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MAXIMUM MASS RATIO OF AM CVn-TYPE BINARY SYSTEMS AND MAXIMUM WHITE DWARF MASS IN ULTRA-COMPACT X-RAY BINARIES

B. Arbutina

Department of Astronomy, Faculty of Mathematics, University of Belgrade
Studentski trg 16, 11000 Belgrade, Serbia

E–mail: *arbo@math.rs*

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SUMMARY: AM CVn-type stars and ultra-compact X-ray binaries are extremely interesting semi-detached close binary systems in which the Roche lobe filling component is a white dwarf transferring mass to another white dwarf, neutron star or a black hole. Earlier theoretical considerations show that there is a maximum mass ratio of AM CVn-type binary systems ($q_{\max} \approx 2/3$) below which the mass transfer is stable. In this paper we derive slightly different value for $q_{\max}$ and more interestingly, by applying the same procedure, we find the maximum expected white dwarf mass in ultra-compact X-ray binaries.

Key words. binaries: close – gravitational waves – white dwarfs

1. INTRODUCTION

AM CVn-type stars and ultra-compact X-ray binaries (UCXB) are extremely interesting semi-detached close binary systems in which the Roche lobe filling component is a white dwarf transferring mass to another white dwarf in the former, and neutron star or a black hole in the latter case. It is believed that mass transfer in such systems is driven by angular momentum loss due to gravitational radiation

$$\frac{\dot{J}}{J} = -\frac{32}{5}\frac{GM_1M_2M}{c^5a^4}, \qquad (1)$$

where M_1 and M_2 are masses of the components, $M = M_1 + M_2$ is the total mass and a is orbital separation.

AM CVn-type stars most likely form from: (i) detached white dwarfs (the so called *double degenerate* or DD systems), (ii) a low-mass helium star which becomes semi-degenerate during mass transfer, or (iii) cataclysmic variables (CV) when after mass loss, the evolved star uncovers the helium-rich core (Nelemans et al. 2001a,b, 2005). The prototype AM CVn (HZ 29) with a period of about 18 minutes was discovered by Smak (1967). Paczyński (1967) immediately realised that this could be a semi-detached double white dwarf. Great interest in the AM CVn-type stars and DD systems nowadays reflects primarily the fact that a large part of astronomical community considers a merger of two white dwarfs as a preferred model for supernovae (SNe) type Ia.[1] Project SPY (*ESO Supernovae Ia Progenitors surveY*, Napiwotzki et al. 2001, 2002, Karl et al. 2003, Nelemans et al. 2005) puts large effort in finding DD system with total mass greater than Chandrasekhar mass with no definite candidates so far (but see Geier et

[1]AM CVn-type stars offer two more CV-like channels for SNe Ia which are believed to be less probable.

al. 2007). Secondly, but not secondarily, AM CVn-type stars, DD systems and UCXB are the primary sources of gravitational waves (GW) at low frequencies ($\sim 10^{-2}$ Hz and less) which are not covered by existing projects for detecting GW such as LIGO (*Laser Interferometer Gravitational-wave Observatory*), GEO600, TAMA300 or VIRGO, but will be by LISA – *Laser Interferometer Space Antenna* (see Stroeer and Vecchio 2006).

Earlier theoretical considerations show that there is a maximum mass ratio of AM CVn-type binary systems ($q_{\max} \approx 2/3$) below which the mass transfer is stable (Postnov and Yungelson 2006). In the following section, we shall derive slightly different value for $q_{\max}$ and more interestingly, by applying the same procedure, we will find the maximum white dwarf mass expected in ultra-compact X-ray binaries.

2. ANALYSIS AND RESULTS

Let us consider a detached close binary system with two white dwarfs. Since the system losses angular momentum due to the emission of GW (although other mechanisms such as magnetic breaking are not excluded) the orbital separation will decrease and so will the Roche lobes of the components. As the mass-radius relation for white dwarfs is approximately $R \propto M^{-1/3}$ in the low mass regime, the secondary will fill out its Roche lobe first and start transferring mass to the companion. The system continues to lose angular momentum but since the mass is transferred from the less massive to the more massive component (presumably conservatively, $\dot{M}_2 = -\dot{M}_1 < 0$) and because of the negative exponent in the above mass-radius relation, the orbit widens as we shall see shortly.

