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Asteroid close encounters and mutual perturbations

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Abstract. Results of the search for close encounters with large asteroids in the last 20 yr are presented. A multistep selection procedure has been used, in combination with accurate numerical integration, to derive the parameters of the close encounters and to estimate their effects on the motion of the perturbed bodies. A total of 10 pairs of asteroids have been found to exhibit measurable effects, thus providing the opportunities for determination of asteroid masses.

Procedures

In our previous paper (Kuzmanoski and Knežević, 1993; hereafter referred to as Paper I), devoted to the search for asteroid close encounters that may serve for determination of asteroid masses, we have developed a combined method for identification of the asteroids which will, in the next 50 yr, have a close encounter with one of the asteroids larger than 100 km in diameter. The method consisted of a multistep selection procedure and an accurate numerical integration, and provided parameters of 208 close encounters with high reliability. We also computed the expected mutual perturbations for the most interesting pairs and found measurable effects in some cases.

The same procedure was applied in the present study to systematically search for close encounters with large asteroids (again with $d \geqslant 100$ km) in the last 20 yr. We found many cases of such close encounters, and in several cases a significant change of motion of perturbed body occurred after the approach.

The method of identification of asteroid pairs which had a close encounter in the recent past is a three-step procedure that consists of the following: (i) by means of a purely geometrical consideration, one finds all the pairs of asteroid orbits close enough to enable a real approach of the bodies; (ii) by using a simple two-body approximation, one checks whether such an approach might occur in a given time span; (iii) finally, a numerical integration in the framework of a simplified dynamic model is performed to determine the parameters of the close encounter (approximate epoch, distance, relative velocity, etc.). Each of these steps significantly reduces the number of cases to examine, providing at the end a reasonable number of pairs which can be investigated in detail.

Computation of the minimum distance between the orbits of the two asteroids was described by Lazović (1967) and Lazović and Kuzmanoski (1978); it involves determination of the true anomalies of the relative nodes and an iterative derivation of the true anomalies of the proximity points on both orbits. In the second stage, one places a primary asteroid in the proximity point on its orbit, and looks for the occurrence of the other body in the vicinity of its proximity point (within a narrow range of the mean anomaly) at instances corresponding to the orbital period of the primary. Because of the fast accumulation of the error in the mean anomaly due to the use of osculating mean motion and two-body propagation (see Paper I for a discussion), in order to make our search as complete as possible, we have used in the first two stages somewhat relaxed criteria for selection of the pairs entering the last stage ($\rho_{\text{orb}} \leqslant 0.2$ AU and $\Delta M \leqslant 5°$). To account for the effects of planetary perturbations, we made use of the ORBIT8V software (kindly provided by A. Milani), which includes four outer perturbing planets and barycentric correction for the indirect effect of the inner ones. The distance criterion $\rho \leqslant 0.02$ AU has been applied for this final stage of selection, resulting in a total of 76 pairs; for 10 of these with the smallest distance and/or involving the largest asteroids, the perturbations were determined by accurate numerical integration in the framework of a realistic dynamic model. Note that, as already explained in Paper I, due to the simplifications introduced in our method, the actual number of interesting close encounters should be larger than that found here.

In all the abovementioned 10 cases, motion of the smal-

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Table 1. Results for 10 pairs. Diameters are taken from asteroid data base provided by E. Tedesco. V_r is the relative velocity, ΔV is the change of the velocity in the perturbed asteroid resulting from the mutual close approach, and r_1 is the heliocentric distance of a perturbing asteroid at the epoch of minimum distance. D_1 and D_2 are given in km, ρ and r_1 in AU, V_r in km/s and ΔV_2 in km/s × 10^{-9}

