diameter D (pc)	Distance $d_{\Sigma'}$ (kpc)	Distance $d_{\Sigma''}$ (kpc)
G34.7−0.4	W44	31	230	3.6×10^{-20}	3.1[a]	28	2.1	2.9
G42.8+0.6		24	3	7.8×10^{-22}	11.0[b]	77	8.1	11.7
G78.2+2.1	γ Cygni	60	340	1.4×10^{-20}	1.2[a]	21	1.4	2.0
G84.2−0.8		18	11	5.1×10^{-21}	4.5[c]	24	6.3	9.0
G89.0+4.7	HB 21	104	220	3.1×10^{-21}	0.8[a]	24	1.3	1.8
G109.1−1.0	CTB 109	28	20	3.8×10^{-21}	3.5[a]	29	4.4	6.3
G111.7−2.1	Cas A	5	2720	1.6×10^{-17}	3.4[c]	5	2.3	3.0
G132.7+1.3	HB 3	80	45	1.1×10^{-21}	2.2[a]	51	2.2	3.2
G166.2+2.5	OA 184	79	11	2.7×10^{-22}	4.5[c]	103	3.4	4.9
G189.1+3.0	IC 443	45	160	1.2×10^{-20}	1.5[a]	20	2.0	2.8
G260.4−3.4	Puppis A	55	130	6.5×10^{-21}	2.2[c]	35	1.9	2.7
G309.8+0.0		22	17	5.3×10^{-21}	3.6[a]	23	5.1	7.3
G315.4−2.3	MSH 14-63	42	49	4.2×10^{-21}	2.8[c]	34	2.9	4.1
G349.7+0.2		2	20	7.5×10^{-19}	22.0[d]	13	13.7	18.7

References: [a]Huang & Thaddeus (1985); [b]Stanimirović et al. (2003); [c]Case & Bhattacharya (1998); [d]Slane et al. (2002).

note that SNRs of another starburst galaxy, NGC 1569, lie on the M82 fit, with fractional errors of 0.05, 0.06 and 0.27.[4]

Another interesting observation is that Cas A also lies on the M82 fit with $f = 0.10$. This suggests a possible connection between young (and even older) Galactic remnants evolving in a dense environment similar to M82 SNRs. To account for the second property (dense environment), we tried to extract a subsample of Galactic SNRs evolving in molecular clouds. A study of shell-type Galactic SNRs associated with large molecular clouds was done by Huang & Thaddeus (1985). We have taken 12 remnants of theirs together with G349.7+0.2 (Slane et al. 2002) and G42.8+0.6 (Stanimirović et al. 2003). The properties of these remnants are given in Table 4. The Σ–*D* relation obtained is

$$\Sigma_{1\ \mathrm{GHz}} = 1.1^{+3.7}_{-0.8} \times 10^{-15} D^{-3.5\pm0.5}\ \mathrm{W\ m^{-2}\ Hz^{-1}\ sr^{-1}}, \tag{7}$$

with 84 per cent fit quality[5] (Fig. 5). Resemblance to the M82 Σ–*D* relation is conspicuous. If we calculate the fractional errors

$$f = \left|\frac{d_{\mathrm{I}} - d_{\Sigma}}{d_{\mathrm{I}}}\right|, \tag{8}$$

where d_{I} is the independently determined distance to the remnant and d_{Σ} the distance derived from the Σ–*D* relation, we obtain $f_{\max} = 0.59$ and $\bar{f} = 0.28$.

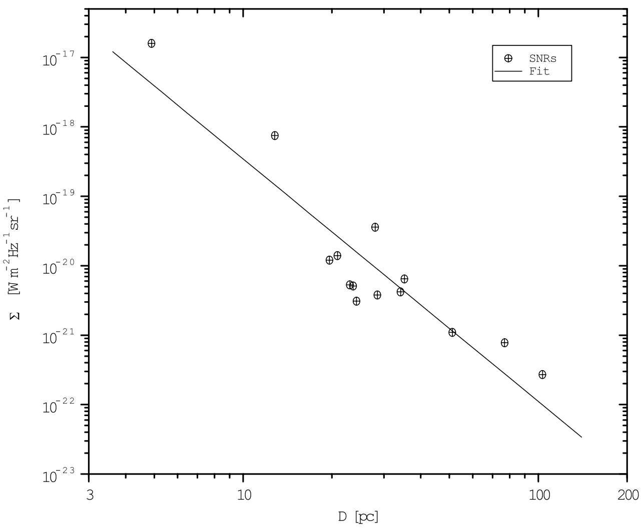

Figure 5. The Σ–*D* relation for a subsample of Galactic SNRs, associated with the large molecular cloud, marked with encircled crosses, where the solid line represents the least-squares fit.