To find $q_{\max}$ in the most simple manner let us begin with the well-known equation for the orbital angular momentum:

$$J = \mu a^2 \Omega \sqrt{1-e^2}, \qquad \Omega^2 = \frac{GM}{a^3}, \tag{2}$$

where $\mu = M_1 M_2/M$ is reduced mass, e is eccentricity and Ω is Keplerian angular velocity. Time derivatives are:

$$\frac{\dot{J}}{J} = \frac{M_1 - M_2}{M_1 M_2}\dot{M}_2 + 2\frac{\dot{a}}{a} + \frac{\dot{\Omega}}{\Omega}, \tag{3}$$

$$2\frac{\dot{\Omega}}{\Omega} = -3\frac{\dot{a}}{a}. \tag{4}$$

By combining the last two equations we obtain:

$$\frac{\dot{J}}{J} = \frac{M_1 - M_2}{M_1 M_2}\dot{M}_2 + \frac{1}{2}\frac{\dot{a}}{a}. \tag{5}$$

For mass transfer to be stable, change in the average radius of the donor corresponds exactly to the change in radius of its inner Roche lobe (IL):

$$\frac{R_2}{R_{\mathrm{IL2}}} = \mathrm{const.} \tag{6}$$

If we use approximate relation (Kopal 1959):

$$\begin{aligned} R_{\mathrm{IL2}} &\approx 0.4622a\Big(\frac{q}{1+q}\Big)^{1/3} \\ &= 0.4622a\Big(\frac{M_2}{M}\Big)^{1/3}, \end{aligned} \tag{7}$$

we have:

$$\frac{\dot{R}_2}{R_2} = \frac{\dot{a}}{a} + \frac{1}{3}\frac{\dot{M}_2}{M_2}, \tag{8}$$

$$\frac{\dot{a}}{a} = \frac{\dot{M}_2}{M_2}\Big(\zeta - \frac{1}{3}\Big), \tag{9}$$

where $\zeta = \frac{\mathrm{d}\ln R_2}{\mathrm{d}\ln M_2}$. Since $\zeta < 0$ and $\dot{M}_2 < 0$ we see that $\dot{a} > 0$ as already said. Eqs. (5) and (9) give:

$$\frac{\dot{J}}{J} = \frac{\dot{M}_2}{M_2}\Big(\frac{1}{2}\zeta - \frac{1}{6} + 1 - q\Big). \tag{10}$$

As the system loses angular momentum, $\dot{J} < 0$, which means that the term in parentheses on the right hand side of Eq. (10) must be positive:

$$\frac{1}{2}\zeta + \frac{5}{6} - q > 0, \tag{11}$$

i.e. it must be $q = M_2/M_1 < \frac{2}{3}$ in the case of low-mass white dwarfs ($\zeta \approx -\frac{1}{3}$). If the secondary is a low-mass helium star, we can use $R_2/R_\odot \approx 0.043(M_2/M_\odot)^{-0.062}$ (Tutukov and Fedorova 1989), and we will obtain a slightly higher value for the $q_{\max}$. A larger mass ratio $q > q_{\max}$ results in an unstable dynamical mass transfer and a probable merger of the components.

