Serb. Astron. J. № 193 (2016), 19 - 26 **UDC 521.16**
DOI: 10.2298/SAJ160613007Z Original scientific paper

INTERACTION BETWEEN YARKOVSKY FORCE AND MEAN-MOTION RESONANCES: SOME SPECIFIC PROPERTIES

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(Received: June 13, 2016; Accepted: August 16, 2016)

SUMMARY: Recently, we analyzed the role of mean-motion resonances in semi-major axis mobility of asteroids, and established a functional relationship that describes the dependence of the average time spent inside the resonance on the strength of this resonance and the semi-major axis drift speed. Here we extend this analyzis in two directions. First, we study the distribution of time delays inside the resonance and found that it could be described by the modified Laplace asymmetric distribution. Second, we analyze how the time spent inside the resonance depends on orbital eccentricity, and propose a relation that allows taking this parameter into account as well.

Key words. minor planets, asteroids – methods: numerical – methods: statistical

1. INTRODUCTION

Gravitational and non-gravitational phenomena influence main-belt asteroids. The most important gravitational mechanisms are orbital resonances. The most important non-gravitational effects are the Yarkovsky and the Yarkovsky-O'Keefe-Radzievskii-Paddack (YORP) forces.

The Main Belt is permeated with mean-motion resonances (MMRs) and secular resonances. The MMRs may cause either slow (e.g. Nesvorný and Morbidelli 1998, Novaković et al. 2010) or fast orbital changes (e.g. Morbidelli et al. 1995, Gladman et al. 1997). These changes depend on the time that an asteroid remains captured inside the resonance, but also on the magnitude of the Yarkovsky effect. In principle, larger asteroids spend longer time in the resonance, allowing a greater diffusion in eccentricity and inclination (Gallardo et al. 2011).

Yarkovsky effect is a radiation effect which acts mainly on the semi-major axes of objects between about 0.1 m and 10 km in the Main Belt (Rubincam 1995, Farinella et al. 1998, Vokrouhlický et al. 2015). Unlike gravitational perturbations, non-gravitational effects depend on a number of parameters, e.g. albedo, thermal characteristics, or rotation state. Thus, as these parameters are often not known, the models involving non-gravitational effects usually fit statistical parameters of a large sample of objects (e.g. Novaković 2010, Bottke et al. 2015).

Some specific issues which could be explained using the Yarkovsky effect are: the cosmic-ray exposure ages of stony and iron meteorites, which are much longer than the dynamical lifetimes of particles delivered from the asteroid belt (Farinella et al. 1998, Morbidelli and Gladman 1998); the overabundance of decameter-sized near-Earth objects (Rubincam 1995, 1998, Vokrouhlický and Farinella 1998); the dynamical evolution of main-belt asteroid fragments and their delivery to Mars and Earth-crossing orbits (Farinella and Vokrouhlický 1999).

For a detailed understanding of the Yarkovsky effect's role in the dynamical evolution of aster-

oids, different analyzes of the interaction among the Yarkovsky-drifting orbits and MMRs should be performed. Some of these analyzes have been conducted and results have been presented in numerous papers (see Vokrouhlický et al. 2015 and references therein).

A new line of research was presented recently by Milić Žitnik and Novaković (2016), who established functional relationship between the time spent inside the resonance, the strength of this resonance and the semi-major axis drift speed.

All these facts were our motivation for studying the effect of different MMRs with Jupiter on an asteroid's semi-major axis mobility due to the Yarkovsky induced drift. Here we present an extended analyzes, and new results on the interaction of Yarkovsky force and MMRs. In particular, we study distribution of time delays inside the resonance as well as their dependence on orbital eccentricity. This work is a natural continuation of the aforementioned research by Milić Zitnik and Novaković (2016).

2. METHODS

In this section we describe the methods used to investigate interplay between the MMRs and the Yarkovsky force. These methods were introduced in Milić Žitnik and Novaković (2015, 2016) and will only briefly be discussed here.

A set of numerical integrations of 66 000 test particles were performed in order to examine the semi-major axis drift delay inside the MMRs. For this purpose a public domain integrator, ORBIT9, was utilized (Milani and Nobili 1988). The orbital motion of test particles was tracked between 40 and 120 Myr, depending on the resonance's strength.

