Seminar of Department of Astronomy - University of Belgrade

Stellar mass Primordial Black Holes

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1 - PBHs from density fluctuations

- Black Holes may have formed in the Early Universe as a consequence of the gravitational collapse of density fluctuations (Hawking 1971, Hawking & Carr 1974, Novikov et al. 1979).
- These **Primordial Black Holes** (PBHs) could have masses ranging from the Planck mass up to $\sim 10^{11} M_\odot$. Here we will consider the (extended) stellar mass range $[0.05 M_\odot - 500 M_\odot]$.
- During inflation, fluctuations of **quantum origin** are stretched to scales much larger than the cosmological horizon becoming causally disconnected from physical processes.
- The inflationary era is followed, respectively, by radiation-dominated and matter-dominated epochs during which these fluctuations can re-enter the cosmological horizon.

For a given physical scale k, the **horizon crossing time** t_k (i.e. the instant when that scale re-enters the cosmological horizon) is given by (Blais et al. 2003):

$$ck = a(t_k)H(t_k)$$

where $a(t_k)$ is the scale factor and $H(t_k)$ the Hubble parameter. The collapse that gives rise to the formation of a PBH is now

possible but only if the amplitude of the density fluctuation:

$$\delta = \frac{\Delta m}{m}$$

is larger than a specific **threshold** value δ_c.

- $\delta_k \geq \delta_c$ – the expansion of the overdense region will, eventually, come to a halt, followed by its collapse leading to the formation of a PBH
- $\delta_k < \delta_c$ – the fluctuation dissipates without forming a PBH

The majority of the PBHs formed at a particular epoch have masses within the order of the horizon mass, M_H, at that epoch (Carr et al. 2003):

$$M_H(t) \sim 10^{15} \left(\frac{t}{10^{-23}\ \mathrm{s}} \right)\ \mathrm{g}.$$

In the case of perturbations with δ only slightly larger than δ_c the PBH masses rather obey the scaling law (Niemeyer & Jedamzik 1999):

$$M_{PBH} \propto M_H \left(\delta - \delta_c\right)^{\gamma}$$

where $\gamma \approx 0.36$ in the case of a radiation-dominated Universe. This scaling law has been found to hold down to $(\delta - \delta_c) \sim 10^{-10}$ (Musco & Miller 2013).

Here we assume: $M_{PBH}(t_k) = M_H(t_k)$.

The probability that a fluctuation crossing the horizon at some instant t_k has of collapsing and forming a PBH can be written as (e.g. Green 2015):

$$\beta(t_k) = \frac{1}{\sqrt{2\pi}\sigma(t_k)} \int_{\delta_c}^{\infty} \exp\left(-\frac{\delta^2}{2\sigma^2(t_k)}\right) d\delta$$

- $\sigma^2(t_k)$ – mass variance at horizon crossing
- δ_c – the threshold for PBH formation

The value of $\beta(t_k)$ can also be regarded as *the fraction of the Universe going into PBHs*.

2 - The threshold for PBH formation

- $\delta_c = 1/3$ – simplified model of an overdense collapsing region for a radiation-dominated Universe (Carr 1975)
- $\delta_c \simeq 0.43 - 0.47$ – numerically solving the relativistic hydrodynamical equations for a radiation-dominated Universe (Musco & Miller 2013, Harada et al. 2013)

The exact value of δ_c depends on the perturbation profile. We consider $\delta_c = 0.43$ (Mexican-Hat perturbation, a very representative one).

δ_c is constant through the radiation-dominated epoch, the exception occurring during cosmological phase transitions, when the value of δ_c decreases (as a consequence of the decrease of the sound speed). This is relevant, since **a lower value of δ_c favours PBH formation** (Carr 2003).

The Standard Model of Particle Physics (SMPP) predicts:

- **Electroweak (EW) phase transition** at temperatures of ~ 100 GeV (when the age of the Universe was $\sim 10^{-10}$ s), responsible for the spontaneous breaking of the EW symmetry. PBHs formed during the epoch of the EW phase transition would have $\sim 10^{-6} M_\odot$.
- **Quantum Chromodynamics (QCD) phase transition** at $T_c = 170$ MeV when quarks and gluons become confined in hadrons. The QCD phase transition occurred when the universe was $\sim 10^{-5}$ s and that corresponds to $M_H \sim 0.5 M_\odot$.

If we want to study the formation of stellar mass PBHs we cannot neglect the effect of the QCD phase transition. In particular we need to know **how the variation of the sound speed during the QCD phase transition will affect the value of δ_c.**

During the radiation-dominated epoch the Universe can be regarded as a diluted gas with its **Equation of State** (EoS) written as (Carr 2003):

$$p = w\rho$$

where p is pressure, ρ is the cosmological density, and the dimensionless quantity w (the *EoS parameter*) is equal to $\frac{1}{3}$, since the sound speed is (Schmid et al. 1999):

$$c_s^2 = \left(\frac{\partial p}{\partial \rho}\right)_S = w = \frac{1}{3}$$

If during the QCD phase transition the Universe becomes matter-dominated (pressureless gas), then we get $w = 0$ and $c_s^2 = 0$.

