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New indications for the solar origin of the 50-day cycle in the atmospheric circulation and Earth's rotation

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Abstract. An analysis of Universal Time, the atmospheric angular momentum, the corona index and the total solar irradiance spectra by the Fourier transform and Wavelet technique, indicates that the Madden-Julian oscillation, which causes the corresponding oscillation in the Earth's rotation, is generated by the variable solar irradiance. The present analysis confirms the existence in the Earth rotation, the atmospheric circulation, and the solar activity of three oscillations with periods around 40, 50 and 70-day found earlier by the first two authors.

Key words: Sun: activity – Earth – solar – terrestrial relations

1. Introduction

The atmospheric 50-day oscillation, known in meteorology as the Madden-Julian oscillation (MJO), was first detected in the tropical atmospheric zonal circulation (Madden & Julian 1971). Later it was confirmed several times in different meteorological parameters, as well as in the Earth's rotation (Feissel & Gambis 1980) and solar activity indices (Djurovic & Pâquet 1988, 1991; Pap et al. 1990); but the mechanism is not yet explained. Different origins of the oscillation are put forward:

1. The first called the Madden-Julian mechanism, connects MJO to the eastward propagation anomalies in tropical convection and zonal wind. The observed oscillation is assumed to result from eastward-propagating, convectively-driven waves in the equatorial zone or from a combination of these tropical waves and Rossby waves.
2. The second mechanism, proposed by Ghil (1987) and Ghil & Childress (1987), relates MJO to an instability of non-zonal westerly flow, caused by the interaction of the jet stream with mountain ranges in the mid-latitudes. A quite different explanation, proposed by Simmons et al. (1983), is also based on circulation disturbances, originating in the atmosphere itself, and topography interactions.
3. Djurovic & Pâquet (1988) assume a cosmic origin for MJO. They consider it to be associated with corpuscular solar radiation, driven by the interplanetary magnetic field, whose sectorial structure corresponds to the cyclic character of the atmospheric disturbances, or that it is associated to the variable ultraviolet radiation, which acts on the atmospheric ozone and triggers the large-scale tropospheric circulation.

From these remarks it is clear that the MJO origin remains open. Besides, the detection of two or three prevailing higher power spectra in the range 40-70 days makes our understanding of the MJO physics difficult.

The large MJO frequency instability was observed by Eubanks et al. (1985). Moreover, Morgan et al. (1985) pointed out separate concentrations of power spectra in the length of day (LOD) and atmospheric angular momentum (AAM) data series at 40 and 50-70 days period.

Graves & Stanford (1987), besides noting the 45-53 day zonal wind oscillation over Easter Island, found an oscillation of meridional wind with periods of 65 and 84 days respectively at 1000 mb altitude.

Dickey et al. (1991) show that MJO in the LOD and AAM variation is most powerful at two distinct periods: 50 days and 40 days. The evidently more intense 50-day oscillation is dominated by the tropical (20^0 N - 20^0 S) zonal circulation while the 40-day oscillation, observed in the LOD and global AAM data, is greater in the zonal circulation over the north hemisphere extratropics.

Djurovic & Pâquet (1988) have even found that the power spectra of LOD, AAM, geomagnetic index Aa, Wolf number and sunspot area data series exhibit maxima over three periods at 40, 50 and 60 to 70 days. The main objectives of the present study of MJO are:

a) to verify the solar origin of the earlier postulated hypothesis.

b) to analyse the MJO frequency variations with the aim to determine whether MJO is composed of two, three, maybe more, dominating oscillations or if it is a single one with large frequency instability.

c) to determine the independence of the MJO intensity from the 11-year cycle of solar activity.

2. Data and computation methods

The data base for the present study consisted of:

- Daily means of total solar irradiance (SR) for the period 1980/02/15 - 1983/11/04, measured by the ACRIM radiometer from the Solar Maximum Mission (SMM) satellite (*Solar-Geophysical Data*, NOAA, Boulder, Colorado). To avoid the interpolation over a large gaps in the series, the data outside this interval are not used.
- Daily values of solar corona index (CI) (intensity of the radiation at 5303 Å) for the period 1964.0-1987.0, published by the Astronomical Institute of Czechoslovakia (Rybansky 1979; Rybansky & Rusin 1983; Rybansky et al. 1988, 1990).
- Daily values of the zonal component of the global atmospheric angular momentum (AAM) for the period 1980.0-1984.0, published by Rosen et al. (1981) and Rosen (personal communication, 1985).
- The 5-day differences of universal times UT1 - TAI (noted UT1) for the period 1962-1992, deduced from UT1-UTC published by the International Earth Rotation Service (IERS, 1993).

