

ABSTRACT

During the last couple of decades of work on the $\Sigma-D$ (radio surface brightness to diameter) relation for supernova remnants (SNRs), it has been generally accepted that no single $\Sigma-D$ relation can be constructed for all SNRs. However, it may still be possible to construct the relations for some classes of SNRs. In our previous paper we analysed $\Sigma-D$ relation(s) for remnants in the dense environments of molecular clouds. The aim of this paper is to examine, in the same context, a class of oxygen-rich SNRs, and to extend the analysis to remnants evolving in lower-density interstellar media, namely Balmer-dominated SNRs. We have obtained good relations with certain similarities to our previous findings – similarities that emphasize, again, the role of ambient density in the evolution of SNRs.

Key words: methods: statistical – supernova remnants – radio continuum: ISM.

1 INTRODUCTION

Many papers have been written in favour of and against the $\Sigma-D$ (surface brightness to diameter) relation for supernova remnants (SNRs), especially concerning its use for distance determination, and it still attracts considerable attention from people working in the field. For a critical overview of the $\Sigma-D$ relation, see, for example, Green (1984, 2004). Notwithstanding all the criticism of the $\Sigma-D$ relation, it remains an important tool in studying and understanding the evolution of SNRs, and even in estimating distances, because there are few other methods available (e.g. Case & Bhattacharya 1998).

During the last couple of decades of work on the $\Sigma-D$ relation for SNRs, it has been generally accepted that no single $\Sigma-D$ relation can be constructed for all SNRs. However, it still might be possible to construct the relations for some classes of SNRs. In our previous paper (Arbutina et al. 2004, hereafter Paper I), we analysed the $\Sigma-D$ relation(s) for remnants in a dense environment of molecular clouds. In this paper, we try to obtain a relation for SNRs evolving in interstellar media (ISMs) of lower density.

1.1 Objectives

The three main objectives of this paper are:

(i) to reanalyse the $\Sigma-D$ relation for SNRs in dense environments;

(ii) to find a homogeneous subsample of SNRs in low-density environments, and to obtain the $\Sigma-D$ relation for this particular class of remnants;

(iii) to examine qualitatively the effect of the ISM density on the $\Sigma-D$ relations for different classes of remnants.

This is important, primarily, for better understanding SNR evolution and, secondarily, for knowing the capabilities and constraints of the $\Sigma-D$ relation as a tool for distance determination.

2 $\Sigma-D$ RELATION AND THE DENSITY OF THE INTERSTELLAR MEDIUM

The surface brightness to diameter relation for SNRs has been generally written as

$$\Sigma = AD^{-\beta},$$

where $A$ depends on the properties of the supernova (SN) explosion and interstellar medium, such as the SN explosion energy, the mass of the SN ejecta, the density of the ISM, the magnetic field strength, etc., while $\beta$ is independent of these properties (or perhaps, weakly dependent).

Early studies of the $\Sigma-D$ relation did not make any distinction between different classes of remnants, mainly due to the lack of observational data, leading to the assumption that all properties, or parameters, of the radio (surface brightness) evolution are, practically, the same for all remnants. However, later studies have shown that these properties may substantially differ from remnant to remnant.

Berkhuijsen (1986) was among the first to emphasize the role of ISM density in the explanation of the observed range of surface brightnesses for a given diameter. We believe that the density of the ISM is of utmost importance in the evolution of SNRs; other parameters are, in one way or another, connected to the ISM density. For example, we know from stellar evolution theory that
massive stars mainly occur and remain in dense environments (such as molecular clouds) owing to their shorter lifetimes, while the longer-lived lower-mass stars tend, on average, to be found in less dense environments. Furthermore, the energy of a SN explosion, or at least its largest part, has to be gravitational by its origin, and thereby it depends on the mass of the star (and indirectly on the environmental density). The magnetic field (which is ‘frozen’ in plasma) may be linked to ISM density, as well.

Taking into consideration all the influences that the density of the ISM may have on the radio evolution of SNRs would be too complicated a task. Therefore, in the subsequent discussion we focus on a more direct connection between the ISM density and the $\Sigma$–$D$ relation.

### 2.1 $\Sigma$–$D$ relation(s) for supernova remnants in a dense environment of molecular clouds

Generally, the dependence of surface brightness on the density of the ISM is assumed to be of the form $\Sigma \propto \rho_s^{1/2} \propto n_H^{1/2}$ (Duric & Seaquist 1986; Berezhko & Volk 2004). This means that the larger the ISM density, the greater the synchrotron emission from the SNR ($\rho_s$ and $n_H$ are the average ambient density and hydrogen number density, respectively).