The evolution of the scale factor for the perturbed region ($s(\tau)$) is (Sobrinho 2011; Sobrinho, Augusto & Gonçalves 2016):

$$\left(\frac{ds}{d\tau}\right)^2 = \frac{8\pi G}{3}\frac{K_s}{1+\delta_k}\left(\frac{1+\delta_k}{s(\tau)^{1+3w}} - \frac{K_k}{K_s}\frac{\delta_k}{a_k^{1+3w_k}}\right)$$

where:

- K_s and K_k are constants to be determined
- a_k is the scale factor value at the horizon crossing time
- w_k is the EoS parameter at the horizon crossing time

The **turnaround point** (t_c) is reached when the perturbed region stops expanding, i.e., when $ds/d\tau = 0$. Thus, we get:

$$s_c^{1+3w_c} = \frac{K_s}{K_k} a_k^{1+3w_k} \left(\frac{1+\delta_k}{\delta_k} \right)$$

which relates the size of the perturbed region at the horizon crossing time with the respective size at the turnaround point. The calculation of the relation K_s/K_k is detailed in Sobrinho, Augusto & Gonçalves (2016).

We assume that whent_c is reached a PBH will form. Hence, we are not taking into account the dynamics between the turnaround point and the instant when the PHB actually arises with the formation of an event horizon (which would require to numerically solve the Hernandez-Misner equations).

If we want to consider the formation of stellar mass PBHs we need to take into account the QCD phase transition.

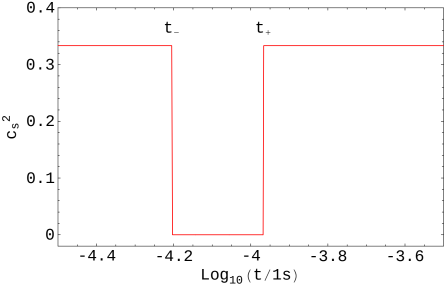
More exactly we need to know how the sound speed behaves during the QCD phase transition.

Unfortunately we don't know exactly which model fits the QCD phase transition. We have considered three different models:

- Bag Model (BM)
- Lattice Fit Model (LFM)
- Crossover Model (CM)

2.1 - Bag Model (BM)

- a high temperature region ($T > T_c$, $t < t_-$) where we have a **Quark Gluon Plasma** (QGP)
- $T = T_c = 170$ MeV ($[t_-, t_+]$): **dust-like phase** where quarks, gluons, and hadrons coexist in equilibrium at constant pressure and temperature and $c_s^2 = 0$
- a low temperature region ($T < T_c$, $t > t_+$) where we have an **Hadron Gas** (HG).



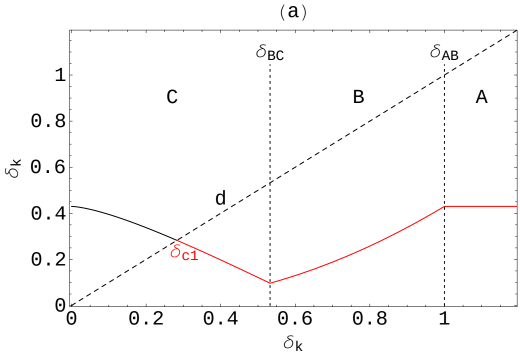
Classes of fluctuations

Class	Horizon crossing	Turnaround
A	quark–gluon	quark–gluon
B	quark–gluon	mixed
C	quark–gluon	hadron
D	mixed	mixed
E	mixed	hadron
F	hadron	hadron

Idea: replace δ_c by $\delta_c(1 - f)$

f $(0 \leq f < 1)$ – fraction of the overdense region spent in the dust-like phase of the transition (Sobrinho, Augusto & Gonçalves 2016).

Figure 1a in "New thresholds for primordial black hole formation during the QCD phase transition", Sobrinho, J. L. G.; Augusto, P.; Gonçalves, A. L., 2016, Monthly Notices of the Royal Astronomical Society, Volume 463, Issue 3.



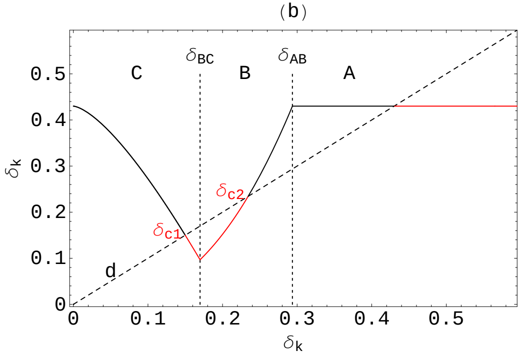
solid line $\longrightarrow (1-f)\delta_c$ [red – PBH formation allowed: $\delta_k \geq (1-f)\delta_c$]

dashed line $\longrightarrow \delta_k$

$t_k = 4.0 \times 10^{-5}$ s

$\delta_{c1} = 0.28$ – **new threshold for PBH formation** (< 0.43)

Figure 1b in “New thresholds for primordial black hole formation during the QCD phase transition", Sobrinho, J. L. G.; Augusto, P.; Gonçalves, A. L., 2016, Monthly Notices of the Royal Astronomical Society, Volume 463, Issue 3.



solid line $\longrightarrow (1-f)\delta_c$ [red – PBH formation allowed: $\delta_k \geq (1-f)\delta_c$]

