Proper elements of near-Earth asteroids: theory, applications, and new results

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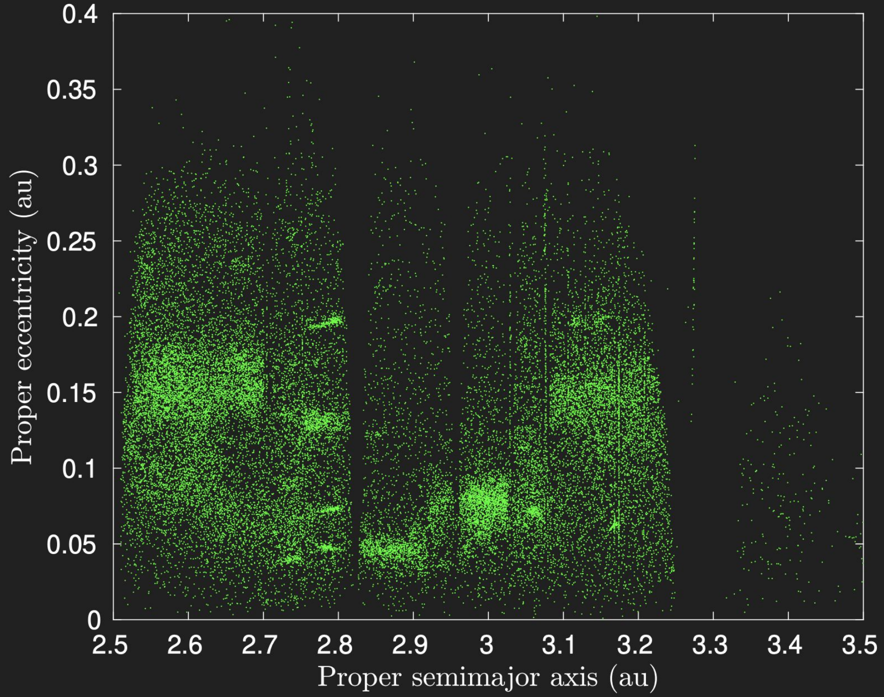
Introduction

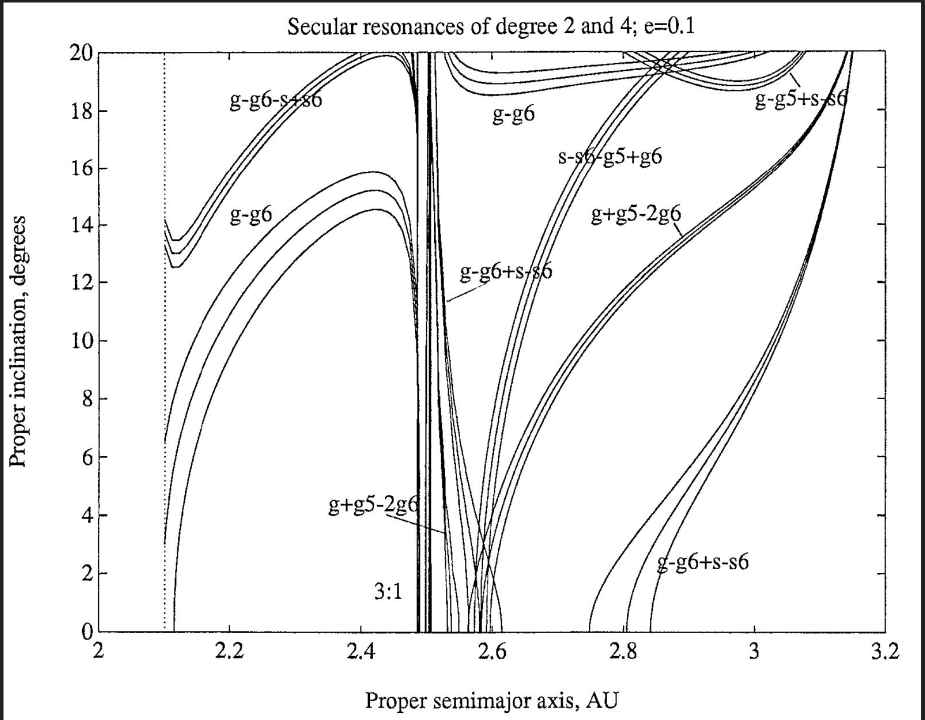
Proper elements are **quasi-integral** of motion of the *N*-body problem

Methods of computation:

- Analytical methods (e.g. *Hirayama 1918, Kozai 1962, Milani & Knežević 1990, 1992, 1994*)
- Semi-analytical methods (e.g. *Williams 1969, 1979, Lemaitre & Morbidelli 1994*)
- Synthetic methods (*Knežević & Milani 2000, 2003*)



Introduction



Secular resonances in the main belt

g: frequency of ϖ

s: frequency of Ω

Linear resonances: $g\text{-}g_p = 0$ or $s\text{-}s_p = 0$

Non-linear resonances: libation of σ = combination of ϖ and Ω fulfilling the D'Alembert rules

[1] Knežević 2022: *Survey of secular resonances in the asteroid belt*, SaJ 204

Introduction

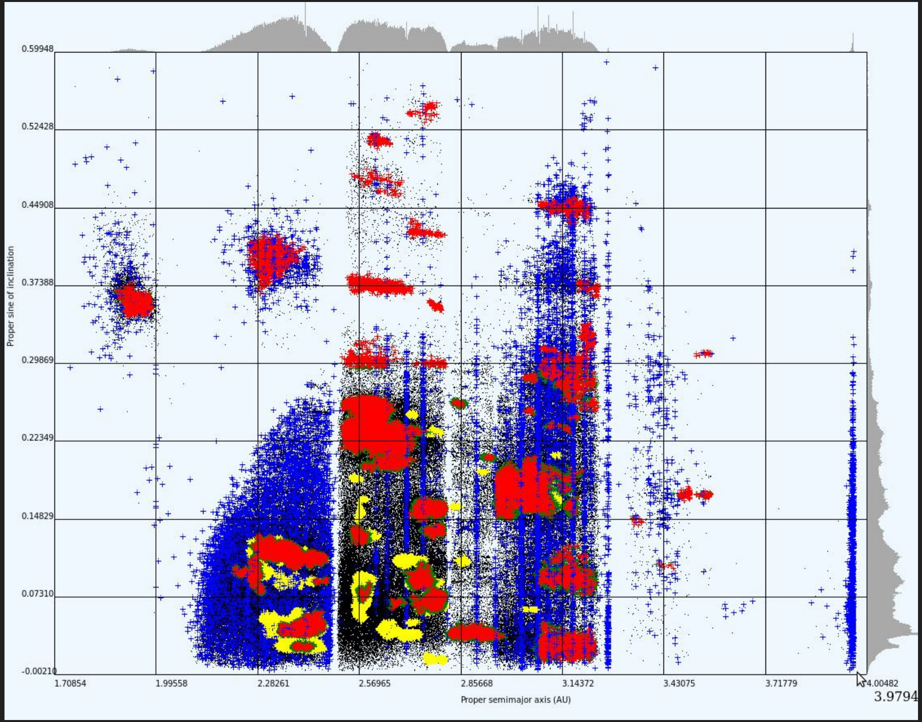
Identification of asteroid families

Typically done in the proper elements space using the metric:

$$d = na_p \sqrt{\frac{5}{4}\left(\frac{\delta a_p}{a_p}\right)^2 + 2(\delta e_p)^2 + 2(\delta \sin(i_p))^2}$$