Ast. 1	Ast. 2	D_1	D_2	JD	ρ	V_r	ΔV_2	r_1
1	2572	913		2441037.0	0.012046	4.786	18157	2.737
1	2660	913	13	2444343.8	0.013519	7.304	10601	2.794
1	3643	913		2441571.7	0.008288	2.764	45696	2.603
2	3131	523		2446038.6	0.011896	10.845	1525	3.043
2	4350	523		2446087.6	0.014509	12.765	1062	2.955
3	1767	244		2445334.0	0.005477	4.535	804	2.757
4	413	501	34	2448433.3	0.010568	8.211	1993	2.545
15	3591	272		2447584.6	0.003795	5.553	1313	2.662
45	673	214	39	2448116.9	0.003764	2.548	1406	2.783
52	1023	312	60	2441102.7	0.006472	3.764	1715	2.832

ler body in a pair was integrated by means of the Radau integrator of order 15 (Everhart, 1985), once under the perturbation of all the major planets except Pluto, and then by adding in the perturbing effect of the larger asteroid [assuming its density to be equal to that of (1) Ceres; the mass of Ceres is taken to be $5.9\times10^{-10}\,M_\odot$, a value recommended by the IAU and accurate enough for our purpose], in a time span beginning 1000 days before the epoch of a close approach and ending up at the year 2000.0.

Results

An example of the results of the selection procedure is shown in Table 1; it contains the data for the final 10 pairs: asteroid numbers, their diameters, epochs of the close encounters, minimum mutual distances, relative velocities, changes of the velocity of the perturbed body due to the close encounter, and the distances from the Sun of the perturbing body at the close encounter. As expected, the close encounters with (1) Ceres were the most effective in changing the motions of the approaching bodies, but there are also some other cases of interaction with other large asteroids in which these changes were quite significant.

In Table 2, the maximum difference in a root mean square sense, $\sqrt{(\Delta\alpha\cos\delta)^2+(\Delta\delta)^2}$, achieved in the time span covered by the integration, between the positions of a perturbed asteroid as computed with and without effects of the corresponding perturbing asteroid, are shown for each pair, together with an epoch of this maximum difference (in Julian days). An interesting finding arises from the comparison of the two tables: the pairs for which the mutual distance and relative velocity at the encounter were the smallest (and the change of the velocity of the

Table 2. Results for 10 pairs. Maximum expected root mean square differences (in arcsec) of perturbed asteroid positions, and the corresponding epochs

Ast. 1	Ast. 2	Diff.	JD
1	2572	26.64	2451237
1	2660	3.69	2451544
1	3643	10.13	2450621
2	3131	0.92	2450939
2	4350	0.30	2451308
3	1767	0.16	2451294
4	413	0.26	2451223
15	3591	0.74	2451535
45	673	0.08	2450777
52	1023	1.68	2451201