The Yarkovsky effect was included in all numerical simulations. The orbit of every test particle was propagated assuming ten different values of $\frac{\mathrm{da}}{\mathrm{dt}}$: from -4×10^{-5} to -2.0×10^{-3} AUMyr^{-1}.

Numerical integrations of the test particles were performed using two different dynamical models, depending on the heliocentric distance of the resonance. The dynamical model that includes four outer planets was used for resonances located more than 2.5 AU from the Sun, while for those located closer than 2.5 AU the dynamical model with seven planets, from Venus to Neptune, was used.

In this study we analyzed eleven isolated MMRs with Jupiter (the most massive planet in the solar system) whose strengths cover a wide range of magnitudes.

The particles were initially located as close as possible to the resonance but outside the resonance. The initial positions of test particles resembled a shape of a given resonance. To measure the time spent inside a resonance it was necessary to determine the moments of entering, t_1, and exiting, t_2, from the resonance. The numerical method used in calculation of these moments is described in Milić Zitnik and Novaković (2016). That is, if Δt and Δa are defined as $\Delta t = t_2 - t_1$, and $\Delta a = a_2 - a_1$, where a_1 and a_2 are semi-major axes at times t_1 and t_2, then the time interval dtr used in performed analyzes is defined as follows (Milić Žitnik and Novaković 2016):

$$dtr = \Delta t - \frac{\Delta a}{(\frac{\mathrm{da}}{\mathrm{dt}})}. \quad (1)$$

Finally, to analyze the distribution of dtr (Eq. 1) we used the asymmetric Laplace statistical distribution (Đorić et al. 2007). The Laplace asymmetric probability density function has two branches, left and right, respectively, defined as:

$$g(x) = \frac{(1-p)}{l} \times \exp\left(\frac{-|x-a|}{l}\right), \; x \leq a, \quad (2a)$$

$$g(x) = \frac{p}{l} \times \exp\left(\frac{-|x-a|}{l}\right), \quad x > a. \quad (2b)$$

where a, the parameter of location, is the only value of x for which $g(x)$ has the global maximum value, where $l > 0$ is the scaling parameter and $0 < p < 1$ is the shape parameter. Please see (Kotz et al. 2001) for a review of the used of the Laplace asymmetric distribution (Eqs. 2a and 2b).

3. RESULTS

In Section 3 we present the results we have obtained by analyzing the distribution of time delays inside the MMRs, and dependence of the time spent inside the resonances on orbital eccentricity. In these analyzis we used the data set of numerical integrations produced by Milić Žitnik and Novaković (2016).

3.1. Distribution of dtr for test objects

One of our most important results is on the time the asteroids spend in MMRs. Test objects spent longer time periods in stronger resonances with smaller Yarkovsky drift than in weaker resonances with greater Yarkovsky drift (Milić Žitnik and Novaković 2016). Weaker (narrower) resonances keep objects inside for less time than stronger ones. So, to study results of interaction between MMRs and secular drift in the semi-major axis, we analyzed the distribution of dtr times.

To this purpose, we produced histograms showing the distributions of dtr in Fig. 1. There are all histograms for the strongest resonance, the 9:4, as a representative example. Distribution of dtr is very similar for all resonances. The histograms suggest that the distribution of dtr is always asymmetric, skewed sometimes more to the left, sometimes more to the right. In order to confirm this characteristic, we calculated the third and the fourth standardized moments, i.e. the skewness γ_1 and the kurtosis γ_2 (see Carruba et al. (2012) for similar application). For almost all values of the Yarkovsky the drift speed and for all MMRs, objects have positive skewness value γ_1 so the distribution of dtr has a tail on the right side which is longer than that on the left side (Table 1). Most of the objects have high and positive kurtosis so that the distribution of dtr has a sharp peak and long fat tails (Table 1).

Table 1. Values of the third and the fourth standardized moments: γ_1 (skewness) and γ_2 (kurtosis) for 11 MMRs and for 10 values of the Yarkovsky drift speed.

