THEORETICAL STUDY OF IRRADIATION EFFECTS IN CLOSE BINARIES

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SUMMARY: The effect of irradiation is studied in a close binary system assuming that the secondary component is a point source, moving in a circular orbit. The irradiation effects are calculated on the atmosphere of the primary component in a 3-dimensional Cartesian coordinate geometry. In treating the reflection effect theoretically, the total radiation (S_{T}) is obtained as the sum of the radiation of 1) the effect of irradiation on the primary component which is calculated by using one dimensional rod model (S_{r}) and 2) the self radiation of the primary component which is calculated by using the solution of radiative transfer equation in spherical symmetry (S_{s}). The radiation field is estimated along the line of sight of the observer at infinity. It is shown how the radiation field changes depending on the position of the secondary component.

Key words. Radiative transfer – binaries: close

1. INTRODUCTION

The atmosphere of a star with a close companion is influenced by two interaction mechanisms. The gravitational interaction results in distortion of the outer layers of the star and the radiative interaction results in the warming of those surface layers of the star which gives rise to reflection effect. The problem of incidence of radiation from a point source is termed the search light problem and was studied by many authors. Peraiah (1982) studied the problem of incidence of radiation from a point source in a binary system and noticed that the middle layers of the atmosphere of the primary component produce the maximum radiation when compared to the inner and outermost layers. Peraiah (1983) also extended this work to the case when secondary component is an extended source in a close binary system. Here he found that outer layers of the atmosphere of the primary component produce maximum radiation as

compared to the inner layers. In a series of papers, Peraiah and Srinivasa Rao (1983), Peraiah and Srinivasa Rao (1998), Srinivasa Rao and Peraiah (2000) and Srinivasa Rao (2003), the effect of reflection on the spectral lines accounting also for the presence of dust was studied. They noticed considerable change in the equivalent width of the line with and without reflection. They also mentioned that reflection effect has to be considered in modeling of the binary systems.

Theoretical studies on the reflection effect by authors like Chandrasekhar (1960), Buerger (1969, 1972) were based on a plane parallel approximation in computing the continuum radiation and line radiation emitted by rotationally and tidally distorted star irradiated by the light of secondary component. Many authors have dealt with on the theory of reflection (see Vaz 1985 for a historical review), including not only contributions to the geometrical and bolometric radiation parts of the problem, but also

those on irradiated stellar atmospheres and on estimation of the bolometric albedo. Vaz and Nordlund (1985) investigated the reflection effect in model atmospheres by introducing an external radiation field with plane parallel approximation. They verified the results in radiative equilibrium for grey atmospheres with exact solutions. For these atmospheres they also studied limb darkening changes due to illumination and suggested the reduction of limb darkening as a function of frequency, incidence angle and relative incident flux. Similarly, Nordlund and Vaz (1990) studied irradiation effects in model atmospheres investigating the dependence of reflection effect on the abundance of heavy elements, on the strength of convection and on the properties of incident radiation flux. Wilson (1990) discussed various classes of close binary systems that require a detailed treatment of reflection effect. The present work implies that, even in systems where the radiative interaction creates only a small change in the atmospheric structure, the variation of emitted flux can be strongly affected. Alencar et al. (1999) studied the influence of reflection effect on gravity brightening exponent (β) using UMA (Uppsala Model Atmosphere) code. They showed that the external illumination increases the value of β. In the case of non-grey atmospheres the gravity brightening exponent depends on the spectral distribution of the incident flux. Recently, Pustynski and Pustynlik (2005) made a detailed study on upper atmospheres of PCB's (Precataclysmic binaries) which are detached binary systems. They used Kubat et al. (1999), Kubat (2000) model calculations for irradiated B type stellar parameters and noticed that the UV radiation penetrates into the upper atmosphere up to a certain optical depth while ionization degree remains close to unity.
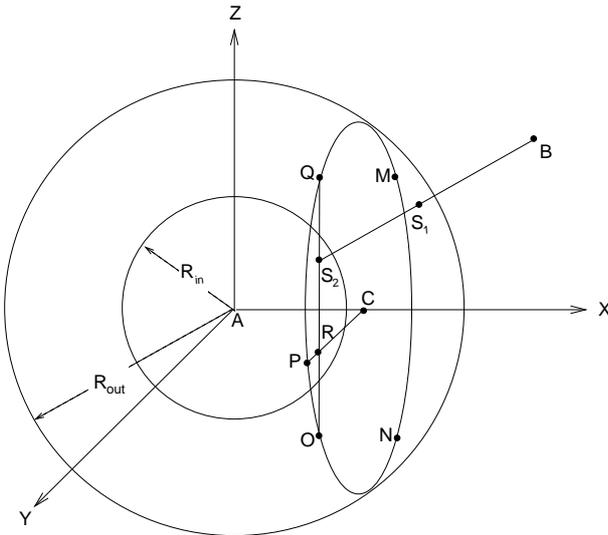
The implications of the present work contain no substantial surprises, but represent a further improvement of understanding through the creation of better and more accurate models of physical systems. With reference to the above studies, it supports the notion that the reflection effect is important in close binary stellar atmospheres. In this paper we study the irradiation from the secondary component in 3-dimensional geometry.

