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Invited review

THE STELLAR ATMOSPHERE PHYSICAL SYSTEM I. PHENOMENOLOGICAL DEFINITION AND REPRESENTATION OF A STELLAR ATMOSPHERE

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SUMMARY: This paper is the first in a series of two that deals with the physical and numerical grounds of stellar atmosphere modelling. After a phenomenological definition of a star and stellar atmosphere, the physics that shapes the stellar atmosphere physical system is discussed and three alternative pictures are considered for its representation.

Key words. Stars: atmospheres – Radiative transfer – Methods: numerical

1. INTRODUCTION

This paper is the first in a series of two, whose aim is to analyse stellar atmosphere modelling both from the physical and the algorithmic standpoint. Here we will set the stage and define stellar atmospheres on physical grounds. The structure of the outer stellar layers, determined by the interactions between the constituting material and the radiation field that permeates the latter, will be analysed from an *operational* point of view. In the second paper we will present an *operative* sequential iterative method for the solution of the stellar atmosphere problem, which may be considered as a paradigm of non-linear and non-local problems. To have such an easy and reliable working tool at hand allows one to test the physical hypotheses introduced in the modelling of astrophysical objects. In other words, it makes possible to set up a veritable *numerical laboratory for computational astrophysics.*

In The Oxford English Dictionary 'operationalism' is defined as *a theory or system which accepts only such concepts as can be described in terms of the operations necessary to determine or prove them*[1]. Modelling any physical system implies to describe the behaviour of a (generally) complex structure in terms of the laws governing each of its elementary components. To achieve that one must resort to a series of operations that includes the ideal dissection of the system into an ensemble of interacting parts, their identification and quantification by means of physical variables, the successive translation of physical magnitudes and their relevant mutual interactions into a system of equations (either continuous or discrete) and eventually their solution. This analytical procedure is possibly the most effective tool we have at hand for scientific inquiry. On the other hand, The Oxford English Dictionary defines the term 'operative' as *characterized by oper-*

[1]The theory, introduced in 1927 by P.W. Bridgman (Bridgman 1927), was labelled with this word although he himself was not pleased by the name.

ating or working; active in producing, or having the power to produce, effects; hence our choice of defining operative the representation of a stellar atmosphere given by the model computed by means of the sequential iterative procedure, to be described in Paper II, in view of its effectiveness.

Phenomenological definitions of star and stellar atmosphere are given in Section 2. A statistical description of radiative transfer (RT) in terms of the photon escape probability (Section 3) paves the way to a definition in Section 4 of stellar atmosphere according to the kinetics of photons. The thickness and border of a stellar atmosphere are likewise defined in Section 5. After the preliminary operational definition in Section 6 of the specific intensity of the radiation field and macroscopic coefficients that characterize the RT equation, the 'stellar atmosphere physical system' is eventually presented in Section 7 where the physical processes that shape the structure of a stellar atmosphere and the relevant equations are briefly discussed. As a conclusion three alternative representations of the stellar atmosphere physical system are shown in Section 8. Some concluding remarks are made in Section 9.

2. PHENOMENOLOGICAL DEFINITION OF STAR AND STELLAR ATMOSPHERE

2.1. Stars as observable physical systems

Although out of our reach, stars can nevertheless be the object of scientific investigation just because they are visible. From observations we can delimitate point-like regions of the sky from which a strong flux of electromagnetic radiation originates. Spectral properties of the observed radiation let us infer also an outward flux of matter. The dynamical effects observed and measured in the neighbourhood of such sources of fluxes suggest that the latter are at the origin of gravitational fields. Upon these considerations we can individualize celestial bodies, whose matter density is enormously higher than that of the surrounding medium and which shine with their own light: the *stars*.

The observed evidence of fluxes implies the existence of gradients of the physical properties inside a star, which are at the origin of *transport phenomena* that tend to establish equilibrium conditions. The energy generated by thermonuclear reactions in the inner core is carried through the stellar layers by two modes of transport: radiative and convective. Their relative weight depends on the thermodynamic conditions point by point inside the star. However, the primary observational fact that stars emit radiation is the clear-cut proof that the former must be always present. The outward acting force due to the radiation pressure is opposed to the gravitational force. The balance between the force due to the total pressure (gas plus radiation) gradient and gravity maintains the structure of the star stationary over very long periods of its life. According to the above phenomenological picture, we may define a star in thermodynamic terms as *a gravitationally bounded open*[2] *concentration of matter and energy.* Such a definition characterizes a system out of equilibrium where irreversible processes take place. Not far of equilibrium, however, transport phenomena are governed by linear phenomenological laws. Thus, at the first order of approximation, a *linear non-equilibrium* approach can be employed.

2.2. A qualitative definition of stellar atmosphere

Which stellar layers does the observed flux of radiation emerge from? Intuitively not from the innermost ones, where the extreme density of stellar material hinders the flow of radiation. On the contrary, it is reasonable to assume that the emergent flux comes from the outermost layers, characterized by an exponential fall of the density that defines the edge of the star (see Section 5). We will consider in a natural way a border region between the dense core, where the stellar material concentrates, and the diluted surrounding medium. The flux of radiation can flow freely through the outer boundary of this region. Thus, we will call *stellar atmosphere this region, where the emergent electromagnetic spectrum forms.* We shall preliminarily remark that the structure of a stellar atmosphere is mainly determined by the physical properties of the stellar interior, firstly by the temperature gradient of the entire star. The latter is responsible for the outward radiative flux. The outer layers do not alter *quantitatively* the outgoing flux. Due to their low density, they cannot absorb and store a large amount of energy, nor the energy produced inside them is comparable with the energy generated in the stellar interior.

The physical conditions inside a stellar atmosphere are governed essentially by the gravitational field generated by the star and the outward radiation flux from the interior. In order to remain in a steady state, the configuration of the atmosphere must be such that radiation can flow outwards. Therefore, two *external* parameters, that is the gravitational acceleration at the surface and the total radiation flux (i.e., the bolometric luminosity of the star), together with an *internal* parameter, namely the chemical composition, determine the physical state of a stellar atmosphere. On the other hand, the spectral features of the emergent radiation, namely the *qualitative* properties of this flux, are determined by interactions between matter and radiation in the outer layers. These processes are responsible for a redistribution in frequency of the radiant energy. Therefore, the emergent spectrum reflects the physical state of the stellar atmosphere where, by definition, it is formed (see Section 4.1). The above statement constitutes the key for the diagnostics of the physical properties of stars: from the theoretical modelling of

[2]Customarily, thermodynamic systems are classified as *open* when both matter and energy fluxes are present, *closed* when only energy fluxes show and *isolated* in absence of both.

stellar atmospheres we try to predict the characteristics of the emergent spectrum via computation of the radiative flux carried through the outer layers, and to compare successively the computed features with the observed ones.