Catalogues:

- https://newton.spacedys.com/astdys/
- http://asteroids.matf.bg.ac.rs/fam/



[2] **Novaković et al. 2022:** *Asteroid families: properties, recent advances, and future opportunities*, CMDA 134

Part 1: Proper elements of NEAs

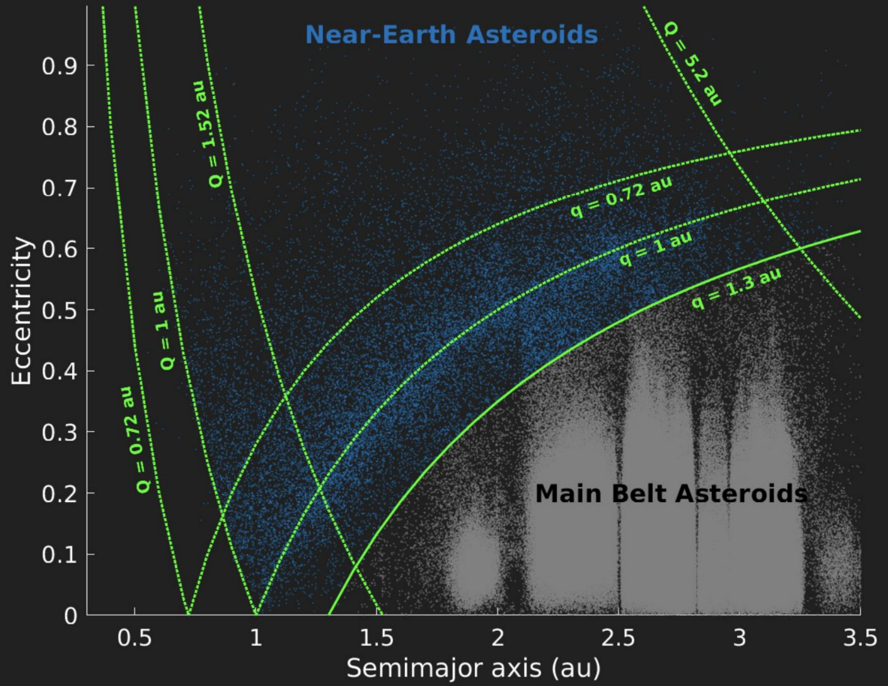
Proper elements of NEAs

Near-Earth Asteroids (NEAs) are objects with q < 1.3 au.

NEAs can **cross** the orbits of planets!

Problems in proper elements computation:

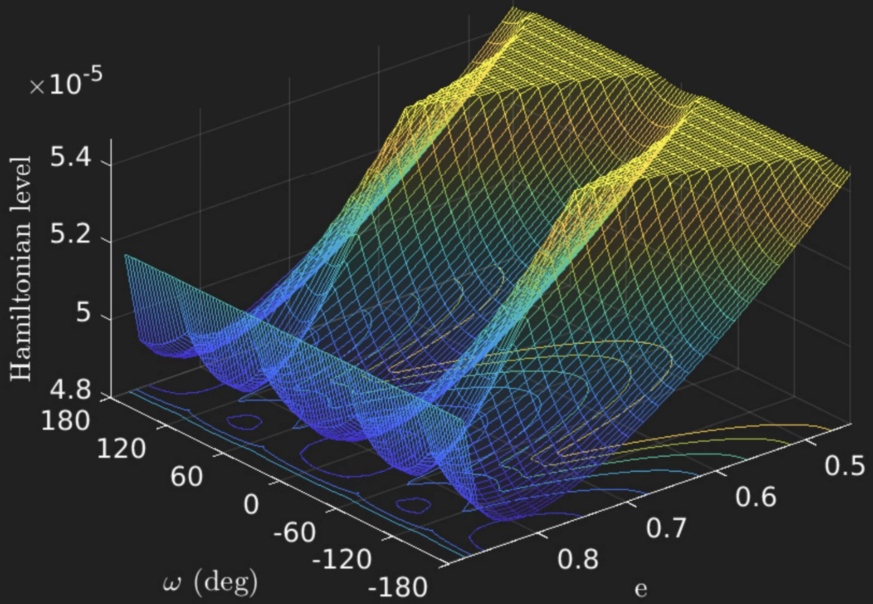
- Orbit crossings cause divergence of series
- Orbit crossings cause divergence of quadratures
- Close encounters with planets shorten the Lyapunov time



Proper elements of NEAs

Kozai secular model:

$$\mathcal{H} = \frac{\kappa^2}{(2\pi)^2} \sum_{i=1}^{N} \int_0^{2\pi} \int_0^{2\pi} \frac{\mu_i}{|\mathbf{r} - \mathbf{r}_i|} \mathrm{d}\ell \mathrm{d}\ell_i$$



With planets on circular orbits, the Delaunay elements *L*, *Z* are constant, the system has 1 DoF

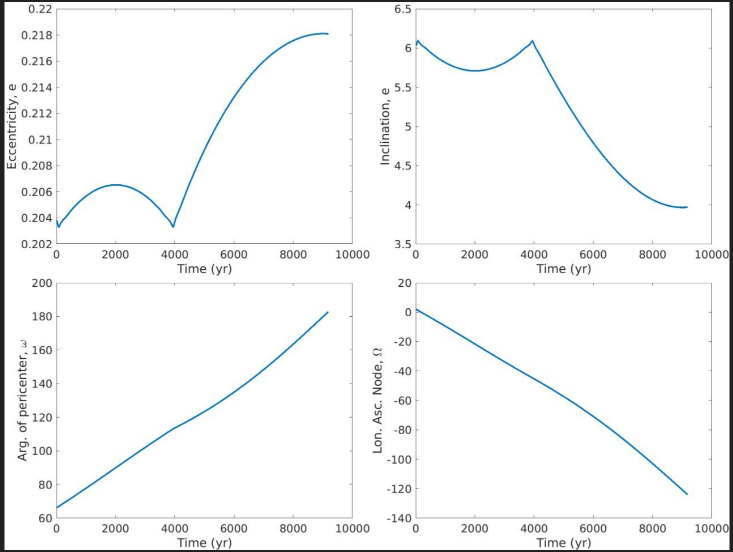
Proper elements: e_{min}, e_{max}, i_{min}, i_{max}