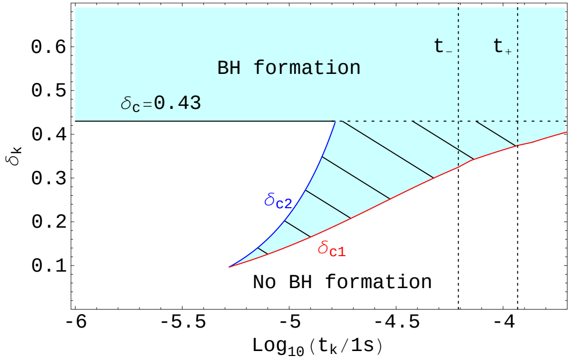
dashed line $\longrightarrow \delta_k$

$t_k = 1.1 \times 10^{-5}$ s

$[\delta_{c1}, \delta_{c2}] = [0.15, 0.23]$ – **new thresholds (new window) for PBH formation** (< 0.43)

δ_c for a QCD BM

Figure 2 in “New thresholds for primordial black hole formation during the QCD phase transition", Sobrinho, J. L. G.; Augusto, P.; Gonçalves, A. L., 2016, Monthly Notices of the Royal Astronomical Society, Volume 463, Issue 3.



Minimum value of δ_c: 0.097

2.2 - Crossover Model (CM)

The sound speed during a QCD Crossover (Schwarz 1998):

$$c_s^2(t) = \left[3 + \frac{\Delta g T(t) \mathrm{sech}\left(\frac{T(t)-T_c}{\Delta T}\right)^2}{\Delta T\left(g_{HG} + g_{QGP} + \Delta g \tanh\left(\frac{T(t)-T_c}{\Delta T}\right)\right)}\right]^{-1}$$

$$T(t) = T_0\left[\exp\left(c\sqrt{\frac{\Lambda}{3}}(t_{SN} - t_0)\right)\left(\frac{t_{eq}}{t_{SN}}\right)^{2/3}\left(\frac{t}{t_{eq}}\right)^{1/2}\right]^{-1}$$

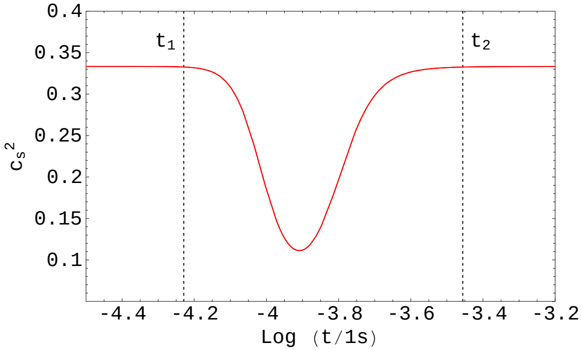
$\Delta T = 0.1T_c$ with $T_c = 170$ MeV; $T_0 = 2.72548$ K

g_{QGP} – number of degrees of freedom for the QGP (61.75).

g_{HG} – number of degrees of freedom for the HG (17.25).

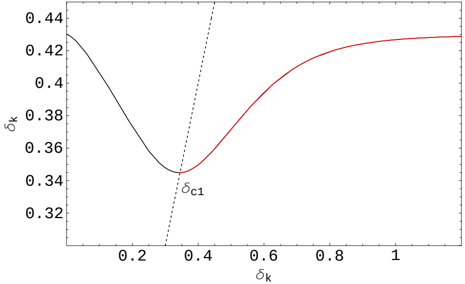
$\Delta g = g_{QGP} - g_{HG}$ (see Sobrinho 2011 for details)

Figure 3 in “New thresholds for primordial black hole formation during the QCD phase transition", Sobrinho, J. L. G.; Augusto, P.; Gonçalves, A. L., 2016, Monthly Notices of the Royal Astronomical Society, Volume 463, Issue 3.



$$f = \frac{3}{2}\left(t_k \frac{1+\delta_k}{\delta_k}\right)^{-3/2} \int_{t_1}^{t_k \frac{1+\delta_k}{\delta_k}} \left(1 - \frac{c_s(t)}{c_{s0}}\right)\sqrt{t}dt$$

Figure 4 in "New thresholds for primordial black hole formation during the QCD phase transition", Sobrinho, J. L. G.; Augusto, P.; Gonçalves, A. L., 2016, Monthly Notices of the Royal Astronomical Society, Volume 463, Issue 3.



solid line $\longrightarrow (1-f)\delta_c$ [red – PBH formation allowed: $\delta_k \geq (1-f)\delta_c$]

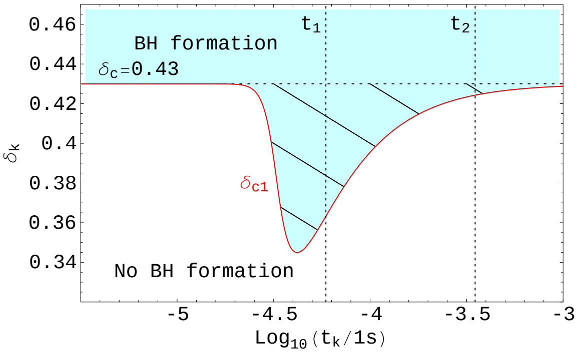
dashed line $\longrightarrow \delta_k$

$t_k = 4.2 \times 10^{-5}$ s

$\delta_{c1} = 0.345$ – **new threshold for PBH formation** (< 0.43)