3. STATISTICAL DESCRIPTION OF RADIATIVE TRANSFER

3.1. Radiative transfer as a kinetic process

We have stressed above the primary role of radiative transfer in the physics of stellar atmospheres. Now, we are going to introduce a statistical description of the propagation of radiation through the stellar material, namely a picture based on the kinetics of photons. The radiation field can be described in terms of a *photon distribution function*, namely a function $f(\boldsymbol{r},\, t;\, \boldsymbol{n},\, \nu)$ such that $f(\boldsymbol{r},\, t;\, \boldsymbol{n},\, \nu)\, d\Omega d\nu$ yields the number of photons per unit volume at location $\boldsymbol{r}$ and time t, with frequencies in the range $(\nu,\, \nu + d\nu)$ and propagating with speed c in direction $\boldsymbol{n}$ into a solid angle $d\Omega$. The basic equation describing particle transport is the *Boltzmann equation*, which determines the space-time evolution of the corresponding distribution function. From this standpoint the RT equation can be considered as a *kinetic equation for photons*[3].

3.2. Photon escape probability

Photons, generated by the thermonuclear reactions in the stellar core, propagate through the stellar material. Along their path they undergo absorption (usually followed by the corresponding process of emission) and scattering processes which divert them almost instantaneously from their initial direction of propagation without a significant alteration of their original frequency. When photons reach the outermost layers, those that propagate outwards may eventually escape from the open boundary of the star. The stochastic nature of these processes leads to the concept of *photon escape probability*: each photon, absorbed and successively reemitted at a certain point inside the atmosphere has a definite probability of leaving the star, either directly or after a series of successive scattering processes[4].

3.3. Stellar opacity and optical depth

Obviously the escape probability depends on the optical properties of the medium, determined frequency by frequency by the opacity of the stellar material. Opacity is specified by a macroscopic extinction coefficient that accounts for both absorption and scattering processes. Let us factorize the monochromatic extinction coefficient $\chi_\nu = k_\nu \rho$ as the product of the total monochromatic extinction cross-section per unit mass k_ν of a single particle times the density ρ of matter. The dimension of χ_ν is $M^{-1}L^2 \times ML^{-3} = L^{-1}$, namely that of the reciprocal of length. In stellar atmospheres the variation of k_ν with frequency can be extremely large. Let us consider now a photon with frequency ν belonging to a beam of similar photons that propagate along a given direction $\boldsymbol{n}$. The reciprocal of χ_ν will give a statistical measure of the distance (measured along $\boldsymbol{n}$) over which such a photon can travel before it is removed from the beam. The parameter $l_\nu \equiv 1/\chi_\nu$ is by definition the *photon mean-free-path*.

The monochromatic optical depth is defined in a customary way by the *differential* relation

$$d\tau_{\nu,\boldsymbol{n}}\,(s) \equiv -\, \chi_\nu\,(s)\; ds \quad . \tag{1}$$

The above optical depth, by definition a dimensionless quantity, is a measure of the attenuation of the beam for a differential path ds along a given direction $\boldsymbol{n}$. We can assume that in most cases the extinction coefficient is independent of the direction of propagation of the absorbed or scattered photon. On the contrary, the optical depth is a *directional* quantity, as it depends on the direction of propagation. The integration of Eq. (1) from an initial point of abscissa s_1 on the s–axis associated with the direction $\boldsymbol{n}$ and pointing towards the outside of the atmosphere to a final point of abscissa $s_2 < s_1$, gives the measure of the integrated attenuation $\Delta\tau_{\nu,\boldsymbol{n}}$ between the two points, namely the *optical thickness*:

$$\Delta\tau_{\nu,\boldsymbol{n}}\,(s) = \int_{s_1}^{s_2} \chi_\nu\,(s')\; ds' \quad . \tag{2}$$

By recalling the definition of the photon mean-free-path, we recognize that $\Delta\tau_{\nu,\boldsymbol{n}}$ is just the number of photon mean-free-paths at frequency ν between s_1 and s_2 along the direction of propagation.

We can define now the *monochromatic optical depth* scale by means of the *integral* relation

$$\tau_{\nu,\boldsymbol{n}}\,(s) \equiv -\int_{S}^{s} \chi_\nu\,(s')\; ds' \quad . \tag{3}$$

The optical depth scale increases inwards from the point of intersection S of the direction of propagation $\boldsymbol{n}$ with the boundary surface of the star[5]. Customarily the optical depth at the surface is assumed

[3]A clear and exhaustive exposition of the problem of kinetic theory and its relation with radiative transfer can be found in Huang (1963).

[4]The problem of quantum exit from a medium and its application to radiative transfer are discussed in Ch. 6 of the seminal book by Sobolev (1963). See also Rybicki (1984).

[5]The surface of a star, as well as the border of its atmosphere, will be defined on physical grounds in Section 5.

to be null. The optical depth $\tau_{\nu,\boldsymbol{n}}\,(s)$ is equal to the optical thickness of the path between s and S along $\boldsymbol{n}$. The relation between the optical depth scale and the geometrical scale is given by Eq. (3).

3.4. Quantitative definition of the escape probability along a given direction

As already mentioned above, each photon at the point P inside the atmosphere has a definite probability of leaving the star in a given direction either directly or after a series of successive scatterings. In order to quantify this escape probability for an individual photon of frequency ν, let us consider the intensity $I_{\nu,\boldsymbol{n}}$ of a beam of similar photons that propagate along the *radial outwards direction* $\boldsymbol{n}$ and cross the surface Σ of unit area centred at P and perpendicular to $\mathbf{n}$. The coordinate of P along $\boldsymbol{n}$ is its radial distance r from the centre of the star.

According to the general law of attenuation, the intensity $I_{\nu,\boldsymbol{n}}$ at P is reduced by the amount

$$dI_{\nu,\boldsymbol{n}} = -\,\chi_\nu\,(r)\;I_{\nu,\boldsymbol{n}}\,(r)\;dr\;, \tag{4}$$

when travelling the differential geometrical path dr along $\boldsymbol{n}$. In terms of the optical depth Eq. (4) becomes

$$dI_{\nu,\boldsymbol{n}} = I_{\nu,\boldsymbol{n}}\,(\tau_{\nu,\boldsymbol{n}})\;d\tau_{\nu,\boldsymbol{n}} \quad . \tag{5}$$

By integration of Eq. (5) between P and the point P_S determined by the intersection of $\boldsymbol{n}$ with the boundary surface, it follows straightforwardly that

$$\frac{I_{P_S}}{I_P} = e^{-\,\tau_{\nu,\boldsymbol{n}}} \quad . \tag{6}$$

The ratio between the intensities I_{P_S} and I_P is the same as the ratio between the number of specific photons that cross the surface Σ and those that cross the corresponding surface Σ_S on the boundary. Then, by definition, this ratio is the probability of escape along $\boldsymbol{n}$ of a photon at optical depth $\tau_{\nu,\boldsymbol{n}}$, that is

$$P_{\boldsymbol{n}}\,(\tau_{\nu,\boldsymbol{n}}) = e^{-\,\tau_{\nu,\boldsymbol{n}}} \quad . \tag{7}$$

The direction $\boldsymbol{n}$ was chosen as the outwards radial direction through P. For all the other outwards directions $\boldsymbol{n}'$ originating at point P the escape probability along these directions will be smaller than $P_{\boldsymbol{n}}\,(\tau_{\nu,\boldsymbol{n}})$ because of the geometrical relation between the differential path dr' along $\boldsymbol{n}'$ and dr along $\boldsymbol{n}$, namely $dr' = dr/\cos\theta$, where θ is the angle formed by $\boldsymbol{n}$ and $\boldsymbol{n}'$. As it holds that $0 \le \theta < \pi/2$, the slant optical depth $d\tau'_{\nu,\boldsymbol{n}} = d\tau_{\nu,\boldsymbol{n}}/\cos\theta$ is always greater than $d\tau_{\nu,\boldsymbol{n}}$.