Beside the above long CI and UT series, their sub-series for the period 1980.0-1984.0 are analysed separately. This is performed to obtain results related to the period for which SR data are available.

For the period 1980-1984, standard deviations of the four series are:

- $s_r = 0.36 Watts m^{-2}$, for SR;
- $s_c = 103$, for CI;
- $s_a = 0.0498 \times 10^6 kg m^2 s^{-1}$, for AAM;
- $s_u = 0.0009 s$, for UT1.

For the long series of CI and UT1 (1964.0 - 1987.0), $s_c = 121$ and $s_u = 0.0013 s$.

To isolate the weak MJO signal, in a first step, the stronger oscillations such as semi-annual, annual and others of larger periods are eliminated. For this purpose, one-side filtering of data by the method of Whittaker-Robinson-Vondrak (WRV) (Whittaker & Robinson 1946; Vondrak 1969) with the smoothing parameter $\epsilon = 10^{-7}$ has been applied. According to the formula of Kun-Yi & Zhou (1981):

$$G = \frac{\epsilon P^6}{64\pi^6 + \epsilon P^6} \tag{1}$$

where $G = A'/A$, with A the amplitude of the sinusoidal input signal of period P (in days), A' the amplitude of the corresponding output signal. The gain factor is G=0.02 for P=50, while G=0.98 for P=180. In consequence, in the residuals "O-C", MJO is practically unaffected while the semi-annual cycle and those of larger periods have been eliminated.

The cyclic decomposition of residuals is simultaneously done by two methods: wavelets (Combes et al. 1990; Daubechies 1990) and direct Fourier transforms. The Fourier method is not appropriate when large frequency fluctuations are expected, but its advantage consists in simple estimation of the statistical significance of results. The wavelet method is known for the identification of the "local" frequency and amplitude variations with time. The two methods thus give complementary information.

To perform the wavelet analysis, Morlet's wavelet is used. It can be expressed as:

$$\begin{aligned} g(t) = {} & e^{i\omega_0 t} \; e^{-t^2/(2\sigma_0^2)} \\ & -\sqrt{2} \; e^{-(\omega_0\sigma_0)^2/4} \; e^{i\omega_0 t} \; e^{-t^2/\sigma_0^2} \end{aligned} \tag{2}$$

The second term in the right-hand side of equation 2 is a corrective one added so as to have a wavelet mean equal to zero; this term is negligible when $(\omega_0 \, \sigma_0) > 5$. Morlet's wavelet is therefore a gaussian function, of width at half maximum proportional to σ_0, modulated at the frequency ω_0 and to which a corrective term is added. Usually ω_0 is taken equal to 5.336 and σ_0 to 1, so that the corrective term becomes negligible. The width of the wavelet is determined by the parameter σ_0: the closer the frequencies we want to distinguish are, the larger σ_0 must be (Antoine et al. 1992; Billiau 1992).

As we are working in two different frequency bands, we have to choose two different values for the σ_0 parameter of Morlet's wavelet. In the first frequency range (from 22 to 56 days), our purpose is to separate a 45-day period from a 50-day one, so we use a σ_0 equal to 6.5 days. In the second frequency interval (from 50 to 80 days), our aim is to distinguish a 60-day period from a 70-day one. Consequently, we choose a σ_0 equal to 4.5 days. Morlet's wavelet parameters for each frequency range, the corresponding minimum and maximum wavelet lengths as well as the corresponding frequency resolutions ($\frac{\Delta f}{f}$) are summarized in Table 1.

3. Results

The results of Fourier transforms applied on the series UT, AAM, CI and SR for the period 1980.0 - 1984.0 are given in Fig. 1. To make the comparison of the results obtained from different series of data easier, instead of the usual presentation of the amplitude A we give, in function of the period, the value of $(A/s)^2$, s being the standard deviation. We assume that the analysed residuals have a white noise distribution.