9:4	8:3	13:6	15:7	11:4	17:8	10:3	16:7	17:7	18:7	17:6
					γ_1					
0.18	0.49	-0.93	0.55	-8.43	0.57	1.70	1.72	2.60	0.67	4.79
1.95	1.96	3.24	2.17	3.84	2.10	2.90	-2.24	8.14	0.42	8.71
1.71	2.01	3.07	1.72	4.84	2.46	0.59	-2.79	0.74	0.33	1.82
3.39	2.39	2.53	1.70	4.11	3.74	0.75	8.08	0.42	0.49	-2.31
1.52	1.84	2.41	3.47	4.62	13.03	0.39	8.60	0.04	-0.21	-2.42
1.87	2.04	2.83	5.31	4.60	2.24	0.39	2.82	0.23	1.26	-2.82
2.06	2.14	2.55	2.00	6.12	11.58	0.68	0.79	0.30	-1.34	-2.67
2.16	1.86	3.22	1.46	7.59	2.09	0.51	1.15	0.23	0.67	-2.18
1.79	2.09	2.59	2.42	9.16	1.40	0.20	0.69	0.19	0.39	-1.77
1.94	2.17	3.97	2.69	7.70	13.07	0.19	0.43	0.34	-0.07	-1.82
					γ_2					
3.66	3.02	4.51	12.73	135.95	3.97	12.07	11.21	11.56	7.44	29.10
8.76	7.50	17.70	13.56	25.95	23.35	27.58	24.60	98.94	4.30	104.08
6.80	9.06	19.28	9.36	35.01	22.84	7.75	13.58	2.97	4.29	16.04
24.17	11.67	14.89	10.81	27.09	42.11	10.08	114.28	2.23	6.52	12.79
5.65	7.37	12.46	35.92	32.75	240.41	8.77	119.21	3.10	28.38	13.14
7.08	8.73	15.08	49.97	30.69	16.58	6.96	18.66	2.58	13.81	13.63
9.06	9.09	12.66	11.94	50.10	193.13	7.49	7.08	3.35	8.96	11.96
10.04	7.50	17.71	11.23	79.71	14.37	9.74	5.55	3.36	9.26	11.08
7.60	9.27	15.57	22.86	110.56	6.95	6.76	4.65	3.57	5.51	8.29
8.99	10.71	28.89	21.96	92.31	247.71	6.74	3.71	4.07	4.57	8.62

Most of the objects crossed the resonance when $dtr \rightarrow \pm 0$. Moreover, histograms of dtr reveal that the dispersion of dtr is smaller for faster Yarkovsky drift speeds and larger for slower Yarkovsky drift speeds. Also, dtr is larger for stronger resonances and smaller for weaker resonances. In histograms it can be seen that the objects spent the longest time period in the strongest resonance with the smallest Yarkovsky drift speed, and the shortest time period in the weakest resonance with the largest Yarkovsky drift speed. That was shown in Fig. 1 for our strongest resonance. This rule is valid for all 11 MMRs and all tested Yarkovsky drift speeds.

To see whether two samples have the same distribution, we took into account 34 histograms of distribution of dtr ($\approx 31\%$ of samples of our histograms) from different resonances, and compared 26 pairs of the histograms. The compared histograms have the same partition on the x-axis (dtr moments) in the same MMR, but different values for the Yarkovsky drift speed. We applied the Kolmogorov-Smirnov test for two samples to test the null hypothesis H0: $P_0 = P_1$, with two values for the level of significance α=0.05 and α=0.01 (see Carruba et al. (2013) for similar application). In the case of α=0.05 we found that the null hypothesis H0 is accepted for 20 pairs of the histograms, while for α=0.01 H0 is accepted for 21 pairs. These results suggest that the data shown in the histograms belong to the same (or very similar) distributions.

3.1.1. Laplace asymmetric distribution

These histograms motivated a further analysis of distribution of dtr for the test objects. From the presented histograms it is visible that this distribution of objects has asymmetric exponential character on both sides with respect to the maximum. After many try-outs whose statistical distribution represents data the best, we found a few potentially good candidates. For example, Maxwell's and Cauchy's four-parameter distribution were almost good enough for the four strongest resonances (not for all values of the Yarkovsky drift speed), but not good for others (the errors of parameters of distributions were too high). As the best solution we adopted the asymmetric Laplace statistical distribution for all resonances and for all Yarkovsky drift speeds.