This suggests that SNRs in dense environments would tend to have, on average, higher surface brightness, in comparison to those in lower-density environments, and should be treated separately. However, it is difficult to decide which remnants should be included or excluded from analysis, only on the basis of high, or low, radio surface brightness, so most of the $\Sigma$–$D$ relations were constructed for all (non-plerionic) SNRs, possibly classifying them only with respect to their diameters.

Recent studies of empirical $\Sigma$–$D$ relations for Galactic and extragalactic SNRs (see Urošević 2002, 2003; Urošević et al. 2005) show $\beta \approx 3.5$ for compact radio SNRs in the starburst galaxy M82, while for remnants in other (normal) galaxies $\beta \approx 2$, indicating a possible break in the relation (if we analyse all remnants together) between presumably young and older SNRs. However, in Paper I we have shown that the result $\beta = 2$ has a statistical rather than a physical cause, so that the shallower $\Sigma$–$D$ relation cannot really be claimed to exist. However, there exists a good relation with $\beta = 3.4$ for M82 SNRs and, moreover, a similar relation with $\beta = 3.5$ for Galactic molecular cloud (GMC) SNRs, with 91 and 84 per cent fit quality, respectively.

Moreover, M82 SNRs may not be that young. Chevalier & Fransson (2001) have discussed the high radio luminosity of a starburst galaxy’s SNRs and have argued that these are correlated with the higher-than-average molecular cloud densities with which these SNRs are interacting (with $n_H \sim 10^3$ cm$^{-3}$, which is two orders of magnitude larger than in the GMC, where $n_H \sim 10$ cm$^{-3}$). Higher density would also lead to much smaller linear diameters, so under these conditions the M82 SNRs would be rather evolved. Because SNRs in dense molecular clouds are likely to rapidly become radiative, Chevalier & Fransson (2001) suggest that they have entered the radiative phase.

The main emphasis in Paper I was on M82 SNRs, because GMC SNRs have unreliable distances and, as a subsample of Galactic remnants, suffer from various selection effects. The M82 sample itself is not free of selection effects (as pointed out by Green 2004), one particularly being the sensitivity limit, which prevents less luminous SNRs (of larger diameters, $D > 4$ pc) from appearing in the sample. 10$^3$ cm$^{-3}$ may be too high, but even in such an extreme environment, larger SNRs can be expected, and they could change the picture. However, Monte Carlo simulations by Urošević et al. (2005) indicate that this relation is not strongly affected by selection effects connected with sensitivity, as is the case with other relations.

Although the $\Sigma$–$D$ relation for GMC SNRs is more limited, we believe that it is giving us some insight into where larger SNRs in a dense environment are to be expected in the $\Sigma$–$D$ plane, and it seems that they would follow the trend defined by the compact M82 SNRs. Another interesting class of remnants that may be connected with dense environments are oxygen-rich SNRs (see Section 2.2). Properties of these remnants are given in Table 1. Presently there are eight known (van den Bergh 1988; Park et al. 2003): three Galactic and five in the Magellanic Clouds (hereafter LMC and SMC denote Large and Small Magellanic Clouds, respectively). Two of these, G 292.0+1.8 and 0540–69.3 in the LMC, show up as plerions (with associated radio pulsars), and are given here only for completeness. The $\Sigma$–$D$ relation obtained for the remaining six is

$$\Sigma_{1 GHz} = 2.2^{+1.1}_{-1.3} \times 10^{-15} D^{-3.3 \pm 0.4} \text{ W m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1},$$

with 96 per cent fit quality. This is, again, similar to the two relations mentioned above. All three classes of remnants in dense environments are plotted together in Fig. 1.

The remnants in Fig. 1 seem to define a relatively broad track in the $\Sigma$–$D$ plane. If the main dependence of surface brightness on ambient density is through the coefficient $A$, all SNRs would...
then follow provisionally parallel tracks in the $\Sigma$D plane, those below the average track evolving in lower-density media, and those above probably having higher SN explosion energy deposits. In Section 2.2, we try to extend our analysis to remnants in the lower-density ISMs.

