Serb. Astron. J. № 182 (2011), 1 - 16 **UDC 524.88**
DOI: 10.2298/SAJ1182001G Invited review

THE CONCEPT OF FRACTAL COSMOS: III. PRESENT STATE

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(Received: November 2, 2010; Accepted: November 2, 2010)

SUMMARY: This is the sequel to the previous accounts on the rise and development of the concept of fractal cosmos, up to year 2001 (Grujić 2001, 2002). Here we give an overview of the present-day state of art, with the emphasis on the latest developments and controversies concerning the model of hierarchical universe. We describe both the theoretical advances and the latest empirical evidence concerning the observation of the large-scale structure of the observable universe. Finally we address a number of epistemological points, putting the fractal paradigm into a broader cosmological frame.

Key words. Cosmology: miscelaneous

1. PROLOGUE

Fractals appear ubiquitous in Nature, Arts and Science. To numerous well known examples (see, Mandelbrot 1983) some new cases may be mentioned, like molecular biology (DNA) (Zuo-Bing 2003), linguistic (Chinese letters) (Johnston 2008), paintings (Taylor et al. 1999), (Mureika 2005), optics (Berry et al. 2001), field theory (Casimir effect) (Funaro 2009), superconductivity (see, e.g. Cartlidge 2010), solid state physics (see, e.g. Pacey 2010), science of material (Farr 2009), to mention some of them only. There is no clear connection at a fundamental level that would make these cases resulting from an underlying principle or fundamental law, but the diversity of cases and their proliferating number points toward something deeper than the mere phenomena. Cosmos turns out the largest object possibly endowed with fractal, or at least hierarchical structure, with intriguing properties, which pose a number of questions, both at the ontological and epistemological levels. Before we go on, it is fair to notice that in many of the quoted instances fractal pictures do not appear as clear cut patterns, and the same holds for the entire Cosmos as well.

In the previous reviews of the fractal cosmology we have shown that the concept of hierarchical structuring was deeply rooted into mind of philosophers and scientists, from the Pre-Socratics to the modern time (Grujić 2001, 2002). It appears somewhat ironic that it was the modern advance on the subject of fractality that has made us appreciate the old achievements of our ancestors. With the risk of assigning our concepts to the past times (Whiggish syndrome) it was only after developing more precise ideas of the fractality that we are able to recognize the old concepts akin to that. It concerns particularly Anaxagoras' *homoeomerias*, which perplexed his contemporaries. Another great contribution to the concept the hierarchical structure of the world was due to Kant, whose ideas on the subject passed almost unnoticed, overshadowed by his cosmogony (Kant 1968). Einstein himself was aware of the hierarchical cosmos, but dismissed the idea on some general grounds (Einstein 1922). The latter, however, should not disturb the "fractal community", since we know the first cosmologist of the previous century used to reject many other models (like those due to Friedman and Lemaitre), not to mention his own cosmological constant Λ. The most significant contribution to the modern paradigm of hierarchical cosmos was due to Charlier in 1922, who put the model on the solid astrophysical grounds (Charlier 1922).

Equally great contribution to the very concept came from mathematicians from the last century. In fact, the first mentioning of fractal concept could be found with Plato, (Plato, Republic, Book VI,509; see, e.g. Jaeger 1973). We first mention Felix Hausdorff, who, at the very beginning of 20th century, introduced a new kind of dimension, now called Hausdorff's dimension and which Mandelbrot used for the very definition of fractal objects (see, e.g. Grujić 2002).

During the latest decade considerable advance was made with regard to the theoretical cosmology and astrophysical evidence of the cosmos at large. Though not epoch making discoveries may be claimed, a number of important results have been achieved in laying down fundamental background of the hierarchical structure in general and fractal cosmology in particular. On the observational side the main event was discovery of the accelerating universe, which has provoked some of the radical amendments to the prevailing Standard Theory of the cosmic dynamics.

On the empirical side three principal goals have been strived for: (i) the upper observational limit R_{up} beyond which a homogeneous distribution of the cosmic matter, mainly galaxies, may be accepted; (ii) the nature of the inhomogeneous distribution particularly in view of the possible hierarchical structure; (iii) the accurate value of the fractal dimension D_f for $R \leq R_{up}$. As for the theoretical side, a number of monographs has been published which have advanced our understanding of the study of cosmic structure in general. We mention first the monograph (Gabrielli et al. 2005) which presents a thorough approach concerning the nature of the statistical methods used in detecting and interpreting the cosmic matter distribution and the books by Nottale, (Nottale 1993, 2011) which lay down the foundations of the fractal structure of space and time at the very fundamental level. We shall discuss these and some other important contributions to the subject later on. In the next Section we discus a number of general theoretical aspects, and in Section 3 a heuristic parallel between a specific case of the atomic dynamics and the Standard Cosmological Model (SCM) is made. In Section 4 theoretical advances have been presented and in Section 5 modeling fractal universe and the relevant observational evidence of the cosmic structure is analyzed and discussed. Section 6 is dedicated to considering some epistemological questions and concluding remarks are given.

2. THEORETICAL CONSIDERATIONS

The case of the fractal cosmology resembles much the story of parallels in geometry: though conceived even in Antiquity, it has been running in parallel with the prevailing ("standard") paradigm since then, with the prospects of " crossing" that have not yet been realized (see, e.g. Baryshev 1999, 2005 and Baryshev and Teerikorpi 2002) for a more popular account). The case of parallels in geometry was "solved", or resolved by abandoning the Euclidian paradigm of "flat space" (fourth postulate about right angles) and allowing for other possibilities, which resulted in constructing hyperbolic and spherical geometries of Lobachevski and Riemann, respectively. Formally, hierarchical cosmos was designed as a response to Eleatic challenge (see, e.g. Kirk et al. 1983) by Anaxagoras, who solved the problem of filling an infinite space with a finite amount of matter (see Grujić 2001 and references therein).

There is no appealing need for a hierarchical Cosmos, as there was no need for non-Euclidian geometries. Yet it is a traditional wisdom that every possibility should be examined and no cosmological model, even paradigm, should be *a priori* rejected. Further, as the observational technic is advancing, this need may appear soon enough that the fractal (or some similar) paradigm becomes compelling.

2.1. Preliminary remarks

Any attempt to conceive the Universe as a Cosmos, that is to ascribe to the totality of our physical environment a particular structure, is based inevitably on a number of premises, upon which one tries to contrive a particular cosmological model. These premises are called, in this particular case, cosmological principles or cosmological postulates. These principles are then used as guiding rules for elaborating fine details of the model under construction (see, e.g. Grujic 2007). Their role appears at least twofold. First, cosmological principles identify the basic ideological status of the cosmologists and second, they introduce simplifications into the formal (mathematical) approach, enabling one to produce final formal solutions of the structure one is searching for.

The essential simplification in contriving Cosmos is to ascribe a particular symmetry to the Universe. The choice is dictated by a number of premises one adopts, intentionally or implicitly. In an abstract sense these symmetries may be divided into two broad classes according to the attributes: homogeneous and inhomogeneous systems.

2.1.1. The concept of homogeneity

The notion of cosmic homogeneity appears subtle and deserves some further elaborations. Before talking about cosmic structuring, one must first specify the scale of the space (physical or otherwise). We know that the universe is not, and can not be homogeneous on the microscopic, mesoscopic and macroscopic scales, for it is the inhomogeneities on these scales which make the nontrivial structuring, including that of life, possible. On the truly cosmic scale, however, the notion of homogeneity becomes vague, if not problematic (as we shall see later on). The issue resembles much that of the theory of gaseous matter phase, which was initially developed for the simple quasi-homogeneous state, close to the thermodynamical equilibrium. Only after this simple case was understood, the theory of inhomogeneous gases, far from the thermodynamical equilibrium, was developed. With this analogy in mind, one may consider the Universe as a cold gas, a collection of galaxies as the principal elementary constituents, playing the role of atoms. We shall, in the following, distinguish three principal cases:

(i) Quasi-homogeneous systems
If one can define an average inter-galactic mutual distance, d_{qh}, on the scale with characteristic length an order of magnitude larger than d_{qh}, and we partition the entire cosmic space into cells of such dimensions, one may speak of a homogeneous distribution if, on the average, each cell contains approximately the same number of galaxies.