The Laplace asymmetric probability density function has two quite distinguishable branches, and was not therefore fully appropriate to describe the distribution of dtr in its original form. Because of that, it was necessary to divide the Laplace density function into two parts with respect to the density function maximum. Instead of two parameters (p, l), four parameters were taken into account, for the left (p_l, l_l) and the right (p_r, l_r) branch, respectively:

$$g(x) = \frac{(1-p_l)}{l_l} \times \exp\left(\frac{-|x-a|}{l_l}\right), \; x \leq a, \quad \text{(3a)}$$

$$g(x) = \frac{p_r}{l_r} \times \exp\left(\frac{-|x-a|}{l_r}\right), \quad x > a. \quad \text{(3b)}$$

This allowed for the best approximation of distribution of dtr with very small errors of (p_l, l_l, p_r, l_r) in almost all cases. Fig. 2 shows the modified Laplace asymmetric density function (Eqs. 3a and 3b) for the 9:4 resonance, as a representative example.

To confirm that the selected distribution is really appropriate for our data, we performed Pearson's chi square test with level of significance $\alpha = 0.05$. Our null hypothesis H_0 that is being tested is "Data have modified asymmetrical Laplace distribution (Eqs. 3a and 3b)". The obtained results show that this distribution is fully appropriate in about 80% of the cases. By analyzing the remaining 20%, we found that practically in all these cases test failed because of a very long tail of data caused by a large single values of dtr. As such values are subjects of significant uncertainties, we subtracted them from the data and repeated Pearson's test. After this modification, all our datasets passed the test. Thus, based on this, we concluded that the adopted modified Laplace asymmetric distribution is appropriate to our data.

The analysis of parameters of this modified Laplace asymmetric probability density function (Eqs. 3a and 3b) gave the following eight most important results.

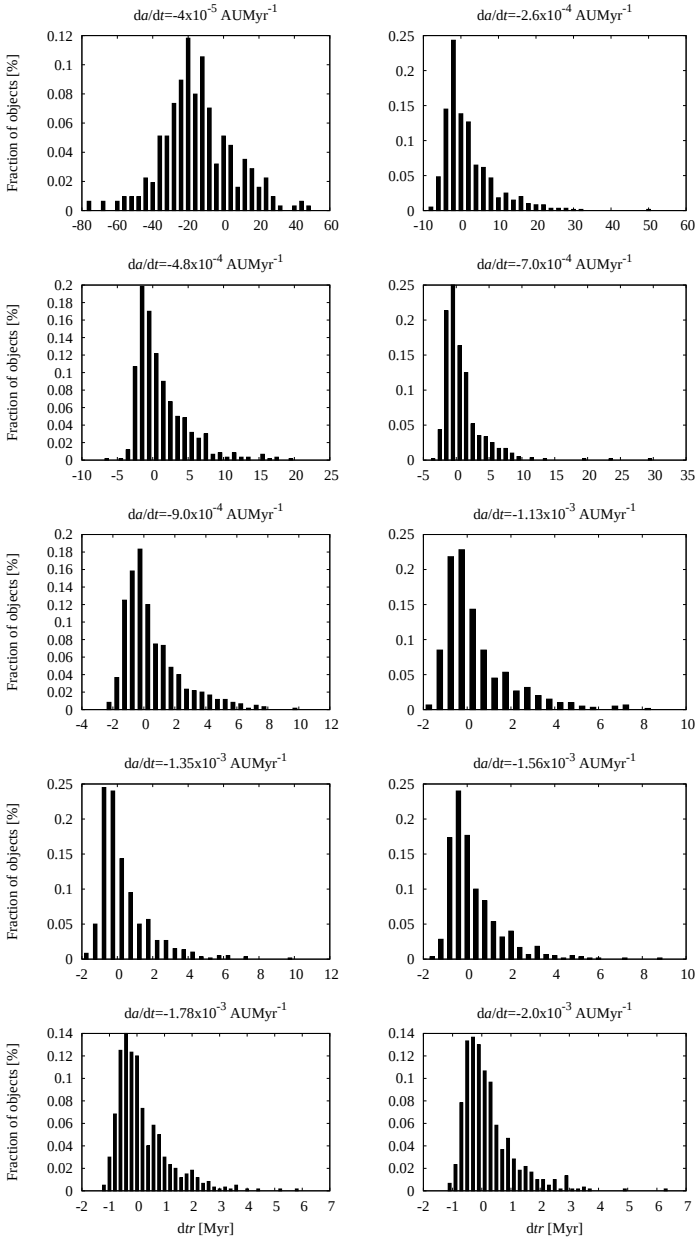

Fig. 1. *Histograms of tests objects as a function of time delay dtr in the strongest examined amongst 11 MMRs, namely the 9:4 resonance.*