4. DEFINITION OF STELLAR ATMOSPHERE ACCORDING TO THE KINETICS OF PHOTONS

4.1. A statistical definition of stellar atmosphere

When photons of frequency ν reach a certain distance from the centre of the star, their escape probability becomes not negligible. As they further progress outwards, their escape probability approaches the unit value. The slab delimited by those layers for which the escape probability is 0.01 and 0.99 respectively (these figures are, of course, arbitrary) can be considered as the atmosphere for *these specific photons*, namely a *monochromatic atmosphere*.

However, the radiation emitted by stars consists of photons within a wide range of frequencies, whose spectral distribution varies greatly frequency by frequency because of the strong dependence on frequency of the opacity of the stellar material. Thus, the points where the escape probability is large and small respectively, namely the points that define the top and the bottom (r_ν^{t} and r_ν^{b}, respectively), can be very different for different frequencies. Therefore, we define the *bottom* of the atmosphere as the region where the escape probability of the photons, whose frequency corresponds to the minimum of χ_ν, is of the order of 0.01. Accordingly, we define the *top* of the atmosphere as the region where the escape probability of the photons with frequency corresponding to the maximum of χ_ν is of the order of 0.99.

The above definitions are *statistical* in character. They are based on the non-local kinetics of photons, namely their space evolution. In the innermost layers, far from the region from where the free escape of photons begins, the space evolution of the photon distribution function for all the frequencies can be described by means of the *diffusion approximation*, typical of non-equilibrium linear thermodynamics. In the outermost layers the density of matter is so low that the latter does not affect the photon distribution function. Consequently, its evolution can be described in terms of a *free-escape kinetics* for all the photons. Thus, a stellar atmosphere can be regarded as a *transition reg*ion between two media, characterized by different kinetic properties of the photons. In this sense the atmosphere constitutes a veritable *boundary layer* for the radiation field. It is precisely in this region where the photon distribution function acquires its qualitative feature, namely *the region where the stellar spectrum forms.*

4.2. Different radiative transfer regimes in the outer stellar layers

The opacity of the stellar material determines, frequency by frequency, the regime of radiative transfer. As shown in Section 3.3, the extinction coefficient χ_ν is defined by the product of the atomic property k_ν and the density $\rho\,(r)$ that varies with depth inside the atmosphere. At great depth, where

$\rho(r)$ is very large, the monochromatic photon mean-free-path is very small so that radiation is trapped and tends locally in a natural way towards thermodynamic equilibrium. The monochromatic flux of radiation is then proportional to the gradient of the temperature (i.e., linear non-equilibrium). This situation defines the *diffusion approximation* regime. Because of the exponential decay of the density and the consequent decrease of χ_ν, the monochromatic photon mean-free-path increases significantly outwards. Therefore, the thermodynamic state of the region where the photons are absorbed will be different from that of the region where they originated. That makes necessary to account for the propagation of the radiation field by means of the corresponding RT equation. The non-equilibrium state of the system establishes a *kinetic regime*. Eventually in the outermost layers, where the stellar material is extremely diluted, the monochromatic photon mean-free-path becomes much larger than the optical thickness measured from the boundary and photons can flow freely into the surrounding medium: *free-escape regime*. The characteristics of the three regimes are quantified in Table 1.

5. THE SURFACE OF A STAR

In order to give a quantitative definition of the *thickness* and *border* of a stellar atmosphere, as well as a physically meaningful definition of stellar radius, we shall make some simplifying hypotheses on structure of the atmosphere. In view of our purposes, we will introduce some simplifying hypotheses. Firstly under the assumption that they are auto gravitating masses of gas, stars and, in particular, their outermost layers have a spherical shape. Hence the hypothesis of spherical symmetry for stellar atmospheres, so that their properties depend only on the radial distance r. Moreover we will consider stellar atmospheres that are stationary in time and chemically homogeneous.

Table 1. The different regimes of monochromatic radiative transfer according to the monochromatic optical depth.

Monochromatic optical depth	Regime
$100 < \tau_\nu$	diffusion approximation
$100 > \tau_\nu > 10^{-3}$ $\tau_\nu \approx 1$	kinetic regime: RT equation effective radiative transfer
$10^{-3} > \tau_\nu$	free escape

5.1. Hydrostatic equilibrium and pressure scale height

The above hypothesis of stationarity leads to the conclusion that the gas must be, at least on the average, in hydrostatic equilibrium (HE). Stellar atmosphere models computed on the bases of all the foregoing hypotheses can be defined 'compact' for the reasons that will clearly appear from the results of the discussion that follows. Such models are fairly representative of the *photosphere*, the region of the atmosphere where the image of the stellar disc forms.

Stellar matter is submitted to gravity, whose acceleration $g(r)$ depends on the total mass inside the sphere of radius r. In principle, it is also submitted to the force due to gas and radiation pressure. However, in order to evaluate the scale height and the size of the atmosphere, we will neglect the latter. This first order of approximation is well justified but for special cases. The HE equation then reads:

$$\frac{dP(r)}{dr} = g(r)\ \rho(r) \quad . \tag{8}$$

As will be proved *a posteriori*, the value of the gravity acceleration $g(r)$ can be assumed constant through the whole atmosphere. Eq. (8) involves the gas pressure $P(r)$ and density $\rho(r)$ that are related via the equation of state

$$P(r) = \frac{k_{\mathrm{B}}}{m_{\mathrm{H}}\ \mu(r)}\ \rho(r)\ T(r) \quad , \tag{9}$$

where $\mu(r)$ is the mean mass of the particles in units of the mass of the hydrogen atom m_{H}, and k_{B} is the Boltzmann constant. Both $\mu(r)$ and $T(r)$ vary slowly inside a stellar atmosphere. The former can vary between 1.5 (when matter is in the neutral state) and 0.5 (when matter is fully ionized). The variation of the latter is in general of order of a factor three between the cooler photospheric layers and the hotter ones at the bottom of the atmosphere. On the contrary, the gradients of both $P(r)$ and $\rho(r)$ are much steeper, by a factor of about one hundred with respect to that of the temperature. Therefore, in the following we will assume a constant temperature, equal to the effective temperature T_{eff} of the star. Under the above assumptions, the equation of state, Eq. (9), becomes

$$P(r) = \frac{k_{\mathrm{B}}\ T_{\mathrm{eff}}}{m_{\mathrm{H}}\ \mu}\ \rho(r) \quad . \tag{10}$$