(ii) Semi-homogeneous systems
If one can determine the length λ so that for scales $d \geq \lambda$ an inhomogeneous system becomes homogeneous, one may speak of a semi-homogeneous structure. Strictly speaking, this definition appears trivial, in the sense of the case (i) above, but, as we shall see in the following in the cosmological structuring, it has a real meaning.

(iii) Essentially inhomogeneous systems
If there is no finite λ in case (ii), i.e. the system is inhomogeneous at all scales, we speak of an essential inhomogeneity. It is exactly the case we shall be considering here with regard to the cosmological models and observations. Another definition of an essentially inhomogeneous system is that nowhere within the system the average density can be determined.

Generally, these definitions need not necessarily refer to the real space or physical systems, but rather apply to the formal, geometrical or mathematical in general, aspects of the real systems. It is in this abstract sense that the hierarchical structure, which may be based on the so-called *scaling symmetry*, can be conceived as *scale homogeneity.* As elaborated elsewhere (cf e.g. Grujic 2007) all present-day cosmological models, or better, paradigms, may be divided into the following classes, according to the symmetry properties:

(i) Time homogeneous
The universe remains the same all the time, whatever its structure may be. Einstein's first cosmological model, the static universe, belongs to this class of time-invariable universe. The original so-called *steady-state* models (Narlikar 1977) described a time-invariant universe, locally and globally alike. With the discovery of the universal expansion (see, e.g. Nussbaumer and Bieri 2009 for a detailed account) the original model was modified so as to account for the global change in time but retaining the local constant matter density. So one may speak of a semi-stationary cosmos.

(ii) Space homogeneous
Majority of cosmological models belong to this class including the above-mentioned Einstein's static cosmological model and Lemaitre's model (see Chap. 4.1).

(iii) Space-time homogeneous
The original de Sitter's model implied homogeneity within the abstract space-time, but as Lemaitre found out later, its spatial part turned out inhomogeneous (see, e.g. Nussbaumer and Bieri 2009).

(iv) Space and time homogeneous
Einstein's first cosmological model satisfied the so-called *Perfect Cosmological Principle*, with the universe remaining the same and homogeneous all the time.

(v) Scale homogeneous (invariant)
This is the case we shall be interested in here. It implies that the matter distribution in the cosmic space is such that a series of scales may me defined with (approximately) the same structure at each level. If we have various discernable levels, but different structuring at different scales, one deals with a hierarchical model. If the same structuring is repeating at each scale, one speaks of a *fractal model.* In the reality situation appears somewhat more complicated, and more complex models, like multi-fractal ones, are invoked to describe the actual state of affair.

2.1.2. The concept of symmetry

Symmetry refers primarily to the geometric properties of real or abstract systems, but the notion has its counterpart in the dynamic, better to say, kinematic sector. Symmetry operations, like translation, rotation etc. do occur in real systems, though not in all of them. It was a great formal achievement of Emmy Noether to link the translation in space with the conservation of impulse, and time homogeneity with the energy conservation (e.g. Goldstein 1981).

Scaling or the so-called self-similarity transformations are abounding in nature. Growing of living creatures, both plant and animal, is a good example of changing size, while (approximate by) keeping the same form. Stalactites and stalagmites are good examples in the nonliving world. After astronomical discovery of the expanding universe in late twenties of the last century (e.g. Nussbaumer and Bieri 2009), we witness that the entire cosmos undergoes global scaling towards ever larger dimensions. As we shall see later on, the hierarchical model has its dynamical extension, just as Friedmann and Lemaitre found for Einstein's original static cosmos. But which physical consequence stems from the scaling symmetry? We shall address this issue later on. Here we shall address a parallel between the Coulombic and self-gravitating systems (see, e.g. Grujic 1993).

3. SMALL-ENERGY SYSTEMS

One defines *dilatation transformations* by

$$r \to \theta r, \tag{1}$$

which expand or shrink the physical system. We call *homogeneous functions* those with the properties (Landau and Lifshitz 1976)

$$V(\theta \mathbf{r}) = \theta^{\lambda} V(\mathbf{r}), \tag{2}$$

where the real numbers θ and λ are the *scaling parameter* and *degree of homogeneity* respectively. A physical system whose potential function has the properties as in Eq. (2), scales under these *homothetic transformations*:
the time as

$$t \to \theta^{3/2} t, \tag{3}$$

the energy as

$$E \to E/\theta\,. \tag{4}$$

A mechanical classical system, whose pairwise inter-constituent interactions are described by:

$$V_{i,j} = \frac{a_{ij}}{r_{ij}^{\lambda}}, \tag{5}$$

has the potential function

$$V = \sum_{i<j} V_{ij}. \tag{6}$$

For $\lambda = 1$ Eq. (6) describes (self-gravitating) Newtonian and Coulombic systems. These systems differ primarily by their respective coefficients a_{ij} which, in the Newtonian case, appear all negative (the law of universal attraction), whereas in the neutral Coulombic systems half of them are positive and half are negative. This difference results in the possibility to form bound-state subsystems. In the Coulombic case constituents (particles) of different sign coefficients can bind to each other, whereas self-gravitating can support all numerically possible bound subsystems. Both classes, however, share a remarkable properties which make them formally indistinguishable. First, pairwise interaction, as described by Eq. (5), have essentially infinite range, which makes them distinctly different from all other sorts of interactions. Second, if a system possesses the (total) energy E, transformations Eq. (1) do not change the shape of the physical trajectories (self-similar transformations), but only expand (or shrink) them, making the constituents move faster or slower (according to Eq. (3)).

Coulombic systems belong to the realm of atoms, whereas the proper representative of the self-gravitating systems appears the Universe itself. The common formal properties of both classes allow one to detect a number of common features, which we shall describe here briefly.

3.1. Lemaitre's universe

In 1927 Belgium cosmologist, reverend George Lemaitre, found remarkable solutions to the Einstein General Relativity cosmological equation, not being aware of the previous results due to Alexander Friedmann (see, e.g. Nussbaumer and Bieri 2009). These dynamical solutions described the universe which expands or shrinks, depending on the initial conditions with the original Einstein's static solution as a special case. But Lemaitre went further than Friedman, contriving a physical model of a dynamical universe. The expansion begins from a state of enormous, but finite, density, and almost infinitely small dimension. This primordial state he called *cosmic atom*, inspired by the remarkable contemporary advance of the atomic physics, due to the application of the newly invented Quantum Mechanics. By invoking the quantum mechanical nature of this initial state universe Lemaitre proposed what we could consider the forerunner of the modern Big Bang cosmological paradigm. By undergoing an explosion, constituents of the primordial atom flew away, making the universe expand, as we observe it today. One remarkable feature of this model is that it recovers space homogeneity (a cornerstone of the majority of cosmological models)- every point in the (physical) space appears equivalent and an observer at any point sees other constituents fly away from him.

In the following we need to note that these constituents are galaxies. Further, according to the present-day theoretical elaborations and observational evidence, our universe seems to possess zero energy (flat, Euclidian universe, in formal terms).