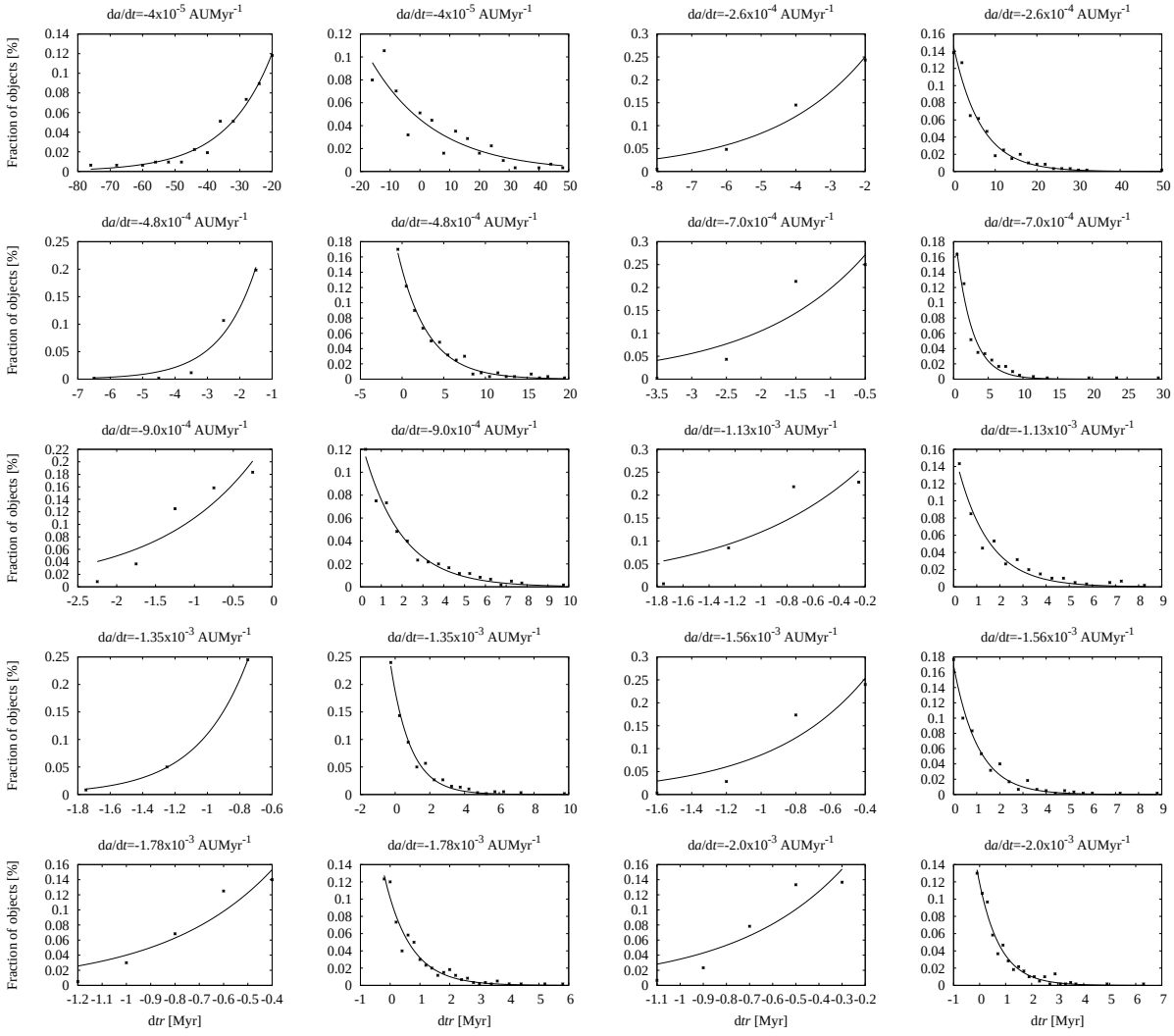

Fig. 2. *The modified Laplace asymmetric distribution of tests objects in the resonance 9:4. Time intervals of dtr are obviously greater for weaker Yarkovsky values, and vice versa.*

