

## THE CONTRIBUTION OF THE DARK MATTER TO THE ROTATION OF SPIRAL GALAXIES AND ITS MASS DISTRIBUTION

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**SUMMARY:** The rotation of a test spiral galaxy with two contributors - the disc and the corona - is considered. The disc is exponential, whereas the corona is the dark subsystem. For the latter several variants of mass distribution are considered. It is found that the homogeneous sphere is almost unavoidable if the circular velocity has a continuous increase, at least in its observable part. On the other hand the rather often applied quasi-isothermal law offers the most satisfactory fit for the case of constant circular velocity though the classical Schuster law may also be used, especially taking into account its simplicity and the consequent possibility of generalising its potential towards the more general case of axial symmetry.

### 1. INTRODUCTION

It is rather well known that for many spiral galaxies the velocity of rotation, i. e. circular velocity, remains roughly constant out to large radii (e. g. Ashman, 1992; Samurović, 1998). In some of them it has been noticed that the circular velocity is even increasing at sufficiently large radii (e. g. Gorbatskij, 1986). This observational fact, as well known, is usually attributed to the presence of the dark matter. In the present paper the structure formed by the dark matter will be called corona. In Ashman's review, for example, a discussion of various mass distributions within the coronae of spiral galaxies is presented. On the other hand one should also take into account that in many instances a concrete knowledge of the potential is required and, therefore, some sufficiently simple functions deserve attention (e. g. Ninković, 1992).

The present study is, thus, concentrated on three different forms of the density-radius depen-

dence in the framework of the assumed spherical shape for the corona.

### 2. THE MASS DISTRIBUTION IN THE CORONA

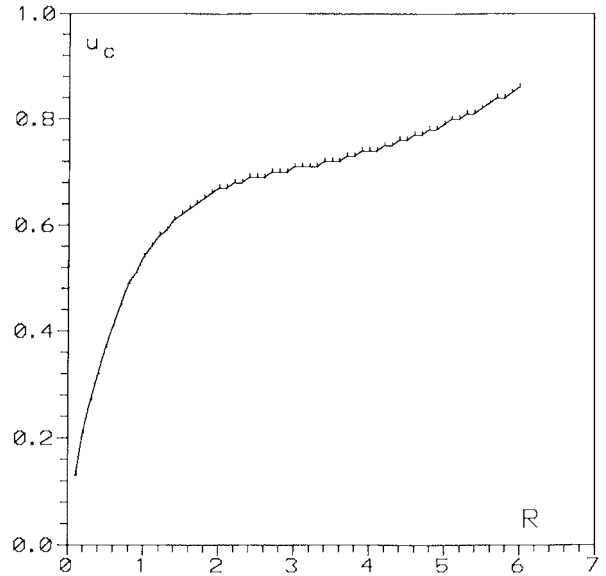
In the present paper for the mass distribution in the corona of a spiral galaxy one assumes it to comply with the generalised Schuster density law, particular case  $\beta = \frac{i}{2}$  (Ninković, 1998). As the three forms of the density-radius dependence, mentioned in the previous section, one considers those involving  $i = 0$ ,  $i = 2$  and  $i = 5$ . These three particular cases are known in the literature as corresponding to homogeneous sphere, quasi-isothermal law and classical Schuster law, respectively. Considering that spherical symmetry is assumed for the corona and that the three cases assumed above yield analytical solutions for the cumulative mass, the calculation of its contribution to the circular velocity is very simple.

### 3. RESULTS

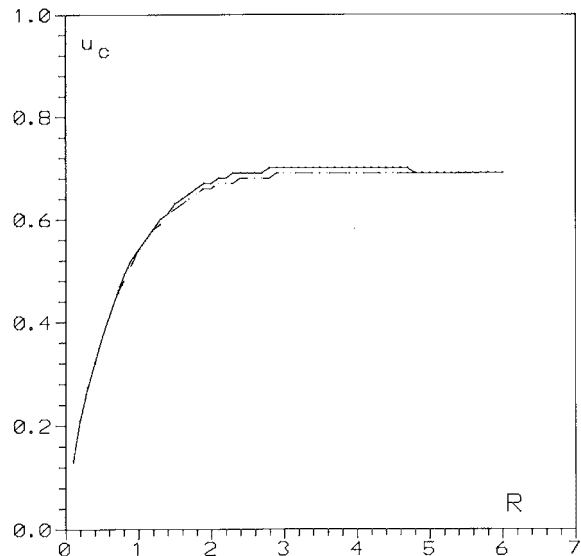
The adequacy of each of the mass distributions indicated in the previous section is examined, as natural, through its fit to a given rotation curve. Two general cases of rotation curves (RC) of a spiral galaxy are considered: RC remaining flat over sufficiently large radii and RC still rising at sufficiently large radii. In order to perform the corresponding calculations one must also take into account the contribution of the seen matter. As well known, this kind of matter is usually split into two principal subsystems: the bulge and the (thin) disc (e. g. Ninković, 1992). However, since the contribution of the former to the total gravitation field of a spiral galaxy is important in its inner parts only, while for the present purpose the more distant parts are of interest, the bulge contribution will be neglected, i. e. only the disc and the corona will be taken into account. For the disc matter it is assumed that it obeys the exponential mass distribution (e. g. Freeman, 1970). The RC (more precisely circular-velocity) given in Freeman's paper is treated here up to the distance of  $6 R_d$  ( $R_d$  scale length of exponential disc - details in Freeman's paper). In this way a set of circular velocity values (disc contribution) taken with step  $0.1 R_d$  expressed in terms of  $GM_d/R_d$  is obtained where  $G$  and  $M_d$  are gravitation constant and total disc mass, respectively.