3.2. Wannier's paradigm

In 1953 American theoretician Gregory Wannier proposed a model for describing ionization of atoms by electronic impacts, close to the ionization threshold (Wannier 1953). For the $n - fold$ ionization of a neutral atom A

$$e + A \to A^{n+} + (n+1)e, \tag{7}$$

near threshold restriction meant that the entire *electron + atom* system possesses small, positive energy. The central idea of the model was an assumption that the impinging electron hits the atomic target, disturbs it and forms with the target a compound system. This transient state system can not be described classically, since the quantum mechanical effects are supposed to play dominant role within a limited space around the target nucleus. Nevertheless, for the restricted problem of finding the dependence of the ionization probability (or cross section) on the entire system energy, can be determined analyzing the system behaviour outside this essentially quantum zone. Wannier's model consists of the inner quantum region, which may be considered as a black box, and the outer, asymptotic region, where emerging particles behave in an essentially classical way, prescribed by the classical electrodynamics and classical mechanics. By applying scaling laws appropriate for this Coulombic system, Wannier was able to derive the so-called threshold law for single and (approximately) double ionization by electron impact. This model has been subsequently used for a number of fragmentation processes with great success, including some non-Coulombic systems (Grujic and Simonovic 1988). Semiclassical and quantum mechanical (where feasible) calculations, as well as the experimental investigations, have corroborated Wannier's paradigm of the near-threshold atomic processes. We note that a complete description of the collisional fragmentation processes, which provides the corresponding cross section, at any energy, requires quantum mechanical approach over the entire physical space (Copeland and Crothers 1985).

3.3. Atomic and cosmic dynamics - a parallel

Two above models share a number of formal similarities (epistemological aspects), but differ substantially in physical domain (ontological aspects). We discuss first the similarities.

Both systems appear covered with two distinct theoretic tools depending on the spatial regions. In the Coulombic case we have a quantum mechanical domain and purely classical one, whereas Standard cosmological model assumes a primordial initial state without inertial particles, but with the (fluctuating) quantum fields (yet to be determined). In the outer region Coulombic systems behave classically. It is interesting that the most effective classical dynamics formalism appears Newton's one, so that this region is Newtonian in every sense. On the other hand, in the cosmological case one does not speak of the outer region since there is no space outside matter. Instead, we follow the evolution of the universe and speak about the *classical phase* of its history. The formal tool for this region is Einstein's General Relativity (GR). In fact, in the asymptotic regime we are living at present, an approximative GR description would suffice (the so-called post Newtonian approximation) so that, apart from quantum regions, it is Newton's theory which rules. Generally, whichever the formal dynamics is accepted, the quantum region appears chaotic one, a sort of black box, from which particles emerge. This stochasticity appears essential in both cases. In Wannier's paradigm it reduces to the quasi-ergodic hypothesis. It is this assumption which enables one to extract the threshold law by considering only the asymptotic region, whereas in the cosmological case, uncontrolable fluctuations of the primordial quantum fields solve (better to say circumvent) the problem of the initial conditions, the weak point of any cosmological model (see, e.g. Hallwell and Hartle 1990).

The parallel between these systems does not go beyond the conceptual similarities. We enumerate here the principal differences.

(i) Since Wannier was interested in finding the multiple-escape cross section small-energy behaviour, he examined those phase-space regions that provided the exits for the escape of all particles to infinity. Since the outgoing electrons experienced the attraction of the positively charged nucleus, but mutual repulsive force, the exit channel consisted of the particles symmetrically positioned, close to apexes of polygons. Those electrons not close enough to the expanding polygon would be pulled back towards the central nucleus (or ion) and would not escape. At the zero energy $E = 0$ configurations leading to the multiple escape form a family of trajectories in the phase-space of a lower dimension than the number of all possible trajectories and the probability of multiple ionization is zero. We note that the asymptotic behaviour of the outgoing particles does not depend on the sign of the energy, provided it is small.

On the other hand, all cosmologically relevant subsystems, like galaxies, experience mutual attraction, and could collide, even form bound subsystems. The Hubble flow may be radically perturbed by local centres of attractions, like the Great Attractor in Virgo cluster. As we shall see later on, it is this possibility of deviating from the uniform overall expansion, which may provide mechanism for forming hierarchical structures. However, in the realm of atomic systems one may have outgoing particles of different sign of charge, like positrons and electrons (ionization by positron impact) (see, e.g. Grujic 1982). In such cases outgoing particles have another exit channel, that of forming bound systems, like positronium $e^- + e^+$. This possibility makes the atomic systems even more akin to the self-gravitating ones, which makes the methods used in the atomic collision theory even more relevant to the study of Newtonian systems.

(ii) Wannier's systems possess the central symmetry in the case of the electron - ion collision, in the exit channel, unlike the cosmological models, which must be endowed with translational symmetry - no cosmic centre. This is the Copernican principle, which has changed our worldview, and which is not to be abandoned. However, one may encounter situations where the similarities with Newtonian systems become prominent enough to consider a common approaches to the evolution of both kinds of systems. For instance, in the case

$$e^- + Ps \rightarrow e^- + e^+ + e^-, \tag{8}$$

where in the exit channel (righthand side of Eq. (8)) one of electrons and positron may recombine into new positronium $(e^- + e^+)$. One may imagine a target which provides many electrons and positrons in the final channel, but the basic difference between the atomic and cosmic systems is that the former are essentially finite, whereas cosmological models deal essentially with infinite (no centre) systems.

In order to stress similarities and differences, we quote another feature of the small-energy atomic dynamical systems. It turns out that only those final configurations with maximum symmetry have chance to expand to infinity. Within these configurations each of the receding constituents experiences a net effective force as if moving in the field of a central charge. If the outgoing particle is negatively charged, like electrons, and the central effective charge appears positive, the resulting force will be the attractive one and the system resembles in that respect a gravitational one. If the effective central charge turns out to be zero, we have in the asymptotic region a free motion, which provides a distinct non-

Wannierian threshold law (Dimitrijevic et al. 1994). Finally, if the effective central charge assumes a negative value, the interaction with the outgoing electrons becomes repulsive and another threshold law follows. The latter case resembles the cosmological model with the dark energy, which pushes galaxies from each other, causing the overall acceleration of the expanding universe as has already been observed (see, e.g. Schneider 2006).

3.4. Microcosmos *versus* megacosmos

We have dwelled over the parallel microcosmos - megacosmos so as to emphasize the unique position cosmology has among all other physical systems studies from the methodological and epistemological point of view. The central dogma of any cosmological model is the Copernican principle, which excludes any special position in the cosmic space (Weak cosmological principle). The universe expands, not from a single point, but from any point. The only variable is the so-called scale parameter $S(t)$, distance between neighbouring galaxies. The effect of galaxy clustering, opposing the overall Hubble flow (global expansion), competes with the effect of the mutually receding cosmic constituents. Hence, the overall uniformity in the cosmic matter distribution, as assumed by majority of the cosmological models, can be neither universal nor holding all the time.

In the case of atomic small-energy systems, as discussed above, recombinations among the constituents in the final channels compete with the total disintegration process and thus determine probability for multiple-escape. This recombination may mean that electrons, for instance, fall back to the central ion (or nucleus), or, in the presence of positrons, for instance, formation of positroniums. Central ions play the role of cosmic attractors. Evidently, the interplay of the Newtonian universal gravitational attraction and the general expansion of the Universe determines in the final account the dynamical structure of the Cosmos. The global expansion serves as the stabilizing force, keeping the galaxies away from each other. In the atomic case it is the inter-particle repulsion which contributes to the final escape to infinity (fragmentation process). In fact, the more realistic comparison would not be between the Universe and an atomic fragmentation process, but between the latter and local instabilities. The latter can result in a massive fall into "great attractors". This phenomenon raises the question of the real meaning of the concept of the universal expansion (the expanding Universe).

