Students wanted: Master and PhD theses possible

Who am I:

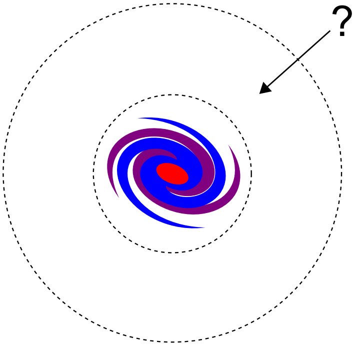
- Dr. Michal Bilek, naucni saradnik at Astronomical Observatory Belgrade, see the contact at the bottom
- Previously: postdoc at Strasbourg and Paris Observatory (with Francoise Combes), ESO visitor, visiting professor at Vienna University

Possible topics:

- Formation and dynamics of galaxies, the missing mass problem
- Analytic models, simulations and observations
- Potentially joint PhD with Strasbourg Observatory

Michal Bílek, AOB Volgina 7, office 24, email: michal.bilek@aob.rs

Gravitational lensing by galaxies as a probe of the law of gravity



Michal Bílek

25 March 2025 / Faculty of Mathematics, Belgrade

Michal Bílek, AOB Volgina 7, office 24, email: michal.bilek@aob.rs

Brief introduction to Λ Cold Dark Matter

- CDM + General relativity + nonzero cosmological constant + inflation = ΛCDM cosmology
- Current standard
- 83% of all matter are hypothetical particles outside of the standard model of particle physics
- Interact only gravitationally
- Their velocity dispersion is $\ll c$

Simulations of structure formation

Video
(Credit: Benedict Diemer)

online:
http://erebos.astro.umd.edu/web/viz/movies/movie10_br03.0_fps50.mp4

Density profile of dark matter halos

- Dark matter forms halos
- They are roughly spherically symmetric
- Halos have the NFW (Navarro-Frenk-White) profile:

$$\rho(r) = \frac{\rho_0}{\frac{r}{r_s}\left(1 + \frac{r}{r_s}\right)^2},$$

- Halos attract baryons, galaxies form from them
- $\rightarrow$ Every galaxy has a $\approx$NFW dark halo

Perhaps the ΛCDM hypothesis is not correct ...

1. Inflation unexpected
2. Hubble tension (Verde 19, Di Valentino 21)
3. σ_8 tension (Abdalla+22)
4. Cosmological constant value
5. Too little ^{7}Li in the Universe (Iliadis+20)
6. Massive galaxy clusters seen too early – El Gordo (Asencio+21)
7. High collisional velocity of the Bullet Cluster (Lee+10,Thompson+12)
8. Large-scale flow of galaxy clusters (Feldman+10,Watkins+23, Whitford+23)
9. Local Void too empty (Peebles 10, Hasbauer+21)
10. Many massive galaxies outside of the Local Sheet (Peebles+10)
11. Disks of satellites (Pawlowski 21, Muller 23)
12. Massive galaxies form too slow in simulations (Eappen+22, Xiao+24)
13. Bulgeless galaxies hardly form in simulations (Kormendy+10, Fisher+11, Brooks+16) and galaxies are overall too thick (Haslbauer+21)
14. Opposite trend of bar frequency with galaxy mass in simulations (Roshan+21)
15. Too fast galaxy bars in simulations (Roshan+21)
16. Too many satellites in simulations (Moore+99,Muller+20)
17. Too few satellites in simulations (Muller+24)
18. Too-big-to-fail problem (Pawlowski+15)
19. The core-cusp problem
20. High diversity of shapes of rotation curves of dwarf galaxies (Ghari+18)
21. Dark matter particles not detected

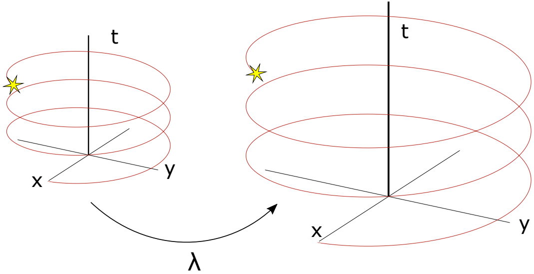
… etc.

Brief introduction to MOND

- Alternative to dark matter as the solution of the missing mass problem (Milgrom 1983)
- *Class* of modified gravity and modified theories
- Newtonian dynamics/general relativity only for strong gravitational field $a \gtrsim a_0 \approx 10^{-10}\,\mathrm{m\,s^{-2}}$
- For weak fields – Objects experience higher acceleration than predicted by Newton, space-time scaling symmetry applies

Brief introduction to MOND

Space-time scaling symmetry:

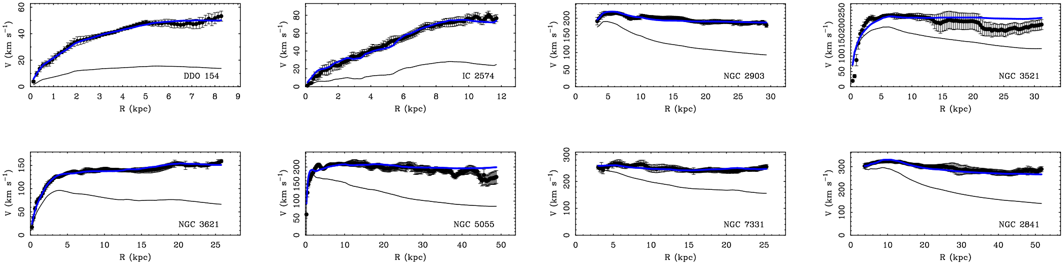


If an orbit is allowed, then the magnified orbit is allowed.

