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Review paper

THE CONCEPT OF FRACTAL COSMOS: I. ANAXAGORAS' COSMOLOGY

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SUMMARY: The concept of a fractal cosmos occupies a prominent position in the modern cosmology. We trace the development of this concept from the presocratic Greece to the present state of affairs. In this first part we consider the original idea due to Anaxagoras and elucidate a number of points with regard to possible interpretation of his cosmological ideas. A comparison has been made with the cosmology of Abderian school and relevance to the modern cosmology discussed.

1. INTRODUCTION

Despite the lack of empirical evidence some presocratic Greek thinkers did not hesitate to speculate on a spacially infinite universe. This should not come as a surprise, since the idea of infinite appears tightly bound with the concept of an universe, though not necessarily with the construct of the cosmos. Ancient thinkers did not reach this idea easily, however, since the concept did not appear a part of an everyday experience. Moreover, in modern sense, the notion of infinity comprises a diversity of meanings, and one may distinguish many possible interpretations within eachone of them. Besides the spatial and temporal infinities, one may speak of a numeral one, which term comprises a set of various (possibly hierarchically ordered) "subinfinities", as the Cantor's set theory defines. Adding the notion of quality one may distinguish numeral infinity within the context of diversity. Likewise, with regard to the notion of quantity, one may speak of an infinity with respect to various graduated attributes of a particular entity, like God. Just as it is difficult to pull the concept of infinity out of both empirical or mental evidence, (see, e.g. Gruünbaum 2000) the idea appears so much inherent in any serious meditation concerning the World as a totality, so that the concept turns out to be a part of any reasonable philosophical and cosmological system that has come to us even from the most remote past. In many cases the concept has been (un)intentionally hidden behind the formal verbal exposition of a religious doctrine, or mythological message, that it is necessary to dig under the superficial appearance of the content to find out the concealed assumptions, in particular that of an infinite attribute. (An interesting model for incorporating God into a finite cosmos was contrived by Dante in his *Comedia Divina*, see e.g. Egginton 1999).

Probably the oldest attempt to tackle the counterintuitive notion of an infinite world was to banish this troublesome idea by fiat. This sort of "solution" was common, more or less, to all religious systems, or mythological constructions. The technique has been to divide The Whole, or Everything into two qualitatively different entities. One is our own, real, conceivable world, the other is "The Rest", which is most often termed God, the term as vague as "omnipotent". Relieved by the usual constraints bound to the "ordinary world", one is enabled to "tackle" most difficult issues, including those connected with the counterintuitive notions, like that of infinity. We mention in this context the (cosmo)genesis of the Bible, which asserts that the (material) world is finite, both in space and time, being the result of an act of creation *ex nihilo*, by the God. The crux of the matter has been tacitly shifted to the God, who is then exempted from any rational analysis. Indian brahmanic tradition does not reach the infinity either, but does speak about temporal eons, whose "astronomical figures" must have impressed European thinkers (as they still do modern cosmologists), and what appears tantamount to our notion of eternity.

Presocratic Greece has witnessed the emergence of a few cosmogonic and cosmological teachings, notably those pertinent to the so-called Eleatic and Abderian schools. Though the former's cosmology was more concerned with logos than with cosmos in the ordinary sense (see, e.g. Kirk *et al.*, 1995, KRS in the following) its "solutions" regarding the most fundamental questions in cosmology marked a significant advance in the field. Eleatic challenge to the common sense, as well as to the attempts to conceive an $\alpha\rho\chi\epsilon$ for the material world, provoked the Abderian response, with the atomic hypothesis of Leukippus and Democritus. In the attempt to explain the apparent diversity of the natural entities as well as the changes these entities undergo, Abderian thinkers postulated the existence of atoms, indivisible constituents of the matter, moving in the void. Making use of this sort of matter discretization Democritus was able to construct both the macroscopic and megascopic (cosmological) levels of experience. The response of Anaxagoras of Clazomenae to the Eleatic challenge (with the proviso that it is hard today to discern what we owe to Leukippus and what to Democritus, we shall in the following quote either the latter, or simply refer to Abderians, see, e.g. KRS), in particular that concerning the alleged absurdity of the infinite divisibility of the matter, was different from the Abderian one, and in a sense more subtle (though it is still unclear who inspired whom in this context, see KRS). Both teachings had many features in common, however, and both have much to say to the modern physical cosmology.

In the next Chapter we shall expound briefly relevant aspects of the modern physical theory of fractal structures, which shall guide the mind when tracing the development of the concept from the presocratic Greece to the modern times. In the Third Chapter we quote essential features of the Abderian atomistic cosmology and analyse the aspects that are relevant to the concept of fractality, as conceived by the modern mathematical physics. Fourth Chapter is devoted to Anaxagoras' concept of "seed" ($o\mu o\iota o\mu\epsilon\rho\eta$) and some modern attempts to reconstruct a possible cosmogony and cosmology that might have been conceived by the thinker of Clazomenae. In the Fifth Chapter we discuss briefly the common features and differences between the atomistic and Anaxagoras' cosmological paradigms.

2. THE CONCEPT OF FRACTAL STRUCTURE

Some features of global properties of space, time and matter turn out very helpful in inferring many specific properties of material entities, without investigating particular details of the latter. These properties are revealed by subjecting the primary entities mentioned to some specific mathematical (geometric) transformations and by making use of a number of general principles. The latter may be of various nature, pertaining to logic, aesthetics, and other criteria, as the case may be. Probably the most general of them is the principle of isonomy ($\iota\sigma o\nu o\mu\iota\alpha$), indifference, equality), as conceived by the ancient Greek thinkers, which requires that if there is no way to distinguish two entities, ontologically or otherwise, these should be considered equivalent. The principle sounds sufficiently tautological to be accepted as reasonable. The power of the principle has been demonstrated in many instances in the modern science, notably in theoretical physics. We mention here the noticeable observable consequence of the concept of identical particles in the formalism of Quantum mechanics, which provides quite real and measurable effects (via Pauli principle, for instance). Homogeneity of space leads to the conservation of the (linear) momentum, isotropy to the angular momentum conservation, whereas the homogeneity of time implies the energy conservation (Noether's theorem). One talks then of translational and rotational symmetry invariance, and these symmetry transformations generate corresponding groups, sets of operators, which act within a generalized mathematical space. The latter may, but not necessarily, be the real (physical) space-time. In the case of physical systems endowed with rigid structure, corresponding symmetry operation leaves the system identical to itself. A particular body may have one or more various symmetries, like rotational, reflectional, inversional, etc., and/or more complicated symmetry operations which are composed of the elementary one mentioned. Besides these rigid-body symmetries, there are others which may include a change of the system dimensions. The latter are called scale transformations, like the homothetic ones, which shrink or expand the system, keeping its shape unaltered. An example of the latter is the Russian matryoshka toy, a series of single-shape figures nested one inside the other. Such systems are endowed with the self-similarity property, or have the selfsimilarity symmetry. If the number of symmetry operations is finite the corresponding group is said to be discrete, otherwise it is continuous. Rigid body groups are example for both discrete and continuous symmetries, whereas space-time transformations are continuous. Many physical objects possess discrete symmetries, like a cube, or continues one, like a sphere. Finite systems can not have translational symmetry, but in many cases this can be assumed to a good approximation, as the case with the solid-state lattice is. An object may be considered to consist of a number of parts, with their shapes and mutual spacial relation fixed. If each of these parts has the same structure as the primary object, and further each of its parts the same structure as the secondary subsystem, etc., we have a complex object which is hierarchically structured. In Fig. 1 we show an illustrative example of the so-called fractal Cantor set, taken from Belić (1990).