Wannier's model for the small-energy system fragmentation applies equally to the self-gravitating few-body systems, as shown in Grujic and Simonovic (1990), which brings the methodologies of the few-body system dynamics even more closely to each other. Both radii of the symmetrically outgoing atomic particles (electrons or positrons, Wannier 1953, for instance) and the scale factor $S(t)$ in the Einstein-de Sitter model (see, e.g. Peebles 1993, Collins et al. 1989), grow in time as $t^{2/3}$. The expansion of the overall intermingled transient system of the incoming body and the target subsystem (compound state) occurs without central (privileged) body making the entire system more democratic than in the atomic case. On the other hand, on an intermediate global scale, "local" deviations from the overall Hubble flow may provide situation similar to that of Wannier's model. We may imagine places like the centre of the Great Attractor, (with overall infall of the cosmic matter towards the attractor centre), from which radical deviations from the global red-shift we experience today may be observed, including the possibility of a systematic blue shift. In a civilization at the technological level similar to ours from the beginning of 20th century the overall outlook of the surrounding cosmos may be verry different from our present-day image of the Universe. We shall return to this local deviation effect when considering the possible mechanism for forming hierarchical structures.

Before we go on, let us note that time evolution of the atomic and cosmic systems differ essentially. In the microscopic world we prepare the initial state of our entire system (impinging particle and target), with the compound, presumably chaotic system being an intermediate phase in the overall evolution. On the other hand the cosmological "initial state" (like Lemaitre's "cosmic atom") appears "put by hand", at least within the Standard model (the Big Bang hypothesis). True, new models which account for the "time before the Big Bang" do appear, like those within the string theory (Gasperini and Veneziano 1993), or various cyclic universe models, but we shall not dwell on it here (see, e.g. Ellis 1999 for a broader discussion).

4. MODELING FRACTAL STRUCTURES

In conceiving space and time Newton postulated strong division between them and the matter, which he inserted into an otherwise empty space, as a universal receptacle of the inert content of the Universe. It is not easy to discern the epistemological aspect of such a postulate from the physical ontology it implied. On the other hand, Leibnitz gave preference to matter, which played more fundamental role than with Newton. To Leibnitz the only observable were the relations between matter constituents of the Universe with notions like absolute space loosing meaning. Finally, Einstein reformulated the issue by going back to the space and time as real observable but subordinated to matter which determines space-time properties. In the following we shall see how the modern cosmological considerations deal this issue with various initial premises giving rise to the observable cosmic features.

4.1. Matter, space and time

Three most fundamental physical entities play the crucial role in conceiving the cosmological structures in a way which may differ significantly from the laboratory/local circumstances. Two principal schools of thought may be discerned in the European culture. The first one may be ascribed to Democritus (the Abderian school) which postulated the empty space (void) filled with atoms. Aristotle rejected the notion of void (*kenon*) and gave primacy to the matter, which is the only reality and the rest just derived quantities. As stressed by Roscoe (2008) Democritus' line was followed by Newton whereas Leibniz, Berkley and Einstein gave preference to Aristotle's relationist approach.

These outlooks gave rise to two distinct, though interrelated approaches: (i) studying the structure of space-time irrespective of the matter, (ii) space-time as a secondary attribute of the inert matter. But the first approach can hardly be considered independent of the matter distribution, if not ontologically then epistemologically. It is from our observations of the matter properties that we conceive the space-time attributes, as we shall see later on. In particular, the eventual fractality of the space-time continuum appears designed after the similar properties of the matter distribution, regardless of the possible mutual influence.

4.1.1. Functions and curves

Dynamics of the inert particles systems is described, at least classically, by functions of mass, space and time. Functions are integrable quantities, though these integrals may differ according to the specific cases. Continuous functions (curves) appear integrable by ordinary integrals of the calculus, whereas discontinuous functions, which can not be represented by curves, need a more general definition of integration, like the Lebesgue's one. Derivatives may be derived from the corresponding integrals, but not all functions appear differentiable, as the case of discontinuous ones illustrates. When integrals are defined, then one may define the first, second etc derivatives in the manner of the standard Newton-Leibnitz calculus but Leibnitz found that a fractal derivative may be defined instead of natural ones (see, e.g. Atanackovic et al. 2008). We note here that the fractal derivatives now find a broad space of application in various fields, including astronomy. Further, not even all continuous curves can be differentiated as the case of the so-called fractal curves shows. It is this situation that introduces need for a more general definition of the derivative, as we shall see later on.

4.1.2. What are fractals?

Definitions of fractal objects are numerous and often vague. The original definition by Benoit Mandelbrot states that objects, curves, functions, or sets are fractal when: "their form is extremely irregular and/or fragmented at all scales" (Roscoe 2008). This definition appears broad enough to encompass many particular situations, but not much helpful in providing precise idea what fractal structure is. Here we enumerate some classes of irregular system which can be classified as fractal.

(i) *Hierarchical structures.* If we may define a number of physical or otherwise scales, with each of them comprising similar objects of the same size, one speaks of the hierarchical structures. Army, state administration, brain are examples of such structuring.

(ii) *Selfsimilar structures.* These are often called fractals in a less strict sense. Objects at the same level are identical, and at different levels possess the same shape. Passing from one object to another at the same level by a number of symmetry transformations leaves the objects unchanged (similarity transformations). Passing from an object at a particular level to another at different level changes the size but not the shape of the object - selfsimilar transformation, or scaling transformation.

If similarity transformations are approximate ones, we speak of quasi-fractals (hierarchical structures are quasi-fractals in this sense). If a parameter which characterizes a fractal at different levels (like the *fractal dimension*) changes from level to level, we speak of the *multifractal structure (multifractals)*. Some fractals have an infinite number of levels, whereas some possess a finite number of them. In fact, only mathematically constructed fractals may have infinitely many levels in principle whereas in nature one finds objects with a few levels. The class of so-called *strange attractors* belongs to the former whereas the snowflakes to the latter case. Of course, quasifractals abound in nature, from coastal lines, bronchial systems, plants etc. It is this approximate fractality which one expects to observe in cosmology, as we shall see later on.

Exact fractals appear rare both in nature and mathematics but are convenient objects to define the most important property of fractality. We shall consider briefly some of them on the so-called fractal curves.

4.1.3. Fractal curves

These mathematical objects serve the best to illustrate the essence of fractality. One of the most used examples is the so-called Koch curve (see, e.g. Figure 3.3 in Roscoe 2008), which may be used to construct snowflakes, for instance. One can not exhibit, of course, any fractal object fully, but just illustrate the iterative procedure for constructing a fractal curve, for instance. Any fractal curve has the *topological dimension* $D_T = 1$, for there is always a way to map it onto a "normal curve", which is one-dimensional. In fact, one of definitions of fractality is that a fractal system has *fractal dimension* greater than its *topological dimension*, $D_F > D_T$. This definition is not quite correct since there are exceptions but may be used as a quick guide when estimating the nature of an object.

The most important parameter of a fractal object is its fractal dimension. In the "normal" physical space the length between any two points on a fractal curve is infinite. The number of segments rises as one increases the resolution ϵ ("zoom"), becoming infinite in the limit $\epsilon \to 0$. If $\aleph_0$ is the cardinal number of a countable set, then in the case of a fractal object $\aleph = 2^{\aleph_0}$.

Deeper structure as revealed by "zooming in" the curve (or any other fractal object) contains "invisible" parts of the curve, as if the latter has been embedded into a higher-dimensional space. It makes, therefore, sense to define a fractal dimension, which is, by definition, higher than the topological one. If within the given segment of a fractal curve we have p (rectilinear) parts, each of length q, then the fractal dimension is given by

$$D_F = \log p / \log q. \tag{9}$$

The length of the segment after n iterations in constructing the curve is

$$L_n = L_0 (p/q)^n. \tag{10}$$

We note that fractality is essentially discrete property and one needs to "zoom" 3^n times to jump to the n-th level. Fractal curves are not differentiable, not being sufficiently smooth for the ordinary concept of differentiation. In a sense, they resemble particle trajectories in a real space according to Nelson's *stochastic quantum mechanics* (Nelson 1966, 1985), with the difference that the latter appear differentiable almost everywhere. Nelson trajectories are essentially stochastic but one may define stochastic fractality too so that the similarities, at least at the heuristic level, do exist.