MOND is non-linear, no principle of superposition, intenal dynamics of an object depends on the strength of the external gravitational field – the External Field Effect (EFE)

Rotation velocities of spiral galaxies

Rotation velocity as a function of radius (Gentile+11)

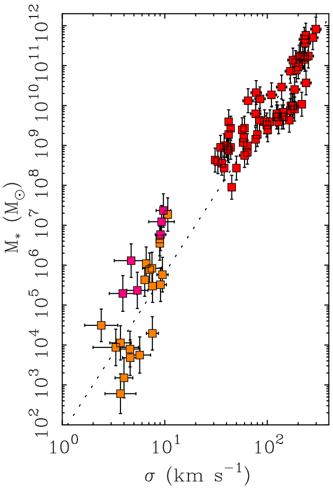


Galaxy masses $3 \times 10^8 - 3 \times 10^{11}$ $M_\odot$

Velocity dispersions of elliptical galaxies

Central velocity dispersion vs. stellar mass

Dotted line = MOND prediction



Famaey & McGaugh (2012)

How to distinguish MOND from dark matter?

Strongest indication of MOND is its ability to model rotation curves. The same can be done with a suitable distribution of dark matter with Newtonian gravity (while not fully seen in simulations yet).

Additional discriminator tests are desirable:

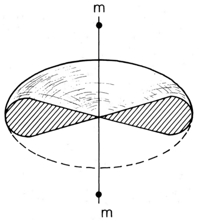
- External field effect (e.g., satellites of galaxies), dynamical friction, wide binary stars, relative velocities of galaxy clusters, growth of cosmological structure, efficiency of formation of tidal dwarf galaxies
- None of them gave clear results, let's try something else...

Phantom (i.e. effective) dark matter halos

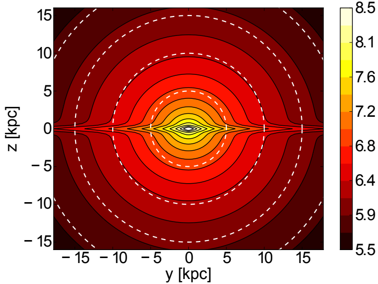
- Phantom dark matter – a mathematical construct, no real particles
- Phantom dark matter = dark matter that we would have to add to the baryonic matter in Newtonian gravity in order to get the gravitational field predicted by MOND
- Can be defined only in some MOND theories

Phantom dark matter has strange properties

- Can have NEGATIVE DENSITY!
- Phantom dark matter concentrates toward the galactic disk
- Phantom halos of galaxies are logarithmic outside
- Phantom halos of galaxies hollow inside
- The halos are different for every theory



Milgrom 84

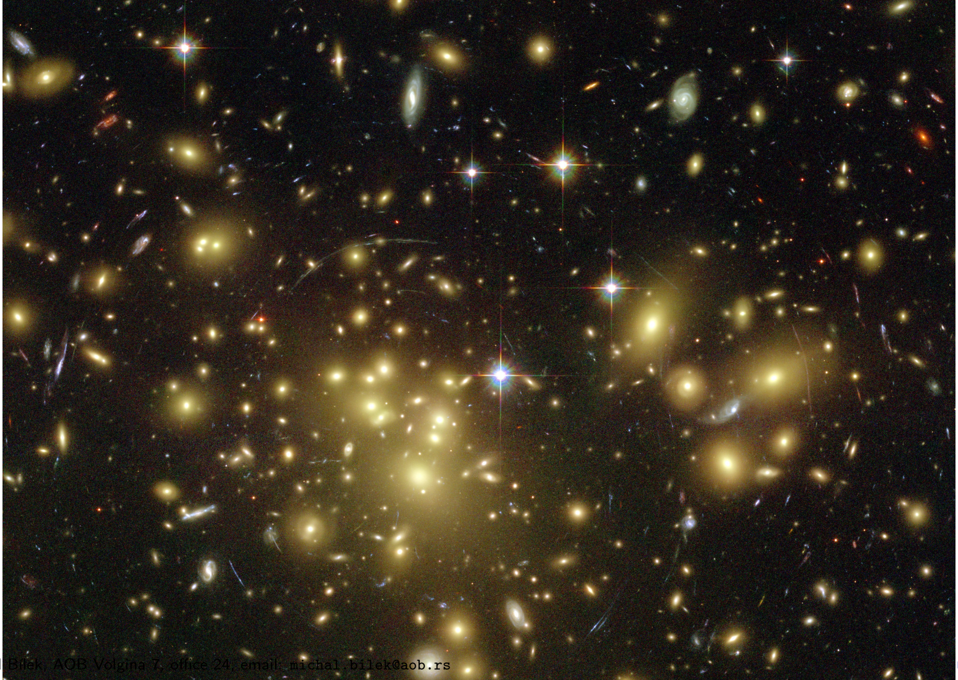


Famaey+12

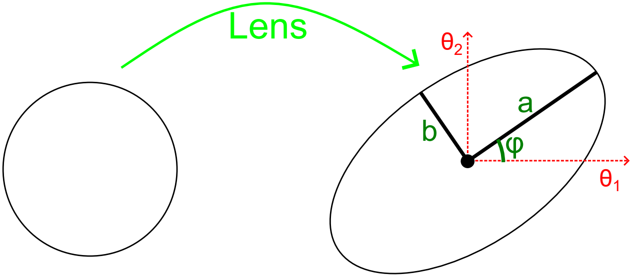
How to learn the real shapes of halos? **Gravitational lensing!**