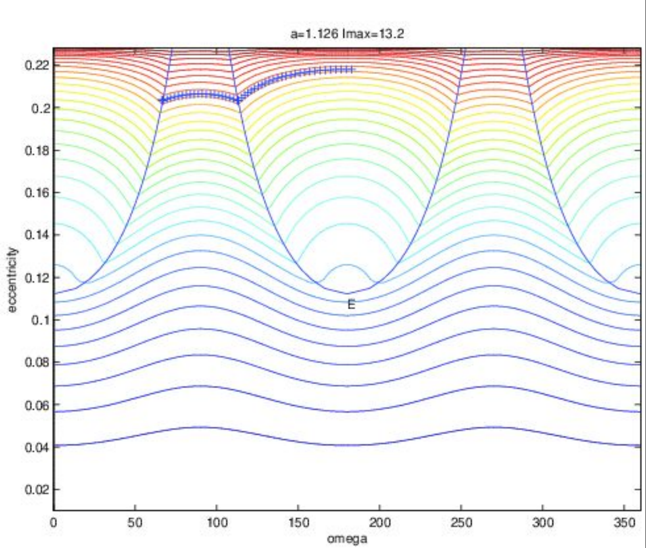
Proper frequencies: s (of Ω) and g (of ϖ)

[3] **Gronchi and Milani 2001:** *Proper Elements for Earth-Crossing Asteroids*, Icarus 2152

Proper elements of NEAs

Example: Bennu





Proper elements:

e_{min} = 0.2033
e_{max} = 0.2181
i_{min} = 3.967
i_{max} = 6.090

Proper frequencies:

g = -4.776 arcsec/yr
s = -51.016 arcsec/yr

Number of crossings:

Venus: 0
Earth: 4
Mars: 0
Jupiter: 0

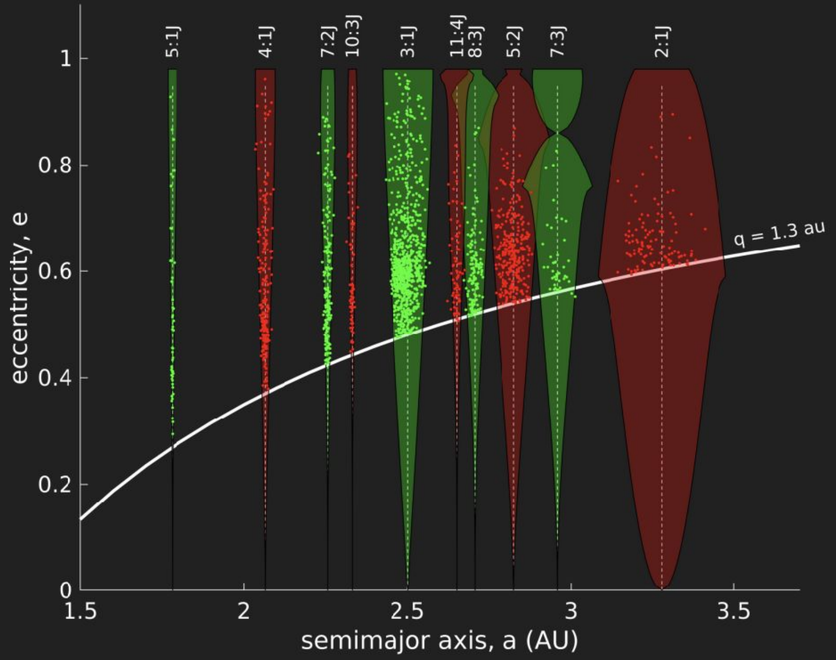
Proper elements of NEAs: mean-motion resonances

Motivations:

- Many NEOs are in a MMR
- MMRs may significantly affect the dynamics
- No theory available for proper elements for resonant NEAs

Goals:

- Define proper elements for resonant NEAs
- Understand the importance of MMR in the long term dynamics of NEAs



[4] **Fenucci et al. 2022:** *Proper elements for resonant planet-crossing asteroids*, CMDA 134

Proper elements of NEAs: mean-motion resonances

Model:

- Circular restricted N-body problem
- κ Gauss constant
- μ_i masses of the planets
- $\varepsilon = \mu_5$ small parameter

Delaunay variables:

$$\begin{cases} L = \kappa\sqrt{a}, \\ G = L\sqrt{1-e^2}, \\ Z = G\cos i, \end{cases} \qquad \begin{cases} \ell = M, \\ g = \omega, \\ z = \Omega. \end{cases}$$

Hamiltonian:

$$\widetilde{\mathcal{H}} = \underbrace{\boxed{-\frac{\kappa^4}{2L^2}}}_{\mathcal{H}_0} + \underbrace{\boxed{\sum_{j=1}^{8} \mathbf{n}_j L_j}}_{\text{Extended space}} + \varepsilon \underbrace{\boxed{\left[-\kappa^2 \sum_{j=1}^{8} \frac{\mu_j}{\mu_5} \left(\frac{1}{|\mathbf{r}-\mathbf{r}_j|} - \frac{\mathbf{r}\cdot\mathbf{r}_j}{|\mathbf{r}_j|^3} \right) \right]}}_{\mathcal{H}_1}$$

Proper elements of NEAs: mean-motion resonances

Assumption: $h{:}h_p$ MMR with the p-th planet, critical argument

$$\sigma = h \underbrace{\boxed{\lambda}}_{\ell+\omega+\Omega} - h_p \underbrace{\boxed{\lambda_p}}_{\ell_p+\omega_p+\Omega_p} - (h-h_p) \underbrace{\boxed{\varpi}}_{\omega+\Omega}$$

Resonant variables (from *Saillenfest et al. 2016*):

$$\begin{cases} \Sigma = \frac{L}{h} \\ \Gamma = hL - h_p L_p \\ U = G - \frac{h}{h_p} L \\ V = Z - \frac{h}{h_p} L \end{cases} \qquad \begin{cases} \sigma = h\lambda - h_p\lambda_p - (h-h_p)\varpi \\ \gamma = c\ell + c_p(\varpi - \ell_p) \\ u = \omega \\ v = \Omega \end{cases}$$