Fractal dimension of a curve, for instance, may depend on the position measured by a parameter along the curve. Thus we have a variable fractal dimension instead of constant one. Equally, different resolution levels may have different fractal dimension in which case one speaks about *multifractality*. These elaborations, however, move us away from the simple geometrical attributes. As we shall see later on, one often tries to find way out of difficulties in recognizing fractal cosmic distribution by resorting to more general notions, like variable or nonunique parameters.

4.2. The Theory of Scaling Relativity of Spacetime

In his comprehensive treatise on the theory of scale transformation Nottale applied the methods developed by him and others (Nottale 2011) to many branches of natural science. We shall here mention some of results relevant to our fractal model of cosmos. The central point of the scale transformation approach is development of the basic operation of the differential calculus. As mentioned before, standard analysis does not apply to fractal object (curves or otherwise).

If we start with a curvilinear coordinate $\mathcal{L}(x, \epsilon)$ and introduce an infinitesimal dilation $\epsilon \longrightarrow \epsilon' = \epsilon(1 + d\rho)$, then the derivative with respect to the scale parameter (we omit the standard coordinate x) is written as

$$\mathcal{L}(\epsilon') = \mathcal{L}(\epsilon + \epsilon d\rho) = (1 + \widetilde{D} d\rho)\mathcal{L}(\epsilon), \tag{11}$$

where the *dilation operator* is defined as

$$\widetilde{D} = \epsilon \frac{\partial}{\partial \epsilon} = \frac{\partial}{\partial ln\epsilon}, \tag{12}$$

which appears as an analogue to the standard infinitesimal calculus.

4.3. Scaling Matter

Space and time are abstract concepts conceived by removing (abstracting) matter from the physical reality. They may be considered ontologically as more fundamental than (inert) matter but, by the very abstracting procedure, they come after matter (epistemological aspect). It was this observation which lead Roscoe (2002, 2008) to invert the procedure when examining the structure of space and time at large.

Distribution of galaxies (and any other object in general) and their clusters etc. on the cosmic scale is usually estimated by counting galaxies within a sphere of radius R with the observer at the centre. The number of encompassing galaxies, taken as point-like elementary constituents, is given by:

$$N_R = AR^D \,. \tag{13}$$

If $D = 3$ we have a homogeneous distribution. For $D = D_F < 3$ we talk of fractal structure. Generally, we have $D_F = 0$ - *point distribution*, $D_F = 1$ - *linear distribution*, $D_F = 2$ - *surface distribution* and $D_F = 3$ -*volumetric*, or *space filling distribution* (see, e.g. Mureika 2007).

Number of objects within a sphere of radius R is given by:

$$N_R = f(R) \,. \tag{14}$$

If Eq. (14) is a monotonous function, then we may write the inverse relation

$$R = g(N) \,. \tag{15}$$

With this starting positions Roscoe (2002) posed the questions (see also Roscoe 2008):

Is it possible to associate a globally inertial space and time with a non-trivial global matter distribution and, if it is, what are the fundamental properties of this distribution? In the context of the simple model analyzed, he finds definitive answers to these questions so that: A globally inertial space/time can be associated with a non-trivial global distribution of matter:

$$M = m_0 (R/R_0)^2 + 2\sqrt{m_0 m_1 / d_0}(R/R_0), \tag{16}$$

where m_0, m_1, d_0, R_0 are undetermined constants. Hence, the answer to the second question is:

This global distribution is necessarily fractal with $D = 2$. Time is likewise derived from the displacements of the particles dr, defining the time interval:

$$dt^2 = < dr \mid dr > / v_0^2 \,. \tag{17}$$

Here, parameter v_0 is not a particle velocity but a universal conversion constant linking the time and space quantities. This universe consists of photon-like mass particles with constant (essentially non-zero) velocity and resembles the Bose-Einstein ensemble. The author concludes that:

> *This result is to be compared with the distribution of galaxies in our directly observable universe, which approximates very closely perfectly inertial conditions, and which appears to be fractal with $D \approx 2$ on the small-to-medium scale at least. If we make the simple assumption that the distribution of ponderable matter traces the distribution of the material vacuum then, given the extreme simplicity of the analyzed model, this latter correspondence between the model's statements and the cosmic reality lends strong support to the idea that our intuitively experienced perceptions of physical space and time are projected out of relationships, and changing relationships, between the particles (whatever these might be) in the material universe in very much the way described.*

This task of inferring the dimension of space via the matter distribution is only a part of the general approach of determining the structure of space-time according to the structuring of the inert matter. The first step was made by Einstein with his idea that it is the matter which determines the global properties of space-time as Euclidean (flat) spherical (closed universe) or hyperbolic ones (open universe) (see e.g. Peebles 1993, Narlikar 1977). We mention that recently, Lu and Hellaby (2007) started the programme of determining numerically the metric of cosmic space within the Lemaitre-Tolman-Bondi model (Peebles 1993) from the available cosmographical data, which should enable one to test the homogeneity assumption rather than to postulate it. The task appears very ambitious indeed but more definite results are expected in the future.

4.4. Dark matter and dark energy

So far we were dealing with the visible cosmic matter which has been estimated to comprise only 5 percent of the total content of the universe. Two other invisible components have been hypothesized, the dark (nonbarionic) matter and dark energy, which share the material content by approximately 0.22 and 0.73 amount respectively (see e.g. Schneider 2006, Spergel 2003). Any realistic proposal for the cosmic structuring must account for these constituents (Schneider 2006, Crittenden 2008, Sahni 2004). Both hypothetical components are usually introduced separately or *ad hoc* but attempts are made to formulate a more general theoretical framework from which the invisible part of the matter emerges (see e.g. Quercellini et al. 2007). Attempts to couple both components are made (Zhao and Li 2008), including the principle of holographic sector (Zimdahl and Pavon 2007).

4.4.1. Dark matter - diabolo ex machina

Dark matter may play significant role in astrophysics in many respects including its influence on structuring the overall inert matter. However, although significant inference into dark matter distribution on small scale, such as that of galaxies, has been achieved (see e.g. Schneider 2006), little is known about the cosmic scale distribution (see e.g. Mureika 2007 and references therein). At present, there are indications that dark matter is also structured according to the fractal pattern with fractal dimension $D_{DM} \approx 1.5 - 2.5$.

Surely dark matter must play a significant role not only within galaxies, but on large scale too (see e.g. Oldershaw 2002). Thus Carati, Cacciatori and Galgani (2009), investigating possible influence of the remote cosmic matter on the test particle, find that the nonvanishing contribution is possible only if discrete and fractal matter distribution is accounted for (Carati et al. 2009). It should be noted here that many approaches to the cosmology do not assume dark matter or/and dark energy as a necessary ingredients, like the quantum cosmology due to Cahill (2007).

Whatever this constituent consists of, it must be the source of gravitational force and thus interact with the rest of the inert matter. The question arises as to the possible structure of this invisible (nonemitting electromagnetic radiation) matter. There is no *a priori* reason for dark matter to differ in this respect from the ordinary component what would imply that if the cosmos is fractal at least partly, dark matter must be fractal too. Moreover, some researchers argue that it was the primordial dark matter which triggered the initial density fluctuations which have grown to the present day observed (fractal) structure (see e.g. Francesco 2008 and references therein). Anyway, considering that dark-matter component dominates the matter sector, it is this hidden part of the universe which governs the overall matter distribution and thus the structure of the visible cosmos (Schneider 2006).