Gravitational (optical) lensing

Credit: Lee Ann Mize

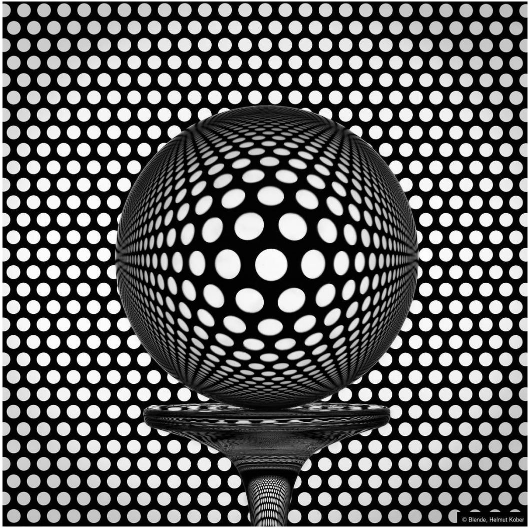


Description of gravitational lensing



- Sky coordinates θ_1, θ_2
- Ellipse has axes a and b, position angle φ
- Complex shear $\gamma = \gamma_1 + i\gamma_2 = \underbrace{\frac{1 - b/a}{1 + b/a}}_{\text{Flattening}} \underbrace{e^{2\varphi i}}_{\text{Orientation}}$ (for weak lensing)

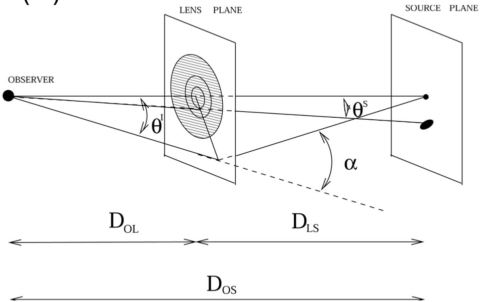
Gravitational (optical) lensing



Credit: Helmut Kober

How to predict gravitational lensing

- Project the lens density $\rho(\boldsymbol{r})$ to a plane perpendicular to the line-of-sight (thin lens approximation) to get surface density $\Sigma(\boldsymbol{\theta})$



Mellier 1999

- Calculate the lensing potential $\psi(\boldsymbol{\theta})$:

$$\Delta\psi(\boldsymbol{\theta}) = \frac{8\pi G D_{OL} D_{LS}}{c^2 D_{OS}} \Sigma(\boldsymbol{\theta})$$

- Complex shear components are

$$\gamma_1 = \frac{1}{2}\left(\frac{\partial^2\psi}{\partial\theta_1^2} - \frac{\partial^2\psi}{\partial\theta_2^2}\right), \quad \gamma_2 = \frac{\partial^2\psi}{\partial\theta_1\theta_2}$$

- Works also vice-versa: $\Sigma(\boldsymbol{\theta})$ from $\gamma(\boldsymbol{\theta})$
- What with modified gravity? Treat Phantom dark matter as real (Milgrom 2012)!

Lensing by point masses and their pairs in MOND

Goals:

- Explore the phantom dark matter halos in detail (QUMOND formulation of MOND)
- Predict maps of gravitational lensing for Euclid. Any strange features there?

Peculiar dark matter halos inferred from gravitational lensing as a manifestation of modified gravity

Michal Bílek[1,2,3]

[1] Observatoire de Paris, LERMA, Collège de France, CNRS, PSL University, Sorbonne University, F-75014, Paris
e-mail: michal.bilek@obspm.fr

[2] Strasbourg University, CNRS, Observatoire astronomique de Strasbourg, F-67000 Strasbourg, France

[3] FZU – Institute of Physics of the Czech Academy of Sciences, Na Slovance 1999/2, Prague 182 21, Czech Republic

Received ...; accepted ...

ABSTRACT

If modified gravity holds, but the weak lensing analysis is done in the standard way, one finds that dark matter halos have peculiar shapes, not following the standard Navarro-Frenk-White profiles, and are fully predictable from the distribution of baryons. Here we study in detail the distribution of the apparent dark matter around point masses, which approximate galaxies and galaxy clusters, and their pairs for the QUMOND MOND gravity, taking an external gravitational acceleration g_e into account. At large radii, the apparent halo of a point mass M is shifted against the direction of the external field. When averaged over all lines-of-sight, the halo has a hollow center, and denoting the by a_0 the MOND acceleration constant, its density behaves like $\rho(r) = \sqrt{Ma_0/G}/(4\pi r^2)$ between the galacticentric radii $\sqrt{GM/a_0}$ and $\sqrt{GMa_0}/g_e$, and like $\rho \propto r^{-7}G^2M^3a_0^3/g_e^5$ further away. Between a pair of point masses, there is a region of a negative apparent dark matter density, whose mass can exceed the baryonic mass of the system. The density of the combined dark matter halo is not a sum of the densities of the halos of the individual points. The density has a singularity near the zero-acceleration point, but remains finite in projection. We compute maps of the surface density and the lensing shear for several configurations of the problem, and derive formulas to scale them to further configurations. In general, for a large subset of MOND theories in their weak field regime, for any configuration of the baryonic mass M with the characteristic size of d, the total lensing density scales as $\rho(\boldsymbol{x}) = \sqrt{Ma_0/G}d^{-2}f\left(\boldsymbol{\alpha}, \boldsymbol{x}/d, g_e d/\sqrt{GMa_0}\right)$, where the vector $\boldsymbol{\alpha}$ describes the geometry of the system. Detecting the difference between QUMOND and cold dark matter halos appears to be possible with the existing instruments.

Key words. Gravitational lensing: weak – Gravitation – Dark matter – Methods: analytical – Methods: observational –

Bílek 2024, A&A, 390, 364

QUMOND and its phantom dark matter

How to calculate gravitational potential form given density $\rho(\boldsymbol{r})$?