Semi-secular Hamiltonian:

$$\mathcal{K} = \underbrace{\boxed{-\frac{\kappa^4}{2(h\Sigma)^2} - \mathrm{n}_p h_p \Sigma}}_{\mathcal{K}_0} + \varepsilon \Bigg[\underbrace{\boxed{\frac{\kappa^2}{\mu_5} \sum_{\substack{j=1 \\ j\neq p}}^{N} \frac{\mu_j}{(2\pi)^2} \int_0^{2\pi} \int_0^{2\pi} \frac{-1}{|\mathbf{r}-\mathbf{r}_j|} \mathrm{d}\ell \mathrm{d}\ell_j}}_{\mathcal{K}_{\text{sec}}} + \underbrace{\boxed{\frac{\kappa^2}{\mu_5} \frac{\mu_p}{2\pi} \int_0^{2\pi} \left(\frac{\mathbf{r}\cdot\mathbf{r}_p}{|\mathbf{r}_p|^3} - \frac{1}{|\mathbf{r}-\mathbf{r}_p|} \right) \mathrm{d}\gamma}}_{\mathcal{K}_{\text{res}}} \Bigg]$$

Remark: crossing singularity is treated as in *Gronchi & Tardioli 2013*

Proper elements of NEAs: mean-motion resonances

Proper elements computation

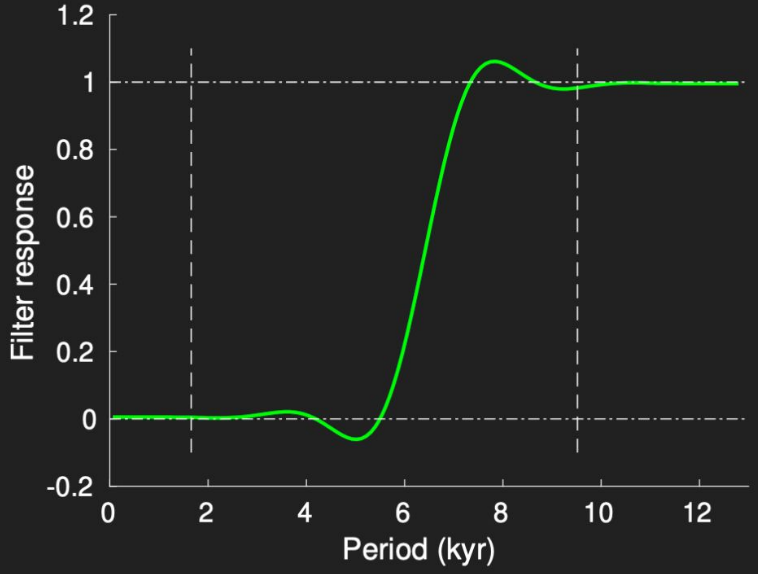
(1) Compute initial elements for the semi-secular Hamiltonian
- Propagate the osculating orbit for short time
- Filter out short periodic oscillations ($P \sim 200$ yr)

(2) Propagate the semi-secular dynamics for 200 ky

(3) Filter out short periodic oscillations ($P \sim 10$ ky)

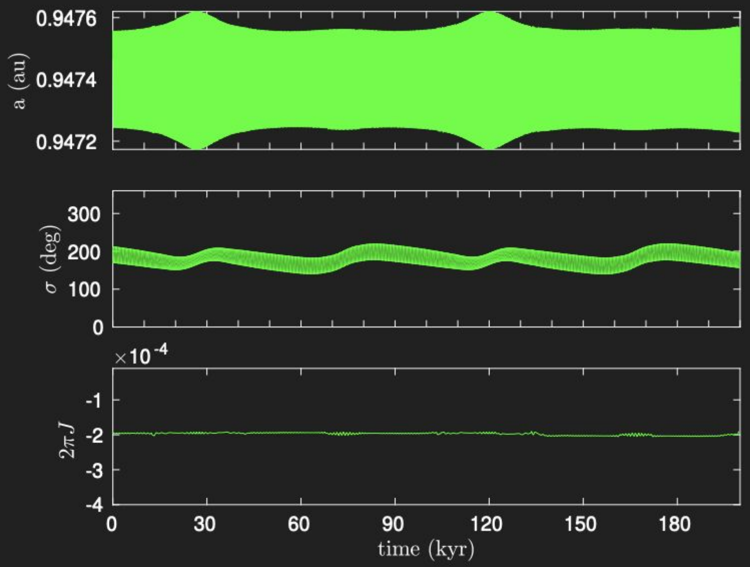
(4) Determine proper frequencies $s, g - s$ of Ω, ω

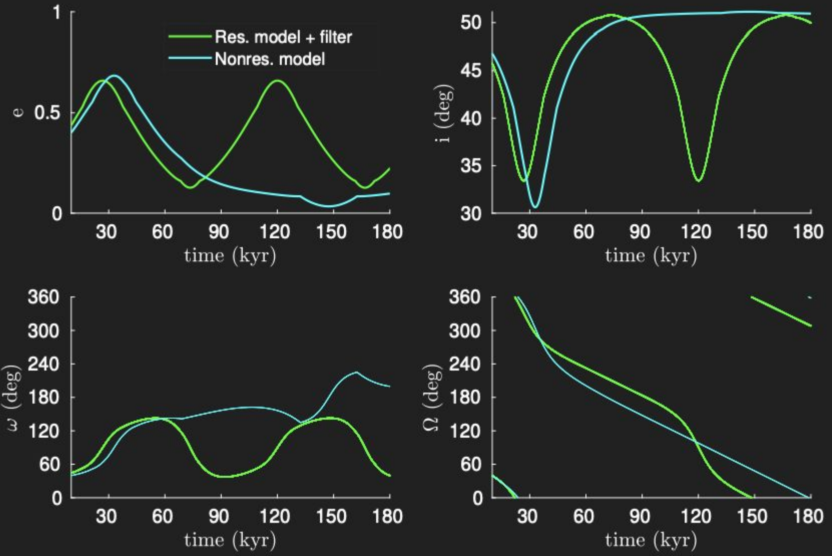
(5) Compute $e_{\text{max}}, e_{\text{min}}, i_{\text{max}}, i_{\text{min}}$



Proper elements of NEAs: mean-motion resonances

Object: (5381) Sekhmet - 2:3V MMR - V/E crosser - $2\pi J$ constant - different res. and non-res. dynamics