2. METHOD OF CALCULATION

We have performed all the calculations in the 3-dimensional X-Y-Z Cartesian geometry as shown in Fig. 1. We assume a spherical shell of the primary star with inner and outer radii R_{in} and R_{out} respectively. The center of the star is at the origin of the coordinates. It is assumed that radiation is incident from a point source at B moving on a circle of radius R in the X-Z plane. The radiation field reflected from the spherical shell of the primary component is calculated. The shell is divided into several circular slices with their centers lying on the X-axis. The selected slice has been divided into line segments for calculating transfer of radiation. Now let us consider such slice MNPQ and the transfer of radiation along the lines such as QS_2RO.
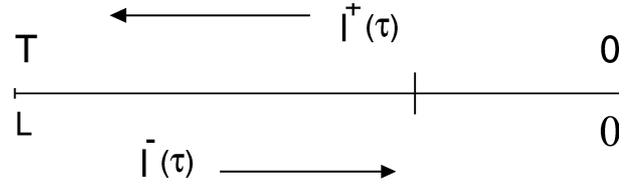
T = Total optical depth

L = Total geometrical depth

Fig. 2. *Schematic diagram showing the rod model.*

It is assumed that the secondary component is situated at point B and a ray from B intersects the outer surface of the primary component at S_1 and passes through the point S_2 on the line QS_2RO. Let the coordinates of the points S_2 and B (in Fig. 1) be (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively. The equation of this line is given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}. \qquad (1)$$

By knowing the coordinates of the points S_2 and B in advance, the coordinates of the point S_1 are obtained by solving Eq. (1) and the equation of the sphere, whose center is at A. The equation of the sphere is given by

$$x^2 + y^2 + z^2 = R_{out}^2. \qquad (2)$$

Fig. 1. *The schematic diagram of the irradiation in 3-dimensional Cartesian co-ordinate system.*

The lengths of the segments are calculated using the distance formula. We need to avoid all points where the incident radiation does not reach, such as the shadow-cone cast by the central star. We calculate the equation of the cone enveloping the sphere

$$x^2 + y^2 + z^2 = R_{in}^2 \qquad (3)$$

and this is given by

$$(x^2 + y^2 + z^2 - R_{in}^2)(x_2^2 + y_2^2 + z_2^2 - R_{in}^2)$$
$$-(xx_2 + yy_2 + zz_2 - R_{in}^2)^2 = 0. \qquad (4)$$

The points that lie in the shadow of this cone should satisfy the relation given by Bali (1988)

$$(x^2 + y^2 + z^2 - R_{in}^2)\ (x_2^2 + y_2^2 + z_2^2 - R_{in}^2)$$
$$-(xx_2 + yy_2 + zz_2 - R_{in}^2)^2 \leq 0 \qquad (5)$$

The points which satisfy above relation are eliminated from the calculations.