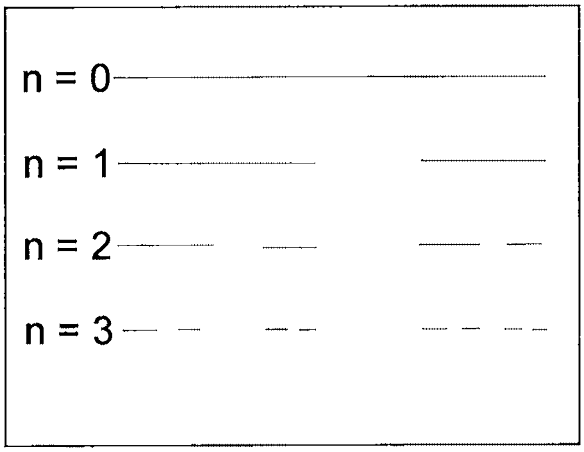

Fig. 1. *Construction of the Cantor's set as an example of the (multi)fractal structure.*

The first level (n=1) is obtained from the auxiliary one (n=0) by removing a segment. The second level (n=2) is obtained from the first one (n=1) by removing segments from both preceding segments, etc. These one-dimensional objects are example of the multifractal structure. If the removed segment were exactly one third of the auxiliary segment and placed in the middle, it would be (simple) fractal (structure). This algorithm for constructing (multi)fractals is strictly deterministic. If, on the other hand, the position and/or the (relative) length of the removed segment were chosen at random, one would have a random (multi)fractal. Also, one can have a random fractal dividing the auxiliary segment into three equal parts and then assigning to each of them random probability to appear in the next step. In the example of fractal Cantor set, considered as random fractal, the probability of the middle segment was set at 0, and of the two wing segments at 1, of course. (One should note that Cantor set is reached only as the limiting case, $n \to \infty$. Obviously, everything said for the fractal Cantor set can be generalized to the structures which are generated by dividing the auxiliary segment into m parts (instead of 2). In this respect Cantor set appears as a special case (bifractal) of the multifractal objects. One can repeat the algorithm backwards, with m = -1, -2,...., levels. It is important to note that in this mathematical structure there is no (natural) unite of length. If one singles out any two-segment cell in the Fig. 1, it is not possible to say which level it belongs to. Further, if one calculates the total length of the segments of a level in the Cantor set and evaluates this sum in the limit $n \to \infty$, one arrives at the zero length. Hence, it turns out that Cantor set has zero measure. Another important feature of a set is its dimension, and one should be able to define it in a proper way. This procedure is straightforward in the case of simple geometrical objects, like straight lines, smooth areas, compact bodies, etc., but there are complex structures where usage of a measure fails. In Fig. 2 we show another fractal (plane) structure, so-called Koch's curves.

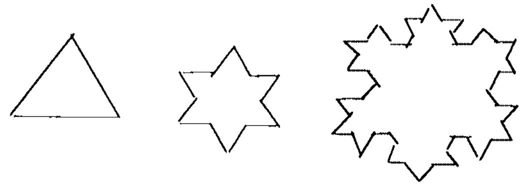

Fig. 2. *Koch's curves (snowflakes).*

One generalizes therefore the notion of dimension, to include more exotic structures. In the case of fractals the so-called Hausdorf's dimension D is used. For the Cantor simple fractal set one has D = 0.631, whereas for the Koch's curve D = 1.262. Hence, within the realm of fractal one encounters noninteger dimensional space, and fractions of the integers appear. It is due to this property that these objects have been dubbed fractals. (Another etymology would be the reference to the way all these structures are constructed - a case for the isonomy). All said above can be generalized to structures embedded in the two-and three-dimensional physical space, as well as in N-dimensional abstract geometrical space. Another important parameter of the fractal structure is the scaling exponent (parameter). If in constructing a (simple) fractal one starts with division of the auxiliary line in Fig. 1 into m segments, then one may write for the length of one of them $l_1 = rl_0$, and for the n-th level $l_n = r^n l_0$. If we have m-fold fractal, then $r_i, i = 1, 2, ..., m$, scaling parameters will appear, instead of a single one. In principle, m need not be a finite number, and one may have an infinite sequence of the scaling exponents. It is evident from the way they are constructed that these mathematical objects have no restriction imposed on the scaling exponents r_i. It is a different matter when one passes to the real physical structures. But before we leave the realm of mathematics, let us turn from these examples, which have mainly heuristic value, to the fractals one encounters within the field of the so-called nonlinear dynamics, where chaotic regimes may be generated. Chaos appears a ubiquitous phenomenon in many classical dynamical systems, like the classical two-electron atom. Although the underlying dynamics is strictly deterministic, the system may be so much sensitive to small changes in the initial parameters, that it is practically impossible to predict the state of the system after not very long time periods. Whether a system is chaotic or not the best way to estimate is to draw the so-called Poincaré section, a plane which is intersected whenever a trajectory which the system traces, during its temporal evolution through the so-called phase-space, passes through the plane. If the (physical, dynamical) system is deterministic and regular, Poincaré section will be filled in a regular manner, with points form-

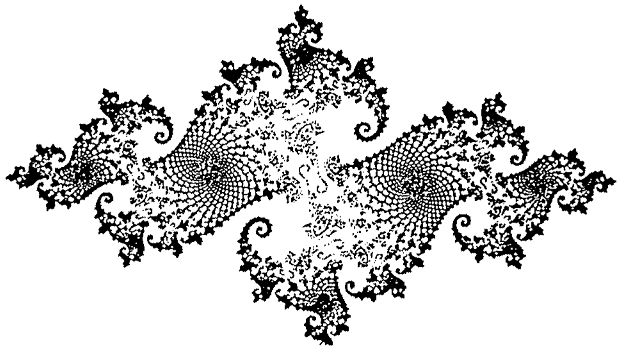

Fig. 3. *"Dragon" attractor (see text).*

ing homogeneous structure. If the system appears governed by random parameters, stochastic effects will direct the points on Poincaré section to appear in random positions, with unpredictable (time) ordering, but the surface will be covered by the points with a homogeneous density, with no discernable structure. Finally, if a system enters a chaotic regime, the points on the Poincaré section will hit the surface in an unpredictable manner again, but after a long enough computing time a discernable structure will appear in the plane. This structure reveals the chaotic behaviour of otherwise deterministic system. Some of the latter may have a part of the Poincaré section densely covered with the points, at the expense of the remaining (almost empty) space. Then one speaks of strange attractors, which reveal a clear, though not a sharp pattern, as the trajectories repeat their passing through the surface, as shown in Fig. 3. This pattern appears recognizable on smaller scales, however, revealing thus a fractal structure. If one magnifies any part of the strange attractor, this part shows the same structure as the entire attractor. And if a part of that part is singled out, the same pattern shows up again and so ad infinitum. In principle, this process may be followed as long as one wishes, better to say as long as the computing time allows. This is the example of an infinite fractal, whose pattern appears ever present. In Fig. 3 we show the so-called "dragon attractor". Strange attractors thus appear as signatures of the chaotic behaviour. The latter is clearly defined in the modern theory of dynamical systems, and is not to be confused with stochastic, random processes, or disordered systems. This distinction will appear important when considering the concept(s) of Chaos in ancient times, in particular from the presocratic Greece. The degree of disorder, as measured by, e.g. entropy of the system, appears maximal for the stochastic processes or disordered structures, with chaotic behaviour placed between the regular (deterministic) and deterministic irregular system evolution. Clearly, disorder (entropy) of the probabilistic (indeterministic) fractal should be larger than for the deterministic fractals. Probabilistic multifractals appear the closest as these semiordered structures approach the completely disordered systems.