The joint solution of Eq. (8) and Eq. (10) yields an exponential decay for both $P(r)$ and $\rho(r)$, with a common scale height given by

$$H_P = \frac{k_{\mathrm{B}}\ T_{\mathrm{eff}}}{g\ m_{\mathrm{H}}\ \mu} \quad . \tag{11}$$

The pressure scale H_P will play a protagonist role in the following discussion on the thickness and border of a stellar atmosphere. It must be stressed that the main features of the structure of an atmosphere in HE depend on the parameter H_P. From its definition, given by Eq. (11), it appears clearly that it depends on two parameters $T_{\rm eff}$ and g. The former is related to the total (*bolometric*) luminosity L of the star. The latter includes the mass M and the radius R of the star.[6] Thus, the three fundamental stellar parameters M, R and L, that characterize unequivocally a star, govern the structure of its atmosphere through the pair of parameters $T_{\rm eff}$ and g.

For hot stars on the main sequence the value of g (in $\rm cm \cdot s^{-1}$) ranges from 10^4 to $1.5 \cdot 10^4$ for decreasing $T_{\rm eff}$; for cool main sequence stars between $1.5 \cdot 10^4$ and $3.5 \cdot 10^4$ as $T_{\rm eff}$ decreases from 10000 K to 3000 K. In the case of giant stars, g takes on values between $2 \cdot 10^3$ and $2 \cdot 10^2$; for hot supergiants between $2 \cdot 10^3$ and $1 \cdot 10^2$; for cool supergiants between 20 and 1. Consequently, the value of $H_P \,/\, R$, namely the ratio of the pressure scale height to the stellar radius, broadly varies between 10^{-4} (main sequence stars) and 10^{-3} (giants and cool supergiants).

Of course the above results hold valid for stationary spherically symmetric stellar atmospheres in hydrostatic equilibrium. The assumption of HE is however justified because it is suggested by the hypothesis of stationarity. The latter is supported by the evidence that the time scales of stellar evolution are much larger than those characteristic of the time derivatives of the equation of structure of a stellar atmosphere.

5.2. Thickness of a stellar atmosphere

Let us compute the monochromatic optical depth $\tau_\nu\,(r)$ along the outward radial direction according to Eq. (1), taking into account the assumption of an outwards exponential decay of density, that is

$$\rho\,(r) = \rho_0 \; e^{\,-\,r\,/\,H_P} \quad . \tag{12}$$

Then Eq. (1) takes on the form

$$d\tau_\nu = -\,k_\nu\;\rho_0\;e^{\,-\,r\,/\,H_P}\;dr \quad , \tag{13}$$

hence

$$\tau_\nu = -\,k_\nu\;\rho_0 \int\limits_\infty^r e^{-\,r'\,/\,H_P}\;dr' = H_P\;\chi_\nu \quad . \tag{14}$$

In Section 4.1 we have defined the top of a monochromatic atmosphere for frequency ν as the layer of radius $r_\nu^{\rm t}$, where the corresponding escape probability is equal to 0.99, so that $\tau_\nu\,(r_\nu^{\rm t}) \;\simeq\; 0.01$. Similarly at the bottom, defined as the point of radius $r_\nu^{\rm b}$ where the escape probability is 0.01, it holds that $\tau_\nu\,\left(r_\nu^{\rm b}\right) \;\simeq\; 4.6$. Consequently

$$\frac{\tau_\nu\,\left(r_\nu^{\rm b}\right)}{\tau_\nu\,\left(r_\nu^{\rm t}\right)} = \frac{4.6}{0.01} = \frac{e^{\,-\,r_\nu^{\rm b}\,/\,H_P}}{e^{\,-\,r_\nu^{\rm t}\,/\,H_P}} \quad , \tag{15}$$

hence

$$r_\nu^{\rm t} \;-\; r_\nu^{\rm b} \quad\approx\quad 6\;H_P \quad . \tag{16}$$

Eq. (16) shows that the geometrical thickness of any *monochromatic* atmosphere, namely the region of kinetic regime for the corresponding monochromatic RT equation, is the same for all frequencies. The qualitative justification is that for optically thin frequencies the spectrum forms in deeper layers where k_ν is multiplied by higher values of $\rho\,(r)$; for optically thick frequencies, whose spectrum forms in shallower layers, the higher value of k_ν is compensated by smaller values of $\rho\,(r)$.

We can now reformulate our previous definition of the top of a stellar atmosphere as the layer where $\tau_{\nu_{\rm M}}\,(r_\nu^{\rm t}) \;\simeq\; 0.01$ for the photons whose opacity is the highest, and the bottom as the layer where $\tau_{\nu_{\rm m}}\,\left(r_\nu^{\rm b}\right) \;\simeq\; 4.6$ for the most transparent radiative transitions. Accordingly, it can be shown that the *geometrical thickness* of the *stellar atmosphere* is

$$r^{\rm t} \;-\; r^{\rm b} \simeq \left(\,6 \;+\; \ln\,\frac{\chi_{\nu_{\rm M}}}{\chi_{\nu_{\rm m}}}\,\right)\;H_P \quad , \tag{17}$$

where $\chi_{\nu_{\rm M}}$ and $\chi_{\nu_{\rm m}}$ are the opacities at the most and least opaque frequency, respectively. By taking into account the typical values of the opacity, the quantity ln $\;(\chi_{\nu_M}\;/\;\chi_{\nu_m})$ is never larger than, say, 10 or 15. Thus, as the geometrical value of $H_P\;/R$ is much less than unity (between 10^{-4} and 10^{-3} as said above), the thickness of a stellar atmosphere is never larger than 1 – 2 % of the stellar radius. These limiting upper values correspond to cool supergiants.

The above result holds true for stars whose outer layers can be described by the 'compact' stellar atmosphere model defined above. On the other hand, when the introduced simplifying hypotheses (i.e., a stationary, spherical symmetric atmosphere in hydrostatic equilibrium and negligible radiation pressure) break down, a very low density extended layer in expansion shows up (e.g., solar and stellar coronae, circumstellar envelopes, stellar winds).

[6] Although the radius is one of the fundamental stellar parameters, its definition is not straightforward. From the observational standpoint, the angular diameter of the stellar disc can be measured in some cases by means of photometric or interferometric techniques. These measures are however difficult. Moreover, the size of the disc observed varies with the wavelength of the radiation measured. On physical grounds the stellar radius is closely related to the star's total luminosity via the Stefan-Boltzmann law. In Section 5.3 we will show that it is meaningful to speak of the radius of a star; in Section 5.4 we will introduce the definition of spectroscopic radius.