From the classical formula of Schuster (1898):

$$P_r\{S(\omega) > \frac{x s^2}{2\pi}\} = e^{-x} \tag{3}$$

Table 1. Wavelet parameters for the two frequency ranges studied

frequency domain	parameter σ_0	parameter ω_0	minimum wavelet length	maximum wavelet length	frequency resolution $\frac{\Delta f}{f}$
22 to 56 days	6.5 days	5.336 cycles/day	522	1305	0.03926
50 to 80 days	4.5 days	5.336 cycles/day	800	1280	0.0625

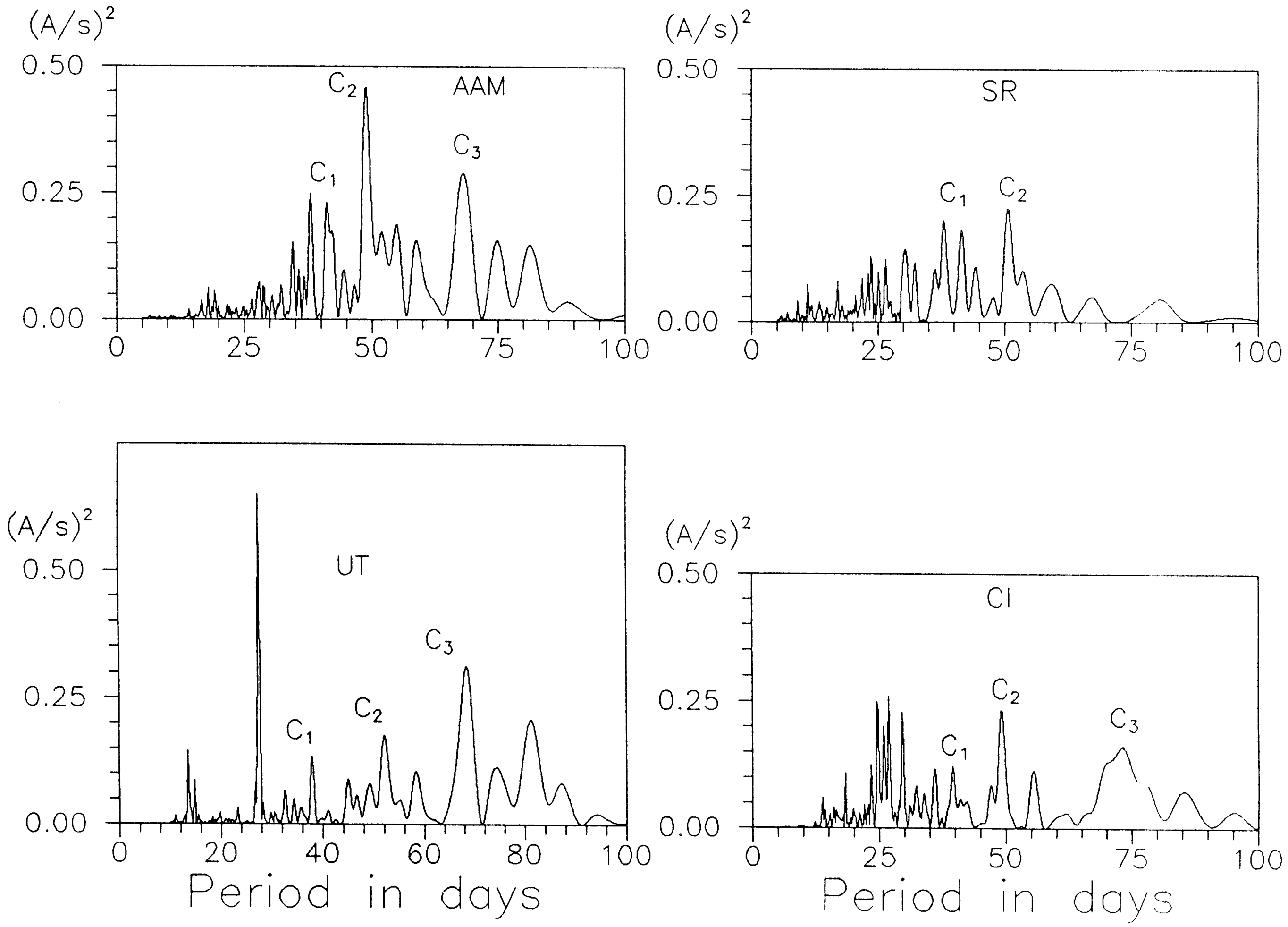

Fig. 1. Fourier transforms of the 4 series (UT1, AAM, SR, CI) for the period 1980/02/15 to 1983/11/04. The peaks indicated C_1, C_2, C_3 correspond to 40,50 and 70 day periods