Fractal objects are ubiquitous in the material world. They appear in physical, chemical, biological, geomorphological, and other areas (see, e.g. Gouyet 1996), including arts (Taylor *et al.* 1999). A typical physical example is the snowflake, which is composed of a particular number of smaller identical replicas, which can be revealed under the microscope. An ordinary size snowflake is formed by aggregation of the smaller ones. Somewhat different type of fractals are various plant leaves, like that of a fern. Some trees may have the fractal patterns that repeat on three or four levels. Many coastlines have fractal shapes, even some mountainous regions of Mediterranean coast, with Clazomenae on the Asia Minor nearing the Aegean Sea, have a discernable selfsimilarity pattern, too. Another example is the system of blood vessels, or the lung. In hydrodynamics fractals are formed after the onset of turbulence, when the ensuing vortices form a complicated fractal structures. Turbulence appears to be one of the most complex (and the least understood) physical macroscopic phenomena, in general (see, e.g. Frisch 1999). The last, but not the least, are astronomical objects on the cosmic scale, which seem to involve, according to some observational inferences, a hierarchical fractal ordering. The underlying physical (we use this term in a general sense) mechanism that guides the process of forming fractals differs from one system to another. In the case of snowflakes it is the nature of chemical bond which shapes the crystal structure of the solid water (ice), whereas in biological structures the mechanism is still poorly understood. Generally, the notion of (un)predictability, inherent in all these fractal objects, both mathematical and physical, appears in many senses. Many physical systems behave in a regular manner for a range of the parameters, but start exhibiting chaotic features for a particular set of numerical values of these parameters. However, if the nonlinear behaviour of a dynamical system results in an unpredictability of the system evolution in time, it is equally not possible to determine in advance the onset of the chaotic motion, relying on the physical properties of the system (like the forces acting between the system constituents, etc.). In this respect, despite the overall advance of the theory, the field of the chaos remains still to a great extent a phenomenological matter. As we have seen there are three distinct classes of fractals, (i) free mathematical (geometrical) constructions, like that of Cantor fractal set, then (ii) fractals that emerge from the numerical computations on the nonlinear dynamical systems, and (iii) real physical fractal objects, like snowflakes. (It is not easy to find the place for the artistic fractals, like those in painting, within this scheme). All these classes differ in an important aspect of fractal parameters, the scaling parameter l. While the latter is a free parameter within the first class, its numerical value appears as a phenomenological quantity in the case of the nonlinear systems, whereas these are the laws of nature that determine the value of this scaling factor, in each particular case of the (physical) class of the fractal objects. Apart from megascopic systems, like that of the cosmos, it is (the value of) Planck constant h that governs ultimately the "distance" between successive fractal levels. Another important distinction between the fractal classes is to be made. While the selfsimilarity (symmetry scaling) is perfect in the first class, it is not ideal within the nonlinear systems (though it may be reached after an infinite computing time), whereas the situation varies from one case to another, when one comes to the real physical objects. The snowflakes may be considered to possess practically ideal geometrical shapes, but some other objects achieve this selfsimilarity only approximately, as the case with the biological structures is (Ben-Jacob 1997). The same holds for the observed geophysical patterns (see, also, Ben-Jacob *at al.* 1994) and cosmic-morphological hierarchical levels (Saar 1994; Einesto *at al.* 1994). There have been attempts to extend the fractal concept to the overall physical reality, from the (sub)atomic to cosmic scales (Oldershaw 1989). Needless to say, the selfsimilarity principle can be applied in this most general case only as a "guide to mind".

3. ABDERIAN COSMOLOGY

It has been widely accepted that the atomistic teachings of Leucippus and Democritus was conceived as a response to the Eleatic school. For our subject the most relevant philosopher of Elea was Zeno and two of his antinomies. The first we mention is his assertion that the plurality as a notion appears incomprehensible to human mind. It includes the question of (in)divisibility of objects, which appears pertinent to both the Abderian and Anaxagoras' solutions. The second is the Stadium aporia, which denies the possibility of motion, basing its argumentation on the continuous structure of the space and time. The latter antinomy will be shown relevant to an interpretation of the kinematics that Anaxagoras' differentiation of the primordial cosmic matter implies and will be discussed in the next Chapter. Here we analyse briefly the question of the (in)divisibility, regarding either purely mathematical or material objects, which appears relevant to both Anaxagoras and Abderians. From Parmenides, whose radical monism rejected even a possibility to consider nonexistence (on epistemological grounds), to Zeno, whose aporias may be interpreted to be construed in order to discourage any attempt to criticize his teacher on rational grounds. In one of his antinomies concerning the (im)possibility of plurality Zeno states:

"(i) If there are many things, it is necessary they are as many as they are,...But if they are as many as they are, they will be limited [in size].

(ii) If there are many things, the things that are unlimited, for there are always others between the things that are, and again others between those. And thus the things that are are unlimited."

As with other Zeno's aporias, it is not clear whether by things mathematical objects are meant only, or they refer (also) to material objects. This antinomy is quite intelligible if one restricts himself to densely packed sets, with cardinal number of the continuum, in modern parlance. In fact, all Zeno's paradoxes may be interpreted as mere transcription of properties of mathematical objects into the material world. In any case, later thinkers understood them to refer to the latter. The thesis that there exists only One has been challenged by atomists by postulating a number of binary divisions, in a sort of a (extending) chain diversifications. The primary division is that of the overall reality into two elements ($\sigma\tau o\iota\chi\epsilon\iota\alpha$): full ($\pi\lambda\eta\rho\epsilon\varsigma$, matter) and nothing ($\kappa\epsilon\nu o\nu$, void), the latter by following Melissus. Void was conceived as a limiting case of rarefication of something, with the air as paradigm. In the final analysis void reduces to the (modern) notion of space, emptiness, but its existence was thought by ancient thinkers as (epistemologically) independent of something, i.e. not conceived as a negation of the latter. In the modern (positivistic) analogue, vacuum, the concept of nothingness appears as elusive as it was in ancient times. The existence of nothing is not only *contradictio in adjecto*, but may be considered an empty notion both in ontological and epistemological sense. In the former aspect, we have evidence that massless fields, like the gravitational one, penetrate any part of the space (or, in a stronger sense, even create the latter), whereas the assertion that a part of a space contains absolutely nothing is not a scientific statement, in Popperian sense.

As the next step Leucippus conceived the material world as appearing on two levels, microcosmos and macrocosmos. The first level of this binary multiplication scheme consists of indivisible objects, atoms, which by their aggregation form macroscopic, everyday reality. Atoms are thus embedded into void, and move freely in it. As for the indivisibility (or indestructibility) of these elementary entities, one should note that this property does not imply atoms must necessarily be deprived of any (internal) structure. Modern physics knows many instances where an "elementary particle", like neutron, is endowed with internal structure, more precisely, may be conceived to contain a number of other "elementary particles", like quarks, for instance. The latter may be unobservable from the point of view of the same theoretical model to which quarks owe their existence. This analogy has, of course, a limited significance, however, since the notion of destruction (or division) has different meaning today from the simple mechanical picture of the ancient, due, mainly, to the mass-energy equivalence.Atoms share the same property of unobservability with the void, but unlike the latter can not be deprived of the logical consistency. They appear unobservable due to their smallness, which makes them exist beyond our perception.

Abderians conceived the universe as consisting of a plurality of worlds (cosmoses), formed by innumerable atoms, the latter being also of any conceivable shape. This property, however, moves atoms further from the concept of pure element, in the traditional sense of $\alpha\epsilon$. By assuming a morphological, as well as in respect to the magnitude, unrestrained plurality, the elegant picture of the original idea is much lost. Such atoms resemble our concept of organic molecules, rather than modern atoms themselves. The structure of atom fits better our notion of elementary particle, like electrons, protons, mesons, etc. Another point of interest here is the mechanism of world creation as conceined by Leucippus (and presumably Democritus) (KRS, p. 17). Leucippus holds that the whole is infinite

"... part of it is full and part void... The worlds come into being as follows. Many bodies of all sorts of shapes move 'by abscission from the infinite' into a great void; they come together there and produce a single whirl, in which, colliding with one another and revolving in all manners of ways, they begin to separate, like to like. But when their multitude prevents them from rotating any longer in equilibrium, those that are fine go out towards surrounding void as if sifted, while the rest 'abide together' and, becoming entangled, unite their motions and make a first spherical structure. This structure stands apart like a 'membrane' which contains in itself all sorts of bodies; and as they whirl around owing to the resistance of the middle, the surrounding membrane becomes thin, while contigous atoms keep flowing together owing to contact with the whirl. So the earth came into being, the atoms that had been borne to the middle abiding together there. Again, the containing membrane is itself increased, owing to the attraction of bodies outside; as it moves around in the whirl it takes in anything it touches. Some of these bodies that get entangled form a structure that is at first moist and muddy, but as they revolve with the whirl of the whole they dry out and then ignite to form the substance of the heavenly bodies."