- In Newtonian gravity:

$$\Delta\phi_N(\boldsymbol{r}) = -4\pi G\rho(\boldsymbol{r}) \tag{1}$$

- In QUMOND:
  1. Solve Eq. 1
  2. Calculate the **phantom dark matter density**

$$\rho_{ph}(\boldsymbol{r}) = -\frac{1}{4\pi G}\nabla\left[\nu\left(\frac{|\nabla\phi_N|}{a_0}\right)\nabla\phi_N\right] \tag{2}$$

  3. Solve Eq. 1 again with $\rho + \rho_{ph}$ instead of ρ to get the QUMOND gravitational potential ϕ_Q

Very simple to get ρ_{ph} for point masses, their superposition, even if they reside in an external field:

Appendix D: Expression for the PDM density around a point mass residing in a homogeneous external field

```
(M*exp((gNz^2*(x^2+y^2+z^2)^3+G^2*M^2*x^2+G^2*M^2*y^2+G^2*M^2*z^2-2*G*M*gNz*z*(x^2+y^2+z^2)
^(3/2))^(1/4)/(a0^(1/2)*(x^2+y^2+z^2)^(3/4)))*(2*gNz^2*z^2*(x^2+y^2+z^2)^3-gNz^2*y^2*(x^2+y^2+
z^2)^3-gNz^2*x^2*(x^2+y^2+z^2)^3+2*G^2*M^2*x^4+2*G^2*M^2*y^4+2*G^2*M^2*z^4+4*G^2*M^2*x^2*y^2+
4*G^2*M^2*x^2*z^2+4*G^2*M^2*y^2*z^2+G*M*gNz*z*(x^2+y^2+z^2)^(5/2)-5*G*M*gNz*z^3*(x^2+y^2+z^2)
^(3/2)-5*G*M*gNz*x^2*z*(x^2+y^2+z^2)^(3/2)-5*G*M*gNz*y^2*z*(x^2+y^2+z^2)^(3/2)))/(8*pi*a0^(1/
2)*(exp((gNz^2*(x^2+y^2+z^2)^3+G^2*M^2*x^2+G^2*M^2*y^2+G^2*M^2*z^2-2*G*M*gNz*z*(x^2+y^2+z^2)
^(3/2))^(1/4)/(a0^(1/2)*(x^2+y^2+z^2)^(3/4)))-1)^2*(x^2+y^2+z^2)^(13/4)*(gNz^2*(x^2+y^2+z^2)^3+
G^2*M^2*x^2+G^2*M^2*y^2+G^2*M^2*z^2-2*G*M*gNz*z*(x^2+y^2+z^2)^(3/2))^(3/4))
```

Appendix E: Expression for the PDM density around an isolated pair of point masses

```
-(((G*M2)/((d+z)^2+r^2)^(3/2)+(G*M1)/(r^2+z^2)^(3/2)-(3*G*M1*z^2)/(r^2+z^2)^(5/2)-(3*G*M2*(2*d+
2*z)^2)/(4*((d+z)^2+r^2)^(5/2)))/(exp(-((abs((G*M2*(2*d+2*z))/(2*((d+z)^2+r^2)^(3/2))+(G*M1*z)
/(r^2+z^2)^(3/2))^2+((G*M2*r)/((d+z)^2+r^2)^(3/2)+(G*M1*r)/(r^2+z^2)^(3/2))^2)^(1/2)/a0)^(1/2))-1)
+(((G*M2*r)/((d+z)^2+r^2)^(3/2)+(G*M1*r)/(r^2+z^2)^(3/2))/(exp(-((abs((G*M2*(2*d+2*z))/(2*((d+z)
^2+r^2)^(3/2))+(G*M1*z)/(r^2+z^2)^(3/2))^2+((G*M2*r)/((d+z)^2+r^2)^(3/2)+(G*M1*r)/(r^2+z^2)^(3/2)
)^2)^(1/2)/a0)^(1/2))-1)+(r*((G*M2)/((d+z)^2+r^2)^(3/2)+(G*M1)/(r^2+z^2)^(3/2)-(3*G*M1*r^2)/(r^2+
z^2)^(5/2)-(3*G*M2*r^2)/((d+z)^2+r^2)^(5/2)))/(exp(-((abs((G*M2*(2*d+2*z))/(2*((d+z)^2+r^2)^(3/
2))+(G*M1*z)/(r^2+z^2)^(3/2))^2+((G*M2*r)/((d+z)^2+r^2)^(3/2)+(G*M1*r)/(r^2+z^2)^(3/2))^2)^(1/
2)/a0)^(1/2))-1)+(r*exp(-((abs((G*M2*(2*d+2*z))/(2*((d+z)^2+r^2)^(3/2))+(G*M1*z)/(r^2+z^2)^(3/
2))^2+((G*M2*r)/((d+z)^2+r^2)^(3/2)+(G*M1*r)/(r^2+z^2)^(3/2))^2)^(1/2)/a0)^(1/2))*(2*((G*M2*r)
/((d+z)^2+r^2)^(3/2)+(G*M1*r)/(r^2+z^2)^(3/2))*((G*M2)/((d+z)^2+r^2)^(3/2)+(G*M1)/(r^2+z^2)^(3/
2)-(3*G*M1*r^2)/(r^2+z^2)^(5/2)-(3*G*M2*r^2)/((d+z)^2+r^2)^(5/2))-2*abs((G*M2*(2*d+2*z))/(2*((d+z)
^2+r^2)^(3/2))+(G*M1*z)/(r^2+z^2)^(3/2))*sign((G*M2*(2*d+2*z))/(2*((d+z)^2+r^2)^(3/2))+(G*M1*z)
/(r^2+z^2)^(3/2))*((3*G*M1*r*z)/(r^2+z^2)^(5/2)+(3*G*M2*r*(2*d+2*z))/(2*((d+z)^2+r^2)^(5/2)))
)*((G*M2*r)/((d+z)^2+r^2)^(3/2)+(G*M1*r)/(r^2+z^2)^(3/2)))/(4*a0*((abs((G*M2*(2*d+2*z))/(2*((d+
z)^2+r^2)^(3/2))+(G*M1*z)/(r^2+z^2)^(3/2))^2+((G*M2*r)/((d+z)^2+r^2)^(3/2)+(G*M1*r)/(r^2+z^2)^(3/
2))^2)^(1/2)/a0)^(1/2)*(abs((G*M2*(2*d+2*z))/(2*((d+z)^2+r^2)^(3/2))+(G*M1*z)/(r^2+z^2)^(3/2))^2+
((G*M2*r)/((d+z)^2+r^2)^(3/2)+(G*M1*r)/(r^2+z^2)^(3/2))^2)^(1/2)*(exp(-((abs((G*M2*(2*d+2*z))/
(2*((d+z)^2+r^2)^(3/2))+(G*M1*z)/(r^2+z^2)^(3/2))^2+((G*M2*r)/((d+z)^2+r^2)^(3/2)+(G*M1*r)/(r^2+
z^2)^(3/2))^2)^(1/2)/a0)^(1/2))-1)^2))/r-(exp(-((abs((G*M2*(2*d+2*z))/(2*((d+z)^2+r^2)^(3/2))+
(G*M1*z)/(r^2+z^2)^(3/2))^2+((G*M2*r)/((d+z)^2+r^2)^(3/2)+(G*M1*r)/(r^2+z^2)^(3/2))^2)^(1/2)/a0)
^(1/2))*(2*((3*G*M1*r*z)/(r^2+z^2)^(5/2)+(3*G*M2*r*(2*d+2*z))/(2*((d+z)^2+r^2)^(5/2)))*((G*M2*r)
/((d+z)^2+r^2)^(3/2)+(G*M1*r)/(r^2+z^2)^(3/2))-2*abs((G*M2*(2*d+2*z))/(2*((d+z)^2+r^2)^(3/2))+
(G*M1*z)/(r^2+z^2)^(3/2))*sign((G*M2*(2*d+2*z))/(2*((d+z)^2+r^2)^(3/2))+(G*M1*z)/(r^2+z^2)^(3/2)
)*((G*M2)/((d+z)^2+r^2)^(3/2)+(G*M1)/(r^2+z^2)^(3/2)-(3*G*M1*z^2)/(r^2+z^2)^(5/2)-(3*G*M2*(2*d+2*z)
^2)/(4*((d+z)^2+r^2)^(5/2))))*((G*M2*(2*d+2*z))/(2*((d+z)^2+r^2)^(3/2))+(G*M1*z)/(r^2+z^2)^(3/2)))
/(4*a0*((abs((G*M2*(2*d+2*z))/(2*((d+z)^2+r^2)^(3/2))+(G*M1*z)/(r^2+z^2)^(3/2))^2+((G*M2*r)/((d+z)
^2+r^2)^(3/2)+(G*M1*r)/(r^2+z^2)^(3/2))^2)^(1/2)/a0)^(1/2)*(abs((G*M2*(2*d+2*z))/(2*((d+z)^2+r^2)
^(3/2))+(G*M1*z)/(r^2+z^2)^(3/2))^2+((G*M2*r)/((d+z)^2+r^2)^(3/2)+(G*M1*r)/(r^2+z^2)^(3/2))^2)^(1/
2)*(exp(-((abs((G*M2*(2*d+2*z))/(2*((d+z)^2+r^2)^(3/2))+(G*M1*z)/(r^2+z^2)^(3/2))^2+((G*M2*r)/((d+
z)^2+r^2)^(3/2)+(G*M1*r)/(r^2+z^2)^(3/2))^2)^(1/2)/a0)^(1/2))-1)^2))/(4*G*pi)
```