where P_r represents the probability, $S(\omega)$ the power spectrum at angular frequency ω, s the standard deviation and x an arbitrarily chosen number. From the known relation:

$$S(\omega) = A^2 \frac{2n+1}{8\pi} \tag{4}$$

where A is the Fourier amplitude, and $(2n+1)$ the number of data, one can deduce:

$$P_r\{(\frac{A}{s})^2 > \frac{4x}{2n+1}\} = e^{-x} \tag{5}$$

Since $2n+1$ is 293 for UT1, 1461 for AAM, 1461 for CI, and 1357 for SR the probability for $(A/s)^2$ to exceed the limit of 0.15, for example, is less than 0.00002.

Therefore, the concentrations of power spectra C_1, C_2, C_3 in Fig. 1 at approximately 40, 50 and 70-day period cannot be explained by noise.

In principle, the deviations of the filtered large amplitude oscillations from the corresponding sinusoidal forms could result in residual peaks whose amplitudes are not negligible. In this sense, the estimation of these effects due to the semi-annual oscillation is particularly informative.

Table 2. The periods (P), the amplitudes $(A/s)^2$ and the phases (Ph for the epoch 1980.0) of the oscillations C_1, C_2, C_3 (Fig. 1)

	Series	P (days)	$(A/s)^2$	Ph	The pair mean
	UT	38.0	0.134	-10^0	-48^0
C_1	AAM	37.9	0.249	-86^0	
	CI	36.0	0.117	115^0	131^0
	SR	38.0	0.201	147^0	
	UT	52.1	0.176	123^0	
	AAM	49.0	0.458	131^0	127^0
C_2	CI	49.0	0.233	-36^0	-70^0
	SR	50.5	0.225	-104^0	
	UT	68.6	0.313	38^0	-26^0
C_3	AAM	68.2	0.290	-90^0	
	CI	73.0	0.160	202^0	202^0
	SR	-	-	-	

Let the amplitude variations of the semi-annual oscillation be of the order of the standard deviation s. The amplitude of the main residual peak will be $(A/s)^2 = 1$. Since the FT selectivity is defined by the sine-function: $f(\lambda) = \frac{sin(\lambda)}{\lambda}$, the pseudo-amplitude δA at angular frequency ω is given by:

$$\delta A = A_j |f(\lambda)| \tag{6}$$

where $\lambda = (\omega - \omega_j)T$, (A_j, ω_j) are the amplitude and the angular frequency of the main peak, where $2T$ is the size of the observation interval (in the same units as $P = \frac{2\pi}{\omega}$). So for P= 40, 50 and 70 days, $\delta A \approx 0.01 A_j$ and the corresponding $(\delta A/s)^2 \approx 1 \text{ x } 10^{-4}$.

Therefore, the peaks C_1, C_2, C_3 could not be attributed to the perturbation of the mentioned lower frequency oscillations.

With the given results, it seems unrealistic to discuss small details as, for example:

1. whether the spectral peak C_1 is unique as in UT series or is it composed of two close peaks as in the other three series;
2. why the peak C_3 does not exist in the spectrum of SR while it is prominent in the *CI* series.

However, on the basis of Fig. 1, one can observe that the power spectra are fairly increased near the 40 and 50-day period. Except in the case of the SR series, the same conclusion also holds for the 60-80-day spectral concentration.

The periods, amplitudes and phases of the concentrations C_1, C_2, C_3 are given in the Table 2. Since C_1 is unique in the *UT* series while double in the other series, the results presented for the other three series are related to the first peak C_1 because it is at the same period ($\approx$ 38 days) as for UT.

Since the amplitudes of the oscillations are small with respect to the noise level, the accuracy of the phases is also small, and any conclusion based on the phase estimation must be accepted with caution.