Part 2: Secular resonances

Secular resonances in the NEA region

Motivation

The location of the secular resonances in the NEA region are not determined

Method of computation:

1. Fix i_0 and define a grid in (a, e_0)
2. Compute s and g-s over the points of the grid
3. Plot the level curves corresponding to

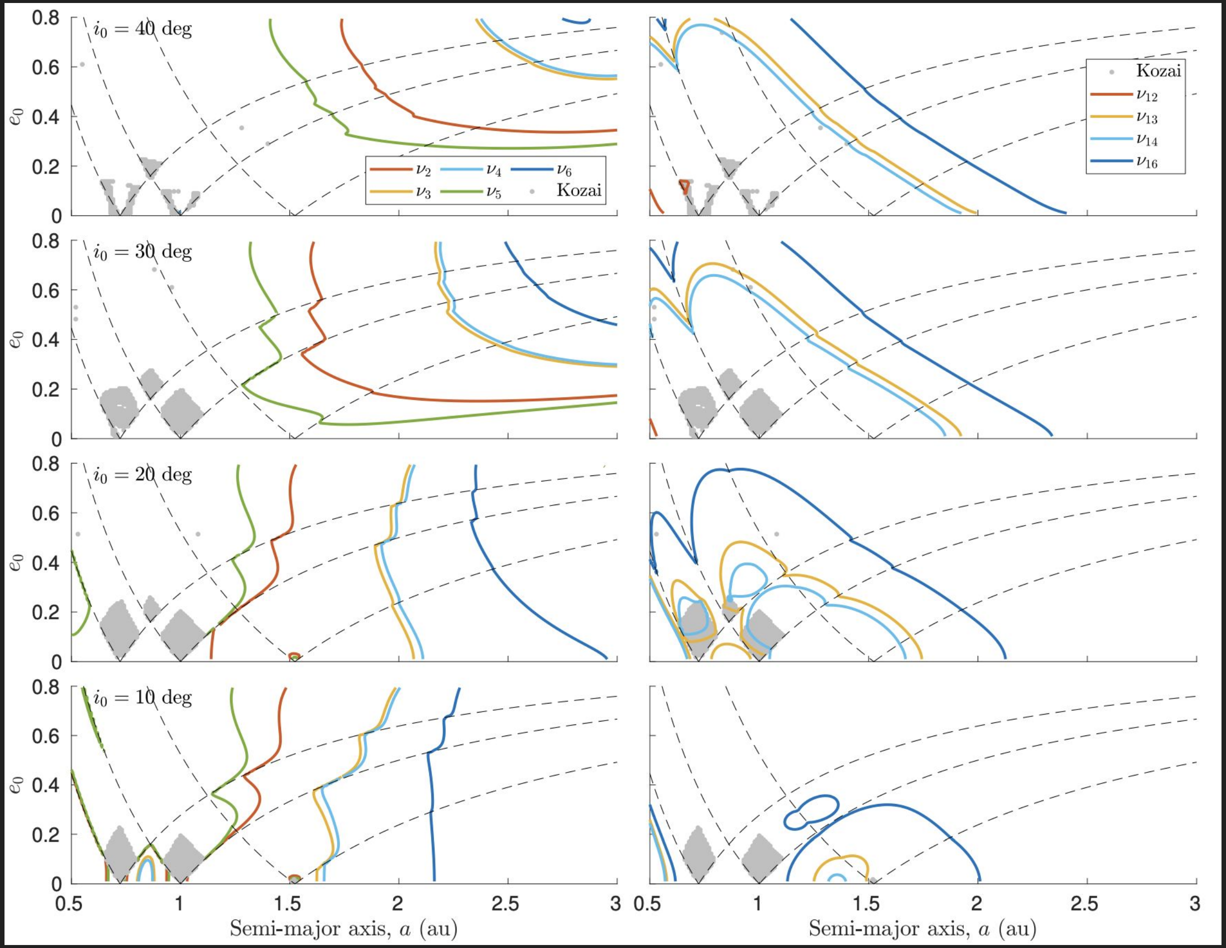
$g\text{-}g_j = 0$ and of $s\text{-}s_j = 0$

Planet	g ($''$ yr^{-1})	s ($''$ yr^{-1})
Venus	7.453	–7.06
Earth	17.368	–18.848
Mars	17.916	–17.751
Jupiter	4.257482	0.0
Saturn	28.2449	–26.347841

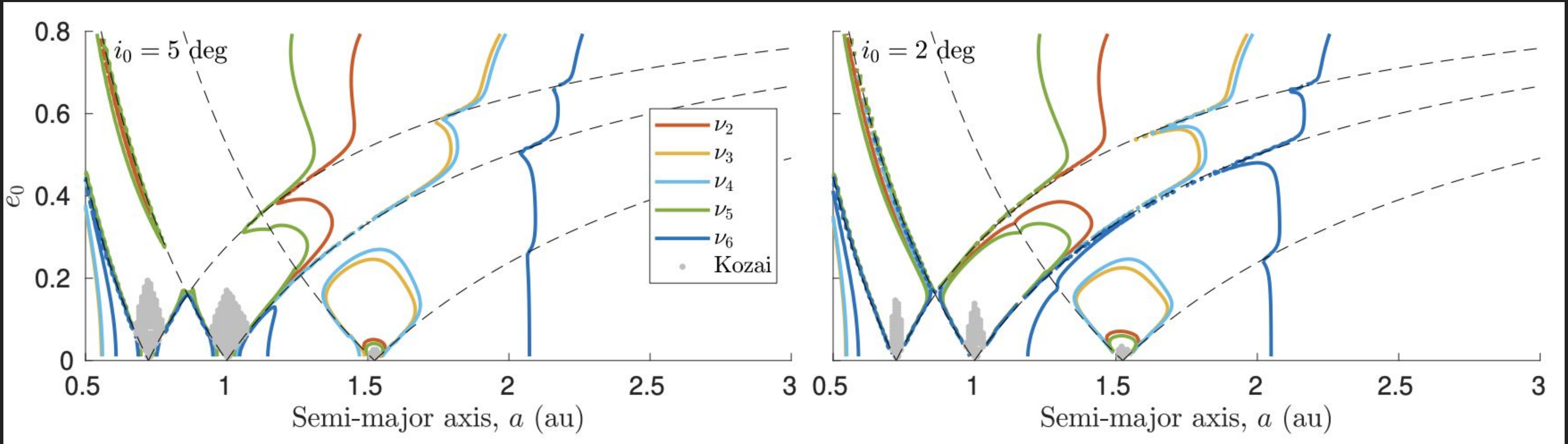
Table: Proper frequencies of planets from Laskar et al. 2011