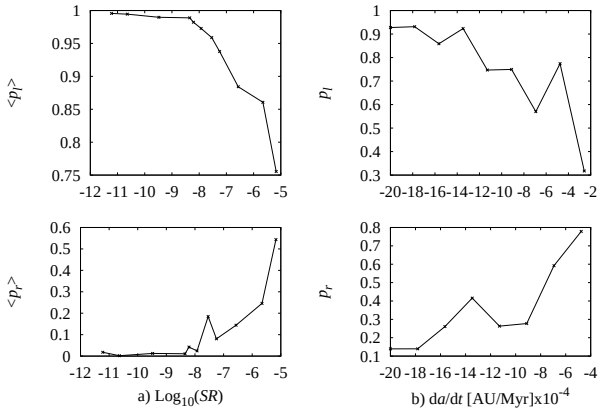

Fig. 3. *a) Dependence between $\langle p_l \rangle$, $\langle p_r \rangle$ and resonance's strength, b) Dependence between p_l, p_r and the Yarkovsky drift speed for the 9:4 resonance.*

The arithmetic mean value of the parameter $\langle p_l \rangle$, which was calculated for all resonances and for all Yarkovsky values, has the highest value for the weakest resonance, and the lowest value for the strongest resonance (Fig. 3.a). The value of $\langle p_l \rangle$ parameter decreases with the strengthening of the resonance.

Contrary to the parameter $\langle p_l \rangle$, the arithmetic mean value of the parameter $\langle p_r \rangle$, which was calculated in all cases, has the highest value for the strongest resonance while the lowest value is found for the weakest resonance (Fig. 3.a). The value of $\langle p_r \rangle$ decreases with the weakening of the resonance. Also, values of p_l increase with the increase of the Yarkovsky speed, while values of p_r increase with the decrease of the Yarkovsky drift speed, in all resonances. In Fig. 3.b these results are presented for the 9:4 resonance. This is valid for all resonances.

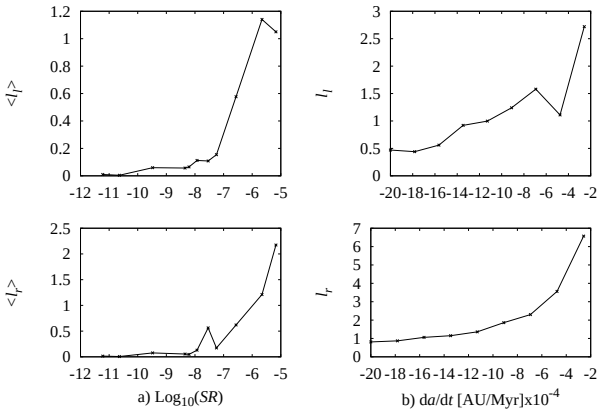

Fig. 4. *a) Dependence between $\langle l_l \rangle$, $\langle l_r \rangle$ and resonance's strength, b) Dependence between l_l, l_r and the Yarkovsky non-gravitational force for the 9:4 resonance.*

The arithmetic mean values of parameters $\langle l_l \rangle$ and $\langle l_r \rangle$, which were calculated for all cases, decrease with the weakening of the resonance (Fig. 4.a). There is an interesting results about the connection between l_l, l_r and the Yarkovsky speed. Their values decrease from the slowest Yarkovsky speed to the fastest Yarkovsky speed in all resonances. As an example, Fig. 4.b shows these results for the 9:4 resonance but the same trend is observed for all resonances.

There evidently exists functional connection between the parameters $\{l_l, l_r, p_l, p_r\}$ and the Yarkovsky drift speed, and the strength of resonances. Functional connection could be described by the following equation:

$$\begin{aligned} \log_{10}(\{l_l, l_r, p_l, p_r\}) &= a \log_{10}(SR) + \\ &+ b \log_{10}\left(\frac{\mathrm{da}}{\mathrm{dt}}\right) + c. \quad (4) \end{aligned}$$

The fitting parameters (a, b, c) could be found numerically applying the least-squares method in fitting the data with the Eq. (4). The fitting parameters that describe best the relation between $\{l_l, l_r, p_l, p_r\}$, SR and $\frac{\mathrm{da}}{\mathrm{dt}}$ are presented in Table 2.