4.4.2. Dark energy - $\Theta\varepsilon o\zeta\ \alpha\pi\emptyset\ \mu\epsilon\chi\alpha\nu\varepsilon\zeta$

This phantom constituent of the universe appears legitimate hair of the (in)famous Einstein's *cosmological constant* Λ. The latter was invented to protect the universe from the (gravitational) collapse which followed from the General relativity (GR) fundamental equation (see e.g. Peebles 1993, Narlikar 1977, Nussbaumer and Bieri 2009). This mathematical barrier to the inherent instability of self-gravitating systems was easy to put by hand but difficult to interpret in physical terms. Or to put it in Mark Twain's terms, nothing easier than that - there exist tens of various "physical incarnations" of this cosmic entity. The most popular present day interpretation imagines the dark energy constituent as a homogeneous fluid, which engenders repulsive force at large distance so that the other material inert constituents repel each other at large mutual separations causing the overall cosmic expansion. The concept suffers a number of deficiencies and is still the subject of extensive speculative investigations. One of these deficiencies is the property of the dark energy to cause the expansion of the Universe, but remaining unchanged itself, in particular keeping the same density. Likewise, it is postulated that the energy causes inert matter to move without influencing the energy which violates Newton's postulate on action and reaction (see e.g. Fahr and Heyl 2007, Bean et al. 2008). Further, all proposed candidates fail to account for the gravitational properties of the vacuum energy (Durrer and Maartens 2007, Elis et al. 2010; see also Thomas 2002).

Two principal questions are posed concerning the hypothetical dark energy: (i) Is it really necessary to invoke such an entity in order to explain the observed cosmic acceleration and (ii) What might be the nature of such constituent and mechanism which supposedly drives the expansion.

The answer to the first question has reached by now an affirmative consensus (see e.g. Schneider 2006 but see White 2007, Ishibashi and Wald 2006 for a concise discussion of possible alternatives), whereas the debate as regards the physical nature of dark energy still goes on (see e.g. Medved 2008, Nayak and Singh 2009). It should be mentioned, however, that a number of authors still question the cosmological origin (or significance) of the observed red shifts which are the central data or the primary source of the observational cosmology. We shall not, however, dwell on it here.

The concept of dark energy (DE) provides the means for the observed acceleration of the visible cosmos but equally puts some restrictions on the possible processes of forming cosmic structures including the fractal one. It has turned out that the presence of DE implies the horizon, with the proper distance to this given by:

$$R_h = \frac{c}{H_0\sqrt{1-\Omega_m}} \approx 3.67\,\mathrm{h}^{-1}\,\mathrm{Gpc},\ \ (\Omega_\Lambda + \Omega_m = 1), \tag{18}$$

where H_0 is the present-day Hubble constant and the fraction of matter sector has been taken $\Omega_m \approx 1/3$ (see e.g. Sahni 2004, also Quercellini et al. 2007).

The presence of the event horizon implies that there exists a "sphere of influence" around any cosmic body with its proper z_h. For those parts of the universe with $z > z_h$, which are causally disconnected, signals appear unreachable from now on. In the case of Cold Dark Mater cosmology with nonzero Λ (ΛCDM) and $\Omega_\Lambda \approx 2\Omega_m \approx 2/3$ we have $z_h \approx 1.8$. According to $N - body$ simulations of the evolution of $LCDM$ universe, it turns out that after 10^{11} years our universe will consist of our Milky Way and Andromeda galaxy only (Sahni 2004). Since the growth of an accelerating universe is frozen, the evolution of mass distribution will stop after approximately $3 \cdot 10^{10}$ years. Hence, formation of the hierarchical levels may stop after a finite elapse of the cosmic time. Of course, all these considerations refer to a particular cosmological paradigm and there are many other models without event horizon, even without DE. We mention here that event horizons are excluded within the string theory, for there they prevent formulation of the $S - matrix$, which is fundamental for the Quantum mechanics.

Many elaborate and sophisticated calculations have been carried out describing the role of the dark energy in the form of various physical fields. To us two questions are of interest here. First, is the concept of this new constituent of our Universe compatible with the fractal model and second, what would be mechanism of coupling the fractal structure with the fluid which, presumably, fills the entire cosmic space.

It turns that not only that the fractal structure goes well with dark energy, but one may easily explain the observed acceleration of the global expansion. We have considered a simple Newtonian model for this matter and have shown that Charlier's model (Charlier 1922) can provide a convenient means for driving the cosmic expansion (Grujic 2004). It relies on the fundamental interaction between inert matter and the surrounding fluid according to Archimedes' hydrostatics. The former experiences anti-gravitational force due to the higher density of the surrounding fluid. These calculations are of qualitative nature, however, and further elaborations are still to be made.

5. THE ORIGIN OF STRUCTURING

The issue of forming fractal (and any other, for that matter) structure has two principal aspects. First, from the epistemological point of view, proposing a convincing mechanism of forming hierarchical (and any other) structure greatly enhances chances that this structure is accepted by the cosmological community. Besides, this issue is a part of the general question of how the structuring process began at the early stages of the universe evolution (ontological aspect). Both aspects are still in the early stages of investigation and it would be premature, if not overambitious, to claim any mechanism to be reliable. In other words, we still have no standard theory of the structure formation. What does not mean there are no attempts to formulate mechanisms which could have brought us to the present cosmic situation.

Generally, two principal classes of approaches have been discerned at present: (i) Formations of small, elementary units, which than join each other in forming larger subsystems and so on (bottom-up approach), (ii) Formation of the bulk cosmic supersystem, which then disintegrates into ever smaller parts (up-down approach) (see e.g. Grujić 2002 and references therein). At present, the first (bottom-up) mechanism appears preferred (see e.g. Yoshida 2009, Mureika 2007).

According to the prevailing Standard Model (Collins et al. 1989) about 380.000 years after the Big Bang and the inflationary phase, after the recombination the Universe became transparent to the electromagnetic radiation. The latter remained unchanged bringing the information *via* Cosmic Microwave Background radiation (CMB) about the initial conditions for processes for the more complex structures to form. The latter process lasted millions of years (possibly billion years) when galaxies and other complex constituents of the present day cosmos have been formed. This period of the structure formation appears the least known (Cosmic Dark Age) and is the subject of intensive investigations at present. The general picture assumes first formation of stars, which then form ever larger structures, like galaxies, clusters, superclusters etc. In fact, both approaches, the bottom-up and up-down imply hierarchical scheme. The question arises, therefore, whether this procedure leaves signature on the present matter distribution.

5.1. Hierarchical cosmos

5.1.1. Why hierarchical cosmos

This question posed above invokes, further, a more general one: Why should the Universe be structured hierarchically, irrespective of the mode of evolution? The answer to this appears easy, in fact too easy, if not trivial. Nature reveals to us (at present) four fundamental forces: gravitational, electromagnetic, strong (nuclear) and weak interactions. Of these only the first one, the gravitational appears operative at the large, cosmic scale. Until the quantum gravity (or something similar) is contrived, gravitation remains the classical interaction, that is, no quantization rules, no natural units ascribed to gravitating systems. True, there are some specific critical quantities, like Chandrasekhar mass (see e.g. Schneider 2006), but these do not affect the overall cosmic structuring. Hence, from planets and stars upward any quantity of mass should be treated on equal footing. In other words, any subsystem of gravitating bodies may be taken as a unit of a larger system. And it is exactly the rationale for modeling hierarchical systems. By picking up a predetermined subunits of mass M_i we single out a particular $i-th$ level of a hierarchical system. How much one may go up or down along this scheme? There is no *a priori* upper limit, whereas the lower one is determined by the dominance of non-gravitational interactions (from planet-size bodies downwards).

We mention here that there have been attempts to push the hierarchical scheme down to atomic and even subatomic levels (see e.g. Oldershaw 2005 and references therein) but not many cosmologists are convinced about it.

5.2. Modeling fractal cosmos

It is the standard procedure of modeling (astro)physical systems to set up guiding principles, on which one elaborates further details, which distinguish one model from the other. The Standard Model is based on the Cosmological Principles, explicitly or not (see e.g. Peebles 1993, Grujic 2007). We quote two of them:

(i) *Strong cosmological principle* - universe is the same everywhere and at any (cosmic) time.

(ii) *Weak cosmological principle* - universe is the same at each point at a fixed cosmic instant.