The transfered radiation field is estimated along the line segments such as $S_1 S_2$ to obtain the source function along the lines QS_2RO. We used the procedure described in Peraiah (1982). A brief description of the method is given in the following section for the sake of completeness.

2.1. Calculation of radiation field due to irradiation

We assume a steady state, monochromatic ray with or without internal sources. In Fig. 2 it is seen that the optical depth is measured in the same direction as the geometrical depth. For a given density distribution, the optical depth is calculated by the relation

$$\tau = \tau(\zeta) = \int_0^L \sigma(\zeta')d\zeta' \ ; \ \tau(L) = T . \qquad (6)$$

In this model we assume that the transfer of radiation takes place as shown in Fig. 2 (along $S_1 S_2$ in Fig. 1) with isotropic scattering ($\mu = \pm 1$, and $p(\tau)$=phase matrix element= $\frac{1}{2}$). The quantities $I^+(\tau)$ and $I^-(\tau)$ represent the specific intensities in the opposite directions. The two equations for oppositely directed rays are written as

$$\frac{dI^+}{d\tau} + I^+ = S^+, \qquad (7)$$

$$\frac{dI^-}{d\tau} + I^- = S^-, \qquad (8)$$

where

$$S^+(\tau) = \omega(\tau)\Big[p(\tau)I^+(\tau) + (1 - p(\tau))I^-(\tau)\Big], \quad (9)$$

$$S^-(\tau) = \omega(\tau)\Big[(1 - p(\tau))I^+(\tau) + p(\tau)I^-(\tau)\Big]. \quad (10)$$

The total source function, including the diffuse radiation field, is given by

$$S_d^+(\tau) = S^+(\tau) + \omega(\tau)\Big[p(\tau)I_1 e^{-\tau}$$
$$+(1 - p(\tau))I_2 e^{-(T-\tau)}\Big], \qquad (11)$$

$$S_d^-(\tau) = S^-(\tau) + \omega(\tau)\Big[(1 - p(\tau))I_1 e^{-\tau}$$
$$+p(\tau)I_2 e^{-(T-\tau)}\Big], \qquad (12)$$

where $S^+(\tau)$ and $S^-(\tau)$ are the source functions at the optical depth τ, with $\omega(\tau)$ being the albedo for single scattering. We set $\omega(\tau) = 1$, which corresponds to pure scattering in the medium. I_1 and I_2 are the incident specific intensities at the boundaries $\tau = 0$ and $\tau = T$, respectively. For isotropic scattering $S_d^+ = S_d^-$, and Eqs. (11) and (12) reduce to

$$S_r = \frac{1}{2}\Big[I^+ + I^-\Big] + \frac{1}{2}\Big[I_1 e^{-\tau} + I_2 e^{-(T-\tau)}\Big], \quad (13)$$

where T is the total optical depth at the point where the source function is calculated. We set $\tau = 0$ at point S_1 (see Fig. 1) where the incident ray enters the medium, and we set $\tau = T$ at the point S_2 where the source function is calculated. Thus

$$I^+(\tau) = I_1 \frac{1 + \Big[T - \tau\Big]\Big[1 - p\Big]}{1 + T\Big[1 - p\Big]}, \qquad (14)$$

and

$$I^-(\tau) = I_1 \frac{\Big[T - \tau\Big]\Big[1 - p\Big]}{1 + T\Big[1 - p\Big]}, \qquad (15)$$

$$I^-(\tau = T) = 0 = I_2, \qquad (16)$$

$$I^+(\tau = 0) = I_1, \qquad (17)$$

therefore

$$I^+(\tau = T) = I_1 \frac{1}{1 + \frac{1}{2}T}, \qquad (18)$$

$$I^-(T) = 0. \qquad (19)$$

At $\tau = T$, the source function (13) becomes,

$$S_r = \frac{1}{2}\Big[I^+ + I^-\Big] + \frac{1}{2}\Big[I_1 e^{-T}\Big]. \qquad (20)$$

Introducing equations (14) and (15) into the above equation with $p = \frac{1}{2}$, we obtain

$$S_r = \frac{1}{2}I_1\Big[\frac{2}{2 + T} + e^{-T}\Big]. \qquad (21)$$

Using the above analysis we can calculate the source functions along the ray.