δ_c for a QCD Crossover

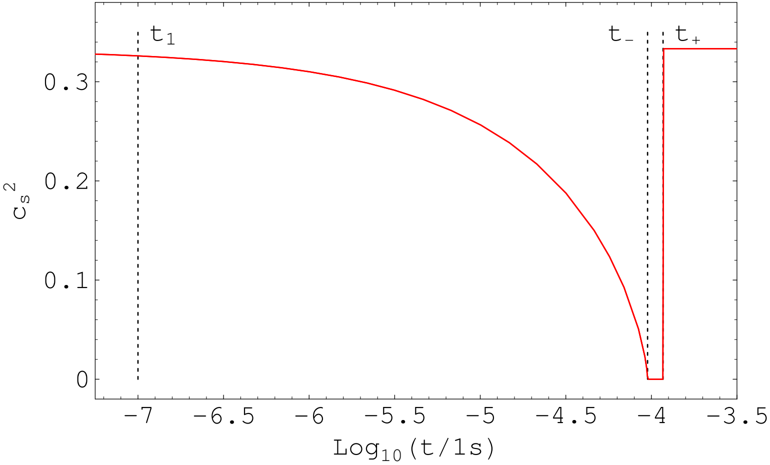
Figure 5 in "New thresholds for primordial black hole formation during the QCD phase transition", Sobrinho, J. L. G.; Augusto, P.; Gonçalves, A. L., 2016, Monthly Notices of the Royal Astronomical Society, Volume 463, Issue 3.



Minimum value of δ_c: 0.345

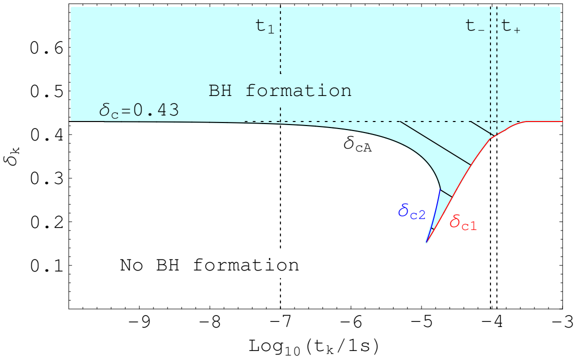
2.3 - Lattice Fit Model

Figure 4 in "New thresholds for primordial black hole formation during the QCD phase transition", Sobrinho, J. L. G.; Augusto, P.; Gonçalves, A. L., 2016, Monthly Notices of the Royal Astronomical Society, Volume 463, Issue 3.



δ_c for a QCD LFM

Figure 4 in “New thresholds for primordial black hole formation during the QCD phase transition", Sobrinho, J. L. G.; Augusto, P.; Gonçalves, A. L., 2016, Monthly Notices of the Royal Astronomical Society, Volume 463, Issue 3.



Minimum value of δ_c: 0.15

3 - Mass variance

(Sobrinho & Augusto 2020)

$$\sigma^2(k) = \int_0^{\frac{k_e}{k}} x^3 \delta_H^2(kx) W_{TH}^2(x) W_{TH}^2(\frac{x}{\sqrt{3}}) dx$$

$$\delta_H^2(k) = \left(\frac{10}{9}\right)^2 \delta_H^2(k_c) \left(\frac{k}{k_c}\right)^{n(k)-1}$$

- W_{TH} – Top-hat window function
- $k_c = 0.05\mathrm{Mpc}^{-1} \approx 1.6 \times 10^{-24}\mathrm{m}^{-1} \rightarrow$ pivot scale (Planck mission)
- $k_e \approx 0.01\mathrm{m}^{-1} \rightarrow$ smallest slace generated by inflation
- $\delta_H^2(k_c) \approx 2.198 \times 10^{-9} \rightarrow$ Planck Collaboration et al. 2016

Spectral index:

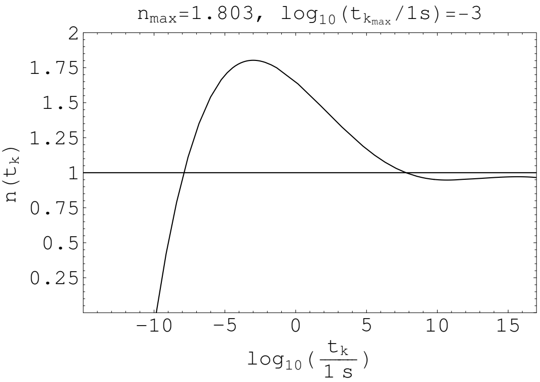
$$n(k) = n_0 + \sum_{i\geq 1} \frac{n_i}{(i+1)!} \left(\ln \frac{k}{k_c} \right)^i$$

$$\left.\begin{aligned} n_0 &= 0.9476 \\ n_1 &= 0.001 \\ n_2 &= 0.022 \end{aligned}\right\} \text{Planck (e.g. Erfani 2014)}$$

$$\left.\begin{aligned} n_3 \\ n_4 \end{aligned}\right\} \text{work on the plane } (n_3, n_4)$$

We are assuming $n_i = 0$ when $i \geq 5$

PBHs in significant numbers $\Longrightarrow$ $n(k) > 1$ (blue spectrum)



$n_3 = 0.0099$, $n_4 = -0.0033$.