Appendix F: Expression for the PDM density around two point masses residing in a homogeneous external field

```
-(((G*M1)/(x^2+y^2+z^2)^(3/2)+(G*M2)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2)-(3*G*M1*z^2)/(x^2+y^2+z^2)^(5/
2)-(3*G*M2*(d+z)^2)/(2*d*z+d^2+x^2+y^2+z^2)^(5/2))/(exp(-(((G*M2*(d+z))/(2*d*z+d^2+x^2+y^2+z^2)
^(3/2)-gNz+(G*M1*z)/(x^2+y^2+z^2)^(3/2))^2+((G*M1*x)/(x^2+y^2+z^2)^(3/2)-gNx+(G*M2*x)/(2*d*z+d^2+
x^2+y^2+z^2)^(3/2))^2+((G*M1*y)/(x^2+y^2+z^2)^(3/2)-gNy+(G*M2*y)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))^2)
^(1/4)/a0^(1/2))-1)+((G*M1)/(x^2+y^2+z^2)^(3/2)+(G*M2)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2)-(3*G*M1*x^2)
/(x^2+y^2+z^2)^(5/2)-(3*G*M2*x^2)/(2*d*z+d^2+x^2+y^2+z^2)^(5/2))/(exp(-(((G*M2*(d+z))/(2*d*z+d^2+
x^2+y^2+z^2)^(3/2)-gNz+(G*M1*z)/(x^2+y^2+z^2)^(3/2))^2+((G*M1*x)/(x^2+y^2+z^2)^(3/2)-gNx+(G*M2*x)
/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))^2+((G*M1*y)/(x^2+y^2+z^2)^(3/2)-gNy+(G*M2*y)/(2*d*z+d^2+x^2+y^2+
z^2)^(3/2))^2)^(1/4)/a0^(1/2))-1)+((G*M1)/(x^2+y^2+z^2)^(3/2)+(G*M2)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2)
-(3*G*M1*y^2)/(x^2+y^2+z^2)^(5/2)-(3*G*M2*y^2)/(2*d*z+d^2+x^2+y^2+z^2)^(5/2))/(exp(-(((G*M2*(d+
z))/(2*d*z+d^2+x^2+y^2+z^2)^(3/2)-gNz+(G*M1*z)/(x^2+y^2+z^2)^(3/2))^2+((G*M1*x)/(x^2+y^2+z^2)
^(3/2)-gNx+(G*M2*x)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))^2+((G*M1*y)/(x^2+y^2+z^2)^(3/2)-gNy+(G*M2*y)
/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))^2)^(1/4)/a0^(1/2))-1)-(exp(-(((G*M2*(d+z))/(2*d*z+d^2+x^2+y^2+
z^2)^(3/2)-gNz+(G*M1*z)/(x^2+y^2+z^2)^(3/2))^2+((G*M1*x)/(x^2+y^2+z^2)^(3/2)-gNx+(G*M2*x)/(2*d*z+
```