2.2. Calculation of self radiation of the primary component

In addition to the irradiation from the secondary component we have the radiation from the primary star itself.

The radiative transfer equation in spherically symmetric approximation reads:

$$\mu\frac{\partial I(r,\mu)}{\partial r} + \frac{1}{r}\frac{\partial}{\partial \mu}\Big[(1-\mu^2)I(r,\mu)\Big]$$
$$= \sigma(r)\Big[S_s(r) - I(r,\mu)\Big], \qquad (22)$$

where

$$S_s(r) = \frac{1}{2}\int_{-1}^{+1} p(r,\mu,\mu')I(r,\mu')d\mu'. \qquad (23)$$

Here $I(r,\mu)$ is the specific intensity of the ray making an angle $\cos^{-1}\mu$ with the radius vector. The quantities $\sigma(r)$ and $S_s(r)$ are the absorption coefficient and the source function respectively and $p(r,\mu,\mu')$ is the phase function (assumed to be isotropic here) normalized in such a way that

$$\frac{1}{2}\int_{-1}^{+1} p(r,\mu,\mu')d\mu' = 1 \qquad (24)$$

where $\quad p(r,\mu,\mu') \geq 0 \quad$ and $\quad -1 \leq \mu, \ \mu' \leq 1$.

2.3. Brief description of the numerical method for solving the radiative transfer equation in spherical symmetry

The solution of radiative transfer Eq. (22) in spherical symmetry is derived by using discrete space theory of radiative transfer (Peraiah and Grant 1973). In general, the following steps are to be followed for obtaining the solution.
(i) We divide the medium into a number of "cells" whose thickness is less than or equal to the critical τ_{crit}. The critical thickness is determined on the basis of physical characteristics of the medium. τ_{crit} ensures the stability and uniqueness of the solution.
(ii) The integration of the radiative transfer equation in spherical symmetry is performed on the "cell" which is 2-dimensional radius - angle grid bounded by $[r_n, r_{n+1}] \times [\mu_{j-\frac{1}{2}}, \mu_{j+\frac{1}{2}}]$ where $\mu_{j+\frac{1}{2}} = \sum_{k=1}^{j} C_k, j = 1, 2 \ldots, J$, and C_k are the weights of Gauss-Legendre formula.
(iii) By using the interaction principle described in Peraiah and Grant (1973), we obtain the reflection and transmission operators over the "cell".
(iv) Finally we combine all the cells by the star algorithm described in Peraiah and Grant (1973) and obtain the radiation field.

2.4. Boundary conditions

The boundary conditions are assumed as follows

$$I_{N+1}^-(\tau = T, \mu_j) = 1 \qquad \text{for all} \quad \mu'_j s \qquad (25)$$

$$I_1^+(\tau = 0, \mu_j) = 0 \qquad \text{for all} \quad \mu'_j s \qquad (26)$$

Eq. (25) represents the incident radiation on the atmosphere, where the radius is minimum, and Eq. (26) represents the boundary condition at maximum radius, for $\omega = 1$.

2.5. Calculation of total source function

The total source function $S_T(x,y,z)$ is given by

$$S_T(x,y,z) = S_r(x,y,z) + S_s(r). \qquad (27)$$

This means that the total source function (S_T) is the sum of the source functions due to self radiation of the primary star (S_s) and the of irradiation from the secondary component which is assumed as point source (S_r).