5.3. Border of the atmosphere

We have defined frequency by frequency the surface of a *monochromatic* atmosphere in terms of the corresponding optical depth τ_ν. From Eq. (13) and Eq. (14) it follows that

$$\frac{H_P}{R}\,\frac{|d\,\tau_\nu|}{\tau_\nu} = \frac{d\,r}{R} \quad . \tag{18}$$

This result shows that *for any frequency* the variation $\Delta\tau_\nu$ at the edge, although large enough to produce sensible spectroscopic effects, does not bring about any appreciable variation of r because $H_P\,/R \ll 1$, as shown above. Hence, we can infer that the border of a stellar atmosphere, as defined in Section 5.2 in terms of $\tau_{\nu_{\rm M}}$, is *sharp* and can define the *optical surface* of the star, i.e. its photosphere.

5.4. The spectroscopic radius

As we have been able to define the sharp surface of any monochromatic atmosphere in terms of corresponding optical depth τ_ν, we can now define in a natural way the spectroscopic radius of such surfaces. By turning inside out the relation between τ_ν and r, we are in position to define a *spectroscopic radius* as

$$R_\nu \equiv r\,(\hat{\tau}_\nu) \quad , \tag{19}$$

namely as the radial distance corresponding to an arbitrarily chosen optical depth $\hat{\tau}_\nu$. Such a definition is however loose and of little practical use, because it defines different spectroscopic radii for different frequencies. Moreover, the choice of $\hat{\tau}_\nu$ must be done on sound grounds. Let us go further towards a suitable definition of *stellar spectroscopic radius.*

Preliminary, it is convenient to introduce a standard reference optical depth scale $\tau_{\rm sd}$ to be used as the natural depth variable for radiative transfer problems and stellar atmosphere modelling. The first step is to define proper mean opacities as the weighted average of the monochromatic opacity. According to the specific RT problem considered, several expressions for the mean opacity have been introduced (e.g., Rosseland, Planck, absorption, flux-weighted).[7] Then, we can construct the scale $\tau_{\rm sd}$ corresponding to the mean opacity chosen.

By using simplified stellar atmosphere models that admit analytical solutions, it can be shown that the distribution with depth of the temperature inside an atmosphere takes on the value equal to $T_{\rm eff}$ at the standard optical depth of the order of unity (e.g., in the Eddington's approximation at an optical depth of 2/3). In more general terms, the emerging intensity reflects the physical state of the atmosphere at about the unit standard optical depth on the line of sight. This suggests to place the *optical surface* of a star at the geometrical depth where $\tau_{\rm st} \approx 1$. At these optical depths the escape probability is of the order of 1/2; more precisely $P\,(\tau_{\rm sd} = 1)$ is equal to 0.4 and $P\,(\tau_{\rm sd} = 2/3)$ is equal to 0.5. Then, it is meaningful to define the *stellar spectroscopic radius* as

$$R_1 \equiv r\,(\tau_{\rm sd} = 1) \quad , \tag{20}$$

namely the radius of the shell at the unit standard optical depth corresponding to the visible photospheric disc.

6. THE SPECIFIC INTENSITY OF THE RADIATION FIELD AND THE RT COEFFICIENTS

Here we will only touch upon the essential phenomenology of the transport of radiant energy. The RT equation accounts for the propagation of a suitable quantity, representative of the radiation field, through a medium. The fundamental physical observable of this process is the energy carried on along a given direction (*radiation beam* or *ray*). An operational definition of this quantity will allow us to introduce the specific intensity of the radiation field, a *macroscopic* quantity suitable for mathematical treatment of radiative transfer, as well as the *macroscopic* coefficients that characterize any specific RT equation.

6.1. Operational definition of specific intensity

For the mathematical treatment of radiative transfer, it is convenient to adopt the Eulerian standpoint, namely to fix our attention on a definite position in space at a given time rather than on a definite ray. We shall therefore introduce a *local* and *directional* variable, starting from the fundamental macroscopic quantity we can actually measure: *the amount of energy carried by the rays*[8].

From the physical point of view, however, it is meaningless to speak of a finite amount of energy localized at a geometrical point or propagating in a geometrical direction (as well as of a pure monochromatic pencil of radiation). Nevertheless such a variable can be defined by means of ideal experiments based, however, on real experiments on the radiant flux, i.e. radiant energy in motion. The definition is given as follows. We measure the amount of radiant energy $\delta E_\nu\,(\boldsymbol{n})$ that flows during a time interval δt through an orientated surface $\boldsymbol{k}\delta S$ around a point P defined by the position vector $\boldsymbol{r}$, inside a solid angle $\delta\Omega$ around the direction of propagation $\boldsymbol{n}$ with frequency in the range(ν, $\nu + \delta\nu$). The experimental laws of radiometry show that the extensive quantity $\delta E_\nu\,(\boldsymbol{n})$ is proportional to each single element in the process of measurement, that is

$$\delta E_\nu\,(\boldsymbol{n}) \propto \boldsymbol{k}\,\cdot\,\boldsymbol{n}\;\delta S\;\delta\Omega\;\delta\nu\;\delta t \quad . \tag{21}$$

[7] See, for instance, Mihalas (1978).

[8] The background of this approach is masterly treated in the book by Planck (1906).

Experience shows also that the limit

$$(\boldsymbol{n} \cdot \boldsymbol{k})^{-1} \lim_{\delta S\, \delta\Omega\, \delta\nu\, \delta t \,\to\, 0} \frac{\delta E_\nu (n)}{\delta S\, \delta\Omega\, \delta\nu\, \delta t} \equiv I(\boldsymbol{r}, t; \boldsymbol{n}, \nu) \tag{22}$$

exists and takes a finite value. This coefficient of proportionality between the measured value of the physical magnitude and the product of the values of the geometrical, spectral and time elements of the process of measurement is by definition the *specific intensity o*f the radiation field. The operational character of this introduction of the concept of specific intensity is exhaustively discussed in Preisendorfer (1965) .

6.2. The macroscopic RT coefficients

Without entering into the details, which can be found in any textbook on stellar atmospheres, we will merely recall that the source and sink terms of the radiative transfer process are determined by the microscopic properties of the interaction between matter and the radiation field, as well as by the equation of state for the stellar material. Here we only wish to stress that the above physics can be incorporated into the RT equation by means of a limited number of macroscopic coefficients that account for true absorption, scattering and emission along any beam of radiation. Analogously to the previous definition of the specific intensity, these coefficients can also be defined operationally as the ratio between an extensive quantity, that is the amount of radiant energy added to or subtracted from the radiation beam, and the parameters of the corresponding process.

For further use in Sections 7 and 8 we shall recall here the definition of the source function as the ratio of the total emission coefficient η_ν^{tot} (thermal emission plus diffusion of the incident photons) to the extinction coefficient $\chi_\nu \equiv a_\nu + \sigma_\nu$, the sum of true absorption and scattering. For most cases of interest, like in the example considered in Section 7.2, the source function can be cast into the form

$$S_\nu = \frac{a_\nu}{a_\nu + \sigma_\nu} B_\nu (T) + \frac{\sigma_\nu}{a_\nu + \sigma_\nu} J_\nu \quad . \tag{23}$$

Such a source function is a weighted average of the Planck function at the local temperature T and the mean intensity of the radiation field J_ν.