### 2.2 $\Sigma$–D relation for supernova remnants in the lower-density interstellar media

Mathewson et al. (1983) classified SNRs into four categories based on their optical features: Balmer-dominated, oxygen-rich, plerionic/composite and evolved SNRs. It has been suggested (see van den Bergh 1988) that Balmer-dominated SNRs are connected to Type Ia SNe – deflagration of a CO white dwarf – and are remnants with high velocity, non-radiative, collisionless shocks interacting with the ISM, while oxygen-rich SNRs originate in the Type Ib event – explosion of a massive O or a Wolf–Rayet (W–R) star – and have emission arising from a shock interacting more with the circumstellar material (CSM) lost by the progenitor in the last stages of stellar evolution. Similarly, plerionic/composite SNRs originate in Type II events – explosion of a massive B star – and derive their energy from the rotational energy losses of a stellar remnant – neutron star (pulsar driven plerions) – plus the shock wave powered shell (in the case of composite type). All of these classes eventually become evolved remnants.

SNe II (and their remnants) are principally different than SNe I, and are not the object of our interest (for a discussion on these SNRs and their environment, see, for example, Guseinov, Anayk & Tagieva 2004). As with oxygen-rich remnants, some do have pulsars and thus may be connected with SNe II, but most of them (six of eight) seem to be shell-type SNRs (perhaps with a radio-quiet neutron star – Cas A, Pup A). They primarily occur in H II and molecular cloud regions, and we have introduced them in Section 2.1. Here we are interested in SNe Ia, i.e. Balmer-dominated (or Tychonic) SNRs, because they may be linked with the lower-density ISM. There are three known Galactic SNRs of this type (Tycho, Kepler and SN1006) and another four in the LMC listed by Mathewson et al. (1983): LHG 10, 14, 26 and 89 (we have used their X-ray source numbers for short). Tuohy et al. (1982) constructed a 408-MHz $\Sigma$–D relation for these seven remnants and showed that it falls systematically below that of the other SNRs.

We have included two more SNRs, SMC 0104–72.3 (IKT 19) (van den Bergh 1988) and SN Ia remnant N103B in the LMC (Hughes et al. 1995), and we have constructed a new relation at 1 GHz. The properties of the remnants in our sample are given in Table 2. The $\Sigma$–D relation obtained is

$$\Sigma_{1 \text{GHz}} = 3.9^{+1.3}_{-0.9} \times 10^{-17} D^{-3.2 \pm 0.6} \text{ W m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1},$$

with 83% per cent fit quality. We define the fractional errors

$$f = \left| \frac{d_I - d_{\Sigma}}{d_I} \right|$$

as an indicator of the applicability of our relation for distance determination. Here, $d_I$ is the independently determined distance to the remnant and $d_{\Sigma}$ is the distance derived from the $\Sigma$–D relation. The maximum and average fractional errors are $f_{\text{max}} = 0.56$ and $f = 0.23$, respectively.

Fig. 2 shows the relation for the Balmer-dominated and oxygen-rich SNRs, for comparison. The two relations have similar slopes, but the former is significantly below the latter, as we would expect for SNRs in low-density environments. To further investigate ISM density influence on the $\Sigma$–D relation, we have tried to find some surrounding density estimates for individual remnants. The results of this search are shown in Table 3.

Although the difference in average surrounding density between the two classes of remnants (Ia and Ib) is clear, the numbers are too uncertain to see the effects of different densities on individual remnants. Some remnants that are significantly above the fit may be connected with the dense environments. N103 B is located at the edge of an H II region (Lewis et al. 2003), and thus may be a SN Ia remnant in a more dense environment, which would explain its higher surface brightness. A similar situation may apply to the SN Ib remnant N132 D, near the 30 Dor nebula (Morse et al. 1996; Blair et al. 2000). There is an exception, however. DEM L71 with a higher surrounding density is below SN1006, which is in a lower-density environment. Because surface brightness is expected to depend on SN energy as well, $\Sigma \propto n_{H} E_{E}$, the explanation may be in a lower energy deposit of this remnant ($E_{E} \approx 4 \times 10^{50} \text{ erg}$; Ghavamian et al. 2003). This would stress the significance of SN energy for dispersion in the $\Sigma$–D relation (Berezhko & Volk 2004).

In addition, young oxygen-rich remnants are likely to interact with especially complex CSM, rather than the ISM. Oxygen-rich remnants seem to have rather high radio surface brightnesses (typically higher than GMC SNRs even). N132 D, mentioned above, has been suggested, for example, to have encountered the wall of the cavity and is now interacting with the high-density material there (Hughes, Hayashi & Koyama 1998). For such remnants, it is the CSM with its complexity that has to be fully appreciated.