3. RESULTS AND DISCUSSION

In our calculations $R_{in} = 10^{12}$cm, $R_{out} = 5 \times 10^{12}$cm, $R = 10^{13}$cm where R_{in} and R_{out} are the inner and outer radius of the primary star, and R is the separation between the two components. A constant electron density of 10^{12}cm^{-3} is assumed. As mentioned earlier, intensities are calculated along the lines such as QS_2RO in a given circular slice. These slices are designated as K= 1, 2, 3,... where the slice K= 1 corresponds to that at $x = R_{out}$, that with K= 11 corresponds to that at $x = 0$, and that with K= 21 corresponds to that at $x = -R_{out}$. Unit incident intensity is given at the surface $r = R_{out}$, the secondary component coordinates which is a point source are set as (x_2, y_2, z_2), and the radiation is incident on the primary component. We consider the following cases and calculate the direction cosines of the lines parallel to the Z-axis and also parallel to the line of sight. We can obtain many possible cases keeping secondary component B in different positions on a circular orbit. We have considered the following cases.

Case 1: $x_2 = R, \quad y_2 = 0, \quad z_2 = 0$;
Case 2: $x_2 = R\sin\frac{\pi}{4}, \quad y_2 = 0, \quad z_2 = R\cos\frac{\pi}{4}$;
Case 3: $x_2 = 0, \quad y_2 = 0, \quad z_2 = R$.

Thus, we have placed the secondary component on the X-axis at a distance R in case 1, in case 2, the secondary component is placed between X and Z axis at the line making $an angle of 45°$ with X-axis and in case 3, the secondary component is placed on the Z-axis at the distance R.