[5] **Fenucci et al. 2023:** *Maps of secular resonances in the NEO region*, A&A 672, A39

Secular resonances in the NEA region: fixed inclination



Secular resonances in the NEA region: fixed inclination



Secular resonances in the NEA region

Conclusions and consequences

- Secular resonances location determined in the NEA region for the first time, for any value of eccentricity and inclination
- All the secular resonances appear well inside the NEA region
- Resonances with *Venus, Earth, Mars, Jupiter*, and *Saturn* (well known) may produce Sun impacting NEAs. This seems to be confirmed by recent numerical simulations [6]

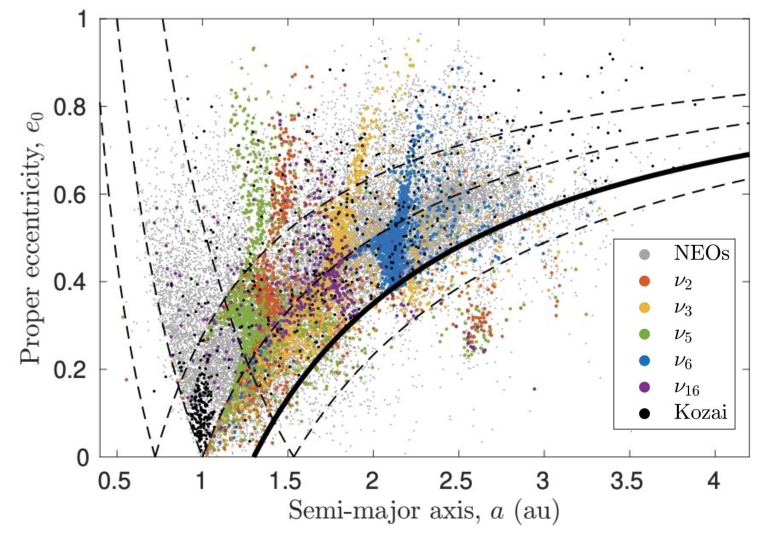


Figure: Known NEAs close to a secular resonance. Proper elements catalogue: https://newton.spacedys.com/neodys/

[6] **Toliou and Granvik 2023:** *Resonant mechanisms that produce near-Sun asteroids*, MNRAS

Part 3: Parent bodies of meteorites

Parent bodies of meteorites

Motivation:

- The current paradigm says that meteorites are born in the main belt as a result of collisions, and they are transported to the Earth by Yarkovsky + MMRs
- Disruptions happen also in the NEA region, but families of NEAs are not known yet [7]

Goal:

- Understand whether some NEA could be the parent body of some known meteorite or not

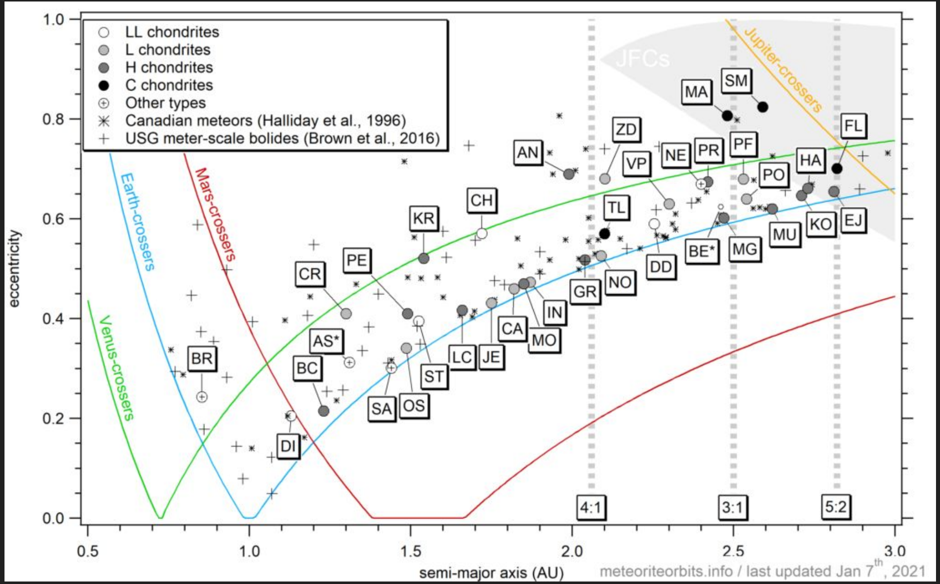
[7] Schunová et al. 2012: *Searching for the first near-Earth object family*, Icarus 220, 2
[8] Carbognani and Fenucci 2023: *Identifying parent bodies of meteorites among near-Earth asteroids*, submitted

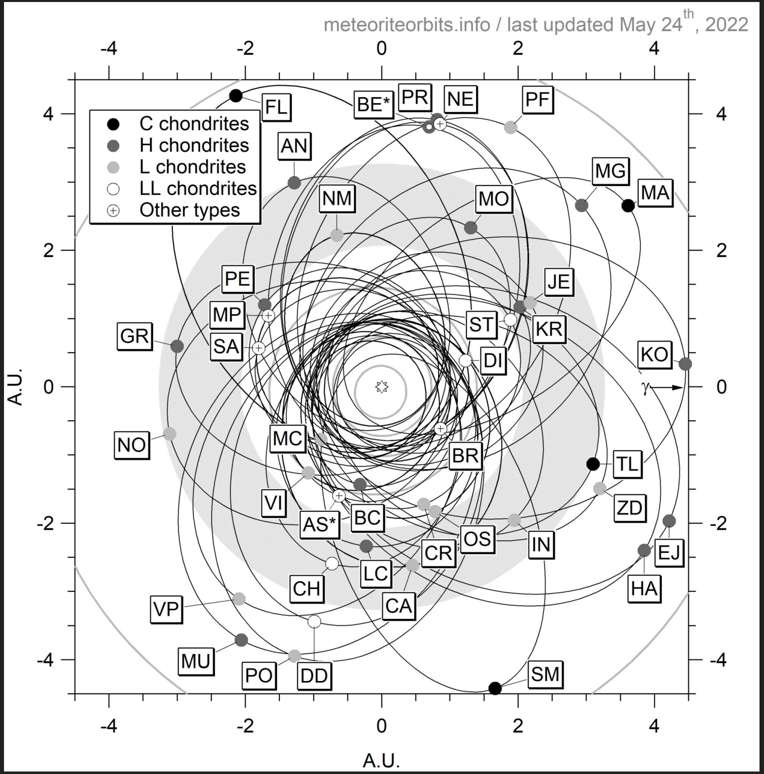
Parent bodies of meteorites

List of all meteorites with an orbit:

https://www.meteoriteorbits.info/

Current number: 37





Parent bodies of meteorites

D_N: distance introduced in [9] based on **geocentric quantities**. In a geocentric frame with:

- *y* facing Earth motion
- *x* opposite to the direction of the Sun
- *z* orthogonal to the ecliptic plane

Then

- U - normalized geocentric velocity
- θ - the angle between the the y axis and the vector of U
- ϕ - the angle between the U vector and the xz plane
- λ - the heliocentric longitude of the Earth at the encounter with the meteor

NOTE1: The angles θ and ϕ are directly related to the radiant

NOTE 2: Encounter conditions of NEAs from secular evolution

$$D_N{}^2 = [U_2 - U_1]^2 + [\cos\theta_1 - \cos\theta_2]^2 + \Delta\xi^2,$$

$$\Delta\xi^2 = \min(\Delta\phi_I^2 + \Delta\lambda_I^2,\ \Delta\phi_{II}^2 + \Delta\lambda_{II}^2)$$

$$\Delta\phi_I = 2\sin\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$\Delta\phi_{II} = 2\sin\left(\frac{180^\circ + \phi_2 - \phi_1}{2}\right)$$

$$\Delta\lambda_I = 2\sin\left(\frac{\lambda_1 - \lambda_2}{2}\right)$$

$$\Delta\lambda_{II} = 2\sin\left(\frac{180^\circ + \lambda_2 - \lambda_1}{2}\right).$$

[9] Valsecchi at al. 1999: *Meteoroid stream identification: a new approach - I. Theory*, MNRAS 304

Parent bodies of meteorites

Searching for possible parent bodies:

1. For each meteorite we computed D_N over the whole population of known Earth crossing NEAs (catalogue from NEODyS)
2. We found **16 couples meteorite-NEA** with $D_N < 0.06$

NOTE: The value of 0.06 is when random associations start to be more likely

Meteorite Name	NEA	Diam. (m)	T_J	C	DN
Pribram	482488	300–600	3.109	0.3	0.047
Peekskill	2014KF22	15–30	4.444	5.9	0.041
Neuschwanstein	482488	300–600	3.109	0.3	0.056
Park Forest	2021WT	30–60	3.178	7.6	0.053
Bunburra Rockhole	2021FB	20–40	6.942	6.4	0.042
Jesenice	2017FZ64	40–100	3.736	7.8	0.055
Košice	2021NV5	10–20	3.234	6.8	0.049
	2019ST2	50–120	3.288	9.0	0.059
Križevci	2022RQ	20–50	4.472	7.9	0.044
	2013BR15	30–60	4.280	9.0	0.056
Sutter's Mill	2016SL2	30–60	3.327	8.7	0.050
Annama	2016RX	20–40	2.943	7.1	0.048
Žd'ár nad Sázavou	2005VE7	500–1200	3.156	0.5	0.048
Creston	2021JN2	40–100	5.328	7.0	0.055
Hamburg	2022UF	10–20	3.110	6.8	0.057
	2021PZ1	20–40	3.279	7.6	0.043

Table: 16 couples meteorite-NEA with $D_N < 0.06$

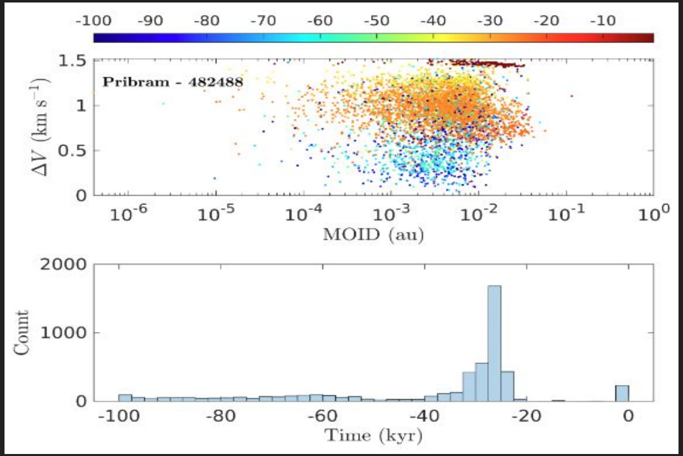
Parent bodies of meteorites

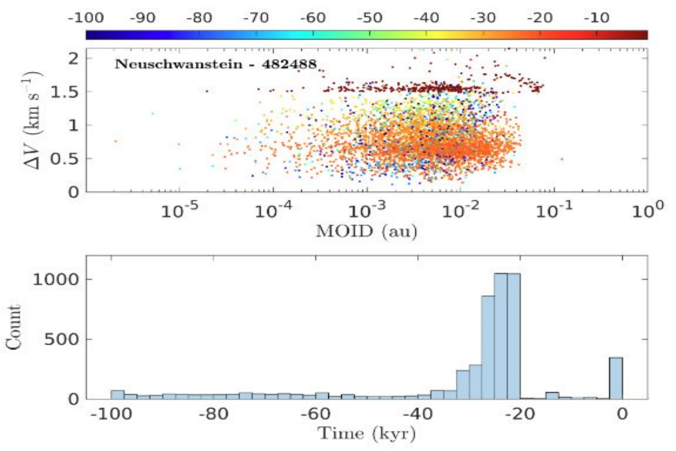
Numerical simulations

- Meteorite: 5000 clones
- NEA: only nominal orbit
- Total integration time: -100 kyr
- Only gravitational model - Mercury integrator [9]

Vicinity of the objects evaluated with MOID and ΔV at MOID, using the minimum of the distance:

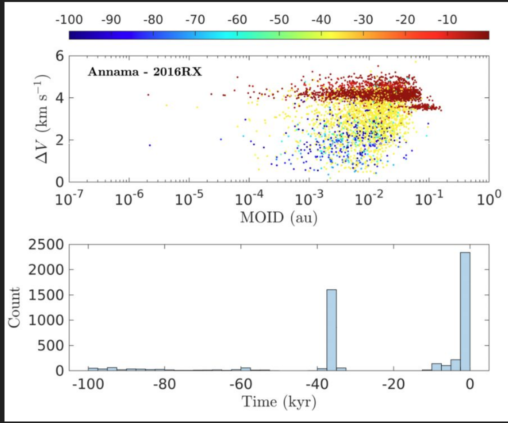
$$d = \sqrt{\left(\frac{\text{MOID}}{\mu(\text{MOID})}\right)^2 + \left(\frac{\Delta V}{\mu(\Delta V)}\right)^2},$$