### 3 Discussion and conclusions

This paper represents an attempt to analyse the dependence of the $\Sigma$–D relation of SNRs on the density of the ISM, with the emphasis

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1. If we analyse the $L$–$D$ correlation, as a test of the $\Sigma$–D relation quality, we obtain the correlation coefficient $|r| = 0.64$ (the fit quality is $r^2$). For oxygen-rich remnants (equation 2), $|r| = 0.90$, $f_{\text{max}} = 0.48$ and $f = 0.23$.

2. However, the SN Ia origin for this remnant is not unquestionable (van der Heyden et al. 2002). As an aside, there are other Ia candidates that should be further investigated (van der Heyden et al. 2004).
on the Σ–D relation for Balmer-dominated (or SNe Ia) remnants, which are, most likely, SNRs evolving in the lower-density ISM. We have obtained a good relation with certain similarities to our previous relations – similarities that emphasize, again, the role of density of the environment in the evolution of SNRs.

The slope of all relations seems quite similar, β ≃ 3.5. This value matches that predicted by Duric & Seaquist (1986); however, their theory may be outdated. Recent theoretical development can be found in Berezhko & Volk (2004). The question is does the Σ–D relation, such as it is, correspond to the evolutionary track of a typical SNR at all? It may be so, if the remnants considered are in the Sedov, or at least, late pre-Sedov (‘free expansion’) or early post-Sedov (radiative) phase, which does not have to be, and surely it is not the case with all remnants. Even for the Sedov phase, the fit should not necessarily be assumed linear (in log–log space).

Then, it would be probably more appropriate to treat these Σ–D relations as a sort of measure of dispersion in the ΣD plane. From this point of view, we found that Σ–D relations for two distinctive classes of remnants, in dense and lower-density environments (assumed to be of Ib and Ia types, respectively), seem to foreshadow two tracks or domains in the ΣD plane, one above the other. These domains may not be strictly parallel, their borders may not be sharp, and they may interleave, but, with the current sample, it seems that they are well defined. Because the ambient density for SNRs can vary from 10^{-3} to 10^{3} cm^{-3}, and the energy is assumed not to deviate significantly from 10^{51} erg, it seems that, on larger scales, the density can be regarded as the most important parameter for the observed dispersion in the Σ–D relation. For SN Ic remnants that may be connected with dense environments, but have larger SN energy, E_{51} ≳ 10^{52} erg, the blast energy is more important, and they would be found further above the Ia and Ib SNRs in the Σ–D plane (as is the case with the hypernova remnant candidates in fig. 2 of Urošević et al. 2005). Our main point is that even though the Σ–D plot for all SNRs shows severe scattering in the ΣD plane, these remnants can be grouped in classes for which dispersion is significantly reduced.
Study of these classes, and their further differentiation, with the aim of gaining better understanding of SNR evolution, may hold some promise.

On smaller scales (e.g. for the Ia class, only) the situation is not clear, yet. The numbers that we work with, in statistical studies of SNRs (both observed and calculated), are far from certain. Even if we overcome this, it would still be hard to bring the density in a quantitative relation to radio surface brightness, because, not only the properties of the environment vary from remnant to remnant, but the environment is probably quite inhomogeneous in the case of a single remnant (e.g. CSM for young massive star SNR), which would add more confusion in statistical studies. However, our qualitative analysis seems to indicate that ISM density cannot, alone, be fully responsible for dispersion here (SN energy may be equally important, if not more).

Still, the sample is too small for firmer conclusions to be made. There have been suggestions that optically identified extragalactic remnants (such as SN 1885A, 1895B, etc.; for upper limits, see fig. 5 in Paper I), which tend not to be detected in radio and X-rays, are the remnants expanding in relatively low-density media (see a concise review by Duric 2000, and references therein). Nevertheless, an ideal subsample of these cannot be included in our analysis (because they do not have radio emission). On the other hand, these optical searches miss the Balmer-dominated SNRs (Pannuti et al. 2000).

As in Paper I, we can conclude that more observations are needed: optical and X-ray observations that will discover new Balmer-dominated (Ia) and oxygen-rich (Ib) SNRs, more accurate radio observations of existing, and radio detection of historical Type I SN remnants (because they do not have radio emission).

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REFERENCES

Green D. A., 2004, BASI, 32, 335

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