Einstein's first cosmological model (static solution of GR equation) implied, albeit implicitly, the Strong Cosmological principle, whereas later modifications due to Friedman, Lemaitre and others, assumed Weak Cosmological principle (dynamic universe).

Both principles may bee reformulated as (i) Distribution homogeneous in space and time; (ii) Distribution uniform in space.

If one treats the elementary constituents of the cosmic matter, galaxies, as massive points, neglecting other degrees of freedom, like (rotational) angular momentum, uniform distribution (even density) implies the cosmic isotropy: universe looks the same whatever direction one chooses for the line of sight. The opposite need not be true: if the universe appears isotropic, it does not follow necessarily it is homogeneous. The fractal cosmos is just the case in point.

In 1930-thies Weyl suggested something that Mandelbrot dubbed later (1983) as the Conditional Cosmological Principle (Mandelbrot 1983):

(iii) *Conditional Cosmological Principle* - universe looks the same seen from any occupied point.

The emphasize here is on "occupied", for Mandelbrot proved that in the case of a fractal structure cosmos appears isotropic observed by a comoving observer from any constituent (which precludes observers from a void), and empty observed from any unoccupied point of space. In the cosmological context, "occupied" means "from a galaxy" not from a point of a cosmic void. This property bears, besides the practical significance (ontological aspect), a remarkable epistemological implications, including the anthropic principle (see e.g. Barrow and Tipler 1986), but we shall not dwell on it here.

5.2.1. Fractal models

As we have seen before, fractality can not be reached by assuming the standard statistical tools which make use of the notion of mass density, for instance. Generally, if a radically new structure is to be designed or revealed, an appropriate set of initial assumptions and principles are to be adopted (Gabrielli et al. 2005; see also e.g. Gaite and Dominguez 2006, Baryshev et al. 1995 for recent reviews).

Mittal and Lohiya (2003) made an attempt to arrive at the fractal cosmic distribution starting from the Conditional Cosmological Principle and assuming from the beginning the fractal cosmic geometry. Defining "hipersurface of homogeneous fractality of dimension D", the authors show that if from any (occupied) point the mass within the sphere around the point increases as R^D, the dynamics of the cosmological scale factor $a(t)$ is described by the same equation as in the standard model with the effective homogeneous density. Hence, the model should be treated on equal footing as the standard model.

In an ambitious approach to an inhomogeneous universe without intrinsic symmetry (making use of the Stephani model), Pompilio and Montuory (2001) examine the case of fractal distribution with specific metric constraints. Abandoning the Copernican principle the authors find that such a universe would undergo a nonuniform expansion. Accounting for the acceleration it turns out that MCB uniformity follows just as in the case of the standard models, like those based on the inhomogeneous metric, like Lemaitre-Tolman-Bondi (LTB).

In a recent paper Calcagni (2009) proposes a field-theoretic approach within fractal spacetime (see also Modesto 2009 where essentially the same results, obtained within the Loop Quantum Gravity approach, have been presented). The system flows from a fixed point within a spacetime of Hausdorff dimension 2, ending in the standard four-dimensional field theory with $D_s = 4$ as the energy decreases. In the classical limit, such systems dissipate energy-momentum in the bulk of integer topological dimension preserving the total energy-momentum. He considers a number of implications of the model, including the cosmological ones, but the work has more heuristic than practical aims. On the contrary, Grujic and Pankovic proposed an analytical fitting formula to account for the evolution of the fractal dimension, as one moves from the close vicinity ($D_f = 1$) to the outer space where the fractal dimension is supposed to become $D_f = 3$ (through $D_f = 2$) (Grujic and Pankovic 2009).

According to the General Relativity (GR) requirements all observations concerning deep space should be carried out along the past light cone. Ribeiro and coworkers (Abdalla et al. 2001) have continued their investigations of the possible outcomes of observing the cosmic structure within this spacetime hypersurface. They find that the so-called apparent fractal conjecture is valid for the perturbed Einstein-de Sitter cosmology (which assumes that the pressure is much smaller than the density (cosmic dust) and space curvature and cosmological constant are zero (Peebles 1993, p. 101) where the dust distribution has a scaling behaviour compatible with the power-law profile of the density-distance correlation, as observed in the galaxy catalogues.

5.2.2. Methodological remarks

As pointed out in (Gabrielli et al. 2005) statistical analysis of inhomogeneous systems, particularly those suspicious to be fractal, must abandon the standard approach which deals with quantities like the average density of the system. Equally, simulation of forming hierarchical structure from a homogeneous substance proceed via the statistical methods appropriate to the nonstandard analysis. Pietronero et al. (2001) carried out simulation of forming structure of selfgravitating system of particles, making use of the function:

$$\Gamma^*(r) = 3\frac{\langle N(< r)\rangle}{4r^3\pi} = \frac{3B}{4\pi}r^{D-3}, \qquad (19)$$

as the conditional average density within the sphere of radius r, where N is the number of galaxies within the sphere and B depends on the lower cut-off. They found the system evolution to proceed towards ever increasing particle concentration into larger clusters, with average density decreasing correspondingly.

5.3. Empirical evidence

Though the old thesis *micrososmos* $\sim$ *macrocosmos* may not be valid, some features of microworld and universe appear common in the epistemological or methodological sense. In both cases direct observations are difficult, if not impossible and an extensive use of models is required in order to get insight into the objective nature of the object to study. The common feature of the research on both scales is the ruling maxim: to notice means to recognize. In the case of the cosmic structure if a particular structuring, different from the uniform galaxy distribution, is to be observed, a proper methodology is to be applied. Since in the case of large-scale structure no quantum effects are to be expected, any particular irregularity must inevitably be approximate. Which implies that if it exists, it is not easy to observe.

The principal problem in estimating a three-dimensional distribution is this very three-dimensionality. The problem reduces mainly in estimating the (radial) distance of remote galaxies, clusters, superclusters etc. This was the central issue of estimating the relationship between the observed spectral shift and the distances of the galaxies which was essential for formulating the famous Hubble law (but see e.g. Nussbaumer and Bieri 2009). The inference of the real large-scale structure is searched from the existing catalogues as primary sources for conceiving cosmic models. It is the application of different methods in analyzing the astronomical data which lead to various interpretations and thus to different models cosmologist choose to ascribe to the observable cosmos. These may be classified into three categories. (i) Those who (often vehemently) deny the fractal (and any hierarchical) structuring, (ii) Those who claim they observe fractal structure, at least up to a certain cosmic distance, if not up to the entire observable universe, (iii) Sceptics, who do not deny hierarchical pattern but do not accept a clear-cut fractal ansatz, allowing for a possible multifractal case instead. We shall quote some of those inferences and interpretations.

Teerikorpi et al. (1998) examined the radial space distribution of KLUN-galaxies up to 200 Mpc and found the data compatible with $D_f = 2.3$ and $D_f = 2.0$. The authors conclude that if $D_f \approx 2$ beyond 200 Mpc the position of our Galaxy would be close to the average in the Universe. Baryshev and Bukhamastova (2004), making use of the two-point conditional column density, estimated from samples LEDA and EDR SDSS $D_f = 2.1 \pm 0.2$ for cylinder lengths of 200 Mpc.

Searching for super-large structure in deep galaxy surveys Nabokov and Baryshev (2010) carried out extensive, multi-colour deep survey of galaxies. With photometric redshift accuracy $0.03(1 + z)$ they found that the distribution in deep surveys like HUDF and FDF is consistent with the existence of super-large structures of luminous matter up to 2000 Mpc.

Many cosmologists, starting with de Sitter (1916, 1917) considered the possibility that the observed cosmological redshift might be of gravitational origin. Baryshev (2008) examined the consequences on the redshift-distance relation for a number of matter distributions. In the case of a homogeneous Universe this relation should be $z_{\rm grav} \sim r^2$, whereas for the fractal distribution with $D_f = 2$ one has $z_{\rm grav} \sim r$. The author argues that the field gravity fractal model could explain satisfactory all three observed phenomena: the cosmic background radiation, the fractal large scale structure and the linear Hubble law, starting with an evolution of an initially homogeneous cold gas.