In order to obtain the circular velocity of a spiral galaxy as a function of the distance to its rotation axis one should simply sum the squares of the circular velocities due to the two contributors. Since three different mass distributions are considered for the corona, in this way one can obtain three different RC. Each one of them corresponds to one of the mass distributions assumed above for the corona since the disc contribution is fixed. Clearly, here arises the question of the parameter ratios. This problem is solved generally by assuming that the disc is dominant in the inner parts; for example at the maximum of the circular velocity due to it, taking place at about  $2 R_d$ , the disc fraction in the circular-velocity square exceeds 86 %. Therefore, only at distances about  $4-5 R_d$  the contributions of the two subsystems will be approximately equal. This assumption is based on the fact that the disc of a spiral galaxy is expected to be maximal, i. e. dominant in the inner parts (e. g. Sackett, 1997).

Now the particular approach will differ for each one of the three mass distributions assumed for the corona. In the case of the homogeneous sphere there is only one parameter - the density. Its amount is found on the basis of the considerations referred to in the preceding paragraph. Considering that in this case the corona contribution to the circular velocity increases continuously, it is clear that equality of the two contributions should be attained sooner than in the other two models, say at about  $4 R_d$ . As for the mass of the corona, expressed in terms of the disc mass, it is possible to specify the lower limit only which, clearly, depends on the radius within which the present problem is studied ( $6 R_d$ ). If the quasi-isothermal sphere is considered, then there are two



**Fig. 1.** An increasing rotation curve: distance unit  $R_d$ , velocity unit  $(\frac{GM_d}{R_d})^{1/2}$ ; corona model homogeneous sphere, its density  $0.0037 \frac{M_d}{R_d^3}$ , the lower limit of corona mass  $3.3 M_d$ .



**Fig. 2.** A flat rotation curve: distance and velocity units as in Fig. 1; solid line: corona as quasi-isothermal sphere with central density  $0.0049 M_d/R_d^3$  and scale length of  $3.43 R_d$ , dashed line: Schuster-model corona, total mass is  $9.3 M_d$ , scale length  $8.57 R_d$ .

parameters - the central density and the characteristic radius (e. g. Ninković, 1998). Both can be obtained on the basis of the requirements given in the preceding paragraph, but it should be taken into account that this time the equality of the two contributions to the square of the circular velocity occurs

at about  $5 R_d$ . As for the limiting radius of the corona, i. e. its total mass (it must be limited in space - e. g. Ninković, 1998), one can obtain its lower limit only and this is the cumulative mass of the corona (e. g. Ninković, 1998) within the range of  $6 R_d$  just as in the homogeneous-sphere case. Finally, in the third case (classical Schuster law is applied), there are also two parameters, this time the characteristic radius and the total mass (e. g. Ninković, 1998). Both can be obtained from the same conditions as in the case of the quasi-isothermal sphere, i. e. the fraction of the disc at the maximum of its contribution to the circular-velocity square will be, as said above, over 86 % and the equality of the two contributions occurs again at about  $5 R_d$ .

The curves obtained according to what is laid out above are presented in Figs. 1-2. As can be seen, the homogeneous sphere yields an increasing circular velocity (at least within the radius enclosing the region subjected to analysis). This could be the case of a spiral galaxy in which the increase of circular velocity has been observed even at distances of several  $R_d$ . Bearing in mind the evident simplicity of the homogeneous sphere it could be recommended as a model for the dark corona in such cases. However, if one treats the case of circular velocity constant over a sufficiently wide area around the axis of rotation, then the other two models of mass distributions are preferred. This especially concerns the quasi isothermal sphere though the classical Schuster law could not be easily rejected. The latter has an advantage since the expressions for the cumulative mass, i. e. potential, offered by it, are very simple; they contain algebraic functions only (e. g. Ninković, 1998) and therefore they can be easily generalised to comprise the more complicated case of axial symmetry. This is important for those cases in which explicit expressions for the potential are necessary (e. g. Ninković, 1992).

#### 4. DISCUSSION AND CONCLUSIONS

Among the three alternative models for the dark corona of a spiral galaxy considered here the homogeneous sphere, taking into account especially its simplicity, appears as the best solution to accounting for a galaxy's rotation curve which still shows rising, even at distances as large as about, for example, 6 disc scale lengths. The other two - the quasi-isothermal sphere and the one conforming to the classical Schuster law appear as satisfactory approximations in the case of a rotation curve flat over a significant area around the axis of rotation. The latter one, though at first glance may seem less suitable, due to its simple potential expression is very applicable. It should be also emphasized that the value yielded by the Schuster sphere for the total mass of the corona is, in fact, the lower limit only because it is inferred from a study of the rotation curve and such studies, as well known, can indicate the lower limits only.

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**ДОПРИНОС ТАМНЕ МАТЕРИЈЕ КРИВОЈ РОТАЦИЈЕ  
СПИРАЛНИХ ГАЛАКСИЈА И ЊЕНА РАСПОДЕЛА МАСЕ**

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*Оригинални научни рад*

Разматра се крива ротације пробне спиралне галаксије са доприносом диска и короне. Диск је експоненцијалан, а корона је таман подсистем. За њу се разматра више варијанти расподеле масе. Нађено је да је хомогена сфера скоро незаобилазна ако кружна брзина бележи сталан раст, бар у оном делу који се може посматрати. С друге стране доста често

коришћени квазиизотермни закон најбоље задовољава случај константне кружне брзине премда класични Шустеров закон може такође да се употреби, нарочито када се има у виду његова једноставност и као последица могућност уопштавања његовог потенцијала на општији случај обртне симетрије.