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AN ALTERNATIVE MASS MODEL FOR GALACTIC DARK CORONAE

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SUMMARY: A spherically symmetric mass distribution with two scale parameters for the dark corona of a (spiral) galaxy as an alternative to the usually applied quasi-isothermal sphere is considered. Examinations of the rotation curve produced by this distribution over a limited interval of the distance to the rotation axis show that it can be a successful alternative to the usual approximation of the quasi-isothermal sphere. This is important taking into account that the potential formula considered in the present paper can be easily generalised towards axial symmetry.

1. INTRODUCTION

In an earlier paper of the present author (Ninković, 1999) the contribution of the dark-matter subsystem (dark corona) to the rotation curve of a spiral galaxy was studied. As its main result one can state that a model usually known as the quasi-isothermal sphere (in the further text QIS) offers the most satisfactory fit for the case of almost constant circular velocity. However, as indicated in that paper, the potential formula involved by in the QIS model is not particularly suitable for use, especially when the possible flattening of the dark corona is taken into account (e. g. Ninković *et al.*, 1999). For this reason in these two earlier papers alternative mass distributions in the form of the Schuster density law (the former paper), resp. the Miyamoto-Nagai potential formula (the latter one), were applied. However, other alternatives to the QIS model also deserve attention, especially when the possibility of a cuspy mass distribution within the dark corona is borne in mind (e. g. Dehnen and Binney, 1998), as well as the corresponding formula yielding a "flattened" potential in conformity with, for example, Miyamoto and Nagai (1975) for the case of the Schuster potential.

2. THE APPROACH

As on alternative model in this paper will be used the so-called generalised isochrone one (e. g. Kuzmin and Veltmann, 1973). For the gravitational potential it yields the following formula

$$\Pi = \frac{G\mathcal{M}}{a + (r^2 + b^2)^{1/2}} \ . \tag{1}$$

Here G is the gravitation constant, $\mathcal{M}$ the total mass of the system (in this particular case the corona), r is the distance to the centre, whereas a and b are two scale parameters. As already shown (Ninković, 1998), when the scale parameter b is much smaller than a, then this potential becomes similar to the cuspy one proposed by Hernquist, whereas if $a = 0$, it is reduced to the potential corresponding to the Schuster density law.

On the other hand, with regard to $r^2 = R^2 + z^2$, where R is the distance to the rotation axis and $|z|$ is the one to the main plane, it is not difficult to conceive the transformation of potential given by Eq. (1) into the corresponding "flattened" form by introducing a third scale length in analogy by Miyamoto and Nagai as already said in Introduction.

3. PROCEDURE AND RESULTS

The contribution of the dark corona to the circular velocity of a spiral galaxy is calculated by two different formulae: that of the QIS model and that corresponding to Eq. (1). In view of the results obtained in Ninković (1999) the QIS model might be assumed as "true" mass distribution within a dark corona. Then, the aim is to achieve a fit as good as possible. The circumstance of three parameters present in Eq. (1) may mean that their values should be combined in various, arbitrary, ways. However, in this problem there are, some specific requirements. For example, there are some constraints to the total mass of the corona indicated by, say, the data on the motion of the satellites of the galaxy under study. Therefore, it seems reasonable to fix the total mass first and then to carry out the fitting procedure with the two scale parameters.

As already said, the Hernquist and Schuster distributions are special cases of Eq. (1). Therefore, the first attempts of fitting a given rotation curve based on the QIS model, for fixed total mass of the corona, could be effected with either of these two models. The corresponding expressions for the circular velocity are equated to the given values (based on the QIS model) and since the total mass is fixed, there is only one remaining parameter (a or b - Eq. (1)). This equating yields, of course, at each point different parameter value. The final step is to calculate the mean value. With the parameter value obtained in this way and the already fixed total mass it is possible to calculate the circular velocities for both special cases (Hernquist and Schuster) and to compare to the values corresponding to the QIS model at the same points. Then the fitting quality will be indicated by the average absolute difference.

This can be more clearly seen from the following numerical example. As a reasonable value for the total mass of the corona the one of 1000 $G\mathcal{M}_\odot$ is assumed. Such an amount seems justified if the usual ones associated with the subsystems in giant galaxies are borne in mind. As the interval within which the fitting is to be examined that of $0 \leq R \leq 30$ (distance unit kpc) is chosen. It is clear that normally the rotation curve of a galaxy is studied closer to the centre and it is well known that at very distant points (say exceeding 30 kpc) such studies can hardly be done. A typical curve resulting from the QIS model is fitted. The application of the Hernquist special case yields a value of 54.61 kpc for the parameter where the circular velocities on the average differ by 24.64 km s^{-1}. On the other hand, when the Schuster special case is applied, the corresponding values are 34.14 kpc, resp. 11.86 km s^{-1}.