In their extensive study of DR5 Sloan Digital Sky Survey, Thieberger and Celerier (2008) examined the sample of 20.000 galaxies extracted from the catalogue of 332.876 galaxies. With the choice $H_0 = 70\,{\rm kms}^{-1}{\rm Mpc}^{-1}$ and maximum distance $d_{\rm max} = 160$ Mpc, the authors carried out correlation dimension calculations which allow for determining the transition scale to eventual homogeneous distribution. Their analysis reveals an increase of the correlation dimension up to $D_2 = 3$ around 70 Mpc. Nevertheless, the authors expect data from larger catalogues, when available, to provide more reliable estimate.

In Francesco et al. (2009) an analysis was carried out of the latest Sloan Digital Sky Survey up to 30 Mpc/h, and large amplitude density fluctuations are found. The two-point correlations appear self-averaging, providing the fractal dimension $D_f = 2.1 \pm 0.1$. The authors find that at larger scales density fluctuations appear too large in magnitude and too extended in space to be self-averaging inside the considered volume. The authors argue that these inhomogeneities are compatible with the fractal correlations but not with the standard theoretical models for the scales lower than 100 Mpc/h.

Antal et al. (2009) carried out statistical analysis of the fluctuations in samples of the Sloan Digital Sky Survey Data Release 7 making use of the conditional galaxy density around each galaxy. They find that the distribution appears clearly distinguishable from a homogeneous spatial galaxy distribution.

Cosmic voids may have fractal distribution with respect to their dimension. According to some surveys this distribution implies $D_f = 2$ (Gaite and Dominguez 2006), but the available data are still inconclusive.

5.3.1. 2 be or not 2 be

The search for actual value of D_f has a specific heuristic motivation, besides the practical interest. Theoretically fractal dimension of the galactic large-scale distribution lies within the interval $(0, 3)$ As we have seen above, theoretical considerations single out $D_f = 2$ as of a particular significance. One of the most significant attributes of the proposed hierarchical universe by Charlier (1922) was the resolution of the cosmic paradoxes, Olbers' (luminosity) (Overduin and Wesson 2008) and the Neumann-Seeliger (gravitational) paradox. Newton was aware of the latter within his concept of an infinite universe where every (gravitating) body was to be subjected to possibly infinite gravitational force. Charlier found that both paradoxes disappear if $D_f \le 2$.

Equally, fractal systems cast their projections onto a plane, as clouds do with their shadows on the surface of Earth. If the fractal dimension of a cloud is $D_f \ge 2$ this shadow is compact, without sunny regions inside. Further, it has been shown that for $D_f = 2$ the linear $z - d$ relation holds (see e.g. Baryshev 1994 and references therein).

At a more fundamental level the fractal dimension $D_f = 2$ appears of the most significant importance. It is not well know that the famous Feynman's line integral formulation of Quantum Mechanics was inspired by Dirac's work (1933) (see, e.g. Park 1979). Since it was published in a less known Soviet journal (Dirac was an eager supporter of the new communist regime in Russia, as many western intellectuals were at the time, in particular following his close friendship with Piotr Kapitza), his idea was ignored by the scientific community, until Feynman realized its value, as he himself used to acknowledge. Elaborating the role of paths in the phase-space, Feynman found that these paths are continuous non-differentiable curves (fractals) and that the most significant contribution to the part integral comes from those paths with $D_f = 2$ (Nottale 2011). In view that line-integral formulation of Quantum Mechanics appears the closest to the classical physics, this feature of the Feynman formulation gains remarkable significance. In fact as Nottale has demonstrated (Nottale 2011) that Schrödinger equation can be derived by scaling transformation, concept of fractality with $D_f = 2$ turns out of a particular importance.

We notice here that many theoretical workers have found $D_f = 2$ fractal dimension natural choice regardless of their models and initial approaches (Calcagni 2009).

On the observational side, all examinations of the catalogues available which have discerned hierarchical distribution argue for $D_f = 2$ dimension (Sylos Labini et al. 2009, Gaite and Dominguez 2006, but see Thieberger and Celerier 2008). That the Universe appears more homogeneous as the scale is enlarged speaks in favour of the bottom-up models, which assume an initial homogeneous distribution with gravitational force forming mass concentrations.

It appears indicative that the actual fractal dimension turns out an integer, one unit smaller than the topological dimension of the Universe. It remains to be seen if this fact has a deeper meaning.

6. EPILOGUE

Observational evidence suggests that our universe displays structuring at various scales. At the lowest level we have stellar systems in the form of galaxy, as our own Milky Way. At a larger scale structural units like voids, filaments and sheets can be discerned in the sky. The question of a regular structure on even higher level has been puzzling astronomers and cosmologists from the beginning of last century. The hierarchical model has been considered as the best candidate for the cosmic regular structure and a lot of theoretical investigations have been devoted to this cosmological paradigm. On the other hand, the current observational evidence can not yet decide between the standard assumption of a uniform galaxy distribution and any other particular structuring, like the fractal paradigm. The latter has a particular intrinsic value not only within cosmological considerations as it appears ubiquitous in nature.

The issue of the hierarchical structuring appears everything but simple. At the lowest level, that of galactic systems, we encounter a diversity of these elementary objects, both concerning the type and/or size. Further, the universe appears a dynamical system and when talking about its structure the issue of the initial conditions arises, as well as the forces which govern the subsequent development of the system. If one adopts the standard Big Bang paradigm, the structuring is determined by the interplay between the overall global dynamics (Hubble flow) and the universal gravitational interaction. It is the latter which makes the set of elementary units a physical system, and not just a collection of noninteracting constituents.

We have argued that the gravitational interaction results naturally in the form in hierarchical structure. But this structure need not be simple, first of all because the units are not identical. This fact and the nature of gravitational force, which does not follow quantum laws, deprives the Universe of a clear-cut regular pattern. Hence, the most we may expect is an approximate, multifractal pattern, which is not easy to observe. It is this fact that still keeps the issue *fractal or nonfractal* unresolved yet. The situation resembles somewhat what one may call the *Stonehenge effect*, the best formulated by Jacquetta Hawkes (1963): *Every age has its own Stonehenge* [*interpretation*]. And as today each branch of science sees in this megalith structure different purpose (including astronomical one), so the modern astronomers and cosmologists see in the available catalogues structures they want to see.

It may turn out that the Universe is Cosmos, with discernable pattern of fractal of $D_f = 2$ dimension appeals to human mind as another *World harmony*, as envisaged by Plato and advocated by many philosophers and astronomers (like Kepler). We are facing a metaphysical question: If nature has chosen not to remain at the (sub)atomic level, what prevents it to evolve towards regular pattern at higher levels? Anaxagoras' ideas about *homoeomeries* (Grujic 2001) might at last turn out relevant to our cosmological insights.

Acknowledgements – I am much indebted to Vladan Pankovic for supplying me with numerous references and helpful discussions. Thanks are due to Milan Ćirković for drawing my attention to a number of points and for useful suggestions. This work could never have been done without support of my wife Ljiljana whose help and devotion have been instrumental in writing this article. The work was supported by the Ministry of Science and Environmental Protection of Serbia, under the contract No ON171020.

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КОНЦЕПТ ФРАКТАЛНОГ КОСМОСА: III. САДАШЊЕ СТАЊЕ

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УДК 524.88
Прегледни рад по позиву

Ово је наставак серије о настанку и развоју концепта фракталног космоса, до 2001. године. Овде дајемо преглед садашњег стања, са нагласком на последњи развој и контроверзе око модела хијерархијског свемира. Описаћемо теоријске помаке као и данашњу емпиријску евиденцију која се тиче структуре на великој скали опсервираног свемира. Најзад размоprићемо и нека од епистемолошких питања, стављајући фракталну пардигму у шири космолошки контекст.