We are interested in situations for which $n(k)$ shows a local **maximum** at some point $k = k_+$ with $n_+ = n(k_+) > 1$.

$$n_+ = n_0 + \frac{n_1}{2} \ln \frac{k_+}{k_c} + \frac{n_2}{6} \left(\ln \frac{k_+}{k_c} \right)^2 + \frac{n_3}{24} \left(\ln \frac{k_+}{k_c} \right)^3 + \frac{n_4}{120} \left(\ln \frac{k_+}{k_c} \right)^4$$

$$\frac{dn(k)}{dk} \Big|_{k=k_+} = 0 \Leftrightarrow \frac{n_1}{2} + \frac{n_2}{3} \ln \frac{k_+}{k_c} + \frac{n_3}{8} \left(\ln \frac{k_+}{k_c} \right)^2 + \frac{n_4}{30} \left(\ln \frac{k_+}{k_c} \right)^3 = 0$$

Given a pair of values (n_+, k_+), or equivalently (n_+, t_+), we can determine the corresponding pair of values (n_3, n_4):

$$n_3 = \frac{-4 \left(24 n_0 - 24 n_+ + 9 n_1 \ln \frac{k_+}{k_c} + 2 n_2 \left(\ln \frac{k_+}{k_c} \right)^2 \right)}{\left(\ln \frac{k_+}{k_c} \right)^3}$$

$$n_4 = \frac{20 \left(18 n_0 - 18 n_+ + 6 n_1 \ln \frac{k_+}{k_c} + n_2 \left(\ln \frac{k_+}{k_c} \right)^2 \right)}{\left(\ln \frac{k_+}{k_c} \right)^4}$$

The path for the calculus of β

$$\beta(t_k) = \frac{1}{\sqrt{2\pi}\sigma(t_k)} \int_{\delta_c}^{\infty} \exp\left(-\frac{\delta^2}{2\sigma^2(t_k)}\right) d\delta$$

$$\sigma^2(k) = \int_0^{\frac{k_e}{k}} x^3 \delta_H^2(kx) W_{TH}^2(x) W_{TH}^2\left(\frac{x}{\sqrt{3}}\right) dx$$

$$W_{TH}(x) = \frac{3}{(x)^3}\left(\sin(x) - x\cos(x)\right)$$

$$\delta_H^2(k_r, t_{k_r}) = \left(\frac{10}{9}\right)^2 \delta_H^2(k_c, t_{k_c}) \left(\frac{k_r}{k_c}\right)^{n(k)-1}$$

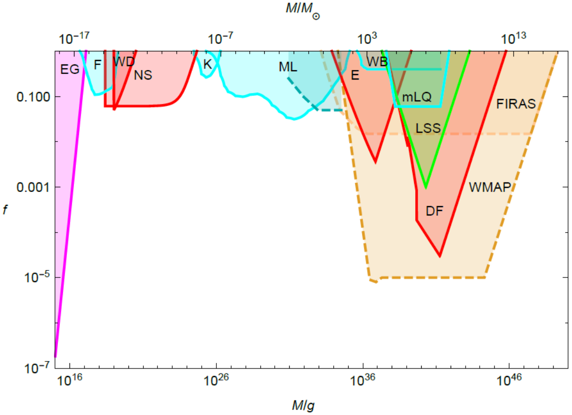
$$n(k) = n_0 + \sum_{i\geq 1} \frac{n_i}{(i+1)!}\left(\ln\frac{k}{k_c}\right)^i$$

$$(n_+, t_+) \longrightarrow (n_+, k_+) \longrightarrow$$

$$n_3 = \frac{-4\left(24n_0 - 24n_+ + 9n_1 \ln\frac{k_+}{k_c} + 2n_2\left(\ln\frac{k_+}{k_c}\right)^2\right)}{\left(\ln\frac{k_+}{k_c}\right)^3}$$

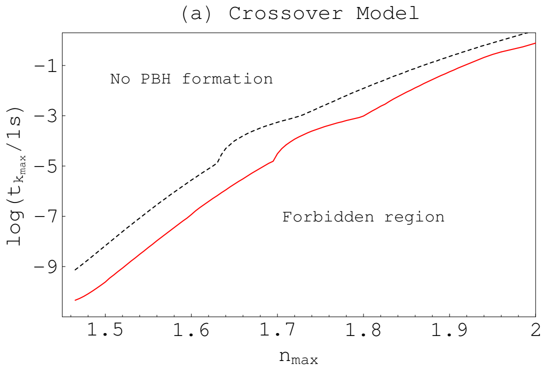
$$n_4 = \frac{20\left(18n_0 - 18n_+ + 6n_1 \ln\frac{k_+}{k_c} + n_2\left(\ln\frac{k_+}{k_c}\right)^2\right)}{\left(\ln\frac{k_+}{k_c}\right)^4}$$

Constraints on PBHs for a variety of evaporation, dynamical, lensing, large-scale structure and accretion effects (Carr et al. 2016)