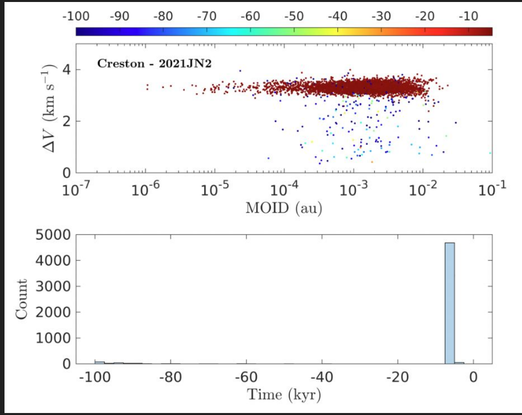


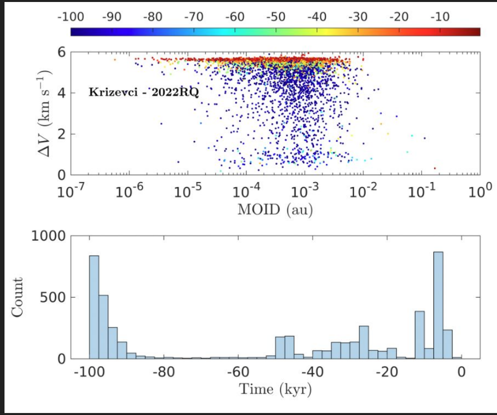


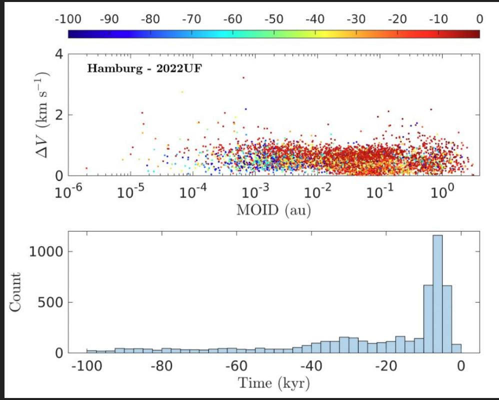
[9] **Fenucci and Novaković 2022:** *Mercury and OrbFit packages for the numerical integration of planetary systems: implementation of the Yarkovsky and YORP effects, SaJ 204*

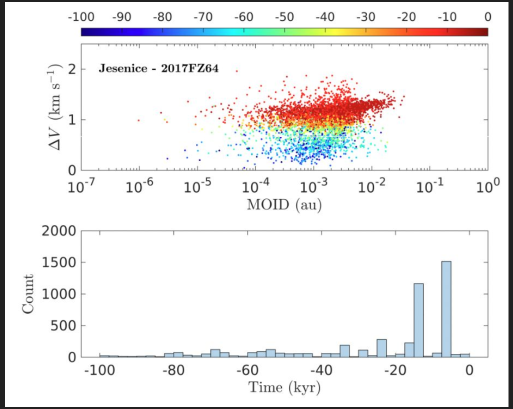
Parent bodies of meteorites

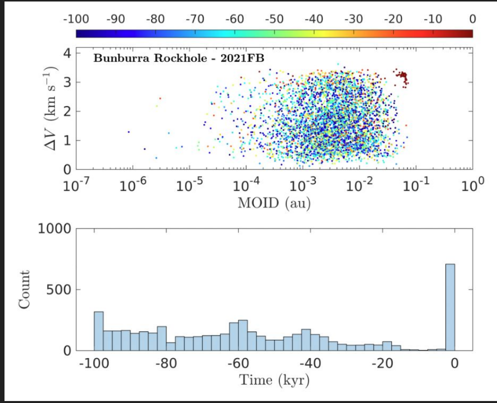




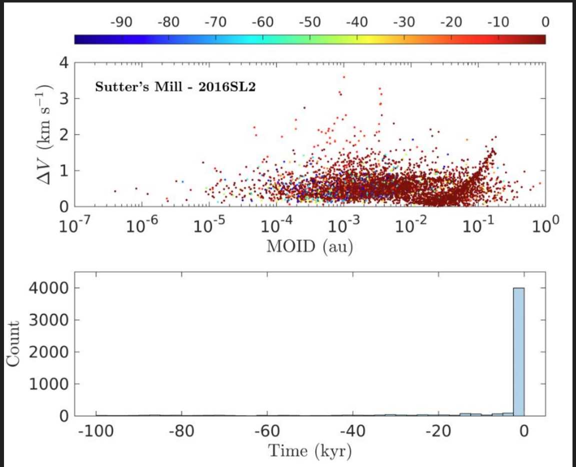


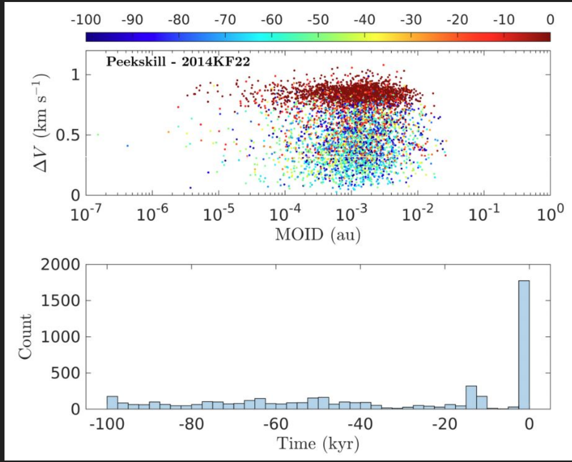






Parent bodies of meteorites





Natural History Museum of Vienna (Austria)

Parent bodies of meteorites

Results and conclusions:

- 10 meteorite-NEA association look dynamically possible
- ΔV are compatible with collisions
- Separation ages ~10 kyr ago
- Current NEA population models suggest a timescale of ~10 kyr for collisions among NEAs
- **~1/3** of meteorites may originate directly in the NEA region

Meteorite Name	NEA	Diam. (m)	T_J	C	DN
Pribram	482488	300–600	3.109	0.3	0.047
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General conclusions

- Proper elements of NEAs are a fast indication of the secular dynamics
- Catalogue maintained only by NEODyS
- No catalogue for NEAs in a MMR, even detection of MMRs is not automatized
- They can be used to identify secular resonances, similar to the case main belt asteroids
- They can be used to search for parent bodies of meteorites, similar to the case of asteroid pairs
- They can be further used to search for NEAs clusters, e.g. families