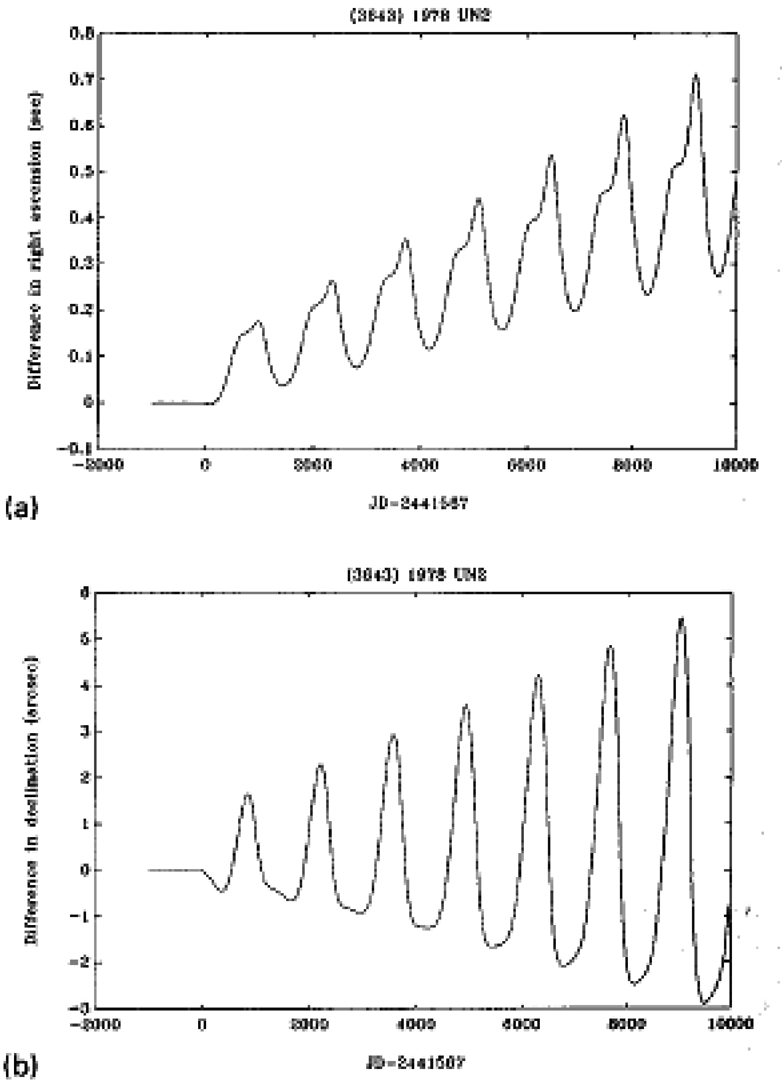

Fig. 1. (a) Difference in right ascension of (3643) 1978 UN_2 for integrations with and without accounting for the perturbing effect by (1) Ceres. Time on the x-axis is given with respect to the epoch of the close encounter. (b) Same as a, but for the declination

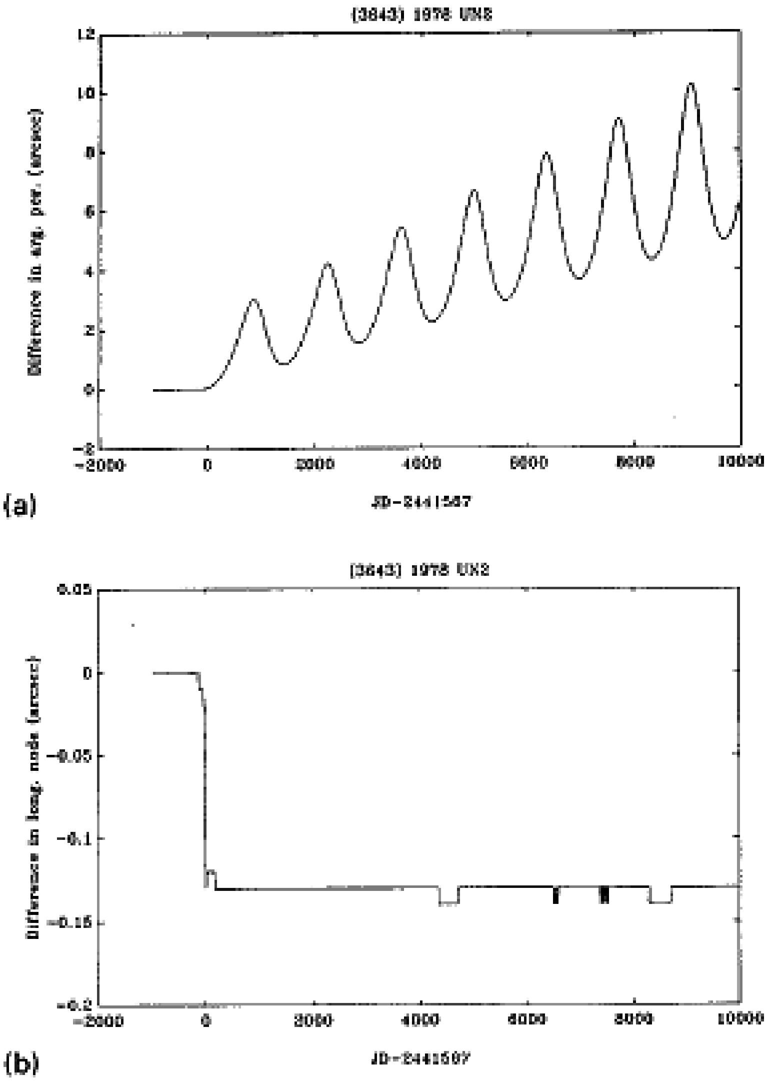

Fig. 2. (a) Same as Fig. 1a, but for the argument of perihelion. (b) Same as Fig. 1a, but for the longitude of node. (c) Same as Fig. 1a, but for the inclination. (d) Same as Fig. 1a, but for the semi-major axis. Note that here differences are given in units to the fifth decimal place

perturbed body, therefore, the largest) do not necessarily exhibit the largest shift in the positions of a perturbed body. Change of the motion due to the close encounter is a more complex phenomenon than suggested by the simple criteria commonly used for the search; although indicative of the possible effects of close encounters, the quantities used as criteria for their identification appear to have an order-of-magnitude reliability only.