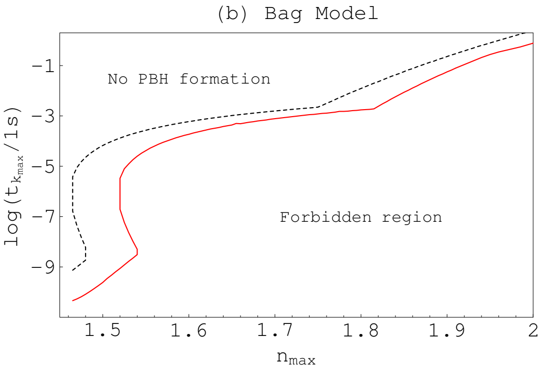
$$f(M) \approx 4.11 \times 10^8 \beta(M) \left(\frac{M}{M_\odot}\right)^{-1/2}$$

Figure 2a in "Stellar mass Primordial Black Holes as Cold Dark Matter", Sobrinho, J. L. G.; Augusto, P., 2020, Monthly Notices of the Royal Astronomical Society, Volume 496, Issue 1.



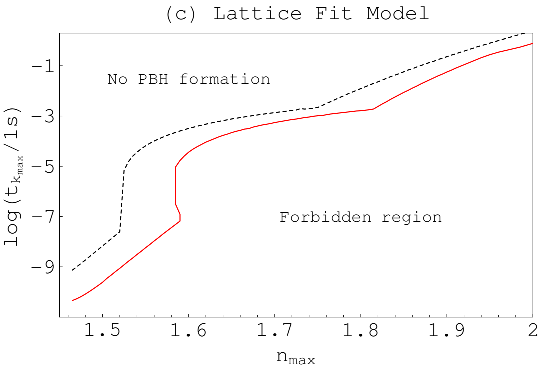
Below the solid curve, PBH formation is not allowed since it would violate the observational constraints. Above the dashed line, PBH formation is allowed although in negligible numbers (less than one PBH within the observable Universe).

Figure 2b in “Stellar mass Primordial Black Holes as Cold Dark Matter", Sobrinho, J. L. G.; Augusto, P., 2020, Monthly Notices of the Royal Astronomical Society, Volume 496, Issue 1.



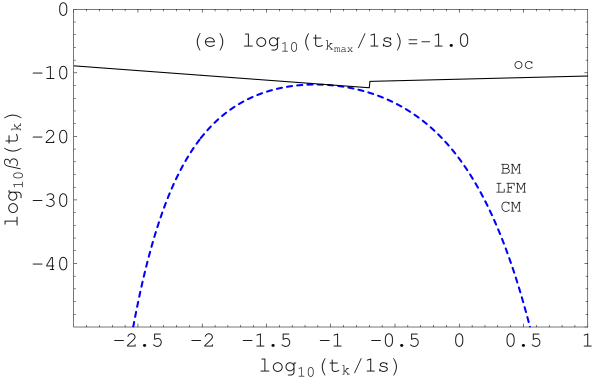
For a given value of $t_{k_{max}}$ the fraction of the Universe going into PBHs, $\beta(t_k)$, will be maximum if the corresponding value of n_{max} is the one located over the solid curve

Figure 2c in "Stellar mass Primordial Black Holes as Cold Dark Matter", Sobrinho, J. L. G.; Augusto, P., 2020, Monthly Notices of the Royal Astronomical Society, Volume 496, Issue 1.



Selected cases: (a) $t_{k_{max}} = 10^{-9}$ s; (b) $t_{k_{max}} = 10^{-7}$ s; (c) $t_{k_{max}} = 10^{-5}$ s; (d) $t_{k_{max}} = 10^{-3}$ s, and (e) $t_{k_{max}} = 10^{-1}$ s.

Figure 3e in "Stellar mass Primordial Black Holes as Cold Dark Matter", Sobrinho, J. L. G.; Augusto, P., 2020, Monthly Notices of the Royal Astronomical Society, Volume 496, Issue 1.



The fluctuations crossed the horizon sufficiently after the QCD epoch. All the three QCD models share the same curve (with $n_{max} = 1.920$), characterized by a **radiation peak**.

The present day value of the **PBH density parameter**:

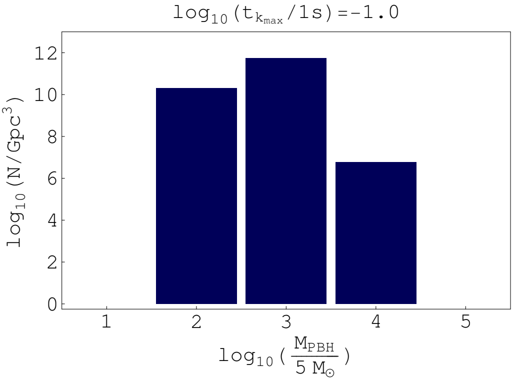
$$\Omega_{PBH}(t_0) = a(t_{eq}) \int_{t_*}^{t'} \frac{\beta(t_k)}{a(t_k)} dt_k$$

where

$$\Omega_{PBH}(t_0, t_k) = \beta(t_k) \frac{a(t_{eq})}{a(t_k)}$$

The present day value of the **PBH number density** for PBHs formed between two given instants t_1 and t_2:

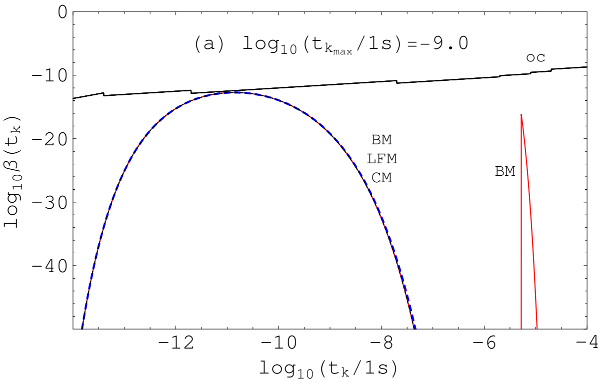
$$n_{PBH}(t_0) = \rho_c(t_0) \int_{t_1}^{t_2} \frac{\Omega_{PBH}(t_0, t_k)}{M_H(t_k)} dt_k$$



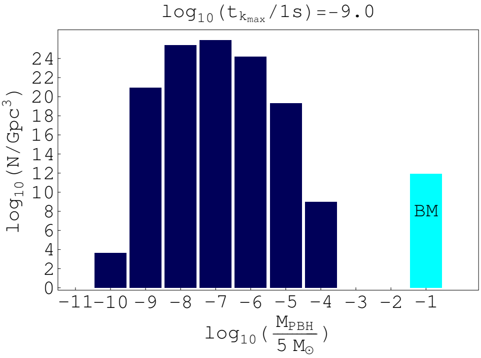
Intermediate mass PBHs $(5 \times 10^2 - 5 \times 10^4 M_\odot)$ with a radiation peak located at $5 \times 10^3 M_\odot$, giving $\sim 10^{11}$ PBHs/Gpc^3.

Contribution to CDM: $\approx$ 0.001%.

Figure 3a in “Stellar mass Primordial Black Holes as Cold Dark Matter", Sobrinho, J. L. G.; Augusto, P., 2020, Monthly Notices of the Royal Astronomical Society, Volume 496, Issue 1.

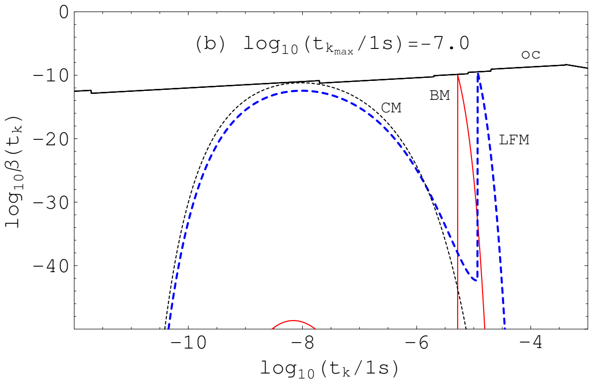


For both the CM and LFM we have the same $\beta(t_k)$ curve (left), which corresponds to a **radiation peak**. These fluctuations crossed the horizon sufficiently before the QCD epoch. If we consider a BM instead, then we cannot neglect the contribution from the QCD and we have, in addition, a **QCD peak**.



PBHs with sub-stellar masses $(5 \times 10^{-10} - 5 \times 10^{-4} M_\odot)$
Peak: $\sim 5 \times 10^{-7} M_\odot$ $(\sim 10^{26}$ PBHs/Gpc$^3)$.
Total: $\sim 10^{26}$ PBHs/Gpc^3
QCD peak (BM only): $\sim 10^{12}$Gpc^3 PBHs with $0.5 M_\odot$.
Contribution to CDM: $\approx 29\%$.

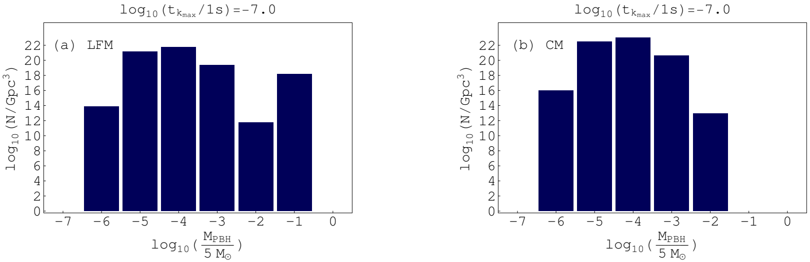
Figure 3b in "Stellar mass Primordial Black Holes as Cold Dark Matter", Sobrinho, J. L. G.; Augusto, P., 2020, Monthly Notices of the Royal Astronomical Society, Volume 496, Issue 1.