Table 3. Mean periods (P), amplitudes $(\frac{A}{s})^2$ and phases (Ph- for Julian Date t_o = 2437669.0) of the peaks C_1, C_2, C_3, computed from annual series. The relative frequency of the peak occurence (fr) and the standard deviations (even rows) are also given

	UT				CI			
	P	$(\frac{A}{s})^2$	Ph	fr	P	$(\frac{A}{s})^2$	Ph	fr
C_1	40.2	0.188	111^0	0.94	38.4	0.338	51^0	0.96
	2.7	.127	103^0		1.8	.203	128^0	
C_2	51.9	.348	100^0	0.94	50.0	.336	285^0	.96
	4.3	.213	87^0		4.7	.201	107^0	
C_3	72.4	.625	101^0	0.94	72.2	.506	128^0	.96
	8.1	.378	109^0		9.0	.378	102^0	

Table 4. Mean $(\frac{A}{s})^2$ of the peaks C_1, C_2, C_3 in CI and UT series around the maxima and minima of solar activity

		C_1	C_2	C_3
CI	max	0.401	0.459	0.761
	min	.347	.287	.385
UT	max	.240	.390	.672
	min	.173	.384	.686

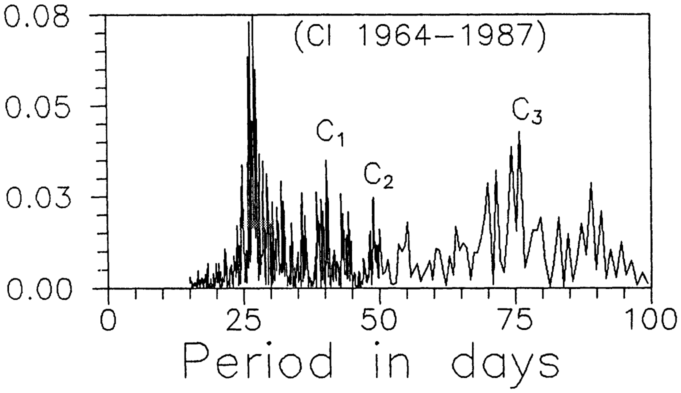

Fig. 2. Fourier spectrum of CI for the period 1964-1987

Having in mind that *AAM* and *UT*, as well as *CI* and *SR* are a priori related, the phase differences in these pairs are randomly produced. It is thus allowed to compare the mean phase of the first pair of series with the mean phase of the second one. We note that for the concentrations C_1, C_2 and C_3, the differences of these means are: $179^0, -197^0, 228^0$ respectively. Therefore, it seems that the corresponding oscillations are in opposite phases.

The results from Fig. 1 suggest two conclusions: (1) MJO is composed of at least three oscillations at 40, 50 and 60 to 80-day period ; (2) the presence of C_1, C_2, C_3 in the spectrum of CI and C_1, C_2 in the spectrum of SR represents an indication that the MJO and the corresponding Earth's rotation perturbations C_2 are related to the variable solar activity.

To obtain additional information about the spectral concentrations C_1, C_2, C_3, the long series of UT (1962.0 - 1993.0) and