7. THE STELLAR ATMOSPHERE PHYSICAL SYSTEM

7.1 The physics that shapes the structure

In the following we will give an outline of the physical information that we have at hand to model the *stellar atmosphere physical system.* First of all, the fundamental observational facts suggest to consider a model consisting of two main components: a *material medium*, constituted by atoms, ions, free electrons and sometimes molecules, and *a radiation field* that permeates the medium and interacts with it. According to convenience, both components will be described either on a macroscopic scale by means of thermodynamic variables, or on a microscopic scale by employing suitable distribution functions of the kinetic formulation.

With regard to the material component, the physical conditions inside a stellar atmosphere are such that the temperature is so high and the gas pressure so low that matter particles can be considered as localized wave-packets, whose dimensions are much smaller than the average inter-particles separation; in other words, they can be treated as classical particles with defined position and momentum, to which there correspond *external* degrees of freedom. But they will also have an *internal structure*, according to quantum physics, with the corresponding *internal* degrees of freedom. For a description of the radiation field consistent with the above model of the material component, we adopt the corpuscular picture: the radiative energy is identified with a flux of photons moving along rays obeying the laws of geometrical optics, which carry on and can exchange with the matter particles their own indivisible amount of energy $\varepsilon = h\nu$. It must be stressed that the complete description of the radiation field would in principle require a continuous range of directions that cover the full solid angle 4π; that is to say, an infinite number of rays. In practice, the choice of a proper numerical quadrature formula makes it possible to reduce the virtually infinite set of directions to a finite one.

The physical state of a stellar atmosphere is governed by the interaction between the above two components; ultimately by the continuous interchange of energy between matter and radiation field. The structure of the system will be described in terms of values at each point of the fundamental variables required to account for the degrees of freedom, both external and internal, of the material component. These values are determined by: (i) the relations among the variables; (ii) the constraints imposed by external conditions; (iii) the global value of the internal energy of the system. We shall be able to express the above conditions by means of an operational formulation of our knowledge of the phenomenological picture, deduced from observations and translated into a series of suitable hypotheses. The evidence that stellar atmospheres are in a steady state over long time intervals implies both the equilibrium among the internal and external forces acting on the matter particles, which results in the condition of *global mechanical equilibrium*, and the *conservation of their total energy*, namely the sum of the internal energy of matter and the energy of the radiation field. Physical information on further conditions of equilibrium (e.g., the hypothesis of local thermodynamic equilibrium) can allow us to derive an *equation of state* for the matter, an equation that couples the mechanical equilibrium with the energy balance. However, we must take into account the fundamental fact, implicit in the definition itself of a stellar atmosphere, of presence of the outward flux of radiative energy. This is the clear-cut evidence that the radiation field cannot be assumed to be in equilibrium: its anisotropy calls for the transport of photons, which is a *non-local process.* The set formed by the laws of conservation and the equation of state, which involves local properties and intensive variables, is not evidently sufficient for a self-consistent mathematical treatment of the problem. Transport processes, *at least radiative transfer,* have to be taken into account.

Transport processes imply the presence of gradients, which are, in turn, the evidence of departures from conditions of global equilibrium of the system. However, if the gradients of the thermodynamic variables are such that the corresponding scale heights are greater than the mean-free-paths of both photons and matter particles, we can assume that the system is locally in thermodynamic equilibrium. That is, we can dissect the stellar atmosphere into 'elemental volumes' of size such that we can meaningfully assign to each of them a unique value of the temperature and the other thermodynamic variables. When we take the successive step from the continuous to the discrete model, our choice of the spatial grid shall be of course consistent with the size of such elemental volumes, as imposed by the physical conditions inside the atmosphere.

7.2. The fundamental equations of the stellar atmosphere theory

In order to achieve a *quantitative* description of the stellar atmosphere physical system, it will be necessary to specify the fundamental variables and the links among them. The required model will then be obtained from the solution of the resulting equations, together with the relevant initial and boundary conditions.

The laws of mass and momentum conservation yield the first two equations (*constitutive equations*): the scalar equation of continuity and the vector equation of motion, which link the density ρ, pressure P and fluid velocity $\boldsymbol{v}$. This is a system of four scalar equations in five unknowns; hence we must seek a new equation in order to close the system. We may resort, whenever it is possible, to an equation of state, but in that way we involve a new unknown, the temperature T. At this point, we have at our disposal the constraint of the conservation of energy, from which we can derive the energy balance equation for matter and the radiation field. Here the temperature plays the major role from the physical point of view. But the bolometric (i.e., frequency-integrated) equation that governs the conservation of the radiant energy, necessarily involves the specific intensity of the radiation field, together with two other quantities derived from the first two moments with respect to direction of the latter, namely the energy density and the radiation flux. That is to say, we must take into account an adequate treatment of the radiation field, through the set of RT equations that yields the values of the specific intensity $I\left(\boldsymbol{r},\, t;\, \boldsymbol{n},\, \nu\right)$ for all directions $\boldsymbol{n}$ and frequencies ν of the mesh of discrete ordinates prescribed by a proper representation of the physical system. Only in such a way we can close the system of fundamental equations.

As an illustrative example, let us consider the case study of a stellar atmosphere, where: (i) the material behaves as a perfect gas endowed with an equation of state; (ii) macroscopic flows of matter are absent (*hydrostatic equilibrium*) so that the radiative transfer is the only mode of energy transport; and (iii) the radiation pressure is negligible. In spite of these crude simplifications, the case considered still retains the fundamental physics of the stellar atmosphere problem and is fully representative of relevant mathematical difficulties.

Under the foregoing hypotheses, the set of constitutive equations reduces to the hydrostatic equilibrium (HE) equation

$$\nabla\, P\,(r) = \rho\,(r)\ \boldsymbol{g} \quad ; \tag{24}$$

where $\boldsymbol{g}$ is the constant gravitational acceleration. If the hypothesis of Local Thermodynamic Equilibrium (LTE) can be assumed, we can write the equation of state (9) hold valid. The energy balance is prescribed by the constraint

$$\int_o^\infty d\nu\ \oint d\boldsymbol{n}\ \left[\,\eta^{\mathrm{tot}}\ \ (r,\nu)\ -\ \chi\,(r,\nu)\ \ I\,(r;\boldsymbol{n},\nu)\right] = 0\,, \tag{25}$$

which states the *radiative equilibrium* (RE) condition. The RT equations, one for each pair of parameters $(\boldsymbol{n},\nu)$, are

$$\boldsymbol{n}\cdot\nabla I\,(r;\boldsymbol{n},\nu) = -\chi\,(r,\nu)\,I\,(r;\boldsymbol{n},\nu) + \eta^{\mathrm{tot}}\,(r,\nu)\ J_\nu\,. \tag{26}$$

The transport coefficients

$$\eta^{tot}\ \ (r,\nu) = a\,(r;\nu)\ \ B_\nu\,[T\,(r)] + \sigma\,(r;\nu)\ J_\nu \tag{27}$$

and

$$\chi\,(r,\nu) = a\,(r;\nu) + \sigma\,(r;\nu) \tag{28}$$

are assumed here to be isotropic.