We will now try to improve the above criterion for stability by using Eggleton's (1983) more precise formula for the average (volume) radius of the Roche lobe:

$$\frac{R_{\mathrm{IL2}}}{a} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1+q^{1/3})}. \tag{12}$$

Derivative of (12) with respect to time is:

$$\begin{aligned} &\frac{a\dot{R}_{\mathrm{IL2}} - R_{\mathrm{IL2}}\dot{a}}{a^2} = \\ &\Bigg(\frac{0.49\frac{2}{3}q^{-1/3}\big[0.6q^{2/3} + \ln(1+q^{1/3})\big]}{\big[0.6q^{2/3} + \ln(1+q^{1/3})\big]^2} - \\ &- \frac{0.49q^{2/3}\big[0.6\frac{2}{3}q^{-1/3} + \frac{\frac{1}{3}q^{-1/3}}{1+q^{1/3}}\big]}{\big[0.6q^{2/3} + \ln(1+q^{1/3})\big]^2}\Bigg)\dot{q}, \end{aligned} \tag{13}$$

that is

$$\begin{aligned} &\frac{\dot{R}_{\mathrm{IL2}}}{R_{\mathrm{IL2}}} - \frac{\dot{a}}{a} = \\ &\frac{2(1+q^{1/3})\ln(1+q^{1/3}) - q^{1/3}}{3q(1+q^{1/3})\big[0.6q^{2/3} + \ln(1+q^{1/3})\big]}\dot{q}, \end{aligned} \tag{14}$$

$$\frac{\dot{R}_{\rm IL2}}{R_{\rm IL2}} - \frac{\dot{a}}{a} = \frac{1}{3}\eta(q)\frac{\dot{M}_2}{M_2}, \tag{15}$$

where:

$$\eta(q) = \frac{\left[2(1+q^{1/3})\ln(1+q^{1/3}) - q^{1/3}\right](1+q)}{(1+q^{1/3})\left[0.6q^{2/3} + \ln(1+q^{1/3})\right]}\dot{q}, \tag{16}$$

and we have used:

$$\dot{q} = q(1+q)\frac{\dot{M}_2}{M_2}. \tag{17}$$

On the other hand, $\zeta \neq$ const for all white dwarf masses $0 < M_2 < M_{\rm Ch}$, but $\zeta = \zeta(M_2)$. If we again use the approximate Eggleton's formula (see Hansen and Kawaler 1994):

$$\frac{R}{R_o} = 2.02\Big[1 - \Big(\frac{M}{M_{\rm Ch}}\Big)^{4/3}\Big]^{1/2}\Big(\frac{M}{M_{\rm Ch}}\Big)^{-1/3}, \tag{18}$$

$$\frac{M_{\rm Ch}}{M_\odot} = 1.456\Big(\frac{2}{\mu_{\rm e}}\Big)^2 \tag{19}$$

$$\frac{R_o}{R_\odot} = 5.585\cdot 10^{-3}\Big(\frac{2}{\mu_{\rm e}}\Big) \tag{20}$$

we have:

$$\ln R_2 = \frac{1}{2}\ln\Big[1 - \Big(\frac{M_2}{M_{\rm Ch}}\Big)^{4/3}\Big] - \frac{1}{3}\ln M_2 + {\rm const}, \tag{21}$$

i.e.:

$$\zeta(M_2) = \frac{{\rm d}\ln R_2}{{\rm d}\ln M_2} = -\frac{1}{3} - \frac{2}{3}\frac{\big(\frac{M_2}{M_{\rm Ch}}\big)^{4/3}}{1 - \big(\frac{M_2}{M_{\rm Ch}}\big)^{4/3}}. \tag{22}$$

Stability condition is now (see Eqs. (10) and (11)):

$$\frac{1}{2}\zeta(M_2) - \frac{1}{6}\eta(q) + 1 - q > 0. \tag{23}$$

If $M_2 \ll M_{\rm Ch}$, $\zeta \to -\frac{1}{3}$ (or -0.062), and by solving numerically the above algebraic equation, we obtain for the maximum mass ratio for AM CVn-type stars:

$$q_{\rm max} = 0.634 - 0.760, \tag{24}$$

depending on whether the secondary is a white dwarf or a low-mass helium star, respectively.