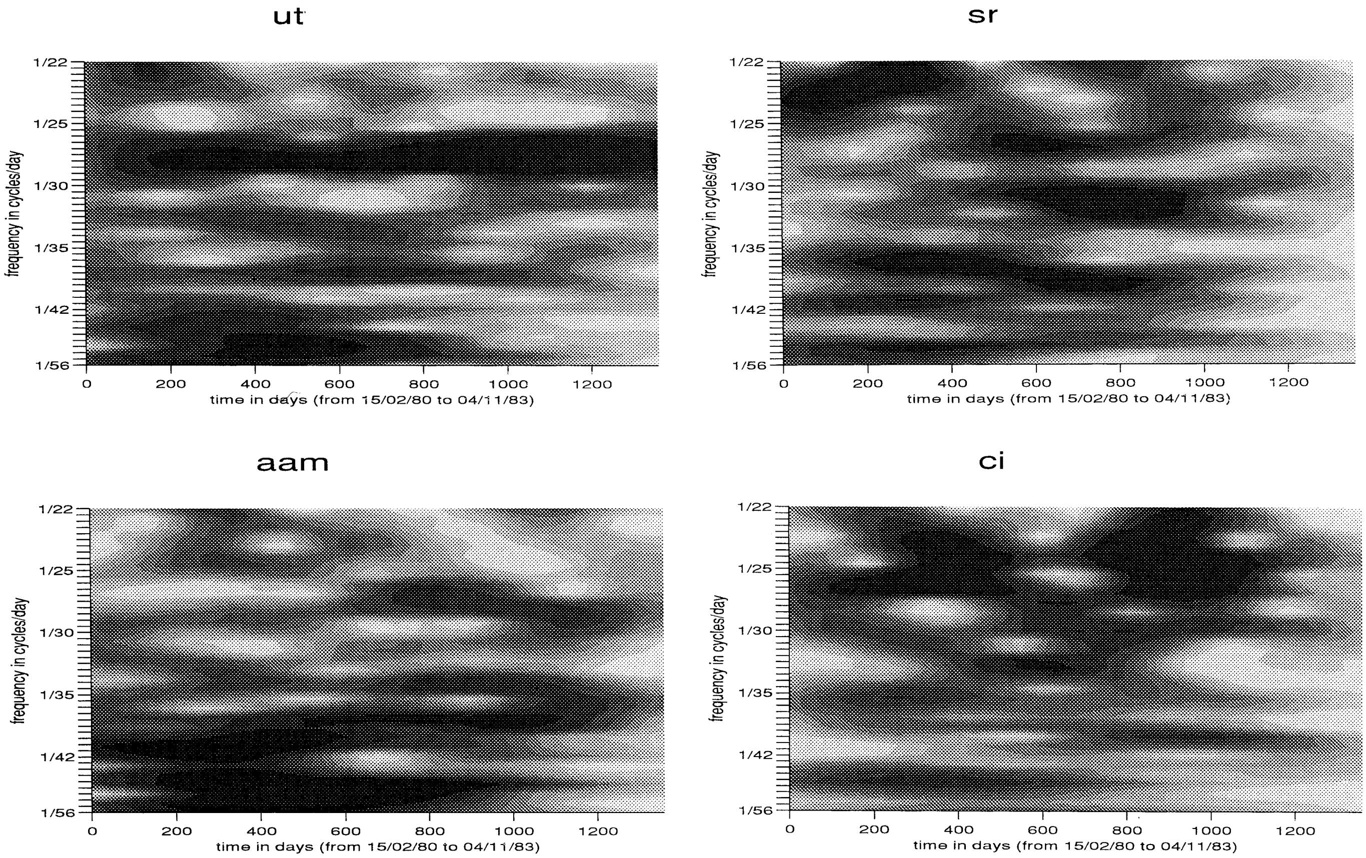

Fig. 3. For the 4 series (UT1, AAM, SR, CI), the wavelet transforms are given for the periods ranging from 22 to 56 days. The darkest parts indicate the strongest amplitudes. The 50-day period appears clearly to change in nominal value with time; the same remark is valid for its amplitude

CI (1964.0 - 1987.0) are subdivided into independent annual series and the whole computational procedure is repeated.

This generates 24 periodograms for CI and 31 periodograms for UT; the results are summarized in Tables 3 and 4.

Since they contain the results only for the clearly pronounced maxima, we observe that three oscillations in both series are present practically every year, the relative frequency of the occurence being 0.94 in UT, and 0.96 in CI.

All amplitudes in Tables 3 and 4 are statistically significant; for example the probability that the smallest one, $A = 0.173$ is due to noise is 0.04.

Large random fluctuations of the phases computed from annual sub-series make any deterministic conclusion impossible.

Having in mind that the 50-day oscillation in solar activity is not widely accepted, the spectrum of CI for the whole interval 1964-1987 is presented in Fig. 2. In this spectrum, the peaks $C_1, C_2 C_3$ also appear well-defined. Therefore, this spectral structure is well pronounced and even remains stable with time.

The results of the wavelet transforms applied on the four series above are given in Fig. 3 for periods ranging from 22 to 56 days. Similar results are given in Fig. 4 for periods 50 to 80 days. The darkest parts indicate the strongest amplitudes, while the clearer ones represent weaker amplitudes; as a good example of a strong and permanent signal, the tidal term Mm of UT1 appears in a broad dark band (Fig. 3-a).

On first inspection of Fig. 3, one notes the existence of dark horizontal bands, which could be understood if the MJO frequency does not continuously change between 40 and 50 days; in the contrary, MJO seems composed of a few prevailing oscillations. The fact that the bands are not strictly separated suggests that the frequencies of the MJO components are variable in time.