3 DISCUSSION

Recent studies of empirical Σ–*D* relations for Galactic and extragalactic SNRs show $\beta \approx 3.5$ for compact radio SNRs in the starburst galaxy M82. For remnants in other (normal) galaxies $\beta \approx 2$, which suggests a possible break in the relation (if we analyse all remnants together) between young and older SNRs (Urošević 2002, 2003). As we concluded in the previous section, the Σ–*D* relation with $\beta = 3.4$ exists for M82 SNRs in our sample, while for all other galaxies the remnants are too scattered in the *LD* plane so that the *L–D* and thereby the true Σ–*D* relation cannot be claimed to exist. In other words, the empirical value $\beta = 2$ has a statistical, rather then physical, cause. A constant luminosity solution for the evolution of older SNRs is of course a possibility, but not the preferable one, as the empirical $\beta = 2$ at first might suggest. This could also mean that the break in the relation might be a fictitious one.

However, theoretical considerations of the Σ–*D* relation (Duric & Seaquist 1986) do predict a change in the slope (in log–log space), with $\beta = 5$ for young and $\beta = 3.5$ for older SNRs. The difference in slope (3.5–2) is the same as that predicted by the theory (5–3.5), but the actual values of the slopes are different. Urošević et al. (2003c) suggest that the shallower evolutionary track predicted for older remnants is completely dominated by different evolutionary tracks

[4] SNRs of the third starburst galaxy in our sample, NGC 2146, have much larger fractional errors (0.79, 0.86 and 0.55) and all lie far above the M82 fit. However, they are probably much more energetic, two of them possibly being the result of hypernova explosions (see Urošević et al. 2003c).

[5] The *L–D* dependence is $L_{1\,\mathrm{GHz}} = 1.1^{+3.8}_{-0.8} \times 10^{26} D^{-1.5\pm0.5}\ \mathrm{erg\ s^{-1}\ Hz^{-1}}$, with a fit quality of 50 per cent.

of individual SNRs. This is presumably caused by the large variation in the density of the ISM. In the case of the much steeper relation for young remnants, the effect is a reduced slope to the lower value of 3.4. Results of the previous subsection indicate that this indeed might have happened, at least in the case of older remnants. As for the M82 case, the situation is not so clear.

There is no doubt that the M82 SNRs differ from all other remnants in our sample, as we have already mentioned. Chevalier & Fransson (2001) have discussed the high radio luminosity of starburst galaxy SNRs and argued that these are correlated with the higher-than-average molecular cloud densities with which these SNRs are interacting. Moreover, the ISM (i.e. molecular clouds) density structure is most likely much more uniform, so that all remnants practically evolve in the same environment [this also refers to our 14 Galactic molecular cloud (GMC) SNRs]. Higher density would also lead to much smaller linear diameters. Because the potential break in the Σ–D relation also depends on ambient density (through shock velocity), it is possible that even M82 SNRs belong to the shallower component. The M82 and the GMC SNRs then practically make one track in the Σ D plane. According to the theory we expect β to depend only on the spectral index and on the parameter that describes the evolution of the magnetic field. The only difference between these two classes is in the coefficient A, which accounts for the different average density of molecular clouds in M82 and our Galaxy. All SNRs then follow parallel tracks in the Σ D plane, those below evolving in lower density media, and those above probably having higher SN explosion energy deposits.

Nevertheless, the above discussion is rather speculative. The theory of Duric & Seaquist (1986) may not be appropriate and even β may depend on density, thus causing different evolutionary tracks for SNRs in lower and higher density ISM. Another possible explanation of the difference in slope, by inclusion of thermal emission, is given in papers by Urošević, Duric & Pannuti (2003a,b). In addition we did not account for the selection effects, one particularly being the sensitivity limit that prevents less luminous SNRs, evolving in lower density environments, from appearing in the sample. Related to this is the Malmquist bias, present in Galactic data sets (or subsets): intrinsically bright SNRs are favoured in any given flux-limited survey because they are sampled from a larger spatial volume. Being aware of all this and knowing the history of the Σ–D relation, we shall try to remain reasonably detached in our conclusions.

4 CONCLUSIONS

In our study we have analysed the L–D correlation (possible dependence of radio luminosity on linear diameter) for supernova remnants in order to see whether the Σ–D relation actually exists and whether determination of SNR distances on the basis of said Σ–D relation is thereby possible. We have obtained good results only for the M82 SNRs and Galactic SNRs associated with molecular clouds. This is consistent with the opinion that no single Σ–D relation can be obtained for all SNRs. However, it might be possible to construct the relation for some classes of SNRs as we have shown. Even though distances to these two particular classes of remnants (M82 and GMC SNRs) may be obtained by other methods, we can still make use of the Σ–D relation for estimation of distances, or for its confirmation at least.

In the end we can conclude the following.

(i) If there is a change in the slope of the Σ–D relation, or if there are different relations for remnants in two distinctive environments (dense and dispersed), then the M82 relation would be the appropriate one for SNRs evolving in dense media (even for young SNRs alone) and it may be used for distance estimates to these kinds of SNRs.

(ii) On the other hand, the resemblance between the M82 and GMC relations and the agreement with the aforementioned theory might indicate that what we have is one (in the sense of slope) Σ–D relation. If this is the case then the M82 relation remains valid for the estimation of distances for all SNRs (compact and evolved) in the dense environment of molecular clouds and further similar relations may be constructed for SNRs in dispersed environments and for remnants arising from stronger SN explosions.

Differentiating between these two possibilities will be aided by future observations.

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