What else can we learn from inequality (23)? Function $\eta > 0$ is slowly varying $\eta(q) \sim \eta(0) = 1$ and since with increasing M_2 function $\zeta < 0$ decreases, the mass ratio q also decreases. For $q = 0$ from (23) we must have

$$\zeta(M_2) > -\frac{5}{3}, \tag{25}$$

i.e.

$$M_2 < \Big(\frac{2}{3}\Big)^{3/4} M_{\rm Ch} \approx 0.738\ M_{\rm Ch} \approx 1.06\ M_\odot. \tag{26}$$

Now we can solve (23) algebraically varying M_2 from 0 to 0.738 $M_{\rm Ch}$ to obtain the relation $q_{\rm max} = q_{\rm max}(M_2)$ or $q_{\rm max} = q_{\rm max}(M)$ (Figs. 1 and 2). In Fig. 2 we showed DD and AM CVn-type systems from Tables 1 and 2. In order to plot DD systems, we needed some estimates for masses of the B components for which only the lower limit is given. The estimates can be made with the help of the mass function:

$$f_1 = \frac{M_2^3\sin^3 i}{(M_1+M_2)^2}, \tag{27}$$

(which is observable) by using some average value for $\sin^3 i$. If orbital planes of binaries are randomly distributed in space, the probability of finding a system with inclination within the interval $(i, i+{\rm d}i)$ is $f_i(i){\rm d}i = \sin i\ {\rm d}i$. This distribution gives $\langle\sin^3 i\rangle = 0.589$. However, one uses more often $\langle\sin^3 i\rangle = 0.679$ ($i \approx 61.5^\circ$) corresponding to distribution function $f_i(i) = (4/\pi)\sin^2 i$, which takes into account the fact that spectroscopic binaries with $i = 0^\circ$ are hard to detect (Trimble 1974). For simplicity, we used $i = 60^\circ$ (some of the masses are pretty uncertain anyway).

We see that all AM CVn-type systems are well within the stability domain. Mass ratios for most of the systems are pretty low, which is a direct consequence of the mass transfer from the less massive to the more massive component. On the other hand, almost all (save one) DD systems are in the instability domain, meaning that when the secondary fills in its Roche lobe, the components will probably merge relatively quickly and possibly form stars like R CrB.

This analysis may be over simplistic in the sense that it implicitly assumes that the secondary rotates synchronously with the orbital revolution and that there is efficient tidal coupling between the accretion disk and the donor, or accretor and the donor if the mass stream hits the companion directly and no accretion disk forms. The additional restriction for stability is to have the initial mass transfer rate below the Eddington limit for the companion. A rigorous treatment has to include a consideration of tidal effects, angular momentum exchange and possible super-Eddington accretion rate which could lead to common envelope formation and subsequent merger (Nelemans et al. 2001b, Marsh et al. 2004, Postnov and Yungelson 2006).

Table 1. DD systems with estimated masses from Nelemans et al. (2005). Mass of the primary* (A) is determined through the model atmosphere and/or gravitational redshift. Mass of the secondary (B) in single-line spectroscopic binaries (Sd1) is the lower limit obtained for inclination $i = 90°$.

System	P [d]	M_A $[M_\odot]$	M_B $[M_\odot]$	Sd
WD0135-052	1.56	0.47	0.52	2
WD0136+768	1.41	0.47	0.37	2
HE0320-1917	0.87	0.29	> 0.35	1
WD0326-273	1.88	0.51	> 0.59	1
WD0957-666	0.06	0.37	0.32	2
WD1013-010	0.43	0.44	> 0.38	1
WD1022+050	1.16	0.39	> 0.28	1
WD1101+364	0.15	0.29	0.35	2
WD1115+166	30.09	0.52	0.43	2
WD1202+608	1.49	0.40	> 0.34	1
WD1204+450	1.6	0.46	0.52	2
WD1210+140	0.64	0.23	> 0.38	1
WD1241-010	3.35	0.31	> 0.37	1
WD1317+453	4.87	0.33	> 0.42	1
WD1349+144	2.12	0.44	0.44	2
HE1414-0848	0.52	0.71	0.55	2
WD1428+373	1.14	0.35	> 0.23	1
HE1511-0448	3.22	0.48	> 0.46	1
WD1704+481	0.14	0.39	0.56	2
WD1713+332	1.12	0.35	> 0.18	1
WD1824+040	6.27	0.43	> 0.52	1
WD2032+188	5.08	0.41	> 0.47	1
HE2209-1444	0.28	0.58	0.58	2
WD2331+290	0.17	0.39	> 0.32	1

*Only here, by primary we mean the brighter and not the more massive component.