As well known, in the case of applying the QIS model the total mass is concentrated within a finite radius, unlike that of Eq. (1) where it is concentrated within an infinite one. This should be borne in mind. Therefore, an extrapolation towards higher values of R is desirable. With the parameter values used here for the QIS case the total mass of 1000 $G\mathcal{M}_\odot$ yields a value of 94 kpc for the limiting radius. An extrapolation towards 90 kpc yields the following: if the Hernquist special case with the value $a = 54.61$ kpc is applied, then the circular-velocity values differ on the average by 50.97 km s^{-1}; otherwise, for the Schuster special case, by retaining the parameter value, i. e. 34.14 kpc, the corresponding average difference will be 14.44 km s^{-1}. It should be said that in the case of the QIS model, the potential after the limiting radius becomes that of point mass immediately, whereas for Eq. (1) it gradually approaches the point-mass case. Due to this at high distances, say greater than 100 kpc, one can expect a sufficiently satisfactory agreement. Therefore, a general conclusion of this part of the present paper may be that the Schuster special case ($a = 0$ - Eq. (1)) yields a better fit and, hence, it should be used as the first approximation. A value of the parameter a different from zero would be introduced as the next step only, for the purpose of improving the fit. Such a procedure leads to reducing of the average difference from 11.86 km s^{-1} ($a = 0$, $b = 34.14$) to 10.3 km s^{-1} for $a = 2.6$ kpc, b remaining the same. After extrapolating Eq. (1) using these two values towards 90 kpc the average difference becomes 11.2 km s^{-1}. In addition, the velocity values for, for example, the QIS curve are largely above 100 km s^{-1} which means that the mentioned average discrepancy is less than 10%. Thus, one may say that model implied by Eq. (1) with all advantages mentioned above, can yield a satisfactory fit of a given rotation curve for a spiral galaxy, more precisely of the contibution to its rotation curve by the dark corona, i. e. at least practically equally good as that offered by the use of the popular QIS model.

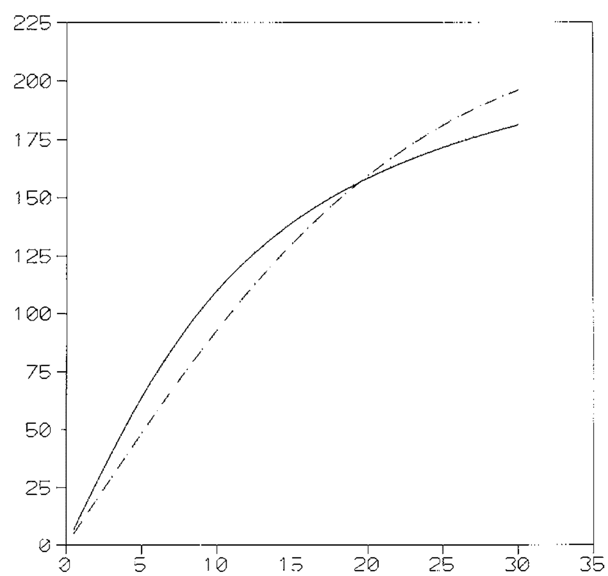

Fig. 1. *A given circular-velocity curve (solid line) based on QIS model - central density $\rho(0) = 0.01\mathcal{M}_\odot$ pc^{-3}, scale parameter equal to 10 kpc - juxtaposed with that based on Eq. (1) (dashed curve) where the values are: total mass 1000 $G\mathcal{M}_\odot$, scale lengths $a = 2.6$ kpc, $b = 34.14$ kpc; velocity unit km s^{-1}, distance unit kpc.*

The case considered here is illustrated in Fig. 1.

4. DISCUSSION AND CONCLUSIONS

It appears that the application of the generalised isochrone potential (Eq. (1)) can be useful for obtaining the contribution to the rotation curve of a (spiral) galaxy's dark corona. The circumstance that it is very desirable to specify the total mass of this corona first and only then to estimate the scale parameters, is not of importance. Normally, one studies a real galaxy, which means that, firstly, its observational rotation curve is known and secondly both the contributions of the observed subsystems (e. g. bulge, disc, etc.) and the total mass of the galaxy as a whole can be estimated independently. Thus in such a case at one's disposal would be the "remaining" rotation curve, i. e. the contribution of the corona with its total mass estimated. On the basis of the results obtained here it is clear that this "remaining" curve can be successfully fitted with model implied by Eq. (1). This model has some advantages compared to the QIS one, for example. As already said, unlike QIS where the density has a cut off, i. e. discontinuity, here the density decreases gradually and there is no need to specify a limiting radius. On the other hand it can be easily generalised towards axial symmetry. Therefore, if the objective is to offer a model for the dark corona of a (spiral) galaxy yielding the gravitational potential analytically with the possibility of its rather simple generalisation towards axial symmetry, this objective, in the present author's opinion, is largely achieved here.

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ЈЕДАН АЛТЕРНАТИВНИ МОДЕЛ МАСЕ ЗА ТАМНЕ КОРОНЕ ГАЛАКСИЈА

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Оригинални научни рад

Разматра се једна сферно симетрична расподела масе са две константе димензије дужине за тамну корону једне (спиралне) галаксије као алтернатива уобичајено примењиваној квази-изотермној сфери. Испитивања криве ротације која потиче од ове расподеле на ограниченом интервалу растојања до осе ротације показују да она може бити успешна алтернатива уобичајеној апроксимацији квази-изотермне сфере. Ово је значајно с обзиром да се формула за потенцијал разматрана у овом раду може лако уопштити на случај обртне симетрије.