If atoms constitute primordial cosmogonical material, the concept of whirl appears the primitive construct of the initial cosmic kinematics. From the modern perspective it is not clear what sets atoms into this sort of motion, in the absence of any concept of (universal) mutual attraction and interaction at distance in general. The vortex as a primordial motion was needed to explain the celestial kinematics, which was circular. But the whirl appears an archetype which goes beyond such a rational explanation. We meet this picture in the ancient mythology, like that beautiful tale of Eurinome and the serpent Ophion emerging from the Chaos in the form of a whirl (Graves, 1966). Vortex appears the only sort of motion that can create a structure out of chaotic matter. It is not the most primitive kind of motion, the translational one being simpler. But the circular movement is an absolute motion, unlike the translational one, owing to the homogeneity of the chaotic matter, which is tantamount to the homogeneity of space in modern parlance. The circular motion is observable from any (inertial) reference frame, and thus may be regarded as a (stationary) structure. It is interesting here to note that experiments with superfluid helium have reproduced vortex structures that simulate kinematics of modern cosmogonical models. The concept of membrane is also significant for a modern physical morphology. It implies a sort of ring around the bulk of the matter, with the latter attributed to Earth. The ring augments by "attracting" ($\kappa\alpha\tau\alpha\tau\eta\nu\epsilon\pi\epsilon\kappa\rho\iota\sigma\iota\nu$) the objects from outside, but the term "collect" would better describe the process. Abderians assumed a many-world universe. Democritus holds the same view as Leucippus about the elements, full and void (see KRS, p. 418)

"... he spoke as if the things that are were in constant motion in the void; and there are innumerable worlds, which differ in size. In some worlds there is no sun and moon, in others they are larger than in our world, and in still others they are more numerous. The intervals between the worlds are unequal; in some parts there are more worlds, in other fewer; some are increasing, some at their height, some decreasing; in some parts they are arising, in others they are missing. They are destroyed by collision one with the other. There are some worlds devoid of living creatures or plants or any moisture."

The idea of many-worlds universe was not exclusively Abderian property and was in the air at the time. This may be regarded as an enormous endeavour of the human mind, that can not be overestimated. Conceptually, it implied a jump from finite to infinite, psychologically a turn from anthropocentrism to a true cosmopolitism (more exactly universalism). With the risk to succumb to Whigish concept of history, it is tempting to ascribe to a number of above assertions corresponding modern interpretations. Which worlds Democritus might have thought of? According to the quoted literal testimony, the most appropriate candidates for those "worlds" are galaxies, which fit best the above description, both with regard to their transient lives and variable separations. In particular the possibility of the worlds collisions appears interesting from the point of view of the modern observational evidence. But such a conjecture would hold only if one could ascribe to Abderians a heliocentric system, instead of then reigning geocentric picture of our planetary system. To them, sun and moon followed the same kinematics and were to be treated on the same footing, apart from the difference in brightness. Hence, Democritus could conceive a planetary system without sun (ie a central star), which in our time would be impossible to do. Anyway, the ancients had no reason to conceive the Milky Way as a three-dimensional celestial structure, instead of two-dimensional star distribution over the celestial sphere. It will come only with the advent of telescope that Kant could afford to speculate on the extragalactic nature of the then observed, "nebulae" (Kant 1755). The speculation that the worlds are scattered with unequal density within the universe appears interesting in view of Anaxagoras' ideas, as we shall discuss later on. It is all the more interesting since this goes, at first sight, against the principle of isonomy, which implies, among other things, the homogeneity of the space. On the other hand, isonomy postulates equivalence of all possible variations, in the absence of acceptable restrictions on the mutual distances, shapes and contents of "other worlds". It seems that Democritus was not satisfied with the infinite universe as a collection of world replicas, but insisted of diversity, as a necessary prerequisite of plurality. The latter, in its turn, was to complete the entire concept of universe, which ought to be infinite both in quantitative and qualitative terms. In fact, his concept of atomic shape and magnitude diversity points in the same direction. Democritus' universe appears a three-level entity. The first, fundamental level belongs to atoms, invisible, indestructible and (albeit implicitly) considered eternal. The kinematics of these particles is chaotic, they collide mutually, and it is this necessity to scatter that required the indestructibility, that is indivisibility. By making aggregates atoms form the macrocosmos of the everyday experience, including the sentient beings like ourselves. Finally, the third, megascopic level of celestial objects and the entire universe (the Whole), which is populated by other worlds, with their own sentient beings, like or unlike ours. The only unifying feature of all these levels of existence are atoms. The overall conceived diversity stemming from the essential diversity of the atoms, hence, on the most fundamental level. Abderians presumably made no attempts to explain why at the observable level objects had different structures and various mutual relations. Their picture of the world was a mechanical one, based on the mere notion of contact interaction. Had they extended the assumed diversity of atomic properties to an analogous plurality of interatomic forces, they would have achieved the idea of the complex structure of the observable world, at least at the macroscopic level. From this the step to the concept of a law of nature would be easily feasible. Instead, their central notion behind the atomic kinematics was chaos, out of which no way to form a structure, like that of everyday objects, could be envisaged. On the other hand, the reluctance of Leucippus and Democritus to resort to "joker type" solution, like that of introducing god(s) must be accounted to their credit. Another important feature of Democritus' cosmology is the transient nature of the objects both at the macroscopic and megascopic levels. Just as the living creatures are born, develop, come to ages and die, so the entire worlds are formed, mature and are destroyed, either by mutual encounters or otherwise. The only eternal and stable are atoms, everything else is just their coming together and/or separations, as an underlying force of becoming and perishing. It has been widely accepted that the concept of atoms has been the most significant scientific idea that has come to us from the ancient Greece. Two questions are in order, however, here. First, is the Abderian (and Epicures') concept of atoms compatible with the modern notion? And second, is the overall internal structure of atomic hypothesis selfconsistent? The answer to the first question is relatively easy. Presocratic atoms correspond to our modern constructs of elementary particles, electrons, protons, mesons etc. Modern ideas of the microworld refer to an countable number of elementary particles, with the latter divided into (possibly) finite number of classes, as indicated above. With the difference that we talk about masses instead of magnitude, whereas the notion of shape of particles appears irrelevant (if not meaningless). But the second question is much more complex, even intriguing and we shall consider it here in some detail. Can atoms be considered $\alpha\epsilon$ in the standard presocratic terminology? They differ in shapes and magnitudes, and, presumably, in Democritus' teaching, in weights as well. The number of both sizes (see KRS, pp. 422-3), on the latter point.and shapes of atoms is infinite, but we have no evidence if these infinities implied that there were no two atoms with equal size and/or shape, or the overall multitude of primary particles was subdivided into classes of subsets with identical atoms. The latter possibility seems to be implied by notions of likeness and congruence, invoked when the separation (within the whirl) and aggregation (when macroscopic bodies are formed), are considered, respectively. Of these diversity of shapes and sizes it is the latter that is important when considering the essence of the atomic hypothesis. What might be the limits of the atomic sizes? In order to answer this question, we ask first another, auxiliary one, viz if the atoms are constituents of the visible objects, why have they not been the object of our everyday experience (or, equivalently, why this idea was not conceived before Abderian doctrine)? The answer to this question sets up the upper limit to atomic size. Atoms are below the threshold of our perception abilities. In absolute terms, this could mean sizes of microscopic particles, in modern sense, like those involved in Brownian motion, as revealed by a medium resolution power microscope. But what about the lower limit of the atomic size? What the phrase "infinite in size" really means? If the size (or anything else ascribed to atoms) appears a continuous property, then one may postulate a lower limit of the atomic magnitude. In the contrary case, with a hypothesis of a discrete distribution of the size, the infinite variation in size would imply an infinitely small primary particle. Even if one stops at an "arbitrary small" finite size, the very idea of atoms as "indivisible" entity would be betrayed. Moreover, the very possibility of an indefinite "dimension" along the "size direction", opens interesting consequences, which are of utmost importance to us here. In a curious sentence we owe to Aetius, we read "it is possible, for there to be an atom the size of the universe" (see, e.g., KRS, p. 415). This allowance, if genuine, implies interesting possibilities when considering creation and structure of the worlds, that Democritus envisages. But in taking this assertion in earnest, one must first substitute "cosmos" for "universe", to avoid paradox. Two essential properties of atoms are their indivisibility and invisibility. Invisibility to whom? Democritus talks about small (sentient) beings sensing small, close and weak and big beings sensing large, remote and strong objects and their properties, respectively. Large atoms (even those as large as a cosmos) may be constituent of a corresponding superlarge cosmos, just as our world is presumably made of invisible atoms. Since there is no absolute standard of length (magnitude), there is no natural dimensional border for a cosmos. Within this interpretation, the unequal (inhomogeneous) distribution of cosmoses and voids between them becomes not only possible, but necessary. This equipartition concerning the size of cosmoses appears then as another manifestation of the principle of isonomy. The universe is, hence, filled with cosmoses, which mutatis mutandi appear replicas of each other, but scaled by a homothetic transformation, to the extent their constituent atoms are scaled by this similarity transformation. The atoms stay indivisible all the time, even those of enormous dimensions (by our standards). This indestructibility precludes any possibility of nesting one cosmos into another, i.e. formation of a fractal cosmos. There is no possibility of under- and over-structure, no hypercosmos unless one allows for an internal, hidden structure within atoms, still indestructible, as is the case of hadrons in modern high energy physics, whose constituents, like quarks and gluons, appear beyond the reach of a direct experience. To summ up, Abderians conceived an infinite universe, both in respect of spacetime and matter content. Out of these infinities, as well as of the infinite diversity of atoms regarding their shapes and magnitudes, arose an infinite collection of (different) worlds, scattered all over the universe, in a random manner. The worlds further come into being and disappear, like living creatures. It is the latter analogy that justifies the maxim of many Greek philosophers that microcosmos is equivalent to macrocosmos. And it is this thesis only that resembles, albeit in a rudimentary form, the principle of selfsimilarity, that was the central dogma of Anaxagoras.