Using the above data, we plotted, in Figs. 3 to 5, the reflected radiation by dash-dot line, emergent radiation (which is self radiation) by dashed line and total radiation by solid line for K= 8, 14 in a scattering medium along Y-axis. In all the three cases we observe that the self radiation is the same and also it is constant throughout the atmosphere of the primary component.
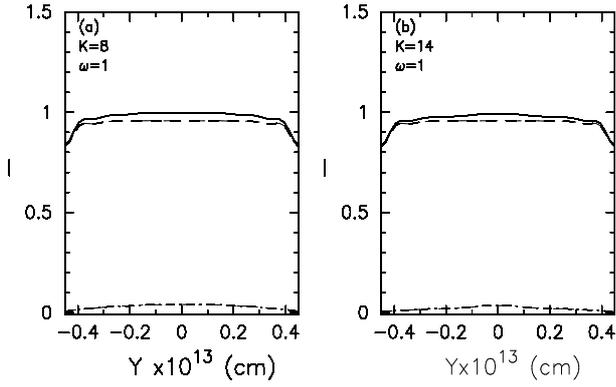
Fig. 3. *The comparison of self radiation represented by the dashed line, reflected radiation represented by the dash-dot line and the total radiation represented by solid line along the Y-axis in the case 1.*
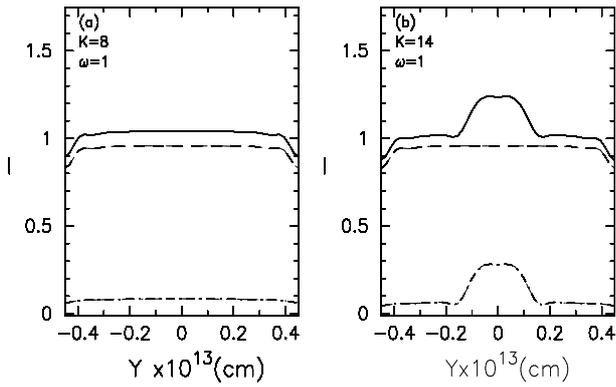
Fig. 4. *The comparison for the case 2. The notation is same as in the case 1.*
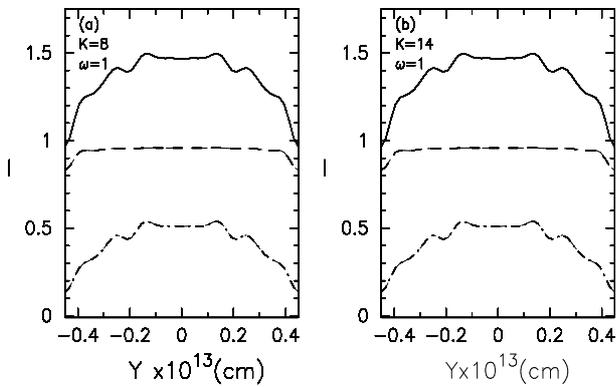
