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THE PRECISION OF TIME REGISTRATION WITH DANJON ASTROLABE

G. Perović[1] and Z. Cvetković[2]

[1]*Faculty of Civil Engineering – Department of Geodesy, Bul. revolucije 73/I, 11000 Belgrade, Yugoslavia*

[2]*Astronomical Observatory, Volgina 7, 11160 Belgrade 74, Yugoslavia*

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SUMMARY: During 1988 at the Munich, Vienna and Graz stations selected stars were observed for the purpose of determining difference in the longitudes. The measurements were performed with Danjon astrolabe by using the method of equal zenith distances. In the present paper the authors analyse the registration precision of 1604 star transits over a given almucantar. They estimate the variance of registration of star transits over a fictive thread as function of declination and parallactic angle before and after elimination of gross errors.

1. INTRODUCTION

For the purpose establishing of the European Longitude Network ELN precise measurements of longitude differences between national reference stations in Germany, Italy, Spain, the Netherlands, France, Potugal and Austria (Kaniuth and Wende, 1980, 1983, Wende 1992) have been performed. These measurements were carried out between 1977 and 1982, those in 1988 with Danjon astrolabe. In determining the longitude with Danjon astrolabe the method of equal zenith distances, i.e. the star-almucantar-transit time was registered at zenith distance $z \approx 30^o$.

In this paper the authors analyse the observational material concerning the determination of longitude differences between the Munich, Vienna and Graz stations from 1988 carried out by W. Wende (this is the part of the material received by the authors thanks to the courtesy of Prof. Dr-ing. Rudolf Sigl and dipl.-ing. Werner Wende). Selected FK5 stars have been observed within $+20°$ and $+70°$ declination range. For each of the 1604 star transits the time registration was performed using 24 contacts.

The present paper is aimed at determining of the registration precision of the times of the star almucantar transits and of the way of controlling of star-transit-registration gross errors.

Since the star-transit registrations must be tested for gross errors, it is necessary to estimate the registration standards. For this the authors use the dependence of time registration on declination and parallactic angle so that as a consequence the variances of star-almucantar-transit registrations are different.

2. THE PRECISION ANALYSIS OF THE REGISTRATION RESULTS

Let Θ_i (i = 1,...,24) be the registration times (contacts) of the almucantar transit of a star. Then, on the basis of 12 contact pairs symmetric with respect to the fictive mean thread, one can calculate

12 "registrations" of the fictive-mean-thread transits for the same star according to the following formula

$$\theta_j = \frac{1}{2} \left(\Theta_{13-j} + \Theta_{12+j}\right) \quad (j = 1, ..12) \; , \qquad (2.1)$$

hereafter the term RESTAFIT is used for **RE**gistrations of **STA**r-**FI**ctive-thread **T**ransits.

For a star transit we calculate the mean value $\bar{\theta}$ from 12 θ_j values, as well as the RESTAFIT-variance estimate

$$m_\theta^2 = \frac{1}{n-1} \sum_{j=1}^{n} (\theta_j - \bar{\theta})^2, \qquad n = 12 \; . \qquad (2.2)$$

We assume that the RESTAFITs θ_j obey a *Gaussian distribution law* with the following parameters: mathematical expectation $\mathrm{M}[\theta_j] = \xi_s$ and variance $\mathrm{D}[\theta_j] = \sigma^2_{\theta_s}$ (s = number of transits). They *differ from star to star* since the accuracy of time registration depends on the star-transit speed, i.e. on declination and paralactic angle.

In order to discover and eliminate those RESTAFITs containing gross errors one should reduce all quantities θ_j to values having the same variance. For this reason *reduced* RESTAFITs are calculated

$$\theta'_j = \theta_j \cdot \cos\delta \sin q \qquad (2.3)$$

having now for each star a Gaussian distribution with *the same variance* estimated for a single transit as

$$m_{\theta'}^2 = \frac{1}{n-1} \sum_{j=1}^{n} (\theta'_j - \bar{\theta}')^2 \; . \qquad (2.4)$$

The calculated values m_θ^2 and $m_{\theta'}^2$ for all transits are ordered in declination δ and paralactic angle q of stars. The mean group values are given: in Table 1 and Fig. 1 for declination, in Table 3 and Fig. 3 for paralactic angle.