In Fig. 4, the situation is less deterministic. One can observe however that the darkest areas in the spectra of UT, AAM and CI are at 65 to 80 day periods. Thus, the results obtained by the two techniques are not discordant.

In the present study the question of the independence of the amplitudes of three oscillations from the phase of the 11-year cycle is also considered.

The average amplitudes around the maxima: 1968-1970, 1979-1981, 1989-1991 and around the minima of the 11-year cycle: 1964-1966, 1975-1977, 1985-1987 are presented in Table 4.

From this table, it follows that the amplitudes of the three oscillations in CI are slightly larger near the maxima than near the minima of solar activity. This conclusion is not mirrored in UT1.

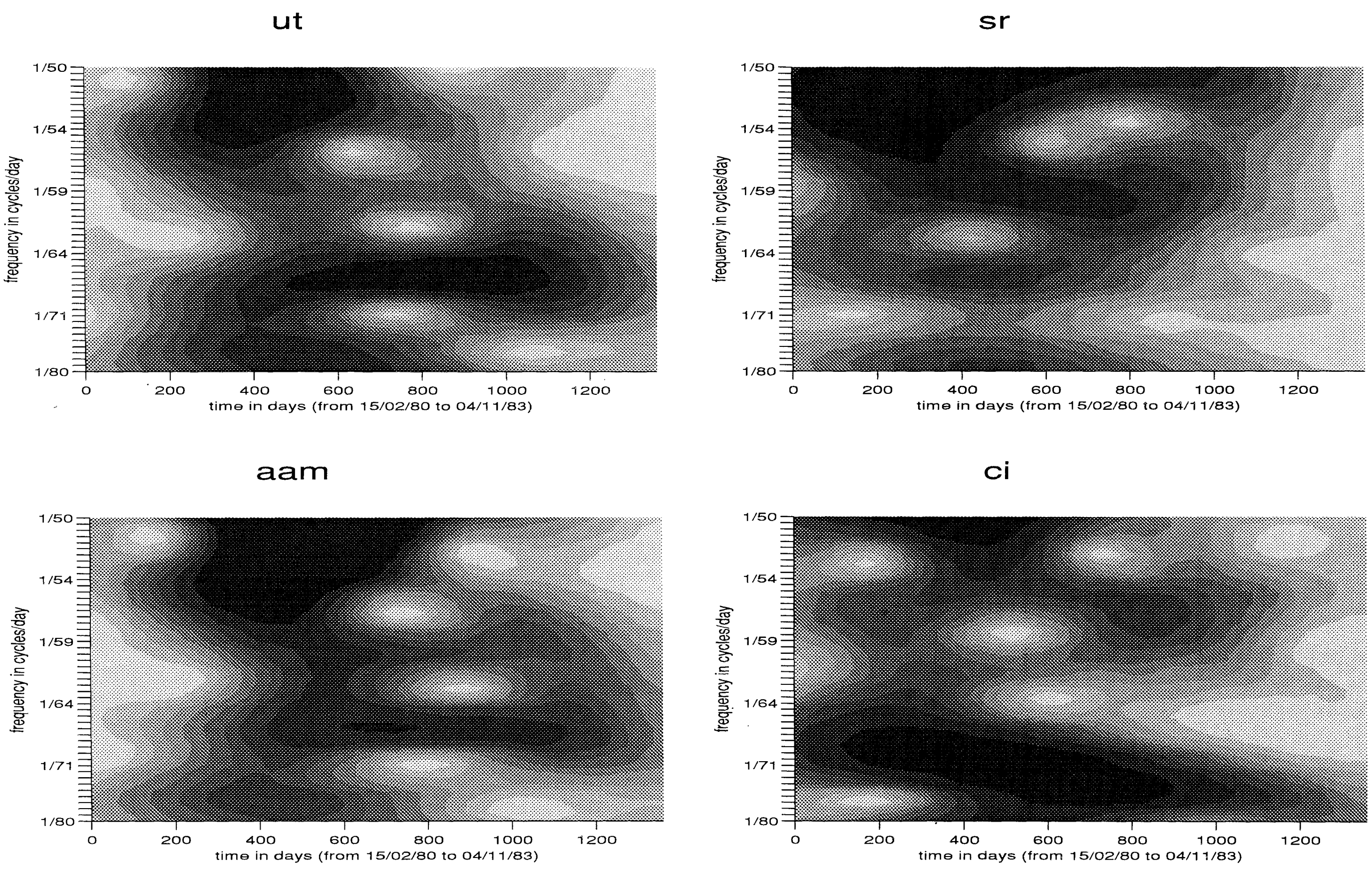

Fig. 4. For the 4 series (UT1, AAM, SR, CI), the wavelet transforms are given for the periods ranging from 50 to 80 days. The darkest parts indicate the strongest amplitudes. In the 4 series, the 60 to 70-day signal is present and exhibits variations as well in amplitude as in frequency