Table 2. AM CVn-type systems with estimated masses from Stroeer and Vecchio (2006).

System	P [s]	M_1 $[M_\odot]$	M_2 $[M_\odot]$
RX J0806.3+1527	321	0.2–0.5	0.13
V407 Vul	569	0.7	0.068
ES Cet	621	0.7	0.062
AM CVn	1029	0.85	0.14
HP Lib	1103	0.57	0.032
CR Boo	1471	0.55	0.023
KL Dra	1500	0.27	0.022
V803 Cen	1612	1.31	0.021
SDSS J0926+3624	1699	0.6	0.02
CP Eri	1701	0.63	0.019
SN2003aw	2028	0.42?	0.015?
SDSS J1240-0159	2242	0.38?	0.015?
GP Com	2794	0.45	0.01
CE 315	3906	0.48	0.006

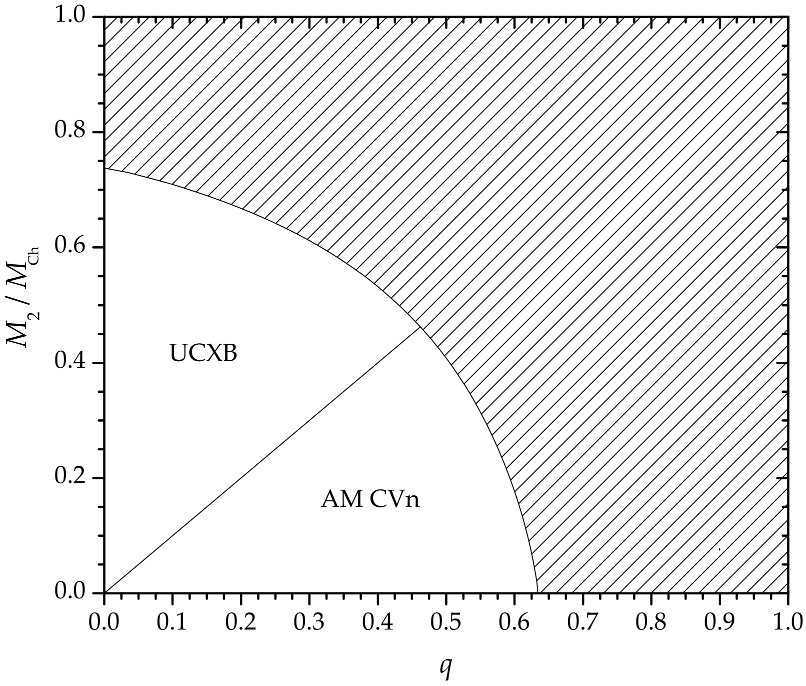

Fig. 1. $q_{\max} = q_{\max}(M_2)$ *relation for white dwarfs. The stability domain for* AM CVn*-type stars and* UCXB *is unsheded. Line* $M_2/M_{\rm Ch} = q$ *represents the limit below which we have two white dwarfs in the system.*