The gross errors within a θ_j are established according the criterion of deviating from the mean value with a known standard of measurements σ_θ (Perović, 1989). The mean value from the standard estimates $m_{\theta'}$ is taken as a measurement standard $\sigma_{\theta'}$ because it is obtained from a large number of measurements (a total of 19 248) and consequently σ_θ is calculated now following (3.1). For a significance level $\alpha = 0.05$ of the criterion 1597 (about 8% of the total) are eliminated.

After the elimination of the gross errors the values m_θ^2 and $m_{\theta'}^2$ are calculated again for all transits and ordered according to declination δ and q of stars. The group mean values are given in: Table 2 and Fig. 2 for declination, and Table 4 and Fig. 4 for paralactic angle respectively.

Table 1. Variance estimate of RESTAFITs depending on declination before gross-errors elimination. N is the number of values within a group.

Group	δ	$m_{\theta'}^2$	m_θ^2	N
1	$20^o - 25^o$	$0^s.000372$	$0^s.001615$	28
2	25 − 30	.000391	.001276	177
3	30 − 35	.000384	.000966	277
4	35 − 40	.000432	.000970	323
5	40 − 45	.000401	.000898	199
6	45 − 50	.000385	.000921	186
7	50 − 55	.000362	.001006	143
8	55 − 60	.000335	.001171	151
9	60 − 65	.000308	.001495	85
10	65 − 70	.000302	.002059	35

Table 2. Variance estimate of RESTAFITs depending on declination after gross-errors elimination. N is the number of values within a group.

Group	δ	$m_{\theta'}^2$	m_θ^2	N
1	$20^o - 25^o$	$0^s.000244$	$0^s.001075$	28
2	25 − 30	.000228	.000748	177
3	30 − 35	.000230	.000578	277
4	35 − 40	.000237	.000535	323
5	40 − 45	.000228	.000511	199
6	45 − 50	.000233	.000557	186
7	50 − 55	.000226	.000628	143
8	55 − 60	.000207	.000720	151
9	60 − 65	.000211	.001027	85
10	65 − 70	.000212	.001425	35

Table 3. Variance estimate of RESTAFITs depending on paralactic angle before gross-errors elimination. N is the number of values within a group.

Group	q	$m_{\theta'}^2$	m_θ^2	N
1	$20^o - 30^o$	$0^s.000346$	$0^s.001769$	9
2	30 − 40	.000394	.001382	143
3	40 − 50	.000376	.000993	255
4	50 − 60	.000415	.000944	362
5	60 − 70	.000408	.000920	314
6	70 − 80	.000385	.000980	204
7	80 − 90	.000331	.001082	173
8	90 − 100	.000343	.001546	100
9	$q > 100$	.000284	.001875	44

Table 4. Variance estimate of RESTAFITs depending on paralactic angle after gross-errors elimination. N is the number of values within a group.

Group	q	$m^2_{\theta'}$	m^2_{θ}	N
1	$20^o - 30^o$	$0\overset{s}{.}000271$	$0\overset{s}{.}001382$	9
2	$30 - 40$	.000231	.000812	143
3	$40 - 50$	.000224	.000595	255
4	$50 - 60$	.000237	.000540	362
5	$60 - 70$	.000230	.000521	314
6	$70 - 80$	.000227	.000579	204
7	$80 - 90$	.000216	.000707	173
8	$90 - 100$	.000215	.000981	100
9	$q > 100$	.000204	.001332	44

In 98 star transits three RESTAFITs are eliminated, in 47 transits 4, in 24 transits 5, in 5 transits 6 (half) and there is one transit where even 7 RESTAFITs are eliminated.

Bartlett's test of variance equality applied to the data from Tables 1-4 for significance level $\alpha = 0.05$ leads to rejecting the equality hypothesis of variances $\sigma_{\theta'}$ before the gross-errors elimination (Tables 1 and 3) and to accepting the same hypothesis concerning variances $\sigma_{\theta'}$ after the gross-errors elimination (Tables 2 and 4).