4. ANAXAGORAS' CONCEPT OF COSMOS

Study of presocratics resembles an archeological, if not detective, work. One is forced often to try to reconstruct one particular doctrine from a restricted number of extant written pieces of evidence, just like one tries to get an idea of a broken ancient vase with the help of few extant pieces. In this respect Anaxagoras' cosmological doctrine is particularly difficult to reconstruct and comprehend. First, the thinker of Clazomenae wrote only one book (On Nature), unlike majority of presocratics. Second, the book seems to be a compendium of author's ideas, rather than a fully developed text. Third, only fragments of his writings have survived (though, presumably, forming a significant part of the original book). (This will be the case with monades, too, Leibniz's counterpart of Anaxagoras' homoeomeries. Leibniz never wrote a full account of his central philosophical concept, apart from the compendium *Monadologie*). But the principal problem with an exegesis of Anaxagoras' doctrine is the very originality of the latter, which stands apart from the rest of the presocratic philosophy. Thus, the principal burden of reconstructing his teaching derives from later commentators, first of all Aristotle and doxographers. In dealing with Anaxagoras' cosmological doctrine three principal questions must be considered. First, was the cosmos conceived by Clazomenian finite or not. Second, in the case one adopts (if not proves) the second alternative, does the infinity implies a single cosmos or a plurality of worlds. And third, what meaning to the concept of plurality may be ascribed, considering the curious solution Anaxagoras might have found for the structure and evolution of the universe. The central construct Anaxagoras conceived to explain the diversity of the observable things was something that we might call seed ($\sigma\pi\epsilon\rho\mu\alpha$), as a paradigm of a concealed essence of (living) beings. His approach was evidently organic, just as it will be the case of Aristotle later. It was Aristotle who coined the term $o\mu oo\epsilon$ (possibly with a mild touch of irony), which, presumably, was never used by Anaxagoras himself. The concept of seed as a carrier of a hidden operative principle was close to Stagirit's construct of entelehy. But the most general principle on which Anaxagoras built his philosophical system was stated as $\epsilon\nu\ \pi\alpha\nu\tau\iota\ \pi\alpha\nu\tau o\varsigma\ \mu o\iota\rho\alpha\ \epsilon\nu\epsilon\sigma\tau\iota\ \pi\lambda\eta\nu\ \nu o\upsilon,\ \epsilon\sigma\tau\iota\nu\ o\iota\sigma\iota\ \delta\epsilon\ \kappa\alpha\iota\ \nu o\upsilon\varsigma\ \epsilon\nu\iota$ ("In everything there is a portion of everything except Mind, and there are some things in which there is Mind as well", see KRS, p. 367, for possible interpretations of $\mu\ o\iota\rho\alpha$, which KRS took for "portion", i.e. a share, not a piece, or particle).

Three further postulates are relevant to considering Anaxagoras' cosmogony. First, matter is infinitely divisible, second, the primordial substance was undifferentiated, and third, it is Mind (o) who does the differentiation and thus forms cosmos. In view of these assumptions, how to interpret the central dogma quoted? The original, undifferentiated matter was a homogeneous mixture of all possible ingredients, as carriers of different qualities. After the action of an external agent ($No\upsilon\varsigma$), differentiated (part of the) primordial substance is no longer homogeneous, but still retains the same feature of every part containing (portions of) everything. The $\sigma\pi\epsilon\rho\mu\alpha\tau\alpha$ then might be taken to be the products of the initial differentiation, from which all further conversion of the primordial matter proceeded. The idea behind this cosmogenesis was probably based on the two-level pattern of the living world. Just as from a seed a living creature arises by growing, so the entire observable cosmos was developed from the seeds. This ansatz, genotype versus phenotype, in modern parlance, was present in the mind of other ancient thinkers, not exclusively European ones. In Indian Upanishads we read the explanation of a Brahman to his pupil, who holds a seed of a fig tree (Chandogya Upanishad 1975):

> [6.12.2] "Verily, my dear, that finest essence which you do not perceive - verily, my dear, from that finest essence this great Nyagrodha [sacred fig] tree thus arises.
>
> [6.12.3] "Believe me, my dear, that which is the finest essence - this whole world has that as its soul. That is Reality [satya]. That is Atman. That art thou, Svetaketu."

where the role of $No\upsilon\varsigma$ is played by the (individual) soul (Atman) as an integral part of the Brahman (World's) soul. This "finest essence" we recognize today as genetic material, which determines our genotype. Hence, to Upanishads' *Tat twam asi* (That are thou) corresponds Anaxagoras' $\pi\alpha\nu\tau\iota\ \pi\alpha\nu\tau o\varsigma\ \mu o\iota\rho\alpha\ \epsilon\nu\epsilon\sigma\tau\iota$ (in everything there is a portion of everything). The idea of homoeomeries came surely from considering living tissues by Clazomenian, which he could not conceive as arising from inorganic material. It is remarkable fact, discovered in the second half of 20ieth century, that every cell in a living organism contains the complete genetic material and thus could reproduce the entire creature (phenotype). This property of the life was not, of course, known to ancient thinkers. Many controversies have been found among modern researchers as to the question what kind of universe Anaxagoras could have envisaged, according to the basic postulates mentioned above (see, e.g. Vizghin 1989). More specifically, did Clazomenian opted for a single cosmos (SW), or did he conceive a plurality of worlds (MW)? And if the latter interpretation is adopted, what was the number of these worlds, finite (number) many worlds (FMW) or infinite (IMW) doctrine? The extant fragments of Anaxagoras' writings appear, unfortunately, incomplete to provide an unambiguous answer. According to Simplicius, fragment B4, Anaxagoras wrote about some other worlds, which contained sun, moon, stars like ours, inhabited with intelligent beings like ours, etc (see. e.g. Diels 1952). Simplicius considered the possibility that Anaxagoras refered to our cosmos in a previous stage of evolution, but rejected such an interpretation on the the ground of the specific terminology of the text. On the other hand he doubted that Anaxagoras by other cosmos meant just another world situated somewhere in the universe, but draws our attention on the precise phrase "sun and moon like ours", not "sun and moon are [like ours]". Hence, according to this exegesis, Anaxagoras was not specific sufficiently on the temporal and spatial correlations of these other worlds relative to our cosmos. So, we shall see, this vagueness leaves much space for speculations on the precise concept Clazomenian might have had of the universe. At first sight it turns out that Anaxagoras opted for MW universe. But to see what sort of structure he ascribed to this universe, and what was the precise meaning of the term cosmos, one has to turn to his fundamental assumptions about the properties of matter and the concept of creation as a differentiation. *Tout court*, one has to consider Anaxagoras' microscopic theory, in modern parlance, of the cosmology and cosmogenesis in particular. We consider first the problem of macrocosmos genesis.