Table 2. Values of fitting parameters (a, b, c) along with their standard errors.

	$a \pm \sigma_a$	$b \pm \sigma_b$	$c \pm \sigma_c$
l_l	0.397±0.014	-0.919±0.055	-0.980±0.205
l_r	0.434±0.019	-0.928±0.077	-0.619±0.284
p_l	-0.018±0.003	0.069±0.012	0.035±0.044
p_r	0.426±0.024	-0.926±0.094	-1.206±0.344

These fitting parameters (a, b, c) of the modified Laplace distribution may be used for determination of dtr for certain number of objects with known Yarkovsky drift speed in the known resonance, that may be very useful for different further investigations on asteroids' motions over MMRs.

3.2. Relation between $\langle dtr \rangle$, SR, $\frac{\mathrm{da}}{\mathrm{dt}}$ and e

A relation between $\langle dtr \rangle$, SR, $\frac{\mathrm{da}}{\mathrm{dt}}$ was established in Milić Žitnik and Novaković (2016). For 9 (out of 10) values of $\frac{\mathrm{da}}{\mathrm{dt}}$ analyzed there, it was revealed that $\langle dtr \rangle$ increases when SR is increasing (the smallest Yarkovsky value had different behaviour). A similar linear dependence is found between $\langle dtr \rangle$ and $\frac{\mathrm{da}}{\mathrm{dt}}$ but with opposite trend, i.e. $\langle dtr \rangle$ decreases while $\frac{\mathrm{da}}{\mathrm{dt}}$ is increasing.

In particular, it was shown by Milić Žitnik and Novaković (2016) that the following equation holds:

$$\langle dtr \rangle = c_1 \ (SR)^{\beta} \ \left(\frac{\mathrm{da}}{\mathrm{dt}}\right)^{\gamma}, \quad (5)$$

with c_1 being a coefficient and β and γ two unknown exponents. In order to estimate the strength of the resonances, SR, we applied the numerical method proposed by Gallardo (2006). The unknown parameters c_1, β and γ in Eq. (5) were found by Milić Žitnik and Novaković (2016) numerically applying the least-squares method of fitting data using the equation:

$$\log_{10}(\langle dtr \rangle) = \beta \log_{10}(SR) + \gamma \log_{10}\left(\frac{\mathrm{da}}{\mathrm{dt}}\right) + c_2. \quad (6)$$

In that way we found that the fitting parameters which describe the best relation between $\langle dtr \rangle$, SR, and $\frac{\mathrm{da}}{\mathrm{dt}}$ are: $\beta = 0.44 \pm 0.03$, $\gamma = -1.09 \pm 0.20$ and $c_2 = 4.35 \pm 0.66$ for $e \sim 0.1$. The results that exclude the five weakest resonances were obtained: $\beta = 0.47 \pm 0.04$, $\gamma = -0.97 \pm 0.15$, $c_2 = 5.11 \pm 0.54$ for $e \sim 0.1$.

The Eq. (6) is valid only for eccentricity of about 0.1 for which SR was estimated. It is well-known that SR depends on eccentricity (Malhotra 1994, Gallardo 2006, Lykawka and Mukai 2007). So, I calculated SR for different values of eccentricity $0.025 \leq e \leq 0.4$ with step of 0.025 (0.4 is the upper value of eccentricity for most of the asteroids). After that, I calculated unknown fitting parameters for these new values of e and SR. The values of β are given in Table 3 $0.025 \leq e \leq 0.4$ because β defines the relation between SR and e. It is clear that β depends on eccentricity linearly, $\beta = ae + b$ so that β increases with the increase of values of e.

The parameters a and b could be found by the least-squares method of fitting the data given in Table 3 as shown in Fig. 5. We found their values to be: $a = 2.06 \pm 0.02$ and $b = 0.24 \pm 0.01$ derived by using all resonances. The parameter γ has the same value for all eccentricity (Table 3) because it depends only on the Yarkovsky drift speed. Values of c_2 depend on eccentricity linearly except for $e = 0.025$, c_2 increases with e.