The final value of the measuring standard is calculated as the weighted mean after eliminating RESTAFITs containing gross errors. Its amount is

$$\sigma_{\theta'} = \bar{m}_{\theta'} = 0\overset{s}{.}0150 \quad .$$

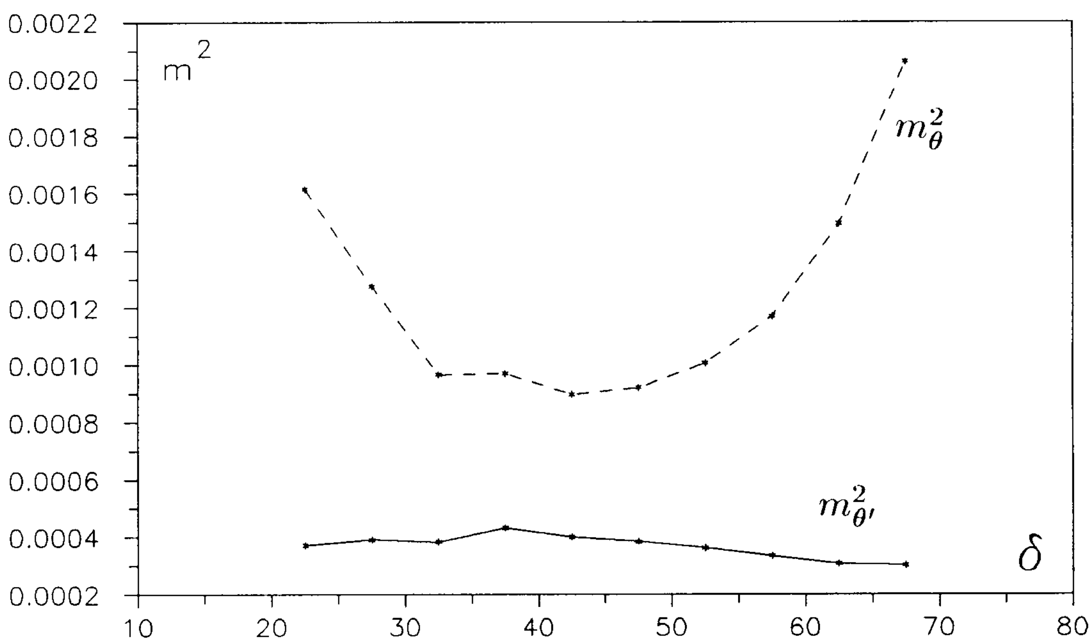

Fig. 1. *Variance estimate for RESTAFITs depending on declination before gross-errors elimination. The dashed line presents m^2_θ; the solid one presents $m^2_{\theta'}$.*

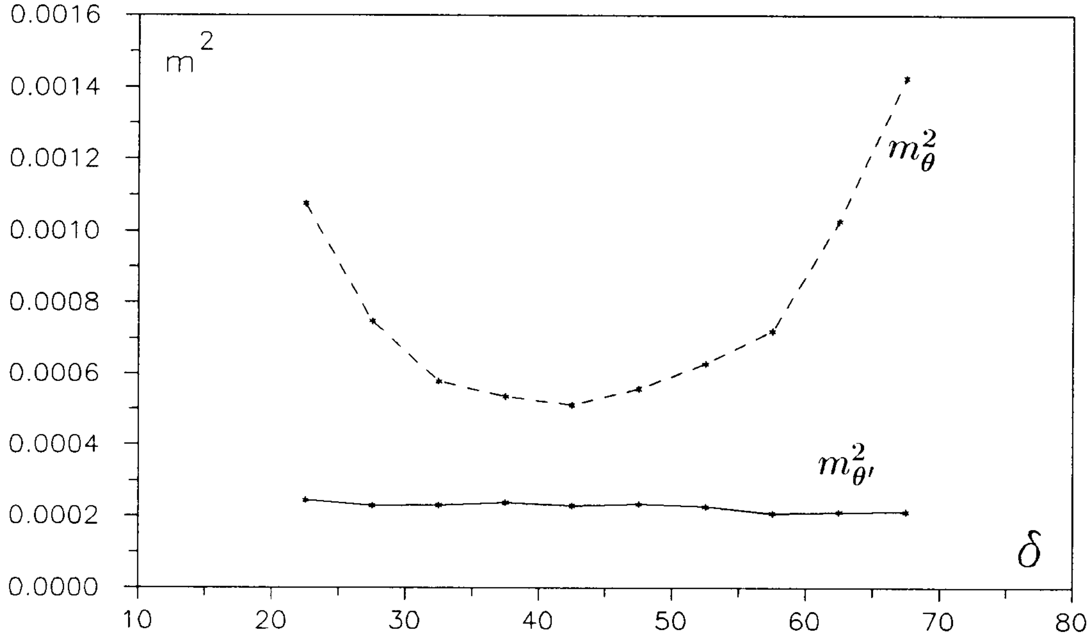

Fig. 2. *Variance estimate for RESTAFITs depending on declination after gross-errors elimination. The dashed line presents* m_θ^2*; the solid one presents* $m_{\theta'}^2$.