Fig. 5. *The comparison for the case 3. The notation is same as in the case 1.*
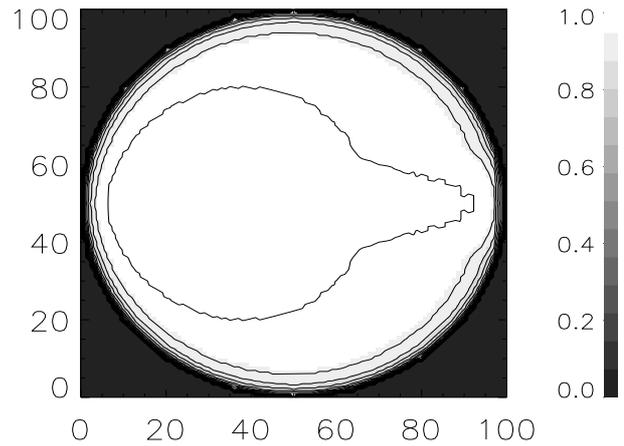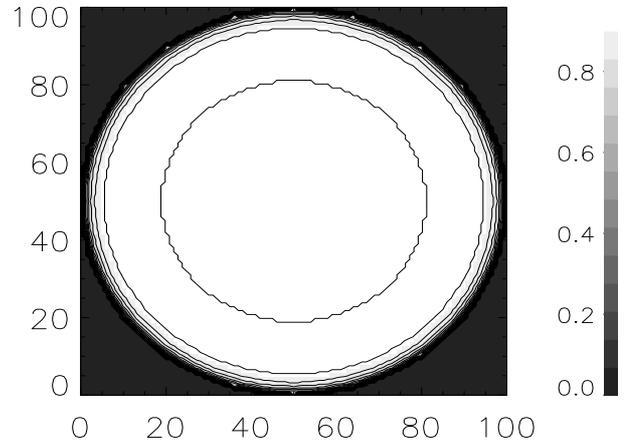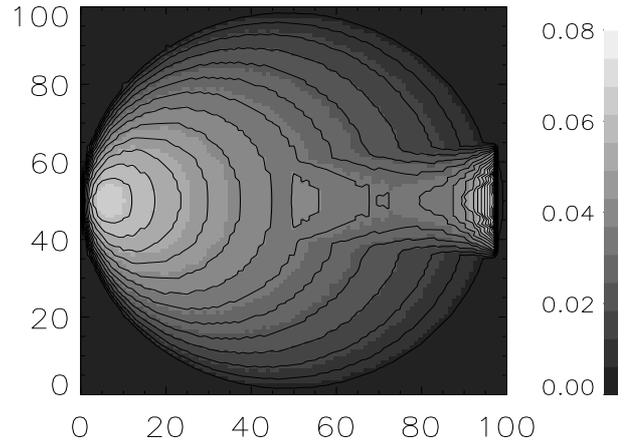
Fig. 6. *A contour map of the brightness distribution on the surface of the primary component for case 1. Here we have considered 100 slices. The dark and bright portions of the contour show no light and the maximum light respectively. The same thing is shown in the grey scale in the side of the figure. Plots shown above are for (a) reflected radiation (b) self radiation (c) total radiation.*
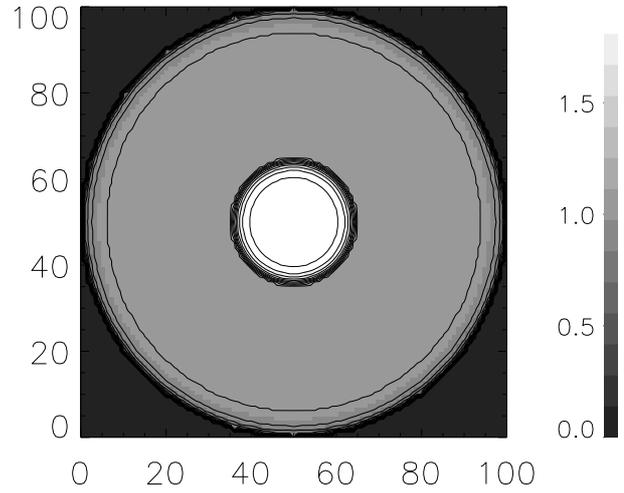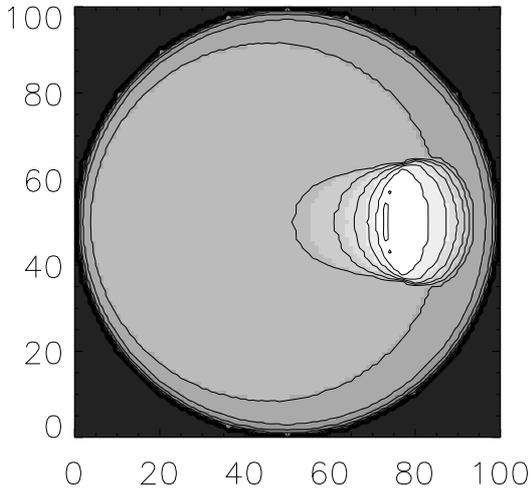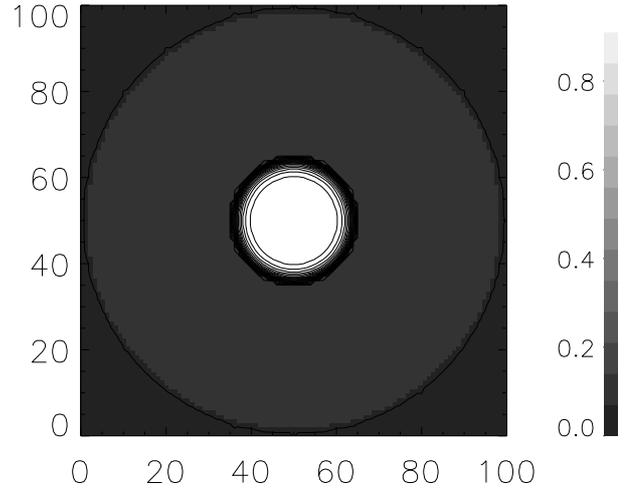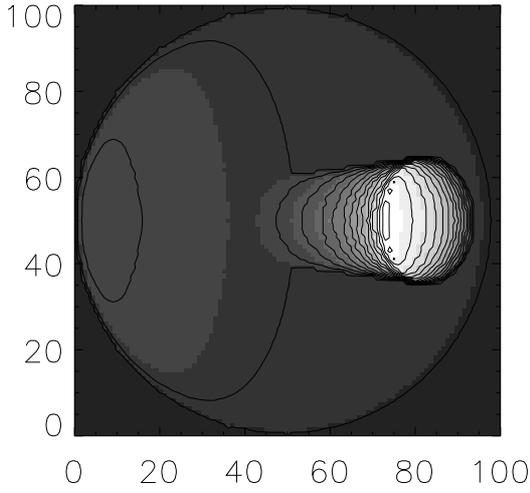
Fig. 7. *Contour maps for (a) reflected and (b) total radiation for case 2.*