CM - radiation peak

LFM - radiation peak + QCD peak

BM - dominated by a QCD peak

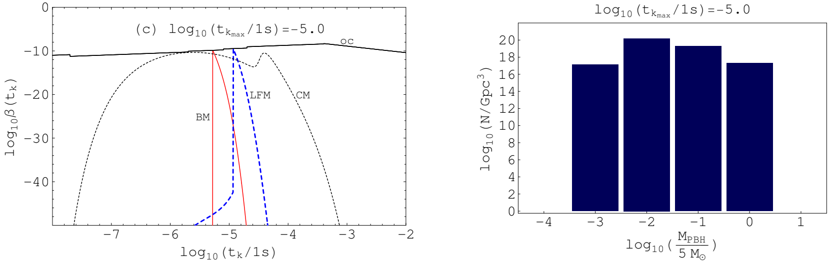


LFM — Radiation peak: $5 \times 10^{-4} M_\odot$, $\sim 10^{22}$ PBHs/Gpc^3
QCD peak: $0.5 M_\odot$, $\sim 10^{18}$ PBHs/Gpc^3
Contribution to CDM: $\approx 2.3\%$

CM — Radiation peak: $5 \times 10^{-4} M_\odot$, $\sim 10^{23}$ PBHs/Gpc^3
Contribution to CDM: $\approx 31\%$

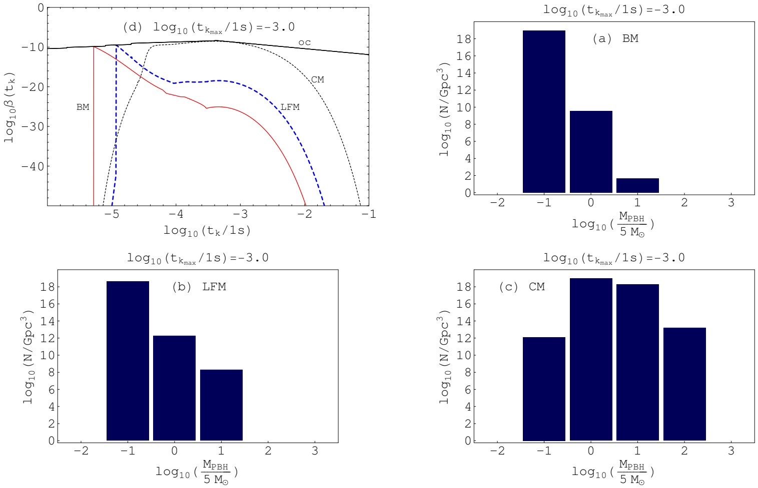
BM — QCD peak: $0.5 M_\odot$, $\sim 10^{20}$ PBHs/Gpc^3
Contribution to CDM: $\approx 0.4\%$

Figure 3c in “Stellar mass Primordial Black Holes as Cold Dark Matter", Sobrinho, J. L. G.; Augusto, P., 2020, Monthly Notices of the Royal Astronomical Society, Volume 496, Issue 1.



CM — Radiation peak: $0.05M_\odot$, $\sim 10^{20}$ PBHs/Gpc^3
QCD peak somehow hidden on the $0.5M_\odot$ bar
Contribution to CDM: $\approx 12\%$

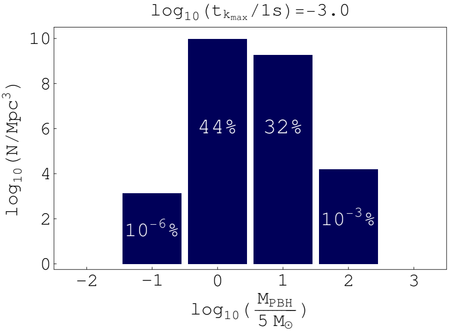
BM & LFM — QCD peak: $0.5M_\odot$, $\sim 10^{18}$ PBHs/Gpc^3
Contribution to CDM: $\approx 0.6\%$



BM QCD peak: $0.5M_{\odot}$, $\sim 10^{19}$ PBHs/Gpc^3, CDM: $\approx 2\%$

LFM QCD peak: $0.5M_{\odot}$, $\sim 10^{18}$ PBHs/Gpc^3, CDM: $\approx 2\%$

Figure 4 in "Stellar mass Primordial Black Holes as Cold Dark Matter", Sobrinho, J. L. G.; Augusto, P., 2020, Monthly Notices of the Royal Astronomical Society, Volume 496, Issue 1.



CM – extended mass spectrum (plateau formed by the radiation and QCD peaks):

- $\sim 10^{12}$ PBHs/Gpc^3 $(0.5M_\odot)$
- $\sim 10^{19}$ PBHs/Gpc^3 $(5M_\odot)$
- $\sim 10^{18}$ PBHs/Gpc^3 $(50M_\odot)$
- $\sim 10^{13}$ PBHs/Gpc^3 $(500M_\odot)$

Conclusions

- Stellar mass PBHs might have formed in the Early Universe
- Stellar mass PBHs could be an important fraction of CDM
- At least some of the BHs mergers observed (gravitational waves) could be of primordial origin
- A monochromatic peak at $\sim 0.5\ M_\odot$ will favour a BM or LFM model, while a broader mass spectrum (5–$50 M_\odot$) will suggest a CM for the QCD.

Some ideas for future work

- Formation of PBH binaries (at the formation epoch)
- Formation of PBH binaries (during the evolution of the Universe)
- Concentration of PBHs around galactic haloes
- Formation of clusters of PBHs
- Find out scenarios that fit with the observed mergers
- Consider other mass ranges (sub-stellar and supermassive)
- .....

Some (other) ideas for future work

Some ideas that can help to refine the results obtained in the previous topics:

- Consider the scaling law for the PBH masses
- Consider non-gaussian distributions for the fluctuations
- Consider other kinds of spcetra for the fluctuations
- Consider other PBH formation mechanisms
- .....

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Hvala na panžji!

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