The system of Eqs. (24) through (26), together with those that define the microscopic state of matter through the formulae for the coefficients μ, η^{tot} and χ may be considered as a model of the physical system under study in terms of continuous mathematics. They are the formal solution of the stellar atmosphere problem. The intrinsic difficulty of its actual solution arises from the structure of the above equations, as shown from the following issues.

(1) The equation of state links the fluid dynamic variables ρ and P with the temperature T and couples the mechanical state of the atmosphere with the energy balance.
(2) The coefficients μ, η^{tot} and χ are functions of the thermodynamic state only in (LTE); otherwise they depend also on the radiation field.
(3) The transport coefficient expresses the dependence, both *explicit* and *implicit*, of the energy balance on the solution of the RT equations.

(4) All RT equations are entangled through the common total emission term η^{tot}.

The coupling of all equations of the system implies that the problem is *non-linear*. Moreover, the fact that the set of specific intensities $\{I\,(r;\,\boldsymbol{n},\,\nu)\}$ depends on transport process makes the problem to be also *non-local*.

8. THREE DIFFERENT REPRESENTATIONS OF A STELLAR ATMOSPHERE

8.1. The equations of the continuous model

Let us get back to the fundamental equations of the simplified stellar atmosphere model in the previous section. We will rewrite them in terms of the monochromatic optical depth τ_ν. By replacing the RT Eq. (26) for the specific intensity $I\,(r;\,\boldsymbol{n},\,\nu)$ with the system formed by the two first moments of the latter with respect to the direction $\boldsymbol{n}$, closed by the relation given by the variable Eddington factor defined as $f_\nu \;\equiv\; K_\nu \;/\; J_\nu$ where J_ν and K_ν are the first and third moment of $I\,(r;\,\boldsymbol{n},\,\nu)$ with respect to $\boldsymbol{n}$, the second order differential equation

$$\frac{\partial^2\;(f_\nu\;J_\nu)}{\partial\tau_\nu^2} = J_\nu - S_\nu \tag{29}$$

is obtained. As the transport coefficients are assumed to be isotropic, it follows straightforwardly from Eq. (25) that

$$\int_0^\infty a_\nu\;\left[J_\nu\;-\;B_\nu\,(T)\right]\;d\nu = 0 \quad , \tag{30}$$

when Eq. (27) and Eq. (28) are taken into account.

The set of Eqs. (9), (24), (29) and (30), grouped in Table 2, may be seen as a representation, in terms of continuous mathematics, of a stellar atmosphere. Their solution, that of course requires the relevant initial and boundary conditions of Eq. (9) and Eq. (29), constitutes a *continuous model* of the physical system under consideration.

Table 2. The four fundamental equations for the simplified stellar atmosphere model under consideration: (i) hydrostatic equilibrium; (ii) equation of state; (iii) RT equations (one for each frequency); (iv) radiative equilibrium constraint.

$$\begin{array}{ll}
\text{(i)} & \dfrac{d\,P}{d\,\tau} = \dfrac{g}{\langle\chi\rangle} \quad ; \\[2ex]
\text{(ii)} & P = N\;k\;T \quad ; \\[1ex]
\text{(iii)} & \dfrac{\partial^{\,2}\;(f_\nu\;J_\nu)}{{\tau_\nu}^{\,2}} = J_\nu\;-\;S_\nu \quad ; \\[2ex]
\text{(iv)} & \displaystyle\int_0^\infty a_\nu\;\left[J_\nu\;-\;B_\nu\,(T)\right]\;d\,\nu = 0 \quad .
\end{array}$$

8.2. The matrix of the complete linearization method

Although we have greatly simplified the physics that governs our idealized stellar atmosphere, the exact mathematical solution of the system of equations in Table 2 is nevertheless unfeasible; therefore a numerical solution by means of a discrete ordinate method is necessary. However, as we are tackling a markedly non-linear problem, the resulting system of discretized equations cannot be linear. In principle, it would be possible to eliminate this drawback through a proper linearization technique. One may write the system of the RT equations plus the constraints of hydrostatic and radiative equilibrium in terms of a linear expansion of the discretized fundamental variables $J_{d,n}$, N_d and T_d around the current solution. The subscripts d and n denote, respectively, the depth and frequency points of the discrete ordinate mesh, of dimension $D\;\times\;N$. The solution is achieved iteratively, starting from an initial guess of the values of the fundamental variables.

The original system of equations can be rearranged into a new one, whose unknowns are the corrections $\delta\;J_{d,n}\;=\;J_{d,n} - J^{\,(0)}_{\,d,n}$, $\delta\;N_d\;=\;N_d - N^{\,(0)}_{\,d}$ and $\delta\;T_d\;=\;T_d\;-\;T^{\,(0)}_{\,d}$ that make the up-dated values consistent with the constraints. The new system can be represented by the matrix equation

$$\begin{pmatrix}
T_1 & 0 & \dots & 0 & U_1 & V_1 \\
0 & T_2 & & & U_2 & V_2 \\
. & & & & . & . \\
. & & & & . & . \\
. & & & 0 & . & . \\
0 & & \dots & T_N & U_N & V_N \\
W_1 & W_2 & \dots & W_N & A & B \\
X_1 & X_2 & \dots & X_N & C & D
\end{pmatrix}
\begin{pmatrix}
\delta\,J_1 \\ \delta\,J_2 \\ \\ \\ \\ \delta\,J_N \\ \delta\,N \\ \delta\,T
\end{pmatrix}
=
\begin{pmatrix}
K_1 \\ K_2 \\ \\ \\ \\ K_N \\ L \\ M
\end{pmatrix} . \tag{31}$$

The elements of the matrix that multiplies the vector formed by the unknown correction terms are matrices of dimension $D \times D$. The first N rows represent the transfer equations at each depth point, the penultimate row represents the constraint of radiative equilibrium and the last row represents the hydrostatic equilibrium equation. The 'error' vector on the right-hand-side is a known term computed by substituting at each step of iteration the current values of the radiation field, particle density and temperature in the discretized transfer and constraint equations[9].

Clearly, the nature and collocation of the matrix elements reflect the organization of the interactions among the fundamental variables. In particular, each of the tridiagonal sub-matrices $\mathbf{T}_i$, $\mathbf{U}_i$ and $\mathbf{V}_i$ can be easily identified with the set of the numerical coefficients that describe, depth by depth, the differential relations among the transfer variables. Thus, Eq. (31) constitutes a *discrete mathematical picture* that shows the plot of the mutual interactions among the different physical processes taking place in the stellar atmosphere.

