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Book review

THEORY OF ORBIT DETERMINATION
Andrea Milani and Giovanni F. Gronchi, "Theory of Orbit Determination", Cambridge University Press, 2010

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If you feel the need for a rational, rigorous, and didactically effective new presentation (p. ix) of the theory of orbit determination then you are a potential reader of this book.

The scope is very broad, spanning from satellite geodesy to planetary systems and proper asteroid orbit determination. Part I (chap. 1-4) contains basic text book material. Chap. 5 and 6 contain the statistical theory of observations. Part III (chap. 7-12) contains the proper orbit determination starting from linking the basic (α, δ)-observations to the derivation of initial orbits. This is a main subject due to the huge increase of observational data in recent surveys. The methods have been tested successfully on millions of simulated observations. Part IV (chap. 13-17) considers tiny effects such as light pressure, aerodynamic drag, relativity and higher harmonics in gravitational fields. Accuracies for artificial satellites observed by radio are at the marvellous ± 1 cm level.

"This book is about making widely available the outcome of the research done by my group over many years" (pag. ix). The results of this group, headed by Andrea Milani, are indeed very striking. The book indicates what should be done nowadays and is indispensable at every library.

Some methods – and even the vocabulary – are particular for this group. The mathematical style introduces new words, like for instance: "attributable time", "a priori penalty", and "triangulated ephemerides". The familiar, 200-years old, "daily motion" is no good anymore and the important concept of Väisälä orbits is also not mentioned at all. This does not facilitate the reading for an uninitiated.

Astronomers have been used to textbooks from those by for instance Watson and Stracke to Herget, just to mention a few, that have the theory clearly explained and illustrated by numerical examples which remove all doubts of what is going on and allows the reader to repeat the computations by himself. This is rarely the case in this book.

Chap. 8 considers the problem of follow-up of two positions observed within a time interval h, where h is of the order of 1-2 hours. The problem is to predict the area in which additional positions can be secured T days later, where T is of order 1-10 days. If the observational mean error is σ, then the accelerations determining the geocentric distance Δ have large errors proportional to σ/h^2. Instead, several trial values for Δ and $\mathrm{d}\Delta/\mathrm{dt}$ are guessed in the so called "admissible region" by a complicated "Delaunay triangulation". For each assumption the position is predicted for a follow-up image frame at time T. It has been shown (Icarus **159** (2002) 339-350) that for each adopted value of Δ the indeterminateness of the velocity along the line of sight gives a segment of a great circle bounded by the energy E = 0. This is where a comet would be found as illustrated by Fig. 3 in this paper. This figure is very similar to Fig. 8.4 in the book where the points on the straight line between nos. 1 and 19 correspond to the constant value $\Delta = 0.4$. Advantage has not been taken of this simple property that a constant value of Δ gives a straight line.

When a possible candidate is found for linking, the accuracy of accelerations is increased by an order of magnitude to σ/hT, which is still too large for orbit determination. Although the arc is now of the order T, the distribution of the observations is far from optimal. The acceleration along the motion may, however, give an indication of $\mathrm{d}\Delta/\mathrm{dt}$, so the range of solutions is a one dimensional manifold depending on Δ. This is illustrated by the straight line in Fig. 8.5, or formulae (8) in the above mentioned paper (if we assume $\mathrm{r}^3 \gg 1$). To find the third position we still have to assume a few trial values for Δ. With a third position, the errors are once again reduced by an order of magnitude to σ/T^2 and elliptical elements make sense. This method is essentially the "generalized Väisälä method", but this is not indicated.

It seems that the preferred method of orbit determination is first to determine a preliminary orbit from three observations by the method of Laplace, and then to improve it by differential corrections. The Laplacean way is, however, strongly criticised by the authority Brian Marsden in Astron. Journ. **90** (1985) 1541-1547. This method is based on the expansion of the geocentric positions in powers of time to the order t^2. The observed positions α and δ are, however, affected by the short period parallax term of order 8.8"$\cdot\sin 2\pi\mathrm{t}/\Delta$ which can not be so expanded.

The so called "dynamical equation" of orbit determination requires the transverse acceleration c of the geocentric motion. The coefficients in the expansion in t must then be obtained by a separate least squares adjustment of the positions and their parallax factors and is thus a sum of two terms, one with the unknown factor $1/\Delta$. The geocentric t^2 term is most easily obtained by correcting for parallax by assuming $1/\Delta = 0$ or 1, which gives respectively the transverse accelerations c(0) and c(1). The dynamical equation is then:

$$\mathrm{c}(0) + (\mathrm{c}(1) - \mathrm{c}(0))/\Delta = \mathrm{k}^2\mathrm{R}(1/\mathrm{R}^3 - 1/\mathrm{r}^3)\sin\psi/\Delta$$

where ψ is the distance from the anti-Sun to the osculating great circle and R and r are heliocentric distances of the Earth and the asteroid respectively. This gives the polynomial equation of degree 8 from which the solution is obtained.

This, seemingly simple, problem occupies several pages in Chap. 9. Topocentric corrections to the method of Laplace has been implemented somehow recently, but its practical advantages are not yet assessed (p. 182).

Conclusion: The book is very valuable by pointing out the kind of problems which must be handled nowadays, so every library should have a copy. The computational results are very impressive, but the question is whether they are due to sheer computing power or rational methods. The above examples using the Laplace method, based on the apparent motion affected by the short period parallax, and the handling of Väisälä orbits seem not to be optimal. It never was the intent to discuss methods of other authors (p. ix) and the style is abstruse, consequently it is not a text-book for beginners and students nor a hand-book. The need for an up-to-date, rational and didactical book remains but the present attempt should be properly appreciated.

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