The mutual independence of the four variables, as usual, might be estimated by the computation of the cross-correlation coefficients (r). However, since the amplitudes of the cyclic fluctuations are small with respect to the noise, one cannot expect to obtain large correlation coefficients. For example for the combination AAM and UT, $r = 0.41$; for all other combinations, the absolute value of r is between 0.10 and 0.20. Computation of the Student variable:

$$t = \frac{(|r|\sqrt{(n-2)})}{\sqrt{(1-r^2)}} \tag{7}$$

shows that they are statistically significant at a high probability level (over 0.95).

4. Discussion of results

The results obtained by the computation of spectra of the 4-year series (Figs.1 to 4) suggest that the Madden-Julian oscillation in the global atmospheric circulation is a consequence of the corresponding oscillation in solar activity. This suggestion is confirmed by the annual periodograms of corona index and universal time series.

The assumption that MJO has a solar origin, based on the results of the present study only, could certainly be considered as speculative.

The existence of the MJO in atmospheric circulation and the existence of the so-called 50-day oscillation in the Earth's rotation cannot be suspected, due to many well documented studies (Dickey et al. 1991 and references therein); the second one represents the answer of the "solid Earth" to the variation of the atmospheric angular momentum. On the other hand Wilson (1982), Pap (1985), Pap et al. (1990), Djurovic & Pâquet (1991; present work) detect the 50-day oscillation in solar activity indices. Of course, if we accept the existence of the 50-day oscillation in solar activity and the atmospheric circulation as real facts, it is still not a proof that they are physically related.

The low level of accuracy of the phase determinations makes impossible to perform a direct approach to this question. However, the existence of other oscillations common to solar activity and atmospheric circulation indirectly increases the probability that MJO is a consequence of the variable solar radiation. So for example it is widely accepted that the quasi-biennial oscillations (QBO), which is the most spectacular phenomenon in equatorial stratosphere circulation, is also well pronounced in different manifestations of variable solar activity (see the references in Djurovic & Pâquet 1990; 1993). Beside that, from sounding rocket and satellite observations, the variable UV radiation of the Sun appears as a main source of upper atmosphere perturbations (Jacchia 1969). This fact could be important, because interactions between upper and lower atmosphere are suspected (Trenberth 1980). Having in mind the whole set of mentioned facts: the concentrations of power spectra in both solar activity and geophysical series at the same frequencies (C_1, C_2, C_3), the existence of other common oscillations (QBO) and solar perturbations on the upper atmosphere together with the suspected interactions with lower atmospheric levels, lead to the hypothesis that MJO solar origin could not be considered an artifact.

The detection of peaks C_1, C_2 and C_3 in the power spectra of CI and AAM represents a new result which is very important, because the corona is the main source of the Sun's UV radiation. It represents an indication that the troposphere is sensitive to upper atmospheric perturbations.

The 50-day oscillation has been also detected in the 10.7-cm solar flux (Pap et al. 1990). Since it is well correlated with the UV radiation from the active regions of the Sun (Anderson 1964), this result also supports our hypothesis of the triggering role of the UV radiation.

5. Conclusions

The power spectra of four analysed series UT, AAM, CI, SR in the range of periods between 35 and 100 days are dispersed, but they exhibit higher energy around 40, 50 and 60-80-day. Two exceptions ought to be mentioned: the 40-day peak is unique in the UT series, while in the other three it is split in two peaks; the 60-80- day peak is not observed in SR spectrum.

The similarity of AAM and CI spectra might be considered as an important argument in favour of the hypothesis that the influence of the solar UV radiation, whose main source is the corona, is propagating into the lower atmosphere. Of course, such a relation between AAM and the UV radiation still needs to be investigated.

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