4.1 Anaxagoras' macro-cosmogenesis

A part of difficulties one encounters when considering Anaxagoras' doctrine is the vagueness of his terminology. This stems partly from the originality of his ideas, and partly from the indirect evidence we have today on the precise meaning and usage of constructs he built into his doctrine. This difficulty one faces right from the start when considering Anaxagoras' concept of cosmogenesis. Generally, one distinguishes two possible alternatives of the latter. Either cosmos has arisen from nothing (*creatio ex nihilo*), or one starts from a tacit assumption that something (universe) existed, and cosmos is brought about by introducing order into disordered matter. We note that both alternative can be derived from a unique term, chaos, which had different meanings in ancient times, even within the same epoch. In one sense, this signifies nothingness, (conceived, vaguely as abyss), but the archaic thinkers usually used the term as signifying an ingredient of the primordial differentiation (see, e.g. KRS, pp. 36-41). In the first usage Chaos was a sort of synonym to infinity, the latter term ($\chi\alpha o\varsigma$) being itself vaguely used, either as a synonymous to infinite (in spatial sense), or indefinite (in qualitative sense) (see, e.g. KRS, pp. 109-110). Hence, chaos by itself could satisfy proponents of both absolute creation and making order within the existing primordial matter. Anaxagoras' position with respect to these ambiguities turns out ambiguous itself. In the fragment B1 we read:

"All things were together, infinite in respect of both number and smallness; for the small too was infinite."

This opening sentence of his book sets up the stage of differentiation of an existing matter and seems to imply cosmogenesis as a process of differentiation, putting in order. But was the primordial substance assumed indeed disordered? In one sense, it was. For what world could be conceived more "chaotic" than the state where "everything is in everything", a total homogeneity? On the other hand, after the differentiation this essential feature of the undifferentiated substance remains, as the guiding principle of any subsequent act of the differentiating agent, $No\upsilon\varsigma$. But in which way does the differentiation proceed? It is clear from the extant fragments and indirect testimonies that Anaxagoras' seeds are "conceptual atoms", both in the sense of indivisibility (with respect to its content) and primary building blocks of the macroscopic matter. Every seed has a portion of everything, but some portions, as carriers of particular quality, are more present than the others. Hence, we have subsets of seeds with equal (or similar) proportions of those qualities, and we arrive at an Orvellian state of affair, where "all seeds are equal, but some seeds are more equal than others". The principal question arises now, as to the relationship between (infinite) divisibility in physical and in the conceptual sense. If seeds are divisible, do they preserve the proportion of its portions or not? If not, at which stage (or scale) of the restructuring of the substance this change occurs? If the seeds retain the same proportions, or internal structure, which would make them "conceptual atoms", then we would have a sort of multifractal structure. But this fractality would be ill defined in the absence of any (physical) atomistic assumption. Would an arbitrary small change in size of a homoeomery, for example, involve equally arbitrary small change in the internal proportions? If it would, we arrive after a number of steps at a different seed. If, on the other hand, every macroscopic object is composed of the homoeomeries of the same type and if this composition is retained at each subsequent level of division, we would have a multifractal structure. But, as we have seen in the Second Chapter, fractals imply discrete structuring of physical matter. Indeed, we witness at our macroscopic level of experience a discontinuous distribution of matter, which enables us, after all, to talk about objects. Natura facit saltus. Besides, we know by direct experience that a part of an egg, for example, does not resemble the egg. If there is a level at which any part of it is a miniature egg, this must be somewhere beyond our sensory threshold. Evidently, if the assertion "everything in everything" is to hold, the implied fractal structure of the matter must be based on a discrete structuring. We know that Anaxagoras was acquainted with Leucippus' doctrine and that Democritus was much younger (presumably by 40 years), (Diels 1952). Though he never says it explicitly, he might have adopted the idea of a discrete structuring, but without lower limit for the divisibility of matter. It is reasonable to assume, at least from the modern perspective, that between Leucippus's two principal assumptions: (i) discretness and (ii) indivisiblity, the former should be considered more fundamental. One might even argue that the notion of $\gamma\alpha$ $\phi o\gamma\alpha$ $\sigma o\mu\alpha\gamma\alpha$ could be considered more general than the term $\alpha\gamma o\mu o$. For the latter might have well been thought of as a unit, endowed with an ivariable set of properties, which preserves its identity throughout the interactions with other units (this is precisely the meaning of the term atom in the contemporary usage, both in Greek, where "atom" is equivalent to "person", or in Latinized version, where an individuum denotes the same). If one assumes the fractal structuring of the substance, with the descrete change of levels, then Anaxagoras' homoeomeries could be taken for atoms, too. For, if divisions of atoms yield the same entities, only at lower (smaller dimension) levels, with the same internal structure, the (geometrical) structure is preserved, i.e. unbroken. Atoms thus would appear divisible (in size) in the physical sense, but indivisible (in shape) in the mathematical sense. Could Anaxagoras be a disguised atomist? We know that he was well acquainted with the contemporary mathematics, in particular with geometry of selfsimilarity. In view of his principle of scale invariance (Mugler 1956, p. 363), the (physical) divisibility of substance becomes irrelevant.