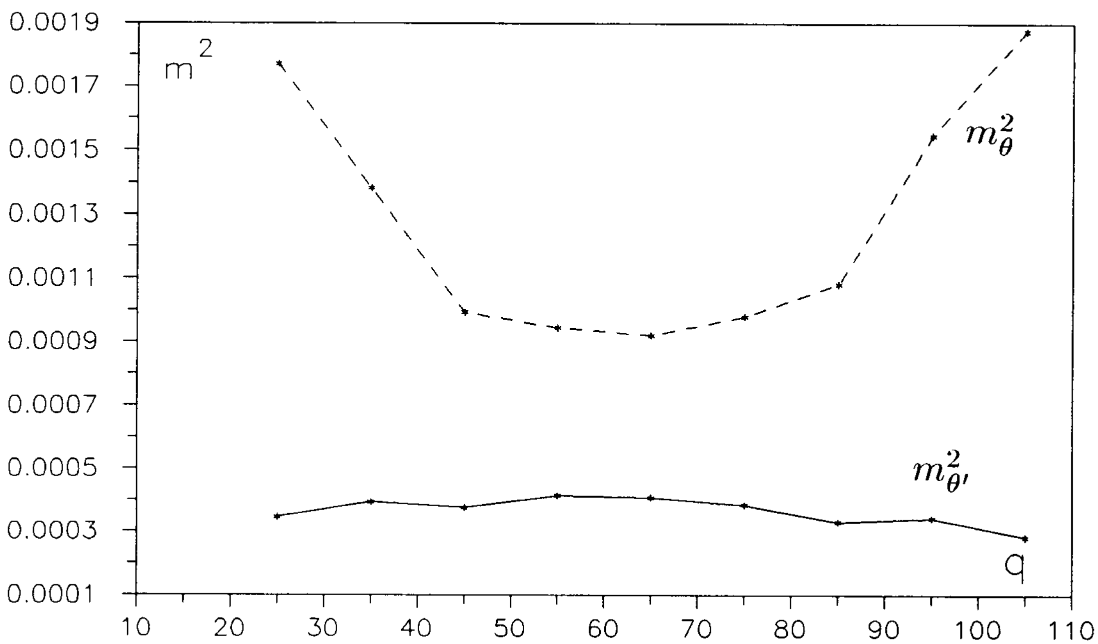

Fig. 3. *Variance estimate for RESTAFITs depending on paralactic angle before gross-errors elimination. The dashed line presents* m_θ^2*; the solid one presents* $m_{\theta'}^2$.