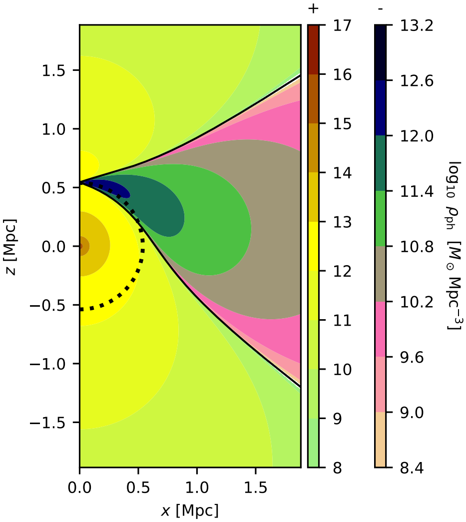


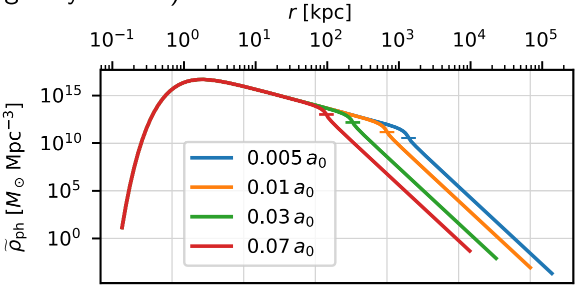
```
d^2+x^2+y^2+z^2)^(3/2))^2+((G*M1*y)/(x^2+y^2+z^2)^(3/2)-gNy+(G*M2*y)/(2*d*z+d^2+x^2+y^2+z^2)^(3/
2))^2)^(1/4)/a0^(1/2))*((G*M1*x)/(x^2+y^2+z^2)^(3/2)-gNx+(G*M2*x)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2)
)*(2*((3*G*M1*x*y)/(x^2+y^2+z^2)^(5/2)+(3*G*M2*x*y)/(2*d*z+d^2+x^2+y^2+z^2)^(5/2))*((G*M1*y)/(x^2+
y^2+z^2)^(3/2)-gNy+(G*M2*y)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))-2*((G*M1*x)/(x^2+y^2+z^2)^(3/2)-gNx+
(G*M2*x)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))*((G*M1)/(x^2+y^2+z^2)^(3/2)+(G*M2)/(2*d*z+d^2+x^2+y^2+z^2)
^(3/2)-(3*G*M1*x^2)/(x^2+y^2+z^2)^(5/2)-(3*G*M2*x^2)/(2*d*z+d^2+x^2+y^2+z^2)^(5/2))+2*((3*G*M1*x*z)
/(x^2+y^2+z^2)^(5/2)+(3*G*M2*x*(2*d+2*z))/(2*(2*d*z+d^2+x^2+y^2+z^2)^(5/2)))*((G*M2*(d+z))/(2*d*z+
d^2+x^2+y^2+z^2)^(3/2)-gNz+(G*M1*z)/(x^2+y^2+z^2)^(3/2))))/(4*a0^(1/2)*(exp(-(((G*M2*(d+z))/(2*d*z+
d^2+x^2+y^2+z^2)^(3/2)-gNz+(G*M1*z)/(x^2+y^2+z^2)^(3/2))^2+((G*M1*x)/(x^2+y^2+z^2)^(3/2)-gNx+
(G*M2*x)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))^2+((G*M1*y)/(x^2+y^2+z^2)^(3/2)-gNy+(G*M2*y)/(2*d*z+d^2+
x^2+y^2+z^2)^(3/2))^2)^(1/4)/a0^(1/2))-1)^2*(((G*M2*(d+z))/(2*d*z+d^2+x^2+y^2+z^2)^(3/2)-gNz+
(G*M1*z)/(x^2+y^2+z^2)^(3/2))^2+((G*M1*x)/(x^2+y^2+z^2)^(3/2)-gNx+(G*M2*x)/(2*d*z+d^2+x^2+y^2+
z^2)^(3/2))^2+((G*M1*y)/(x^2+y^2+z^2)^(3/2)-gNy+(G*M2*y)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))^2)^(3/4)
)-(exp(-(((G*M2*(d+z))/(2*d*z+d^2+x^2+y^2+z^2)^(3/2)-gNz+(G*M1*z)/(x^2+y^2+z^2)^(3/2))^2+((G*M1*x)
/(x^2+y^2+z^2)^(3/2)-gNx+(G*M2*x)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))^2+((G*M1*y)/(x^2+y^2+z^2)^(3/2)
-gNy+(G*M2*y)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))^2)^(1/4)/a0^(1/2))*((G*M1*y)/(x^2+y^2+z^2)^(3/2)-gNy+
(G*M2*y)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))*(2*((3*G*M1*x*y)/(x^2+y^2+z^2)^(5/2)+(3*G*M2*x*y)/(2*d*z+
d^2+x^2+y^2+z^2)^(5/2))*((G*M1*x)/(x^2+y^2+z^2)^(3/2)-gNx+(G*M2*x)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2)
)-2*((G*M1*y)/(x^2+y^2+z^2)^(3/2)-gNy+(G*M2*y)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))*((G*M1)/(x^2+y^2+z^2)
^(3/2)+(G*M2)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2)-(3*G*M1*y^2)/(x^2+y^2+z^2)^(5/2)-(3*G*M2*y^2)/(2*d*z+
d^2+x^2+y^2+z^2)^(5/2))+2*((3*G*M1*y*z)/(x^2+y^2+z^2)^(5/2)+(3*G*M2*y*(2*d+2*z))/(2*(2*d*z+d^2+
x^2+y^2+z^2)^(5/2)))*((G*M2*(d+z))/(2*d*z+d^2+x^2+y^2+z^2)^(3/2)-gNz+(G*M1*z)/(x^2+y^2+z^2)^(3/2)
)))/(4*a0^(1/2)*(exp(-(((G*M2*(d+z))/(2*d*z+d^2+x^2+y^2+z^2)^(3/2)-gNz+(G*M1*z)/(x^2+y^2+z^2)^(3/
2))^2+((G*M1*x)/(x^2+y^2+z^2)^(3/2)-gNx+(G*M2*x)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))^2+((G*M1*y)/(x^2+
y^2+z^2)^(3/2)-gNy+(G*M2*y)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))^2)^(1/4)/a0^(1/2))-1)^2*(((G*M2*(d+z)
)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2)-gNz+(G*M1*z)/(x^2+y^2+z^2)^(3/2))^2+((G*M1*x)/(x^2+y^2+z^2)^(3/2)
-gNx+(G*M2*x)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))^2+((G*M1*y)/(x^2+y^2+z^2)^(3/2)-gNy+(G*M2*y)/(2*d*z+
d^2+x^2+y^2+z^2)^(3/2))^2)^(3/4))-(exp(-(((G*M2*(d+z))/(2*d*z+d^2+x^2+y^2+z^2)^(3/2)-gNz+(G*M1*z)
/(x^2+y^2+z^2)^(3/2))^2+((G*M1*x)/(x^2+y^2+z^2)^(3/2)-gNx+(G*M2*x)/(2*d*z+d^2+x^2+y^2+z^2)^(3/
2))^2+((G*M1*y)/(x^2+y^2+z^2)^(3/2)-gNy+(G*M2*y)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))^2)^(1/4)/a0^(1/
2))*(2*((3*G*M1*x*z)/(x^2+y^2+z^2)^(5/2)+(3*G*M2*x*(2*d+2*z))/(2*(2*d*z+d^2+x^2+y^2+z^2)^(5/2)))
*((G*M1*x)/(x^2+y^2+z^2)^(3/2)-gNx+(G*M2*x)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))+2*((3*G*M1*y*z)/(x^2+
y^2+z^2)^(5/2)+(3*G*M2*y*(2*d+2*z))/(2*(2*d*z+d^2+x^2+y^2+z^2)^(5/2)))*((G*M1*y)/(x^2+y^2+z^2)
^(3/2)-gNy+(G*M2*y)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))-2*((G*M2*(d+z))/(2*d*z+d^2+x^2+y^2+z^2)^(3/2)
-gNz+(G*M1*z)/(x^2+y^2+z^2)^(3/2))*((G*M1)/(x^2+y^2+z^2)^(3/2)+(G*M2)/(2*d*z+d^2+x^2+y^2+z^2)^(3/
2)-(3*G*M1*z^2)/(x^2+y^2+z^2)^(5/2)-(3*G*M2*(d+z)^2)/(2*d*z+d^2+x^2+y^2+z^2)^(5/2)))*((G*M2*(d+z)
)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2)-gNz+(G*M1*z)/(x^2+y^2+z^2)^(3/2)))/(4*a0^(1/2)*(exp(-(((G*M2*(d+z)
)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2)-gNz+(G*M1*z)/(x^2+y^2+z^2)^(3/2))^2+((G*M1*x)/(x^2+y^2+z^2)^(3/2)
-gNx+(G*M2*x)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))^2+((G*M1*y)/(x^2+y^2+z^2)^(3/2)-gNy+(G*M2*y)/(2*d*z+
d^2+x^2+y^2+z^2)^(3/2))^2)^(1/4)/a0^(1/2))-1)^2*(((G*M2*(d+z))/(2*d*z+d^2+x^2+y^2+z^2)^(3/2)-gNz+
(G*M1*z)/(x^2+y^2+z^2)^(3/2))^2+((G*M1*x)/(x^2+y^2+z^2)^(3/2)-gNx+(G*M2*x)/(2*d*z+d^2+x^2+y^2+
z^2)^(3/2))^2+((G*M1*y)/(x^2+y^2+z^2)^(3/2)-gNy+(G*M2*y)/(2*d*z+d^2+x^2+y^2+z^2)^(3/2))^2)^(3/4)))
/(4*G*pi)
```