What was exactly the role of Anaxagoras' $No\upsilon\varsigma$? Or, more precisely, what was the nature of the process of differentiation of the undifferentiated substance? If the world is to be hierarchicaly structured, what would be the point of departure and in which direction, towards the smaller or larger dimensions? Clazomenian is not specific on these points. He assumes the existence of seeds, "countless in number and in no respect like one another", which appear already differentiated. Since the biological flavour of the spermata implies "growing", it is reasonable to assume that homoeomeries are to be taken as bulding blocks of the more complex structure, and we have a process "upwards", toward our macroscopic level of experience. In this sense, seeds play the role of atoms, presumably subdivided into plurality of classes. The role of $No\upsilon\varsigma$ then would be just to put together a finite number of spermata so as to make aggregates appear. On the other hand Anaxagoras talks about seeds that are charactarized by the same combinations of opposites [qualities] tending together towards their appropriate place in te universe. This would mean, by implication, that similar seeds lump together as well, in a spontaneous manner, which would imply that the (role of) $No\upsilon\varsigma$ is superfluous, in the last instance. Anaxagoras' $No\upsilon\varsigma$ is a corporeal entity, omnipresent if it is to do differentiation, but on the other hand, not present in the things except in the living beings (which makes it equivalent to the Atman of Upanishads, see above). As Simplicius qoutes "it is the finest of all things and purest, it has all knowledge about everything and the gratest power". Hence, Mind cannot be confused with Law (of Nature), a product of our mind. On the other hand, spontaneous gathering of like seeds points toward natural laws that govern those movements. This eclectic outlook of Anaxagoras' doctrine puts him in an ambiguous position regarding rational thinking as such. Hence differentiation leading to the macroscopic discernable object consists, in fact, of a selective gathering of homoeomeries of similar structure (proportions of qualities). But what about the (primordial) substance below the reference (auxiliary) level? If $No\upsilon\varsigma$ carries out differentiation towards the smaller and smaller dimensions, what would be the relation of such a process with respect to differentiation towards the upper levels? Evidently, this can not be the same process, that would make the primordial substance more complex as one sinks deeply into smaller and smaller levels. The only process one might think of would be creation of very seeds out of completely disordered substance. But this process of coarse graining, in modern parlance, consists essentially of separation, rather than aggregation. By concentrating a particular property into a class of seeds, the primordial homogeneity is destroyed. On the other hand, the very process of concentrating at different points within the homogeneous substance is an aggregation itself. Since the principle of an infinite divisibility holds all the time, this process goes on *ad infinitum*, towards ever smaller dimensions, repeating at each (presumably discrete) level. If one adopts this algorithm for structuring material world, starting from a reference level, it is only natural to assume that both processes, towards the "upper" and "lower" levels, take place simultaneously, for the sake of isonomy. In the absence of any "natural" unit, the auxiliary starting level appears undetermined. This is so in particular in view of the level being infinitely "distant" from both infinitely small and infinitely large, assumed by Anaxagoras to exist both (at least conceptually). Every (physical) process involves a specific algorithm and a particular kinematics of the temporal evolution. If one adopts the above procedure, what would be the speed of differentiations, as described above? This question bears a direct relevance to the age of Anaxagoras' cosmos, as a response to the eternal question about the (in)finite duration of the world. Put in another way, the question is whether we live in a completed cosmos, or the "cosmization" of the universe, is still going on? Before attempting to speculate on this point, we shall first quote the model proposed by Mugler. He has made an attempt to reproduce the procedure that $No\upsilon\varsigma$ might have carried out (Mugler 1956, pp. 339-341, 361-362). According to this scheme, Anaxagoras' cosmos consists of the central (differentiated) part of the universe. With a tacit assumption that the latter is finite, differentiation starts at a point and proceeds outwards, in a spherical symmetric manner. Instead of fractal levels Mugler considered a particular property to be differentiated from the primordial substance and reversing the procedure backwards in time tried to estimate the length of time necessary for reaching the zero-instant, which could be considered to be the absolute beginning. If one ascribes the same time-period for reaching the phase with double-magnitude property, argues Mugler, the zero-point is reached in an infinite past. If, however, these time intervals are reduced according to the geometric progression (just as Aristotle suggested as a solution to the Zeno's Stadium aporia), the temporal distance from the beginning of differentiation remains finite. And vice versa. What is the answer to the question of speed of structuring in real physical terms? There have not been, to our knowledge, observational estimates of the effect within the physical chemistry today. One may resort, on the other hand, to computational procedures, which can be regarded as simulation of the real processes. The time spent to compute successive levels in calculating Cantor's fractal set, for instance, increases at least according to the geometric progression. When computing chaotic structures, like those involving strange attractors, the time consuming increases even faster, exponentially (Belic, private communication). Thus, according to modern inference $Nov\varsigma$ would have been spending an infinite time in the process of turning the primordial substance into an ordered cosmos, starting from any level inwards. (Involving random multifractals instead of the simple deterministic ones would imply even slower rate of structuring). According to this picture, Anaxagoras' cosmos not only had an infinite past, but its beginning is not even possible to define in the outward direction. Mugler did not consider the outward process within the fractal scheme, but turned to the standard model of putting the matter into order starting from a particular amount of already differentiated matter. By considering the (real time) procedure towards the upper levels (this cosmogonical model will be adopted by Kant, with a number of further elaborations), and takin tint account that the amount of matter to be worked out increases with time, as $Nov\varsigma$ proceeds from the surface of a sphere outwards, the speed of differentiation along a radius decreases with time and asymptotically reaches zero (just as in the case of the flat-space modern Friedmanian cosmological models). The cosmos attains, thus, a finite radius, R_0, in an infinite future. This appears counterintuitive when compared with the inward process considered previously according to modern inferences, but is seemingly consistent with Mugler's picture of the same inward procedure. However, his model is inconsistent. In following the inward process Mugler starts with an finite amount of matter, whereas in the opposite direction new matter is continuously involved. If infinities are involved in both directions, they are surely of different nature. All these considerations remain, inevitably, speculative and superfluous, in the absence of any model of the dynamics of the process of differentiation. And as long as $Nov\varsigma$ assumes the role of Law of nature, no quantitative estimates are possible. Anaxagoras was surely acquainted with the iterative methods of constructing curves like circle, as the limit of inscribed polygons (Mugler 1956), which will take the form of the so-called method of exhaustion at the hands of Eudoxos and Archimedes, but all these possible analogies with geometrical procedures could not replace a direct application of mathematics. Besides, the very analogy with geometry keeps the whole procedure out of reach of kinematics, where the time appears an essential ingredient. It should be mentioned here that the concept of an infinite regression inwards, towards ever smaller dimensions of space, would lose its grounds within the modern quantum field theory of space and time. Beyond the so-called Planck's length, $l_P \approx 10^{-33}\ cm$, space is supposed to lose all its essential attributes, like the topological structure and metric. Equally, there is a lower limit to time intervals, Planck's intervals, $t_P \approx 5 \times 10^{-44}\ s$ (but not to the mass). Hence, even if one ascribes any meaning to the recursive iteration inwards, this would not bring us to a definite conclusion. Anyway, in the absence of a cosmological model in modern sense, all speculations ascribed to Anaxagoras remain purely academic. Another important point to be considered regarding Clazomenian's picture of the world is the exact model he had in mind when talking about evolution of his cosmos. According to Mugler's interpretation the cosmos will never be finished, and we live in a transient state of affair ("... le processus cosmique consiste moins à créer des formes nouvelles qu' à faire croitre celles qu' existent déjà,...", Mugler 1956, p. 371). This amounts to ascribing to the homothetic similarity transformations a kinematic role. This interpretation is based on an anecdotic answer which Anaxagoras gave to someone enquiring whether mountains at Lampsacus will ever become sea: "Yes, but it will take time for that". Whether the question was refering to Anaxagoras' doctrine, or to the fact that sea shells are found in many mountains (or, simply, this may be an ironic question, with the response "on equal footing"), remains uncertain. Anyhow, the interpretation which links the homothetic transformation with a change of shape of (physical) geography is unacceptable, since the similarity transformations preserve the shape of objects and it is not clear how the sea can "overrun" mountains in the process of overall growth. Contrary to Mugler's interpretation Vizghin (1989) holds that one should stick to Simplicius' comment and consequently "other worlds" refer not to preceding stages of the contemporary cosmos, but rather to the plurality of worlds implied by Anaxagoras' fractal universe.

4.2 Megacosmos genesis

If the process of matter differentiation concerns the physico-chemical structure of the substance, and thus explains the macrocosmos structure, formation of the world at the celestial level refers to the cosmogony in the proper sense. Anaxagoras relies on the rotational motion as the primordial mode of structuring the megacosmos, like Democritus, but invoking again $Nov\varsigma$ as the Demiurg. However, his concept of the circular motion involved in the cosmogenesis appears far from clear and much controversies still remain regarding the exact model Anaxagoras had in mind. While Abderians took whirls as the primitive constructs, Clazomenian talks rather about circular motion (KRS, p. 363).

"Mind controlled also the whole rotation, so that it began to rotate in the beginning. And it began to rotate from a small area, but it now rotates over a wider and will rotate over wider still. And the things that mingled and separated and divided off, all are known to Mind. And all things that were to be - those that were and those that are now and those that shall be - Mind arranged them all, including the rotation in which are now rotating the stars, the sun and moon, the air and the other that are being separated off. And this rotation caused the separating off. And the dense is separated off from the rare, the hot from the cold, the bright from the dark and the dry from the moist. But there are many portion of many things, and nothing is altogether separated off nor divided one from the other except Mind. Mind is all alike, both the greater and the smaller quantities of it, while nothing else is like anything else, but each single body is and was most plainly those things of which it contains most.

And when Mind initiated motion, from all that was moved Mind was separated, and as much as Mind moved was all divided off; and as things moved and were divided off, the rotation greatly increased the process of dividing."

".... as these things rotated thus and were separated off by the force and speed is like the speed of nothing that now exists among men, but is altogether many times as fast.

But Mind, which ever is, is assuredly even now where everything else is too, in the surrounding mass and in the things that have been either aggregated or separated."