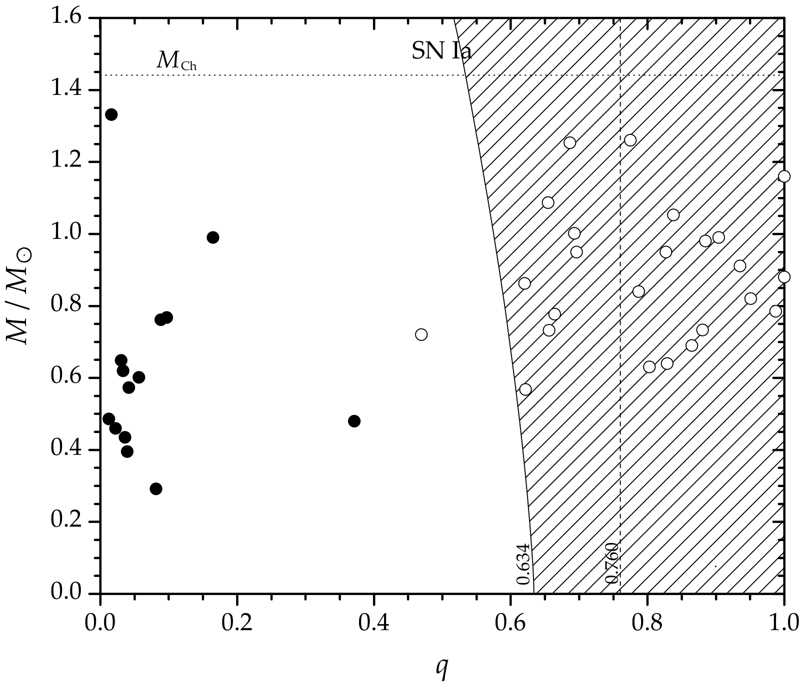

Fig. 2. *Known* DD *systems (empty circles) and* AM CVn*-type stars (filled circles). Curve separating unshaded and shaded regions represents the* $q_{\max} = q_{\max}(M)$ *relation for white dwarfs. The dashed vertical line* $q = q_{\max}$ *is the stability limit if the secondary is a low-mass helium star. The dotted horizontal line is the Chandrasekhar limit above which we could potentially have* SN Ia *from a merger.*

3. CONCLUSIONS

In this paper we derived the maximum mass ratio for AM CVn-type binary systems below which the mass transfer should be stable for a low-mass white dwarf and a low-mass helium star secondary respectively:

$$q_{\max} = 0.634 - 0.760. \tag{28}$$

More interestingly, by applying the same procedure, we found the maximum expected white dwarf mass in ultra-compact X-ray binaries (Arbutina 2009):

$$M_{\max} = \left(\frac{2}{3}\right)^{3/4} M_{\rm Ch} \approx 0.738\ M_{\rm Ch} \approx 1.06\ M_{\odot}. \tag{29}$$

This implies that UCXB with white dwarfs more massive than $M_{\max}$ should not be observed because once the white dwarf fills in its Roche lobe, the components will probably merge relatively quickly due to the unstable mass transfer.

Inclusion of a more accurate mass-radius relation i.e. equation of state for white dwarfs and considering tidal effects and the effects of super-Eddington accretion rate on the stability of mass transfer we leave for future work.

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МАКСИМАЛНИ ОДНОС МАСА ЗА ДВОЈНЕ СИСТЕМЕ ТИПА AM CVn И МАКСИМАЛНА МАСА БЕЛОГ ПАТУЉКА У УЛТРА-КОМПАКТНИМ X-ДВОЈНИМ СИСТЕМИМА

B. Arbutina

Department of Astronomy, Faculty of Mathematics, University of Belgrade
Studentski trg 16, 11000 Belgrade, Serbia

E–mail: *arbo@math.rs*

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Оригинални научни рад

Звезде типа AM CVn и ултра-компактни X-двојни системи су изузетно занимљиви полуконтактни тесно двојни системи у којима је компонента која испуњава Рошов овал бели патуљак који претаче масу на другог белог патуљка, неутронску звезду или црну рупу. Ранија теоријска разматрања показала су да за системе типа AM CVn постоји максимални однос маса ($q_{\max} \approx 2/3$) испод којег је трансфер масе стабилан. У овом раду изведена је кориговања вредност за $q_{\max}$, и што је интересантније, примењујући исти поступак добијена је максимална маса за белог патуљка у ултра-компактним X-двојним системима.