Michal Bílek, AOB Volgina 7, office 24, email: michal.bilek@aob.rs

Phantom dark matter for point mass in a homogeneous external field

(external field comes, e.g., form a distant galaxy cluster)



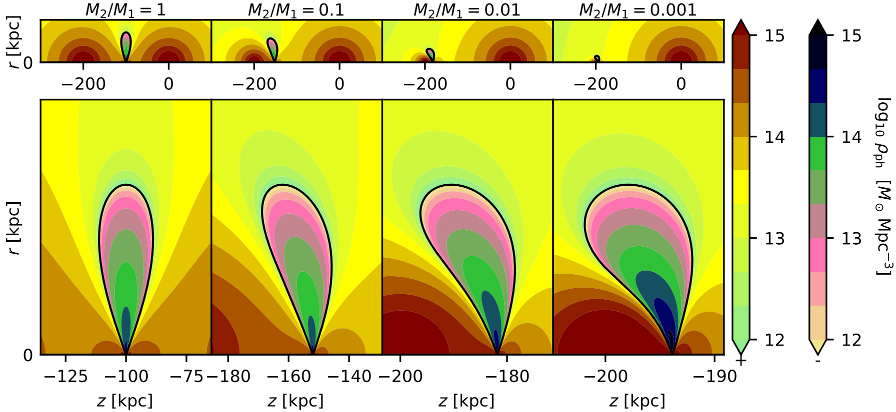


- Not NFW halo!
- Hollow center under the radius $\sqrt{\frac{GM}{a_0}}$
- Intermediate part $\tilde{\rho}_{ph}(r) \approx \frac{1}{4\pi G} r^{-2}$
- $\tilde{\rho}_{ph}(r) \propto r^{-7}$ beyond the radius $r = \sqrt{\frac{GMa_0}{g_{ext}}}$

Dark matter ring observed?