Fig. 8. *Contour maps for (a) reflected and (b) total radiation for case 3.*

Case 1: In Fig. 3(a, b) the irradiation from the secondary component is small, compared to the self radiation of the primary star. This is due to the fact that we have considered secondary component as a point source which is at a distance of R= 10^{13} cm from the primary component. The contribution of reflected radiation is small and there is no significant quantitative difference between self radiation and the total radiation curves in Fig. 3(a, b) for K=8 and 14. Thus, the self radiation is a dominant factor in this case but still the effect of irradiation can not be neglected, because the effect of irradiation can be seen in contour maps of Fig. 6(a, b, c) when the secondary component is placed on the X-axis.

The surface of the primary component shows changes in intensity which is seen in Fig. 6(b). We can also observe irradiation effect on the primary component which can be seen in Fig. 6(a) facing towards the secondary component. Fig. 6(c) shows the surface brightness distribution of the total radia-

tion (self+irradiation). In Figs. 6-8, the grey scale represents the variations of surface brightness which is given at the side of each plot.

Case 2: In Fig. 4(a, b) we can see that the reflected radiation is almost constant in the atmosphere of the primary component and a considerable amount of irradiation is added from the secondary component to the self radiation of the primary component in Fig. 4(a). The total radiation behaves as the self radiation, but one can clearly see that the curves are separated in Fig. 4(a) because the irradiation contributed more substantially.

In Fig. 4(b) we see that reflected radiation is stronger in the range $|Y| \leq 0.2$ than outside this range, it contributes a significant amount of radiation going towards the center of the star due to scattering processes and is a dominant factor in that range. The total radiation almost behaves like the reflected radiation in the range $|Y| \leq 0.2$. This happens because

more photons are scattered in the interior of the primary component.

We can also observe similar features in the contour maps of Fig. 7(a, b) which shows reflected and total radiation (since self radiation contour is the same as in Fig. 6(b) we have not repeated it here). Due to the angle of incidence, the shadow cone casted by the central star is changed, brightness distribution is shown in Fig. 7(a) and total radiation in 7(b).

Case 3: In Fig. 5(a, b) we see that for K= 8 and K= 14, the reflected, self and total radiation are the same by symmetry. In comparison with cases 1 and 2, more radiation is reflected in this case. This is because primary component receives the radiation directly from the secondary component. It is also observed that intermediate regions have combined radiation fields (i.e. irradiation which is coming towards the centre of the star and the self radiation of the primary star). Due to this the maximum radiation occurs in the central regions of the primary component. We also plotted the contour maps for reflected and for total radiation distribution in this case in Fig. 8(a, b).

4. CONCLUSION

We calculated the radiation field when irradiation comes from a point source of close binary system in 3-dimensional geometry. The irradiation effects are studied in 3 cases when the secondary component is placed at the distance R from the primary component: 1) on the X-axis 2) making an angle $\frac{\pi}{4}$ with X-axis and 3) on Z axis. We analyzed how the effect of irradiation from the secondary on the primary varies depending upon the position of the secondary component. We intend to study the irradiation effects when secondary component has an extended surface and the effect of gravity darkening in a binary system in XYZ-geometry.

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ТЕОРИЈСКА СТУДИЈА ЕФЕКТА ОЗРАЧИВАЊА У ТЕСНОМ ДВОЈНОМ СИСТЕМУ

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У раду се разматра ефекат озрачивања у тесном двојном систему у коме се секундарна компонента апроксимира тачкастим извором. Ефекат озрачивања атмосфере примарне компоненте разматран је у тродимензионалном Декартовом координатном систему. Претпостављено је да се секундарна компонента креће по кружној путањи. При теоријском разматрању ефекта рефлексије укупно зрачење (S_T) дато је као збир две компоненте: 1) зрачења које настаје као последица озрачивања примарне компоненте (S_r), а које је рачунато коришћењем модела "једнодимензионалног лењира", и 2) сопственог зрачења примарне компоненте (S_s), које је рачунато на основу решења једначине преноса зрачења у сферној симетрији. Процењено је поље зрачења дуж правца визуре посматрача у бесконачности. Показано је како се поље зрачења мења зависно од положаја секундарне компоненте.