Out of these few fragments modern commentators have tried to construe a selfconsistent cosmological model, which Clazomenian might have had in mind. But from these passages it is not clear, first of all, whether they refer to the differentiation at the microscopic level or to the celestial system. Besides, it contains again the same uncertainty as to the question whether Anaxagoras was talking about world evolution (diachronic aspect) or about "a snap shot" state of affairs (synchronic aspect). Mugler (1956, p. 346) opts for the second alternative, while Vizghin (1989) interprets the text as implying a slowing of the rotation with time. In the latter case, as the system expands, rotation decreases, which should imply that Anaxagoras was refering to the peripheric velocity. This sort of differential rotation, in modern terminology, is typical of the vortex velocity field, which the ancients were surely well acquainted with (water whirls, etc.). In the celestial mechanics such motion is present in (spiral) galaxies, with the speed of tangential motion of the stars in the spiral arms first rising with the distance from the centre to acquire almost constant value, and then slowly vanishing. The slowing down of the peripheral velocity, up to the rest, has been considered by some authors to have been assumed by Anaxagoras as a prerequisite to the many-world universe (multiverses, in modern parlance). According to this picture, cosmogenesis might start at different points in the infinitely large mass of primordial undifferentiated substance. If those centres were sufficiently separated, the cosmogenesis would not interfere and a plurality of worlds would be possible.

All these inferences, though intriguing from the modern point of view, remain highly speculative. We consider here some alternatives, which could explain the Anaxagoras' cosmogenesis in a less exotic manner. What would be the rationale for assuming greater speed of rotation within the inner, smaller region? Since Anaxagoras rightly assumes that the speed of motion implies a corresponding force, created rotational velocities impart strong forces, necessary for separating "portions" in the primordial substance. Anaxagoras was surely aware of the stiffness of dense objects, like metallic ones and the (chemical) force needed for keeping the object together. It is significant that he treated air and other separately from the rest of matter, as if the standard explanations for the differentiation did not apply to these rarefied entities. He even envisaged that Mind was composed of the finest substance and was thus of a corporeal nature. The past tense associated with initial (fast) rotation implies simply that Anaxagoras pictures his overall cosmogony as starting from microcosmic level and proceeding towards larger, macroscopic and megascopic scales (i.e., outwards). This idea fits well the modern concept of fundamental forces, which decrease as one moves from the field of elementary particles to the gravitational (celestial) systems. But if the same mechanism of separating things off is assumed at both micro- and macrocosmic levels, what would be the structure of the megacosmos, in view of the supposed fractal structuring of the microcosmos? It seems that Anaxagoras imagined the totality of reality as obeying the unique principle "everything in everything" and that the same fractal structure holds throughout all scales of the universe (Simplicius, Fr.

6). And since the portions of the great and of the small are equal in number, so too all things would be in everything. Nor is it possible that they should exist apart, but all things have a portion of everything. Since it is not possible that there should be a smallest part, nothing can be put apart nor come-to-be all by itself, but as things were originally, so they must be now too, all together. In all things there are many ingredients, equal in number in the greater and in the smaller of things that are being separated off.

Anaxagoras does not mention the upper domain to this general principle, but had he imagined that the megacosmos was out of the domain he would have been surely explicit on the matter. This in particular as this would go contrary to the principle of isonomy, so dear to many Greek thinkers (see, e.g. Naddaf 1998). He was not explicit on the megacosmos either, but without observational evidence on the large-scale structure of the universe we have today, it is understandable that his interest did not go further from the first supposed higher level. So, Anaxagoras' plurality of megascopic world remained presumably at the heavenly spheres level, just as was the case with Abderians.

The problem of stability

Anaxagoras conceived a dynamic universe. According to Simplicius the opening sentence of his book was "All things were together", setting the stage for a cosmogony. But this cosmogenesis implied an evolution, carried out by $No\upsilon\varsigma$, with the time involved, albeit implicitly. This was not an act of (instantaneous) creation by Demiurg, but a process. This assumption brings in a problem of an underlying dynamics (or, at least, kinematics), as we discussed above. At the same time it offers a solution of the problem which many contemporary thinkers did not even recognized, the question of the cosmic stability. Though the ancient thinkers were not particularly concerned with this problem, in the absence of any dynamics governing the behaviour of discernable objects, some of them, like Empedocles, conceived the cosmos as subject to change, though not necessarily to (an unidirectional) evolution. While Empedocles contrived a sort of cyclic evolution, Anaxagoras opted for a linear time, with an infinite process of structure creation, with a whirl as the driving mechanism, also (see, e.g. KRS, p. 296). One could consider Anaxagoras' cosmogony as pertaining to Empedocles' half-cycle, with $No\upsilon\varsigma$ playing the role of the Strife, the separating agens. In differentiating the primordial substance $No\upsilon\varsigma$ not only brings about the order, but ensures that this order lasts (though not necessarily any particular created part of the universe). In this sense, his solution of the cosmic stability resembles the modern concept of steady-state model, as advocated by Bondy, Gould and Hoyle (see, e.g. Narlikar 1977). This idea of dynamic sup porting of the cosmic structure should be compared with the theological concept of God's permanent support of the existing universe, keeping it from lapsing into nothingness (see, e.g. Grünbaum 2000, p. 24). The idea of the "permanent creation" not only ensures the longevity of the cosmos, but *ipso facto* pushes its end to the infinite future. Anaxagoras' (mega)cosmos has the beginning, but has no end. This beginning was ill-defined, as we saw, but so is it within the modern cosmology, too.

5. SUMMARY AND CONCLUDING REMARKS

Anaxagoras seems to have conceived a world picture that was both eclectic and original. It amalgamated Parmenides' concept of the overall Unity and Democritus' doctrine of plurality. His fractal cosmology, with the selfsimilarity implied, describes a single-pattern cosmos, with (physical) replicas proliferating at the same level, but also on an infinity of different scales, forming a sort of hierarchical structure. It is an infinite cosmos, like that of Abderians, but it did not require an infinite amount of matter, nor an infinite space. As for the temporal span, Anaxagoras assumed the cosmos of a finite age, but probably of unlimited future duration. Clazomenian's concept of an unlimited world appears thus close to the present-day cosmological paradigm, which deals with the cosmos without ends, both with regards to the space-time environment and (the forms of) the matter, but renounces the requirement of a quantitative infinite content of the material substance of our universe. This concept of a limited without boundaries Anaxagoras realised by turning to the "internal structure" of the space, rather than to an infinite reservoir of space and matter. In the modern terminology, Clazomenian took refuge from the counterintuitive construct of infinity in the topology of space, providing a new insight into the eternal dichotomy: finite vs infinite universe. He was not able to resolve the problem of the temporal aspect of the cosmogony implied by the concept of an infinite divisibility of matter, but his modern commentators failed at the same point too. Another feature of his cosmology that makes appeal to the modern reader is the unique treatment of the microcosmic and macrocosmic genesis and structuring. As we witness in the last decades attempts of unification of the elementary particle physics and (physical) cosmology, Anaxagoras' (and Abderian, in this context) approach looks even more modern than many later cosmologies. Generally, Anaxagoras' world picture shares many common features with the Abderian one, though it is still difficult to disentangle the mutual influences (and inspirations) concerning these two doctrines.

We have argued that some modern scholars have gone too far in reading modern ideas into Anaxagoras' cosmology, in particular cosmogony, but in the absence of reliable sources and in view of Anaxagoras' own lapidarity (and vagueness) in the extant fragments, this freedom in interpretation appears somewhat understandable. On the other hand, modern cosmological studies (and speculations), resemble very much those of the Presocratics, in the spirit if not in methods, and in a sense appear a revival of the latter (Disney, preprint; see, also, Ćirković, preprint). In the next paper we shall consider the history of the concept of fractal world, up to the 20^{th} century, and then the modern, most recent developments along these lines in the theoretical and observational cosmology.

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КОНЦЕПТ ФРАКТАЛНОГ КОСМОСА: I. АНАКСАГОРИНА КОСМОЛОГИЈА

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Прегледни чланак

Концепт фракталног космоса заузима истакнуто место у модерној космологији. Овде ћемо пратити развој овог концепта од Пресократовске Хеладе до данашњег дана. У овом првом делу размотрићемо оригиналну идеју коју је предложио Анаксагора и анализираћемо неке од могућих интерпретација његових космолошких идеја. Направљена је паралела се космологијом Абдерске школе и дискутована релевантност идеје за савремену космологију.