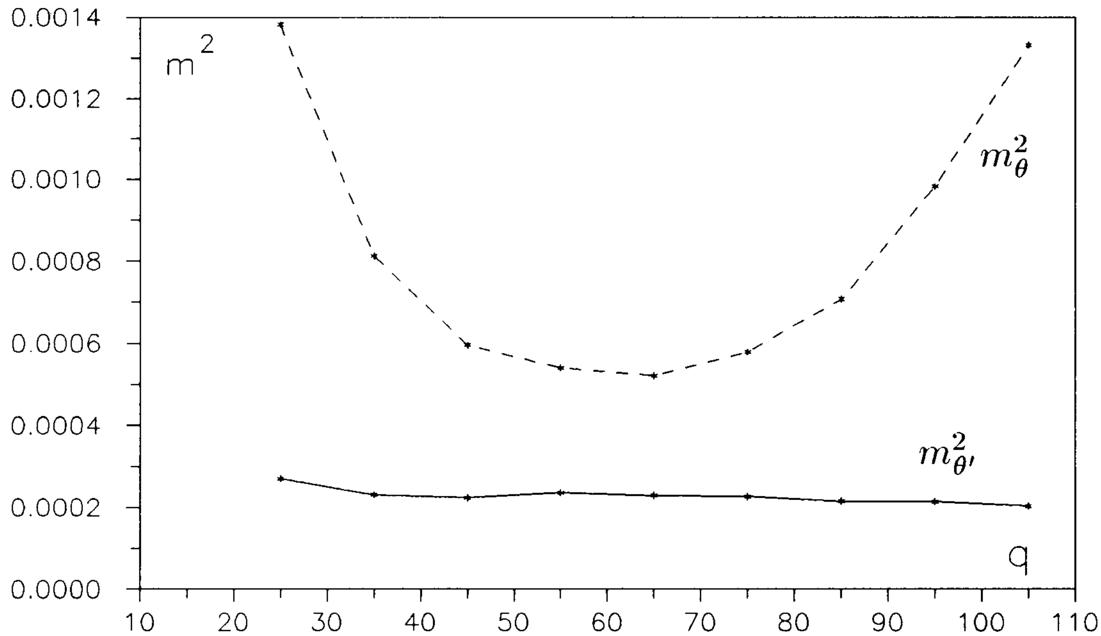

Fig. 4. *Variance estimate for RESTAFITs depending on paralactic angle after gross-errors elimination. The dashed line presents* m_θ^2*; the solid one presents* $m_{\theta'}^2$*.*

3. THE CONTROL OF GROSS ERRORS

The gross errors within a RESTAFIT are controled by testing the deviations from the mean value with known observing standard. Here it is necessary either to reduce RESTAFITs θ_j to θ'_j which have the same standard $\sigma_{\theta'}$ or for every star transit to calculate σ_θ by using the formula

$$\sigma_\theta = \frac{\sigma_{\theta'}}{\cos\delta \sin q} \quad , \tag{3.1}$$

and then to test the deviations from $\bar{\theta}_j$.

Now for a star with, for example, $\delta = 45^o$ and $q = 60^o$ it would be $\sigma_\theta = 0\overset{s}{.}0245$.

4. CONCLUSION

By reducing the RESTAFITs θ_j whose variances $\sigma^2_{\theta_s}$ depend on star declinations δ and paralactic angles q, to the quantities θ'_j with the same variance $\sigma^2_{\theta'}$ for every star it is possible to evaluate the variances of registrations of star almucantar transits on the basis of transit registrations for all stars. In this way one obtains

$$\sigma_{\theta'} = 0\overset{s}{.}0150 \quad \text{with} \quad \text{f} = 16\ 047 \text{ degrees of freedom.}$$

The testing shows that the given material is very *homogeneous* because the total of 19 248 RESTAFITs contains only 1597 (about 8%) with gross errors (for a significance level of 0.05).

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REFERENCES

Kaniuth, K. and Wende, W.: 1980, *Weröff. Deutsche Geod. Komm.*, Reihe B, München, **250**, 1-44.

Kaniuth, K. and Wende, W.: 1983, *Weröff. Bayer. Komm. f.d. Int. Erdmessung*, Astronom.-Geod. Arbeiten, München, **44**, 5-35.

Perović, G.: 1989, *Račun izravnanja, Kniga I, Teorija grešaka*, Naučna knjiga, Beograd, 134-140.

Wende, W.: 1992, *Weröff. Bayer. Komm. f.d. Int. Erdmessung*, Astronom.-Geod. Arbeiten, München, **50**, 7-25.

ПРЕЦИЗНОСТ РЕГИСТРАЦИЈЕ ВРЕМЕНА ДАНЖОНОВИМ АСТРОЛАБОМ

Г. Перовић[1] и З. Цветковић[2]

[1]*Грађевински факултет – Институт за геодезију, Бул. револуције 73/I, 11000 Београд, Југославија*

[2]*Астрономска опсерваторија, Волгина 7, 11160 Београд 74, Југославија*

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Оригинални научни рад

У току 1988. године на станицама Минхен, Беч и Грац посматране су одабране звезде ради одређивања разлика лонгитуда. Посматрања су обављена Данжоновим астролабом, по методи једнаких зенитних даљина. У раду је анализирана прецизност регистрације тренутака 1604 пролаза звезда кроз дати алмукантарат. Приказана је оцена дисперзије регистрације пролаза звезда кроз фиктивни конац у зависности од деклинације и паралактичког угла пре и после елиминације грубих грешака.