8.3. Flowchart of the iterative sequential approach

In the foregoing complete linearization procedure the constitutive and the RT equations are solved simultaneously, under the implicit assumption that each variable interacts with all the others, and that all of them can be held on an equal footing. The mathematical translation of this principle is the matrix equation Eq. (31) whose solution implies the numerical inversion of a matrix of huge dimension (very often larger than 10^4). Hence a severe problem arises because such a brute force approach brings about not only a high risk of numerical instabilities but, above all, the possibility of achieving a solution that is spurious from the physical point of view.

Against such an 'equalitarian' treatment of the fundamental variables, it must be remarked that different processes taking place in a stellar atmosphere are characterized by distinct height scales, and the strength of the couplings among the different phenomena varies notably case by case. The natural alternative to complete linearization is a sequential procedure. Again, an iterative scheme will be necessary: an initial guess is made for the initial values of a variable chosen among the fundamental ones. The values of the others are then obtained by solving all the equations of the problem in succession, except for the RE equation. The latter is employed to check whether the current values of the variables satisfy the constraint. If not, the RE equation, taken as a transcendental equation for T, has to be used as the *temperature corrector*. The procedure is then iterated until the test is fulfilled. This iterative sequential approach will be the gist of Paper II.

Fig. 1 shows the flowchart of an iterative sequential procedure that can be designed for the solution of the specific stellar atmosphere problem under consideration. The algorithm is organized into macro-blocks. The first one, the *mechanical* macro-block, includes the constitutive equations and the equation of state, employed here to yield the description of the macroscopic state of matter. The second one is the *energy* macro-block, which contains the RT equations and the RE constraint. The equation of state is used once more to couple the former with the latter.

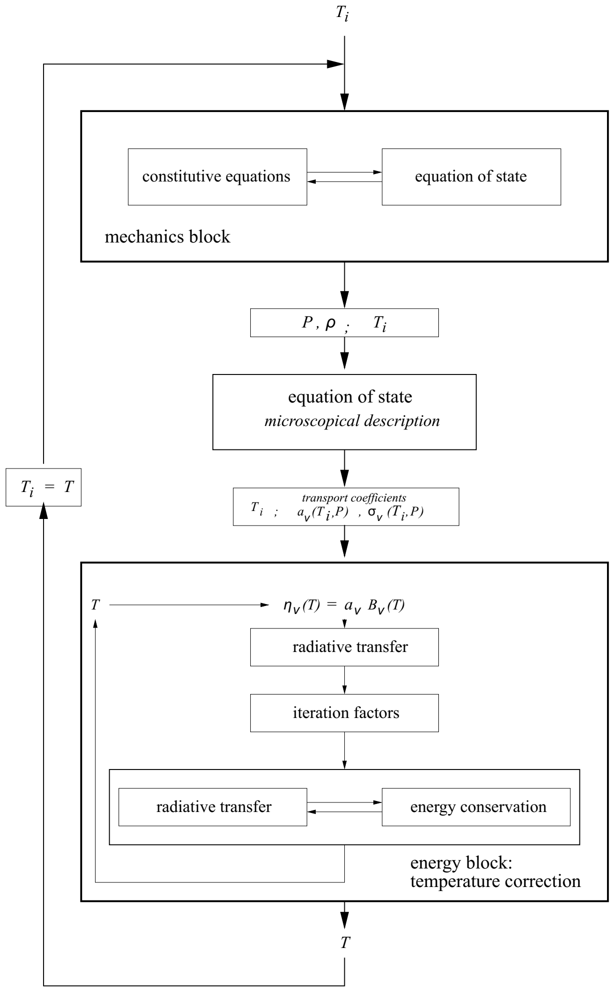

Fig. 1 *Flowchart of the iterative sequential procedure. The temperature is chosen as the protagonist variable of the process. Starting from an initial guess of the run of its values with depth, the other thermodynamic variables are obtained in the mechanical macro-block by the iterative solution of the constitutive equations coupled with the state equation. The macroscopic RT parameters are then computed for use in the energy macro-block where the solution of the RT equations under the constraint of energy conservation, obtained by a series of internal iterations, yields the up-to-dated values of the temperature for the next loop of the complete iteration.*

[9] For more details, see *A linearization method* in Section 7-2 of Mihalas (1978).

The analysis of the different nature and strength of the coupling between the constitutive equations and the equation of state on the one hand, and that between the RT equations and the energy conservation constraint on the other, will be duly studied in Paper II. Here we will merely note that the flowchart in Fig. 1 is an *icon* of the structure of the stellar atmosphere physical system. The flowchart can be straightforwardly translated into a numerical algorithm, which turns out to be *operative* because it faithfully reflects the plot of interactions among the physical processes.

9. CONCLUSIONS

Table 2, the matrix equation Eq. (31) and Fig. 1 are three different and alternative pictures of the stellar atmosphere physical system. They correspond, respectively, to a *continuous model*, a *discrete model* and an *algorithmic representation*. Although their languages are different, the physical content is of course the same. The reach of these representations, in view of the actual computation of a stellar atmosphere model, is however not equal. In practice, only the second and the third one can lead to an effective numerical solution; that is to a working tool for the diagnostics of stellar atmospheres.

Severe difficulties arise from the non-linear and non-local nature of the stellar atmosphere problem. The entanglement of different physical processes and constraints brings about the coupling among different parts of the whole system. These couplings reflect on links among the distinct components of the mathematical model and, ultimately, among the individual blocks that constitute the numerical algorithm. In order to classify the different kinds of interactions that take place among the distinct components, we may consider the range of action of the different processes considered. Short range interactions will individualize a sequence of regions, each governed by a local physics. On the other hand, long range interactions will imply the transfer of physical information among regions that are far apart by means of non-local transport processes. Quite arbitrarily, we may define the latter as a case of strong coupling and the former as a weak one. From the mathematical point of view, different degrees of strength of the physical interactions reflect into stronger or weaker ties among the fundamental variables of the model. These definitions will be sharpened in Paper II, where we will consider the stellar atmosphere problem as a paradigm of non-linear and non-local problems. We will show there, by means of an illustrative example, how it is possible to lessen the strong coupling between the RT equations and the constraint of energy conservation, by improving the exchange of information between the two parts of a simultaneous solution of the above equations.

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ЗВЕЗДАНА АТМОСФЕРА КАО ФИЗИЧКИ СИСТЕМ. I ФЕНОМЕНОЛОШКА ДЕФИНИЦИЈА И ОПИС ЗВЕЗДАНЕ АТМОСФЕРЕ

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Прегледни рад по позиву

Ово је први од два рада који се баве физичким и нумеричким основама моделовања звезданих атмосфера. Након увођења феноменолошких дефиниција звезде и звездане атмосфере, разматрана је физика која обликује звездану атмосферу као физички систем и дата су три алтернативна приступа њеном описивању.