Note: uncertain, but illustrates the principle

Phantom dark matter for isolated pair of point masses

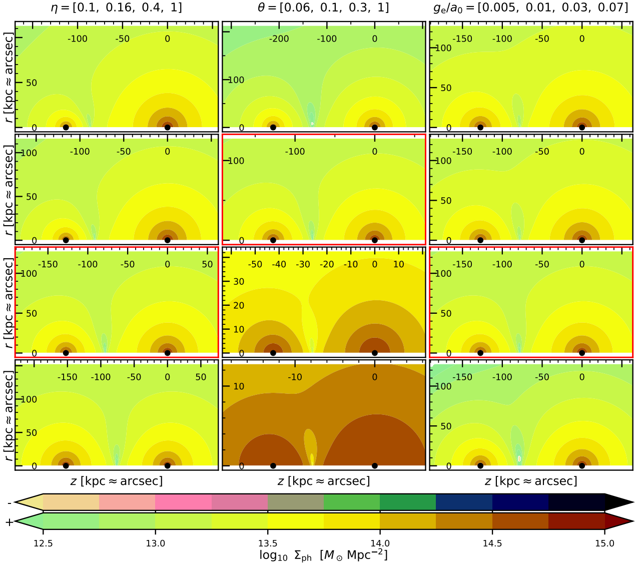


- Near the zero-acceleration point, $\rho_{ph} \to \pm\infty$!

$$\rho_{ph} \propto \frac{(z - z_0)^2 - r^2}{[4(z - z_0)^2 + r^2]^{5/4}}$$

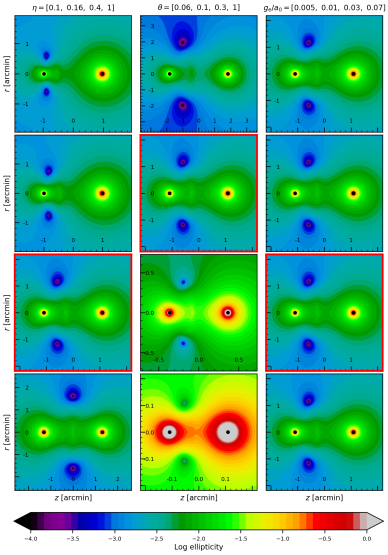
- Unfortunately, surface density is finite and low, so no strong lensing
- Mass of negative region can exceed the baryonic mass of the system!

Surface density – phantom halos repel each other

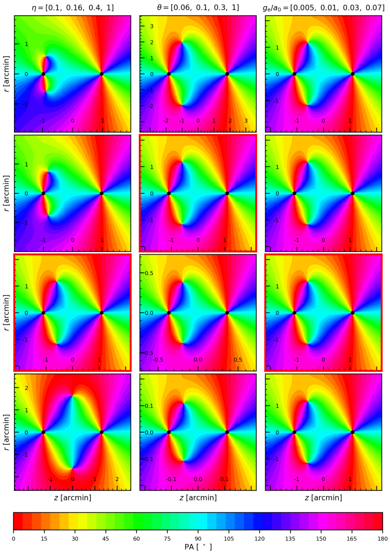


Surface density drop between the halos – unexpected for particle dark matter

Lensing maps – prediction for Euclid



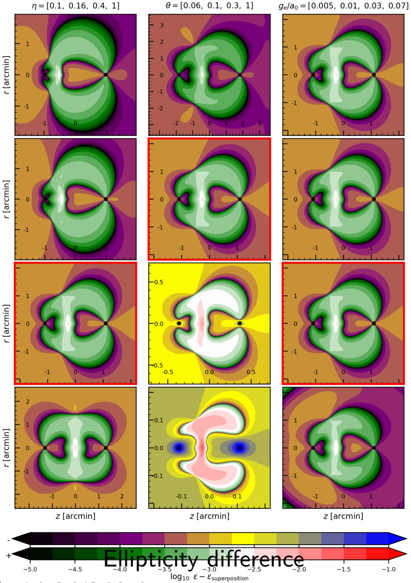
Ellipticity



Positional angle

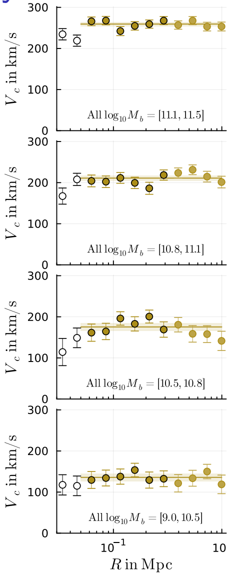
Michal Bílek, AOB Volgina 7, office 24, email: michal.bilek@aob.rs

Difference with respect to superposition density



Will the special MOND effects be observable by Euclid?

- $\delta\epsilon = \frac{0.2}{\sqrt{N_b}}$
- Central cavities: Maybe.
- Halos of isolated galaxies are not NFW: observed already!
- The ellipticity peak between two point masses: probably no.
- “Repelling halos” of galaxies in pairs: yes



Mistele+24

Future plans (for students)

- Gravitational lensing around a pair of galaxies in the ΛCDM cosmology
- Phantom dark matter halos of disk galaxies
- Phantom dark matter in cosmic voids
- Include projection effects
- Repeat for other MOND formulations
- Analyze real data (Final Euclid data release planned to 2031)