Table 3. Values of β, γ, c_2 for $0.025 \leq e \leq 0.4$ with their standards errors.

e	$\beta \pm \sigma_\beta$	$\gamma \pm \sigma_\gamma$	$c_2 \pm \sigma_{c_2}$
0.025	0.327±0.024	-1.092±0.207	4.460±0.680
0.05	0.348±0.025	-1.092±0.207	4.325±0.675
0.075	0.392±0.028	-1.092±0.204	4.332±0.665
0.1	0.441±0.030	-1.092±0.201	4.347±0.656
0.125	0.494±0.034	-1.092±0.199	4.374±0.649
0.150	0.546±0.037	-1.092±0.197	4.399±0.645
0.175	0.598±0.040	-1.092±0.196	4.418±0.642
0.200	0.650±0.043	-1.092±0.196	4.432±0.641
0.225	0.702±0.047	-1.092±0.196	4.440±0.641
0.250	0.754±0.050	-1.092±0.196	4.445±0.643
0.275	0.805±0.054	-1.092±0.197	4.447±0.646
0.300	0.858±0.058	-1.092±0.198	4.450±0.649
0.325	0.911±0.062	-1.092±0.200	4.455±0.654
0.350	0.966±0.067	-1.092±0.201	4.464±0.659
0.375	1.023±0.071	-1.092±0.202	4.482±0.663
0.400	1.084±0.076	-1.092±0.203	4.511±0.667

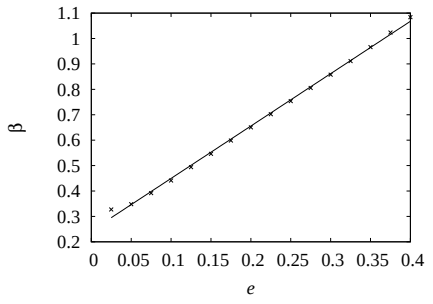

Fig. 5. *Dependence between e and β for resonance's strength calculated for* $0.025 \leq e \leq 0.4$.

4. CONCLUSIONS AND IMPLICATIONS

This paper presents the functional relation between the average time spent inside a resonance $\langle dtr \rangle$, the strength of a resonance SR, eccentricity e, the semi-major axis drift speed $\frac{\mathrm{da}}{\mathrm{dt}}$, with corrected and generalized Eq. (6) that is valid for $0.025 \leq e \leq 0.4$:

$\log_{10}(\langle dtr \rangle) = (2.06e + 0.24)\log_{10}(SR) - 1.09\log_{10}(\frac{\mathrm{da}}{\mathrm{dt}}) + c_2$.

Then, it would be easy to calculate the average time that an object spent inside an MMR, $\langle dtr \rangle$, with given the resonance's strength, the Yarkovsky drift speed, and an object's eccentricity.

The modified Laplace statistical distribution could be used for generating dtr for certain number of objects with a particular Yarkovsky drift speed in MMRs. These results may be easily implemented in different Monte-Carlo methods aiming to simulate migration of asteroids across the MMRs in the Main Belt.

Work on the remaining topics will continue and will include other MMRs as well as a wider range of Yarkovsky drift speeds.

Acknowledgements – I would like to thank Prof. Valerio Carruba, the referee, for his useful suggestions and comments which helped to improve this article. This work was funded by the Ministry of Education, Science and Technological Development of the Republic of Serbia, under the Project 176011.

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ИНТЕРАКЦИЈА ИЗМЕЂУ СИЛЕ ЈАРКОВСКОГ И РЕЗОНАНЦИ У СРЕДЊЕМ КРЕТАЊУ: НЕКЕ СПЕЦИФИЧНЕ ОСОБИНЕ

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УДК 521.16
Оригинални научни рад

Недавно смо анализирали улогу резонанци у средњем кретању у промени велике полуосе астероида и утврдили смо функционалну везу која описује зависност просечног времена проведеног у резонанци и снаге ове резонанце и брзине промене велике полуосе. Овде смо проширили ову анализу у два правца. Прво, проучавали смо расподелу времена кашњења у резонанци и пронашли да би могла да се опише модификованом Лапласовом асиметричном расподелом. Друго, анализирали смо како време проведено у резонанци зависи од орбиталног ексцентрицитета и предложили релацију која узима овај параметар у обзир.