As an illustration of our results, we plotted the computed differences, due to the close encounter with (1) Ceres, of coordinates and orbital elements of asteroid (3643) 1978 UN_2 in Figs 1 and 2, respectively (the figure for eccentricity is missing since no differences in eccentricity exceeding the computational accuracy threshold were detected). It is easily seen that the orbit and the motion of this asteroid change due only to Ceres' perturbations, and that post-encounter orbit behaves in a manner easily distinguishable from the pre-encounter one.

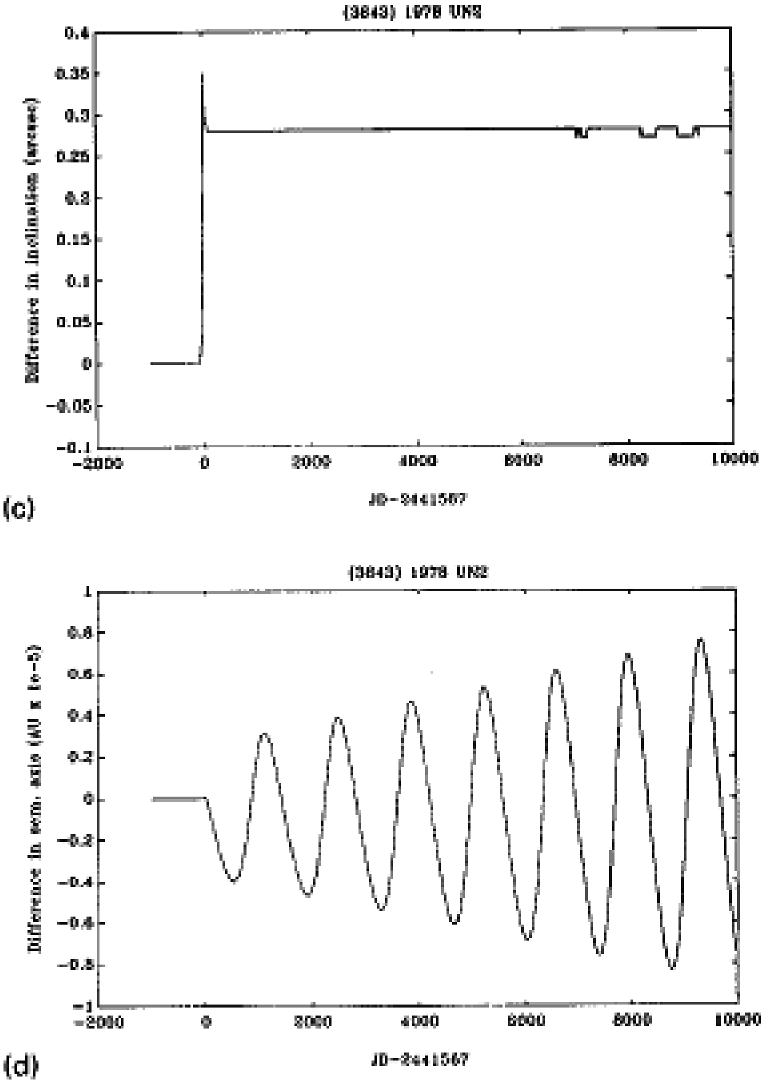

Fig. 2 (*continued*)

In conclusion, one can state that, providing the precise positions of perturbed asteroids are derived from past and future observations (taking into account recently achieved improvements of asteroid astrometry and the improvements expected in the near future), the masses of the perturbing asteroids can hopefully be determined and/or improved in almost all cases considered here (and many others).

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